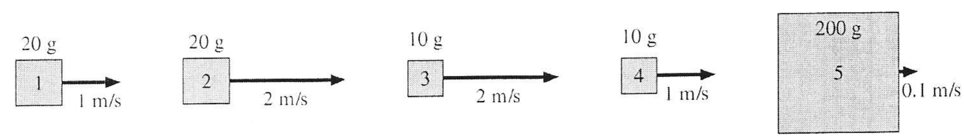
9 Impulse and Momentum

9.1 Momentum and Impulse

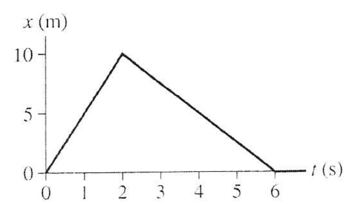
9.2 Solving Impulse and Momentum Problems

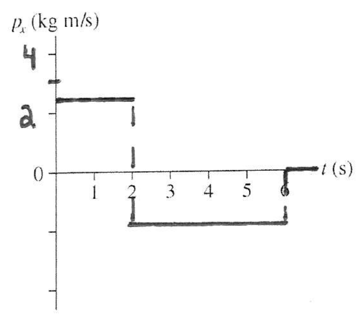
1. Rank in order, from largest to smallest, the momenta $(p_x)_1$ to $(p_x)_5$.



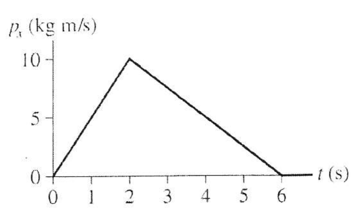
Order: $(P_x)_2 > (P_x)_1 = (P_x)_3 = (P_x)_5 > (P_x)_4$

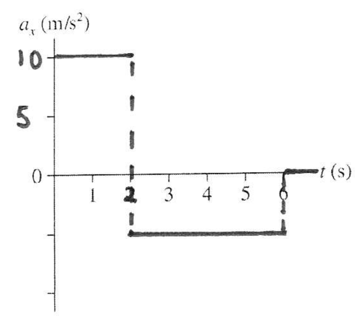
2. The position-versus-time graph is shown for a 500 g object. Draw the corresponding momentum-versus-time graph. Supply an appropriate scale on the vertical axis.





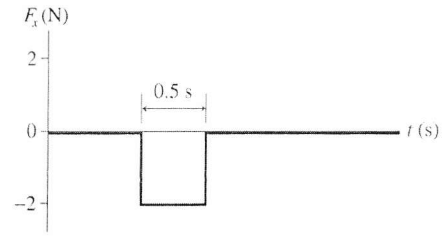
3. The momentum-versus-time graph is shown for a 500 g object. Draw the corresponding acceleration-versus-time graph. Supply an appropriate scale on the vertical axis.





4. A 2 kg object is moving to the right with a speed of 1 m/s when it experiences an impulse due to the force shown in the graph. What is the object's speed and direction after the impulse?

a.

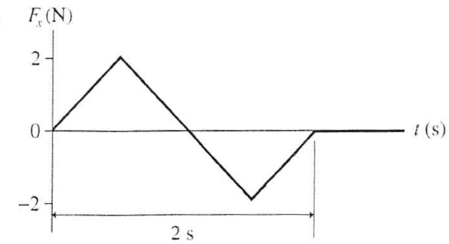


$J_x = -2\text{N} \cdot 0.5\text{s} = -1\text{N}\cdot\text{s} = \Delta P_x$

$\Delta V_x = \frac{\Delta P_x}{m} = \frac{-1\text{ N}\cdot\text{s}}{2\text{ kg}} = -0.5\frac{\text{m}}{\text{s}}$

$V_{fx} = V_{ix} + \Delta V_x = 1\frac{\text{m}}{\text{s}} - 0.5\frac{\text{m}}{\text{s}} =$ **0.5 m/s right**

b.



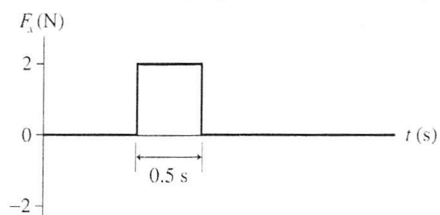
$J_x = \frac{1}{2}\cdot 2\text{N}\cdot 1\text{s} - \frac{1}{2}\cdot 2\text{N}\cdot 1\text{s} = 0$

$\Delta V_x = 0$

$V_{fx} =$ **1 m/s to the right**

5. A 2 kg object is moving to the left with a speed of 1 m/s when it experiences an impulse due to the force shown in the graph. What is the object's speed and direction after the impulse?

a.

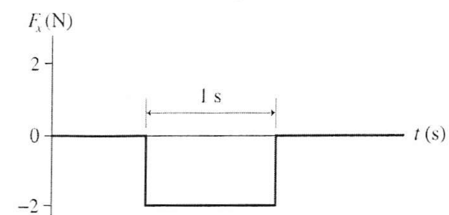


$J_x = 2\text{N}\cdot 0.5\text{s} = 1\text{ N}\cdot\text{s} = \Delta P_x$

$\Delta V_x = \frac{\Delta P_x}{m} = \frac{1\text{ N}\cdot\text{s}}{2\text{ kg}} = 0.5\frac{\text{m}}{\text{s}}$

$V_{fx} = V_{ix} + \Delta V_x = -1\frac{\text{m}}{\text{s}} + 0.5\frac{\text{m}}{\text{s}} =$ **−0.5 m/s (left)**

b.

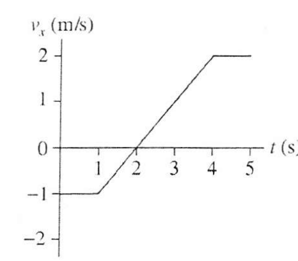


$J_x = -2\text{ N}\cdot 1\text{s} = -2\text{ N}\cdot\text{s} = \Delta P_x$

$\Delta V_x = \frac{\Delta P_x}{m} = \frac{-2\text{ N}\cdot\text{s}}{2\text{ kg}} = -1\frac{\text{m}}{\text{s}}$

$V_{fx} = V_{ix} + \Delta V_x = -1\frac{\text{m}}{\text{s}} - 1\frac{\text{m}}{\text{s}} =$ **−2 m/s**

6. A 2 kg object has the velocity graph shown.

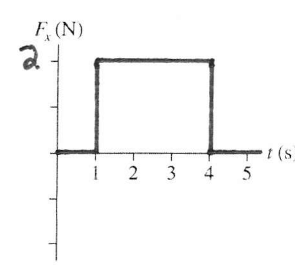


a. What is the object's initial momentum? $P_x = -2\text{ kg}\frac{\text{m}}{\text{s}}$

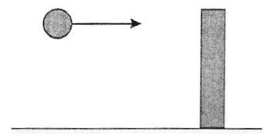
b. What is the object's final momentum? $P_x = 4\text{ kg}\frac{\text{m}}{\text{s}}$

c. What impulse does the object experience? $J_x = 6\text{ kg}\frac{\text{m}}{\text{s}}$

d. Draw the graph showing the force on the object.



7. A carnival game requires you to knock over a wood post by throwing a ball at it. You're offered a very bouncy rubber ball and a very sticky clay ball of equal mass. Assume that you can throw them with equal speed and equal accuracy. You only get one throw.



a. Which ball will you choose? Why?

Choose the bouncy rubber ball. Because the rubber ball bounces back, it experiences a greater change in momentum than the clay ball at the same initial speed. Therefore, it imparts a greater impulse on the post and is more likely to knock it over.

b. Let's think about the situation more carefully. Both balls have the same initial momentum p_{ix} just before hitting the post. The clay ball sticks, the rubber ball bounces off with essentially no loss of speed. In terms of p_{ix}, what is the final momentum of each ball?

Clay ball: $p_{fx} =$ 0 Rubber ball: $p_{fx} =$ $-P_{fx}$

Hint: Momentum has a sign. Did you take the sign into account?

c. What is the *change* in the momentum of each ball?

Clay ball: $\Delta p_x =$ $-P_{fx}$ Rubber ball: $\Delta p_x =$ $-2P_{fx}$

d. Which ball experiences a larger impulse during the collision? Explain.

The bouncy rubber ball experiences a greater impulse as can be seen from its greater change in momentum.

e. From Newton's third law, the impulse that the ball exerts on the post is equal in magnitude, although opposite in direction, to the impulse that the post exerts on the ball. Which ball exerts the larger impulse on the post?

The rubber ball exerts a larger impulse because of the greater impulse on it.

f. Don't change your answer to part a, but are you still happy with that answer? If not, how would you change your answer? Why?

Still happy, we hope.

8. A falling rubber ball bounces on the floor.

a. Use the language of force, acceleration, and action/reaction to describe what happens.

The ball exerts a force on the floor and the floor exerts an equal, but opposite, force back on the ball. The ball's acceleration is large, though the floor's is negligible because of its huge mass. It is difficult to analyze the forces and the acceleration without knowing more.

b. Use the language of impulse and momentum to describe what happens.

Because of the impulse from the floor, the rubber ball's momentum changes by more than the magnitude of its initial momentum. The floor receives an equal impulse in the opposite direction but has no measurable velocity due to its large mass.

9. A small, light ball S and a large, heavy ball L move toward each other, collide, and bounce apart.

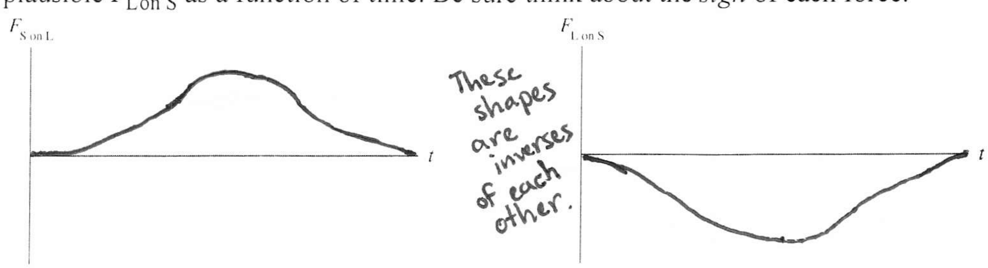
a. Compare the force that S exerts on L during the collision to the force that L exerts on S. That is, is $F_{S\text{ on }L}$ larger, smaller, or equal to $F_{L\text{ on }S}$? Explain.

$|F_{S\text{ on }L}| = |F_{L\text{ on }S}|$. These two forces are an action-reaction pair, equal and opposite by Newton's third law.

b. Compare the time interval during which S experiences a force to the time interval during which L experiences a force. Are they equal, or is one longer than the other?

The time intervals are also equal.

c. Sketch a graph showing a *plausible* $F_{S\text{ on }L}$ as a function of time and another graph showing a plausible $F_{L\text{ on }S}$ as a function of time. Be sure think about the *sign* of each force.



d. Compare the impulse delivered to S to the impulse delivered to L. Explain.

The forces are equal and are exerted over the same time interval. Therefore, the impulses are equal, but opposite in direction.

e. Compare the momentum change of S to the momentum change of L.

The momentum changes are equal in magnitude, but opposite in direction for each.

f. Compare the velocity change of S to the velocity change of L.

The velocity changes are not equal. S experiences a much greater change because its mass is much smaller.

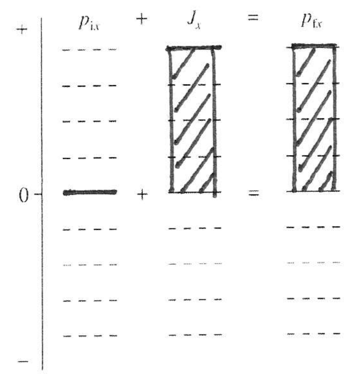
g. What is the change in the *sum* of the momenta of the two balls? Is it positive, negative, or zero?

There is no change in the sum of the momenta of the two balls, because their changes are equal and opposite.

Exercises 10–12: Draw a momentum bar chart to show the momenta and impulse for the situation described.

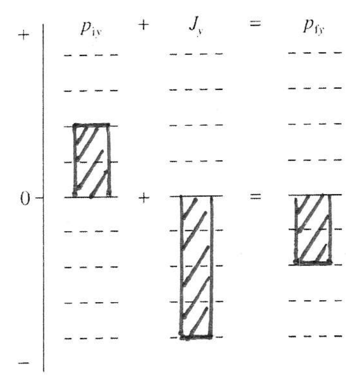
10. A compressed spring shoots a ball to the right. The ball was initially at rest.

All of the final momentum comes from the impulse provided by the spring.



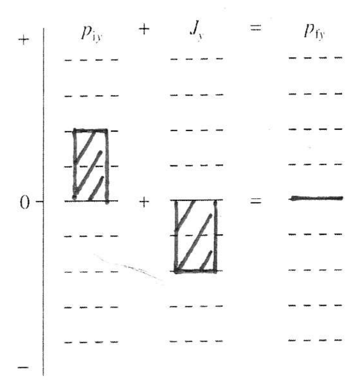
11. A rubber ball is tossed straight up and bounces off the ceiling. Consider only the collision with the ceiling.

For a perfectly elastic collision with the ceiling, the change in momentum is twice the final momentum. The impulse is provided by the ceiling.



12. A clay ball is tossed straight up and sticks to the ceiling. Consider only the collision with the ceiling.

The ceiling provides an impulse equal and opposite to the initial momentum.



9.3 Conservation of Momentum

13. A golf club continues forward after hitting the golf ball. Is momentum conserved in the collision? Explain, making sure you are careful to identify the "system."

At the moment of collision it may be reasonable to consider the momentum of just the club and ball as conserved. Just before and after the collision, it would be necessary to include the golfer and even the earth as part of the system.

14. As you release a ball, it falls—gaining speed and momentum. Is momentum conserved?

a. Answer this question from the perspective of choosing the ball alone as the system.

The momentum of the ball is not conserved. The weight is an unbalanced force from an agent external to the system.

b. Answer this question from the perspective of choosing ball + earth as the system.

If the earth is included, the momentum is conserved. While the ball is gaining downward momentum, the earth is gaining an equal magnitude of upward momentum. (but it can't be measured due to the earth's mass.)

15. Two particles collide, one of which was initially moving and the other initially at rest.

a. Is it possible for *both* particles to be at rest after the collision? Give an example in which this happens, or explain why it can't happen.

Both particles cannot be at rest immediately after the collision. If they were both at rest, then some momentum would have to be lost to a third object that is part of the system.

b. Is it possible for *one* particle to be at rest after the collision? Give an example in which this happens, or explain why it can't happen.

If the masses are equal and the collision is elastic, the moving particle will stop and give all of its momentum to the previously resting particle. A good example of this appears when a billiard ball rolls directly into another resting billiard ball.

16. A tennis ball traveling to the left at speed v_{Bi} is hit by a tennis racket moving to the right at speed v_{Ri}.
PSS 9.1 Although the racket is swung in a circular arc, its forward motion during the collision with the ball is so small that we can consider it to be moving in a straight line. Further, we can invoke the *impulse approximation* to neglect the steady force of the arm on the racket during the brief duration of its collision with the ball. Afterward, the ball is returned to the right at speed v_{Bf}. What is the racket's speed after it hits the ball? The masses of the ball and racket are m_B and m_R, respectively.

a. Begin by drawing a before-and-after pictorial representation as described in Tactics Box 9.1. You can assume that the racket continues in the forward direction but at a reduced speed.

b. Define the system. That is, what object or objects should be inside the system so that it is an *isolated system* whose momentum is conserved?

The racket and the ball.

c. Write an expression for P_{ix}, the total momentum of the system before the collision. Your expression should be written using the quantities given in the problem statement. Notice, however, that you're given *speeds*, but momentum is defined in terms of *velocities*. Based on your coordinate system and the directions of motion, you may need to give a negative momentum to one or more objects.

$$P_{ix} = P_{\text{ball}_i\,(B)} + P_{\text{racket}_i\,(R)} = -m_B v_{B_i} + m_R v_{R_i}$$

d. Now write an expression for P_{fx}, the total momentum of the system after the collision.

$$P_{fx} = m_R v_{R_f} + m_B v_{B_f}$$

e. If you chose the system correctly, its momentum is conserved. So equate your expressions for the initial and final total momentum, and then solve for what you want to find.

$$-m_B v_{B_i} + m_R v_{R_i} = m_R v_{R_f} + m_B v_{B_f}$$

$$v_{R_i} - \frac{m_B(v_{B_i} + v_{B_f})}{m_R} = v_{R_f}$$

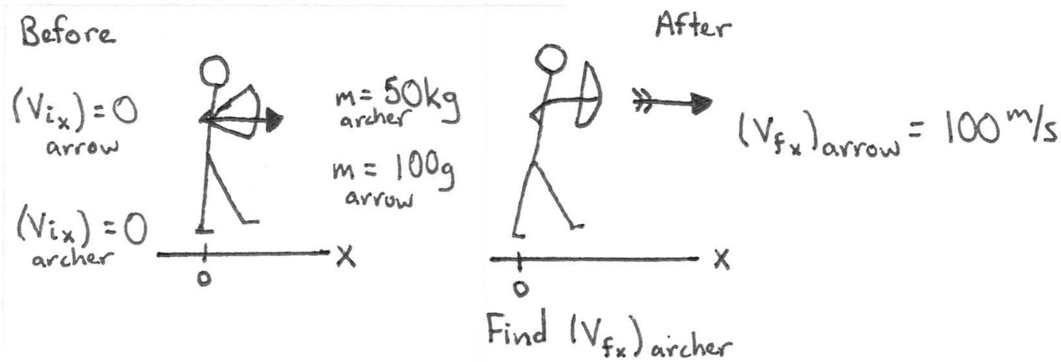
9.4 Inelastic Collisions

9.5 Explosions

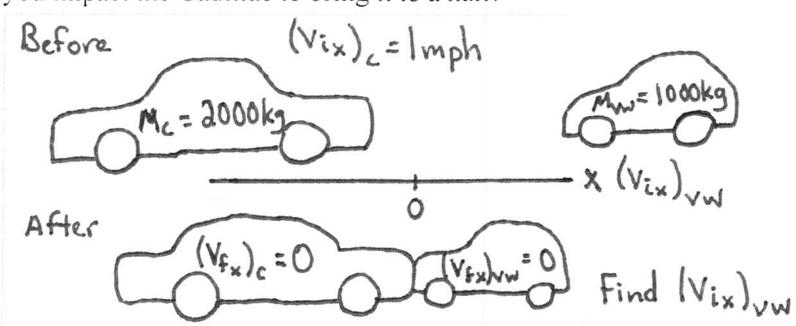
Exercises 17–19: Prepare a pictorial representation for these problems, but *do not* solve them.

- Draw pictures of "before" and "after."
- Define symbols relevant to the problem.
- List known information, and identify the desired unknown.

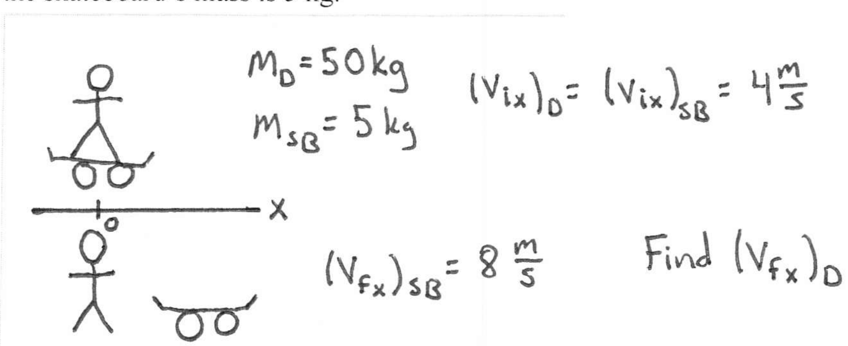
17. A 50 kg archer, standing on frictionless ice, shoots a 100 g arrow at a speed of 100 m/s. What is the recoil speed of the archer?



18. The parking brake on a 2000 kg Cadillac has failed, and it is rolling slowly, at 1 mph, toward a group of small innocent children. As you see the situation, you realize there is just time for you to drive your 1000 kg Volkswagen head-on into the Cadillac and thus to save the children. With what speed should you impact the Cadillac to bring it to a halt?

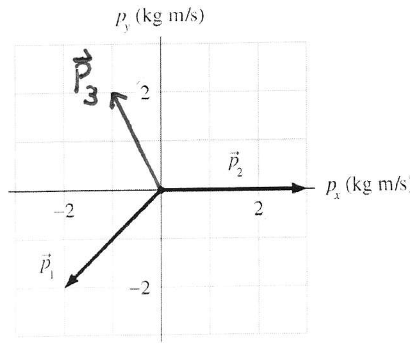


19. Dan is gliding on his skateboard at 4 m/s. He suddenly jumps backward off the skateboard, kicking the skateboard forward at 8 m/s. How fast is Dan going as his feet hit the ground? Dan's mass is 50 kg and the skateboard's mass is 5 kg.

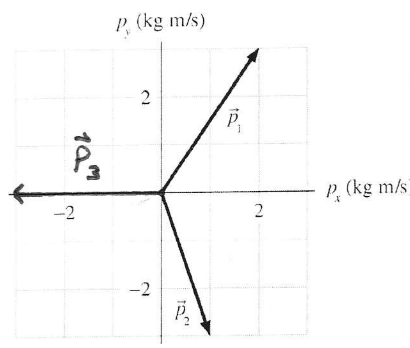


9.6 Momentum in Two Dimensions

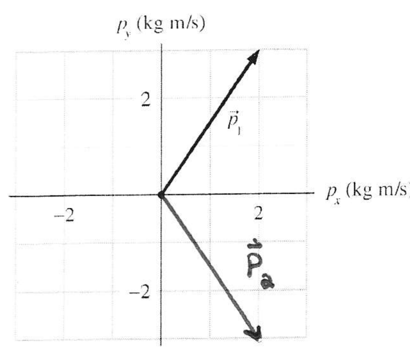
20. An object initially at rest explodes into three fragments. The momentum vectors of two of the fragments are shown. Draw the momentum vector $\vec{p}_3$ of the third fragment.



21. An object initially at rest explodes into three fragments. The momentum vectors of two of the fragments are shown. Draw the momentum vector $\vec{p}_3$ of the third fragment.



22. A 500 g ball traveling to the right at 8.0 m/s collides with and bounces off another ball. The figure shows the momentum vector $\vec{p}_1$ of the first ball after the collision. Draw the momentum vector $\vec{p}_2$ of the second ball.



23. A 500 g ball traveling to the right at 4.0 m/s collides with and bounces off another ball. The figure shows the momentum vector $\vec{p}_1$ of the first ball after the collision. Draw the momentum vector $\vec{p}_2$ of the second ball.