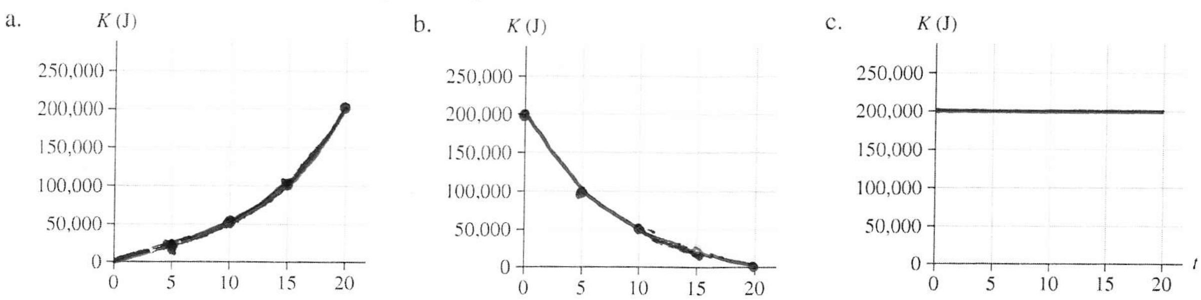
10 Energy

10.2 Kinetic Energy and Gravitational Potential Energy

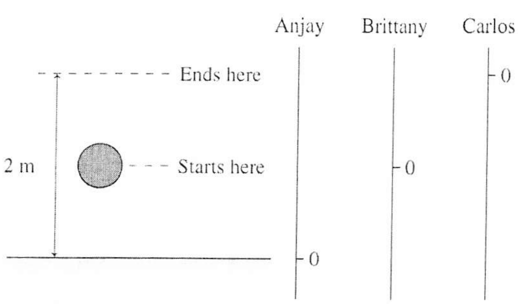
10.3 A Closer Look at Gravitational Potential Energy

1. On the axes below, draw graphs of the kinetic energy of
   a. A 1000 kg car that uniformly accelerates from 0 to 20 m/s in 20 s.
   b. A 1000 kg car moving at 20 m/s that brakes to a halt with uniform deceleration in 20 s.
   c. A 1000 kg car that drives once around a 130-m-diameter circle at a speed of 20 m/s.

   Calculate K at several times, plot the points, and draw a smooth curve between them.



2. Below we see a 1 kg object that is initially 1 m above the ground and rises to a height of 2 m. Anjay, Brittany, and Carlos each measure its position, but each of them uses a different coordinate system. Fill in the table to show the initial and final gravitational potential energies and ΔU as measured by our three aspiring scientists.



	U_i	U_f	ΔU
Anjay	9.8 J	19.6 J	+9.8 J
Brittany	0	9.8 J	+9.8 J
Carlos	-9.8 J	0	+9.8 J

3. A roller coaster car rolls down a frictionless track, reaching speed v_f at the bottom.

   a. If you want the car to go twice as fast at the bottom, by what factor must you increase the height of the track?

   $U_i = K_f$ $\quad K_f = \frac{1}{2}m(2v_i)^2 = 4k$ $\quad$ so you must increase the height by a factor of 4.

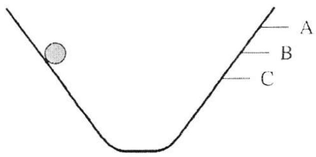
   $mgh = \frac{1}{2}mv^2$ $\quad K_f \rightarrow 4K_f$ when $V \rightarrow 2V$

   $mgh \rightarrow mg(4h)$

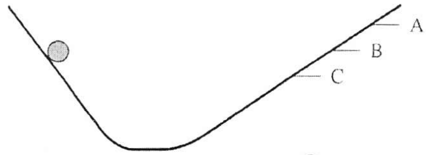
   b. Does your answer to part a depend on whether the track is straight or not? Explain.

   No, the gravitational potential energy depends only on the height.

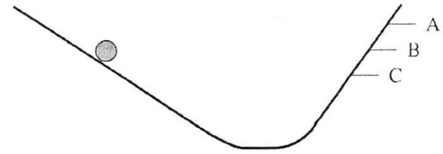
4. Below are shown three frictionless tracks. A ball is released from rest at the position shown on the left. To which point does the ball make it on the right before reversing direction and rolling back? Point B is the same height as the starting position.



Makes it to B



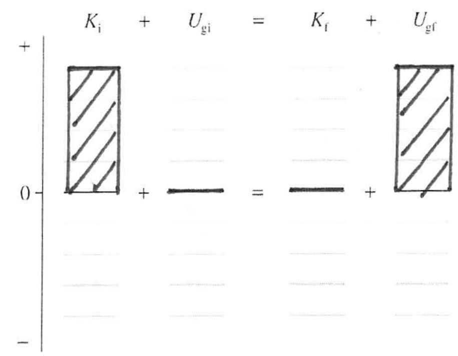
Makes it to B



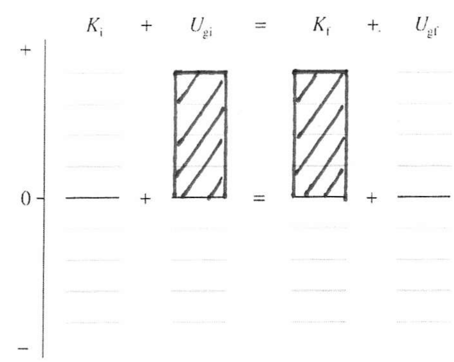
Makes it to B

Exercises 5–7: Draw an energy bar chart to show the energy transformations for the situation described.

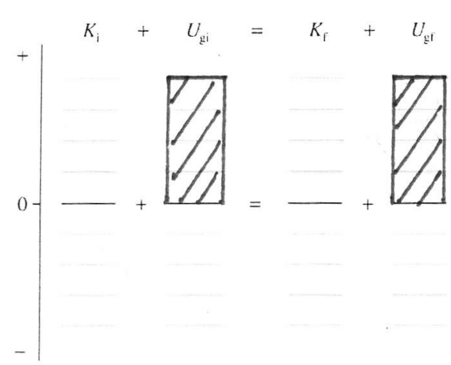
5. A car runs out of gas and coasts up a hill until finally stopping.



6. A pendulum is held out at 45° and released from rest. A short time later it swings through the lowest point on its arc.

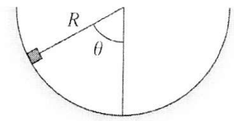


7. A ball starts from rest on the top of one hill, rolls without friction through a valley, and just barely makes it to the top of an adjacent hill.

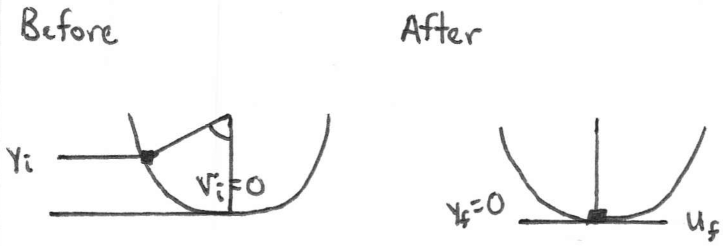


8. A small cube of mass m slides back and forth in a frictionless, hemispherical bowl of radius R. Suppose the cube is released at angle θ. What is the cube's speed at the bottom of the bowl?

PSS 10.1



a. Begin by drawing a before-and-after visual overview. Let the cube's initial position and speed be y_i and v_i. Use a similar notation for the final position and speed.



b. At the initial position, are either K_i or U_{gi} zero? If so, which? $K_i = 0$

c. At the final position, are either K_f or U_{gf} zero? If so, which? $U_{gf} = 0$

d. Does thermal energy need to be considered in this situation? Why or why not?

No. The bowl is frictionless.

e. Write the conservation of energy equation in terms of position and speed variables, omitting any terms that are zero.

$$mgy_i = \frac{1}{2}mv_f^2$$

f. You're given not the initial position but the initial angle. Do the geometry and trigonometry to find y_i in terms of R and θ.

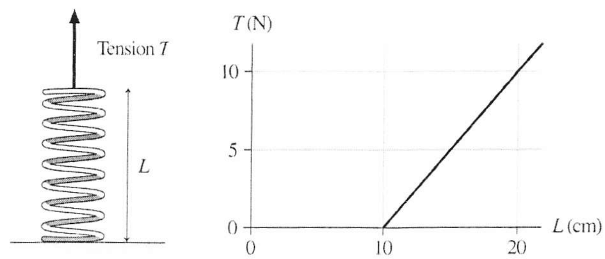
$$y_i = R(1 - \cos\theta)$$

g. Use your result of part f in the energy conservation equation, and then finish solving the problem.

$$\cancel{m}gR(1-\cos\theta) = \frac{1}{2}\cancel{m}v_f^2 \quad \text{so} \quad \sqrt{2gR(1-\cos\theta)} = v_f$$

10.4 Restoring Forces and Hooke's Law

9. A spring is attached to the floor and pulled straight up by a string. The string's tension is measured. The graph shows the tension in the string as a function of the spring's length L.



a. Does this spring obey Hooke's Law? Explain why or why not.

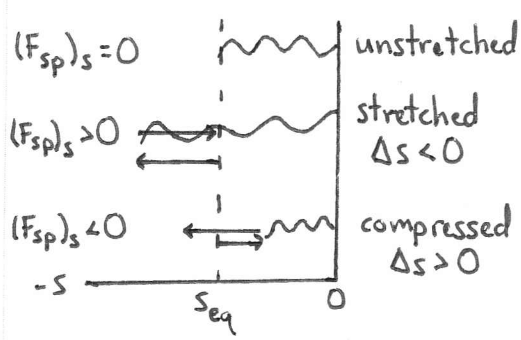
Yes, the plot is linear.

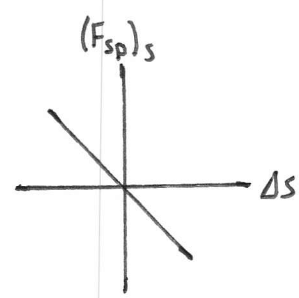
$\Delta T = k \Delta L$

b. If it does, what is the spring constant?

$$k = \frac{\Delta T}{\Delta L} = \frac{10\,N}{10\,cm} = 1\,\frac{N}{cm} = \underline{100\,\frac{N}{m}}$$

10. Draw a figure analogous to Figure 10.15 in the textbook for a spring that is attached to a wall on the *right* end. Use the figure to show that F and Δs always have opposite signs.





11. A spring has an unstretched length of 10 cm. It exerts a restoring force F when stretched to a length of 11 cm.

a. For what length of the spring is its restoring force $3F$?

$F_{sp} = -k\Delta x$ so for $F \to 3F$, $\Delta x \to 3\Delta x = 3\text{cm}$

$10\text{cm} + 3\Delta x = \underline{13\text{ cm}}$

b. At what compressed length is the restoring force $2F$?

$F \to -2F \quad \Delta x \to -2\Delta x = -2\text{cm}$

$10\text{cm} - 2\text{cm} = \underline{8\text{cm}}$

12. The left end of a spring is attached to a wall. When Bob pulls on the right end with a 200 N force, he stretches the spring by 20 cm. The same spring is then used for a tug-of-war between Bob and Carlos. Each pulls on his end of the spring with a 200 N force.

a. How far does Bob's end of the spring move? Explain.

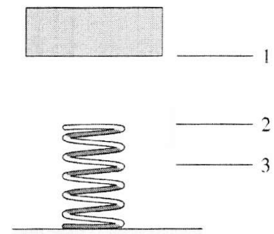
10cm Though the spring stretched 20 cm originally, its center moved by 10cm. In this case, Carlos provides the opposing force previously provided by the wall, except that he moves also.

b. How far does Carlos's end of the spring move? Explain.

-10cm The total stretch under a 200 N tension must still be 20cm.

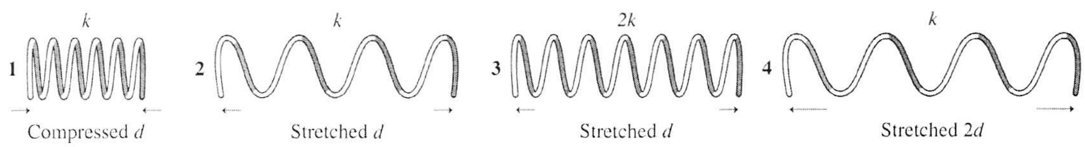
10.5 Elastic Potential Energy

13. A heavy object is released from rest at position 1 above a spring. It falls and contacts the spring at position 2. The spring achieves maximum compression at position 3. Fill in the table below to indicate whether each of the quantities are +, –, or 0 during the intervals 1→2, 2→3, and 1→3.



	1→2	2→3	1→3
ΔK	+	–	0
ΔU_g	–	–	–
ΔU_s	0	+	+

14. Rank in order, from most to least, the amount of elastic potential energy $(U_s)_1$ to $(U_s)_4$ stored in each of these springs.



Order: $(U_s)_4 > (U_s)_3 > (U_s)_2 = (U_s)_1$

Explanation:

$U_s = \frac{1}{2}k(\Delta s)^2$ Increasing the stretch by a factor of 2 increases the stored energy by a factor of 4.

15. A spring gun shoots out a plastic ball at speed v_0. The spring is then compressed twice the distance it was on the first shot.

a. By what factor is the spring's potential energy increased?

$\frac{1}{2}k(2\Delta s)^2 = 4[\frac{1}{2}k(\Delta s)^2]$

$\Rightarrow \underline{4x}$

b. By what factor is the ball's launch speed increased? Explain.

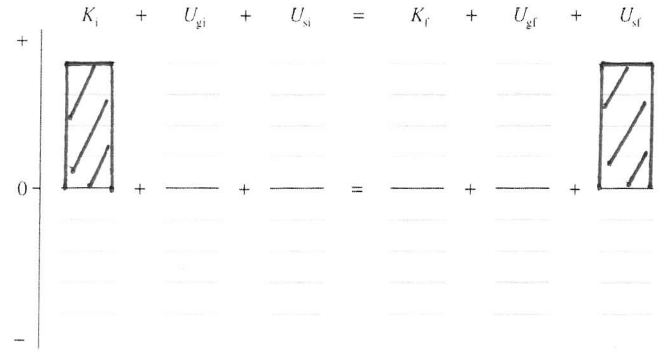
$\frac{1}{2}k(2\Delta s)^2 = \frac{1}{2}m(2v)^2$

$\Rightarrow \underline{2x}$

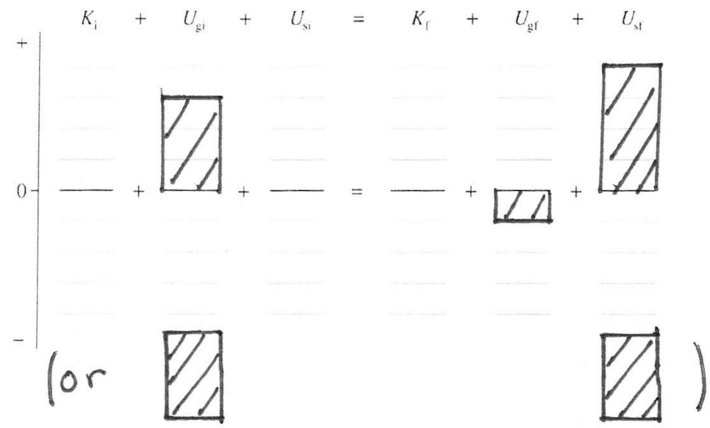
Both the speed and Δs are squared in the energy expressions.

Exercises 16–17: Draw an energy bar chart to show the energy transformations for the situation described.

16. A bobsled sliding across frictionless, horizontal ice runs into a giant spring. A short time later the spring reaches its maximum compression.

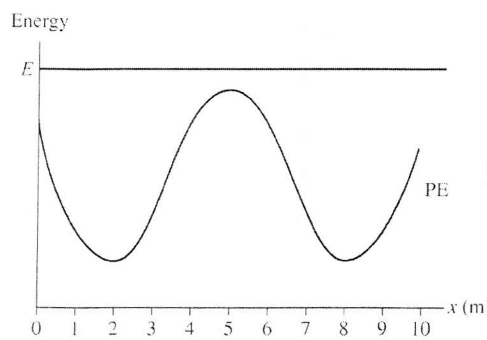


17. A brick is held above a spring that is standing on the ground. The brick is released from rest, and a short time later the spring reaches its maximum compression.



10.6 Energy Diagrams

18. A particle with the potential energy shown in the graph is moving to the right at $x = 0$ m with total energy E.



a. At what value or values of x is the particle's speed a maximum?

At 2m and 8m.

b. At what value or values of x is the particle's speed a minimum?

At 5m.

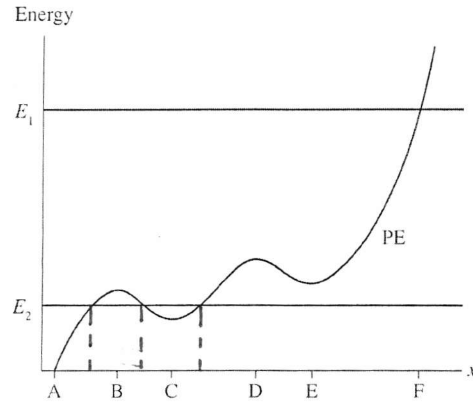
c. At what value or values of x is the potential energy a maximum?

At 5m.

d. Does this particle have a turning point in the range of x covered by the graph? If so, where?

The particle does not have a turning point on this graph.

19. The figure shows a potential-energy curve. Suppose a particle with total energy E_1 is at position A and moving to the right.



a. For each of the following regions of the x-axis, does the particle speed up, slow down, maintain a steady speed, or change direction?

A to B slows down

B to C speeds up

C to D slows down

D to E speeds up

E to F slows down

b. Where is the particle's turning point? F

c. For a particle that has total energy E_2, what are the possible motions and where do they occur along the x-axis?

The particle could be moving between $x=0$ and the point indicated by the dashed line between A and B. The particle could be oscillating about point C between the nearest dashed lines.

d. What position or positions are points of stable equilibrium? For each, would a particle in equilibrium at that point have total energy $\le E_2$, between E_2 and E_1, or $\ge E_1$?

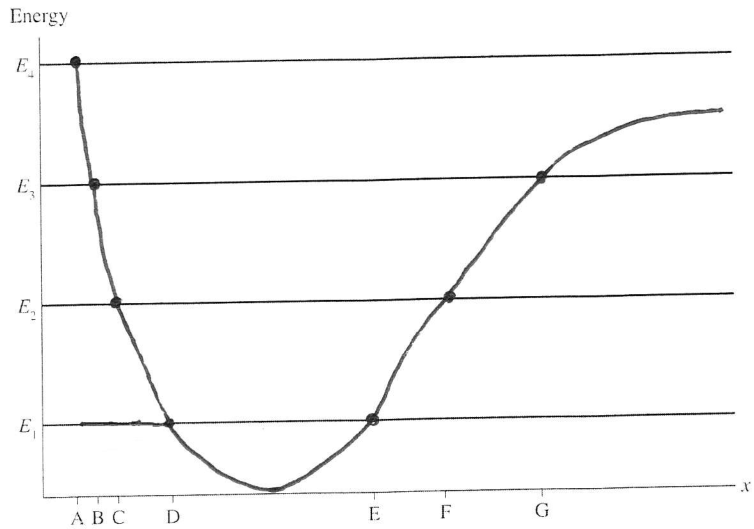
C and E are points of stable equilibrium. At C, the total energy could be $< E_2$ or between E_1 and E_2. At E, the total energy must be between E_1 and E_2.

e. What position or positions are points of unstable equilibrium? For each, would a particle in equilibrium at that point have total energy $\le E_2$, between E_2 and E_1, or $\ge E_1$?

B and D are unstable equilibrium points. The particle would have an energy between E_1 and E_2.

20. Below are a set of axes on which you are going to draw a potential-energy curve. By doing experiments, you find the following information:
   - A particle with energy E_1 oscillates between positions D and E.
   - A particle with energy E_2 oscillates between positions C and F.
   - A particle with energy E_3 oscillates between positions B and G.
   - A particle with energy E_4 enters from the right, bounces at A, then never returns.

   Draw a potential-energy curve that is consistent with this information.



10.7 Elastic Collisions

21. Ball 1 with an initial speed of 14 m/s has a perfectly elastic collision with ball 2 that is initially at rest. Afterward, the speed of ball 2 is 21 m/s.

   a. What will be the speed of ball 2 if the initial speed of ball 1 is doubled?

$(V_{fx})_2 = \frac{2m_1}{m_1+m_2}(V_{ix})_1$. Therefore, doubling $(v_{ix})_1$ will double $(V_{fx})_2$

$$(V_{fx})_2 = 2(21\,\tfrac{m}{s}) = \underline{42\,\tfrac{m}{s}}$$

   b. What will be the speed of ball 2 if the mass of ball 1 is doubled?

From the previous part a. $21\,\frac{m}{s} = \frac{2m_1}{m_1+m_2}(14\,\frac{m}{s})$ multiply by $\frac{\frac{1}{m_1}}{\frac{1}{m_1}}$

$\boxed{24\,\frac{m}{s}}$

$$21\,\tfrac{m}{s} = \frac{2}{1+\frac{m_2}{m_1}}(14\,\tfrac{m}{s}) \quad \text{solve for } \frac{m_2}{m_1}$$

$$\frac{m_2}{m_1} = \frac{1}{3}$$

Doubling m_1 yields $\frac{m_2}{m_1} = \frac{1}{6}$, then $(V_{fx})_2 = \frac{2}{1+\frac{1}{6}}(14\,\frac{m}{s}) = \boxed{24\,\frac{m}{s}}$