10

ENERGY

Conceptual Questions

10.1. Kinetic energy depends on speed. Potential energy depends on position.

10.2. No, kinetic energy can never be negative. Kinetic energy is energy of motion. Motion may stop, but it can't be negative. Speed has no direction and cannot be negative. Yes, gravitational potential energy can be negative. Potential energy depends upon position, which can be positive or negative.

10.3. We must calculate the new kinetic energy and compare it to the original value. Originally, $K = \frac{1}{2}mv^2$. With a velocity of $3v$, $K' = \frac{1}{2}m(3v)^2 = 9\left(\frac{1}{2}mv^2\right) = 9K$. The kinetic energy increases by a factor of 9.

10.4. We have

$$K_A = 8K_B$$
$$\frac{1}{2}m_A v_A^2 = 8\left(\frac{1}{2}m_B v_B^2\right)$$

Since $m_A = \frac{1}{2}m_B$,

$$\frac{1}{2}\left(\frac{1}{2}m_B\right)v_A^2 = 8\left(\frac{1}{2}m_B v_B^2\right)$$
$$\Rightarrow \frac{v_A}{v_B} = 4$$

10.5. Conservation of energy tells us that $U_i = K_f$, since the car starts at rest. Originally, this means that in rolling down a track of height h,

$$mgh = \frac{1}{2}mv_0^2$$

To go twice as fast at the bottom, we must find the height h' such that

$$mgh' = \frac{1}{2}m(2v_0)^2$$
$$\Rightarrow mgh' = 4\left(\frac{1}{2}mv_0^2\right).$$

So $h' = 4h$. You must increase the track height by a factor of 4.

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10.6. $v_a = v_b = v_c$. They each start with the same kinetic energy and they each have the same change in potential energy, so they end with the same kinetic energy and, thus, the same speed.

10.7. $v_a = v_b = v_c$. The balls start off with the same kinetic energy and have the same change in potential energy, so their final kinetic energy is the same.

10.8. (a) We identify the equilibrium position $s_e = 10$ cm. At $s = 11$ cm, $\Delta s = s - s_e = 11\text{ cm} - 10\text{ cm} = 1\text{ cm}$, and $F_{sp} = F = -k\Delta s = -k(1\text{ cm})$. To get $F_{sp} = 3F$, we must have $3F = -k(\Delta s)'$, which means $(\Delta s)' = 3$ cm. So the spring must have length $10\text{ cm} + 3\text{ cm} = 13$ cm.
(b) Note that the direction of the force is reversed when the spring is compressed.
To get $F_{sp} = -2F$, we must have $-2F = -k(\Delta s)'$, which means $(\Delta s)' = -2$ cm. So the spring must have length $10\text{ cm} - 2\text{ cm} = 8$ cm.

10.9. Note that Carlos takes the place of the wall, and that the force on the spring is still 200 N. The spring still stretches 20 cm.

10.10. $(U_s)_d > (U_s)_c > (U_s)_b = (U_s)_a$. $U_s = \frac{1}{2}k(\Delta s)^2$. Increasing the stretch by a factor of 2 increases the stored energy by a factor of 4. Doubling k doubles the stored energy.

10.11. The original spring stores energy $U = \frac{1}{2}k(1.0\text{ cm})^2$. For a spring with spring constant $k' = 2k$,

$$U' = \frac{1}{2}k'(\Delta s)^2 = \frac{1}{2}(2k)(\Delta s)^2$$

If $U' = U$, then

$$\frac{1}{2}k(1.0\text{ cm})^2 = \frac{1}{2}(2k)(\Delta s)^2$$
$$\Rightarrow \Delta s = \frac{1}{\sqrt{2}}\text{ cm} = 0.71\text{ cm}$$

10.12. Energy conservation tells us that the initial potential energy stored in the spring is equal to the final kinetic energy of the ball.

$$\frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_0^2$$

When the spring is compressed twice as far,

$$\frac{1}{2}k(2\Delta s)^2 = \frac{1}{2}m(2v_0)^2$$

So the ball speed increases by a factor of 2.

10.13. (a) At $x = 6$ m the particle has the most kinetic energy. The kinetic energy is the difference between the total energy (TE) and potential energy (PE) curves. At $x = 3$ m the particle's speed is locally a maximum, but is not as fast as at $x = 6$ m.
(b) The turning points for the particle with total energy (TE) shown are at $x = 2$ m and $x = 8$ m.
(c) The particle could remain at rest in stable equilibrium at $x = 3$ m and $x = 6$ m. The particle could also remain at rest in unstable equilibrium at $x = 1$ m and $x = 4$ m.

10.14. The problem can be divided into three parts: (1) from when the first ball is released and to just before it hits the stationary ball, (2) the two balls collide, and (3) the two balls swing up together just after the collision to their highest point. Energy is conserved in parts (1) and (3) as the balls swing like pendulums, but during the collision in

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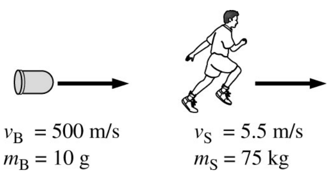
part (2) momentum is conserved but energy is not. So both energy and momentum conservation are each separately used as you work through each part of the problem.

Exercises and Problems

Section 10.2 Kinetic Energy and Gravitational Potential Energy

10.1. Model: We will use the particle model for the bullet (B) and the running student (S).
Visualize:



Solve: For the bullet,

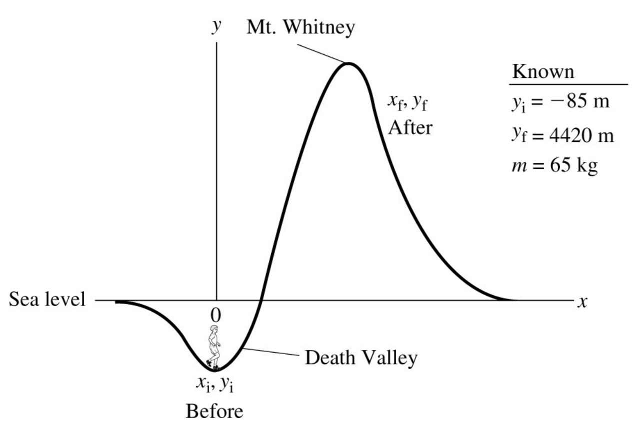
$$K_B = \frac{1}{2}m_B v_B^2 = \frac{1}{2}(0.010\text{ kg})(500\text{ m/s})^2 = 1250\text{ J}$$

For the running student,

$$K_S = \frac{1}{2}m_S v_S^2 = \frac{1}{2}(75\text{ kg})(5.5\text{ m/s})^2 = 206\text{ J}$$

Thus, the bullet has the larger kinetic energy.
Assess: Kinetic energy depends not only on mass but also on the square of the velocity. The above calculation shows this dependence. Although the mass of the bullet is 7500 times smaller than the mass of the student, its speed is more than 90 times larger.

10.2. Model: Model the hiker as a particle.
Visualize:



The origin of the coordinate system chosen for this problem is at sea level so that the hiker's position in Death Valley is $y_0 = -8.5$ m.
Solve: The hiker's change in potential energy from the bottom of Death Valley to the top of Mt. Whitney is

$$\Delta U = U_{gf} - U_{gi} = mgy_f - mgy_i = mg(y_f - y_i)$$
$$= (65\text{ kg})(9.8\text{ m/s}^2)[4420\text{ m} - (-85\text{ m})] = 2.9\times10^6\text{ J}$$

Assess: Note that ΔU is independent of the origin of the coordinate system.

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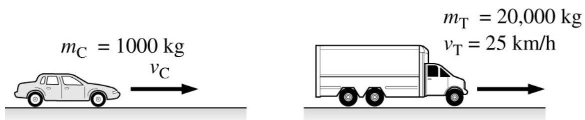
10.3. Model: Model the compact car (C) and the truck (T) as particles.
Visualize:

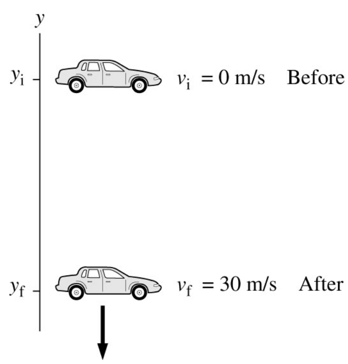


Solve: For the kinetic energy of the compact car and the kinetic energy of the truck to be equal,

$$K_C = K_T \Rightarrow \frac{1}{2}m_C v_C^2 = \frac{1}{2}m_T v_T^2 \Rightarrow v_C = \sqrt{\frac{m_T}{m_C}}v_T = \sqrt{\frac{20{,}000\text{ kg}}{1000\text{ kg}}}(25\text{ km/h}) = 112\text{ km/h}$$

Assess: A smaller mass needs a greater velocity for its kinetic energy to be the same as that of a larger mass.

10.4. Model: Model the car (C) as a particle. This is an example of free fall, and therefore the sum of kinetic and potential energy does not change as the car falls.
Visualize:



Solve: (a) The kinetic energy of the car is

$$K_C = \frac{1}{2}m_C v_C^2 = \frac{1}{2}(1500\text{ kg})(30\text{ m/s})^2 = 6.75\times10^5\text{ J}$$

The car's kinetic energy is 6.8×10^5 J.
(b) Let us relabel K_C as K_f and place our coordinate system at $y_f = 0$ m so that the car's potential energy U_{gf} is zero, its velocity is v_f, and its kinetic energy is K_f. At position y_i, $v_i = 0$ m/s or $K_i = 0$ J, and the only energy the car has is $U_{gi} = mgy_i$. Since the sum $K + U_g$ is unchanged by motion, $K_f + U_{gf} = K_i + U_{gi}$. This means

$$K_f + mgy_f = K_i + mgy_i \Rightarrow K_f + 0 = K_i + mgy_i$$

$$\Rightarrow y_i = \frac{(K_f - K_i)}{mg} = \frac{(6.75\times10^5\text{ J} - 0\text{ J})}{(1500\text{ kg})(9.8\text{ m/s}^2)} = 46\text{ m}$$

(c) From part (b),

$$y_i = \frac{(K_f - K_i)}{mg} = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{mg} = \frac{(v_f^2 - v_i^2)}{2g}$$

Free fall does *not* depend upon the mass.

10.5. Model: This is a case of free fall, so the sum of the kinetic and gravitational potential energy does not change as the ball rises and falls.
Visualize:

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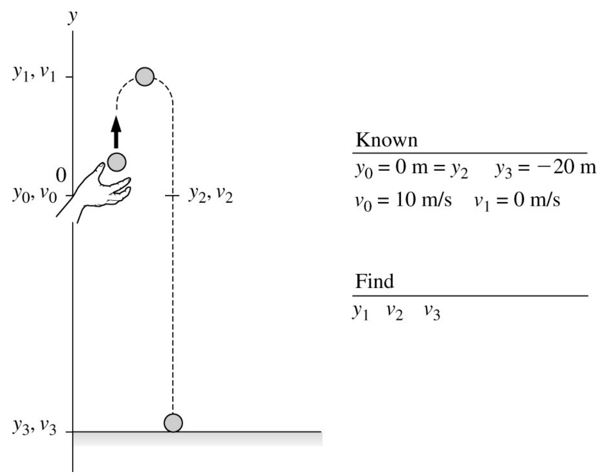


The figure shows a ball's before-and-after pictorial representation for the three situations in parts (a), (b), and (c).

Solve: The quantity $K + U_g$ is the same during free fall: $K_f + U_{gf} = K_i + U_{gi}$. We have

(a) $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$

$$\Rightarrow y_1 = (v_0^2 - v_1^2)/2g = [(10 \text{ m/s})^2 - (0 \text{ m/s})^2]/(2 \times 9.8 \text{ m/s}^2) = 5.10 \text{ m}$$

5.1 m is therefore the maximum height of the ball above the window. This is 25.1 m above the ground.

(b) $\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_0^2 + mgy_0$

Since $y_2 = y_0 = 0$, we get for the magnitudes $v_2 = v_0 = 10$ m/s.

(c) $\frac{1}{2}mv_3^2 + mgy_3 = \frac{1}{2}mv_0^2 + mgy_0 \Rightarrow v_3^2 + 2gy_3 = v_0^2 + 2gy_0 \Rightarrow v_3^2 = v_0^2 + 2g(y_0 - y_3)$

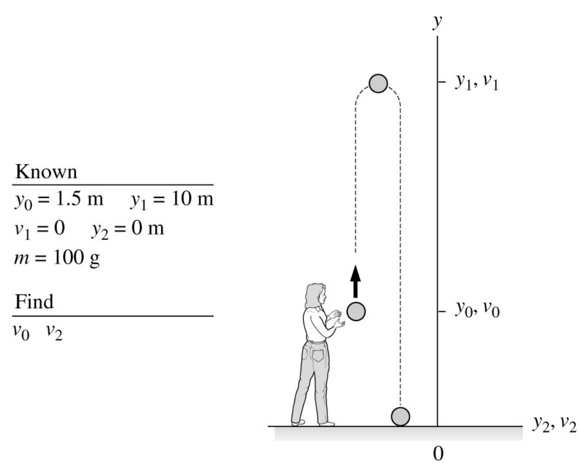
$$\Rightarrow v_3^2 = (10 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)[0 \text{ m} - (-20 \text{ m})] = 492 \text{ m}^2/\text{s}^2$$

This means the magnitude of v_3 is equal to 22 m/s.

Assess: Note that the ball's speed as it passes the window on its way down is the same as the speed with which it was tossed up, but in the opposite direction.

10.6. Model: This is a problem of free fall. The sum of the kinetic and gravitational potential energy for the ball, considered as a particle, does not change during its motion.

Visualize:



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The figure shows the ball's before-and-after pictorial representation for the two situations described in parts (a) and (b).
Solve: The quantity $K+U_g$ is the same during free fall. Thus, $K_f + U_{gf} = K_i + U_{gi}$.

(a) $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0 \Rightarrow v_0^2 = v_1^2 + 2g(y_1 - y_0)$

$\Rightarrow v_0^2 = (0\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(10\text{ m} - 1.5\text{ m}) = 166.6\text{ m}^2/\text{s}^2 \Rightarrow v_0 = 12.9\text{ m/s} \approx 13\text{ m/s}$

(b) $\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_0^2 + mgy_0 \Rightarrow v_2^2 = v_0^2 + 2g(y_0 - y_2)$

$\Rightarrow v_2^2 = 166.6\text{ m}^2/\text{s}^2 + 2(9.8\text{ m/s}^2)(1.5\text{ m} - 0\text{ m}) \Rightarrow v_2 = 14\text{ m/s}$

Assess: An increase in speed from 12.9 m/s to 14.0 m/s as the ball falls through a distance of 1.5 m is reasonable. Also, note that mass does not appear in the calculations that involve free fall.

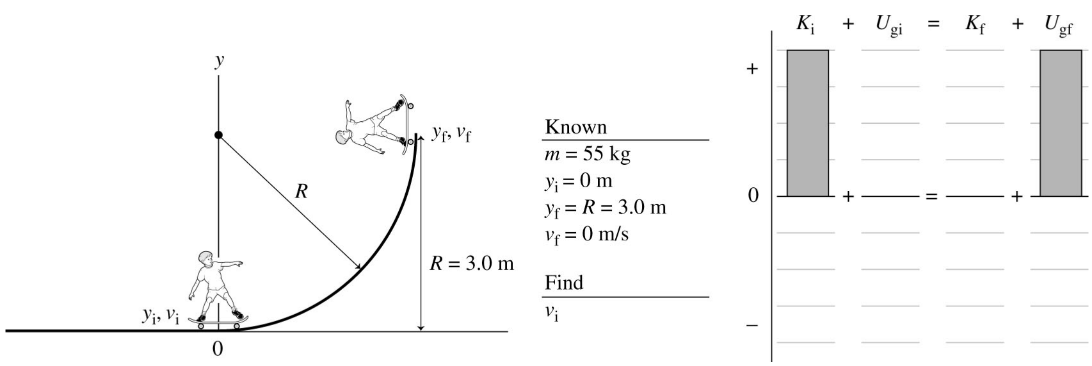
10.7. Model: Model the mother and the son as particles.
Visualize: $m_{\text{mother}} = 4\,m_{\text{son}}$.
Solve: The energy conservation equation $K_{\text{mother}} = K_{\text{son}}$ is

$$\frac{1}{2}m_{\text{mother}}v_{\text{mother}}^2 = \frac{1}{2}m_{\text{son}}v_{\text{son}}^2 \Rightarrow (4\,m_{\text{son}})v_{\text{mother}}^2 = m_{\text{son}}v_{\text{son}}^2 \Rightarrow \frac{v_{\text{son}}}{v_{\text{mother}}} = 2.0$$

Assess: The result $v_{\text{son}} = 2v_{\text{mother}}$, combined with the fact that $m_{\text{son}} = \frac{1}{4}m_{\text{mother}}$, is a consequence of the way kinetic energy is defined: It is directly proportional to the mass and to the square of the speed.

Section 10.3 A Closer Look at Gravitational Potential Energy

10.8. Model: Model the skateboarder as a particle. Assuming that the track offers no rolling friction, the sum of the skateboarder's kinetic and gravitational potential energy does not change during his rolling motion.
Visualize:



The vertical displacement of the skateboarder is equal to the radius of the track.
Solve: The quantity $K+U_g$ is the same at the upper edge of the quarter-pipe track as it was at the bottom. The energy conservation equation $K_f + U_{gf} = K_i + U_{gi}$ is

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i \Rightarrow v_i^2 = v_f^2 + 2g(y_f - y_i)$$

$$v_i^2 = (0\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(3.0\text{ m} - 0\text{ m}) = 58.8\text{ m}^2/\text{s}^2 \Rightarrow v_i = 7.7\text{ m/s}$$

Assess: Note that we did not need to know the skateboarder's mass, as is the case with free-fall motion.

10.9. Model: Model the puck as a particle. Since the ramp is frictionless, the sum of the puck's kinetic and gravitational potential energy does not change during its sliding motion.
Visualize:

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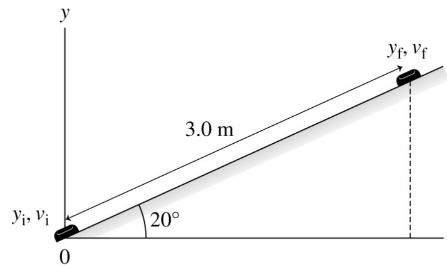
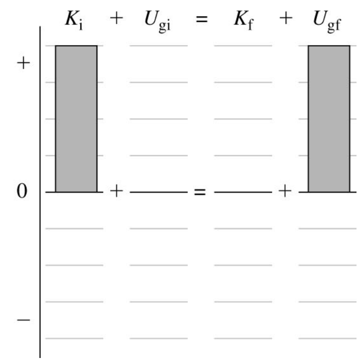


Known
$m = 100$ g
$y_i = 0$ m
$y_f = (3.0 \text{ m}) \sin 20°$
$= 1.03$ m
$v_f = 0$ m/s

Find
v_i



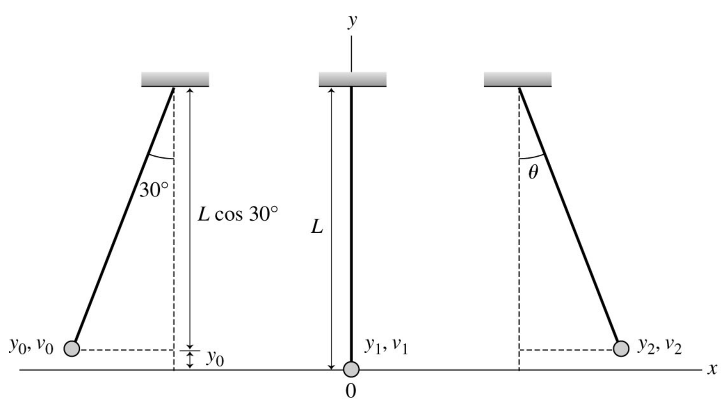
Solve: The quantity $K + U_g$ is the same at the top of the ramp as it was at the bottom. The energy conservation equation $K_f + U_{gf} = K_i + U_{gi}$ is

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i \Rightarrow v_i^2 = v_f^2 + 2g(y_f - y_i)$$

$$\Rightarrow v_i^2 = (0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(1.03 \text{ m} - 0 \text{ m}) = 20.2 \text{ m}^2/\text{s}^2 \Rightarrow v_i = 4.5 \text{ m/s}$$

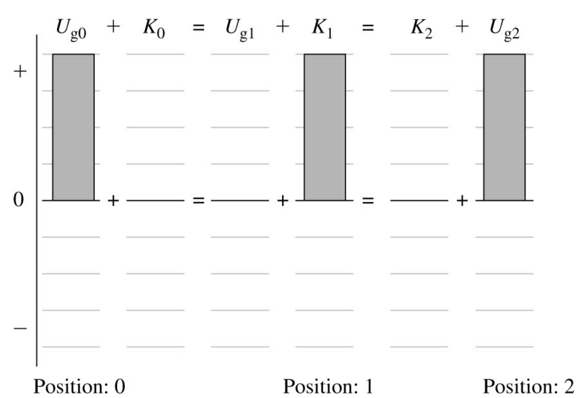
Assess: An initial push with a speed of $4.5 \text{ m/s} \approx 10$ mph to cover a distance of 3.0 m up a $20°$ ramp seems reasonable.

10.10. Model: In the absence of frictional and air-drag effects, the sum of the kinetic and gravitational potential energy does not change as the pendulum swings from one side to the other.
Visualize:



Known
$m = 500$ g
$L = 75$ cm
$y_1 = 0$
$v_0 = 0 = v_2$

Find
v_1 θ



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The figure shows the pendulum's before-and-after pictorial representation for the two situations described in parts (a) and (b).

Solve: (a) The quantity $K+U_g$ is the same at the lowest point of the trajectory as it was at the highest point. Thus, $K_1+U_{g1}=K_0+U_{g0}$ means

$$\frac{1}{2}mv_1^2+mgy_1=\frac{1}{2}mv_0^2+mgy_0 \Rightarrow v_1^2+2gy_1=v_0^2+2gy_0$$

$$\Rightarrow v_1^2+2g(0\text{ m})=(0\text{ m/s})^2+2gy_0 \Rightarrow v_1=\sqrt{2gy_0}$$

From the pictorial representation, we find that $y_0=L-L\cos30°$. Thus,

$$v_1=\sqrt{2gL(1-\cos30°)}=\sqrt{2(9.8\text{ m/s}^2)(0.75\text{ m})(1-\cos30°)}=1.403\text{ m/s}$$

The speed at the lowest point is 1.4 m/s.

(b) Since the quantity $K+U_g$ does not change, $K_2+U_{g2}=K_1+U_{g1}$. We have

$$\frac{1}{2}mv_2^2+mgy_2=\frac{1}{2}mv_1^2+mgy_1 \Rightarrow y_2=(v_1^2-v_2^2)/2g$$

$$\Rightarrow y_2=[(1.403\text{ m/s})^2-(0\text{ m/s})^2]/(2\times9.8\text{ m/s}^2)=0.100\text{ m}$$

Since $y_2=L-L\cos\theta$, we obtain

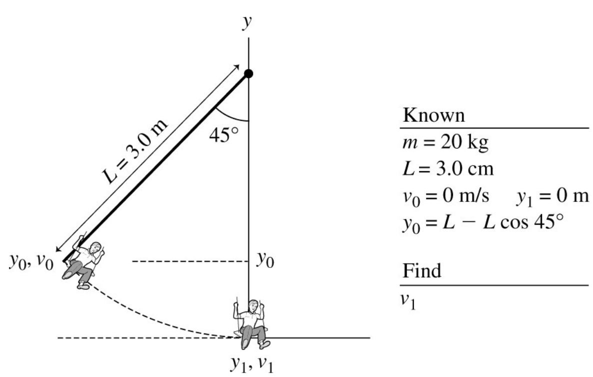
$$\cos\theta=\frac{L-y_2}{L}=\frac{(0.75\text{ m})-(0.10\text{ m})}{(0.75\text{ m})}=0.8667 \Rightarrow \theta=\cos^{-1}(0.8667)=30°$$

That is, the pendulum swings to the other side by 30°.

Assess: The swing angle is the same on either side of the rest position. This result is a consequence of the fact that the sum of the kinetic and gravitational potential energy does not change. This is shown as well in the energy bar chart in the figure.

10.11. Model: Model the child and swing as a particle, and assume the chain to be massless. In the absence of frictional and air-drag effects, the sum of the kinetic and gravitational potential energy does not change during the swing's motion.

Visualize:



Solve: The quantity $K+U_g$ is the same at the highest point of the swing as it is at the lowest point. That is, $K_0+U_{g0}=K_1+U_{g1}$. It is clear from this equation that maximum kinetic energy occurs where the gravitational potential energy is the least. This is the case at the lowest position of the swing. At this position, the speed of the swing and child will also be maximum. The above equation is

$$\frac{1}{2}mv_0^2+mgy_0=\frac{1}{2}mv_1^2+mgy_1 \Rightarrow v_1^2=v_0^2+2g(y_0-y_1)$$

$$\Rightarrow v_1^2=(0\text{ m/s})^2+2g(y_0-0\text{ m}) \Rightarrow v_1=\sqrt{2gy_0}$$

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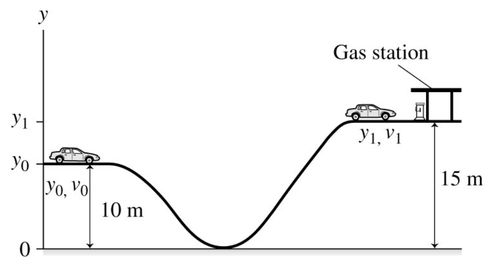
We see from the pictorial representation that

$$y_0 = L - L\cos 45° = (3.0 \text{ m}) - (3.0 \text{ m})\cos 45° = 0.879 \text{ m}$$

$$\Rightarrow v_1 = \sqrt{2gy_0} = \sqrt{2(9.8 \text{ m/s}^2)(0.879 \text{ m})} = 4.2 \text{ m/s}$$

Assess: We did not need to know the swing's or the child's mass. Also, a maximum speed of 4.2 m/s is reasonable.

10.12. Model: Model the car as a particle with zero rolling friction and no air resistance. The sum of the kinetic and gravitational potential energy, therefore, does not change during the car's motion.
Visualize:



Solve: The initial energy of the car is

$$K_0 + U_{g0} = \frac{1}{2}mv_0^2 + mgy_0 = \frac{1}{2}(1500 \text{ kg})(10.0 \text{ m/s})^2 + (1500 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m}) = 2.22\times10^5 \text{ J}$$

The car increases its height to 15 m at the gas station. The conservation of energy equation $K_0 + U_{g0} = K_1 + U_{g1}$ is

$$2.22\times10^5 \text{ J} = \frac{1}{2}mv_1^2 + mgy_1 \Rightarrow 2.22\times10^5 \text{ J} = \frac{1}{2}(1500 \text{ kg})v_1^2 + (1500 \text{ kg})(9.8 \text{ m/s}^2)(15 \text{ m})$$

$$\Rightarrow v_1 = 1.4 \text{ m/s}$$

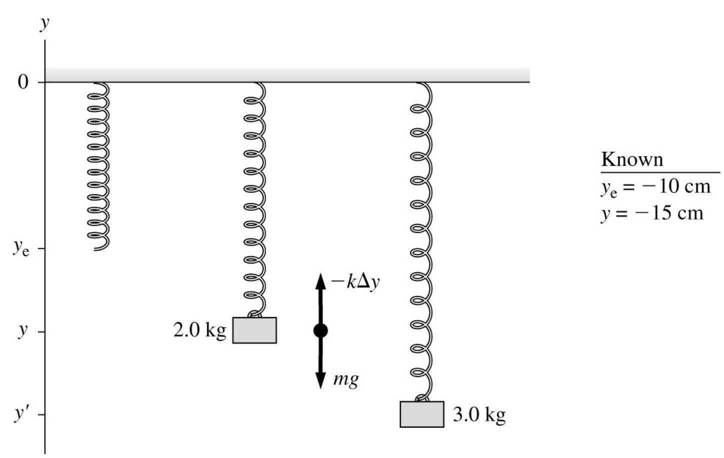
Assess: A lower speed at the gas station is reasonable because the car has decreased its kinetic energy and increased its potential energy compared to its starting values.

Section 10.4 Restoring Forces and Hooke's Law

10.13. Model: Assume that the spring is ideal and obeys Hooke's law.
Visualize: According to Hooke's law, the spring force acting on a mass (*m*) attached to the end of a spring is given as $F_{sp} = k\Delta x,$ where Δx is the change in length of the spring. If the mass *m* is at rest, then F_{sp} is also equal to the gravitational force $F_G = mg.$
Solve: We have $F_{sp} = k\Delta x = mg.$ We want a 0.100 kg mass to give $\Delta x = 0.010$ m. This means

$$k = mg/\Delta x = (0.100 \text{ kg})(9.8 \text{ N/m})/(0.010 \text{ m}) = 98 \text{ N/m}$$

10.14. Model: Assume an ideal spring that obeys Hooke's law.
Visualize:



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Solve: **(a)** The spring force on the 2.0 kg mass is $F_{sp} = -k\Delta y$. Notice that Δy is negative, so F_{sp} is positive. This force is equal to mg, because the 2.0 kg mass is at rest. We have $-k\Delta y = mg$. Solving for k:

$$k = -(mg/\Delta y) = -(2.0 \text{ kg})(9.8 \text{ m/s}^2)/(-0.15 \text{ m} - (-0.10 \text{ m})) = 392 \text{ N/m}$$

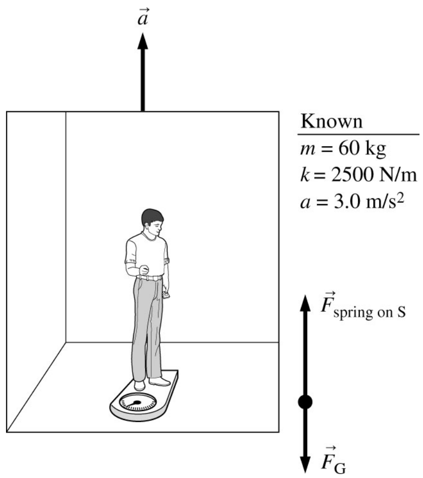
The spring constant is 3.9×10^2 N/m.
(b) Again using $-k\Delta y = mg$:

$$\Delta y = -mg/k = -(3.0 \text{ kg})(9.8 \text{ m/s}^2)/(392 \text{ N/m})$$

$$y' - y_e = -0.075 \text{ m} \Rightarrow y' = y_e - 0.075 \text{ m} = -0.10 \text{ m} - 0.075 \text{ m} = -0.175 \text{ m} = -17.5 \text{ cm}$$

The length of the spring is 17.5 cm when a mass of 3.0 kg is attached to the spring. The *position* of the end of the spring is negative because it is below the origin, but length must be a positive number.

10.15. Model: Model the student (S) as a particle and the spring as obeying Hooke's law.
Visualize:



Solve: According to Newton's second law the force on the student is

$$\Sigma(F_{\text{on S}})_y = F_{\text{spring on S}} - F_G = ma_y$$

$$\Rightarrow F_{\text{spring on S}} = F_G + ma_y = mg + ma_y = (60 \text{ kg})(9.8 \text{ m/s}^2 + 3.0 \text{ m/s}^2) = 768 \text{ N}$$

Since $F_{\text{spring on S}} = F_{\text{S on spring}} = k\Delta y$, $k\Delta y = 768$ N. This means $\Delta y = (768 \text{ N})/(2500 \text{ N/m}) = 0.31$ m.

10.16. Model: Assume the spring is ideal and obeys Hooke's law.
Visualize: The stretch produced by hanging mass m is $L_1 - L_0$.
Solve: For a hanging mass of m

$$L_1 = L_0 + (L_1 - L_0)$$

If we double the hanging mass to $2m$, then

$$L_2 = L_0 + 2(L_1 - L_0)$$

and, indeed, for any mass nm,

$$L_n = L_0 + n(L_1 - L_0)$$

In particular for $3m$

$$L_3 = L_0 + 3(L_1 - L_0) = -2L_0 + 3L_1$$

Assess: Even though one term is negative the answer won't be because $L_1 > L_0$.

10.17. Model: Assume that the spring is ideal and obeys Hooke's law. We also model the 5.0 kg mass as a particle.
Visualize: We will use the subscript s for the scale and sp for the spring.

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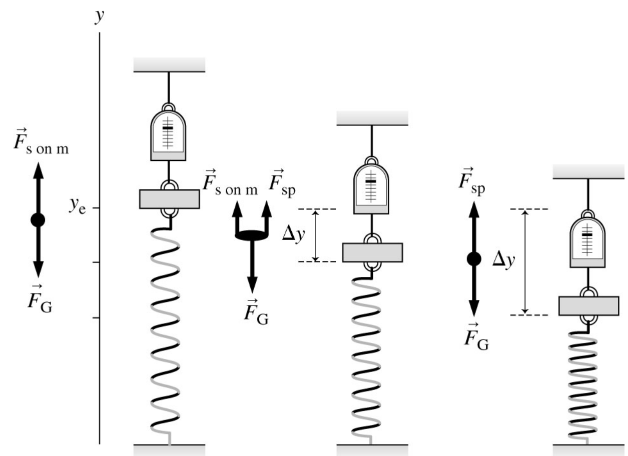

Solve: **(a)** The scale reads the upward force $F_{\text{s on m}}$ that it applies to the mass. Newton's second law gives

$$\sum (F_{\text{on m}})_y = F_{\text{s on m}} - F_G = 0 \Rightarrow F_{\text{s on m}} = F_G = mg = (5.0\text{ kg})(9.8\text{ m/s}^2) = 49\text{ N}$$

(b) In this case, the force is

$$\sum (F_{\text{on m}})_y = F_{\text{s on m}} + F_{sp} - F_G = 0 \Rightarrow 20\text{ N} + k\Delta y - mg = 0$$

$$\Rightarrow k = (mg - 20\text{ N})/\Delta y = (49\text{ N} - 20\text{ N})/0.02\text{ m} = 1450\text{ N/m}$$

The spring constant for the lower spring is 1.45×10^3 N/m.

(c) In this case, the force is

$$\sum (F_{\text{on m}})_y = F_{sp} - F_G = 0 \Rightarrow k\Delta y - mg = 0$$

$$\Rightarrow \Delta y = mg/k = (49\text{ N})/(1450\text{ N/m}) = 0.0338\text{ m} = 3.4\text{ cm}$$

Section 10.5 Elastic Potential Energy

10.18. Model: Assume an ideal spring that obeys Hooke's law.

Solve: The elastic potential energy of a spring is defined as $U_s = \frac{1}{2}k(\Delta s)^2$, where Δs is the magnitude of the stretching or compression relative to the unstretched or uncompressed length. $\Delta U_s = 0$ when the spring is at its equilibrium length and $\Delta s = 0$. We have $U_s = 200$ J and $k = 1000$ N/m. Solving for Δs:

$$\Delta s = \sqrt{2U_s/k} = \sqrt{2(200\text{ J})/1000\text{ N/m}} = 0.632\text{ m}$$

10.19. Model: Assume the spring is ideal and obeys Hooke's law. Then the potential energy of a stretched spring is $U_{sp} = \dfrac{1}{2}k(\Delta s)^2$.

Visualize: Use ratios to solve this problem. Use primed variables for the new situation with the spring stretched three times as far.

Solve:

$$\frac{U'_{sp}}{U_{sp}} = \frac{\frac{1}{2}k(\Delta s')^2}{\frac{1}{2}k(\Delta s)^2} = \frac{\frac{1}{2}k(3\Delta s)^2}{\frac{1}{2}k(\Delta s)^2} = 3^2 = 9$$

$$U'_{sp} = 9U_{sp} = 9(2.0\text{ J}) = 18\text{ J}$$

Assess: The stored energy scales with the square of the spring stretch.

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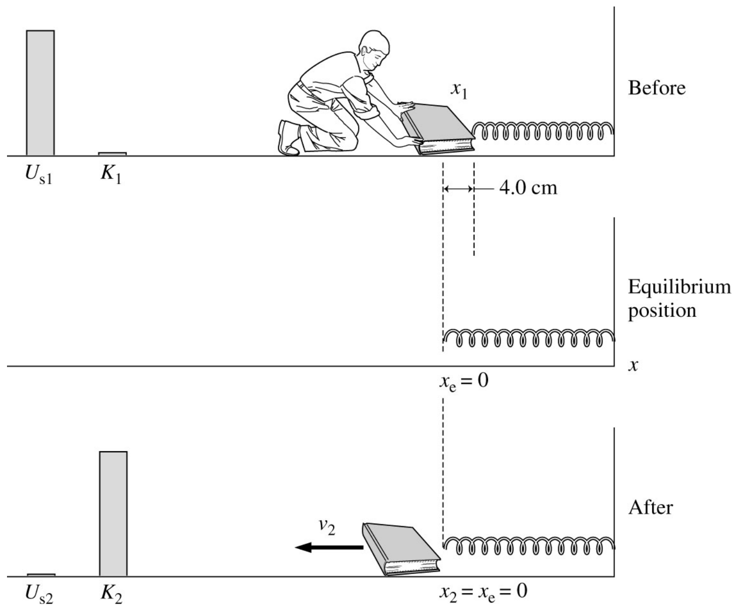
10.20. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, so the mechanical energy $K + U_s$ is conserved. Also model the book as a particle.

Visualize:



The figure shows a before-and-after pictorial representation. The compressed spring will push on the book until the spring has returned to its equilibrium length. We put the origin of our coordinate system at the equilibrium position of the free end of the spring. The energy bar chart shows that the potential energy of the compressed spring is entirely transformed into the kinetic energy of the book.

Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

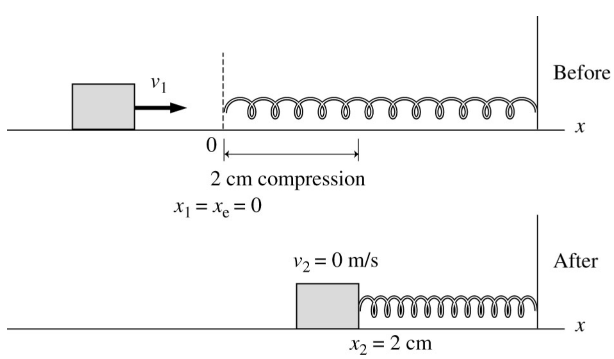
Using $x_2 = x_e = 0$ m and $v_1 = 0$ m/s, this simplifies to

$$\frac{1}{2}mv_2^2 = \frac{1}{2}k(x_1 - 0\text{ m})^2 \Rightarrow v_2 = \sqrt{\frac{kx_1^2}{m}} = \sqrt{\frac{(1250\text{ N/m})(0.040\text{ m})^2}{(0.500\text{ kg})}} = 2.0\text{ m/s}$$

Assess: This problem cannot be solved using constant-acceleration kinematic equations. The acceleration is not a constant in this problem, since the spring force, given as $F_s = -k\Delta x$, is directly proportional to Δx or $|x - x_e|$.

10.21. Model: Assume an ideal spring that obeys Hooke's law. Since there is no friction, the mechanical energy $K + U_s$ is conserved. Also, model the block as a particle.

Visualize:



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The figure shows a before-and-after pictorial representation. We have put the origin of our coordinate system at the equilibrium position of the free end of the spring. This gives us $x_1 = x_e = 0$ cm and $x_2 = 2.0$ cm.

Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

Using $v_2 = 0$ m/s, $x_1 = x_e = 0$ m, and $x_2 - x_e = 0.020$ m, we get

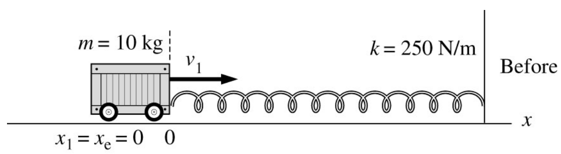
$$\frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 \Rightarrow \Delta x = (x_2 - x_e) = \sqrt{\frac{m}{k}}v_1$$

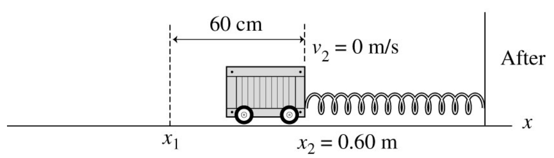
That is, the compression is directly proportional to the velocity v_1. When the block collides with the spring with twice the earlier velocity $(2v_1)$, the compression will also be doubled to $2(x_2 - x_e) = 2(2.0 \text{ cm}) = 4.0$ cm.

Assess: This problem shows the power of using energy conservation over using Newton's laws in solving problems involving nonconstant acceleration.

10.22. Model: Model the grocery cart as a particle and the spring as an ideal that obeys Hooke's law. We will also assume zero rolling friction during the compression of the spring, so that mechanical energy is conserved.

Visualize:





The figure shows a before-and-after pictorial representation. The "before" situation is when the cart hits the spring in its equilibrium position. We put the origin of our coordinate system at this equilibrium position of the free end of the spring. This gives $x_1 = x_e = 0$ and $(x_2 - x_e) = 60$ cm.

Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

Using $v_2 = 0$ m/s, $(x_2 - x_e) = 0.60$ m, and $x_1 = x_e = 0$ m gives:

$$\frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{\frac{k}{m}}(x_2 - x_e) = \sqrt{\frac{250 \text{ N/m}}{10 \text{ kg}}}(0.60 \text{ m}) = 3.0 \text{ m/s}$$

10.23. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, and thus the mechanical energy $K + U_s + U_g$ is conserved.

Visualize:

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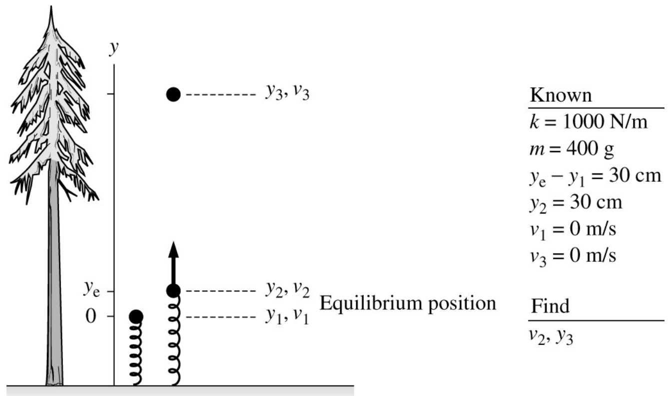


We place the origin of our coordinate system at the spring's compressed position $y_1 = 0$. The rock leaves the spring with velocity v_2 as the spring reaches its equilibrium position.

Solve: (a) The conservation of mechanical energy equation is

$$K_2 + U_{s2} + U_{g2} = K_1 + U_{s1} + U_{g1} \qquad \frac{1}{2}mv_2^2 + \frac{1}{2}k(y_2 - y_e)^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(y_1 - y_e)^2 + mgy_1$$

Using $y_2 = y_e$, $y_1 = 0$ m, and $v_1 = 0$ m/s, this simplifies to

$$\frac{1}{2}mv_2^2 + 0 \text{ J} + mgy_2 = 0 \text{ J} + \frac{1}{2}k(y_1 - y_e)^2 + 0$$

$$\frac{1}{2}(0.400 \text{ kg})v_2^2 + (0.400 \text{ kg})(9.8 \text{ m/s}^2)(0.30 \text{ m}) = \frac{1}{2}(1000 \text{ N/m})(0.30 \text{ m})^2 \Rightarrow v_2 = 14.8 \text{ m/s}$$

(b) Let us use the conservation of mechanical energy equation once again to find the highest position (y_3) of the rock where its speed (v_3) is zero:

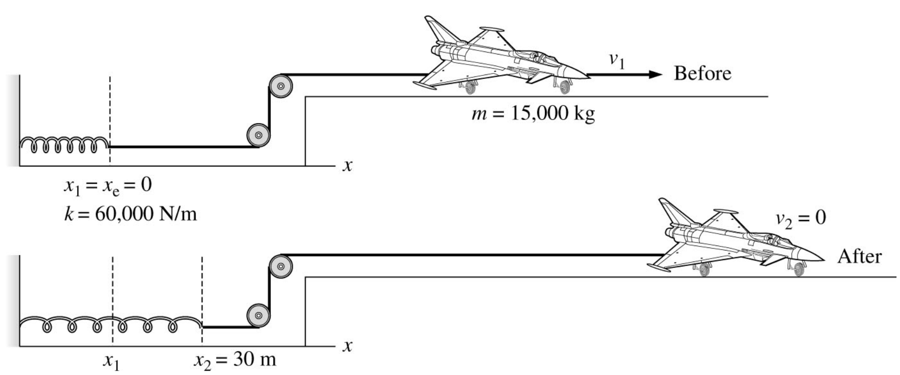
$$K_3 + U_{g3} = K_2 + U_{g2} \Rightarrow \frac{1}{2}mv_3^2 + mgy_3 = \frac{1}{2}mv_2^2 + mgy_2$$

$$\Rightarrow 0 + g(y_3 - y_2) = \frac{1}{2}v_2^2 \Rightarrow (y_3 - y_2) = \frac{v_2^2}{2g} = \frac{(14.8 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 11.2 \text{ m}$$

If we assume the spring's length to be 0.5 m, then the distance between ground and fruit is $11.2 \text{ m} + 0.5 \text{ m} = 11.7 \text{ m}$. This is much smaller than the distance of 15 m between fruit and ground. So, the rock does not reach the fruit, and the contestants go hungry.

10.24. Model: Model the jet plane as a particle, and the spring as an ideal that obeys Hooke's law. We will also assume zero rolling friction during the stretching of the spring, so that mechanical energy is conserved.

Visualize:



The figure shows a before-and-after pictorial representation. The "before" situation occurs just as the jet plane lands on the aircraft carrier and the spring is in its equilibrium position. We put the origin of our coordinate system at the right free end of the spring. This gives $x_1 = x_e = 0$ m. Since the spring stretches 30 m to stop the plane, $x_2 - x_e = 30$ m.

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Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ for the spring-jet plane system is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

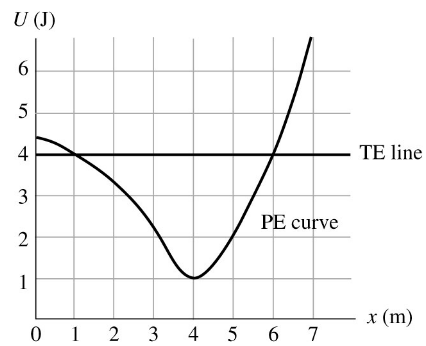
Using $v_2 = 0$ m/s, $x_1 = x_e = 0$ m, and $x_2 - x_e = 30$ m yields

$$\frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{\frac{k}{m}}(x_2 - x_1) = \sqrt{\frac{60{,}000\text{ N/m}}{15{,}000\text{ kg}}}(30\text{ m}) = 60\text{ m/s}$$

Assess: A landing speed of 60 m/s or ≈ 120 mph is reasonable.

Section 10.6 Energy Diagrams

10.25. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.
Visualize:



The particle is released from rest at $x = 1.0$ m. That is, $K = 0$ at $x = 1.0$ m. Since the total energy is given by $E = K + U$, we can draw a horizontal total energy (TE) line through the point of intersection of the potential energy curve (PE) and the $x = 1.0$ m line. The distance from the PE curve to the TE line is the particle's kinetic energy. These values are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ does not change.

Solve: (a) We have $E = 4.0$ J and this energy is a constant. For $x < 1.0, U > 4.0$ J and, therefore, K must be negative to keep E the same (note that $K = E - U$ or $K = 4.0\text{ J} - U$). Since negative kinetic energy is unphysical, the particle cannot move to the left. That is, the particle will move to the right of $x = 1.0$ m.

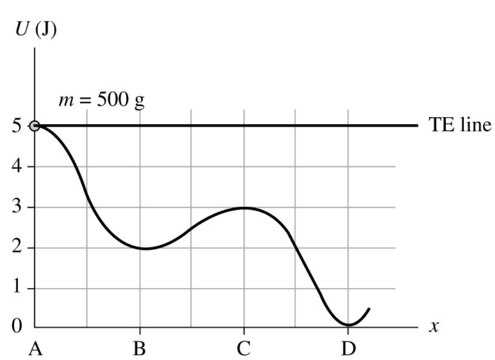
(b) The expression for the kinetic energy is $E - U$. This means the particle has maximum speed or maximum kinetic energy when U is minimum. This happens at $x = 4.0$ m. Thus,

$$K_{max} = E - U_{min} = (4.0\text{ J}) - (1.0\text{ J}) = 3.0\text{ J} \qquad \frac{1}{2}mv_{max}^2 = 3.0\text{ J} \Rightarrow v_{max} = \sqrt{\frac{2(3.0\text{ J})}{m}} = \sqrt{\frac{8.0\text{ J}}{0.020\text{ kg}}} = 17.3\text{ m/s}$$

The particle possesses this speed at $x = 4.0$ m.

(c) The total energy (TE) line intersects the potential energy (PE) curve at $x = 1.0$ m and $x = 6.0$ m. These are the turning points of the motion.

10.26. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.
Visualize:



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The particle with a mass of 500 g is released from rest at A. That is, $K = 0$ at A. Since $E = K + U = 0\text{ J} + U$, we can draw a horizontal TE line through $U = 5.0$ J. The distance from the PE curve to the TE line is the particle's kinetic energy. These values are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ does not change.

Solve: The kinetic energy is given by $E - U$, so we have

$$\frac{1}{2}mv^2 = E - U \Rightarrow v = \sqrt{2(E-U)/m}$$

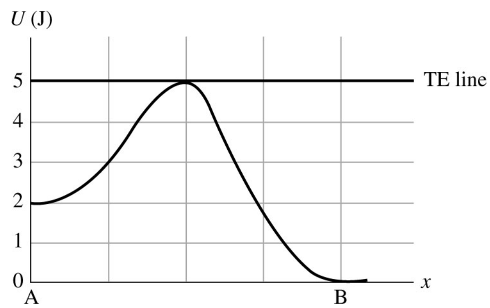
Using $U_B = 2.0\text{ J}$, $U_C = 3.0\text{ J}$, and $U_D = 0\text{ J}$, we get

$$v_B = \sqrt{2(5.0\text{ J} - 2.0\text{ J})/0.500\text{ kg}} = 3.5\text{ m/s} \qquad v_C = \sqrt{2(5.0\text{ J} - 3.0\text{ J})/0.500\text{ kg}} = 2.8\text{ m/s}$$

$$v_D = \sqrt{2(5.0\text{ J} - 0\text{ J})/0.500\text{ kg}} = 4.5\text{ m/s}$$

10.27. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.

Visualize:



For the speed of the particle at A that is needed to reach B to be a minimum, the particle's kinetic energy as it reaches the top must be zero. Similarly, the minimum speed at B for the particle to reach A obtains when the particle just makes it to the top with zero kinetic energy.

Solve: (a) The energy equation $K_A + U_A = K_{top} + U_{top}$ is

$$\frac{1}{2}mv_A^2 + U_A = 0\text{ J} + U_{top}$$

$$\Rightarrow v_A = \sqrt{2(U_{top} - U_A)/m} = \sqrt{2(5.0\text{ J} - 2.0\text{ J})/0.100\text{ kg}} = 7.7\text{ m/s}$$

(b) To go from point B to point A, $K_B + U_B = K_{top} + U_{top}$ is

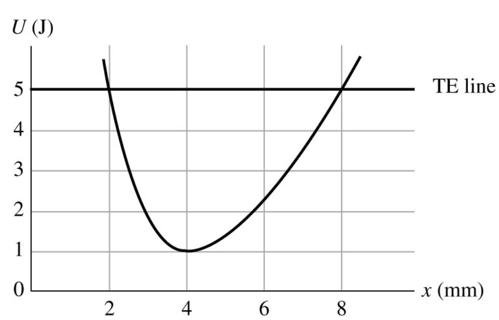
$$\frac{1}{2}mv_B^2 + U_B = 0\text{ J} + U_{top}$$

$$\Rightarrow v_B = \sqrt{2(U_{top} - U_B)/m} = \sqrt{2(5.0\text{ J} - 0\text{ J})/0.100\text{ kg}} = 10.0\text{ m/s}$$

Assess: The particle requires a higher kinetic energy to reach A from B than to reach B from A.

10.28. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.

Visualize:



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Since the particle oscillates between $x = 2.0$ mm and $x = 8.0$ mm, the speed of the particle is zero at these points. That is, for these values of *x*, $E = U = 5.0$ J, which defines the total energy (TE) line. The distance from the potential energy (PE) curve to the TE line is the particle's kinetic energy. These values are transformed as the position changes, but the sum $K + U$ does not change.

Solve: The equation for total energy $E = U + K$ means $K = E - U$, so that *K* is maximum when *U* is minimum. We have

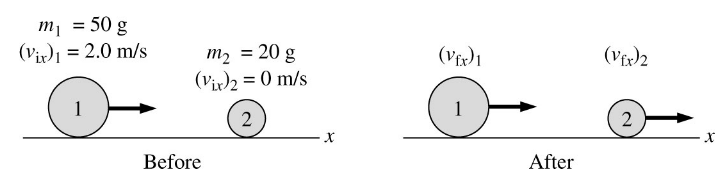
$$K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = 5.0\text{ J} - U_{\text{min}}$$

$$\Rightarrow v_{\text{max}} = \sqrt{2(5.0\text{ J} - U_{\text{min}})/m} = \sqrt{2(5.0\text{ J} - 1.0\text{ J})/0.0020\text{ kg}} = 63\text{ m/s}$$

Section 10.7 Elastic Collisions

10.29. Model: We assume this is a one-dimensional collision that obeys the conservation laws of momentum and mechanical energy.

Visualize:



Note that momentum conservation alone is not sufficient to solve this problem because the two final velocities $(v_{fx})_1$ and $(v_{fx})_2$ are unknowns and cannot be determined from one equation.

Solve: Momentum conservation: $m_1(v_{ix})_1 + m_2(v_{ix})_2 = m_1(v_{fx})_1 + m_2(v_{fx})_2$

Energy conservation: $\frac{1}{2}m_1(v_{ix})_1^2 + \frac{1}{2}m_2(v_{ix})_2^2 = \frac{1}{2}m_1(v_{fx})_1^2 + \frac{1}{2}m_2(v_{fx})_2^2$

These two equations can be solved for $(v_{fx})_1$ and $(v_{fx})_2$, as shown by Equations 10.39 through 10.43, to give

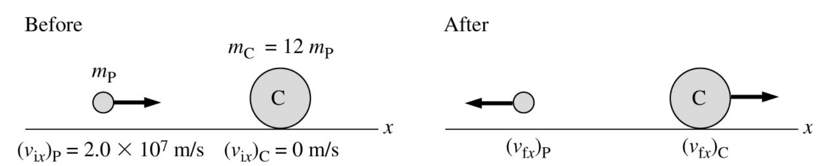
$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 = \frac{50\text{ g} - 20\text{ g}}{50\text{ g} + 20\text{ g}}(2.0\text{ m/s}) = 0.86\text{ m/s}$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1 = \frac{2(50\text{ g})}{50\text{ g} + 20\text{ g}}(2.0\text{ m/s}) = 2.9\text{ m/s}$$

Assess: These velocities are of a reasonable magnitude. Since both these velocities are positive, both balls move along the +*x*-direction.

10.30. Model: This is a case of a perfectly elastic collision between a proton and a carbon atom. The collision obeys the momentum as well as the energy conservation law.

Visualize:



Solve: Momentum conservation: $m_P(v_{ix})_P + m_C(v_{ix})_C = m_P(v_{fx})_P + m_C(v_{fx})_C$

Energy conservation: $\frac{1}{2}m_P(v_{ix})_P^2 + \frac{1}{2}m_C(v_{ix})_C^2 = \frac{1}{2}m_P(v_{fx})_P^2 + \frac{1}{2}m_C(v_{fx})_C^2$

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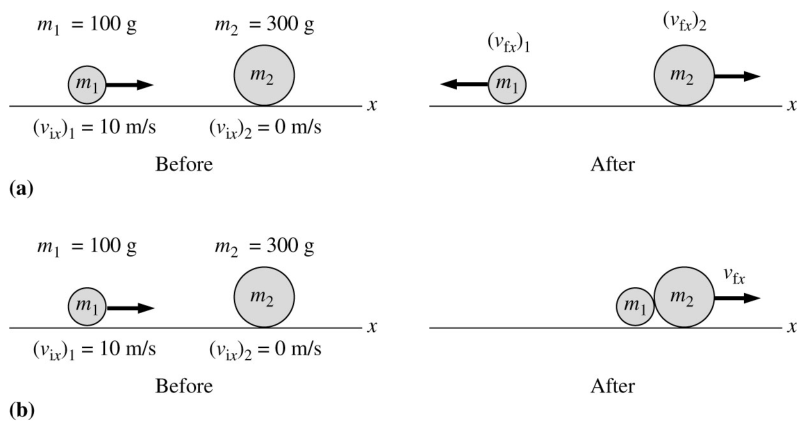
These two equations can be solved, as described in the text through Equations 10.38 to 10.42:

$$(v_{fx})_P = \frac{m_P - m_C}{m_P + m_C} m_P + m_C (v_{ix})_P = \left(\frac{m_P - 12m_P}{m_P + 12m_P} \right) (2.0 \times 10^7 \text{ m/s}) = -1.69 \times 10^7 \text{ m/s}$$

$$(v_{fx})_C = \frac{2m_P}{m_P + m_C} (v_{ix})_P = \left(\frac{2m_P}{m_P + 12m_P} \right) (2.0 \times 10^7 \text{ m/s}) = 3.1 \times 10^6 \text{ m/s}$$

After the elastic collision the proton rebounds at 1.69×10^7 m/s and the carbon atom moves forward at 3.08×10^6 m/s.

10.31. Model: In this case of a one-dimensional collision, the momentum conservation law is obeyed whether the collision is perfectly elastic or perfectly inelastic. Assume ball 1 is initially moving right, in the positive direction.
Visualize:



Solve: In the case of a perfectly elastic collision, the two velocities $(v_{fx})_1$ and $(v_{fx})_2$ can be determined by combining the conservation equations of momentum and mechanical energy. By contrast, a perfectly inelastic collision involves only one final velocity v_{fx} and can be determined from just the momentum conservation equation.

(a) Momentum conservation: $m_1(v_{ix})_1 + m_2(v_{ix})_2 = m_1(v_{fx})_1 + m_2(v_{fx})_2$

Energy conservation: $\frac{1}{2}m_1(v_{ix})_1^2 + \frac{1}{2}m_2(v_{ix})_2^2 = \frac{1}{2}m_1(v_{fx})_1^2 + \frac{1}{2}m_2(v_{fx})_2^2$

These two equations can be solved as shown in Equations 10.38 through 10.42:

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 = \frac{(100 \text{ g}) - (300 \text{ g})}{(100 \text{ g}) + (300 \text{ g})}(10 \text{ m/s}) = -5.0 \text{ m/s}$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1 = \frac{2(100 \text{ g})}{(100 \text{ g}) + (300 \text{ g})}(10 \text{ m/s}) = +5.0 \text{ m/s}$$

(b) For the inelastic collision, both balls travel with the same final speed v_{fx}. The momentum conservation equation $p_{fx} = p_{ix}$ is

$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$

$$\Rightarrow v_{fx} = \left(\frac{100 \text{ g}}{100 \text{ g} + 300 \text{ g}} \right)(10 \text{ m/s}) + 0 \text{ m/s} = 2.5 \text{ m/s}$$

Assess: In the case of the perfectly elastic collision, the two balls bounce off each other with a speed of 5.0 m/s. In the case of the perfectly inelastic collision, the balls stick together and move together at 2.5 m/s.

10.32. Model: This is the case of a perfectly inelastic collision. Momentum is conserved because no external force acts on the system $(\text{clay} + \text{brick})$. We also represent our system as a particle.

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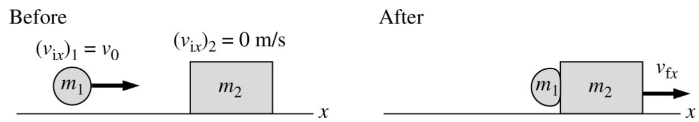
Visualize:



Solve: **(a)** The conservation of momentum equation $p_{fx} = p_{ix}$ is

$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$

Using $(v_{ix})_1 = v_0$ and $(v_{ix})_2 = 0,$ we get

$$v_{fx} = \frac{m_1}{m_1 + m_2}(v_{ix})_1 = \frac{0.050 \text{ kg}}{(1.0 \text{ kg} + 0.050 \text{ kg})}(v_{ix})_1 = 0.0476(v_{ix})_1 = 0.0476\, v_0$$

The brick is moving with speed $0.048v_0$.

(b) The initial and final kinetic energies are given by

$$K_i = \frac{1}{2}m_1(v_{ix})_1^2 + \frac{1}{2}m_2(v_{ix})_2^2 = \frac{1}{2}(0.050 \text{ kg})v_0^2 + \frac{1}{2}(1.0 \text{ kg})(0 \text{ m/s})^2 = (0.025 \text{ kg})v_0^2$$

$$K_f = \frac{1}{2}(m_1 + m_2)v_{fx}^2 = \frac{1}{2}(1.0 \text{ kg} + 0.050 \text{ kg})(0.0476)^2 v_0^2 = 0.00119\, v_0^2$$

The percent of energy lost $= \left(\frac{K_i - K_f}{K_i}\right) \times 100\% = \left(1 - \frac{0.00119}{0.025}\right) \times 100\% = 95\%$

Exercises and Problems

10.33. Model: We will take the system to be the person plus the earth.

Visualize: When a person drops from a certain height, the initial potential energy is transformed to kinetic energy. When the person hits the ground, if they land rigidly upright, we assume that all of this energy is transformed into elastic potential energy of the compressed leg bones. The maximum energy that can be absorbed by the leg bones is 200 J; this limits the maximum height.

Solve: **(a)** The initial potential energy can be at most 200 J, so the height h of the jump is limited by $mgh = 200$ J For $m = 60$ kg, this limits the height to

$$h = 200 \text{ J/mg} = 200 \text{ J}/(60 \text{ kg})(9.8 \text{ m/s}^2) = 0.34 \text{ m}$$

(b) If some of the energy is transformed to other forms than elastic energy in the bones, the initial height can be greater. If a person flexes her legs on landing, some energy is transformed to thermal energy. This allows for a greater initial height.

Assess: There are other tissues in the body with elastic properties that will absorb energy as well, so this limit is quite conservative.

10.34. Model: Model your vehicle as a particle. Assume zero rolling friction, so that the sum of your kinetic and gravitational potential energy does not change as the vehicle coasts down the hill.

Visualize:

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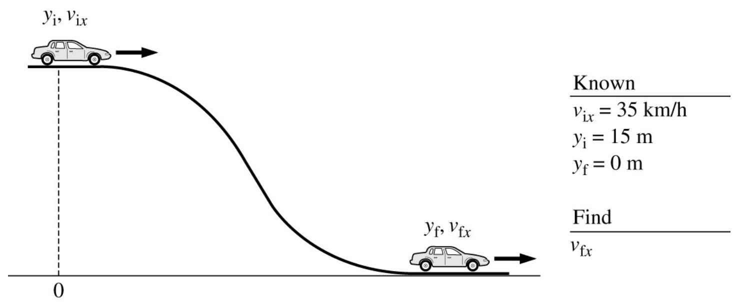
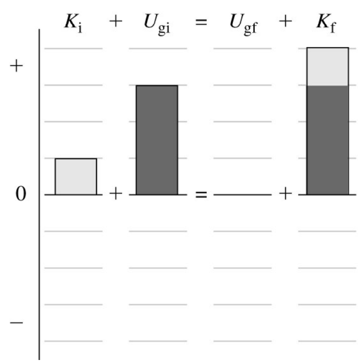




The figure shows a before-and-after pictorial representation. Note that neither the shape of the hill nor the angle of the downward slope is given, since these are not needed to solve the problem. All we need is the change in potential energy as you and your vehicle descend to the bottom of the hill. Also note that

$$35\text{ km/hr} = (35{,}000\text{ m}/3600\text{ s}) = 9.722\text{ m/s}$$

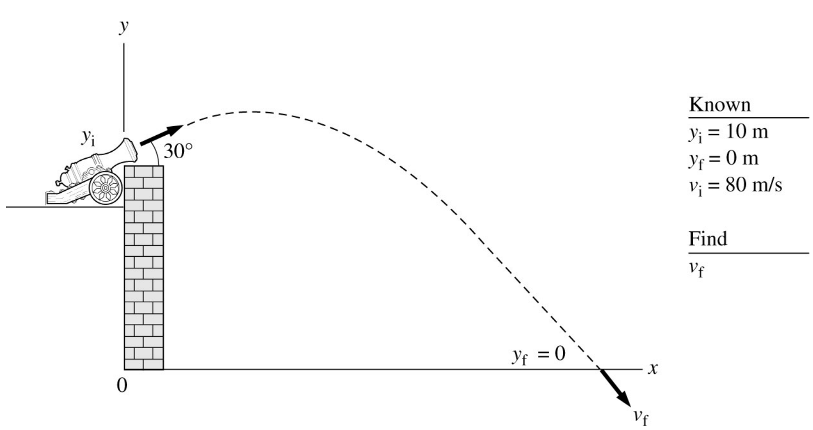
Solve: Using $y_f = 0$ and the equation $K_i + U_{gi} = K_f + U_{gf}$ we get

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f \Rightarrow v_i^2 + 2gy_i = v_f^2$$

$$\Rightarrow v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{(9.722\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(15\text{ m})} = 19.7\text{ m/s} = 71\text{ km/h}$$

You are driving over the speed limit. Yes, you will get a ticket.
Assess: A speed of 19.7 m/s or 71 km/h at the bottom of the hill, when your speed at the top of the hill was 35 km/s, is reasonable. From the energy bar chart, we see that the initial potential energy is completely transformed into the final kinetic energy.

10.35. Model: This is case of free fall, so the sum of the kinetic and gravitational potential energy does not change as the cannon ball falls.
Visualize:



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The figure shows a before-and-after pictorial representation. To express the gravitational potential energy, we put the origin of our coordinate system on the ground below the fortress.

Solve: Using $y_f = 0$ and the equation $K_i + U_{gi} = K_f + U_{gf}$ we get

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f \Rightarrow v_i^2 + 2gy_i = v_f^2$$

$$v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{(80\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(10\text{ m})} = 81\text{ m/s}$$

Assess: Note that we did not need to use the tilt angle of the cannon, because kinetic energy is a scalar. Also note that using the energy conservation equation, we can find only the magnitude of the final velocity, not the direction of the velocity vector.

10.36. Model: Assume the spring is ideal and obeys Hooke's law. Then the potential energy of a stretched spring is $U_{sp} = \frac{1}{2}k(\Delta s)^2$.

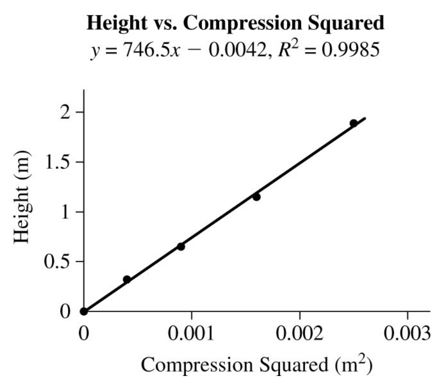
Visualize: The kinetic energy at the beginning is zero, and it is also zero at maximum height, so the spring potential energy at the beginning equals the gravitational potential energy at maximum height. We are given $k = 950\text{N/m}$.

Solve:

$$(U_s)_i = (U_g)_f \Rightarrow \frac{1}{2}k(\Delta s)^2 = mgy$$

Putting this in $y = mx + b$ form $y = \frac{k}{2mg}(\Delta s)^2$ leads us to believe that a graph of y vs. $(\Delta s)^2$ would produce a straight line whose slope is $k/2mg$ and whose intercept is zero.

Compression (m)	Height (m)	Compression Squared (m^2)
0	0	0
0.02	32	0.0004
0.03	65	0.0009
0.04	115	0.0016
0.05	189	0.0025



The spreadsheet tells us the fit is very good and that the slope is 746.5m^{-1}.

$$\frac{k}{2mg} = 746.5\text{ m}^{-1} \Rightarrow m = \frac{k}{2g(746.5\text{ m}^{-1})} = \frac{950\text{ N/m}}{2(9.8\text{ m/s}^2)(746.5\text{ m}^{-1})} = 0.065\text{ kg} = 65\text{ g}$$

Assess: 65 g seems reasonable, and we were happy to get a very small intercept on our best-fit line.

10.37. Model: For the ice cube sliding around the inside of a smooth pipe, the sum of the kinetic and gravitational potential energy does not change.

Visualize:

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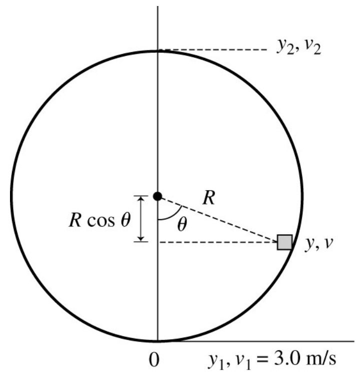


We use a coordinate system with the origin at the bottom of the pipe, that is, $y_1 = 0$. The radius R of the pipe is 10 cm, and therefore $y_{top} = y_2 = 2R = 0.20$ m. At an arbitrary angle θ, measured counterclockwise from the bottom of the circle, $y = R - R\cos\theta$.

Solve: **(a)** The energy conservation equation $K_2 + U_{g2} = K_1 + U_{g1}$ is

$$\Rightarrow \frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + mgy_1$$

$$\Rightarrow v_2 = \sqrt{v_1^2 + 2g(y_1 - y_2)} = \sqrt{(3.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(0 \text{ m} - 0.20 \text{ m})} = 2.25 \text{ m/s} \approx 2.3 \text{ m/s}$$

(b) Expressing the energy conservation equation as a function of θ:

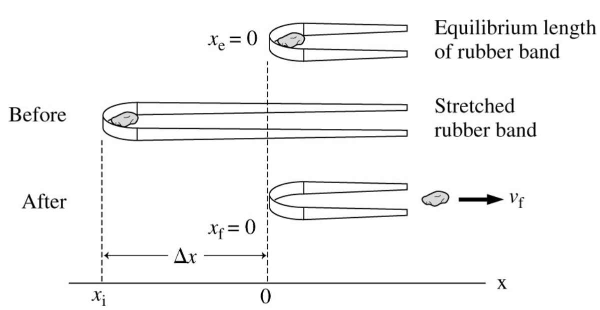
$$K(\theta) + U_g(\theta) = K_1 + U_{g1} \Rightarrow \frac{1}{2}mv^2(\theta) + mgy(\theta) = \frac{1}{2}mv_1^2 + 0 \text{ J}$$

$$\Rightarrow v(\theta) = \sqrt{v_1^2 - 2gy(\theta)} = \sqrt{v_1^2 - 2gR(1-\cos\theta)}$$

Using $v_1 = 3.0$ m/s, $g = 9.8$ m/s^2, and $R = 0.10$ m, we get $v(\theta) = \sqrt{9 - 1.96(1-\cos\theta)}$ (m/s)

Assess: Beginning with a speed of 3.0 m/s at the bottom, the marble's potential energy increases and kinetic energy decreases as it gets toward the top of the circle. At the top, its speed is 2.25 m/s. This is reasonable since some of the kinetic energy has been transformed into the marble's potential energy.

10.38. Model: Assume that the rubber band behaves similar to a spring. Also, model the rock as a particle.
Visualize:



Solve: **(a)** The rubber band is stretched to the left since a positive spring force on the rock due to the rubber band results from a negative displacement of the rock. That is, $(F_{sp})_x = -kx$, where x is the rock's displacement from the equilibrium position due to the spring force F_{sp}.

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(b) Since the F_{sp} versus x graph is linear with a negative slope and can be expressed as $F_{sp} = -kx$, the rubber band obeys Hooke's law.
(c) From the graph, $|\Delta F_{sp}| = 20$ N for $|\Delta x| = 10$ cm. Thus,

$$k = \frac{|\Delta F_{sp}|}{|\Delta x|} = \frac{20\text{ N}}{0.10\text{ m}} = 200\text{ N/m} = 2.0\times10^2\text{ N/m}$$

(d) The conservation of energy equation $K_f + U_{sf} = K_i + U_{si}$ for the rock is

$$\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 \Rightarrow \frac{1}{2}mv_f^2 + \frac{1}{2}k(0\text{ m})^2 = \frac{1}{2}m(0\text{ m/s})^2 + \frac{1}{2}kx_i^2$$

$$v_f = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{200\text{ N/m}}{0.050\text{ kg}}}(0.30\text{ m}) = 19\text{ m/s}$$

Assess: Note that x_i is Δx, which is the displacement relative to the equilibrium position, and x_f is the equilibrium position of the rubber band, which is equal to zero.

10.39. Model: We will assume the knee extensor tendon behaves according to Hooke's Law and stretches in a straight line.
Visiualize: The elastic energy stored in a spring is given by $U_s = \frac{1}{2}k(\Delta s)^2$.
Solve: For athletes,

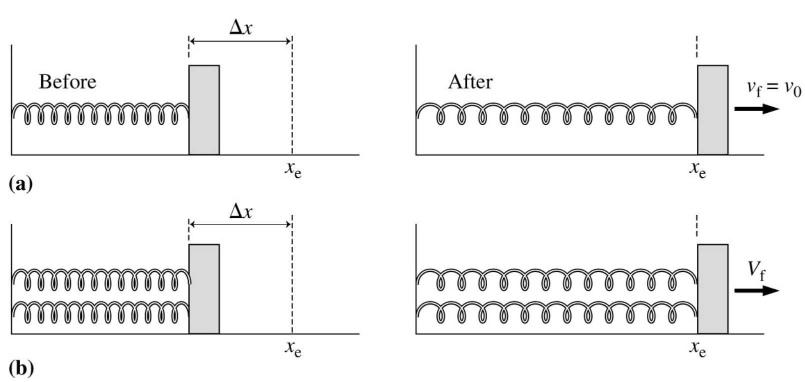
$$U_{s,\text{athlete}} = \tfrac{1}{2}k(\Delta s)^2 = \frac{1}{2}(33{,}000\text{ N/m})(0.041\text{ m})^2 = 27.7\text{ J}$$

For non-athletes,

$$U_{s,\text{non-athlete}} = \tfrac{1}{2}k(\Delta s)^2 = \frac{1}{2}(33{,}000\text{ N/m})(0.033\text{ m})^2 = 18.0\text{ J}$$

The difference in energy stored between athletes and non-athletes is therefore 9.7 J.
Assess: Notice the energy stored by athletes is over 1.5 times the energy stored by non-athletes.

10.40. Model: Model the block as a particle and the springs as ideal springs obeying Hooke's law. There is no friction, hence the mechanical energy $K + U_s$ is conserved.
Visualize:



Note that $x_f = x_e$ and $x_i - x_e = \Delta x$. The before-and-after pictorial representations show that we put the origin of the coordinate system at the equilibrium position of the free end of the springs.
Solve: The conservation of energy equation $K_f + U_{sf} = K_i + U_{si}$ for the single spring is

$$\frac{1}{2}mv_f^2 + \frac{1}{2}k(x_f - x_e)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(x_i - x_e)^2$$

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Using the value for v_f given in the problem, we get

$$\frac{1}{2}mv_0^2 + 0\text{ J} = 0\text{ J} + \frac{1}{2}k(\Delta x)^2 \Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}k(\Delta x)^2$$

Conservation of energy for the two-spring case:

$$\frac{1}{2}mV_f^2 + 0\text{ J} = 0\text{ J} + \frac{1}{2}k(x_i - x_e)^2 + \frac{1}{2}k(x_i - x_e)^2 \quad \frac{1}{2}mV_f^2 = k(\Delta x)^2$$

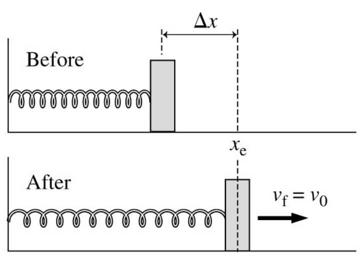
Using the result of the single-spring case,

$$\frac{1}{2}mV_f^2 = mv_0^2 \Rightarrow V_f = \sqrt{2}v_0$$

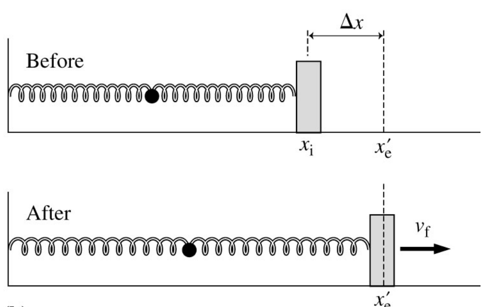
Assess: The block separates from the spring at the equilibrium position of the spring.

10.41. Model: Model the block as a particle and the springs as ideal springs obeying Hooke's law. There is no friction, hence the mechanical energy $K + U_s$ is conserved.

Visualize:



(a)



(b)

The springs in both cases have the same compression Δx. We put the origin of the coordinate system at the equilibrium position of the free end of the spring for the single-spring case (a), and at the free end of the two connected springs for the two-spring case (b).

Solve: The conservation of energy for the single-spring case:

$$K_f + U_{sf} = K_i + U_{si} \Rightarrow \frac{1}{2}mv_f^2 + \frac{1}{2}k(x_f - x_e)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(x_i - x_e)^2$$

Using $x_f = x_e = 0$ m, $v_i = 0$ m/s, and $v_f = v_0$, this equation simplifies to

$$\frac{1}{2}mv_0^2 = \frac{1}{2}k(\Delta x)^2$$

Conservation of energy in the case of the two springs in series, where each spring compresses by $\Delta x/2$, is

$$K_f + U_{sf} = K_i + U_{si} \Rightarrow \frac{1}{2}mV_f^2 + 0 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(\Delta x/2)^2 + \frac{1}{2}k(\Delta x/2)^2$$

Using $x_f = x_e' = 0$ m and $v_i = 0$ m/s, we get

$$\frac{1}{2}mV_f^2 = \frac{1}{2}\left[\frac{1}{2}k(\Delta x)^2\right]$$

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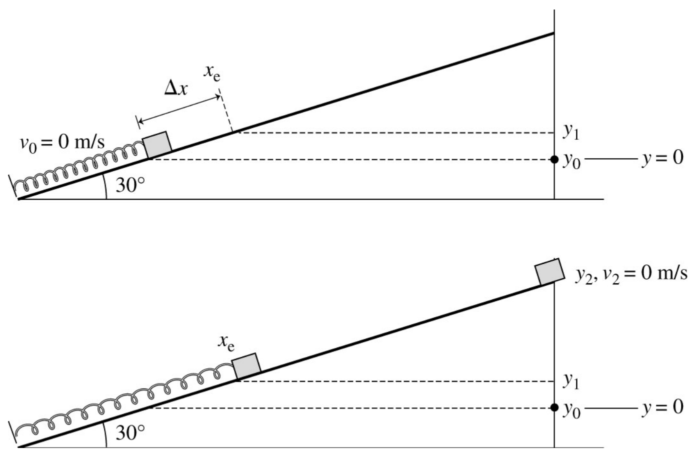
Comparing the two results we see that $V_f = v_0/\sqrt{2}$.

Assess: The block pushes on the spring until the spring has returned to its equilibrium length.

10.42. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, and therefore the mechanical energy $K + U_s + U_g$ is conserved.

Visualize:



The figure shows a before-and-after pictorial representation. We have chosen to place the origin of the coordinate system at the position where the ice cube has compressed the spring 10 cm. That is, $y_0 = 0$.

Solve: (a) The energy conservation equation $K_2 + U_{s2} + U_{g2} = K_0 + U_{s0} + U_{g0}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_e - x_e)^2 + mgy_2 = \frac{1}{2}mv_0^2 + \frac{1}{2}k(x - x_e)^2 + mgy_0$$

Using $v_2 = 0$ m/s, $y_0 = 0$ m, and $v_0 = 0$ m/s,

$$mgy_2 = \frac{1}{2}k(x - x_e)^2 \Rightarrow y_2 = \frac{k(\Delta x)^2}{2\,mg} = h$$

(b) Insert the values given

$$h = \frac{k(\Delta x)^2}{2\,mg} = \frac{(25\text{ N/m})(0.10\text{ m})^2}{2(0.050\text{ kg})(9.8\text{ m/s}^2)} = 25.5\text{ cm} \approx 26\text{ cm}$$

Assess: The net effect of the launch is to transform the potential energy stored in the spring into gravitational potential energy. The block has kinetic energy as it comes off the spring, but we did not need to know this energy to solve the problem. The answer is independent of the angle of the slope.

10.43. Model: Model the two packages as particles. Momentum is conserved in both inelastic and elastic collisions. Kinetic energy is conserved only in a perfectly elastic collision.

Visualize:

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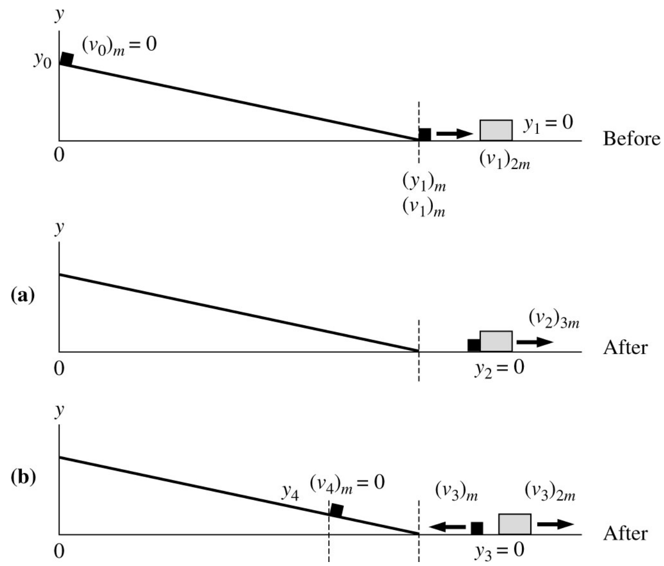

Solve: For a package with mass m the conservation of energy equation is

$$K_1 + U_{g1} = K_0 + U_{g0} \Rightarrow \frac{1}{2}m(v_1)_m^2 + mgy_1 = \frac{1}{2}m(v_0)_m^2 + mgy_0$$

Using $(v_0)_m = 0$ m/s and $y_1 = 0$ m,

$$\frac{1}{2}m(v_1)_m^2 = mgy_0 \Rightarrow (v_1)_m = \sqrt{2gy_0} = \sqrt{2(9.8 \text{ m/s}^2)(3.0 \text{ m})} = 7.668 \text{ m/s}$$

(a) For the perfectly inelastic collision the conservation of momentum equation is

$$p_{fx} = p_{ix} \Rightarrow (m + 2m)(v_2)_{3m} = m(v_1)_m + (2m)(v_1)_{2m}$$

Using $(v_1)_{2m} = 0$ m/s, we get

$$(v_2)_{3m} = (v_1)_m/3 = 2.56 \text{ m/s}$$

The packages move off together at a speed of 2.6 m/s.
(b) For the elastic collision, the mass m package rebounds with velocity

$$(v_3)_m = \frac{m - 2m}{m + 2m}(v_1)_m = -\frac{1}{3}(7.668 \text{ m/s}) = -2.56 \text{ m/s}$$

The negative sign with $(v_3)_m$ shows that the package with mass m rebounds and goes to the position y_4. We can determine y_4 by applying the conservation of energy equation as follows. For a package of mass m:

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}m(v_4)_m^2 + mgy_4 = \frac{1}{2}m(v_3)_m^2 + mgy_3$$

Using $(v_3)_m = -2.55$ m/s, $y_3 = 0$ m, and $(v_4)_m = 0$ m/s, we get

$$mgy_4 = \frac{1}{2}m(-2.56 \text{ m/s})^2 \Rightarrow y_4 = 33 \text{ cm}$$

10.44. Model: Model the marble and the steel ball as particles. We will assume an elastic collision between the marble and the ball, and apply the conservation of momentum and the conservation of energy equations. We will also assume zero rolling friction between the marble and the incline.
Visualize:

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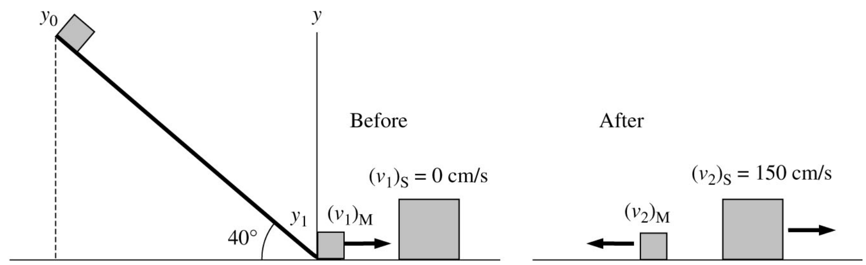
Known
$m_M = 100$ g $\quad m_S = 200$ g
$(v_0)_M = 0$ m/s $\quad (v_1)_S = 0$ m/s
$(v_2)_S = 150$ cm/s
$y_1 = 0$ m

Find
y_0



This is a two-part problem. In the first part, we will apply the conservation of energy equation to find the marble's speed as it exits onto a horizontal surface. We have put the origin of our coordinate system on the horizontal surface just where the marble exits the incline. In the second part, we will consider the elastic collision between the marble and the steel ball.

Solve: The conservation of energy equation $K_1 + U_{g1} = K_0 + U_{g0}$ gives us:

$$\frac{1}{2}m_M(v_1)_M^2 + m_M g y_1 = \frac{1}{2}m_M(v_0)_M^2 + m_M g y_0$$

Using $(v_0)_M = 0$ m/s and $y_1 = 0$ m, we get $\frac{1}{2}(v_1)_M^2 = g y_0 \Rightarrow (v_1)_M = \sqrt{2 g y_0}$. When the marble collides with the steel ball, the elastic collision gives the ball velocity

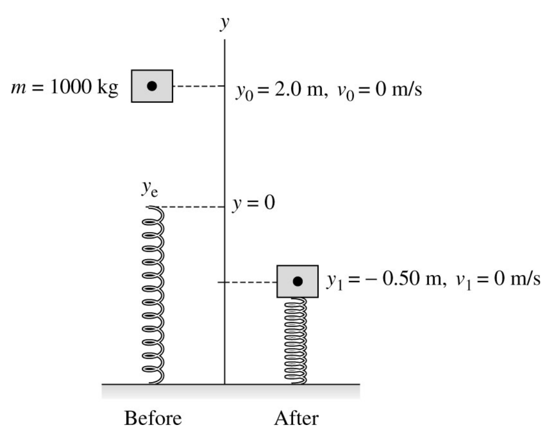
$$(v_2)_S = \frac{2m_M}{m_M + m_S}(v_1)_M = \frac{2m_M}{m_M + m_S}\sqrt{2 g y_0}$$

Solving for y_0 gives

$$y_0 = \frac{1}{2g}\left[\frac{m_M + m_S}{2m_M}(v_2)_S\right]^2 = 0.258 \text{ m} = 25.8 \text{ cm}$$

10.45. Model: Assume an ideal spring that obeys Hooke's law. Since this is a free-fall problem, the mechanical energy $K + U_g + U_s$ is conserved. Also, model the safe as a particle.

Visualize:



We have chosen to place the origin of our coordinate system at the free end of the spring, which is neither stretched nor compressed. The safe gains kinetic energy as it falls. The energy is then converted into elastic potential energy as the safe compresses the spring. The only two forces are gravity and the spring force, which are both conservative, so energy is conserved throughout the process. This means that the initial energy—as the safe is released—equals the final energy—when the safe is at rest and the spring is fully compressed.

Solve: The conservation of energy equation $K_1 + U_{g1} + U_{s1} = K_0 + U_{g0} + U_{s0}$ is

$$\frac{1}{2}mv_1^2 + mg(y_1 - y_e) + \frac{1}{2}k(y_1 - y_e)^2 = \frac{1}{2}mv_0^2 + mg(y_0 - y_e) + \frac{1}{2}k(y_e - y_e)^2$$

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Using $v_1 = v_0 = 0$ m/s and $y_e = 0$ m, the above equation simplifies to

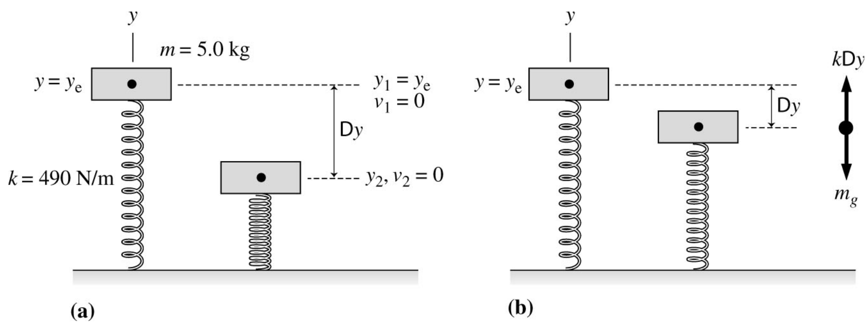
$$mgy_1 + \frac{1}{2}ky_1^2 = mgy_0$$

$$\Rightarrow k = \frac{2mg(y_0 - y_1)}{y_1^2} = \frac{2(1000\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m} - (-0.50\text{ m}))}{(-0.50\text{ m})^2} = 1.96\times10^5\text{ N/m} \approx 2.0\times10^5\text{ N/m}$$

Assess: By equating energy at these two points, we do not need to find how fast the safe was moving when it hit the spring.

10.46. Model: Assume an ideal spring that obeys Hooke's law. There is no friction and hence the mechanical energy $(v_{fx})_1 = -1.2\text{ m/s} + 1.0\text{ m/s} = -0.2\text{ m/s}$ is conserved.

Visualize:



Solve: (a) When releasing the block suddenly, $K_2 + U_{s2} + U_{g2} = K_1 + U_{s1} + U_{g1}$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(y_2 - y_e)^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(y_1 - y_e)^2 + mgy_1$$

Using $v_2 = 0$ m/s, $v_1 = 0$ m/s, and $y_1 = y_e$, we get

$$0\text{ J} + \frac{1}{2}(490\text{ N/m})(y_2 - y_1)^2 + mgy_2 = 0\text{ J} + 0\text{ J} + mgy_1 \Rightarrow (245\text{ N/m})(y_2 - y_1)^2 = mg(y_1 - y_2)$$

$$\Rightarrow (245\text{ N/m})(y_1 - y_2)^2 = (5.0\text{ kg})(9.8\text{ m/s}^2)(y_1 - y_2) \Rightarrow (y_1 - y_2) = 0.20\text{ m}$$

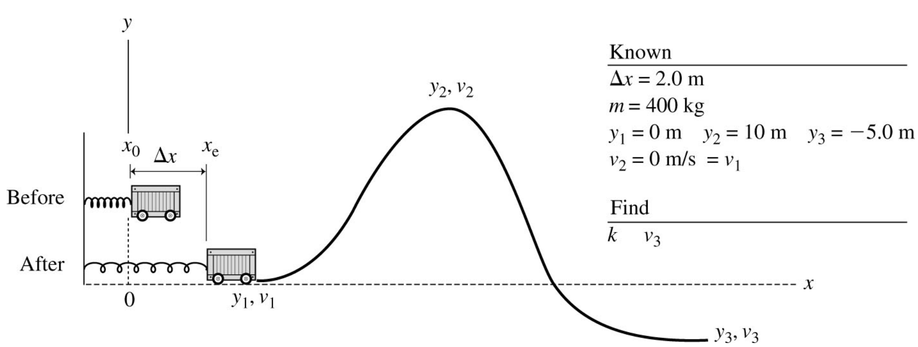
(b) When lowering the block gently until it rests on the spring, the block reaches a point of static equilibrium.

$$F_{net} = k\Delta y - mg = 0 \Rightarrow \Delta y = \frac{mg}{k} = \frac{(5.0\text{ kg})(9.8\text{ m/s}^2)}{490\text{ N/m}} = 0.10\text{ m}$$

(c) In part (b), at a point 0.10 m down, the forces balance. But in part (a) the block has kinetic energy as it reaches 0.10 m. So the block continues on past the equilibrium point until all the gravitational potential energy is stored in the spring.

10.47. Model: Assume an ideal spring that obeys Hooke's law. Also assume zero rolling friction between the roller coaster and the track, and a particle model for the roller coaster. Since no friction is involved, the mechanical energy $K + U_s + U_g$ is conserved.

Visualize:



We have chosen to place the origin of the coordinate system on the end of the spring that is compressed and touches the roller coaster car.

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Solve: (a) The energy conservation equation for the car going to the top of the hill is

$$K_2 + U_{g2} + U_{s2} = K_0 + U_{g0} + U_{s0} \quad \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}k(x_e - x_e)^2 = \frac{1}{2}mv_0^2 + mgy_0 + \frac{1}{2}k(x_0 - x_e)^2$$

Noting that $y_0 = 0$ m, $v_2 = 0$ m/s, $v_1 = 0$ m/s, and $|x_0 - x_e| = 2.0$ m, we obtain

$$0 \text{ J} + mgy_2 + 0 \text{ J} = 0 \text{ J} + 0 \text{ J} + \frac{1}{2}k(2.0 \text{ m})^2$$

$$\Rightarrow k = \frac{2mgy_2}{(2.0 \text{ m})^2} = \frac{2(400 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m})}{(2.0 \text{ m})^2} = 1.96 \times 10^4 \text{ N/m}$$

We now increase this value for *k* by 10% for safety, giving a value of 2.156×10^4 N/m $\approx 2.2 \times 10^4$ N/m.

(b) The energy conservation equation $K_3 + U_{g3} + U_{s3} = K_0 + U_{g0} + U_{s0}$ is

$$\frac{1}{2}mv_3^2 + mgy_3 + \frac{1}{2}k(x_e - x_e)^2 = \frac{1}{2}mv_0^2 + mgy_0 + \frac{1}{2}k(x_0 - x_e)^2$$

Using $y_3 = -5.0$ m, $v_0 = 0$ m/s, $y_0 = 0$ m, and $|x_0 - x_e| = 2.0$ m, we get

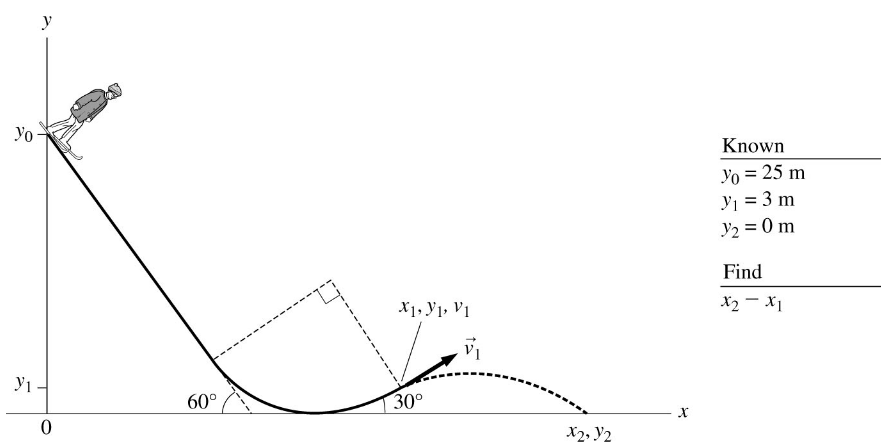
$$\frac{1}{2}mv_3^2 + mg(-5.0 \text{ m}) + 0 \text{ J} = 0 \text{ J} + 0 \text{ J} + \frac{1}{2}k(x_0 - x_e)^2$$

$$\frac{1}{2}(400 \text{ kg})v_3^2 - (400 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m}) = \frac{1}{2}(2.156 \times 10^4 \text{ N/m})(2.0 \text{ m})^2$$

$$\Rightarrow v_3 = 17.7 \text{ m/s} \approx 18 \text{ m/s}$$

10.48. Model: Since there is no friction, the sum of the kinetic and gravitational potential energy does not change. Model Julie as a particle.

Visualize:



We place the coordinate system at the bottom of the ramp directly below Julie's starting position. From geometry, Julie launches off the end of the ramp at a 30º angle.

Solve: Energy conservation: $K_1 + U_{g1} = K_0 + U_{g0} \Rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$

Using $v_0 = 0$ m/s, $y_0 = 25$ m, and $y_1 = 3$ m, the above equation simplifies to

$$\frac{1}{2}mv_1^2 + mgy_1 = mgy_0 \Rightarrow v_1 = \sqrt{2g(y_0 - y_1)} = \sqrt{2(9.8 \text{ m/s}^2)(25 \text{ m} - 3 \text{ m})} = 20.77 \text{ m/s}$$

We can now use kinematic equations to find the touchdown point from the base of the ramp. First we'll consider the vertical motion:

$$y_2 = y_1 + v_{1y}(t_2 - t_1) + \frac{1}{2}a_y(t_2 - t_1)^2 \quad 0 \text{ m} = 3 \text{ m} + (v_1 \sin 30°)(t_2 - t_1) + \frac{1}{2}(-9.8 \text{ m/s}^2)(t_2 - t_1)^2$$

$$\Rightarrow (t_2 - t_1)^2 - \frac{(20.77 \text{ m/s})\sin 30°}{(4.9 \text{ m/s}^2)}(t_2 - t_1) - \frac{(3 \text{ m})}{(4.9 \text{ m/s}^2)} = 0$$

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$$(t_2 - t_1)^2 - (2.119\text{ s})(t_2 - t_1) - (0.6122\text{ s}^2) = 0 \Rightarrow (t_2 - t_1) = 2.377\text{ s}$$

For the horizontal motion:

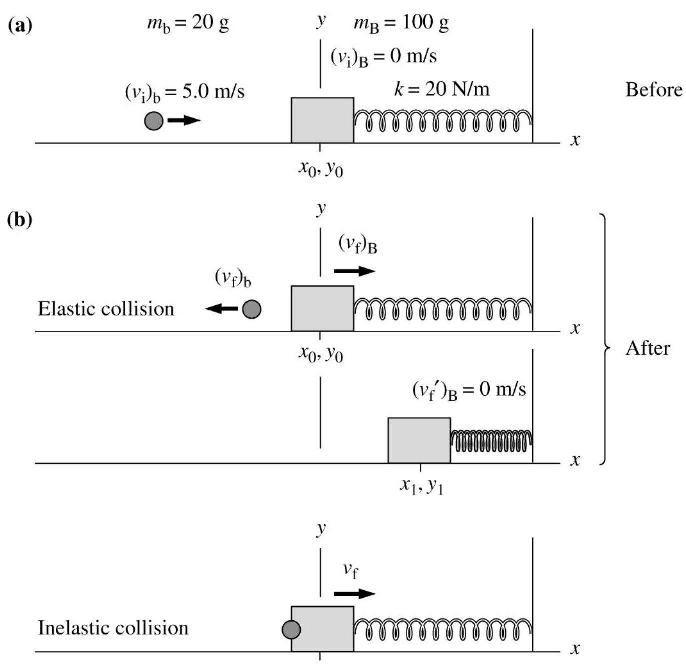
$$x_2 = x_1 + v_{1x}(t_2 - t_1) + \frac{1}{2}a_x(t_2 - t_1)^2$$

$$x_2 - x_1 = (v_1\cos30°)(t_2 - t_1) + 0\text{ m} = (20.77\text{ m/s})(\cos30°)(2.377\text{ s}) = 43\text{ m}$$

Assess: Note that we did not have to make use of the information about the circular arc at the bottom that carries Julie through a 90° turn.

10.49. Model: We assume the spring to be ideal and to obey Hooke's law. We also treat the block (B) and the ball (b) as particles. In the case of an elastic collision, both the momentum and kinetic energy equations apply. On the other hand, for a perfectly inelastic collision only the equation of momentum conservation is valid.

Visualize:



Place the origin of the coordinate system on the block that is attached to one end of the spring. The before-and-after pictorial representations of the elastic and perfectly inelastic collision are shown in figures (a) and (b), respectively.

Solve: (a) For an elastic collision, the ball's rebound velocity is

$$(v_f)_b = \frac{m_b - m_B}{m_b + m_B}(v_i)_b = \frac{-80\text{ g}}{120\text{ g}}(5.0\text{ m/s}) = -3.33\text{ m/s}$$

The ball's speed is 3.3 m/s.

(b) An elastic collision gives the block speed

$$(v_f)_B = \frac{2m_B}{m_b + m_B}(v_i)_b = \frac{40\text{ g}}{120\text{ g}}(5.0\text{ m/s}) = 1.667\text{ m/s}$$

To find the maximum compression of the spring, we use the conservation equation of mechanical energy for the block + spring system. That is $K_1 + U_{s1} = K_0 + U_{s0}$:

$$\frac{1}{2}m_B(v_f')_B^2 + \frac{1}{2}k(x_1 - x_0)^2 = \frac{1}{2}m_B(v_f)_B^2 + \frac{1}{2}k(x_0 - x_0)^2 \quad 0 + k(x_1 - x_0)^2 = m_B(v_f)_B^2 + 0$$

$$(x_1 - x_0) = \sqrt{(0.100\text{ kg})(1.667\text{ m/s})^2/(20\text{ N/m})} = 11.8\text{ cm}$$

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(c) Momentum conservation $p_f = p_i$ for the perfectly inelastic collision means

$$(m_B + m_B)v_f = m_b(v_i)_b + m_B(v_i)_B$$

$$(0.100 \text{ kg} + 0.020 \text{ kg})v_f = (0.020 \text{ kg})(5.0 \text{ m/s}) + 0 \text{ m/v} \Rightarrow v_f = 0.833 \text{ m/s}$$

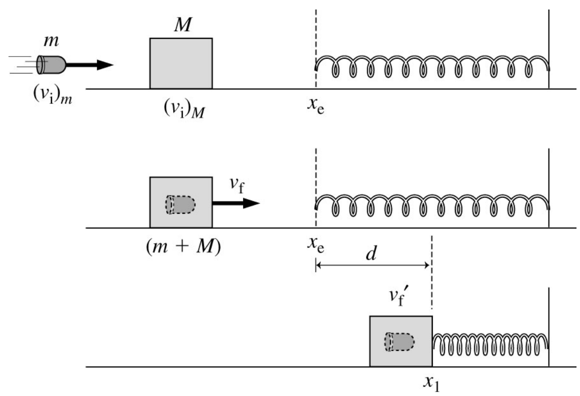
The maximum compression in this case can now be obtained using the conservation of energy equation $K_1 + U_{s1} = K_0 + U_{s0}$:

$$0 \text{ J} + (1/2)k(\Delta x)^2 = (1/2)(m_B + m_b)v_f^2 + 0 \text{ J}$$

$$\Rightarrow \Delta x = \sqrt{\frac{m_B + m_b}{k}}v_f = \sqrt{\frac{0.120 \text{ kg}}{20 \text{ N/m}}}(0.833 \text{ m/s}) = 0.0645 \text{ m} = 6.5 \text{ cm}$$

10.50. Model: Assume an ideal spring that obeys Hooke's law. We treat the bullet and the block in the particle model. For a perfectly inelastic collision, the momentum is conserved. Furthermore, since there is no friction, the mechanical energy of the system $(\text{bullet} + \text{block} + \text{spring})$ is conserved.

Visualize:



We place the origin of our coordinate system at the end of the spring that is not anchored to the wall.

Solve: (a) Momentum conservation for perfectly inelastic collision states $p_f = p_i$. This means

$$(m+M)v_f = m(v_i)_m + M(v_i)_M \Rightarrow (m+M)v_f = mv_B + 0 \text{ kg m/s} \Rightarrow v_f = \left(\frac{m}{m+M}\right)v_B$$

where we have used v_B for the initial speed of the bullet. The mechanical energy conservation equation $K_1 + U_{s1} = K_e + U_{se}$ as the bullet embedded block compresses the spring is:

$$\frac{1}{2}m(v_f')^2 + \frac{1}{2}k(x_1 - x_e)^2 = \frac{1}{2}(m+M)(v_f)^2 + \frac{1}{2}k(x_e - x_e)^2$$

$$0 \text{ J} + \frac{1}{2}kd^2 = \frac{1}{2}(m+M)\left(\frac{m}{m+M}\right)^2 v_B^2 + 0 \text{ J} \Rightarrow v_B = \sqrt{\frac{(m+M)kd^2}{m^2}}$$

(b) Using the above formula with $m = 5.0$ g, $M = 2.0$ kg, $k = 50$ N/m, and $d = 10$ cm,

$$v_B = \sqrt{(0.0050 \text{ kg} + 2.0 \text{ kg})(50 \text{ N/m})(0.10 \text{ m})^2/(0.0050)^2} = 2.0\times10^2 \text{ m/s}$$

(c) The fraction of energy lost is

$$\frac{\frac{1}{2}mv_B^2 - \frac{1}{2}(m+M)v_f^2}{\frac{1}{2}mv_B^2} = 1 - \frac{m+M}{m}\left(\frac{v_f}{v_B}\right)^2 = 1 - \frac{m+M}{m}\left(\frac{m}{m+M}\right)^2$$

$$= 1 - \frac{m}{m+M} = 1 - \frac{0.0050 \text{ kg}}{(0.0050 \text{ kg} + 2.0 \text{ kg})} = 0.9975$$

That is, during the perfectly inelastic collision 99.75% of the bullet's energy is lost. The energy is dissipated inside the block. Although it is common to say, "The energy is lost to heat," in the next chapter we'll see that it is more accurate to say, "The energy is transformed to thermal energy."

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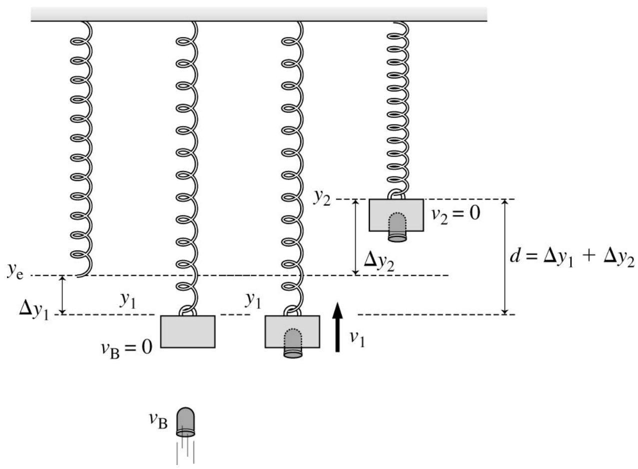
10.51. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, hence the mechanical energy $K + U_g + U_s$ is conserved.

Visualize:



We have chosen to place the origin of the coordinate system on the free end of the spring that is neither stretched nor compressed, that is, at the equilibrium position of the end of the unstretched spring. The bullet's mass is m and the block's mass is M.

Solve: (a) The energy conservation equation $K_2 + U_{s2} + U_{g2} = K_1 + U_{s1} + U_{g1}$ becomes

$$\frac{1}{2}(m+M)v_2^2 + \frac{1}{2}k(y_2 - y_e)^2 + (m+M)g(y_2 - y_e) = \frac{1}{2}(m+M)v_1^2 + \frac{1}{2}k(y_1 - y_e)^2 - (m+M)g(y_1 - y_e)$$

Noting $v_2 = 0$ m/s, we can rewrite the above equation as

$$k(\Delta y_2)^2 + 2(m+M)g(\Delta y_2 + \Delta y_1) = (m+M)v_1^2 + k(\Delta y_1)^2$$

Let us express v_1 in terms of the bullet's initial speed v_B by using the momentum conservation equation $p_f = p_i$ which is $(m+M)v_1 = mv_B + Mv_{block}$. Since $v_{block} = 0$ m/s, we have

$$v_1 = \left(\frac{m}{m+M}\right)v_B$$

We can also find the magnitude of y_1 from the equilibrium condition $k(y_1 - y_e) = Mg$.

$$\Delta y_1 = \frac{Mg}{k}$$

With these substitutions for v_1 and Δy_1, the energy conservation equation simplifies to

$$k(\Delta y_2)^2 + 2(m+M)g(\Delta y_1 + \Delta y_2) = \frac{m^2 v_B^2}{(m+M)} + k\left(\frac{Mg}{k}\right)^2$$

$$\Rightarrow v_B^2 = 2\left(\frac{m+M}{m}\right)^2 g(\Delta y_1 + \Delta y_2) - \frac{(m+M)}{m^2}\frac{M^2 g^2}{k} + k\left(\frac{m+M}{m^2}\right)(\Delta y_2)^2$$

We still need to include the spring's maximum compression (d) into this equation. We assume that $d = \Delta y_1 + \Delta y_2$, that is, maximum compression is measured from the initial position (y_1) of the block. Thus, using $\Delta y_2 = d - \Delta y_1 =$ $(d - Mg/k)$, we have

$$v_B = \left[2\left(\frac{m+M}{m}\right)^2 gd - \left(\frac{m+M}{m^2}\right)\frac{M^2 g^2}{k} + k\left(\frac{m+M}{m^2}\right)(d - Mg/k)^2\right]^{1/2}$$

(b) Using $m = 0.010$ kg, $M = 2.0$ kg, $k = 50$ N/m, and $d = 0.45$ m,

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$$v_B^2 = 2\left(\frac{2.010 \text{ kg}}{0.010 \text{ kg}}\right)^2 (9.8 \text{ m/s}^2)(0.45 \text{ m}) - (2.010 \text{ kg})\left(\frac{2.0 \text{ kg}}{0.010 \text{ kg}}\right)^2 (9.8 \text{ m/s}^2)^2/(50 \text{ N/m})$$

$$+ (50 \text{ N/m})(2.010 \text{ kg})\frac{1}{(0.010 \text{ kg})^2}[0.45 \text{ m} - (2.0 \text{ kg})\times(9.8 \text{ m/s}^2)/50 \text{ N/m}]^2$$

$$\Rightarrow v_B = 453 \text{ m/s}$$

The bullet has a speed of 4.5×10^2 m/s.

Assess: This is a reasonable speed for the bullet.

10.52. Model: The track is frictionless.

Visualize: $v_c = \sqrt{Rg}$

Solve: (a) First use conservation of momentum during the collision, then conservation of energy as the combined block goes to the top of the loop.

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$mv_m + 0 = (m+M)v_{tot}$$

$$v_m = \frac{m+M}{m}v_{tot}$$

Now use the conservation of energy. v_{tot} is the speed of the combined block just after the collision.

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}(m+M)v_{tot}^2 = (m+M)g(2R) + \frac{1}{2}(m+M)(v_c)^2$$

Cancel $(m+M)$ and replace v_c with $\sqrt{Rg}$.

$$v_{tot}^2 = 4Rg + Rg = 5Rg \Rightarrow v_{tot} = \sqrt{5Rg}$$

$$v_m = \frac{m+M}{m}v_{tot} = \frac{m+M}{m}\sqrt{5Rg}$$

(b) First use conservation of momentum and kinetic energy during the collision, then conservation of mechanical energy as the big block goes to the top of the loop. Call the speed of M just after the elastic collision V and the speed of m just after the collision v'_m.

$$\sum \vec{p}_i = \sum \vec{p}_f$$

We drop the vectors because this is one-dimensional motion, but v' may be negative.

$$mv_m + 0 = mv' + MV$$

$$v_m = v' + \frac{M}{m}V$$

Now use the conservation of energy as the block goes to the top of the loop.

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}mv_m'^2 + \frac{1}{2}MV^2 = Mg(2R) + \frac{1}{2}mv_m'^2 + \frac{1}{2}M\left(\sqrt{Rg}\right)^2$$

Subtract $\frac{1}{2}mv_m'^2$ from both sides, cancel M, and solve for V.

$$V = \sqrt{4Rg + Rg} = \sqrt{5Rg}$$

Now go back to the conservation of kinetic energy in the elastic collision.

$$\sum K_i = \sum K_f$$

$$\frac{1}{2}mv_m^2 = \frac{1}{2}mv_m'^2 + \frac{1}{2}MV^2$$

Cancel $\frac{1}{2}$ and divide by m.

$$v_m^2 = v_m'^2 + \frac{M}{m}5Rg$$

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Find $v_m'^2$ from the momentum equation.

$$v_m^2 = \left(v_m - \frac{M}{m}V\right)^2 + \frac{M}{m}5Rg$$

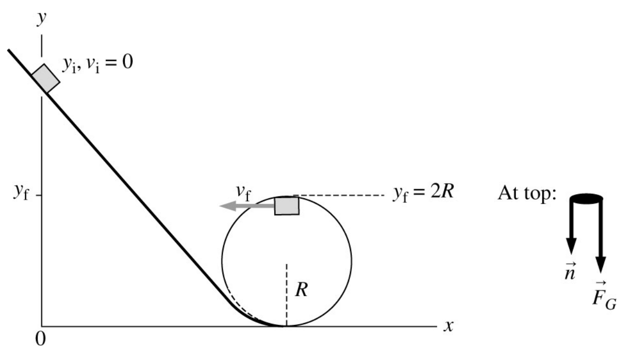
$$v_m^2 = v_m^2 - 2\frac{M}{m}v_m V + \left(\frac{M}{m}V\right)^2 + \frac{M}{m}5Rg$$

Subtract v_m^2 from both sides, cancel $\frac{M}{m}$, and solve for v_m.

$$v_m = \frac{1}{2}\frac{M}{m}V + \frac{1}{2}\frac{5Rg}{\sqrt{5Rg}} = \frac{1}{2}\frac{M}{m}\sqrt{5Rg} + \frac{1}{2}\sqrt{5Rg} = \frac{1}{2}\frac{m+M}{m}\sqrt{5Rg}$$

Assess: We expected the initial speed needed to be greater for the inelastic case because kinetic energy isn't conserved in the collision.

10.53. Model: This is a two-part problem. In the first part, we will find the critical velocity for the block to go over the top of the loop without falling off. Since there is no friction, the sum of the kinetic and gravitational potential energy is conserved during the block's motion. We will use this conservation equation in the second part to find the minimum height the block must start from to make it around the loop.
Visualize:



We place the origin of our coordinate system directly below the block's starting position on the frictionless track.
Solve: The free-body diagram on the block implies

$$F_G + n = \frac{mv_c^2}{R}$$

For the block to just stay on track, $n = 0$. Thus the critical velocity v_c is

$$F_G = mg = \frac{mv_c^2}{R} \Rightarrow v_c^2 = gR$$

The block needs kinetic energy $\frac{1}{2}mv_c^2 = \frac{1}{2}mgR$ to go over the top of the loop. We can now use the conservation of mechanical energy equation to find the minimum height h.

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Using $v_f = v_c = \sqrt{gR}, y_f = 2R, v_i = 0$ m/s, and $y_i = h,$ we obtain

$$\frac{1}{2}gR + g(2R) = 0 + gh \Rightarrow h = 2.5R$$

10.54. Model: Model Lisa (L) and the bobsled (B) as particles. We will assume the ramp to be frictionless, so that the mechanical energy of the system (Lisa + bobsled + spring) is conserved. Furthermore, during the collision, as Lisa leaps onto the bobsled, the momentum of the Lisa + bobsled system is conserved. We will also assume the spring to be an ideal one that obeys Hooke's law.

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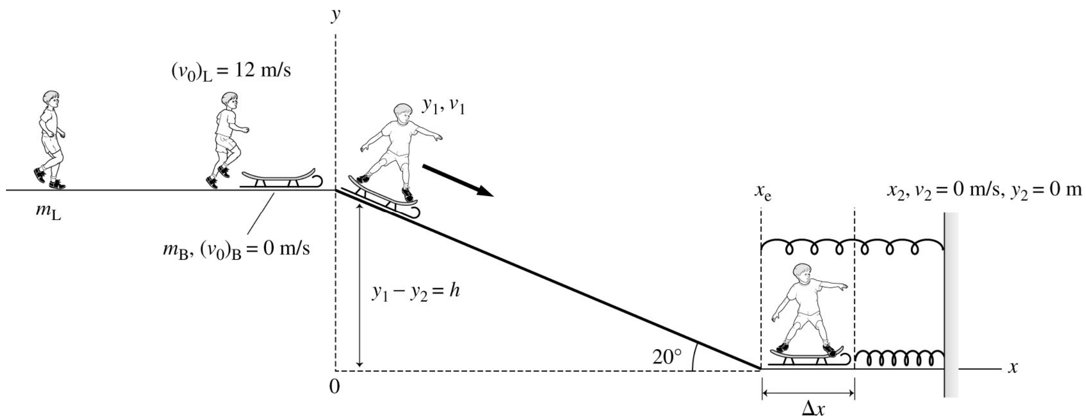
Visualize:



We place the origin of our coordinate system directly below the bobsled's initial position.

Solve: (a) Momentum conservation in Lisa's collision with bobsled states $p_1 = p_0$, or

$$(m_L + m_B)v_1 = m_L(v_0)_L + m_B(v_0)_B \Rightarrow (m_L + m_B)v_1 = m_L(v_0)_L + 0$$

$$\Rightarrow v_1 = \left(\frac{m_L}{m_L + m_B}\right)(v_0)_L = \left(\frac{40 \text{ kg}}{40 \text{ kg} + 20 \text{ kg}}\right)(12 \text{ m/s}) = 8.0 \text{ m/s}$$

The energy conservation equation: $K_2 + U_{s2} + U_{g2} = K_1 + U_{s1} + U_{g1}$ is

$$\frac{1}{2}(m_L + m_B)v_2^2 + \frac{1}{2}k(x_2 - x_e)^2 + (m_L + m_B)gy_2 = \frac{1}{2}(m_L + m_B)v_1^2 + \frac{1}{2}k(x_e - x_e)^2 + (m_L + m_B)gy_1$$

Using $v_2 = 0$ m/s, $k = 2000$ N/m, $y_2 = 0$ m, $y_1 = (50 \text{ m})\sin 20° = 17.1$ m, $v_1 = 8.0$ m/s, and $(m_L m_B) = 60$ kg, we get

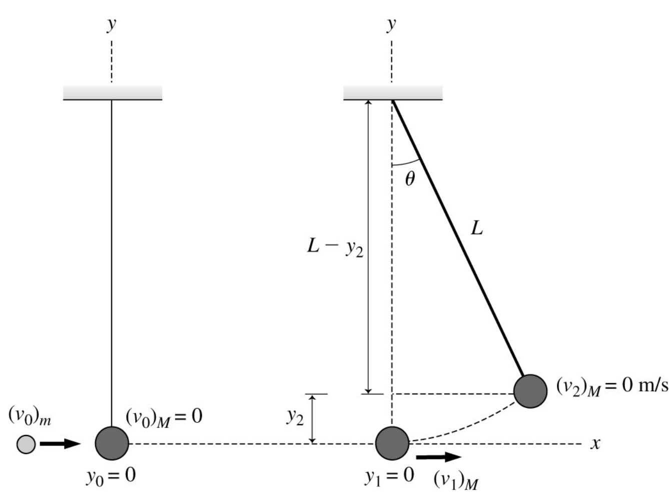
$$0 \text{ J} + \frac{1}{2}(2000 \text{ N/m})(x_2 - x_e)^2 + 0 \text{ J} = \frac{1}{2}(60 \text{ kg})(8.0 \text{ m/s})^2 + 0 \text{ J} + (60 \text{ kg})(9.8 \text{ m/s}^2)(17.1 \text{ m})$$

Solving this equation yields $(x_2 - x_e) = 3.5$ m.

(b) As long as the ice is slippery enough to be considered frictionless, we know from conservation of mechanical energy that the speed at the bottom depends only on the vertical descent Δy. Only the ramp's height h is important, not its shape or angle.

10.55. Model: We can divide this problem into two parts. First, we have an elastic collision between the 20 g ball (m) and the 100 g ball (M). Second, the 100 g ball swings up as a pendulum.

Visualize:



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The figure shows three distinct moments of time: the time before the collision, the time after the collision but before the two balls move, and the time the 100 g ball reaches its highest point. We place the origin of our coordinate system on the 100 g ball when it is hanging motionless.

Solve: For a perfectly elastic collision, the ball moves forward with speed

$$(v_1)_M = \frac{2m_m}{m_m + m_M}(v_0)_m = \frac{1}{3}(v_0)_m$$

In the second part, the sum of the kinetic and gravitational potential energy is conserved as the 100 g ball swings up after the collision. That is, $K_2 + U_{g2} = K_1 + U_{g1}$. We have

$$\frac{1}{2}M(v_2)_M^2 + Mgy_2 = \frac{1}{2}M(v_1)_M^2 + Mgy_1$$

Using $(v_2)_M = 0$ J, $(v_1)_M = \frac{(v_0)_m}{3}$, $y_1 = 0$ m, and $y_2 = L - L\cos\theta$, the energy equation simplifies to

$$g(L - L\cos\theta) = \frac{1}{2}\frac{(v_0)_m^2}{9}$$

$$\Rightarrow (v_0)_m = \sqrt{18\text{ g } L(1-\cos\theta)} = \sqrt{18(9.8\text{ m/s}^2)(1.0\text{ m})(1-\cos 50°)} = 7.9\text{ m/s}$$

10.56. Model: Model the balls as particles. We will use the Galilean transformation of velocities to analyze the problem of elastic collisions. We will transform velocities from the lab frame L to a frame M in which one ball is at rest. This allows us to apply equations to the case of a perfectly elastic collision in M, find the final velocities of the balls in M, and then transform these velocities back to the lab frame L.

Visualize: Let M be the frame of the 200 g ball. Denoting masses as $m_1 = 100$ g and $m_2 = 200$ g, the initial velocities in the L frame are $(v_{ix})_{1L} = 4$ m/s and $(v_{ix})_{2L} = -3$ m/s.

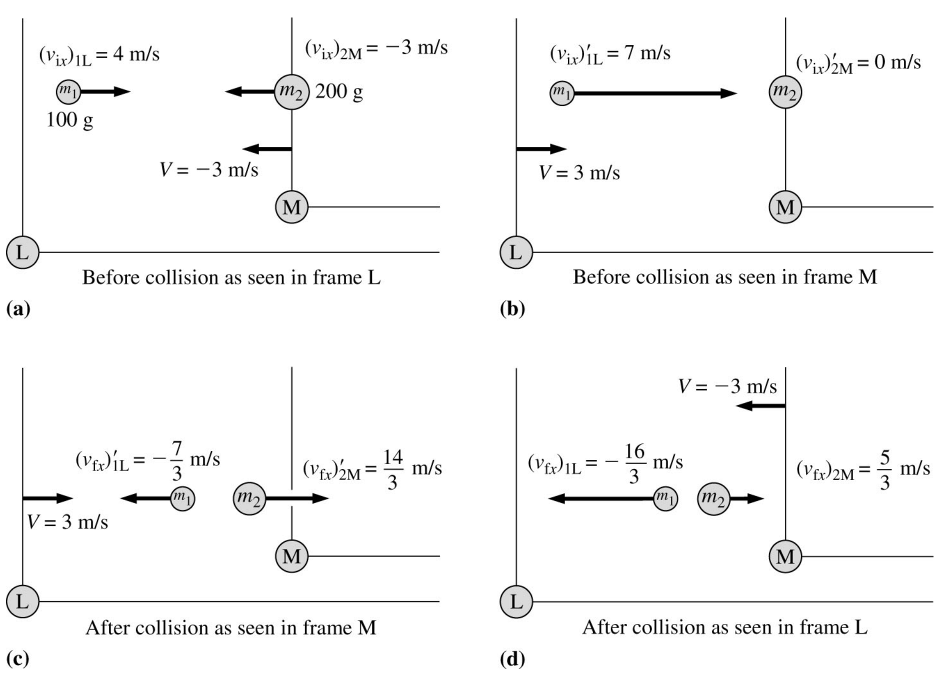

Figure (a) shows the before-collision situation as seen in frame L, and figure (b) shows the before-collision situation as seen in frame M. The after-collision velocities in M are shown in figure (c), and figure (d) indicates velocities in L after they have been transformed to frame L from M.

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Solve: (a) In the L frame, $(v_{ix})_{1L} = 4$ m/s and $(v_{ix})_{2L} = -3$ m/s. M is the reference frame of the 200 g ball, so $(v_x)_{ML} = -3$ m/s. The velocities of the two balls in this frame can be obtained using the Galilean transformation of velocities $(v_x)_{OM} = (v_x)_{OL} + (v_x)_{LM}$. So,

$$(v_{ix})_{1M} = (v_{ix})_{1L} - (v_{ix})_{ML} = 4 \text{ m/s} - (-3 \text{ m/s}) = 7 \text{ m/s} \qquad (v_{ix})_{2M} = (v_{ix})_{2L} - (v_{ix})_{ML} = -3 \text{ m/s} - (-3 \text{ m/s}) = 0 \text{ m/s}$$

Figure (b) shows the "before" situation, where ball 2 is at rest.
Now we can use Equations 10.42 to find the after-collision velocities in frame M:

$$(v_{fx})_{1M} = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_{1M} = \frac{100 \text{ g} - 200 \text{ g}}{100 \text{ g} + 200 \text{ g}}(7 \text{ m/s}) = -\frac{7}{3} \text{ m/s}$$

$$(v_{fx})_{2M} = \frac{2m_1}{m_1 + m_2}(v_{ix})_{1M} = \frac{2(100) \text{ g}}{100 \text{ g} + 200 \text{ g}}(7 \text{ m/s}) = \frac{14}{3} \text{ m/s}$$

Finally, we need to apply the reverse Galilean transformation $(v_x)_{OM} = (v_x)_{OL} + (v_x)_{LM}$ with the same $(v_x)_{LM}$, to transform the after-collision velocities back to the lab frame S:

$$(v_{fx})_{1L} = (v_{fx})_{1M} + (v_x)_{ML} = -\frac{7}{3}\text{m/s} - 3 \text{ m/s} = -5.33 \text{ m/s}$$

$$(v_{fx})_{2L} = (v_{fx})_{2M} + (v_x)_{ML} = \frac{14}{3}\text{m/s} - 3 \text{ m/s} = 1.67 \text{ m/s}$$

Figure (d) shows the "after" situation in the lab frame. The 100 g ball is moving left at 5.3 m/s; the 200 g ball is moving right at 1.7 m/s.

(b) The momentum conservation equation $p_{fx} = p_{ix}$ for a perfectly inelastic collision is

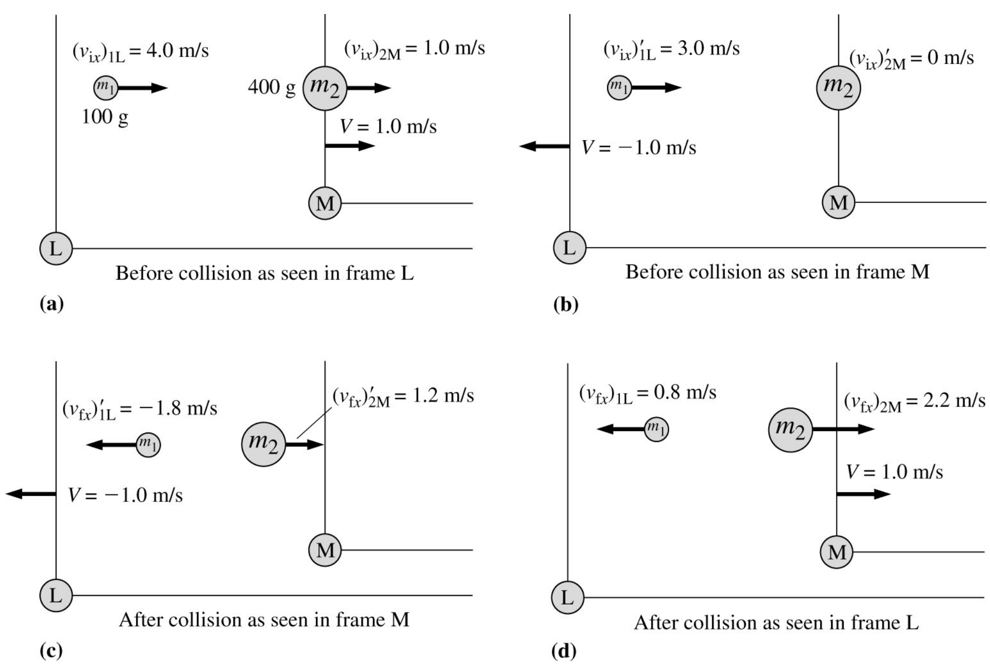
$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$

$$(0.100 \text{ kg} + 0.200 \text{ kg})v_{fx} = (0.100 \text{ kg})(4.0 \text{ m/s}) + (0.200 \text{ kg})(-3.0 \text{ m/s}) \Rightarrow v_{fx} = -0.667 \text{ m/s}$$

Both balls are moving left at 0.67 m/s.

10.57. Model: Model the balls as particles. We will use the Galilean transformation of velocities (Equation 10.43) to analyze the problem of elastic collisions. We will transform velocities from the lab frame L to a frame M in which one ball is at rest. This allows us to apply Equations 10.43 to a perfectly elastic collision in M. After finding the final velocities of the balls in M, we can then transform these velocities back to the lab frame L.

Visualize: Let M be the frame of the 400 g ball. Denoting masses as $m_1 = 100$ g and $m_2 = 400$ g, the initial velocities in the S frame are $(v_{ix})_{1L} = +4.0$ m/s and $(v_{ix})_{2L} = +1.0$ m/s.



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Figures (a) and (b) show the before-collision situations in frames L and M, respectively. The after-collision velocities in M are shown in figure (c). Figure (d) indicates velocities in L after they have been transformed to L from M.

Solve: In frame L, $(v_{ix})_{1L} = 4.0$ m/s and $(v_{ix})_{2L} = 1.0$ m/s. Because M is the reference frame of the 400 g ball, $(v_x)_{ML} = 1.0$ m/s. The velocities of the two balls in this frame can be obtained using the Galilean transformation of velocities $(v_x)_{OM} = (v_x)_{OL} - (v_x)_{ML}$. So,

$$(v_{ix})_{1M} = (v_{ix})_{1L} - (v_x)_{ML} = 4.0 \text{ m/s} - 1.0 \text{ m/s} = 3.0 \text{ m/s} \qquad (v_{ix})_{2M} = (v_{ix})_{2L} - (v_x)_{ML} = 1.0 \text{ m/s} - 1.0 \text{ m/s} = 0 \text{ m/s}$$

Figure (b) shows the "before" situation in frame M where the ball 2 is at rest.

Now we can use Equations 10.43 to find the after-collision velocities in frame M.

$$(v_{fx})_{1M} = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_{1M} = \frac{100 \text{ g} - 400 \text{ g}}{100 \text{ g} + 400 \text{ g}}(3.0 \text{ m/s}) = -1.80 \text{ m/s}$$

$$(v_{fx})_{2M} = \frac{2m_1}{m_1 + m_2}(v_{ix})_{1M} = \frac{2(100 \text{ g})}{100 \text{ g} + 400 \text{ g}}(3.0 \text{ m/s}) = 1.20 \text{ m/s}$$

Finally, we need to apply the reverse Galilean transformation $(v_x)_{OM} = (v_x)_{OL} + (v_x)_{LM}$ with the same $(v_x)_{LM}$, to transform the after-collision velocities back to the lab frame L.

$$(v_{fx})_{1L} = (v_{fx})_{1M} + (v_x)_{ML} = -1.80 \text{ m/s} + 1.0 \text{ m/s} = -0.80 \text{ m/s}$$

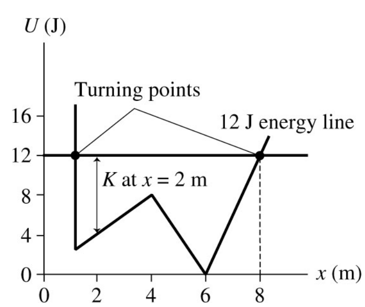
$$(v_{fx})_{2L} = (v_{fx})_{2M} + (v_x)_{ML} = 1.20 \text{ m/s} + 1.0 \text{ m/s} = 2.20 \text{ m/s}$$

Figure (d) shows the "after" situation in frame L. The 100 g ball moves left at 0.80 m/s, the 400 g ball right at 2.2 m/s.

Assess: The magnitudes of the after-collision velocities are similar to the magnitudes of the before-collision velocities.

10.58. Model: Use the model of the conservation of mechanical energy.

Visualize:



Solve: **(a)** The turning points occur where the total energy line crosses the potential energy curve. For $E = 12$ J, this occurs at the points $x = 1$ m and $x = 8$ m.

(b) The equation for kinetic energy $K = E - U$ gives the distance between the potential energy curve and total energy line. $U = 4$ J at $x = 2$ m, so $K = 12 \text{ J} - 4 \text{ J} = 8$ J. The speed corresponding to this kinetic energy is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(8 \text{ J})}{0.5 \text{ kg}}} = 5.7 \text{ m/s}$$

(c) Maximum speed occurs for minimum *U*. This occurs at $x = 6$ m where $U = 0$ J and $K = 12$ J. The speed at this point is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(12 \text{ J})}{0.500 \text{ kg}}} = 6.9 \text{ m/s}$$

(d) The particle leaves $x = 1$ m with $v = 6.3$ m/s. It gradually slows down, reaching $x = 4$ m with a speed of 4.0 m/s. After $x = 4$ m, it speeds up again, achieving a speed of 6.9 m/s as it crosses $x = 6$ m. Then it slows again, coming instantaneously to a halt ($v = 0$ m/s) at the $x = 8$ m turning point. Now it will reverse direction and move back to the left.

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(e) If the particle has $E = 4$ J it cannot cross the 8 J potential energy "mountain" in the center. It can either oscillate back and forth over the range $1.0 \text{ m} \le x \le 2 \text{ m}$ or over the range $5 \text{ m} \le x \le 6.7 \text{ m}$.

10.59. Solve: (a) The equilibrium positions are located at points where $\frac{dU}{dx} = 0.$

$$\frac{dU}{dx} = 0 = 1 + 2\cos(2x) \Rightarrow \cos(2x) = -\frac{1}{2}$$
$$\Rightarrow x = \frac{1}{2}\cos^{-1}\left(-\frac{1}{2}\right)$$

Note that $-\frac{1}{2}$ is in radians and x is in meters. The function $\cos^{-1}\left(-\frac{1}{2}\right)$ may have values $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Thus there are two values of x,

$$x_1 = \frac{\pi}{3} \text{ and } x_2 = \frac{2\pi}{3}$$

within the interval $0 \text{ m} \le x \le \pi \text{ m}$.

(b) A point of stable equilibrium corresponds to a local minimum, while a point of unstable equilibrium corresponds to a local maximum. Compute the concavity of $U(x)$ at the equilibrium positions to determine their stability.

$$\frac{d^2U}{dx^2} = -4\sin(2x)$$

At $x_1 = \frac{\pi}{3}$, $\frac{d^2U}{dx^2}(x_1) = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$. Since $\frac{d^2U}{dx^2}(x_1) < 0$, $x_1 = \frac{\pi}{3}$ is a local maximum, so $x_1 = \frac{\pi}{3}$ is a point of unstable equilibrium.

At $x_2 = \frac{2\pi}{3}$, $\frac{d^2U}{dx^2}(-x_2) = -4\left(-\frac{\sqrt{3}}{2}\right) = +2\sqrt{3}$. Since $\frac{d^2U}{dx^2} > 0$, $x_2 = \frac{2\pi}{3}$ is a local minimum, so $x_2 = \frac{2\pi}{3}$ is a point of stable equilibrium.

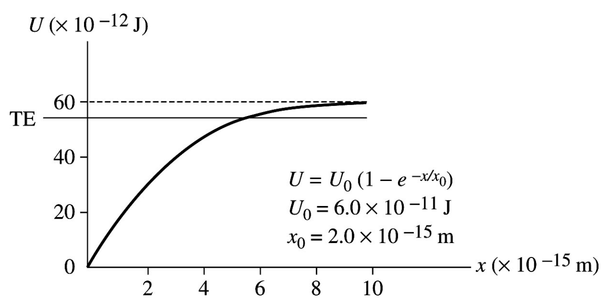
10.60. Model: The potential energy of two nucleons interacting via the strong force is

$$U = U_0[1 - e^{-x/x_0}]$$

where x is the distance between the centers of the two nucleons, $x_0 = 2.0\times10^{-15}$ m, and $U_0 = 6.0\times10^{-11}$ J.

Visualize: Nucleons are protons and neutrons, and they are held together in the nucleus by a force called the *strong force*. This force exists between nucleons at very small separations.

Solve: (a)



(b) For $x = 5.0\times10^{-15}$ m, $U = 55.1\times10^{-12}$ J. This energy is represented by a total energy line.

(c) Due to conservation of total energy, the potential energy when $x = 5.0\times10^{-15}$ J is transformed into kinetic energy until $x = $ twice the radius $= 1.0\times10^{-15}$ m. At this separation, $u = 23.6\times10^{-12}$ J. Thus,

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 + 23.6\times10^{-12} \text{ J} = 55.1\times10^{-12} \text{ J} \Rightarrow v = 1.94\times10^8 \text{ m/s}$$

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Assess: A speed of 1.94×10^{8} m/s is approximately 0.65 c where c is the speed of light. This speed is understandable for the present model.

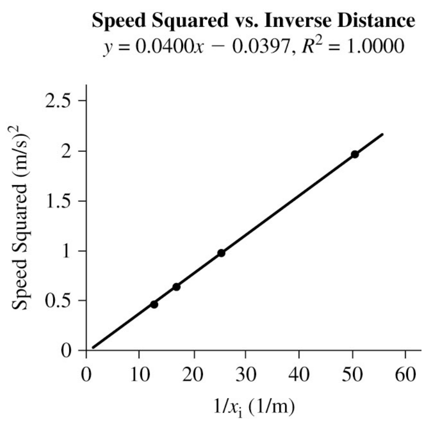
10.61. Model: Assume $U = c/x$. Use conservation of energy.
Visualize: Measure x from the post. The glider is released from position x_1 with initial speed $v_1 = 0$. It is repelled by the post and speeds up to speed v_f at $x_f = 1.0$. m.
Solve: (a) Energy conservation $K_i + U_i = K_f + U_f$ gives

$$\frac{1}{2}mv_f^2 + \frac{c}{x_f} = \frac{1}{2}mv_i^2 + \frac{c}{x_i}$$

Rearranging and using $v_i = 0$,

$$v_f^2 = \left(\frac{2c}{m}\right)\frac{1}{x_i} - \frac{2c}{mx_f}$$

If our hypothesis about the potential energy is correct, then a graph of v_f^2 versus $1/x_i$ should be linear with slope $2c/m$ and y-intercept $-2c/mx_f$. If this turns out to be true we can use the slope to determine c.



The linear fit is seen to be extremely good, so the potential-energy hypothesis is supported.
(b) From the slope of the line,

$$c = \frac{m}{2}\times(\text{slope}) = -\frac{0.050\text{ kg}}{2}(0.04000\text{ m}^3/\text{s}^2) = 0.0010\text{ J m} = 1.0\times10^{-3}\text{ J m}$$

The units of J m follow from $1\text{ kg m}^2/\text{s}^2 = 1\text{ J}$. The same value for c can also be found from the y-intercept.
Assess: The units of J m are also seen to be necessary so that c divided by x, a distance of meters, gives an energy.

10.62. A 2.5 kg ball is thrown upward at a speed of 4.0 m/s from a height of 82 cm above a vertical spring. When the ball comes down it lands on and compresses the spring. If the spring has a spring constant of $k = 600$ N/m, by how much is it compressed?

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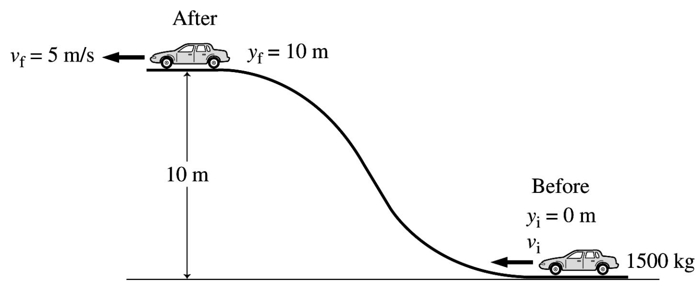
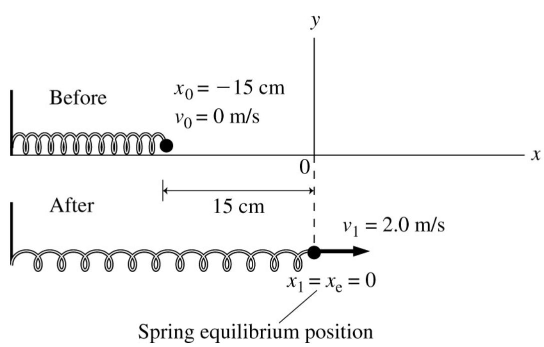
10.63. (a) A 1500 kg auto coasts up a 10.0 m high hill and reaches the top with a speed of 5.0 m/s. What initial speed must the auto have had at the bottom of the hill?
(b)



We place the origin of our coordinate system at the bottom of the hill.
(c) The solution of the equation is $v_i = 14.9\text{ m/s} \approx 15\text{ m/s}$. This is approximately 30 mph and is a reasonable value for the speed at the bottom of the hill.

10.64. (a) A spring gun is compressed 15 cm to launch a 200 g ball on a horizontal, frictionless surface. The ball has a speed of 2.0 m/s as it loses contact with the spring. Find the spring constant of the gun.
(b)



We place the origin of our coordinate system on the free end of the spring in the equilibrium position. Because the surface is frictionless, the mechanical energy for the system $(\text{ball} + \text{spring})$ is conserved.
(c) The conservation of energy equation is

$$K_f + U_{sf} = K_i + U_{si}$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}k(0\text{ m})^2 = \frac{1}{2}m(0\text{ m/s})^2 + \frac{1}{2}k(-0.15\text{ m})^2$$

$$(0.200\text{ kg})(2.0\text{ m/s})^2 = k(-0.15\text{ m})^2$$

$$k = 36\text{ N/m}$$

10.65. (a) A 100 g lump of clay traveling at 3.0 m/s strikes and sticks to a 200 g lump of clay at rest on a frictionless surface. The combined lumps smash into a horizontal spring with $k = 3.0$ N/m. The other end of the spring is firmly anchored to a fixed post on the surface. How far will the spring compress?

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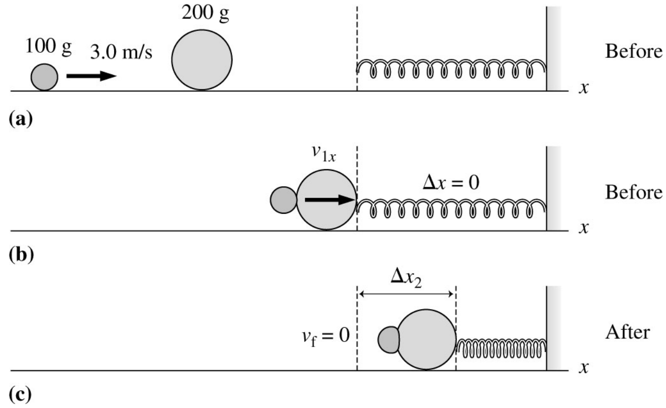
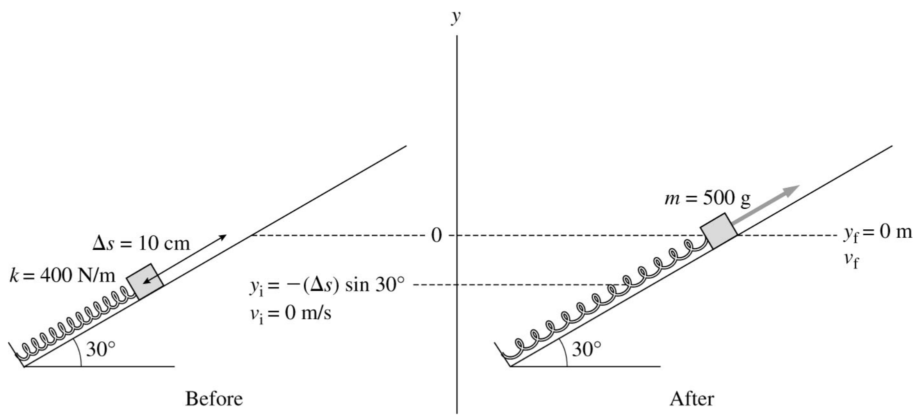
(b)



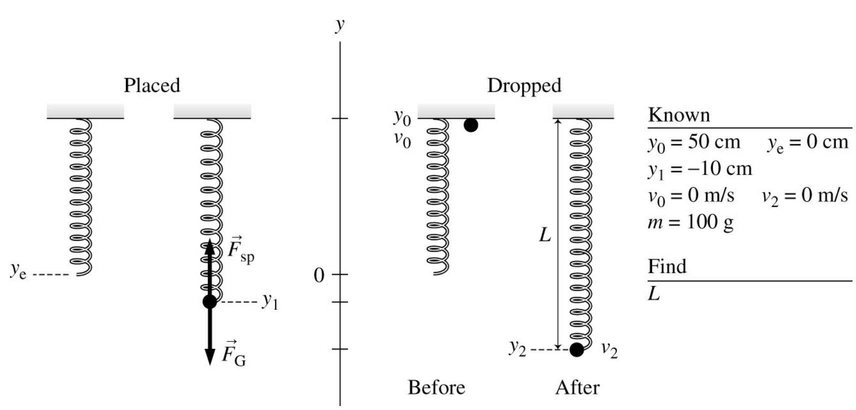
(c) Solving the conservation of momentum equation we get $v_{1x} = 1.0$ m/s. Substituting this value into the conservation of energy equation yields $\Delta x_2 = 32$ cm.

10.66. (a) A spring with spring constant 400 N/m is anchored at the bottom of a frictionless 30° incline. A 500 g block is pressed against the spring, compressing the spring by 10 cm, then released. What is the speed with which the block is launched up the incline?
(b) The origin is placed at the end of the uncompressed spring. This is the point from which the block is launched as the spring expands.



(c) Solving the energy conservation equation, we get $v_f = 2.6$ m/s.

10.67. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, so the mechanical energy $K + U_g + U_s$ is conserved.
Visualize:



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Place the origin of the coordinate system at the end of the unstretched spring, making $y_e = 0$ m.

Solve: The clay is in static equilibrium while resting in the pan. The net force on it is zero. We can start by using this to find the spring constant.

$$F_{sp} = F_G \Rightarrow -k(y_1 - y_e) = -ky_1 = mg \Rightarrow k = -\frac{mg}{y_1} = -\frac{(0.10 \text{ kg})(9.8 \text{ m/s}^2)}{-0.10 \text{ m}} = 9.8 \text{ N/m}$$

Now apply conservation of energy. Initially, the spring is unstretched and the clay ball is at the ceiling. At the end, the spring has maximum stretch and the clay is instantaneously at rest. Thus

$$K_2 + (U_g)_2 + (U_s)_2 = K_0 + (U_g)_0 + (U_s)_0 \Rightarrow \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 = \tfrac{1}{2}mv_0^2 + mgy_0 + 0 \text{ J}$$

Since $v_0 = 0$ m/s and $v_2 = 0$ m/s, this equation becomes

$$mgy_2 + \tfrac{1}{2}ky_2^2 = mgy_0 \Rightarrow y_2^2 + \frac{2mg}{k}y_2 - \frac{2mgy_0}{k} = 0$$

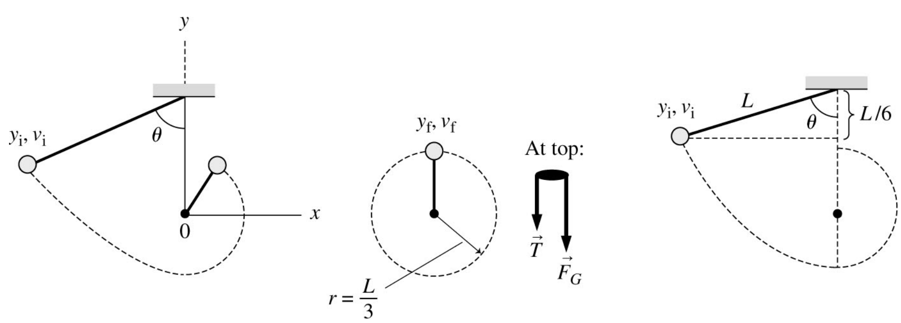
$$y_2^2 + 0.20y_2 - 0.10 = 0$$

The numerical values were found using known values of m, g, k, and y_0. The two solutions to this quadratic equation are $y_2 = 0.231$ m and $y_2 = -0.432$ m. The point we're looking for is below the origin, so we need the negative root. The distance of the pan from the ceiling is

$$L = |y_2| + 50 \text{ cm} = 93 \text{ cm}$$

10.68. Model: This is a two-part problem. In the first part, we will find the critical velocity for the ball to go over the top of the peg without the string going slack. Using the energy conservation equation, we will then obtain the gravitational potential energy that gets transformed into the critical kinetic energy of the ball, thus determining the angle θ.

Visualize:



We place the origin of our coordinate system on the peg. This choice will provide a reference to measure gravitational potential energy. For θ to be minimum, the ball will just go over the top of the peg.

Solve: The two forces in the free-body force diagram provide the centripetal acceleration at the top of the circle. Newton's second law at this point is

$$F_G + T = \frac{mv^2}{r}$$

where T is the tension in the string. The critical speed v_c at which the string goes slack is found when $T \to 0$. In this case,

$$mg = \frac{mv_C^2}{r} \Rightarrow v_C^2 = gr = gL/3$$

The ball should have kinetic energy at least equal to

$$\frac{1}{2}mv_C^2 = \frac{1}{2}mg\left(\frac{L}{3}\right)$$

for the ball to go over the top of the peg. We will now use the conservation of mechanical energy equation to get the minimum angle θ. The equation for the conservation of energy is

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

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Using $v_f = v_c, y_f = \frac{1}{3}L, v_i = 0$, and the above value for v_C^2, we get

$$\frac{1}{2}mg\frac{L}{3} + mg\frac{L}{3} = mgy_i \Rightarrow y_i = \frac{L}{2}$$

That is, the ball is a vertical distance $\frac{1}{2}L$ above the peg's location or a distance of

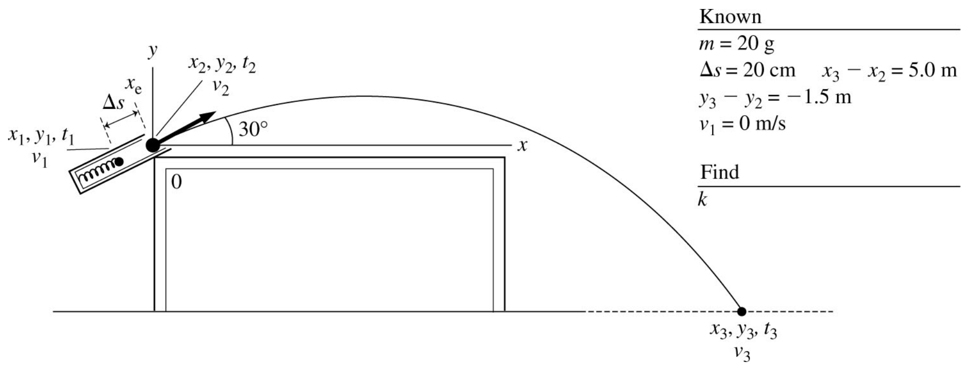
$$\left(\frac{2L}{3} - \frac{L}{2}\right) = \frac{L}{6}$$

below the point of suspension of the pendulum, as shown in the figure on the right. Thus,

$$\cos\theta = \frac{L/6}{L} = \frac{1}{6} \Rightarrow \theta = 80.4°$$

10.69. Model: Assume an ideal spring that obeys Hooke's law. The mechanical energy $K + U_s + U_g$ is conserved during the launch of the ball.

Visualize:



This is a two-part problem. In the first part, we use projectile equations to find the ball's velocity v_2 as it leaves the spring. This will yield the ball's kinetic energy as it leaves the spring.

Solve: Using the equations of kinematics,

$$x_3 = x_2 + v_{2x}(t_3 - t_2) + \frac{1}{2}a_x(t_3 - t_2)^2 \Rightarrow 5.0 \text{ m} = 0 \text{ m} + (v_2\cos30°)(t_3 - 0 \text{ s}) + 0 \text{ m}$$

$$(v_2\cos30°)t_3 = 5.0 \text{ m} \Rightarrow t_3 = (5.0 \text{ m}/v_2\cos30°)$$

$$y_3 = y_2 + v_{2y}(t_3 - t_2) + \frac{1}{2}a_y(t_3 - t_2)^2$$

$$-1.5 \text{ m} = 0 + (v_2\sin30°)(t_3 - 0 \text{ s}) + \frac{1}{2}(9.8 \text{ m/s}^2)(t_3 - 0 \text{ s})^2$$

Substituting the value for t_3, $(-1.5 \text{ m}) = (v_2\sin30°)\left(\frac{5.0 \text{ m}}{v_2\cos30°}\right) - (4.9 \text{ m/s}^2)\left(\frac{5.0 \text{ m}}{v_2\cos30°}\right)^2$

$$\Rightarrow (-1.5 \text{ m}) = +(2.887 \text{ m}) - \frac{163.33}{v_2^2} \Rightarrow v_2 = 6.102 \text{ m/s}$$

The conservation of energy equation $K_2 + U_{s2} + U_{g2} = K_1 + U_{s1} + U_{g1}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(0 \text{ m})^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(\Delta s)^2 + mgy_1$$

Using $y_2 = 0$ m, $v_1 = 0$ m/s, $\Delta s = 0.20$ m, and $y_1 = -(\Delta s)\sin g30°$, we get

$$\frac{1}{2}mv_2^2 + 0 \text{ J} + 0 \text{ J} = 0 \text{ J} + \frac{1}{2}k(\Delta s)^2 - mg(\Delta s)\sin30° \quad (\Delta s)^2 k = mv_2^2 + 2mg(\Delta s)\sin30°$$

$$(0.20 \text{ m})^2 k = (0.020 \text{ kg})(6.102 \text{ m/s})^2 + 2(0.020 \text{ kg})(9.8 \text{ m/s}^2)(0.20)(0.5) \Rightarrow k = 19.6 \text{ N/m}$$

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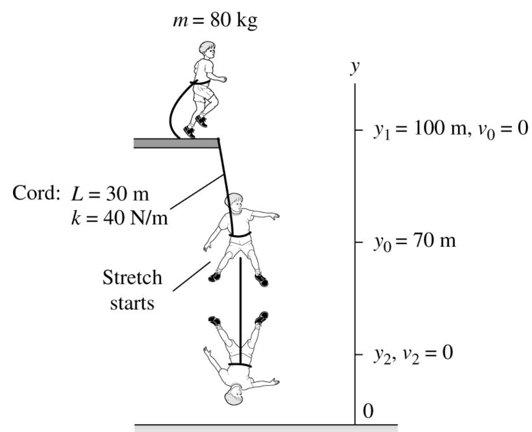
The final answer rounds to 20 N/m.

Assess: Note that $y_1 = -(\Delta s)\sin 30°$ is with a minus sign and hence the gravitational potential energy mgy_1 is $-mg(\Delta s)\sin 30°$.

10.70. Model: Choose yourself + spring + earth as the system. There are no forces from outside this system, so it is an isolated system. The interaction forces within the system are the spring force of the bungee cord and the gravitational force. These are both conservative forces, so mechanical energy is conserved.

Visualize:



We can equate the system's initial energy, as you step off the bridge, to its final energy when you reach the lowest point. We do *not* need to compute your speed at the point where the cord starts to stretch. We do, however, need to note that the end of the *unstretched* cord is at $y_0 = y_1 - 30\text{ m} = 70\text{ m}$, so $U_{2s} = \frac{1}{2}k(y_2 - y_0)^2$. Also note that $U_{1s} = 0$, since the cord is not stretched. The energy conservation equation is

$$K_2 + U_{2g} + U_{2s} = K_1 + U_{1g} + U_{1s} \Rightarrow 0\text{ J} + mgy_2 + \frac{1}{2}k(y_2 - y_0)^2 = 0\text{ J} + mgy_1 + 0\text{ J}$$

Multiply out the square of the binomial and rearrange:

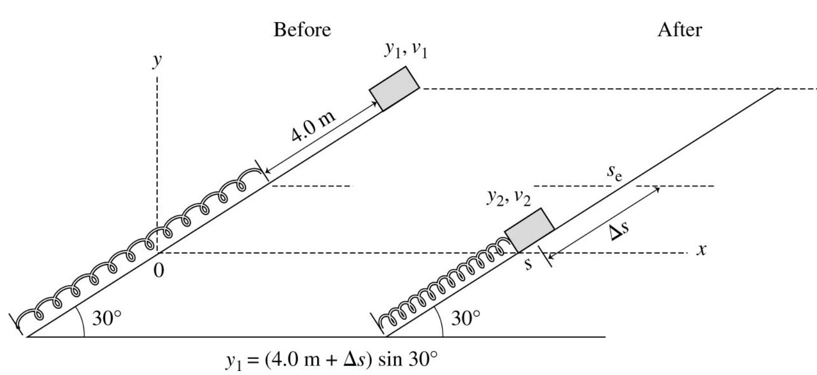
$$mgy_2 + \frac{1}{2}ky_2^2 - ky_0y_2 + \frac{1}{2}ky_0^2 = mgy_1$$

$$\Rightarrow y_2^2 + \left(\frac{2mg}{k} - 2y_0\right)y_2 + \left(y_0^2 - \frac{2mgy_1}{k}\right) = y_2^2 - 100.8y_2 + 980 = 0$$

This is a quadratic equation with roots 89.9 m and 10.9 m. The first is not physically meaningful because it is a height above the point where the cord started to stretch. So we find that your distance from the water when the bungee cord stops stretching is 10.9 m which is 11 m to two sig figs.

10.71. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, hence the mechanical energy $K + U_g + U_s$ is conserved.

Visualize:



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We have chosen to place the origin of the coordinate system at the point of maximum compression. We will use lengths along the ramp with the variable *s* rather than *x*.

Solve: (a) The conservation of energy equation $K_2 + U_{g2} + U_{s2} = K_1 + U_{g1} + U_{s1}$ is

$$\frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_1^2 + mgy_1 + k(0\text{ m})^2$$

$$\frac{1}{2}m(0\text{ m/s})^2 + mg(0\text{ m}) + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}m(0\text{ m/s})^2 + mg(4.0\text{ m} + \Delta s)\sin30° + 0\text{ J}$$

$$\frac{1}{2}(250\text{ N/m})(\Delta s)^2 = (10\text{ kg})(9.8\text{ m/s}^2)(4.0\text{ m} + \Delta s)\left(\frac{1}{2}\right)$$

This gives the quadratic equation:

$$(125\text{ N/m})(\Delta s)^2 - (49\text{ kg}\cdot\text{m/s}^2)\Delta s - 196\text{ kg}\cdot\text{m}^2/\text{s}^2 = 0$$

$$\Rightarrow \Delta s = 1.46\text{ m and } -1.07\text{ m (unphysical)}$$

The maximum compression is 1.46 m which rounds to 1.5 m.

(b) We will now apply the conservation of mechanical energy to a point where the vertical position is *y* and the block's velocity is *v*. We place the origin of our coordinate system on the free end of the spring when the spring is neither compressed nor stretched.

$$\frac{1}{2}mv^2 + mgy + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}k(0\text{ m})^2$$

$$\frac{1}{2}mv^2 + mg(-\Delta s\ \sin30°) + \frac{1}{2}k(\Delta s)^2 = 0\text{ J} + mg(4.0\text{ m}\ \sin30°) + 0\text{ J}$$

$$\frac{1}{2}k(\Delta s)^2 - (mg\ \sin30°)\Delta s + \frac{1}{2}mv^2 - mg\sin30°(4.0\text{ m}) = 0$$

To find the compression where *v* is maximum, take the derivative of this equation with respect to Δs:

$$\frac{1}{2}k\ 2(\Delta s) - (mg\ \sin30°) + \frac{1}{2}m\ 2v\frac{dv}{d\Delta s} - 0 = 0$$

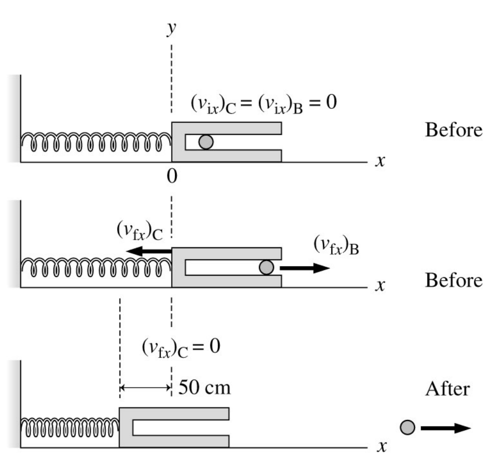
Since $\frac{dv}{d\Delta s} = 0$ at the maximum, we have

$$\Delta s = (mg\ \sin30°)/k = (10\text{ kg})(9.8\text{ m/s}^2)(0.5)/(250\text{ N/m}) = 19.6\text{ cm}$$

This is 20 cm to two sig figs.

10.72. Model: Assume an ideal, massless spring that obeys Hooke's law. Let us also assume that the cannon (C) fires balls (B) horizontally and that the spring is directly behind the cannon to absorb all motion.

Visualize:



The before-and-after pictorial representation is shown, with the origin of the coordinate system located at the spring's free end when the spring is neither compressed nor stretched. This free end of the spring is just behind the cannon.

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Solve: The momentum conservation equation $p_{fx} = p_{ix}$ is

$$m_B(v_{fx})_B + m_C(v_{fx})_C = m_B(v_{ix})_B + m_C(v_{ix})_C$$

Since the initial momentum is zero,

$$(v_{fx})_B = -\frac{m_C}{m_B}(v_{fx})_C = -\left(\frac{200\text{ kg}}{10\text{ kg}}\right)(v_{fx})_C = -20(v_{fx})_C$$

The mechanical energy conservation equation for the cannon + spring $K_f + U_{sf} = K_i + U_{si}$ is

$$\frac{1}{2}m(v_f)_C^2 + \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m(v_i)_C^2 + 0\text{ J} \Rightarrow 0\text{ J} + \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m(v_{fx})_C^2$$

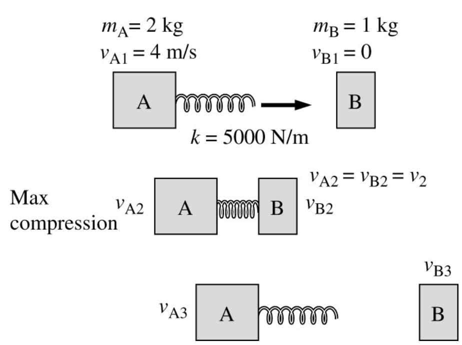
$$\Rightarrow (v_{fx})_C = \pm\sqrt{\frac{k}{m}}\Delta x = \pm\sqrt{\frac{(20{,}000\text{ N/m})}{200\text{ kg}}}(0.50\text{ m}) = \pm 5.0\text{ m/s}$$

To make this velocity physically correct, we retain the minus sign with $(v_{fx})_C$. Substituting into the momentum conservation equation yields:

$$(v_{fx})_B = -20(-5.0\text{ m/s}) = 100\text{ m/s}$$

10.73. Model: This is a collision between two objects, and momentum is conserved in the collision. In addition, because the interaction force is a spring force and the surface is frictionless, energy is also conserved.

Visualize:



Let part 1 refer to the time before the collision starts, part 2 refer to the instant when the spring is at maximum compression, and part 3 refer to the time after the collision. Notice that just for an instant, when the spring is at maximum compression, the two blocks are moving side by side and have *equal* velocities: $v_{A2} = v_{B2} = v_2$. This is an important observation.

Solve: First relate part 1 to part 2. Conservation of energy is

$$\frac{1}{2}m_A v_{A1}^2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 + \frac{1}{2}k(\Delta x_{max})^2 = \frac{1}{2}(m_A + m_B)v_2^2 + \frac{1}{2}k(\Delta x_{max})^2$$

where Δx_{max} is the spring's compression. U_g is not in the equation because there are no elevation changes. Also note that K_2 is the sum of the kinetic energies of *all* moving objects. Both v_2 and Δx_{max} are unknowns. Now add the conservation of momentum:

$$m_A v_{A1} = m_A v_{A2} + m_B v_{B2} = (m_A + m_B)v_2 \Rightarrow v_2 = \frac{m_A v_{A1}}{m_A + m_B} = 2.667\text{ m/s}$$

Substitute this result for v_2 into the energy equation to find:

$$\frac{1}{2}k(\Delta x_{max})^2 = \frac{1}{2}m_A v_{A1}^2 - \frac{1}{2}(m_A + m_B)v_2^2$$

$$\Rightarrow \Delta x_{max} = \sqrt{\frac{m_A v_{A1}^2 - (m_A + m_B)v_2^2}{k}} = 0.046\text{ m} = 4.6\text{ cm}$$

Notice how *both* conservation laws were needed to solve this problem.

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(b) Again, both energy and momentum are conserved between "before" and "after." Energy is

$$\frac{1}{2}m_\text{A}v_\text{A1}^2 = \frac{1}{2}m_\text{A}v_\text{A3}^2 + \frac{1}{2}m_\text{B}v_\text{B3}^2 \Rightarrow 16 = v_\text{A3}^2 + \frac{1}{2}v_\text{B3}^2$$

The spring is no longer compressed, so the energies are purely kinetic. Momentum is

$$m_\text{A}v_\text{A1} = m_\text{A}v_\text{A3} + m_\text{B}v_\text{B3} \Rightarrow 8 = 2v_\text{A3} + v_\text{B3}$$

We have two equations in two unknowns. From the momentum equation, we can write $v_\text{B3} = 2(4 - v_\text{A3})$ and use this in the energy equation to obtain:

$$16 = v_\text{A3}^2 + \frac{1}{2}\cdot 4(4 - v_\text{A3}^2)^2 = 3v_\text{A3}^2 - 16v_\text{A3} + 32 \Rightarrow 3v_\text{A3}^2 - 16v_\text{A3} + 16 = 0$$

This is a quadratic equation for v_A3 with roots $v_\text{A3} = (4\text{ m/s}, 1.33\text{ m/s})$. Using $v_\text{B3} = 2(4 - v_\text{A3})$, these two roots give $v_\text{B3} = (0\text{ m/s}, 5.333\text{ m/s})$. The first pair of roots corresponds to a "collision" in which A misses B, so each keeps its initial speed. That's not the situation here. We want the second pair of roots, from which we learn that the blocks' speeds after the collision are $v_\text{A3} = 1.33\text{ m/s}$ and $v_\text{B3} = 5.3\text{ m/s}$.

10.74. Model: Mechanical energy and momentum are conserved during the expansion of the spring.
Visualize: Please refer to Figure CP10.74.
Solve: Example 10.8 is a very similar problem, except that the objects are initially at rest. We can use the solution from Example 10.8 for this problem in a reference frame S′ in which the two carts are initially at rest, then transform the answer to the frame S in which the carts are initially moving.
Thus in the S′ frame,

$$(v_\text{fx})_2' = \sqrt{\frac{k(\Delta x_\text{i})^2}{m_2(1 + m_2/m_1)}}$$

$$(v_\text{fx})_1' = -\frac{m_2}{m_1}(v_\text{fx})_2'$$

Let the 100 g cart be Cart 1 and the 300 g cart be Cart 2. With $k = 120\text{ N/m}$ and $\Delta x_\text{i} = 4.0\text{ cm}$,

$$(v_\text{fx})_2' = 0.40\text{ m/s},\ (v_\text{fx})_1' = -1.2\text{ m/s}$$

An object at rest in the S′ frame is traveling to the right at 1.0 m/s in the S frame. The equation of transformation is therefore

$$v_x = v_x' + 1.0\text{ m/s}$$

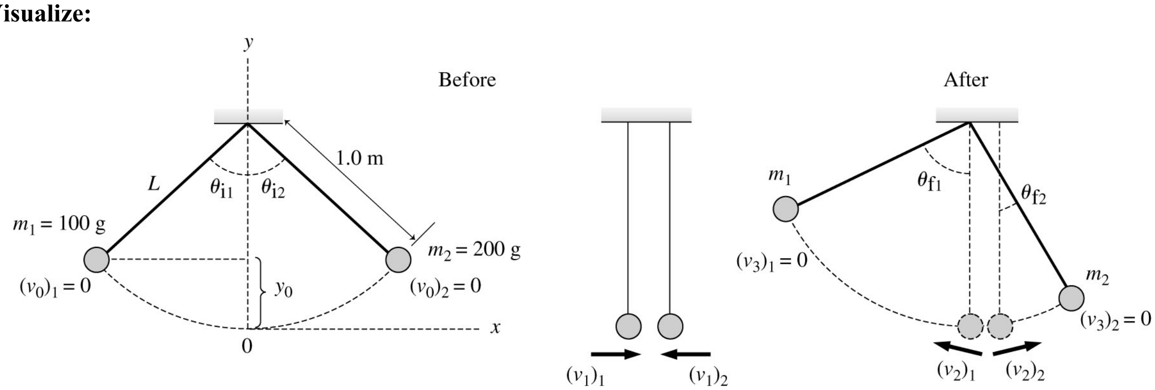
In the S frame, the velocities of the carts are

$$(v_\text{fx})_1 = -1.2\text{ m/s} + 1.0\text{ m/s} = -0.2\text{ m/s}$$

$$(v_\text{fx})_2 = 0.40\text{ m/s} + 1.0\text{ m/s} = 1.4\text{ m/s}$$

Assess: Cart 1 is moving slowly to the left while the heavier Cart 2 is moving quickly to the right.

10.75. Model: Model the balls as particles, and assume a perfectly elastic collision. After the collision is over, the balls swing out as pendulums. The sum of the kinetic energy and gravitational potential energy does not change as the balls swing out.
Visualize:



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In the pictorial representation we have identified before-and-after quantities for both the collision and the pendulum swing. We have chosen to place the origin of the coordinate system at a point where the two balls at rest barely touch each other.

Solve: As the ball with mass m_1, whose string makes an angle θ_{i1} with the vertical, swings through its equilibrium position, it lowers its gravitational energy from $m_1 g y_0 = m_1 g(L - L\cos\theta_{i1})$ to zero. This change in potential energy transforms into a change in kinetic energy. That is,

$$m_1 g(L - L\cos\theta_{i1}) = \frac{1}{2} m_1 (v_1)_1^2 \Rightarrow (v_1)_1 = \sqrt{2gL(1-\cos\theta_{i1})}$$

Similarly, $(v_1)_2 = \sqrt{2gL(1-\cos\theta_{i2})}$. Using $\theta_{i1} = \theta_{i2} = 45°$, we get $(v_1)_1 = 2.396 \text{ m/s} = (v_1)_2$. Both balls are moving at the point where they have an elastic collision. Since our analysis of elastic collisions was for a situation in which ball 2 is initially at rest, we need to use the Galilean transformation to change to a frame S′ in which ball 2 is at rest. Ball 2 is at rest in a frame that moves with ball 2, so choose S′ to have $V = -2.396$ m/s, with the minus sign because this frame (like ball 2) is moving to the left. In this frame, ball 1 has velocity $(v_1')_1 = (v_1)_1 - V = 2.396 \text{ m/s} + 2.396 \text{ m/s} = 4.792 \text{ m/s}$ and ball 2 is at rest. The elastic collision causes the balls to move with velocities

$$(v_2')_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_1')_1 = -\frac{1}{3}(4.792 \text{ m/s}) = -1.597 \text{ m/s}$$

$$(v_2')_2 = \frac{2m_1}{m_1 + m_2}(v_1')_1 = \frac{2}{3}(4.792 \text{ m/s}) = 3.194 \text{ m/s}$$

We can now use $v = v' + V$ to transform these back into the laboratory frame:

$$(v_2)_1 = -1.597 \text{ m/s} - 2.396 \text{ m/s} = -3.99 \text{ m/s}$$
$$(v_2)_2 = 3.195 \text{ m/s} - 2.396 \text{ m/s} = 0.799 \text{ m/s}$$

Having determined the velocities of the two balls after collision, we will once again use the conservation equation $K_f + U_{gf} = K_i + U_{gi}$ for each ball to solve for the θ_{f1} and θ_{f2}.

$$\frac{1}{2} m_1 (v_3)_1^2 + m_1 g L(1-\cos\theta_{f1}) = \frac{1}{2} m_1 (v_2)_1^2 + 0 \text{ J}$$

Using $(v_3)_1 = 0$, this equation simplifies to

$$gL(1-\cos\theta_{f1}) = \frac{1}{2}(-3.99 \text{ m/s})^2 \Rightarrow \cos\theta_{f1} = 1 - \frac{1}{2gL}(-3.99 \text{ m/s})^2 \Rightarrow \theta_{f1} = 79.3°$$

The 100 g ball rebounds to 79º. Similarly, for the other ball:

$$\frac{1}{2} m_2 (v_3)_2^2 + m_2 g L(1-\cos\theta_{f2}) = \frac{1}{2} m_2 (v_2)_2^2 + 0 \text{ J}$$

Using $(v_3)_2 = 0$, this equation becomes

$$\cos\theta_{f2} = 1 - \left(\frac{1}{2gL}\right)(0.799)^2 \Rightarrow \theta_{f2} = 14.7°$$

The 200 g ball rebounds to 14.7°.

10.76. Model: Model the sled as a particle. Because there is no friction, the sum of the kinetic and gravitational potential energy is conserved during motion.

Visualize:

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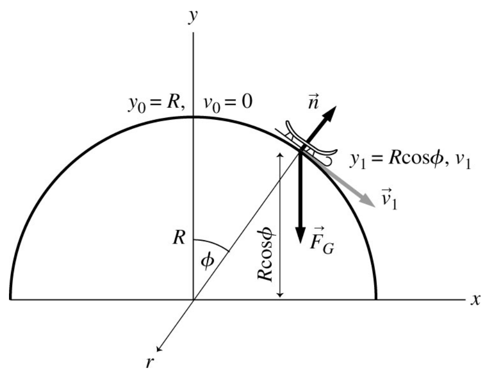

Place the origin of the coordinate system at the center of the hemisphere. Then $y_0 = R$ and, from geometry, $y_1 = R\cos\phi.$

Solve: The energy conservation equation $K_1 + U_1 = K_0 + U_0$ is

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0 \Rightarrow \frac{1}{2}mv_1^2 + mgR\cos\phi = mgR \Rightarrow v_1 = \sqrt{2gR(1-\cos\phi)}$$

(b) If the sled is on the hill, it is moving in a circle and the *r*-component of $\vec{F}_{\text{net}}$ has to point to the center with magnitude $F_{\text{net}} = mv^2/R.$ Eventually the speed gets so large that there is not enough force to keep it in a circular trajectory, and that is the point where it flies off the hill. Consider the sled at angle $\phi.$ Establish an *r*-axis pointing toward the center of the circle, as we usually do for circular motion problems. Newton's second law along this axis requires:

$$(F_{\text{net}})_r = F_G\cos\phi - n = mg\cos\phi - n = ma_r = \frac{mv^2}{R}$$

$$\Rightarrow n = mg\cos\phi - \frac{mv^2}{R} = m\left(g\cos\phi - \frac{v^2}{R}\right)$$

The normal force decreases as *v* increases. But *n* can't be negative, so the fastest speed at which the sled stays on the hill is the speed v_{max} that makes $n \to 0.$ We can see that $v_{\text{max}} = \sqrt{gR\cos\phi}.$

(c) We now know the sled's speed at angle $\phi,$ and we know the maximum speed it can have while remaining on the hill. The angle at which *v* reaches v_{max} is the angle ϕ_{max} at which the sled will fly off the hill. Combining the two expressions for v_1 and v_{max} gives:

$$\sqrt{2gR(1-\cos\phi)} = \sqrt{gR\cos\phi} \Rightarrow 2R(1-\cos\phi_{\text{max}}) = R\cos\phi_{\text{max}}$$

$$\Rightarrow \cos\phi_{\text{max}} = \frac{2}{3} \Rightarrow \phi_{\text{max}} = \cos^{-1}\left(\frac{2}{3}\right) = 48°$$

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