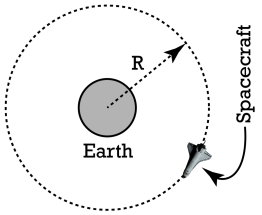
Flipping Physics Lecture Notes:

2018 #1 Free Response Question - AP Physics 1 - Exam Solution
http://www.flippingphysics.com/ap1-2018-frq1.html



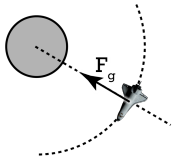
Note: Figure not drawn to scale.

A spacecraft of mass m is in a clockwise circular orbit of radius R around Earth, as shown in the figure above. The mass of Earth is M_E.

(a) In the figure below, draw and label the forces (not components) that act on the spacecraft. Each force must be represented by a distinct arrow starting on, and pointing away from, the spacecraft.

Draw the free body diagram with 1 force, the force of gravity, coming from the center of the spacecraft and pointed towards the center of the Earth.

A comment about grading. 2 out of 7 points for this problem are for this free body diagram (or force diagram) which has only one force in it. Free Body Diagrams are important!! Draw them carefully and clearly, and only include forces.



Note: Figure not drawn to scale.

(b) i. Derive an equation for the orbital period T of the spacecraft in terms of m, M_E, R, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

$$\sum F_{in} = F_g = ma_c \Rightarrow \frac{Gm_s m_E}{r^2} = m_s\left(\frac{v_t^2}{r}\right) \Rightarrow \frac{Gm_s m_E}{R^2} = \frac{m_s v_t^2}{R} \Rightarrow \frac{Gm_E}{R} = v_t^2$$

$$v_t = \frac{\Delta x}{\Delta t} = \frac{C}{T} = \frac{2\pi R}{T}$$

$$\Rightarrow \frac{Gm_E}{R} = \left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 R^2}{T^2} \Rightarrow T = \sqrt{\frac{4\pi^2 R^3}{Gm_E}}$$

More comments about grading. Do not add anything to the free body diagram answer from part (a). I know it is tempting. Do not do it! Also, make sure your answer is only in terms of the variables indicated. For example, it cannot be in terms of the speed of the spacecraft because that is not a given variable.

Alternate Solution:

$$\sum F_{in} = F_g = ma_c \Rightarrow \frac{Gm_s m_E}{r^2} = m_s r\omega^2 \ \& \ \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

$$\Rightarrow \frac{Gm_E}{R^2} = R\left(\frac{2\pi}{T}\right)^2 \Rightarrow \frac{Gm_E}{R^3} = \frac{4\pi^2}{T^2} \Rightarrow T = \sqrt{\frac{4\pi^2 R^3}{Gm_E}}$$

(b) ii A second spacecraft of mass 2m is placed in a circular orbit with the same radius R. Is the orbital period of the second spacecraft greater than, less than, or equal to the orbital period of the first spacecraft?

____ Greater than ____ Less than __X__ Equal to Briefly explain your reasoning.

The mass of the spacecraft cancelled out of our equation for the orbital period of the spacecraft derived in part (b) i, so the mass of the spacecraft does not affect period.

More, more comments about grading. As long as your explanation is consistent with your answer, you can get the full point for part (b) ii, even if you got part (b) i incorrect. Please answer every part of every question.

(c) The first spacecraft is moved into a new circular orbit that has a radius greater than R, as shown in the figure. Is the speed of the spacecraft in the new orbit greater than, less than, or equal to the original speed?

____ Greater than __X__ Less than ____ Equal to Briefly explain your reasoning.

Going back to the middle of our solution to part (b) i: $\frac{Gm_E}{R} = v_t^2$

We have a relationship between orbital speed and orbital radius which shows that as orbital radius increases, orbital speed must decrease. So, the spacecraft's new orbital speed is less than the original speed.