Flipping Physics Lecture Notes:

Free Response Question #5 - AP Physics 1 - 2015 Exam Solutions
http://www.flippingphysics.com/ap1-2015-frq5.html

Part (a): We know the velocity of a wave equals its frequency times its wavelength, $v = f\lambda$. Each of the four strings has the same length, L, and is vibrating at its fundamental frequency, therefore the wavelength for each string is the same. Each string is vibrating at a difference frequency, therefore the velocity of the wave on each string must be different. The velocity of a wave on a string is given as $v = \sqrt{\frac{F_T}{m/L}}$. Because the mass, M, attached to the end of each string is the same, the force of tension, F_T, will also be the same. Therefore, the mass per unit length, m/L, or linear mass density, for each string must be different.

Part (b): Frequency, f, is on the y-axis and $\frac{1}{m/L}$ is on the x-axis. Let's combine equations from part (a) and solve for the relationship between the x and y-axis variables:

$$v = f\lambda = \sqrt{\frac{F_T}{m/L}} \Rightarrow f^2\lambda^2 = \frac{F_T}{m/L} \Rightarrow f^2 = \left(\frac{F_T}{\lambda^2}\right)\left(\frac{1}{m/L}\right)$$

In order to get a linear relationship, we would need to graph frequency squared, f^2, as a funciton of $\frac{1}{m/L}$. This is because $f^2 = \left(\frac{F_T}{\lambda^2}\right)\left(\frac{1}{m/L}\right)$ and the slope intercept form of a line is $y = mx + b$. The y-intercept, b, would be zero and the slope, m, would be $\frac{F_T}{\lambda^2}$. So, *no* the graph would not be linear.

Part (c): Because antinodes are defined as the location on a standing wave with the greatest maximum amplitude and every point on the string takes the same amount of time to go through one full cycle, antinodes are the locations of "greatest average vertical speed". At it's first harmonic or fundamental frequency, the string will have two nodes, one on either end, and one antinode in the middle, halfway between the two nodes. The second harmonic increases both of those numbers by one. So, we now have three nodes, one node on either end and one node in the middle. And two antinodes, each antinode located halfway between nodes. So the locations of "greatest average vertical speed" are at the antinodes, one fourth of L toward the middle from both ends.