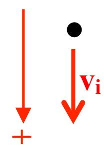
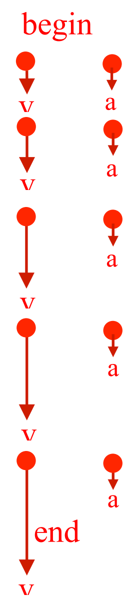
Particle Models in Two Dimensions Worksheet 1:
Free-Fall Kinematics

Note: For questions in which the object only moves downward, some teachers may elect to make downward (+) to reduce the number of (-) signs students have to deal with. For question 1, parallel solutions are provided. For #5 and #6 down is (+).

1. A ball is thrown downward with an initial speed of 20 m/s on Earth.
   a. Make a labeled diagram (specify the (+) direction), then make a motion map of the situation.





   b. What is the acceleration of the ball?

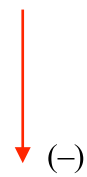
   Ignoring air friction, the ball accelerates at g which is -10 m/s/s

   c. Calculate the displacement during the first 4.0 s.

$v_i = 20$ m/s
$a = g = 10\,\frac{m}{s^2}$

$$\Delta y = \tfrac{1}{2}at^2 + v_i\Delta t$$
$$\Delta y = 5\tfrac{m}{s^2}(4.0s)^2 + (20\tfrac{m}{s})4.0s = 160m$$

or, if



$$\Delta y = -5\tfrac{m}{s^2}(4.0s)^2 + -20\tfrac{m}{s}(4.0s) = -160m$$

   d. Calculate the time required to reach a speed of 50 m/s.

$v_i = 20$ m/s
$a = g = 10\,\frac{m}{s^2}$
$v_f = 50$ m/s

$$v_f - v_i = a\Delta t \Rightarrow t = \frac{v_f - v_i}{a}$$
$$t = \frac{50\frac{m}{s} - (20\frac{m}{s})}{10\frac{m}{s^2}} = 3.0s$$

or

$$t = \frac{-50\frac{m}{s} - (-20\frac{m}{s})}{-10\frac{m}{s^2}} = 3.0s$$

   e. Calculate the time required to fall 300 m (Hint: factor the quadratic or use the quadratic formula).

$v_i = 20$ m/s
$a = g = 10\,\frac{m}{s^2}$
$\Delta y = 300$m

$$\Delta y = \tfrac{1}{2}at^2 + v_i\Delta t \Rightarrow \tfrac{1}{2}at^2 + v_i\Delta t - \Delta y = 0$$
$$\tfrac{1}{2}(10\tfrac{m}{s^2})t^2 + 20\tfrac{m}{s}\Delta t - 300m = 0 \Rightarrow (t-6)(t+10) = 0$$
$$t = 6.0s$$

   if down is (–), students can still factor the quadratic by multiplying by -1 to make the leading term (+)

$$-1\left(\tfrac{1}{2}(-10\tfrac{m}{s^2})t^2 - 20\tfrac{m}{s}\Delta t + 300m = 0\right)$$
$$(t-6)(t+10) = 0$$
$$t = 6.0s$$

f. Calculate the speed after falling 100 m.

$v_i = 20$ m/s
$a = g = 10\ \frac{m}{s^2}$
$\Delta y = 100$m

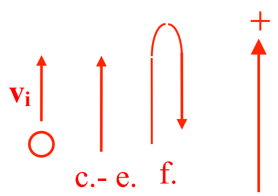
$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v_f^2 = \left(20\,\tfrac{m}{s}\right)^2 + 2(10\,\tfrac{m}{s^2})(100m)$$
$$v_f^2 = 2400\,\tfrac{m^2}{s^2} \Rightarrow v_f = \pm\sqrt{2400\,\tfrac{m^2}{s^2}} = 49\,\tfrac{m}{s}$$

or

$$v_f^2 = \left(-20\,\tfrac{m}{s}\right)^2 + 2(-10\,\tfrac{m}{s^2})(-100m)$$
$$v_f^2 = 2400\,\tfrac{m^2}{s^2} \Rightarrow v_f = -49\,\tfrac{m}{s}$$

2. A rock is thrown upward with an initial speed of 15 m/s on Earth.

a. Make a well-labeled diagram of the situation.



b. Make a list of given quantities and quantities to find, labeled with units and appropriate algebraic signs (+, -).

$v_i = 15$ m/s
$a = g = -10\ \frac{m}{s^2}$
$\Delta y = ?$
$\Delta t = ?$
$v_f = 0$

At this time only two values are known. When parts c – f are done, the other knowns can be listed for the problem.

c. What is the acceleration of the rock?

Ignoring air friction, the ball accelerates at g which is -10 m/s/s

d. Calculate the rock's height after 1.0 sec.

$v_i = 15$ m/s
$a = g = -10\ \frac{m}{s^2}$
$\Delta t = 1.0$s
$\Delta y = ?$

$$\Delta y = \tfrac{1}{2}at^2 + v_i\Delta t \Rightarrow \Delta y = \tfrac{1}{2}(-10\,\tfrac{m}{s^2})(1.0s)^2 + (15\,\tfrac{m}{s})1.0s = 5.0m$$

e. Calculate the time required to reach an upward speed of 3.0 m/s.

$v_i = 15$ m/s
$a = g = -10\ \frac{m}{s^2}$
$v_f = 3.0$ m/s
$\Delta t = ?$

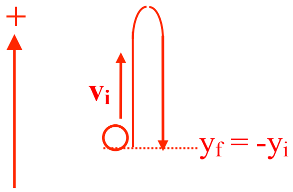
$$v_f = v_i + a\Delta t \Rightarrow \Delta t = \frac{v_f = v_i}{a} \Rightarrow \Delta t = \frac{3.0\,\frac{m}{s} - 15\,\frac{m}{s}}{-10\,\frac{m}{s^2}} = 1.2s$$

f. Calculate the time required to reach a downward speed of 5.0 m/s.

$v_i = 15$ m/s
$a = g = -10\ \frac{m}{s^2}$
$v_f = -5.0$ m/s
$\Delta t = ?$

$$v_f = v_i + a\Delta t \Rightarrow \Delta t = \frac{v_f = v_i}{a} \Rightarrow \Delta t = \frac{-5.0\,\frac{m}{s} - 15\,\frac{m}{s}}{-10\,\frac{m}{s^2}} = 2.0s$$

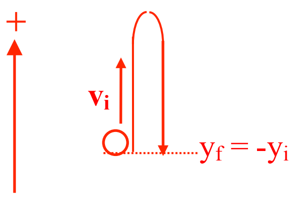
3. A ball punted vertically has a hang time of 3.8 seconds. What was its initial velocity? Make a well-labeled diagram of the situation. Make a list of given quantities and quantities to find, labeled with units and appropriate algebraic signs (+, -).



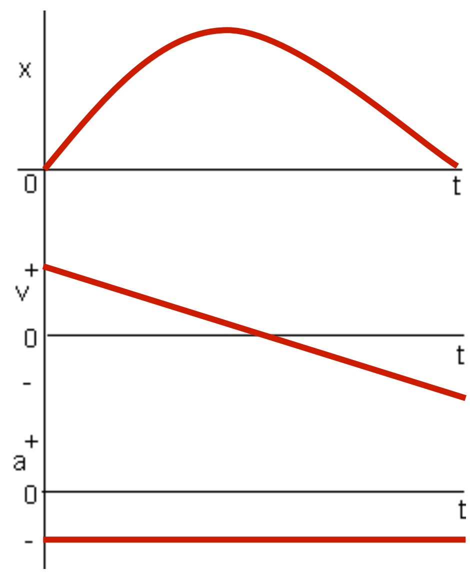
$\Delta t = 3.8s$
$a = g = -10\,\frac{m}{s^2}$
$\Delta y = 0$
$v_i = ?$

$$\Delta y = \tfrac{1}{2}at^2 + v_i\Delta t \Rightarrow 0 = \tfrac{1}{2}(-10\,\tfrac{m}{s^2})(3.8s)^2 + v_i(3.8s)$$
$$v_i = 5\,\tfrac{m}{s^2}(3.8s) = 19\,\tfrac{m}{s}$$

4. A rock is thrown straight up with an initial speed of 22 m/s. How long will it be in the air before it returns to the thrower? Graph the vertical position, velocity, and acceleration of the rock on the axes provided. Make a well-labeled diagram of the situation. Make a list of given quantities and quantities to find, labeled with units and appropriate algebraic signs (+, -).

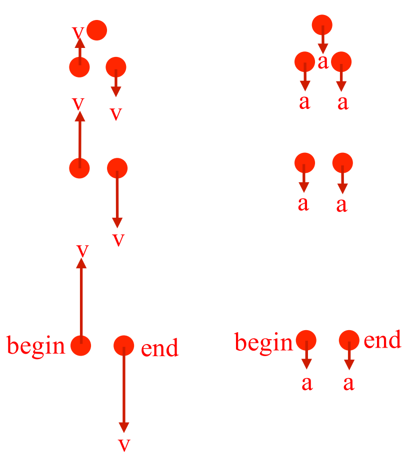


$a = g = -10\,\frac{m}{s^2}$
$\Delta y = 0$
$v_i = 22$ m/s
$\Delta t = ?$



$$\Delta y = \tfrac{1}{2}at^2 + v_i\Delta t \Rightarrow 0 = \tfrac{1}{2}(-10\,\tfrac{m}{s^2})t^2 + (22\,\tfrac{m}{s})t$$
$$(5\,\tfrac{m}{s^2})t = 22\,\tfrac{m}{s} \Rightarrow t = 4.4s$$

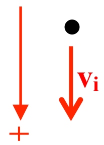
b. Draw a velocity and an acceleration motion map for the trip.



All a = g

The motion is actually up and down but the top 3 are so close, the top dot was centered.

5. A student throws a baseball off a 120 m high bridge with an initial downward speed of 10 m/s.
   a. How long does it take the ball to hit the ground below?



$a = g = 10\,\frac{m}{s^2}$

$\Delta y = 120m$

$v_i = 10$ m/s

$\Delta t = ?$

$$v_f^2 = v_i^2 + 2a\Delta y$$

$$v_f^2 = \left(10\,\tfrac{m}{s}\right)^2 + 2(10\,\tfrac{m}{s^2})(120m)$$

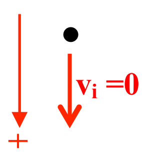
$$v_f^2 = 2500\,\tfrac{m^2}{s^2} \Rightarrow v_f = \pm\sqrt{2500\,\tfrac{m^2}{s^2}} = 50\,\tfrac{m}{s}$$

$$v_f = v_i + a\Delta t \Rightarrow \Delta t = \frac{v_f = v_i}{a} \Rightarrow \Delta t = \frac{50\,\frac{m}{s} - 10\,\frac{m}{s}}{10\,\frac{m}{s^2}} = 4.0s$$

   b. How fast is the ball going at the moment of impact?

   50 m/s (see 1st part of a.)

6. When a kid drops a rock off the edge of a cliff, it takes 4.0 s to reach the ground below.
   When she throws the rock down, it strikes the ground in 3.0 s.
   What initial speed did she give the rock?



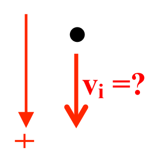
Dropped:

$a = g = 10\,\frac{m}{s^2}$

$v_i = 0$

$\Delta t = 4.0s$

$\Delta y = ?$



Thrown:

$a = g = 10\,\frac{m}{s^2}$

$\Delta t = 3.0s$

$\Delta y = 80m$

$v_i = ?$

$$\Delta y = \tfrac{1}{2}at^2 + v_i\Delta t$$

$$\Delta y = \tfrac{1}{2}(-10\,\tfrac{m}{s^2})(4.0s)^2 + 0$$

$$\Delta y = 80m$$

$$\Delta y = \tfrac{1}{2}at^2 + v_i\Delta t \Rightarrow v_i = \frac{\Delta y - \frac{1}{2}at^2}{\Delta t}$$

$$v_i = \frac{80m - \frac{1}{2}(10\,\frac{m}{s^2})(3.0s)^2}{3.0s} = \frac{80m - 45m}{3.0s}$$

$$v_i = 11.7\,\tfrac{m}{s} \Rightarrow 12\,\tfrac{m}{s}$$