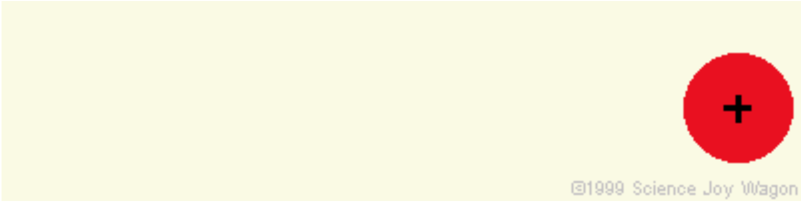
Electrical Energy and Electric Potential

AP Physics C

Electric Fields and WORK



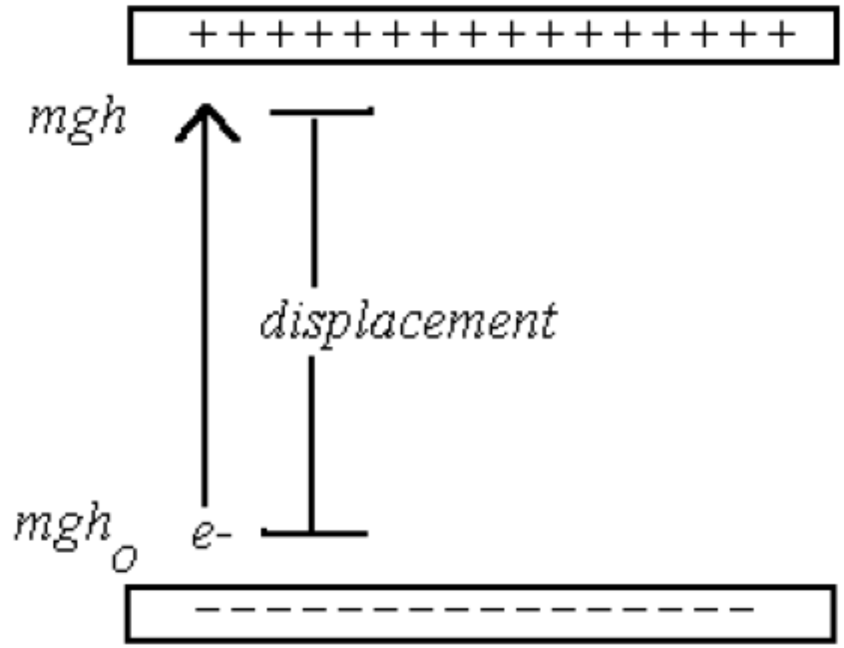
In order to bring two like charges near each other work must be done. In order to separate two opposite charges, work must be done. Remember that whenever work gets done, energy changes form.

As the monkey does work on the positive charge, he increases the energy of that charge. The closer he brings it, the more electrical potential energy it has. When he releases the charge, work gets done on the charge which changes its energy from electrical potential energy to kinetic energy. Every time he brings the charge back, he does work on the charge. If he brought the charge closer to the other object, it would have more electrical potential energy. If he brought 2 or 3 charges instead of one, then he would have had to do more work so he would have created more electrical potential energy. Electrical potential energy could be measured in Joules just like any other form of energy.

Electric Fields and WORK

Consider a negative charge moving in between 2 oppositely charged parallel plates initial KE=0 Final KE= 0, therefore in this case Work = ΔPE

We call this ELECTRICAL potential energy, U_E, and it is equal to the amount of work done by the ELECTRIC FORCE, caused by the ELECTRIC FIELD over distance, d, which in this case is the plate separation distance.



Is there a symbolic relationship with the FORMULA for gravitational potential energy?

Electric Potential

$$U_g = mgh$$

Here we see the equation for gravitational potential energy.

$$U_g \rightarrow U_E (or\ W)$$

Instead of gravitational potential energy we are talking about ELECTRIC POTENTIAL ENERGY

$$m \rightarrow q$$

A charge will be in the field instead of a mass

$$g \rightarrow E$$

The field will be an ELECTRIC FIELD instead of a gravitational field

$$h \rightarrow x \rightarrow d$$

The displacement is the same in any reference frame and use various symbols

$$U_E (W) = qEd$$

$$\boxed{\frac{W}{q}} = Ed$$

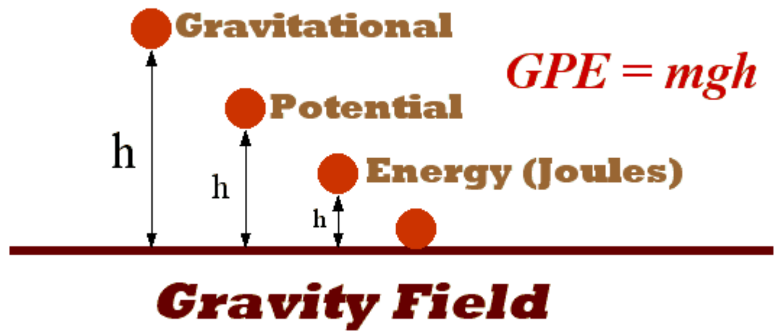
Putting it all together!

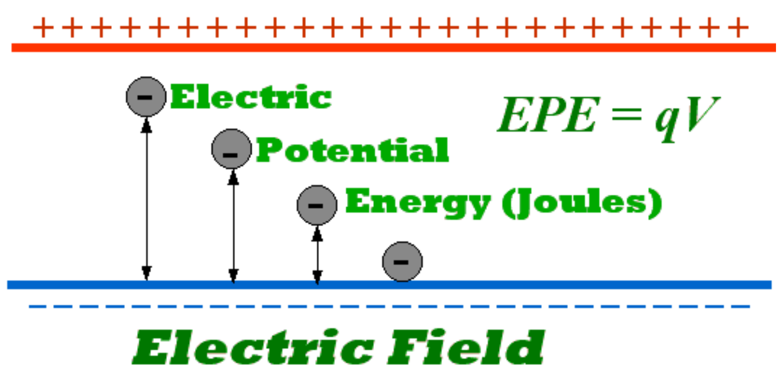
Question: What does the LEFT side of the equation mean in words? The amount of Energy per charge!

Energy per charge

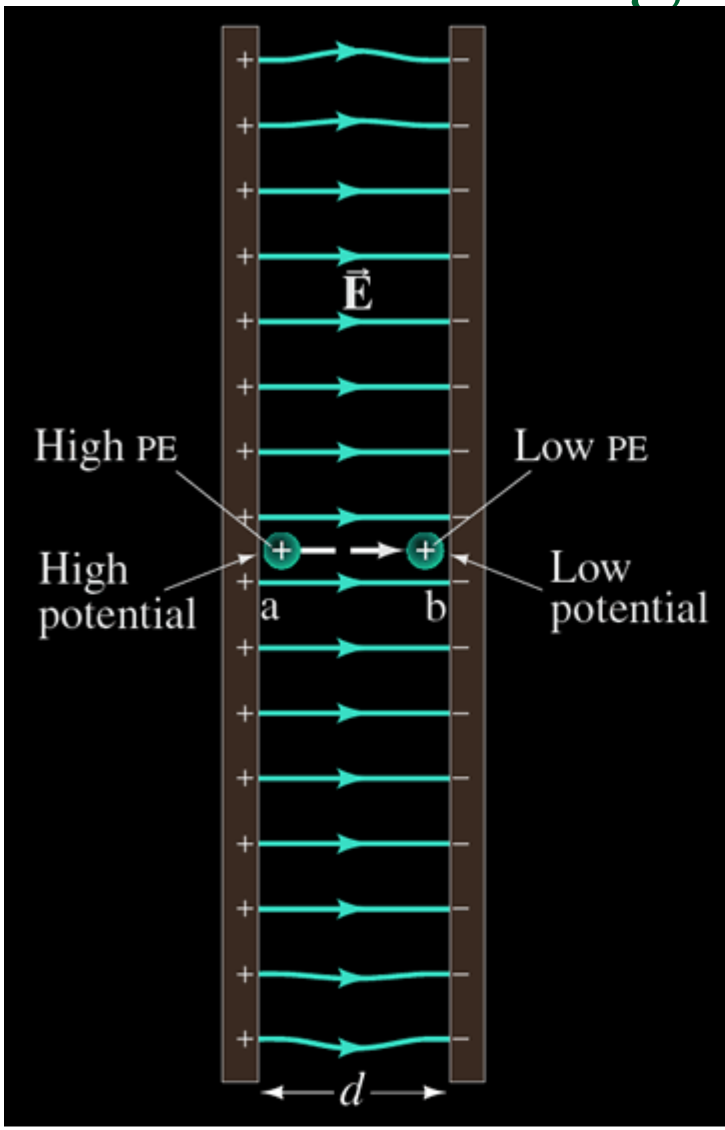
The amount of energy per charge has a specific name and it is called, VOLTAGE or ELECTRIC POTENTIAL (difference). Why the "difference"?

$$\Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$$





Understanding "Difference"

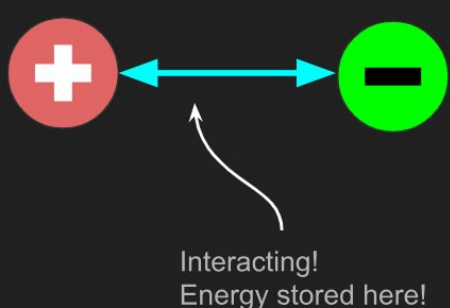


Let's say we have a proton placed between a set of charged plates. If the proton is held fixed at the positive plate, the ELECTRIC FIELD will apply a FORCE on the proton (charge). Since like charges repel, the proton is considered to have a high potential (voltage) similar to being above the ground. It moves towards the negative plate or low potential (voltage). The plates are charged using a battery source where one side is positive and the other is negative. The positive side is at 9V, for example, and the negative side is at 0V. So basically the charge travels through a "change in voltage" much like a falling mass experiences a "change in height. **(Note: The electron does the opposite)**

Electric Potential vs Electric Potential Energy

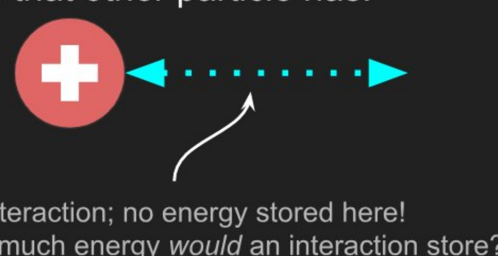
Electric Potential Energy

Energy stored in the electric field due to an interaction between two particles



Electric Potential

Description of space around **one** particle describing the energy that *would* be stored in an interaction with another particle per coulomb of charge that other particle has.



BEWARE!!!!!!

***W* is *Electric Potential Energy* (Joules)**
is <u>not</u>
***V* is *Electric Potential* (Joules/Coulomb)**
a.k.a Voltage, Potential Difference

The "other side" of that equation?

$$U_g = mgh$$

$$U_g \rightarrow U_E (or\, W)$$

$$m \rightarrow q$$

$$g \rightarrow E$$

$$h \rightarrow x \rightarrow d$$

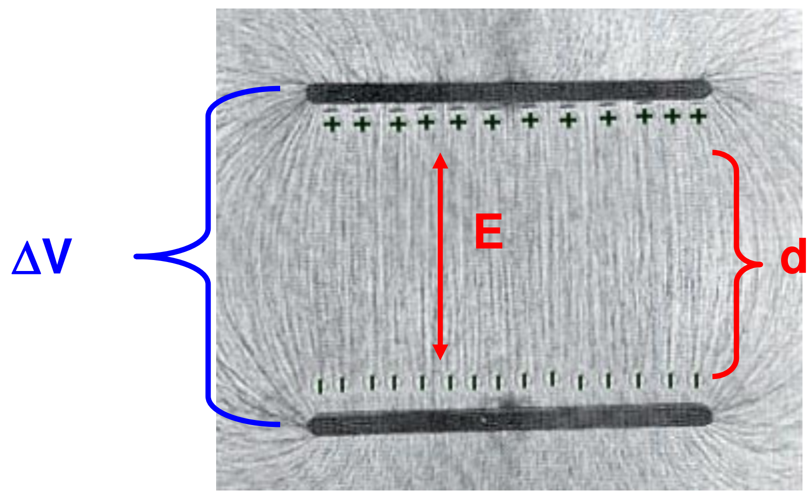
$$U_E(W) = qEd$$

$$\frac{W}{q} = \boxed{Ed}$$

Since the amount of energy per charge is called Electric Potential, or Voltage, the product of the electric field and displacement is also VOLTAGE

This makes sense as it is applied usually to a set of PARALLEL PLATES.

ΔV=Ed



Example

A pair of oppositely charged, parallel plates are separated by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field strength between the plates? (b) What is the magnitude of the force on an electron between the plates?

$$d = 0.00533m$$

$$\Delta V = 600V$$

$$E = ?$$

$$q_{e^-} = 1.6x10^{-19}C$$

$$\Delta V = Ed$$

$$600 = E(0.0053)$$

$$E = \textbf{113,207.55 N/C}$$

$$E = \frac{F_e}{q} = \frac{F_e}{1.6x10^{-19}C}$$

$$F_e = \textbf{1.81x10}^{-14}\textbf{ N}$$

Example

Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V

$$q_{p^+} = 1.6x10^{-19}C$$

$$m_{p^+} = 1.67x10^{-27}kg$$

$$V = 120V$$

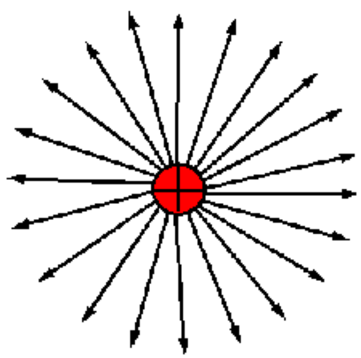
$$v = ?$$

$$\Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$$

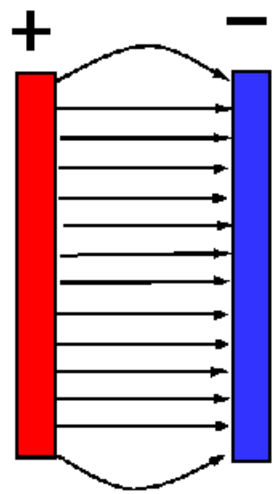
$$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6x10^{-19})(120)}{1.67x10^{-27}}} = 1.52x10^5 \text{ m/s}$$

Electric Potential of a Point Charge

Up to this point we have focused our attention solely to that of a set of parallel plates. But those are not the ONLY thing that has an electric field. Remember, point charges have an electric field that surrounds them.



A



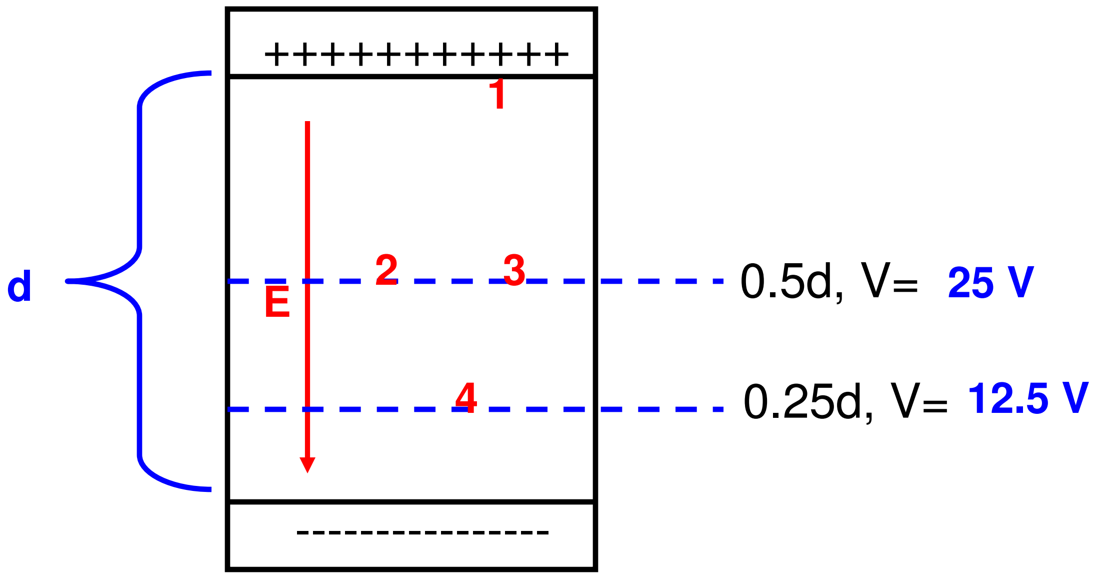
B

So imagine placing a TEST CHARGE out way from the point charge. Will it experience a change in electric potential energy? **YES!**

Thus is also must experience a change in electric potential as well.

Electric Potential

Let's use our "plate" analogy. Suppose we had a set of parallel plates symbolic of being "above the ground" which has potential difference of 50V and a CONSTANT Electric Field.



ΔV = ? From 1 to 2

25 V

ΔV = ? From 2 to 3

0 V

ΔV = ? From 3 to 4

12.5 V

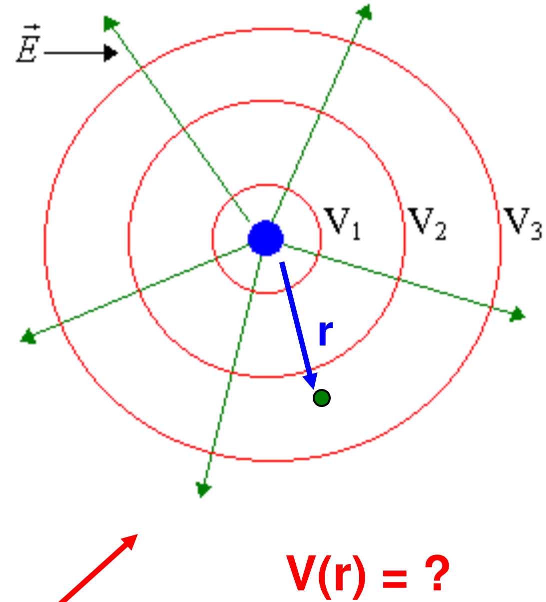
ΔV = ? From 1 to 4

37.5 V

Notice that the "ELECTRIC POTENTIAL" (Voltage) DOES NOT change from 2 to 3. They are symbolically at the same height and thus at the same voltage. The line they are on is called an EQUIPOTENTIAL LINE. What do you notice about the orientation between the electric field lines and the equipotential lines?

Equipotential Lines

So let's say you had a positive charge. The electric field lines move AWAY from the charge. The equipotential lines are perpendicular to the electric field lines and thus make concentric circles around the charge. As you move AWAY from a positive charge the potential decreases. So V1>V2>V3.



Now that we have the direction or visual aspect of the equipotential line understood the question is how can we determine the potential at a certain distance away from the charge?

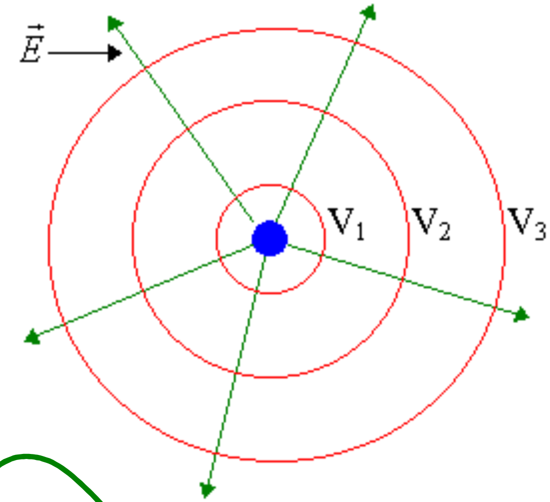
Electric Potential

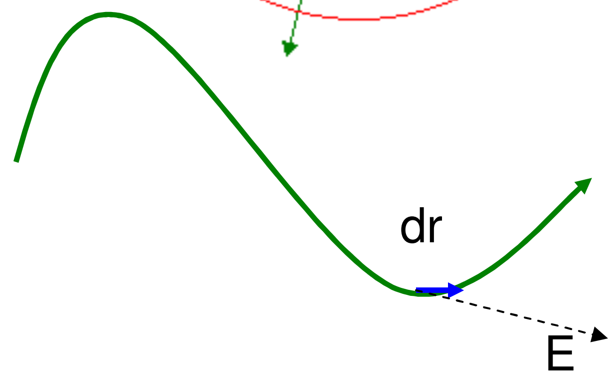
In the last slide is stated, "As you move AWAY from a positive charge the potential decreases". Since this is true we can say:

$$-\Delta V = Ed = Er, d = r$$

$$\Delta V = -\int E\, dr$$

The expression MUST be negative as a positive point charge moves towards a decreasing potential yet in the SAME direction a the electric field. A negative point, on the other hand, moves towards increasing potential yet in the OPPOSITE direction of the electric field.





In the case where the path or field varies we must define the path of a single dr, determine the "E" at that point and use integration to sum up over the entire path

Electric Potential of a Point Charge

$$-\Delta V = Ed = Er, d = r$$

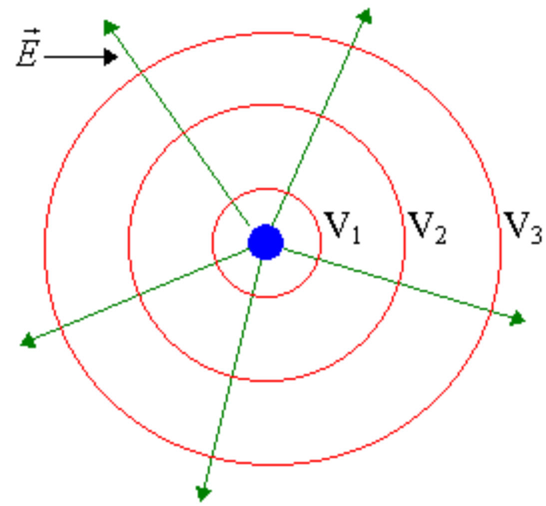
$$\Delta V = -\int E\, dr$$

$$V(r) - V(\infty) = -\int_{r=\infty}^{r} \left(\frac{Q}{4\pi\varepsilon_o r^2}\right) dr$$

$$V(r) = -\frac{Q}{4\pi\varepsilon_o} \int_{r=\infty}^{r} \frac{1}{r^2} dr$$

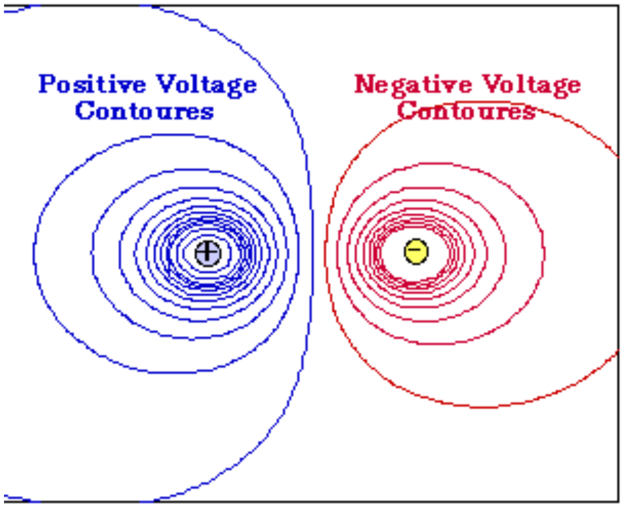
$$V(r) = -\frac{Q}{4\pi\varepsilon_o} \left[-\frac{1}{r} \right] \Big|_{r=\infty}^{r}$$

$$V(r) = \frac{1}{4\pi\varepsilon_o} \frac{Q}{r}$$

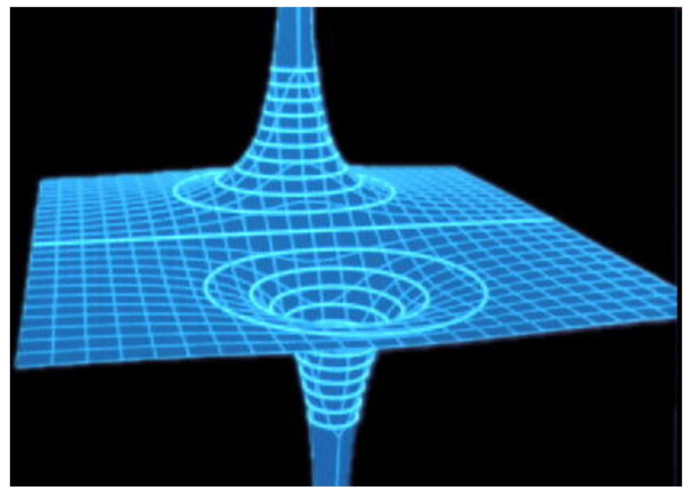


There are a few things you must keep in mind about electric potentials. They can be positive or negative, yet the sign has NOTHING to due with direction as electric potentials are SCALARS.

Electric Potential of a Point Charge



This is what you would see if you mapped 2 oppositely charged points charges. The view is like that of looking down from above. The equipotentials look like concentric circles.



This is what you would see if you rotated the above picture and looked at it as if your view was from the side. The positive point charge creates a HILL whereas the negative point charge creates a valley.

So the question is: *How would you find the voltage (electric potential) at a give position due to BOTH charges?*

Electric Potential of a Point Charge

$$\Delta V = \frac{W}{q} = \frac{F_E x}{q}; \quad x = r$$

$$\Delta V = \frac{F_E r}{q}; \quad F_E = k\frac{Qq}{r^2}$$

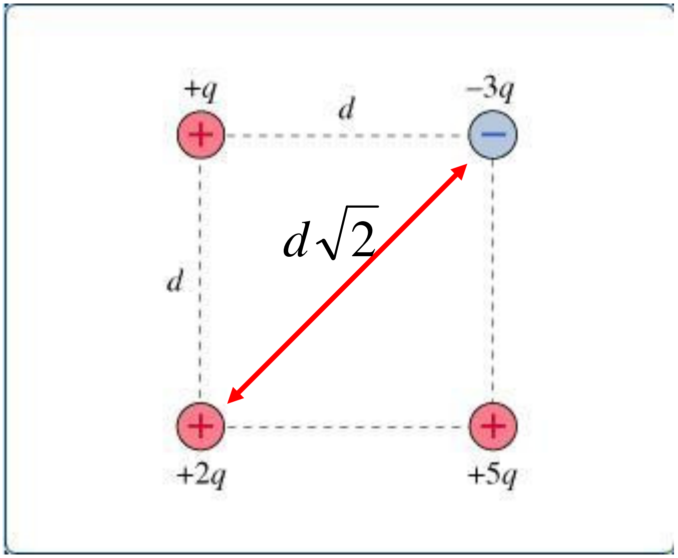
$$\Delta V = k\frac{Qqr}{qr^2} \rightarrow k\sum\frac{Q}{r}$$

Why the "sum" sign?

Voltage, unlike Electric Field, is NOT a vector! So if you have MORE than one charge you don't need to use vectors. Simply add up all the voltages that each charge contributes since voltage is a SCALAR.

WARNING! You must use the "sign" of the charge in this case.

Potential of a point charge



Suppose we had 4 charges each at the corners of a square with sides equal to ***d***.

If I wanted to find the potential at the CENTER I would SUM up all of the individual potentials.

Thus the distance from a corner to the center will equal:

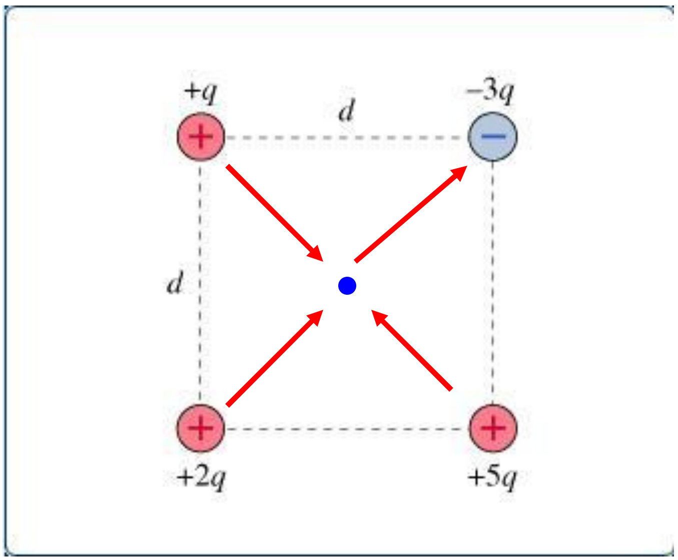
$$r = \frac{d\sqrt{2}}{2}$$

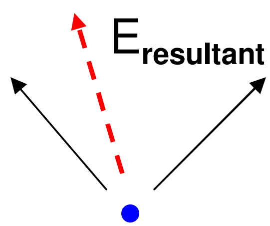
$$V_{center} = k\sum\frac{Q}{r}$$

$$V_{center} = k\sum\left(\frac{q}{r} + \frac{2q}{r} + \frac{-3q}{r} + \frac{5q}{r}\right)$$

$$V_{center} = k\left(\frac{5q}{r}\right) \rightarrow k\left(\frac{10q}{d\sqrt{2}}\right) \rightarrow k\frac{5\sqrt{2}}{d}$$

Electric field at the center? (Not so easy)



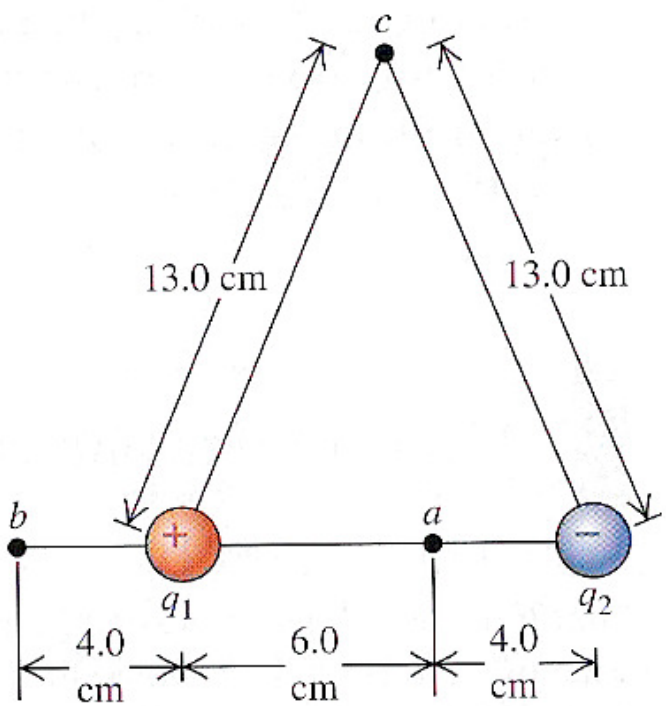


- If they had asked us to find the electric field, we first would have to figure out the visual direction, use vectors to break individual electric fields into components and use the Pythagorean Theorem to find the resultant and inverse tangent to find the angle

- So, yea….Electric Potentials are NICE to deal with!

Example

An electric dipole consists of two charges q_1 = +12nC and q_2 = -12nC, placed 10 cm apart as shown in the figure. Compute the potential at points a,b, and c.

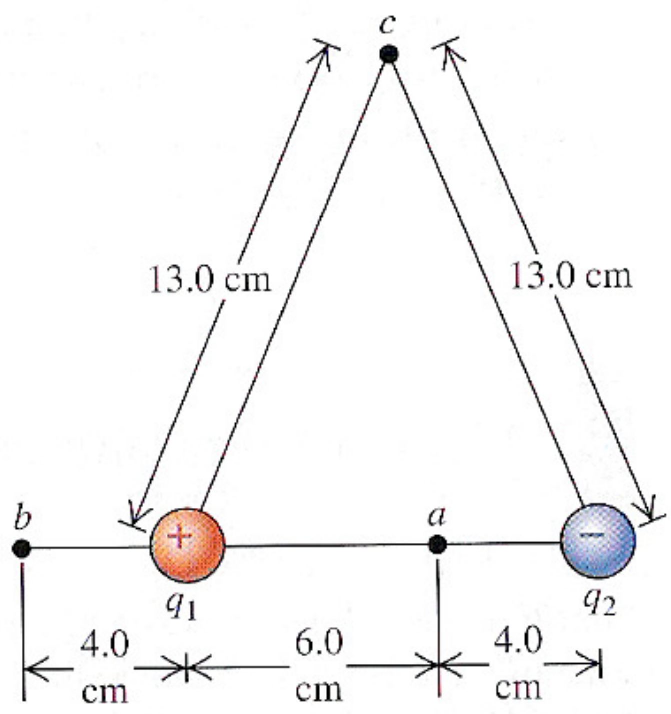


$$V_a = k\sum\left(\frac{q_1}{r_a} + \frac{q_2}{r_a}\right)$$

$$V_a = 8.99x10^9\left(\frac{12x10^{-9}}{0.06} + \frac{-12x10^{-9}}{0.04}\right)$$

$$V_a = $$ **-899 V**

Example cont'



$$V_b = k\sum\left(\frac{q_1}{r_b} + \frac{q_2}{r_b}\right)$$

$$V_b = 8.99x10^9\left(\frac{12x10^{-9}}{0.04} + \frac{-12x10^{-9}}{0.14}\right)$$

$V_b =$ **1926.4 V**

$V_c =$ **0 V**

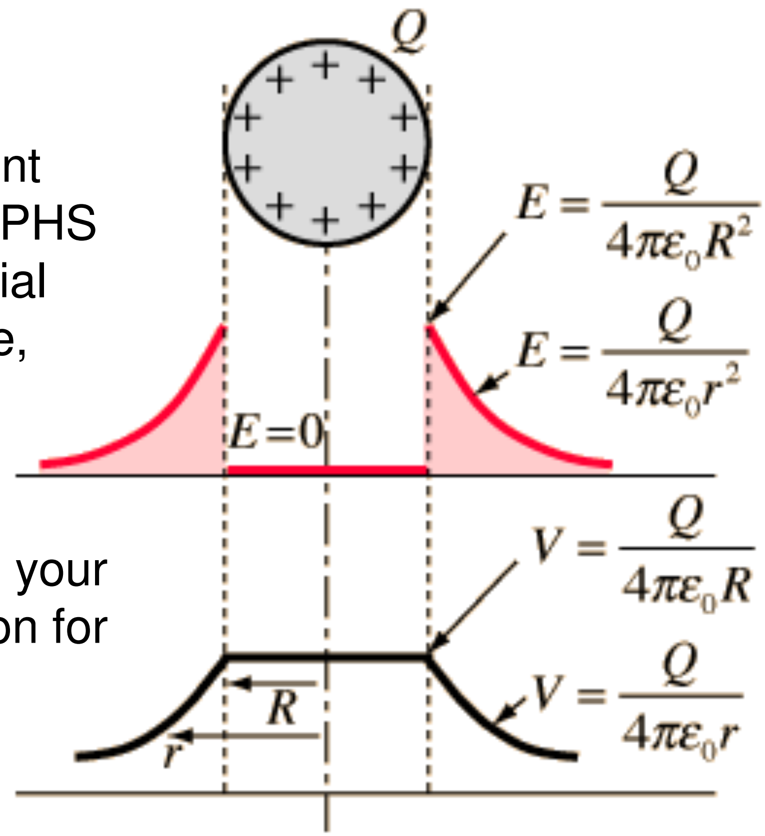
Since direction isn't important, the electric potential at "c" is zero. The electric field however is NOT. The electric field would point to the right.

Electric Potentials and Gauss' Law

Suppose you had a charged conducting sphere.

This figure provides us with an excellent visual representation of what the GRAPHS for the electric field and electric potential look like as you approach, move inside, and move away from the sphere.

Since the sphere behaves as a point charge (due to ENCLOSING IT within your chosen Gaussian surface), the equation for the electric potential is the same.



But what about a cylinder or sheet?

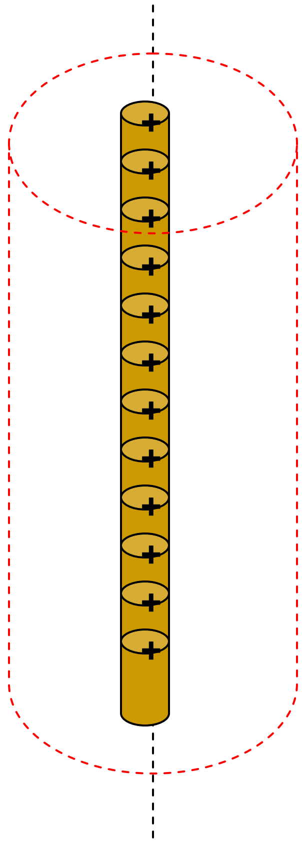
Electric Potential for Cylinders

$$E\oint da = \frac{q_{enc}}{\varepsilon_o} \qquad E(2\pi rL) = \frac{q_{enc}}{\varepsilon_o}$$

$$E(2\pi rL) = \frac{\lambda L}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi r\varepsilon_o}$$

Using Gauss' Law we derived and equation to define the electric field as we move radially away from the charged cylinder. Electric Potential?



$$\Delta V = -\int E\,dr$$

$$V(b) - V(a) = -\int_a^b \left(\frac{\lambda}{2\pi r\varepsilon_o}\right)dr$$

$$V(b) - V(a) = \frac{\lambda}{2\pi\varepsilon_o}\int_b^a \frac{1}{r}dr$$

You can get a POSITIVE expression by switching your limits, thus eliminating the minus sign!

$$V(r) = -\frac{\lambda}{2\pi\varepsilon_o}\left[\ln\frac{|a|}{|b|}\right] \rightarrow \boxed{V(r) = \frac{\lambda}{2\pi\varepsilon_o}\ln|a| - \ln|b|}$$

The electric potential function for a cylinder.

Electric Potential for Conducting Sheets

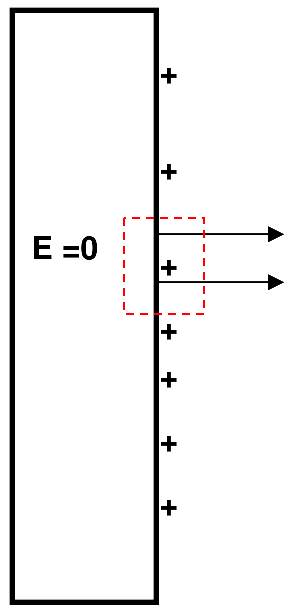
$$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$$

$$EA = \frac{Q}{\varepsilon_o}$$

$$\sigma = \frac{Q}{A}, \quad EA = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{\varepsilon_o}$$

Using Gauss' Law we derived and equation to define the electric field as we move radially away from the charged sheet or plate. Electric Potential?



$$\Delta V = -\int E\, dr$$

$$V(b) - V(a) = -\int_a^b \left(\frac{\sigma}{\varepsilon_o}\right) dr$$

$$V(b) - V(a) = \int_b^a \left(\frac{\sigma}{\varepsilon_o}\right) dr$$

$$V(b) - V(a) = \frac{\sigma}{\varepsilon_o}(a - b), \quad a - b = d$$

$$\Delta V = \frac{\sigma}{\varepsilon_o} d = Ed = \frac{Qd}{\varepsilon_o A}$$

This expression will be particularly useful later

In summary

You can use Gauss' Law to derive electric field functions for conducting/insulating: spheres (points), cylinders (rods), or sheets (plates). If you INTEGRATE that function you can then derive the electric potential function.

$$E\oint da = \frac{q_{enc}}{\varepsilon_o} \qquad \Delta V = -\int E dr$$