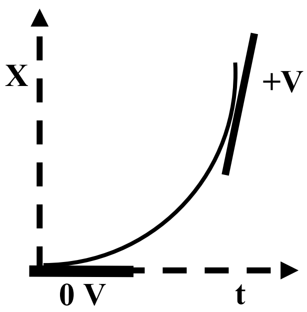
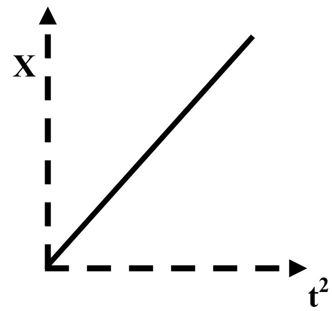
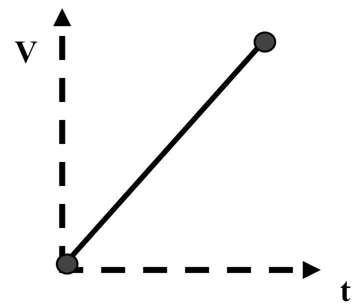
Unit 3 Notes for Kinematic Equations - From the lab, the students have the following graphs.



$X = m*t^2 + b$



$X = m*t^2 + b$



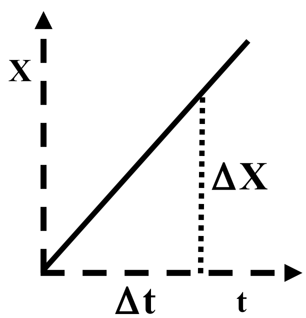
$V = m*t + b$

Post-lab extension

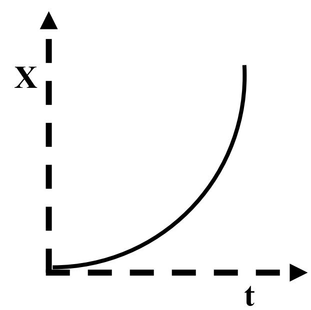
Discussion

Contrast the **x** vs **t** graph for this lab with the one obtained in unit II.

Unit II



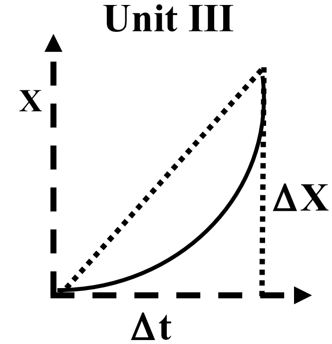
Unit III



One can speak of the average velocity as the __**slope**__ of the graph (above left) because the slope of a straight line is constant. It doesn't matter which two points are used to determine the slope.

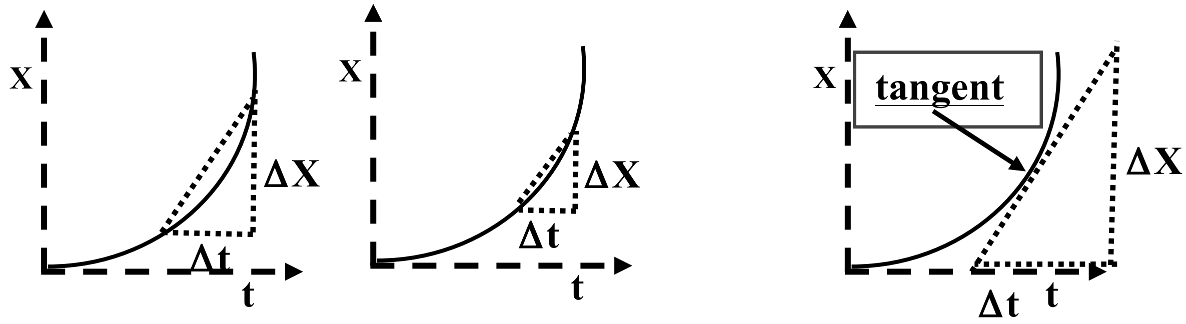
On the other hand, one could speak of the average velocity of the object in the graph at right, but since the object started very slowly and steadily increased its speed, the term average velocity has **little meaning**.

Unit III



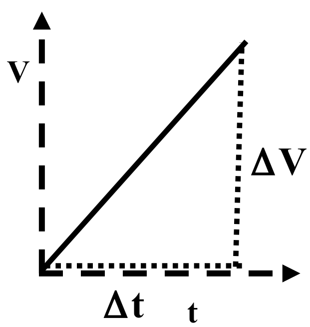
Average Velocity = $\Delta X / \Delta t$

What would be more useful is to have a way of describing the object's speed at a **given instant**. To develop this idea, you must show that, as you shrink the time interval Δt over which you calculate the average velocity, the secant (line intersecting the curve at two points) more closely resembles the curve during that interval.



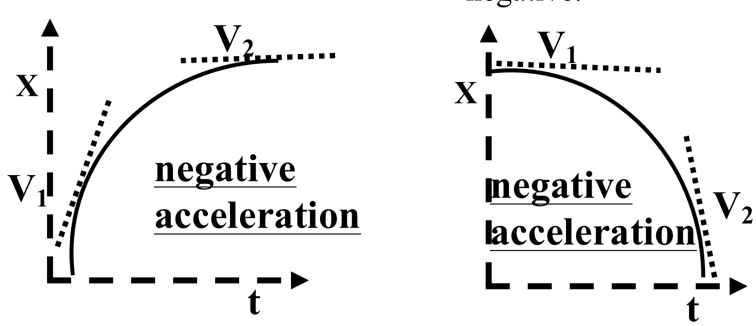
That is, the slope of the __**secant**____ gives the average velocity for that interval. As the interval gets shorter and shorter, the secant more closely approximates the curve. Thus, the average velocity of this interval becomes a more and more reasonable estimate of how fast the object is moving *at any instant* during this interval.

As one shrinks the interval, Δt to zero, the __**secant**__ becomes a tangent; the slope of the tangent is the average velocity at this instant, or simply the ***instantaneous velocity*** at that clock reading.



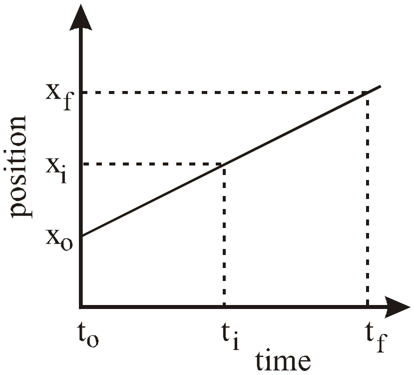
$V_f = V_i + a\Delta t$

A plot of instantaneous velocity (v, instead of v-bar) vs time should yield a straight line. The slope of this line is $\frac{\Delta\vec{v}}{\Delta\vec{t}} \equiv \bar{a}$. That is, the change in velocity during a given time interval is defined to be the *average acceleration*. The equation for the line can be written as $\vec{v} = \vec{a}t + \vec{v}_0$, where $\vec{V}_0$ is the y-intercept. It is important to define the acceleration this way, and then show examples of **x** vs **t** graphs in which the acceleration is negative.



In both cases, $v_2 - v_1$ is negative, yet very different situations are being represented. We advise against the use of the term ***deceleration***, because students invariably think that this term implies negative acceleration ***means*** slowing down; the two conditions are not synonymous.

Generalizing the linear v-t equation for any time interval t_i to t_f yields $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$ The development of this expression is provided below to clarify the use of Δt as opposed to t.



Constant <u>Velocity</u> means the forces are balanced.

UNIT 2 Constant Velocity

The slope is defined to be average velocity.

$$\boxed{\frac{\Delta\vec{x}}{\Delta t} \equiv \bar{\vec{v}}} \qquad \textit{Eq.1}$$

Equation of the line

$$\boxed{\vec{x} = \vec{v}t + \vec{x}_0} \qquad \textit{Eq.2}$$

Generalize the equation for the interval t_i to t_f.

At t_f:

$$\vec{x}_f = \vec{v}t_f + \vec{x}_0 \qquad \textit{Eq.3}$$

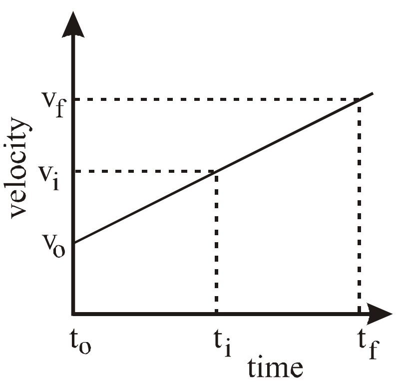
At t_i:

$$\vec{x}_i = \vec{v}t_i + \vec{x}_0 \qquad \textit{Eq.4}$$

Subtract equation 4 from 3:

$$\vec{x}_f - \vec{x}_i = \vec{v}(t_f - t_i) + \vec{x}_0 - \vec{x}_0$$

$$\boxed{\vec{x}_f = \vec{x}_i + \vec{v}\Delta t \quad \textit{Eq.5} \longrightarrow \textbf{\textit{NO ACCELERATION}}}$$



Constant <u>Acceleration</u> means there is a Net Force.

UNIT 3 Constant Acceleration

The slope is defined to be average acceleration.

$$\boxed{\frac{\Delta\vec{v}}{\Delta t} \equiv \bar{\vec{a}}} \qquad \textit{Eq.6}$$

Equation of the line

$$\boxed{\vec{v} = \vec{a}t + \vec{v}_0} \qquad \textit{Eq.7}$$

Generalize the equation for the interval t_i to t_f.

At t_f:

$$\vec{v}_f = \vec{a}t_f + \vec{v}_0 \qquad \textit{Eq.8}$$

At t_i:

$$\vec{v}_i = \vec{a}t_i + \vec{v}_0 \qquad \textit{Eq.9}$$

Subtract equation 9 from 8:

$$\vec{v}_f - \vec{v}_i = \vec{a}(t_f - t_i) + \vec{v}_0 - \vec{v}_0$$

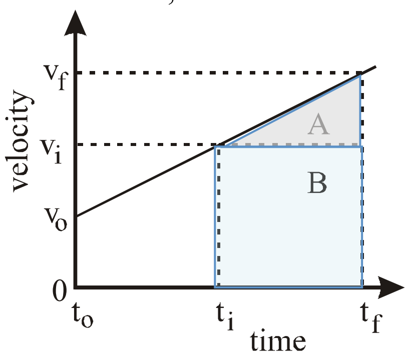
$$\boxed{\vec{v}_f = \vec{v}_i + \vec{a}\Delta t \quad \textit{Eq.10}}$$

CONSTANT ACCELERATION

Post-lab Extension (Development of Kinematic Expressions)

Developing the remaining kinematic equations involves finding the area under a v-t graph and algebraic combination of equations.

The displacement of a uniformly accelerating object is equivalent to the area under the v-t graph. In this situation, we are interested in the displacement during the time interval t_i to t_f.



Area of region B
length x width **area of a rectangle**

The velocity at the horizontal axis is zero;

$$(\vec{v}_i - 0)\cdot(t_f - t_i) = \vec{v}_i\Delta t$$

Area of region A:
1/2 *height x base* **area of a triangle**

$$\tfrac{1}{2}\cdot(\vec{v}_f - \vec{v}_i)\cdot(t_f - t_i)$$

$$\tfrac{1}{2}\cdot\Delta\vec{v}\cdot\Delta t \quad \textit{substitute } \vec{v} = \vec{a}\Delta t$$

$$\tfrac{1}{2}\cdot\vec{a}\Delta t\cdot\Delta t$$

½ * a(Δt)²

The total displacement is equal to A + B.

$$\boxed{\Delta\vec{x} = \tfrac{1}{2}\vec{a}\Delta t^2 + \vec{v}_i\Delta t}$$

Rearranging:

$$\boxed{\vec{x}_f = \vec{x}_i + \vec{v}_i\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2} \qquad \textit{Eq.11}$$

Combining equations 6 and 11 produces a time-independent kinematics expression.

$$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} \qquad \textit{Eq. 6}$$

Rearrange:

$$\Delta t = \frac{\Delta\vec{v}}{\vec{a}} \;;\; \Delta t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}} \qquad \textit{Eq. 12}$$

$$\vec{x}_f = \vec{x}_i + \vec{v}_i\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2 \qquad \textit{Eq. 11}$$

$$\Delta\vec{x} = \vec{v}_i\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2$$

Substitute equation 12 into equation 11:

$$\Delta\vec{x} = \vec{v}_i\left[\frac{\vec{v}_f - \vec{v}_i}{\vec{a}}\right] + \tfrac{1}{2}\vec{a}\left[\frac{\vec{v}_f - \vec{v}_i}{\vec{a}}\right]^2$$

Multiply both sides by $2\vec{a}$

$$2\vec{a}\Delta\vec{x} = 2\vec{v}_i(\vec{v}_f - \vec{v}_i) + (\vec{v}_f - \vec{v}_i)^2$$

Multiply out the terms on the right.

$$2\vec{a}\Delta\vec{x} = 2\vec{v}_i\vec{v}_f - 2\vec{v}_i^{\,2} + \vec{v}_f^{\,2} - 2\vec{v}_i\vec{v}_f + \vec{v}_i^{\,2}$$

Simplify the right side of equation

$$2\vec{a}\Delta\vec{x} = -\vec{v}_i^{\,2} + \vec{v}_f^{\,2}$$

Rearrange:

$$\boxed{\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\Delta\vec{x} \quad \textit{Eq. 13}}$$

Summary of mathematical models:

$\mathbf{a \equiv \Delta V/\Delta t}$	Eq. 6	definition of average acceleration
$\mathbf{V_f = V_i + a\Delta t}$	Eq. 10	generalized equation for any t_i to t_f interval parabolic equation for an **x**-t graph
$\mathbf{X_f = X_i + V_i\Delta t + ½ a\Delta t^2}$	Eq. 11	generalized equation for any t_i to t_f interval
$\mathbf{V_f^2 = V_i^2 + 2a\Delta X}$	Eq. 13	algebraic combination of equations 3 and 5

$\mathbf{V_f = V_i + a\Delta t}$	**NO DISPLACEMENT (ΔX) NEEDED**
$\mathbf{X_f = X_i + V_i \Delta t + ½ a\Delta t^2}$	**NO FINAL VELOCITY (V_f) NEEDED**
$\mathbf{V_f^2 = V_i^2 + 2a\Delta X}$	**NO TIME NEEDED**
$\mathbf{\Delta X = ½ (V_f + V_i) \Delta t}$	**NO ACCELERATION NEEDED, but you do have acceleration**

$$V_f = V_i + a\Delta t$$
$$m/s = m/s + m/s^2 * s$$
$$m/s = m/s + m/s$$
$$m/s = m/s$$

$$X_f = X_i + V_i\Delta t + ½ a\Delta t^2$$
$$m = m + m/s * s + m/s^2 * s^2$$
$$m = m + m + m$$
$$m = m$$

$$V_f^2 = V_i^2 + 2a\Delta X$$
$$m^2/s^2 = m^2/s^2 + m/s^2 * m$$
$$m^2/s^2 = m^2/s^2 + m^2/s^2$$
$$m^2/s^2 = m^2/s^2$$

$$\Delta X = ½ (V_f + V_i) \Delta t$$
$$m = (m/s + m/s) * s$$
$$m = (m/s) * s$$
$$m = m$$