8

DYNAMICS II: MOTION IN A PLANE

Conceptual Questions

8.1. For an object in uniform circular motion the speed is constant (that's what uniform means), the angular velocity is constant, and the magnitude of the net force is constant. The velocity and centripetal acceleration are both vectors whose magnitude is constant but whose direction changes.

8.2. The free-body diagram (a) is correct. The forces acting on the car at the bottom of the hill are the downward gravitational force and an upward normal force. The car can be considered to be in circular motion about a point above the bottom center of the valley, which requires a net force toward the center of the circle. In this case, the circle center is above the car, so the normal force is greater than the gravitational force.

8.3. $T_c > T_a = T_d > T_b$. Use $T = \frac{mv^2}{r}$. For (a), $T_a = \frac{mv^2}{r}$. For (b), $T_b = \frac{mv^2}{2r} = \frac{1}{2}T_a$. For (c), . $T_c = \frac{(2m)v^2}{r} = 2T_a$..

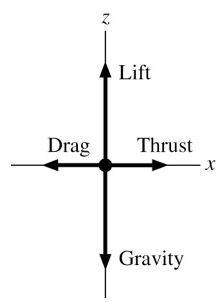
For (d), $T_d = \frac{(2m)v^2}{2r} = T_a$.

8.4. The tension in the vine at the lowest part of Tarzan's swing is greater than the gravitational force on Tarzan. If Tarzan is at rest on the vine, just hanging, the tension in the vine is equal to the gravitational force on Tarzan. But when Tarzan is swinging he is in circular motion, with the center of the circle at the top end of the vine, and the vine must provide the additional centripetal force necessary to move him in a circle.

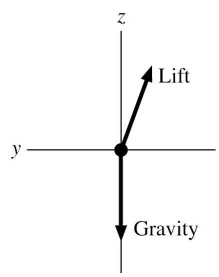
8.5. **(a)** The difference in the tension between A and B is due to the centripetal force $(F_r)_{net} = \frac{mv^2}{r}$. Since the velocity v is the same for both, the greater radius for B means that the tension in case A is greater than for case B.
(b) In this case, we use $(F_r)_{net} = m\omega^2 r$. The angular velocity ω is the same for both A and B, so the larger radius for B means that the tension is case A is less than the tension for case B.

8.6. Neither Sally nor Raymond is completely correct. Both have partially correct descriptions, but are missing key points. In order to speed up, there must be a nonzero acceleration parallel to the track. In order to move in a circle, there must be a nonzero centripetal acceleration. Since both of these are required, the net force points somewhere between the forward direction (parallel to the track) and the center of the circle.

8.7. **(a)** The plane is in dynamic equilibrium, so the net force on the plane is zero.
(b) The vertical forces cancel, and so do the horizontal forces, so the net force is zero. The plane is traveling in the positive x direction.



(c) As seen from behind, with the velocity and positive x direction into the page.

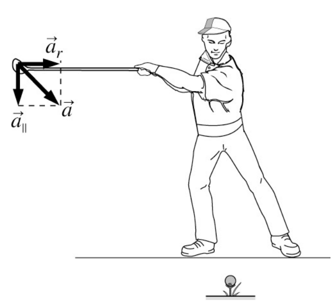


(d) The net force must be toward the center of the circle during a turn at constant speed and altitude. Note that the radial component of the lift force provides the centripetal force while the vertical component balances the gravitational force. The velocity is into the page.

8.8. Yes, the bug is weightless because it, like the projectile it is riding in, is in free fall around the planet.

8.9. When the gravitational force on the ball is greater than the required centripetal force $\frac{mv^2}{r}$, the ball is no longer in circular motion. As the figure shows, at the top of the circle the net force on the ball is $(F_r)_{\text{net}} = F_G + T = \frac{mv^2}{r}$. When the string goes slack, $T = 0$, leaving $F_G = \frac{mv^2}{r}$. If the velocity is not high enough to make this equality true, the equation above becomes an inequality, $F_G > \frac{mv^2}{r}$, and the ball begins to fall downward since the net force downward is greater than the centripetal force required for circular motion.

8.10.

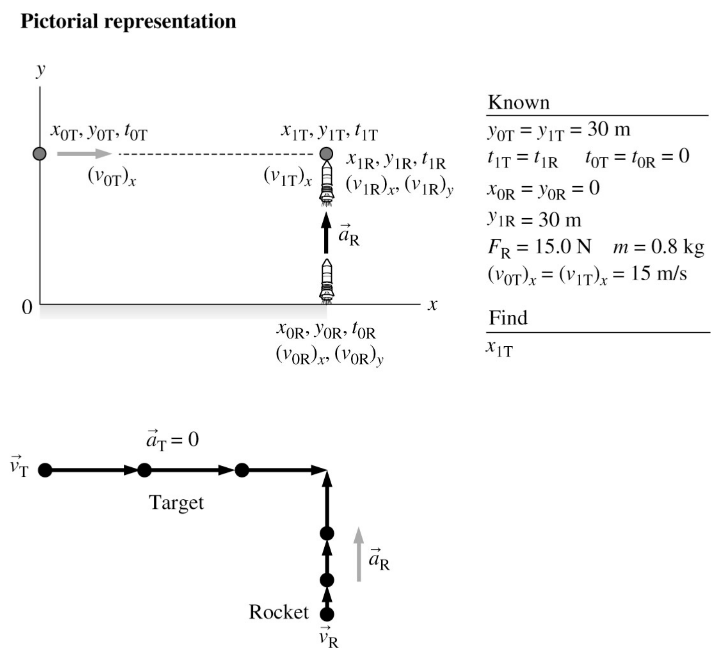


The golfer is swinging the club in circular motion. The club is speeding up as it swings. This motion requires a linear acceleration in the direction of motion of the club to speed it up and a centripetal acceleration to maintain circular motion. The vector sum yields a total acceleration pointing approximately toward the golfer's feet (c), as shown in the figure.

Exercises and Problems

Section 8.1 Dynamics in Two Dimensions

8.1. Model: The model rocket and the target will be treated as particles. The kinematics equations in two dimensions apply.
Visualize:



Solve: For the rocket, Newton's second law along the y-direction is

$$(F_{net})_y = F_R - mg = ma_R$$
$$\Rightarrow a_R = \frac{1}{m}(F_R - mg) = \frac{1}{0.8\ \text{kg}}[15\ \text{N} - (0.8\ \text{kg})(9.8\ \text{m/s}^2)] = 8.95\ \text{m/s}^2$$

Using the kinematic equation $y_{1R} = y_{0R} + (v_{0R})_y(t_{1R} - t_{0R}) + \frac{1}{2}a_R(t_{1R} - t_{0R})^2$,

$$30\ \text{m} = 0\ \text{m} + 0\ \text{m} + \tfrac{1}{2}(8.95\ \text{m/s}^2)(t_{1R} - 0\ \text{s})^2 \Rightarrow t_{1R} = 2.589\ \text{s}$$

For the target (noting $t_{1T} = t_{1R}$),

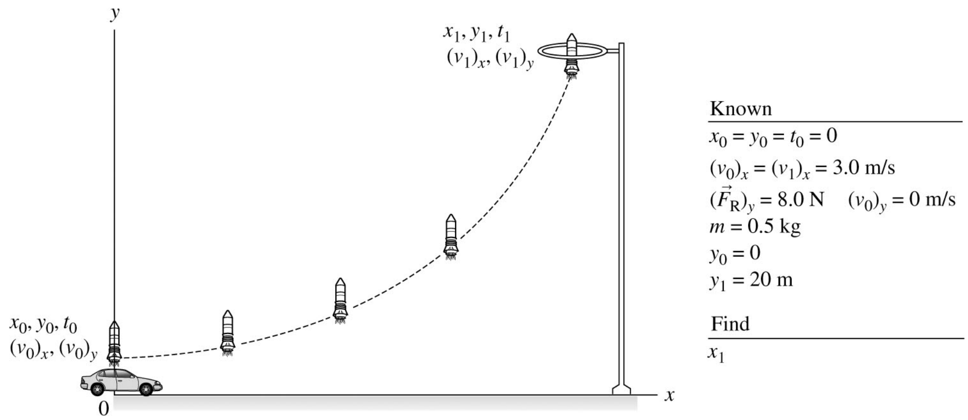
$$x_{1T} = x_{0T} + (v_{0T})_x(t_{1T} - t_{0T}) + \tfrac{1}{2}a_T(t_{1T} - t_{0T})^2 = 0\ \text{m} + (15\ \text{m/s})(2.589\ \text{s} - 0\ \text{s}) + 0\ \text{m} = 39\ \text{m}$$

You should launch when the target is 39 m away.
Assess: The rocket is to be fired when the target is at x_{0T}. For a net acceleration of approximately $9\ \text{m/s}^2$ in the vertical direction and a time of 2.6 s to cover a vertical distance of 30 m, a horizontal distance of 39 m is reasonable.

8.2. Model: The model rocket will be treated as a particle. Kinematic equations in two dimensions apply. Air resistance is neglected.

Visualize:

Pictorial representation



The horizontal velocity of the rocket is equal to the speed of the car, which is 3.0 m/s.

Solve: For the rocket, Newton's second law along the y-direction is:

$$(F_{\text{net}})_y = F_R - mg = ma_R \Rightarrow a_y = \frac{1}{0.5\text{ kg}}[(8.0\text{ N}) - (0.5\text{ kg})(9.8\text{ m/s}^2)] = 6.2\text{ m/s}^2$$

Thus using $y_1 = y_0 + (v_0)_y(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$(20\text{ m}) = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(6.2\text{ m/s}^2)(t_{1R} - 0\text{ s})^2 \Rightarrow (20\text{ m}) = (3.1\text{ m/s}^2)t_1^2 \Rightarrow t_1 = 2.54\text{ s}$$

Since t_1 is also the time for the rocket to move horizontally up to the hoop,

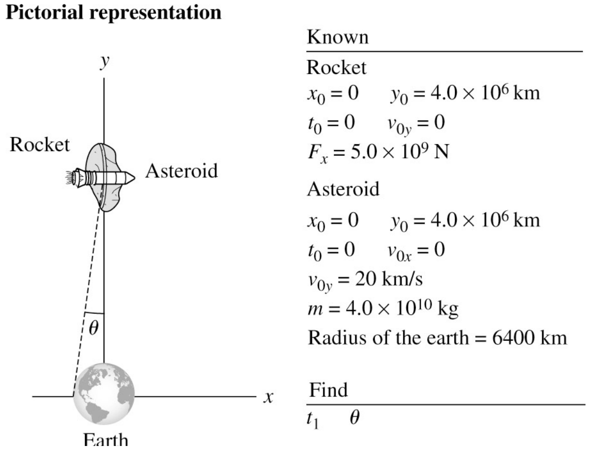
$$x_1 = x_0 + (v_0)_x(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 = 0\text{ m} + (3.0\text{ m/s})(2.54\text{ s} - 0\text{ s}) + 0\text{ m} = 7.6\text{ m}$$

Assess: In view of the rocket's horizontal speed of 3.0 m/s and its vertical thrust of 8.0 N, the above-obtained value for the horizontal distance is reasonable.

8.3. Model: The asteroid and the giant rocket will be treated as particles undergoing motion according to the constant-acceleration equations of kinematics.

Visualize:

Pictorial representation



Solve: **(a)** The time it will take the asteroid to reach the earth is

$$\frac{\text{displacement}}{\text{velocity}} = \frac{4.0 \times 10^6\text{ km}}{20\text{ km/s}} = 2.0 \times 10^5\text{ s} = 56\text{ h}$$

(b) The angle of a line that just misses the earth is

$$\tan\theta = \frac{R}{y_0} \Rightarrow \theta = \tan^{-1}\left(\frac{R}{y_0}\right) = \tan^{-1}\left(\frac{6400\text{ km}}{4.0\times 10^6\text{ km}}\right) = 0.092°$$

(c) When the rocket is fired, the horizontal acceleration of the asteroid is

$$a_x = \frac{5.0\times 10^9\text{ N}}{4.0\times 10^{10}\text{ kg}} = 0.125\text{ m/s}^2$$

(Note that the mass of the rocket is much smaller than the mass of the asteroid and can therefore be ignored completely.) The velocity of the asteroid after the rocket has been fired for 300 s is

$$v_x = v_{0x} + a_x(t - t_0) = 0\text{ m/s} + (0.125\text{ m/s}^2)(300\text{ s} - 0\text{ s}) = 37.5\text{ m/s}$$

After 300 s, the vertical velocity is $v_y = 2\times 10^4$ m/s and the horizontal velocity is $v_x = 37.5$ m/s. The deflection due to this horizontal velocity is

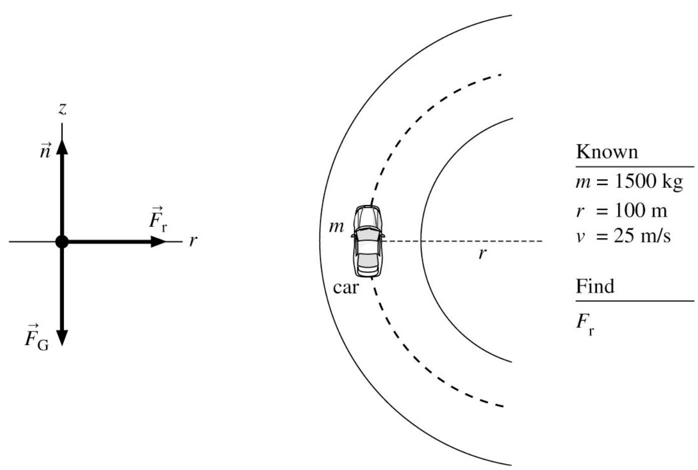
$$\tan\theta = \frac{v_x}{v_y} \Rightarrow \theta = \tan^{2\ 1}\left(\frac{37.5\text{ m/s}}{2\times 10^4\text{ m/s}}\right) = 0.107°$$

That is, the earth is saved.

Section 8.2 Uniform Circular Motion

8.4. Model: We are using the particle model for the car in uniform circular motion on a flat circular track. There must be friction between the tires and the road for the car to move in a circle.
Visualize:

Pictorial representation



Solve: The centripetal acceleration is

$$a_r = \frac{v^2}{r} = \frac{(25\text{ m/s})^2}{100\text{ m}} = 6.25\text{ m/s}^2$$

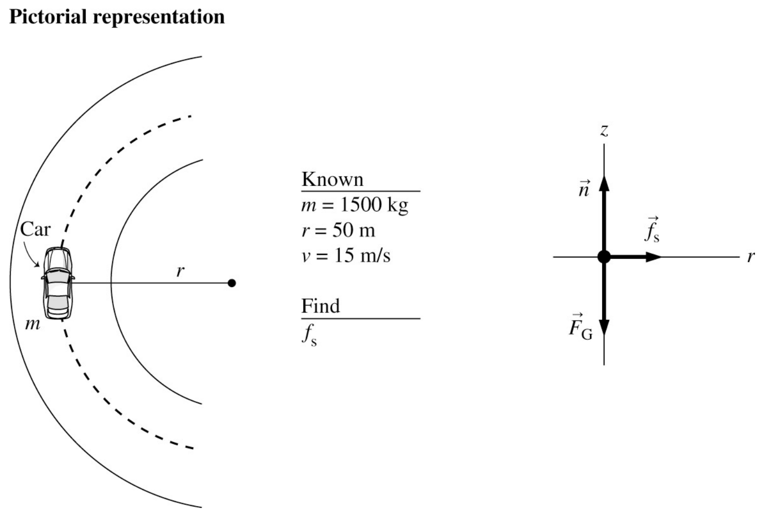
The acceleration points to the center of the circle, so the net force is

$$F_r = ma = (1500\text{ kg})(6.25\text{ m/s}^2,\ \text{toward center})$$
$$= (9380\text{ N, toward center}) \approx (9400\text{ N, toward center})$$

This force is provided by static friction

$$f_s = F_r = 9.4\text{ kN}$$

8.5. Model: We will use the particle model for the car which is in uniform circular motion.
Visualize:

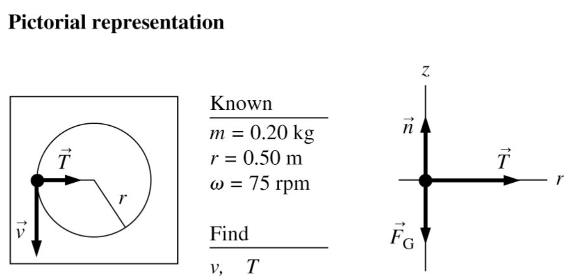


Solve: The centripetal acceleration of the car is

$$a_r = \frac{v^2}{r} = \frac{(15\text{ m/s})^2}{50\text{ m}} = 4.5\text{ m/s}^2$$

The acceleration is due to the force of static friction. The force of friction is $f_s = ma_r = (1500\text{ kg})(4.5\text{ m/s}^2) = 6750\text{ N} = 6.8\text{ kN}$.
Assess: The model of static friction is $(f_s)_{max} = n\mu_s = mg\mu_s \approx mg \approx 15{,}000\text{ N}$ since $\mu_s \approx 1$ for a dry road surface. We see that $f_s < (f_s)_{max}$, which is reasonable.

8.6. Model: Treat the block as a particle attached to a massless string that is swinging in a circle on a frictionless table.
Visualize:



Solve: (a) The angular velocity and speed are

$$\omega = 75\frac{\text{rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 471.2\text{ rad/min} \quad v_t = r\omega = (0.50\text{ m})(471.2\text{ rad/min}) \times \frac{1\text{min}}{60\text{ s}} = 3.93\text{ m/s}$$

The tangential velocity is 3.9 m/s.
(b) The radial component of Newton's second law is

$$\sum F_r = T = \frac{mv^2}{r}$$

Thus

$$T = (0.20\text{ kg})\frac{(3.93\text{ m/s})^2}{0.50\text{ m}} = 6.2\text{ N}$$

8.7. Solve: Newton's second law is $F_r = ma_r = mr\omega^2$. Substituting into this equation yields:

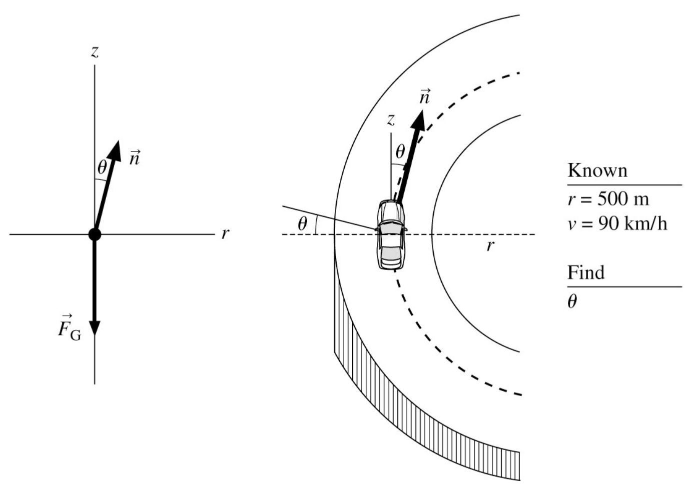
$$\omega = \sqrt{\frac{F_r}{mr}} = \sqrt{\frac{8.2\times10^{-8}\ \text{N}}{(9.1\times10^{-31}\ \text{kg})(5.3\times10^{-11}\ \text{m})}}$$

$$= 4.37\times10^{16}\ \text{rad/s} = 4.37\times10^{16}\frac{\text{rad}}{\text{s}}\times\frac{1\ \text{rev}}{2\pi\ \text{rad}} = 6.6\times10^{15}\ \text{rev/s}$$

Assess: This is a very high number of revolutions per second.

8.8. Model: The vehicle is to be treated as a particle in uniform circular motion.
Visualize:

Pictorial representation



On a banked road, the normal force on a vehicle has a horizontal component that provides the necessary centripetal acceleration. The vertical component of the normal force balances the gravitational force.
Solve: From the physical representation of the forces in the *r*-*z* plane, Newton's second law can be written

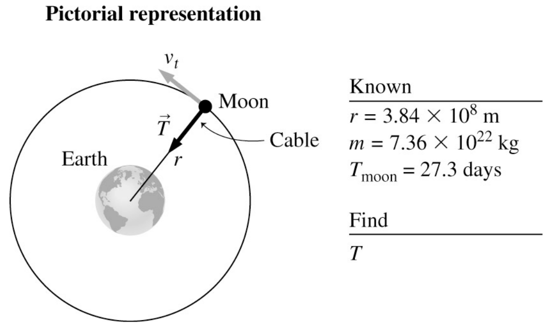
$$\sum F_r = n\sin\theta = \frac{mv^2}{r} \qquad \sum F_z = n\cos\theta - mg = 0 \Rightarrow n\cos\theta = mg$$

Dividing the two equations and making the conversion $90\ \text{km/h} = 25\ \text{m/s}$ yields:

$$\tan\theta = \frac{v^2}{rg} = \frac{(25\ \text{m/s})^2}{(9.8\ \text{m/s}^2)500\ \text{m}} = 0.128 \Rightarrow \theta = 7.3^\circ$$

Assess: Such a banking angle for a speed of approximately 55 mph is clearly reasonable and within our experience as well.

8.9. Model: The motion of the moon around the earth will be treated through the particle model. The circular motion is uniform.
Visualize:



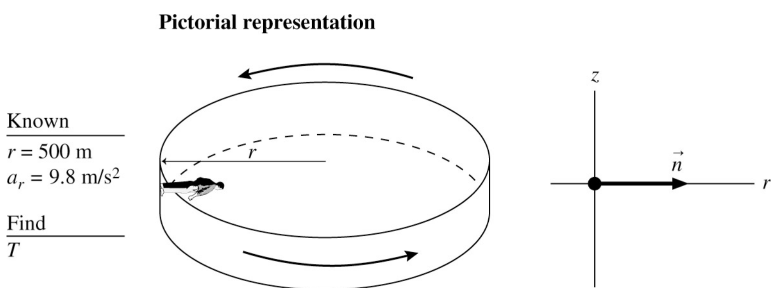
Solve: The tension in the cable provides the centripetal acceleration. Newton's second law is

$$\sum F_r = T = mr\omega^2 = mr\left(\frac{2\pi}{T_{moon}}\right)^2$$

$$= (7.36\times10^{22}\text{ kg})(3.84\times10^8\text{ m})\left[\frac{2\pi}{27.3\text{ days}}\times\frac{1\text{ day}}{24\text{ h}}\times\frac{1\text{ h}}{3600\text{ s}}\right]^2 = 2.01\times10^{20}\text{ N}$$

Assess: This is a tremendous tension, but clearly understandable in view of the moon's large mass and the large radius of circular motion around the earth. This is the same answer we'll get later with Newton's law of universal gravitation.

8.10. Model: Model the person as a particle in uniform circular motion.
Visualize:



Solve: The only force acting on the passengers is the normal force of the wall. Newton's second law along the r-axis is:

$$\sum F_r = n = mr\omega^2$$

To create "normal" gravity, the normal force by the inside surface of the space station equals mg. Therefore,

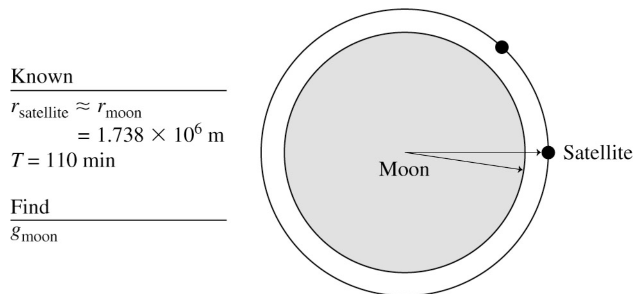
$$mg = mr\omega^2 \Rightarrow \omega = \frac{2\pi}{T} = \sqrt{\frac{g}{r}} \Rightarrow T = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{500\text{ m}}{9.8\text{ m/s}^2}} = 45\text{ s}$$

Assess: This is a fast rotation. The tangential speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi\,(500\text{ m})}{45\text{ s}} = 70\text{ m/s} \approx 140\text{ mph}$$

Section 8.3 Circular Orbits

8.11. Model: The satellite is considered to be a particle in uniform circular motion around the moon.
Visualize:



Solve: The radius of the moon is 1.738×10^6 m and the satellite's distance from the center of the moon is the same quantity. The angular velocity of the satellite is

$$\omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{110 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 9.52 \times 10^{-4} \text{ rad/s}$$

and the centripetal acceleration is

$$a_r = r\omega^2 = (1.738\times10^6 \text{ m})(9.52\times10^{-4} \text{ rad/s})^2 = 1.58 \text{ m/s}^2$$

The acceleration of a body in orbit is the local "g" experienced by that body.

8.12. Model: The earth is considered to be a particle in uniform circular motion around the sun.

Solve: The earth orbits the sun in 365 days and is 1.5×10^{11} m from the sun. The angular velocity and centripetal acceleration are

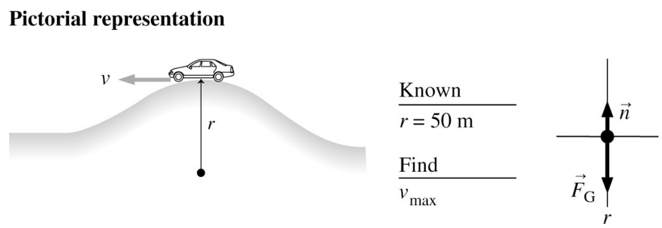
$$\omega = \frac{2\pi \text{ rad}}{365 \text{ days}}\times\frac{1 \text{ day}}{24 \text{ h}}\times\frac{1\text{h}}{3600 \text{ s}} = 2.0\times10^{-7} \text{ rad/s}$$

$$a_r = g = r\omega^2 = (1.5\times10^{11} \text{ m})(2.0\times10^{-7} \text{ rad/s})^2 = 6.0\times10^{-3} \text{ m/s}^2$$

Assess: The smallness of this acceleration due to gravity is essentially due to the large earth-sun distance.

Section 8.4 Fictitious Forces

8.13. Model: Use the particle model for the car which is undergoing circular motion.
Visualize:



Solve: The car is in circular motion with the center of the circle below the car. Newton's second law at the top of the hill is

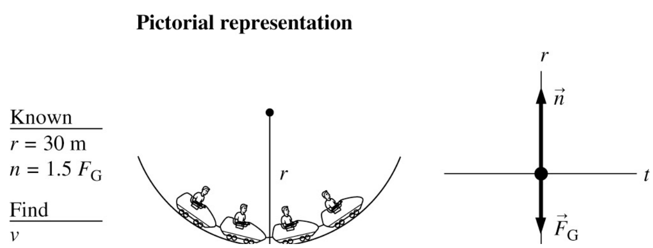
$$\sum F_r = (F_G)_r - n_r = mg - n = ma_r = \frac{mv^2}{r} \Rightarrow v^2 = r\left(g - \frac{n}{m}\right)$$

Maximum speed is reached when $n = 0$ and the car is beginning to lose contact with the road.

$$v_{max} = \sqrt{rg} = \sqrt{(50 \text{ m})(9.8 \text{ m/s}^2)} = 22 \text{ m/s}$$

Assess: A speed of 22 m/s is equivalent to 49 mph, which seems like a reasonable value.

8.14. Model: The passengers are particles in circular motion.
Visualize:



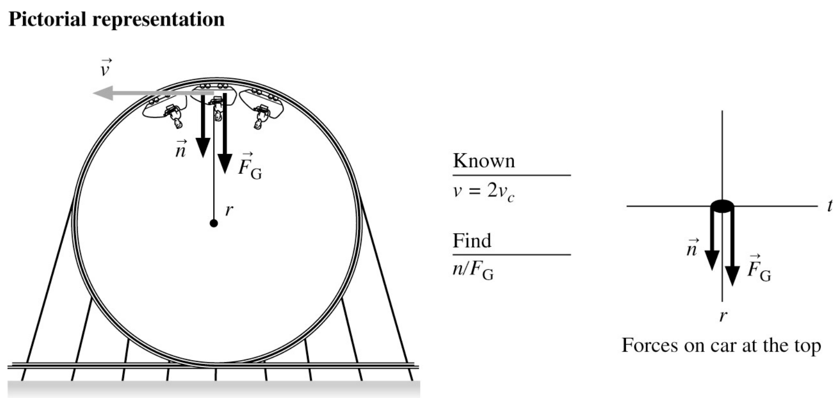
Solve: The center of the circle of motion of the passengers is directly above them. There must be a net force pointing up that provides the needed centripetal acceleration. The normal force on the passengers is their weight. Ordinarily their weight is F_G, so if their weight increases by 50%, $n = 1.5F_G$. Newton's second law at the bottom of the dip is

$$\sum F_r = n - F_G = (1.5-1)F_G = 0.5mg = \frac{mv^2}{r}$$

$$\Rightarrow v = \sqrt{0.5gr} = \sqrt{0.5(9.8 \text{ m/s}^2)(30 \text{ m})} = 12 \text{ m/s}$$

Assess: A speed of 12.1 m/s is 27 mph, which seems very reasonable.

8.15. Model: Model the roller coaster car as a particle at the top of a circular loop-the-loop undergoing uniform circular motion.
Visualize:

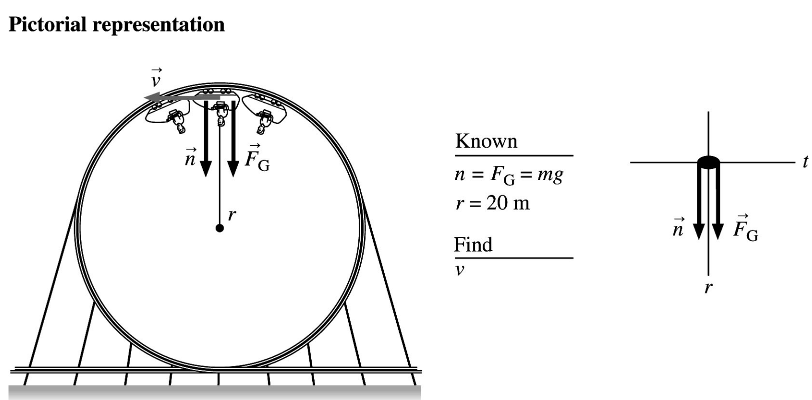


Notice that the r-axis points downward, toward the center of the circle.
Solve: The critical speed occurs when n goes to zero and F_G provides all the centripetal force pulling the car in the vertical circle. At the critical speed $mg = mv_c^2/r$, therefore $v_c = \sqrt{rg}$. Since the car's speed is twice the critical speed, $v_t = 2v_c$ and the centripetal force is

$$\sum F_r = n + F_G = \frac{mv^2}{r} = \frac{m(4v_c^2)}{r} = \frac{m(4rg)}{r} = 4mg$$

Thus the normal force is $n = 3mg$. Consequently, $n/F_G = 3$.

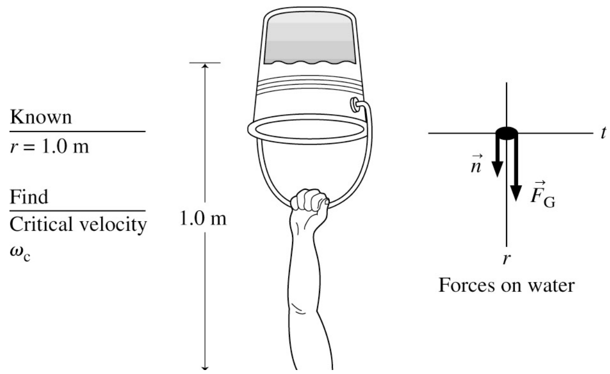
8.16. Model: Model the roller coaster car as a particle undergoing uniform circular motion along a loop.
Visualize:



Notice that the r-axis points downward, toward the center of the circle.
Solve: In this problem the normal force is equal to the gravitational force: $n = F_G = mg$. We have

$$\sum F_r = n + F_G = \frac{mv^2}{r} = mg + mg \Rightarrow v = \sqrt{2rg} = \sqrt{2(20\text{ m})(9.8\text{ m/s}^2)} = 19.8\text{ m/s} \approx 20\text{ m/s}$$

8.17. Model: Model the bucket of water as a particle in uniform circular motion.
Visualize:



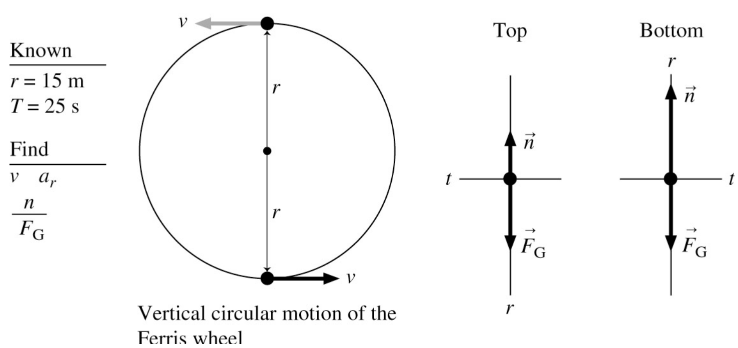
Solve: Let us say the distance from the bucket handle to the top of the water in the bucket is 35 cm. This makes the shoulder to water distance $65\text{ cm} + 35\text{ cm} = 1.00\text{ m}$. The minimum angular velocity for swinging a bucket of water in a vertical circle without spilling any water corresponds to the case when the speed of the bucket is critical. In this case, $n = 0\text{ N}$ when the bucket is in the top position of the circular motion. We get

$$\sum F_r = n + F_G = 0\text{ N} + mg = \frac{mv_c^2}{r} = mr\omega_c^2$$

$$\Rightarrow \omega_c = \sqrt{g/r} = \sqrt{\frac{9.8\text{ m/s}^2}{1.00\text{ m}}} = 3.13\text{ rad/s} = 3.13\text{ rad/s} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 30\text{ rpm}$$

8.18. Model: Use the particle model for yourself while in uniform circular motion.
Visualize:

Pictorial representation



Solve: **(a)** The speed and acceleration are

$$v = \frac{2\pi r}{T} = \frac{2\pi(15\text{ m})}{25\text{ s}} = 3.77\text{ m/s} \quad a_r = \frac{v^2}{r} = \frac{(3.77\text{ m/s})^2}{15\text{ m}} = 0.95\text{ m/s}^2$$

So the speed is 3.8 m/s and the centripetal acceleration is 0.95 m/s^2.

(b) The weight $w = m$, the normal force. On the ground, your weight is the same as the gravitational force F_G. Newton's second law at the top is

$$\sum F_r = F_G - n = ma_r = \frac{mv^2}{r}$$

$$\Rightarrow n = w = m\left(g - \frac{v^2}{r}\right) = m\left(9.8\text{ m/s}^2 - \frac{(3.77\text{ m/s})^2}{15\text{ m}}\right) = m(8.85\text{ m/s}^2)$$

$$\Rightarrow \frac{w}{F_G} = \frac{8.85\text{ m/s}^2}{9.8\text{ m/s}^2} = 0.90$$

(c) Newton’s second law at the bottom is

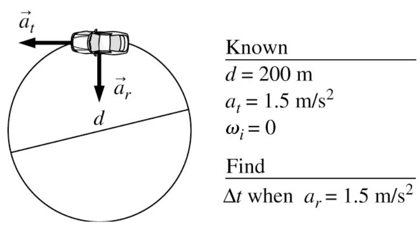
$$\sum F_r = n - F_G = ma_r = \frac{mv^2}{r}$$

$$\Rightarrow n = w = m\left(g + \frac{v^2}{r}\right) = m\left(9.8\ \text{m/s}^2 + \frac{(3.77\ \text{m/s}^2)}{15\ \text{m}}\right) = (10.75\ \text{m/s}^2)m$$

$$\Rightarrow \frac{w}{F_G} = \frac{10.75\ \text{m/s}^2}{9.8\ \text{m/s}^2} = 1.1$$

Section 8.5 Nonuniform Circular Motion

8.19. Model: Use the particle model for the car, which is undergoing nonuniform circular motion.
Visualize:



Solve: The car is in circular motion with radius $r = \frac{d}{2} = 100\ \text{m}$. We require

$$a_r = \omega^2 r = 1.5\ \text{m/s}^2 \Rightarrow \omega = \sqrt{\frac{1.5\ \text{m/s}^2}{r}} = \sqrt{\frac{1.5\ \text{m/s}^2}{100\ \text{m}}} = 0.122\ \text{s}^{-1}$$

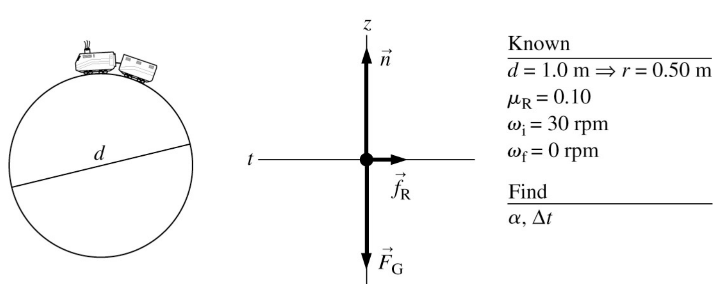
The definition of the angular velocity can be used to determine the time Δt using the angular acceleration $\alpha = \frac{a_t}{r} = \frac{1.5\ \text{m/s}^2}{100\ \text{m}} = 1.5 \times 10^{-2}\ \text{s}^{-2}$.

$$\omega = \omega_i + \alpha \Delta t$$

$$\Rightarrow \Delta t = \frac{\omega - \omega_i}{\alpha} = \frac{0.122\ \text{s}^{-1} - 0\ \text{s}^{-1}}{0.015\ s^{-2}} = 8.2\ \text{s}$$

8.20. Model: The train is a particle undergoing nonuniform circular motion.
Visualize:

Pictorial representation



Solve: (a) Newton’s second law in the vertical direction is

$$(F_{net})_y = n - F_G = 0$$

from which $n = mg$. The rolling friction is $f_R = \mu_R n = \mu_R mg$. This force provides the tangential acceleration

$$a_t = -\frac{f_R}{m} = -\mu_R g$$

The angular acceleration is

$$\alpha = \frac{a_t}{r} = \frac{-\mu_R g}{r} = \frac{-(0.10)(9.8 \text{ m/s}^2)}{0.50 \text{ m}} = -1.96 \text{ rad/s}^2$$

The magnitude is 2.0 rad/s^2.

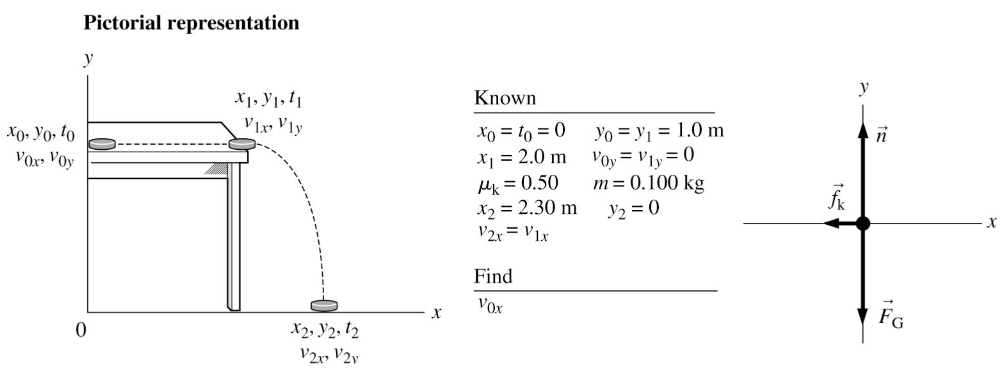
(b) The initial angular velocity is $30\left(\frac{\text{rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = 3.14 \text{ rad/s}$. The time to come to a stop due to the rolling friction is

$$\Delta t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 3.14 \text{ rad/s}}{-1.96 \text{ rad/s}^2} = 1.6 \text{ s}$$

Assess: The original angular speed of π rad/s means the train goes around the track one time every 2 seconds, so a stopping time of less than 2 s is reasonable.

Problems

8.21. Model: The object is treated as a particle in the model of kinetic friction with its motion governed by constant-acceleration kinematics.
Visualize:



Solve: The velocity v_{1x} as the object sails off the edge is related to the initial velocity v_{0x} by $v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0)$. Using Newton's second law to determine a_x while sliding gives

$$\sum F_x = -f_k = ma_x \Rightarrow \sum F_y = n - mg = 0 \text{ N} \Rightarrow n = mg$$

Using this result and the model of kinetic friction $(f_k = \mu_k n)$, the x-component equation can be written as $-\mu_k mg = ma_x$. This implies

$$a_x = -\mu_k g = -(0.50)(9.8 \text{ m/s}^2) = -4.9 \text{ m/s}^2$$

Kinematic equations for the object's free fall can be used to determine v_{1x}:

$$y_2 = y_1 + v_{1y}(t_2 - t_1) + \tfrac{1}{2}(-g)(t_2 - t_1)^2 \Rightarrow 0 \text{ m} = 1.0 \text{ m} + 0 \text{ m} - \frac{g}{2}(t_2 - t_1)^2 \Rightarrow (t_2 - t_1) = 0.4518 \text{ s}$$

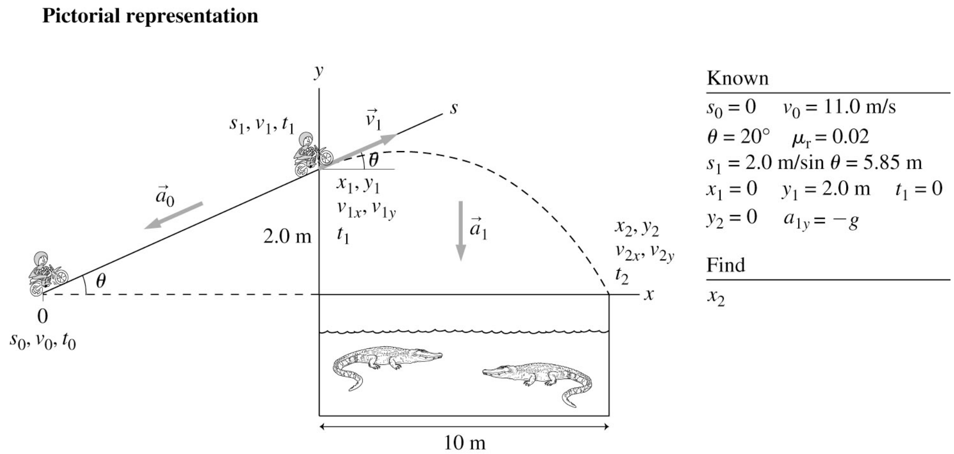
$$x_2 = x_1 + v_{1x}(t_2 - t_1) = 2.30 \text{ m} = 2.0 \text{ m} + v_{1x}(0.4518 \text{ s}) \Rightarrow v_{1x} = 0.664 \text{ m/s}$$

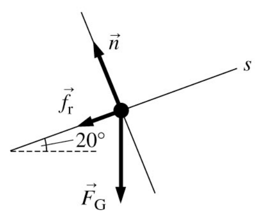
Having determined v_{1x} and a_x, we can go back to the velocity equation $v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0)$:

$$(0.664 \text{ m/s})^2 = v_{0x}^2 + 2(-4.9 \text{ m/s}^2)(2.0 \text{ m}) \Rightarrow v_{0x} = 4.5 \text{ m/s}$$

Assess: $v_{0x} = 4.5$ m/s is about 10 mph and is a reasonable speed.

8.22. Model: Treat the motorcycle and rider as a particle.
Visualize: This is a two-part problem. Use an s-axis parallel to the slope for the first part, regular xy-coordinates for the second. The motorcycle's final velocity at the top of the ramp is its initial velocity as it becomes airborne.





Solve: The motorcycle's acceleration on the ramp is given by Newton's second law:

$$(F_{net})_s = -f_r - mg\sin 20° = -\mu_r n - mg\sin 20° = -\mu_r mg\cos 20° - mg\sin 20° = ma_0$$

$$a_0 = -g(\mu_r\cos 20° + \sin 20°) = -(9.8\text{ m/s}^2)((0.02)\cos 20° + \sin 20°) = -3.536\text{ m/s}^2$$

The length of the ramp is $s_1 = (2.0\text{ m})/\sin 20° = 5.85\text{ m}$. We can use kinematics to find its speed at the top of the ramp:

$$v_1^2 = v_0^2 + 2a_0(s_1 - s_0) = v_0^2 + 2a_0 s_1$$

$$\Rightarrow v_1 = \sqrt{(11.0\text{ m/s})^2 + 2(-3.536\text{ m/s}^2)(5.85\text{ m})} = 8.92\text{ m/s}$$

This is the motorcycle's initial speed into the air, with velocity components $v_{1x} = v_1\cos 20° = 8.38\text{ m/s}$ and $v_{1y} = v_1\sin 20° = 3.05\text{ m/s}$. We can use the y-equation of projectile motion to find the time in the air:

$$y_2 = 0\text{ m} = y_1 + v_{1y}t_2 + \tfrac{1}{2}a_{1y}t_2^2 = 2.0\text{ m} + (3.05\text{ m/s})t_2 - (4.90\text{ m/s}^2)t_2^2$$

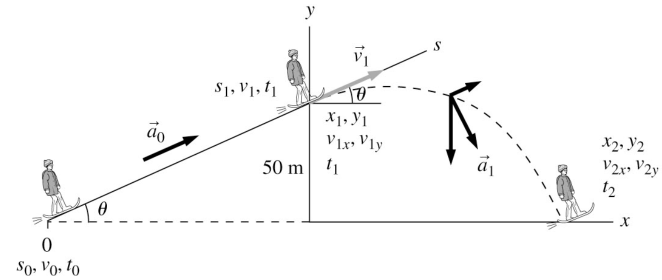
This quadratic equation has roots $t_2 = -0.399$ s (unphysical) and $t_2 = 1.021$ s. The x-equation of motion is thus

$$x_2 = x_1 + v_{1x}t_2 = 0\text{ m} + (8.38\text{ m/s})t_2 = 8.56\text{ m}$$

$8.56\text{ m} < 10.0\text{ m}$, so it looks like crocodile food.

8.23. Model: Treat Sam as a particle.
Visualize: This is a two-part problem. Use an s-axis parallel to the slope for the first part, regular xy-coordinates for the second. Sam's final velocity at the top of the slope is his initial velocity as he becomes airborne.

Pictorial representation

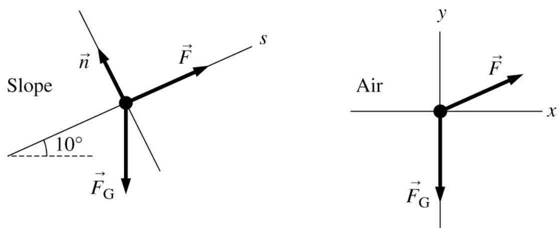


Known

$m = 75$ kg $\theta = 10°$
$s_0 = v_0 = 0$
$s_1 = 50 \text{ m}/\sin\theta = 288$ m
$x_1 = 0$ $y_1 = 50$ m $t_1 = 0$
$y_2 = 0$ $F = 200$ N

Find

x_2



Solve: Sam's acceleration up the slope is given by Newton's second law:

$$(F_{net})_s = F - mg\sin 10° = ma_0$$

$$a_0 = \frac{F}{m} - g\sin 10° = \frac{200 \text{ N}}{75 \text{ kg}} - (9.8 \text{ m/s}^2)\sin 10° = 0.965 \text{ m/s}^2$$

The length of the slope is $s_1 = (50 \text{ m})/\sin 10° = 288 \text{ m}$. His velocity at the top of the slope is

$$v_1^2 = v_0^2 + 2a_0(s_1 - s_0) = 2a_0 s_1 \Rightarrow v_1 = \sqrt{2(0.965 \text{ m/s}^2)(288 \text{ m})} = 23.6 \text{ m/s}$$

This is Sam's initial speed into the air, giving him velocity components $v_{1x} = v_1\cos 10° = 23.2 \text{ m/s}$ and $v_{1y} = v_1 \sin 10° = 410 \text{ m/s}$. This is not projectile motion because Sam experiences both the force of gravity *and* the thrust of his skis. Newton's second law for Sam's acceleration is

$$a_{1x} = \frac{(F_{net})_x}{m} = \frac{(200 \text{ N})\cos 10°}{75 \text{ kg}} = 2.63 \text{ m/s}^2$$

$$a_{1y} = \frac{(F_{net})_y}{m} = \frac{(200 \text{ N})\sin 10° - (75 \text{ kg})(9.80 \text{ m/s}^2)}{75 \text{ kg}} = -9.34 \text{ m/s}^2$$

The y-equation of motion allows us to find out how long it takes Sam to reach the ground:

$$y_2 = 0 \text{ m} = y_1 + v_{1y}t_2 + \tfrac{1}{2}a_{1y}t_2^2 = 50 \text{ m} + (4.10 \text{ m/s})t_2 - (4.67 \text{ m/s}^2)t_2^2$$

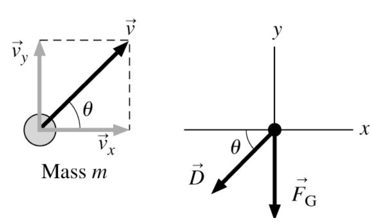
This quadratic equation has roots $t_2 = -2.86 \text{ s}$ (unphysical) and $t_2 = 3.74 \text{ s}$. The x-equation of motion—this time with an acceleration—is

$$x_2 = x_1 + v_{1x}t_2 + \tfrac{1}{2}a_{1x}t_2^2 = 0 \text{ m} + (23.2 \text{ m/s})t_2 - \tfrac{1}{2}(2.63 \text{ m/s}^2)t_2^2 = 105 \text{ m}$$

Sam lands 105 m from the base of the cliff.

8.24. Visualize:

Pictorial representation



Solve: From Chapter 6 the drag on a projectile is $D = \left(\frac{1}{4}Av^2, \text{direction opposite to motion}\right)$, where A is the cross-sectional area. Using the free-body diagram above, apply Newton's second law to each direction.
In the *x*-direction,

$$a_x = \frac{(F_{\text{net}})_x}{m} = 2\,\frac{D\cos\theta}{n} = 2\,\frac{\frac{1}{4}Av^2\cos\theta}{m}$$

$$a_y = \frac{(F_{\text{net}})_y}{m} = 2\,\frac{D\sin\theta - F_G}{m} = 2\,\frac{\frac{1}{4}Av^2\sin\theta}{m} - g$$

Since $v = \sqrt{v_x^2 + v_y^2}$, $v_x = v\cos\theta$, and $v_y = v\sin\theta$, we can rewrite these as

$$a_x = 2\,\frac{A(v\cos\theta)v}{4\text{m}} = 2\,\frac{Av_x\sqrt{v_x^2 + v_y^2}}{4\text{m}}$$

$$a_y = 2\,\frac{A(v\sin\theta)v}{4\text{m}} - g = -g - \frac{Av_y\sqrt{v_x^2 + v_y^2}}{4\text{m}}$$

8.25. Model: Use the particle model and the constant-acceleration equations of kinematics for the rocket.
Solve: (a) The acceleration of the rocket in the launch direction is obtained from Newton's second law $F = ma$:

$$140{,}700\ \text{N} = (5000\ \text{kg})a \Rightarrow a = 28.14\ \text{m/s}^2$$

Therefore, $a_x = a\cos 44.7° = 20.0\ \text{m/s}^2$ and $a_y = a\sin 44.7° = 19.8\ \text{m/s}^2$. The net acceleration in the *y*-direction is thus

$$(a_{\text{net}})_y = a_y - g = (19.8 - 9.8)\ \text{m/s}^2 = 10.0\ \text{m/s}^2$$

With this acceleration, we can write the equations for the *x*- and *y*-motions of the rocket.

$$y = y_0 + v_{0y}(t - t_0) + \tfrac{1}{2}(a_{\text{net}})_y(t - t_0)^2 = 0\ \text{m} + 0\ \text{m} + \tfrac{1}{2}(10.0\ \text{m/s}^2)t^2 = (5.00\ \text{m/s}^2)t^2$$

$$x = x_0 + v_{0x}(t - t_0) + \tfrac{1}{2}(a_{\text{net}})_x(t - t_0)^2 = 0\ \text{m} + 0\ \text{m} + \tfrac{1}{2}(20.0\ \text{m/s}^2)t^2 = (10.0\ \text{m/s}^2)t^2$$

From these two equations,

$$\frac{x}{y} = \frac{(10.0\ \text{m/s}^2)t^2}{(5.00\ \text{m/s}^2)t^2} = 2$$

The equation that describes the rocket's trajectory is $y = \frac{1}{2}x$.

(b) It is a straight line with a slope of $\frac{1}{2}$.

(c) In general,

$$v_y = v_{0y} + (a_{\text{net}})_y(t_1 - t_0) = 0 + (10.0\ \text{m/s}^2)t_1$$

$$v_x = v_{0x} + (a_{\text{net}})_x(t_1 - t_0) = 0 + (20.0\ \text{m/s}^2)t_1$$

$$v = \sqrt{(10.0\ \text{m/s}^2)^2 t_1^2 + (20.0\ \text{m/s}^2)^2 t_1^2} = (22.36\ \text{m/s}^2)t_1$$

The time required to reach the speed of sound is calculated as follows:

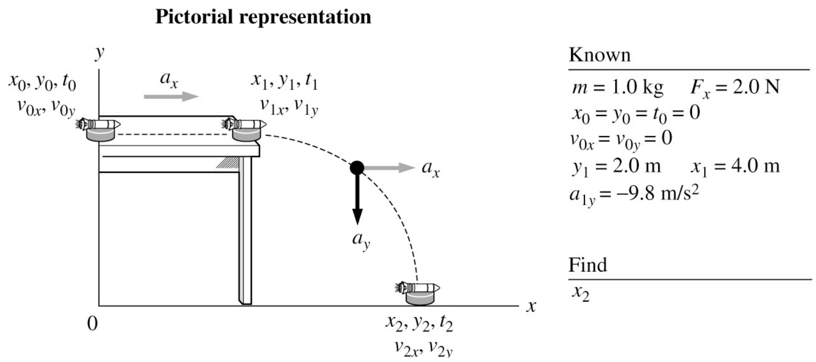
$$330\ \text{m/s} = (22.36\ \text{m/s}^2)t_1 \Rightarrow t_1 = 14.76\ \text{s}$$

We can now obtain the elevation of the rocket. From the *y*-equation,

$$y = (5.00\ \text{m/s}^2)t_1^2 = (5.00\ \text{m/s}^2)(14.76\ \text{s})^2 = 1090\ \text{m}$$

8.26. Model: The hockey puck will be treated as a particle whose motion is determined by constant-acceleration kinematic equations. We break this problem in two parts, the first pertaining to motion on the table and the second to free fall.

Visualize:



Solve: Newton's second law is:

$$F_x = ma_x \Rightarrow a_x = \frac{F_x}{m} = \frac{2.0\text{ N}}{1.0\text{ kg}} = 2.0\text{ m/s}^2$$

The kinematic equation $v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0)$ yields:

$$v_{1x}^2 = 0\text{ m}^2/\text{s}^2 + 2(2.0\text{ m/s}^2)(4.0\text{ m}) \Rightarrow v_{1x} = 4.0\text{ m/s}$$

Let us now find the time of free fall $(t_2 - t_1)$:

$$y_2 = y_1 + v_{1y}(t_2 - t_1) + \tfrac{1}{2}a_{1y}(t_2 - t_1)^2$$

$$\Rightarrow 0\text{ m} = 2.0\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t_2 - t_1)^2 \Rightarrow (t_2 - t_1) = 0.639\text{ s}$$

Having obtained v_{1x} and $(t_2 - t_1)$, we can now find $(x_2 - x_1)$ as follows:

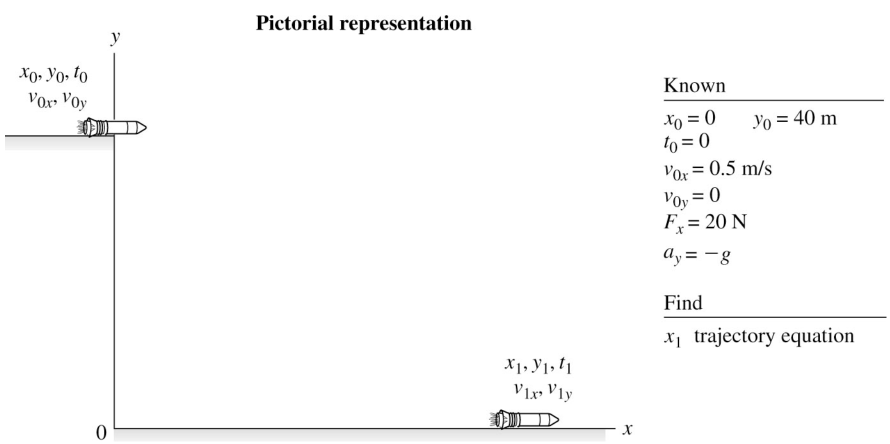
$$x_2 = x_1 + v_{1x}(t_2 - t_1) + \tfrac{1}{2}a_x(t_2 - t_1)^2$$

$$\Rightarrow x_2 - x_1 = (4.0\text{ m/s})(0.639\text{ s}) + \tfrac{1}{2}(2.0\text{ m/s}^2)(0.639\text{ s})^2 = 3.0\text{ m}$$

Assess: For a modest horizontal thrust of 2.0 N, a landing distance of 3.0 m is reasonable.

8.27. Model: The model rocket is treated as a particle and its motion is determined by constant-acceleration kinematic equations.

Visualize:



Solve: As the rocket is accidentally bumped $v_{0x} = 0.5\text{ m/s}$ and $v_{0y} = 0\text{ m/s}$. On the other hand, when the engine is fired

$$F_x = ma_x \Rightarrow a_x = \frac{F_x}{m} = \frac{20\text{ N}}{0.500\text{ kg}} = 40\text{ m/s}^2$$

(a) Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$0\text{ m} = 40\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)t_1^2 \Rightarrow t_1 = 2.857\text{ s}$$

The distance from the base of the wall is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 = 0\text{ m} + (0.5\text{ m/s})(2.857\text{s}) + \tfrac{1}{2}(40\text{ m/s}^2)(2.857\text{ s})^2 = 165\text{ m}$$

(b) The x- and y-equations are

$$y = y_0 + v_{0y}(t - t_0) + \tfrac{1}{2}a_y(t - t_0)^2 = 40 - 4.9t^2$$

$$x = x_0 + v_{0x}(t - t_0) + \tfrac{1}{2}a_x(t - t_0)^2 = 0.5t + 20t^2$$

Except for a brief interval near $t = 0$, $20t^2$ $0.5t$. Thus $x \approx 20t^2$, or $t^2 = x/20$. Substituting this into the y-equation gives

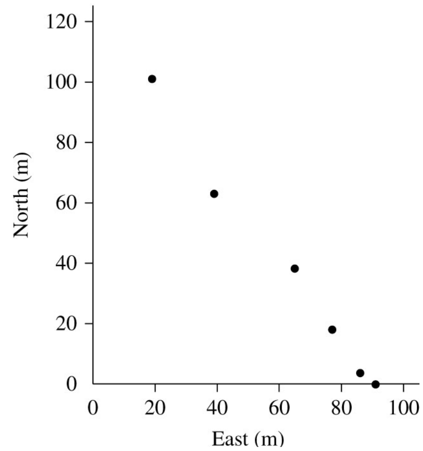
$$y = 40 - 0.245x$$

This is the equation of a straight line, so the rocket follows a linear trajectory to the ground.

8.28. Model: Model the plane as a particle with constant a_x and constant a_y.

Visualize: The plane is taking off toward the northwest as we can see by plotting the x-y data.

Position at various times

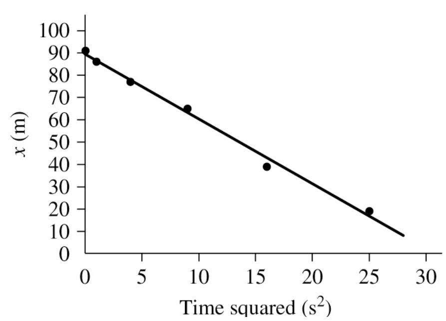


But we analyze the data in each direction separately and then apply $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$.

Solve: In each direction we apply the kinematic equation $s(t) = s_0 + (v_0)_s t + \frac{1}{2}a_s t^2$. With $v_0 = 0$ we can graph s vs. t^2 and expect to get a straight line whose slope is $\frac{1}{2}a_s$ and whose intercept is s_0.

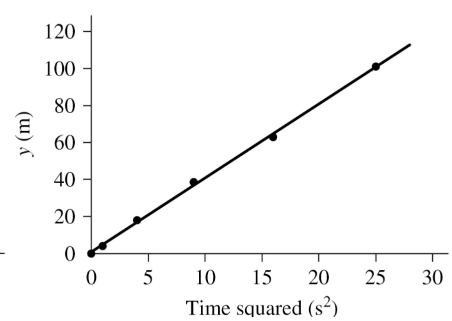
***x* vs. t^2**

$y = -2.9017x + 89.43$, $R^2 = 0.9934$



***y* vs. t^2**

$y = 3.9961x + 0.868$, $R^2 = 0.9986$



From the x vs. t^2 graph we see that $\frac{1}{2}a_x = -2.90\text{ m/s}^2 \Rightarrow a_x = -5.8\text{ m/s}^2$.

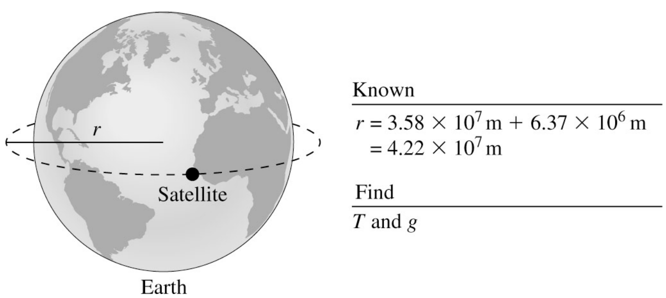
From the y vs. t^2 graph we see that $\frac{1}{2}a_y = 4.00\text{ m/s}^2 \Rightarrow a_y = 8.0\text{ m/s}^2.$

$$a = |a| = \sqrt{a_x^2 + a_y^2} = \sqrt{(-5.8\text{ m/s}^2)^2 + (8.0\text{ m/s}^2)^2} = 9.9\text{ m/s}^2$$

Assess: The intercepts of the best-fit lines are close to what the data table has.

8.29. Model: Assume the particle model for the satellite in circular motion.
Visualize:

Pictorial representation



To be in a geosynchronous orbit means rotating at the same rate as the earth, which is 24 hours for one complete rotation. Because the altitude of the satellite is 3.58×10^7 m, $r = 3.58\times10^7$ m, $r_e = 3.58\times10^7\text{ m} + 6.37\times10^6\text{ m} = 4.22\times10^7$ m.

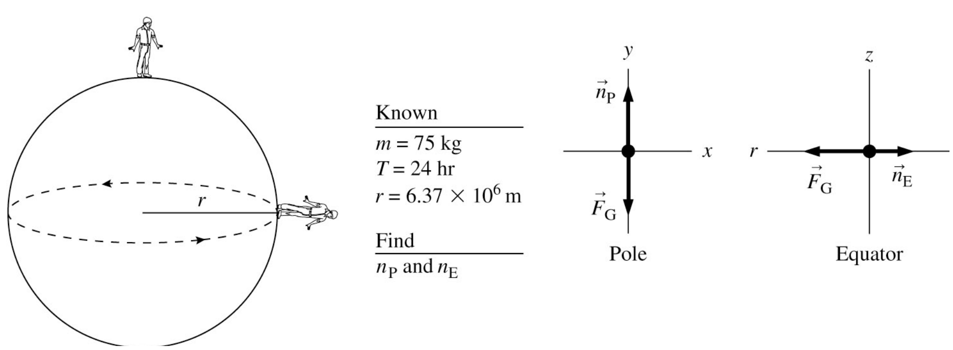
Solve: (a) The period (T) of the satellite is 24.0 hours.
(b) The acceleration due to gravity is

$$g = a_r = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 = (4.22\times10^7\text{ m})\left(\frac{2\pi}{24.0\text{ hr}}\times\frac{1\text{ hr}}{3600\text{ s}}\right)^2 = 0.223\text{ m/s}^2$$

(c) There is no normal force on a satellite, so the weight is zero. It is in free fall.

8.30. Model: Treat the man as a particle. The man at the equator undergoes uniform circular motion as the earth rotates.
Visualize:

Pictorial representation



Solve: The scale reads the man's weight $F_G = n$, the force of the scale pushing up against his feet. At the north pole, where the man is in static equilibrium,

$$n_P = F_G = mg = 735\text{ N}$$

At the equator, there must be a net force toward the center of the earth to keep the man moving in a circle. The r-axis points toward the center, so

$$\sum F_r = F_G - n_E = m\omega^2 r \Rightarrow n_E = mg - m\omega^2 r = n_P - m\omega^2 r$$

The equator scale reads less than the north pole scale by the amount $m\omega^2 r$. The man's angular velocity is that of the equator, or

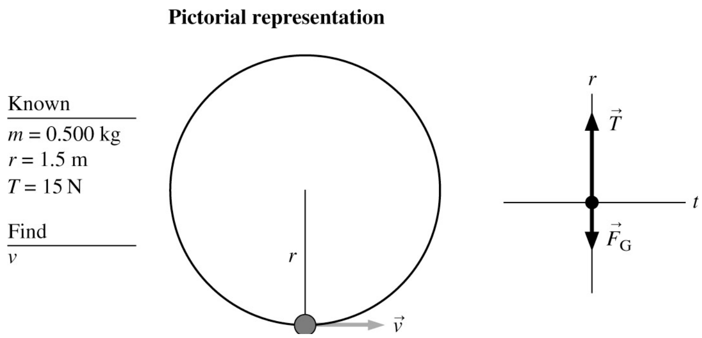
$$\omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{24 \text{ hours} \times (3600 \text{ s/1 h})} = 7.27 \times 10^{-5} \text{ rad/s}$$

Thus the north pole scale reads more than the equator scale by

$$\Delta w = (75\text{ kg})(7.27\times10^{-5}\text{ rad/s})^2(6.37\times10^6\text{ m}) = 2.5\text{ N}$$

Assess: The man at the equator appears to have lost $\Delta m = \Delta w/g \approx 0.25\text{ kg}$, or the equivalent of $\approx \frac{1}{2}$ lb.

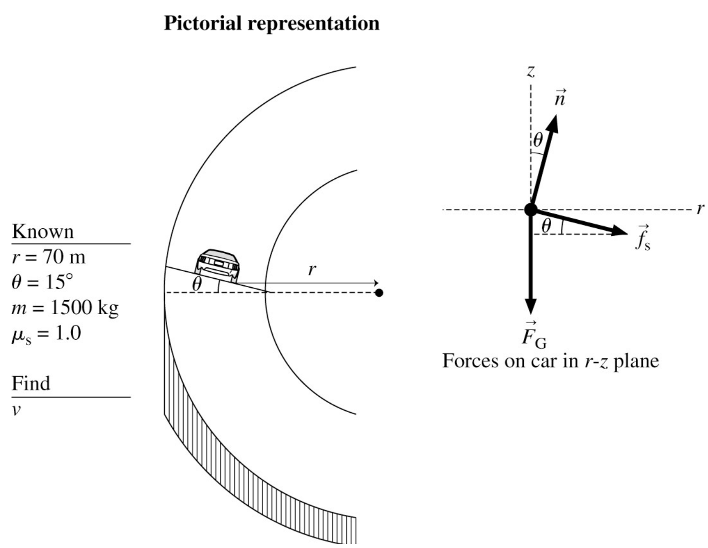
8.31. Model: Model the ball as a particle which is in a vertical circular motion.
Visualize:



Solve: At the bottom of the circle,

$$\sum F_r = T - F_G = \frac{mv^2}{r} \Rightarrow (15\text{ N}) - (0.500\text{ kg})(9.8\text{ m/s}^2) = \frac{(0.500\text{ kg})v^2}{(1.5\text{ m})} \Rightarrow v = 5.5\text{ m/s}$$

8.32. Model: We will use the particle model for the car, which is undergoing uniform circular motion on a banked highway, and the model of static friction.
Visualize:



Note that we need to use the coefficient of static friction μ_s, which is 1.0 for rubber on concrete.

Solve: Newton's second law for the car is

$$\sum F_r = f_s\cos\theta + n\sin\theta = \frac{mv^2}{r} \quad \sum F_z = n\cos\theta - f_s\sin\theta - F_G = 0\text{ N}$$

Maximum speed is when the static friction force reaches its maximum value $(f_s)_{max} = \mu_s n$. Then

$$n(\mu_s\cos15^\circ + \sin15^\circ) = \frac{mv^2}{r} \quad n(\cos15^\circ - \mu_s\sin15^\circ) = mg$$

Dividing these two equations and simplifying, we get

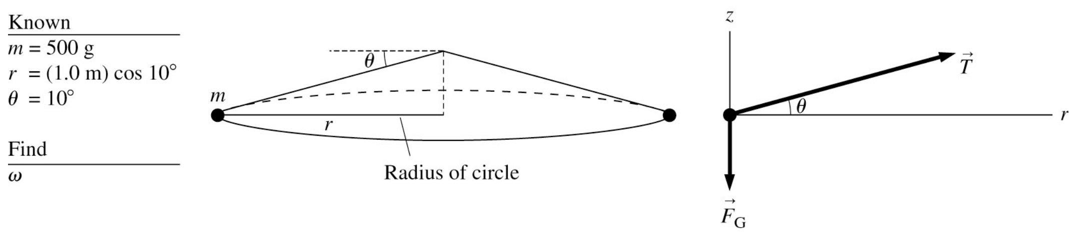
$$\frac{\mu_s + \tan 15^\circ}{1 - \mu_s \tan 15^\circ} = \frac{v^2}{gr} \Rightarrow v = \sqrt{gr\frac{\mu_s + \tan 15^\circ}{1 - \mu_s \tan 15^\circ}}$$

$$= \sqrt{(9.80\text{ m/s}^2)(70\text{ m})\frac{(1.0 + 0.268)}{(1 - 0.268)}} = 34\text{ m/s}$$

Assess: The above value of $34\text{ m/s} \approx 70\text{ mph}$ is reasonable.

8.33. Model: Use the particle model for the rock, which is undergoing uniform circular motion.
Visualize: L is the hypotenuse of the right triangle. The radius of the circular motion is $r = L\cos\theta$.

Pictorial representation



Solve:
(a) Apply Newton's second law in the z- and r-directions.

$$\sum F_z = T\sin\theta - mg = 0 \Rightarrow T = \frac{mg}{\sin\theta}$$

$$\sum F_r = T\cos\theta = m\omega^2 r = m\omega^2(L\cos\theta) \Rightarrow T = m\omega^2 L$$

Set the two expressions for T equal to each other and solve for ω.

$$\frac{mg}{\sin\theta} = m\omega^2 L \Rightarrow \omega = \sqrt{\frac{g}{L\sin\theta}}$$

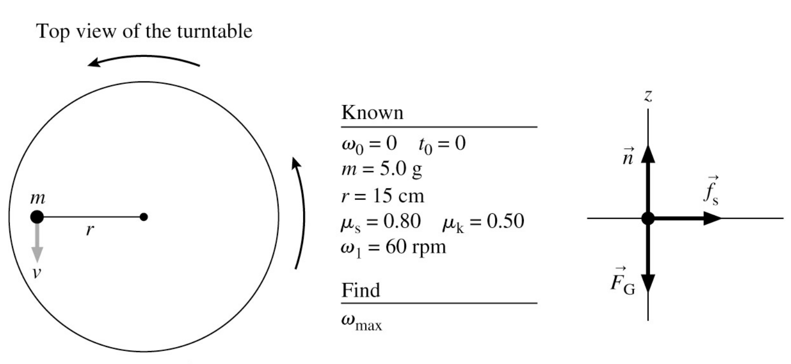
(b) Insert $L = 1.0\text{ m}$ and $\theta = 10^\circ$.

$$\omega = \sqrt{\frac{g}{L\sin\theta}} = \sqrt{\frac{9.8\text{ m/s}^2}{(1.0\text{ m})\sin 10^\circ}} = 7.51\text{ rad/s}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 72\text{ rpm}$$

Assess: Notice that the mass canceled out of the equation so the 500 g was unnecessary information. In other words, the answer, 72 rpm, would be the same regardless of the mass.
The dependencies of ω on g, L, and θ seem to be in the right directions.

8.34. Model: Use the particle model and static friction model for the coin, which is undergoing circular motion.
Visualize:

Pictorial representation



Solve: The force of static friction is $f_s = \mu_s n = \mu_s mg$. This force is equivalent to the maximum centripetal force that can be applied without sliding. That is,

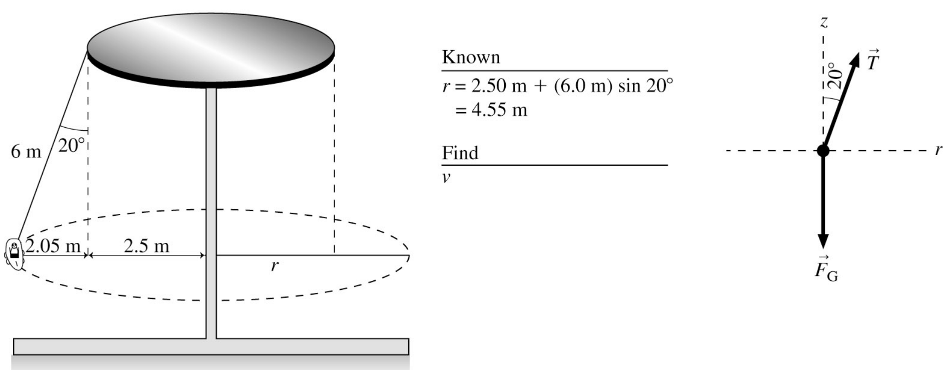
$$\mu_s mg = m\frac{v_t^2}{r} = m(r\omega_{max}^2) \Rightarrow \omega_{max} = \sqrt{\frac{\mu_s g}{r}} = \sqrt{\frac{(0.80)(9.8\text{ m/s}^2)}{0.15\text{ m}}} = 7.23\text{ rad/s}$$

$$= 7.23\frac{\text{rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 69\text{ rpm}$$

So, the coin will stay still on the turntable.
Assess: A rotational speed of approximately 1 rev per second for the coin to stay stationary seems reasonable.

8.35. Model: Use the particle model for the car, which is in uniform circular motion.
Visualize:

Pictorial representation



Solve: Newton's second law is

$$\sum F_r = T\sin 20° = ma_r = \frac{mv^2}{r} \quad \sum F_z = T\cos 20° - F_G = 0\text{ N}$$

These equations can be written as

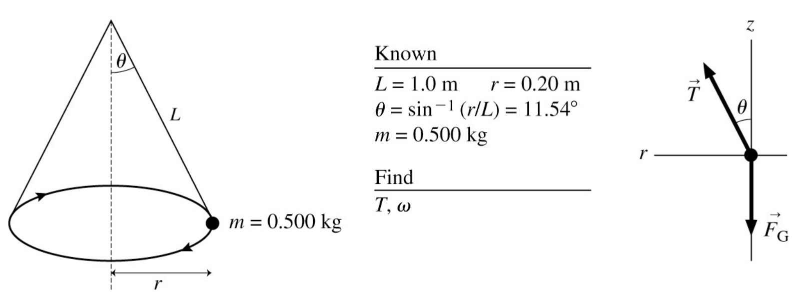
$$T\sin 20° = \frac{mv^2}{r} \quad T\cos 20° = mg$$

Dividing these two equations gives

$$\tan 20° = v^2/rg \Rightarrow v = \sqrt{rg\tan 20°} = \sqrt{(4.55\text{ m})(9.8\text{ m/s}^2)\tan 20°} = 4.03\text{ m/s} \approx 4\text{ m/s}$$

8.36. Use the particle model for the ball, which is undergoing uniform circular motion.
Visualize: We are given L, r, and m, so our answers must be in terms of those variables. L is the hypotenuse of the right triangle. The ball moves in a *horizontal* circle of radius $r = L\cos\theta$. The acceleration and net force point toward the center of the circle, not along the string.

Pictorial representation



Solve:
(a) Apply Newton's second law in the z-direction.

$$\sum F_z = T\cos\theta - mg = 0 \Rightarrow T = \frac{mg}{\cos\theta}$$

From the right triangle $\cos\theta = \sqrt{L^2 - r^2}/L$.

$$T = \frac{mg}{\cos\theta} = \frac{mgL}{\sqrt{L^2 - r^2}}$$

(b) Apply Newton's second law in the r-direction.

$$\sum F_r = T\sin\theta = m\omega^2 r = m\omega^2(L\sin\theta) \Rightarrow T = m\omega^2 L.$$

Set the two expressions for T equal to each other, cancel m and one L, and solve for ω.

$$\frac{mgL}{\sqrt{L^2 - r^2}} = m\omega^2 L \Rightarrow \omega = \sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$$

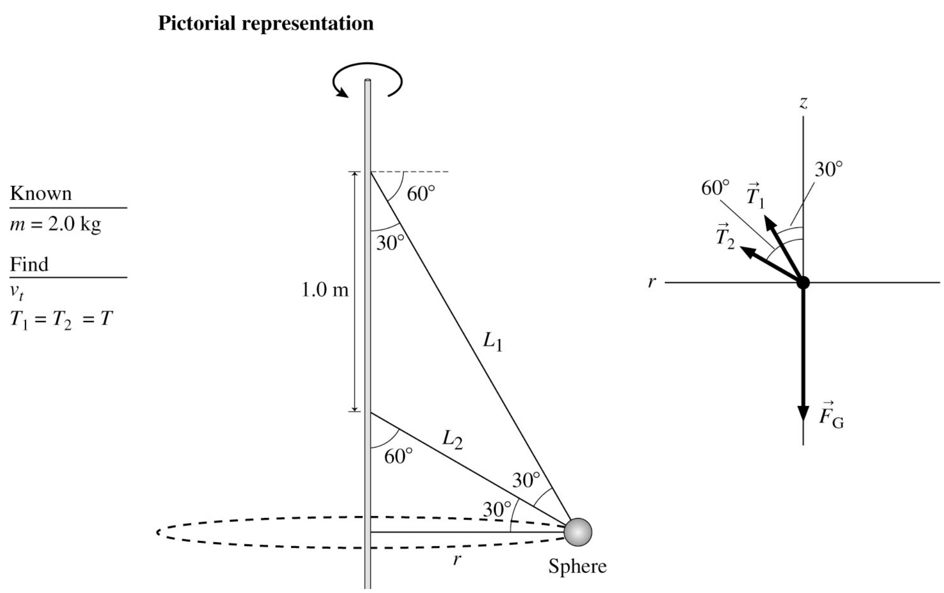
(c) Insert $L = 1.0\text{ m}$, $r = 0.20\text{ m}$ and $m = 0.50\text{ kg}$.

$$T = \frac{mgL}{\sqrt{L^2 - r^2}} = \frac{(0.50\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m})}{\sqrt{(1.0\text{ m})^2 - (0.20\text{ m})^2}} = 5.0\text{ N}$$

$$\omega = \sqrt{\frac{g}{\sqrt{L^2 - r^2}}} = \sqrt{\frac{9.8\text{ m/s}^2}{\sqrt{(1.0\text{ m})^2 - (0.20\text{ m})^2}}} = 3.163\text{ rad/s}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 30\text{ rpm}$$

Assess: Notice that the mass canceled out of the equation for ω, but not for T, so the 500 g was necessary information.

8.37. Model: Assume the particle model for a sphere in circular motion at constant speed.
Visualize:



Solve: **(a)** Newton's second law along the r and z axes is:

$$\sum F_r = T_1\sin30° + T_2\sin60° = \frac{mv_t^2}{r} \quad \sum F_z = T_1\cos30° + T_2\cos60° - F_G = 0\text{ N}$$

Since we want $T_1 = T_2 = T$, these two equations become

$$T(\sin30° + \sin60°) = \frac{mv_t^2}{r} \quad T(\cos30° + \cos60°) = mg$$

Since $\sin 30° + \sin 60° = \cos 30° + \cos 60°$,

$$mg = \frac{mv_t^2}{r} \Rightarrow v_t = \sqrt{rg}$$

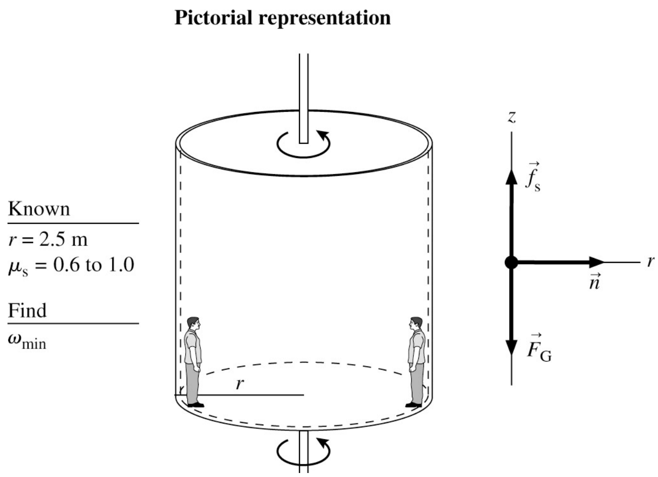
The triangle with sides L_1, L_2, and 1.0 m is isosceles, so $L_2 = 1.0\text{ m}$ and $r = L_2 \cos 30°$. Thus

$$\sqrt{L_2 \cos 30° g} = \sqrt{(1.0\text{ m})\cos 30° g} = \sqrt{(0.866\text{ m})(9.8\text{ m/s}^2)} = 2.9\text{ m/s}$$

(b) The tension is

$$T = \frac{mg}{\cos 30° + \cos 60°} = \frac{(2.0\text{ kg})(9.8\text{ m/s}^2)}{0.866 + 0.5} = 14.3\text{ N} \approx 14\text{ N}$$

8.38. Model: Consider the passenger to be a particle and use the model of static friction.
Visualize:



Solve: The passengers stick to the wall if the static friction force is sufficient to support the gravitational force on them: $f_s = F_G$. The minimum angular velocity occurs when static friction reaches its maximum possible value $(f_s)_{max} = \mu_s n$. Although clothing has a range of coefficients of friction, it is the clothing with the smallest coefficient $(\mu_s = 0.60)$ that will slip first, so this is the case we need to examine. Assuming that the person is stuck to the wall, Newton's second law is

$$\sum F_r = n = m\omega^2 r \qquad \sum F_z = f_s - w = 0 \Rightarrow f_s = mg$$

The minimum frequency occurs when

$$f_s = (f_s)_{max} = \mu_s n = \mu_s m r \omega_{min}^2$$

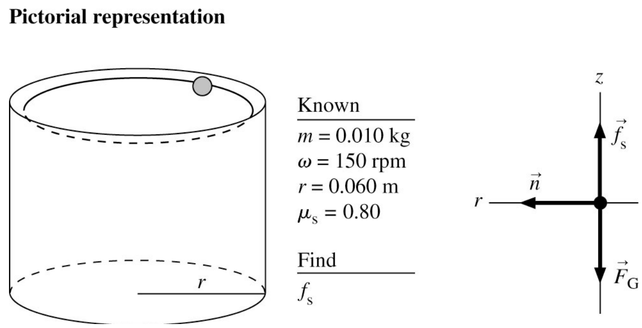
Using this expression for f_s in the z-equation gives

$$f_s = \mu_s m r \omega_{min}^2 = mg$$

$$\Rightarrow \omega_{min} = \sqrt{\frac{g}{\mu_s r}} = \sqrt{\frac{9.80\text{ m/s}^2}{0.60(2.5\text{ m})}} = 2.56\text{ rad/s} = 2.56\text{ rad/s} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 24\text{ rpm}$$

Assess: Note the velocity does not depend on the mass of the individual. Therefore, the minimum mass sign is not necessary.

8.39. Model: Use the particle model for the marble in uniform circular motion.
Visualize:



Solve: The marble will roll in a horizontal circle if the static friction force is sufficient to support the gravitational on it: $f_s = F_G$. If $mg > (f_s)_{max}$ then static friction is not sufficient and the marble will slip down the side as it rolls around the circumference. The r-equation of Newton's second law is

$$\sum F_r = n = mr\omega^2 = (0.010\text{ kg})(0.060\text{ m})\left(150\text{ rpm}\times\frac{2\pi\text{ rad}}{1\text{ rev}}\times\frac{1\text{ min}}{60\text{ s}}\right)^2 = 0.148\text{ N}$$

Thus the maximum possible static friction is $(f_s)_{max} = \mu_s n = (0.80)(0.148\text{ N}) = 0.118\text{ N}.$ The friction force needed to support a 10 g marble is $f_s = mg = 0.098\text{ N}.$ We see that $f_s < (f_s)_{max}$, therefore friction is sufficient and the marble spins in a horizontal circle.

Assess: In reality, rolling friction will cause the marble to gradually slow down until $(f_s)_{max} < mg.$ At that point, it will begin to slip down the inside wall.

8.40. Model: Assume uniform circular motion.

Visualize: We expect the centripetal acceleration to be very large because ω is large. This will produce a significant force even though the mass difference of 10 mg is so small.

A preliminary calculation will convert the mass difference to kg: $10\text{ mg} = 1.0\times10^{-5}\text{ kg}.$ If the two samples are equally balanced then the shaft doesn't feel a net force in the horizontal plane. However, the mass difference of 10 mg is what causes the force.

We'll do another preliminary calculation to convert $\omega = 70{,}000$ rpm into rad/s.

$$\omega = 70{,}000\text{ rpm} = 70{,}000\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 7330\text{ rad/s}$$

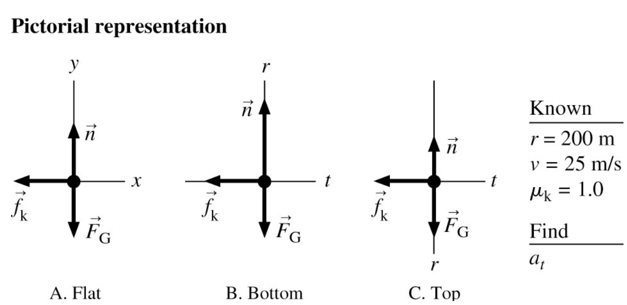
Solve: The centripetal acceleration is given by Equation 6.9 and the net force by Newton's second law.

$$F_{net} = (\Delta m)(a) = (\Delta m)(\omega^2 r) = (1.0\times10^{-5}\text{ kg})(7330\text{ rad/s})^2(0.12\text{ m}) = 64\text{ N}$$

Assess: As we expected, the centripetal acceleration is large. The force is not huge (because of the small mass difference) but still enough to worry about. The net force scales with this mass difference, so if the mistake were bigger it could be enough to shear off the shaft.

8.41. Model: Use the particle model for the car and the model of kinetic friction.

Visualize:



Solve: We will apply Newton's second law to all three cars.
Car A:

$$\sum F_x = n_x + (f_k)_x + (F_G)_x = 0\text{ N} - f_k + 0\text{ N} = ma_x$$
$$\sum F_y = n_y + (f_k)_y + y_y = n + 0\text{ N} - mg = 0\text{ N}$$

The y-component equation means $n = mg$. Since $f_k = \mu_k n$, we have $f_k = \mu_k mg$. From the x-component equation,

$$a_x = \frac{-f_k}{m} = \frac{-\mu_k mg}{m} = -\mu_k g = -9.8\text{ m/s}^2$$

Car B: Car B is in circular motion with the center of the circle above the car.

$$\sum F_r = n_r + (f_k)_r + (F_G)_r = n + 0\text{ N} - mg = ma_r = \frac{mv^2}{r}$$
$$\sum F_t = n_t + (f_k)_t + (F_G)_t = 0\text{ N} - f_k + 0\text{ N} = +ma_t$$

From the r-equation

$$n = mg + \frac{mv^2}{r} \Rightarrow f_k = \mu_k n = \mu_k m\left(g + \frac{v^2}{r}\right)$$

Substituting back into the t-equation,

$$a_t = -\frac{f_k}{m} = -\frac{\mu_k m}{m}\left(g + \frac{v^2}{r}\right) = -\mu_k\left(9.8\text{ m/s}^2 + \frac{(25\text{ m/s})^2}{200\text{ m}}\right) = -12.9\text{ m/s}^2$$

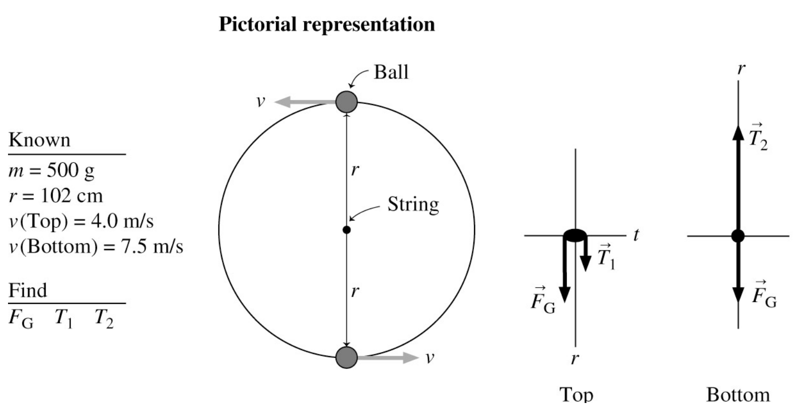
Car C: Car C is in circular motion with the center of the circle below the car.

$$\sum F_r = n_r + (f_k)_r + (F_G)_r = -n + 0\text{ N} + mg = ma_r = \frac{mv^2}{r}$$
$$\sum F_t = n_t + (f_k)_t + (F_G)_t = 0\text{ N} - f_k + 0\text{ N} = ma_t$$

From the r-equation $n = m(g - v^2/r)$. Substituting this into the t-equation yields

$$a_t = \frac{-f_k}{m} = \frac{-\mu_k n}{m} = -\mu_k(g - v^2/r) = -6.7\text{ m/s}^2$$

8.42. Model: Model the ball as a particle that is moving in a vertical circle.
Visualize:



Solve: **(a)** The ball's gravitational force $F_G = mg = (0.500\text{ kg})(9.8\text{ m/s}^2) = 4.9\text{ N}$.
(b) Newton's second law at the top is

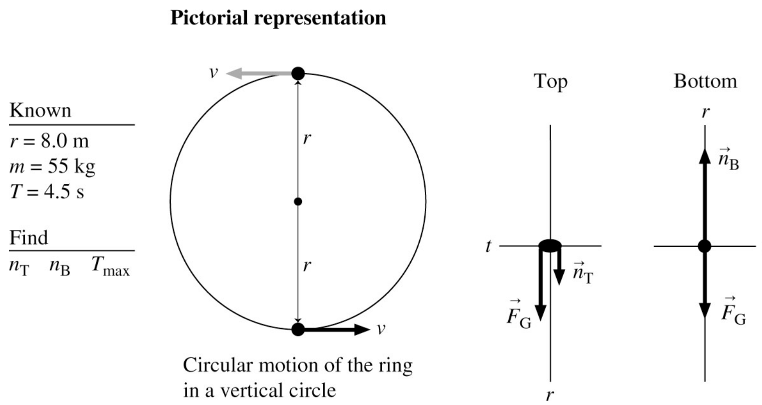
$$\sum F_r = T_1 + F_G = ma_r = m\frac{v^2}{r}$$
$$\Rightarrow T_1 = m\left(\frac{v^2}{r} - g\right) = (0.500\text{ kg})\left[\frac{(4.0\text{ m/s})^2}{1.02\text{ m}} - 9.8\text{ m/s}^2\right] = 2.9\text{ N}$$

(c) Newton's second law at the bottom is

$$\sum F_r = T_2 - F_G = \frac{mv^2}{r}$$

$$\Rightarrow T_2 = m\left(g + \frac{v^2}{r}\right) = (0.500\text{ kg})\left[9.8\text{ m/s}^2 + \frac{(7.5\text{ m/s})^2}{1.02\text{ m}}\right] = 32\text{ N}$$

8.43. Model: Model a passenger as a particle rotating in a vertical circle.
Visualize:



Solve: (a) Newton's second law at the top is

$$\sum F_r = n_T + F_G = ma_r = \frac{mv^2}{r} \Rightarrow n_T + mg = \frac{mv^2}{r}$$

The speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi(8.0\text{ m})}{4.5\text{ s}} = 11.17\text{ m/s}$$

$$\Rightarrow n_T = m\left(\frac{v^2}{r} - g\right) = (55\text{ kg})\left[\frac{(11.17\text{ m/s})^2}{8.0\text{ m}} - 9.8\text{ m/s}^2\right] = 319\text{ N}$$

That is, the ring pushes on the passenger with a force of 3.2×10^2 N at the top of the ride. Newton's second law at the bottom:

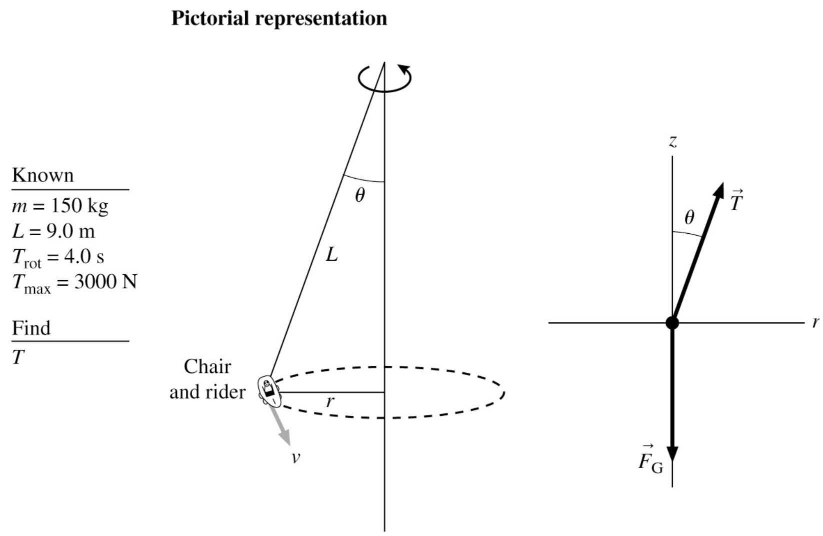
$$\sum F_r = n_B - F_G = ma_r = \frac{mv^2}{r} \Rightarrow n_B = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right)$$

$$= (55\text{ kg})\left[\frac{(11.17\text{ m/s})^2}{8.0\text{ m}} + 9.8\text{ m/s}^2\right] = 1397\text{ N}$$

Thus the force with which the ring pushes on the rider when she is at the bottom of the ring is 1.4 kN.
(b) To just stay on at the top, $n_T = 0$ N in the r-equation at the top in part (a). Thus,

$$mg = \frac{mv^2}{r} = mr\omega^2 = mr\left(\frac{2\pi}{T_{max}}\right)^2 \Rightarrow T_{max} = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{8.0\text{ m}}{9.8\text{ m/s}^2}} = 5.7\text{ s}$$

8.44. Model: Model the chair and the rider as a particle in uniform circular motion.
Visualize:



Solve: Newton's second law along the r-axis is

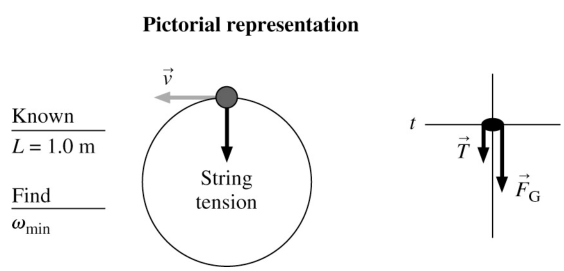
$$\sum F_r = T_r + (F_G)_r = ma_r \Rightarrow T\sin\theta + 0\text{ N} = mr\omega^2$$

Since $r = L\sin\theta$, this equation becomes

$$T = mL\omega^2 = (150\text{ kg})(9.0\text{ m})\left[\frac{2\pi}{4.0}\frac{\text{rad}}{\text{s}}\right]^2 = 3330\text{ N}$$

Thus, the 3000 N chain is *not* strong enough for the ride.

8.45. Model: Model the ball as a particle in motion in a vertical circle.
Visualize:



Solve: (a) *If* the ball moves in a complete circle, then there is a tension force T when the ball is at the top of the circle. The tension force adds to the gravitational force to cause the centripetal acceleration. The forces are along the r-axis, and the center of the circle is below the ball. Newton's second law at the top is

$$(F_{net})_r = T + F_G = T + mg = \frac{mv^2}{L}$$

$$\Rightarrow v_{top} = \sqrt{Lg + \frac{LT}{m}}$$

The tension T can't become negative, so $T = 0\text{ N}$ gives the minimum speed v_{min} at which the ball moves in a circle. If the speed is less than v_{min}, then the string will go slack and the ball will fall out of the circle before it reaches the top. Thus,

$$v_{min} = \sqrt{Lg} \Rightarrow \omega_{min} = \frac{v_{min}}{L} = \frac{\sqrt{Lg}}{L} = \sqrt{\frac{g}{L}}$$

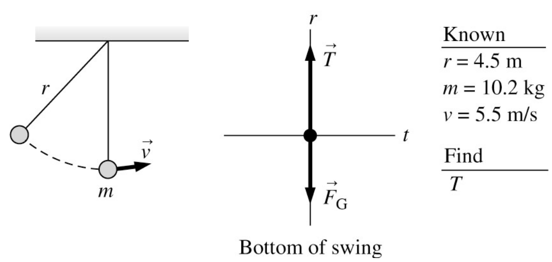
(b) Insert $L = 1.0$ m.

$$\omega_{\min} = \sqrt{\frac{g}{r}} = \sqrt{\frac{(9.8\ \text{m/s}^2)}{(1.0\ \text{m})}} = 3.13\ \text{rad/s} = 30\ \text{rpm}$$

Assess: Notice that the mass doesn't appear in the answer, so $\omega_{\min}$ is independent of the mass.

8.46. Model: The ball is a particle on a massless rope in circular motion about the point where the rope is attached to the ceiling.
Visualize:

Pictorial representation



Solve: Newton's second law in the radial direction is

$$(\sum F_r) = T - F_G = T - mg = \frac{mv^2}{r}$$

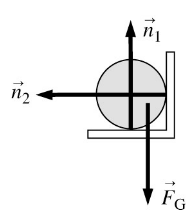
Solving for the tension in the rope and evaluating,

$$T = m\left(g + \frac{v^2}{r}\right) = (10.2\ \text{kg})\left(9.8\ \text{m/s}^2 + \frac{(5.5\ \text{m/s})^2}{4.5\ \text{m}}\right) = 168\ \text{N}$$

Assess: The tension in the rope is greater than the gravitational force on the ball in order to keep the ball moving in a circle.

8.47. Model: Model the ball as a particle in uniform circular motion. Rolling friction is ignored.
Visualize:

Pictorial representation



Solve: The track exerts both an upward normal force and an inward normal force. From Newton's second law,

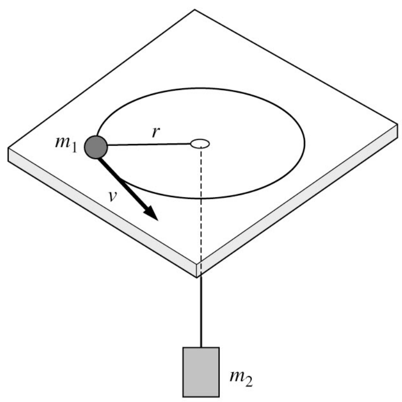
$$n_1 = mg = (0.030\ \text{kg})(9.8\ \text{m/s}^2) = 0.294\ \text{N, up}$$

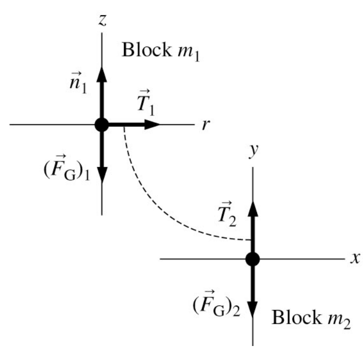
$$n_2 = mr\omega^2 = (0.030\ \text{kg})(0.20\ \text{m})\left[\frac{60\ \text{rev}}{\text{min}} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} \times \frac{1\ \text{min}}{60\ \text{s}}\right]^2 = 0.2369\ \text{N, in}$$

$$F_{\text{net}} = \sqrt{n_1^2 + n_2^2} = \sqrt{(0.294\ \text{N})^2 + (0.2369\ \text{N})^2} = 0.38\ \text{N}$$

8.48. Model: Masses m_1 and m_2 are considered particles. The string is assumed to be massless.
Visualize:

Pictorial representation





Solve: The tension in the string causes the centripetal acceleration of the circular motion. If the hole is smooth, it acts like a pulley. Thus tension forces T_1 and T_2 act as if they were an action/reaction pair. Mass m_1 is in circular motion of radius r, so Newton's second law for m_1 is

$$\sum F_r = T_1 = \frac{m_1 v^2}{r}$$

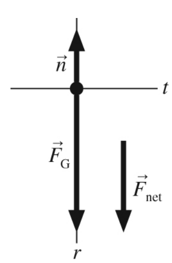
Mass m_2 is at rest, so the y-equation of Newton's second law is

$$\sum F_y = T_2 - m_2 g = 0\text{ N} \Rightarrow T_2 = m_2 g$$

Newton's third law tells us that $T_1 = T_2$. Equating the two expressions for these quantities:

$$\frac{m_1 v^2}{r} = m_2 g \Rightarrow v = \sqrt{\frac{m_2 r g}{m_1}}$$

8.49. Model: Model yourself as a particle in circular motion atop a leg of length L.
Visualize: Set up the coordinate system so that the r-direction is down, toward the center of the circle.



Solve:
(a) Apply Newton's second law in the downward vertical direction, which is the $+r$-direction.

$$\sum F_r = mg - n = \frac{mv^2}{L}$$

Notice that $n < mg$. Your body tries to "lift off" as it pivots over your foot, decreasing the normal force exerted on you by the ground. The normal force becomes smaller as you walk faster, but n cannot be less than zero. Thus the maximum possible walking speed v_{max} occurs when $n = 0$. Setting $n = 0$,

$$mg = \frac{mv_{max}^2}{L} \Rightarrow v_{max} = \sqrt{gL}$$

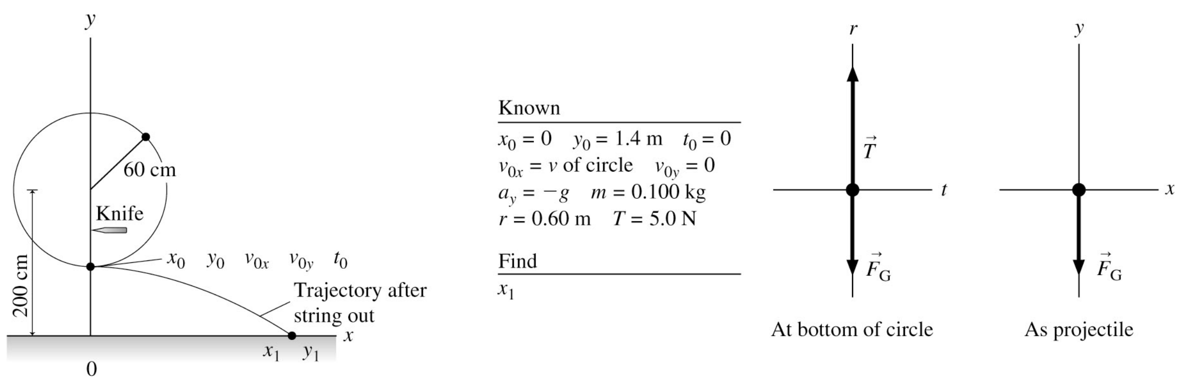
(b) Insert $L = 0.70\ \text{m}$.

$$v_{max} = \sqrt{gL} = \sqrt{(9.8\ \text{m/s}^2)(0.70\ \text{m})} = 2.6\ \text{m/s} = 5.9\ \text{mph}$$

Assess: The answer of 5.9 mph is faster than the "normal" walking speed of 3 mph, but we expect v_{max} to be greater than the normal speed. This seems reasonable. The units check out.
The maximum walking speed depends on L, so taller people can walk faster than shorter people.

8.50. Model: Model the ball as a particle swinging in a vertical circle, then as a projectile.
Visualize:

Pictorial representation



Solve: Initially, the ball is moving in a circle. Once the string is cut, it becomes a projectile. The final circular-motion velocity is the initial velocity for the projectile. The free-body diagram for circular motion is shown at the bottom of the circle. Since $T > F_G$, there is a net force toward the center of the circle that causes the centripetal acceleration. The r-equation of Newton's second law is

$$(F_{net})_r = T - F_G = T - mg = \frac{mv^2}{r}$$

$$\Rightarrow v_{bottom} = \sqrt{\frac{r}{m}(T - mg)} = \sqrt{\frac{0.60\ \text{m}}{0.100\ \text{kg}}}[5.0\ \text{N} - (0.10\ \text{kg})(9.8\ \text{m/s}^2)] = 4.91\ \text{m/s}$$

As a projectile the ball starts at $y_0 = 1.4\ \text{m}$ with $v_0 = 4.91\hat{i}\ \text{m/s}$. The equation for the y-motion is

$$y_1 = 0\ \text{m} = y_0 + v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2 = y_0 - \tfrac{1}{2}gt_1^2$$

This is easily solved to find that the ball hits the ground at time

$$t_1 = \sqrt{\frac{2y_0}{g}} = 0.535\ \text{s}$$

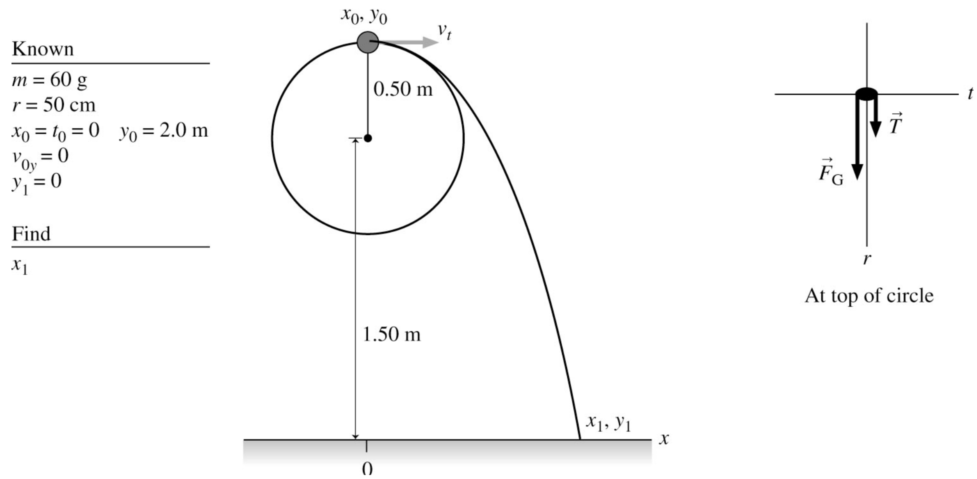
During this time interval it travels a horizontal distance

$$x_1 = x_0 + v_{0x}t_1 = (4.91\ \text{m/s})(0.535\ \text{s}) = 2.63\ \text{m}$$

So the ball hits the floor 2.6 m to the right of the point where the string was cut.

8.51. Model: Use the particle model for a ball in motion in a vertical circle and then as a projectile.
Visualize:

Pictorial representation



Solve: For the circular motion, Newton's second law along the r-direction is

$$\sum F_r = T + F_G = \frac{mv_t^2}{r}$$

Since the string goes slack as the particle makes it over the top, $T = 0\text{ N}$. That is,

$$F_G = mg = \frac{mv_t^2}{r} \Rightarrow v_t = \sqrt{gr} = \sqrt{(9.8\text{ m/s}^2)(0.5\text{ m})} = 2.21\text{ m/s}$$

The ball begins projectile motion as the string is released. The time it takes for the ball to hit the floor can be found as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow 0\text{ m} = 2.0\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t_1 - 0\text{ s})^2 \Rightarrow t_1 = 0.639\text{ s}$$

The place where the ball hits the ground is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) = 0\text{ m} + (+2.21\text{ m/s})(0.639\text{ s} - 0\text{ s}) = +1.41\text{ m}$$

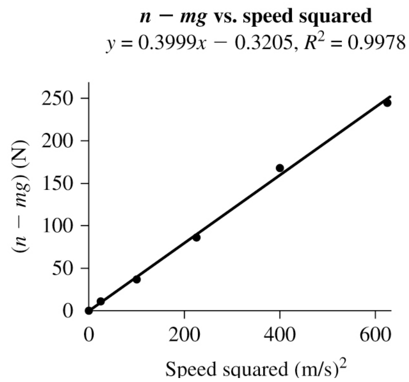
The ball hits the ground 1.4 m to the right of the point beneath the center of the circle.

8.52. Model: Model yourself as a particle in circular motion.
Visualize: Apply Newton's second law for circular motion in the vertical direction. The upward normal force is the scale reading. We are given $mg = 588\text{ N}$. This means $m = 60\text{ kg}$. We seek r.
Solve:

$$\sum F = n - mg = \frac{mv^2}{r}$$

We can easily get the spreadsheet to subtract $mg = 588\text{ N}$ from each of the scale readings on the left side of the equation. Then if we graph $n - mg$ vs. v^2 we should get a straight line whose slope is m/r.

Speed (m/s)	Scale reading (N)	$(n - mg)$ (N)	Speed squared $(m/s)^2$
0	588	0	0
5	599	11	25
10	625	37	100
15	674	86	225
20	756	168	400
25	834	246	625

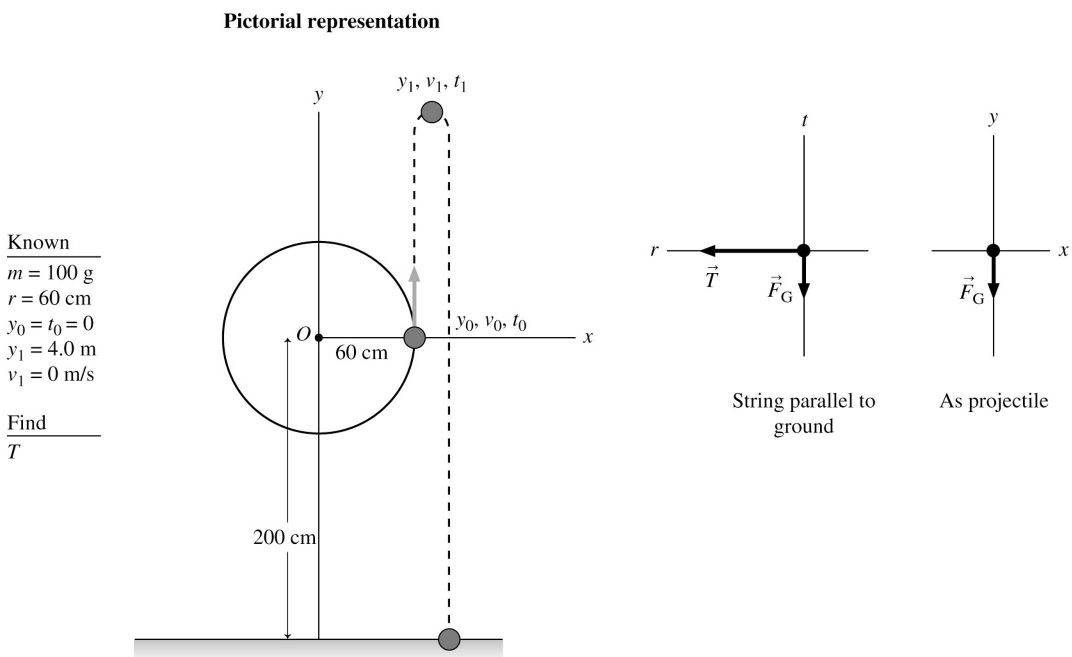


We see from the spreadsheet that the fit is very good and that the slope is $m/r = 0.3999$ kg/m.
With $m = 60$ kg,

$$\frac{m}{r} = 0.3999\text{ kg/m} \Rightarrow r = \frac{60\text{ kg}}{0.3999\text{ kg/m}} = 150\text{ m}$$

Assess: This radius of curvature does not seem extreme.

8.53. Model: Model the ball as a particle undergoing circular motion in a vertical circle.
Visualize:



Solve: Initially, the ball is moving in circular motion. Once the string breaks, it becomes a projectile. The final circular-motion velocity is the initial velocity for the projectile, which we can find by using the kinematic equation

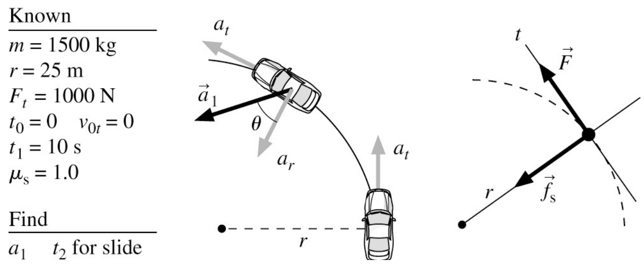
$$v_1^2 = v_0^2 + 2a_y(y_1 - y_0) \Rightarrow 0\text{ m}^2/\text{s}^2 = (v_0)^2 + 2(-9.8\text{ m/s}^2)(4.0\text{ m} - 0\text{ m}) \Rightarrow v_0 = 8.85\text{ m/s}$$

This is the speed of the ball as the string broke. The tension in the string at that instant can be found by using the r-component of the net force on the ball:

$$\sum F_r = T = m\left(\frac{v_{0y}^2}{r}\right) \Rightarrow T = (0.100\text{ kg})\frac{(8.85\text{ m/s})^2}{0.60\text{ m}} = 13\text{ N}$$

8.54. Model: Model the car as a particle on a circular track.
Visualize:

Pictorial representation



Solve: **(a)** Newton's second law along the *t*-axis is

$$\sum F_t = F_t = ma_t \Rightarrow 1000\text{ N} = (1500\text{ kg})a_t \Rightarrow a_t = 2/3\text{ m/s}^2$$

With this tangential acceleration, the car's tangential velocity after 10 s will be

$$v_{1t} = v_{0t} + a_t(t_1 - t_0) = 0\text{ m/s} + (2/3\text{ m/s}^2)(10\text{ s} - 0\text{ s}) = 20/3\text{ m/s}$$

The radial acceleration at this instant is

$$a_r = \frac{v_{1t}^2}{r} = \frac{(20/3\text{ m/s})^2}{25\text{ m}} = \frac{16}{9}\text{ m/s}^2$$

The car's acceleration at 10 s has magnitude

$$a_1 = \sqrt{a_t^2 + a_r^2} = \sqrt{(2/3\text{ m/s}^2)^2 + (16/9\text{ m/s}^2)^2} = 1.90\text{ m/s}^2 \qquad \theta = \tan^{-1}\frac{a_t}{a_r} = \tan^{-1}\left(\frac{2/3}{16/9}\right) = 21°$$

where the angle is measured from the *r*-axis.

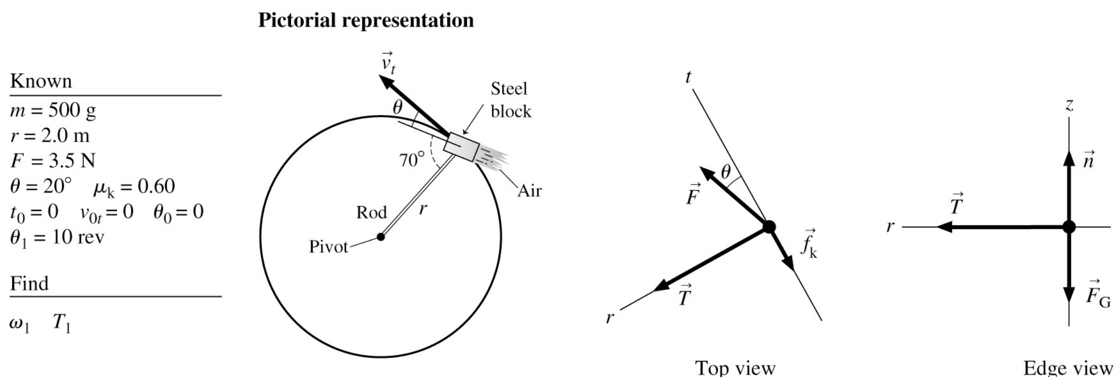
(b) The car will begin to slide out of the circle when the static friction reaches its maximum possible value $(f_s)_{max} = \mu_s n$. That is,

$$\sum F_r = (f_s)_{max} = \mu_s n = \mu_s mg = \frac{mv_{2t}^2}{r} \Rightarrow v_{2t} = \sqrt{rg} = \sqrt{(25\text{ m})(9.8\text{ m/s}^2)} = 15.7\text{ m/s}$$

In the above equation, $n = mg$ follows from Newton's second law along the *z*-axis. The time when the car begins to slide can now be obtained as follows:

$$v_{2t} = v_{0t} + a_t(t_2 - t_0) \Rightarrow 15.7\text{ m/s} = 0\text{ m/s} + (2/3\text{ m/s}^2)(t_2 - 0) \Rightarrow t_2 = 24\text{ s}$$

8.55. Model: Model the steel block as a particle and use the model of kinetic friction.
Visualize:



Solve: **(a)** The components of thrust (F) along the *r*-, *t*-, and *z*-directions are

$$F_r = F\sin 20° = (3.5\text{ N})\sin 20° = 1.20\text{ N} \qquad F_t = F\cos 20° = (3.5\text{ N})\cos 20° = 3.29\text{ N} \qquad F_z = 0\text{ N}$$

Newton's second law is

$$(F_{net})_r = T + F_r = mr\omega^2 \quad (F_{net})_t = F_t - f_k = ma_t$$
$$(F_{net})_z = n - mg = 0\text{ N}$$

The *z*-component equation means $n = mg$. The force of friction is

$$f_k = \mu_k n = \mu_k mg = (0.60)(0.500\text{ kg})(9.8\text{ m/s}^2) = 2.94\text{ N}$$

Substituting into the *t*-component of Newton's second law

$$(3.29\text{ N}) - (2.94\text{ N}) = (0.500\text{ kg})a_t \Rightarrow a_t = 0.70\text{ m/s}^2$$

Having found a_t, we can now find the tangential velocity after $10 \text{ revolutions} = 20\pi$ rad as follows:

$$\theta_1 = \frac{1}{2}\left(\frac{a_t}{r}\right)t_1^2 \Rightarrow t_1 = \sqrt{\frac{2r\theta_1}{a_t}} = 18.95 \text{ s}$$

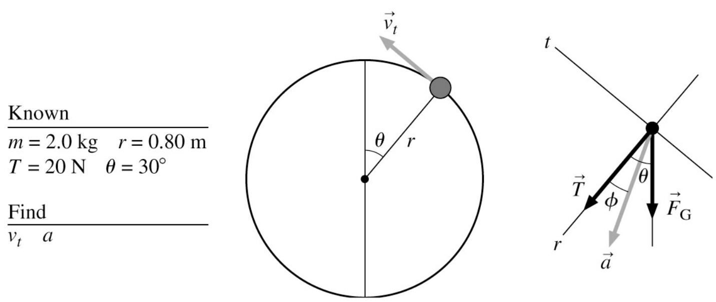
$$\omega_1 = \omega_0 + \left(\frac{a_t}{r}\right)t_1 = 6.63 \text{ rad/s}$$

The block's angular velocity after 10 rev is 6.6 rad/s.
(b) Substituting ω_1 into the r-component of Newton's second law yields:

$$T_1 + F_r = mr\omega_1^2 \Rightarrow T_1 + (1.20 \text{ N}) = (0.500 \text{ kg})(2.0 \text{ m})(6.63 \text{ rad/s})^2 \Rightarrow T_1 = 44 \text{ N}$$

8.56. Model: Assume the particle model for a ball in vertical circular motion.
Visualize:

Pictorial representation



Solve: (a) Newton's second law in the r- and t-directions is

$$(F_{\text{net}})_r = T + mg\cos\theta = ma_r = \frac{mv_t^2}{r} \quad (F_{\text{net}})_t = -mg\sin\theta = ma_t$$

Substituting into the r-component,

$$(20 \text{ N}) + (2.0 \text{ kg})(9.8 \text{ m/s}^2)\cos 30° = (2.0 \text{ kg})\frac{v_t^2}{(0.80 \text{ m})} \Rightarrow v_t = 3.85 \text{ m/s}$$

The tangential velocity is 3.8 m/s.
(b) Substituting into the t-component,

$$-(9.8 \text{ m/s}^2)\sin 30° = a_t \Rightarrow a_t = -4.9 \text{ m/s}^2$$

The radial acceleration is

$$a_r = \frac{v_t^2}{r} = \frac{(3.85 \text{ m/s})^2}{0.80 \text{ m}} = 18.5 \text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(18.5 \text{ m/s}^2)^2 + (-4.9 \text{ m/s}^2)^2} = 19.1 \text{ m/s}^2 \approx 19 \text{ m/s}^2$$

The angle of the acceleration vector from the r-axis is

$$\text{o} = \tan^{2\ 1}\frac{|a_t|}{a_r} = \tan^{2\ 1}\frac{4.9}{18.5} = 14.8° \approx 15°$$

The angle is below the r-axis.

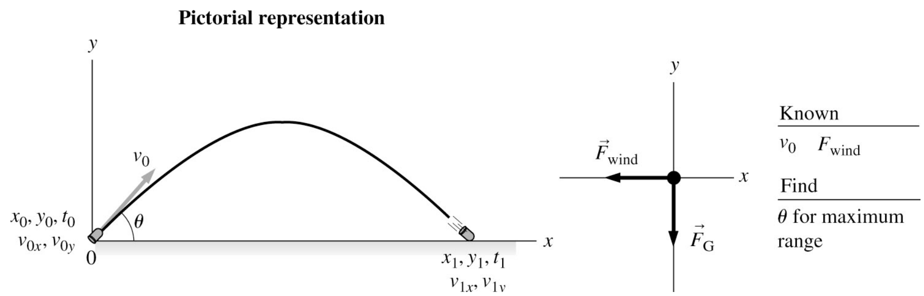
8.57. Solve: (a) You are spinning a lead fishing weight in a horizontal 1.0 m diameter circle on the ice of a pond when the string breaks. You know that the test weight (breaking force) of the line is 60 N and that the lead weight has a mass of 0.30 kg. What was the weight's angular velocity in rad/s and in rpm?

(b) $$\omega^2 = \frac{60 \text{ N}}{(0.3 \text{ kg})(0.5 \text{ m})} \Rightarrow \omega = 20 \text{ rad/s} \times \frac{\text{rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{\text{min}} = 191 \text{ rpm}$$

8.58. Solve: (a) At what speed does a 1500 kg car going over a hill with a radius of 200 m have a weight of 11,760 N?
(b) The weight is the normal force.

$$2940\text{ N} = \frac{1500\text{ kg } v^2}{200\text{ m}} \Rightarrow v = 19.8\text{ m/s}$$

8.59. Model: Assume the particle model and apply the constant-acceleration kinematic equations.
Visualize:



Solve: (a) Newton's second law for the projectile is

where F_{wind} is shortened to F. For the y-motion:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow 0\text{ m} = 0\text{ m} + (v_0\sin\theta)t_1 - \tfrac{1}{2}gt_1^2 \Rightarrow t_1 = 0\text{ s and } t_1 = \frac{2v_0\sin\theta}{g}$$

Using the above expression for t_1 and defining the range as R we get from the x motion:

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2$$

$$\Rightarrow x_1 - x_0 = R = v_{0x}t_1 + \tfrac{1}{2}\left(-\frac{F}{m}\right)t_1^2 = (v_0\cos\theta)\left(\frac{2v_0\sin\theta}{g}\right) - \frac{F}{2m}\left(\frac{2v_0\sin\theta}{g}\right)^2$$

$$= \frac{2v_0^2}{g}\cos\theta\sin\theta - \frac{2v_0^2F}{mg^2}\sin^2\theta$$

We will now maximize R as a function of θ by setting the derivative equal to 0:

$$\frac{dR}{d\theta} = \frac{2v_0^2}{g}(\cos^2\theta - \sin^2\theta) - \frac{2Fv_0^2}{mg^2}2\sin\theta\cos\theta = 0$$

$$\Rightarrow \cos^2\theta - \sin^2\theta = \cos 2\theta = \left(\frac{2Fv_0^2}{mg^2}\right)\left(\frac{g}{2v_0^2}\right)\sin 2\theta \Rightarrow \tan 2\theta = \frac{mg}{F}$$

Thus the angle for maximum range is $\theta = \tfrac{1}{2}\tan^{-1}(mg/F)$.

(b) We have

$$\frac{mg}{F} = \frac{(0.50\text{ kg})(9.8\text{ m/s}^2)}{0.60\text{ N}} = 8.167 \Rightarrow \theta = \tfrac{1}{2}\tan^{-1}(8.167) = 41.51°$$

The maximum range without air resistance is

$$R' = \frac{2v_0^2\sin 45°\cos 45°}{g} = \frac{v_0^2}{g}$$

Therefore, we can write the equation for the range R as

$$R = 2R'\sin 41.51°\cos 41.51° - \frac{2F}{mg}R'\sin^2 41.51° = R'(0.9926 - 0.1076) = 0.885R'$$

$$\Rightarrow \frac{R}{R'} = 0.8850 \Rightarrow \frac{R' - R}{R'} = 1 - 0.8850 = 0.115$$

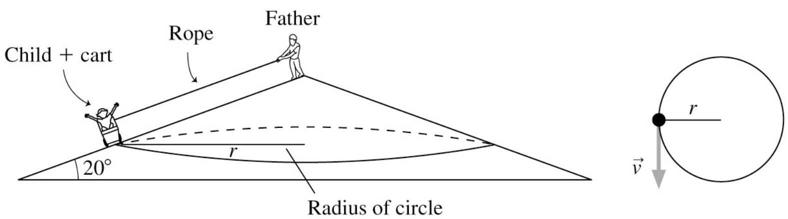
Thus R is reduced from R' by 11.5%.

Assess: The condition for maximum range $(\tan 2\theta = mg/F)$ means $2\theta \to 90°$ as $F \to 0$. That is, $\theta = 45°$ when $F = 0$, as is to be expected.

8.60. Model: Use the particle model for the (cart + child) system which is in uniform circular motion.

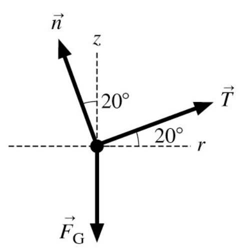
Visualize:

Pictorial representation



Known
$m = 25$ kg
$r = (2.0)\cos 20° = 1.88$ m
$\omega = 13.5$ rpm

Find
T



Forces in the r-z plane

Solve: Newton's second law along r and z directions can be written:

$$\sum F_r = T\cos 20° - n\sin 20° = ma_r \quad \sum F_z = T\sin 20° - n\cos 20° - mg = 0$$

The cart's centripetal acceleration is

$$a_r = r\omega^2 = (2.0\cos 20°\text{ m})\left(14\,\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}}\right)^2 = 4.04\text{ m/s}^2$$

The above force equations can be rewritten as

$$0.94T - 0.342n = (25\text{ kg})(4.04\text{ m/s}^2) = 101\text{ N}$$

$$0.342T + 0.94n = (25\text{ kg})(9.8\text{ m/s}^2) = 245\text{ N}$$

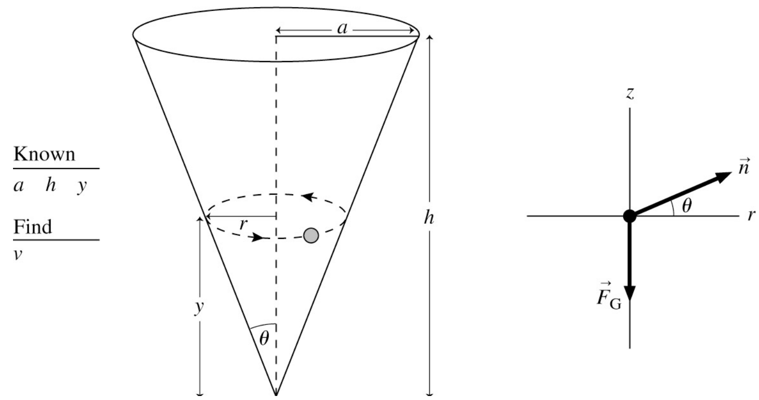
Solving these two equations yields $T = 179\text{ N} \approx 180\text{ N}$ for the tension in the rope.

Assess: In view of the (child + cart) weight of 245 N, a tension of 179 N is reasonable.

8.61. Model: Use the particle model for the ball, which is in uniform circular motion.

Visualize:

Pictorial representation



Solve: From Newton's second law along *r* and *z* directions,

$$\sum F_r = n\cos\theta = \frac{mv^2}{r} \qquad \sum F_z = n\sin\theta - mg = 0 \Rightarrow n\sin\theta = mg$$

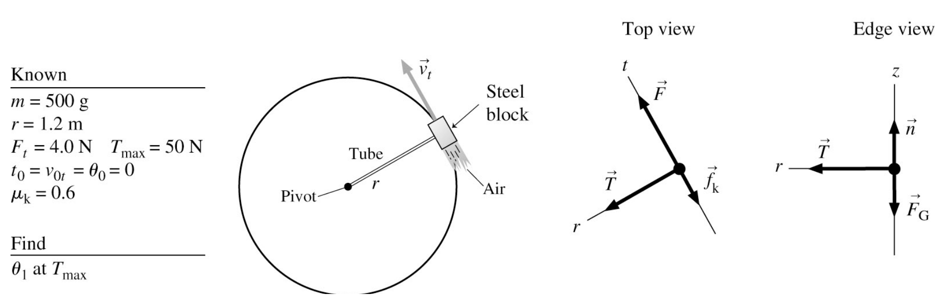
Dividing the two force equations gives

$$\tan\theta = \frac{gr}{v^2}$$

From the geometry of the cone, $\tan\theta = r/y$. Thus

$$\frac{r}{y} = \frac{gr}{v^2} \Rightarrow v = \sqrt{gy}$$

8.62. Model: Model the block as a particle and use the model of kinetic friction.
Visualize:



Solve: The only radial force is tension, so we can use Newton's second law to find the angular velocity ω_{max} at which the tube breaks:

$$\sum F_r = T = m\omega^2 r \Rightarrow \omega_{max} = \sqrt{\frac{T_{max}}{mr}} = \sqrt{\frac{50\text{ N}}{(0.50\text{ kg})(1.2\text{ m})}} = 9.12\text{ rad/s}$$

The compressed air and friction exert tangential forces, and the second law along the tangential direction is

$$\sum F_t = F_t - f_k = F_t - \mu_k n = F_t - \mu_k mg = ma_t$$

$$a_t = \frac{F_t}{m} - \mu_k g = \frac{4.0\text{ N}}{0.50\text{ kg}} - (0.60)(9.80\text{ m/s}^2) = 2.12\text{ m/s}^2$$

The time needed to accelerate to 9.12 rad/s is given by

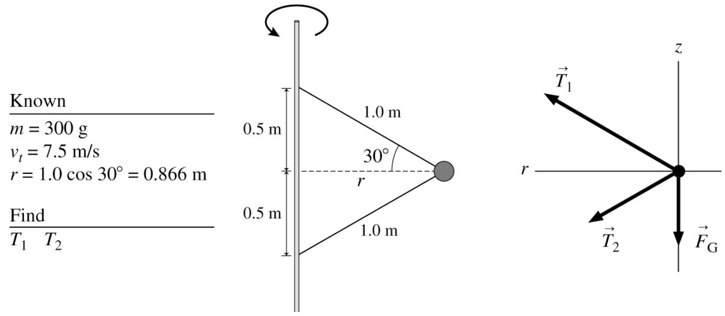
$$\omega_1 = \omega_{max} = 0 + \left(\frac{a_t}{r}\right)t_1 \Rightarrow t_1 = \frac{r\omega_{max}}{a_t} = \frac{(1.2)(9.12\text{ rad/s})}{2.12\text{ m/s}^2} = 5.16\text{ s}$$

During this interval, the block turns through angle

$$\Delta\theta = \theta_1 - \theta_0 = \omega_0 t_1 + \frac{1}{2}\left(\frac{a_t}{r}\right)t_1^2 = 0 + \frac{1}{2}\left(\frac{2.12\text{ m/s}^2}{1.2\text{ m}}\right)(5.16\text{ s})^2 = 23.52\text{ rad} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = 3.7\text{ rev}$$

8.63. Model: Use the particle model for a sphere revolving in a horizontal circle.
Visualize:

Pictorial representation



Solve: Newton's second law in the r- and z-directions is

$$\sum(F)_r = T_1\cos30° + T_2\cos30° = \frac{mv_t^2}{r} \qquad \sum(F)_z = T_1\sin30° - T_2\sin30° - F_G = 0\text{ N}$$

Using $r = (1.0\text{ m})\cos30° = 0.886\text{ m}$, these equations become

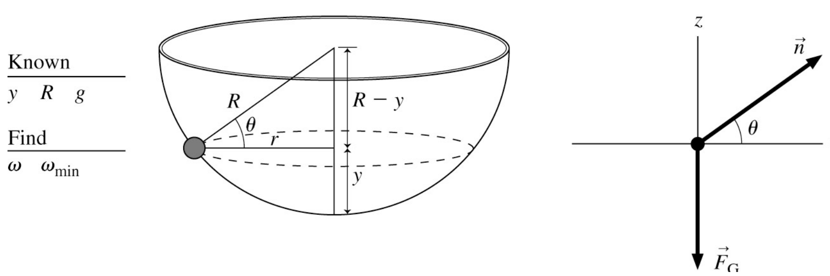
$$T_1 + T_2 = \frac{mv_t^2}{r\cos30°} = \frac{(0.300\text{ kg})(7.5\text{ m/s})^2}{(0.866\text{ m})(0.866)} = 22.5\text{ N}$$

$$T_1 - T_2 = \frac{mg}{\sin30°} = \frac{(0.300\text{ kg})(9.8\text{ m/s}^2)}{(0.5)} = 5.88\text{ N}$$

Solving for T_1 and T_2 yields $T_1 = 14.2\text{ N} \approx 14\text{ N}$ and $T_2 = 8.3\text{ N}$.

8.64. Model: Use the particle model for the ball.
Visualize:

Pictorial representation



Solve: (a) Newton's second law along the r- and z-directions is

$$\sum F_r = n\cos\theta = mr\omega^2 \qquad \sum F_z = n\sin\theta - F_G = 0\text{ N}$$

Using $F_G = mg$ and dividing these equations yields:

$$\tan\theta = \frac{g}{r\omega^2} = \frac{R-y}{r}$$

where you can see from the figure that $\tan\theta = (R-y)/r$. Thus $\omega = \sqrt{\frac{g}{R-y}}$.

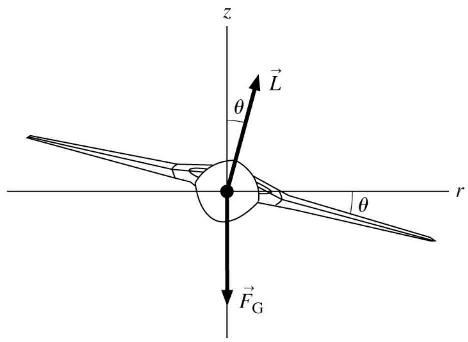
(b) ω will be minimum when $(R-y)$ is maximum or when $y = 0$ m. Then $\omega_{min} = \sqrt{g/R}$.

(c) Substituting into the above expression,

$$\omega = \sqrt{\frac{g}{R-y}} = \sqrt{\frac{9.8\text{ m/s}^2}{0.20\text{ m} - 0.10\text{ m}}} = 9.9\ \frac{\text{rad}}{\text{s}} \times \frac{60\text{ s}}{1\text{ min}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = 95\text{ rpm}$$

8.65. Model: Use the particle model for the airplane.
Visualize:

Pictorial representation



Known
$\theta = 10°$
$v = 400$ mph

Find
$2r$

Solve: In level flight, the lift force L balances the gravitational force. When turning, the plane banks so that the radial component of the lift force can create a centripetal acceleration. Newton's second law along the *r*- and *z*-directions is

$$\sum F_r = L\sin\theta = \frac{mv_t^2}{r} \qquad \sum F_z = L\cos\theta - mg = 0\text{ N}$$

These can be written:

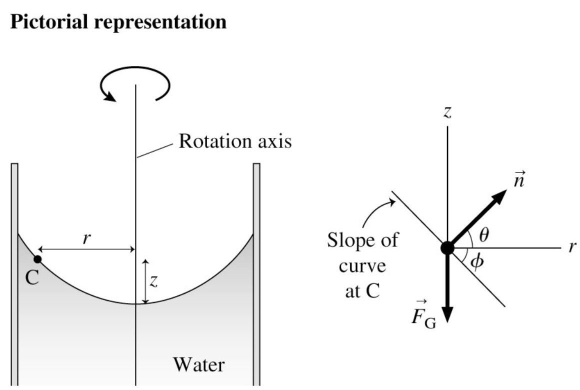
$$\sin\theta = \frac{mv^2}{rL} \quad \cos\theta = \frac{mg}{L}$$

Dividing the two equations gives:

$$\tan\theta = \frac{v^2}{gr} \Rightarrow r = \frac{v^2}{g\tan\theta} = \frac{\left[400\frac{\text{miles}}{\text{hour}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1610\text{ m}}{1\text{ mile}}\right]^2}{\left[9.8\text{ m/s}^2\right]\tan 10°} = 18.5\text{ km}$$

The diameter of the airplane's path around the airport is $2 \times 18.5\text{ km} = 37\text{ km}$.

8.66. Model: Use the particle model for a small volume of water on the surface.
Visualize:



Solve: Consider a particle of water of mass *m* at point C on the surface. Newton's second law along the *r*- and *z*-directions is

$$(F_{\text{net}})_r = n\cos\theta = mr\omega^2 \Rightarrow \cos\theta = \frac{mr\omega^2}{n}$$

$$(F_{\text{net}})_z = n\sin\theta - mg = 0\text{ N} \Rightarrow \sin\theta = \frac{mg}{n}$$

Dividing both equations gives $\tan\theta = g/r\omega^2$. For a parabola $z = ar^2$. This means

$$\frac{dz}{dr} = 2ar = \text{slope of the curve at } C = \tan\phi = \tan(90° - \theta) = \frac{1}{\tan\theta} \Rightarrow \tan\theta = \frac{1}{2ar}$$

Equating the two equations for $\tan\theta$, we get

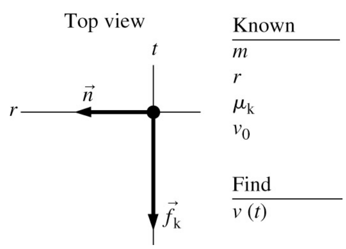
$$\frac{1}{2ar} = \frac{g}{r\omega^2} \Rightarrow a = \frac{\omega^2}{2g}$$

Thus the surface is described by the equation

$$z = \frac{\omega^2}{2g}r^2$$

which is the equation of a parabola.

8.67. Model: Model the block as a particle in circular motion.
Visualize: Apply Newton's second law in the *r*- and *t*-directions. We seek $v(t)$ where v is the tangential speed.



Solve:

$$\sum F_r = n = ma_r = \frac{mv^2}{r} \qquad \sum F_t = -f_k = ma_t$$

Use $n = mv^2/r$ in $f_k = \mu_k n$.

$$-f_k = -\mu_k n = -\mu_k \frac{mv^2}{r} = ma_t$$

Cancel m and use $a_t = \frac{dv}{dt}$.

$$-\frac{\mu_k}{r} v^2 = \frac{dv}{dt}$$

Separate variables and integrate.

$$-\frac{\mu_k}{r}\int_0^t dt = \int_{v_0}^v \frac{dv}{v^2}$$

$$-\frac{\mu_k}{r}[t]_0^t = \left[\frac{-1}{v}\right]_{v_0}^v$$

$$-\frac{\mu_k}{r}t = \frac{1}{v_0} - \frac{1}{v}$$

Solve for v which is a function of t.

$$\frac{1}{v} = \frac{1}{v_0} + \frac{\mu_k}{r}t \Rightarrow v = \left(\frac{1}{v_0} + \frac{\mu_k}{r}t\right)^{-1} = \frac{rv_0}{r + v_0\mu_k t}$$

Assess: The dependencies seem to go in the right direction: as t increases $v(t)$ decreases; as μ_k increases $v(t)$ decreases. The m canceled out, so the result does not depend on the mass.