A Progression of Static Equilibrium Laboratory Exercises

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A Progression of Static Equilibrium Laboratory Exercises

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Although simple architectural structures like bridges, catwalks, cantilevers, and Stonehenge have been integral in human societies for millennia, as have levers and other simple tools, modern students of introductory physics continue to grapple with Newton's conditions for static equilibrium. As formulated in typical introductory physics textbooks,[1-4] these two conditions appear as

$$\sum \mathbf{F} = 0 \tag{1}$$

and

$$\sum \boldsymbol{\tau} = 0, \tag{2}$$

where each torque $\boldsymbol{\tau}$ is defined as the cross product between the lever arm vector **r** and the corresponding applied force **F**,

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}, \tag{3}$$

having magnitude,

$$\tau = Fr \sin \theta. \tag{4}$$

The angle θ here is between the two vectors **F** and **r**. In Eq. (1), upward (downward) forces are considered positive (negative). In Eq. (2), counterclockwise (clockwise) torques are considered positive (negative). Equation (1) holds that the vector sum of the external forces acting on an object must be zero to prevent linear accelerations; Eq. (2) states that the vector sum of torques due to external *forces about any axis* must be zero to prevent angular accelerations. In our view these conditions can be problematic for students because a) the equations contain the unfamiliar summation notation Σ, b) students are uncertain of the role of torques in causing rotations, and c) it is not clear why the sum of torques is zero regardless of the choice of axis. Gianino[5] describes an experiment using MBL and a force sensor to convey the meaning of torque as applied to a rigid-body lever system without exploring quantitative aspects of the conditions for static equilibrium.

In this paper, we describe a suite of labs that reinforce the notions of the conditions for static equilibrium, emphasizing simplicity at first and growing in complexity. The experiments described below always employ simple equipment such as rulers, strings, masses, and force-measuring devices.

Forces applied at right angles

Experiment Ia. The bridge or catwalk

The first experiment is a relatively simple lab involving forces acting only at right angles to a body, such that $\theta = 90^\circ$ and thus $\sin \theta = 1$. The situation is analogous to a bridge or catwalk with two supports and carrying a single load.

Two Dual-Range Force Sensors from Vernier Software and Technology[6] (with front hooks and rear adjustment screws removed) are placed on the lab table with the remaining flat plate sensors facing upward [see Fig. 1(a)]. The force sensors may be calibrated by placing a known weight on each flat

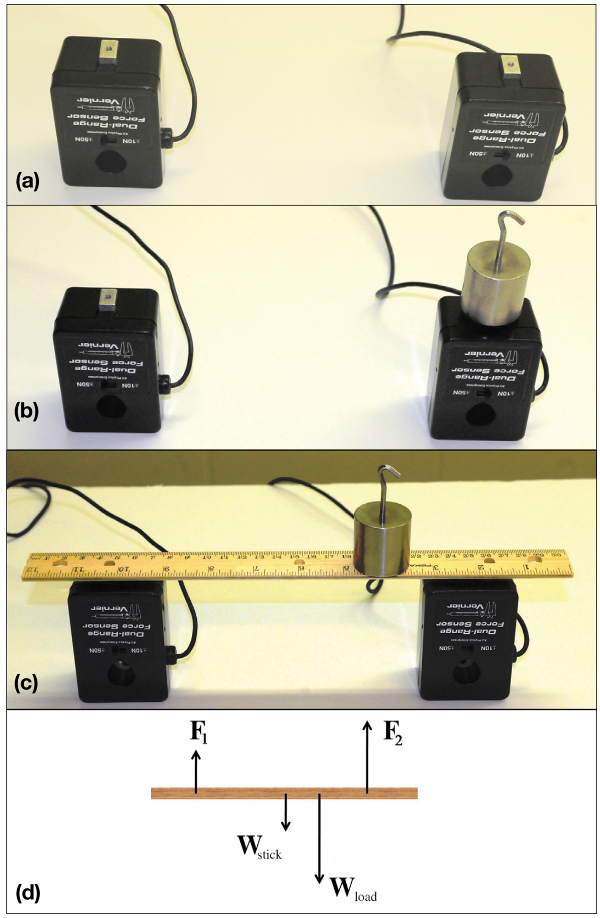

Fig. 1. Static equilibrium bridge setup. (a) Force sensors with hooks removed used for bridge support. (b) Calibrating force sensor. (c) The bridge plus load. (d) Free-body diagram of the ruler. The experiment demonstrates that the vector sums of both forces and torques are zero. Positive torques include $\tau_1 = +F_1 r_1$ and $\tau_2 = +F_2 r_2$. Negative torques include $\tau_{stick} = -W_{stick}\, r_{stick}$ and $\tau_{load} = -W_{load}\, r_{load}$ when summed about the 0-cm mark of the ruler. Note that we use W_{load} for convenience here and throughout, but the normal force on the ruler due to the load is what is actually required—the two are equal in magnitude due to Newton's laws as applied to the load weight in equilibrium.

plate [see Fig. 1(b)]. The force sensors support a horizontal meterstick from below at two widely spaced locations such that lever-arm distances from the 0-cm mark are easily measured by the metric scale. Figure 1(c) shows the less expensive, more compact alternative—a 1.0-ft/30.5-cm imperial/metric ruler available at discount stores; however, a meterstick is interesting and preferable to use as it contributes more

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Table I. Static equilibrium data for a bridge setup with torques summed about one end. The sign of the force vectors is determined by whether the force is directed upward (+) or downward (-). The sign of each torque vector is determined by whether it acts counterclockwise (+) or clockwise (-) about the 0-cm point.

Force name	Force vector, F (N)	Lever arm magnitude, r (m)	Torque vector, $\pm Fr$ (N·m)
F_1	+0.575	0.050	+0.0288
W_{stick}	-0.135	0.150	-0.0203
W_{load}	-1.960	0.200	-0.392
F_2	+1.535	0.250	+0.384
$F_{net} = \sum F = +0.015 \approx 0.00$		$\tau_{net} = \sum \tau = +0.00025 \approx 0.000$	

Table II. Static equilibrium data with torques summed about center of ruler. Forces are identical to those of Table I. Torque signs are determined by their directions about the center of the ruler.

Force name	Lever arm magnitude, r (m)	Torque, $\pm Fr$ (N·m)
F_1	0.100	-0.0575
W_{stick}	0.010	-0.0014
W_{load}	0.050	-0.098
F_2	0.100	+0.1535
$\sum \tau_i = -0.0004 \approx 0.000$	%Err = 0.3%	

of its own weight to the problem. A load mass is placed on top of the ruler (meterstick), typically closer to one force sensor than the other, with its location measurable on the metric scale. Using a slotted, cylindrical weight enables the experimenter to accurately place the center of mass of the cylinder directly above a particular, visible ruler marking. Mainardi[7] describes a similar setup utilizing a thin board and platform scales for demonstration purposes, Yoder and Cook[8] implement wireless force sensors for a torque board demonstration, and Belloni[9] uses a meterstick as a demonstration of the static ladder problem. The forces and lever-arm distances are all recorded as in Table I.

Given a ruler length of approximately 30.5 cm, the location of the center of gravity of the ruler is approximately 0.1525 m from the 0-cm mark, as may be verified by balancing the ruler, but the placement of predrilled holes may alter this somewhat. The ruler we purchased came with three holes and we drilled a fourth to put the center of gravity closer to the middle at 0.152 m.

Have students verify that the vector forces approximately sum to zero by comparing the magnitude of the net force F_{net} with the sum of only positive, upward forces, $F_+ = F_1 + F_2$, i.e.,

$$\%Err = \frac{|F_{net}|}{F_+} \times 100\% = 0.7\%.$$

The vector sum of the torques is shown to be approximately zero by comparing the net torque τ_{net} with the sum of only positive, counterclockwise torques, τ_{CCW}, i.e.,

$$\%Err = \frac{|\tau_{net}|}{\tau_{CCW}} \times 100\% = 0.06\%.$$

To illustrate that the choice of rotation axis is arbitrary, have students recalculate the lever-arm distances relative to some other point such as the 15-cm approximate center of the ruler as in Table II, making certain that the signs of the torques are corrected.

For a more kinesthetic form of this experiment, have students perform the bridge measurements using two force plates[10] (or bathroom scales) supporting a beam with a student providing the load (see Fig. 2). If only a single force platform is available, one can still demonstrate that the sum of torques is zero if the sum is made about the end without the force platform. A student can even lay prone on the board to determine the student's center of gravity as described by Serway and Vuille.[1]

Fig. 2. A student on a beam supported by two force plates[10] becomes an example of a load on a bridge.

Experiment Ib. The cantilever or diving board

In this experiment, force sensor 1 has its hook replaced, is inverted, and is recalibrated using a known weight hung from the hook. It is good practice to recalibrate any force sensor whenever its orientation is changed as the weight of the sensing device can combine significantly with the applied force. Clamp sensor 1 into position above the ruler with the hook pushing down near one end and support the meterstick near its middle from below with the second sensor plate. Place a load weight near the far end as shown in Fig. 3(a). The free-body diagram is shown in Fig. 3(b). The analysis is similar to that of Experiment Ia (the bridge) with the exception that F_1

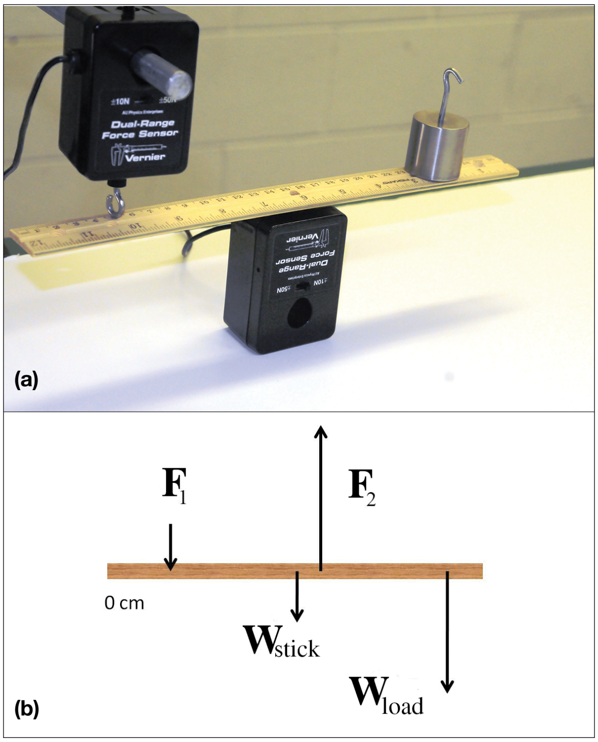

Fig. 3. (a) Cantilever system for static equilibrium experiments. One force sensor is inverted and must be recalibrated. (b) The corresponding free-body diagram includes both an upward and downward applied force.

Table III. Static equilibrium data for a cantilever (diving board) system. Torque signs are determined by directions about the 0-cm mark.

Force name	Force vector, F (N)	Lever arm magnitude, r (m)	Torque vector, $\pm Fr$ (N·m)
F_1	-1.991	0.050	-0.09955
W_{stick}	-0.135	0.150	-0.02025
F_2	+4.085	0.150	+0.61275
W_{load}	-1.960	0.250	-0.4900
$\sum F_i = -0.001 \approx 0.00$		$\sum \tau_i = +0.003 \approx 0.00$	

is now negative and the corresponding torque applied by F_1 about the 0-cm mark is negative (see Table III).

Forces applied at arbitrary angles – the model arm

This experiment illustrating the second condition of static equilibrium may be carried out with realism using the model of the human arm, including two force sensors and an angle sensor, marketed by PASCO scientific.[11] Alternatively, one may utilize simple equipment, as shown in Fig. 4 (a), allowing student imaginations to complete the anatomical picture of this class III lever. The metric ruler with at least three holes is used to represent the forearm. A vertical rod representing the upper arm has a screw in the lower end attached with the minimum of frictional torque to a hole at one end of the ruler, forming the elbow joint. A string is tied through one of the next two holes of the ruler attaching to a force sensor mounted above. Attaching the force sensor string to the hole nearest the elbow joint provides a more realistic model of the arm, with the biceps muscle attaching to the forearm via the tendon close to the elbow joint, but this makes the string tension angle not too different from 90^{o}[1] and thus less interesting for analysis. Using the hole near the center of the forearm allows the string tension to be applied at a lower, more acute angle. We mount the force sensor by passing a short, horizontal rod loosely through the hole designed for such purposes such that the sensor can rotate freely to aim directly along the line of the string since it measures tensile forces only. A load of approximately 200 g hangs from a string through the hole at the opposite end of the ruler. By sliding the vertical upper arm up or down through its support clamp at the top, one can affect the tilt of the forearm.

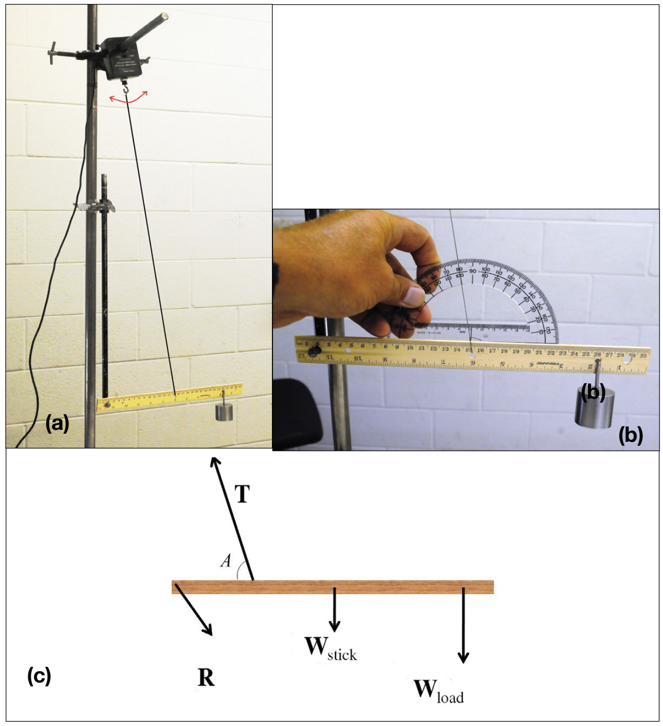

Fig. 4. (a) A model of the human forearm. A horizontal ruler represents the forearm. The vertical rod represents the upper arm. At the intersection of the vertical rod and horizontal ruler, the elbow joint, modeled by a relatively low-friction machine screw and nut, is the point about which torques are summed. The distal end of the biceps muscle is represented by a string. The tension in the string is measured by a force sensor allowed to swivel to align with the string. (b) The angle A of the string is measured using a protractor and the level of the forearm is checked by comparing with wall blocks in the background. (c) Free-body diagram of the model horizontal forearm.

Experiment IIa. Vertical upper arm with horizontal forearm

The apparatus and its free-body diagram are shown in Figs. 4 (a)–(c). Here the forearm is horizontal, as noted with either a level or comparison to bricks in the wall; thus, the string tension force acts at an acute angle, but both the weight of the forearm and load act perpendicularly to their respective lever arms. The angle A between the string tension force and the lever arm is measured using a protractor as shown in Fig. 4(b) with the string passing through the protractor's central hole. The reaction force **R** of the elbow joint on the forearm is not measurable in this configuration, so we ignore the first condition for static equilibrium and apply only the second condition by summing the torques about the elbow joint where **R** exerts no torque. Note that the positive torque due to the tension force **T** is now slightly complicated due to the angle A between the force and the corresponding lever-arm vector, $\mathbf{r}_T$, such that three factors are included:

$$\tau_T = +Tr_T \sin A.$$

If a rotating clamp is used at the top of the rod representing the upper arm, it becomes possible to rotate the upper arm away from the vertical while maintaining a horizontal forearm, thereby lowering the angle of the string representing the biceps muscle. Students can appreciate how producing torques at a low angle requires an increased amount of biceps muscle tension.

Experiment IIb. Tilted forearm

When the forearm is tilted by loosening the upper arm clamp and vertically raising/lowering the upper arm, gravitational forces act at an angle B, differing from 90°. Figure 5 shows this arrangement. Both angles A and B must be measured and included in torque calculations. Results are presented in Table IV. Each torque will now be computed from the three factors of Eq. (4). The precision of this arrangement is expected to be less than in previous arrangements, as two angle measurements (A and B) are required and each is only accurate to about 1° with the protractor.

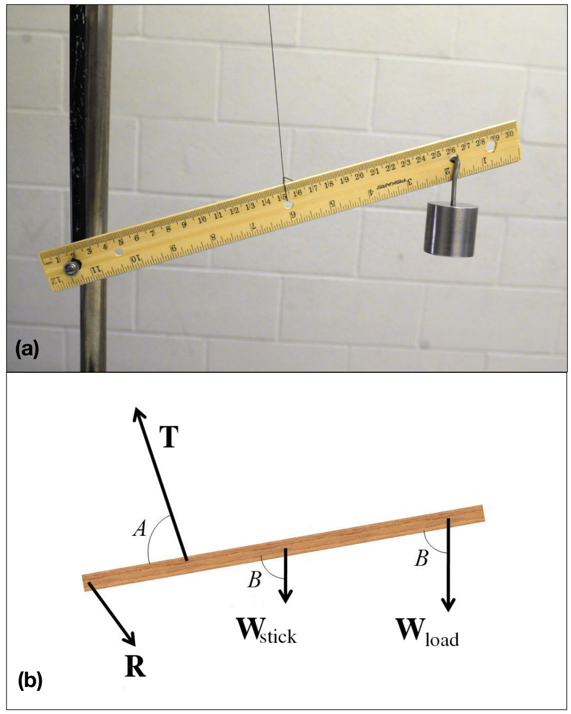

Fig. 5. (a) Model forearm tilted upward above the horizontal. Both angles, *A* and *B*, must now be measured. (b) Free-body diagram for model of a tilted forearm.

Table IV. Static equilibrium data for a tilted model forearm.

Force name	Force magnitude, F (N)	Angle θ	Lever arm magnitude, r (m)	Torque vector, $\pm Fr \sin\theta$ (N·m)
T	3.661	A = 100°	0.136	+0.4903
W_{stick}	0.135	B = 70°	0.135	-0.0171
W_{load}	1.960	B = 70°	0.245	-0.4512
				$\sum \tau_i = +0.02 \approx 0.0$

Conclusion

We have presented a variety of experiments, differing in both complexity and application, that reinforce and build upon students' conceptual understanding of static equilibrium. Engineering and architecture students will appreciate the bridge/cantilever experiments. High school students may find the plank/force plate experiments more engaging, whereas life science and pre-health profession majors will find the model of the human arm to be instructive. Teachers should not allow a lack of technology in their laboratory to interfere with the performance of these experiments as spring balances, mechanical weighing balances, platform scales, and bathroom scales can take the place of force sensors and force plates. Lab writeups with embedded video describing these and other elementary labs may be found at www.andrews.edu/phys .

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