9

Impulse and Momentum

Conceptual Questions

9.1. The velocities and masses vary from object to object, so there is no choice but to compute $p_x = mv_x$ for each one and then compare:

$$p_{1x} = (20\text{ g})(1\text{ m/s}) = 20\text{ g m/s}$$
$$p_{2x} = (20\text{ g})(2\text{ m/s}) = 40\text{ g m/s}$$
$$p_{3x} = (10\text{ g})(2\text{ m/s}) = 20\text{ g m/s}$$
$$p_{4x} = (10\text{ g})(1\text{ m/s}) = 10\text{ g m/s}$$
$$p_{5x} = (200\text{ g})(0.1\text{ m/s}) = 20\text{ g m/s}$$

So the answer is $p_{2x} > p_{1x} = p_{3x} = p_{5x} > p_{4x}$.

9.2. Impulse gives a measure of the effect of a force acting over a period of time. During the time a net force acts on an object, the object will accelerate and its velocity will change. Impulse gives a measure of the effect of a force without specifying the detailed time dependence of the force.

9.3. An isolated system is a collection of interacting objects for which all outside forces (i.e., forces from objects outside the system) are balanced; that is, in an isolated system, all external forces cancel each other out.

9.4. When the question talks about forces, times, and momenta, we immediately think of the impulse-momentum theorem, which tells us that, to change the momentum of an object, we must exert a net external force on it over a time interval: $\Delta\vec{p} = \vec{F}_{\text{avg}}\Delta t$. Because equal forces are exerted over equal times, the impulses are equal and the changes in momentum are equal. Because both carts start from rest, the change in momentum of each is the same as the final momentum of each, so their final momenta are equal. Notice that, to answer the question, we do not need to know the mass of either cart, or even the specific time interval (as long as it is the same for both carts).

9.5. The impulse-momentum theory tells us that the change in momentum of an object is related to the net force on the object and the length of time the force was applied. Mathematically, $\Delta\vec{p} = \vec{F}_{\text{avg}}\Delta t$. The same force applied to the two carts results in a larger acceleration for the less massive plastic cart (Newton's second law), enabling it to travel the 1-m distance in a shorter time. Therefore, the plastic cart has a smaller change in momentum than the lead cart. Because the final momentum of each cart is equal to their change in momentum (zero initial momentum), the final momentum of the plastic cart is less than that of the lead cart.

9.6. In this story, Carlos is correct. During the short time of the bullet-block collision other forces are negligible compared to the force between the bullet and block, so in the impulse approximation momentum is conserved. When the bullet bounces off of the steel block, the bullet's final momentum is backward. To balance that, the steel block must be moving forward faster than the case in which the bullet embeds itself in the wooden block (in which case the bullet has a final momentum in the forward direction).

9.7. The impulse-momentum theory states that a change in an object's momentum results when a net force is applied to the object for some time interval; $\Delta\vec{p} = \vec{F}_{avg}\Delta t.$ Stopping a hard ball requires changing its momentum, and this change can be accomplished with a small force over a long time interval or a large force over a short time interval. The padding in a glove lets the time interval during which the ball is stopped be long, resulting in a smaller force on the glove and on your hand.

9.8. The impulse-momentum theory states that a change in an object's momentum results when a net force is applied to the object for some time interval; $\Delta\vec{p} = \vec{F}_{avg}\Delta t.$ Stopping an automobile requires changing its momentum from some to none. This change can be accomplished with a small force over a long time interval or a large force over a short time interval. The crumple zone that collapses during an automobile collision lengthens the time interval during which the automobile is stopped, resulting in a smaller force on the passengers as they also come to a stop.

9.9. The impulse is equal to the change in momentum, so

$$\Delta p_x = mv_{fx} - mv_{ix} = 4 \text{ Ns}$$

The final velocity is thus

$$v_{fx} = v_{ix} + \frac{4 \text{ Ns}}{m} = 1 \text{ m/s} + \frac{4 \text{ Ns}}{2 \text{ kg}} = 3 \text{ m/s}$$

Since the velocity is positive, the object is moving to the right.

9.10. The impulse is equal to the change in momentum, so

$$\Delta p_x = mv_{fx} - mv_{ix} = -4 \text{ Ns}$$

The final velocity is thus

$$v_{fx} = v_{ix} - \frac{4 \text{ Ns}}{m} = 1 \text{ m/s} - \frac{4 \text{ Ns}}{2 \text{ kg}} = -1 \text{ m/s}$$

Since the velocity is negative, the object is now moving to the left with a speed of 1 m/s. Note that the impulse was negative, which decreases the initially positive velocity.

9.11. The club and ball form a system. The interaction force when the club and ball collide is very large compared to other forces at the time of collision, such as gravity and the force of the golfer on the club. So, in this impulse approximation, momentum is conserved during the collision. After the club hits the ball, it will give the ball some of its momentum. The club can continue to move forward as long as the momentum the ball obtains is less than the initial momentum of the club. Note that the momentum conservation is only valid if we consider the short time between just before and just after the collision. The wider we make the time window, the more time gravity and the golfer have to influence the motion of the club and ball, so that momentum conservation would no longer hold for the club-ball system.

9.12. The impulse one ball receives is equal to the average force on it from the other ball multiplied by the time during which the force is applied. But by Newton's third law the force that the rubber ball exerts on the steel ball is equal to the force the steel ball exerts on the rubber ball. So both balls receive the same amount of impulse, although the impulses are in opposite directions.

9.13. **(a)** Both particles cannot be at rest immediately after the collision. If they were both at rest, then the sum of the momenta after the collision would be zero, and since momentum is conserved in collisions, it would have had to be zero before as well (and it wasn't).
(b) If the masses are equal and the collision is elastic, the moving particle will stop and give all of its momentum to the previously resting particle. A good example of this appears when a billiard ball collides head-on with another billiard ball that is at rest.

We say momentum is conserved in all collisions because we assume that both colliding objects are part of the system and we assume the "impulse approximation" prevails, meaning that other forces can be neglected during the short time interval of the collision. In part (a), if the system contained a third particle that participated in the collision, then it is possible for the first two particles to end up at rest if the momentum were carried off by the third particle.

9.14. (a) Let Paula and Ricardo be a system. Initially, their total momentum is zero since they are at rest. After they push off each other, the total momentum must still be zero, so Ricardo and Paula must have equal but opposite momenta.
(b) Momentum $p = mv$. Since Paula is less massive than Ricardo, her speed must be higher than Ricardo's for her to have the same momentum as Ricardo.

Exercises and Problems

Section 9.1 Momentum and Impulse

9.1. Model: Model the car and the baseball as particles.

Solve: (a) The momentum $p = mv = (3000\text{ kg})(15\text{ m/s}) = 4.5 \times 10^4\text{ kg m/s}$.

(b) The momentum $p = mv = (0.20\text{kg})(40\text{ m/s}) = 8.0\text{ kg m/s}$.

9.2. Model: Model the bicycle and its rider as a particle. Also model the car as a particle.
Solve: From the definition of momentum,

$$p_{\text{car}} = p_{\text{bicycle}} \Rightarrow m_{\text{car}} v_{\text{car}} = m_{\text{bicycle}} v_{\text{bicycle}} \Rightarrow v_{\text{bicycle}} = \left(\frac{m_{\text{car}}}{m_{\text{bicycle}}}\right) v_{\text{car}} = \left(\frac{1500\text{ kg}}{100\text{ kg}}\right)(5.0\text{ m/s}) = 75\text{ m/s}$$

Assess: This is a very high speed $(\approx 168\text{ mph})$. This problem shows the importance of mass in comparing two momenta.

9.3. Visualize: Please refer to Figure EX9.3.

Solve: The impulse J_x is defined in Equation 9.6 as $J_x = \int_{t_i}^{t_f} F_x(t)\,dt =$ area under the $F_x(t)$ curve between t_i and t_f. The area under the force-time curve in Figure EX9.3 is

$J_x = (2\text{ ms})(1000\text{ N}) + \frac{1}{2}(6\text{ ms} - 2\text{ ms})(1000\text{ N}) = 4\text{ N s}.$

9.4. Model: The particle is subjected to an impulsive force.
Visualize: Please refer to Figure EX9.4.
Solve: Using Equation 9.6, the impulse is the area under the force-time curve. From 0 to 2 ms the impulse is

$$\int F(t)\,dt = \tfrac{1}{2}(-500\text{ N})(2 \times 10^{-3}\text{ s}) = -0.5\text{ Ns}$$

From 2 to 8 ms the impulse is

$$\int F(t)\,dt = \tfrac{1}{2}(+2000\text{ N})(8\text{ ms} - 2\text{ ms}) = +6.0\text{ Ns}$$

From 8 ms to 10 ms the impulse is

$$\int F(t)\,dt = \tfrac{1}{2}(-500\text{ N})(10\text{ ms} - 8\text{ ms}) = -0.5\text{ Ns}$$

Thus, from 0 s to 10 ms the impulse is $(-0.5 + 6.0 - 0.5)\text{ Ns} = 5\text{ Ns}$.

9.5. Visualize: Please refer to Figure EX9.5.

Solve: The impulse is defined in Equation 9.6 as $J_x = \int_{t_i}^{t_f} F_x(t)\,dt =$ area under the $F_x(t)$ curve between t_i and t_f.

For the force-time curve shown in Figure EX9.5, the impulse is $6.0 \text{ Ns} = \frac{1}{2}(F_{max})(8.0 \text{ ms}) \Rightarrow F_{max} = 1.5 \times 10^3$ N.

9.6. Model: Model the object as a particle and the interaction as a collision.
Visualize: Please refer to Figure EX9.6.
Solve: The momentum bar chart tells us the final momentum $(p_{fx} = 2 \text{ kg m/s})$ and the impulse $(J_x = 6 \text{ kg m/s})$. Using the impulse-momentum theorem $p_{fx} = p_{ix} + J_x$, we can find the initial momentum:

$$p_{ix} = p_{fx} - J_x = 2 \text{ kg m/s} - 6 \text{ kg m/s} = -4 \text{ kg m/s}$$

Since $p_{ix} = mv_{ix}$ we have $v_{ix} = p_{ix}/m = (-4 \text{ kg m/s})/(0.05 \text{ kg}) = -80 \text{ m/s}$. The speed is thus 80 m/s and the direction is to the left.

9.7. Model: Model the object as a particle and the interaction as a collision.
Visualize: Please refer to Figure EX9.7.
Solve: The object is initially moving to the right (positive momentum) and ends up moving to the left (negative momentum). Using the impulse-momentum theorem $p_{fx} = p_{ix} + J_x$,

$$-2 \text{ kg m/s} = +6 \text{ kg m/s} + J_x \quad \Rightarrow \quad J_x = -8 \text{ kg m/s} = -8 \text{ Ns}$$

Since $J_x = F_{avg}\Delta t$, we have

$$F_{avg}\Delta t = -8 \text{ Ns} \quad \Rightarrow \quad F_{avg} = \frac{-8 \text{ Ns}}{10 \text{ ms}} = -8 \times 10^2 \text{ N}$$

Thus, the force is 800 N to the left.

Section 9.2 Solving Impulse and Momentum Problems

9.8. Model: Model the object as a particle and the interaction with the force as a collision.
Visualize: Please refer to Figure EX9.8.
Solve: Using the equations

$$p_{fx} = p_{ix} + J_x \text{ and } J_x = \int_{t_i}^{t_f} F_x(t)\,dt = \text{area under force curve}$$

$$(2.0 \text{ kg})v_{fx} = (2.0 \text{ kg})(1.0 \text{ m/s}) + (\text{area under the force curve})$$

$$v_{fx} = (1.0 \text{ m/s}) + \frac{1}{2.0 \text{ kg}}(1.0 \text{ s})(2.0 \text{ N}) = 2.0 \text{ m/s}$$

Becaue v_{fx} is positive, the object moves to the right at 2.0 m/s.
Assess: For an object with positive velocity, a positive impulse increases the object's speed. The opposite is true for an object with negative velocity.

9.9. Model: Model the object as a particle and the interaction with the force as a collision.
Visualize: Please refer to Figure EX9.9.
Solve: Using the equations

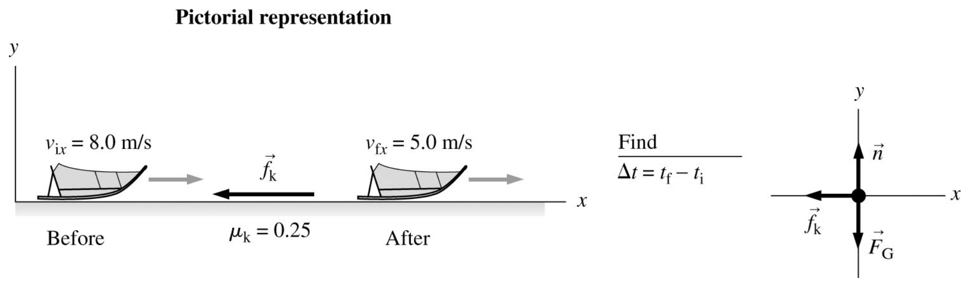
$$p_{fx} = p_{ix} + J_x \text{ and } J_x = \int_{t_i}^{t_f} F_x(t)\,dt = \text{area under force curve}$$

$$(2.0 \text{ kg})v_{fx} = (2.0 \text{ kg})(1.0 \text{ m/s}) + (\text{area under the force curve})$$

$$v_{fx} = (1.0 \text{ m/s}) + \frac{1}{2.0 \text{ kg}}(-8.0 \text{ N})(0.50 \text{ s}) = -1.0 \text{ m/s}$$

Because v_{fx} is negative, the object is now moving to the left at 1.0 m/s.
Assess: The direction of the velocity has reversed.

9.10. Model: Use the particle model for the sled, the model of kinetic friction, and the impulse-momentum theorem.
Visualize:



Note that the force of kinetic friction f_k imparts a negative impulse to the sled.

Solve: Using $\Delta p_x = J_x$, we have

$$p_{fx} - p_{ix} = \int_{t_i}^{t_f} F_x(t)dt = -f_k \int_{t_i}^{t_f} dt = -f_k \Delta t \quad \Rightarrow \quad mv_{fx} - mv_{ix} = -\mu_k n \Delta t = -\mu_k mg \Delta t$$

We have used the model of kinetic friction $f_k = \mu_k n$, where μ_k is the coefficient of kinetic friction and n is the normal (contact) force by the surface. The force of kinetic friction is independent of time and was therefore taken out of the impulse integral. Thus,

$$\Delta t = \frac{1}{\mu_k g}(v_{ix} - v_{fx}) = \frac{(8.0 \text{ m/s} - 5.0 \text{ m/s})}{(0.25)(9.8 \text{ m/s}^2)} = 1.2 \text{ s}$$

9.11. Model: Model the rocket as a particle, and use the impulse-momentum theorem. The only force acting on the rocket is due to its own thrust.
Visualize: Please refer to Figure EX9.11.
Solve: (a) The impulse is

$$J_x = \int F_x(t)\,dt = \text{area of the graph of } F_x(t) \text{ between } t = 0 \text{ s and } t = 30 \text{ s} = \tfrac{1}{2}(1000 \text{ N})(30 \text{ s}) = 1.5 \times 10^4 \text{ Ns}$$

(b) From the impulse-momentum theorem, $p_{fx} = p_{ix} + J_x(t)$, so the momentum or velocity increases as long as J_x is positive. If J_x becomes negative, the speed will stop increasing and start decreasing, so this point will be the maximum. For the current problem, the impluse is always positive, so the speed increases continuously. The maximum is therefore at the end of the impluse $(t = 30 \text{ s})$. The speed at this point is

$$mv_{fx} = mv_{ix} + 1.5 \times 10^4 \text{ Ns} \quad \Rightarrow \quad (425 \text{ kg})v_{fx} = (425 \text{ kg})(75 \text{ m/s}) + 1.5 \times 10^4 \text{ Ns} \quad \Rightarrow \quad v_{fx} = 110 \text{ m/s}$$

9.12. Model: Model the ball as a particle, and its interaction with the wall as a collision in the impulse approximation.
Visualize: Please refer to Figure EX9.12.
Solve: Using the equations

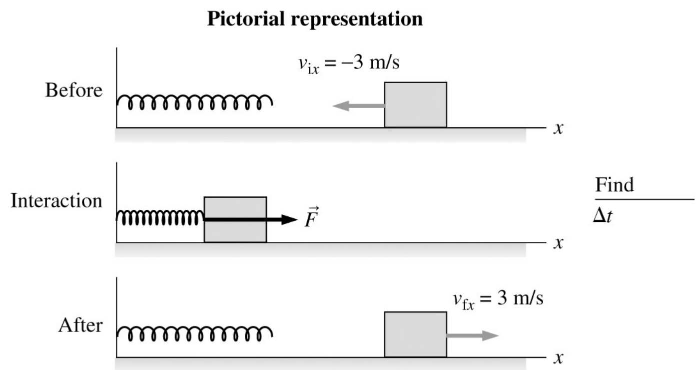
$$p_{fx} = p_{ix} + J_x \text{ and } J_x = \int_{t_i}^{t_f} F_x(t)dt = \text{area under force curve}$$

$$(0.250 \text{ kg})v_{fx} = (0.250 \text{ kg})(-10 \text{ m/s}) + (500 \text{ N})(8.0 \text{ ms})$$

$$v_{fx} = (-10 \text{ m/s}) + \left(\frac{4.0 \text{ N}}{0.250 \text{ kg}}\right) = 6.0 \text{ m/s}$$

Assess: The ball's final velocity is positive, indicating it has turned around.

9.13. Model: Model the glider cart as a particle, and its interaction with the spring as a collision. Assume a frictionless track.
Visualize:

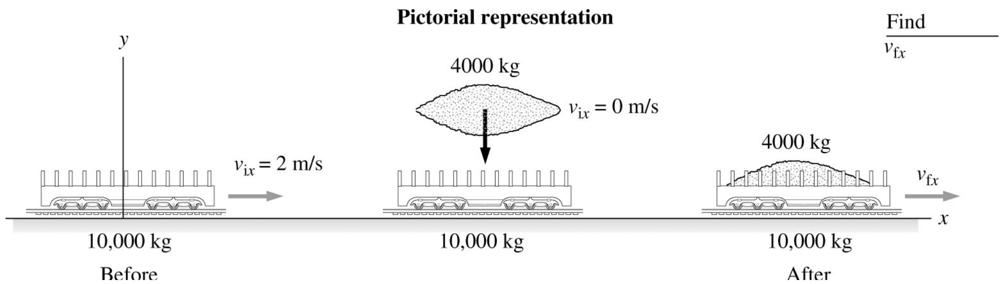


Solve: Using the impulse-momentum theorem $p_{fx} - p_{ix} = \int F(t)\,dt$,

$$(0.60\text{ kg})(3\text{ m/s}) - (0.60\text{ kg})(-3\text{ m/s}) = \text{area under force curve} = \tfrac{1}{2}(36\text{ N})(\Delta t) \quad\Rightarrow\quad \Delta t = 0.2\text{ s}$$

Section 9.3 Conservation of Momentum

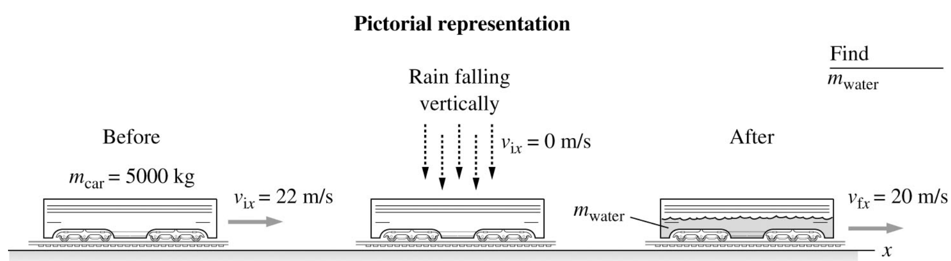
9.14. Model: Choose car + gravel to be the system. Ignore friction in the impulse approximation.
Visualize:



Solve: There are no *external* horizontal forces on the car + gravel system, so the horizontal momentum is conserved. This means $p_{fx} = p_{ix}$. Hence,

$$(10{,}000\text{ kg} + 4000\text{ kg})v_{fx} = (10{,}000\text{ kg})(2.0\text{ m/s}) + (4000\text{ kg})(0.0\text{ m/s}) \quad\Rightarrow\quad v_{fx} = 1.4\text{ m/s}$$

9.15. Model: Choose car + rainwater to be the system.
Visualize:

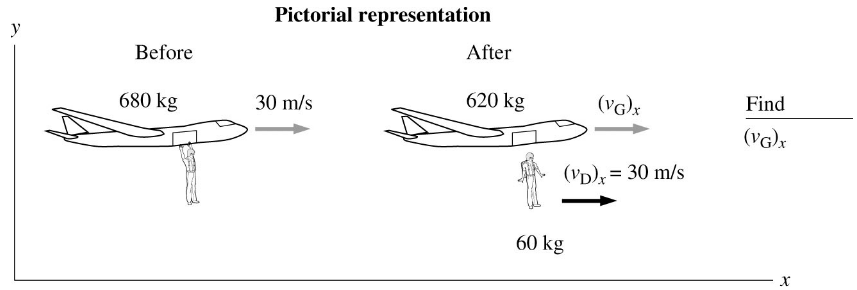


There are no *external* horizontal forces on the car + water system, so the horizontal momentum is conserved.
Solve: Conservation of momentum gives $p_{fx} = p_{ix}$. Hence,

$$(m_{car} + m_{water})(20\text{ m/s}) = (m_{car})(22\text{ m/s}) + (m_{water})(0\text{ m/s})$$

$$(5000\text{ kg} + m_{\text{water}})(20\text{ m/s}) = (5000\text{ kg})(22\text{ m/s}) \quad \Rightarrow \quad m_{\text{water}} = 5.0 \times 10^2\text{ kg}$$

9.16. Model: Choose skydiver + glider to be the system in the impulse approximation. Ignore air resistance.
Visualize:



Note that there are no *external* forces in the x-direction (ignoring friction in the impulse approximation), implying conservation of momentum along the x-direction.
Solve: The momentum conservation equation $p_{fx} = p_{ix}$ gives

$$(680\text{ kg} - 60\text{ kg})(v_G)_x + (60\text{ kg})(v_D)_x = (680\text{ kg})(30\text{ m/s})$$

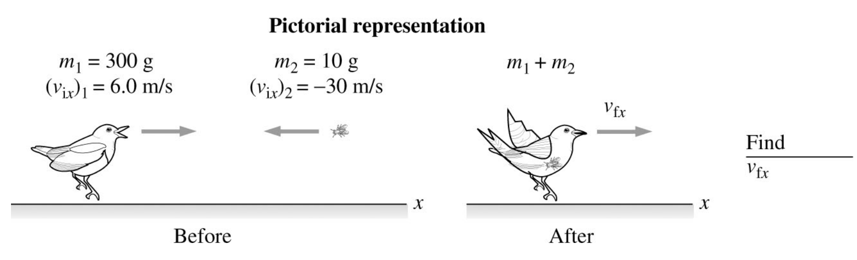
Immediately after release, the skydiver's horizontal velocity is still $(v_D)_x = 30$ m/s because he experiences no net horizontal force. Thus

$$(620\text{ kg})(v_G)_x + (60\text{ kg})(30\text{ m/s}) = (680\text{ kg})(30\text{ m/s}) \quad \Rightarrow \quad (v_G)_x = 30\text{ m/s}$$

Assess: The skydiver's motion in the vertical direction has *no* influence on the glider's horizontal motion. Notice that we did not need to invoke conservation of momentum to solve this problem. Because there are no external horizontal forces acting on either the skydiver or the glider, neither will change their horizontal speed when the skydiver lets go!

Section 9.4 Inelastic Collisions

9.17. Model: We will define our system to be bird + bug. This is the case of an inelastic collision because the bird and bug move together after the collision. Horizontal momentum is conserved because there are no external forces acting on the system during the collision in the impulse approximation.
Visualize:

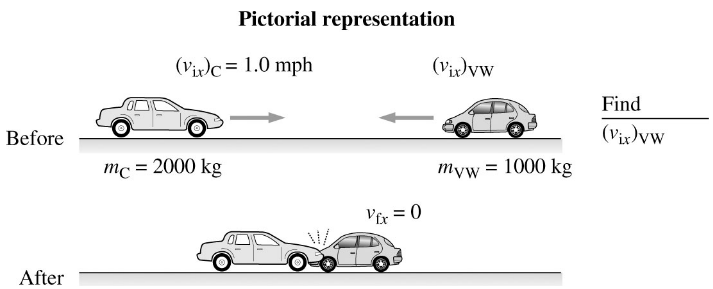


Solve: The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2 \quad \Rightarrow \quad (300\text{ g} + 10\text{ g})v_{fx} = (300\text{ g})(6.0\text{ m/s}) + (10\text{ g})(-30\text{ m/s}) \quad \Rightarrow \quad v_{fx} = 4.8\text{ m/s}$$

Assess: We left masses in grams, rather than convert to kilograms, because the mass units cancel out from both sides of the equation. Note that $(v_{ix})_2$ is negative because the bug is flying to the left.

9.18. Model: The two cars are not an isolated system because of external frictional forces. But during the collision friction is not going to be significant. Within the impulse approximation, the momentum of the Cadillac + Volkswagen system will be conserved in the collision.
Visualize:

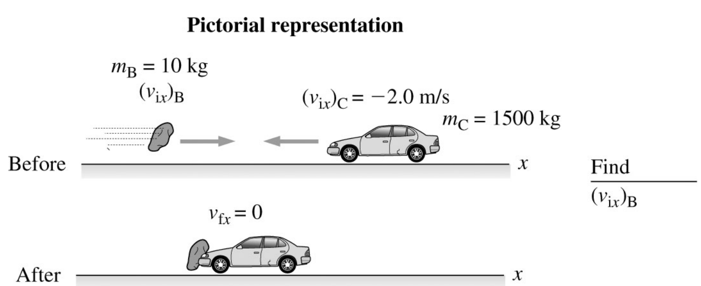


Solve: The momentum conservation equation $p_{fx} = p_{ix}$ gives

$$(m_C + m_{VW})v_{fx} = m_C(v_{ix})_C + m_{VW}(v_{ix})_{VW}$$
$$0 \text{ kg mph} = (2000 \text{ kg})(1.0 \text{ mph}) + (1000 \text{ kg})(v_{ix})_{VW} \quad \Rightarrow \quad (v_{ix})_{VW} = -2.0 \text{ mph}$$

so you need a *speed* of 2.0 mph.

9.19. Model: Because of external friction and drag forces, the car and the blob of sticky clay are not exactly an isolated system. But during the collision, friction and drag are not going to be significant. The momentum of the system will be conserved in the collision, within the impulse approximation.
Visualize:



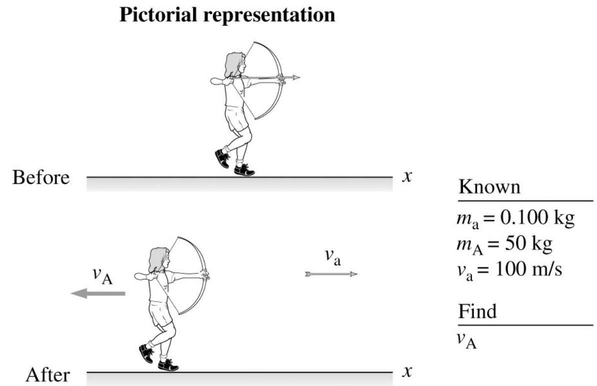
Solve: The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$(m_C + m_B)(v_f)_x = m_B(v_{ix})_B + m_C(v_{ix})_C$$
$$0 \text{ kg m/s} = (10 \text{ kg})(v_{ix})_B + (1500 \text{ kg})(-2.0 \text{ m/s}) \quad \Rightarrow \quad (v_{ix})_B = 3.0 \times 10^2 \text{ m/s}$$

Assess: This speed of the blob is around 600 mph, which is very large. However, a very large speed is *expected* in order to stop a car with only 10 kg of clay.

Section 9.5 Explosions

9.20. Model: We will define our system to be archer + arrow. The force of the archer (A) on the arrow (a) is equal to the force of the arrow on the archer. These are internal forces within the system. The archer is standing on frictionless ice, and the normal force by ice on the system balances the weight force. Thus $\vec{F}_{ext} = \vec{0}$ on the system, and momentum is conserved.
Visualize:



The initial momentum p_{ix} of the system is zero, because the archer and the arrow are at rest. The final moment p_{fx} must also be zero.

Solve: We have $M_A v_A + m_a v_a = 0$ kg m/s. Therefore,

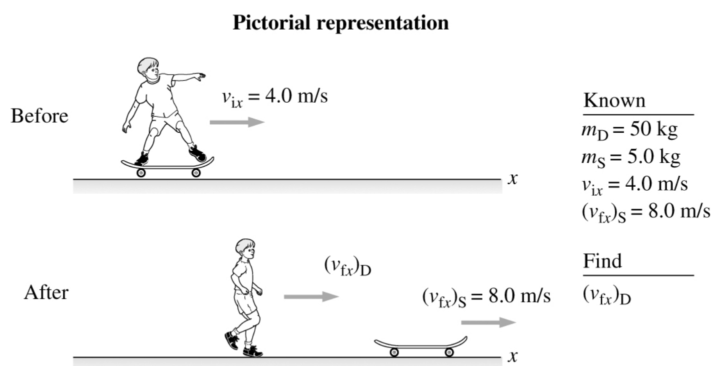
$$v_A = \frac{-m_a v_a}{m_A} = \frac{-(0.100 \text{ kg})(100 \text{ m/s})}{50 \text{ kg}} = -0.20 \text{ m/s}$$

The archer's recoil *speed* is 0.20 m/s.

Assess: It is the total final momentum that is zero, although the individual momenta are nonzero. Since the arrow has forward momentum, the archer will have backward momentum.

9.21. Model: We will define our system to be Dan + skateboard, and their interaction as an explosion. While friction is present between the skateboard and the ground, it is negligible in the impulse approximation.

Visualize:



The system has nonzero initial momentum p_{ix}. As Dan (D) jumps backward off the gliding skateboard (S), the skateboard will move forward so that the final total momentum of the system p_{fx} is equal to p_{ix}.

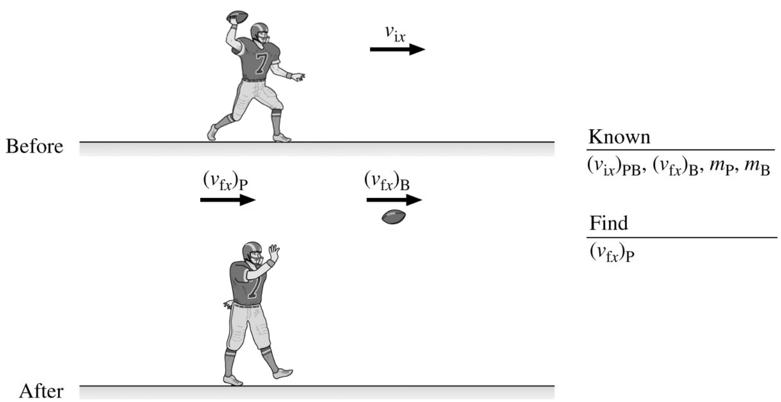
Solve: We have $m_S(v_{fx})_S + m_D(v_{fx})_D = (m_S + m_D)v_{ix}$. Thus,

$$(5.0 \text{ kg})(8.0\ m/\text{s}) + (50 \text{ kg})(v_{fx})_D = (5.0 \text{ kg} + 50 \text{ kg})(4.0\ m/\text{s}) \quad \Rightarrow \quad (v_{fx})_D = 3.6 \text{ m/s}$$

Assess: Although Dan jumps backward from the skateboard, he still winds up going forward relative to the ground.

9.22. Model: We will define our system to be the football player (P) and the football (B). Their interaction is an explosion because the force involved is internal to the P + B system. There are no external horizontal forces present on either of the two, so horizontal momentum is conserved.

Visualize:



The system has nonzero initial momentum p_{ix}, which must be conserved.

Solve: (a) The final velocity of the ball is $(v_{fx})_B = 15.0$ m/s. Equating the initial and final momentum gives $m_P(v_{fx})_P + m_B(v_{fx})_B = (m_B + m_P)v_{ix}$. Solving for $(v_{fx})_P$ gives

$$(v_{fx})_P = \frac{(m_B + m_P)v_{ix} - m_B(v_{fx})_B}{m_P} = \frac{(0.450 \text{ kg} + 70.0 \text{ kg})(2.00 \text{ m/s}) - (0.450 \text{ kg})(15.0 \text{ m/s})}{70.0 \text{ kg}} = 1.92 \text{ m/s}$$

(b) The final velocity of the ball is $(v_{fx})_B = (v_{fx})_P + 15.0 \text{ m/s}$. Inserting this into the equation for conservation of momentum and solving for $(v_{fx})_B$ gives

$$m_P(v_{fx})_P + m_B\left[(v_{fx})_P + 15.0 \text{ m/s}\right] = (m_B + m_P)v_{ix}.$$

$$(v_{fx})_P = \frac{(m_B + m_P)v_{ix} - m_B(15.0 \text{ m/s})}{m_P + m_B} = \frac{(0.450 \text{ kg} + 70.0 \text{ kg})(2.00 \text{ m/s}) - (0.450 \text{ kg})(15.0 \text{ m/s})}{70.0 \text{ kg} + 0.450 \text{ kg}} = 1.90 \text{ m/s}$$

Assess: In part (b), the final velocity of the ball is greater than in part (a), so the player's final velocity is slightly less so that momentum is conserved.

Section 9.6 Momentum in Two Dimensions

9.23. Model: Assume that the momentum is conserved in the collision.
Visualize: Please refer to Figure EX9.23.
Solve: Applying conservation of momentum in the x- and y-directions yields

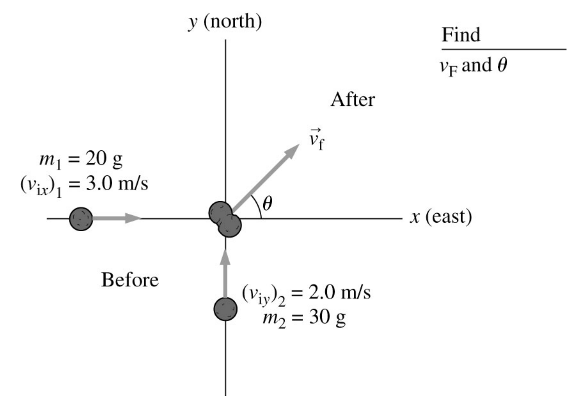
$$(p_{fx})_1 + (p_{fx})_2 = (p_{ix})_1 + (p_{ix})_2 \quad \Rightarrow \quad (p_{fx})_1 + 0 \text{ kg m/s} = -2 \text{ kg m/s} + 4 \text{ kg m/s} \quad \Rightarrow \quad (p_{fx})_1 = 2 \text{ kg m/s}$$
$$(p_{fy})_1 + (p_{fy})_2 = (p_{iy})_1 + (p_{iy})_2 \quad \Rightarrow \quad (p_{fy})_1 - 1 \text{ kg m/s} = 2 \text{ kg m/s} + 1 \text{ kg m/s} \quad \Rightarrow \quad (p_{fy})_1 = 4 \text{ kg m/s}$$

Thus, the final momentum of particle 1 is $(\vec{p}_f)_1 = (2\hat{i} + 4\hat{j})$ kg m/s.
Assess: As a check, the vector sum of the initial momenta should equal the vector sum of the final momenta.

9.24. Model: Assume that the momentum is conserved in the explosion. Since the object is initially at rest, its total initial momentum is zero. After it explodes the total momentum of the fragments must also be zero.
Solve: With $\vec{p}_1 = (-2,\ 2)$ kg m/s and $\vec{p}_2 = (3,\ 0)$ kg m/s, the requirement that $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$ means that $\vec{p}_3 = (-1,\ -2)$ kg m/s.
Assess: Do a vector sum of the momenta to see if they add to zero.

9.25. Model: This problem deals with the conservation of momentum in two dimensions in an inelastic collision.
Visualize:

Pictorial representation



Solve: The conservation of momentum equation $\vec{p}_{before} = \vec{p}_{after}$ gives

$$m_1(v_{ix})_1 + m_2(v_{ix})_2 = (m_1 + m_2)v_{fx},\ m_1(v_{iy})_1 + m_2(v_{iy})_2 = (m_1 + m_2)v_{fy}$$

Substituting in the given values, we find

$$(0.020 \text{ kg})(3.0 \text{ m/s}) + 0.0 \text{ kg m/s} = (0.020 \text{ kg} + 0.030 \text{ kg})v_f \cos\theta$$

$$0.0 \text{ kg m/s} + (0.030 \text{ kg})(2.0 \text{ m/s}) = (0.020 \text{ kg} + 0.030 \text{ kg})v_f \sin\theta$$

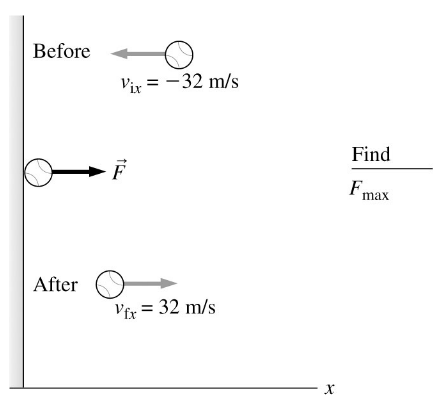
$$v_f \cos\theta = 1.2 \text{ m/s},\ v_f \sin\theta = 1.2 \text{ m/s}$$

$$v_f = \sqrt{(1.2 \text{ m/s})^2 + (1.2 \text{ m/s})^2} = 1.7 \text{ m/s},\ \theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}(1) = 45°$$

The ball of clay moves 45° north of east at 1.7 m/s.

9.26. Model: Model the tennis ball as a particle, and its interaction with the wall as a collision.
Visualize:

Pictorial representation



The force increases to F_{max} during the first two ms, stays at F_{max} for two ms, and then decreases to zero during the last two ms. The graph shows that F_x is positive, so the force acts to the right.

Solve: Using the impulse-momentum theorem $p_{fx} = p_{ix} + J_x$, we find

$$(0.06 \text{ kg})(32 \text{ m/s}) = (0.06 \text{ kg})(-32 \text{ m/s}) + \int_0^{6 \text{ ms}} F_x(t)dt$$

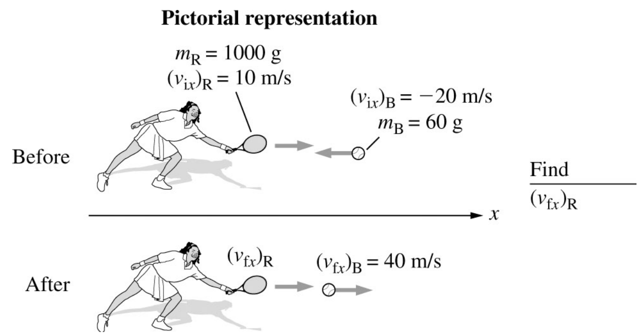
The impulse is

$$J_x = \int_0^{6\text{ ms}} F_x(t)dx = \text{area under force curve} = \frac{1}{2}F_{\text{max}}(0.0020\text{ s}) + F_{\text{max}}(0.0020\text{ s}) + \frac{1}{2}F_{\text{max}}(0.0020\text{ s}) = (0.0040\text{ s})F_{\text{max}}$$

$$F_{\text{max}} = \frac{(0.060\text{ kg})(32\text{ m/s}) + (0.060\text{ kg})(32\text{ m/s})}{0.0040\text{ s}} = 9.6 \times 10^2\text{ N}$$

9.27. Model: Let the system be ball + racket. During the collision of the ball and racket, momentum is conserved because all external interactions are insignificantly small.

Visualize:



Solve: (a) The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_R(v_{fx})_R + m_B(v_{fx})_B = m_R(v_{ix})_R + m_B(v_{ix})_B$$

$$(1.000\text{ kg})(v_{fx})_R + (0.060\text{ kg})(40\text{ m/s}) = (1.000\text{ kg})(10\text{ m/s}) + (0.060\text{ kg})(-20\text{ m/s}) \Rightarrow (v_{fx})_R = 6.4\text{ m/s}$$

(b) The impulse on the ball is calculated from $(p_{fx})_B = (p_{ix})_B + J_x$ as follows:

$$(0.060\text{ kg})(40\text{ m/s}) = (0.060\text{ kg})(-20\text{ m/s}) + J_x \Rightarrow J_x = 3.6\text{ N s} = \int F\,dt = F_{\text{avg}}\Delta t$$

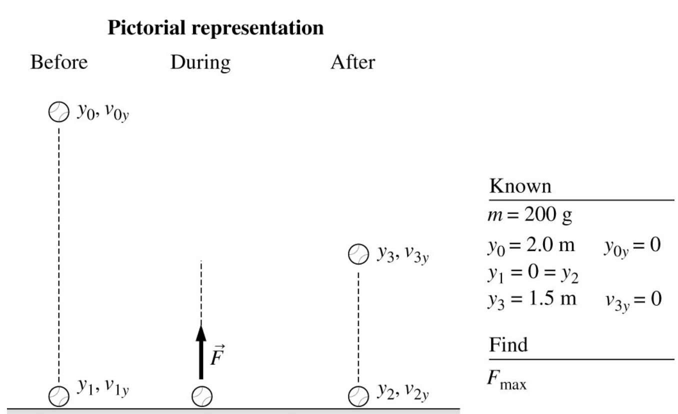
$$F_{\text{avg}} = \frac{3.6\text{ Ns}}{10\text{ ms}} = 3.6 \times 10^2\text{ N}$$

Let us now compare this force with the gravitational force on the ball $(F_G)_B = m_B g = (0.060\text{ kg})(9.8\text{ m/s}^2) = 0.588$ N. We find $F_{\text{avg}} = 612(F_G)_B$.

Assess: This is a significant force and is reasonable because the impulse due to this force not only changes the direction of the ball also but changes the speed of the ball from approximately 45 mph to 90 mph.

9.28. Model: Model the ball as a particle that is subjected to an impulse when it is in contact with the floor. We shall also use constant-acceleration kinematic equations. During the collision, ignore any forces other than the interaction between the floor and the ball in the impulse approximation.

Visualize:



Solve: To find the ball's velocity just before and after it hits the floor:

$$v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0) = 0 \text{ m}^2/\text{s}^2 + 2(-9.8 \text{ m/s}^2)(0 - 2.0 \text{ m}) \Rightarrow v_{1y} = -6.261 \text{ m/s}$$

$$v_{3y}^2 = v_{2y}^2 + 2a_y(y_3 - y_2) \Rightarrow 0 \text{ m}^2/\text{s}^2 = v_{2y}^2 + 2(-9.8 \text{ m/s}^2)(1.5 \text{ m} - 0 \text{ m}) \Rightarrow v_{2y} = 5.422 \text{ m/s}$$

The force exerted by the floor on the ball can be found from the impulse-momentum theorem:

$$mv_{2y} = mv_{1y} + \int F\,dt = mv_{1y} + \text{area under the force curve}$$

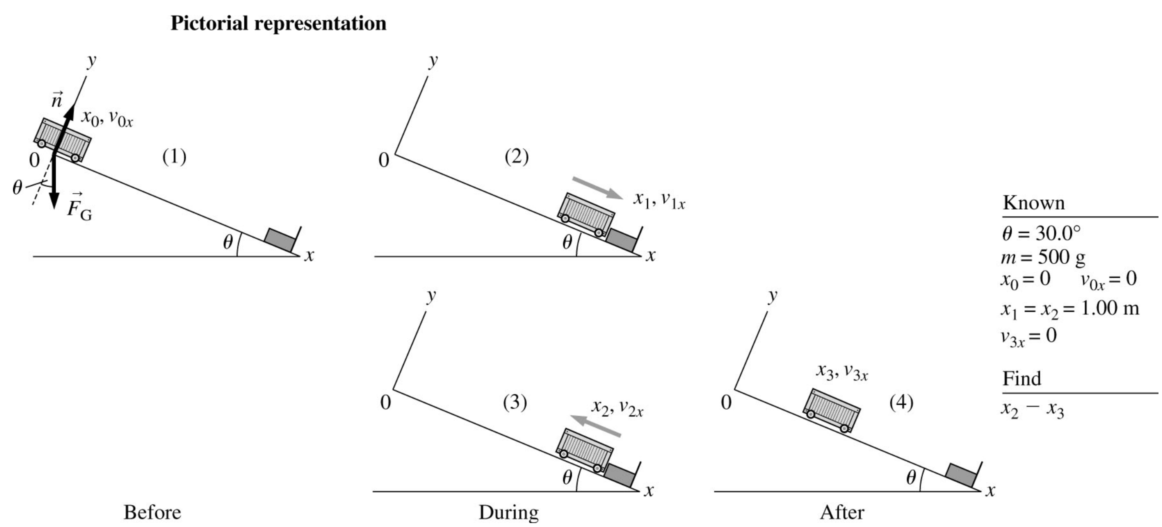
$$(0.200 \text{ kg})(5.422 \text{ m/s}) = -(0.200 \text{ kg})(6.261 \text{ m/s}) + \tfrac{1}{2}F_{\text{max}}(5.0 \times 10^{-3} \text{ s})$$

$$F_{\text{max}} = 9.3 \times 10^2 \text{ N}$$

Assess: A maximum force of 9.3×10^2 N exerted by the floor is reasonable. This force is the same order of magnitude as the force of the racket on the tennis ball in the previous problem.

9.29. Model: Model the cart as a particle sliding down a frictionless ramp. The cart is subjected to an impulsive force when it comes in contact with a rubber block at the bottom of the ramp. We shall use the impulse-momentum theorem and the constant-acceleration kinematic equations.

Visualize:



Solve: From the free-body diagram on the cart, Newton's second law applied to the system before the collision gives

$$\sum(F)_x = F_G \sin\theta = ma_x \Rightarrow a_x = \frac{mg\sin\theta}{m} = g\sin 30.0° = \frac{9.81 \text{ m/s}^2}{2} = 4.905 \text{ m/s}^2$$

Using this acceleration, we can find the cart's speed just before its contact with the rubber block:

$$v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0) = 0 \text{ m}^2/\text{s}^2 + 2(4.905 \text{ m / s}^2)(1.00 \text{ m} - 0 \text{ m}) \Rightarrow v_{1x} = 3.132 \text{ m/s}$$

Now we can use the impulse-momentum theorem to obtain the velocity just after the collision:

$$mv_{2x} = mv_{1x} + \int F_x\,dt = mv_{1x} + \text{area under the force graph}$$

$$(0.500 \text{ kg})v_{2x} = (0.500 \text{ kg})(3.13 \text{ m/s}) - \tfrac{1}{2}(200 \text{ N})(26.7 \times 10^{23} \text{ s}) \Rightarrow v_{2x} = -2.208 \text{ m/s}$$

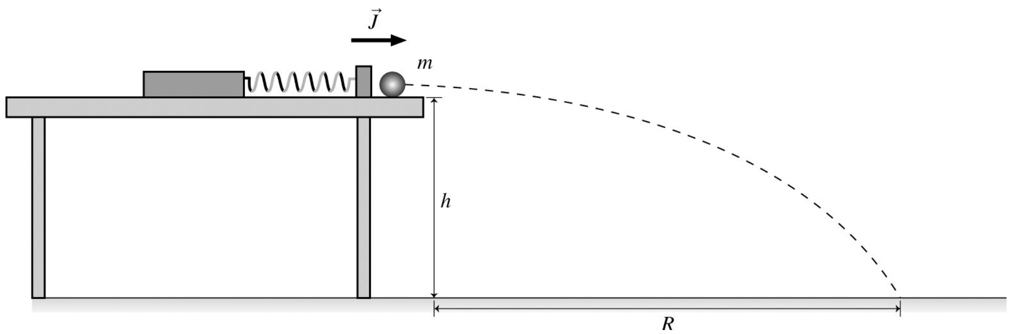
Note that the given force graph is positive, but in this coordinate system the impulse of the force is to the left (i.e., up the slope). That is the reason to put a minus sign while evaluating the $\int F_x\,dt$ integral.

We can once again use a kinematic equation to find how far the cart will roll back up the ramp:

$$v_{3x}^2 = v_{2x}^2 + 2a_x(x_3 - x_2) \Rightarrow (0 \text{ m/s})^2 = (-2.208 \text{ m/s})^2 + 2(-4.905 \text{ m/s}^2)(x_3 - x_2) \Rightarrow (x_3 - x_2) = 0.497 \text{ m}$$

9.30. Model: Model the balls as particles and ignore air resistance and friction on the table. Apply the impulse-momentum theorem and the constant-acceleration kinematic Equations 4.12.

Visualize:



Solve: (a) Once the ball leaves the table, the time it takes for it to hit the ground is

$$h = y_f - y_i = -\tfrac{1}{2}g\Delta t^2 \Rightarrow \Delta t = \sqrt{\frac{2h}{g}}$$

The spring imparts an impulse J to a stationary ball, so the final momentum the ball is equal to the impulse J. The horizontal velocity of the ball as it leaves the table is then

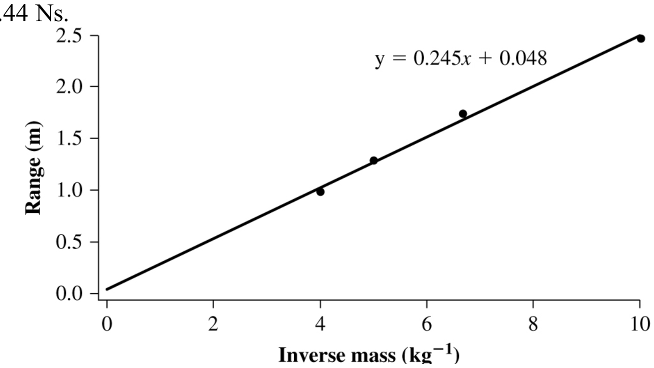
$$mv_{ix} = J \Rightarrow v_{ix} = J/m$$

This velocity remains constant during the free fall, so the range is given by

$$R = v_{ix}\Delta t = \left(\frac{J}{m}\right)\sqrt{\frac{2h}{g}}$$

(b) To obtain a linear slope, you should graph the range R as a function of $1/m$. The slope of the line will be $J\sqrt{\frac{2h}{g}}$.

(c) The data are plotted in the figure below. From the best-fit line, we find a slope of $s = 0.245$ kg m. This gives an impluse of $J = s\sqrt{\frac{g}{2h}} = 0.44$ Ns.



Assess: Experimentally it would be important to keep h large enough so the values of R are great enough to be measured accurately.

9.31. Model: Apply Equation 9.7 and Newton's second law. Assume that the force from the flower, while it acts, is much greater than all the other forces acting, so they can be neglected.

Solve: Newton's second law tells us that the average force used to expel the grains is

$$F_{avg} = ma = (1.0 \times 10^{-10} \text{ kg})(2.5 \times 10^4 \text{ m/s}^2) = 2.5 \times 10^{-6} \text{ N}$$

Inserting this into Equation 9.7 gives

$$J = F_{avg}\Delta t = (2.5 \times 10^{-6} \text{ N})(3.0 \times 10^4 \text{ s}) = 7.5 \times 10^{-10} \text{ kg m/s}$$

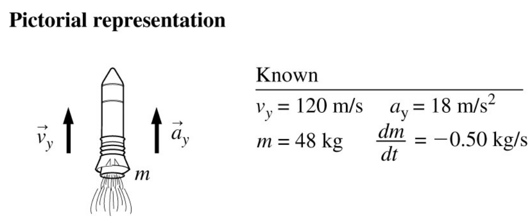
Assess: The impulse seems very small, but remember that the pollen grains have *very* little mass.

9.32. Solve: Using Newton's second law for the x-direction, $F_x = dp_x/dt$. Therefore,

$$F_x = \frac{d}{dt}(6t^2 \text{ kg m/s}) = 12t \text{ N}$$

Assess: The x-component of the net force on an object is equal to the time rate of change of the x-component of the object's momentum.

9.33. Visualize:



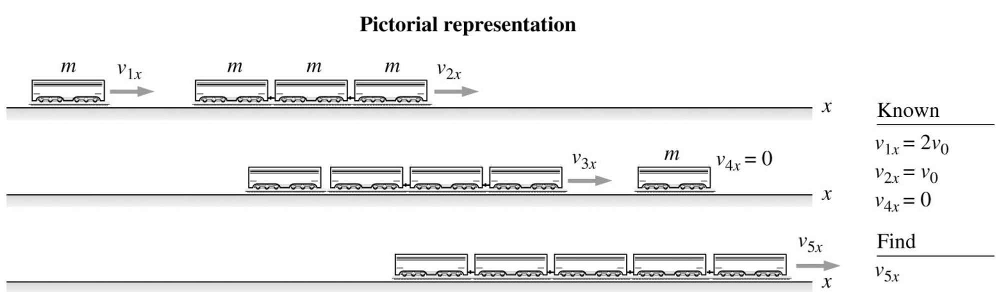
Solve: Using Newton's second law for the y-direction and the chain rule,

$$(F_{\text{net}})_y = \frac{dp_y}{dt} = \frac{d}{dt}(mv_y) = \frac{dm}{dt}(v_y) + m\left(\frac{dv_y}{dt}\right)$$
$$= (-0.50 \text{ kg/s})(120 \text{ m/s}) + (48 \text{ kg})(18 \text{ m/s}^2)$$
$$= 8.0 \times 10^2 \text{ N}$$

Assess: Since the rocket is losing mass, $dm/dt < 0$. The time derivative of the velocity is the acceleration.

9.34. Model: Model the train cars as particles. Since the train cars stick together, we are dealing with perfectly inelastic collisions. Momentum is conserved in the collisions of this problem in the impulse approximation, in which we ignore external forces during the time of the collision.

Visualize:



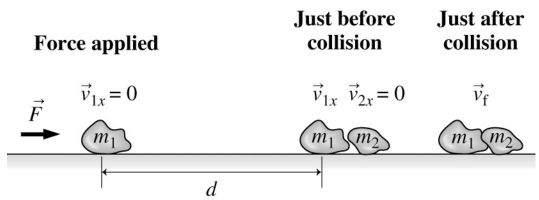
Solve: The initial momentum is $5mv_0$ before any collisions occur. Since momentum is conserved, the final momentum must be the same, so

$$5mv_0 = 5mv_{5x} \quad \Rightarrow \quad v_{5x} = v_0$$

Assess: Think of the fourth car colliding with the stationary fifth car. The final speed would be v_0, which is the same as that of the three-car train, so our result is reasonable.

9.35. Model: The two blobs of clay will be modeled as particles. Their collision is completely inelastic and conserves momentum. Newton's second law and the equations for constant-acceleration kinematics will apply.

Visualize:



Solve: Newton's second law tells us that the accelearation of the first blob is $a = F/m_1$. Therefore, after covering a distance d, the speed of blob 1 is

$$v_{1x}^2 = 2ad \quad \Rightarrow \quad v_{1x} = \sqrt{2ad} = \sqrt{2Fd/m_1}$$

Momentum is conserved in the collision with blob 2, so

$$m_1 v_{1x} = v_f(m_1 + m_2) \quad \Rightarrow \quad v_f = \frac{m_1 v_{1x}}{(m_1 + m_2)} = \frac{m_1}{(m_1 + m_2)}\sqrt{\frac{2Fd}{m_1}}$$

Assess: We find that v_f increases as F and d increase, but decreases as m_2 increases, which is physically reasonable.

9.36. Model: Model the gliders as particles and apply conservation of momentum. The 200 g glider will be labeled 1, the 300 g glider will be labeled 2, and the 400 g glider will be labeled 3.
Visualize: See Figure P9.36.
Solve: To apply conservation of momentum, we need to calculate the speed of two gliders from the figure. Differentiating the position-versus-time curves gives:

$$v_{1x} = \frac{dy}{dt} = 0.53 \text{ m/s}, \; v_{2x} = \frac{dy}{dt} = -0.41 \text{ m/s}$$

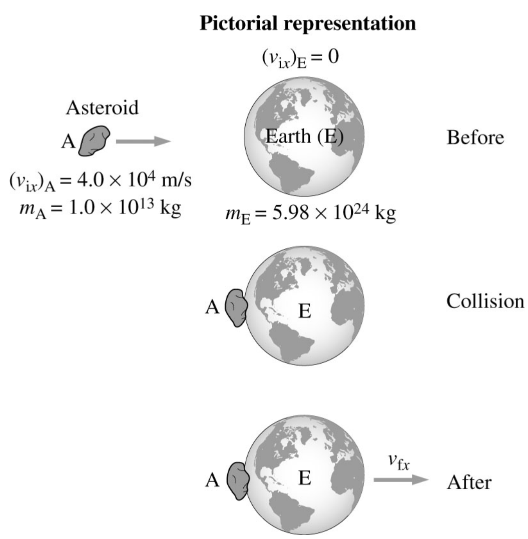
The intial momentum is zero because the three gliders are stationary. Therefore, conservation of momentum gives

$$m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} = 0$$

$$v_{3x} = \frac{-1}{m_3}(m_1 v_{1x} + m_2 v_{2x}) = \frac{-1}{0.40 \text{ kg}}[(0.20 \text{ kg})(0.53 \text{ m/s}) + (0.30 \text{ kg})(-0.41 \text{ m/s})] = 0.043 \text{ m/s}$$

Thus, the 400 g glider flies of the right (because $v_{3x} > 0$) at a speed of 0.043 m/s.

9.37. Model: Model the earth (E) and the asteroid (A) as particles. Earth + asteroid is our system. Since the two stick together during the collision, this is a case of a perfectly inelastic collision. Momentum is conserved in the collision since no significant external force acts on the system.
Visualize:



Solve: **(a)** The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_A(v_{ix})_A + m_E(v_{ix})_E = (m_A + m_E)v_{fx}$$

$$(1.0\times10^{13}\text{ kg})(4.0\times10^4\text{ m/s}) + 0\text{ kg m/s} = (1.0\times10^{13}\text{ kg} + 5.98\times10^{24}\text{ kg})v_{fx} \Rightarrow v_{fx} = 6.7\times10^{-8}\text{ m/s}$$

(b) The speed of the earth going around the sun is

$$v_E = \frac{2\pi r}{T} = \frac{2\pi(1.50\times10^{11}\text{ m})}{3.15\times10^7\text{ s}} = 3.0\times10^4\text{ m/s}$$

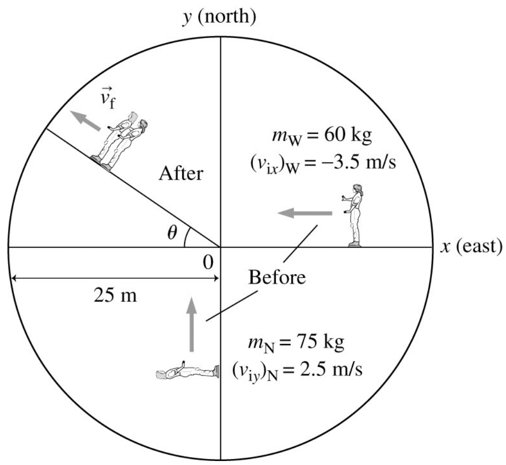
Thus, $v_{fx}/v_E = 2.2\times10^{-12} = 2.2\times10^{-10}$, .

Assess: The earth's recoil speed is insignificant compared to its orbital speed because of its large mass.

9.38. Model: Model the skaters as particles. The two skaters, one traveling north (N) and the other traveling west (W), are the system. Since the two skaters hold together after the "collision," this is a case of a perfectly inelastic collision in two dimensions. Momentum is conserved since no significant external force in the *x*-*y* plane acts on the system during the "collision."

Visualize:

Pictorial representation



Solve: **(a)** Applying conservation of momentum in the *x*-direction gives

$$(m_N + m_W)v_{fx} = m_N(v_{ix})_N + m_W(v_{ix})_W \Rightarrow (75\text{ kg} + 60\text{ kg})v_{fx} = 0\text{ kg m/s} + (60\text{ kg})(-3.5\text{ m/s})$$

$$v_{fx} = -1.556\text{ m/s}$$

Applying conservation of momentum in the *y*-direction gives

$$(m_N + m_W)v_{fy} = m_N(v_{iy})_N + m_W(v_{iy})_W \Rightarrow (75\text{ kg} + 60\text{ kg})v_{fy} = (75\text{ kg})(2.5\text{ m/s}) + 0\text{ kg m/s}$$

$$v_{fy} = 1.389\text{ m/s}$$

The final speed is therefore

$$v_f = \sqrt{(v_{fx})^2 + (v_{fy})^2} = 2.085\text{ m/s}$$

The time to glide to the edge of the rink is

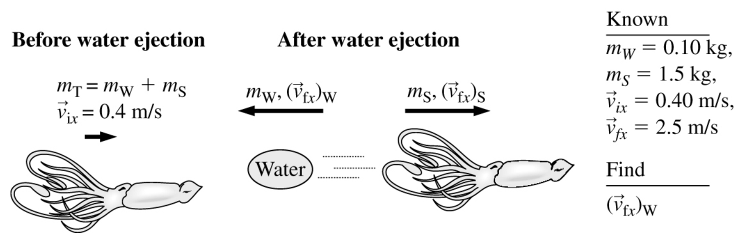
$$\frac{\text{radius of the rink}}{v_f} = \frac{25\text{ m}}{2.085\text{ m/s}} = 12\text{ s}$$

(b) The location is $\theta = \tan^{-1}(v_{fy}/v_{fx}) = 42°$ north of west.

Assess: A time of 12 s in covering a distance of 25 m at a speed of ≈ 2 m/s is reasonable.

9.39. Model: Model the squid and the water ejected as particles and ignore drag forces during the short time interval over which the water is expelled (the impulse approximation). Because the external forces are negligible, momentum will be conserved.

Visualize:



Solve: Applying conservation of momentum gives

$$m_T v_{1x} = m_W (v_{fx})_W + m_S (v_{fx})_S$$

$$(v_{fx})_W = \frac{1}{m_W}[m_T v_{1x} - m_S (v_{fx})_S] = \frac{1}{0.10\text{ kg}}[(1.6\text{ kg})(0.4\text{ m/s}) - (1.5\text{ kg})(2.5\text{ m/s})] = -31.1\text{ m/s}$$

This water is ejected in the direction opposite the squid's initial velocity, so the speed with which the water is ejected relative to the squid is

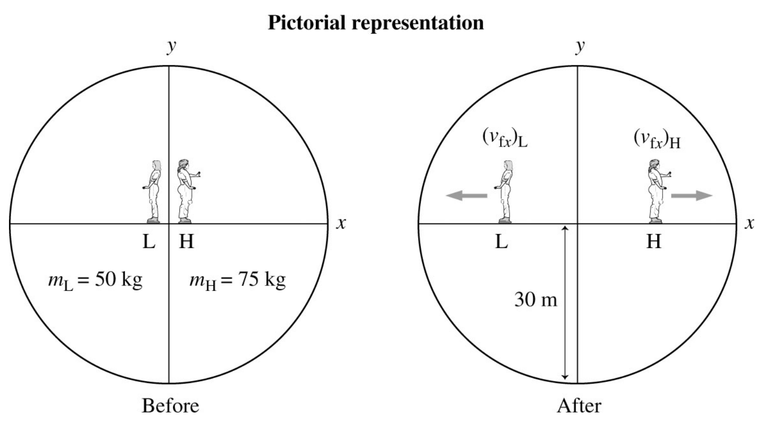
$$v_{rel} = (v_{fx})_W - (v_{fx})_S = -31.1\text{ m / s} - 0.4\text{ m / s} = -31.5\text{ m / s}$$

or 32 m/s to two significant figures.

Assess: The ejected water must move much faster than the squid because its mass is much less than that of the squid, so our result is reasonable.

9.40. Model: This problem deals with a case that is the opposite of a collision. The two ice skaters, heavier and lighter, will be modeled as particles. The skaters (or particles) move apart after pushing off against each other. During the "explosion," the total momentum of the system is conserved.

Visualize:



Solve: The initial momentum is zero. Thus the conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_H (v_{fx})_H + m_L (v_{fx})_L = 0\text{ kg m/s} \Rightarrow (75\text{ kg})(v_{fx})_H + (50\text{ kg})(v_{fx})_L = 0\text{ kg m/s}$$

Using the observation that the heavier skater takes 20 s to cover a distance of 30 m, we find $(v_{fx})_H$ = (30 m)/(20 s) = 1.5 m/s. Thus,

$$(75\text{ kg})(1.5\text{ m/s}) + (50\text{ kg})(v_{fx})_L = 0\text{ kg m/s} \Rightarrow (v_{fx})_L = -2.25\text{ m/s}$$

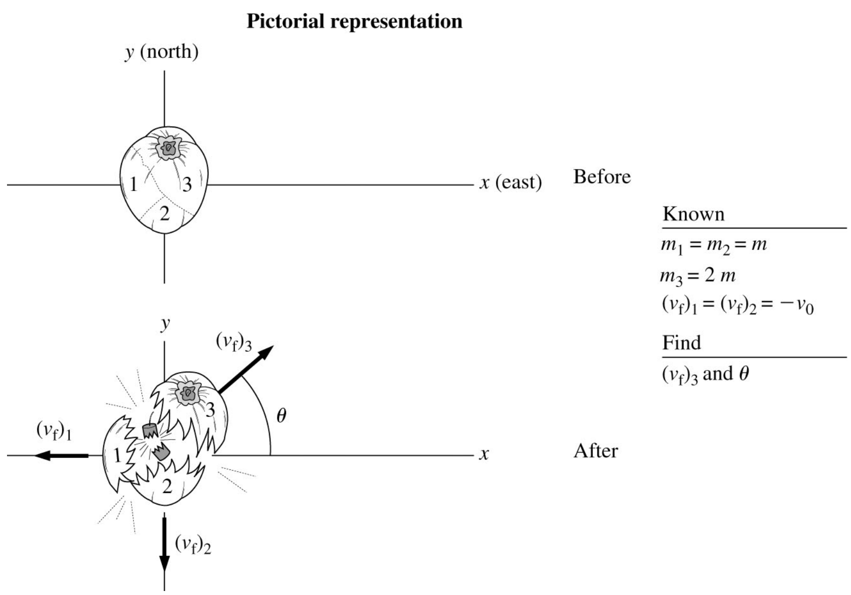
Thus, the time for the lighter skater to reach the edge is

$$\frac{30\text{ m}}{(v_{fx})_L} = \frac{30\text{ m}}{2.25\text{ m/s}} = 13\text{ s}$$

Assess: Conservation of momentum leads to a higher speed for the lighter skater, and hence a shorter time to reach the edge of the ice rink.

9.41. Model: This problem deals with a case that is the opposite of a collision. Our system is comprised of three coconut pieces that are modeled as particles. During the explosion, the total momentum of the system is conserved in the x- and y-directions.

Visualize:



Solve: The initial momentum is zero. From $p_{fx} = p_{ix}$, we get

$$+m_1(v_f)_1 + m_3(v_f)_3\cos\theta = 0\text{ kg m/s} \Rightarrow (v_f)_3\cos\theta = \frac{-m_1(v_f)_1}{m_3} = \frac{-m(-v_0)}{2m} = \frac{v_0}{2} = (v_{fx})_3$$

From $p_{fy} = p_{iy}$, we get

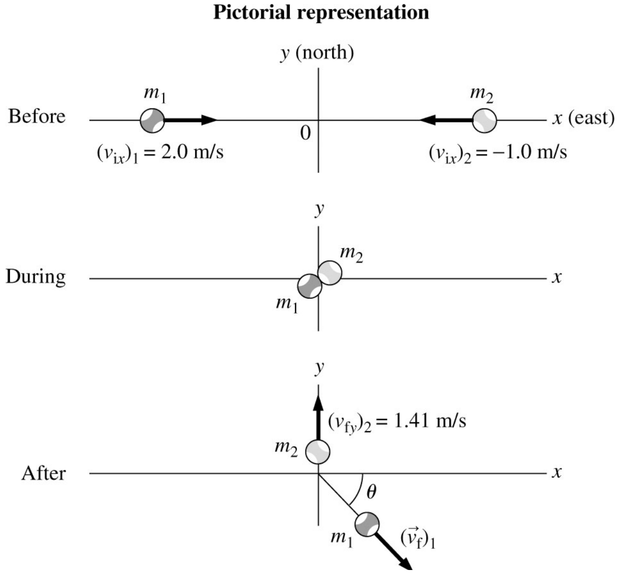
$$+m_2(v_f)_2 + m_3(v_f)_3\sin\theta = 0\text{ kg m/s} \Rightarrow (v_f)_3\sin\theta = \frac{-m_2(v_f)_2}{m_3} = \frac{-m(-v_0)}{2m} = \frac{v_0}{2} = (v_{fy})_3$$

$$(v_f)_3 = \sqrt{\left(\frac{v_0}{2}\right)^2 + \left(\frac{v_0}{2}\right)^2} = \frac{v_0}{\sqrt{2}}, \quad \theta = \tan^{-1}(1) = 45°$$

The speed to the third piece is $\frac{v_0}{\sqrt{2}}$ at 45° east of north.

9.42. Model: The billiard balls will be modeled as particles. The two balls, m_1 (moving east) and m_2 (moving west), together are our system. This is an isolated system because any frictional force during the brief collision period is going to be insignificant. Within the impulse approximation, the momentum of our system will be conserved in the collision.

Visualize:



Note that $m_1 = m_2 = m$.

Solve: The equation $p_{fx} = p_{ix}$ yields:

$$m_1(v_{fx})_1 + m_2(v_{fx})_2 = m_1(v_{ix})_1 + m_2(v_{ix})_2 \quad \Rightarrow \quad m_1(v_f)_1 \cos\theta + 0 \text{ kg m/s} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$

$$(v_f)_1 \cos\theta = (v_{ix})_1 + (v_{ix})_2 = 2.0 \text{ m/s} - 1.0 \text{ m/s} = 1.0 \text{ m/s}$$

The equation $p_{fy} = p_{iy}$ yields:

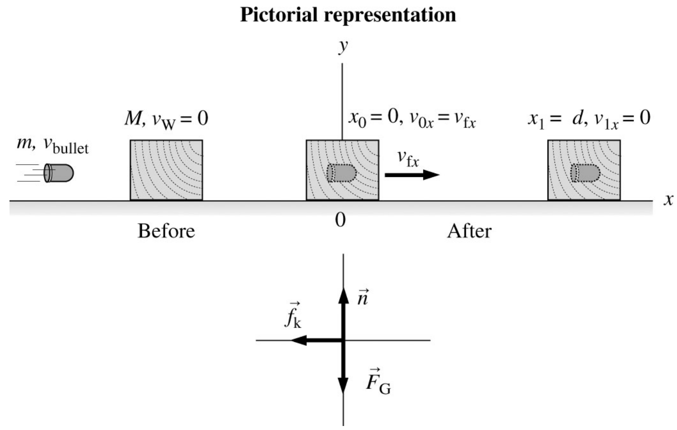
$$+m_1(v_{fy})_1 \sin\theta + m_2(v_{fy})_2 = 0 \text{ kg m/s} \quad \Rightarrow \quad (v_f)_1 \sin\theta = -(v_{fy})_2 = -1.41 \text{ m/s}$$

$$(v_f)_1 = \sqrt{(1.0 \text{ m/s})^2 + (-1.41 \text{ m/s})^2} = 1.7 \text{ m/s}, \ \theta = \tan^{-1}\left(\frac{1.41 \text{ m/s}}{1.0 \text{ m/s}}\right) = 55°$$

The angle is below +x axis, or south of east.

9.43. Model: Model the bullet and block as particles. This is an isolated system because any frictional force during the brief collision period is going to be insignificant. Within the impulse approximation, the momentum of our system will be conserved in the collision. After the collision, we will consider the frictional force and apply Newton's second law and kinematic equations to find the distance traveled by the block + bullet.

Visualize:



Solve: **(a)** Applying conservation of momentum to the collision gives

$$mv_{bullet} + Mv_W = (m + M)v_{fx} \quad \Rightarrow \quad v_{bullet} = \frac{m + M}{m} v_{fx}$$

The speed v_{fx} can be found from the kinematics equation

$$v_{1x}^2 = v_{0x}^2 + 2ad = v_{fx}^2 + 2ad \quad \Rightarrow \quad v_{fx} = \sqrt{-2ad}$$

The acceleration in the x-direction may be found using Newton's second law and the friction model. Because the block does not accelerate in the y-direction, the normal force must be the same magnitude as the force due to gravity (Newton's second law). Thus, the frictional force is $f_k = -\mu_k n = -\mu_k (m + M)g$, where the negative sign indicates that the force acts in the negative x-direction. Newton's second law then gives the acceleration of the block as

$$a = F_{net}/(m + M) = -\mu_k (m + M)g/(m + M) = -\mu_k g$$

Inserting this into the expression for v_{fx} gives

$$v_{fx} = \sqrt{-2ad} = \sqrt{2\mu_k gd}$$

Finally, we insert this expression for v_{fx} into the expression for the bullet's velocity to find

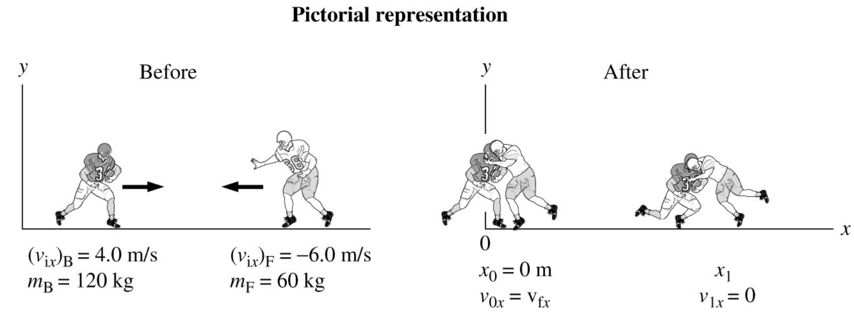
$$v_{bullet} = \frac{m + M}{m} \sqrt{2\mu_k gd}$$

(b) Inserting the given quantities and using $\mu_k = 0.20$ for wood on wood from Table 6.1 gives

$$v_{bullet} = \frac{0.010 \text{ kg} + 10 \text{ kg}}{0.010 \text{ kg}} \sqrt{2(0.20)(9.8 \text{ m/s}^2)(0.050 \text{ m})} = 4.4 \times 10^2 \text{ m/s}$$

Assess: If we let the bullet's mass go to zero, we see that the bullet's speed goes to infinity, which is reasonable because a zero-mass bullet would need an infinite speed to make the block move. If the bullet's mass goes to infinity, the bullet's speed would go to $\sqrt{2\mu_k gd}$, which is just the result for the initial speed of an object that decelerates to a stop at a constant rate $(\mu_k g)$ over a distance d. In other words, the block becomes insignificant compared to the infinite-mass bullet.

9.44. Model: This is a two-part problem. First, we have an inelastic collision between Fred (F) and Brutus (B). Fred and Brutus are an isolated system. The momentum of the system during collision is conserved since no significant external force acts on the system. The second part involves the dynamics of the Fred + Brutus system sliding on the ground.
Visualize:



Note that the collision is head-on and therefore one-dimensional.

Solve: The equation $p_{fx} = p_{ix}$ gives

$$(m_F + m_B)v_{fx} = m_F(v_{ix})_F + m_B(v_{ix})_B \Rightarrow (60\text{ kg} + 120\text{ kg})v_{fx} = (60\text{ kg})(-6.0\text{ m/s}) + (120\text{ kg})(4.0\text{ m/s})$$
$$v_{fx} = 0.667\text{ m/s}$$

The positive value indicates that the motion is in the direction of Brutus.
The model of kinetic friction yields:

$$f_k = -\mu_k n = -\mu_k(m_F + m_B)g = (m_F + m_B)a_x \Rightarrow a_x = -\mu_k g$$

Using the kinematic equation $v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0)$, we get

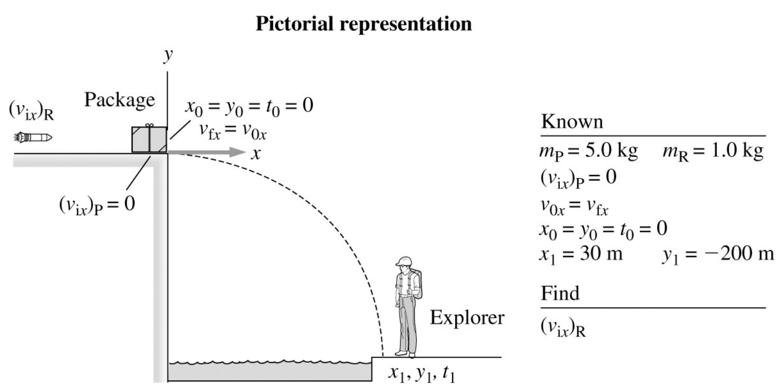
$$v_{1x}^2 = v_{0x}^2 - 2\mu_k g\, x_1 \Rightarrow 0\text{m}^2/\text{s}^2 = v_{fx}^2 - 2(0.30)(9.8\text{ m/s}^2)x_1$$
$$0\text{m}^2/\text{s}^2 = (0.667\text{ m/s})^2 - (5.9\text{ m/s}^2)x_1 \Rightarrow x_1 = 7.6\text{ cm}$$

They slide 7.6 cm in the direction Brutus was running.

Assess: After the collision, Fred and Brutus slide with a small speed but with a good amount of kinetic friction. A stopping distance of 7.6 cm is reasonable.

9.45. Model: Model the package and the rocket as particles. This is a two-part problem. First we have an inelastic collision between the rocket (R) and the package (P). During the collision, momentum is conserved since no significant external force acts on the rocket and the package. However, as soon as the package + rocket system leaves the cliff they become a projectile motion problem.

Visualize:



Solve: The minimum velocity after collision that the package + rocket must have to reach the explorer is v_{0x}, which can be found as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \;\Rightarrow\; -200\text{m} = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)t_1^2 \;\Rightarrow\; t_1 = 6.389\text{ s}$$

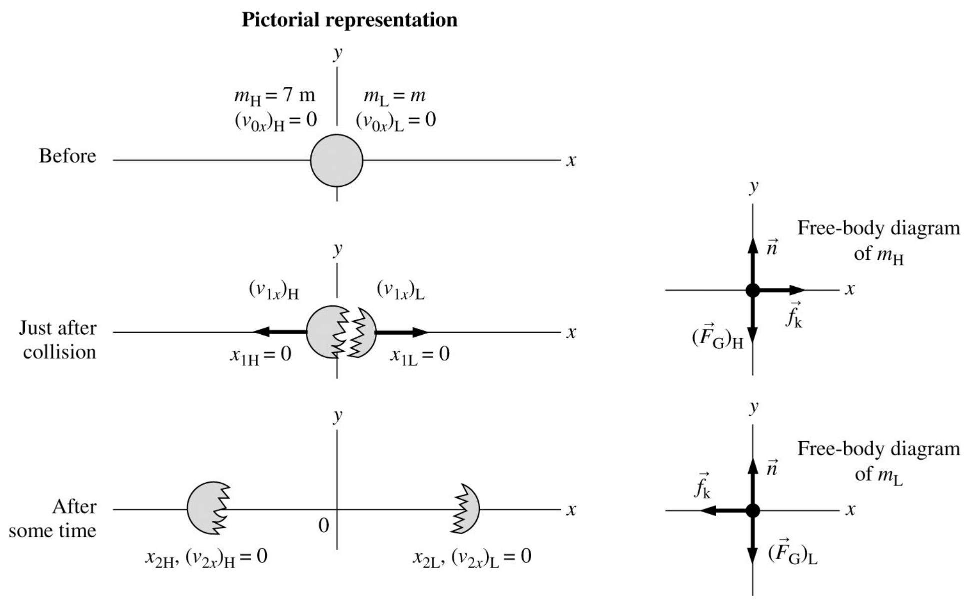
With this time, we can now find v_{0x} using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$. We obtain

$$30\text{ m} = 0\text{ m} + v_{0x}(6.389\text{ s}) + 0\text{ m} \;\Rightarrow\; v_{0x} = 4.696\text{ m/s} = v_{\text{fx}}$$

We now use the momentum conservation equation $p_{\text{fx}} = p_{\text{ix}}$ which can be written

$$(m_\text{R} + m_\text{P})v_{\text{fx}} = m_\text{R}(v_{\text{ix}})_\text{R} + m_\text{P}(v_{\text{ix}})_\text{P}$$
$$(1.0\text{ kg} + 5.0\text{ kg})(4.696\text{ m/s}) = (1.0\text{ kg})(v_{\text{ix}})_\text{R} + (5.0\text{ kg})(0\text{ m/s}) \;\Rightarrow\; (v_{\text{ix}})_\text{R} = 28\text{ m/s}$$

9.46. Model: This is a two-part problem. First, we have an explosion that creates two particles. The momentum of the system, comprised of two fragments, is conserved in the explosion. Second, we will use kinematic equations and the model of kinetic friction to find the displacement of the lighter fragment.
Visualize:



Solve: The initial momentum is zero. Using momentum conservation $p_{\text{fx}} = p_{\text{ix}}$ during the explosion,

$$m_\text{H}(v_{1x})_\text{H} + m_\text{L}(v_{1x})_\text{L} = m_\text{H}(v_{0x})_\text{H} + m_\text{L}(v_{0x})_\text{L} \;\Rightarrow\; 7m(v_{1x})_\text{H} + m(v_{1x})_\text{L} = 0\text{ kg m/s} \;\Rightarrow\; (v_{1x})_\text{H} = -\left(\tfrac{1}{7}\right)(v_{1x})_\text{L}$$

Because m_H slides to $x_{2\text{H}} = -8.2$ m before stopping, we have

$$f_\text{k} = \mu_\text{k} n_\text{H} = \mu_\text{k} w_\text{H} = \mu_\text{k} m_\text{H} g = m_\text{H} a_\text{H} \;\Rightarrow\; a_\text{H} = \mu_\text{k} g$$

Using kinematics,

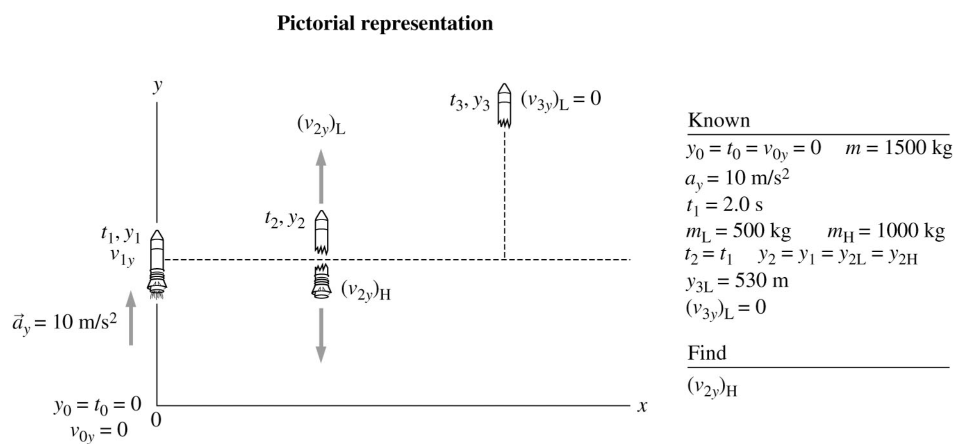
$$(v_{2x})_\text{H}^2 = (v_{1x})_\text{H}^2 + 2a_\text{H}(x_{2\text{H}} - x_{1\text{H}}) \;\Rightarrow\; 0\text{m}^2/\text{s}^2 = \left(\tfrac{1}{7}\right)^2 (v_{1x})_\text{L}^2 + 2\mu_\text{k} g(-8.2\text{ m} - 0\text{ m})$$
$$(v_{1x})_\text{L} = -88.74\sqrt{\mu_\text{k}}\text{ m/s}$$

How far does m_L slide? Using the information obtained above in the following kinematic equation,

$$(v_{2x})_\text{L}^2 = (v_{1x})_\text{L}^2 + 2a_\text{L}(x_{2\text{L}} - x_{1\text{L}}) \;\Rightarrow\; 0\text{m}^2/\text{s}^2 = \mu_\text{k}(88.74)^2 - 2\mu_\text{k} g x_{2\text{L}} \;\Rightarrow\; x_{2\text{L}} = 4.0 \times 10^2\text{ m}$$

Assess: Note that a_H is positive, but a_L is negative, and both are equal in magnitude to $\mu_k g$. Also, x_{2H} is negative but x_{2L} is positive.

9.47. Model: We will model the two fragments of the rocket after the explosion as particles. We assume the explosion separates the two parts in a vertical manner. This is a three-part problem. In the first part, we will use kinematic equations to find the vertical position where the rocket breaks into two pieces. In the second part, we will apply conservation of momentum to the system (that is, the two fragments) in the explosion. In the third part, we will again use kinematic equations to find the velocity of the heavier fragment just after the explosion.
Visualize:



Solve: The rocket accelerates for 2.0 s from rest, so

$$v_{1y} = v_{0y} + a_y(t_1 - t_0) = 0 \text{ m/s} + (10 \text{ m/s}^2)(2.0 \text{ s} - 0 \text{ s}) = 20 \text{ m/s}$$

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 = 0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}(10 \text{ m/s}^2)(2.0 \text{ s})^2 = 20 \text{ m}$$

At the explosion the equation $p_{fy} = p_{iy}$ is

$$m_L(v_{2y})_L + m_H(v_{2y})_H = (m_L + m_H)v_{1y} \quad \Rightarrow \quad (500 \text{ kg})(v_{2y})_L + (1000 \text{ kg})(v_{2y})_H = (1500 \text{ kg})(20 \text{ m/s})$$

To find $(v_{2y})_H$ we must first find $(v_{2y})_L$, the velocity after the explosion of the upper section. Using kinematics,

$$(v_{3y})_L^2 = (v_{2y})_L^2 + 2(-9.8 \text{ m/s}^2)(y_{3L} - y_{2L}) \quad \Rightarrow \quad (v_{2y})_L = \sqrt{2(9.8 \text{ m/s}^2)(530 \text{ m} - 20 \text{ m})} = 99.98 \text{ m/s}$$

Now, going back to the momentum conservation equation we get

$$(500 \text{ kg})(99.98 \text{ m/s}) + (1000 \text{ kg})(v_{2y})_H = (1500 \text{ kg})(20 \text{ m/s}) \quad \Rightarrow \quad (v_{2y})_H = -20 \text{ m/s}$$

The negative sign indicates downward motion.

9.48. Model: Let the system be bullet + target. No external horizontal forces act on this system, so the horizontal momentum is conserved. Model the bullet and the target as particles. Since the target is much more massive than the bullet, it is reasonable to assume that the target undergoes no significant motion during the brief interval in which the bullet passes through it.
Visualize:

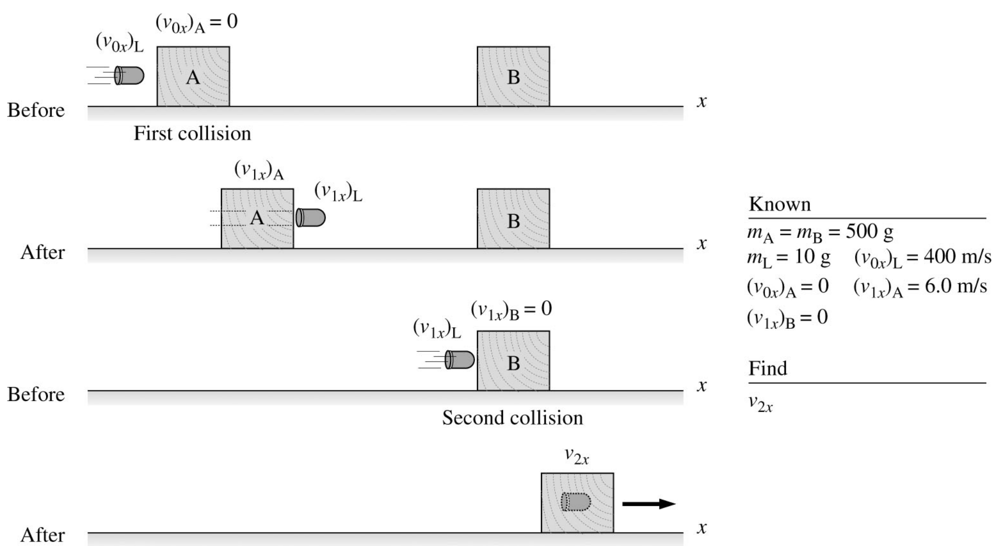
Pictorial representation

Solve: Use the conservation of momentum equation $p_{1x} = p_{0x}$ to find

$$m_T(v_{1x})_T + m_B(v_{1x})_B = m_T(v_{0x})_T + m_B(v_{0x})_B = 0 + m_B(v_{0x})_B$$

$$(v_{1x})_T = \frac{m_B(v_{0x})_B - m_B(v_{1x})_B}{m_T} = \frac{0.025\text{ kg}}{350\text{ kg}}[(1200\text{ m/s}) - (900\text{ m/s})] = 0.021\text{ m/s}$$

9.49. Model: Model the two blocks (A and B) and the bullet (L) as particles. This is a two-part problem. First, we have a collision between the bullet and the first block (A). Momentum is conserved since no external force acts on the system (bullet + block A). The second part of the problem involves a perfectly inelastic collision between the bullet and block B. Momentum is again conserved for this system (bullet + block B).
Visualize:



Solve: For the first collision the equation $p_{fx} = p_{ix}$ is

$$m_L(v_{1x})_L + m_A(v_{1x})_A = m_L(v_{0x})_L + m_A(v_{0x})_A$$

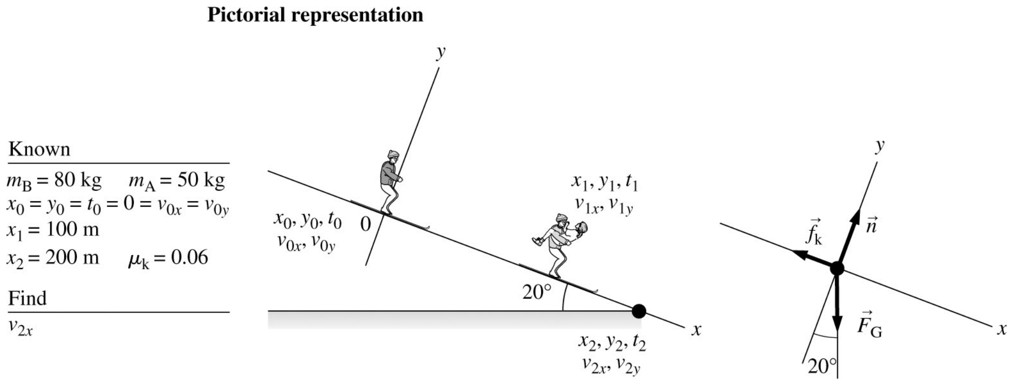
$$(0.010\text{ kg})(v_{1x})_L + (0.500\text{ kg})(6.0\text{ m/s}) = (0.010\text{ kg})(400\text{ m/s}) + 0\text{ kg m/s} \Rightarrow (v_{1x})_L = 100\text{ m/s}$$

The bullet emerges from the first block at 100 m/s. For the second collision the equation $p_{fx} = p_{ix}$ is

$$(m_L + m_B)v_{2x} = m_L(v_{1x})_L \Rightarrow (0.010\text{ kg} + 0.500\text{ kg})v_{2x} = (0.010\text{ kg})(100\text{ m/s}) \Rightarrow v_{2x} = 2.0\text{ m/s}$$

9.50. Model: Model Brian (B) along with his wooden skis as a particle. The "collision" between Brian and Ashley lasts for a short time, and during this time no significant external forces act on the Brian + Ashley system. Within the impulse approximation, we can then assume momentum conservation for our system. After finding the velocity of the system immediately after the collision, we will apply constant-acceleration kinematic equations and the model of kinetic friction to find the final speed at the bottom of the slope.

Visualize:



Solve: Brian skiing down for 100 m:

$$(v_{1x})_B^2 = (v_{0x})_B^2 + 2a_x(x_{1B} - x_{0B}) = 0\text{m}^2/\text{s}^2 + 2a_x(100 \text{ m} - 0 \text{ m}) \quad \Rightarrow \quad (v_{1x})_B = \sqrt{(200 \text{ m})a_x}$$

To obtain a_x, we apply Newton's second law to Brian in the x and y directions as follows:

$$\sum(F_{\text{on B}})_x = w_B \sin\theta - f_k = m_B a_x \sum(F_{\text{on B}})_y = n - w_B \cos\theta = 0 \text{ N} \quad \Rightarrow \quad n = w\cos\theta$$

From the model of kinetic friction, $f_k = \mu_k n = \mu_k w_B \cos\theta.$ The x-equation thus becomes

$$w_B \sin\theta - \mu_k w_B \cos\theta = m_B a_x$$

$$a_x = g(\sin\theta - \mu_k \cos\theta) = (9.8 \text{ m/s}^2)[\sin 20° - (0.060)\cos 20°] = 2.80 \text{ m/s}^2$$

Using this value of a_x, $(v_{1x})_B = \sqrt{(200 \text{ m})(2.80 \text{ m/s}^2)} = 23.7 \text{ m/s}.$ In the collision with Ashley the conservation of momentum equation $p_{fx} = p_{ix}$ is

$$(m_B + m_A)v_{2x} = m_B(v_{1x})_B \quad \Rightarrow \quad v_{2x} = \frac{m_B}{m_B + m_A}(v_{1x})_B = \frac{80 \text{ kg}}{80 \text{ kg} + 50 \text{ kg}}(23.66 \text{ m/s}) = 14.56 \text{ m/s}$$

Brian + Ashley skiing down the slope:

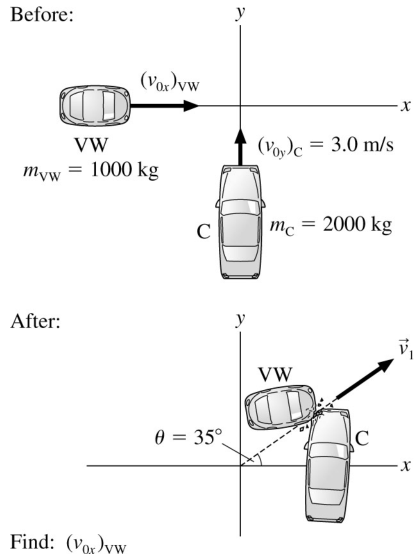
$$v_{3x}^2 = v_{2x}^2 + 2a_x(x_3 - x_2) = (14.56\text{m/s})^2 + 2(2.80 \text{ m/s}^2)(100\text{m}) \quad \Rightarrow \quad v_{3x} = 28 \text{ m/s}$$

That is, Brian + Ashley arrive at the bottom of the slope with a speed of 28 m/s. Note that we have used the same value of a_x in the first and the last parts of this problem. This is because a_x is independent of mass.

Assess: A speed of approximately 60 mph on a ski slope of 200 m length and 20° slope is reasonable.

9.51. Model: This is an inelastic collision. The total momentum of the Volkswagen + Cadillac system is conserved.

Visualize:



Solve: Apply conservation of momentum in the x- and y-directions.

$$(m_C + m_{VW})v_{1x} = (m_C + m_{VW})v_1 \cos\theta = m_{VW}(v_{0x})_{VW}$$

$$(m_C + m_{VW})v_{1y} = (m_C + m_{VW})v_1 \sin\theta = m_C(v_{0y})_C$$

From the y-equation, we find

$$v_1 = \frac{m_C(v_{0y})_C}{(m_C + m_{VW})\sin\theta} = \frac{(2000\text{ kg})(3.0\text{ m/s})}{(3000\text{ kg})\sin 35°} = 3.49\text{ m/s}$$

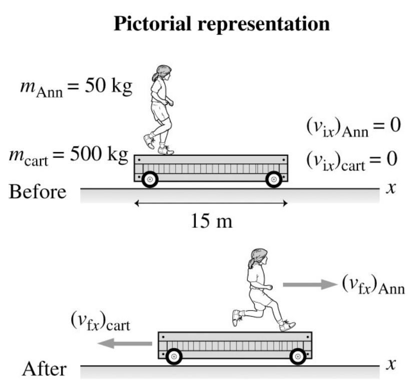
Inserting this value into the x-equation gives

$$(v_{0x})_{VW} = \frac{(m_C + m_{VW})v_1 \cos\theta}{m_{VW}} = \frac{(3000\text{ kg})(3.49\text{ m/s})\cos 35°}{1000\text{ kg}} = 8.6\text{ m/s}$$

Assess: The speed of the VW was nearly three times that of the Cadillac, which is reasonable since the much heavier Cadillac was deflected 55° from its original direction of travel.

9.52. Model: Model Ann and cart as particles. The initial momentum is $p_i = 0$ kg m/s in a coordinate system attached to the ground. As Ann begins running to the right, the cart will have to recoil to the left to conserve momentum.

Visualize:



Solve: The difficulty with this problem is that we are given Ann's velocity of 5.0 m/s relative to the cart. If the cart is also moving with velocity v_{cart} then Ann's velocity relative to the ground is not 5.0 m/s. Using the Galilean transformation equation for velocity, Ann's velocity relative to the ground is

$$(v_{fx})_{Ann} = (v_{fx})_{cart} + 5.0\text{ m/s}$$

Now, the momentum conservation equation $p_{ix} = p_{fx}$ is

$$0 \text{ kg m/s} = m_{Ann}(v_{fx})_{Ann} + m_{cart}(v_{fx})_{cart} \Rightarrow 0 \text{ kg m/s} = (50 \text{ kg})[(v_{fx})_{cart} + 5.0 \text{ m/s}] + (500 \text{ kg})(v_{fx})_{cart}$$

$$(v_{fx})_{cart} = -0.45 \text{ m/s}$$

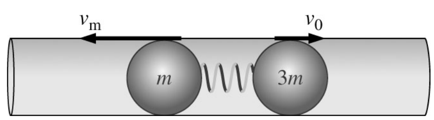
Using the recoil velocity $(v_{fx})_{cart}$ relative to the ground, we find Ann's velocity relative to the ground to be

$$(v_{fx})_{Ann} = 5.00 \text{ m/s} - 0.45 \text{ m/s} = 4.55 \text{ m/s}$$

The distance Ann runs relative to the ground is $\Delta x = (v_{fx})_{Ann}\Delta t$, where Δt is the time it takes to reach the end of the cart. Relative to the cart, which is 15 m long, Ann's velocity is 5 m/s. Thus, $\Delta t = (15 \text{ m})/(5.0 \text{ m/s}) = 3.0 \text{ s}$. Her distance over the ground during this interval is

$$\Delta x = (v_{fx})_{Ann}\Delta t = (4.55 \text{ m/s})(3.0 \text{ s}) = 14 \text{ m}$$

9.53. Model: Assume that the tube is frictionless and horizontal, and ignore air resistance. Model the two balls as particles.
Visualize:



Solve: The initial momentum of the system is zero because both balls are stationary. Therefore, conservation of momentum tells us that the final momentum of the system must be zero:

$$mv_m + 3mv_0 = 0 \Rightarrow v_m = -3v_0$$

Thus, the speed of the lighter ball is $3v_0$.
Assess: The negative sign indicates that the lighter ball moves in the direction opposite the larger ball. The result is reasonable because the lighter ball is 1/3 the mass of the heavier one.
9.54. Model: Change in momentum is given by the impulse-momentum theorem (Equation 9.8).
Solve: Using $\Delta p = mv_{fx} - mv_{ix} = J_x = \int_{t_i}^{t_f} F_x(t)\,dx$ with $v_{ix} = 0$, the velocity after the force has been applied is

$$\begin{aligned} v_{fx} &= \frac{1}{m}\int_{t_i}^{t_f} F_x(t)\,dx = \frac{10 \text{ N}}{0.25 \text{ kg}} \int_{0.0 \text{ s}}^{2.0 \text{ s}} \sin(2\pi t/4.0 \text{ s})\,dt \\ &= (40 \text{ N/kg})\left[-\frac{4.0 \text{ s}}{2\pi}\cos\left(\frac{2\pi t}{4.0 \text{ s}}\right)\Bigg|_{0.0 \text{ s}}^{2.0 \text{ s}}\right] \\ &= -(25.5 \text{ m/s})[\cos(\pi) - \cos(0)] \\ &= -(25.5 \text{ m/s})(-1-1) \\ &= 51 \text{ m/s} \end{aligned}$$

Assess: The force is applied for half the period of 4.0 s. During that time, $\sin\left(\frac{2\pi t}{4.0 \text{ s}}\right)$ is positive, so an object initially at rest acquires a positive velocity.

9.55. Model: Apply the impulse-momentum theorem (Equation 9.8) to find the initial velocity.

$$\Delta p = \int F\,dt$$

$$v_{fx} = v_{ix} + \frac{1}{m}\int F\,dt$$

$$= -5.0 \text{ m/s} + \frac{1}{0.500 \text{ kg}}\int_{-2}^{2}(4 - t^2)\,dt$$

$$= -5.0 \text{ m/s} + \frac{1}{0.500 \text{ kg}}\left(4t - \frac{1}{3}t^3\right)_{-2}^{2}$$

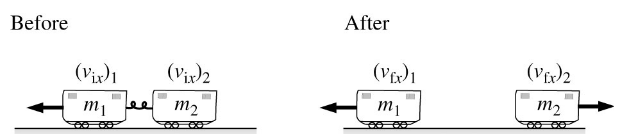
$$= -5.0 \text{ m/s} + \frac{1}{0.500 \text{ kg}}\left[8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right)\right]$$

$$= 16 \text{ m/s}$$

Assess: The impulse is large enough to reverse the direction of motion of the particle.

9.56. Model: The two railcars make up a system. The impulse approximation is used while the spring is expanding, so friction can be ignored.
Visualize:

Pictorial representation



Known
$(v_{ix})_1 = (v_{ix})_2 = 0$
$m_1 = 30$ tons $m_2 = 90$ tons
$(v_{fx})_2 - (v_{fx})_1 = 4.0$ m/s

Find
$(v_{fx})_1$

Solve: Since the cars are at rest initially, the total momentum of the system is zero. Conservation of momentum gives

$$0 = m_1(v_{fx})_1 + m_2(v_{fx})_2$$

We are only told that the relative velocity of the two cars after the spring expands is 4.0 m/s, so

$$(v_{fx})_2 - (v_{fx})_1 = 4.0 \text{ m/s}$$

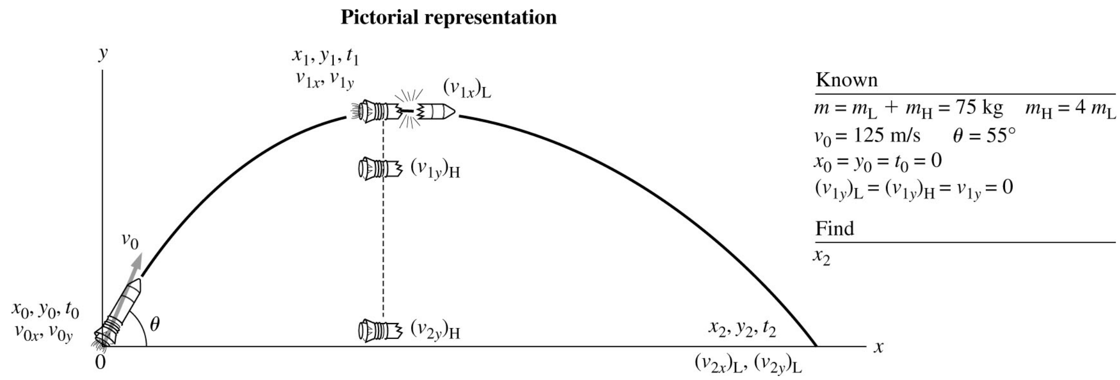
Substitute $(v_{fx})_2 = (v_{fx})_1 + 4.0$ m/s into the conservation of momentum equation, then solve for $(v_{fx})_1$:

$$0 = m_1(v_{fx})_1 + m_2[(v_{fx})_1 + 4.0 \text{ m/s}]$$

$$(v_{fx})_1 = -\frac{m_2(4.0 \text{ m/s})}{(m_1 + m_2)} = -\frac{(90 \text{ tons})(4.0 \text{ m/s})}{(30 \text{ tons} + 90 \text{ tons})} = -3.0 \text{ m/s}$$

so the speed of the 30 ton car relative to the ground is 3.0 m/s.
Assess: The other more massive railcar has a velocity $(v_{fx})_2 = (v_{fx})_1 + 4.0 \text{ m/s} = 1.0 \text{ m/s}.$ A slower speed for the more massive car makes sense.

9.57. Model: This is a three-part problem. In the first part, the shell, treated as a particle, is launched as a projectile and reaches its highest point. We will use constant-acceleration kinematic equations for this part. The shell, which is our system, then explodes at the highest point. During this brief explosion time, momentum is conserved. In the third part, we will again use the kinematic equations to find the horizontal distance between the landing of the lighter fragment and the origin.
Visualize:



Solve: The initial velocity is

$$v_{0x} = v\cos\theta = (125 \text{ m/s})\cos 55° = 71.7 \text{ m/s}$$
$$v_{0y} = v\sin\theta = (125 \text{ m/s})\sin 55° = 102.4 \text{ m/s}$$

At the highest point, $v_{1y} = 0$ m/s and $v_{1x} = 71.7$ m/s. The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_L(v_{1x})_L + m_H(v_{1x})_H = (m_L + m_H)v_{1x}$$

The heavier particle falls straight down, so $(v_{1x})_H = 0$ m/s. Thus,

$$(15 \text{ kg})(v_{1x})_L + 0 \text{ kg m/s} = (15 \text{ kg} + 60 \text{ kg})(71.7 \text{ m/s}) \quad \Rightarrow \quad (v_{1x})_L = 358 \text{ m/s}$$

That is, the velocity of the smaller fragment immediately after the explosion is 358 m/s and this velocity is in the horizontal *x*-direction. Note that $(v_{1y})_L = 0$ m/s. To find x_2, we will first find the displacement $x_1 - x_0$ and then $x_2 - x_1$. For $x_1 - x_0$,

$$v_{1y} = v_{0y} + a_y(t_1 - t_0) \quad \Rightarrow \quad 0 \text{m/s} = (102.4 \text{ m/s}) + (-9.8 \text{ m/s}^2)(t_1 - 0 \text{ s}) \quad \Rightarrow \quad t_1 = 10.45 \text{ s}$$
$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 \quad \Rightarrow \quad x_1 - x_0 = (71.7 \text{ m/s})(10.45 \text{ s}) + 0 \text{ m} = 749 \text{ m}$$

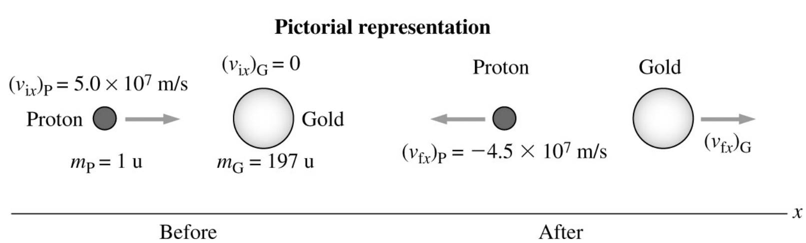
For $x_2 - x_1$:

$$x_2 = x_1 + (v_{1x})_L(t_2 - t_1) + \tfrac{1}{2}a_x(t_2 - t_1)^2 \Rightarrow x_2 - x_1 = (358 \text{ m/s})(10.45 \text{ s}) + 0 \text{ m} = 3741 \text{ m}$$
$$x_2 = (x_2 - x_1) + (x_1 - x_0) = 3741 \text{ m} + 749 \text{ m} = 4490 \text{ m} = 4.5 \text{ km}$$

Assess: Note that the time of ascent to the highest point is equal to the time of descent to the ground, that is, $t_1 - t_0 = t_2 - t_1$.

9.58. Model: Model the proton (P) and the gold atom (G) as particles. The two constitute our system, and momentum is conserved in the collision between the proton and the gold atom.

Visualize:



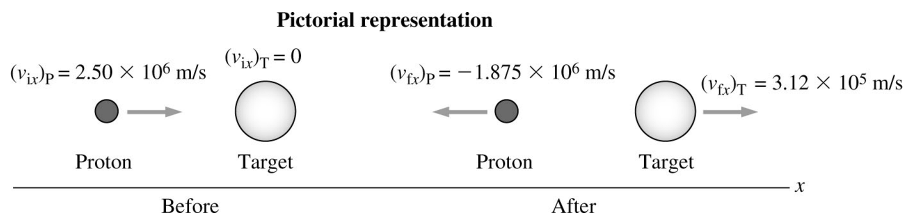
Solve: The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_G(v_{fx})_G + m_P(v_{fx})_P = m_P(v_{ix})_P + m_G(v_{ix})_G$$

$$(197\text{ u})(v_{\text{fx}})_{\text{G}} + (1\text{ u})(-0.90 \times 5.0 \times 10^7\text{ m/s}) = (1\text{ u})(5.0 \times 10^7\text{ m/s}) + 0\text{ u m/s}$$

$$(v_{\text{fx}})_{\text{G}} = 4.8 \times 10^5\text{ m/s}$$

9.59. Model: Model the proton (P) and the target nucleus (T) as particles. The proton and the target nucleus make our system and in the collision between them momentum is conserved. This is due to the impulse approximation because the collision lasts a very short time and the external forces acting on the system during this time are not significant.
Visualize:



Solve: The conservation of momentum equation $p_{\text{fx}} = p_{\text{ix}}$ gives

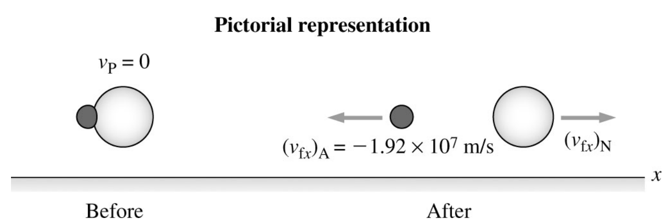
$$m_{\text{T}}(v_{\text{fx}})_{\text{T}} + m_{\text{P}}(v_{fx})_{\text{P}} = m_{\text{T}}(v_{\text{ix}})_{\text{T}} + m_{\text{P}}(v_{\text{ix}})_{\text{P}}$$

$$m_{\text{T}}(3.12 \times 10^5\text{ m/s}) + (1\text{ u})(-0.750 \times 2.50 \times 10^6\text{ m/s}) = 0\text{ u m/s} + (1\text{ u})(2.50 \times 10^6\text{ m/s})$$

$$m_{\text{T}} = 14.0\text{ u}$$

Assess: This is the mass of the nucleus of a nitrogen atom.

9.60. Model: This problem deals with an "explosion" in which a ^{214}Po nucleus (P) decays into an alpha-particle (A) and a daughter nucleus (N). During the "explosion" or decay, the total momentum of the system is conserved.
Visualize:

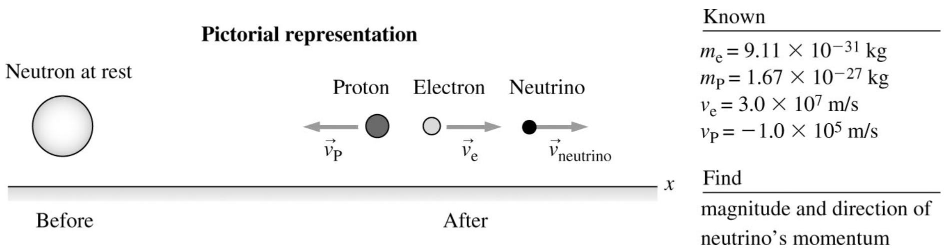


Solve: Conservation of mass requires the daughter nucleus to have mass $m_{\text{N}} = 214\text{ u} - 4\text{ u} = 210\text{ u}$. The conservation of momentum equation $p_{\text{fx}} = p_{\text{ix}}$ gives

$$m_{\text{N}}(v_{\text{fx}})_{\text{N}} + m_{\text{A}}(v_{\text{fx}})_{\text{A}} = (m_{\text{N}} + m_{\text{A}})v_{\text{P}} \quad \Rightarrow \quad (210\text{ u})(v_{\text{fx}})_{\text{N}} + (4\text{ u})(-1.92 \times 10^7\text{m/s}) = 0\text{ u m/s}$$

$$(v_{\text{fx}})_{\text{N}} = 3.66 \times 10^5\text{ m/s}$$

9.61. Model: The neutron's decay is an "explosion" of the neutron into several pieces. The neutron is an isolated system, so its momentum should be conserved. The observed decay products, the electron and proton, move in opposite directions.
Visualize:



Solve: **(a)** The initial momentum is $p_{ix} = 0$ kg m/s. The final momentum $p_{fx} = m_e v_e + m_p v_p$ is

$$p_{fx} = 2.73 \times 10^{-23} \text{ kg m/s} - 1.67 \times 10^{-22} \text{ kg m/s} = -1.4 \times 10^{-22} \text{ kg m/s}$$

No, momentum does not seem to be conserved.

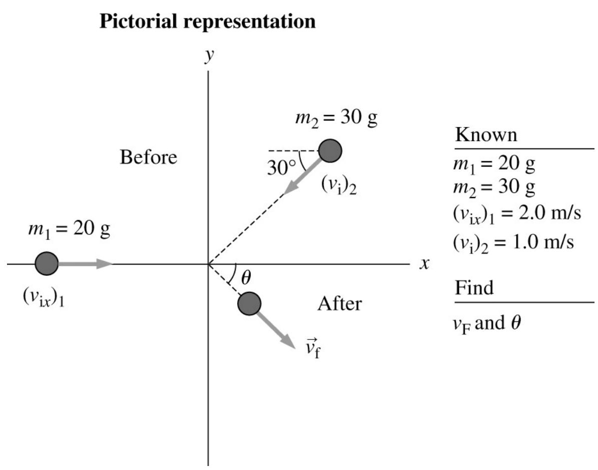
(b) and **(c)** If the neutrino is needed to conserve momentum, then $p_e + p_P + p_{neutrino} = 0$ kg m/s. This requires

$$p_{neutrino} = -(p_e + p_P) = +1.4 \times 10^{-22} \text{ kg m/s}$$

The neutrino must "carry away" 1.4×10^{-22} kg m/s of momentum in the same direction as the electron.

9.62. Model: Model the two balls of clay as particles. Our system comprises these two balls. Momentum is conserved in the perfectly inelastic collision.

Visualize:



Solve: Applying conservation of momentum in the x-direction gives

$$p_{fx} = p_{ix} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$
$$= (0.020 \text{ kg})(2.0 \text{ m/s}) - (0.030 \text{ kg})(1.0 \text{ m/s})\cos 30° = 0.0140 \text{ kg m/s}$$

The y-component of the final momentum is

$$p_{fy} = p_{iy} = m_1(v_{iy})_1 + m_2(v_{iy})_2$$
$$= (0.02 \text{ kg})(0 \text{ m/s}) - (0.03 \text{ kg})(1.0 \text{ m/s})\sin 30° = -0.0150 \text{ kg m/s}$$
$$p_f = \sqrt{(0.014 \text{ kg m/s})^2 + (-0.015 \text{ kg m/s})^2} = 0.0205 \text{ kg m/s}$$

Since $p_f = (m_1 + m_2)v_f = 0.0205$kg m/s, the final speed is

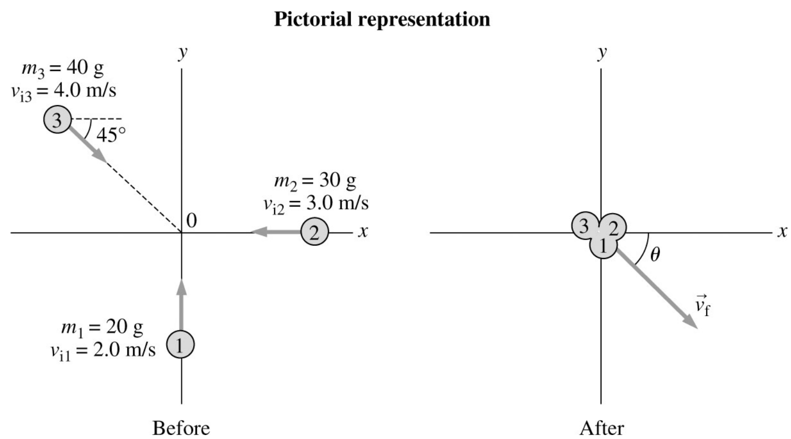
$$v_f = \frac{0.0205 \text{ kg m/s}}{(0.02 + 0.03) \text{ kg}} = 0.41 \text{ m/s}$$

and the direction is

$$\theta = \tan^{-1}\frac{|p_{fy}|}{p_{fx}} = \tan^{-1}\frac{0.015}{0.014} = 47° \text{ south of east}$$

9.63. Model: Model the three balls of clay as particle 1 (moving north), particle 2 (moving west), and particle 3 (moving southeast). The three stick together during their collision, which is perfectly inelastic. The momentum of the system is conserved.

Visualize:



Solve: The three initial momenta are

$$\vec{p}_{i1} = m_1\vec{v}_{i1} = (0.020 \text{ kg})(2.0 \text{ m/s})\hat{j} = 0.040\hat{j} \text{ kg m/s}$$

$$\vec{p}_{i2} = m_2\vec{v}_{i2} = (0.030 \text{ kg})(-3.0 \text{ m/s }\hat{i}) = -0.090\hat{i} \text{ kg m/s}$$

$$\vec{p}_{i3} = m_3\vec{v}_{i3} = (0.040 \text{ kg})[(4.0 \text{ m/s})\cos45°\hat{i} - (4.0 \text{ m/s})\sin45°\hat{j}] = (0.113\hat{i} - 0.113\hat{j}) \text{ kg m/s}$$

Since $\vec{p}_f = \vec{p}_i = \vec{p}_{i1} + \vec{p}_{i2} + \vec{p}_{i3}$, we have

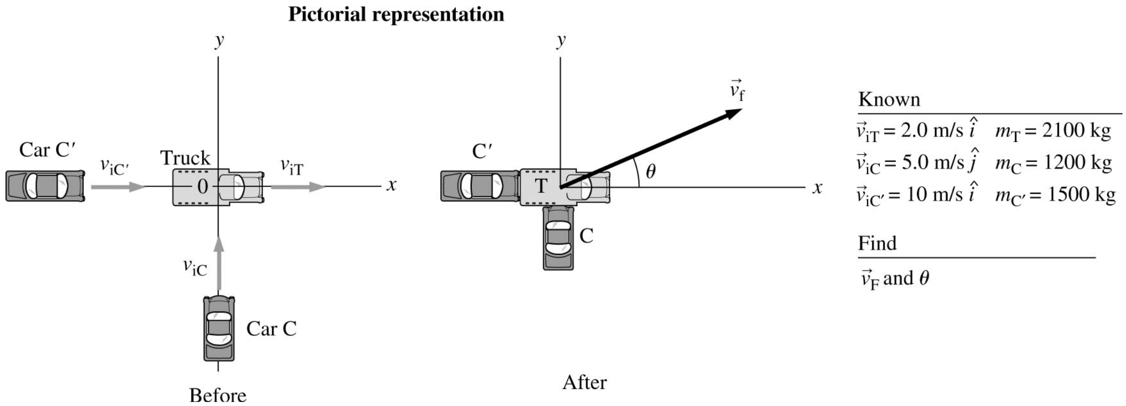
$$(m_1 + m_2 + m_3)\vec{v}_f = (0.023\hat{i} - 0.073\hat{j}) \text{ kg m/s} \quad \Rightarrow \quad \vec{v}_f = (0.256\hat{i} - 0.811\hat{j}) \text{ m/s}$$

$$v_f = \sqrt{(0.256 \text{ m/s})^2 + (-0.811 \text{ m/s})^2} = 0.85 \text{ m/s}$$

$$\theta = \tan^{-1}\frac{|v_{fy}|}{v_{fx}} = \tan^{-1}\frac{0.811}{0.256} = 72° \text{ below the } x\text{-axis.}$$

9.64. Model: Model the truck (T) and the two cars (C and C') as particles. The three forming our system stick together during their collision, which is perfectly inelastic. Since no significant external forces act on the system during the brief collision time, the momentum of the system is conserved.

Visualize:



Solve: The three momenta are

$$\vec{p}_{iT} = m_T \vec{v}_{iT} = (2100 \text{ kg})(2.0 \text{ m/s})\hat{i} = 4200\hat{i} \text{ kg m/s}$$

$$\vec{p}_{iC} = m_C \vec{v}_{iC} = (1200 \text{ kg})(5.0 \text{ m/s})\hat{j} = 6000\hat{j} \text{ kg m/s}$$

$$\vec{p}_{iC'} = m_{C'} \vec{v}_{iC'} = (1500 \text{ kg})(10 \text{ m/s})\hat{i} = 15{,}000\hat{i} \text{ kg m/s}$$

$$\vec{p}_f = \vec{p}_i = \vec{p}_{iT} + \vec{p}_{iC} + \vec{p}_{iC'} = (19{,}200\hat{i} + 6000\hat{j}) \text{ kg m/s}$$

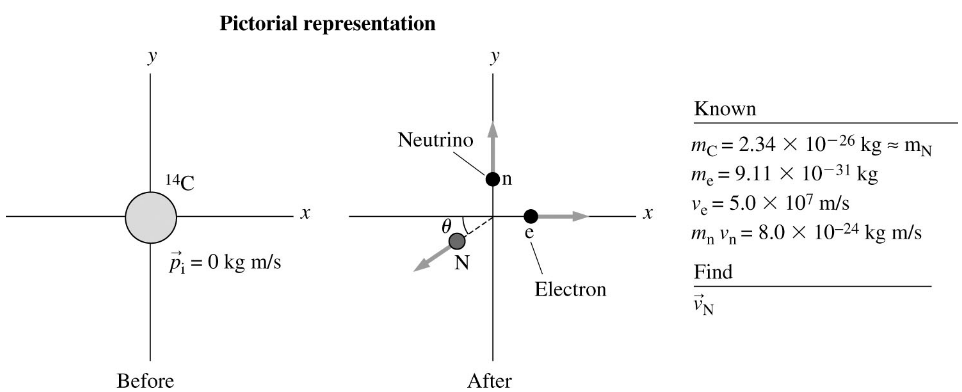
$$p_f = (m_T + m_C + m_{C'})v_f = \sqrt{(19{,}200 \text{ kg m/s})^2 + (6000 \text{ kg m/s})^2}$$

$$v_f = 4.2 \text{ m/s}, \; \theta = \tan^{-1}\frac{p_y}{p_x} = \tan^{-1}\frac{6000}{19{,}200} = 17^\circ \text{ above the } +x\text{-axis}$$

Assess: A speed of 4.2 m/s for the entangled three vehicles is reasonable since the individual speeds of the cars and the truck before entanglement were of the same order of magnitude.

9.65. Model: The ^{14}C atom undergoes an "explosion" and decays into a nucleus, an electron, and a neutrino. Momentum is conserved in the process of "explosion" or decay.

Visualize:



Solve: The conservation of momentum equation $\vec{p}_f = \vec{p}_i = 0$ kg m/s gives

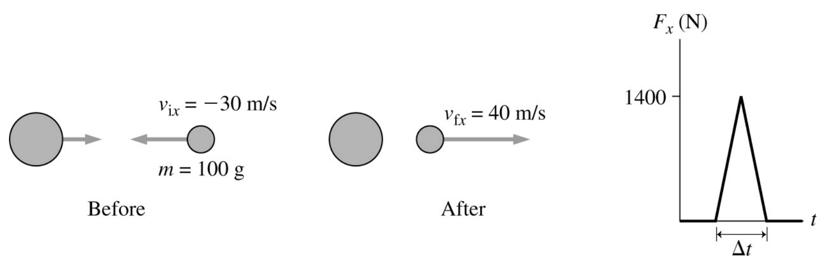
$$\vec{p}_e + \vec{p}_n + \vec{p}_N = 0 \text{ N} \;\Rightarrow\; \vec{p}_N = -(\vec{p}_e + \vec{p}_n) = -m_e\vec{v}_e - m_n\vec{v}_n$$

$$= -(9.11 \times 10^{-31} \text{ kg})(5.0 \times 10^7 \text{ m/s})\hat{i} - (8.0 \times 10^{-24} \text{ kg m/s})\hat{j} = -(45.55 \times 10^{-24}\hat{i} + 8.0 \times 10^{-24}\hat{j}) \text{ kg m/s}$$

$$p_N = m_N v_N = \sqrt{(45.55 \times 10^{-24})^2 + (8.0 \times 10^{-24})^2} \text{ kg m/s}$$

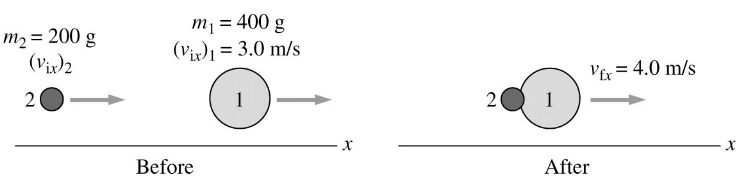
$$(2.34 \times 10^{-26} \text{ kg})v_N = 4.62 \times 10^{-23} \text{ kg m/s} \;\Rightarrow\; v_N = 2.0 \times 10^3 \text{ m/s}$$

9.66. **(a)** A 100 g ball traveling to the left at 30 m/s is batted back to the right at 40 m/s. The force curve for the force of the bat on the ball can be modeled as a triangle with a maximum force of 1400 N. How long is the ball in contact with the bat?
(b)



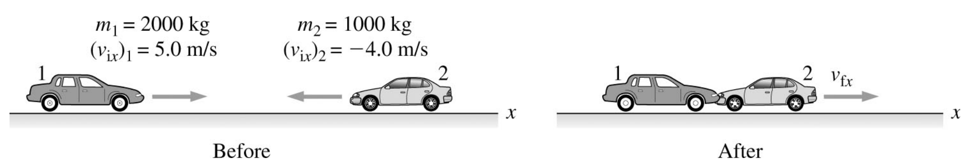
(c) The solution is $\Delta t = 0.100 \text{ s} = 10 \text{ ms}$.

9.67. **(a)** A 200 g ball of clay traveling to the right overtakes and collides with a 400 g ball of clay traveling to the right at 3.0 m/s. The balls stick and move forward at 4.0 m/s. What was the speed of the 200 g ball of clay?
(b)



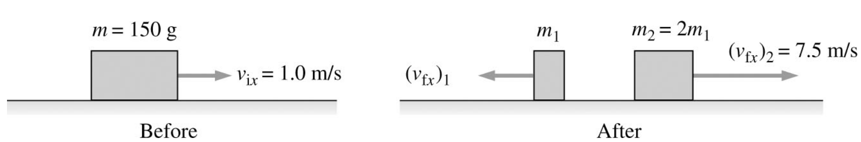
(c) The solution is $(v_{ix})_2 = 6.0 \text{ m/s}$.

9.68. **(a)** A 2000 kg auto traveling east at 5.0 m/s suffers a head-on collision with a small 1000 kg auto traveling west at 4.0 m/s. They lock bumpers and stick together after the collision. What will be the speed and direction of the combined wreckage after the collision?
(b)



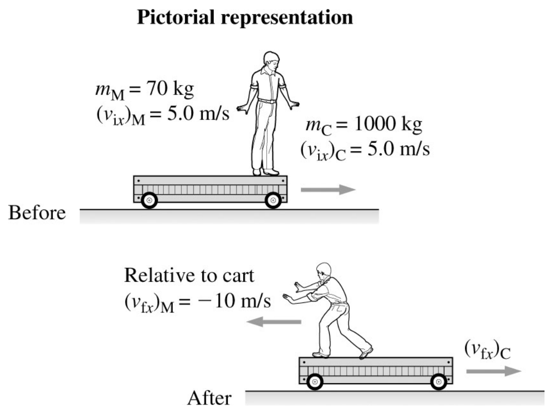
(c) The solution is $v_{fx} = 2.0 \text{ m/s}$ along the $+x$ direction.

9.69. **(a)** A 150 g spring-loaded toy is sliding across a frictionless floor at 1.0 m/s. It suddenly explodes into two pieces. One piece, which has twice the mass of the second piece, continues to slide in the forward direction at 7.5 m/s. What is the speed and direction of the second piece?
(b)



(c) The solution is $(v_{fx})_1 = -12 \text{ m/s}$. The minus sign tells us that the second piece moves backward at 12 m/s.

9.70. Model: The cart + man (C + M) is our system. It is an isolated system, and momentum is conserved.
Visualize:



Solve: The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_M(v_{fx})_M + m_C(v_{fx})_C = m_M(v_{ix})_M + m_C(v_{ix})_C$$

Note that $(v_{fx})_M$ and $(v_{fx})_C$ are the final velocities of the man and the cart relative to the ground. What is given in this problem is the velocity of the man relative to the moving cart. The man's velocity relative to the ground is

$$(v_{fx})_M = (v_{fx})_C - 10 \text{ m/s}$$

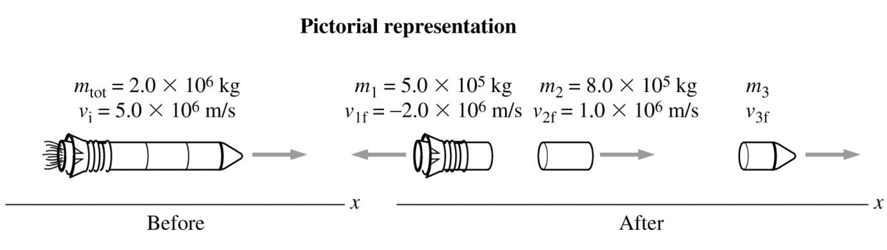
With this form for $(v_{fx})_M$, we rewrite the momentum conservation equation as

$$m_M[(v_{fx})_C - 10 \text{ m/s}] + m_C(v_{fx})_C = m_M(5.0 \text{ m/s}) + m_C(5.0 \text{ m/s})$$

$$(70 \text{ kg})[(v_{fx})_C - 10 \text{ m/s}] + (1000 \text{ kg})(v_{fx})_C = (1000 \text{ kg} + 70 \text{ kg})(5.0 \text{ m/s})$$

$$(v_{fx})_C[1000 \text{ kg} + 70 \text{ kg}] = (1070 \text{ kg})(5.0 \text{ m/s}) + (70 \text{ kg})(10 \text{ m/s}) \Rightarrow (v_{fx})_C = 5.7 \text{ m/s}$$

9.71. Model: This is an isolated system, so momentum is conserved in the explosion. Momentum is a *vector* quantity, so the direction of the initial velocity vector $\vec{v}_1$ establishes the direction of the momentum vector. The final momentum vector, after the explosion, must still point in the +*x*-direction. The two known pieces continue to move along this line and have no *y*-components of momentum. The missing third piece cannot have a *y*-component of momentum if momentum is to be conserved, so it must move along the *x*-axis—either straight forward or straight backward. We can use conservation laws to find out.
Visualize:



Solve: From the conservation of mass, the mass of piece 3 is

$$m_3 = m_{total} - m_1 - m_2 = 7.0 \times 10^5 \text{ kg}$$

To conserve momentum along the *x*-axis, we require

$$p_i = m_{total}v_i = p_f = p_{1f} + p_{2f} + p_{3f} = m_1v_{1f} + m_2v_{2f} + p_{3f}$$

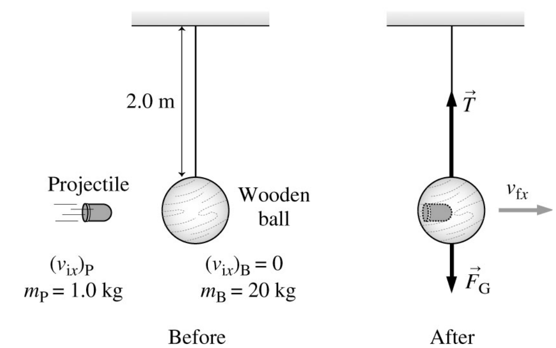
$$p_{3f} = m_{total}v_i - m_1v_{1f} - m_2v_{2f} = +1.02 \times 10^{13} \text{ kg m/s}$$

Because $p_{3f} > 0$, the third piece moves in the $+x$-direction, that is, straight forward. Because we know the mass m_3, we can find the velocity of the third piece as follows:

$$v_{3f} = \frac{p_{3f}}{m_3} = \frac{1.02 \times 10^{13} \text{ kg m/s}}{7.0 \times 10^5 \text{ kg}} = 1.5 \times 10^7 \text{ m/s}$$

The third piece moves to the right with a speed of 1.5×10^7 m/s.

9.72. Model: The projectile + wood ball are our system. In the collision, momentum is conserved.
Visualize:



Solve: The momentum conservation equation $p_{fx} = p_{ix}$ is

$$(m_P + m_B)v_{fx} = m_P(v_{ix})_P + m_B(v_{ix})_B \Rightarrow (1.0 \text{ kg} + 20 \text{ kg})v_{fx} = (1.0 \text{ kg})(v_{ix})_P + 0 \text{ kg m/s}$$

$$(v_{ix})_P = 21v_{fx}$$

We therefore need to determine v_{fx}. Newton's second law for circular motion is

$$T - F_G = T - (m_P + m_B)g = \frac{(m_P + m_B)v_{fx}^2}{r}$$

Using $T_{max} = 400$ N, this equation gives

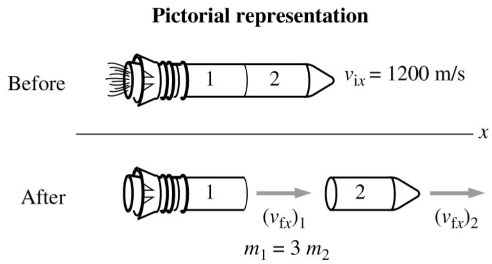
$$400\text{N} - (1.0 \text{ kg} + 20 \text{ kg})(9.8 \text{ m/s}) = \frac{(1.0 \text{ kg} + 20 \text{ kg})v_{fx}^2}{2.0 \text{ m}} \Rightarrow (v_{fx})_{max} = 4.3 \text{ m/s}$$

Going back to the momentum conservation equation,

$$(v_{ix})_P = 21v_{fx} = (21)(4.3 \text{ m/s}) = 90 \text{ m/s}$$

That is, the largest speed this projectile can have without causing the cable to break is 90 m/s.

9.73. Model: This is an "explosion" problem and momentum is conserved. The two-stage rocket is our system.
Visualize:



Solve: Relative to the ground, the conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_1(v_{fx})_1 + m_2(v_{fx})_2 = (m_1 + m_2)v_{1x}$$

$$3\text{ m}_2(v_{fx})_1 + m_2(v_{fx})_2 = (4\text{ m}_2)(1200\text{ m/s}) \Rightarrow 3(v_{fx})_1 + (v_{fx})_2 = 4800\text{ kg m/s}$$

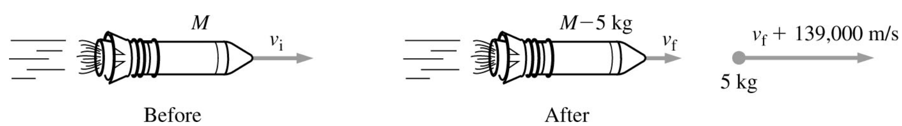
The fact that the first stage is pushed backward at 35 m/s relative to the second can be written

$$(v_{fx})_1 = -35\text{ m/s} + (v_{fx})_2$$

Substituting this form of $(v_{fx})_1$ in the conservation of momentum equation,

$$3[-35\text{ m/s} + (v_{fx})_2] + (v_{fx})_2 = 4800\text{ kg m/s} \Rightarrow (v_{fx})_2 = 1.2\text{ km/s}$$

9.74. Model: Let the system be rocket + bullet. This is an isolated system, so momentum is conserved.
Visualize: The fact that the bullet's velocity relative to the rocket is 139,000 can be written $(v_f)_B = (v_f)_R + 139{,}000$ m/s.



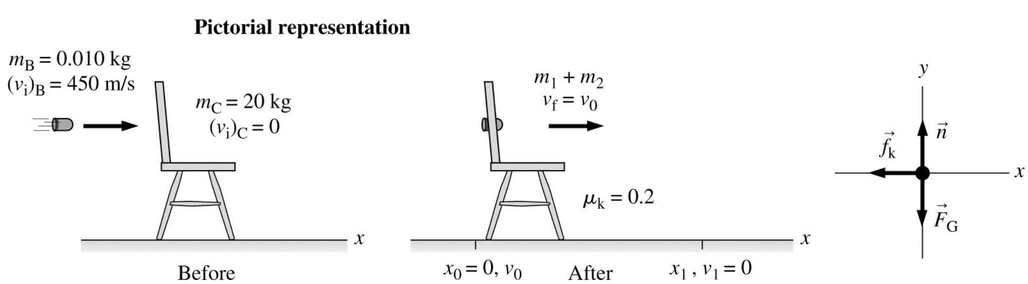
Solve: Consider the firing of one bullet when the rocket has mass *M* and velocity v_i. The conservation of momentum equation $p_f = p_i$ gives

$$(M - 5\text{kg})v_f + (5\text{ kg})(v_f + 139{,}000\text{ m/s}) = Mv_i$$

$$\Delta v = v_f - v_i = -\frac{5\text{ kg}}{M}(139{,}000\text{ m/s})$$

The rocket starts with mass $M = 2000$ kg, which is much larger than 5 kg. If only a few bullets are needed, *M* will not change significantly as the rocket slows. If we assume that *M* remains constant at 2000 kg, the loss of speed per bullet is $\Delta v = -347.5\text{ m/s} = -1250\text{ km/h}$. Thus exactly 8 bullets will reduce the speed by 10,000 km/h, from 25,000 km/h to 15,000 km/h. If you're not sure that treating *M* as a constant is valid, you can calculate Δv for each bullet and reduce *M* by 5 kg after each shot. The loss of mass causes Δv to increase slightly for each bullet. An eight-step calculation then finds that 8 bullets will slow the rocket to 14,900 km/h. Seven bullets wouldn't be enough, and 9 would slow the rocket far too much.

9.75. Visualize:



Solve: Ladies and gentlemen of the jury, how far would the chair slide if it was struck with a bullet from my client's gun? We know the bullet's velocity as it leaves the gun is 450 m/s. The bullet travels only a small distance to the chair, so we will neglect any speed loss due to air resistance. The bullet and chair can be considered an isolated system during the brief interval of the collision. The bullet embedded itself in the chair, so this was a perfectly inelastic collision. Momentum conservation allows us to calculate the velocity of the chair *immediately* after the collision as follows:

$$p_{ix} = p_{fx} \quad \Rightarrow \quad m_B(v_i)_B = (m_B + m_C)v_f \quad \Rightarrow \quad v_f = \frac{m_B(v_i)_B}{m_B + m_C} = \frac{(450 \text{ m/s})(0.010 \text{ kg})}{20.01 \text{ kg}} = 0.225 \text{ m/s}$$

This is the velocity *immediately* after the collision when the chair starts to slide but before it covers any distance. For the purpose of the problem in dynamics, call this the initial velocity v_0. The free-body diagram of the chair shows three forces. Newton's second law applied to the chair (with the embedded bullet) is

$$a_x = a = \frac{(F_{net})_x}{m_{tot}} = \frac{-f_k}{m_{tot}} = -\frac{\mu_k n}{m_{tot}}, \; a_y = 0 \text{ m/s}^2 = \frac{(F_{net})_y}{m_{tot}} = \frac{n - m_{tot}g}{m_{tot}}$$

where we've used the friction model in the x-equation. The y-equation yields $n = m_{tot}g$, and the x-equation yields $a = -\mu_k g = -1.96 \text{ m/s}^2$. We know the coefficient of kinetic friction because it is a wood chair sliding on a wood floor. Finally, we have to determine the stopping distance of the chair. The motion of the chair ends with $v_1 = 0$ m/s after sliding a distance Δx, so

$$v_1^2 = 0 \text{ m}^2/\text{s}^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad \Delta x = -\frac{v_0^2}{2a} = -\frac{(0.225 \text{ m/s})^2}{2(-1.96 \text{ m/s}^2)} = 0.013 \text{ m} = 1.3 \text{ cm}$$

If the bullet lost any speed in the air before hitting the chair, the sliding distance would be even less. So you can see that the *most* the chair could slide if it had been struck by a bullet from my client's gun would be 1.3 cm. But in actuality, the chair slid 3 cm, more than twice as far. The murder weapon, ladies and gentlemen, was a much more powerful gun than the one possessed by my client. I rest my case.