Ch 13 Newton's Theory of Gravity Notes and Ideas

$$\boldsymbol{F_G = G\frac{m_1 m_2}{r^2}} \qquad \textbf{G = 6.67 x 10}^{\textbf{-11}}\ \textbf{N*m}^{\textbf{2}}\textbf{/kg}^{\textbf{2}}$$

To derive 'g'; $a = \frac{F_{Net}}{M_{Total}}$; F_{net} is F_g or m*g so $a = \frac{\cancel{M_{Total}} * g}{\cancel{M_{Total}}}$

so g = a for an object (m_1) at a certain position,

$\cancel{m_1} g = G\frac{\cancel{m_1} M_{planet}}{r^2}$ → $\left[g = G\frac{M_{planet}}{r^2}\right]$

only works if r > planets radius (surface)

g = 9.83 m/s^2 w/o spinning Earth, but 9.81 m/s^2 since Earth ***is*** spinning $\left(\frac{2\pi r}{day}\right)$

- There are several ways to think about U_g from a planet. Basically, at ∞, there is <u>no</u> force, no interaction, so U_g = 0 J. To push an object away from the Earth takes work (energy), but since force is pushing away it is **<u>negative</u>** work…
- Also, since U_g = 0 @ ∞, but U_g increases as distance increases, U_g must be negative for points less than ∞.

Either way $W = F * d$ ➔ $G\frac{m_1 m_2}{r^{\cancel{2}}} * \cancel{r}$ ➔ $\left[U_g = -G\frac{M_1 M_2}{r}\right]$ see page 362.

To find escape velocity (page 363 Example 13.2); we need the change in potential energy to be equal to the object initial kinetic energy.

$E_i = E_f$ $\quad K_1 + U_1 = K_2 + U_2$ $\qquad 0\ J + 0\ J = \frac{1}{2}\cancel{m_1} v^2 - G\frac{\cancel{m_1} M_{Planet}}{R}$

$\frac{1}{2}v^2 = G\frac{M_{Planet}}{R}$ ➔ $v^2 = \frac{2GM_{Planet}}{R}$ ➔ $v_{esc} = \sqrt{\frac{2GM_{Planet}}{R}}$

To find an orbital speed @ a certain position we need to realize that the gravitational force will cause a centripetal acceleration.

$F_{Net} = M_{Satellite} * a$ $\qquad a = \frac{v^2}{r}$ → For centripetal acceleration

↓

$F_{Gravitational}$ so $F_g = G\frac{\cancel{M_{Satellite}}M_{Earth}}{r^2} = \cancel{M_{Satellite}} * \frac{v_{Orbital}{}^2}{\cancel{r}}$

$\frac{GM_{Earth}}{r} = v_{Orbital}{}^2$ ➔ $v_{Orbital} = \sqrt{\frac{GM_{Earth}}{r}}$

For Keplers' 3rd Law

$V = \frac{x}{t}$ → $V = \frac{2\pi r}{T}$ ➔ $\left(\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}\right)^2$ → $\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$ rearrange to solve for T^2 $\qquad T^2 = \frac{4\pi^2 r^3}{GM}$ ➔ Kepler's 3rd Law

Finally …

The kinetic energy of a satellite = $\frac{1}{2} m_{Satellite} v^2$,

but $v_{Orbital}{}^2 = \frac{GM_{Earth}}{r}$ so … $K = \frac{1}{2} m_{Satellite}\left(G\frac{GM_{Earth}}{r}\right)$

$K = G\frac{M_{Satellite}M_{Earth}}{2r}$ Since $U_g = -G\frac{M_{Satelitte}M_{Earth}}{r}$ the K is always ½ of the U_g for a satellite in stable orbit.

Therefore the total mechanical energy will ***ALWAYS*** be negative and ***ALWAYS*** be ½ of U_g.

$$\mathbf{E_{Mech} = K + U_g = \tfrac{1}{2} U_g}$$