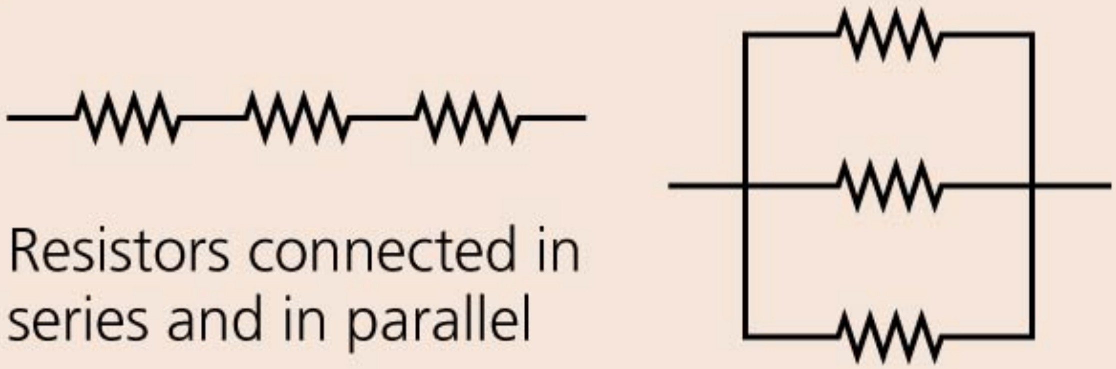
Combining Resistors

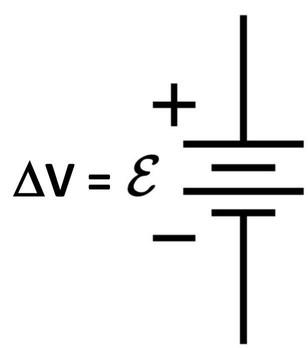
Resistors often occur **in series** or **in parallel.**



Resistors connected in series and in parallel

You'll learn that these combinations of resistors can be "simplified" by replacing them with one **equivalent resistor.**

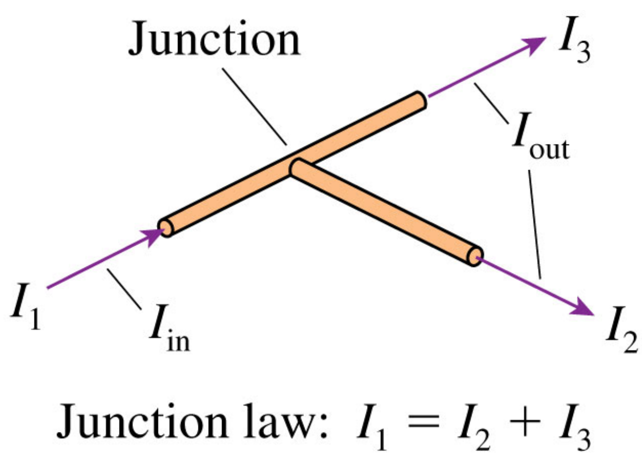
- A circuit diagram replaces pictures of the circuit elements with symbols.
- The longer line at one end of the battery symbol represents the positive terminal of the battery.
- The battery's emf is shown beside the battery.
- $+$ and $-$ symbols, even though somewhat redundant, are shown beside the terminals.



For a *junction*, the law of conservation of current requires that:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

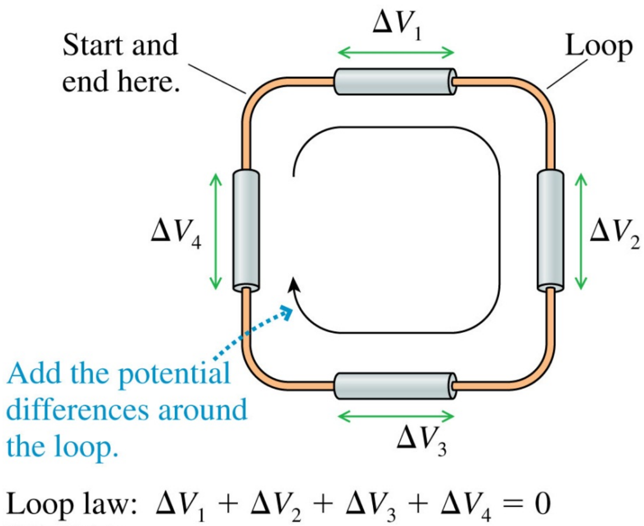
where the Σ symbol means summation. This basic conservation statement is called **Kirchhoff's junction law.**



- For any path that starts and ends at the same point:

$$\Delta V_{\text{loop}} = \sum (\Delta V)_i = 0$$

- The sum of all the potential differences encountered while moving around a loop or closed path is zero.
- This statement is known as **Kirchhoff's loop law.**



TACTICS BOX 31.1 **Using Kirchhoff's loop law**

❶ **Draw a circuit diagram.** Label all known and unknown quantities.

❷ **Assign a direction to the current.** Draw and label a current arrow I to show your choice.

- If you know the actual current direction, choose that direction.
- If you don't know the actual current direction, make an arbitrary choice. All that will happen if you choose wrong is that your value for I will end up negative.

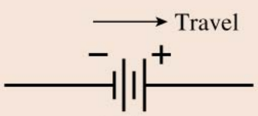
TACTICS BOX 31.1 **Using Kirchhoff's loop law**

❸ **"Travel" around the loop.** Start at any point in the circuit, then go all the way around the loop in the direction you assigned to the current in step 2. As you go through each circuit element, ΔV is interpreted to mean

$$\Delta V = V_{\text{downstream}} - V_{\text{upstream}}$$

- For an ideal battery in the negative-to-positive direction:

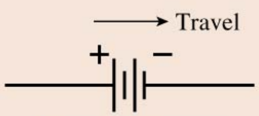
$$\Delta V_{\text{bat}} = +\mathcal{E}$$



Potential increases

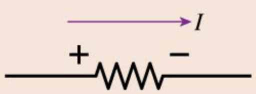
- For an ideal battery in the positive-to-negative direction:

$$\Delta V_{\text{bat}} = -\mathcal{E}$$



Potential decreases

- For a resistor: $\Delta V_{\text{res}} = -\Delta V_R = -IR$

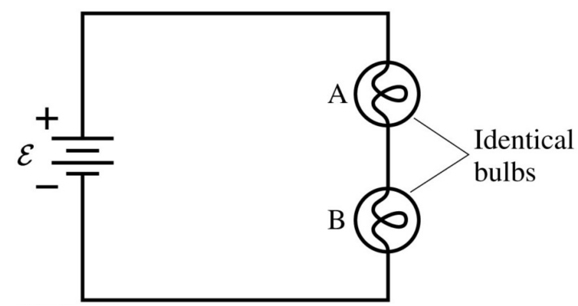


Potential decreases

❹ **Apply the loop law:** $\sum (\Delta V)_i = 0$

Exercises 4–7

- The figure shows two identical lightbulbs in a circuit.
- The current through both bulbs is *exactly the same!*
- It's not the current that the bulbs use up, it's *energy.*



- The battery creates a potential difference, which supplies potential energy to the charges.
- As the charges move through the lightbulbs, they lose some of their potential energy, transferring the energy to the bulbs.

- The power supplied by a battery is:

$$P_{bat} = I\Delta V \quad \text{(power delivered by an emf)}$$

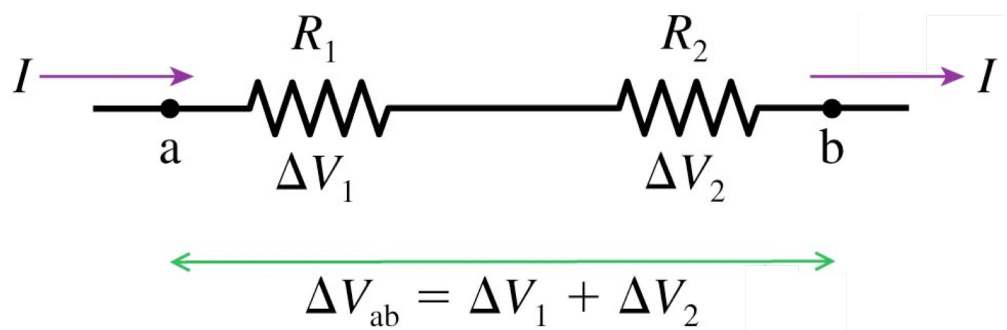
- The units of power are J/s or W.
- The power dissipated by a resistor is: $\boldsymbol{P_R = I\Delta V_R}$

- Or, in terms of the potential drop across the resistor:

$$P_R = I\,\Delta V_R = I^2 R = \frac{(\Delta V_R)^2}{R} \qquad \text{(power dissipated by a resistor)}$$

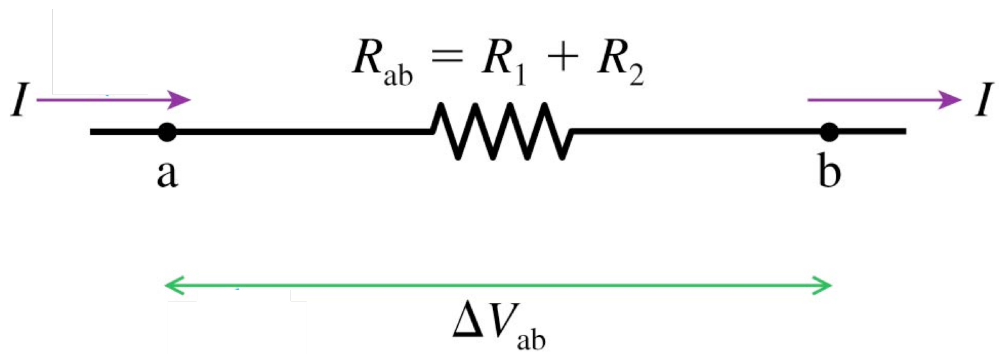
- The figure below shows two resisters connected *in series* between points a and b.
- The total potential difference between points a and b is the sum of the individual potential differences across R_1 and R_2:

$$\Delta V_{ab} = \Delta V_1 + \Delta V_2 = IR_1 + IR_2 = I(R_1 + R_2)$$



- Suppose we replace R_1 and R_2 with a single resistor with the same current I and the same potential difference ΔV_{ab}.
- Ohm's law gives resistance between points a and b:

$$R_{ab} = \frac{\Delta V_{ab}}{I} = \frac{I(R_1 + R_2)}{I} = R_1 + R_2$$



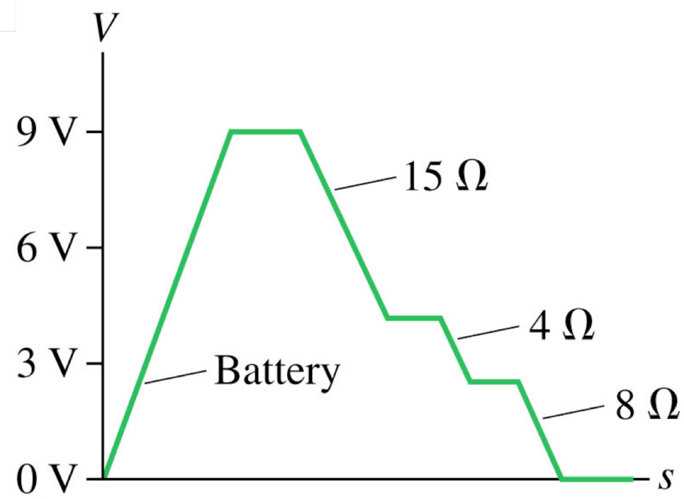
- Resistors that are aligned end to end, *with no junctions between them,* are called **series resistors** or, sometimes, resistors "in series."
- The current I is the same through all resistors placed in series.
- If we have N resistors in series, their **equivalent resistance** is:

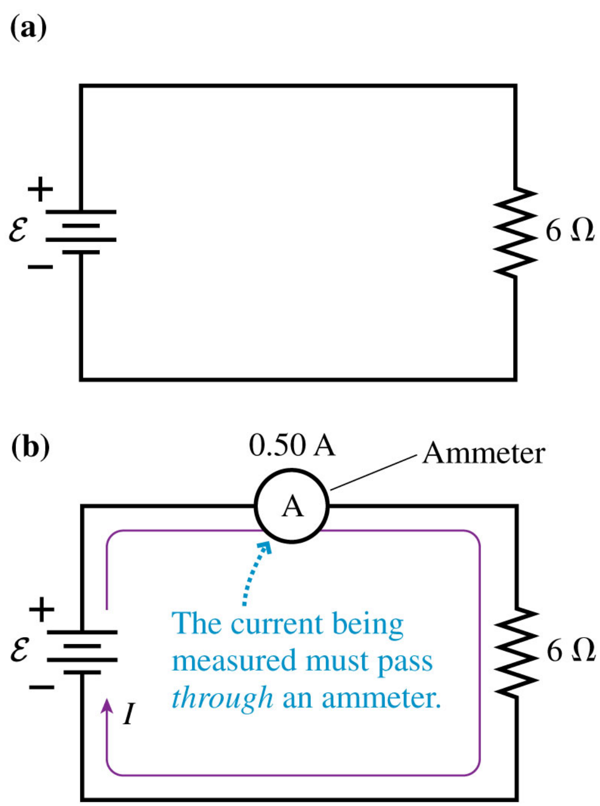
$$R_{eq} = R_1 + R_2 + \cdots + R_N \qquad \text{(series resistors)}$$

The behavior of the circuit will be unchanged if the N series resistors are replaced by the single resistor R_{eq}.

EXAMPLE 31.5 **A series resistor circuit**

b. $I = 0.333$ A is the current in each of the three resistors in the original circuit. Thus the potential differences across the resistors are $\Delta V_{res\ 1} = -IR_1 = -5.0$ V, $\Delta V_{res\ 2} = -IR_2 = -1.3$ V, and $\Delta V_{res\ 3} = -IR_3 = -2.7$ V for the 15 Ω, the 4 Ω, and the 8 Ω resistors, respectively. The figure below shows that the potential increases by 9 V due to the battery's emf, then decreases by 9 V in three steps.

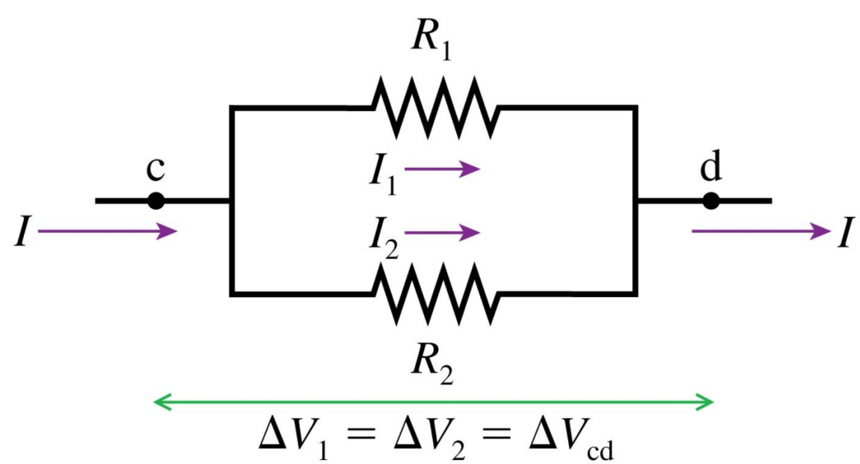




- Figure (a) shows a simple one-resistor circuit.
- We can measure the current by breaking the connection and inserting an ammeter *in series.*
- The resistance of the ammeter is negligible.
- The potential difference across the resistor must be $\Delta V_R = IR = 3.0\ \text{V}$.
- So the battery's emf must be $3.0\ \text{V}$.

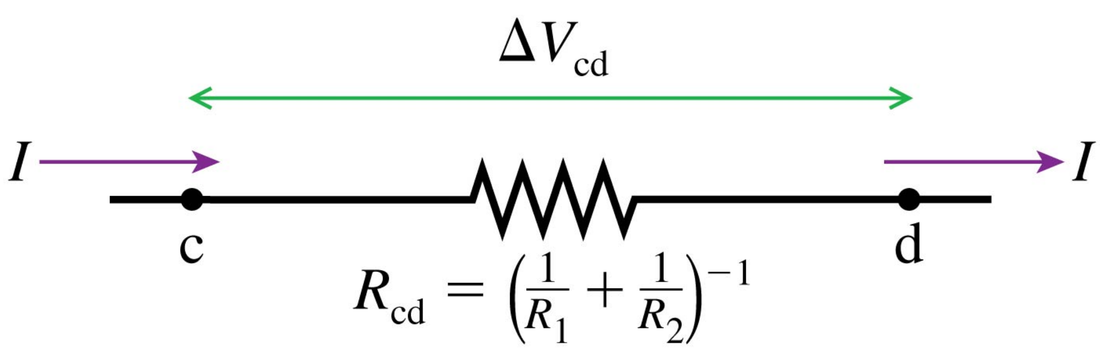
- The figure below shows two resisters connected *in parallel* between points c and d.
- By Kirchhoff's junction law, the input current is the sum of the current through each resistor: $I = I_1 + I_2$.

$$I = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \frac{\Delta V_{cd}}{R_1} + \frac{\Delta V_{cd}}{R_2} = \Delta V_{cd}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



- Suppose we replace R_1 and R_2 with a single resistor with the same current I and the same potential difference ΔV_{cd}.
- Ohm's law gives resistance between points c and d:

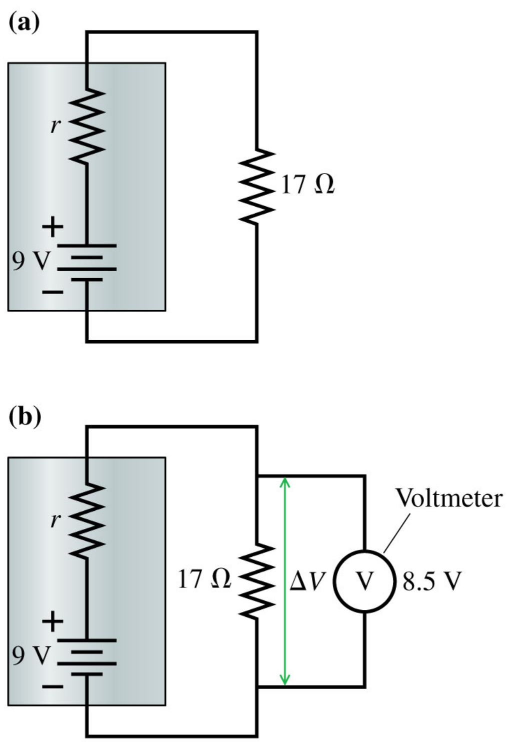
$$R_{\text{cd}} = \frac{\Delta V_{\text{cd}}}{I} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$



- Resistors connected *at both ends* are called **parallel resistors** or, sometimes, resistors "in parallel."
- The left ends of all the resistors connected in parallel are held at the same potential V_1, and the right ends are all held at the same potential V_2.
- The potential differences ΔV are the *same* across all resistors placed in parallel.
- If we have N resistors in parallel, their **equivalent resistance** is:

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1} \qquad \text{(parallel resistors)}$$

The behavior of the circuit will be unchanged if the N parallel resistors are replaced by the single resistor R_{eq}.



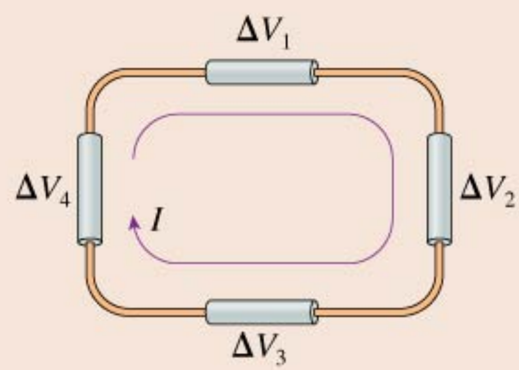
- Figure (a) shows a simple circuit with a resistor and a real battery.
- We can measure the potential difference across the resistor by connecting a voltmeter *in parallel* across the resistor.
- The resistance of the voltmeter must be very high.
- The internal resistance is:

$$r = \frac{\Delta V_{bat} - \Delta V_R}{\Delta V_R} R = \frac{0.5\ \text{V}}{8.5\ \text{V}} 17\ \Omega = 1.0\ \Omega$$

Kirchhoff's loop law

For a closed loop:

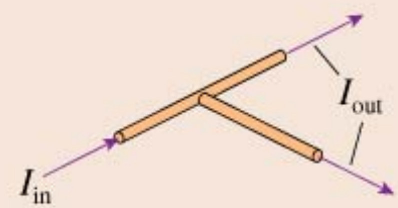
- Assign a direction to the current I.
- $\sum(\Delta V)_i = 0$



Kirchhoff's junction law

For a junction:

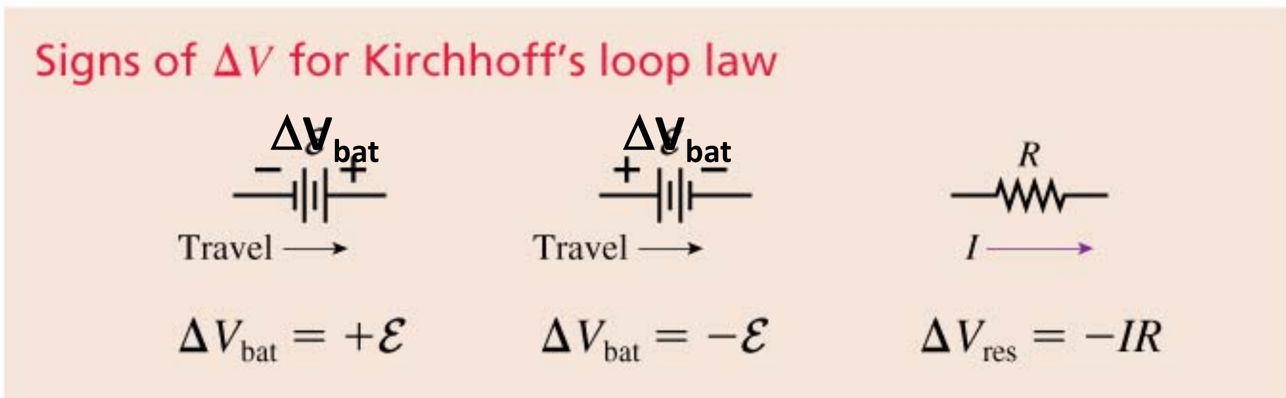
- $\sum I_{in} = \sum I_{out}$



Ohm's Law

A potential difference ΔV between the ends of a conductor with resistance R creates a current

$$I = \frac{\Delta V}{R}$$



ε =Electromotive Force

The energy used by a circuit is supplied by the emf $\mathcal{E}$ of the battery through the energy transformations

$$E_{\text{chem}} \rightarrow U \rightarrow K \rightarrow E_{\text{th}}$$

The battery *supplies* energy at the rate

$$P_{\text{bat}} = IV$$

The resistors *dissipate* energy at the rate

$$P_{\text{R}} = I\Delta V_{\text{R}} = I^2 R = \frac{(\Delta V_{\text{R}})^2}{R}$$