College Physics for AP® Courses

openstax COLLEGE™

OpenStax College
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Table of Contents

PREFACE

The OpenStax College *Physics: AP® Edition* program has been developed with several goals in mind: accessibility, customization, and student engagement—all while encouraging science students toward high levels of academic scholarship. Instructors and students alike will find that this program offers a strong foundation in physics in an accessible format. Welcome!

About OpenStax College

OpenStax College is a nonprofit organization committed to improving student access to quality learning materials. Our free textbooks are developed and peer-reviewed by educators to ensure they are readable, accurate, and meet the scope and sequence requirements of today's high school courses. Unlike traditional textbooks, OpenStax College resources live online and are owned by the community of educators using them. Through our partnerships with companies and foundations committed to reducing costs for students, OpenStax College is working to improve access to education for all. OpenStax College is an initiative of Rice University and is made possible through the generous support of several philanthropic foundations.

OpenStax College resources provide quality academic instruction. Three key features set our materials apart from others: they can be customized by instructors for each class, they are a "living" resource that grows online through contributions from science educators, and they are available for free or at minimal cost.

Customization

OpenStax College learning resources are designed to be customized for each course. Our textbooks provide a solid foundation on which instructors can build, and our resources are conceived and written with flexibility in mind. Instructors can simply select the sections most relevant to their curricula and create a textbook that speaks directly to the needs of their classes and students. Teachers are encouraged to expand on existing examples by adding unique context via geographically localized applications and topical connections. This customization feature will help bring physics to life for students and will ensure that your textbook truly reflects the goals of your course.

Curation

To broaden access and encourage community curation, OpenStax College *Physics: AP® Edition* is "open source" licensed under a Creative Commons Attribution (CC-BY) license. The scientific community is invited to submit examples, emerging research, and other feedback to enhance and strengthen the material and keep it current and relevant for today's students. Submit your suggestions to info@openstaxcollege.org, and find information on edition status, alternate versions, errata, and news on the StaxDash at **http://openstaxcollege.org (http://openstaxcollege.org)** .

Cost

Our textbooks are available for free online and in low-cost print and e-book editions.

About *OpenStax College Physics: AP® Edition*

In 2012, OpenStax College published *College Physics* as part of a series that offers free and open college textbooks for higher education. *College Physics* was quickly adopted for science courses all around the country, and as word about this valuable resource spread, advanced placement teachers around the country started utilizing the book in AP® courses too.

Physics: AP® Edition is the result of an effort to better serve these teachers and students. Based on *College Physics*—a program based on the teaching and research experience of numerous physicists—*Physics: AP® Edition* focuses on and emphasizes the new AP® curriculum's concepts and practices.

Alignment to the AP® curriculum

The new AP® Physics curriculum framework outlines the two full-year physics courses AP® Physics 1: Algebra-Based and AP® Physics 2: Algebra-Based. These two courses replaced the one-year AP® Physics B course, which over the years had become a fast-paced survey of physics facts and formulas that did not provide in-depth conceptual understanding of major physics ideas and the connections between them.

The new AP® Physics 1 and 2 courses focus on the big ideas typically included in the first and second semesters of an algebra-based, introductory college-level physics course, providing students with the essential knowledge and skills required to support future advanced course work in physics. The AP® Physics 1 curriculum includes mechanics, mechanical waves, sound, and electrostatics. The AP® Physics 2 curriculum focuses on thermodynamics, fluid statics, dynamics, electromagnetism, geometric and physical optics, quantum physics, atomic physics, and nuclear physics. Seven unifying themes of physics called the Big Ideas each include three to seven Enduring Understandings (EU), which are themselves composed of Essential Knowledge (EK) that provides details and context for students as they explore physics.

AP® Science Practices emphasize inquiry-based learning and development of critical thinking and reasoning skills. Inquiry usually uses a series of steps to gain new knowledge, beginning with an observation and following with a hypothesis to explain the observation; then experiments are conducted to test the hypothesis, gather results, and draw conclusions from data. The

AP® framework has identified seven major science practices, which can be described by short phrases: using representations and models to communicate information and solve problems; using mathematics appropriately; engaging in questioning; planning and implementing data collection strategies; analyzing and evaluating data; justifying scientific explanations; and connecting concepts. The framework's Learning Objectives merge content (EU and EK) with one or more of the seven science practices that students should develop as they prepare for the AP® Physics exam.

Each chapter of OpenStax College *Physics: AP® Edition* begins with a *Connection for AP® Courses* introduction that explains how the content in the chapter sections align to the Big Ideas, Enduring Understandings, and Essential Knowledge in the AP® framework. *Physics: AP® Edition* contains a wealth of information and the *Connection for AP® Courses* sections will help you distill the required AP® content from material that, although interesting, exceeds the scope of an introductory-level course.

Each section opens with the program's learning objectives as well as the AP® learning objectives and science practices addressed. We have also developed *Real World Connections* features and *Applying the Science Practices* features that highlight concepts, examples, and practices in the framework.

Pedagogical Foundation and Features

OpenStax College *Physics: AP® Edition* is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize. Our features include:

- **Connections for AP® Courses** introduce each chapter and explain how its content addresses the AP® curriculum.
- **Worked examples** promote both analytical and conceptual skills. They are introduced using an application of interest followed by a strategy that emphasizes the concepts involved, a mathematical solution, and a discussion.
- **Problem-solving strategies** are presented independently and subsequently appear at crucial points in the text where students can benefit most from them.
- **Misconception Alerts** address common misconceptions that students may bring to class.
- **Take Home Investigations** provide the opportunity for students to apply or explore what they have learned with a hands-on activity.
- **Real World Connections** highlight important concepts and examples in the AP® framework.
- **Applying the Science Practices** includes activities and challenging questions that engage students while they apply the AP® science practices.
- **Things Great and Small** explain macroscopic phenomena (such as air pressure) with submicroscopic phenomena (such as atoms bouncing off walls).
- **Simulations** direct students to further explore the physics concepts they have learned about in the module through the interactive PHeT physics simulations developed by the University of Colorado.

Assessment

Physics: AP® Edition offers a wealth of assessment options that include:

- **End-of-Module Problems** include conceptual questions that challenge students' ability to explain what they have learned conceptually, independent of the mathematical details, and problems and exercises that challenge students to apply both concepts and skills to solve mathematical physics problems.
- **Integrated Concept Problems** challenge students to apply concepts and skills to solve a problem.
- **Unreasonable Results** encourage students to analyze the answer with respect to how likely or realistic it really is.
- **Construct Your Own Problem** requires students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem's solution, and finally discuss the meaning of the result.
- **Test Prep for AP® Courses** consists of end-of-module problems that include assessment items with the format and rigor found in the AP® exam to help prepare students.

About Our Team

Physics: AP® Edition would not be possible if not for the tremendous contributions of the authors and community reviewing team.

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Additional Resources

Preparing for the AP® Physics 1 Exam

Rice Online's dynamic new course, available on edX, is fully integrated with Physics for AP® Courses for free. Developed by nationally recognized Rice Professor Dr. Jason Hafner and AP® Physics teachers Gigi Nevils-Noe and Matt Wilson the course combines innovative learning technologies with engaging, professionally-produced Concept Trailers™, inquiry based labs, practice problems, lectures, demonstrations, assessments, and other compelling resources to promote engagement and long-term retention of AP® Physics 1 concepts and application. Learn more at online.rice.edu.

Other learning resources (powerpoint slides, testbanks, online homework etc) are updated frequently and can be viewed by going to **https://openstaxcollege.org**.

To the AP® Physics Student

The fundamental goal of physics is to discover and understand the "laws" that govern observed phenomena in the world around us. Why study physics? If you plan to become a physicist, the answer is obvious—introductory physics provides the foundation for your career; or if you want to become an engineer, physics provides the basis for the engineering principles used to solve applied and practical problems. For example, after the discovery of the photoelectric effect by physicists, engineers developed photocells that are used in solar panels to convert sunlight to electricity. What if you are an aspiring medical doctor? Although the applications of the laws of physics may not be obvious, their understanding is tremendously valuable. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; cancer radiotherapy uses ionizing radiation. What if you are planning a nonscience career? Learning physics provides you with a well-rounded education and the ability to make important decisions, such as evaluating the pros and cons of energy production sources or voting on decisions about nuclear waste disposal.

This AP® Physics 1 course begins with **kinematics,** the study of motion without considering its causes. Motion is everywhere: from the vibration of atoms to the planetary revolutions around the Sun. Understanding motion is key to understanding other concepts in physics. You will then study **dynamics,** which considers the forces that affect the motion of moving objects and systems. Newton’s laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of the principles under which nature functions. One of the most remarkable simplifications in physics is that only four distinct forces account for all known phenomena. Your journey will continue as you learn about energy. Energy plays an essential role both in everyday events and in scientific phenomena. You can likely name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. The next stop is learning about oscillatory motion and waves. All oscillations involve force and energy: you push a child in a swing to get the motion started and you put energy into a guitar string when you pluck it. Some oscillations create waves. For example, a guitar creates sound waves. You will conclude this first physics course with the study of static electricity and electric currents. Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Similarly, lightning results from air movements under certain weather conditions.

In AP® Physics 2 course you will continue your journey by studying **fluid dynamics,** which explains why rising smoke curls and twists and how the body regulates blood flow. The next stop is **thermodynamics,** the study of heat transfer—energy in transit—that can be used to do work. Basic physical laws govern how heat transfers and its efficiency. Then you will learn more about electric phenomena as you delve into **electromagnetism.** An electric current produces a magnetic field; similarly, a magnetic field produces a current. This phenomenon, known as **magnetic induction,** is essential to our technological society. The generators in cars and nuclear plants use magnetism to generate a current. Other devices that use magnetism to induce currents include pickup coils in electric guitars, transformers of every size, certain microphones, airport security gates, and damping mechanisms on sensitive chemical balances. From electromagnetism you will continue your journey to **optics,** the study of light. You already know that visible light is the type of electromagnetic waves to which our eyes respond. Through vision, light can evoke deep emotions, such as when we view a magnificent sunset or glimpse a rainbow breaking through the clouds. Optics is concerned with the generation and propagation of light. The **quantum mechanics, atomic physics,** and **nuclear physics** are at the end of your journey. These areas of physics have been developed at the end of the 19th and early 20th centuries and deal with submicroscopic objects. Because these objects are smaller than we can observe directly with our senses and generally must be observed with the aid of instruments, parts of these physics areas may seem foreign and bizarre to you at first. However, we have experimentally confirmed most of the ideas in these areas of physics.

AP® Physics is a challenging course. After all, you are taking physics at the introductory college level. You will discover that some concepts are more difficult to understand than others; most students, for example, struggle to understand rotational motion and angular momentum or particle-wave duality. The AP® curriculum promotes depth of understanding over breadth of content, and to make your exploration of topics more manageable, concepts are organized around seven major themes called the **Big Ideas** that apply to all levels of physical systems and interactions between them (see web diagram below). Each Big Idea identifies **Enduring Understandings** (EU), **Essential Knowledge** (EK), and **illustrative examples** that support key concepts and content. Simple descriptions define the focus of each Big Idea.

- Big Idea 1: Objects and systems have properties.
- Big Idea 2: Fields explain interactions.
- Big Idea 3: The interactions are described by forces.
- Big Idea 4: Interactions result in changes.
- Big Idea 5: Changes are constrained by conservation laws.
- Big Idea 6: Waves can transfer energy and momentum.
- Big Idea 7: The mathematics of probability can to describe the behavior of complex and quantum mechanical systems.

Doing college work is not easy, but completion of AP® classes is a reliable predictor of college success and prepares you for subsequent courses. The more you engage in the subject, the easier your journey through the curriculum will be. Bring your enthusiasm to class every day along with your notebook, pencil, and calculator. Prepare for class the day before, and review concepts daily. Form a peer study group and ask your teacher for extra help if necessary. The AP® lab program focuses on more open-ended, student-directed, and inquiry-based lab investigations designed to make you think, ask questions, and analyze data like scientists. You will develop critical thinking and reasoning skills and apply different means of communicating information. By the time you sit for the AP® exam in May, you will be fluent in the language of physics; because you have been doing real science, you will be ready to show what you have learned. Along the way, you will find the study of the world around us to be one of the most relevant and enjoyable experiences of your high school career.

Irina Lyublinskaya, PhD
Professor of Science Education

To the AP® Physics Teacher

The AP® curriculum was designed to allow instructors flexibility in their approach to teaching the physics courses. OpenStax College *Physics: AP® Edition* helps you orient students as they delve deeper into the world of physics. Each chapter includes a Connection for AP® Courses introduction that describes the AP® Physics Big Ideas, Enduring Understandings, and Essential Knowledge addressed in that chapter.

Each section starts with specific AP® learning objectives and includes essential concepts, illustrative examples, and science practices, along with suggestions for applying the learning objectives through take home experiments, virtual lab investigations, and activities and questions for preparation and review. At the end of each section, students will find the Test Prep for AP® courses with multiple-choice and open-response questions addressing AP® learning objectives to help them prepare for the AP® exam.

OpenStax College *Physics: AP® Edition* has been written to engage students in their exploration of physics and help them relate what they learn in the classroom to their lives outside of it. Physics underlies much of what is happening today in other sciences and in technology. Thus, the book content includes interesting facts and ideas that go beyond the scope of the AP® course. The AP® Connection in each chapter directs students to the material they should focus on for the AP® exam, and what content—although interesting—is not part of the AP® curriculum.

Physics is a beautiful and fascinating science. It is in your hands to engage and inspire your students to dive into an amazing world of physics, so they can enjoy it beyond just preparation for the AP® exam.

Irina Lyublinskaya, PhD
Professor of Science Education

The concept map showing major links between Big Ideas and Enduring Understandings is provided below for visual reference.

1 INTRODUCTION: THE NATURE OF SCIENCE AND PHYSICS

Figure 1.1 Galaxies are as immense as atoms are small. Yet the same laws of physics describe both, and all the rest of nature—an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature's apparent complexity. (credit: NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for Astrophysics)

Chapter Outline

Connection for AP® Courses

What is your first reaction when you hear the word “physics”? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people's regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, “invisibility cloaks” that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

Chapter 1 introduces many fundamental skills and understandings needed for success with the AP® Learning Objectives. While this chapter does not directly address any Big Ideas, its content will allow for a more meaningful understanding when these Big Ideas are addressed in future chapters. For instance, the discussion of models, theories, and laws will assist you in understanding the concept of fields as addressed in Big Idea 2, and the section titled 'The Evolution of Natural Philosophy into Modern Physics' will help prepare you for the statistical topics addressed in Big Idea 7.

This chapter will also prepare you to understand the Science Practices. In explicitly addressing the role of models in representing and communicating scientific phenomena, Section 1.1 supports Science Practice 1. Additionally, anecdotes about historical investigations and the inset on the scientific method will help you to engage in the scientific questioning referenced in Science Practice 3. The appropriate use of mathematics, as called for in Science Practice 2, is a major focus throughout sections 1.2, 1.3, and 1.4.

1.1 Physics: An Introduction

Figure 1.2 The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

Learning Objectives

By the end of this section, you will be able to:

- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone (**Figure 1.3**). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.

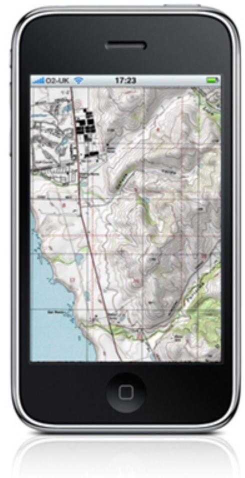


Figure 1.3 The Apple “iPhone” is a common smart phone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images)

Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See **Figure 1.4** and **Figure 1.5**.) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes (**Figure 1.6** and **Figure 1.7**). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

Figure 1.4 The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

Figure 1.5 These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)

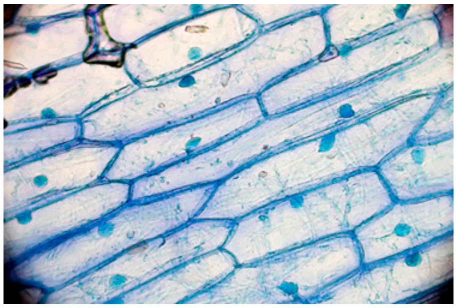


Figure 1.6 Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)

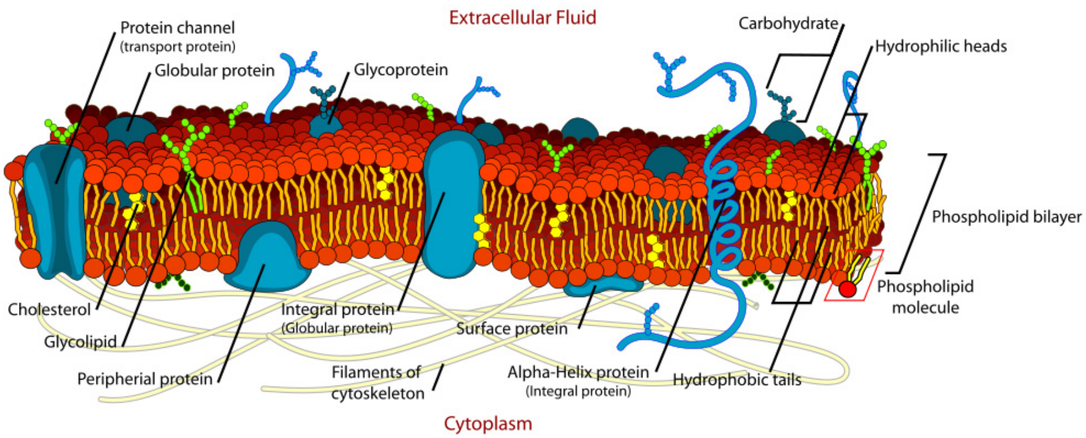


Figure 1.7 An artist's rendition of the the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See **Figure 1.8** and **Figure 1.9**.) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.

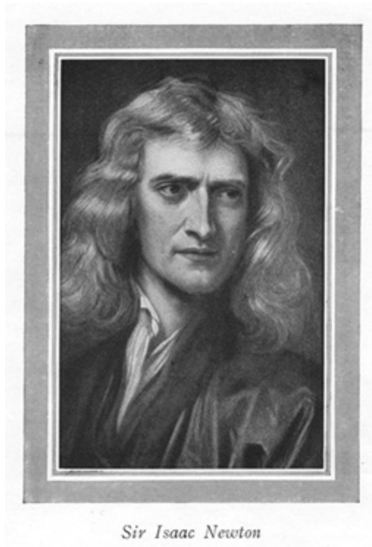


Figure 1.8 **Isaac Newton** (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: *Britain's Heritage of Science*. London, 1917.)

Figure 1.9 **Marie Curie** (1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See **Figure 1.10**.) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.

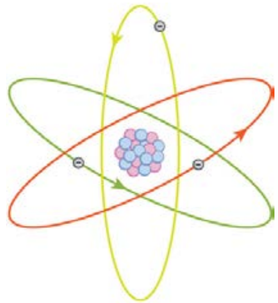


Figure 1.10 What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

Models, Theories, and Laws

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

The Scientific Method

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See **Figure 1.11**, **Figure 1.12**, and **Figure 1.13**.) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

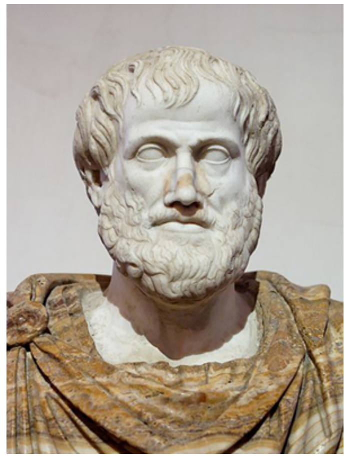


Figure 1.11 Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow (2006)/Ludovisi Collection)

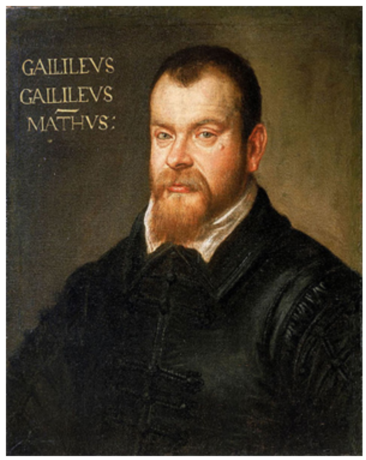


Figure 1.12 **Galileo Galilei** (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto)

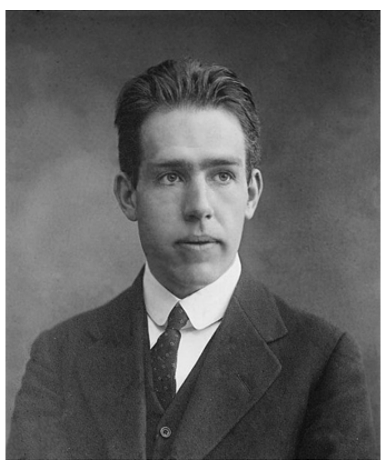


Figure 1.13 **Niels Bohr** (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division)

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually “picture” the atom.

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.

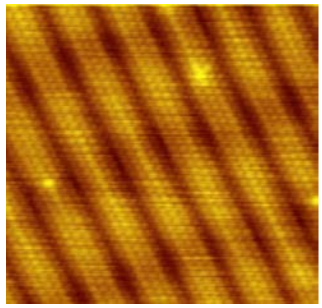


Figure 1.14 Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics,* and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

Check Your Understanding

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

Solution

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

PhET Explorations: Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

PhET Interactive Simulation

Figure 1.15 Equation Grapher (http://cnx.org/content/m54764/1.2/equation-grapher_en.jar)

1.2 Physical Quantities and Units

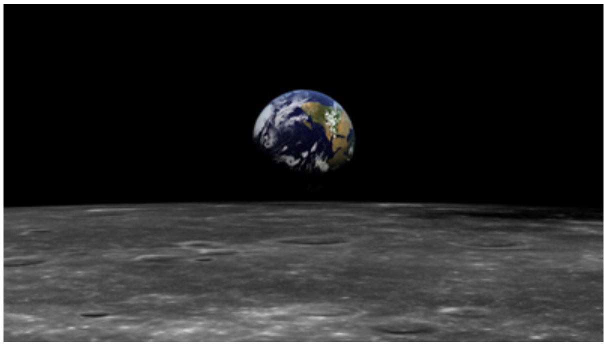


Figure 1.16 The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

Learning Objectives

By the end of this section, you will be able to:

- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See **Figure 1.17**.)

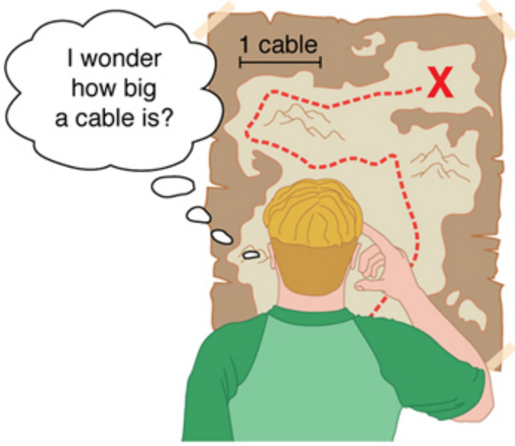


Figure 1.17 Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym “SI” is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

Table 1.1 gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Table 1.1 Fundamental SI Units

Length	Mass	Time	Electric Charge
meter (m)	kilogram (kg)	second (s)	coulomb (c)

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric charge. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric current, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See **Figure 1.18**.) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.

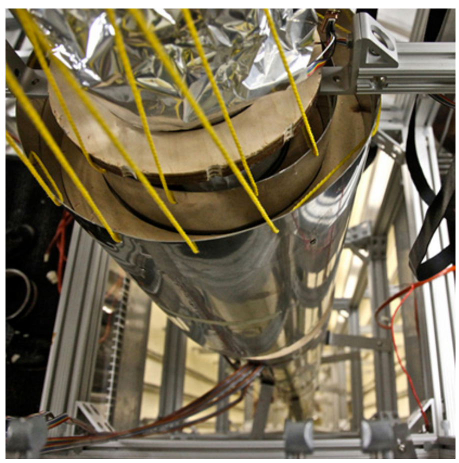


Figure 1.18 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See **Figure 1.19**.) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass.

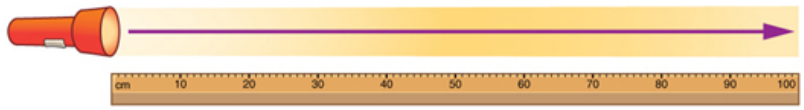


Figure 1.19 The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

Electric current and its accompanying unit, the ampere, will be introduced in **Introduction to Electric Current, Resistance, and Ohm's Law** when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. **Table 1.2** gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example, 10^1, 10^2, 10^3, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as $4.5\times10^2.$ Thus, the numbers 800 and 450 are of the same order of magnitude: $10^2.$ Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of $10^{-9}\text{ m},$ while the diameter of the Sun is on the order of $10^9\text{ m}.$

The Quest for Microscopic Standards for Basic Units

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

Table 1.2 Metric Prefixes for Powers of 10 and their Symbols

Prefix	Symbol	Value[1]	Example (some are approximate)			
exa	E	10^{18}	exameter	Em	$10^{18}\ \text{m}$	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	$10^{15}\ \text{s}$	30 million years
tera	T	10^{12}	terawatt	TW	$10^{12}\ \text{W}$	powerful laser output
giga	G	10^{9}	gigahertz	GHz	$10^{9}\ \text{Hz}$	a microwave frequency
mega	M	10^{6}	megacurie	MCi	$10^{6}\ \text{Ci}$	high radioactivity
kilo	k	10^{3}	kilometer	km	$10^{3}\ \text{m}$	about 6/10 mile
hecto	h	10^{2}	hectoliter	hL	$10^{2}\ \text{L}$	26 gallons
deka	da	10^{1}	dekagram	dag	$10^{1}\ \text{g}$	teaspoon of butter
—	—	10^{0} (=1)				
deci	d	10^{-1}	deciliter	dL	$10^{-1}\ \text{L}$	less than half a soda
centi	c	10^{-2}	centimeter	cm	$10^{-2}\ \text{m}$	fingertip thickness
milli	m	10^{-3}	millimeter	mm	$10^{-3}\ \text{m}$	flea at its shoulders
micro	µ	10^{-6}	micrometer	µm	$10^{-6}\ \text{m}$	detail in microscope
nano	n	10^{-9}	nanogram	ng	$10^{-9}\ \text{g}$	small speck of dust
pico	p	10^{-12}	picofarad	pF	$10^{-12}\ \text{F}$	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	$10^{-15}\ \text{m}$	size of a proton
atto	a	10^{-18}	attosecond	as	$10^{-18}\ \text{s}$	time light crosses an atom

Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in **Table 1.3**. Examination of this table will give you some feeling for the range of possible topics and numerical values. (See **Figure 1.20** and **Figure 1.21**.)

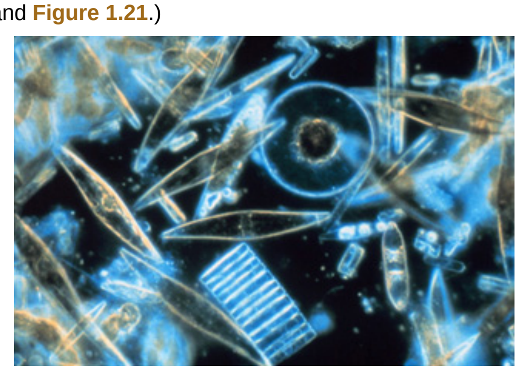


Figure 1.20 Tiny phytoplankton swims among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (credit: Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)

1. See Appendix A for a discussion of powers of 10.

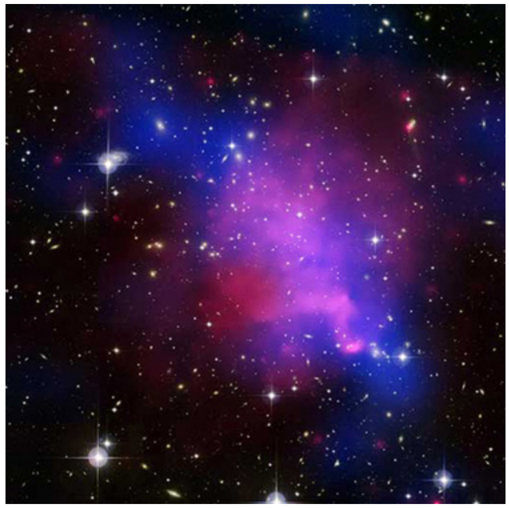


Figure 1.21 Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (credit: NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$80\cancel{\text{m}}\times\frac{1\text{ km}}{1000\cancel{\text{m}}}=0.080\text{ km}. \tag{1.1}$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Click **Appendix C** for a more complete list of conversion factors.

Table 1.3 Approximate Values of Length, Mass, and Time

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron $(9.11 \times 10^{-31}\ \text{kg})$	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom $(1.67 \times 10^{-27}\ \text{kg})$	10^{-22}	Mean life of an extremely unstable nucleus
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day $(8.64 \times 10^{4}\ \text{s})$
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (y) $(3.16 \times 10^{7}\ \text{s})$
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of the Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from the Earth to the Sun	10^{23}	Mass of the Moon $(7.35 \times 10^{22}\ \text{kg})$	10^{17}	Age of the Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of the Earth $(5.97 \times 10^{24}\ \text{kg})$	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way galaxy	10^{30}	Mass of the Sun $(1.99 \times 10^{30}\ \text{kg})$		
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from the Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Example 1.1 Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}. \tag{1.2}$$

(2) Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0\text{ km}}{20.0\text{ min}} = 0.500\frac{\text{km}}{\text{min}}. \tag{1.3}$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr . Thus,

$$\text{average speed} = 0.500\frac{\text{km}}{\text{min}}\times\frac{60\text{ min}}{1\text{ h}} = 30.0\frac{\text{km}}{\text{h}}. \tag{1.4}$$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\text{km}}{\text{min}}\times\frac{1\text{ hr}}{60\text{ min}} = \frac{1}{60}\frac{\text{km}\cdot\text{hr}}{\text{min}^2}, \tag{1.5}$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\text{Average speed} = 30.0\frac{\text{km}}{\text{h}}\times\frac{1\text{ h}}{3{,}600\text{ s}}\times\frac{1{,}000\text{ m}}{1\text{ km}}, \tag{1.6}$$

$$\text{Average speed} = 8.33\frac{\text{m}}{\text{s}}. \tag{1.7}$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module **Accuracy, Precision, and Significant Figures** will help you answer these questions.

Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different “weights and measures.” Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Check Your Understanding

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Solution

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Check Your Understanding

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Solution

The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

1.3 Accuracy, Precision, and Significant Figures

Figure 1.22 A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The “known masses” are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki)

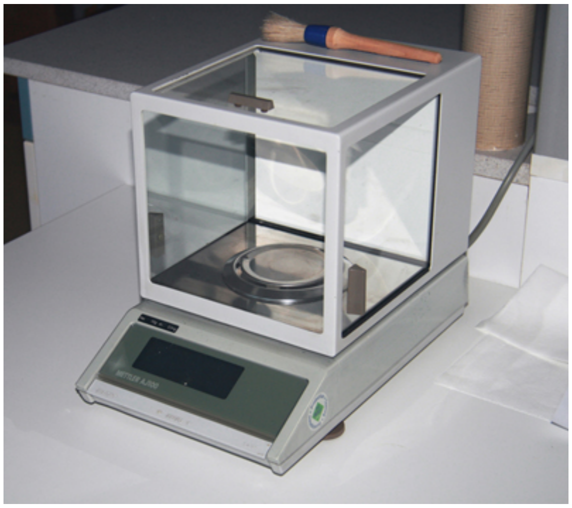


Figure 1.23 Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

Learning Objectives

By the end of this section, you will be able to:

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In **Figure 1.24**, you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in **Figure 1.25**, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

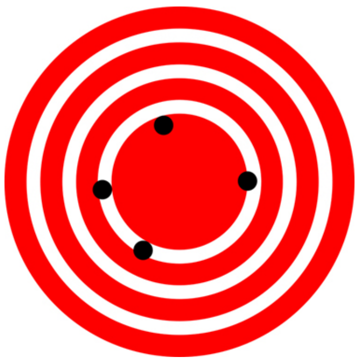


Figure 1.24 A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil)

Figure 1.25 In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil)

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as δA ("delta A"), so the measurement result would be recorded as $A \pm \delta A$. In our paper example, the length of the paper could be expressed as $11\text{ in.} \pm 0.2.$

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Making Connections: Real-World Connections—Fevers or Chills?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were $3.0^\circ C$? If the child's temperature reading was $37.0^\circ C$ (which is normal body temperature), the "true" temperature could be anywhere from a hypothermic $34.0^\circ C$ to a dangerously high $40.0^\circ C$. A thermometer with an uncertainty of $3.0^\circ C$ would be useless.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty, δA , the **percent uncertainty** (%unc) is defined to be

$$\%\text{ unc} = \frac{\delta A}{A} \times 100\%. \tag{1.8}$$

Example 1.2 Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5 lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: 4.8 lb
- Week 2 weight: 5.3 lb
- Week 3 weight: 4.9 lb
- Week 4 weight: 5.4 lb

You determine that the weight of the 5 lb bag has an uncertainty of $\pm 0.4\text{ lb}$. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A , is 5 lb. The uncertainty in this value, δA , is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$\%\text{ unc} = \frac{\delta A}{A} \times 100\%. \tag{1.9}$$

Solution

Plug the known values into the equation:

$$\%\text{ unc} = \frac{0.4\text{ lb}}{5\text{ lb}} \times 100\% = 8\%. \tag{1.10}$$

Discussion

We can conclude that the weight of the apple bag is $5\text{ lb} \pm 8\%$. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that *the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent* *uncertainties in the items used to make the calculation*. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0 m^2 and has an uncertainty of 3%. (Expressed as an area this is 0.36 m^2, which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter.)

Check Your Understanding

A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of $\pm 0.05 \text{ s}$. Runners on the track coach's team regularly clock 100 m sprints of 11.49 s to 15.01 s. At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

Solution
No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7 cm. You could not express this value as 36.71 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6 cm and 36.7 cm, and he or she must estimate the value of the last digit. Using the method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7 cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

Check Your Understanding

Determine the number of significant figures in the following measurements:

a. 0.0009
b. 15,450.0
c. 6×10^3
d. 87.990
e. 30.42

Solution

(a) 1; the zeros in this number are placekeepers that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 1; the value 10^3 signifies the decimal place, not the number of measured values

(d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) 4; any zeros located in between significant figures in a number are also significant

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

1. For multiplication and division: *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation*. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area has if the radius has only two—say, $r = 1.2\text{ m}$. Then,

$$A = \pi r^2 = (3.1415927...)\times(1.2\text{ m})^2 = 4.5238934\text{ m}^2 \tag{1.11}$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or

$$A=4.5\text{ m}^2, \tag{1.12}$$

even though π is good to at least eight digits.

2. For addition and subtraction: *The answer can contain no more decimal places than the least precise measurement.* Suppose that you buy 7.56 kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052 kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{r} 7.56\text{ kg} \\ -\ 6.052\text{ kg} \\ \underline{+13.7\text{ kg}} \\ 15.208\text{ kg} \end{array} = 15.2\text{ kg}. \tag{1.13}$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

Check Your Understanding

Perform the following calculations and express your answer using the correct number of significant digits.

(a) A woman has two bags weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?

(b) The force F on an object is equal to its mass m multiplied by its acceleration a . If a wagon with mass 55 kg accelerates at a rate of 0.0255 m/s^2 , what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

Solution

(a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.

(b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.

PhET Explorations: Estimation

Explore size estimation in one, two, and three dimensions! Multiple levels of difficulty allow for progressive skill improvement.

PhET Interactive Simulation

Figure 1.26 Estimation (http://cnx.org/content/m54766/1.7/estimation_en.jar)

1.4 Approximation

Learning Objectives

By the end of this section, you will be able to:

- Make reasonable approximations based on given data.

On many occasions, physicists, other scientists, and engineers need to make **approximations** or “guesstimates” for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

Example 1.3 Approximate the Height of a Building

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

Strategy

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

Solution

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2 m tall), then we can estimate the total height of the building to be

$$\frac{2\text{ m}}{1\text{ person}}\times\frac{2\text{ person}}{1\text{ story}}\times 39\text{ stories} = 156\text{ m}. \tag{1.14}$$

Discussion

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand lengths equal the width of your desk? What other measurements can you approximate besides length?

Example 1.4 Approximating Vast Numbers: a Trillion Dollars

Figure 1.27 A bank stack contains one-hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? (credit: Andrew Magill)

The U.S. federal deficit in the 2008 fiscal year was a little greater than $10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

$$\begin{aligned}\text{volume of stack} &= \text{length}\times\text{width}\times\text{height},\\ \text{volume of stack} &= 6\text{ in.}\times 3\text{ in.}\times 0.5\text{ in.},\\ \text{volume of stack} &= 9\text{ in.}^3.\end{aligned} \tag{1.15}$$

(2) Calculate the number of stacks. Note that a trillion dollars is equal to $\$1\times10^{12},$ and a stack of one-hundred $\$100$ bills is equal to $\$10{,}000,$ or $\$1\times10^4$. The number of stacks you will have is:

$$\$1\times10^{12}(\text{a trillion dollars})/\ \$1\times10^4\text{ per stack} = 1\times10^8\text{ stacks}. \tag{1.16}$$

(3) Calculate the area of a football field in square inches. The area of a football field is $100\text{ yd}\times 50\text{ yd},$ which gives $5{,}000\text{ yd}^2.$ Because we are working in inches, we need to convert square yards to square inches:

$$\begin{aligned}\text{Area} = 5{,}000\text{ yd}^2\times\frac{3\text{ ft}}{1\text{ yd}}\times\frac{3\text{ ft}}{1\text{ yd}}\times\frac{12\text{ in.}}{1\text{ ft}}\times\frac{12\text{ in.}}{1\text{ ft}} = 6{,}480{,}000\text{ in.}^2,\\ \text{Area}\approx 6\times10^6\text{ in.}^2.\end{aligned} \tag{1.17}$$

This conversion gives us $6\times10^6\text{ in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

(4) Calculate the total volume of the bills. The volume of all the $\$100$ -bill stacks is $9\text{ in.}^3/\text{stack}\times10^8\text{ stacks} = 9\times10^8\text{ in.}^3.$

(5) Calculate the height. To determine the height of the bills, use the equation:

$$\begin{aligned}\text{volume of bills} &= \text{area of field}\times\text{height of money:}\\ \text{Height of money} &= \frac{\text{volume of bills}}{\text{area of field}},\\ \text{Height of money} &= \frac{9\times10^8\text{in.}^3}{6\times10^6\text{in.}^2} = 1.33\times10^2\text{in.},\\ \text{Height of money} &\approx 1\times10^2\text{in.} = 100\text{ in.}\end{aligned} \tag{1.18}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

$$100\text{ in.}\times\frac{1\text{ ft}}{12\text{ in.}} = 8.33\text{ ft}\approx 8\text{ ft}. \tag{1.19}$$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough "guesstimates" versus carefully calculated approximations?

Check Your Understanding

Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

Solution

An average male is about two meters tall. It would take approximately 15 men laid out end to end to cover the length, and about 7 to cover the width. That gives an approximate area of 420 m^2.

Glossary

accuracy: the degree to which a measured value agrees with correct value for that measurement

approximation: an estimated value based on prior experience and reasoning

classical physics: physics that was developed from the Renaissance to the end of the 19th century

conversion factor: a ratio expressing how many of one unit are equal to another unit

derived units: units that can be calculated using algebraic combinations of the fundamental units

English units: system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units: units that can only be expressed relative to the procedure used to measure them

kilogram: the SI unit for mass, abbreviated (kg)

law: a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

meter: the SI unit for length, abbreviated (m)

method of adding percents: the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

metric system: a system in which values can be calculated in factors of 10

model: representation of something that is often too difficult (or impossible) to display directly

modern physics: the study of relativity, quantum mechanics, or both

order of magnitude: refers to the size of a quantity as it relates to a power of 10

percent uncertainty: the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

physical quantity : a characteristic or property of an object that can be measured or calculated from other measurements

physics: the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

precision: the degree to which repeated measurements agree with each other

quantum mechanics: the study of objects smaller than can be seen with a microscope

relativity: the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

scientific method: a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

second: the SI unit for time, abbreviated (s)

SI units : the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

significant figures: express the precision of a measuring tool used to measure a value

theory: an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

uncertainty: a quantitative measure of how much your measured values deviate from a standard or expected value

units : a standard used for expressing and comparing measurements

Section Summary

1.1 Physics: An Introduction

- Science seeks to discover and describe the underlying order and simplicity in nature.

- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

1.2 Physical Quantities and Units

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

1.3 Accuracy, Precision, and Significant Figures

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

1.4 Approximation

Scientists often approximate the values of quantities to perform calculations and analyze systems.

Conceptual Questions

1.1 Physics: An Introduction

1. Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

2. How does a model differ from a theory?

3. If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

4. What determines the validity of a theory?

5. Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

6. Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

7. Classical physics is a good approximation to modern physics under certain circumstances. What are they?

8. When is it *necessary* to use relativistic quantum mechanics?

9. Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

1.2 Physical Quantities and Units

10. Identify some advantages of metric units.

1.3 Accuracy, Precision, and Significant Figures

11. What is the relationship between the accuracy and uncertainty of a measurement?

12. Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

Problems & Exercises

1.2 Physical Quantities and Units

1. The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

2. A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/h speed limit?

3. Show that $1.0 \text{ m/s} = 3.6 \text{ km/h}$. Hint: Show the explicit steps involved in converting $1.0 \text{ m/s} = 3.6 \text{ km/h}.$

4. American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

5. Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

6. What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

7. Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

8. The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

9. Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

10. (a) Refer to Table 1.3 to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

1.3 Accuracy, Precision, and Significant Figures

Express your answers to problems in this section to the correct number of significant figures and proper units.

11. Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

12. A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

13. (a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90 km/h? (b) Convert this range to miles per hour. $(1 \text{ km} = 0.6214 \text{ mi})$

14. An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

15. (a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

16. A can contains 375 mL of soda. How much is left after 308 mL is removed?

17. State how many significant figures are proper in the results of the following calculations: (a) $(106.7)(98.2) / (46.210)(1.01)$ (b) $(18.7)^2$ (c) $\left(1.60\times10^{-19}\right)(3712)$.

18. (a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

19. (a) If your speedometer has an uncertainty of 2.0 km/h at a speed of 90 km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads 60 km/h, what is the range of speeds you could be going?

20. (a) A person's blood pressure is measured to be $120 \pm 2 \text{ mm Hg}$. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80 mm Hg?

21. A person measures his or her heart rate by counting the number of beats in 30 s. If 40 ± 1 beats are counted in $30.0 \pm 0.5 \text{ s}$, what is the heart rate and its uncertainty in beats per minute?

22. What is the area of a circle 3.102 cm in diameter?

23. If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22 mi marathon?

24. A marathon runner completes a 42.188 km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

25. The sides of a small rectangular box are measured to be $1.80 \pm 0.01 \text{ cm}$, $2.05 \pm 0.02 \text{ cm, and } 3.1 \pm 0.1 \text{ cm}$ long. Calculate its volume and uncertainty in cubic centimeters.

26. When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where $1 \text{ lbm} = 0.4539 \text{ kg}$. (a) If there is an uncertainty of 0.0001 kg in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

27. The length and width of a rectangular room are measured to be $3.955 \pm 0.005 \text{ m}$ and $3.050 \pm 0.005 \text{ m}$. Calculate the area of the room and its uncertainty in square meters.

28. A car engine moves a piston with a circular cross section of $7.500 \pm 0.002 \text{ cm}$ diameter a distance of $3.250 \pm 0.001 \text{ cm}$ to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

1.4 Approximation

29. How many heartbeats are there in a lifetime?

30. A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?

31. How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic nucleus is on the order of 10^{-22} s.)

32. Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of 10^{-27} kg and the mass of a bacterium is on the order of 10^{-15} kg.)

Figure 1.28 This color-enhanced photo shows *Salmonella typhimurium* (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? (credit: Rocky Mountain Laboratories, NIAID, NIH)

33. Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?

34. (a) What fraction of Earth's diameter is the greatest ocean depth? (b) The greatest mountain height?

35. (a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?

36. Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

2 KINEMATICS

Figure 2.1 The motion of an American kestrel through the air can be described by the bird's displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

Chapter Outline

Connection for AP® Courses

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. Even in inanimate objects, there is a continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?*

Understanding motion will not only provide answers to these questions, but will be key to understanding more advanced concepts in physics. For example, the discussion of force in Chapter 4 will not fully make sense until you understand acceleration. This relationship between force and acceleration is also critical to understanding Big Idea 3.

Additionally, this unit will explore the topic of reference frames, a critical component to quantifying how things move. If you have ever waved to a departing friend at a train station, you are likely familiar with this idea. While you see your friend move away from you at a considerable rate, those sitting with her will likely see her as not moving. The effect that the chosen reference frame has on your observations is substantial, and an understanding of this is needed to grasp both Enduring Understanding 3.A and Essential Knowledge 3.A.1.

Our formal study of physics begins with **kinematics,** which is defined as the *study of motion without considering its causes*. In one- and two-dimensional kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. Later, in two-dimensional kinematics, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion), for example, that of a car rounding a curve.

The content in this chapter supports:

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A All forces share certain common characteristics when considered by observers in inertial reference frames.

Essential Knowledge 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.

2.1 Displacement

Figure 2.2 These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Learning Objectives

By the end of this section, you will be able to:

- Define position, displacement, distance, and distance traveled in a particular frame of reference.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. (See **Figure 2.3**.) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See **Figure 2.4**.)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word “displacement” implies that an object has moved, or has been displaced.

Displacement

Displacement is the *change in position* of an object:

$$\Delta x = x_\mathrm{f} - x_0, \tag{2.1}$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means "change in" whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_f.

Note that the SI unit for displacement is the meter (m) (see **Physical Quantities and Units**), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

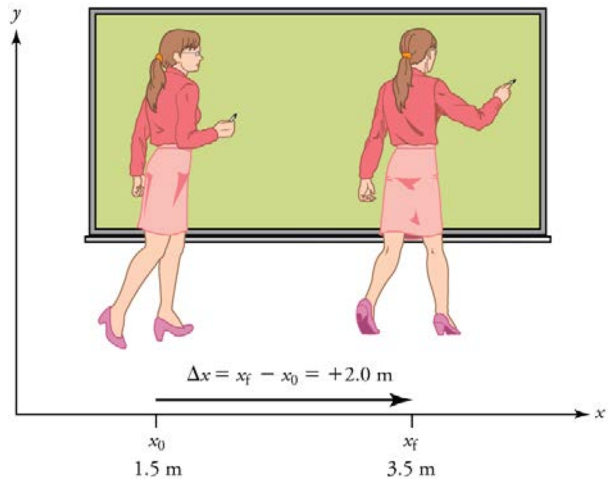


Figure 2.3 A professor paces left and right while lecturing. Her position relative to the blackboard is given by x. The $+2.0\text{ m}$ displacement of the professor relative to the blackboard is represented by an arrow pointing to the right.

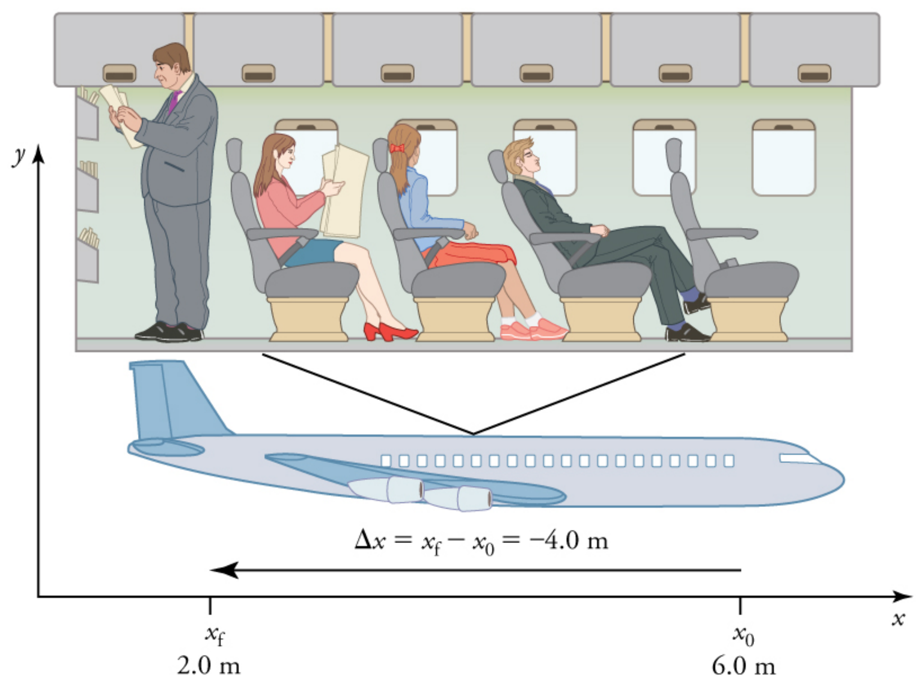


Figure 2.4 A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The −4 m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in **Figure 2.3**.

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5\text{ m}$ and her final position is $x_f = 3.5\text{ m}$. Thus her displacement is

$$\Delta x = x_f - x_0 = 3.5\text{ m} - 1.5\text{ m} = +2.0\text{ m}. \tag{2.2}$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0\text{ m}$ and his final position is $x_f = 2.0\text{ m}$, so his displacement is

$$\Delta x = x_\mathrm{f} - x_0 = 2.0\ \mathrm{m} - 6.0\ \mathrm{m} = -4.0\ \mathrm{m}. \tag{2.3}$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Check Your Understanding

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Solution

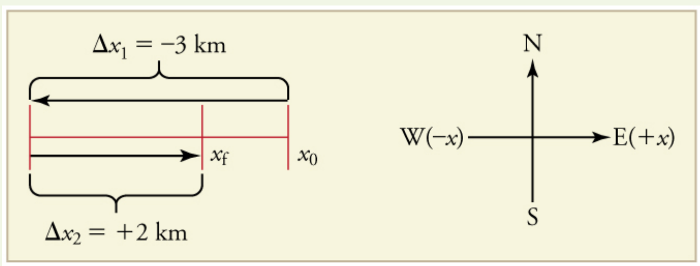


Figure 2.5

(a) The rider's displacement is $\Delta x = x_\mathrm{f} - x_0 = -1\ \mathrm{km}$. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3\ \mathrm{km} + 2\ \mathrm{km} = 5\ \mathrm{km}$.

(c) The magnitude of the displacement is $1\ \mathrm{km}$.

2.2 Vectors, Scalars, and Coordinate Systems

Figure 2.6 The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x -coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.2** The student is able to design an experimental investigation of the motion of an object.

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus $(+)$ or minus $(-)$ sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20°C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20°C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in **Figure 2.6**, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

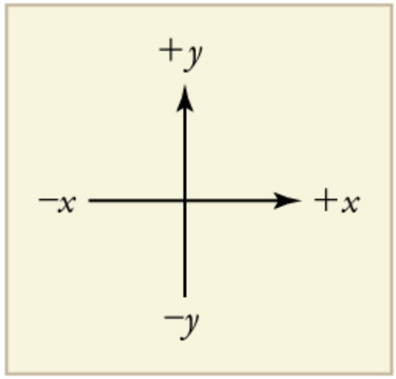


Figure 2.7 It is usually convenient to consider motion upward or to the right as positive $(+)$ and motion downward or to the left as negative $(-)$.

Check Your Understanding

A person's speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Solution

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

Switching Reference Frames

A fundamental tenet of physics is that information about an event can be gathered from a variety of reference frames. For example, imagine that you are a passenger walking toward the front of a bus. As you walk, your motion is observed by a fellow bus passenger and by an observer standing on the sidewalk.

Both the bus passenger and sidewalk observer will be able to collect information about you. They can determine how far you moved and how much time it took you to do so. However, while you moved at a consistent pace, both observers will get different results. To the passenger sitting on the bus, you moved forward at what one would consider a normal pace, something similar to how quickly you would walk outside on a sunny day. To the sidewalk observer though, you will have moved much quicker. Because the bus is also moving forward, the distance you move forward against the sidewalk each second increases, and the sidewalk observer must conclude that you are moving at a greater pace.

To show that you understand this concept, you will need to create an event and think of a way to view this event from two different frames of reference. In order to ensure that the event is being observed simultaneously from both frames, you will need an assistant to help out. An example of a possible event is to have a friend ride on a skateboard while tossing a ball. How will your friend observe the ball toss, and how will those observations be different from your own?

Your task is to describe your event and the observations of your event from both frames of reference. Answer the following questions below to demonstrate your understanding. For assistance, you can review the information given in the ‘Position' paragraph at the start of Section 2.1.

1. What is your event? What object are both you and your assistant observing?
2. What do *you* see as the event takes place?
3. What does *your assistant* see as the event takes place?
4. How do your reference frames cause you and your assistant to have two different sets of observations?

2.3 Time, Velocity, and Speed

Figure 2.8 The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

Learning Objectives

By the end of this section, you will be able to:

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner's speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed in **Physical Quantities and Units**, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple— **time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** $\boldsymbol{\Delta t}$ is the difference between the ending time and beginning time,

$$\Delta t = t_{\text{f}} - t_0, \tag{2.4}$$

where Δt is the change in time or elapsed time, t_{f} is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ , means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_{\text{f}} \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $(t_0 = 0)$
- the symbol t is used for elapsed time unless otherwise specified $(\Delta t = t_{\text{f}} \equiv t)$

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Average Velocity

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_0}{t_\text{f} - t_0}, \tag{2.5}$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}. \tag{2.6}$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the minus sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\text{ m}}{5\text{ s}} = -0.8\text{ m/s}. \tag{2.7}$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

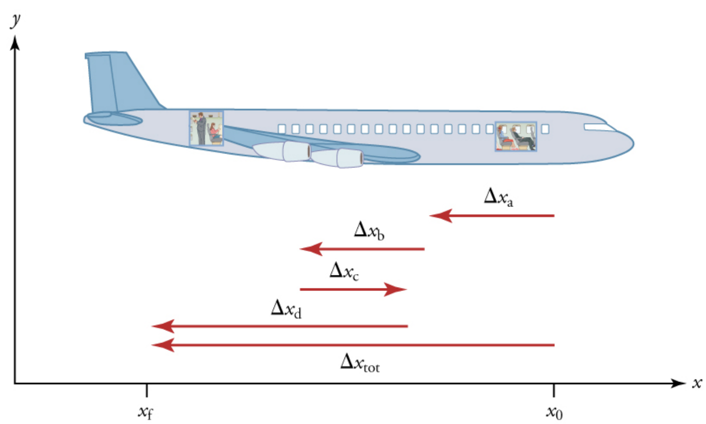


Figure 2.9 A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms “speed” and “velocity” interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

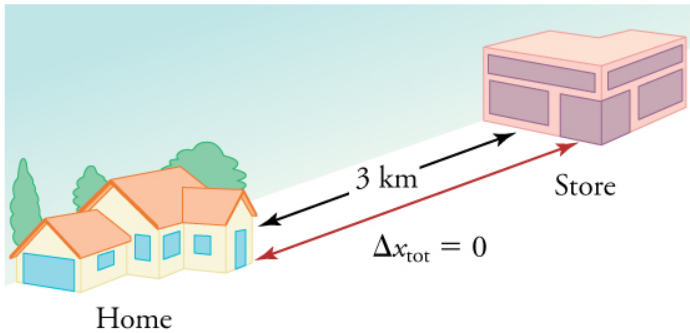


Figure 2.10 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in **Figure 2.11**. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

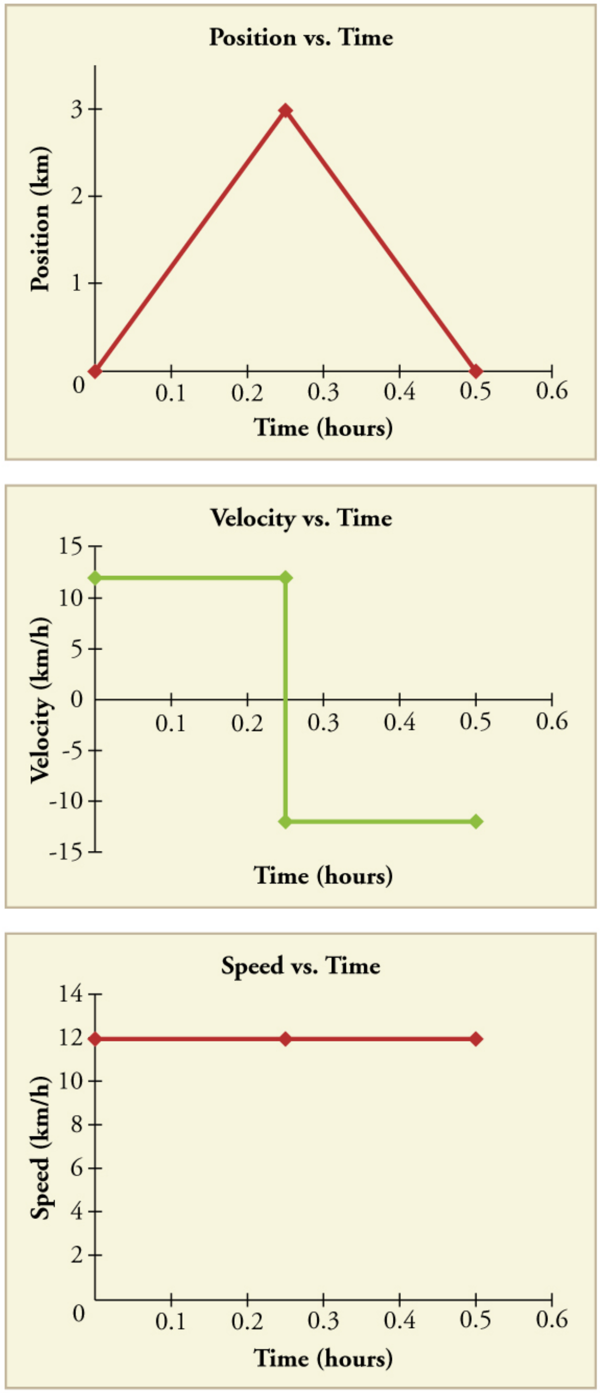


Figure 2.11 Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Check Your Understanding

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Solution

(a) The average velocity of the train is zero because $x_{\text{f}} = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{\text{distance}}{\text{time}} = \frac{80 \text{ miles}}{105 \text{ minutes}} \tag{2.8}$$

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s} \tag{2.9}$$

2.4 Acceleration

Figure 2.12 A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between instantaneous acceleration and average acceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Average Acceleration

Average Acceleration is *the rate at which velocity changes*,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\text{f}} - v_0}{t_{\text{f}} - t_0}, \tag{2.10}$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are m/s^2 , meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object's acceleration is in the same direction of its motion, the object will speed up. However, when an object's acceleration is opposite to the direction of its motion, the object will slow down. Speeding up and slowing down should not be confused with a positive and negative acceleration. The next two examples should help to make this distinction clear.

Figure 2.13 A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Making Connections: Car Motion

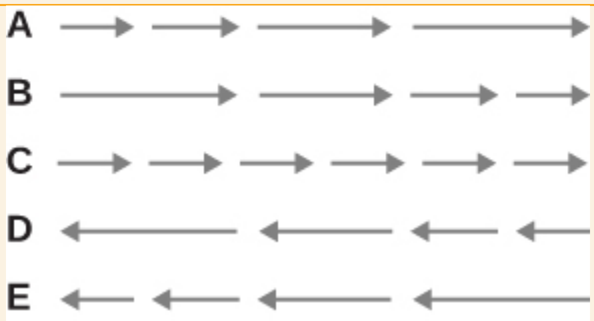


Figure 2.14 Above are arrows representing the motion of five cars (A–E). In all five cases, the positive direction should be considered to the right of the page.

Consider the acceleration and velocity of each car in terms of its direction of travel.

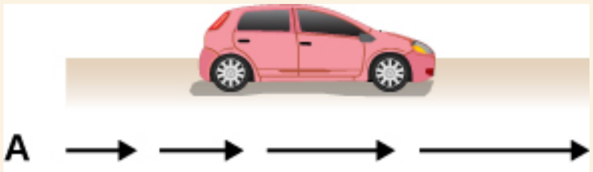


Figure 2.15 Car A is speeding up.

Because the positive direction is considered to the right of the paper, Car A is moving with a positive velocity. Because it is speeding up while moving with a positive velocity, its acceleration is also considered positive.

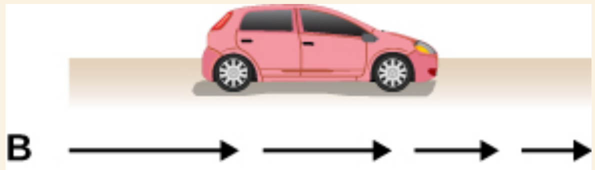


Figure 2.16 Car B is slowing down.

Because the positive direction is considered to the right of the paper, Car B is also moving with a positive velocity. However, because it is slowing down while moving with a positive velocity, its acceleration is considered negative. (This can be viewed in a mathematical manner as well. If the car was originally moving with a velocity of +25 m/s, it is finishing with a speed less than that, like +5 m/s. Because the change in velocity is negative, the acceleration will be as well.)

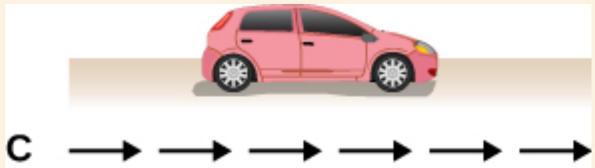


Figure 2.17 Car C has a constant speed.

Because the positive direction is considered to the right of the paper, Car C is moving with a positive velocity. Because all arrows are of the same length, this car is not changing its speed. As a result, its change in velocity is zero, and its acceleration must be zero as well.

Figure 2.18 Car D is speeding up in the opposite direction of Cars A, B, C.

Because the car is moving opposite to the positive direction, Car D is moving with a negative velocity. Because it is speeding up while moving in a negative direction, its acceleration is negative as well.

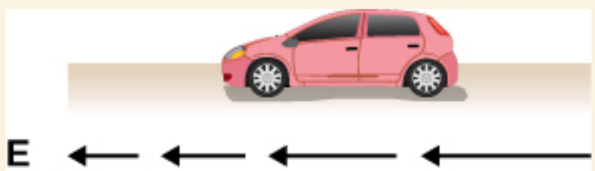


Figure 2.19 Car E is slowing down in the same direction as Car D and opposite of Cars A, B, C.

Because it is moving opposite to the positive direction, Car E is moving with a negative velocity as well. However, because it is slowing down while moving in a negative direction, its acceleration is actually positive. As in example B, this may be more easily understood in a mathematical sense. The car is originally moving with a large negative velocity (−25 m/s) but slows to a final velocity that is less negative (−5 m/s). This change in velocity, from −25 m/s to −5 m/s, is actually a positive change ($v_f - v_i = -5\text{ m/s} - -25\text{ m/s}$ of 20 m/s. Because the change in velocity is positive, the acceleration must also be positive.

Making Connection - Illustrative Example

The three graphs below are labeled A, B, and C. Each one represents the position of a moving object plotted against time.

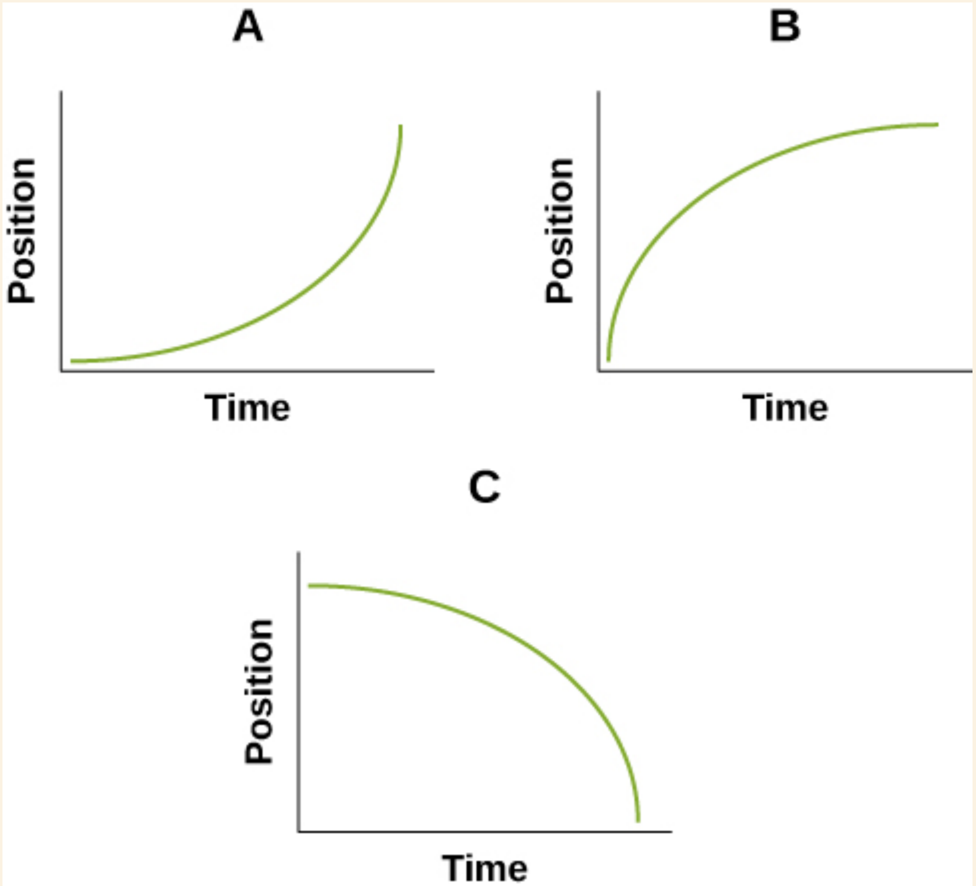


Figure 2.20 Three position and time graphs: A, B, and C.

As we did in the previous example, let's consider the acceleration and velocity of each object in terms of its direction of travel.

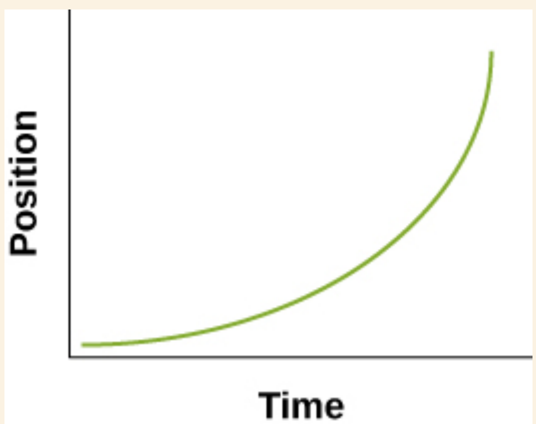


Figure 2.21 Graph A of Position (y axis) vs. Time (x axis).

Object A is continually increasing its position in the positive direction. As a result, its velocity is considered positive.

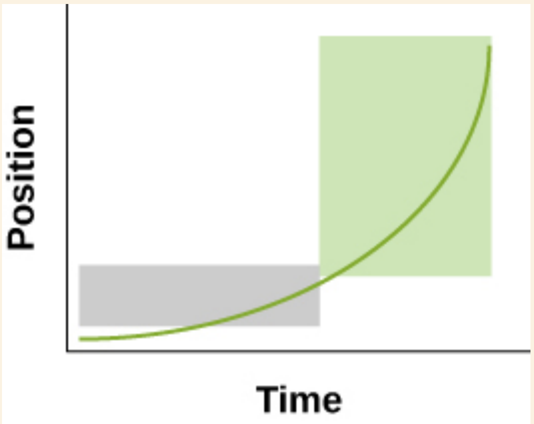


Figure 2.22 Breakdown of Graph A into two separate sections.

During the first portion of time (shaded grey) the position of the object does not change much, resulting in a small positive velocity. During a later portion of time (shaded green) the position of the object changes more, resulting in a larger positive velocity. Because this positive velocity is increasing over time, the acceleration of the object is considered positive.

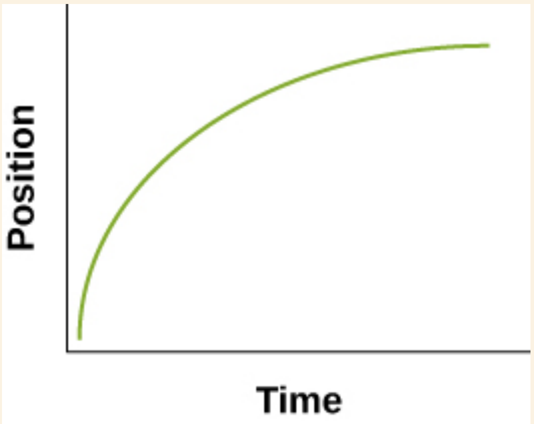


Figure 2.23 Graph B of Position (y axis) vs. Time (x axis).

As in case A, Object B is continually increasing its position in the positive direction. As a result, its velocity is considered positive.

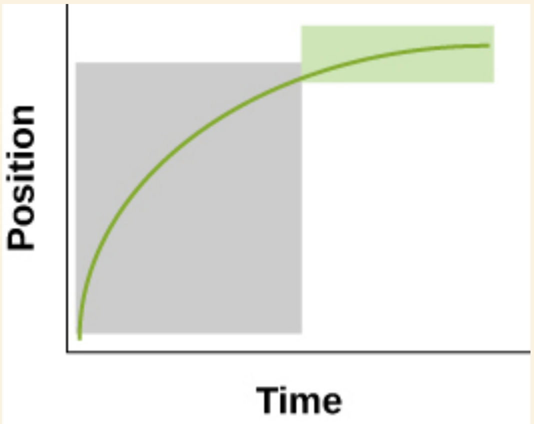


Figure 2.24 Breakdown of Graph B into two separate sections.

During the first portion of time (shaded grey) the position of the object changes a large amount, resulting in a large positive velocity. During a later portion of time (shaded green) the position of the object does not change as much, resulting in a smaller positive velocity. Because this positive velocity is decreasing over time, the acceleration of the object is considered negative.

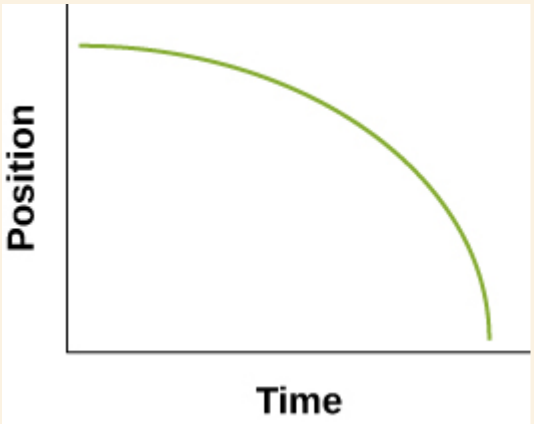


Figure 2.25 Graph C of Position (y axis) vs. Time (x axis).

Object C is continually decreasing its position in the positive direction. As a result, its velocity is considered negative.

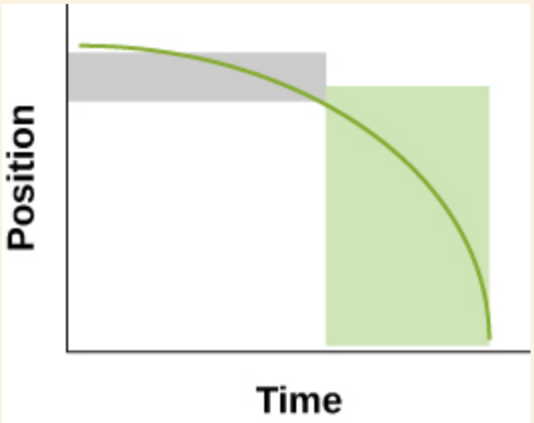


Figure 2.26 Breakdown of Graph C into two separate sections.

During the first portion of time (shaded grey) the position of the object does not change a large amount, resulting in a small negative velocity. During a later portion of time (shaded green) the position of the object changes a much larger amount, resulting in a larger negative velocity. Because the velocity of the object is becoming more negative during the time period, the change in velocity is negative. As a result, the object experiences a negative acceleration.

Example 2.1 Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

Figure 2.27 (credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

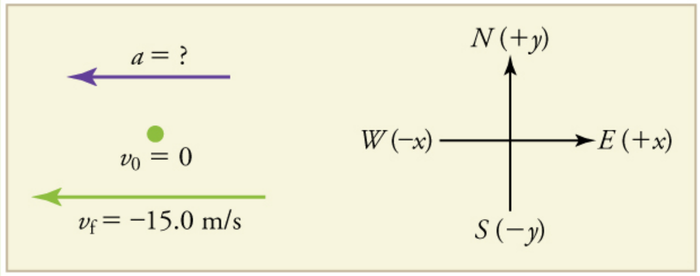


Figure 2.28

We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\text{ m/s}$ (the minus sign indicates direction toward the west), $\Delta t = 1.80\text{ s}$.

2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0\text{ m/s}$.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\text{ m/s}}{1.80\text{ s}} = -8.33\text{ m/s}^2. \quad (2.11)$$

Discussion

The minus sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in **Time, Velocity, and Speed**—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. **Figure 2.29** shows graphs of instantaneous acceleration versus time for two very different motions. In **Figure 2.29**(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 m/s^2). In **Figure 2.29**(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\text{ m/s}^2$ and -2.0 m/s^2, respectively.

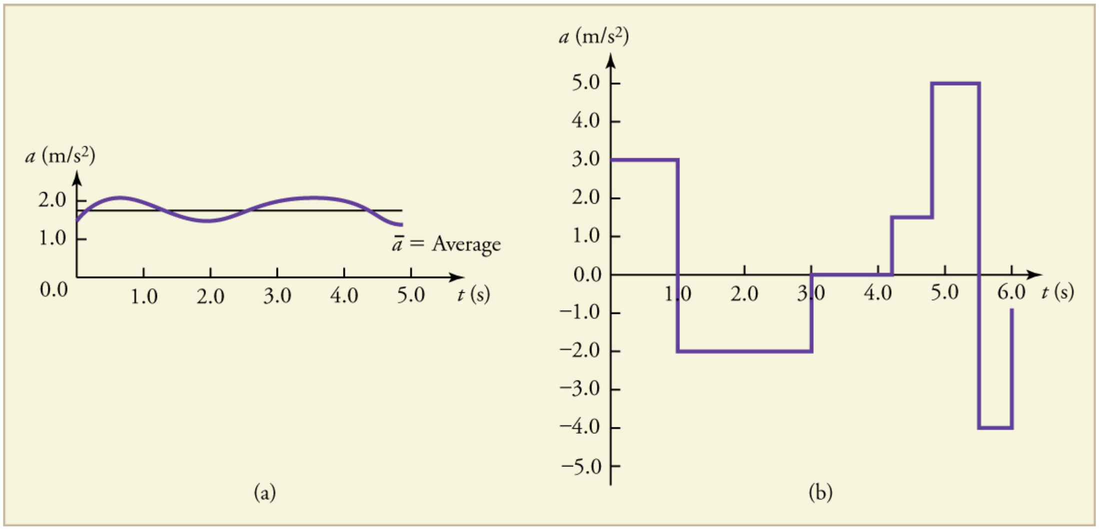


Figure 2.29 Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in **Figure 2.30**. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

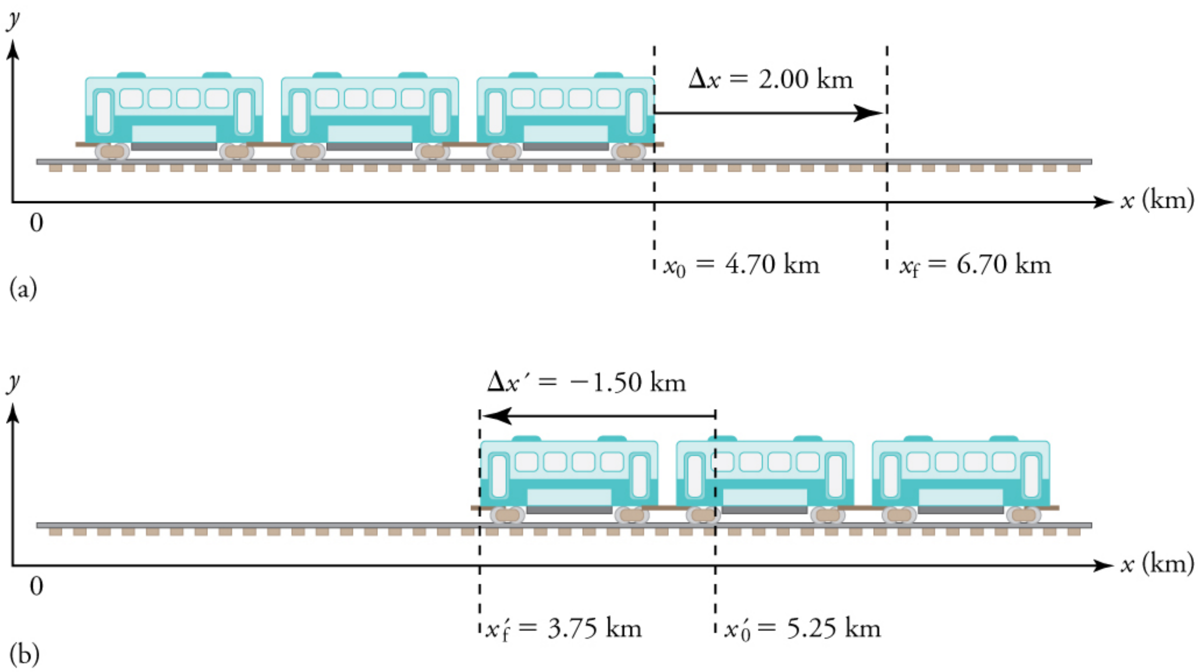


Figure 2.30 One-dimensional motion of a subway train considered in **Example 2.2**, **Example 2.3**, **Example 2.4**, **Example 2.5**, **Example 2.6**, and **Example 2.7**. Here we have chosen the x -axis so that + means to the right and $-$ means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_f . Its displacement Δx is +2.0 km. (b) The train moves to the left from x'_0 to x'_f . Its displacement $\Delta x'$ is -1.5 km . (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

Example 2.2 Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of **Figure 2.30**?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_\text{f} - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_\text{f} = 6.70\text{ km}$ and $x_0 = 4.70\text{ km}$ for part (a), and $x'_\text{f} = 3.75\text{ km}$ and $x'_0 = 5.25\text{ km}$ for part (b).

2. Solve for displacement in part (a).

$$\Delta x = x_\text{f} - x_0 = 6.70\text{ km} - 4.70\text{ km} = +2.00\text{ km} \quad (2.12)$$

3. Solve for displacement in part (b).

$$\Delta x' = x'_\text{f} - x'_0 = 3.75\text{ km} - 5.25\text{ km} = -1.50\text{ km} \quad (2.13)$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a minus sign.

Example 2.3 Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in Figure 2.30?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in Example 2.2. Distance traveled is the total length of the path traveled between the two positions. (See Displacement.) In the case of the subway train shown in Figure 2.30, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was $-1.5\text{ km}.$ Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

Example 2.4 Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in Figure 2.30(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:

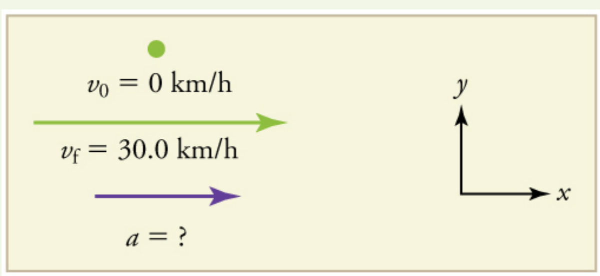


Figure 2.31

This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_\text{f} = 30.0\ \text{km/h}$, and $\Delta t = 20.0\ \text{s}$.

2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0\ \text{km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0\ \text{km/h}}{20.0\ \text{s}} \tag{2.14}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See **Physical Quantities and Units** for more guidance.)

$$\bar{a} = \left(\frac{+30\ \text{km/h}}{20.0\ \text{s}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 0.417\ \text{m/s}^2 \tag{2.15}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

Example 2.5 Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in **Figure 2.30**(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy

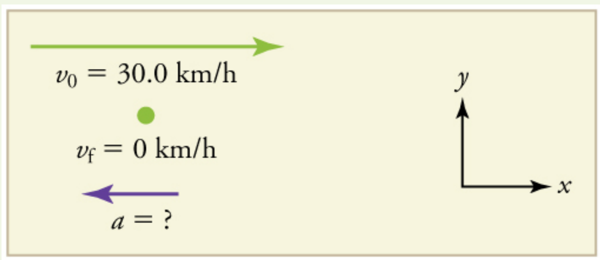


Figure 2.32

In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0\ \text{km/h}$, $v_\text{f} = 0\ \text{km/h}$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00\ \text{s}$.

2. Solve for the change in velocity, Δv.

$$\Delta v = v_\text{f} - v_0 = 0 - 30.0\ \text{km/h} = -30.0\ \text{km/h} \tag{2.16}$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0\ \text{km/h}}{8.00\ \text{s}} \tag{2.17}$$

4. Convert the units to meters and seconds.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0\ \text{km/h}}{8.00\ \text{s}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = -1.04\ \text{m/s}^2. \tag{2.18}$$

Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in **Example 2.4** and **Example 2.5** are displayed in **Figure 2.33**. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)

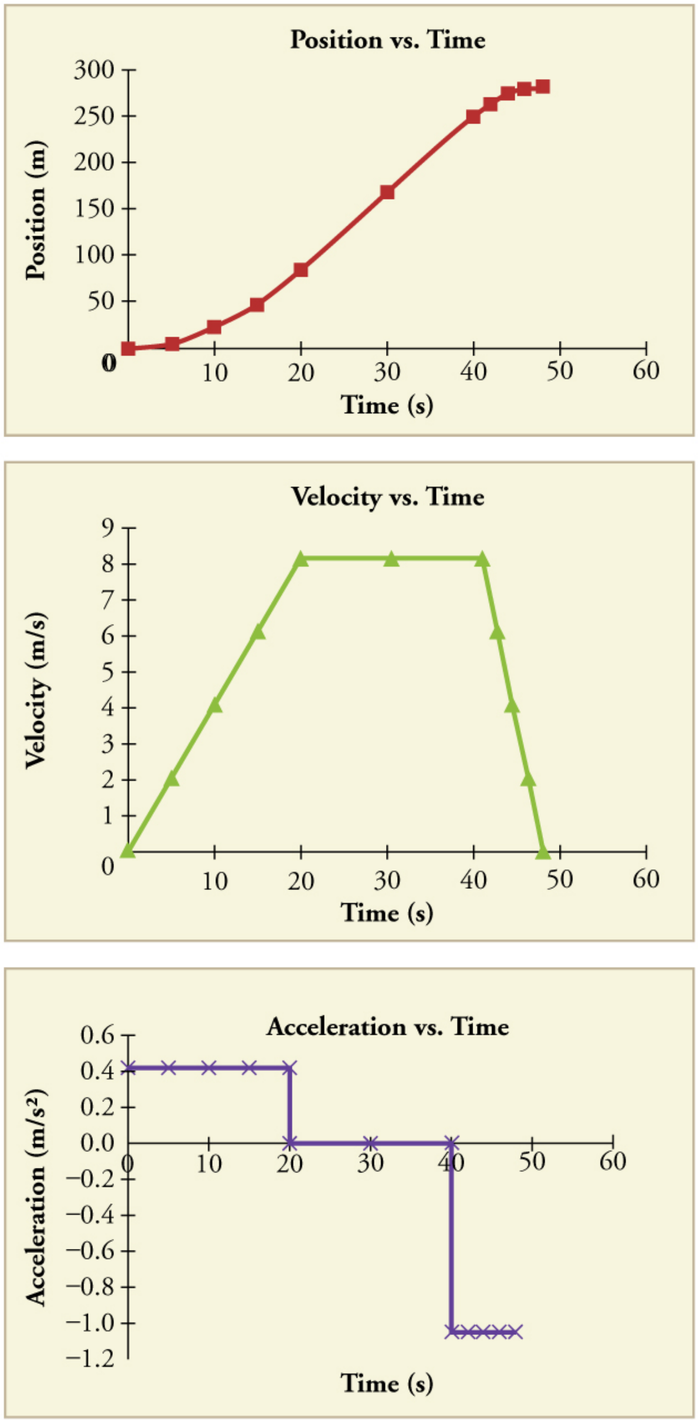


Figure 2.33 (a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Example 2.6 Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of Example 2.2, and shown again below, if it takes 5.00 min to make its trip?

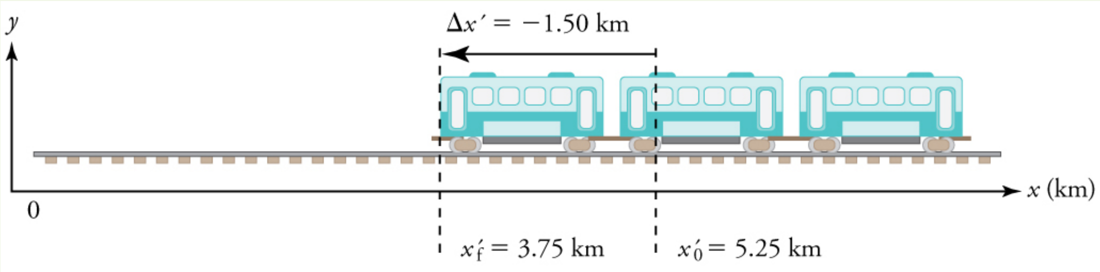


Figure 2.34

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $x'_{\text{f}} = 3.75\text{ km}$, $x'_0 = 5.25\text{ km}$, $\Delta t = 5.00\text{ min}$.

2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be -1.5 km in Example 2.2.

3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50\text{ km}}{5.00\text{ min}} \tag{2.19}$$

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \left(\frac{-1.50\text{ km}}{5.00\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = -18.0\text{ km/h} \tag{2.20}$$

Discussion

The negative velocity indicates motion to the left.

Example 2.7 Calculating Deceleration: The Subway Train

Finally, suppose the train in Figure 2.34 slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:

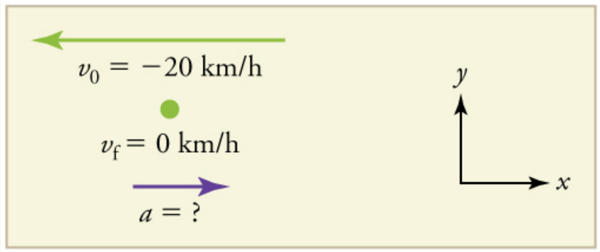


Figure 2.35

As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20\text{ km/h}$, $v_{\text{f}} = 0\text{ km/h}$, $\Delta t = 10.0\text{ s}$.

2. Calculate Δv . The change in velocity here is actually positive, since

$$\Delta v = v_{\text{f}} - v_0 = 0 - (-20\text{ km/h}) = +20\text{ km/h}. \tag{2.21}$$

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0\text{ km/h}}{10.0\text{ s}} \tag{2.22}$$

4. Convert units.

$$\bar{a} = \left(\frac{+20.0\text{ km/h}}{10.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = +0.556\text{ m/s}^2 \tag{2.23}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in **Example 2.5**, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in **Example 2.7**, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in **Figure 2.34** is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

Check Your Understanding

An airplane lands on a runway traveling east. Describe its acceleration.

Solution
If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

PhET Explorations: Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

PhET Interactive Simulation

Figure 2.36 Moving Man (http://cnx.org/content/m54772/1.3/moving-man_en.jar)

2.5 Motion Equations for Constant Acceleration in One Dimension

Figure 2.37 Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

Learning Objectives

By the end of this section, you will be able to:

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, or graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_{\text{f}} - t_0$, taking $t_0 = 0$ means that $\Delta t = t_{\text{f}}$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity*. We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\left.\begin{array}{rcl} \Delta t &=& t \\ \Delta x &=& x - x_0 \\ \Delta v &=& v - v_0 \end{array}\right\} \tag{2.24}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = \text{constant}, \tag{2.25}$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}. \tag{2.26}$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}. \tag{2.27}$$

Solving for x yields

$$x = x_0 + \bar{v}t, \tag{2.28}$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2}\ (\text{constant } a). \tag{2.29}$$

The equation $\bar{v} = \frac{v_0 + v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30\text{ km/h} + 60\text{ km/h}}{2} = 45\text{ km/h}, \tag{2.30}$$

which seems logical.

Example 2.8 Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.

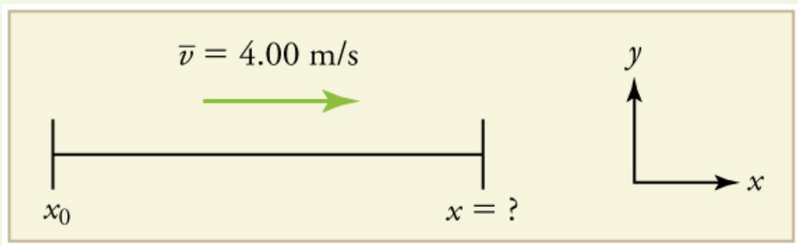


Figure 2.38

The final position x is given by the equation

$$x = x_0 + \bar{v}t. \tag{2.31}$$

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00\text{ m/s}$, $\Delta t = 2.00\text{ min}$, and $x_0 = 0\text{ m}$.

2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00\text{ m/s})(120\text{ s}) = 480\text{ m} \tag{2.32}$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

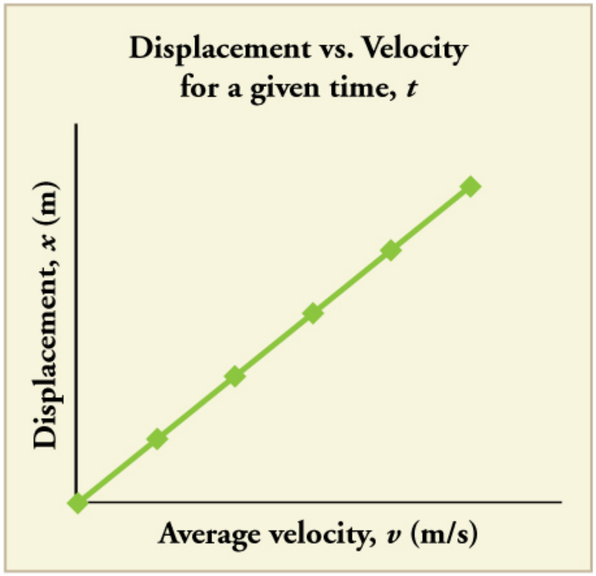


Figure 2.39 There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will move twice as far as the other object.

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \frac{\Delta v}{\Delta t} \tag{2.33}$$

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t} \text{ (constant } a\text{).} \tag{2.34}$$

Solving for v yields

$$v = v_0 + at \text{ (constant } a\text{).} \tag{2.35}$$

Example 2.9 Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.

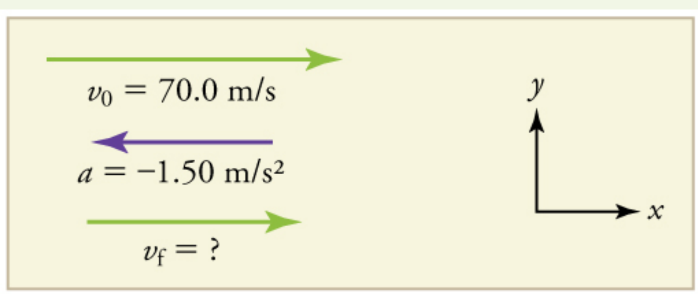


Figure 2.40

Solution

1. Identify the knowns. $v_0 = 70.0 \text{ m/s}$, $a = -1.50 \text{ m/s}^2$, $t = 40.0 \text{ s}$.

2. Identify the unknown. In this case, it is final velocity, v_f.

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + at$.

4. Plug in the known values and solve.

$$v = v_0 + at = 70.0\text{ m/s} + \left(-1.50\text{ m/s}^2\right)(40.0\text{ s}) = 10.0\text{ m/s} \tag{2.36}$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

Figure 2.41 The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

Making Connections: Real-World Connection

Figure 2.42 The Space Shuttle *Endeavor* blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Solving for Final Position When Velocity is Not Constant ($a \neq 0$)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + at. \tag{2.37}$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at. \tag{2.38}$$

Since $\frac{v_0 + v}{2} = \bar{v}$ for constant acceleration, then

$$\bar{v} = v_0 + \frac{1}{2}at. \tag{2.39}$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \text{ (constant } a\text{)}. \tag{2.40}$$

Example 2.10 Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of 26.0 m/s^2. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

Figure 2.43 U.S. Army Top Fuel pilot Tony "The Sarge" Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.

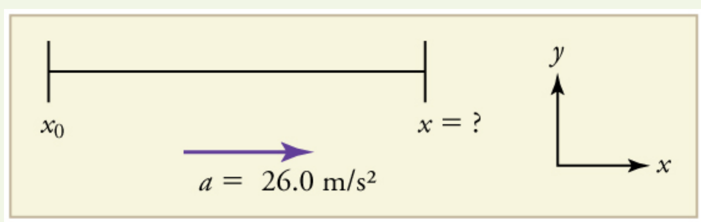


Figure 2.44

We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as 26.0 m/s^2 and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$$x = x_0 + v_0 t + \frac{1}{2}at^2. \tag{2.41}$$

Since the initial position and velocity are both zero, this simplifies to

$$x = \frac{1}{2}at^2. \tag{2.42}$$

Substituting the identified values of a and t gives

$$x = \frac{1}{2}\left(26.0 \text{ m/s}^2\right)(5.56 \text{ s})^2, \tag{2.43}$$

yielding

$$x = 402 \text{ m}. \tag{2.44}$$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0t + \frac{1}{2}at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In **Example 2.10**, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0t + \frac{1}{2}at^2$ becomes $x = x_0 + v_0t$

Solving for Final Velocity when Velocity Is Not Constant ($a \neq 0$)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + at$ for t, we get

$$t = \frac{v - v_0}{a}. \tag{2.45}$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

$$v^2 = v_0^2 + 2a(x - x_0) \text{ (constant}a\text{)}. \tag{2.46}$$

Example 2.11 Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in **Example 2.10** without using information about time.

Strategy

Draw a sketch.

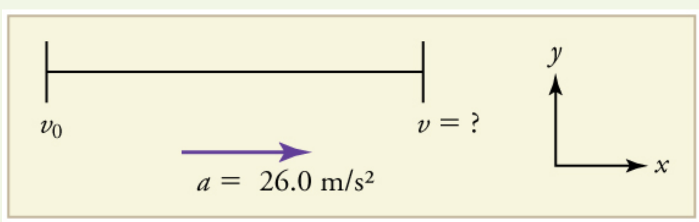


Figure 2.45

The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402 \text{ m}$ (this was the answer in **Example 2.10**). Finally, the average acceleration was given to be $a = 26.0 \text{ m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

$$v^2 = 0 + 2\left(26.0 \text{ m/s}^2\right)(402 \text{ m}). \tag{2.47}$$

Thus

$$v^2 = 2.09\times10^4\ \mathrm{m^2/s^2}. \tag{2.48}$$

To get v, we take the square root:

$$v = \sqrt{2.09\times10^4\ \mathrm{m^2/s^2}} = 145\ \mathrm{m/s}. \tag{2.49}$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Summary of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t \tag{2.50}$$

$$\bar{v} = \frac{v_0 + v}{2} \tag{2.51}$$

$$v = v_0 + at \tag{2.52}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{2.53}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{2.54}$$

Example 2.12 Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of $7.00\ \mathrm{m/s^2}$, whereas on wet concrete it can decelerate at only $5.00\ \mathrm{m/s^2}$. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.

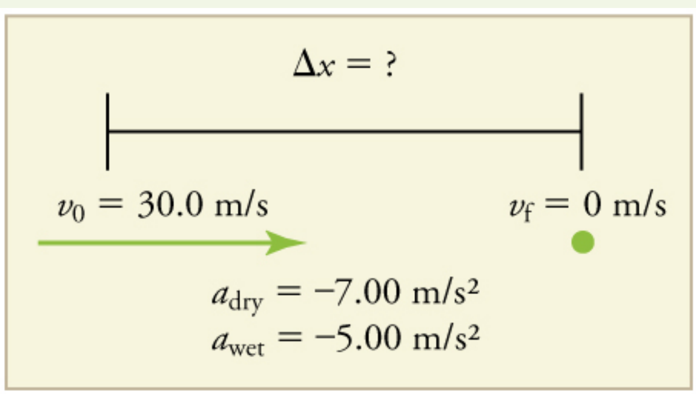


Figure 2.46

In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0\text{ m/s}$; $v = 0$; $a = -7.00\text{ m/s}^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx , or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a(x - x_0). \tag{2.55}$$

This equation is best because it includes only one unknown, x . We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x , but they require us to know the stopping time, t , which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x .

$$x - x_0 = \frac{v^2 - v_0^2}{2a} \tag{2.56}$$

4. Enter known values.

$$x - 0 = \frac{0^2 - (30.0\text{ m/s})^2}{2\left(-7.00\text{ m/s}^2\right)} \tag{2.57}$$

Thus,

$$x = 64.3\text{ m on dry concrete.} \tag{2.58}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is -5.00 m/s^2 . The result is

$$x_{\text{wet}} = 90.0\text{ m on wet concrete.} \tag{2.59}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0\text{ m/s}$; $t_{\text{reaction}} = 0.500\text{ s}$; $a_{\text{reaction}} = 0$. We take $x_{0-\text{reaction}}$ to be 0. We are looking for x_{reaction} .

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x , which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$$x = 0 + (30.0\text{ m/s})(0.500\text{ s}) = 15.0\text{ m.} \tag{2.60}$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}} \tag{2.61}$$

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet

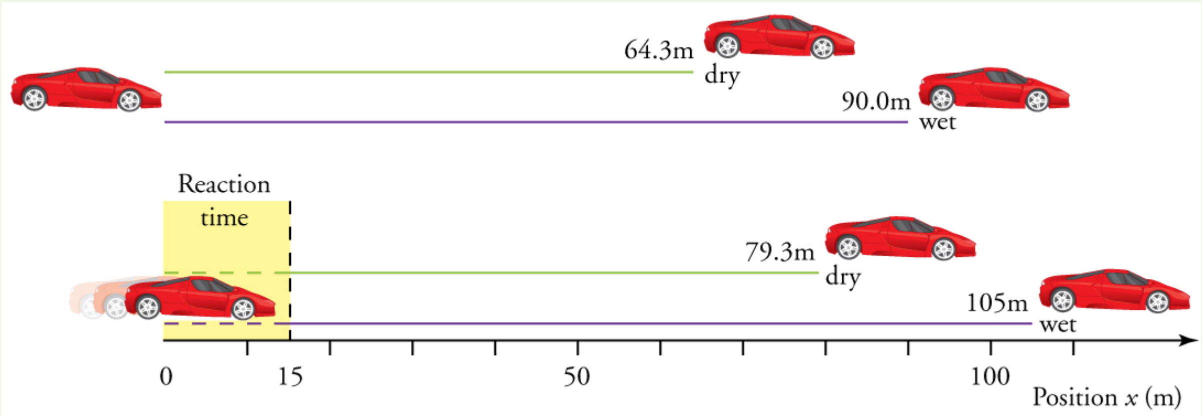


Figure 2.47 The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

Example 2.13 Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at 2.00 m/s^2, how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.

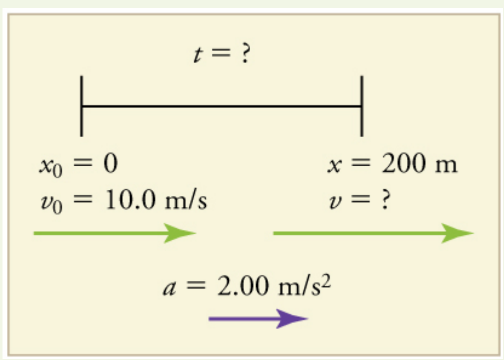


Figure 2.48

We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10\text{ m/s}$; $a = 2.00\text{ m/s}^2$; and $x = 200\text{ m}$.

2. We need to solve for t. Choose the best equation. $x = x_0 + v_0 t + \frac{1}{2}at^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

$$200\text{ m} = 0\text{ m} + (10.0\text{ m/s})t + \frac{1}{2}\left(2.00\text{ m/s}^2\right)t^2 \quad (2.62)$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = t \text{ s}$, where t is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2. \tag{2.63}$$

5. Use the quadratic formula to solve for t.

(a) Rearrange the equation to get 0 on one side of the equation.

$$t^2 + 10t - 200 = 0 \tag{2.64}$$

This is a quadratic equation of the form

$$at^2 + bt + c = 0, \tag{2.65}$$

where the constants are $a = 1.00, b = 10.0, \text{ and } c = -200$.

(b) Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{2.66}$$

This yields two solutions for t, which are

$$t = 10.0 \text{ and} -20.0. \tag{2.67}$$

In this case, then, the time is $t = t$ in seconds, or

$$t = 10.0 \text{ s and } - 20.0 \text{ s}. \tag{2.68}$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

$$t = 10.0 \text{ s}. \tag{2.69}$$

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. **Problem-Solving Basics** discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v / \Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Check Your Understanding

A manned rocket accelerates at a rate of 20 m/s^2 during launch. How long does it take the rocket reach a velocity of 400 m/s?

Solution

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.

$$v = v_0 + at \tag{2.70}$$

Rearrange to solve for t.

$$t = \frac{v - v}{a} = \frac{400 \text{ m/s} - 0 \text{ m/s}}{20 \text{ m/s}^2} = 20 \text{ s} \tag{2.71}$$

2.6 Problem-Solving Basics for One Dimensional Kinematics

Figure 2.49 Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

Learning Objectives

By the end of this section, you will be able to:

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at 0.40 m/s^2 for 100 s, his final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$v = v_0 + at = 0 + \left(0.40\text{ m/s}^2\right)(100\text{ s}) = 40\text{ m/s}. \tag{2.72}$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$\left(\frac{40\text{ m}}{\text{s}}\right)\left(\frac{3.28\text{ ft}}{\text{m}}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{60\text{ s}}{\text{min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = 89\text{ mph} \tag{2.73}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at 0.40 m/s^2, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of 0.40 m/s^2 for 100 s (almost two minutes).

2.7 Falling Objects

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, or graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.2** The student is able to design an experimental investigation of the motion of an object. **(S.P. 4.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

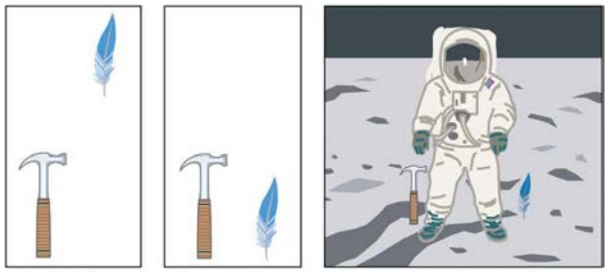


Figure 2.50 A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only 1.67 m/s^2.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, g. It is constant at any given location on Earth and has the average value

$$g = 9.80\text{ m/s}^2. \tag{2.74}$$

Although g varies from 9.78 m/s^2 to 9.83 m/s^2, depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80 m/s^2 will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80\text{ m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80\text{ m/s}^2$.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Kinematic Equations for Objects in Free-Fall where Acceleration = -*g*

$$v = v_0 - gt \tag{2.75}$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \tag{2.76}$$

$$v^2 = v_0^2 - 2g(y - y_0) \tag{2.77}$$

Example 2.14 Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to Earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.

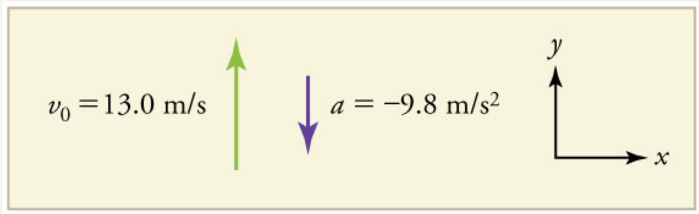


Figure 2.51

We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$; and $t = 1.00\text{ s}$.

2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2}at^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.

3. Plug in the known values and solve for y_1.

$$y = 0 + (13.0\text{ m/s})(1.00\text{ s}) + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)(1.00\text{ s})^2 = 8.10\text{ m} \tag{2.78}$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$; and $t = 1.00\text{ s}$. We also know from the solution above that $y_1 = 8.10\text{ m}$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - gt$ (from $v = v_0 + at$, where $a = \text{gravitational acceleration} = -g$).

3. Plug in the knowns and solve.

$$v_1 = v_0 - gt = 13.0\text{ m/s} - \left(9.80\text{ m/s}^2\right)(1.00\text{ s}) = 3.20\text{ m/s} \tag{2.79}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00\text{ s}$. However, it has slowed from its original 13.0 m/s, as expected.

Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00\text{ s}$ and 3.00 s are the same as those above. The results are summarized in Table 2.1 and illustrated in Figure 2.52.

Table 2.1 Results

Time, *t*	Position, *y*	Velocity, *v*	Acceleration, *a*
1.00 s	8.10 m	3.20 m/s	$-9.80\ \text{m/s}^2$
2.00 s	6.40 m	−6.60 m/s	$-9.80\ \text{m/s}^2$
3.00 s	−5.10 m	−16.4 m/s	$-9.80\ \text{m/s}^2$

Graphing the data helps us understand it more clearly.

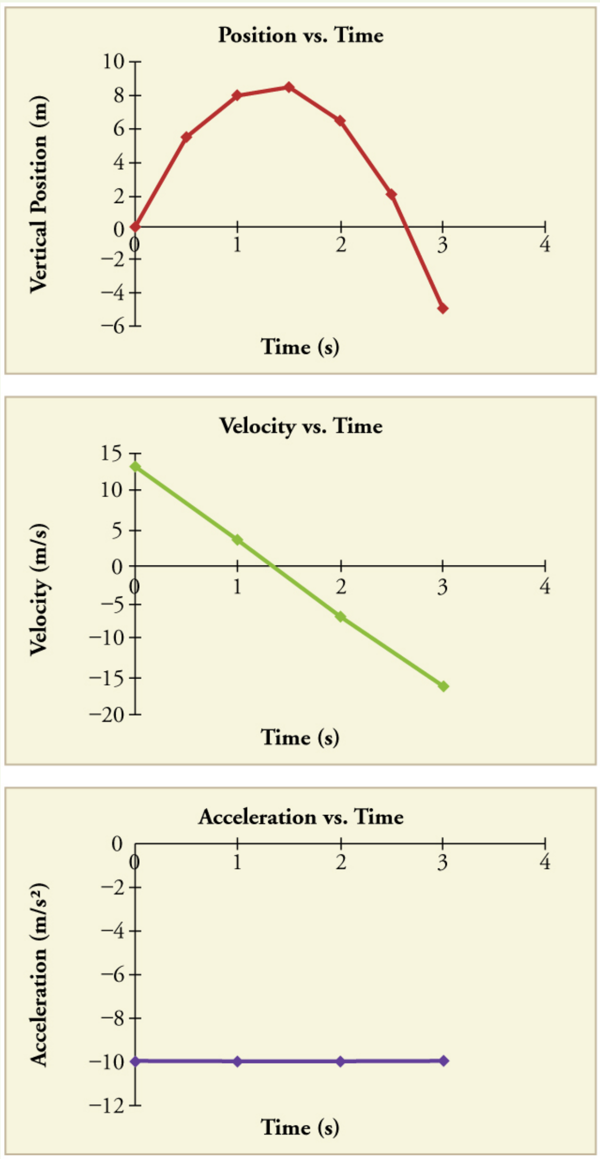


Figure 2.52 Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still -9.80 m/s^2. Its acceleration is -9.80 m/s^2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Example 2.15 Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.

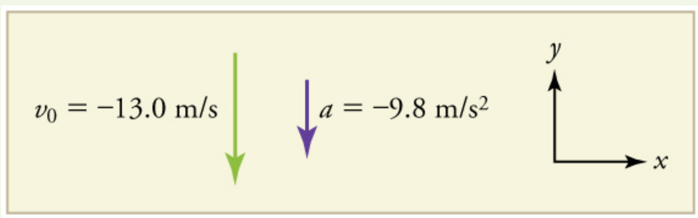


Figure 2.53

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10 \text{ m}$; $v_0 = -13.0 \text{ m/s}$; $a = -g = -9.80 \text{ m/s}^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

$$v^2 = (-13.0 \text{ m/s})^2 + 2\left(-9.80 \text{ m/s}^2\right)(-5.10 \text{ m} - 0 \text{ m}) = 268.96 \text{ m}^2/\text{s}^2, \tag{2.80}$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$v = \pm 16.4 \text{ m/s}. \tag{2.81}$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4 \text{ m/s}. \tag{2.82}$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See **Example 2.14** and **Figure 2.54**(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from **Example 2.14**) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20 \text{ m/s}$

is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.

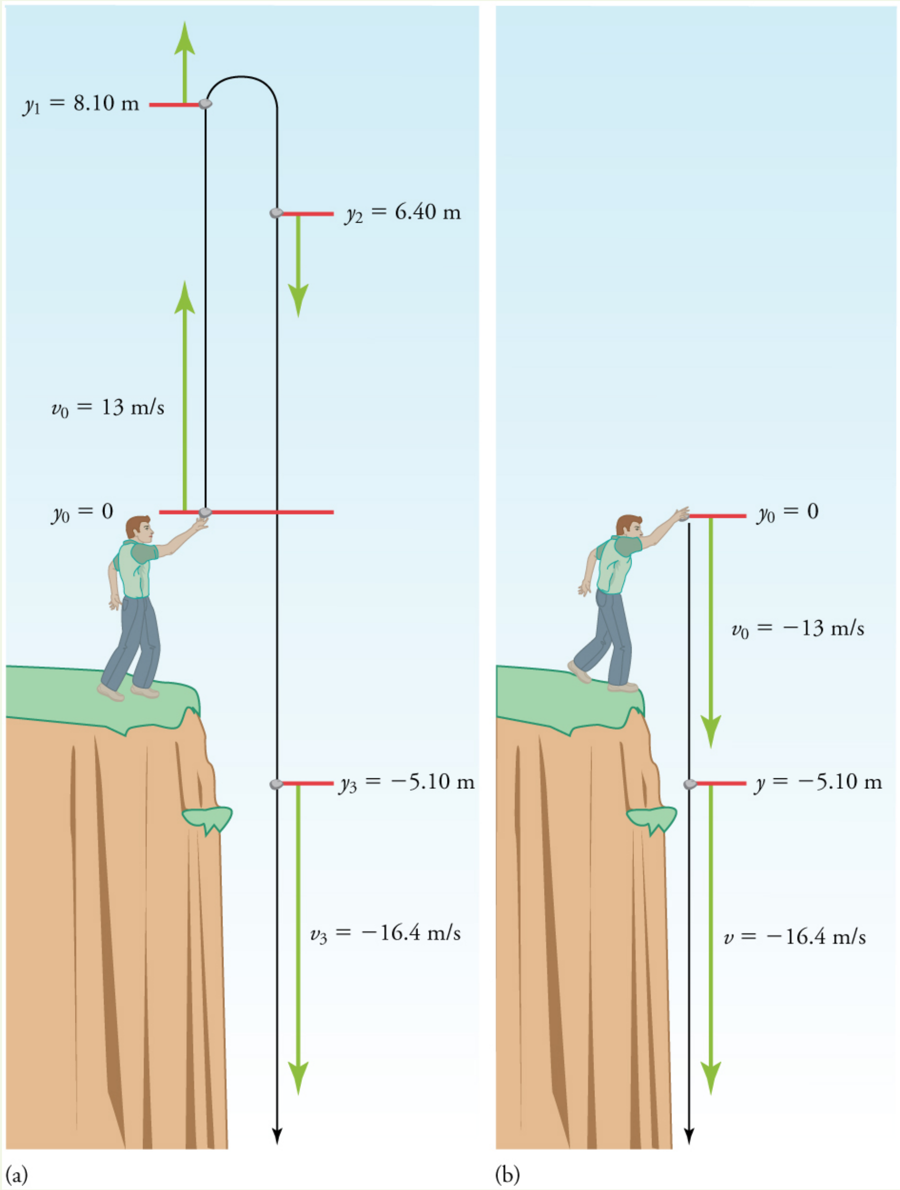
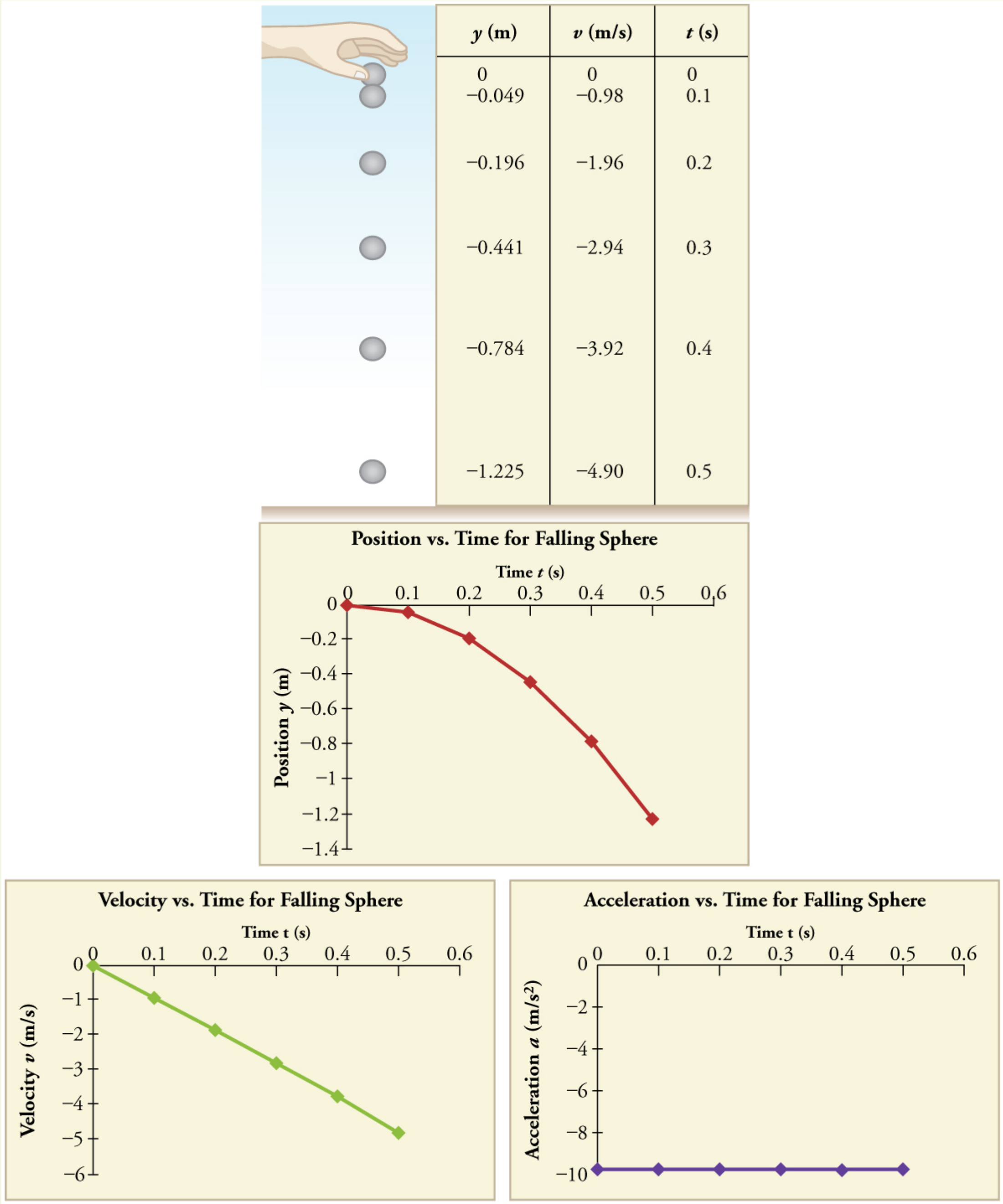


Figure 2.54 (a) A person throws a rock straight up, as explored in **Example 2.14**. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in **Example 2.15**. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In **Example 2.14**, the rock is thrown up with an initial velocity of 13.0 m/s. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is -13.0 m/s. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10 \text{ m}$ to be the same whether we have thrown it upwards at $+13.0 \text{ m/s}$ or thrown it downwards at -13.0 m/s. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example 2.16 Find *g* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, Figure 2.55. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

Figure 2.55 Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.

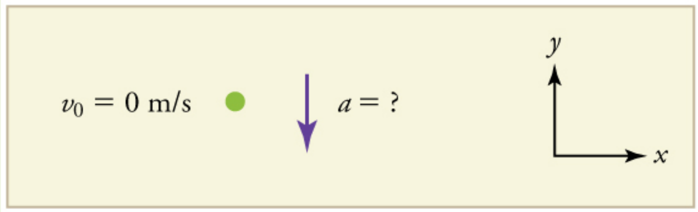


Figure 2.56

We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = –1.0000\text{ m}$; $t = 0.45173$; $v_0 = 0$.

2. Choose the equation that allows you to solve for a using the known values.

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \tag{2.83}$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y = y_0 + \frac{1}{2}at^2. \tag{2.84}$$

Solving for a gives

$$a = \frac{2(y - y_0)}{t^2}. \tag{2.85}$$

4. Substitute known values yields

$$a = \frac{2(-1.0000\text{ m} - 0)}{(0.45173\text{ s})^2} = -9.8010\text{ m/s}^2, \tag{2.86}$$

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010\text{ m/s}^2. \tag{2.87}$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of 9.80 m/s^2, so 9.8010 m/s^2 makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of 9.80 m/s^2; it represents the local value for the acceleration due to gravity.

Applying the Science Practices: Finding Acceleration Due to Gravity

While it is well established that the acceleration due to gravity is quite nearly 9.8 m/s^2 at all locations on Earth, you can verify this for yourself with some basic materials.

Your task is to find the acceleration due to gravity at your location. Achieving an acceleration of precisely 9.8 m/s^2 will be difficult. However, with good preparation and attention to detail, you should be able to get close. Before you begin working, consider the following questions.

What measurements will you need to take in order to find the acceleration due to gravity?

What relationships and equations found in this chapter may be useful in calculating the acceleration?

What variables will you need to hold constant?

What materials will you use to record your measurements?

Upon completing these four questions, record your procedure. Once recorded, you may carry out the experiment. If you find that your experiment cannot be carried out, you may revise your procedure.

Once you have found your experimental acceleration, compare it to the assumed value of 9.8 m/s^2. If error exists, what were the likely sources of this error? How could you change your procedure in order to improve the accuracy of your findings?

Check Your Understanding

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Solution

We know that initial position $y_0 = 0$, final position $y = -30.0\text{ m}$, and $a = -g = -9.80\text{ m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain

$$\begin{aligned} y &= 0 + 0 - \frac{1}{2}gt^2 \\ t^2 &= \frac{2y}{-g} \\ t &= \pm\sqrt{\frac{2y}{-g}} = \pm\sqrt{\frac{2(-30.0\text{ m})}{-9.80\text{ m/s}^2}} = \pm\sqrt{6.12\text{ s}^2} = 2.47\text{ s} \approx 2.5\text{ s} \end{aligned} \tag{2.88}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

PhET Explorations: Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

Figure 2.57 Equation Grapher (http://cnx.org/content/m54775/1.5/equation-grapher_en.jar)

2.8 Graphical Analysis of One Dimensional Motion

Learning Objectives

By the end of this section, you will be able to:

- Describe a straight-line graph in terms of its slope and *y*-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of displacement, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in Figure 2.58, a straight-line graph has the general form

$$y = mx + b. \tag{2.89}$$

Here m is the **slope**, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the ***y*-intercept**, which is the point at which the line crosses the vertical axis.

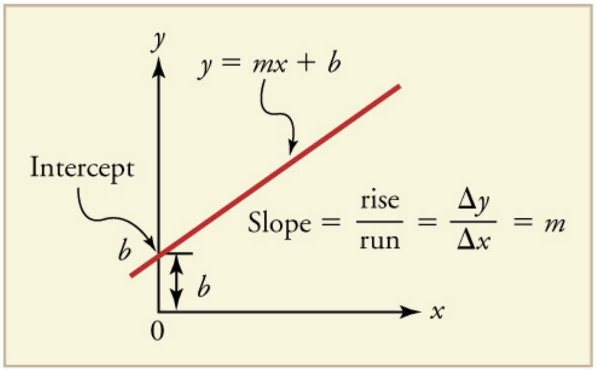


Figure 2.58 A straight-line graph. The equation for a straight line is $y = mx + b$.

Graph of Displacement vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as displacement, depend upon. A graph of displacement versus time would, thus, have x on the vertical axis and t on the horizontal axis. **Figure 2.59** is just such a straight-line graph. It shows a graph of displacement versus time for a jet-powered car on a very flat dry lake bed in Nevada.

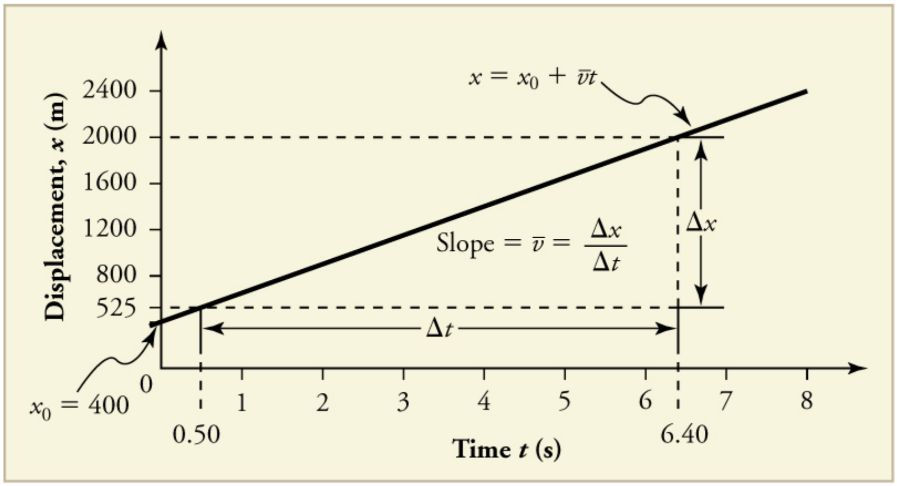


Figure 2.59 Graph of displacement versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is displacement at time zero—that is, x_0 . Substituting these symbols into $y = mx + b$ gives

$$x = \bar{v}t + x_0 \tag{2.90}$$

or

$$x = x_0 + \bar{v}t. \tag{2.91}$$

Thus a graph of displacement versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation.

The Slope of *x* vs. *t*

The slope of the graph of displacement x vs. time t is velocity v .

$$\text{slope} = \frac{\Delta x}{\Delta t} = v \tag{2.92}$$

Notice that this equation is the same as that derived algebraically from other motion equations in **Motion Equations for Constant Acceleration in One Dimension**.

From the figure we can see that the car has a displacement of 400 m at time 0.650 m at t = 1.0 s, and so on. Its displacement at times other than those listed in the table can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Example 2.17 Determining Average Velocity from a Graph of Displacement versus Time: Jet Car

Find the average velocity of the car whose position is graphed in **Figure 2.59**.

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in displacement and run = change in time, so that

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}. \tag{2.93}$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000\text{ m} - 525\text{ m}}{6.4\text{ s} - 0.50\text{ s}}, \tag{2.94}$$

yielding

$$\bar{v} = 250\text{ m/s}. \tag{2.95}$$

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when a is constant but $a \neq 0$

The graphs in **Figure 2.60** below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.

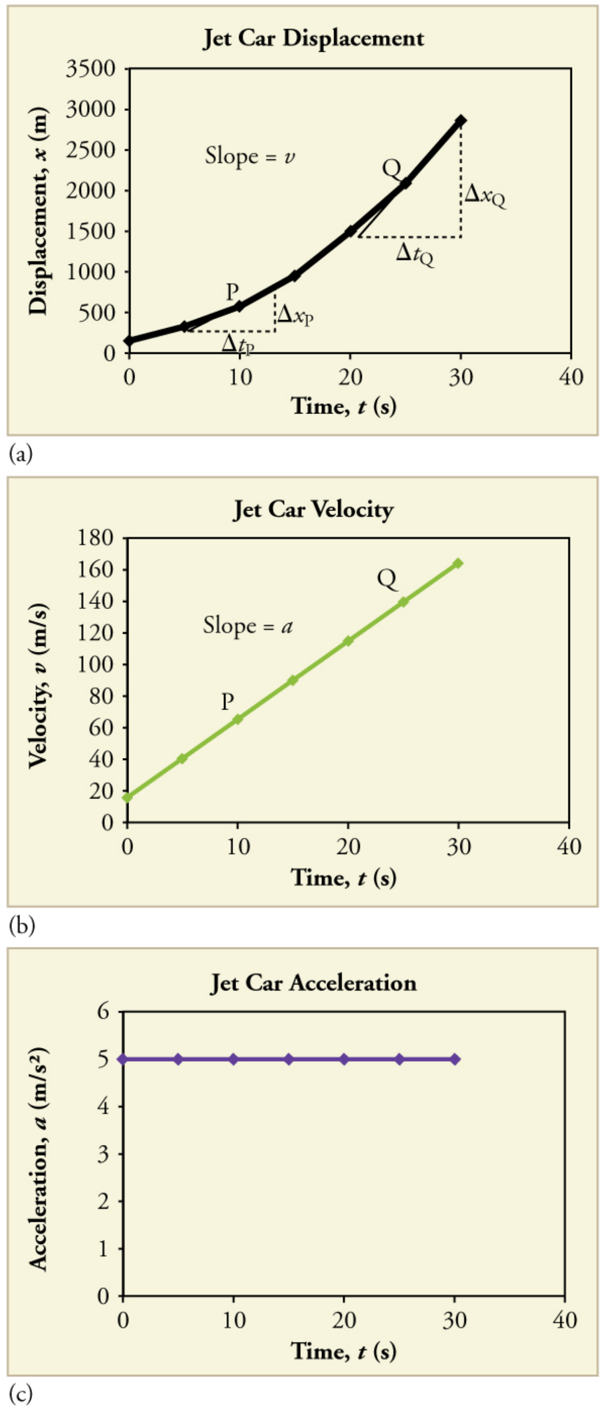


Figure 2.60 Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an x vs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of 5.0 m/s^2 over the time interval plotted.

Figure 2.61 A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of displacement versus time in Figure 2.60(a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a displacement-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure 2.60(a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in Figure 2.60(b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in Figure 2.60(c).

Example 2.18 Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.

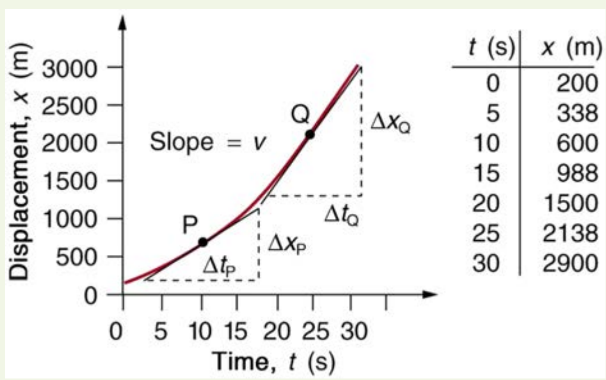


Figure 2.62 The slope of an x vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in Figure 2.62, where Q is the point at $t = 25\text{ s}$.

Solution

1. Find the tangent line to the curve at $t = 25\text{ s}$.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, v.

$$\text{slope} = v_\text{Q} = \frac{\Delta x_\text{Q}}{\Delta t_\text{Q}} = \frac{(3120\text{ m} - 1300\text{ m})}{(32\text{ s} - 19\text{ s})} \tag{2.96}$$

Thus,

$$v_\text{Q} = \frac{1820\text{ m}}{13\text{ s}} = 140\text{ m/s}. \tag{2.97}$$

Discussion

This is the value given in this figure's table for v at $t = 25\text{ s}$. The value of 140 m/s for v_{Q} is plotted in **Figure 2.62**. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

The Slope of *v* vs. *t*

The slope of a graph of velocity v vs. time t is acceleration a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = a \tag{2.98}$$

Since the velocity versus time graph in **Figure 2.60**(b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in **Figure 2.60**(c).

Additional general information can be obtained from **Figure 2.62** and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

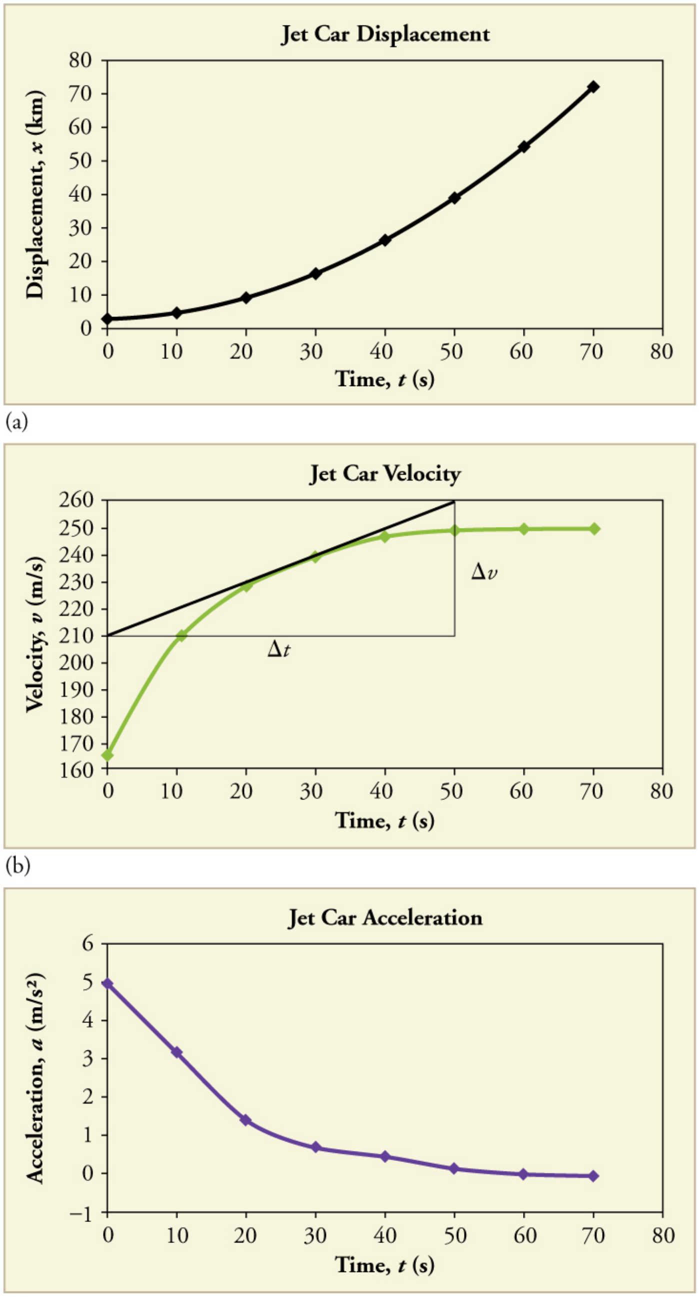
$$v = v_0 + at. \tag{2.99}$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in **Motion Equations for Constant Acceleration in One Dimension**.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in **Figure 2.63**. Time again starts at zero, and the initial displacement and velocity are 2900 m and 165 m/s, respectively. (These were the final displacement and velocity of the car in the motion graphed in **Figure 2.60**.) Acceleration gradually decreases from 5.0 m/s^2 to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until $t = 55\text{ s}$, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.



(c)

Figure 2.63 Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in **Figure 2.60** ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

Example 2.19 Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in **Figure 2.63**(b).

Strategy

The slope of the curve at $t = 25\text{ s}$ is equal to the slope of the line tangent at that point, as illustrated in **Figure 2.63**(b).

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = \frac{(260\text{ m/s} - 210\text{ m/s})}{(51\text{ s} - 1.0\text{ s})} \tag{2.100}$$

$$a = \frac{50 \text{ m/s}}{50 \text{ s}} = 1.0 \text{ m/s}^2. \tag{2.101}$$

Discussion

Note that this value for a is consistent with the value plotted in **Figure 2.63**(c) at $t = 25 \text{ s}$.

A graph of displacement versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Check Your Understanding

A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?

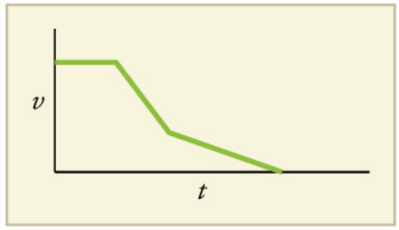


Figure 2.64

Solution

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

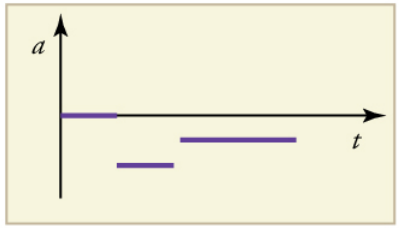


Figure 2.65

Glossary

acceleration: the rate of change in velocity; the change in velocity over time

acceleration due to gravity: acceleration of an object as a result of gravity

average acceleration: the change in velocity divided by the time over which it changes

average speed: distance traveled divided by time during which motion occurs

average velocity: displacement divided by time over which displacement occurs

deceleration: acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

dependent variable: the variable that is being measured; usually plotted along the y -axis

displacement: the change in position of an object

distance: the magnitude of displacement between two positions

distance traveled: the total length of the path traveled between two positions

elapsed time: the difference between the ending time and beginning time

free-fall: the state of movement that results from gravitational force only

independent variable: the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

instantaneous acceleration: acceleration at a specific point in time

instantaneous speed: magnitude of the instantaneous velocity

instantaneous velocity: velocity at a specific instant, or the average velocity over an infinitesimal time interval

kinematics: the study of motion without considering its causes

model: simplified description that contains only those elements necessary to describe the physics of a physical situation

position: the location of an object at a particular time

scalar: a quantity that is described by magnitude, but not direction

slope: the difference in y-value (the rise) divided by the difference in x-value (the run) of two points on a straight line

time: change, or the interval over which change occurs

vector: a quantity that is described by both magnitude and direction

y-intercept: the y- value when $x = 0$, or when the graph crosses the y-axis

Section Summary

2.1 Displacement

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be

$$\Delta x = x_{\text{f}} - x_0,$$

where x_0 is the initial position and x_{f} is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.

- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

2.2 Vectors, Scalars, and Coordinate Systems

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

2.3 Time, Velocity, and Speed

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

$$\Delta t = t_{\text{f}} - t_0,$$

where t_{f} is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.

- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_{\text{f}} - x_0}{t_{\text{f}} - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.

- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

2.4 Acceleration

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\mathrm{f}} - v_0}{t_{\mathrm{f}} - t_0}.$$

- The SI unit for acceleration is $\mathrm{m/s}^2$.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

2.5 Motion Equations for Constant Acceleration in One Dimension

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

$$\left.\begin{array}{rcl} \Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0 \end{array}\right\}$$

- The following kinematic equations for motion with constant a are useful:

$$x = x_0 + \bar{v}\,t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, y is substituted for x .

2.6 Problem-Solving Basics for One Dimensional Kinematics

- *The six basic problem solving steps for physics are:*
  Step 1. Examine the situation to determine which physical principles are involved.

  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

  Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

  Step 4. Find an equation or set of equations that can help you solve the problem.

  Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

  Step 6. Check the answer to see if it is reasonable: Does it make sense?

2.7 Falling Objects

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g , which averages

$$g = 9.80\ \mathrm{m/s}^2.$$

- Whether the acceleration *a* should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80\ \mathrm{m/s}^2$ is negative. In the opposite case, $a = +\mathrm{g} = 9.80\ \mathrm{m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a .
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

2.8 Graphical Analysis of One Dimensional Motion

- Graphs of motion can be used to analyze motion.

- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Conceptual Questions

2.1 Displacement

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

3. Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \mu\text{m/s}\ \left(50\times10^{-6}\ \text{m/s}\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

2.2 Vectors, Scalars, and Coordinate Systems

4. A student writes, "*A bird that is diving for prey has a speed of* $-10\ m/s$." What is wrong with the student's statement? What has the student actually described? Explain.

5. What is the speed of the bird in **Exercise 2.4**?

6. Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

7. A weather forecast states that the temperature is predicted to be -5°C the following day. Is this temperature a vector or a scalar quantity? Explain.

2.3 Time, Velocity, and Speed

8. Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

9. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

10. Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

11. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

12. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

2.4 Acceleration

13. Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

14. Is it possible for velocity to be constant while acceleration is not zero? Explain.

15. Give an example in which velocity is zero yet acceleration is not.

16. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

17. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

2.6 Problem-Solving Basics for One Dimensional Kinematics

18. What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

19. What is the last thing you should do when solving a problem? Explain.

2.7 Falling Objects

20. What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

21. An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

22. Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

23. If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

24. The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

25. How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

2.8 Graphical Analysis of One Dimensional Motion

26. (a) Explain how you can use the graph of position versus time in **Figure 2.66** to describe the change in velocity over time. Identify (b) the time (t_a , t_b , t_c , t_d , or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.

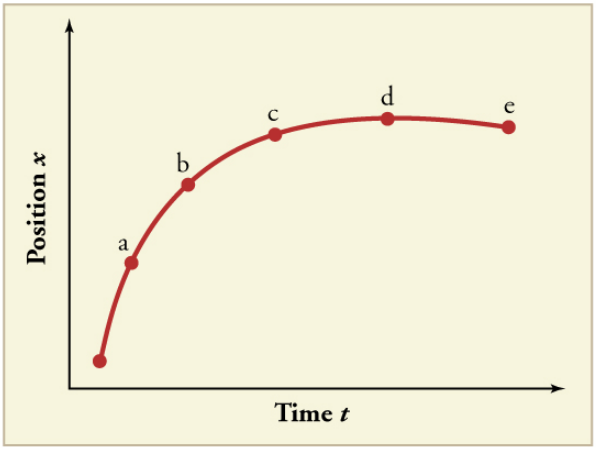


Figure 2.66

27. (a) Sketch a graph of velocity versus time corresponding to the graph of displacement versus time given in **Figure 2.67**. (b) Identify the time or times (t_a , t_b , t_c , etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?

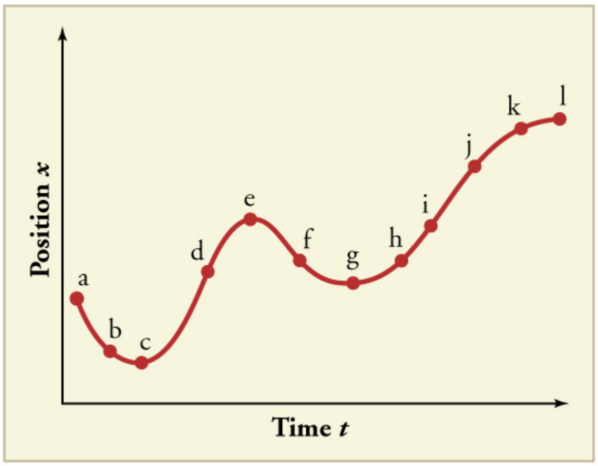


Figure 2.67

28. (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in **Figure 2.68**. (b) Based on the graph, how does acceleration change over time?

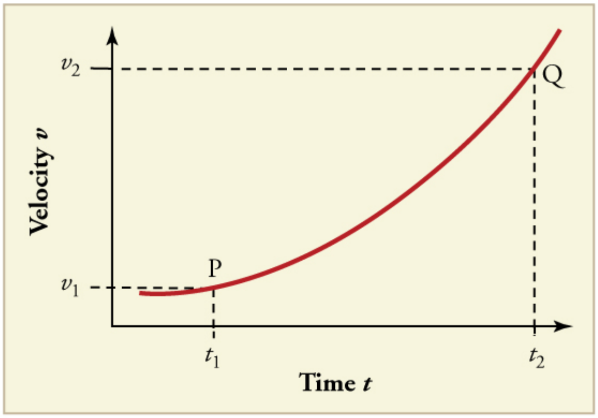


Figure 2.68

29. (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure 2.69. (b) Identify the time or times (t_a , t_b , t_c , etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?

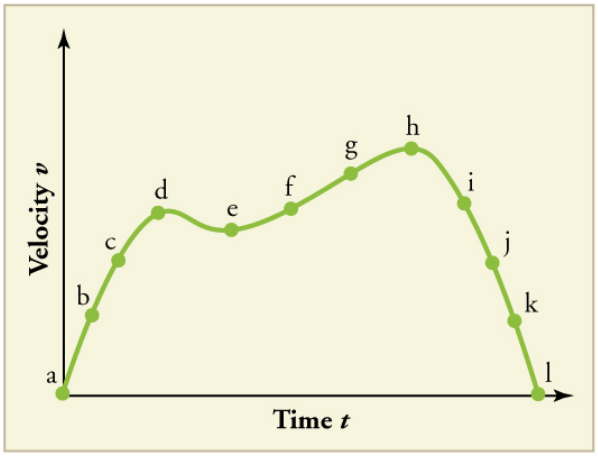


Figure 2.69

30. Consider the velocity vs. time graph of a person in an elevator shown in Figure 2.70. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

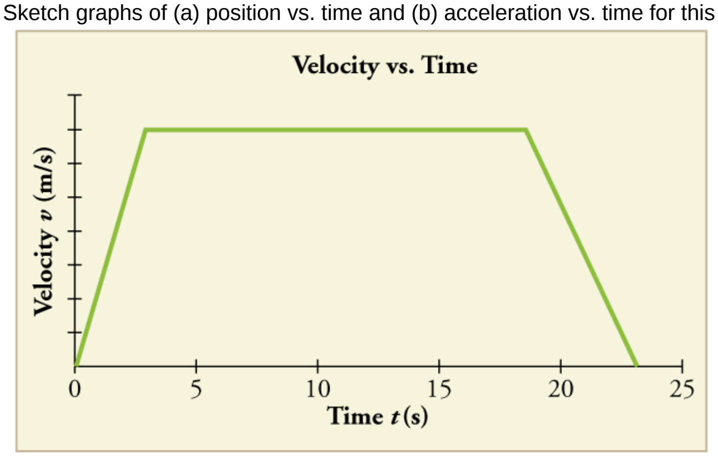


Figure 2.70

31. A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

Problems & Exercises

2.1 Displacement

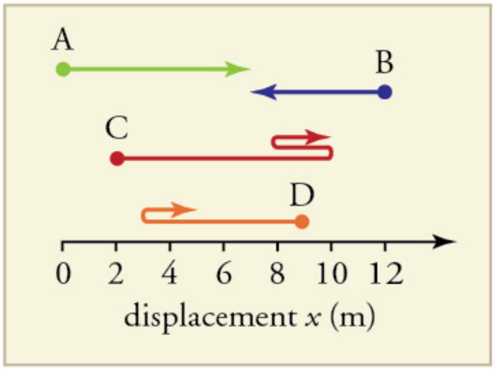


Figure 2.71

1. Find the following for path A in **Figure 2.71**: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2. Find the following for path B in **Figure 2.71**: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

3. Find the following for path C in **Figure 2.71**: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

4. Find the following for path D in **Figure 2.71**: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2.3 Time, Velocity, and Speed

5. (a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?

6. A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

7. The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

8. Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

9. On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

10. Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84\times10^6\text{ m}$ (1%)?

11. A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0° south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

12. The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

13. Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light $(3.00\times10^8\text{ m/s})$.

14. A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

15. The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06\times10^{-10}\text{ m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20\times10^6\text{ m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

2.4 Acceleration

16. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

17. Professional Application

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of g (9.80 m/s^2) by taking its ratio to the acceleration of gravity.

18. A commuter backs her car out of her garage with an acceleration of 1.40 m/s^2. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

19. Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of g (9.80 m/s^2)?

2.5 Motion Equations for Constant Acceleration in One Dimension

20. An Olympic-class sprinter starts a race with an acceleration of 4.50 m/s^2 . (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

21. A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10\times10^4 \text{ m/s}^2$, and 1.85 ms $(1 \text{ ms} = 10^{-3} \text{ s})$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

22. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20\times10^5 \text{ m/s}^2$ for $8.10\times10^{-4} \text{ s}$. What is its muzzle velocity (that is, its final velocity)?

23. (a) A light-rail commuter train accelerates at a rate of 1.35 m/s^2 . How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of 1.65 m/s^2 . How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2 ?

24. While entering a freeway, a car accelerates from rest at a rate of 2.40 m/s^2 for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

25. At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of 2.00 m/s^2 . (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

26. Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

27. In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes $3.33\times10^{-2} \text{ s}$, calculate the distance over which the puck accelerates.

28. A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

29. Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of 0.0500 m/s^2 for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of 0.550 m/s^2 , how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

30. A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

31. A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of 0.350 m/s^2 , how far will it travel before becoming airborne? (b) How long does this take?

32. Professional Application:

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in m/s^2 and in multiples of $g \left(g = 9.80 \text{ m/s}^2\right)$. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g ?

33. An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

34. In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

35. Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

36. An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of 0.150 m/s^2 as it goes through. The station is 210 m long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

37. Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in **Example 2.10** and **Example 2.11**. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

38. A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of 0.500 m/s^2 for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

39. In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

40. (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt “coasted” across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

2.7 Falling Objects

Assume air resistance is negligible unless otherwise stated.

41. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.

42. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

43. A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

44. A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

45. A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

46. A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

47. (a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

48. A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

49. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

50. A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

51. Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

52. An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

53. There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

54. A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 1.30 s to go past the window. What was the ball's initial velocity?

55. Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

56. A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00 \times 10^{-5} \text{ s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

57. A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

58. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50\times10^{-3}\text{ s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

2.8 Graphical Analysis of One Dimensional Motion

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

59. (a) By taking the slope of the curve in Figure 2.72, verify that the velocity of the jet car is 115 m/s at $t = 20\text{ s}$. (b) By taking the slope of the curve at any point in Figure 2.73, verify that the jet car's acceleration is 5.0 m/s^2.

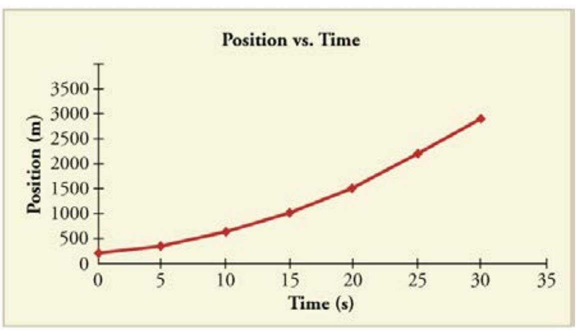


Figure 2.72

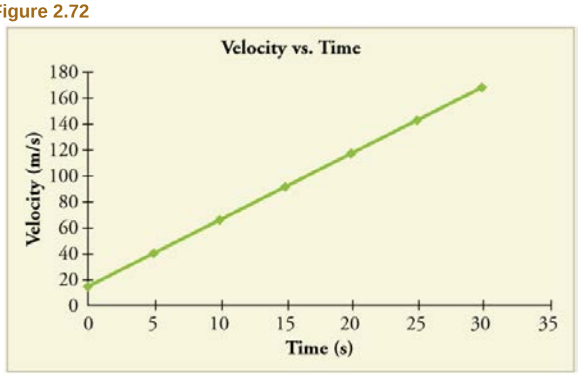


Figure 2.73

60. Using approximate values, calculate the slope of the curve in Figure 2.74 to verify that the velocity at $t = 10.0\text{ s}$ is 0.208 m/s. Assume all values are known to 3 significant figures.

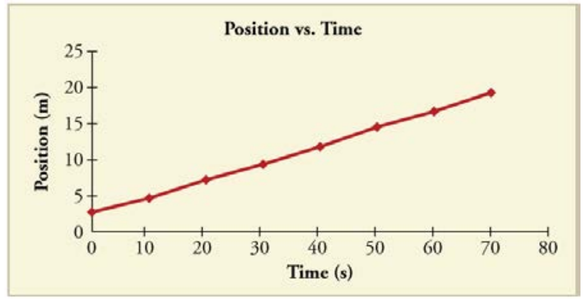


Figure 2.74

61. Using approximate values, calculate the slope of the curve in Figure 2.74 to verify that the velocity at $t = 30.0\text{ s}$ is 0.238 m/s. Assume all values are known to 3 significant figures.

62. By taking the slope of the curve in Figure 2.75, verify that the acceleration is 3.2 m/s^2 at $t = 10\text{ s}$.

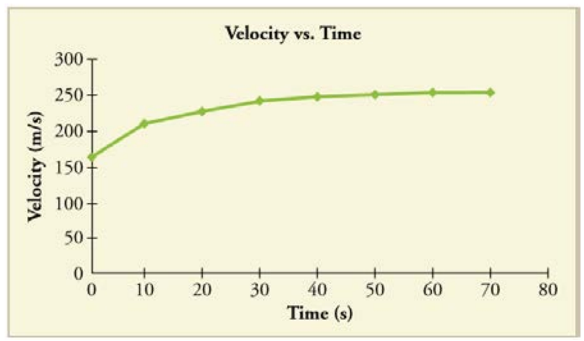


Figure 2.75

63. Construct the displacement graph for the subway shuttle train as shown in Figure 2.30(a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

64. (a) Take the slope of the curve in Figure 2.76 to find the jogger's velocity at $t = 2.5\text{ s}$. (b) Repeat at 7.5 s. These values must be consistent with the graph in Figure 2.77.

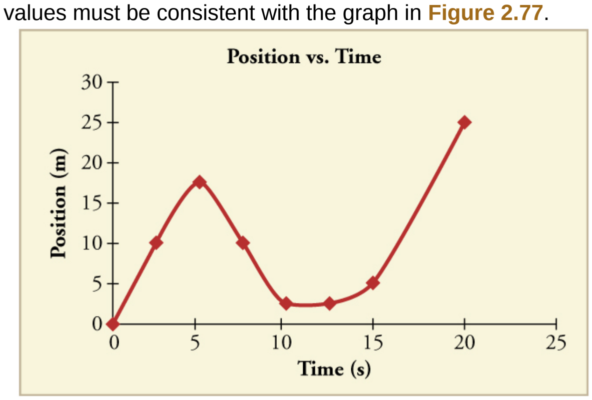


Figure 2.76

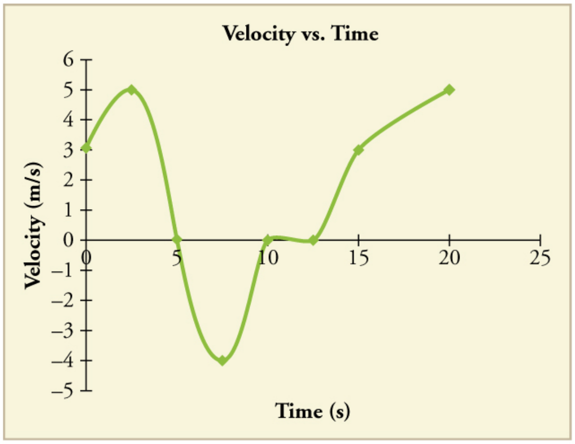


Figure 2.77

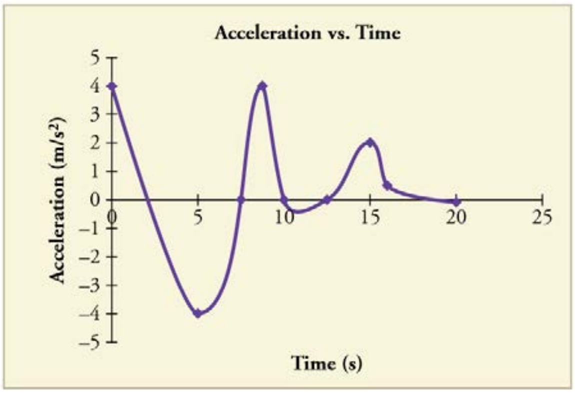


Figure 2.78

65. A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See Figure 2.79). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5\text{ s}$? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?

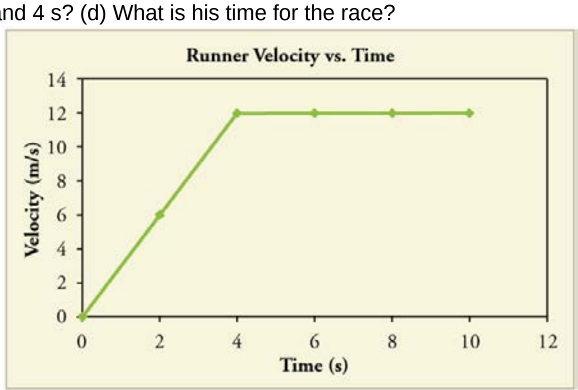


Figure 2.79

66. Figure 2.80 shows the displacement graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.

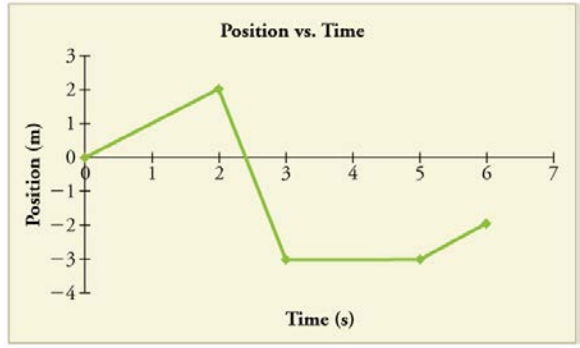


Figure 2.80

Test Prep for AP® Courses

2.1 Displacement

1. Which of the following statements comparing position, distance, and displacement is correct?

a. An object may record a distance of zero while recording a non-zero displacement.
b. An object may record a non-zero distance while recording a displacement of zero.
c. An object may record a non-zero distance while maintaining a position of zero.

d. An object may record a non-zero displacement while maintaining a position of zero.

2.2 Vectors, Scalars, and Coordinate Systems

2. A student is trying to determine the acceleration of a feather as she drops it to the ground. If the student is looking to achieve a positive velocity and positive acceleration, what is the *most sensible* way to set up her coordinate system?

a. Her hand should be a coordinate of zero and the upward direction should be considered positive.
b. Her hand should be a coordinate of zero and the downward direction should be considered positive.
c. The floor should be a coordinate of zero and the upward direction should be considered positive.
d. The floor should be a coordinate of zero and the downward direction should be considered positive.

2.3 Time, Velocity, and Speed

3. A group of students has two carts, *A* and *B*, with wheels that turn with negligible friction. The two carts travel along a straight horizontal track and eventually collide. Before the collision, cart *A* travels to the right and cart *B* is initially at rest. After the collision, the carts stick together.

a. Describe an experimental procedure to determine the velocities of the carts before and after the collision, including all the additional equipment you would need. You may include a labeled diagram of your setup to help in your description. Indicate what measurements you would take and how you would take them. Include enough detail so that another student could carry out your procedure.
b. There will be sources of error in the measurements taken in the experiment both before and after the collision. Which velocity will be more greatly affected by this error: the velocity prior to the collision or the velocity after the collision? Or will both sets of data be affected equally? Justify your answer.

2.4 Acceleration

4.

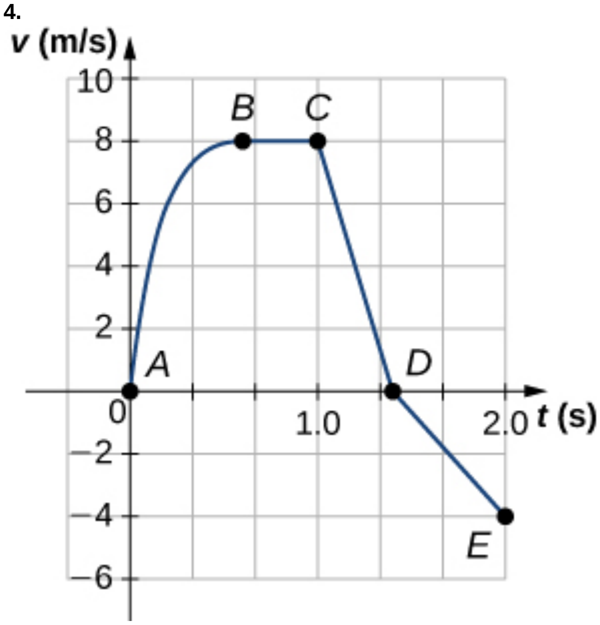


Figure 2.81 Graph showing Velocity vs. Time of a cart. A cart is constrained to move along a straight line. A varying net force along the direction of motion is exerted on the cart. The cart's velocity *v* as a function of time *t* is shown in the graph. The five labeled points divide the graph into four sections.

Which of the following correctly ranks the magnitude of the average acceleration of the cart during the four sections of the graph?

a. $a_{CD} > a_{AB} > a_{BC} > a_{DE}$
b. $a_{BC} > a_{AB} > a_{CD} > a_{DE}$
c. $a_{AB} > a_{BC} > a_{DE} > a_{CD}$
d. $a_{CD} > a_{AB} > a_{DE} > a_{BC}$

5. Push a book across a table and observe it slow to a stop.

Draw graphs showing the book's position vs. time and velocity vs. time if the direction of its motion is considered positive.

Draw graphs showing the book's position vs. time and velocity vs. time if the direction of its motion is considered negative.

2.5 Motion Equations for Constant Acceleration in One Dimension

6. A group of students is attempting to determine the average acceleration of a marble released from the top of a long ramp. Below is a set of data representing the marble's position with respect to time.

Position (cm)	Time (s)
0.0	0.0
0.3	0.5
1.25	1.0
2.8	1.5
5.0	2.0
7.75	2.5
11.3	3.0

Use the data table above to construct a graph determining the acceleration of the marble. Select a set of data points from the table and plot those points on the graph. Fill in the blank column in the table for any quantities you graph other than the given data. Label the axes and indicate the scale for each. Draw a best-fit line or curve through your data points.

Using the best-fit line, determine the value of the marble's acceleration.

2.7 Falling Objects

7. Observing a spacecraft land on a distant asteroid, scientists notice that the craft is falling at a rate of 5 m/s. When it is 100 m closer to the surface of the asteroid, the craft reports a velocity of 8 m/s. According to their data, what is the approximate gravitational acceleration on this asteroid?

a. 0 m/s^2
b. 0.03 m/s^2
c. 0.20 m/s^2
d. 0.65 m/s^2
e. 33 m/s^2

3 TWO-DIMENSIONAL KINEMATICS

Figure 3.1 Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain's Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedia Commons)

Chapter Outline

Connection for AP® Courses

Most instances of motion in everyday life involve changes in displacement and velocity that occur in more than one direction. For example, when you take a long road trip, you drive on different roads in different directions for different amounts of time at different speeds. How can these motions all be combined to determine information about the trip such as the total displacement and average velocity? If you kick a ball from ground level at some angle above the horizontal, how can you describe its motion? To what maximum height does the object rise above the ground? How long is the object in the air? How much horizontal distance is covered before the ball lands? To answer questions such as these, we need to describe motion in two dimensions.

Examining two-dimensional motion requires an understanding of both the scalar and the vector quantities associated with the motion. You will learn how to combine vectors to incorporate both the magnitude and direction of vectors into your analysis. You will learn strategies for simplifying the calculations involved by choosing the appropriate reference frame and by treating each dimension of the motion separately as a one-dimensional problem, but you will also see that the motion itself occurs in the same way regardless of your chosen reference frame (Essential Knowledge 3.A.1).

This chapter lays a necessary foundation for examining interactions of objects described by forces (Big Idea 3). Changes in direction result from acceleration, which necessitates force on an object. In this chapter, you will concentrate on describing motion that involves changes in direction. In later chapters, you will apply this understanding as you learn about how forces cause these motions (Enduring Understanding 3.A). The concepts in this chapter support:

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A All forces share certain common characteristics when considered by observers in inertial reference frames.

Essential Knowledge 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.

3.1 Kinematics in Two Dimensions: An Introduction

Learning Objectives

By the end of this section, you will be able to:

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.2** The student is able to design an experimental investigation of the motion of an object. **(S.P. 4.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

Figure 3.2 Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in **Figure 3.3**.

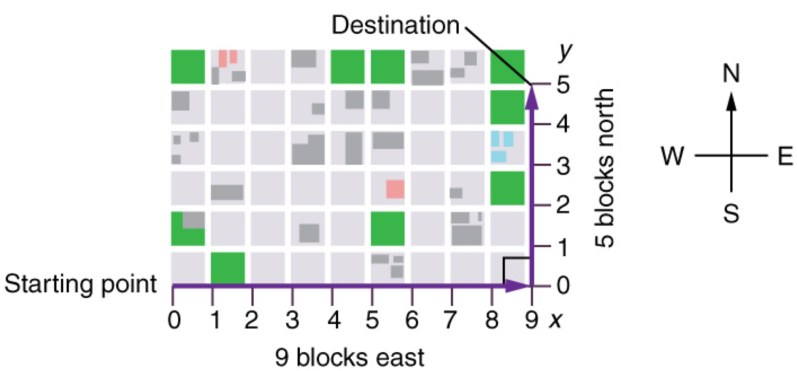


Figure 3.3 A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used to find the straight-line distance.

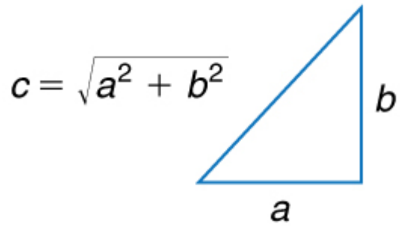


Figure 3.4 The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for c : $c = \sqrt{a^2 + b^2}$.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9\text{ blocks})^2 + (5\text{ blocks})^2} = 10.3\text{ blocks}$, considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that "9" and "5" have only one significant digit, they are discrete numbers. In this case "9 blocks" is the same as "9.0 or 9.00 blocks." We have decided to use three significant figures in the answer in order to show the result more precisely.)

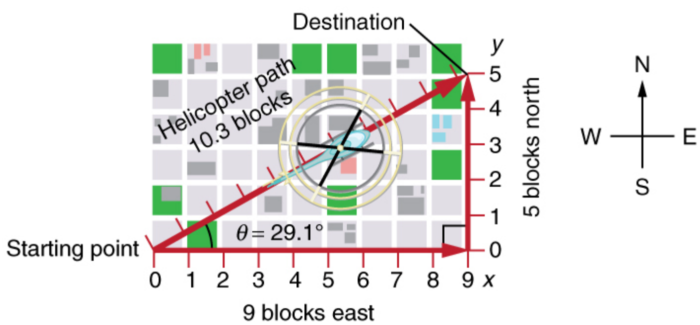


Figure 3.5 The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in **Figure 3.5** is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in **Figure 3.3** and **Figure 3.5**. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in **Figure 3.5**. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in **Vector Addition and Subtraction: Graphical Methods** and **Vector Addition and Subtraction: Analytical Methods**.)

The Independence of Perpendicular Motions

The person taking the path shown in **Figure 3.5** walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

Independence of Motion

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

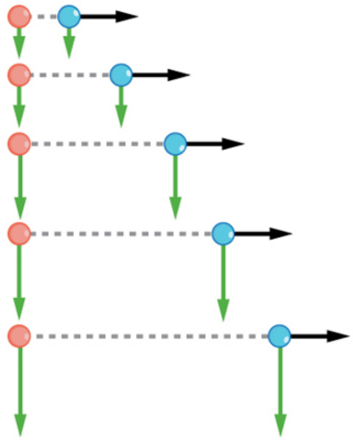


Figure 3.6 This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

Applying the Science Practices: Independence of Horizontal and Vertical Motion or Maximum Height and Flight Time

Choose one of the following experiments to design:

Design an experiment to confirm what is shown in **Figure 3.6**, that the vertical motion of the two balls is independent of the horizontal motion. As you think about your experiment, consider the following questions:

- How will you measure the horizontal and vertical positions of each ball over time? What equipment will this require?
- How will you measure the time interval between each of your position measurements? What equipment will this require?
- If you were to create separate graphs of the horizontal velocity for each ball versus time, what do you predict it would look like? Explain.
- If you were to compare graphs of the vertical velocity for each ball versus time, what do you predict it would look like? Explain.
- If there is a significant amount of air resistance, how will that affect each of your graphs?

Design a two-dimensional ballistic motion experiment that demonstrates the relationship between the maximum height reached by an object and the object's time of flight. As you think about your experiment, consider the following questions:

- How will you measure the maximum height reached by your object?
- How can you take advantage of the symmetry of an object in ballistic motion launched from ground level, reaching maximum height, and returning to ground level?
- Will it make a difference if your object has no horizontal component to its velocity? Explain.
- Will you need to measure the time at multiple different positions? Why or why not?
- Predict what a graph of travel time versus maximum height will look like. Will it be linear? Parabolic? Horizontal? Explain the shape of your predicted graph qualitatively or quantitatively.
- If there is a significant amount of air resistance, how will that affect your measurements and your results?

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in **Vector Addition and Subtraction: Graphical Methods** and **Vector Addition and Subtraction: Analytical Methods**. We will find such techniques to be useful in many areas of physics.

PhET Explorations: Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

PhET Interactive Simulation

Figure 3.7 Ladybug Motion 2D (http://cnx.org/content/m54779/1.2/ladybug-motion-2d_en.jar)

3.2 Vector Addition and Subtraction: Graphical Methods

Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

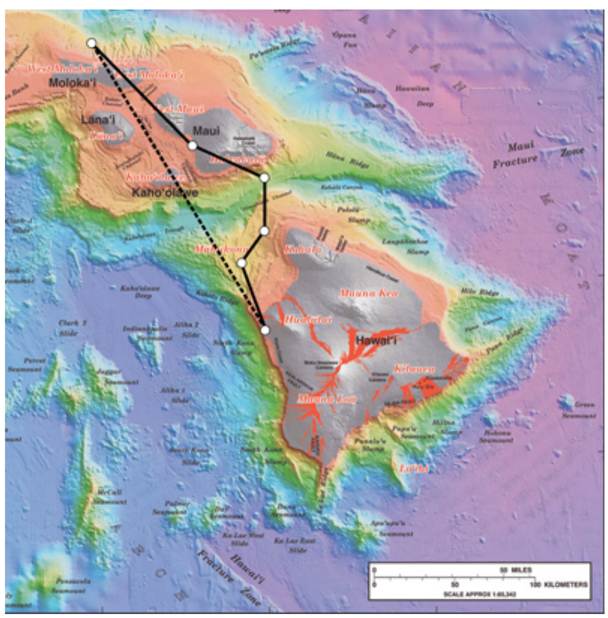


Figure 3.8 Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

Vectors in Two Dimensions

A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

Figure 3.9 shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in **Kinematics in Two Dimensions: An Introduction**. We shall use the notation that a boldface symbol, such as $\mathbf{D}$, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

Vectors in this Text

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector $\mathbf{F}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.

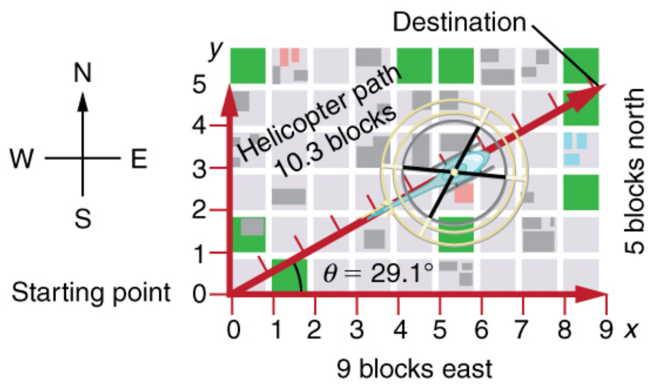


Figure 3.9 A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle $29.1°$ north of east.

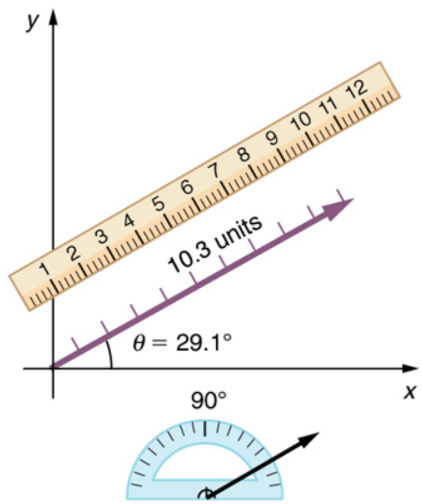


Figure 3.10 To describe the resultant vector for the person walking in a city considered in **Figure 3.9** graphically, draw an arrow to represent the total displacement vector $\mathbf{D}$. Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is $29.1°$ north of east.

Vector Addition: Head-to-Tail Method

The **head-to-tail method** is a graphical way to add vectors, described in **Figure 3.11** below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.

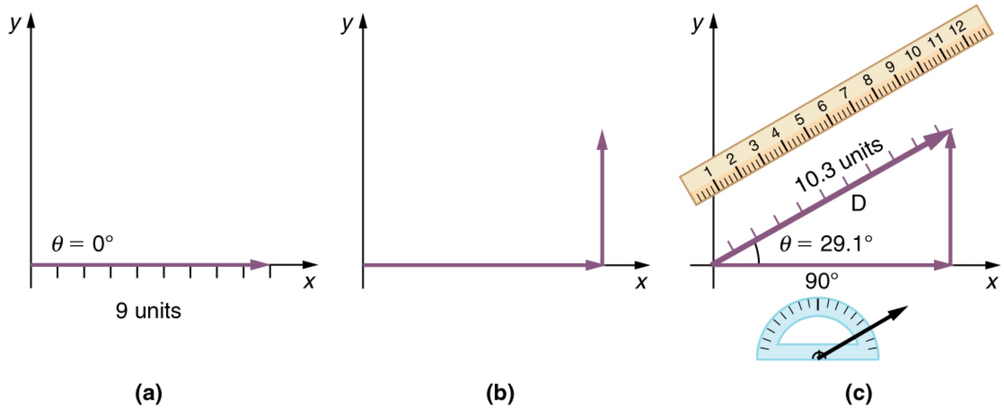


Figure 3.11 **Head-to-Tail Method:** The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in Figure 3.9. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or **resultant vector** $\mathbf{D}$. The length of the arrow $\mathbf{D}$ is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1° .

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor*.

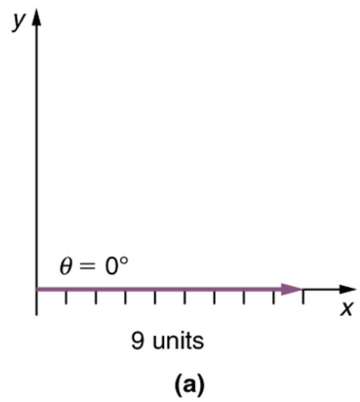


Figure 3.12

Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector*.

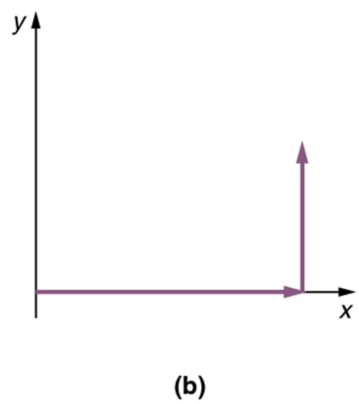


Figure 3.13

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail*.

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.

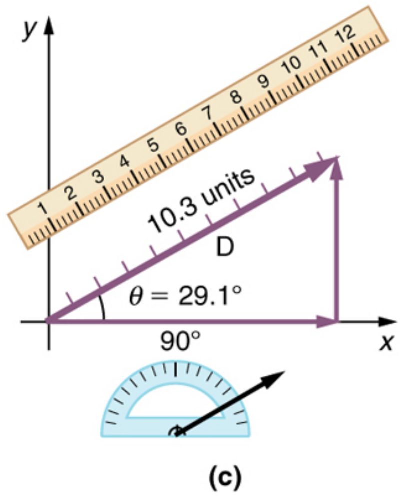


Figure 3.14

Step 5. To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Example 3.1 Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction $49.0°$ north of east. Then, she walks 23.0 m heading $15.0°$ north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

Strategy

Represent each displacement vector graphically with an arrow, labeling the first $\mathbf{A}$, the second $\mathbf{B}$, and the third $\mathbf{C}$, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted $\mathbf{R}$.

Solution

(1) Draw the three displacement vectors.

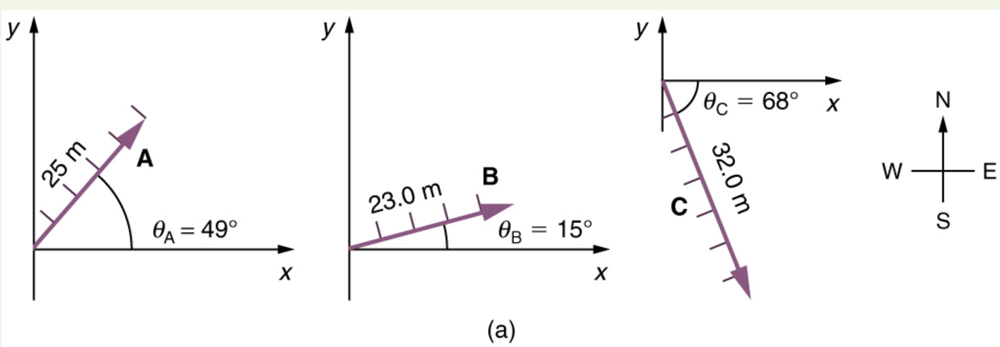


Figure 3.15

(2) Place the vectors head to tail retaining both their initial magnitude and direction.

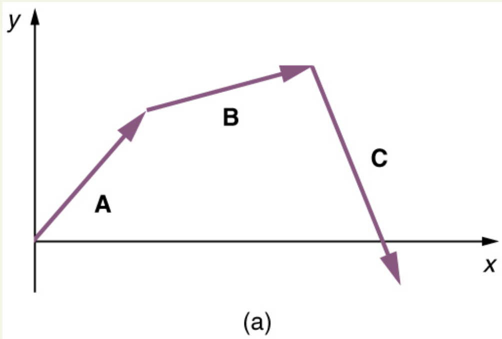


Figure 3.16

(3) Draw the resultant vector, $\mathbf{R}$.

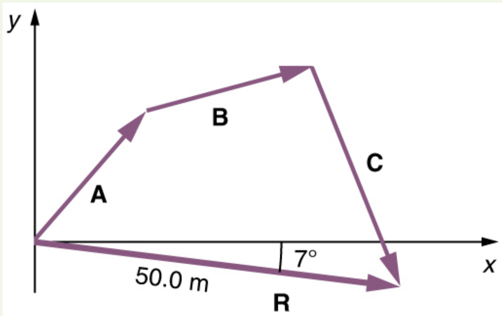


Figure 3.17

(4) Use a ruler to measure the magnitude of $\mathbf{R}$, and a protractor to measure the direction of $\mathbf{R}$. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.

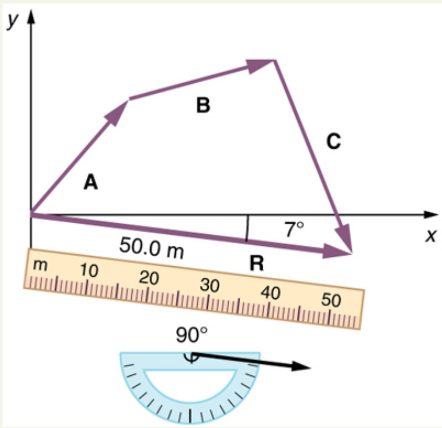


Figure 3.18

In this case, the total displacement $\mathbf{R}$ is seen to have a magnitude of 50.0 m and to lie in a direction 7.0° south of east. By using its magnitude and direction, this vector can be expressed as $R = 50.0 \text{ m}$ and $\theta = 7.0^\circ$ south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in **Figure 3.19** and we will still get the same solution.

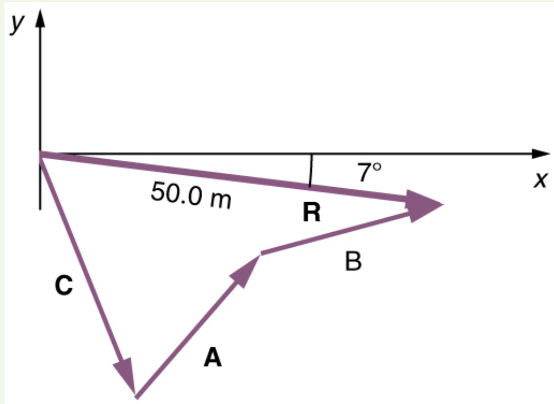


Figure 3.19

Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}. \tag{3.1}$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add $2 + 3$ or $3 + 2$, for example).

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract $\mathbf{B}$ from $\mathbf{A}$, written $\mathbf{A} - \mathbf{B}$, we must first define what we mean by subtraction. The *negative* of a vector $\mathbf{B}$ is defined to be $-\mathbf{B}$; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in **Figure 3.20**. In other words, $\mathbf{B}$ has the same length as $-\mathbf{B}$, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.

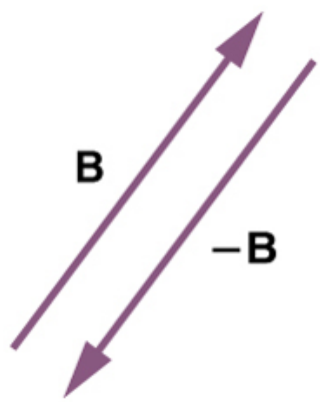


Figure 3.20 The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So $\mathbf{B}$ is the negative of $-\mathbf{B}$; it has the same length but opposite direction.

The *subtraction* of vector $\mathbf{B}$ from vector $\mathbf{A}$ is then simply defined to be the addition of $-\mathbf{B}$ to $\mathbf{A}$. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}). \tag{3.2}$$

This is analogous to the subtraction of scalars (where, for example, $5 - 2 = 5 + (-2)$). Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

Example 3.2 Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.

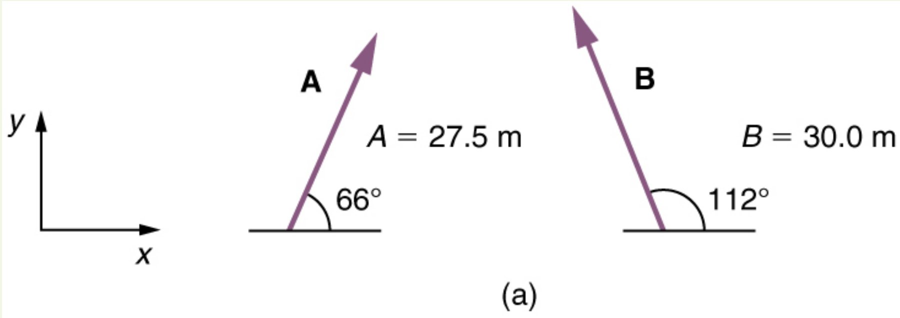


Figure 3.21

Strategy

We can represent the first leg of the trip with a vector $\mathbf{A}$, and the second leg of the trip with a vector $\mathbf{B}$. The dock is located at a location $\mathbf{A} + \mathbf{B}$. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction $180° - 112° = 68°$ south of east. We represent this as $-\mathbf{B}$, as shown below. The vector $-\mathbf{B}$ has the same magnitude as $\mathbf{B}$ but is in the opposite direction. Thus, she will end up at a location $\mathbf{A} + (-\mathbf{B})$, or $\mathbf{A} - \mathbf{B}$.

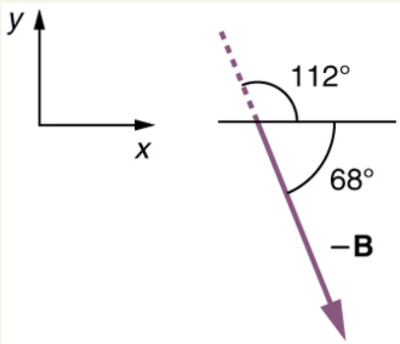


Figure 3.22

We will perform vector addition to compare the location of the dock, $\mathbf{A} + \mathbf{B}$, with the location at which the woman mistakenly arrives, $\mathbf{A} + (-\mathbf{B})$.

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors $\mathbf{A}$ and $-\mathbf{B}$.

(2) Place the vectors head to tail.

(3) Draw the resultant vector $\mathbf{R}$.

(4) Use a ruler and protractor to measure the magnitude and direction of $\mathbf{R}$.

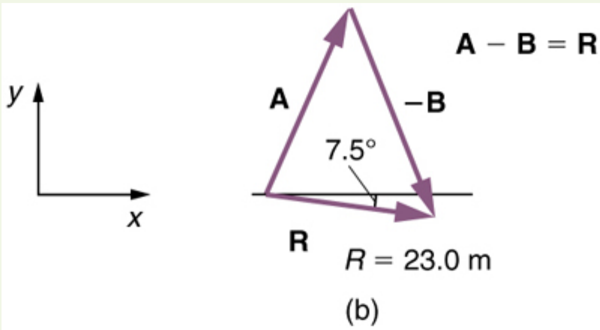


Figure 3.23

In this case, $R = 23.0 \text{ m}$ and $\theta = 7.5°$ south of east.

(5) To determine the location of the dock, we repeat this method to add vectors $\mathbf{A}$ and $\mathbf{B}$. We obtain the resultant vector $\mathbf{R}'$:

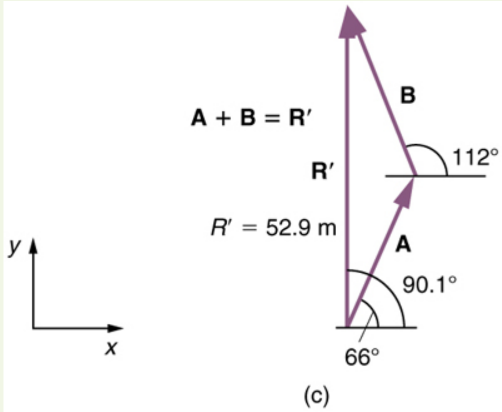


Figure 3.24

In this case $R = 52.9\text{ m}$ and $\theta = 90.1^\circ$ north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk $3 \times 27.5\text{ m}$, or 82.5 m, in a direction 66.0° north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector $\mathbf{A}$ is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of $c\ A$,
- if c is positive, the direction of the vector does not change,
- if c is negative, the direction is reversed.

In our case, $c = 3$ and $A = 27.5\text{ m}$. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x- and y*-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction 29.0° north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in **Projectile Motion**, and much more when we cover **forces** in **Dynamics: Newton's Laws of Motion**. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in **Vector Addition and Subtraction: Analytical Methods** are ideal for finding vector components.

PhET Explorations: Maze Game

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

PhET Interactive Simulation

Figure 3.25 Maze Game (http://cnx.org/content/m54781/1.2/maze-game_en.jar)

3.3 Vector Addition and Subtraction: Analytical Methods

Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like $\mathbf{A}$ in Figure 3.26, we may wish to find which two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it.

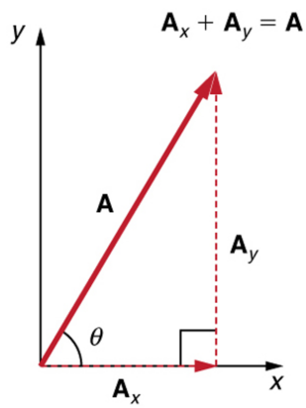


Figure 3.26 The vector $\mathbf{A}$, with its tail at the origin of an *x*, *y*-coordinate system, is shown together with its *x*- and *y*-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of $\mathbf{A}$ along the *x*- and *y*-axes. The three vectors $\mathbf{A}$, $\mathbf{A}_x$, and $\mathbf{A}_y$ form a right triangle:

$$\mathbf{A}_x + \mathbf{A}_\mathbf{y} = \mathbf{A}. \tag{3.3}$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $\mathbf{A}_x = 3\text{ m}$ east, $\mathbf{A}_y = 4\text{ m}$ north, and $\mathbf{A} = 5\text{ m}$ north-east, then it is true that the vectors $\mathbf{A}_x + \mathbf{A}_\mathbf{y} = \mathbf{A}$. However, it is *not* true that the sum of the magnitudes of the vectors is also equal. That is,

$$3\text{ m} + 4\text{ m} \neq 5\text{ m} \tag{3.4}$$

Thus,

$$A_x + A_y \neq A \tag{3.5}$$

If the vector $\mathbf{A}$ is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its *x*- and *y*-components, we use the following relationships for a right triangle.

$$A_x = A \cos \theta \tag{3.6}$$

and

$$A_y = A \sin \theta. \tag{3.7}$$

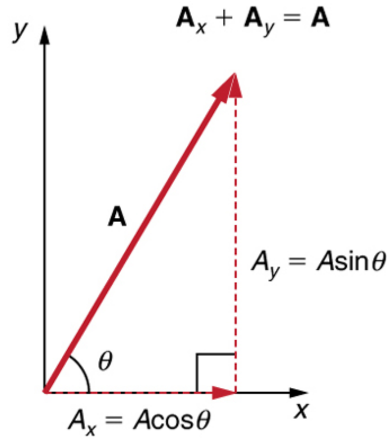


Figure 3.27 The magnitudes of the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x = A \cos \theta$ and $A_y = A \sin \theta$.

Suppose, for example, that $\mathbf{A}$ is the vector representing the total displacement of the person walking in a city considered in **Kinematics in Two Dimensions: An Introduction** and **Vector Addition and Subtraction: Graphical Methods**.

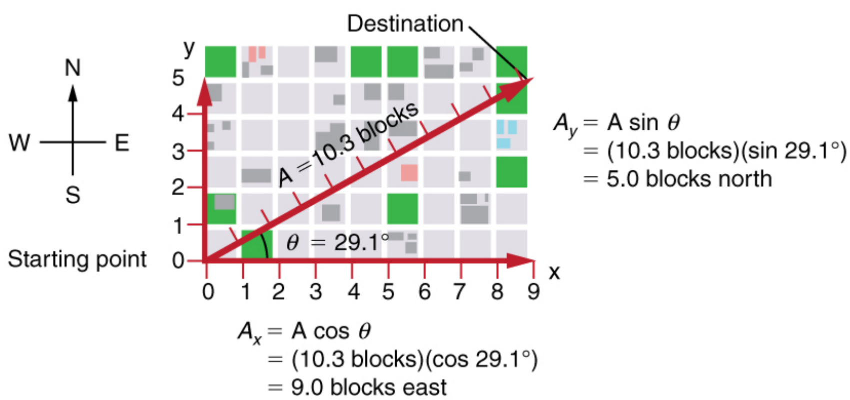


Figure 3.28 We can use the relationships $A_x = A \cos \theta$ and $A_y = A \sin \theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then $A = 10.3$ blocks and $\theta = 29.1^\circ$, so that

$$A_x = A \cos \theta = (10.3 \text{ blocks})(\cos 29.1^\circ) = 9.0 \text{ blocks} \tag{3.8}$$

$$A_y = A \sin \theta = (10.3 \text{ blocks})(\sin 29.1^\circ) = 5.0 \text{ blocks}. \tag{3.9}$$

Calculating a Resultant Vector

If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then $\mathbf{A}$ can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, we use the following relationships:

$$A = \sqrt{A_x{}^2 + A_y{}^2} \tag{3.10}$$

$$\theta = \tan^{-1}(A_y / A_x). \tag{3.11}$$

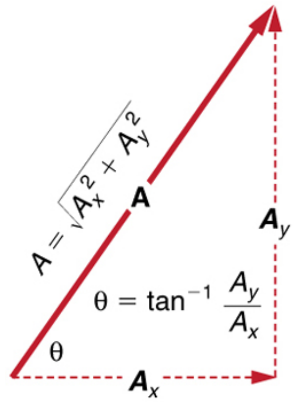


Figure 3.29 The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined.

Note that the equation $A = \sqrt{A_x^2 + A_y^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if A_x and A_y are 9 and 5 blocks, respectively, then $A = \sqrt{9^2 + 5^2} = 10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = \tan^{-1}(5/9) = 29.1^\circ$, as before.

Determining Vectors and Vector Components with Analytical Methods

Equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y / A_x)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider **Figure 3.30**, in which the vectors $\mathbf{A}$ and $\mathbf{B}$ are added to produce the resultant $\mathbf{R}$.

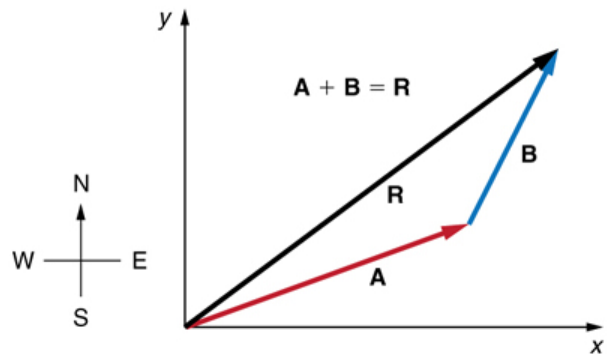


Figure 3.30 Vectors $\mathbf{A}$ and $\mathbf{B}$ are two legs of a walk, and $\mathbf{R}$ is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

If $\mathbf{A}$ and $\mathbf{B}$ represent two legs of a walk (two displacements), then $\mathbf{R}$ is the total displacement. The person taking the walk ends up at the tip of $\mathbf{R}$. There are many ways to arrive at the same point. In particular, the person could have walked first in the *x*-direction and then in the *y*-direction. Those paths are the *x*- and *y*-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $A = \sqrt{A_x{}^2 + A_y{}^2}$ and $\theta = \tan^{-1}(A_y / A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.* Use the equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to find the components. In **Figure 3.31**, these components are A_x, A_y, B_x, and B_y. The angles that vectors $\mathbf{A}$ and $\mathbf{B}$ make with the *x*-axis are θ_A and θ_B, respectively.

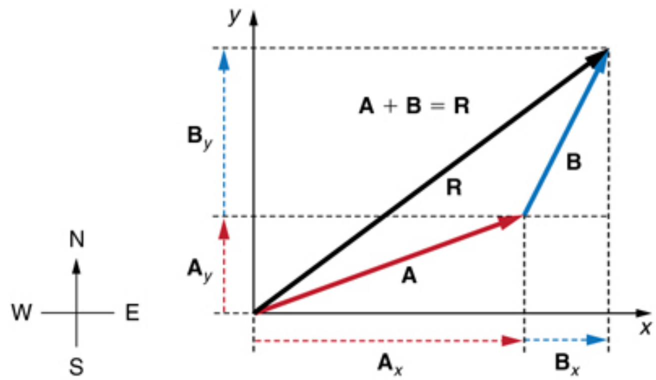


Figure 3.31 To add vectors $\mathbf{A}$ and $\mathbf{B}$, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$ and $\mathbf{B}_y$ shown in the image.

Step 2. *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis.* That is, as shown in **Figure 3.32**,

$$R_x = A_x + B_x \tag{3.12}$$

and

$$R_y = A_y + B_y. \tag{3.13}$$

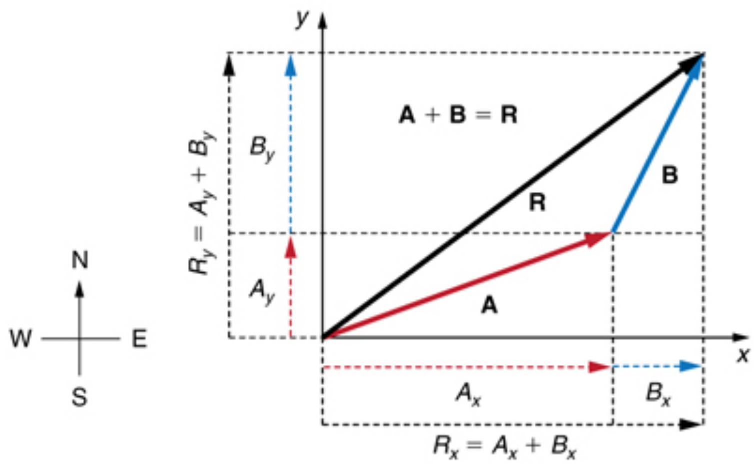


Figure 3.32 The magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude R_y of the resultant vector in the vertical direction.

Components along the same axis, say the *x*-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the *y*-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of $\mathbf{R}$ are known, its magnitude and direction can be found.

Step 3. *To get the magnitude R of the resultant, use the Pythagorean theorem:*

$$R = \sqrt{R_x^2 + R_y^2}. \tag{3.14}$$

Step 4. *To get the direction of the resultant:*

$$\theta = \tan^{-1}(R_y / R_x). \tag{3.15}$$

The following example illustrates this technique for adding vectors using perpendicular components.

Example 3.3 Adding Vectors Using Analytical Methods

Add the vector $\mathbf{A}$ to the vector $\mathbf{B}$ shown in Figure 3.33, using perpendicular components along the *x*- and *y*-axes. The *x*- and *y*-axes are along the east–west and north–south directions, respectively. Vector $\mathbf{A}$ represents the first leg of a walk in which a person walks 53.0 m in a direction 20.0^o north of east. Vector $\mathbf{B}$ represents the second leg, a displacement of 34.0 m in a direction 63.0^o north of east.

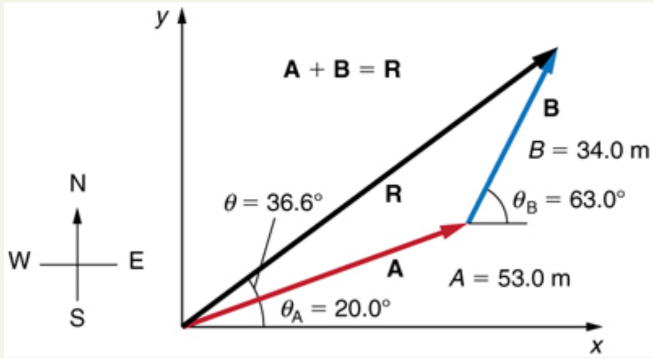


Figure 3.33 Vector $\mathbf{A}$ has magnitude 53.0 m and direction $20.0\ ^\text{o}$ north of the *x*-axis. Vector $\mathbf{B}$ has magnitude 34.0 m and direction 63.0^o north of the *x*-axis. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

Strategy

The components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes. Note that $A = 53.0\text{ m}$, $\theta_\text{A} = 20.0^\text{o}$, $B = 34.0\text{ m}$, and $\theta_\text{B} = 63.0^\text{o}$. We find the *x*-components by using $A_x = A\cos\theta$, which gives

$$\begin{aligned} A_x &= A\cos\theta_\text{A} = (53.0\text{ m})(\cos 20.0^\text{o}) \\ &= (53.0\text{ m})(0.940) = 49.8\text{ m} \end{aligned} \tag{3.16}$$

and

$$\begin{aligned} B_x &= B\cos\theta_\text{B} = (34.0\text{ m})(\cos 63.0^\text{o}) \\ &= (34.0\text{ m})(0.454) = 15.4\text{ m}. \end{aligned} \tag{3.17}$$

Similarly, the *y*-components are found using $A_y = A\sin\theta_\text{A}$:

$$\begin{aligned} A_y &= A\sin\theta_\text{A} = (53.0\text{ m})(\sin 20.0^\text{o}) \\ &= (53.0\text{ m})(0.342) = 18.1\text{ m} \end{aligned} \tag{3.18}$$

and

$$\begin{aligned} B_y &= B\sin\theta_\text{B} = (34.0\text{ m})(\sin 63.0\ ^\text{o}\ ) \\ &= (34.0\text{ m})(0.891) = 30.3\text{ m}. \end{aligned} \tag{3.19}$$

The *x*- and *y*-components of the resultant are thus

$$R_x = A_x + B_x = 49.8\text{ m} + 15.4\text{ m} = 65.2\text{ m} \tag{3.20}$$

and

$$R_y = A_y + B_y = 18.1\text{ m}{+}30.3\text{ m} = 48.4\text{ m}. \tag{3.21}$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2\text{ m}} \tag{3.22}$$

so that

$$R = 81.2\text{ m}. \tag{3.23}$$

Finally, we find the direction of the resultant:

$$\theta = \tan^{-1}(R_y / R_x) = +\tan^{-1}(48.4 / 65.2). \tag{3.24}$$

Thus,

$$\theta = \tan^{-1}(0.742) = 36.6^\circ . \tag{3.25}$$

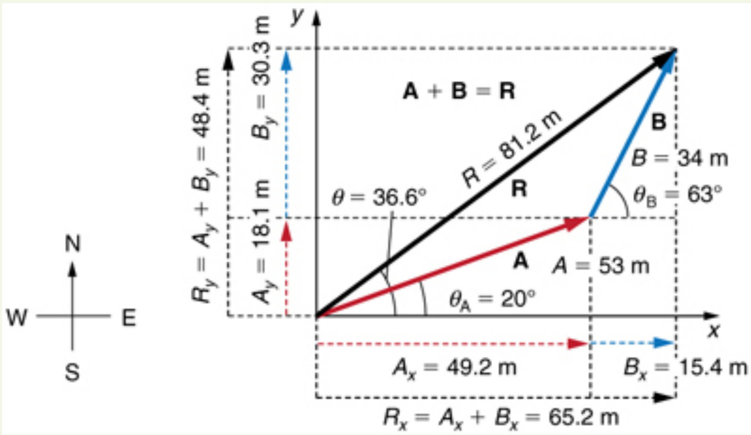


Figure 3.34 Using analytical methods, we see that the magnitude of $\mathbf{R}$ is 81.2 m and its direction is 36.6° north of east.

Discussion

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $\mathbf{A} - \mathbf{B} \equiv \mathbf{A} + (-\mathbf{B})$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition*. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The *x*- and *y*-components of the resultant $\mathbf{A} - \mathbf{B} = \mathbf{R}$ are thus

$$R_x = A_x + (-B_x) \tag{3.26}$$

and

$$R_y = A_y + \left(-B_y\right) \tag{3.27}$$

and the rest of the method outlined above is identical to that for addition. (See **Figure 3.35**.)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, **Projectile Motion**, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.

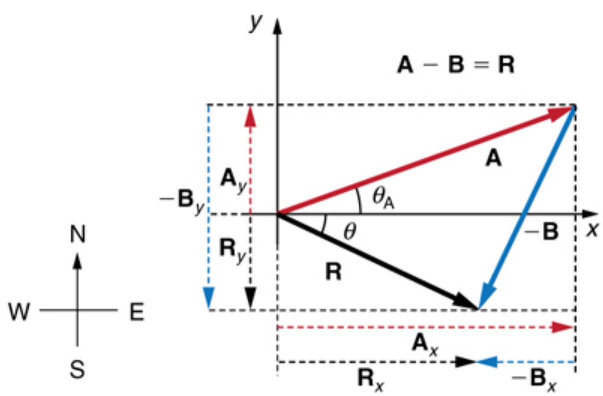


Figure 3.35 The subtraction of the two vectors shown in **Figure 3.30**. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The method of subtraction is the same as that for addition.

PhET Explorations: Vector Addition

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

PhET Interactive Simulation

Figure 3.36 Vector Addition (http://cnx.org/content/m54783/1.2/vector-addition_en.jar)

3.4 Projectile Motion

Learning Objectives

By the end of this section, you will be able to:

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

The information presented in this section supports the following AP® learning objectives:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

Projectile motion is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects, as covered in **Problem-Solving Basics for One-Dimensional Kinematics**, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

The most important fact to remember here is that *motions along perpendicular axes are independent* and thus can be analyzed separately. This fact was discussed in **Kinematics in Two Dimensions: An Introduction**, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. **Figure 3.37** illustrates the notation for displacement, where $\mathbf{s}$ is defined to be the total displacement and $\mathbf{x}$ and $\mathbf{y}$ are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are *s*, *x*, and *y*. (Note that in the last section we used the notation $\mathbf{A}$ to represent a vector with components $\mathbf{A}_x$ and $\mathbf{A}_y$. If we continued this format, we would call displacement $\mathbf{s}$ with components $\mathbf{s}_x$ and $\mathbf{s}_y$. However, to simplify the notation, we will simply represent the component vectors as $\mathbf{x}$ and $\mathbf{y}$.)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the *x*- and *y*-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $a_y = -g = -9.80\text{ m/s}^2$. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical, $a_x = 0$. Both accelerations are constant, so the kinematic equations can be used.

Review of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t \tag{3.28}$$

$$\bar{v} = \frac{v_0 + v}{2} \tag{3.29}$$

$$v = v_0 + at \tag{3.30}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{3.31}$$

$$v^2 = v_0^2 + 2a(x - x_0). \tag{3.32}$$

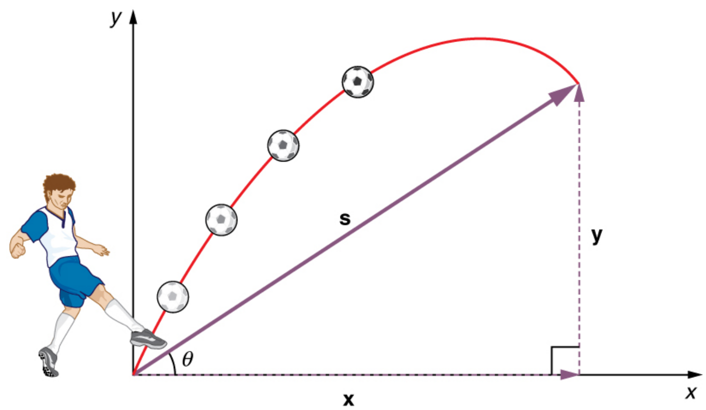


Figure 3.37 The total displacement $\mathbf{s}$ of a soccer ball at a point along its path. The vector $\mathbf{s}$ has components $\mathbf{x}$ and $\mathbf{y}$ along the horizontal and vertical axes. Its magnitude is s, and it makes an angle θ with the horizontal.

Given these assumptions, the following steps are then used to analyze projectile motion:

Step 1. *Resolve or break the motion into horizontal and vertical components along the x- and y-axes.* These axes are perpendicular, so $A_x = A\cos\theta$ and $A_y = A\sin\theta$ are used. The magnitude of the components of displacement $\mathbf{s}$ along these axes are x and $y.$ The magnitudes of the components of the velocity $\mathbf{v}$ are $v_x = v\cos\theta$ and $v_y = v\sin\theta,$ where v is the magnitude of the velocity and θ is its direction, as shown in **Figure 3.38**. Initial values are denoted with a subscript 0, as usual.

Step 2. *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

$$\text{Horizontal Motion}(a_x = 0) \tag{3.33}$$

$$x = x_0 + v_x t \tag{3.34}$$

$$v_x = v_{0x} = v_x = \text{velocity is a constant.} \tag{3.35}$$

$$\text{Vertical Motion}(\text{assuming positive is up } a_y = -g = -9.80\text{m/s}^2) \tag{3.36}$$

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t \tag{3.37}$$

$$v_y = v_{0y} - gt \tag{3.38}$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \tag{3.39}$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0). \tag{3.40}$$

Step 3. *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time t. The problem solving procedures here are the same as for one-dimensional **kinematics** and are illustrated in the solved examples below.

Step 4. *Recombine the two motions to find the total displacement* $\mathbf{s}$ *and velocity* $\mathbf{v}$. Because the *x* - and *y* -motions are perpendicular, we determine these vectors by using the techniques outlined in the **Vector Addition and Subtraction: Analytical Methods** and employing $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y / A_x)$ in the following form, where θ is the direction of the displacement $\mathbf{s}$ and θ_v is the direction of the velocity $\mathbf{v}$:

Total displacement and velocity

$$s = \sqrt{x^2 + y^2} \tag{3.41}$$

$$\theta = \tan^{-1}(y / x) \tag{3.42}$$

$$v = \sqrt{v_x^2 + v_y^2} \tag{3.43}$$

$$\theta_v = \tan^{-1}(v_y / v_x). \tag{3.44}$$

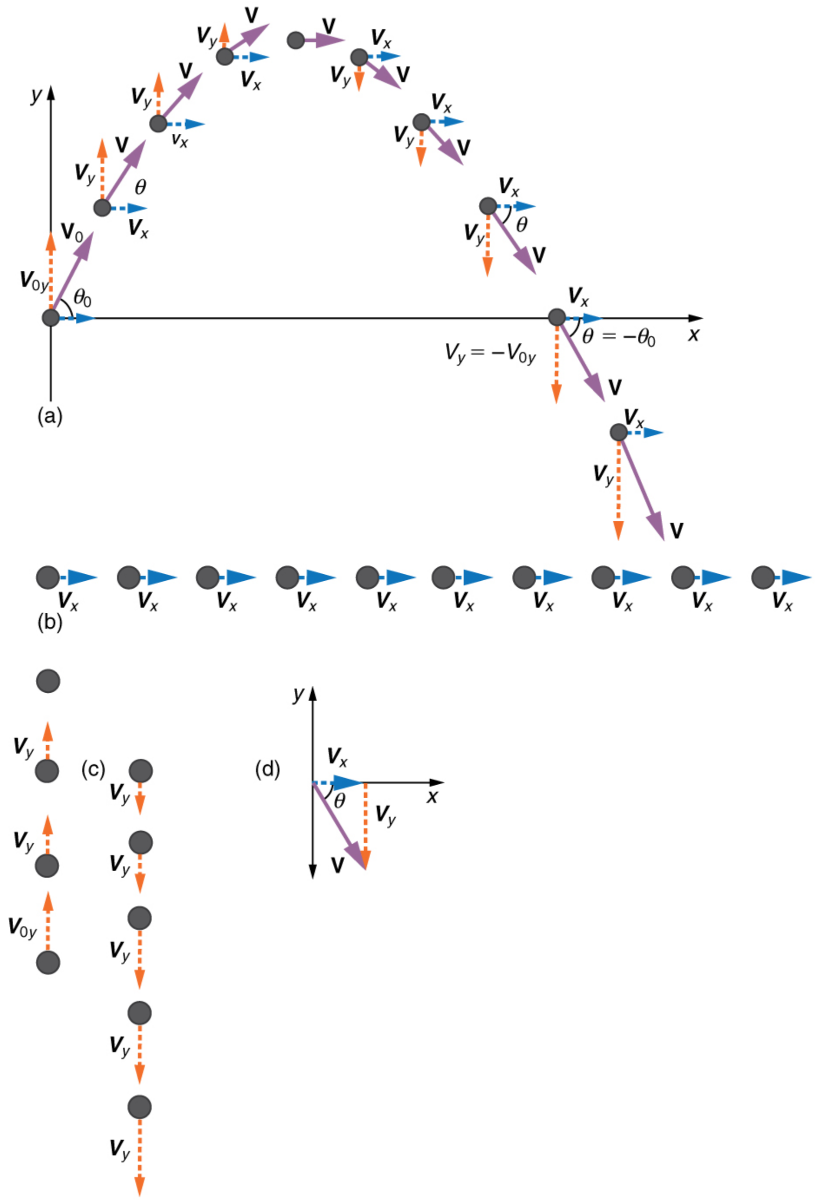


Figure 3.38 (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The *x* - and *y* -motions are recombined to give the total velocity at any given point on the trajectory.

Example 3.4 A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of $75.0°$ above the horizontal, as illustrated in **Figure 3.39**. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

Strategy

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -g$. We can then define x_0 and y_0 to be zero and solve for the desired quantities.

Solution for (a)

By "height" we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities as well as the initial position, we use the following equation to find y:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0). \tag{3.45}$$

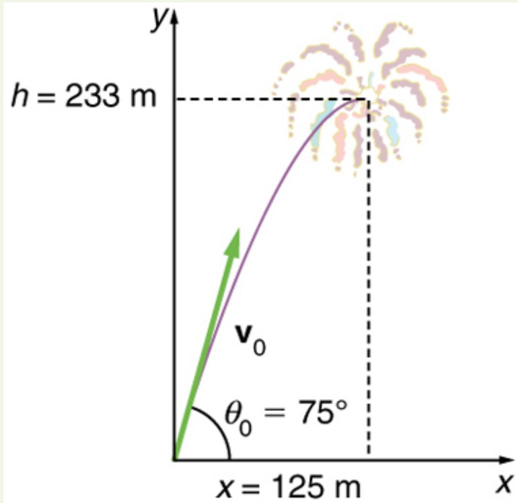


Figure 3.39 The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Because y_0 and v_y are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2gy. \tag{3.46}$$

Solving for y gives

$$y = \frac{v_{0y}^2}{2g}. \tag{3.47}$$

Now we must find v_{0y}, the component of the initial velocity in the *y*-direction. It is given by $v_{0y} = v_0 \sin\theta$, where v_{0y} is the initial velocity of 70.0 m/s, and $\theta_0 = 75.0°$ is the initial angle. Thus,

$$v_{0y} = v_0 \sin\theta_0 = (70.0\text{ m/s})(\sin 75°) = 67.6\text{ m/s}. \tag{3.48}$$

and y is

$$y = \frac{(67.6\text{ m/s})^2}{2(9.80\text{ m/s}^2)}, \tag{3.49}$$

so that

$$y = 233\text{m}. \tag{3.50}$$

Discussion for (a)

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

Solution for (b)

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. Because y_0 is zero, this equation reduces to simply

$$y = \frac{1}{2}(v_{0y} + v_y)t. \tag{3.51}$$

Note that the final vertical velocity, v_y, at the highest point is zero. Thus,

$$\begin{aligned} t &= \frac{2y}{(v_{0y} + v_y)} = \frac{2(233\text{ m})}{(67.6\text{ m/s})} \\ &= 6.90\text{ s}. \end{aligned} \tag{3.52}$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, and solving the quadratic equation for t.)

Solution for (c)

Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero:

$$x = v_x t, \tag{3.53}$$

where v_x is the *x*-component of the velocity, which is given by $v_x = v_0 \cos\theta_0$. Now,

$$v_x = v_0 \cos\theta_0 = (70.0\text{ m/s})(\cos 75.0°) = 18.1\text{ m/s}. \tag{3.54}$$

The time t for both motions is the same, and so x is

$$x = (18.1\text{ m/s})(6.90\text{ s}) = 125\text{ m}. \tag{3.55}$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for y is valid for any projectile motion where air resistance is negligible. Call the maximum height $y = h$; then,

$$h = \frac{v_{0y}^2}{2g}. \tag{3.56}$$

This equation defines the *maximum height of a projectile* and depends only on the vertical component of the initial velocity.

Defining a Coordinate System

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, g, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, g takes a positive value.

Example 3.5 Calculating Projectile Motion: Hot Rock Projectile

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle $35.0°$ above the horizontal, as shown in **Figure 3.40**. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?

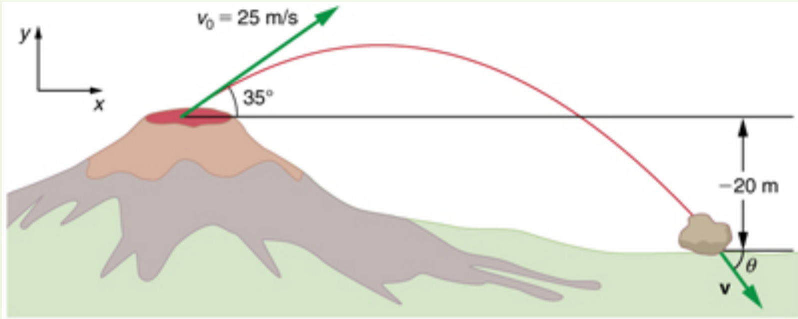


Figure 3.40 The trajectory of a rock ejected from the Kilauea volcano.

Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for t first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain v and θ_v at the final time t determined in the first part of the example.

Solution for (a)

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2. \tag{3.57}$$

If we take the initial position y_0 to be zero, then the final position is $y = -20.0\text{ m}.$ Now the initial vertical velocity is the vertical component of the initial velocity, found from $v_{0y} = v_0 \sin\theta_0$ = (25.0 m/s)($\sin 35.0°$) = 14.3 m/s . Substituting known values yields

$$-20.0\text{ m} = (14.3\text{ m/s})t - \left(4.90\text{ m/s}^2\right)t^2. \tag{3.58}$$

Rearranging terms gives a quadratic equation in t :

$$\left(4.90\text{ m/s}^2\right)t^2 - (14.3\text{ m/s})t - (20.0\text{ m}) = 0. \tag{3.59}$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are $a = 4.90$, $b = -14.3$, and $c = -20.0.$ Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{3.60}$$

This equation yields two solutions: $t = 3.96$ and $t = -1.03$. (It is left as an exercise for the reader to verify these solutions.) The time is $t = 3.96\text{ s}$ or -1.03 s . The negative value of time implies an event before the start of motion, and so we discard it. Thus,

$$t = 3.96\text{ s}. \tag{3.61}$$

Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities v_x and v_y and combine them to find the total velocity v and the angle θ_0 it makes with the horizontal. Of course, v_x is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

$$v_x = v_0 \cos\theta_0 = (25.0\text{ m/s})(\cos 35°) = 20.5\text{ m/s}. \tag{3.62}$$

The final vertical velocity is given by the following equation:

$$v_y = v_{0y} - gt, \tag{3.63}$$

where v_{0y} was found in part (a) to be 14.3 m/s . Thus,

$$v_y = 14.3\text{ m/s} - (9.80\text{ m/s}^2)(3.96\text{ s}) \tag{3.64}$$

so that

$$v_y = -24.5\text{ m/s}. \tag{3.65}$$

To find the magnitude of the final velocity v we combine its perpendicular components, using the following equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5\text{ m/s})^2 + (-24.5\text{ m/s})^2}, \tag{3.66}$$

which gives

$$v = 31.9\text{ m/s}. \tag{3.67}$$

The direction θ_v is found from the equation:

$$\theta_v = \tan^{-1}(v_y / v_x) \tag{3.68}$$

so that

$$\theta_v = \tan^{-1}(-24.5/20.5) = \tan^{-1}(-1.19). \tag{3.69}$$

Thus,

$$\theta_v = -50.1^\circ . \tag{3.70}$$

Discussion for (b)

The negative angle means that the velocity is 50.1° below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See **Figure 3.40**.)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define **range** to be the horizontal distance R traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.

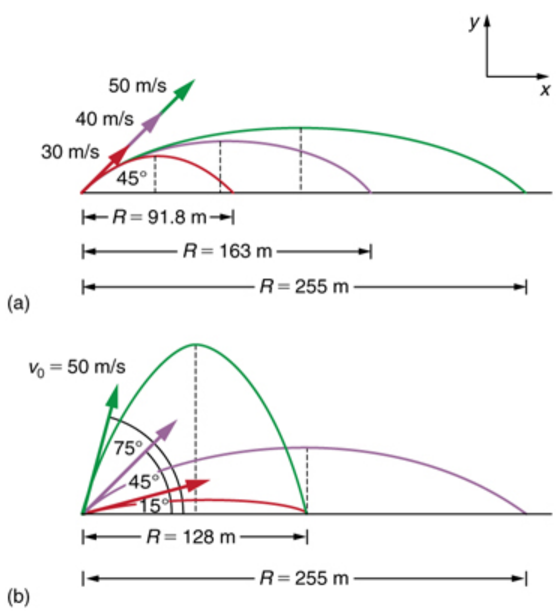


Figure 3.41 Trajectories of projectiles on level ground. (a) The greater the initial speed v_0 , the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for 15° and 75° , although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in **Figure 3.41**(a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in **Figure 3.41**(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45°$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately $38°$. Interestingly, for every initial angle except $45°$, there are two angles that give the same range—the sum of those angles is $90°$. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on *level ground* for which air resistance is negligible is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g}, \tag{3.71}$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See **Figure 3.42**.) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In **Addition of Velocities**, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.

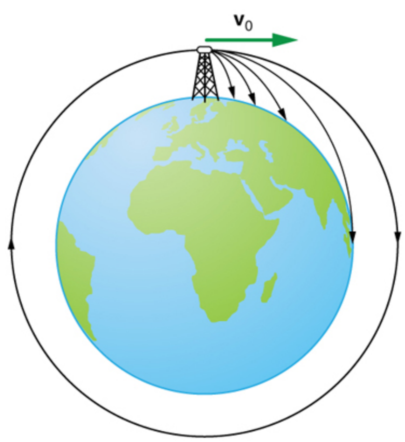


Figure 3.42 Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

PhET Explorations: Projectile Motion

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.

PhET Interactive Simulation

Figure 3.43 Projectile Motion (http://cnx.org/content/m54787/1.2/projectile-motion_en.jar)

3.5 Addition of Velocities

Learning Objectives

By the end of this section, you will be able to:

- Apply principles of vector addition to determine relative velocity.
- Explain the significance of the observer in the measurement of velocity.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves *diagonally* relative to the shore, as in **Figure 3.44**. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in **Figure 3.45**. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.

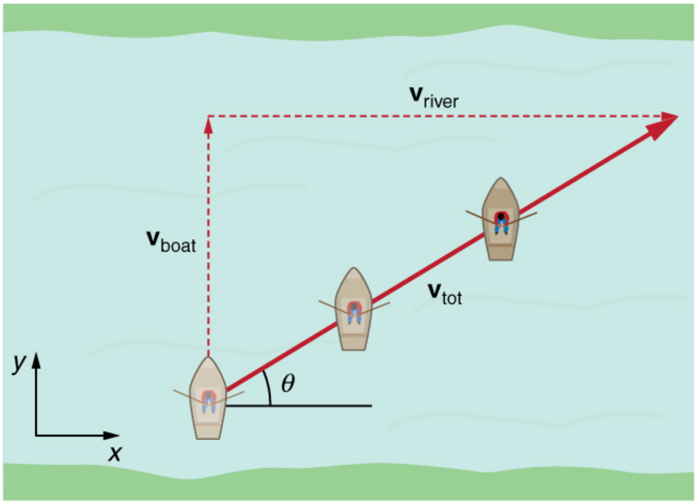


Figure 3.44 A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.

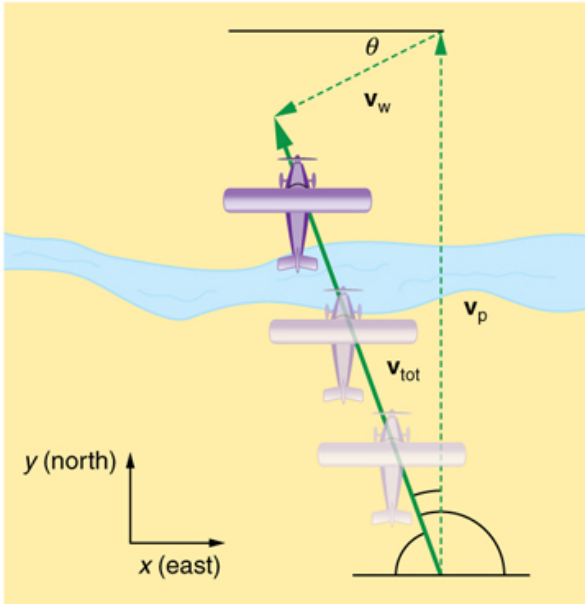


Figure 3.45 An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a **velocity** relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object *relative to the observer* is the sum of these velocity vectors, as indicated in **Figure 3.44** and **Figure 3.45**. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of **vector addition** discussed in **Vector Addition and Subtraction: Graphical Methods** and **Vector Addition and Subtraction: Analytical Methods** apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the *x*- and *y*-axes of an appropriately chosen coordinate system:

$$v_x = v\cos\theta \tag{3.72}$$

$$v_y = v\sin\theta \tag{3.73}$$

$$v = \sqrt{v_x^2 + v_y^2} \tag{3.74}$$

$$\theta = \tan^{-1}(v_y / v_x). \tag{3.75}$$

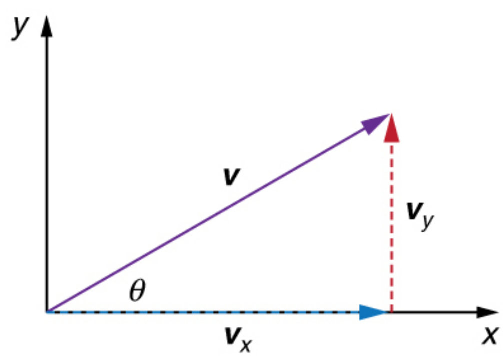


Figure 3.46 The velocity, v , of an object traveling at an angle θ to the horizontal axis is the sum of component vectors $\mathbf{v}_x$ and $\mathbf{v}_y$.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

Take-Home Experiment: Relative Velocity of a Boat

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Example 3.6 Adding Velocities: A Boat on a River

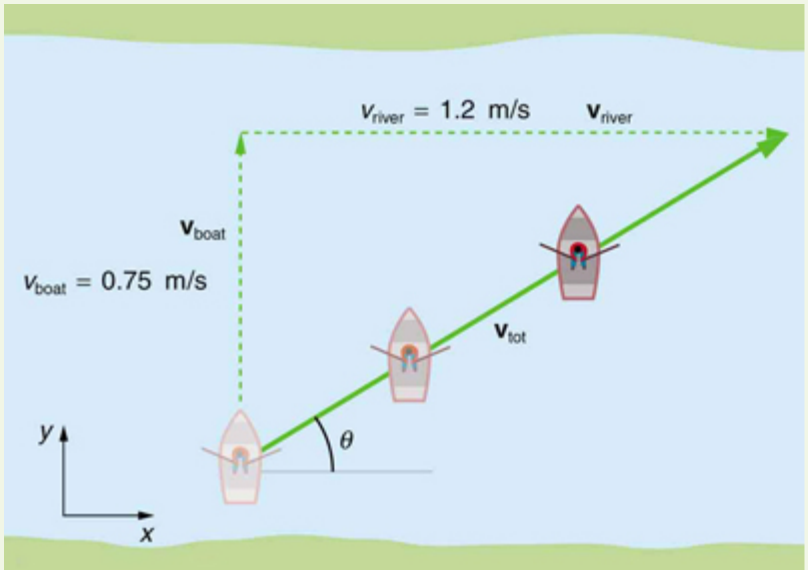


Figure 3.47 A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?

Refer to **Figure 3.47**, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, $\mathbf{v}_{\text{tot}}$. The velocity of the boat, $\mathbf{v}_{\text{boat}}$, is 0.75 m/s in the y-direction relative to the river and the velocity of the river, $\mathbf{v}_{\text{river}}$, is 1.20 m/s to the right.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river, as shown in **Figure 3.47**. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}(v_y / v_x)$ directly.

Solution

The magnitude of the total velocity is

$$v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}, \tag{3.76}$$

where

$$v_x = v_{\text{river}} = 1.20 \text{ m/s} \tag{3.77}$$

and

$$v_y = v_{\text{boat}} = 0.750 \text{ m/s}. \tag{3.78}$$

Thus,

$$v_{\text{tot}} = \sqrt{(1.20 \text{ m/s})^2 + (0.750 \text{ m/s})^2} \tag{3.79}$$

yielding

$$v_{\text{tot}} = 1.42 \text{ m/s}. \tag{3.80}$$

The direction of the total velocity θ is given by:

$$\theta = \tan^{-1}(v_y / v_x) = \tan^{-1}(0.750 / 1.20). \tag{3.81}$$

This equation gives

$$\theta = 32.0^{\text{o}}. \tag{3.82}$$

Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with **Figure 3.47**. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only 32.0^{o}) the total velocity has relative to the riverbank.

Example 3.7 Calculating Velocity: Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in **Figure 3.48**. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction 20.0^{o} west of north.

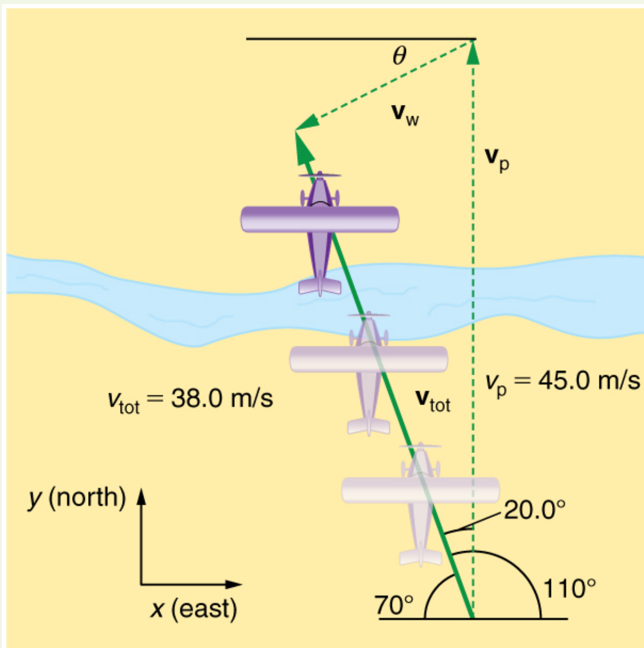


Figure 3.48 An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity $\mathbf{v}_{\text{tot}}$ and that it is the sum of two other velocities, $\mathbf{v}_{\text{w}}$ (the wind) and $\mathbf{v}_{\text{p}}$ (the plane relative to the air mass). The quantity $\mathbf{v}_{\text{p}}$ is known, and we are asked to find $\mathbf{v}_{\text{w}}$. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of $\mathbf{v}_{\text{w}}$, then we can combine them to solve for its magnitude and direction. As shown in **Figure 3.48**, we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to $\mathbf{v}_{\text{p}}$). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in **Vector Addition and Subtraction: Analytical Methods**.)

Solution

Because $\mathbf{v}_{\text{tot}}$ is the vector sum of the $\mathbf{v}_{\text{w}}$ and $\mathbf{v}_{\text{p}}$, its *x*- and *y*-components are the sums of the *x*- and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{\text{p}x} = 0$ and $v_{\text{p}y} = v_{\text{p}}$. That is,

$$v_{\text{tot}x} = v_{\text{w}x} \tag{3.83}$$

and

$$v_{\text{tot}y} = v_{\text{w}y} + v_{\text{p}}. \tag{3.84}$$

We can use the first of these two equations to find $v_{\text{w}x}$:

$$v_{\text{w}x} = v_{\text{tot}x} = v_{\text{tot}} \cos 110^\circ. \tag{3.85}$$

Because $v_{\text{tot}} = 38.0 \text{ m / s}$ and $\cos 110^\circ = -0.342$ we have

$$v_{\text{w}x} = (38.0 \text{ m/s})(–0.342)=–13.0 \text{ m/s}. \tag{3.86}$$

The minus sign indicates motion west which is consistent with the diagram.

Now, to find $v_{\text{w}y}$ we note that

$$v_{\text{tot}y} = v_{\text{w}y} + v_{\text{p}} \tag{3.87}$$

Here $v_{\text{tot}y} = v_{\text{tot}} \sin 110^\circ$; thus,

$$v_{\text{w}y} = (38.0 \text{ m/s})(0.940) - 45.0 \text{ m/s} = -9.29 \text{ m/s}. \tag{3.88}$$

This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity $v_{\text{w}x}$ and $v_{\text{w}y}$ are known, we can find the magnitude and direction of $\mathbf{v}_{\text{w}}$. First, the magnitude is

$$\begin{aligned} v_{\text{w}} &= \sqrt{v_{\text{w}x}^2 + v_{\text{w}y}^2} \\ &= \sqrt{(-13.0 \text{ m/s})^2 + (-9.29 \text{ m/s})^2} \end{aligned} \tag{3.89}$$

so that

$$v_{\text{w}} = 16.0 \text{ m/s}. \tag{3.90}$$

The direction is:

$$\theta = \tan^{-1}(v_{\text{w}y} / v_{\text{w}x}) = \tan^{-1}(-9.29 / -13.0) \tag{3.91}$$

giving

$$\theta = 35.6^\circ. \tag{3.92}$$

Discussion

The wind's speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in **Figure 3.48**. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*. These velocities are called **relative velocities**. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of **relativity**, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton. **Classical relativity** is limited to situations where speeds are less than about 1% of the speed of light—that is, less than $3{,}000 \text{ km/s}$. Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops his binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (See **Figure 3.49**.) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the curved path shown in **Figure 3.49**. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

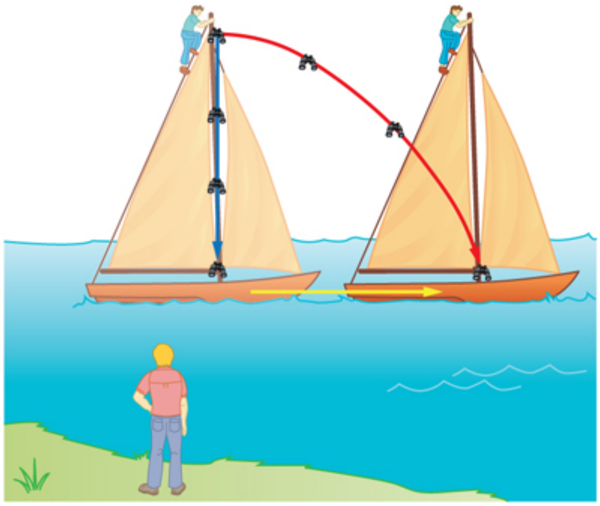


Figure 3.49 Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

Example 3.8 Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?

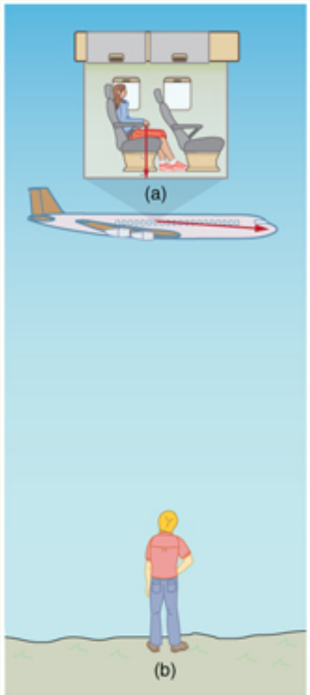


Figure 3.50 The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/

s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

$$v_y{}^2 = v_{0y}{}^2 - 2g(y - y_0). \tag{3.93}$$

Substituting known values into the equation, we get

$$v_y{}^2 = 0^2 - 2(9.80\text{ m/s}^2)(-1.50\text{ m} - 0\text{ m}) = 29.4\text{ m}^2/\text{s}^2 \tag{3.94}$$

yielding

$$v_y = -5.42\text{ m/s}. \tag{3.95}$$

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42\text{ m/s}$, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260\text{ m/s}$. The *x*- and *y*-components of velocity can be combined to find the magnitude of the final velocity:

$$v = \sqrt{v_x{}^2 + v_y{}^2}. \tag{3.96}$$

Thus,

$$v = \sqrt{(260\text{ m/s})^2 + (-5.42\text{ m/s})^2} \tag{3.97}$$

yielding

$$v = 260.06\text{ m/s}. \tag{3.98}$$

The direction is given by:

$$\theta = \tan^{-1}(v_y / v_x) = \tan^{-1}(-5.42 / 260) \tag{3.99}$$

so that

$$\theta = \tan^{-1}(-0.0208) = -1.19^{\text{o}}. \tag{3.100}$$

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is *not* $(260 - 5.42)\text{ m/s}$; rather, it is 260.06 m/s. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see *very* different paths. (See **Figure 3.50**.) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

Making Connections: Relativity and Einstein

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

PhET Explorations: Motion in 2D

Try the new "Ladybug Motion 2D" simulation for the latest updated version. Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

PhET Interactive Simulation

Figure 3.51 Motion in 2D (http://cnx.org/content/m54798/1.2/motion-2d_en.jar)

Glossary

air resistance: a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

analytical method: the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

classical relativity: the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s

commutative: refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

component (of a 2-d vector): a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

direction (of a vector): the orientation of a vector in space

head (of a vector): the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the “tip”

head-to-tail method: a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

kinematics: the study of motion without regard to mass or force

magnitude (of a vector): the length or size of a vector; magnitude is a scalar quantity

motion: displacement of an object as a function of time

projectile: an object that travels through the air and experiences only acceleration due to gravity

projectile motion: the motion of an object that is subject only to the acceleration of gravity

range: the maximum horizontal distance that a projectile travels

relative velocity: the velocity of an object as observed from a particular reference frame

relativity: the study of how different observers moving relative to each other measure the same phenomenon

resultant: the sum of two or more vectors

resultant vector: the vector sum of two or more vectors

scalar: a quantity with magnitude but no direction

tail: the start point of a vector; opposite to the head or tip of the arrow

trajectory: the path of a projectile through the air

vector: a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

vector addition: the rules that apply to adding vectors together

velocity: speed in a given direction

Section Summary

3.1 Kinematics in Two Dimensions: An Introduction

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

3.2 Vector Addition and Subtraction: Graphical Methods

- The **graphical method of adding vectors** $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A}+\mathbf{B}=\mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\mathbf{B}$ from $\mathbf{A}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})=\mathbf{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is **commutative** such that $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$.
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector $\mathbf{A}$ is multiplied by a scalar quantity c, the magnitude of the product is given by cA. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.

3.3 Vector Addition and Subtraction: Analytical Methods

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors $\mathbf{A}$ and $\mathbf{B}$ using the analytical method are as follows:

  Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

$$\begin{aligned} A_x &= A\cos\theta \\ B_x &= B\cos\theta \end{aligned}$$

  and

$$\begin{aligned} A_y &= A\sin\theta \\ B_y &= B\sin\theta. \end{aligned}$$

  Step 2: Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, $\mathbf{R}$:

$$R_x = A_x + B_x$$

  and

$$R_y = A_y + B_y.$$

  Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector $\mathbf{R}$:

$$R = \sqrt{R_x^2 + R_y^2}.$$

  Step 4: Use a trigonometric identity to determine the direction, θ, of $\mathbf{R}$:

$$\theta = \tan^{-1}(R_y / R_x).$$

3.4 Projectile Motion

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:
  1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position $\mathbf{s}$ are given by the quantities x and y, and the components of the velocity $\mathbf{v}$ are given by $v_x = v\cos\theta$ and $v_y = v\sin\theta$, where v is the magnitude of the velocity and θ is its direction.
  2. Analyze the motion of the projectile in the horizontal direction using the following equations:

$$\text{Horizontal motion}(a_x = 0)$$

$$x = x_0 + v_x t$$

$$v_x = v_{0x} = \mathbf{v}_\text{x} = \text{velocity is a constant.}$$

3. Analyze the motion of the projectile in the vertical direction using the following equations:

$$\text{Vertical motion(Assuming positive direction is up; } a_y = -g = -9.80\ \text{m/s}^2)$$

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

4. Recombine the horizontal and vertical components of location and/or velocity using the following equations:

$$s = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y / x)$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta_\text{v} = \tan^{-1}(v_y / v_x).$$

- The maximum height h of a projectile launched with initial vertical velocity v_{0y} is given by

$$h = \frac{v_{0y}^2}{2g}.$$

- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

3.5 Addition of Velocities

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as

$$v_x = v \cos \theta$$

$$v_y = v \sin \theta$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1}(v_y / v_x).$$

- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

Conceptual Questions

3.2 Vector Addition and Subtraction: Graphical Methods

1. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

2. Give a specific example of a vector, stating its magnitude, units, and direction.

3. What do vectors and scalars have in common? How do they differ?

4. Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?

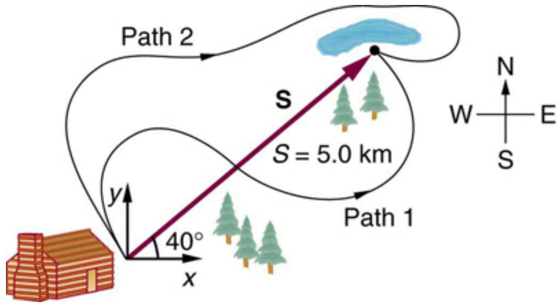


Figure 3.52

5. If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in Figure 3.53. What other information would he need to get to Sacramento?

Figure 3.53

6. Suppose you take two steps $\mathbf{A}$ and $\mathbf{B}$ (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point $\mathbf{A} + \mathbf{B}$ the sum of the lengths of the two steps?

7. Explain why it is not possible to add a scalar to a vector.

8. If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

3.3 Vector Addition and Subtraction: Analytical Methods

9. Suppose you add two vectors $\mathbf{A}$ and $\mathbf{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

10. Give an example of a nonzero vector that has a component of zero.

11. Explain why a vector cannot have a component greater than its own magnitude.

12. If the vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, what is the component of $\mathbf{A}$ along the direction of $\mathbf{B}$? What is the component of $\mathbf{B}$ along the direction of $\mathbf{A}$?

3.4 Projectile Motion

13. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither $0°$ nor $90°$): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than $t = 0$? (d) Can the speed ever be the same as the initial speed at a time other than $t = 0$?

14. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

15. For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

16. During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

3.5 Addition of Velocities

17. What frame or frames of reference do you instinctively use when driving a car? When flying in a commercial jet airplane?

18. A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn't he need to keep his eyes on the ball?

19. If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?

20. The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger's frame of reference. Draw its path as viewed by a stationary observer.

21. A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

Problems & Exercises

3.2 Vector Addition and Subtraction: Graphical Methods

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

1. Find the following for path A in Figure 3.54: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

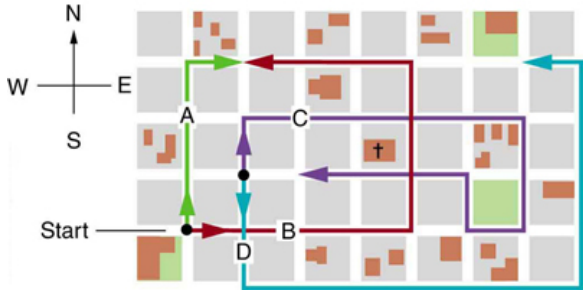


Figure 3.54 The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

2. Find the following for path B in Figure 3.54: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

3. Find the north and east components of the displacement for the hikers shown in Figure 3.52.

4. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 3.55, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

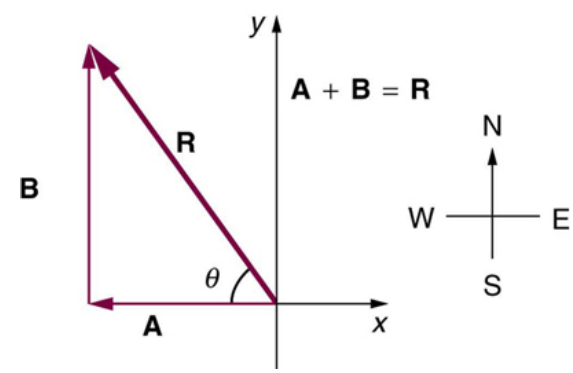


Figure 3.55 The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

5. Suppose you first walk 12.0 m in a direction 20° west of north and then 20.0 m in a direction 40.0° south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 3.56, then this problem finds their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

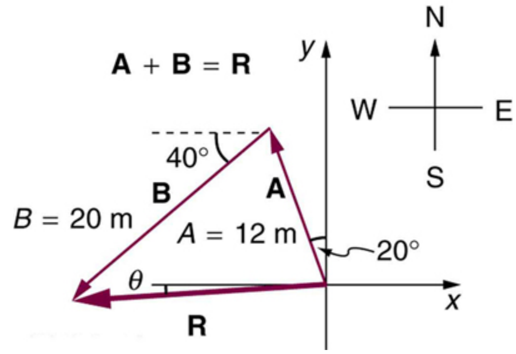


Figure 3.56

6. Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg $\mathbf{B}$, which is 20.0 m in a direction exactly 40° south of west, and then leg $\mathbf{A}$, which is 12.0 m in a direction exactly 20° west of north. (This problem shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.)

7. (a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0° north of east (which is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, to finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0° south of west and then 12.0 m in a direction 20.0° east of south (which is equivalent to subtracting $\mathbf{A}$ from $\mathbf{B}$ —that is, to finding $\mathbf{R}'' = \mathbf{B} - \mathbf{A} = -\mathbf{R}'$). Show that this is the case.

8. Show that the *order* of addition of three vectors does not affect their sum. Show this property by choosing any three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, all having different lengths and directions. Find the sum $\mathbf{A} + \mathbf{B} + \mathbf{C}$ then find their sum when added in a different order and show the result is the same. (There are five other orders in which $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ can be added; choose only one.)

9. Show that the sum of the vectors discussed in Example 3.2 gives the result shown in Figure 3.24.

10. Find the magnitudes of velocities v_{A} and v_{B} in Figure 3.57

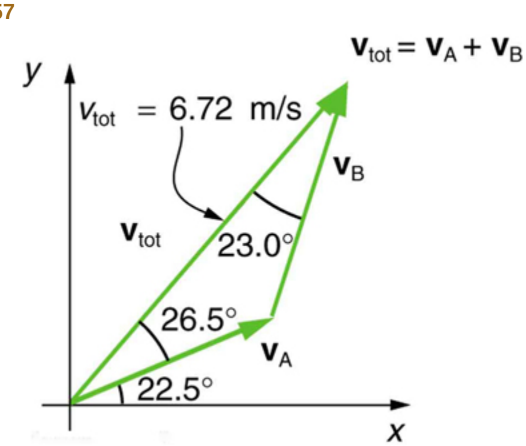


Figure 3.57 The two velocities $\mathbf{v}_{\text{A}}$ and $\mathbf{v}_{\text{B}}$ add to give a total $\mathbf{v}_{\text{tot}}$.

11. Find the components of v_{tot} along the *x*- and *y*-axes in Figure 3.57.

12. Find the components of v_{tot} along a set of perpendicular axes rotated 30° counterclockwise relative to those in **Figure 3.57**.

3.3 Vector Addition and Subtraction: Analytical Methods

13. Find the following for path C in **Figure 3.58**: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

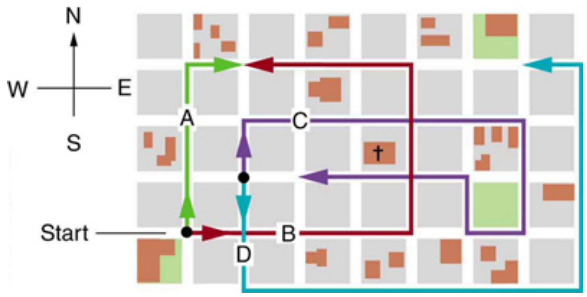


Figure 3.58 The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

14. Find the following for path D in **Figure 3.58**: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

15. Find the north and east components of the displacement from San Francisco to Sacramento shown in **Figure 3.59**.

Figure 3.59

16. Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in **Figure 3.60**, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

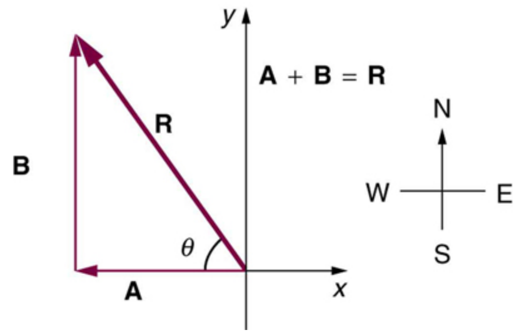


Figure 3.60 The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

17. Repeat **Exercise 3.16** using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result—that is, $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}$.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

18. You drive 7.50 km in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.

19. Do **Exercise 3.16** again using analytical techniques and change the second leg of the walk to 25.0 m straight south. (This is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$) (b) Repeat again, but now you first walk 25.0 m north and then 18.0 m east. (This is equivalent to subtract $\mathbf{A}$ from $\mathbf{B}$ —that is, to find $\mathbf{A} = \mathbf{B} + \mathbf{C}$. Is that consistent with your result?)

20. A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors $\mathbf{A}$ from $\mathbf{B}$ in **Figure 3.61**. She then correctly calculates the length and orientation of the third side $\mathbf{C}$. What is her result?

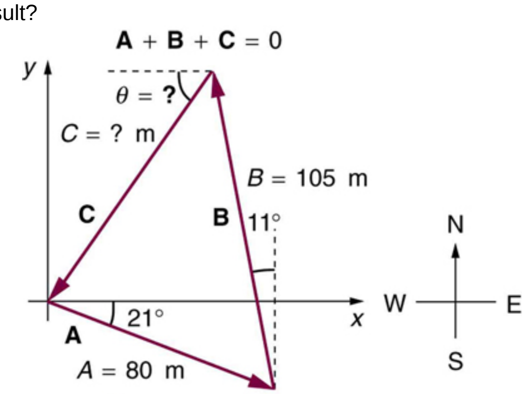


Figure 3.61

21. You fly 32.0 km in a straight line in still air in the direction 35.0° south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. These are the components of the displacement along a different set of axes—one rotated 45°.

22. A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ in Figure 3.62, and then correctly calculates the length and orientation of the fourth side $\mathbf{D}$. What is his result?

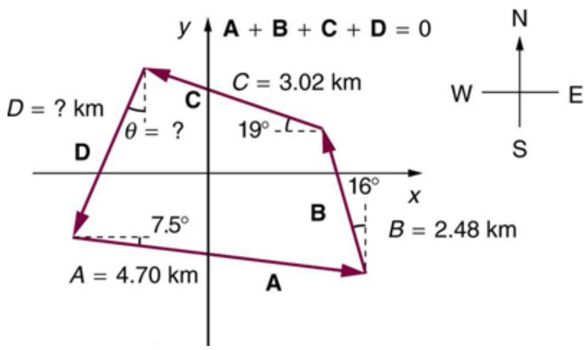


Figure 3.62

23. In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: 2.50 km 45.0° north of west; then 4.70 km 60.0° south of east; then 1.30 km 25.0° south of west; then 5.10 km straight east; then 1.70 km 5.00° east of north; then 7.20 km 55.0° south of west; and finally 2.80 km 10.0° north of east. What is his final position relative to the island?

24. Suppose a pilot flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east as shown in Figure 3.63. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.

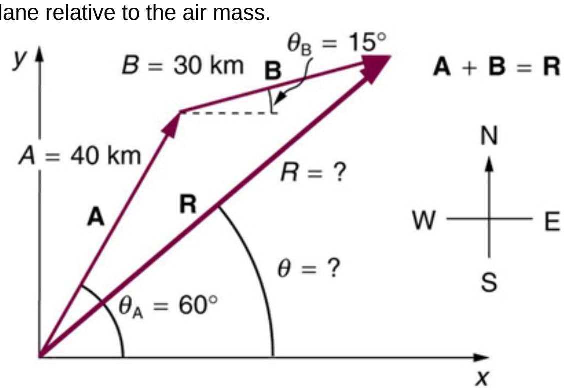


Figure 3.63

3.4 Projectile Motion

25. A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

26. A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c)What maximum height is attained by the ball?

27. A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

28. (a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a 32° ramp at a speed of $40.0 \text{ m/s} \ (144 \text{ km/h})$. How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

29. An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

30. A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?

31. Verify the ranges for the projectiles in Figure 3.41(a) for $\theta = 45^\circ$ and the given initial velocities.

32. Verify the ranges shown for the projectiles in Figure 3.41(b) for an initial velocity of 50 m/s at the given initial angles.

33. The cannon on a battleship can fire a shell a maximum distance of 32.0 km. (a) Calculate the initial velocity of the shell. (b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.) (c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is $6.37 \times 10^3 \text{ km}$. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

34. An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of $60°$ above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?

35. In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, g. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

36. The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

37. Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The service line is 11.9 m from the net, which is 0.91 m high. What is the angle θ such that the ball just crosses the net? Will the ball land in the service box, whose out line is 6.40 m from the net?

38. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of $25°$ relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?

39. Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

40. An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.

41. An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle $30.0°$ below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.

42. Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be $40°$ above the horizontal.

43. Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

44. The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 7.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

45. In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of $38.0°$ above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at $45°$ when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus, $38°$ will give a longer range than $45°$ in the shot put.)

46. A basketball player is running at 5.00 m/s directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

47. A football player punts the ball at a $45.0°$ angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

48. Prove that the trajectory of a projectile is parabolic, having the form $y = ax + bx^2$. To obtain this expression, solve the equation $x = v_{0x}t$ for t and substitute it into the expression for $y = v_{0y}t - (1/2)gt^2$ (These equations describe the x and y positions of a projectile that starts at the origin.) You should obtain an equation of the form $y = ax + bx^2$ where a and b are constants.

49. Derive $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$ for the range of a projectile on level ground by finding the time t at which y becomes zero and substituting this value of t into the expression for $x - x_0$, noting that $R = x - x_0$

50. Unreasonable Results (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

51. Construct Your Own Problem Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

3.5 Addition of Velocities

52. Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979. (a) He flew for 169 min at an average velocity of 3.53 m/s in a direction 45° south of east. What was his total displacement? (b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air? (c) What was his total displacement relative to the air mass?

53. A seagull flies at a velocity of 9.00 m/s straight into the wind. (a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind? (b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km? (c) Discuss how the wind affects the total round-trip time compared to what it would be with no wind.

54. Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?

55. Verify that the coin dropped by the airline passenger in the **Example 3.8** travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.

56. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of 25.0° relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball *relative to the quarterback* ?

57. A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to the Earth?

58. (a) A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction 5.0° south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction 15° south of east. What is the velocity of the airplane relative to the Earth? (b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.

59. (a) In what direction would the ship in **Exercise 3.57** have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains 7.00 m/s ? (b) What would its speed be relative to the Earth?

60. (a) Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20° south of east (as in **Exercise 3.58**). Its direction of motion relative to the Earth is 45.0° south of west, while its direction of travel relative to the air is 5.00° south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?

61. A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship: (a) relative to the ship and (b) relative to a stationary observer on shore. (c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

62. The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction 30.0° east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of 50.0° south of west relative to the Earth. What is the velocity of the wind relative to the water?

63. The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. **Figure 3.64** illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities: (a) relative to galaxy 2 and (b) relative to galaxy 5. The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.

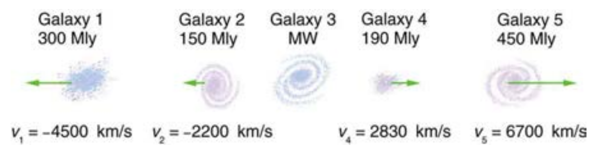


Figure 3.64 Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across.

64. (a) Use the distance and velocity data in **Figure 3.64** to find the rate of expansion as a function of distance.

(b) If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.

65. An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. He reaches the opposite side at a distance 40 m downstream from his starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?

66. A ship sailing in the Gulf Stream is heading 25.0° west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is 4.80 m/s 5.00° west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)

67. An ice hockey player is moving at 8.00 m/s when he hits the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a 90.0° angle relative to his path as shown in **Figure 3.65**. What angle must the puck's velocity make relative to the player (in his frame of reference) to hit the center of the goal?

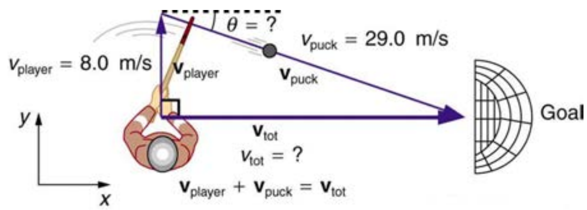


Figure 3.65 An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal.

68. Unreasonable Results Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth. (a) At what velocity must the supplies be launched? (b) What is unreasonable about this velocity? (c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height? (d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

69. Unreasonable Results A commercial airplane has an air speed of 280 m/s due east and flies with a strong tailwind. It travels 3000 km in a direction 5^{o} south of east in 1.50 h. (a) What was the velocity of the plane relative to the ground? (b) Calculate the magnitude and direction of the tailwind's velocity. (c) What is unreasonable about both of these velocities? (d) Which premise is unreasonable?

70. Construct Your Own Problem Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.

Test Prep for AP® Courses

3.1 Kinematics in Two Dimensions: An Introduction

1. A ball is thrown at an angle of 45 degrees above the horizontal. Which of the following best describes the acceleration of the ball from the instant after it leaves the thrower's hand until the time it hits the ground?
a. Always in the same direction as the motion, initially positive and gradually dropping to zero by the time it hits the ground
b. Initially positive in the upward direction, then zero at maximum height, then negative from there until it hits the ground
c. Always in the opposite direction as the motion, initially positive and gradually dropping to zero by the time it hits the ground
d. Always in the downward direction with the same constant value

2. In an experiment, a student launches a ball with an initial horizontal velocity at an elevation 2 meters above ground. The ball follows a parabolic trajectory until it hits the ground. Which of the following accurately describes the graph of the ball's vertical acceleration versus time (taking the downward direction to be negative)?
a. A negative value that does not change with time
b. A gradually increasing negative value (straight line)
c. An increasing rate of negative values over time (parabolic curve)
d. Zero at all times since the initial motion is horizontal

3. A student wishes to design an experiment to show that the acceleration of an object is independent of the object's velocity. To do this, ball A is launched horizontally with some initial speed at an elevation 1.5 meters above the ground, ball B is dropped from rest 1.5 meters above the ground, and ball C is launched vertically with some initial speed at an elevation 1.5 meters above the ground. What information would the student need to collect about each ball in order to test the hypothesis?

3.2 Vector Addition and Subtraction: Graphical Methods

4. A ball is launched vertically upward. The vertical position of the ball is recorded at various points in time in the table shown.

Table 3.1

Height (m)	Time (sec)
0.490	0.1
0.882	0.2
1.176	0.3
1.372	0.4
1.470	0.5
1.470	0.6
1.372	0.7

Which of the following correctly describes the graph of the ball's vertical velocity versus time?

a. Always positive, steadily decreasing
b. Always positive, constant
c. Initially positive, steadily decreasing, becoming negative at the end
d. Initially zero, steadily getting more and more negative

5.

Table 3.2

Height (m)	Time (sec)
0.490	0.1
0.882	0.2
1.176	0.3
1.372	0.4
1.470	0.5
1.470	0.6
1.372	0.7

A ball is launched at an angle of 60 degrees above the horizontal, and the vertical position of the ball is recorded at various points in time in the table shown, assuming the ball was at a height of 0 at time t = 0.

a. Draw a graph of the ball's vertical velocity versus time.
b. Describe the graph of the ball's horizontal velocity.
c. Draw a graph of the ball's vertical acceleration versus time.

3.4 Projectile Motion

6. In an experiment, a student launches a ball with an initial horizontal velocity of 5.00 meters/sec at an elevation 2.00 meters above ground. Draw and clearly label with appropriate values and units a graph of the ball's horizontal velocity vs. time and the ball's vertical velocity vs. time. The graph should cover the motion from the instant after the ball is launched until the instant before it hits the ground. Assume the downward direction is negative for this problem.

4 DYNAMICS: FORCE AND NEWTON'S LAWS OF MOTION

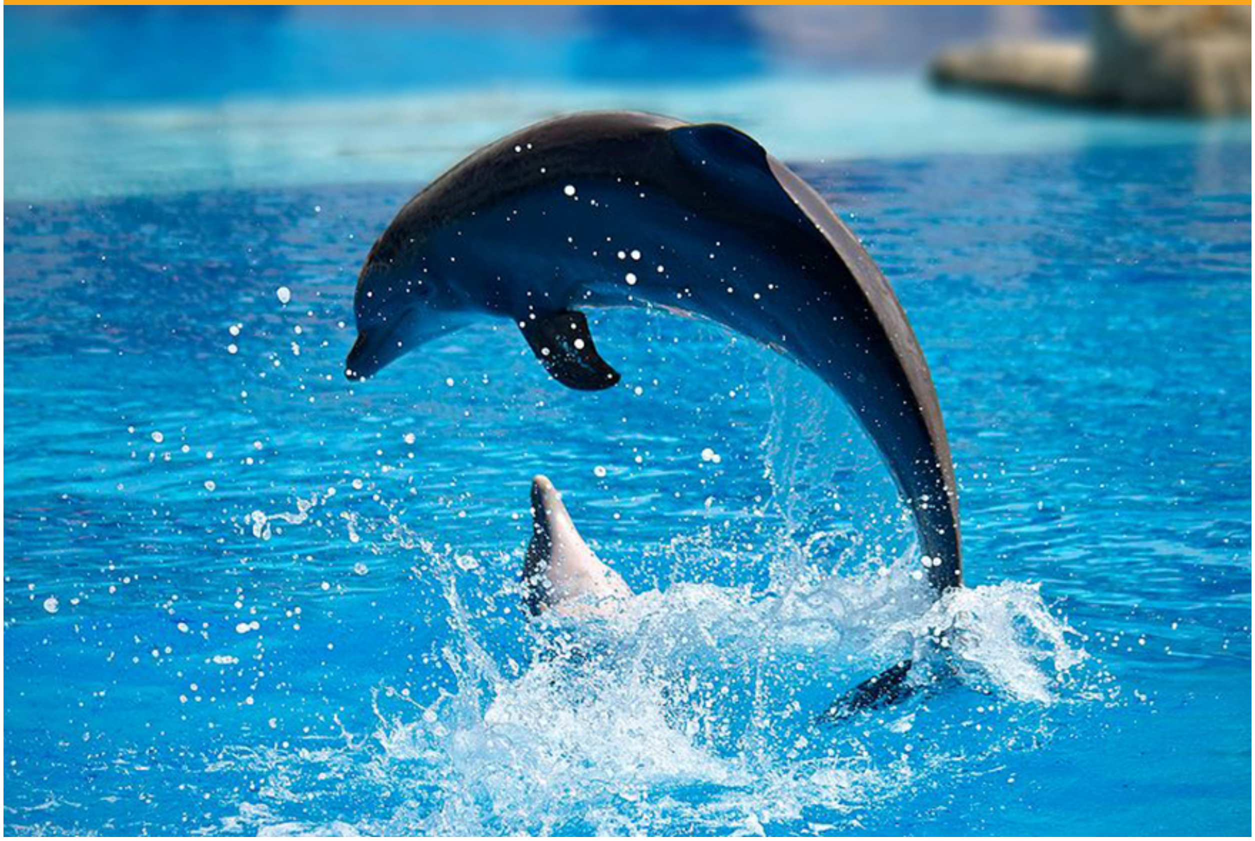


Figure 4.1 Newton's laws of motion describe the motion of the dolphin's path. (credit: Jin Jang)

Chapter Outline

Connection for AP® Courses

Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a jumping dolphin, a leaping pole vaulter, a bird in flight, or an orbiting satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to situations on Earth as well as in space.

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic, with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo Galilei (1564–1647).

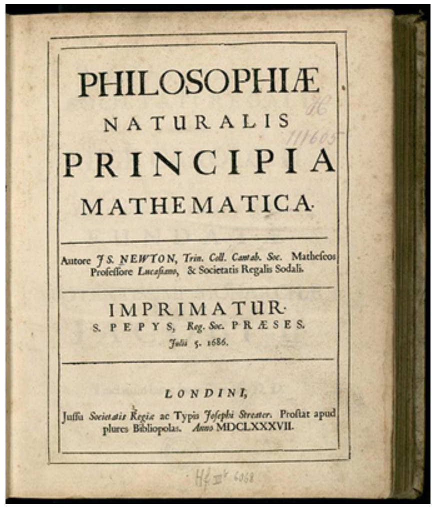


Figure 4.2 Isaac Newton's monumental work, *Philosophiae Naturalis Principia Mathematica*, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects. (credit: Service commun de la documentation de l'Université de Strasbourg)

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than "logical" argument. Galileo's use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formulation of what is now called Newton's first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made by Newton working alone, without the benefit of the usual interactions that take place among scientists today.

Newton's laws are introduced along with Big Idea 3, that interactions can be described by forces. These laws provide a theoretical basis for studying motion depending on interactions between the objects. In particular, Newton's laws are applicable to all forces in inertial frames of references (Enduring Understanding 3.A). We will find that all forces are vectors; that is, forces always have both a magnitude and a direction (Essential Knowledge 3.A.2). Furthermore, we will learn that all forces are a result of interactions between two or more objects (Essential Knowledge 3.A.3). These interactions between any two objects are described by Newton's third law, stating that the forces exerted on these objects are equal in magnitude and opposite in direction to each other (Essential Knowledge 3.A.4).

We will discover that there is an empirical cause-effect relationship between the net force exerted on an object of mass m and its acceleration, with this relationship described by Newton's second law (Enduring Understanding 3.B). This supports Big Idea 1, that inertial mass is a property of an object or a system. The mass of an object or a system is one of the factors affecting changes in motion when an object or a system interacts with other objects or systems (Essential Knowledge 1.C.1). Another is the net force on an object, which is the vector sum of all the forces exerted on the object (Essential Knowledge 3.B.1). To analyze this, we use free-body diagrams to visualize the forces exerted on a given object in order to find the net force and analyze the object's motion (Essential Knowledge 3.B.2).

Thinking of these objects as systems is a concept introduced in this chapter, where a system is a collection of elements that could be considered as a single object without any internal structure (Essential Knowledge 5.A.1). This will support Big Idea 5, that changes that occur to the system due to interactions are governed by conservation laws. These conservation laws will be the focus of later chapters in this book. They explain whether quantities are conserved in the given system or change due to transfer to or from another system due to interactions between the systems (Enduring Understanding 5.A).

Furthermore, when a situation involves more than one object, it is important to define the system and analyze the motion of a whole system, not its elements, based on analysis of external forces on the system. This supports Big Idea 4, that interactions between systems cause changes in those systems. All kinematics variables in this case describe the motion of the center of mass of the system (Essential Knowledge 4.A.1, Essential Knowledge 4.A.2). The internal forces between the elements of the system do not affect the velocity of the center of mass (Essential Knowledge 4.A.3). The velocity of the center of mass will change only if there is a net external force exerted on the system (Enduring Understanding 4.A).

We will learn that some of these interactions can be explained by the existence of fields extending through space, supporting Big Idea 2. For example, any object that has mass creates a gravitational field in space (Enduring Understanding 2.B). Any material object (one that has mass) placed in the gravitational field will experience gravitational force (Essential Knowledge 2.B.1).

Forces may be categorized as contact or long-distance (Enduring Understanding 3.C). In this chapter we will work with both. An example of a long-distance force is gravitation (Essential Knowledge 3.C.1). Contact forces, such as tension, friction, normal force, and the force of a spring, result from interatomic electric forces at the microscopic level (Essential Knowledge 3.C.4).

It was not until the advent of modern physics early in the twentieth century that it was discovered that Newton's laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the size of most molecules (about 10^{-9} m in diameter). These constraints define the realm of classical mechanics, as discussed in **Introduction to the Nature of Science and Physics**. At the beginning of the twentieth century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, quantum theory. Quantum theory does not have the constraints present in classical physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity in **Special Relativity**, are in the realm of classical physics.

The development of special relativity and empirical observations at atomic scales led to the idea that there are four basic forces that account for all known phenomena. These forces are called fundamental (Enduring Understanding 3.G). The properties of gravitational (Essential Knowledge 3.G.1) and electromagnetic (Essential Knowledge 3.G.2) forces are explained in more detail.

Big Idea 1 Objects and systems have properties such as mass and charge. Systems may have internal structure.

Essential Knowledge 1.C.1 Inertial mass is the property of an object or a system that determines how its motion changes when it interacts with other objects or systems.

Big Idea 2 Fields existing in space can be used to explain interactions.

Enduring Understanding 2.A A field associates a value of some physical quantity with every point in space. Field models are useful for describing interactions that occur at a distance (long-range forces) as well as a variety of other physical phenomena.

Essential Knowledge 2.A.1 A vector field gives, as a function of position (and perhaps time), the value of a physical quantity that is described by a vector.

Essential Knowledge 2.A.2 A scalar field gives the value of a physical quantity.

Enduring Understanding 2.B A gravitational field is caused by an object with mass.

Essential Knowledge 2.B.1 A gravitational field *g* at the location of an object with mass *m* causes a gravitational force of magnitude *mg* to be exerted on the object in the direction of the field.

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A All forces share certain common characteristics when considered by observers in inertial reference frames.

Essential Knowledge 3.A.2 Forces are described by vectors.

Essential Knowledge 3.A.3 A force exerted on an object is always due to the interaction of that object with another object.

Essential Knowledge 3.A.4 If one object exerts a force on a second object, the second object always exerts a force of equal magnitude on the first object in the opposite direction.

Enduring Understanding 3.B Classically, the acceleration of an object interacting with other objects can be predicted by using $a = \sum F / m$.

Essential Knowledge 3.B.1 If an object of interest interacts with several other objects, the net force is the vector sum of the individual forces.

Essential Knowledge 3.B.2 Free-body diagrams are useful tools for visualizing the forces being exerted on a single object and writing the equations that represent a physical situation.

Enduring Understanding 3.C At the macroscopic level, forces can be categorized as either long-range (action-at-a-distance) forces or contact forces.

Essential Knowledge 3.C.1 Gravitational force describes the interaction of one object that has mass with another object that has mass.

Essential Knowledge 3.C.4 Contact forces result from the interaction of one object touching another object, and they arise from interatomic electric forces. These forces include tension, friction, normal, spring (Physics 1), and buoyant (Physics 2).

Enduring Understanding 3.G Certain types of forces are considered fundamental.

Essential Knowledge 3.G.1 Gravitational forces are exerted at all scales and dominate at the largest distance and mass scales.

Essential Knowledge 3.G.2 Electromagnetic forces are exerted at all scales and can dominate at the human scale.

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding 4.A The acceleration of the center of mass of a system is related to the net force exerted on the system, where $a = \sum F / m$.

Essential Knowledge 4.A.1 The linear motion of a system can be described by the displacement, velocity, and acceleration of its center of mass.

Essential Knowledge 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

Essential Knowledge 4.A.3 Forces that systems exert on each other are due to interactions between objects in the systems. If the interacting objects are parts of the same system, there will be no change in the center-of-mass velocity of that system.

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.1 A system is an object or a collection of objects. The objects are treated as having no internal structure.

4.1 Development of Force Concept

Learning Objectives

By the end of this section, you will be able to:

- Understand the definition of force.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.2** The student is able to challenge a claim that an object can exert a force on itself. **(S.P. 6.1)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in Figure 4.3, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in Figure 4.3(a) for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods. These ideas were developed in Two-Dimensional Kinematics.

By definition, force is always the result of an interaction of two or more objects. No object possesses force on its own. For example, a cannon does not possess force, but it can exert force on a cannonball. Earth does not possess force on its own, but exerts force on a football or on any other massive object. The skaters in Figure 4.3 exert force on one another as they interact.

No object can exert force on itself. When you clap your hands, one hand exerts force on the other. When a train accelerates, it exerts force on the track and vice versa. A bowling ball is accelerated by the hand throwing it; once the hand is no longer in contact with the bowling ball, it is no longer accelerating the bowling ball or exerting force on it. The ball continues moving forward due to inertia.

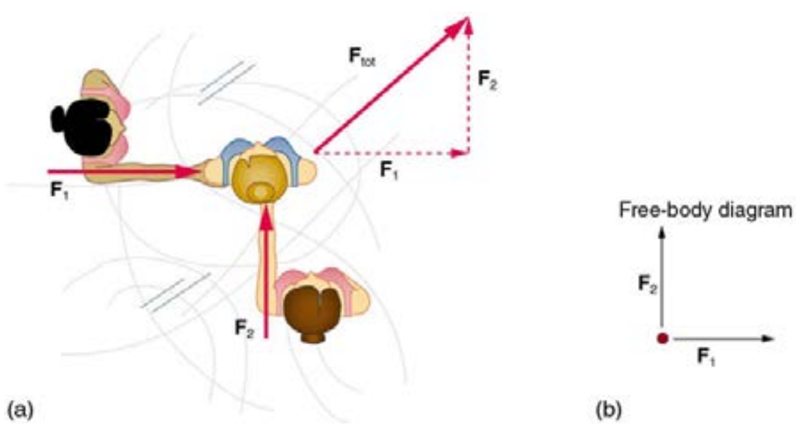


Figure 4.3 Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

Figure 4.3(b) is our first example of a **free-body diagram**, which is a technique used to illustrate all the **external forces** acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown, because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton’s laws of motion.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in **Figure 4.4**, and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. (One that we will encounter in **Magnetism** is the magnetic force between two wires carrying electric current.) Some alternative definitions of force will be given later in this chapter.

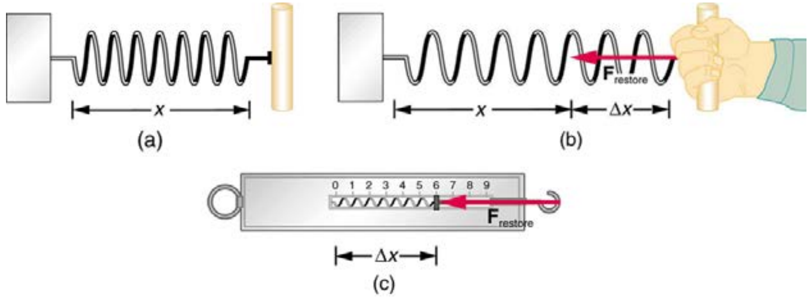


Figure 4.4 The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance Δx, the spring exerts a restoring force, $\mathbf{F}_{\text{restore}}$, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force $\mathbf{F}_{\text{restore}}$ is exerted on whatever is attached to the hook. Here $\mathbf{F}_{\text{restore}}$ has a magnitude of 6 units in the force standard being employed.

Take-Home Experiment: Force Standards

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

4.2 Newton's First Law of Motion: Inertia

Learning Objectives

By the end of this section, you will be able to:

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton's first law of motion** states, however, is the following:

Newton's First Law of Motion

There exists an inertial frame of reference such that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb “remains.” We can think of this law as preserving the status quo of motion.

The first law of motion postulates the existence of at least one frame of reference which we call an inertial reference frame, relative to which the motion of an object not subject to forces is a straight line at a constant speed. An inertial reference frame is any reference frame that is not itself accelerating. A car traveling at constant velocity is an inertial reference frame. A car slowing down for a stoplight, or speeding up after the light turns green, will be accelerating and is not an inertial reference frame. Finally, when the car goes around a turn, which is due to an acceleration changing the direction of the velocity vector, it is not an inertial reference frame. Note that Newton’s laws of motion are only valid for inertial reference frames.

Rather than contradicting our experience, **Newton’s first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)* in an inertial reference frame. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, "What is the cause?" Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as "Why does a tiger have stripes?" would have been answered in Aristotelian fashion, "That is the nature of the beast." True perhaps, but not a useful insight.

Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**.

An object with a small mass will exhibit less inertia and be more affected by other objects. An object with a large mass will exhibit greater inertia and be less affected by other objects. This inertial mass of an object is a measure of how difficult it is to alter the uniform motion of the object by an external force.

Roughly speaking, mass is a measure of the amount of "stuff" (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

Check Your Understanding

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

Solution

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

4.3 Newton's Second Law of Motion: Concept of a System

Learning Objectives

By the end of this section, you will be able to:

- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.

Newton's second law of motion is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest. For example, in **Figure 4.5**(a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at **Figure 4.5**(a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external*. Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.

When we describe the acceleration of a system, we are modeling the system as a single point which contains all of the mass of that system. The point we choose for this is the point about which the system's mass is evenly distributed. For example, in a rigid object, this center of mass is the point where the object will stay balanced even if only supported at this point. For a sphere or disk made of homogenous material, this point is of course at the center. Similarly, for a rod made of homogenous material, the center of mass will be at the midpoint.

For the rider in the wagon in **Figure 4.5**, the center of mass is probably between the rider's hips. Due to internal forces, the rider's hand or hair may accelerate slightly differently, but it is the acceleration of the system's center of mass that interests us. This is true whether the system is a vehicle carrying passengers, a bowl of grapes, or a planet. When we draw a free-body diagram of a system, we represent the system's center of mass with a single point and use vectors to indicate the forces exerted on that center of mass. (See **Figure 4.5**.)

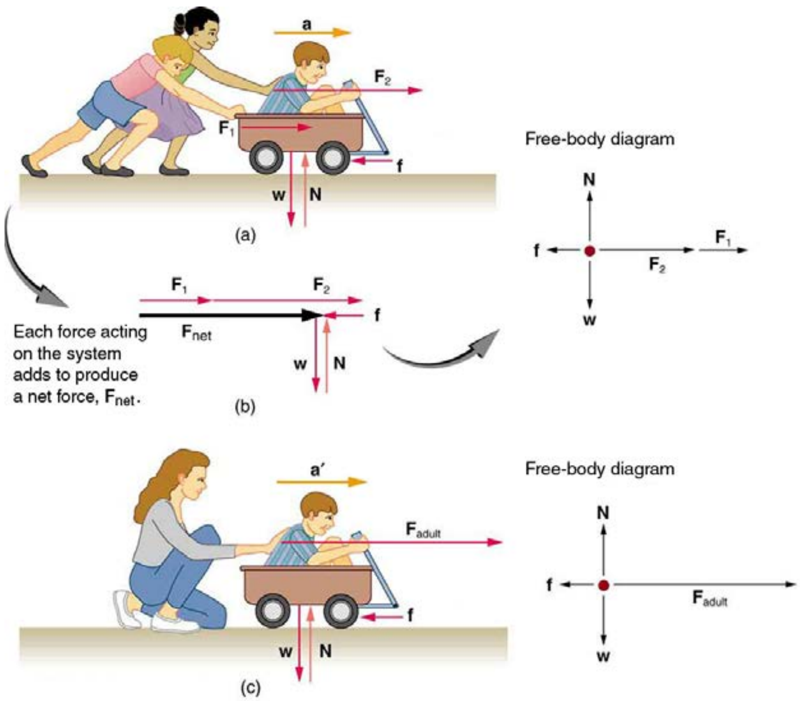


Figure 4.5 Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight $\mathbf{w}$ of the system and the support of the ground $\mathbf{N}$ are also shown for completeness and are assumed to cancel. The vector $\mathbf{f}$ represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force, $\mathbf{F}_{\text{net}}$. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger acceleration ($\mathbf{a}' > \mathbf{a}$) when an adult pushes the child.

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in **Figure 4.5**. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight $\mathbf{w}$ and the support of the ground $\mathbf{N}$, and the horizontal force $\mathbf{f}$ represents the force of friction. These will be discussed in more detail in later sections. For now, we will define **friction** as a force that opposes the motion past each other of objects that are touching. **Figure 4.5**(b) shows how vectors representing the external forces add together to produce a net force, $\mathbf{F}_{\text{net}}$.

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality

$$\mathbf{a} \propto \mathbf{F}_{\text{net}}, \tag{4.1}$$

where the symbol $\propto$ means "proportional to," and $\mathbf{F}_{\text{net}}$ is the **net external force**. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors, and are covered in **Two-Dimensional Kinematics**.) This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in **Figure 4.6**, the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

$$\mathbf{a} \propto \frac{1}{m} \tag{4.2}$$

where m is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.

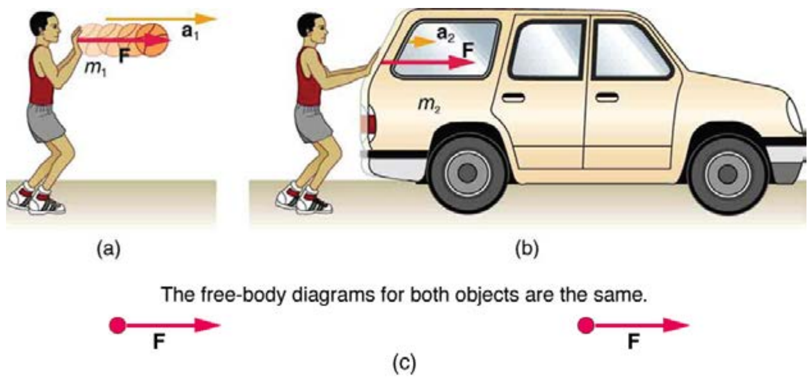


Figure 4.6 The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

Both of these proportionalities have been experimentally verified repeatedly and consistently, for a broad range of systems and scales. Thus, it has been **experimentally** found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

Applying the Science Practices: Testing the Relationship Between Mass, Acceleration, and Force

Plan three simple experiments using objects you have at home to test relationships between mass, acceleration, and force.

(a) Design an experiment to test the relationship between mass and acceleration. What will be the independent variable in your experiment? What will be the dependent variable? What controls will you put in place to ensure force is constant?

(b) Design a similar experiment to test the relationship between mass and force. What will be the independent variable in your experiment? What will be the dependent variable? What controls will you put in place to ensure acceleration is constant?

(c) Design a similar experiment to test the relationship between force and acceleration. What will be the independent variable in your experiment? What will be the dependent variable? Will you have any trouble ensuring that the mass is constant?

What did you learn?

Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}. \tag{4.3}$$

This is often written in the more familiar form

$$\mathbf{F}_{\text{net}} = m\mathbf{a}. \tag{4.4}$$

When only the magnitude of force and acceleration are considered, this equation is simply

$$F_{\text{net}} = ma. \tag{4.5}$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

Applying the Science Practices: Systems and Free-Body Diagrams

First, consider a person on a sled sliding downhill. What is the system in this situation? Try to draw a free-body diagram describing this system, labeling all the forces and their directions. Which of the forces are internal? Which are external?

Next, consider a person on a sled being pushed along level ground by a friend. What is the system in this situation? Try to draw a free-body diagram describing this system, labelling all the forces and their directions. Which of the forces are internal? Which are external?

Units of Force

$\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the **newton** (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of 1m/s^2. That is, since $\mathbf{F}_{\text{net}} = m\mathbf{a}$,

$$1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2. \tag{4.6}$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight** $\mathbf{w}$. Weight can be denoted as a vector $\mathbf{w}$ because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration g. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass m falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude w. Newton's second law states that the magnitude of the net external force on an object is $F_{\text{net}} = ma$.

Since the object experiences only the downward force of gravity, $F_{\text{net}} = w$. We know that the acceleration of an object due to gravity is g, or $a = g$. Substituting these into Newton's second law gives

Weight

This is the equation for *weight*—the gravitational force on a mass m:

$$w = mg. \tag{4.7}$$

Since $g = 9.80\text{ m/s}^2$ on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

$$w = mg = (1.0\text{ kg})(9.80\text{ m/s}^2) = 9.8\text{ N}. \tag{4.8}$$

Recall that g can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity g varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only 1.67 m/s^2. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and "microgravity," they are really referring to the phenomenon we call "free-fall" in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our “weight” in kilograms, but never in the correct units of newtons.

Common Misconceptions: Mass vs. Weight

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the “slug” in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (m) multiplied by the acceleration due to gravity (g). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is $1.67\ \text{m/s}^2$ (which is much less than the acceleration due to gravity on Earth, $9.80\ \text{m/s}^2$). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you “weigh” much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are “losing weight,” they really mean that they are losing “mass” (which in turn causes them to weigh less).

Take-Home Experiment: Mass and Weight

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same “mass” on Earth as on the Moon?

Example 4.1 What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?

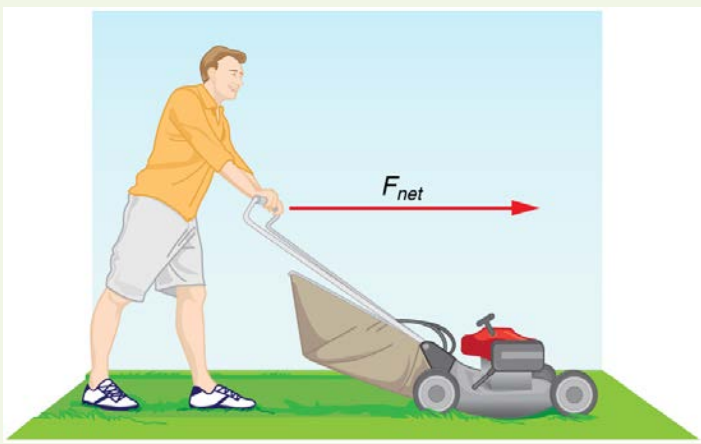


Figure 4.7 The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right?

Strategy

Since $\mathbf{F}_{\text{net}}$ and m are given, the acceleration can be calculated directly from Newton’s second law as stated in $\mathbf{F}_{\text{net}} = m\mathbf{a}$.

Solution

The magnitude of the acceleration a is $a = \dfrac{F_{\text{net}}}{m}$. Entering known values gives

$$a = \frac{51\ \text{N}}{24\ \text{kg}} \tag{4.9}$$

Substituting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

$$a = \frac{51\ \text{kg} \cdot \text{m/s}^2}{24\ \text{kg}} = 2.1\ \text{m/s}^2. \tag{4.10}$$

Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

Example 4.2 What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust $\mathbf{T}$, for the four-rocket propulsion system shown in **Figure 4.8**. The sled's initial acceleration is $49\ \text{m/s}^2,$ the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.

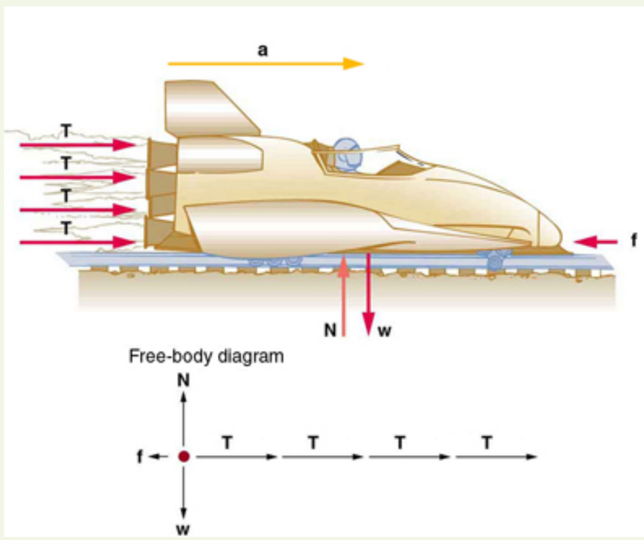


Figure 4.8 A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust $\mathbf{T}$. As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force $\mathbf{N}$ on the system that is equal in magnitude and opposite in direction to its weight, $\mathbf{w}$. The system here is the sled, its rockets, and rider, so none of the forces *between* these objects are considered. The arrow representing friction ($\mathbf{f}$) is drawn larger than scale.

Strategy

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$F_{\text{net}} = ma, \tag{4.11}$$

where F_{net} is the net force along the horizontal direction. We can see from **Figure 4.8** that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

$$F_{\text{net}} = 4T - f. \tag{4.12}$$

Substituting this into Newton's second law gives

$$F_{\text{net}} = ma = 4T - f. \tag{4.13}$$

Using a little algebra, we solve for the total thrust 4*T*:

$$4T = ma + f. \tag{4.14}$$

Substituting known values yields

$$4T = ma + f = (2100\text{ kg})(49\text{ m/s}^2) + 650\text{ N}. \tag{4.15}$$

So the total thrust is

$$4T = 1.0\times10^5\text{ N}, \tag{4.16}$$

and the individual thrusts are

$$T = \frac{1.0\times10^5\text{ N}}{4} = 2.6\times10^4\text{ N}. \tag{4.17}$$

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 g 's. (Recall that g , the acceleration due to gravity, is 9.80 m/s^2 . When we say that an acceleration is 45 g 's, it is $45\times9.80\text{ m/s}^2$, which is approximately 440 m/s^2 .) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

Applying the Science Practices: Sums of Forces

Recall that forces are vector quantities, and therefore the net force acting on a system should be the vector sum of the forces.

(a) Design an experiment to test this hypothesis. What sort of a system would be appropriate and convenient to have multiple forces applied to it? What features of the system should be held constant? What could be varied? Can forces be arranged in multiple directions so that, while the hypothesis is still tested, the resulting calculations are not too inconvenient?

(b) Another group of students has done such an experiment, using a motion capture system looking down at an air hockey table to measure the motion of the 0.10-kg puck. The table was aligned with the cardinal directions, and a compressed air hose was placed in the center of each side, capable of varying levels of force output and fixed so that it was aimed at the center of the table.

Table 4.1

Forces	Measured acceleration (magnitudes)
3 N north, 4 N west	48 ± 4 m/s^2
5 N south, 12 N east	132 ± 6 m/s^2
6 N north, 12 N east, 4 N west	99 ± 3 m/s^2

Given the data in the table, is the hypothesis confirmed? What were the directions of the accelerations?

4.4 Newton's Third Law of Motion: Symmetry in Forces

Learning Objectives

By the end of this section, you will be able to:

- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.1** The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. **(S.P. 6.4, 7.2)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.A.4.1** The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. **(S.P. 1.4, 6.2)**
- **3.A.4.2** The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. **(S.P. 6.4, 7.2)**
- **3.A.4.3** The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. **(S.P. 1.4)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**
- **4.A.2.1** The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. **(S.P. 6.4)**
- **4.A.2.2** The student is able to evaluate using given data whether all the forces on a system or whether all the parts of a system have been identified. **(S.P. 5.3)**
- **4.A.3.1** The student is able to apply Newton's second law to systems to calculate the change in the center-of-mass velocity when an external force is exerted on the system. **(S.P. 2.2)**

There is a passage in the musical *Man of la Mancha* that relates to Newton’s third law of motion. Sancho, in describing a fight with his wife to Don Quixote, says, “Of course I hit her back, Your Grace, but she’s a lot harder than me and you know what they say, ‘Whether the stone hits the pitcher or the pitcher hits the stone, it’s going to be bad for the pitcher.’” This is exactly what happens whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or throwing a ball, confirm this. It is precisely stated in **Newton’s third law of motion**.

Newton’s Third Law of Motion

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as “action-reaction,” where the force exerted is the action and the force experienced as a consequence is the reaction. Newton’s third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton’s third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in **Figure 4.9**. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then $\mathbf{F}_{\text{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of $\mathbf{F}_{\text{wall on feet}}$. In contrast, the force $\mathbf{F}_{\text{feet on wall}}$ acts on the wall and not on our system of interest. Thus $\mathbf{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathbf{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.

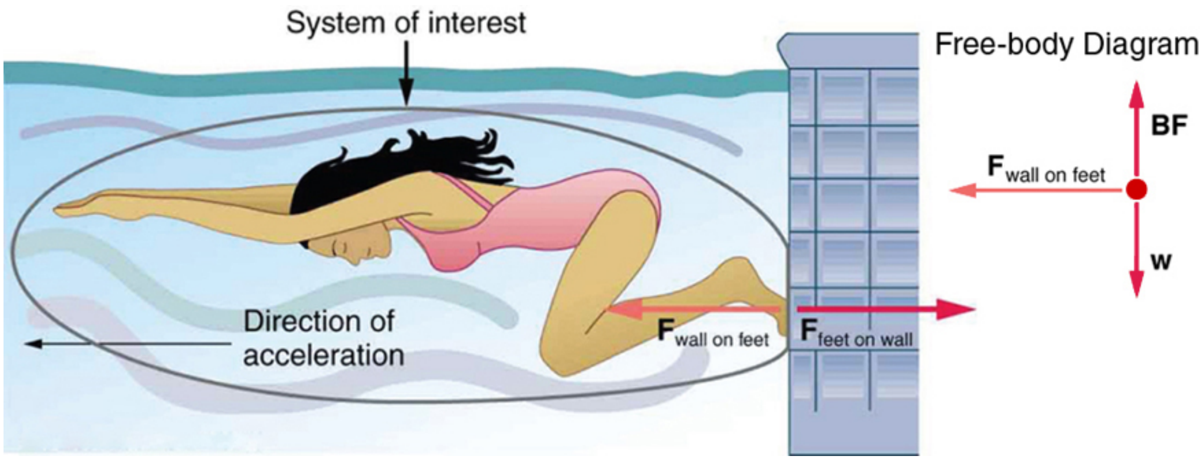


Figure 4.9 When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to $\mathbf{F}_{\text{feet on wall}}$. This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force $\mathbf{F}_{\text{wall on feet}}$ on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that $\mathbf{F}_{\text{feet on wall}}$ does not act on this system (the swimmer) and, thus, does not cancel $\mathbf{F}_{\text{wall on feet}}$. Thus the free-body diagram shows only $\mathbf{F}_{\text{wall on feet}}$, $\mathbf{w}$, the gravitational force, and $\mathbf{BF}$, the buoyant force of the water supporting the swimmer's weight. The vertical forces $\mathbf{w}$ and $\mathbf{BF}$ cancel since there is no vertical motion.

Similarly, when a person stands on Earth, the Earth exerts a force on the person, pulling the person toward the Earth. As stated by Newton's third law of motion, the person also exerts a force that is equal in magnitude, but opposite in direction, pulling the Earth up toward the person. Since the mass of the Earth is so great, however, and $F = ma$, the acceleration of the Earth toward the person is not noticeable.

Other examples of Newton's third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called **thrust**. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases. Helicopters similarly create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski. In a situation similar to Sancho's, professional cage fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

Example 4.3 Getting Up To Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in **Figure 4.10**. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

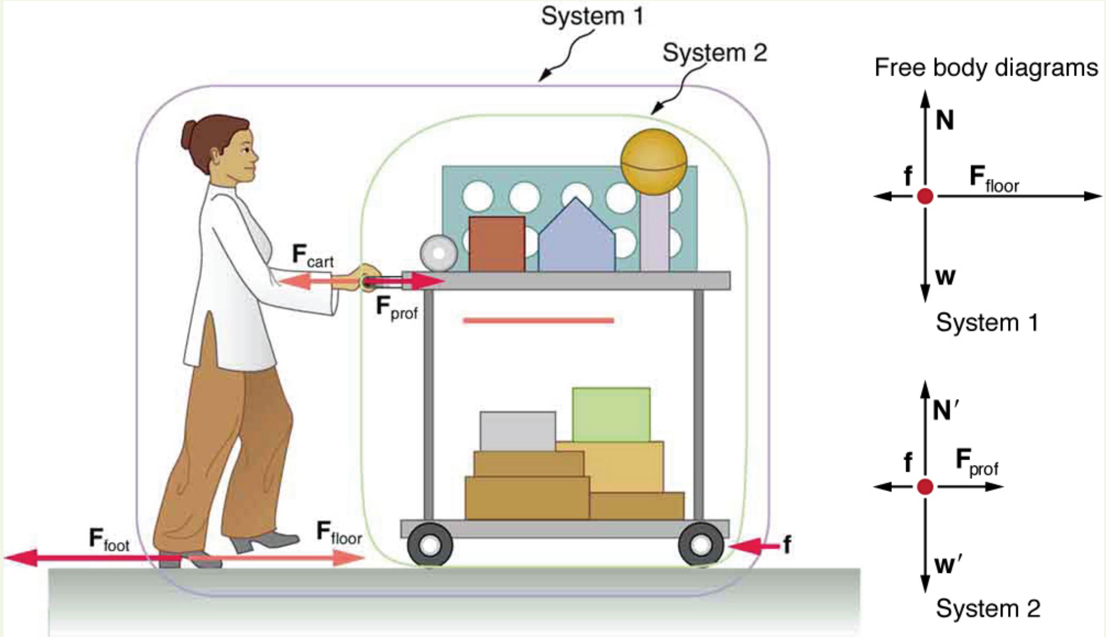


Figure 4.10 A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for $\mathbf{f}$, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for **Example 4.4**, since it asks for the acceleration of the entire group of objects. Only $\mathbf{F}_{\text{floor}}$ and $\mathbf{f}$ are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for this example so that $\mathbf{F}_{\text{prof}}$ will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in **Figure 4.10**. The professor pushes backward with a force $\mathbf{F}_{\text{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\mathbf{F}_{\text{floor}}$ of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction. As noted, $\mathbf{f}$ opposes the motion and is thus in the opposite direction of $\mathbf{F}_{\text{floor}}$. Note that we do not include the forces $\mathbf{F}_{\text{prof}}$ or $\mathbf{F}_{\text{cart}}$ because these are internal forces, and we do not include $\mathbf{F}_{\text{foot}}$ because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

$$a = \frac{F_{\text{net}}}{m}. \tag{4.18}$$

The net external force on System 1 is deduced from **Figure 4.10** and the discussion above to be

$$F_{\text{net}} = F_{\text{floor}} - f = 150\text{ N} - 24.0\text{ N} = 126\text{ N}. \tag{4.19}$$

The mass of System 1 is

$$m = (65.0 + 12.0 + 7.0)\text{ kg} = 84\text{ kg}. \tag{4.20}$$

These values of F_{net} and m produce an acceleration of

$$\begin{aligned} a &= \frac{F_{\text{net}}}{m}, \\ a &= \frac{126\text{ N}}{84\text{ kg}} = 1.5\text{ m/s}^2. \end{aligned} \tag{4.21}$$

Discussion

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Example 4.4 Force on the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in **Figure 4.10** using data from the previous example if needed.

Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in **Figure 4.10**), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, $\mathbf{F}_{\text{prof}}$, is an external force acting on System 2. $\mathbf{F}_{\text{prof}}$ was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

Solution

Newton's second law can be used to find $\mathbf{F}_{\text{prof}}$. Starting with

$$a = \frac{F_{\text{net}}}{m} \tag{4.22}$$

and noting that the magnitude of the net external force on System 2 is

$$F_{\text{net}} = F_{\text{prof}} - f, \tag{4.23}$$

we solve for F_{prof}, the desired quantity:

$$F_{\text{prof}} = F_{\text{net}} + f. \tag{4.24}$$

The value of f is given, so we must calculate net F_{net}. That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$F_{\text{net}} = ma, \tag{4.25}$$

where the mass of System 2 is 19.0 kg (m = 12.0 kg + 7.0 kg) and its acceleration was found to be $a = 1.5 \text{ m/s}^2$ in the previous example. Thus,

$$F_{\text{net}} = ma, \tag{4.26}$$

$$F_{\text{net}} = (19.0 \text{ kg})(1.5 \text{ m/s}^2) = 29 \text{ N}. \tag{4.27}$$

Now we can find the desired force:

$$F_{\text{prof}} = F_{\text{net}} + f, \tag{4.28}$$

$$F_{\text{prof}} = 29 \text{ N} + 24.0 \text{ N} = 53 \text{ N}. \tag{4.29}$$

Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

PhET Explorations: Gravity Force Lab

Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force.

PhET Interactive Simulation

Figure 4.11 Gravity Force Lab (http://cnx.org/content/m54849/1.2/gravity-force-lab_en.jar)

4.5 Normal, Tension, and Other Examples of Force

Learning Objectives

By the end of this section, you will be able to:

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

The information presented in this section supports the following AP® learning objectives and science practices:

- **2.B.1.1** The student is able to apply $F = mg$ to calculate the gravitational force on an object with mass m in a gravitational field of strength g in the context of the effects of a net force on objects and systems. **(S.P. 2.2, 7.2)**
- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.1** The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. **(S.P. 6.4, 7.2)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.A.4.1** The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. **(S.P. 1.4, 6.2)**
- **3.A.4.2** The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. **(S.P. 6.4, 7.2)**
- **3.A.4.3** The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. **(S.P. 1.4)**
- **3.B.1.3** The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. **(S.P. 1.5, 2.2)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

Normal Force

Weight (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in **Figure 4.12**(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in **Figure 4.12**(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

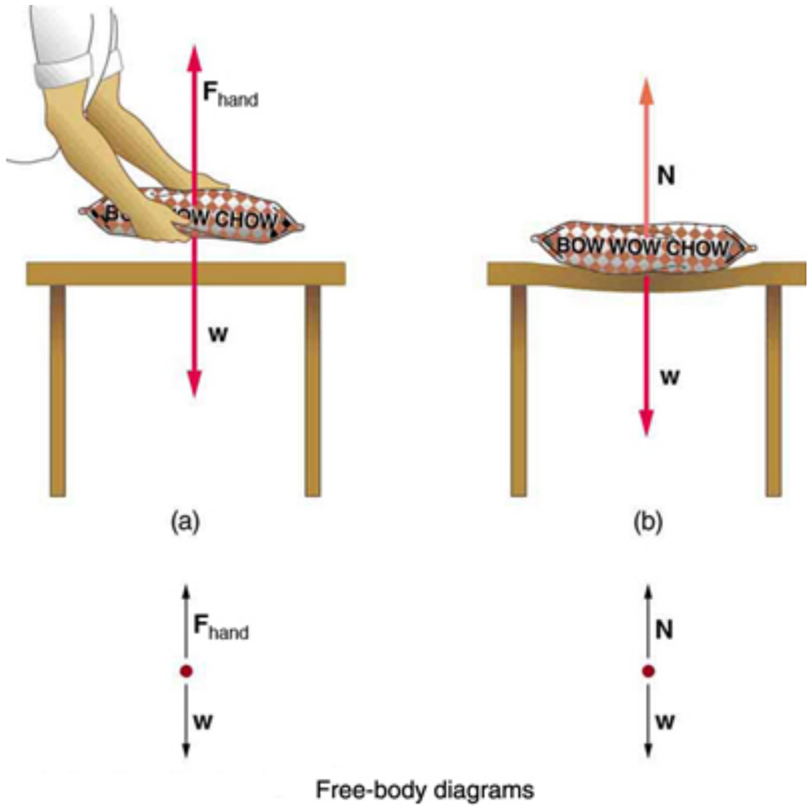


Figure 4.12 (a) The person holding the bag of dog food must supply an upward force $\mathbf{F}_{\text{hand}}$ equal in magnitude and opposite in direction to the weight of the food $\mathbf{w}$. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force $\mathbf{N}$ equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol $\mathbf{N}$. (This is not the unit for force N.) The word *normal* means perpendicular to a surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

Common Misconception: Normal Force (N) vs. Newton (N)

In this section we have introduced the quantity normal force, which is represented by the variable $\mathbf{N}$. This should not be confused with the symbol for the newton, which is also represented by the letter N. These symbols are particularly important to distinguish because the units of a normal force ($\mathbf{N}$) happen to be newtons (N). For example, the normal force $\mathbf{N}$ that the floor exerts on a chair might be $\mathbf{N} = 100\text{ N}$. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work (W) and the unit watts (W).

Example 4.5 Weight on an Incline, a Two-Dimensional Problem

Consider the skier on a slope shown in Figure 4.13. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be 45.0 N?

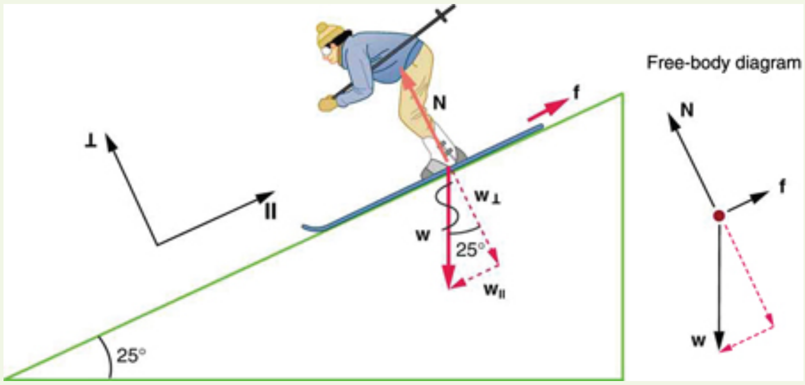


Figure 4.13 Since motion and friction are parallel to the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). $\mathbf{N}$ is perpendicular to the slope and **f** is parallel to the slope, but $\mathbf{w}$ has components along both axes, namely $\mathbf{w}_{\perp}$ and $\mathbf{w}_{\parallel}$. $\mathbf{N}$ is equal in magnitude to $\mathbf{w}_{\perp}$, so that there is no motion perpendicular to the slope, but f is less than $w_{\parallel}$, so that there is a downslope acceleration (along the parallel axis).

Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating *two* connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols $\perp$ and $\parallel$ to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier's weight, friction, and the support of the slope, respectively labeled $\mathbf{w}$, $\mathbf{f}$, and $\mathbf{N}$ in **Figure 4.13**. $\mathbf{N}$ is always perpendicular to the slope, and $\mathbf{f}$ is parallel to it. But $\mathbf{w}$ is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining $w_{\parallel}$ to be the component of weight parallel to the slope and $w_{\perp}$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of the weight parallel to the slope is $w_{\parallel} = w \sin(25^{\circ}) = mg \sin(25^{\circ})$, and the magnitude of the component of the weight perpendicular to the slope is $w_{\perp} = w \cos(25^{\circ}) = mg \cos(25^{\circ})$.

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope $w_{\parallel}$ and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$a_{\parallel} = \frac{F_{\text{net}\,\parallel}}{m} \tag{4.30}$$

where $F_{\text{net}\,\parallel} = w_{\parallel} = mg \sin(25^{\circ})$, assuming no friction for this part, so that

$$a_{\parallel} = \frac{F_{\text{net}\,\parallel}}{m} = \frac{mg \sin(25^{\circ})}{m} = g \sin(25^{\circ}) \tag{4.31}$$

$$(9.80 \text{ m/s}^2)(0.4226) = 4.14 \text{ m/s}^2 \tag{4.32}$$

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$F_{\text{net}\,\parallel} = w_{\parallel} - f, \tag{4.33}$$

and substituting this into Newton's second law, $a_{\parallel} = \dfrac{F_{\text{net}\,\parallel}}{m}$, gives

$$a_{\parallel} = \frac{F_{\text{net}\,\parallel}}{m} = \frac{w_{\parallel} - f}{m} = \frac{mg\sin(25^\circ) - f}{m}. \tag{4.34}$$

We substitute known values to obtain

$$a_{\parallel} = \frac{(60.0\text{ kg})(9.80\text{ m/s}^2)(0.4226) - 45.0\text{ N}}{60.0\text{ kg}}, \tag{4.35}$$

which yields

$$a_{\parallel} = 3.39\text{ m/s}^2, \tag{4.36}$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is $a = g\sin\theta$, *regardless of mass*. This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

Resolving Weight into Components

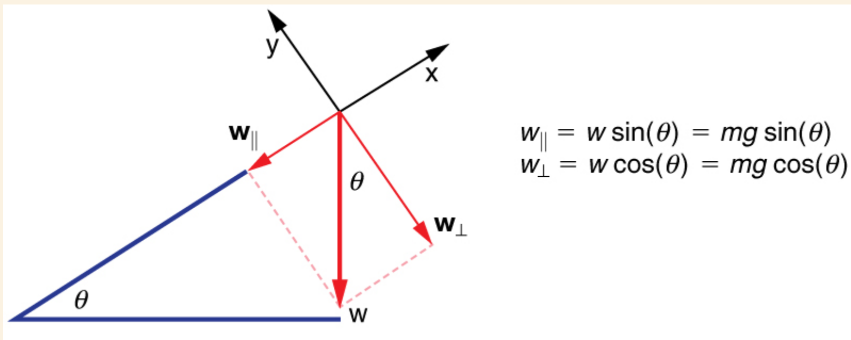


Figure 4.14 An object rests on an incline that makes an angle θ with the horizontal.

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, $\mathbf{w}_{\perp}$, and a force acting parallel to the plane, $\mathbf{w}_{\parallel}$. The perpendicular force of weight, $\mathbf{w}_{\perp}$, is typically equal in magnitude and opposite in direction to the normal force, $\mathbf{N}$. The force acting parallel to the plane, $\mathbf{w}_{\parallel}$, causes the object to accelerate down the incline. The force of friction, $\mathbf{f}$, opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$w_{\parallel} = w\sin(\theta) = mg\sin(\theta) \tag{4.37}$$

and

$$w_{\perp} = w\cos(\theta) = mg\cos(\theta). \tag{4.38}$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle θ of the incline is the same as the angle formed between $\mathbf{w}$ and $\mathbf{w}_{\perp}$. Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

$$\begin{aligned} \cos(\theta) &= \frac{w_{\perp}}{w} \\ w_{\perp} &= w\cos(\theta) = mg\cos(\theta) \end{aligned} \tag{4.39}$$

$$\begin{aligned} \sin(\theta) &= \frac{w_{\parallel}}{w} \\ w_{\parallel} &= w\sin(\theta) = mg\sin(\theta) \end{aligned} \tag{4.40}$$

Take-Home Experiment: Force Parallel

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word “tension” comes from a Latin word meaning “to stretch.” Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: “You can’t push a rope.” The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in **Figure 4.15**.

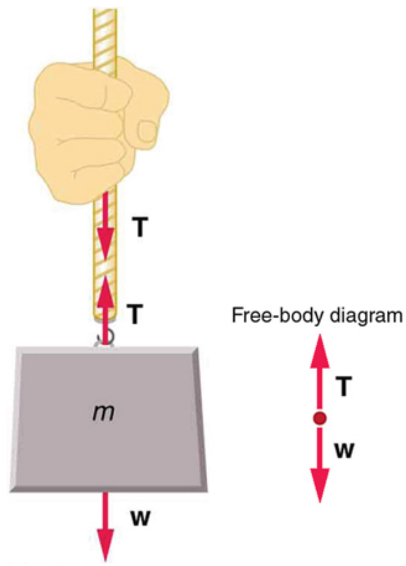


Figure 4.15 When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force $\mathbf{T}$, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton’s third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton’s second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight $\mathbf{w}$ and the tension $\mathbf{T}$ supplied by the rope. Thus,

$$F_{\text{net}} = T - w = 0, \tag{4.41}$$

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

$$T = w = mg. \tag{4.42}$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$T = mg = (5.00\text{ kg})(9.80\text{ m/s}^2) = 49.0\text{ N}. \tag{4.43}$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in **Figure 4.16** (a) and (b).

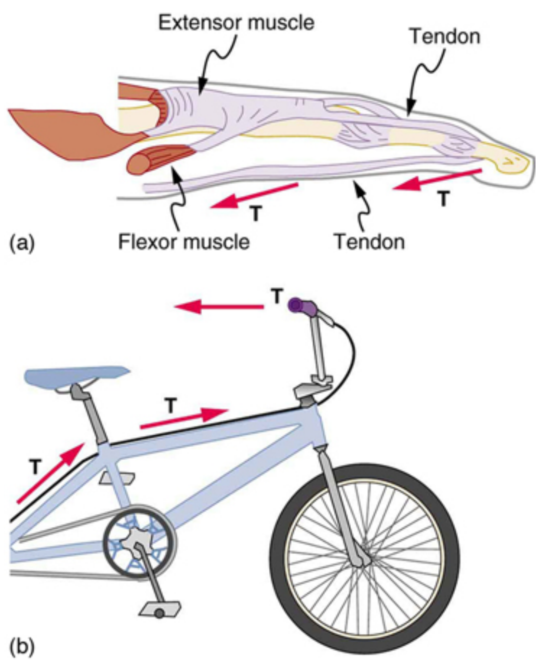


Figure 4.16 (a) Tendons in the finger carry force $\mathbf{T}$ from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension $\mathbf{T}$ from the handlebars to the brake mechanism. Again, the direction but not the magnitude of $\mathbf{T}$ is changed.

Example 4.6 What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in **Figure 4.17**.

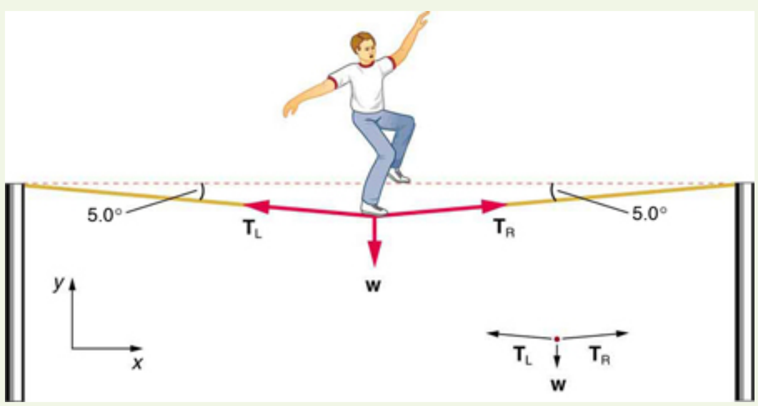


Figure 4.17 The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.

Strategy

As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight $\mathbf{w}$ and the two tensions $\mathbf{T}_\mathrm{L}$ (left tension) and $\mathbf{T}_\mathrm{R}$ (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions T_L and T_R must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are T_L and T_R. Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the x -axis and the vertical the y -axis.

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.

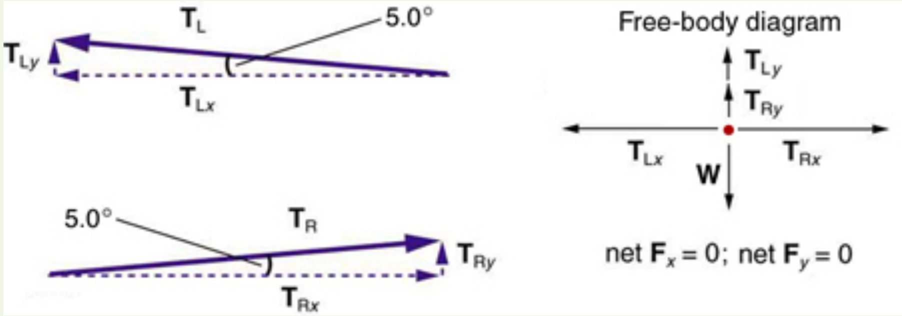


Figure 4.18 When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in T being much greater than w.

Consider the horizontal components of the forces (denoted with a subscript x):

$$F_{\text{net}x} = T_{\text{L}x} - T_{\text{R}x}. \tag{4.44}$$

The net external horizontal force $F_{\text{net}x} = 0$, since the person is stationary. Thus,

$$\begin{aligned} F_{\text{net}x} = 0 &= T_{\text{L}x} - T_{\text{R}x} \\ T_{\text{L}x} &= T_{\text{R}x}. \end{aligned} \tag{4.45}$$

Now, observe **Figure 4.18**. You can use trigonometry to determine the magnitude of T_{L} and T_{R}. Notice that:

$$\begin{aligned} \cos(5.0^\circ) &= \frac{T_{\text{L}x}}{T_{\text{L}}} \\ T_{\text{L}x} &= T_{\text{L}} \cos(5.0^\circ) \\ \cos(5.0^\circ) &= \frac{T_{\text{R}x}}{T_{\text{R}}} \\ T_{\text{R}x} &= T_{\text{R}} \cos(5.0^\circ). \end{aligned} \tag{4.46}$$

Equating $T_{\text{L}x}$ and $T_{\text{R}x}$:

$$T_{\text{L}} \cos(5.0^\circ) = T_{\text{R}} \cos(5.0^\circ). \tag{4.47}$$

Thus,

$$T_{\text{L}} = T_{\text{R}} = T, \tag{4.48}$$

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton's second law implies that net $F_y = 0$. Thus, as illustrated in the free-body diagram in **Figure 4.18**,

$$F_{\text{net}y} = T_{\text{L}y} + T_{\text{R}y} - w = 0. \tag{4.49}$$

Observing **Figure 4.18**, we can use trigonometry to determine the relationship between $T_{\text{L}y}$, $T_{\text{R}y}$, and T. As we determined from the analysis in the horizontal direction, $T_{\text{L}} = T_{\text{R}} = T$:

$$\begin{aligned} \sin(5.0^\circ) &= \frac{T_{\text{L}y}}{T_{\text{L}}} \\ T_{\text{L}y} = T_{\text{L}} \sin(5.0^\circ) &= T \sin(5.0^\circ) \\ \sin(5.0^\circ) &= \frac{T_{\text{R}y}}{T_{\text{R}}} \\ T_{\text{R}y} = T_{\text{R}} \sin(5.0^\circ) &= T \sin(5.0^\circ). \end{aligned} \tag{4.50}$$

Now, we can substitute the values for T_{Ly} and T_{Ry}, into the net force equation in the vertical direction:

$$\begin{aligned} F_{\text{net}y} &= T_{Ly} + T_{Ry} - w = 0 \\ F_{\text{net}y} &= T\sin(5.0^\circ) + T\sin(5.0^\circ) - w = 0 \\ 2\,T\sin(5.0^\circ) - w &= 0 \\ 2\,T\sin(5.0^\circ) &= w \end{aligned} \tag{4.51}$$

and

$$T = \frac{w}{2\sin(5.0^\circ)} = \frac{mg}{2\sin(5.0^\circ)}, \tag{4.52}$$

so that

$$T = \frac{(70.0\text{ kg})(9.80\text{ m/s}^2)}{2(0.0872)}, \tag{4.53}$$

and the tension is

$$T = 3900\text{ N}. \tag{4.54}$$

Discussion

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in **Figure 4.19**. As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:

$$T = \frac{w}{2\sin(\theta)}. \tag{4.55}$$

We can extend this expression to describe the tension T created when a perpendicular force ($\mathbf{F}_\perp$) is exerted at the middle of a flexible connector:

$$T = \frac{F_\perp}{2\sin(\theta)}. \tag{4.56}$$

Note that θ is the angle between the horizontal and the bent connector. In this case, T becomes very large as θ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., $\theta = 0$ and $\sin\theta = 0$). (See **Figure 4.19**.)

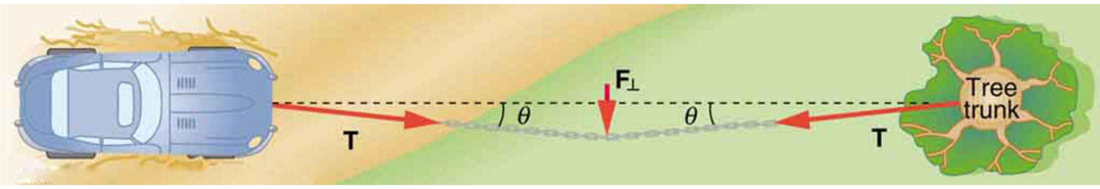


Figure 4.19 We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by $T = \dfrac{F_\perp}{2\sin(\theta)}$; since θ is small, T is very large. This situation is analogous to the tightrope walker shown in **Figure 4.17**, except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where $\mathbf{F}_\perp$ is applied.

Figure 4.20 Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)

Extended Topic: Real Forces and Inertial Frames

There is another distinction among forces in addition to the types already mentioned. Some forces are real, whereas others are not. *Real forces* are those that have some physical origin, such as the gravitational pull. Contrastingly, *fictitious forces* are those that arise simply because an observer is in an accelerating frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's northern hemisphere, then to an observer on Earth it will appear to experience a force to the west that has no physical origin. Of course, what is happening here is that Earth is rotating toward the east and moves east under the satellite. In Earth's frame this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). An **inertial frame of reference** is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On the large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are considered in inertial frames.

All the forces discussed in this section are real forces, but there are a number of other real forces, such as lift and thrust, that are not discussed in this section. They are more specialized, and it is not necessary to discuss every type of force. It is natural, however, to ask where the basic simplicity we seek to find in physics is in the long list of forces. Are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as will be seen in the next (extended) section and in the treatment of modern physics later in the text.

PhET Explorations: Forces in 1 Dimension

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

PhET Interactive Simulation

Figure 4.21 Forces in 1 Dimension (http://cnx.org/content/m54857/1.4/forces-1d_en.jar)

4.6 Problem-Solving Strategies

Learning Objectives

By the end of this section, you will be able to:

- Apply a problem-solving procedure to solve problems using Newton's laws of motion

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.B.1.1** The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. **(S.P. 6.4, 7.2)**
- **3.B.1.3** The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. **(S.P. 1.5, 2.2)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton’s laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

Problem-Solving Strategy for Newton’s Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton’s laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation*. Such a sketch is shown in **Figure 4.22**(a). Then, as in **Figure 4.22**(b), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).

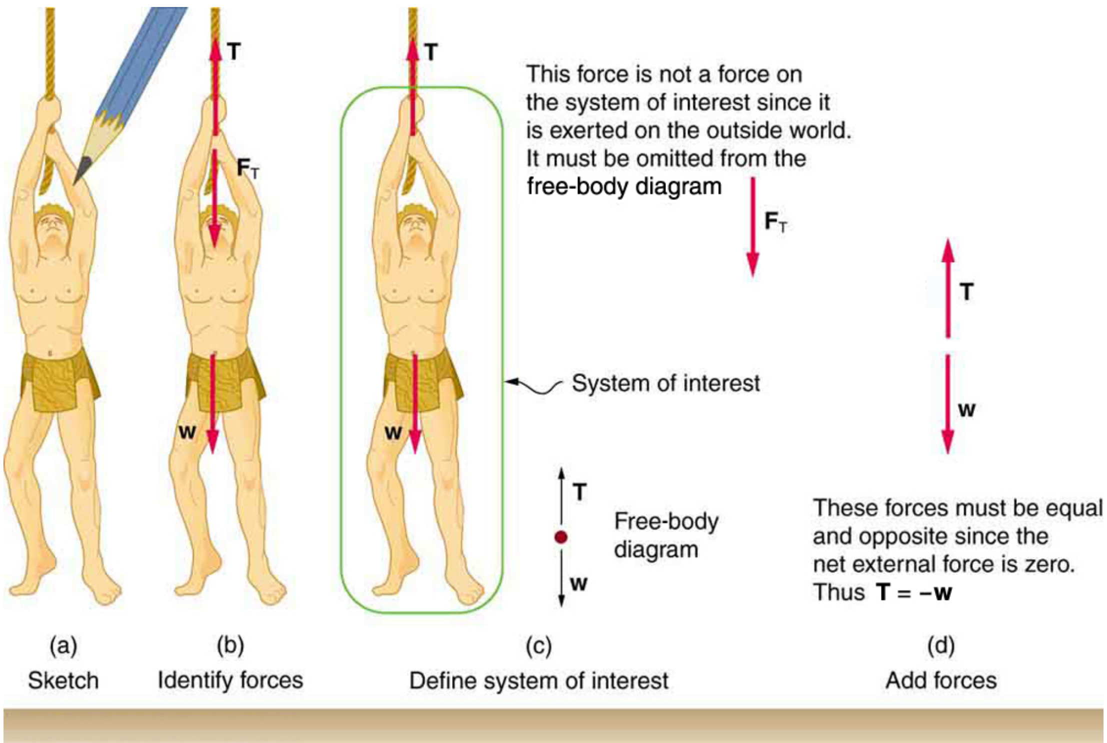


Figure 4.22 (a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces. $\mathbf{T}$ is the tension in the vine above Tarzan, $\mathbf{F}_\mathrm{T}$ is the force he exerts on the vine, and $\mathbf{w}$ is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man’s mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram. $\mathbf{F}_\mathrm{T}$ is no longer shown, because it is not a force acting on the system of interest; rather, $\mathbf{F}_\mathrm{T}$ acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that $\mathbf{T} = -\mathbf{w}$, if Tarzan is stationary.

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest*. This decision is a crucial step, since Newton’s second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to employ Newton’s second law. (See **Figure 4.22**(c).) Newton’s third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a **free-body diagram**. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. Figure 4.22(c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem*. This is done in Figure 4.22(d) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known.

Applying Newton's Second Law

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation: $F_{\text{net}} = ma$.

For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

$$F_{\text{net }x} = ma, \tag{4.57}$$

$$F_{\text{net }y} = 0. \tag{4.58}$$

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable*. In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

4.7 Further Applications of Newton's Laws of Motion

Learning Objectives

By the end of this section, you will be able to:

- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.1** The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. **(S.P. 6.4, 7.2)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.B.1.1** The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. **(S.P. 6.4, 7.2)**
- **3.B.1.3** The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. **(S.P. 1.5, 2.2)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

Example 4.7 Drag Force on a Barge

Suppose two tugboats push on a barge at different angles, as shown in Figure 4.23. The first tugboat exerts a force of $2.7\times10^5\ \text{N}$ in the *x*-direction, and the second tugboat exerts a force of $3.6\times10^5\ \text{N}$ in the *y*-direction.

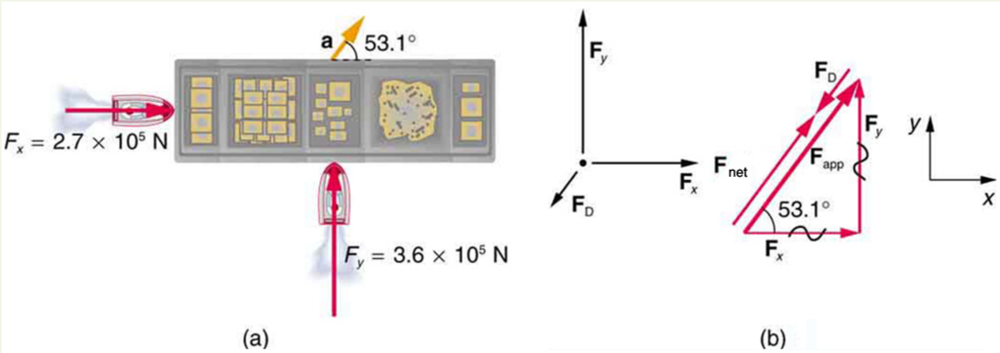


Figure 4.23 (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the *x*- and *y*-axes are in the same direction as $\mathbf{F}_x$ and $\mathbf{F}_y$. The problem quickly becomes a one-dimensional problem along the direction of $\mathbf{F}_{\text{app}}$, since friction is in the direction opposite to $\mathbf{F}_{\text{app}}$.

If the mass of the barge is $5.0\times10^6\ \text{kg}$ and its acceleration is observed to be $7.5\times10^{-2}\ \text{m/s}^2$ in the direction shown, what is the drag force of the water on the barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

Strategy

The directions and magnitudes of acceleration and the applied forces are given in Figure 4.23**(a)**. We will define the total force of the tugboats on the barge as $\mathbf{F}_{\text{app}}$ so that:

$$\mathbf{F}_{\text{app}}=\mathbf{F}_x+\mathbf{F}_y \tag{4.59}$$

Since the barge is flat bottomed, the drag of the water $\mathbf{F}_\text{D}$ will be in the direction opposite to $\mathbf{F}_{\text{app}}$, as shown in the free-body diagram in Figure 4.23(b). The system of interest here is the barge, since the forces on *it* are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force $\mathbf{F}_{\text{app}}$, and then apply Newton's second law to solve for the drag force $\mathbf{F}_\text{D}$.

Solution

Since $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular, the magnitude and direction of $\mathbf{F}_{\text{app}}$ are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

$$\begin{aligned} F_{\text{app}} &= \sqrt{\text{F}_x^2+\text{F}_y^2} \\ F_{\text{app}} &= \sqrt{(2.7\times10^5\ \text{N})^2+(3.6\times10^5\ \text{N})^2} \;=\; 4.5\times10^5\ \text{N}. \end{aligned} \tag{4.60}$$

The angle is given by

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{F_y}{F_x}\right) \\ \theta &= \tan^{-1}\left(\frac{3.6\times10^5\ \text{N}}{2.7\times10^5\ \text{N}}\right)=53^\circ, \end{aligned} \tag{4.61}$$

which we know, because of Newton's first law, is the same direction as the acceleration. $\mathbf{F}_\text{D}$ is in the opposite direction of $\mathbf{F}_{\text{app}}$, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as $\mathbf{F}_{\text{app}}$, but its magnitude is slightly less than $\mathbf{F}_{\text{app}}$. The problem is now one-dimensional. From Figure 4.23**(b)**, we can see that

$$F_{\text{net}}=F_{\text{app}}-F_\text{D}. \tag{4.62}$$

But Newton's second law states that

$$F_{\text{net}} = ma. \tag{4.63}$$

Thus,

$$F_{\text{app}} - F_{\text{D}} = ma. \tag{4.64}$$

This can be solved for the magnitude of the drag force of the water F_{D} in terms of known quantities:

$$F_{\text{D}} = F_{\text{app}} - ma. \tag{4.65}$$

Substituting known values gives

$$\text{F}_{\text{D}} = (4.5\times10^{5}\text{ N}) - (5.0\times10^{6}\text{ kg})(7.5\times10^{-2}\text{ m/s}^{2}) = 7.5\times10^{4}\text{ N}. \tag{4.66}$$

The direction of $\mathbf{F}_{\text{D}}$ has already been determined to be in the direction opposite to $\mathbf{F}_{\text{app}}$, or at an angle of 53^{o} south of west.

Discussion

The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where F_{D} is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

Example 4.8 Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in **Figure 4.24**. Find the tension in each wire, neglecting the masses of the wires.

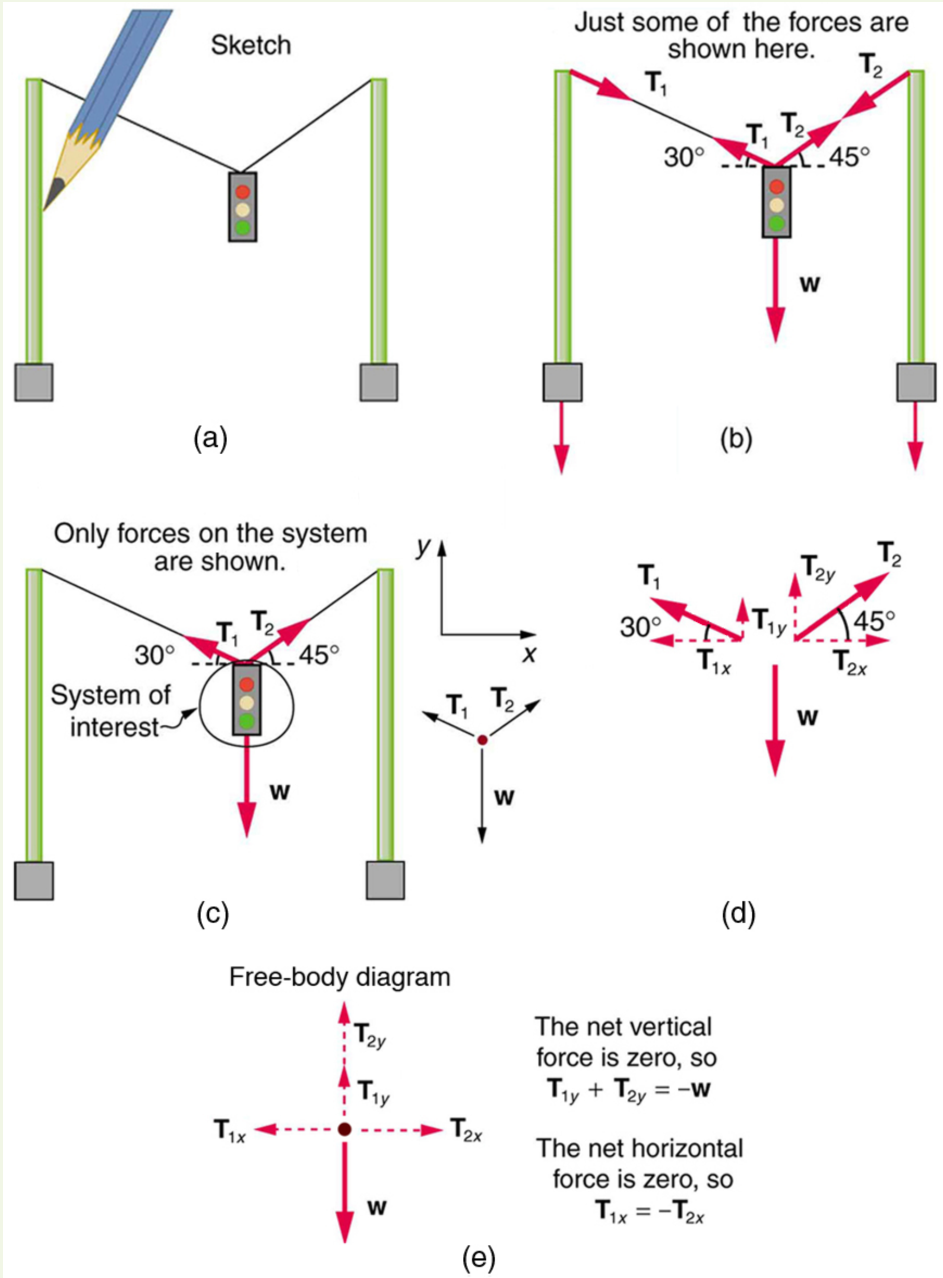


Figure 4.24 A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (y) and horizontal (x) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in **Figure 4.24**(c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem (T_1 and T_2), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or *x*-axis:

$$F_{\text{net}x} = T_{2x} - T_{1x} = 0. \tag{4.67}$$

Thus, as you might expect,

$$T_{1x} = T_{2x}. \tag{4.68}$$

This gives us the following relationship between T_1 and T_2:

$$T_1 \cos(30°) = T_2 \cos(45°). \tag{4.69}$$

Thus,

$$T_2 = (1.225)T_1. \tag{4.70}$$

Note that T_1 and T_2 are not equal in this case, because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1, because it is exerted more vertically than T_1.

Now consider the force components along the vertical or *y*-axis:

$$F_{\text{net } y} = T_{1y} + T_{2y} - w = 0. \tag{4.71}$$

This implies

$$T_{1y} + T_{2y} = w. \tag{4.72}$$

Substituting the expressions for the vertical components gives

$$T_1 \sin(30°) + T_2 \sin(45°) = w. \tag{4.73}$$

There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:

$$T_1(0.500) + (1.225T_1)(0.707) = w = mg, \tag{4.74}$$

which yields

$$(1.366)T_1 = (15.0 \text{ kg})(9.80 \text{ m/s}^2). \tag{4.75}$$

Solving this last equation gives the magnitude of T_1 to be

$$T_1 = 108 \text{ N}. \tag{4.76}$$

Finally, the magnitude of T_2 is determined using the relationship between them, T_2 = 1.225 T_1, found above. Thus we obtain

$$T_2 = 132 \text{ N}. \tag{4.77}$$

Discussion

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

Example 4.9 What Does the Bathroom Scale Read in an Elevator?

Figure 4.25 shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of 1.20 m/s^2, and (b) if the elevator moves upward at a constant speed of 1 m/s.

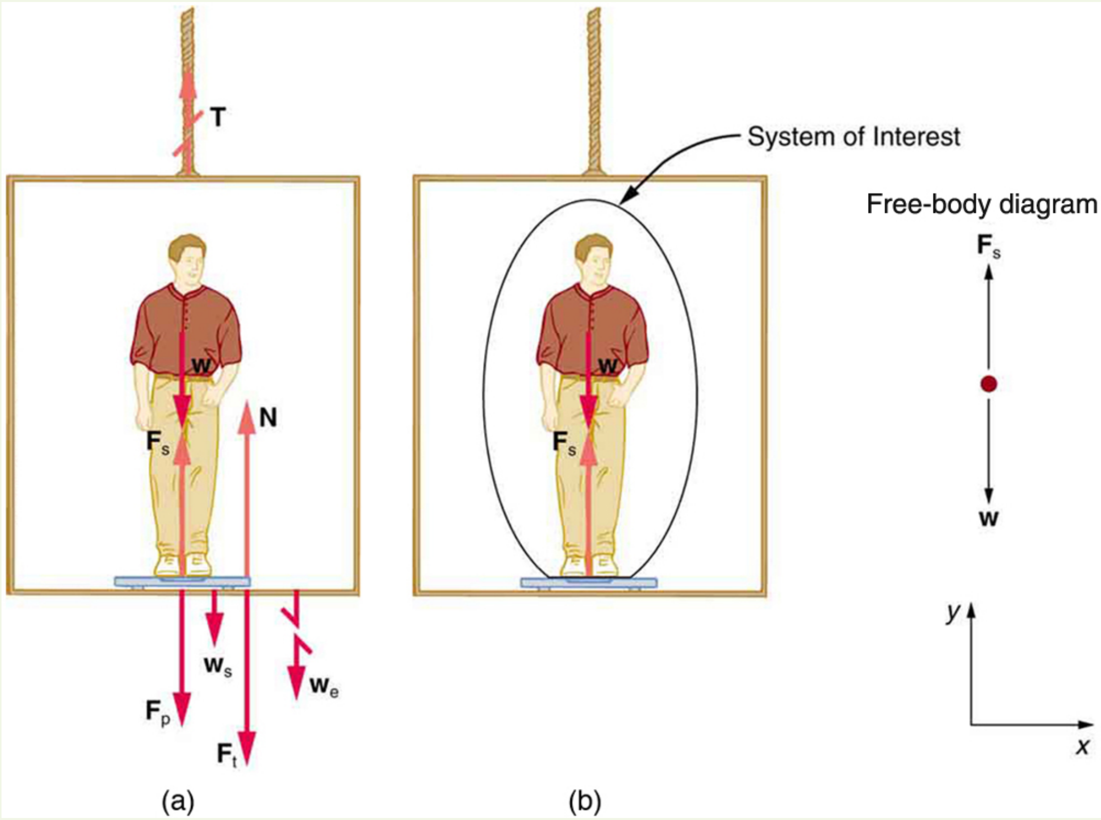


Figure 4.25 (a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. $\mathbf{T}$ is the tension in the supporting cable, $\mathbf{w}$ is the weight of the person, $\mathbf{w}_\text{s}$ is the weight of the scale, $\mathbf{w}_\text{e}$ is the weight of the elevator, $\mathbf{F}_\text{s}$ is the force of the scale on the person, $\mathbf{F}_\text{p}$ is the force of the person on the scale, $\mathbf{F}_\text{t}$ is the force of the scale on the floor of the elevator, and $\mathbf{N}$ is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.

Strategy

If the scale is accurate, its reading will equal F_p, the magnitude of the force the person exerts downward on it. **Figure 4.25**(a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in **Figure 4.25**(b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight $\mathbf{w}$ and the upward force of the scale $\mathbf{F}_\text{s}$. According to Newton's third law $\mathbf{F}_\text{p}$ and $\mathbf{F}_\text{s}$ are equal in magnitude and opposite in direction, so that we need to find F_s in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

$$F_\text{net} = ma. \tag{4.78}$$

From the free-body diagram we see that $F_\text{net} = F_\text{s} - w$, so that

$$F_\text{s} - w = ma. \tag{4.79}$$

Solving for F_s gives an equation with only one unknown:

$$F_\text{s} = ma + w, \tag{4.80}$$

or, because $w = mg$, simply

$$F_\text{s} = ma + mg. \tag{4.81}$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.

Solution for (a)

In this part of the problem, $a = 1.20 \text{ m/s}^2$, so that

$$F_\text{s} = (75.0 \text{ kg})(1.20 \text{ m/s}^2) + (75.0 \text{ kg})(9.80 \text{ m/s}^2), \tag{4.82}$$

yielding

$$F_\text{s} = 825 \text{ N}. \tag{4.83}$$

Discussion for (a)

This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$\begin{aligned} F_\text{net} &= ma = 0 = F_\text{s} - w \\ F_\text{s} &= w = mg \\ F_\text{s} &= (75.0 \text{ kg})(9.80 \text{ m/s}^2) \\ F_\text{s} &= 735 \text{ N}. \end{aligned} \tag{4.84}$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

Solution for (b)

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$, and $\Delta v = 0$.

Thus,

$$F_\text{s} = ma + mg = 0 + mg. \tag{4.85}$$

Now

$$F_\text{s} = (75.0 \text{ kg})(9.80 \text{ m/s}^2), \tag{4.86}$$

which gives

$$F_\text{s} = 735 \text{ N}. \tag{4.87}$$

Discussion for (b)

The scale reading is 735 N, which equals the person’s weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, a is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person’s weight. If the elevator is in free-fall and accelerating downward at g , then the scale reading will be zero and the person will *appear* to be weightless.

Integrating Concepts: Newton’s Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton’s laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

Problem-Solving Strategy

Step 1. *Identify which physical principles are involved*. Listing the givens and the quantities to be calculated will allow you to identify the principles involved.
Step 2. *Solve the problem using strategies outlined in the text*. If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

Example 4.10 What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

Strategy

1. To solve an *integrated concept problem*, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers *acceleration* along a straight line. This is a topic of *kinematics*. Part (b) deals with *force*, a topic of *dynamics* found in this chapter.
2. The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

Solution for (a)

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is $\Delta v = 8.00\text{ m/s}$. We are given the elapsed time, and so $\Delta t = 2.50\text{ s}$. The unknown is acceleration, which can be found from its definition:

$$a = \frac{\Delta v}{\Delta t}. \tag{4.88}$$

Substituting the known values yields

$$\begin{aligned} a &= \frac{8.00\text{ m/s}}{2.50\text{ s}} \\ &= 3.20\text{ m/s}^2. \end{aligned} \tag{4.89}$$

Discussion for (a)

This is an attainable acceleration for an athlete in good condition.

Solution for (b)

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,

$$F_{\text{net}} = ma. \tag{4.90}$$

Substituting the known values of m and a gives

$$\begin{aligned} F_{\text{net}} &= (70.0\text{ kg})(3.20\text{ m/s}^2) \\ &= 224\text{ N}. \end{aligned} \tag{4.91}$$

Discussion for (b)

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

4.8 Extended Topic: The Four Basic Forces—An Introduction

Learning Objectives

By the end of this section, you will be able to:

- Understand the four basic forces that underlie the processes in nature.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.C.4.1** The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces. **(S.P. 6.1)**
- **3.C.4.2** The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions. **(S.P. 6.2)**
- **3.G.1.1** The student is able to articulate situations when the gravitational force is the dominant force and when the electromagnetic, weak, and strong forces can be ignored. **(S.P. 7.1)**

One of the most remarkable simplifications in physics is that only four distinct forces account for all known phenomena. In fact, nearly all of the forces we experience directly are due to only one basic force, called the electromagnetic force. (The gravitational force is the only force we experience directly that is not electromagnetic.) This is a tremendous simplification of the myriad of *apparently* different forces we can list, only a few of which were discussed in the previous section. As we will see, the basic forces are all thought to act through the exchange of microscopic carrier particles, and the characteristics of the basic forces are determined by the types of particles exchanged. Action at a distance, such as the gravitational force of Earth on the Moon, is explained by the existence of a **force field** rather than by "physical contact."

The *four basic forces* are the gravitational force, the electromagnetic force, the weak nuclear force, and the strong nuclear force. Their properties are summarized in **Table 4.2**. Since the weak and strong nuclear forces act over an extremely short range, the size of a nucleus or less, we do not experience them directly, although they are crucial to the very structure of matter. These forces determine which nuclei are stable and which decay, and they are the basis of the release of energy in certain nuclear reactions. Nuclear forces determine not only the stability of nuclei, but also the relative abundance of elements in nature. The properties of the nucleus of an atom determine the number of electrons it has and, thus, indirectly determine the chemistry of the atom. More will be said of all of these topics in later chapters.

Concept Connections: The Four Basic Forces

The four basic forces will be encountered in more detail as you progress through the text. The gravitational force is defined in **Uniform Circular Motion and Gravitation**, electric force in **Electric Charge and Electric Field**, magnetic force in **Magnetism**, and nuclear forces in **Radioactivity and Nuclear Physics**. On a macroscopic scale, electromagnetism and gravity are the basis for all forces. The nuclear forces are vital to the substructure of matter, but they are not directly experienced on the macroscopic scale.

Table 4.2 Properties of the Four Basic Forces[1]

Force	Approximate Relative Strengths	Range	Attraction/Repulsion	Carrier Particle
Gravitational	10^{-38}	∞	attractive only	Graviton
Electromagnetic	10^{-2}	∞	attractive and repulsive	Photon
Weak nuclear	10^{-13}	$< 10^{-18}\text{ m}$	attractive and repulsive	W^+, W^-, Z^0
Strong nuclear	1	$< 10^{-15}\text{ m}$	attractive and repulsive	gluons

The gravitational force is surprisingly weak—it is only because gravity is always attractive that we notice it at all. Our weight is the gravitational force due to the *entire* Earth acting on us. On the very large scale, as in astronomical systems, the gravitational force is the dominant force determining the motions of moons, planets, stars, and galaxies. The gravitational force also affects the nature of space and time. As we shall see later in the study of general relativity, space is curved in the vicinity of very massive bodies, such as the Sun, and time actually slows down near massive bodies.

Take a good look at the ranges for the four fundamental forces listed in **Table 4.2**. The range of the strong nuclear force, 10^{-15} m, is approximately the size of the nucleus of an atom; the weak nuclear force has an even shorter range. At scales on the order of 10−10 m, approximately the size of an atom, both nuclear forces are completely dominated by the electromagnetic force. Notice that this scale is still utterly tiny compared to our everyday experience. At scales that we do experience daily, electromagnetism tends to be negligible, due to its attractive and repulsive properties canceling each other out. That leaves gravity, which is usually the strongest of the forces at scales above ~10^{-4} m, and hence includes our everyday activities, such as throwing, climbing stairs, and walking.

Electromagnetic forces can be either attractive or repulsive. They are long-range forces, which act over extremely large distances, and they nearly cancel for macroscopic objects. (Remember that it is the *net* external force that is important.) If they did not cancel, electromagnetic forces would completely overwhelm the gravitational force. The electromagnetic force is a combination of electrical forces (such as those that cause static electricity) and magnetic forces (such as those that affect a compass needle). These two forces were thought to be quite distinct until early in the 19th century, when scientists began to discover that they are different manifestations of the same force. This discovery is a classical case of the *unification of forces*. Similarly, friction, tension, and all of the other classes of forces we experience directly (except gravity, of course) are due to electromagnetic interactions of atoms and molecules. It is still convenient to consider these forces separately in specific applications, however, because of the ways they manifest themselves.

1. The graviton is a proposed particle, though it has not yet been observed by scientists. See the discussion of gravitational waves later in this section. The particles W^+, W^-, and Z^0 are called vector bosons; these were predicted by theory and first observed in 1983. There are eight types of gluons proposed by scientists, and their existence is indicated by meson exchange in the nuclei of atoms.

Concept Connections: Unifying Forces

Attempts to unify the four basic forces are discussed in relation to elementary particles later in this text. By “unify” we mean finding connections between the forces that show that they are different manifestations of a single force. Even if such unification is achieved, the forces will retain their separate characteristics on the macroscopic scale and may be identical only under extreme conditions such as those existing in the early universe.

Physicists are now exploring whether the four basic forces are in some way related. Attempts to unify all forces into one come under the rubric of Grand Unified Theories (GUTs), with which there has been some success in recent years. It is now known that under conditions of extremely high density and temperature, such as existed in the early universe, the electromagnetic and weak nuclear forces are indistinguishable. They can now be considered to be different manifestations of one force, called the *electroweak* force. So the list of four has been reduced in a sense to only three. Further progress in unifying all forces is proving difficult—especially the inclusion of the gravitational force, which has the special characteristics of affecting the space and time in which the other forces exist.

While the unification of forces will not affect how we discuss forces in this text, it is fascinating that such underlying simplicity exists in the face of the overt complexity of the universe. There is no reason that nature must be simple—it simply is.

Action at a Distance: Concept of a Field

All forces act at a distance. This is obvious for the gravitational force. Earth and the Moon, for example, interact without coming into contact. It is also true for all other forces. Friction, for example, is an electromagnetic force between atoms that may not actually touch. What is it that carries forces between objects? One way to answer this question is to imagine that a **force field** surrounds whatever object creates the force. A second object (often called a *test object*) placed in this field will experience a force that is a function of location and other variables. The field itself is the “thing” that carries the force from one object to another. The field is defined so as to be a characteristic of the object creating it; the field does not depend on the test object placed in it. Earth’s gravitational field, for example, is a function of the mass of Earth and the distance from its center, independent of the presence of other masses. The concept of a field is useful because equations can be written for force fields surrounding objects (for gravity, this yields $w = mg$ at Earth’s surface), and motions can be calculated from these equations.

(See **Figure 4.26**.)

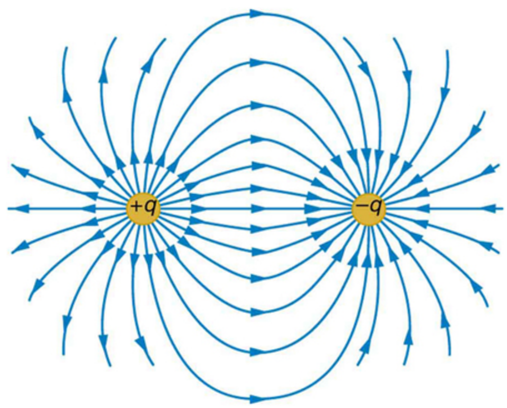


Figure 4.26 The electric force field between a positively charged particle and a negatively charged particle. When a positive test charge is placed in the field, the charge will experience a force in the direction of the force field lines.

Concept Connections: Force Fields

The concept of a *force field* is also used in connection with electric charge and is presented in **Electric Charge and Electric Field**. It is also a useful idea for all the basic forces, as will be seen in **Particle Physics**. Fields help us to visualize forces and how they are transmitted, as well as to describe them with precision and to link forces with subatomic carrier particles.

Making Connections: Vector and Scalar Fields

These fields may be either scalar or vector fields. Gravity and electromagnetism are examples of vector fields. A test object placed in such a field will have both the magnitude and direction of the resulting force on the test object completely defined by the object’s location in the field. We will later cover examples of scalar fields, which have a magnitude but no direction.

The field concept has been applied very successfully; we can calculate motions and describe nature to high precision using field equations. As useful as the field concept is, however, it leaves unanswered the question of what carries the force. It has been proposed in recent decades, starting in 1935 with Hideki Yukawa’s (1907–1981) work on the strong nuclear force, that all forces are transmitted by the exchange of elementary particles. We can visualize particle exchange as analogous to macroscopic phenomena such as two people passing a basketball back and forth, thereby exerting a repulsive force without touching one another. (See **Figure 4.27**.)

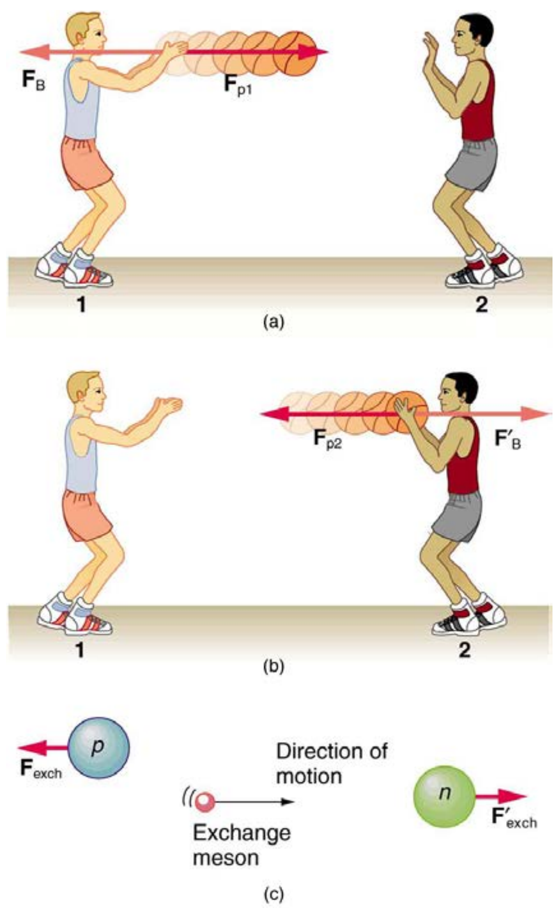


Figure 4.27 The exchange of masses resulting in repulsive forces. (a) The person throwing the basketball exerts a force $\mathbf{F}_{\text{p1}}$ on it toward the other person and feels a reaction force $\mathbf{F}_{\text{B}}$ away from the second person. (b) The person catching the basketball exerts a force $\mathbf{F}_{\text{p2}}$ on it to stop the ball and feels a reaction force $\mathbf{F'}_{\text{B}}$ away from the first person. (c) The analogous exchange of a meson between a proton and a neutron carries the strong nuclear forces $\mathbf{F}_{\text{exch}}$ and $\mathbf{F'}_{\text{exch}}$ between them. An attractive force can also be exerted by the exchange of a mass—if person 2 pulled the basketball away from the first person as he tried to retain it, then the force between them would be attractive.

This idea of particle exchange deepens rather than contradicts field concepts. It is more satisfying philosophically to think of something physical actually moving between objects acting at a distance. Table 4.2 lists the exchange or **carrier particles**, both observed and proposed, that carry the four forces. But the real fruit of the particle-exchange proposal is that searches for Yukawa's proposed particle found it *and* a number of others that were completely unexpected, stimulating yet more research. All of this research eventually led to the proposal of quarks as the underlying substructure of matter, which is a basic tenet of GUTs. If successful, these theories will explain not only forces, but also the structure of matter itself. Yet physics is an experimental science, so the test of these theories must lie in the domain of the real world. As of this writing, scientists at the CERN laboratory in Switzerland are starting to test these theories using the world's largest particle accelerator: the Large Hadron Collider. This accelerator (27 km in circumference) allows two high-energy proton beams, traveling in opposite directions, to collide. An energy of 14 trillion electron volts will be available. It is anticipated that some new particles, possibly force carrier particles, will be found. (See Figure 4.28.) One of the force carriers of high interest that researchers hope to detect is the Higgs boson. The observation of its properties might tell us why different particles have different masses.

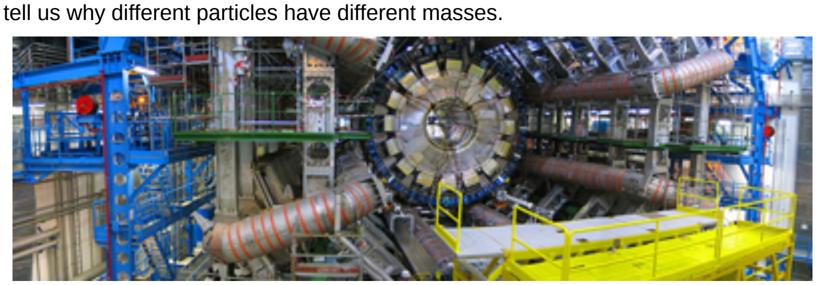


Figure 4.28 The world's largest particle accelerator spans the border between Switzerland and France. Two beams, traveling in opposite directions close to the speed of light, collide in a tube similar to the central tube shown here. External magnets determine the beam's path. Special detectors will analyze particles created in these collisions. Questions as broad as what is the origin of mass and what was matter like the first few seconds of our universe will be explored. This accelerator began preliminary operation in 2008. (credit: Frank Hommes)

Tiny particles also have wave-like behavior, something we will explore more in a later chapter. To better understand force-carrier particles from another perspective, let us consider gravity. The search for gravitational waves has been going on for a number of years. Almost 100 years ago, Einstein predicted the existence of these waves as part of his general theory of relativity. Gravitational waves are created during the collision of massive stars, in black holes, or in supernova explosions—like shock waves. These gravitational waves will travel through space from such sites much like a pebble dropped into a pond sends out ripples—except these waves move at the speed of light. A detector apparatus has been built in the U.S., consisting of two large installations nearly 3000 km apart—one in Washington state and one in Louisiana! The facility is called the Laser Interferometer Gravitational-Wave Observatory (LIGO). Each installation is designed to use optical lasers to examine any slight shift in the relative positions of two masses due to the effect of gravity waves. The two sites allow simultaneous measurements of these small effects to be separated from other natural phenomena, such as earthquakes. Initial operation of the detectors began in 2002, and work is proceeding on increasing their sensitivity. Similar installations have been built in Italy (VIRGO), Germany (GEO600), and Japan (TAMA300) to provide a worldwide network of gravitational wave detectors.

International collaboration in this area is moving into space with the joint EU/US project LISA (Laser Interferometer Space Antenna). Earthquakes and other Earthly noises will be no problem for these monitoring spacecraft. LISA will complement LIGO by looking at much more massive black holes through the observation of gravitational-wave sources emitting much larger wavelengths. Three satellites will be placed in space above Earth in an equilateral triangle (with 5,000,000-km sides) (**Figure 4.29**). The system will measure the relative positions of each satellite to detect passing gravitational waves. Accuracy to within 10% of the size of an atom will be needed to detect any waves. The launch of this project might be as early as 2018.

"I'm sure LIGO will tell us something about the universe that we didn't know before. The history of science tells us that any time you go where you haven't been before, you usually find something that really shakes the scientific paradigms of the day. Whether gravitational wave astrophysics will do that, only time will tell." —David Reitze, LIGO Input Optics Manager, University of Florida

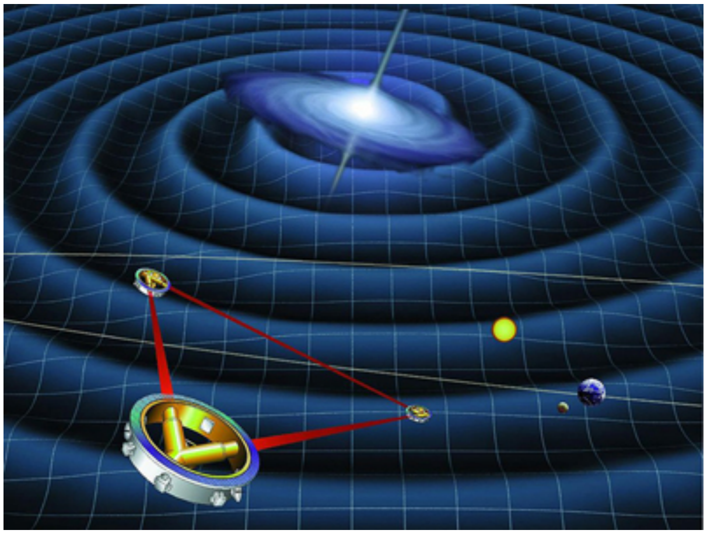


Figure 4.29 Space-based future experiments for the measurement of gravitational waves. Shown here is a drawing of LISA's orbit. Each satellite of LISA will consist of a laser source and a mass. The lasers will transmit a signal to measure the distance between each satellite's test mass. The relative motion of these masses will provide information about passing gravitational waves. (credit: NASA)

The ideas presented in this section are but a glimpse into topics of modern physics that will be covered in much greater depth in later chapters.

Glossary

acceleration: the rate at which an object's velocity changes over a period of time

carrier particle: a fundamental particle of nature that is surrounded by a characteristic force field; photons are carrier particles of the electromagnetic force

dynamics: the study of how forces affect the motion of objects and systems

external force: a force acting on an object or system that originates outside of the object or system

force: a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

force field: a region in which a test particle will experience a force

free-body diagram: a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

free-fall: a situation in which the only force acting on an object is the force due to gravity

friction: a force past each other of objects that are touching; examples include rough surfaces and air resistance

inertia: the tendency of an object to remain at rest or remain in motion

inertial frame of reference: a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

law of inertia: see Newton's first law of motion

mass: the quantity of matter in a substance; measured in kilograms

net external force: the vector sum of all external forces acting on an object or system; causes a mass to accelerate

Newton's first law of motion: in an inertial frame of reference, a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

Newton's second law of motion: the net external force $\mathbf{F}_{\text{net}}$ on an object with mass m is proportional to and in the same direction as the acceleration of the object, $\mathbf{a}$, and inversely proportional to the mass; defined mathematically as $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$

Newton's third law of motion: whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

normal force: the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

system: defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

tension: the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

thrust: a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

weight: the force $\mathbf{w}$ due to gravity acting on an object of mass m; defined mathematically as: $w = mg$, where $\mathbf{g}$ is the magnitude and direction of the acceleration due to gravity

Section Summary

4.1 Development of Force Concept

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- **External forces** are any outside forces that act on a body. A **free-body diagram** is a drawing of all external forces acting on a body.

4.2 Newton's First Law of Motion: Inertia

- **Newton's first law of motion** states that in an inertial frame of reference a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

4.3 Newton's Second Law of Motion: Concept of a System

- Acceleration, $\mathbf{a}$, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$.
- This is often written in the more familiar form: $\mathbf{F}_{\text{net}} = m\mathbf{a}$.
- The weight $\mathbf{w}$ of an object is defined as the force of gravity acting on an object of mass m. The object experiences an acceleration due to gravity $\mathbf{g}$:

$$\mathbf{w} = m\mathbf{g}.$$

- If the only force acting on an object is due to gravity, the object is in free fall.

- Friction is a force that opposes the motion past each other of objects that are touching.

4.4 Newton's Third Law of Motion: Symmetry in Forces

- **Newton’s third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A **thrust** is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

4.5 Normal, Tension, and Other Examples of Force

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, $\mathbf{N}$.
- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object:

$$N = mg.$$

- When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular ($\mathbf{w}_{\perp}$) and parallel ($\mathbf{w}_{\parallel}$) to the surface of the plane. These components can be calculated using:

$$w_{\parallel} = w \sin(\theta) = mg \sin(\theta)$$

$$w_{\perp} = w \cos(\theta) = mg \cos(\theta).$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, $\mathbf{T}$. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:

$$T = mg.$$

- In any inertial frame of reference (one that is not accelerated or rotated), Newton’s laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

4.6 Problem-Solving Strategies

- To solve problems involving Newton’s laws of motion, follow the procedure described:
  1. Draw a sketch of the problem.
  2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
  3. Write Newton’s second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the x -direction) then $F_{\text{net } x} = 0$. If the object does accelerate in that direction, $F_{\text{net } x} = ma$.
  4. Check your answer. Is the answer reasonable? Are the units correct?

4.7 Further Applications of Newton's Laws of Motion

- Newton’s laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $F_{\text{net}} = ma$ or $F_{\text{net}} = 0$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

4.8 Extended Topic: The Four Basic Forces—An Introduction

- The various types of forces that are categorized for use in many applications are all manifestations of the *four basic forces* in nature.
- The properties of these forces are summarized in **Table 4.2**.
- Everything we experience directly without sensitive instruments is due to either electromagnetic forces or gravitational forces. The nuclear forces are responsible for the submicroscopic structure of matter, but they are not directly sensed because of their short ranges. Attempts are being made to show all four forces are different manifestations of a single unified force.
- A force field surrounds an object creating a force and is the carrier of that force.

Conceptual Questions

4.1 Development of Force Concept

1. Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

2. What properties do forces have that allow us to classify them as vectors?

4.2 Newton's First Law of Motion: Inertia

3. How are inertia and mass related?

4. What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

4.3 Newton's Second Law of Motion: Concept of a System

5. Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

6. Why can we neglect forces such as those holding a body together when we apply Newton’s second law of motion?

7. Explain how the choice of the “system of interest” affects which forces must be considered when applying Newton’s second law of motion.

8. Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

9. A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.

10. A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

11. (a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

12. If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

13. If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

14. The gravitational force on the basketball in **Figure 4.6** is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

4.4 Newton's Third Law of Motion: Symmetry in Forces

15. When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)

16. A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the “ballistocardiograph.” What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

17. Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton’s laws of motion apply?

18. Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton’s third law applies when one is fired. Can you safely stand close behind one when it is fired?

19. An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton’s laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

20. Newton’s third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the “system of interest” affects whether one such pair of forces cancels.

4.5 Normal, Tension, and Other Examples of Force

21. If a leg is suspended by a traction setup as shown in **Figure 4.30**, what is the tension in the rope?

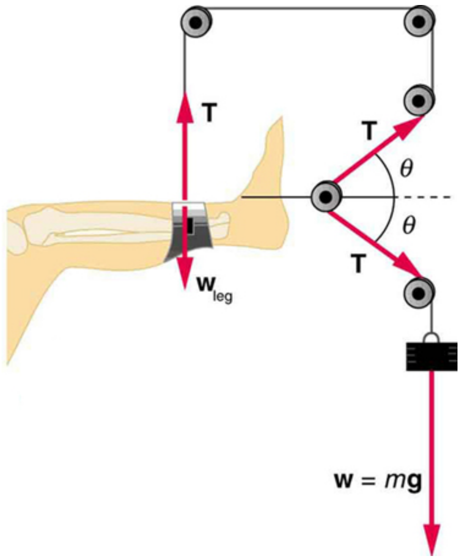


Figure 4.30 A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force *T* without changing its magnitude.

22. In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See **Figure 4.30**.) (Note that the tibia is the shin bone shown in this image.)

4.7 Further Applications of Newton's Laws of Motion

23. To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g . Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

24. A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

4.8 Extended Topic: The Four Basic Forces—An Introduction

25. Explain, in terms of the properties of the four basic forces, why people notice the gravitational force acting on their bodies if it is such a comparatively weak force.

26. What is the dominant force between astronomical objects? Why are the other three basic forces less significant over these very large distances?

27. Give a detailed example of how the exchange of a particle can result in an *attractive* force. (For example, consider one child pulling a toy out of the hands of another.)

Problems & Exercises

4.3 Newton's Second Law of Motion: Concept of a System

You may assume data taken from illustrations is accurate to three digits.

1. A 63.0-kg sprinter starts a race with an acceleration of 4.20 m/s^2. What is the net external force on him?

2. If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

3. A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

4. Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be 0.893 m/s^2. (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.

5. In **Figure 4.7**, the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?

6. The same rocket sled drawn in **Figure 4.31** is decelerated at a rate of 196 m/s^2. What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.

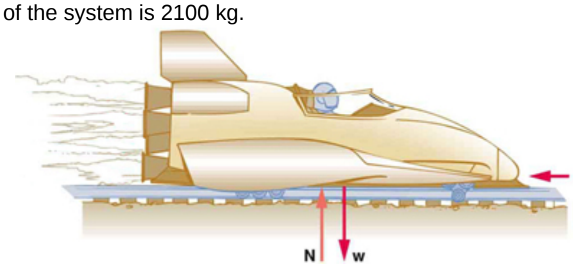


Figure 4.31

7. (a) If the rocket sled shown in **Figure 4.32** starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust T is 2.4×10^4 N, and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

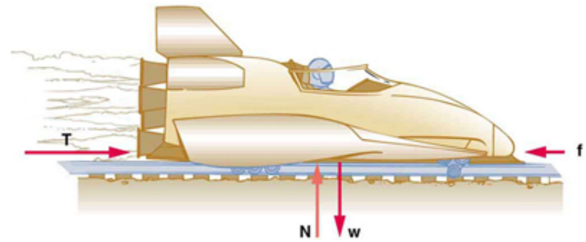


Figure 4.32

8. What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

9. Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (b) Draw a free-body diagram, including all forces acting on the system. (c) Calculate the acceleration. (d) What would the acceleration be if friction were 15.0 N?

10. A powerful motorcycle can produce an acceleration of 3.50 m/s^2 while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?

11. The rocket sled shown in **Figure 4.33** accelerates at a rate of 49.0 m/s^2. Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

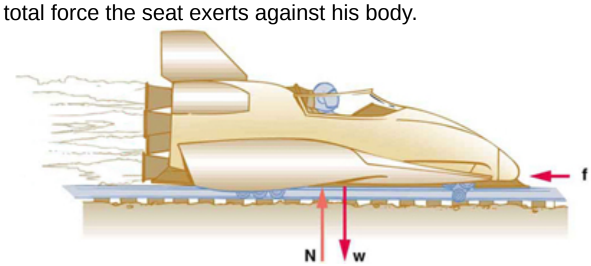


Figure 4.33

12. Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of 201 m/s^2. In this problem, the forces are exerted by the seat and restraining belts.

13. The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?

14. Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

4.4 Newton's Third Law of Motion: Symmetry in Forces

15. What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at $2.40\times10^4\ \text{m/s}^2$? What is the magnitude of the force exerted on the ship by the artillery shell?

16. A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at $1.20\ \text{m/s}^2$ backward. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and write the net force equation.

4.5 Normal, Tension, and Other Examples of Force

17. Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?

18. What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at $7.50\ \text{m/s}^2$? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

19. (a) Calculate the tension in a vertical strand of spider web if a spider of mass $8.00\times10^{-5}\ \text{kg}$ hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in Figure 4.17. The strand sags at an angle of 12° below the horizontal. Compare this with the tension in the vertical strand (find their ratio).

20. Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of $1.50\ \text{m/s}^2$?

21. Show that, as stated in the text, a force $\mathbf{F}_\perp$ exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in Figure 4.17) gives rise to a tension of magnitude $T = \dfrac{F_\perp}{2\sin(\theta)}$.

22. Consider the baby being weighed in Figure 4.34. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension T_1 in the cord attaching the baby to the scale? (c) What is the tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.

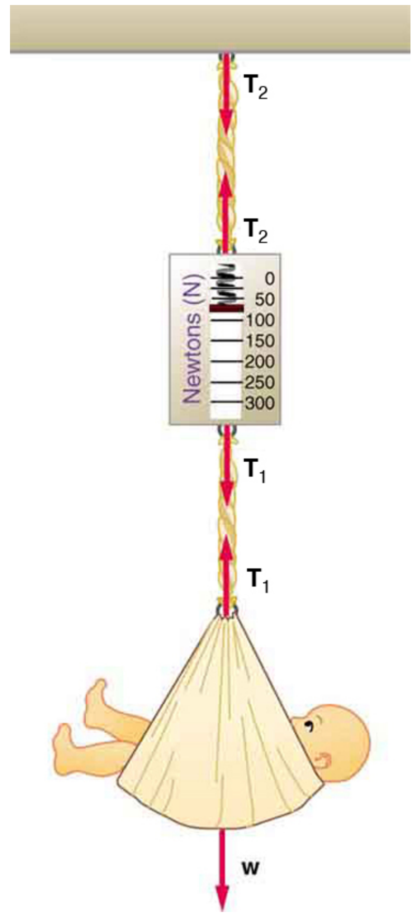


Figure 4.34 A baby is weighed using a spring scale.

4.6 Problem-Solving Strategies

23. A 5.00×10^5-kg rocket is accelerating straight up. Its engines produce $1.250\times10^7\ \text{N}$ of thrust, and air resistance is $4.50\times10^6\ \text{N}$. What is the rocket's acceleration? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

24. The wheels of a midsize car exert a force of 2100 N backward on the road to accelerate the car in the forward direction. If the force of friction including air resistance is 250 N and the acceleration of the car is $1.80\ \text{m/s}^2$, what is the mass of the car plus its occupants? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. For this situation, draw a free-body diagram and write the net force equation.

25. Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

26. When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

27. A freight train consists of two $8.00\times10^4\text{-kg}$ engines and 45 cars with average masses of $5.50\times10^4\ \text{kg}$. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00\times10^{-2}\ \text{m/s}^2$ if the force of friction is $7.50\times10^5\ \text{N}$, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

28. Commercial airplanes are sometimes pushed out of the passenger loading area by a tractor. (a) An 1800-kg tractor exerts a force of $1.75\times10^4\ \text{N}$ backward on the pavement, and the system experiences forces resisting motion that total 2400 N. If the acceleration is $0.150\ \text{m/s}^2$, what is the mass of the airplane? (b) Calculate the force exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane. (c) Draw two sketches showing the systems of interest used to solve each part, including the free-body diagrams for each.

29. A 1100-kg car pulls a boat on a trailer. (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900-N force on the road and produces an acceleration of $0.550\ \text{m/s}^2$? The mass of the boat plus trailer is 700 kg. (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?

30. (a) Find the magnitudes of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ that add to give the total force $\mathbf{F}_{\text{tot}}$ shown in Figure 4.35. This may be done either graphically or by using trigonometry. (b) Show graphically that the same total force is obtained independent of the order of addition of $\mathbf{F}_1$ and $\mathbf{F}_2$. (c) Find the direction and magnitude of some other pair of vectors that add to give $\mathbf{F}_{\text{tot}}$. Draw these to scale on the same drawing used in part (b) or a similar picture.

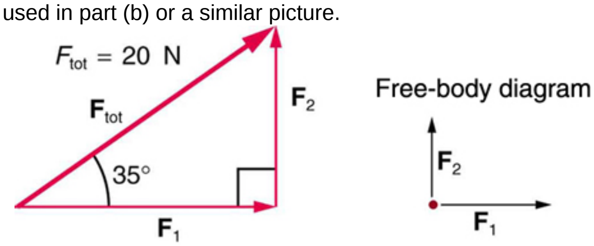


Figure 4.35

31. Two children pull a third child on a snow saucer sled exerting forces $\mathbf{F}_1$ and $\mathbf{F}_2$ as shown from above in Figure 4.36. Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of $\mathbf{F}_1$ and $\mathbf{F}_2$.

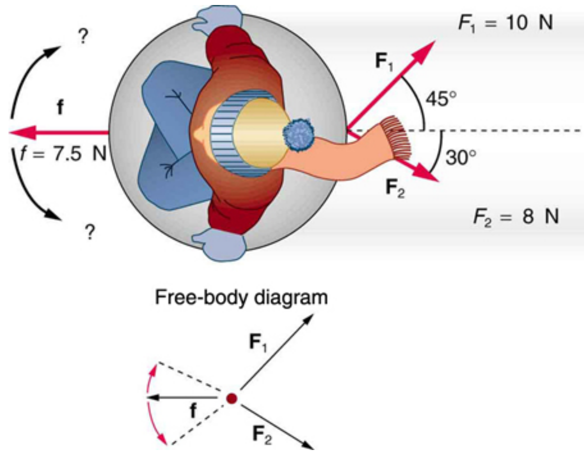


Figure 4.36 An overhead view of the horizontal forces acting on a child's snow saucer sled.

32. Suppose your car was mired deeply in the mud and you wanted to use the method illustrated in Figure 4.37 to pull it out. (a) What force would you have to exert perpendicular to the center of the rope to produce a force of 12,000 N on the car if the angle is 2.00°? In this part, explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. (b) Real ropes stretch under such forces. What force would be exerted on the car if the angle increases to 7.00° and you still apply the force found in part (a) to its center?

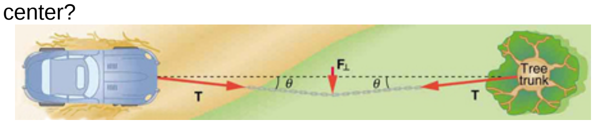


Figure 4.37

33. What force is exerted on the tooth in Figure 4.38 if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.

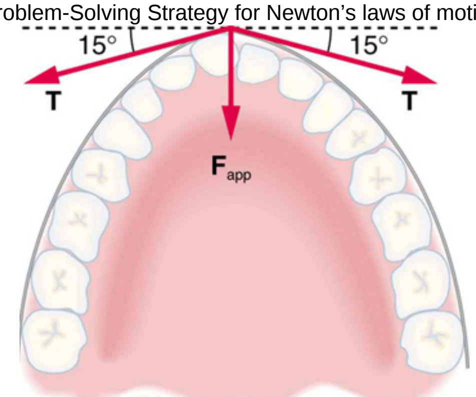


Figure 4.38 Braces are used to apply forces to teeth to realign them. Shown in this figure are the tensions applied by the wire to the protruding tooth. The total force applied to the tooth by the wire, $\mathbf{F}_{\text{app}}$, points straight toward the back of the mouth.

34. Figure 4.39 shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between

Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.

Figure 4.39 Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywhere in the rope?

35. A nurse pushes a cart by exerting a force on the handle at a downward angle 35.0° below the horizontal. The loaded cart has a mass of 28.0 kg, and the force of friction is 60.0 N. (a) Draw a free-body diagram for the system of interest. (b) What force must the nurse exert to move at a constant velocity?

36. Construct Your Own Problem Consider the tension in an elevator cable during the time the elevator starts from rest and accelerates its load upward to some cruising velocity. Taking the elevator and its load to be the system of interest, draw a free-body diagram. Then calculate the tension in the cable. Among the things to consider are the mass of the elevator and its load, the final velocity, and the time taken to reach that velocity.

37. Construct Your Own Problem Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Include a free-body diagram of the appropriate system of interest as the basis for your analysis. Show vector forces and their components and explain the choice of coordinates. Among the things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.

38. Unreasonable Results (a) Repeat Exercise 4.29, but assume an acceleration of 1.20 m/s^2 is produced. (b) What is unreasonable about the result? (c) Which premise is unreasonable, and why is it unreasonable?

39. Unreasonable Results (a) What is the initial acceleration of a rocket that has a mass of $1.50\times10^{6} \text{ kg}$ at takeoff, the engines of which produce a thrust of $2.00\times10^{6} \text{ N}$? Do not neglect gravity. (b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.) (c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)

4.7 Further Applications of Newton's Laws of Motion

40. A flea jumps by exerting a force of $1.20\times10^{-5} \text{ N}$ straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of $0.500\times10^{-6} \text{ N}$ on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is $6.00\times10^{-7} \text{ kg}$. Do not neglect the gravitational force.

41. Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in Figure 4.40. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?

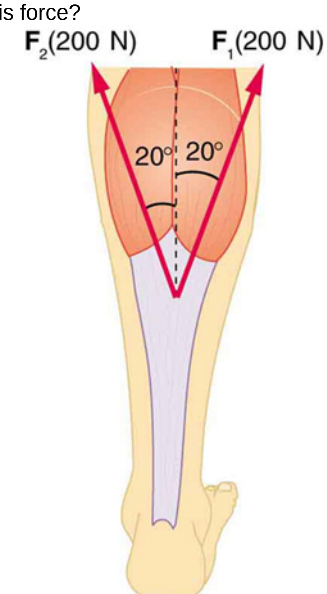


Figure 4.40 Achilles tendon

42. A 76.0-kg person is being pulled away from a burning building as shown in Figure 4.41. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

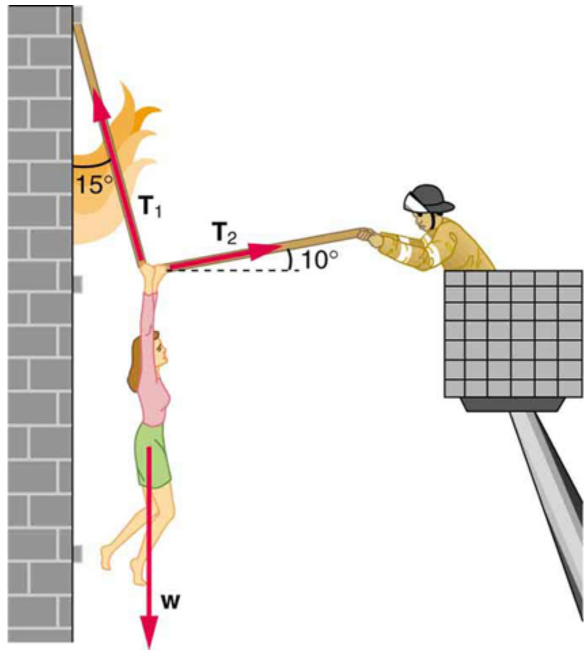


Figure 4.41 The force $\mathbf{T}_2$ needed to hold steady the person being rescued from the fire is less than her weight and less than the force $\mathbf{T}_1$ in the other rope, since the more vertical rope supports a greater part of her weight (a vertical force).

43. Integrated Concepts A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

44. Integrated Concepts When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?

45. Integrated Concepts A large rocket has a mass of $2.00\times10^{6}\ \text{kg}$ at takeoff, and its engines produce a thrust of $3.50\times10^{7}\ \text{N}$. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

46. Integrated Concepts A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

47. Integrated Concepts A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

48. Integrated Concepts Repeat Exercise 4.47 for a shell fired at an angle 10.0° from the vertical.

49. Integrated Concepts An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of $1.20\ \text{m/s}^2$ for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of $0.600\ \text{m/s}^2$ for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

50. Unreasonable Results (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of $0.400\ \text{m/s}^2$ for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

51. Unreasonable Results A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

4.8 Extended Topic: The Four Basic Forces—An Introduction

52. (a) What is the strength of the weak nuclear force relative to the strong nuclear force? (b) What is the strength of the weak nuclear force relative to the electromagnetic force? Since the weak nuclear force acts at only very short distances, such as inside nuclei, where the strong and electromagnetic forces also act, it might seem surprising that we have any knowledge of it at all. We have such knowledge because the weak nuclear force is responsible for beta decay, a type of nuclear decay not explained by other forces.

53. (a) What is the ratio of the strength of the gravitational force to that of the strong nuclear force? (b) What is the ratio of the strength of the gravitational force to that of the weak nuclear force? (c) What is the ratio of the strength of the gravitational force to that of the electromagnetic force? What do your answers imply about the influence of the gravitational force on atomic nuclei?

54. What is the ratio of the strength of the strong nuclear force to that of the electromagnetic force? Based on this ratio, you might expect that the strong force dominates the nucleus, which is true for small nuclei. Large nuclei, however, have sizes greater than the range of the strong nuclear force. At these sizes, the electromagnetic force begins to affect nuclear stability. These facts will be used to explain nuclear fusion and fission later in this text.

Test Prep for AP® Courses

4.1 Development of Force Concept

1.

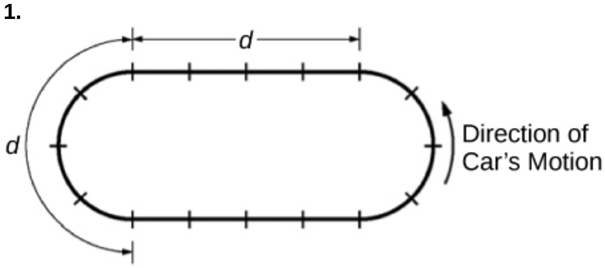


Figure 4.42 The figure above represents a racetrack with semicircular sections connected by straight sections. Each section has length *d*, and markers along the track are spaced *d*/4 apart. Two people drive cars counterclockwise around the track, as shown. Car *X* goes around the curves at constant speed *v*c, increases speed at constant acceleration for half of each straight section to reach a maximum speed of 2*v*c, then brakes at constant acceleration for the other half of each straight section to return to speed *v*c. Car *Y* also goes around the curves at constant speed *v*c, increases its speed at constant acceleration for one-fourth of each straight section to reach the same maximum speed 2*v*c, stays at that speed for half of each straight section, then brakes at constant acceleration for the remaining fourth of each straight section to return to speed *v*c.

(a) On the figures below, draw an arrow showing the direction of the net force on each of the cars at the positions noted by the dots. If the net force is zero at any position, label the dot with 0.

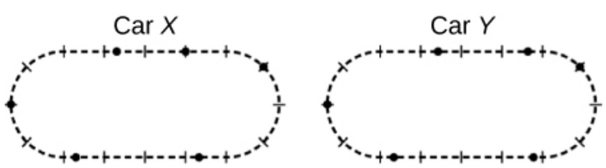


Figure 4.43

The position of the six dots on the Car Y track on the right are as follows:

The first dot on the left center of the track is at the same position as it is on the Car X track.

The second dot is just slight to the right of the Car X dot (less than a dash) past three perpendicular hash marks moving to the right.

The third dot is about one and two-thirds perpendicular hash marks to the right of the center top perpendicular has mark.

The fourth dot is in the same position as the Car X figure (one perpendicular hash mark above the center right perpendicular hash mark).

The fifth dot is about one and two-third perpendicular hash marks to the right of the center bottom perpendicular hash mark.

The sixth dot is in the same position as the Car Y dot (one and two third perpendicular hash marks to the left of the center bottom hash mark).

(b)

i. Indicate which car, if either, completes one trip around the track in less time, and justify your answer qualitatively without using equations.

ii. Justify your answer about which car, if either, completes one trip around the track in less time quantitatively with appropriate equations.

2. Which of the following is an example of a body exerting a force on itself?

a. a person standing up from a seated position
b. a car accelerating while driving
c. both of the above
d. none of the above

3. A hawk accelerates as it glides in the air. Does the force causing the acceleration come from the hawk itself? Explain.

4. What causes the force that moves a boat forward when someone rows it?

a. The force is caused by the rower's arms.
b. The force is caused by an interaction between the oars and gravity.
c. The force is caused by an interaction between the oars and the water the boat is traveling in.
d. The force is caused by friction.

4.4 Newton's Third Law of Motion: Symmetry in Forces

5. What object or objects commonly exert forces on the following objects in motion? (a) a soccer ball being kicked, (b) a dolphin jumping, (c) a parachutist drifting to Earth.

6. A ball with a mass of 0.25 kg hits a gym ceiling with a force of 78.0 N. What happens next?

a. The ball accelerates downward with a force of 80.5 N.
b. The ball accelerates downward with a force of 78.0 N.
c. The ball accelerates downward with a force of 2.45 N.
d. It depends on the height of the ceiling.

7. Which of the following is true?

a. Earth exerts a force due to gravity on your body, and your body exerts a smaller force on the Earth, because your mass is smaller than the mass of the Earth.
b. The Moon orbits the Earth because the Earth exerts a force on the Moon and the Moon exerts a force equal in magnitude and direction on the Earth.
c. A rocket taking off exerts a force on the Earth equal to the force the Earth exerts on the rocket.
d. An airplane cruising at a constant speed is not affected by gravity.

8. Stationary skater A pushes stationary skater B, who then accelerates at 5.0 m/s^2. Skater A does not move. Since forces act in action-reaction pairs, explain why Skater A did not move?

9. The current in a river exerts a force of 9.0 N on a balloon floating in the river. A wind exerts a force of 13.0 N on the balloon in the opposite direction. Draw a free-body diagram to show the forces acting on the balloon. Use your free-body diagram to predict the effect on the balloon.

10. A force is applied to accelerate an object on a smooth icy surface. When the force stops, which of the following will be true? (Assume zero friction.)

a. The object's acceleration becomes zero.
b. The object's speed becomes zero.
c. The object's acceleration continues to increase at a constant rate.
d. The object accelerates, but in the opposite direction.

11. A parachutist's fall to Earth is determined by two opposing forces. A gravitational force of 539 N acts on the parachutist. After 2 s, she opens her parachute and experiences an air resistance of 615 N. At what speed is the parachutist falling after 10 s?

12. A flight attendant pushes a cart down the aisle of a plane in flight. In determining the acceleration of the cart relative to the plane, which factor do you not need to consider?

a. The friction of the cart's wheels.
b. The force with which the flight attendant's feet push on the floor.
c. The velocity of the plane.
d. The mass of the items in the cart.

13. A landscaper is easing a wheelbarrow full of soil down a hill. Define the system you would analyze and list all the forces that you would need to include to calculate the acceleration of the wheelbarrow.

14. Two water-skiers, with masses of 48 kg and 61 kg, are preparing to be towed behind the same boat. When the boat accelerates, the rope the skiers hold onto accelerates with it and exerts a net force of 290 N on the skiers. At what rate will the skiers accelerate?

a. 10.8 m/s^2
b. 2.7 m/s^2
c. 6.0 m/s^2 and 4.8 m/s^2
d. 5.3 m/s^2

15. A figure skater has a mass of 40 kg and her partner's mass is 50 kg. She pushes against the ice with a force of 120 N, causing her and her partner to move forward. Calculate the pair's acceleration. Assume that all forces opposing the motion, such as friction and air resistance, total 5.0 N.

4.5 Normal, Tension, and Other Examples of Force

16. An archer shoots an arrow straight up with a force of 24.5 N. The arrow has a mass of 0.4 kg. What is the force of gravity on the arrow?

a. 9.8 m/s^2
b. 9.8 N
c. 61.25 N
d. 3.9 N

17. A cable raises a mass of 120.0 kg with an acceleration of 1.3 m/s2. What force of tension is in the cable?

18. A child pulls a wagon along a grassy field. Define the system, the pairs of forces at work, and the results.

19. Two teams are engaging in a tug–of-war. The rope suddenly snaps. Which statement is true about the forces involved?

a. The forces exerted by the two teams are no longer equal; the teams will accelerate in opposite directions as a result.
b. The forces exerted by the players are no longer balanced by the force of tension in the rope; the teams will accelerate in opposite directions as a result.
c. The force of gravity balances the forces exerted by the players; the teams will fall as a result
d. The force of tension in the rope is transferred to the players; the teams will accelerate in opposite directions as a result.

20. The following free-body diagram represents a toboggan on a hill. What acceleration would you expect, and why?

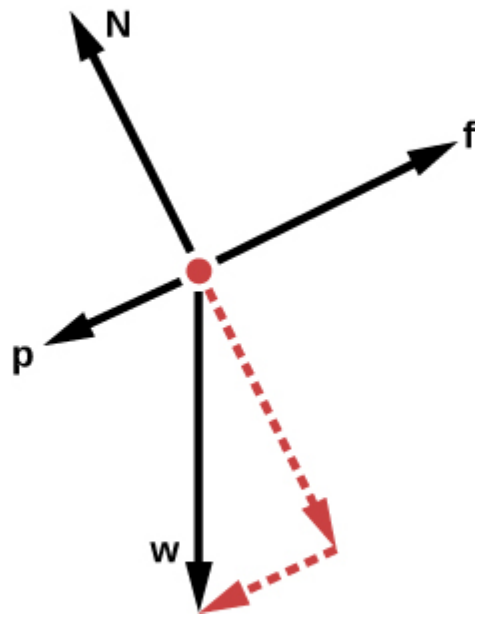


Figure 4.44

a. Acceleration down the hill; the force due to being pushed, together with the downhill component of gravity, overcomes the opposing force of friction.
b. Acceleration down the hill; friction is less than the opposing component of force due to gravity.
c. No movement; friction is greater than the force due to being pushed.
d. It depends on how strong the force due to friction is. *p*

21. Draw a free-body diagram to represent the forces acting on a kite on a string that is floating stationary in the air. Label the forces in your diagram.

22. A car is sliding down a hill with a slope of 20°. The mass of the car is 965 kg. When a cable is used to pull the car up the slope, a force of 4215 N is applied. What is the car's acceleration, ignoring friction?

4.6 Problem-Solving Strategies

23. A toboggan with two riders has a total mass of 85.0 kg. A third person is pushing the toboggan with a force of 42.5 N at the top of a hill with an angle of 15°. The force of friction on the toboggan is 31.0 N. Which statement describes an accurate free-body diagram to represent the situation?

a. An arrow of magnitude 10.5 N points down the slope of the hill.
b. An arrow of magnitude 833 N points straight down.
c. An arrow of magnitude 833 N points perpendicular to the slope of the hill.
d. An arrow of magnitude 73.5 N points down the slope of the hill.

24. A mass of 2.0 kg is suspended from the ceiling of an elevator by a rope. What is the tension in the rope when the elevator (i) accelerates upward at 1.5 m/s^2? (ii) accelerates downward at 1.5 m/s^2?

a. (i) 22.6 N; (ii) 16.6 N
b. Because the mass is hanging from the elevator itself, the tension in the rope will not change in either case.
c. (i) 22.6 N; (ii) 19.6 N
d. (i) 16.6 N; (ii) 19.6 N

25. Which statement is true about drawing free-body diagrams?

a. Drawing a free-body diagram should be the last step in solving a problem about forces.
b. Drawing a free-body diagram helps you compare forces quantitatively.
c. The forces in a free-body diagram should always balance.
d. Drawing a free-body diagram can help you determine the net force.

4.7 Further Applications of Newton's Laws of Motion

26. A basketball player jumps as he shoots the ball. Describe the forces that are acting on the ball and on the basketball player. What are the results?

27. Two people push on a boulder to try to move it. The mass of the boulder is 825 kg. One person pushes north with a force of 64 N. The other pushes west with a force of 38 N. Predict the magnitude of the acceleration of the boulder. Assume that friction is negligible.

28.

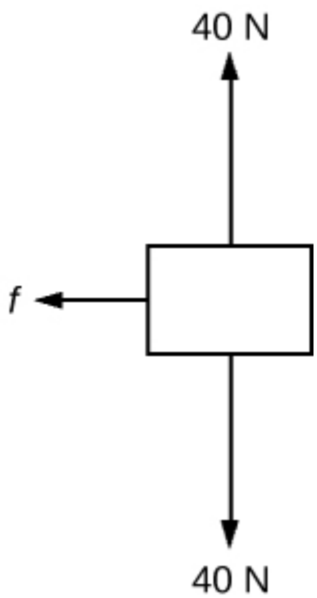


Figure 4.45 The figure shows the forces exerted on a block that is sliding on a horizontal surface: the gravitational force of 40 N, the 40 N normal force exerted by the surface, and a frictional force exerted to the left. The coefficient of friction between the block and the surface is 0.20. The acceleration of the block is most nearly

a. 1.0 m/s^2 to the right
b. 1.0 m/s^2 to the left
c. 2.0 m/s^2 to the right
d. 2.0 m/s^2 to the left

4.8 Extended Topic: The Four Basic Forces—An Introduction

29. Which phenomenon correctly describes the direction and magnitude of normal forces?
a. electromagnetic attraction
b. electromagnetic repulsion
c. gravitational attraction
d. gravitational repulsion

30. Explain which of the four fundamental forces is responsible for a ball bouncing off the ground after it hits, and why this force has this effect.

31. Which of the basic forces best explains tension in a rope being pulled between two people? Is the acting force causing attraction or repulsion in this instance?
a. gravity; attraction
b. electromagnetic; attraction
c. weak and strong nuclear; attraction
d. weak and strong nuclear; repulsion

32. Explain how interatomic electric forces produce the normal force, and why it has the direction it does.

33. The gravitational force is the weakest of the four basic forces. In which case can the electromagnetic, strong, and weak forces be ignored because the gravitational force is so strongly dominant?
a. a person jumping on a trampoline
b. a rocket blasting off from Earth
c. a log rolling down a hill
d. all of the above

34. Describe a situation in which gravitational force is the dominant force. Why can the other three basic forces be ignored in the situation you described?

5 FURTHER APPLICATIONS OF NEWTON'S LAWS: FRICTION, DRAG, AND ELASTICITY

Figure 5.1 Total hip replacement surgery has become a common procedure. The head (or ball) of the patient's femur fits into a cup that has a hard plastic-like inner lining. (credit: National Institutes of Health, via Wikimedia Commons)

Chapter Outline

Connection for AP® Courses

Have you ever wondered why it is difficult to walk on a smooth surface like ice? The interaction between you and the surface is a result of forces that affect your motion. In the previous chapter, you learned Newton's laws of motion and examined how net force affects the motion, position and shape of an object. Now we will look at some interesting and common forces that will provide further applications of Newton's laws of motion.

The information presented in this chapter supports learning objectives covered under Big Idea 3 of the AP Physics Curriculum Framework, which refer to the nature of forces and their roles in interactions among objects. The chapter discusses examples of specific contact forces, such as friction, air or liquid drag, and elasticity that may affect the motion or shape of an object. It also discusses the nature of forces on both macroscopic and microscopic levels (Enduring Understanding 3.C and Essential Knowledge 3.C.4). In addition, Newton's laws are applied to describe the motion of an object (Enduring Understanding 3.B) and to examine relationships between contact forces and other forces exerted on an object (Enduring Understanding 3.A, 3.A.3 and

Essential Knowledge 3.A.4). The examples in this chapter give you practice in using vector properties of forces (Essential Knowledge 3.A.2) and free-body diagrams (Essential Knowledge 3.B.2) to determine net force (Essential Knowledge 3.B.1).

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A All forces share certain common characteristics when considered by observers in inertial reference frames.

Essential Knowledge 3.A.2 Forces are described by vectors.

Essential Knowledge 3.A.3 A force exerted on an object is always due to the interaction of that object with another object.

Essential Knowledge 3.A.4 If one object exerts a force on a second object, the second object always exerts a force of equal magnitude on the first object in the opposite direction.

Enduring Understanding 3.B Classically, the acceleration of an object interacting with other objects can be predicted by using

$$\vec{a} = \frac{\sum \vec{F}}{m}.$$

Essential Knowledge 3.B.1 If an object of interest interacts with several other objects, the net force is the vector sum of the individual forces.

Essential Knowledge 3.B.2 Free-body diagrams are useful tools for visualizing forces being exerted on a single object and writing the equations that represent a physical situation.

Enduring Understanding 3.C At the macroscopic level, forces can be categorized as either long-range (action-at-a-distance) forces or contact forces.

Essential Knowledge 3.C.4 Contact forces result from the interaction of one object touching another object, and they arise from interatomic electric forces. These forces include tension, friction, normal, spring (Physics 1), and buoyant (Physics 2).

5.1 Friction

Learning Objectives

By the end of this section, you will be able to:

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitudes of static and kinetic frictional forces.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.C.4.1** The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces. **(S.P. 6.1)**
- **3.C.4.2** The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions. **(S.P. 6.2)**

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

Friction

Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

Kinetic Friction

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

Figure 5.2 is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

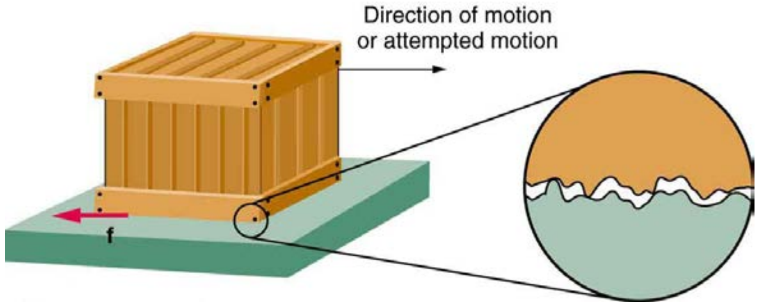


Figure 5.2 Frictional forces, such as f , always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction $\mathbf{f_s}$** is

$$f_\text{s} \leq \mu_\text{s}N, \tag{5.1}$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

Magnitude of Static Friction

Magnitude of static friction f_s is

$$f_\text{s} \leq \mu_\text{s}N, \tag{5.2}$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_\text{s} N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{\text{s(max)}}$, the object will move. Thus

$$f_{\text{s(max)}} = \mu_\text{s}N. \tag{5.3}$$

Once an object is moving, the **magnitude of kinetic friction $\mathbf{f_k}$** is given by

$$f_\text{k} = \mu_\text{k}N, \tag{5.4}$$

where μ_k is the coefficient of kinetic friction. A system in which $f_\text{k} = \mu_\text{k}N$ is described as a system in which *friction behaves simply*.

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_k is given by

$$f_{\text{k}} = \mu_{\text{k}} N, \tag{5.5}$$

where μ_{k} is the coefficient of kinetic friction.

As seen in **Table 5.1**, the coefficients of kinetic friction are less than their static counterparts. That values of μ in **Table 5.1** are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

Table 5.1 Coefficients of Static and Kinetic Friction

System	Static friction μ_{s}	Kinetic friction μ_{k}
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100\text{ kg})(9.80\text{ m/s}^2) = 980\text{ N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{\text{s(max)}} = \mu_{\text{s}} N = (0.45)(980\text{ N}) = 440\text{ N}$ to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($f_{\text{k}} = \mu_{\text{k}} N = (0.30)(980\text{ N}) = 290\text{ N}$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Take-Home Experiment

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (**Figure 5.3**). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

Figure 5.3 Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to mover freely over the skin.

Example 5.1 Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force N as $f_{\text{k}} = \mu_{\text{k}}N$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in **Figure 5.4**.)

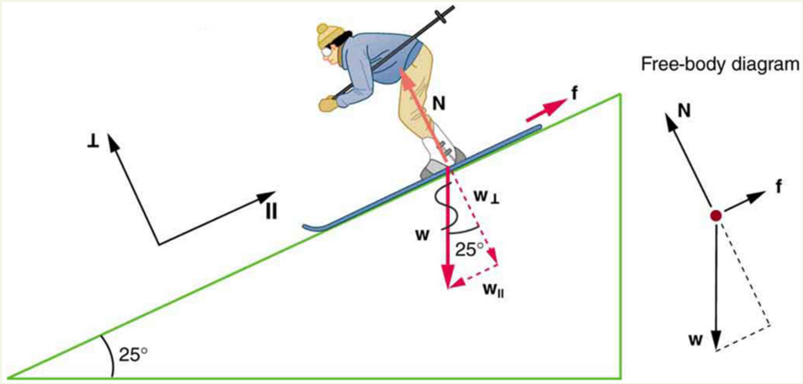


Figure 5.4 The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). $\mathbf{N}$ (the normal force) is perpendicular to the slope, and $\mathbf{f}$ (the friction) is parallel to the slope, but $\mathbf{w}$ (the skier's weight) has components along both axes, namely $\mathbf{w}_{\perp}$ and $\mathbf{W}_{//}$. $\mathbf{N}$ is equal in magnitude to $\mathbf{w}_{\perp}$, so there is no motion perpendicular to the slope. However, $\mathbf{f}$ is less than $\mathbf{W}_{//}$ in magnitude, so there is acceleration down the slope (along the *x*-axis).

That is,

$$N = w_{\perp} = w\cos 25^{\circ} = mg\cos 25^{\circ}. \tag{5.6}$$

Substituting this into our expression for kinetic friction, we get

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} mg\cos 25^{\circ}, \tag{5.7}$$

which can now be solved for the coefficient of kinetic friction μ_{k}.

Solution

Solving for μ_{k} gives

$$\mu_{\mathrm{k}} = \frac{f_{\mathrm{k}}}{N} = \frac{f_{\mathrm{k}}}{w\cos 25^{\circ}} = \frac{f_{\mathrm{k}}}{mg\cos 25^{\circ}}. \tag{5.8}$$

Substituting known values on the right-hand side of the equation,

$$\mu_{\mathrm{k}} = \frac{45.0\ \mathrm{N}}{(62\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(0.906)} = 0.082. \tag{5.9}$$

Discussion

This result is a little smaller than the coefficient listed in **Table 5.1** for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_{\mathrm{k}} = \mu_{\mathrm{k}} mg\cos\theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

Take-Home Experiment

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in **Example 5.1**, the kinetic friction on a slope $f_{\mathrm{k}} = \mu_{\mathrm{k}} mg\cos\theta$. The component of the weight down the slope is equal to $mg\sin\theta$ (see the free-body diagram in **Figure 5.4**). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

$$f_{\mathrm{k}} = Fg_x \tag{5.10}$$

$$\mu_{\mathrm{k}} mg\cos\theta = mg\sin\theta. \tag{5.11}$$

Solving for μ_{k}, we find that

$$\mu_{\mathrm{k}} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta. \tag{5.12}$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_{k}. Note that the coin will not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for μ_{k} and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

Making Connections: Submicroscopic Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

Figure 5.5 illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.

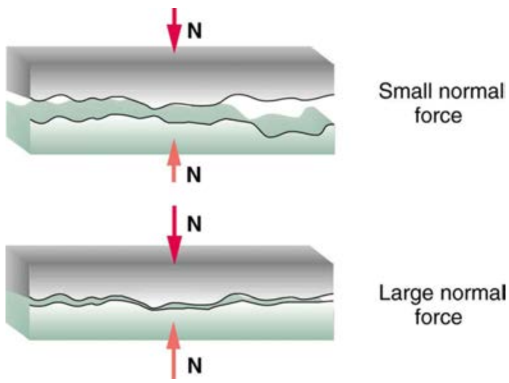


Figure 5.5 Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. **Figure 5.6** shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

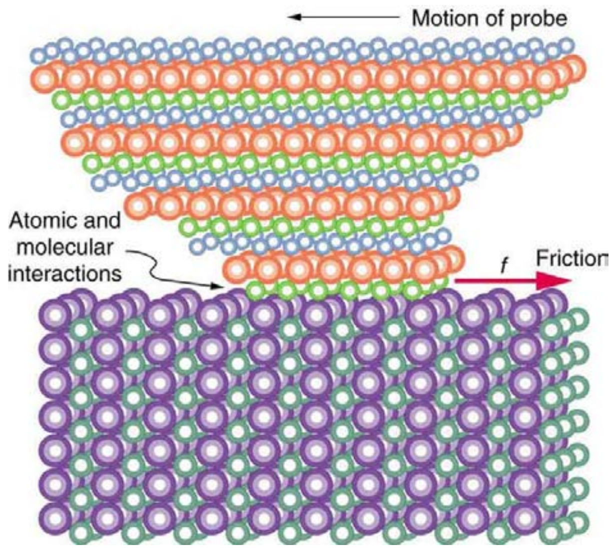


Figure 5.6 The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

PhET Explorations: Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

PhET Interactive Simulation

Figure 5.7 Forces and Motion (http://cnx.org/content/m54899/1.2/forces-and-motion_en.jar)

5.2 Drag Forces

Learning Objectives

By the end of this section, you will be able to:

- Define drag force and model it mathematically.
- Discuss the applications of drag force.
- Define terminal velocity.
- Perform calculations to find terminal velocity.

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion. Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_{D} is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $F_D \propto v^2$. When taking into account other factors, this relationship becomes

$$F_{\text{D}} = \frac{1}{2}\text{C}\rho A v^2, \tag{5.13}$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as $F_{\text{D}} = bv^2$, where b is a constant equivalent to $0.5C\rho A$. We have set the exponent for these equations as 2 because, when an object is moving at high velocity

Although measurable, this is not a significant decrease in volume considering that the force per unit area is about 500 atmospheres (1 million pounds per square foot). Liquids and solids are extraordinarily difficult to compress.

Conversely, very large forces are created by liquids and solids when they try to expand but are constrained from doing so—which is equivalent to compressing them to less than their normal volume. This often occurs when a contained material warms up, since most materials expand when their temperature increases. If the materials are tightly constrained, they deform or break their container. Another very common example occurs when water freezes. Water, unlike most materials, expands when it freezes, and it can easily fracture a boulder, rupture a biological cell, or crack an engine block that gets in its way.

Other types of deformations, such as torsion or twisting, behave analogously to the tension, shear, and bulk deformations considered here.

Glossary

deformation: change in shape due to the application of force

drag force: F_{D}, found to be proportional to the square of the speed of the object; mathematically

$$F_{\text{D}} \propto v^2$$

$$F_{\text{D}} = \frac{1}{2}C\rho Av^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid

friction: a force that opposes relative motion or attempts at motion between systems in contact

Hooke's law: proportional relationship between the force F on a material and the deformation ΔL it causes, $F = k\Delta L$

kinetic friction: a force that opposes the motion of two systems that are in contact and moving relative to one another

magnitude of kinetic friction: $f_{\text{k}} = \mu_{\text{k}}N$, where μ_{k} is the coefficient of kinetic friction

magnitude of static friction: $f_{\text{s}} \le \mu_{\text{s}}N$, where μ_{s} is the coefficient of static friction and N is the magnitude of the normal force

shear deformation: deformation perpendicular to the original length of an object

static friction: a force that opposes the motion of two systems that are in contact and are not moving relative to one another

Stokes' law: $F_{\text{s}} = 6\pi r\eta v$, where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity

strain: ratio of change in length to original length

stress: ratio of force to area

tensile strength: measure of deformation for a given tension or compression

Section Summary

5.1 Friction

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction f_{s} between systems stationary relative to one another is given by

  $$f_{\text{s}} \le \mu_{\text{s}}N,$$

  where μ_{s} is the coefficient of static friction, which depends on both of the materials.
- The kinetic friction force f_{k} between systems moving relative to one another is given by

  $$f_{\text{k}} = \mu_{\text{k}}N,$$

  where μ_{k} is the coefficient of kinetic friction, which also depends on both materials.

5.2 Drag Forces

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity v in air, the drag force is given by

  $$F_D = \frac{1}{2}C\rho Av^2,$$

  where C is the drag coefficient (typical values are given in **Table 5.2**), A is the area of the object facing the fluid, and ρ is the fluid density.
- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is given by Stokes' law,

  $$F_s = 6\pi\eta rv,$$

  where r is the radius of the object, η is the fluid viscosity, and v is the object's velocity.

5.3 Elasticity: Stress and Strain

- Hooke's law is given by

  $$F = k\Delta L,$$

  where ΔL is the amount of deformation (the change in length), F is the applied force, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. The relationship between the deformation and the applied force can also be written as

  $$\Delta L = \frac{1}{Y}\frac{F}{A}L_0,$$

  where Y is *Young's modulus*, which depends on the substance, A is the cross-sectional area, and L_0 is the original length.
- The ratio of force to area, $\frac{F}{A}$, is defined as *stress*, measured in N/m^2.
- The ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as *strain* (a unitless quantity). In other words,

  $$\text{stress} = Y \times \text{strain}.$$

- The expression for shear deformation is

  $$\Delta x = \frac{1}{S}\frac{F}{A}L_0,$$

  where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A.
- The relationship of the change in volume to other physical quantities is given by

  $$\Delta V = \frac{1}{B}\frac{F}{A}V_0,$$

  where B is the bulk modulus, V_0 is the original volume, and $\frac{F}{A}$ is the force per unit area applied uniformly inward on all surfaces.

Conceptual Questions

5.1 Friction

1. Define normal force. What is its relationship to friction when friction behaves simply?

2. The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

3. When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

4. When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

5.2 Drag Forces

5. Athletes such as swimmers and bicyclists wear body suits in competition. Formulate a list of pros and cons of such suits.

6. Two expressions were used for the drag force experienced by a moving object in a liquid. One depended upon the speed, while the other was proportional to the square of the speed. In which types of motion would each of these expressions be more applicable than the other one?

7. As cars travel, oil and gasoline leaks onto the road surface. If a light rain falls, what does this do to the control of the car? Does a heavy rain make any difference?

8. Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

5.3 Elasticity: Stress and Strain

9. The elastic properties of the arteries are essential for blood flow. Explain the importance of this in terms of the characteristics of the flow of blood (pulsating or continuous).

10. What are you feeling when you feel your pulse? Measure your pulse rate for 10 s and for 1 min. Is there a factor of 6 difference?

11. Examine different types of shoes, including sports shoes and thongs. In terms of physics, why are the bottom surfaces designed as they are? What differences will dry and wet conditions make for these surfaces?

12. Would you expect your height to be different depending upon the time of day? Why or why not?

13. Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

14. Explain why pregnant women often suffer from back strain late in their pregnancy.

15. An old carpenter's trick to keep nails from bending when they are pounded into hard materials is to grip the center of the nail firmly with pliers. Why does this help?

16. When a glass bottle full of vinegar warms up, both the vinegar and the glass expand, but vinegar expands significantly more with temperature than glass. The bottle will break if it was filled to its tightly capped lid. Explain why, and also explain how a pocket of air above the vinegar would prevent the break. (This is the function of the air above liquids in glass containers.)

Problems & Exercises

5.1 Friction

1. A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

2. (a) When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

3. (a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

4. Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

5. (a) If half of the weight of a small $1.00\times10^3\ \text{kg}$ utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

6. A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

7. Consider the 65.0-kg ice skater being pushed by two others shown in Figure 5.21. (a) Find the direction and magnitude of $\mathbf{F}_{\text{tot}}$, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of $\mathbf{F}_{\text{tot}}$? (c) What is her acceleration assuming she is already moving in the direction of $\mathbf{F}_{\text{tot}}$? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)

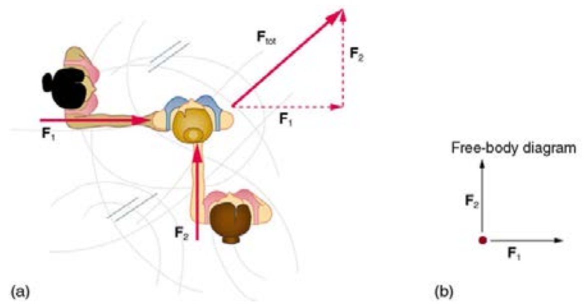


Figure 5.21

8. Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g\sin\theta$. (Note that this acceleration is independent of mass.)

9. Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_\text{k} = \mu_\text{k}N$) is $a = g(\sin\theta - \mu_\text{k}\cos\theta)$. Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small $(\mu_\text{k} = 0)$.

10. Calculate the deceleration of a snow boarder going up a 5.0°, slope assuming the coefficient of friction for waxed wood on wet snow. The result of Exercise 5.9 may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in Problem-Solving Strategies.

11. (a) Calculate the acceleration of a skier heading down a 10.0° slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of Exercise 5.9 to be useful. Explicitly show how you follow the steps in the Problem-Solving Strategies.

12. If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1}\ \mu_\text{s}$. You may use the result of the previous problem. Assume that $a = 0$ and that static friction has reached its maximum value.

13. Calculate the maximum deceleration of a car that is heading down a 6° slope (one that makes an angle of 6° with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_\text{s} = 0.100$, the same as for shoes on ice.

14. Calculate the maximum acceleration of a car that is heading up a 4° slope (one that makes an angle of 4° with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

15. Repeat **Exercise 5.14** for a car with four-wheel drive.

16. A freight train consists of two $8.00\times10^5\text{-kg}$ engines and 45 cars with average masses of $5.50\times10^5\ \text{kg}$. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00\times10^{-2}\ \text{m/s}^2$ if the force of friction is $7.50\times10^5\ \text{N}$, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

17. Consider the 52.0-kg mountain climber in **Figure 5.22**. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?

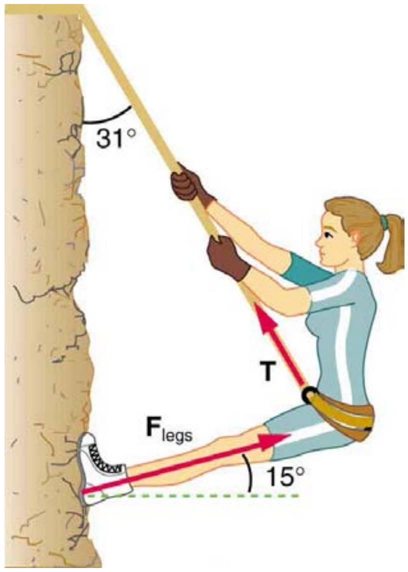


Figure 5.22 Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.

18. A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in **Figure 5.23**(a). (a) Calculate the minimum force F he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

19. Repeat **Exercise 5.18** with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in **Figure 5.23**(b).

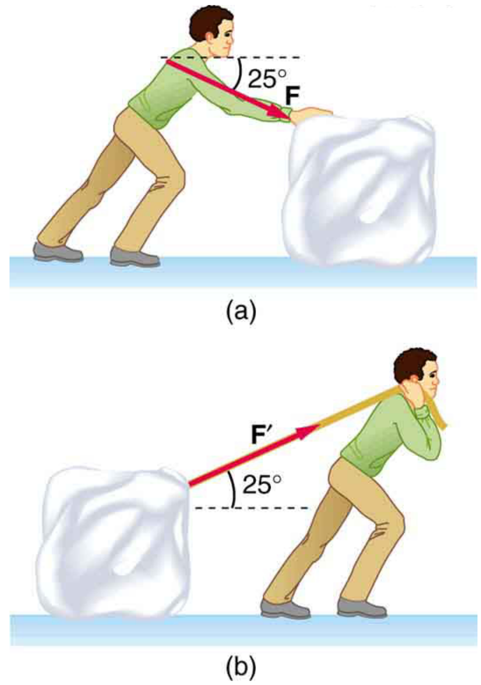


Figure 5.23 Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?

5.2 Drag Forces

20. The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid. Find the terminal velocity (in meters per second and kilometers per hour) of an 80.0-kg skydiver falling in a pike (headfirst) position with a surface area of $0.140\ \text{m}^2$.

21. A 60-kg and a 90-kg skydiver jump from an airplane at an altitude of 6000 m, both falling in the pike position. Make some assumption on their frontal areas and calculate their terminal velocities. How long will it take for each skydiver to reach the ground (assuming the time to reach terminal velocity is small)? Assume all values are accurate to three significant digits.

22. A 560-g squirrel with a surface area of $930\ \text{cm}^2$ falls from a 5.0-m tree to the ground. Estimate its terminal velocity. (Use a drag coefficient for a horizontal skydiver.) What will be the velocity of a 56-kg person hitting the ground, assuming no drag contribution in such a short distance?

23. To maintain a constant speed, the force provided by a car's engine must equal the drag force plus the force of friction of the road (the rolling resistance). (a) What are the magnitudes of drag forces at 70 km/h and 100 km/h for a Toyota Camry? (Drag area is $0.70\ \text{m}^2$) (b) What is the magnitude of drag force at 70 km/h and 100 km/h for a Hummer H2? (Drag area is $2.44\ \text{m}^2$) Assume all values are accurate to three significant digits.

24. By what factor does the drag force on a car increase as it goes from 65 to 110 km/h?

25. Calculate the speed a spherical rain drop would achieve falling from 5.00 km (a) in the absence of air drag (b) with air drag. Take the size across of the drop to be 4 mm, the density to be $1.00\times10^3\ \text{kg/m}^3$, and the surface area to be πr^2.

26. Using Stokes' law, verify that the units for viscosity are kilograms per meter per second.

27. Find the terminal velocity of a spherical bacterium (diameter $2.00\ \mu m$) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be $1.10\times10^3\ kg/m^3$.

28. Stokes' law describes sedimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density $7.8\times10^3\ kg/m^3$, diameter $3.0\ mm$) is dropped in a container of motor oil. It takes 12 s to fall a distance of 0.60 m. Calculate the viscosity of the oil.

5.3 Elasticity: Stress and Strain

29. During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.

30. During a wrestling match, a 150 kg wrestler briefly stands on one hand during a maneuver designed to perplex his already moribund adversary. By how much does the upper arm bone shorten in length? The bone can be represented by a uniform rod 38.0 cm in length and 2.10 cm in radius.

31. (a) The “lead” in pencils is a graphite composition with a Young's modulus of about $1\times10^9\ N/m^2$. Calculate the change in length of the lead in an automatic pencil if you tap it straight into the pencil with a force of 4.0 N. The lead is 0.50 mm in diameter and 60 mm long. (b) Is the answer reasonable? That is, does it seem to be consistent with what you have observed when using pencils?

32. TV broadcast antennas are the tallest artificial structures on Earth. In 1987, a 72.0-kg physicist placed himself and 400 kg of equipment at the top of one 610-m high antenna to perform gravity experiments. By how much was the antenna compressed, if we consider it to be equivalent to a steel cylinder 0.150 m in radius?

33. (a) By how much does a 65.0-kg mountain climber stretch her 0.800-cm diameter nylon rope when she hangs 35.0 m below a rock outcropping? (b) Does the answer seem to be consistent with what you have observed for nylon ropes? Would it make sense if the rope were actually a bungee cord?

34. A 20.0-m tall hollow aluminum flagpole is equivalent in strength to a solid cylinder 4.00 cm in diameter. A strong wind bends the pole much as a horizontal force of 900 N exerted at the top would. How far to the side does the top of the pole flex?

35. As an oil well is drilled, each new section of drill pipe supports its own weight and that of the pipe and drill bit beneath it. Calculate the stretch in a new 6.00 m length of steel pipe that supports 3.00 km of pipe having a mass of 20.0 kg/m and a 100-kg drill bit. The pipe is equivalent in strength to a solid cylinder 5.00 cm in diameter.

36. Calculate the force a piano tuner applies to stretch a steel piano wire 8.00 mm, if the wire is originally 0.850 mm in diameter and 1.35 m long.

37. A vertebra is subjected to a shearing force of 500 N. Find the shear deformation, taking the vertebra to be a cylinder 3.00 cm high and 4.00 cm in diameter.

38. A disk between vertebrae in the spine is subjected to a shearing force of 600 N. Find its shear deformation, taking it to have the shear modulus of $1\times10^9\ N/m^2$. The disk is equivalent to a solid cylinder 0.700 cm high and 4.00 cm in diameter.

39. When using a pencil eraser, you exert a vertical force of 6.00 N at a distance of 2.00 cm from the hardwood-eraser joint. The pencil is 6.00 mm in diameter and is held at an angle of 20.0° to the horizontal. (a) By how much does the wood flex perpendicular to its length? (b) How much is it compressed lengthwise?

40. To consider the effect of wires hung on poles, we take data from **Example 4.8**, in which tensions in wires supporting a traffic light were calculated. The left wire made an angle 30.0° below the horizontal with the top of its pole and carried a tension of 108 N. The 12.0 m tall hollow aluminum pole is equivalent in strength to a 4.50 cm diameter solid cylinder. (a) How far is it bent to the side? (b) By how much is it compressed?

41. A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is, $\Delta V/V_0 = 2\times10^{-3}$) relative to the space available. Calculate the magnitude of the normal force exerted by the juice per square centimeter if its bulk modulus is $1.8\times10^9\ N/m^2$, assuming the bottle does not break. In view of your answer, do you think the bottle will survive?

42. (a) When water freezes, its volume increases by 9.05% (that is, $\Delta V/V_0 = 9.05\times10^{-2}$). What force per unit area is water capable of exerting on a container when it freezes? (It is acceptable to use the bulk modulus of water in this problem.) (b) Is it surprising that such forces can fracture engine blocks, boulders, and the like?

43. This problem returns to the tightrope walker studied in **Example 4.6**, who created a tension of $3.94\times10^3\ N$ in a wire making an angle 5.0° below the horizontal with each supporting pole. Calculate how much this tension stretches the steel wire if it was originally 15 m long and 0.50 cm in diameter.

44. The pole in **Figure 5.24** is at a 90.0° bend in a power line and is therefore subjected to more shear force than poles in straight parts of the line. The tension in each line is $4.00\times10^4\ N$, at the angles shown. The pole is 15.0 m tall, has an 18.0 cm diameter, and can be considered to have half the strength of hardwood. (a) Calculate the compression of the pole. (b) Find how much it bends and in what direction. (c) Find the tension in a guy wire used to keep the pole straight if it is attached to the top of the pole at an angle of 30.0° with the vertical. (Clearly, the guy wire must be in the opposite direction of the bend.)

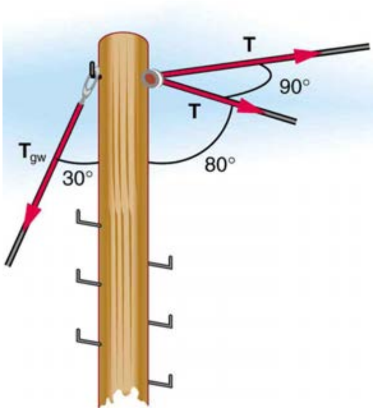


Figure 5.24 This telephone pole is at a 90° bend in a power line. A guy wire is attached to the top of the pole at an angle of 30° with the vertical.

Test Prep for AP® Courses

5.1 Friction

1. When a force of 20 N is applied to a stationary box weighing 40 N, the box does not move. This means the coefficient of static friction

a. is equal to 0.5.
b. is greater than 0.5.
c. is less than 0.5.
d. cannot be determined.

2. A 2-kg block slides down a ramp which is at an incline of 25º. If the frictional force is 4.86 N, what is the coefficient of friction? At what incline will the box slide at a constant velocity? Assume g = 10 m/s^2.

3. A block is given a short push and then slides with constant friction across a horizontal floor. Which statement best explains the direction of the force that friction applies on the moving block?

a. Friction will be in the same direction as the block's motion because molecular interactions between the block and the floor will deform the block in the direction of motion.
b. Friction will be in the same direction as the block's motion because thermal energy generated at the interface between the block and the floor adds kinetic energy to the block.
c. Friction will be in the opposite direction of the block's motion because molecular interactions between the block and the floor will deform the block in the opposite direction of motion.
d. Friction will be in the opposite direction of the block's motion because thermal energy generated at the interface between the block and the floor converts some of the block's kinetic energy to potential energy.

4. A student pushes a cardboard box across a carpeted floor and afterwards notices that the bottom of the box feels warm. Explain how interactions between molecules in the cardboard and molecules in the carpet produced this heat.

6 GRAVITATION AND UNIFORM CIRCULAR MOTION

Figure 6.1 This Australian Grand Prix Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly—the latter completing many revolutions, the former only part of one (a circular arc). The same physical principles are involved in each. (credit: Richard Munckton)

Chapter Outline

Connection for AP® Courses

Many motions, such as the arc of a bird's flight or Earth's path around the Sun, are curved. Recall that Newton's first law tells us that motion is along a straight line at constant speed unless there is a net external force. We will therefore study not only motion along curves, but also the forces that cause it, including gravitational forces. This chapter supports Big Idea 3 that interactions between objects are described by forces, and thus change in motion is a result of a net force exerted on an object. In this chapter, this idea is applied to uniform circular motion. In some ways, this chapter is a continuation of **Dynamics: Newton's Laws of Motion** as we study more applications of Newton's laws of motion.

This chapter deals with the simplest form of curved motion, **uniform circular motion**, which is motion in a circular path at constant speed. As an object moves on a circular path, the magnitude of its velocity remains constant, but the direction of the velocity is changing. This means there is an acceleration that we will refer to as a “centripetal” acceleration caused by a net external force, also called the “centripetal” force (Enduring Understanding 3.B). The centripetal force is the net force totaling all external forces acting on the object (Essential Knowledge 3.B.1). In order to determine the net force, a free-body diagram may be useful (Essential Knowledge 3.B.2).

Studying this topic illustrates most of the concepts associated with rotational motion and leads to many new topics we group under the name *rotation*. This motion can be described using kinematics variables (Essential Knowledge 3.A.1), but in addition to linear variables, we will introduce angular variables. We use various ways to describe motion, namely, verbally, algebraically and graphically (Learning Objective 3.A.1.1). Pure *rotational motion* occurs when points in an object move in circular paths centered on one point. Pure *translational motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving over ice. Some combinations of both types of motion are conveniently described with fictitious forces which appear as a result of using a non-inertial frame of reference (Enduring Understanding 3.A).

Furthermore, the properties of uniform circular motion can be applied to the motion of massive objects in a gravitational field. Thus, this chapter supports Big Idea 1 that gravitational mass is an important property of an object or a system.

We have experimental evidence that gravitational and inertial masses are equal (Enduring Understanding 1.C), and that gravitational mass is a measure of the strength of the gravitational interaction (Essential Knowledge 1.C.2). Therefore, this chapter will support Big Idea 2 that fields existing in space can be used to explain interactions, because any massive object creates a gravitational field in space (Enduring Understanding 2.B). Mathematically, we use Newton's universal law of gravitation to provide a model for the gravitational interaction between two massive objects (Essential Knowledge 2.B.2). We will discover that this model describes the interaction of one object with mass with another object with mass (Essential Knowledge 3.C.1), and also that gravitational force is a long-range force (Enduring Understanding 3.C).

The concepts in this chapter support:

Big Idea 1 Objects and systems have properties such as mass and charge. Systems may have internal structure.

Enduring Understanding 1.C Objects and systems have properties of inertial mass and gravitational mass that are experimentally verified to be the same and that satisfy conservation principles.

Essential Knowledge 1.C.2 Gravitational mass is the property of an object or a system that determines the strength of the gravitational interaction with other objects, systems, or gravitational fields.

Essential Knowledge 1.C.3 Objects and systems have properties of inertial mass and gravitational mass that are experimentally verified to be the same and that satisfy conservation principles.

Big Idea 2 Fields existing in space can be used to explain interactions.

Enduring Understanding 2.B A gravitational field is caused by an object with mass.

Essential Knowledge 2.B.2. The gravitational field caused by a spherically symmetric object with mass is radial and, outside the object, varies as the inverse square of the radial distance from the center of that object.

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A All forces share certain common characteristics when considered by observers in inertial reference frames.

Essential Knowledge 3.A.1. An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.

Essential Knowledge 3.A.3. A force exerted on an object is always due to the interaction of that object with another object.

Enduring Understanding 3.B Classically, the acceleration of an object interacting with other objects can be predicted by using $a = \sum F / m$.

Essential Knowledge 3.B.1 If an object of interest interacts with several other objects, the net force is the vector sum of the individual forces.

Essential Knowledge 3.B.2 Free-body diagrams are useful tools for visualizing forces being exerted on a single object and writing the equations that represent a physical situation.

Enduring Understanding 3.C At the macroscopic level, forces can be categorized as either long-range (action-at-a-distance) forces or contact forces.

Essential Knowledge 3.C.1. Gravitational force describes the interaction of one object that has mass with another object that has mass.

6.1 Rotation Angle and Angular Velocity

Learning Objectives

By the end of this section, you will be able to:

- Define arc length, rotation angle, radius of curvature, and angular velocity.
- Calculate the angular velocity of a car wheel spin.

In **Kinematics**, we studied motion along a straight line and introduced such concepts as displacement, velocity, and acceleration. **Two-Dimensional Kinematics** dealt with motion in two dimensions. Projectile motion is a special case of two-dimensional kinematics in which the object is projected into the air, while being subject to the gravitational force, and lands a distance away. In this chapter, we consider situations where the object does not land but moves in a curve. We begin the study of uniform circular motion by defining two angular quantities needed to describe rotational motion.

Rotation Angle

When objects rotate about some axis—for example, when the CD (compact disc) in Figure 6.2 rotates about its center—each point in the object follows a circular arc. Consider a line from the center of the CD to its edge. Each **pit** used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation and is analogous to linear distance. We define the **rotation angle** $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta s}{r}. \tag{6.1}$$

Figure 6.2 All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle $\Delta\theta$ in a time Δt .

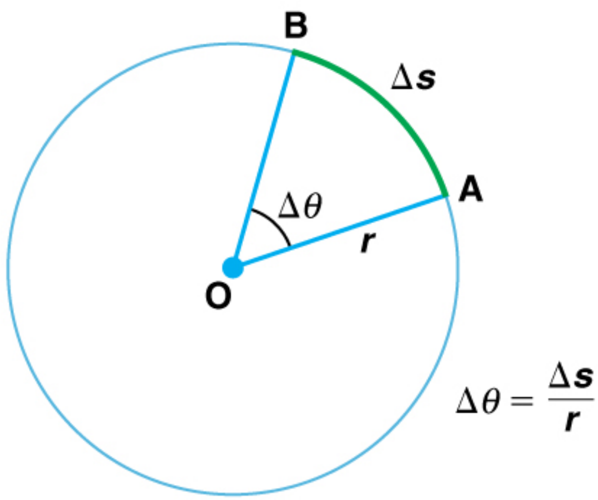


Figure 6.3 The radius of a circle is rotated through an angle $\Delta\theta$. The arc length Δs is described on the circumference.

The **arc length** Δs is the distance traveled along a circular path as shown in Figure 6.3 Note that r is the **radius of curvature** of the circular path.

We know that for one complete revolution, the arc length is the circumference of a circle of radius r . The circumference of a circle is $2\pi r$. Thus for one complete revolution the rotation angle is

$$\Delta\theta = \frac{2\pi r}{r} = 2\pi. \tag{6.2}$$

This result is the basis for defining the units used to measure rotation angles, $\Delta\theta$ to be **radians** (rad), defined so that

$$2\pi \text{ rad} = 1 \text{ revolution}. \tag{6.3}$$

A comparison of some useful angles expressed in degrees and radians is shown in Table 6.1.

Table 6.1 Comparison of Angular Units

Degree Measures	Radian Measure
30°	$\frac{\pi}{6}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
180°	π

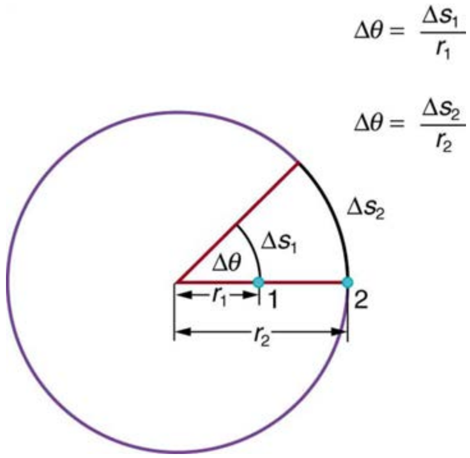


Figure 6.4 Points 1 and 2 rotate through the same angle ($\Delta\theta$), but point 2 moves through a greater arc length (Δs) because it is at a greater distance from the center of rotation (r) .

If $\Delta\theta = 2\pi$ rad, then the CD has made one complete revolution, and every point on the CD is back at its original position. Because there are 360° in a circle or one revolution, the relationship between radians and degrees is thus

$$2\pi \text{ rad} = 360^{\circ} \tag{6.4}$$

so that

$$1 \text{ rad} = \frac{360^{\circ}}{2\pi} \approx 57.3^{\circ}. \tag{6.5}$$

Angular Velocity

How fast is an object rotating? We define **angular velocity** ω as the rate of change of an angle. In symbols, this is

$$\omega = \frac{\Delta\theta}{\Delta t}, \tag{6.6}$$

where an angular rotation $\Delta\theta$ takes place in a time Δt . The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity ω is analogous to linear velocity v . To get the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length Δs in a time Δt , and so it has a linear velocity

$$v = \frac{\Delta s}{\Delta t}. \tag{6.7}$$

From $\Delta\theta = \frac{\Delta s}{r}$ we see that $\Delta s = r\Delta\theta$. Substituting this into the expression for v gives

$$v = \frac{r\Delta\theta}{\Delta t} = r\omega. \tag{6.8}$$

We write this relationship in two different ways and gain two different insights:

$$v = r\omega \text{ or } \omega = \frac{v}{r}. \tag{6.9}$$

The first relationship in $v = r\omega \text{ or } \omega = \frac{v}{r}$ states that the linear velocity v is proportional to the distance from the center of rotation, thus, it is largest for a point on the rim (largest r), as you might expect. We can also call this linear speed v of a point on the rim the *tangential speed*. The second relationship in $v = r\omega \text{ or } \omega = \frac{v}{r}$ can be illustrated by considering the tire of a moving car. Note that the speed of a point on the rim of the tire is the same as the speed v of the car. See **Figure 6.5**. So the faster the car moves, the faster the tire spins—large v means a large ω, because $v = r\omega$. Similarly, a larger-radius tire rotating at the same angular velocity (ω) will produce a greater linear speed (v) for the car.

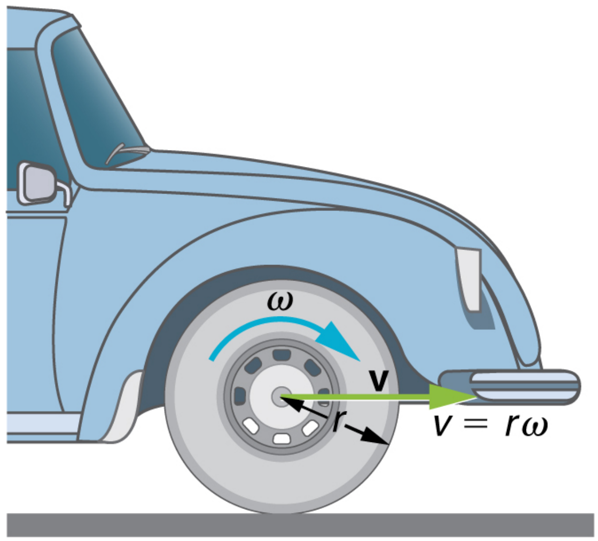


Figure 6.5 A car moving at a velocity v to the right has a tire rotating with an angular velocity ω .The speed of the tread of the tire relative to the axle is v, the same as if the car were jacked up. Thus the car moves forward at linear velocity $v = r\omega$, where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

Example 6.1 How Fast Does a Car Tire Spin?

Calculate the angular velocity of a 0.300 m radius car tire when the car travels at 15.0 m/s (about 54 km/h). See **Figure 6.5**.

Strategy

Because the linear speed of the tire rim is the same as the speed of the car, we have $v = 15.0 \text{ m/s}.$ The radius of the tire is given to be $r = 0.300 \text{ m}.$ Knowing v and r, we can use the second relationship in $v = r\omega, \omega = \frac{v}{r}$ to calculate the angular velocity.

Solution

To calculate the angular velocity, we will use the following relationship:

$$\omega = \frac{v}{r}. \tag{6.10}$$

Substituting the knowns,

$$\omega = \frac{15.0 \text{ m/s}}{0.300 \text{ m}} = 50.0 \text{ rad/s}. \tag{6.11}$$

Discussion

When we cancel units in the above calculation, we get 50.0/s. But the angular velocity must have units of rad/s. Because radians are actually unitless (radians are defined as a ratio of distance), we can simply insert them into the answer for the angular velocity. Also note that if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular velocity

$$\omega = (15.0 \text{ m/s}) / (1.20 \text{ m}) = 12.5 \text{ rad/s}. \tag{6.12}$$

Both ω and v have directions (hence they are angular and linear *velocities*, respectively). Angular velocity has only two directions with respect to the axis of rotation—it is either clockwise or counterclockwise. Linear velocity is tangent to the path, as illustrated in **Figure 6.6**.

Take-Home Experiment

Tie an object to the end of a string and swing it around in a horizontal circle above your head (swing at your wrist). Maintain uniform speed as the object swings and measure the angular velocity of the motion. What is the approximate speed of the object? Identify a point close to your hand and take appropriate measurements to calculate the linear speed at this point. Identify other circular motions and measure their angular velocities.

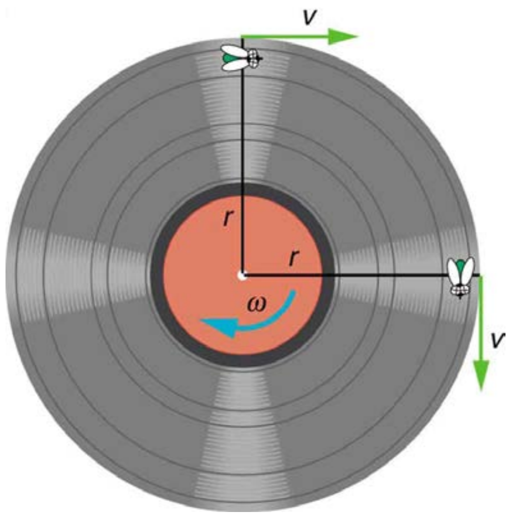


Figure 6.6 As an object moves in a circle, here a fly on the edge of an old-fashioned vinyl record, its instantaneous velocity is always tangent to the circle. The direction of the angular velocity is clockwise in this case.

PhET Explorations: Ladybug Revolution

PhET Interactive Simulation

Figure 6.7 Ladybug Revolution (http://cnx.org/content/m54992/1.2/rotation_en.jar)

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

6.2 Centripetal Acceleration

Learning Objectives

By the end of this section, you will be able to:

- Establish the expression for centripetal acceleration.
- Explain the centrifuge.

We know from kinematics that acceleration is a change in velocity, either in its magnitude or in its direction, or both. In uniform circular motion, the direction of the velocity changes constantly, so there is always an associated acceleration, even though the magnitude of the velocity might be constant. You experience this acceleration yourself when you turn a corner in your car. (If you hold the wheel steady during a turn and move at constant speed, you are in uniform circular motion.) What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we examine the direction and magnitude of that acceleration.

Figure 6.8 shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the **centripetal acceleration**(a_c); centripetal means "toward the center" or "center seeking."

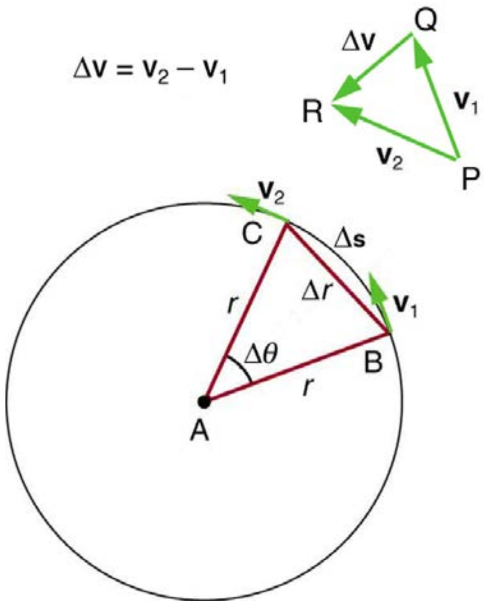


Figure 6.8 The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta \mathbf{v}$ is seen to point directly toward the center of curvature. (See small inset.) Because $\mathbf{a}_c = \Delta \mathbf{v} / \Delta t$, the acceleration is also toward the center; $\mathbf{a}_c$ is called centripetal acceleration. (Because $\Delta\theta$ is very small, the arc length Δs is equal to the chord length Δr for small time differences.)

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? Note that the triangle formed by the velocity vectors and the one formed by the radii r and Δs are similar. Both the triangles ABC and PQR are isosceles triangles (two equal sides). The two equal sides of the velocity vector triangle are the speeds $v_1 = v_2 = v$. Using the properties of two similar triangles, we obtain

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}. \tag{6.13}$$

Acceleration is $\Delta v / \Delta t$, and so we first solve this expression for Δv :

$$\Delta v = \frac{v}{r}\Delta s. \tag{6.14}$$

Then we divide this by Δt , yielding

$$\frac{\Delta v}{\Delta t} = \frac{v}{r}\times\frac{\Delta s}{\Delta t}. \tag{6.15}$$

Finally, noting that $\Delta v / \Delta t = a_c$ and that $\Delta s / \Delta t = v$, the linear or tangential speed, we see that the magnitude of the centripetal acceleration is

$$a_c = \frac{v^2}{r}, \tag{6.16}$$

which is the acceleration of an object in a circle of radius r at a speed v . So, centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that a_c is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that a_c is greater for tighter turns, as you have probably noticed.

It is also useful to express a_c in terms of angular velocity. Substituting $v = r\omega$ into the above expression, we find $a_c = (r\omega)^2 / r = r\omega^2$. We can express the magnitude of centripetal acceleration using either of two equations:

$$a_c = \frac{v^2}{r};\ a_c = r\omega^2. \tag{6.17}$$

Recall that the direction of a_c is toward the center. You may use whichever expression is more convenient, as illustrated in examples below.

A **centrifuge** (see **Figure 6.9**b) is a rotating device used to separate specimens of different densities. High centripetal acceleration significantly decreases the time it takes for separation to occur, and makes separation possible with small samples. Centrifuges are used in a variety of applications in science and medicine, including the separation of single cell suspensions such as bacteria, viruses, and blood cells from a liquid medium and the separation of macromolecules, such as DNA and protein, from a solution. Centrifuges are often rated in terms of their centripetal acceleration relative to acceleration due to gravity (g); maximum centripetal acceleration of several hundred thousand g is possible in a vacuum. Human centrifuges, extremely large centrifuges, have been used to test the tolerance of astronauts to the effects of accelerations larger than that of Earth's gravity.

Example 6.2 How Does the Centripetal Acceleration of a Car Around a Curve Compare with That Due to Gravity?

What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed. See **Figure 6.9**(a).

Strategy

Because v and r are given, the first expression in $a_{\mathrm{c}} = \frac{v^2}{r}; a_{\mathrm{c}} = r\omega^2$ is the most convenient to use.

Solution

Entering the given values of $v = 25.0\text{ m/s}$ and $r = 500\text{ m}$ into the first expression for a_{c} gives

$$a_{\mathrm{c}} = \frac{v^2}{r} = \frac{(25.0\text{ m/s})^2}{500\text{ m}} = 1.25\text{ m/s}^2. \tag{6.18}$$

Discussion

To compare this with the acceleration due to gravity $(g = 9.80\text{ m/s}^2)$, we take the ratio of $a_{\mathrm{c}} / g = \left(1.25\text{ m/s}^2\right) / \left(9.80\text{ m/s}^2\right) = 0.128$. Thus, $a_{\mathrm{c}} = 0.128\text{ g}$ and is noticeable especially if you were not wearing a seat belt.

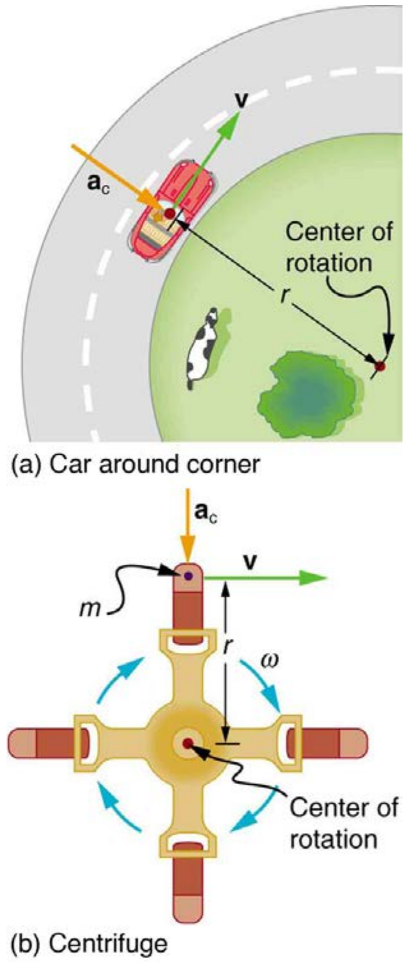


Figure 6.9 (a) The car following a circular path at constant speed is accelerated perpendicular to its velocity, as shown. The magnitude of this centripetal acceleration is found in **Example 6.2**. (b) A particle of mass in a centrifuge is rotating at constant angular velocity . It must be accelerated perpendicular to its velocity or it would continue in a straight line. The magnitude of the necessary acceleration is found in **Example 6.3**.

Example 6.3 How Big Is the Centripetal Acceleration in an Ultracentrifuge?

Calculate the centripetal acceleration of a point 7.50 cm from the axis of an **ultracentrifuge** spinning at $7.5 \times 10^4 \text{ rev/min}$. Determine the ratio of this acceleration to that due to gravity. See **Figure 6.9**(b).

Strategy

The term rev/min stands for revolutions per minute. By converting this to radians per second, we obtain the angular velocity ω. Because r is given, we can use the second expression in the equation $a_\text{c} = \frac{v^2}{r}; a_\text{c} = r\omega^2$ to calculate the centripetal acceleration.

Solution

To convert $7.50\times10^4 \text{ rev / min}$ to radians per second, we use the facts that one revolution is $2\pi\text{rad}$ and one minute is 60.0 s. Thus,

$$\omega = 7.50\times10^4 \ \frac{\text{rev}}{\text{min}}\times\frac{2\pi \text{ rad}}{1 \text{ rev}}\times\frac{1 \text{ min}}{60.0 \text{ s}} = 7854 \text{ rad/s}. \tag{6.19}$$

Now the centripetal acceleration is given by the second expression in $a_\text{c} = \frac{v^2}{r}; a_\text{c} = r\omega^2$ as

$$a_\text{c} = r\omega^2. \tag{6.20}$$

Converting 7.50 cm to meters and substituting known values gives

$$a_\text{c} = (0.0750 \text{ m})(7854 \text{ rad/s})^2 = 4.63\times10^6 \text{ m/s}^2. \tag{6.21}$$

Note that the unitless radians are discarded in order to get the correct units for centripetal acceleration. Taking the ratio of a_{c} to g yields

$$\frac{a_{\mathrm{c}}}{g} = \frac{4.63\times10^{6}}{9.80} = 4.72\times10^{5}. \tag{6.22}$$

Discussion

This last result means that the centripetal acceleration is 472,000 times as strong as g. It is no wonder that such high ω centrifuges are called ultracentrifuges. The extremely large accelerations involved greatly decrease the time needed to cause the sedimentation of blood cells or other materials.

Of course, a net external force is needed to cause any acceleration, just as Newton proposed in his second law of motion. So a net external force is needed to cause a centripetal acceleration. In **Centripetal Force**, we will consider the forces involved in circular motion.

PhET Explorations: Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

PhET Interactive Simulation

Figure 6.10 Ladybug Motion 2D (http://cnx.org/content/m54995/1.2/ladybug-motion-2d_en.jar)

6.3 Centripetal Force

Learning Objectives

By the end of this section, you will be able to:

- Calculate coefficient of friction on a car tire.
- Calculate ideal speed and angle of a car on a turn.

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: net $\mathrm{F} = ma$. For uniform circular motion, the acceleration is the centripetal acceleration— $a = a_c$. Thus, the magnitude of centripetal force $\mathrm{F_c}$ is

$$\mathrm{F_c} = ma_{\mathrm{c}}. \tag{6.23}$$

By using the expressions for centripetal acceleration a_c from $a_c = \frac{v^2}{r}; a_c = r\omega^2$, we get two expressions for the centripetal force $\mathrm{F_c}$ in terms of mass, velocity, angular velocity, and radius of curvature:

$$F_c = m\frac{v^2}{r}; F_c = mr\omega^2. \tag{6.24}$$

You may use whichever expression for centripetal force is more convenient. Centripetal force F_{c} is always perpendicular to the path and pointing to the center of curvature, because $\mathbf{a}_c$ is perpendicular to the velocity and pointing to the center of curvature.

Note that if you solve the first expression for r, you get

$$r = \frac{mv^2}{F_c}. \tag{6.25}$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.

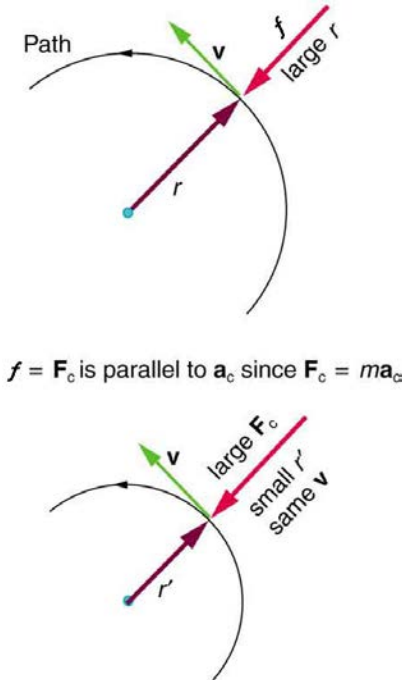


Figure 6.11 The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the $\mathrm{F_c}$, the smaller the radius of curvature r and the sharper the curve. The second curve has the same v , but a larger $\mathrm{F_c}$ produces a smaller r' .

Example 6.4 What Coefficient of Friction Do Car Tires Need on a Flat Curve?

(a) Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.

(b) Assuming an unbanked curve, find the minimum static coefficient of friction, between the tires and the road, static friction being the reason that keeps the car from slipping (see Figure 6.12).

Strategy and Solution for (a)

We know that $F_{\text{c}} = \frac{mv^2}{r}$. Thus,

$$F_{\text{c}} = \frac{mv^2}{r} = \frac{(900\text{ kg})(25.0\text{ m/s})^2}{(500\text{ m})} = 1125\text{ N}. \tag{6.26}$$

Strategy for (b)

Figure 6.12 shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is $\mu_{\text{s}}N$, where μ_{s} is the static coefficient of friction and N is the normal force. The normal force equals the car's weight on level ground, so that $N = mg$. Thus the centripetal force in this situation is

$$F_{\text{c}} = f = \mu_{\text{s}}N = \mu_{\text{s}}mg. \tag{6.27}$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the first expression for F_{c} from the equation

$$\left.\begin{array}{l} F_{\text{c}} = m\frac{v^2}{r} \\ F_{\text{c}} = mr\omega^2 \end{array}\right\}, \tag{6.28}$$

$$m\frac{v^2}{r} = \mu_{\text{s}}mg. \tag{6.29}$$

We solve this for μ_{s} , noting that mass cancels, and obtain

$$\mu_s = \frac{v^2}{rg}. \tag{6.30}$$

Solution for (b)

Substituting the knowns,

$$\mu_s = \frac{(25.0\text{ m/s})^2}{(500\text{ m})(9.80\text{ m/s}^2)} = 0.13. \tag{6.31}$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

Discussion

We could also solve part (a) using the first expression in $\left.\begin{array}{l} F_c = m\frac{v^2}{r} \\ F_c = mr\omega^2 \end{array}\right\}$, because m, v, and r are given. The coefficient of friction found in part (b) is much smaller than is typically found between tires and roads. The car will still negotiate the curve if the coefficient is greater than 0.13, because static friction is a responsive force, being able to assume a value less than but no more than $\mu_s N$. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less as will be discussed below.

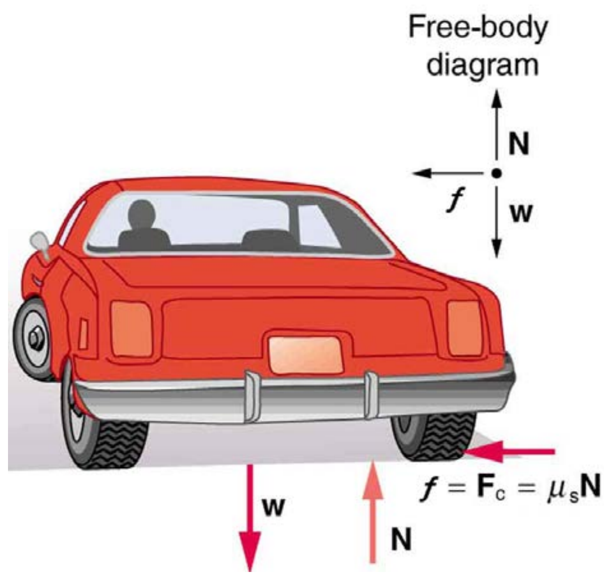


Figure 6.12 This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

Let us now consider **banked curves**, where the slope of the road helps you negotiate the curve. See **Figure 6.13**. The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for θ for an ideally banked curve and consider an example related to it.

For **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

Figure 6.13 shows a free body diagram for a car on a frictionless banked curve. If the angle θ is ideal for the speed and radius, then the net external force will equal the necessary centripetal force. The only two external forces acting on the car are its weight $\mathbf{w}$ and the normal force of the road $\mathbf{N}$. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude mv^2/r. Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, and so this must equal the centripetal force—that is,

$$N \sin \theta = \frac{mv^2}{r}. \tag{6.32}$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From the figure, we see that the vertical component of the normal force is $N \cos \theta$, and the only other vertical force is the car's weight. These must be equal in magnitude; thus,

$$N \cos \theta = mg. \tag{6.33}$$

Now we can combine the last two equations to eliminate N and get an expression for θ, as desired. Solving the second equation for $N = mg/(\cos \theta)$, and substituting this into the first yields

$$mg\frac{\sin \theta}{\cos \theta} = \frac{mv^2}{r} \tag{6.34}$$

$$\begin{aligned} mg \tan(\theta) &= \frac{mv^2}{r} \\ \tan \theta &= \frac{v^2}{rg.} \end{aligned} \tag{6.35}$$

Taking the inverse tangent gives

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) \text{(ideally banked curve, no friction)}. \tag{6.36}$$

This expression can be understood by considering how θ depends on v and r. A large θ will be obtained for a large v and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless. Note that θ does not depend on the mass of the vehicle.

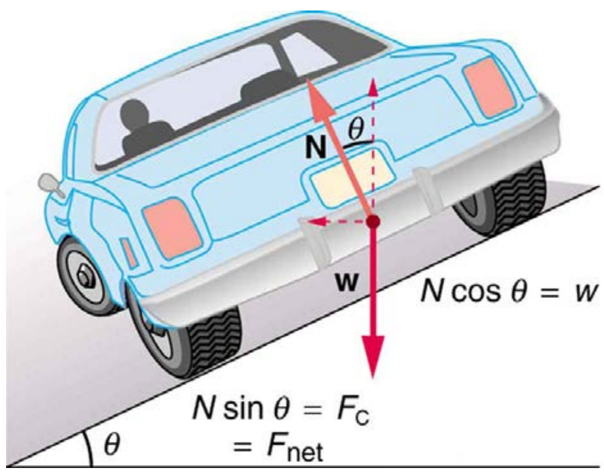


Figure 6.13 The car on this banked curve is moving away and turning to the left.

Example 6.5 What Is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as the Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100 m radius curve banked at 65.0° should be driven if the road is frictionless.

Strategy

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

Solution

Starting with

$$\tan \theta = \frac{v^2}{rg} \tag{6.37}$$

we get

$$v = (rg\ \tan\theta)^{1/2}. \tag{6.38}$$

Noting that tan 65.0º = 2.14, we obtain

$$\begin{aligned} v &= \left[(100\text{ m})(9.80\text{ m/s}^2)(2.14)\right]^{1/2} \\ &= 45.8\text{ m/s}. \end{aligned} \tag{6.39}$$

Discussion

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Calculations similar to those in the preceding examples can be performed for a host of interesting situations in which centripetal force is involved—a number of these are presented in this chapter's Problems and Exercises.

Take-Home Experiment

Ask a friend or relative to swing a golf club or a tennis racquet. Take appropriate measurements to estimate the centripetal acceleration of the end of the club or racquet. You may choose to do this in slow motion.

PhET Explorations: Gravity and Orbits

Move the sun, earth, moon and space station to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it!

PhET Interactive Simulation

Figure 6.14 Gravity and Orbits (http://cnx.org/content/m55002/1.2/gravity-and-orbits_en.jar)

6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

Learning Objectives

By the end of this section, you will be able to:

- Discuss the inertial frame of reference.
- Discuss the non-inertial frame of reference.
- Describe the effects of the Coriolis force.

What do taking off in a jet airplane, turning a corner in a car, riding a merry-go-round, and the circular motion of a tropical cyclone have in common? Each exhibits fictitious forces—unreal forces that arise from motion and may *seem* real, because the observer’s frame of reference is accelerating or rotating.

When taking off in a jet, most people would agree it feels as if you are being pushed back into the seat as the airplane accelerates down the runway. Yet a physicist would say that *you* tend to remain stationary while the *seat* pushes forward on you, and there is no real force backward on you. An even more common experience occurs when you make a tight curve in your car—say, to the right. You feel as if you are thrown (that is, *forced*) toward the left relative to the car. Again, a physicist would say that *you* are going in a straight line but the *car* moves to the right, and there is no real force on you to the left. Recall Newton’s first law.

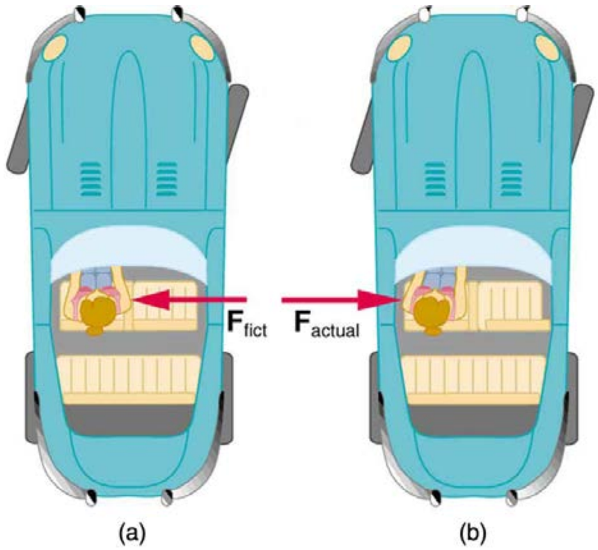


Figure 6.15 (a) The car driver feels herself forced to the left relative to the car when she makes a right turn. This is a fictitious force arising from the use of the car as a frame of reference. (b) In the Earth's frame of reference, the driver moves in a straight line, obeying Newton's first law, and the car moves to the right. There is no real force to the left on the driver relative to Earth. There is a real force to the right on the car to make it turn.

We can reconcile these points of view by examining the frames of reference used. Let us concentrate on people in a car. Passengers instinctively use the car as a frame of reference, while a physicist uses Earth. The physicist chooses Earth because it is very nearly an inertial frame of reference—one in which all forces are real (that is, in which all forces have an identifiable physical origin). In such a frame of reference, Newton's laws of motion take the form given in **Dynamics: Newton's Laws of Motion** The car is a **non-inertial frame of reference** because it is accelerated to the side. The force to the left sensed by car passengers is a **fictitious force** having no physical origin. There is nothing real pushing them left—the car, as well as the driver, is actually accelerating to the right.

Let us now take a mental ride on a merry-go-round—specifically, a rapidly rotating playground merry-go-round. You take the merry-go-round to be your frame of reference because you rotate together. In that non-inertial frame, you feel a fictitious force, named **centrifugal force (**not to be confused with centripetal force**)**, trying to throw you off. You must hang on tightly to counteract the centrifugal force. In Earth's frame of reference, there is no force trying to throw you off. Rather you must hang on to make yourself go in a circle because otherwise you would go in a straight line, right off the merry-go-round.

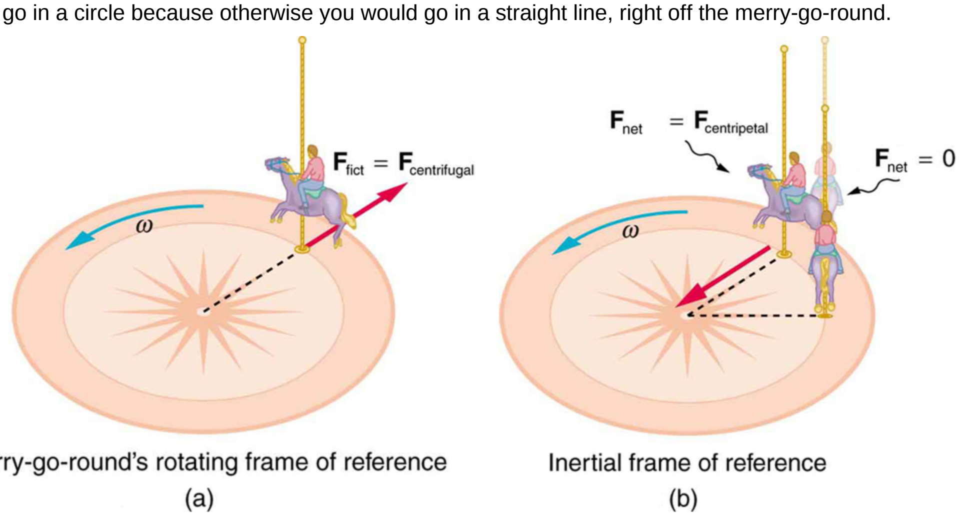


Figure 6.16 (a) A rider on a merry-go-round feels as if he is being thrown off. This fictitious force is called the centrifugal force—it explains the rider's motion in the rotating frame of reference. (b) In an inertial frame of reference and according to Newton's laws, it is his inertia that carries him off and not a real force (the unshaded rider has $F_{\text{net}} = 0$ and heads in a straight line). A real force, $F_{\text{centripetal}}$, is needed to cause a circular path.

This inertial effect, carrying you away from the center of rotation if there is no centripetal force to cause circular motion, is put to good use in centrifuges (see **Figure 6.17**). A centrifuge spins a sample very rapidly, as mentioned earlier in this chapter. Viewed from the rotating frame of reference, the fictitious centrifugal force throws particles outward, hastening their sedimentation. The greater the angular velocity, the greater the centrifugal force. But what really happens is that the inertia of the particles carries them along a line tangent to the circle while the test tube is forced in a circular path by a centripetal force.

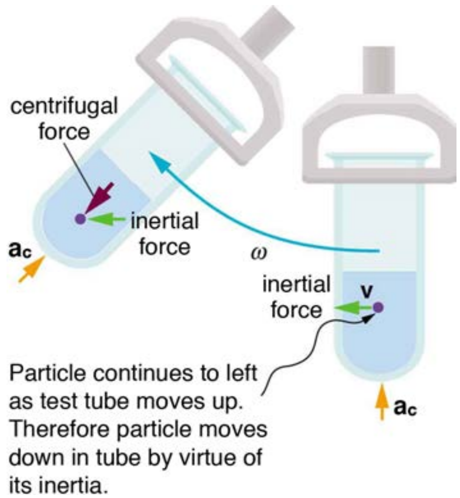


Figure 6.17 Centrifuges use inertia to perform their task. Particles in the fluid sediment come out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles will come into contact with the test tube walls, which will then supply the centripetal force needed to make them move in a circle of constant radius.

Let us now consider what happens if something moves in a frame of reference that rotates. For example, what if you slide a ball directly away from the center of the merry-go-round, as shown in **Figure 6.18**? The ball follows a straight path relative to Earth (assuming negligible friction) and a path curved to the right on the merry-go-round's surface. A person standing next to the merry-go-round sees the ball moving straight and the merry-go-round rotating underneath it. In the merry-go-round's frame of reference, we explain the apparent curve to the right by using a fictitious force, called the **Coriolis force**, that causes the ball to curve to the right. The fictitious Coriolis force can be used by anyone in that frame of reference to explain why objects follow curved paths and allows us to apply Newton's Laws in non-inertial frames of reference.

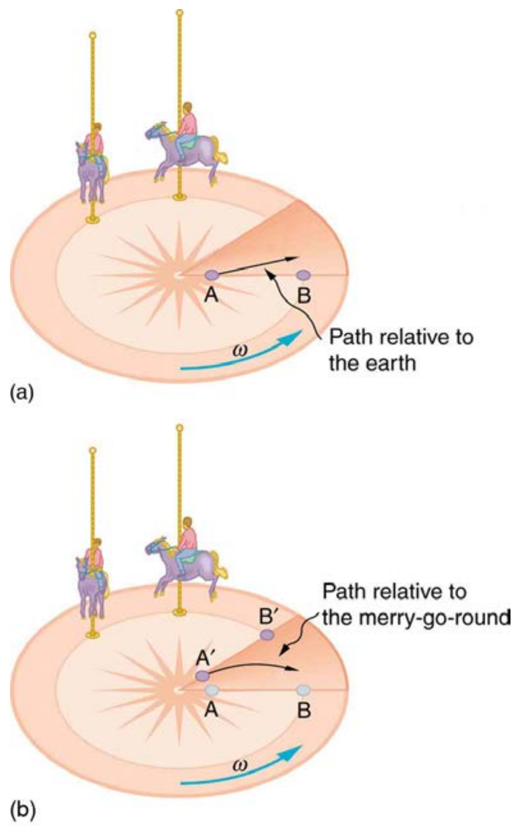


Figure 6.18 Looking down on the counterclockwise rotation of a merry-go-round, we see that a ball slid straight toward the edge follows a path curved to the right. The person slides the ball toward point B, starting at point A. Both points rotate to the shaded positions (A' and B') shown in the time that the ball follows the curved path in the rotating frame and a straight path in Earth's frame.

Up until now, we have considered Earth to be an inertial frame of reference with little or no worry about effects due to its rotation. Yet such effects *do* exist—in the rotation of weather systems, for example. Most consequences of Earth's rotation can be qualitatively understood by analogy with the merry-go-round. Viewed from above the North Pole, Earth rotates counterclockwise, as does the merry-go-round in **Figure 6.18**. As on the merry-go-round, any motion in Earth's northern hemisphere experiences a

Coriolis force to the right. Just the opposite occurs in the southern hemisphere; there, the force is to the left. Because Earth's angular velocity is small, the Coriolis force is usually negligible, but for large-scale motions, such as wind patterns, it has substantial effects.

The Coriolis force causes hurricanes in the northern hemisphere to rotate in the counterclockwise direction, while the tropical cyclones (what hurricanes are called below the equator) in the southern hemisphere rotate in the clockwise direction. The terms hurricane, typhoon, and tropical storm are regionally-specific names for tropical cyclones, storm systems characterized by low pressure centers, strong winds, and heavy rains. **Figure 6.19** helps show how these rotations take place. Air flows toward any region of low pressure, and tropical cyclones contain particularly low pressures. Thus winds flow toward the center of a tropical cyclone or a low-pressure weather system at the surface. In the northern hemisphere, these inward winds are deflected to the right, as shown in the figure, producing a counterclockwise circulation at the surface for low-pressure zones of any type. Low pressure at the surface is associated with rising air, which also produces cooling and cloud formation, making low-pressure patterns quite visible from space. Conversely, wind circulation around high-pressure zones is clockwise in the northern hemisphere but is less visible because high pressure is associated with sinking air, producing clear skies.

The rotation of tropical cyclones and the path of a ball on a merry-go-round can just as well be explained by inertia and the rotation of the system underneath. When non-inertial frames are used, fictitious forces, such as the Coriolis force, must be invented to explain the curved path. There is no identifiable physical source for these fictitious forces. In an inertial frame, inertia explains the path, and no force is found to be without an identifiable source. Either view allows us to describe nature, but a view in an inertial frame is the simplest and truest, in the sense that all forces have real origins and explanations.

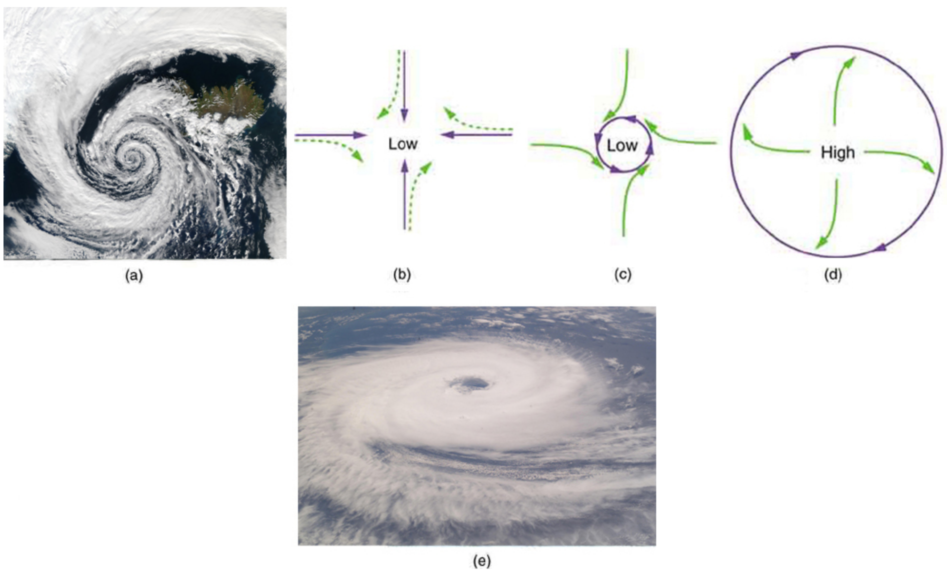


Figure 6.19 (a) The counterclockwise rotation of this northern hemisphere hurricane is a major consequence of the Coriolis force. (credit: NASA) (b) Without the Coriolis force, air would flow straight into a low-pressure zone, such as that found in tropical cyclones. (c) The Coriolis force deflects the winds to the right, producing a counterclockwise rotation. (d) Wind flowing away from a high-pressure zone is also deflected to the right, producing a clockwise rotation. (e) The opposite direction of rotation is produced by the Coriolis force in the southern hemisphere, leading to tropical cyclones. (credit: NASA)

6.5 Newton's Universal Law of Gravitation

Learning Objectives

By the end of this section, you will be able to:

- Explain Earth's gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.
- Understand the Cavendish experiment.

The information presented in this section supports the following AP® learning objectives and science practices:

- **2.B.2.1** The student is able to apply $g = \dfrac{GM}{r^2}$ to calculate the gravitational field due to an object with mass *M,* where the field is a vector directed toward the center of the object of mass *M.* **(S.P. 2.2)**
- **2.B.2.2** The student is able to approximate a numerical value of the gravitational field (g) near the surface of an object from its radius and mass relative to those of the Earth or other reference objects. **(S.P. 2.2)**
- **3.A.3.4.** The student is able to make claims about the force on an object due to the presence of other objects with the same property: mass, electric charge. **(S.P. 6.1, 6.4)**

What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth's gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth's surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See **Figure 6.20**. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others. This was one of the earliest examples of a theory derived from empirical evidence doing more than merely describing those empirical results; it made claims about the fundamental workings of the universe.

Figure 6.20 According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton's apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, **Newton's universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

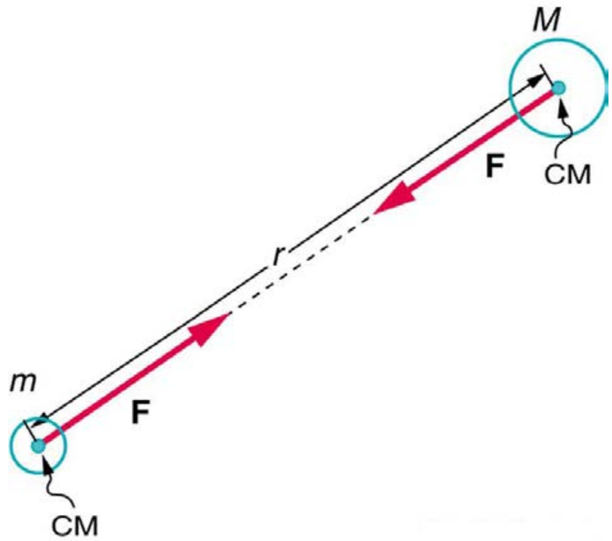


Figure 6.21 Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law.

Misconception Alert

The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the **center of mass** (CM), which will be further explored in **Linear Momentum and Collisions**. For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton's universal law of gravitation is

$$F = G\frac{mM}{r^2}, \tag{6.40}$$

where F is the magnitude of the gravitational force and G is a proportionality factor called the **gravitational constant**. G is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be

$$G = 6.673 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \tag{6.41}$$

in SI units. Note that the units of G are such that a force in newtons is obtained from $F = G\frac{mM}{r^2}$, when considering masses in kilograms and distance in meters. For example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of $6.673 \times 10^{-11}\ \text{N}$. This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of $6 \times 10^{24}\ \text{kg}$.

The experiment to measure *G* was first performed by Cavendish, and is explained in more detail later. The fundamental concept it is based on is having a known mass on a spring with a known force (or spring) constant. Then, a second known mass is placed at multiple known distances from the first, and the amount of stretch in the spring resulting from the gravitational attraction of the two masses is measured.

Recall that the acceleration due to gravity g is about $9.80\ \text{m/s}^2$ on Earth. We can now determine why this is so. The weight of an object *mg* is the gravitational force between it and Earth. Substituting *mg* for F in Newton's universal law of gravitation gives

$$mg = G\frac{mM}{r^2}, \tag{6.42}$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See **Figure 6.22**. The mass m of the object cancels, leaving an equation for g:

$$g = G\frac{M}{r^2}. \tag{6.43}$$

Substituting known values for Earth's mass and radius (to three significant figures),

$$g = \left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\times\frac{5.98\times10^{24}\text{ kg}}{(6.38\times10^6\text{ m})^2}, \tag{6.44}$$

and we obtain a value for the acceleration of a falling body:

$$g = 9.80\text{ m/s}^2. \tag{6.45}$$

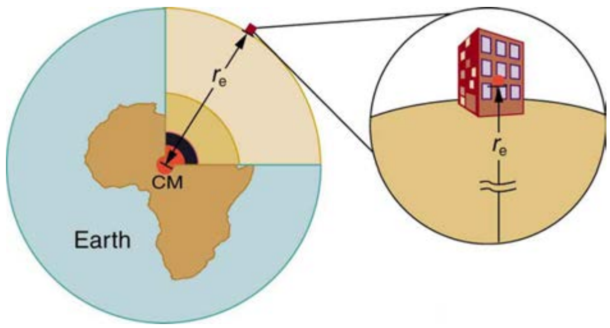


Figure 6.22 The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

Gravitational Mass and Inertial Mass

Notice that, in Equation 6.40, the mass of the objects under consideration is directly proportional to the gravitational force. More mass means greater forces, and vice versa. However, we have already seen the concept of mass before in a different context.

In Chapter 4, you read that mass is a measure of inertia. However, we normally measure the mass of an object by measuring the force of gravity (F) on it.

How do we know that inertial mass is identical to gravitational mass? Assume that we compare the mass of two objects. The objects have inertial masses m_1 and m_2. If the objects balance each other on a pan balance, we can conclude that they have the same gravitational mass, that is, that they experience the same force due to gravity, F. Using Newton's second law of motion, $F = ma$, we can write $m_1 a_1 = m_2 a_2$.

If we can show that the two objects experience the same acceleration due to gravity, we can conclude that $m_1 = m_2$, that is, that the objects' inertial masses are equal.

In fact, Galileo and others conducted experiments to show that, when factors such as wind resistance are kept constant, all objects, regardless of their mass, experience the same acceleration due to gravity. Galileo is famously said to have dropped two balls of different masses off the leaning tower of Pisa to demonstrate this. The balls accelerated at the same rate. Since acceleration due to gravity is constant for all objects on Earth, regardless of their mass or composition, i.e., $a_1 = a_2$, then $m_1 = m_2$. Thus, we can conclude that inertial mass is identical to gravitational mass. This allows us to calculate the acceleration of free fall due to gravity, such as in the orbits of planets.

Take-Home Experiment

Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

Making Connections: Gravitation, Other Forces, and General Relativity

Attempts are still being made to understand the gravitational force. As we shall see in **Particle Physics**, modern physics is exploring the connections of gravity to other forces, space, and time. General relativity alters our view of gravitation, leading us to think of gravitation as bending space and time.

Applying the Science Practices: All Objects Have Gravitational Fields

We can use the formula developed above, $g = \frac{GM}{r^2}$, to calculate the gravitational fields of other objects.

For example, the Moon has a radius of 1.7 × 10^6 m and a mass of 7.3 × 10^{22} kg. The gravitational field on the surface of the Moon can be expressed as

$$g = G\frac{M}{r^2}$$

$$= \left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\times\frac{7.3\times10^{22}\ \text{kg}}{\left(1.7\times10^6\ \text{m}\right)^2}$$

$$= 1.685\ \text{m/s}^2$$

This is about 1/6 of the gravity on Earth, which seems reasonable, since the Moon has a much smaller mass than Earth does.

A person has a mass of 50 kg. The gravitational field 1.0 m from the person's center of mass can be expressed as

$$g = G\frac{M}{r^2}$$

$$= \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\times\frac{50\ \text{kg}}{(1\ \text{m})^2}$$

$$= 3.34\times10^{-9}\ \text{m/s}^2$$

This is less than one millionth of the gravitational field at the surface of Earth.

In the following example, we make a comparison similar to one made by Newton himself. He noted that if the gravitational force caused the Moon to orbit Earth, then the acceleration due to gravity should equal the centripetal acceleration of the Moon in its orbit. Newton found that the two accelerations agreed "pretty nearly."

Example 6.6 Earth's Gravitational Force Is the Centripetal Force Making the Moon Move in a Curved Path

(a) Find the acceleration due to Earth's gravity at the distance of the Moon.

(b) Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit about a fixed Earth), and compare it with the value of the acceleration due to Earth's gravity that you have just found.

Strategy for (a)

This calculation is the same as the one finding the acceleration due to gravity at Earth's surface, except that r is the distance from the center of Earth to the center of the Moon. The radius of the Moon's nearly circular orbit is $3.84\times10^8\ \text{m}$.

Solution for (a)

Substituting known values into the expression for g found above, remembering that M is the mass of Earth not the Moon, yields

$$\begin{aligned} g &= G\frac{M}{r^2} = \left(6.67\times10^{-11}\text{N}\cdot\text{m}^2\over\text{kg}^2\right)\times\frac{5.98\times10^{24}\ \text{kg}}{(3.84\times10^8\ \text{m})^2} \\ &= 2.70\times10^{-3}\ \text{m/s.}^2 \end{aligned} \tag{6.46}$$

Strategy for (b)

Centripetal acceleration can be calculated using either form of

$$\left.\begin{aligned} a_c &= \frac{v^2}{r} \\ a_c &= r\omega^2 \end{aligned}\right\}. \tag{6.47}$$

We choose to use the second form:

$$a_c = r\omega^2, \tag{6.48}$$

where ω is the angular velocity of the Moon about Earth.

Solution for (b)

Given that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days, (d) and using

$$1\text{ d}\times 24\frac{\text{hr}}{\text{d}}\times 60\frac{\text{min}}{\text{hr}}\times 60\frac{\text{s}}{\text{min}} = 86{,}400\text{ s} \tag{6.49}$$

we see that

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\text{ rad}}{(27.3\text{ d})(86{,}400\text{ s/d})} = 2.66\times 10^{-6}\frac{\text{rad}}{\text{s}}. \tag{6.50}$$

The centripetal acceleration is

$$\begin{aligned} a_c &= r\omega^2 = (3.84\times 10^8\text{ m})(2.66\times 10^{-6}\text{ rad/s})^2 \\ &= 2.72\times 10^{-3}\text{ m/s.}^2 \end{aligned} \tag{6.51}$$

The direction of the acceleration is toward the center of the Earth.

Discussion

The centripetal acceleration of the Moon found in (b) differs by less than 1% from the acceleration due to Earth's gravity found in (a). This agreement is approximate because the Moon's orbit is slightly elliptical, and Earth is not stationary (rather the Earth-Moon system rotates about its center of mass, which is located some 1700 km below Earth's surface). The clear implication is that Earth's gravitational force causes the Moon to orbit Earth.

Why does Earth not remain stationary as the Moon orbits it? This is because, as expected from Newton's third law, if Earth exerts a force on the Moon, then the Moon should exert an equal and opposite force on Earth (see **Figure 6.23**). We do not sense the Moon's effect on Earth's motion, because the Moon's gravity moves our bodies right along with Earth but there are other signs on Earth that clearly show the effect of the Moon's gravitational force as discussed in **Satellites and Kepler's Laws: An Argument for Simplicity**.

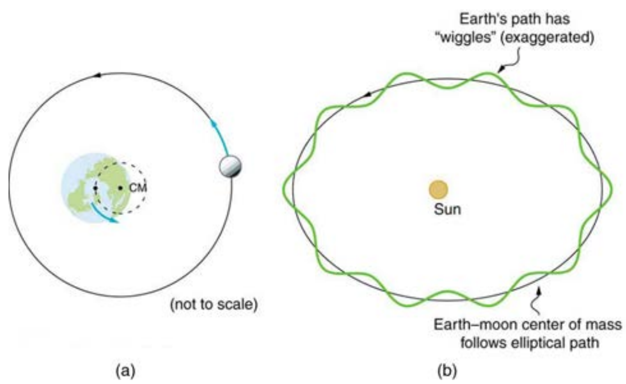


Figure 6.23 (a) Earth and the Moon rotate approximately once a month around their common center of mass. (b) Their center of mass orbits the Sun in an elliptical orbit, but Earth's path around the Sun has "wiggles" in it. Similar wiggles in the paths of stars have been observed and are considered direct evidence of planets orbiting those stars. This is important because the planets' reflected light is often too dim to be observed.

Tides

Ocean tides are one very observable result of the Moon's gravity acting on Earth. **Figure 6.24** is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).

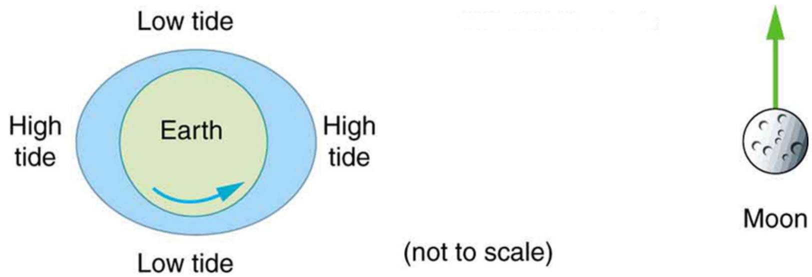


Figure 6.24 The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the water on the far side. The distances and sizes are not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day at any location, because Earth rotates under the tidal bulge.

The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a 90° angle to the Earth-Moon alignment.

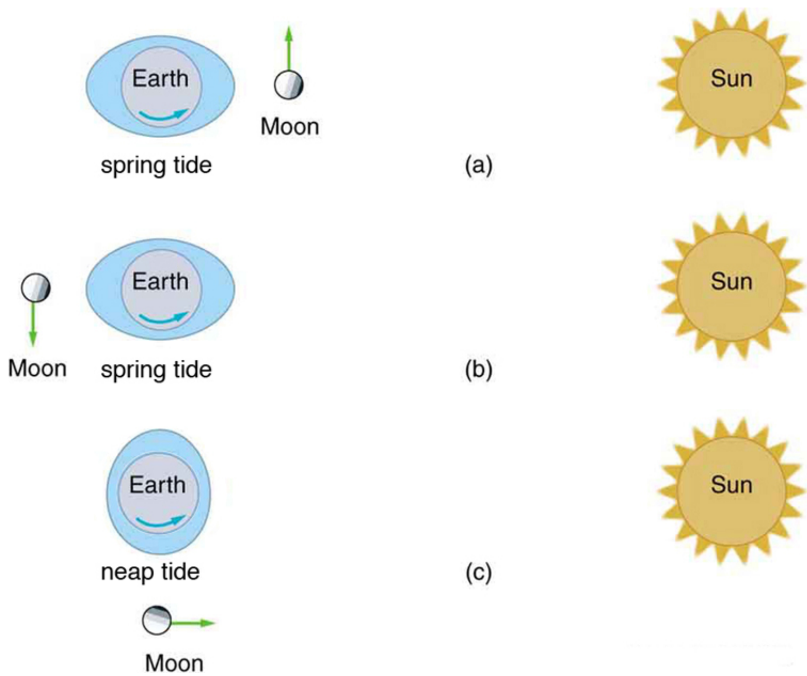


Figure 6.25 (a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at 90° to the Earth-Moon alignment. Note that this figure is not drawn to scale.

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see **Figure 6.26**). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.

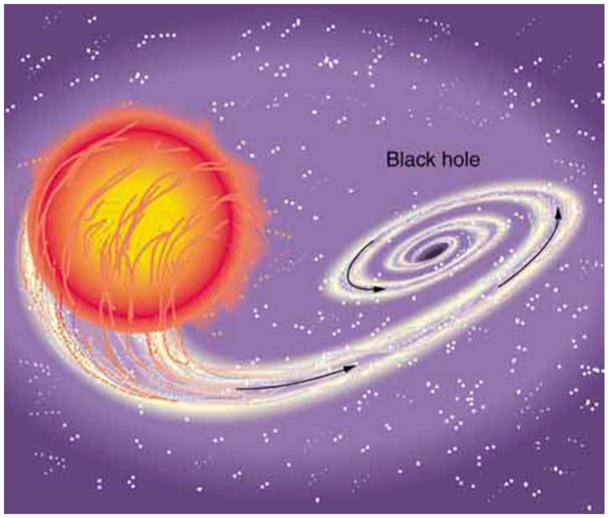


Figure 6.26 A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth.

”Weightlessness” and Microgravity

In contrast to the tremendous gravitational force near black holes is the apparent gravitational field experienced by astronauts orbiting Earth. What is the effect of “weightlessness” upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn't mean that an astronaut is not being acted upon by the gravitational force. There is no “zero gravity” in an astronaut's orbit. The term just means that the astronaut is in free-fall, accelerating with the acceleration due to gravity. If an elevator cable breaks, the passengers inside will be in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks.

Figure 6.27 Astronauts experiencing weightlessness on board the International Space Station. (credit: NASA)

Microgravity refers to an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

The Cavendish Experiment: Then and Now

As previously noted, the universal gravitational constant G is determined experimentally. This definition was first done accurately by Henry Cavendish (1731–1810), an English scientist, in 1798, more than 100 years after Newton published his universal law of gravitation. The measurement of G is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most), using apparatus like that in **Figure 6.28**. Remarkably, his value for G differs by less than 1% from the best modern value.

One important consequence of knowing G was that an accurate value for Earth's mass could finally be obtained. This was done by measuring the acceleration due to gravity as accurately as possible and then calculating the mass of Earth M from the relationship Newton's universal law of gravitation gives

$$mg = G\frac{mM}{r^2}, \tag{6.52}$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See **Figure 6.21**. The mass m of the object cancels, leaving an equation for g :

$$g = G\frac{M}{r^2}. \tag{6.53}$$

Rearranging to solve for M yields

$$M = \frac{gr^2}{G}. \tag{6.54}$$

So M can be calculated because all quantities on the right, including the radius of Earth r , are known from direct measurements. We shall see in **Satellites and Kepler's Laws: An Argument for Simplicity** that knowing G also allows for the determination of astronomical masses. Interestingly, of all the fundamental constants in physics, G is by far the least well determined.

The Cavendish experiment is also used to explore other aspects of gravity. One of the most interesting questions is whether the gravitational force depends on substance as well as mass—for example, whether one kilogram of lead exerts the same gravitational pull as one kilogram of water. A Hungarian scientist named Roland von Eötvös pioneered this inquiry early in the 20th century. He found, with an accuracy of five parts per billion, that the gravitational force does not depend on the substance. Such experiments continue today, and have improved upon Eötvös' measurements. Cavendish-type experiments such as those of Eric Adelberger and others at the University of Washington, have also put severe limits on the possibility of a fifth force and have verified a major prediction of general relativity—that gravitational energy contributes to rest mass. Ongoing measurements there use a torsion balance and a parallel plate (not spheres, as Cavendish used) to examine how Newton's law of gravitation works over sub-millimeter distances. On this small-scale, do gravitational effects depart from the inverse square law? So far, no deviation has been observed.

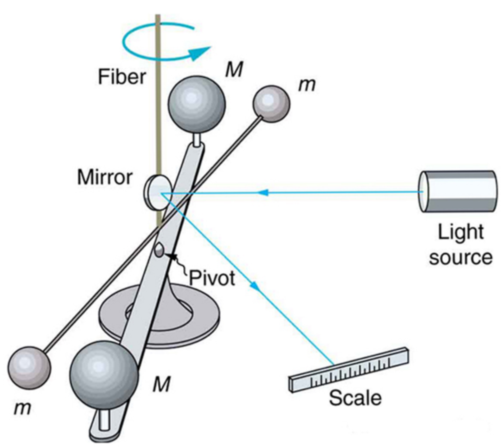


Figure 6.28 Cavendish used an apparatus like this to measure the gravitational attraction between the two suspended spheres (m) and the two on the stand (M) by observing the amount of torsion (twisting) created in the fiber. Distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

Figure 6.31(b) represents the modern or Copernican model. In this model, a small set of rules and a single underlying force explain not only all motions in the solar system, but all other situations involving gravity. The breadth and simplicity of the laws of physics are compelling. As our knowledge of nature has grown, the basic simplicity of its laws has become ever more evident.

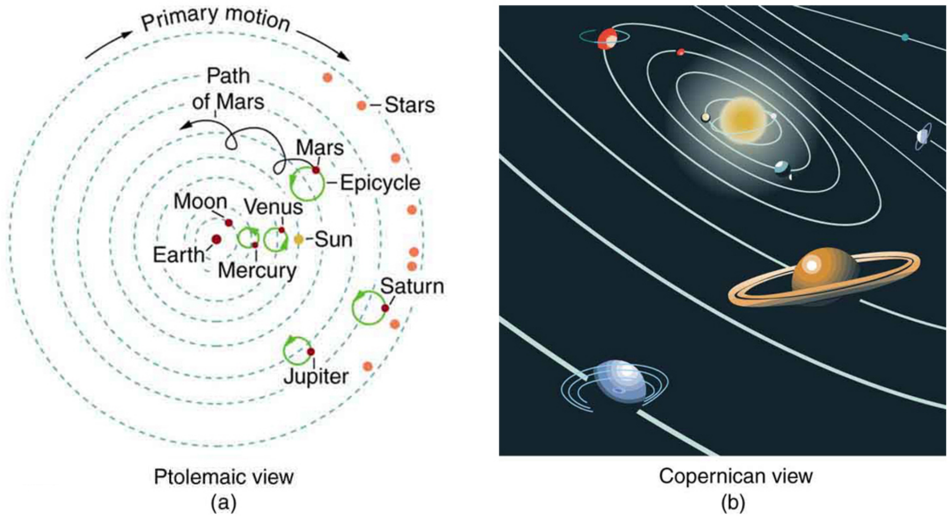


Figure 6.31 (a) The Ptolemaic model of the universe has Earth at the center with the Moon, the planets, the Sun, and the stars revolving about it in complex superpositions of circular paths. This geocentric model, which can be made progressively more accurate by adding more circles, is purely descriptive, containing no hints as to what are the causes of these motions. (b) The Copernican model has the Sun at the center of the solar system. It is fully explained by a small number of laws of physics, including Newton's universal law of gravitation.

Glossary

angular velocity: ω , the rate of change of the angle with which an object moves on a circular path

arc length: Δs , the distance traveled by an object along a circular path

banked curve: the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

center of mass: the point where the entire mass of an object can be thought to be concentrated

centrifugal force: a fictitious force that tends to throw an object off when the object is rotating in a non-inertial frame of reference

centripetal acceleration: the acceleration of an object moving in a circle, directed toward the center

centripetal force: any net force causing uniform circular motion

Coriolis force: the fictitious force causing the apparent deflection of moving objects when viewed in a rotating frame of reference

fictitious force: a force having no physical origin

gravitational constant, *G*: a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

ideal angle: the angle at which a car can turn safely on a steep curve, which is in proportion to the ideal speed

ideal banking: the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

ideal speed: the maximum safe speed at which a vehicle can turn on a curve without the aid of friction between the tire and the road

microgravity: an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

Newton's universal law of gravitation: every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

non-inertial frame of reference: an accelerated frame of reference

pit: a tiny indentation on the spiral track moulded into the top of the polycarbonate layer of CD

radians: a unit of angle measurement

radius of curvature: radius of a circular path

rotation angle: the ratio of the arc length to the radius of curvature on a circular path:

$$\Delta\theta = \frac{\Delta s}{r}$$

ultracentrifuge: a centrifuge optimized for spinning a rotor at very high speeds

uniform circular motion: the motion of an object in a circular path at constant speed

Section Summary

6.1 Rotation Angle and Angular Velocity

- Uniform circular motion is motion in a circle at constant speed. The rotation angle $\Delta\theta$ is defined as the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta s}{r},$$

  where arc length Δs is distance traveled along a circular path and r is the radius of curvature of the circular path. The quantity $\Delta\theta$ is measured in units of radians (rad), for which

$$2\pi \text{ rad} = 360^\circ = 1 \text{ revolution.}$$

- The conversion between radians and degrees is $1 \text{ rad} = 57.3^\circ$.
- Angular velocity ω is the rate of change of an angle,

$$\omega = \frac{\Delta\theta}{\Delta t},$$

  where a rotation $\Delta\theta$ takes place in a time Δt. The units of angular velocity are radians per second (rad/s). Linear velocity v and angular velocity ω are related by

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

6.2 Centripetal Acceleration

- Centripetal acceleration a_c is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity v and has the magnitude

$$a_\text{c} = \frac{v^2}{r};\ a_\text{c} = r\omega^2.$$

- The unit of centripetal acceleration is m/s^2.

6.3 Centripetal Force

- Centripetal force F_c is any force causing uniform circular motion. It is a “center-seeking” force that always points toward the center of rotation. It is perpendicular to linear velocity v and has magnitude

$$F_\text{c} = ma_\text{c},$$

  which can also be expressed as

$$\left.\begin{array}{c} F_\text{c} = m\frac{v^2}{r} \\ \text{or} \\ F_\text{c} = mr\omega^2 \end{array}\right\},$$

6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

- Rotating and accelerated frames of reference are non-inertial.
- Fictitious forces, such as the Coriolis force, are needed to explain motion in such frames.

6.5 Newton's Universal Law of Gravitation

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is

$$F = G\frac{mM}{r^2},$$

where F is the magnitude of the gravitational force. G is the gravitational constant, given by $G = 6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

- Newton's law of gravitation applies universally.

6.6 Satellites and Kepler's Laws: An Argument for Simplicity

- Kepler's laws are stated for a small mass m orbiting a larger mass M in near-isolation. Kepler's laws of planetary motion are then as follows:

  Kepler's first law

  The orbit of each planet about the Sun is an ellipse with the Sun at one focus.

  Kepler's second law

  Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.

  Kepler's third law

  The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

  where T is the period (time for one orbit) and r is the average radius of the orbit.

- The period and radius of a satellite's orbit about a larger body M are related by

$$T^2 = \frac{4\pi^2}{GM}r^3$$

  or

$$\frac{r^3}{T^2} = \frac{G}{4\pi^2}M.$$

Conceptual Questions

6.1 Rotation Angle and Angular Velocity

1. There is an analogy between rotational and linear physical quantities. What rotational quantities are analogous to distance and velocity?

6.2 Centripetal Acceleration

2. Can centripetal acceleration change the speed of circular motion? Explain.

6.3 Centripetal Force

3. If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

4. Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

5. If centripetal force is directed toward the center, why do you feel that you are ‘thrown' away from the center as a car goes around a curve? Explain.

6. Race car drivers routinely cut corners as shown in **Figure 6.32**. Explain how this allows the curve to be taken at the greatest speed.

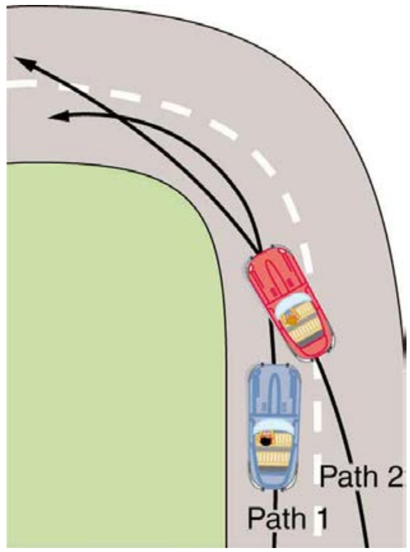


Figure 6.32 Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner) whenever possible because it allows them to take the curve at the highest speed.

7. A number of amusement parks have rides that make vertical loops like the one shown in **Figure 6.33**. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:

(a) The car goes over the top at faster than this speed?

(b)The car goes over the top at slower than this speed?

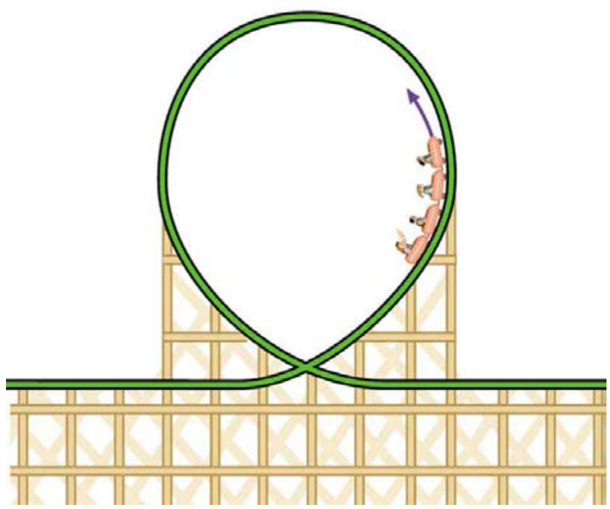


Figure 6.33 Amusement rides with a vertical loop are an example of a form of curved motion.

8. What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in **Figure 6.33** under the following circumstances:

(a) The car goes over the top at such a speed that the gravitational force is the only force acting?

(b) The car goes over the top faster than this speed?

(c) The car goes over the top slower than this speed?

9. As a skater forms a circle, what force is responsible for making her turn? Use a free body diagram in your answer.

10. Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown in **Figure 6.34** will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.

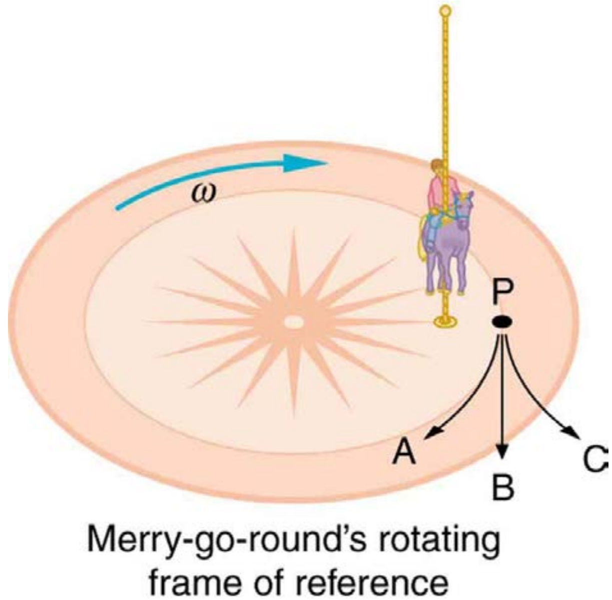


Figure 6.34 A child riding on a merry-go-round releases her lunch box at point P. This is a view from above the clockwise rotation. Assuming it slides with negligible friction, will it follow path A, B, or C, as viewed from Earth's frame of reference? What will be the shape of the path it leaves in the dust on the merry-go-round?

11. Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

12. Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.

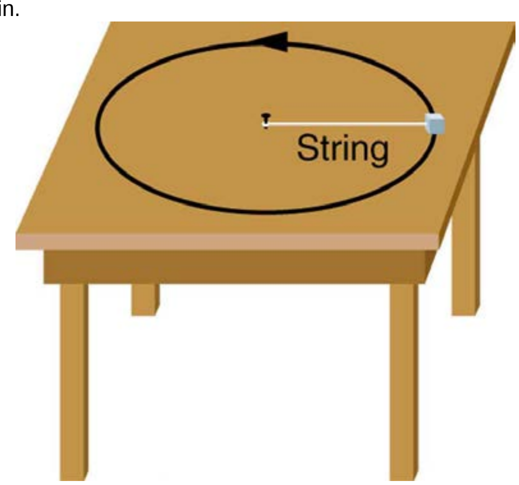


Figure 6.35 A mass attached to a nail on a frictionless table moves in a circular path. The force stretching the string is real and not fictional. What is the physical origin of the force on the string?

6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

13. When a toilet is flushed or a sink is drained, the water (and other material) begins to rotate about the drain on the way down. Assuming no initial rotation and a flow initially directly straight toward the drain, explain what causes the rotation and which direction it has in the northern hemisphere. (Note that this is a small effect and in most toilets the rotation is caused by directional water jets.) Would the direction of rotation reverse if water were forced up the drain?

14. Is there a real force that throws water from clothes during the spin cycle of a washing machine? Explain how the water is removed.

15. In one amusement park ride, riders enter a large vertical barrel and stand against the wall on its horizontal floor. The barrel is spun up and the floor drops away. Riders feel as if they are pinned to the wall by a force something like the gravitational force. This is a fictitious force sensed and used by the riders to explain events in the rotating frame of reference of the barrel. Explain in an inertial frame of reference (Earth is nearly one) what pins the riders to the wall, and identify all of the real forces acting on them.

16. Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

17. Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not 9.80 m/s^2. Who do you agree with and why?

18. A non-rotating frame of reference placed at the center of the Sun is very nearly an inertial one. Why is it not exactly an inertial frame?

6.5 Newton's Universal Law of Gravitation

19. Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

20. Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not 9.80 m/s^2. Who do you agree with and why?

21. Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.

22. Newton's laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

6.6 Satellites and Kepler's Laws: An Argument for Simplicity

23. In what frame(s) of reference are Kepler's laws valid? Are Kepler's laws purely descriptive, or do they contain causal information?

Problems & Exercises

6.1 Rotation Angle and Angular Velocity

1. Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?

2. Microwave ovens rotate at a rate of about 6 rev/min. What is this in revolutions per second? What is the angular velocity in radians per second?

3. An automobile with 0.260 m radius tires travels 80,000 km before wearing them out. How many revolutions do the tires make, neglecting any backing up and any change in radius due to wear?

4. (a) What is the period of rotation of Earth in seconds? (b) What is the angular velocity of Earth? (c) Given that Earth has a radius of $6.4\times10^6\text{ m}$ at its equator, what is the linear velocity at Earth's surface?

5. A baseball pitcher brings his arm forward during a pitch, rotating the forearm about the elbow. If the velocity of the ball in the pitcher's hand is 35.0 m/s and the ball is 0.300 m from the elbow joint, what is the angular velocity of the forearm?

6. In lacrosse, a ball is thrown from a net on the end of a stick by rotating the stick and forearm about the elbow. If the angular velocity of the ball about the elbow joint is 30.0 rad/s and the ball is 1.30 m from the elbow joint, what is the velocity of the ball?

7. A truck with 0.420-m-radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?

8. Integrated Concepts When kicking a football, the kicker rotates his leg about the hip joint.

(a) If the velocity of the tip of the kicker's shoe is 35.0 m/s and the hip joint is 1.05 m from the tip of the shoe, what is the shoe tip's angular velocity?

(b) The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s?

(c) Find the maximum range of the football, neglecting air resistance.

9. Construct Your Own Problem

Consider an amusement park ride in which participants are rotated about a vertical axis in a cylinder with vertical walls. Once the angular velocity reaches its full value, the floor drops away and friction between the walls and the riders prevents them from sliding down. Construct a problem in which you calculate the necessary angular velocity that assures the riders will not slide down the wall. Include a free body diagram of a single rider. Among the variables to consider are the radius of the cylinder and the coefficients of friction between the riders' clothing and the wall.

6.2 Centripetal Acceleration

10. A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00 m radius, at how many revolutions per minute will the riders be subjected to a centripetal acceleration whose magnitude is 1.50 times that due to gravity?

11. A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If he completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of his centripetal acceleration as he runs the curved portion of the track?

12. Taking the age of Earth to be about 4×10^9 years and assuming its orbital radius of 1.5×10^{11} has not changed and is circular, calculate the approximate total distance Earth has traveled since its birth (in a frame of reference stationary with respect to the Sun).

13. The propeller of a World War II fighter plane is 2.30 m in diameter.

(a) What is its angular velocity in radians per second if it spins at 1200 rev/min?

(b) What is the linear speed of its tip at this angular velocity if the plane is stationary on the tarmac?

(c) What is the centripetal acceleration of the propeller tip under these conditions? Calculate it in meters per second squared and convert to multiples of g.

14. An ordinary workshop grindstone has a radius of 7.50 cm and rotates at 6500 rev/min.

(a) Calculate the magnitude of the centripetal acceleration at its edge in meters per second squared and convert it to multiples of g.

(b) What is the linear speed of a point on its edge?

15. Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip.

(a) Calculate the magnitude of the centripetal acceleration at the tip of a 4.00 m long helicopter blade that rotates at 300 rev/min.

(b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s).

16. Olympic ice skaters are able to spin at about 5 rev/s.

(a) What is their angular velocity in radians per second?

(b) What is the centripetal acceleration of the skater's nose if it is 0.120 m from the axis of rotation?

(c) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius?

(d) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.

17. What percentage of the acceleration at Earth's surface is the acceleration due to gravity at the position of a satellite located 300 km above Earth?

18. Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating:

(a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min.

(b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth's orbit and approximate it as being circular).

19. A rotating space station is said to create "artificial gravity"—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise and maintain muscle and bone strength more naturally than in non-rotating space environments. If the space station is 200 m in diameter, what angular velocity would produce an "artificial gravity" of 9.80 m/s^2 at the rim?

20. At takeoff, a commercial jet has a 60.0 m/s speed. Its tires have a diameter of 0.850 m.

(a) At how many rev/min are the tires rotating?

(b) What is the centripetal acceleration at the edge of the tire?

(c) With what force must a determined $1.00\times10^{-15}\text{ kg}$ bacterium cling to the rim?

(d) Take the ratio of this force to the bacterium's weight.

21. Integrated Concepts

Riders in an amusement park ride shaped like a Viking ship hung from a large pivot are rotated back and forth like a rigid pendulum. Sometime near the middle of the ride, the ship is momentarily motionless at the top of its circular arc. The ship then swings down under the influence of gravity.

(a) Assuming negligible friction, find the speed of the riders at the bottom of its arc, given the system's center of mass travels in an arc having a radius of 14.0 m and the riders are near the center of mass.

(b) What is the centripetal acceleration at the bottom of the arc?

(c) Draw a free body diagram of the forces acting on a rider at the bottom of the arc.

(d) Find the force exerted by the ride on a 60.0 kg rider and compare it to her weight.

(e) Discuss whether the answer seems reasonable.

22. Unreasonable Results

A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child's center of mass.

(a) What is the magnitude of the centripetal acceleration of the child at the low point?

(b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg?

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent?

6.3 Centripetal Force

23. (a) A 22.0 kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force must she exert to stay on if she is 1.25 m from its center?

(b) What centripetal force does she need to stay on an amusement park merry-go-round that rotates at 3.00 rev/min if she is 8.00 m from its center?

(c) Compare each force with her weight.

24. Calculate the centripetal force on the end of a 100 m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.

25. What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

26. What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?

27. (a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s, assuming it is ideally banked?

(b) Calculate the centripetal acceleration.

(c) Does this acceleration seem large to you?

28. Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen in Figure 6.36. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force), and the vertical normal force (which must equal the system's weight).

(a) Show that θ (as defined in the figure) is related to the speed v and radius of curvature r of the turn in the same way as for an ideally banked roadway—that is, $\theta = \tan^{-1} v^2 / rg$

(b) Calculate θ for a 12.0 m/s turn of radius 30.0 m (as in a race).

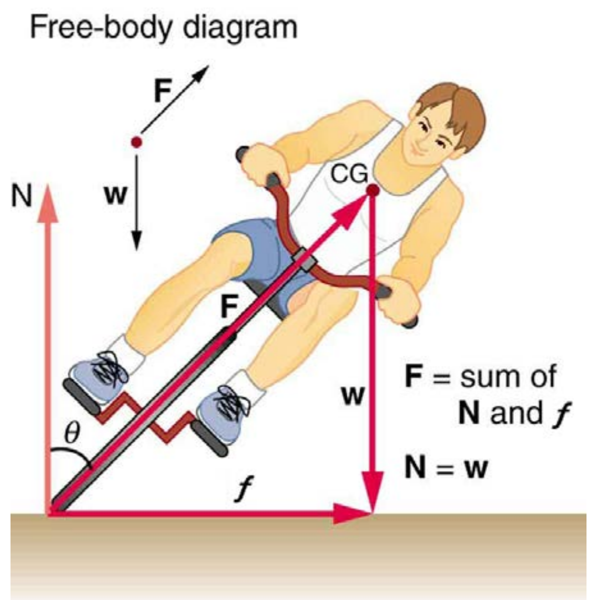


Figure 6.36 A bicyclist negotiating a turn on level ground must lean at the correct angle—the ability to do this becomes instinctive. The force of the ground on the wheel needs to be on a line through the center of gravity. The net external force on the system is the centripetal force. The vertical component of the force on the wheel cancels the weight of the system while its horizontal component must supply the centripetal force. This process produces a relationship among the angle θ, the speed v, and the radius of curvature r of the turn similar to that for the ideal banking of roadways.

29. A large centrifuge, like the one shown in Figure 6.37(a), is used to expose aspiring astronauts to accelerations similar to those experienced in rocket launches and atmospheric reentries.

(a) At what angular velocity is the centripetal acceleration $10\ g$ if the rider is 15.0 m from the center of rotation?

(b) The rider's cage hangs on a pivot at the end of the arm, allowing it to swing outward during rotation as shown in **Figure 6.37**(b). At what angle θ below the horizontal will the cage hang when the centripetal acceleration is $10\ g$? (Hint: The arm supplies centripetal force and supports the weight of the cage. Draw a free body diagram of the forces to see what the angle θ should be.)

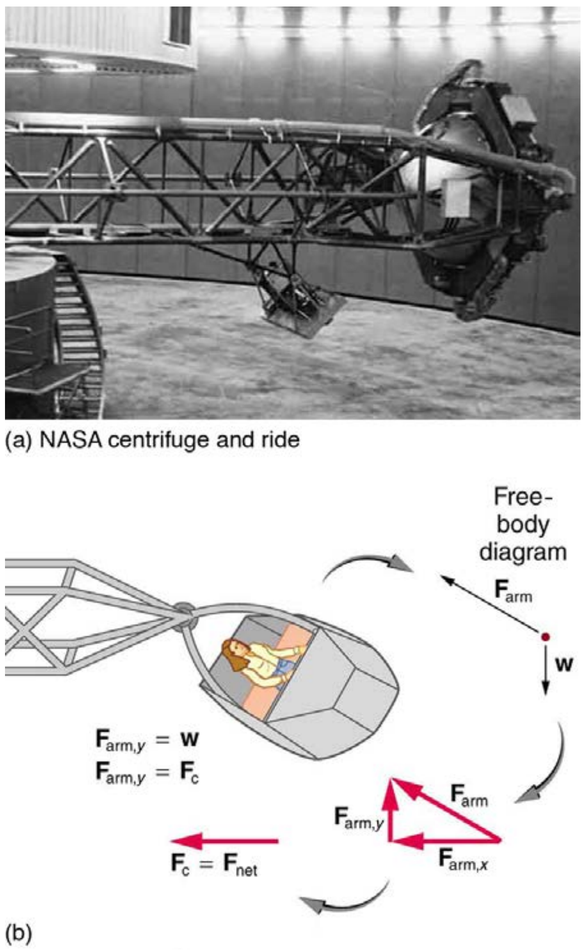


Figure 6.37 (a) NASA centrifuge used to subject trainees to accelerations similar to those experienced in rocket launches and reentries. (credit: NASA) (b) Rider in cage showing how the cage pivots outward during rotation. This allows the total force exerted on the rider by the cage to be along its axis at all times.

30. Integrated Concepts

If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a real problem on icy mountain roads). (a) Calculate the ideal speed to take a 100 m radius curve banked at 15.0º. (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?

31. Modern roller coasters have vertical loops like the one shown in **Figure 6.38**. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g?

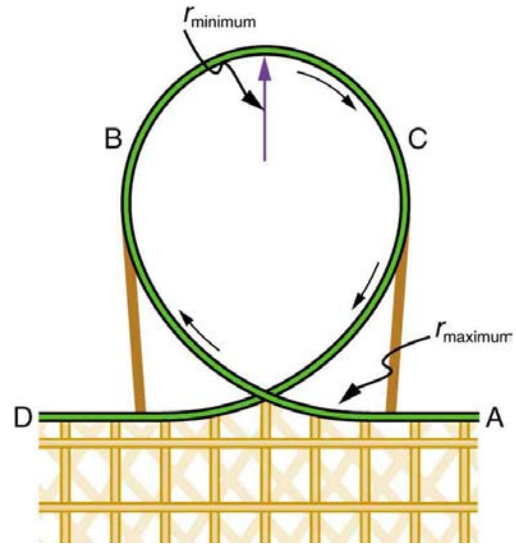


Figure 6.38 Teardrop-shaped loops are used in the latest roller coasters so that the radius of curvature gradually decreases to a minimum at the top. This means that the centripetal acceleration builds from zero to a maximum at the top and gradually decreases again. A circular loop would cause a jolting change in acceleration at entry, a disadvantage discovered long ago in railroad curve design. With a small radius of curvature at the top, the centripetal acceleration can more easily be kept greater than g so that the passengers do not lose contact with their seats nor do they need seat belts to keep them in place.

32. Unreasonable Results

(a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s.

(b) What is unreasonable about the result?

(c) Which premises are unreasonable or inconsistent?

6.5 Newton's Universal Law of Gravitation

33. (a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is $9.830\ \text{m/s}^2$ and the radius of the Earth is 6371 km from pole to pole.

(b) Compare this with the accepted value of $5.979\times10^{24}\ \text{kg}$.

34. (a) Calculate the magnitude of the acceleration due to gravity on the surface of Earth due to the Moon.

(b) Calculate the magnitude of the acceleration due to gravity at Earth due to the Sun.

(c) Take the ratio of the Moon's acceleration to the Sun's and comment on why the tides are predominantly due to the Moon in spite of this number.

35. (a) What is the acceleration due to gravity on the surface of the Moon?

(b) On the surface of Mars? The mass of Mars is $6.418\times10^{23}\ \text{kg}$ and its radius is $3.38\times10^{6}\ \text{m}$.

36. (a) Calculate the acceleration due to gravity on the surface of the Sun.

(b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)

37. The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.)

(a) Calculate the magnitude of the acceleration due to the Moon's gravity at that point.

(b) Calculate the magnitude of the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a). Comment on whether or not they are equal and why they should or should not be.

38. Solve part (b) of **Example 6.6** using $a_c = v^2 / r$.

39. Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational.

(a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child).

(b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some $6.29\times10^{11}\text{ m}$ away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)

40. The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:

(a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are $4.50\times10^{12}\text{ m}$ apart, as they are at present. The mass of Pluto is $1.4\times10^{22}\text{ kg}$.

(b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about $2.50\times10^{12}\text{ m}$ apart, and compare it with that due to Pluto. The mass of Uranus is $8.62\times10^{25}\text{ kg}$.

41. (a) The Sun orbits the Milky Way galaxy once each $2.60\text{ x }10^8\text{ y}$, with a roughly circular orbit averaging $3.00\text{ x }10^4$ light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun?

(b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?

42. Unreasonable Result

A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight.

(a) Calculate the mass of the mountain.

(b) Compare the mountain's mass with that of Earth.

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)

6.6 Satellites and Kepler's Laws: An Argument for Simplicity

43. A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). Calculate the radius of such an orbit based on the data for the moon in **Table 6.2**.

44. Calculate the mass of the Sun based on data for Earth's orbit and compare the value obtained with the Sun's actual mass.

45. Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass.

46. Find the ratio of the mass of Jupiter to that of Earth based on data in **Table 6.2**.

47. Astronomical observations of our Milky Way galaxy indicate that it has a mass of about 8.0×10^{11} solar masses. A star orbiting on the galaxy's periphery is about 6.0×10^4 light years from its center. (a) What should the orbital period of that star be? (b) If its period is 6.0×10^7 instead, what is the mass of the galaxy? Such calculations are used to imply the existence of “dark matter” in the universe and have indicated, for example, the existence of very massive black holes at the centers of some galaxies.

48. Integrated Concepts

Space debris left from old satellites and their launchers is becoming a hazard to other satellites. (a) Calculate the speed of a satellite in an orbit 900 km above Earth's surface. (b) Suppose a loose rivet is in an orbit of the same radius that intersects the satellite's orbit at an angle of 90° relative to Earth. What is the velocity of the rivet relative to the satellite just before striking it? (c) Given the rivet is 3.00 mm in size, how long will its collision with the satellite last? (d) If its mass is 0.500 g, what is the average force it exerts on the satellite? (e) How much energy in joules is generated by the collision? (The satellite's velocity does not change appreciably, because its mass is much greater than the rivet's.)

49. Unreasonable Results

(a) Based on Kepler's laws and information on the orbital characteristics of the Moon, calculate the orbital radius for an Earth satellite having a period of 1.00 h. (b) What is unreasonable about this result? (c) What is unreasonable or inconsistent about the premise of a 1.00 h orbit?

50. Construct Your Own Problem

On February 14, 2000, the NEAR spacecraft was successfully inserted into orbit around Eros, becoming the first artificial satellite of an asteroid. Construct a problem in which you determine the orbital speed for a satellite near Eros. You will need to find the mass of the asteroid and consider such things as a safe distance for the orbit. Although Eros is not spherical, calculate the acceleration due to gravity on its surface at a point an average distance from its center of mass. Your instructor may also wish to have you calculate the escape velocity from this point on Eros.

Test Prep for AP® Courses

6.5 Newton's Universal Law of Gravitation

1. Jupiter has a mass approximately 300 times greater than Earth's and a radius about 11 times greater. How will the gravitational acceleration at the surface of Jupiter compare to that at the surface of the Earth?

a. Greater
b. Less
c. About the same
d. Not enough information

2. Given Newton's universal law of gravitation (Equation 6.40), under what circumstances is the force due to gravity maximized?

3. In the formula $g = \frac{GM}{r^2}$, what does *G* represent?

a. The acceleration due to gravity
b. A gravitational constant that is the same everywhere in the universe
c. A gravitational constant that is inversely proportional to the radius
d. The factor by which you multiply the inertial mass to obtain the gravitational mass

4. Saturn's moon Titan has a radius of 2.58×10^6 m and a measured gravitational field of 1.35 m/s^2. What is its mass?

5. A recently discovered planet has a mass twice as great as Earth's and a radius twice as large as Earth's. What will be the approximate size of its gravitational field?

a. 19 m/s^2
b. 4.9 m/s^2
c. 2.5 m/s^2
d. 9.8 m/s^2

6. 4. Earth is 1.5×10^{11} m from the Sun. Mercury is 5.7×10^{10} m from the Sun. How does the gravitational field of the Sun on Mercury (g_{SM}) compare to the gravitational field of the Sun on Earth (g_{SE})?

7 WORK, ENERGY, AND ENERGY RESOURCES

Figure 7.1 How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben, Germany, Wikimedia Commons)

Chapter Outline

Connection for AP® Courses

Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods to the energy we use to run our cars and the sunlight that warms us on the beach. You can also cite examples of what people call “energy” that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics.

There is no simple and accurate scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define energy as the ability to do **work**, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form. The work-energy theorem supports Big Idea 3, that interactions between objects are described by forces. In particular, exerting a force on an object may do work on it, changing it's energy (Enduring Understanding 3.E). The work-energy theorem, introduced in this chapter, establishes the relationship between work done on an object by an external force and changes in the object’s kinetic energy (Essential Knowledge 3.E.1).

Similarly, systems can do work on each other, supporting Big Idea 4, that interactions between systems can result in changes in those systems—in this case, changes in the total energy of the system (Enduring Understanding 4.C). The total energy of the system is the sum of its kinetic energy, potential energy, and microscopic internal energy (Essential Knowledge 4.C.1). In this chapter students learn how to calculate kinetic, gravitational, and elastic potential energy in order to determine the total mechanical energy of a system. The transfer of mechanical energy into or out of a system is equal to the work done on the system by an external force with a nonzero component parallel to the displacement (Essential Knowledge 4.C.2).

An important aspect of energy is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is "conserved." **Conservation of energy** (as physicists call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $E = mc^2$). This is one of the most important applications of Big Idea 5, that changes that occur as a result of interactions are constrained by conservation laws. Specifically, there are many situations where conservation of energy (Enduring Understanding 5.B) is both a useful concept and starting point for calculations related to the system. Note, however, that conservation doesn't necessarily mean that energy in a system doesn't change. Energy may be transferred into or out of the system, and the change must be equal to the amount transferred (Enduring Understanding 5.A). This may occur if there is an external force or a transfer between external objects and the system (Essential Knowledge 5.A.3). Energy is one of the fundamental quantities that are conserved for all systems (Essential Knowledge 5.A.2). The chapter introduces concepts of kinetic energy and potential energy. Kinetic energy is introduced as an energy of motion that can be changed by the amount of work done by an external force. Potential energy can only exist when objects interact with each other via conservative forces according to classical physics (Essential Knowledge 5.B.3). Because of this, a single object can only have kinetic energy and no potential energy (Essential Knowledge 5.B.1). The chapter also introduces the idea that the energy transfer is equal to the work done on the system by external forces and the rate of energy transfer is defined as power (Essential Knowledge 5.B.5).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

The concepts in this chapter support:

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.E A force exerted on an object can change the kinetic energy of the object.

Essential Knowledge 3.E.1 The change in the kinetic energy of an object depends on the force exerted on the object and on the displacement of the object during the interval that the force is exerted.

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding 4.C Interactions with other objects or systems can change the total energy of a system.

Essential Knowledge 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy.

Essential Knowledge 4.C.2 Mechanical energy (the sum of kinetic and potential energy) is transferred into or out of a system when an external force is exerted on a system such that a component of the force is parallel to its displacement. The process through which the energy is transferred is called work.

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved.

Essential Knowledge 5.A.3 An interaction can be either a force exerted by objects outside the system or the transfer of some quantity with objects outside the system.

Enduring Understanding 5.B The energy of a system is conserved.

Essential Knowledge 5.B.1 Classically, an object can only have kinetic energy since potential energy requires an interaction between two or more objects.

Essential Knowledge 5.B.3 A system with internal structure can have potential energy. Potential energy exists within a system if the objects within that system interact with conservative forces.

Essential Knowledge 5.B.5 Energy can be transferred by an external force exerted on an object or system that moves the object or system through a distance; this energy transfer is called work. Energy transfer in mechanical or electrical systems may occur at different rates. Power is defined as the rate of energy transfer into, out of, or within a system.

7.1 Work: The Scientific Definition

Learning Objectives

By the end of this section, you will be able to:

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement of an object determine whether the work done on the object is positive, negative, or zero.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.B.5.1** The student is able to design an experiment and analyze data to examine how a force exerted on an object or system does work on the object or system as it moves through a distance. **(S.P. 4.2, 5.1)**

- **5.B.5.2** The student is able to design an experiment and analyze graphical data in which interpretations of the area under a force-distance curve are needed to determine the work done on or by the object or system. **(S.P. 4.5, 5.1)**
- **5.B.5.3** The student is able to predict and calculate from graphical data the energy transfer to or work done on an object or system from information about a force exerted on the object or system through a distance. **(S.P. 1.5, 2.2, 6.4)**

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done on an object, a force must be exerted on that object and there must be motion or displacement of that object in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion and the distance through which the force acts*. For a constant force, this is expressed in equation form as

$$W = \mid \mathbf{F} \mid (\cos\theta) \mid \mathbf{d} \mid , \tag{7.1}$$

where W is work, $\mathbf{d}$ is the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$, as in **Figure 7.2**. We can also write this as

$$W = Fd\cos\theta. \tag{7.2}$$

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

What is Work?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = Fd\cos\theta, \tag{7.3}$$

where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$.

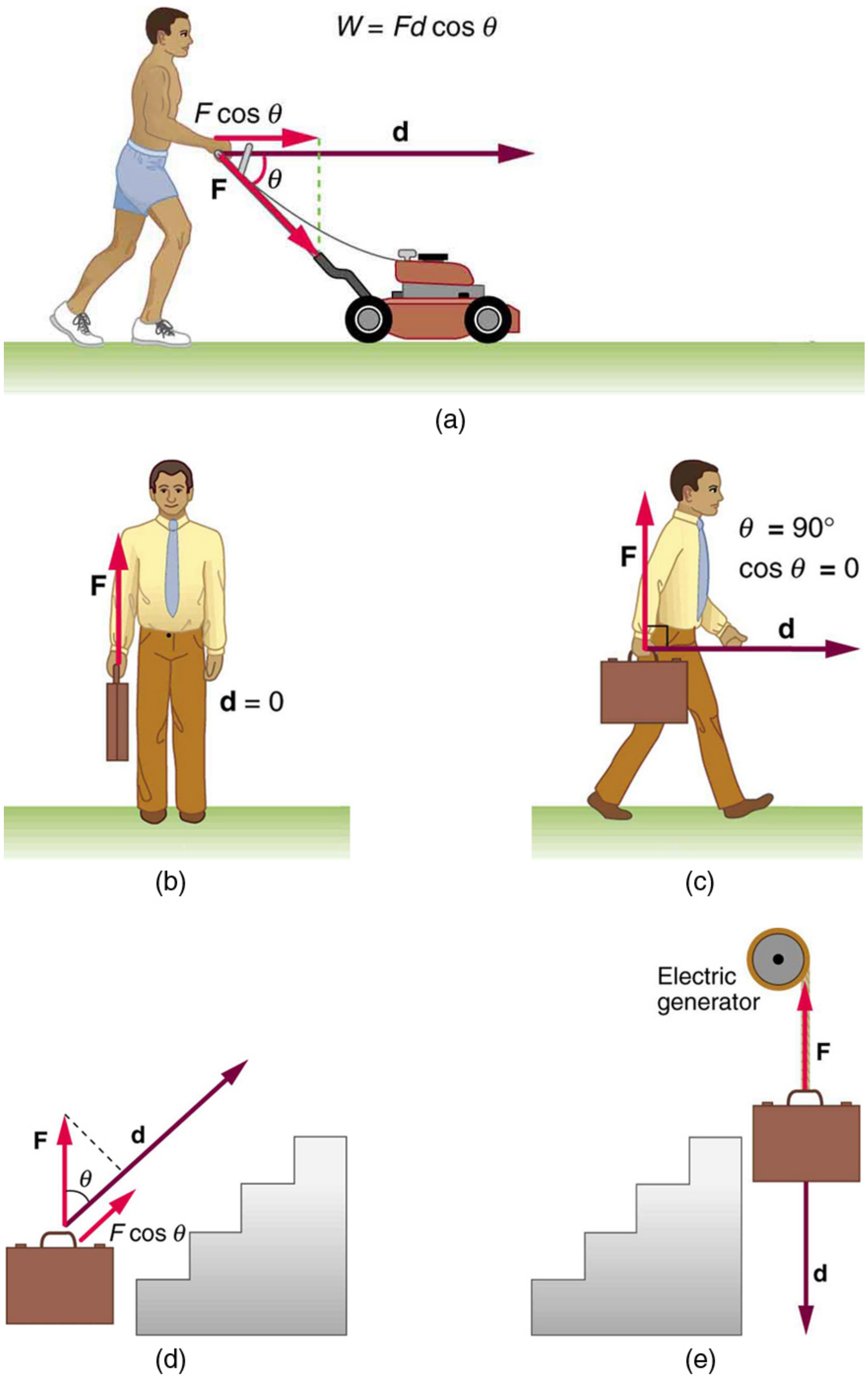


Figure 7.2 Examples of work. (a) The work done by the force $\mathbf{F}$ on this lawn mower is $Fd\cos\theta$. Note that $F\cos\theta$ is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no motion. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work *is* done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force $\mathbf{F}$ in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because $\mathbf{F}$ and $\mathbf{d}$ are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in **Figure 7.2**. The person holding the briefcase in **Figure 7.2**(b) does no work, for example. Here $d = 0$, so $W = 0$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the "briefcase-Earth system"—see **Gravitational Potential Energy** for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in **Figure 7.2**(c) does no work on it, because the force is perpendicular to the motion. That is, $\cos 90^\circ = 0$, and so $W = 0$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in **Figure 7.2**(d), work *is* done—energy is transferred to the briefcase. Finally, in **Figure 7.2**(e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase's weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes $\theta = 180^\circ$, and $\cos 180^\circ = -1$; therefore, W is negative.

Real World Connections: When Work Happens

Note that work as we define it is not the same as effort. You can push against a concrete wall all you want, but you won't move it. While the pushing represents effort on your part, the fact that you have not changed the wall's state in any way indicates that you haven't done work. If you did somehow push the wall over, this would indicate a change in the wall's state, and therefore you would have done work.

This can also be shown with **Figure 7.2**(a): as you push a lawnmower against friction, both you and friction are changing the lawnmower's state. However, only the component of the force parallel to the movement is changing the lawnmower's state. The component perpendicular to the motion is trying to push the lawnmower straight into Earth; the lawnmower does not move into Earth, and therefore the lawnmower's state is not changing in the direction of Earth.

Similarly, in **Figure 7.2**(c), both your hand and gravity are exerting force on the briefcase. However, they are both acting perpendicular to the direction of motion, hence they are not changing the condition of the briefcase and do no work. However, if the briefcase were dropped, then its displacement would be parallel to the force of gravity, which would do work on it, changing its state (it would fall to the ground).

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and $1\text{ J} = 1\text{ N}\cdot\text{m} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example 7.1 Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in **Figure 7.2**(a) if he exerts a constant force of 75.0 N at an angle 35° below the horizontal and pushes the mower 25.0 m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of $10{,}000\text{ kJ}$ (about 2400 kcal) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1°C, and is equivalent to 4.184 J, while one *food calorie* (1 kcal) is equivalent to 4184 J.

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = Fd\cos\theta$. The force, angle, and displacement are given, so that only the work W is unknown.

Solution

The equation for the work is

$$W = Fd\cos\theta. \tag{7.4}$$

Substituting the known values gives

$$\begin{aligned} W &= (75.0\text{ N})(25.0\text{ m})\cos(35.0^\circ) \\ &= 1536\text{ J} = 1.54\times10^3\text{ J}. \end{aligned} \tag{7.5}$$

Converting the work in joules to kilocalories yields $W = (1536\text{ J})(1\text{ kcal}/4184\text{ J}) = 0.367\text{ kcal}$. The ratio of the work done to the daily consumption is

$$\frac{W}{2400\text{ kcal}} = 1.53\times10^{-4}. \tag{7.6}$$

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we "work" all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Applying the Science Practices: Boxes on Floors

Plan and design an experiment to determine how much work you do on a box when you are pushing it over different floor surfaces. Make sure your experiment can help you answer the following questions: What happens on different surfaces? What happens if you take different routes across the same surface? Do you get different results with two people pushing on perpendicular surfaces of the box? What if you vary the mass in the box? Remember to think about both your effort in any given instant (a proxy for force exerted) and the total work you do. Also, when planning your experiments, remember that in any given set of trials you should only change one variable.

You should find that you have to exert more effort on surfaces that will create more friction with the box, though you might be surprised by which surfaces the box slides across easily. Longer routes result in your doing more work, even though the box ends up in the same place. Two people pushing on perpendicular sides do less work for their total effort, due to the forces and displacement not being parallel. A more massive box will take more effort to move.

Applying the Science Practices: Force-Displacement Diagrams

Suppose you are given two carts and a track to run them on, a motion detector, a force sensor, and a computer that can record the data from the two sensors. Plan and design an experiment to measure the work done on one of the carts, and compare your results to the work-energy theorem. Note that the motion detector can measure both displacement and velocity versus time, while the force sensor measures force over time, and the carts have known masses. Recall that the work-energy theorem states that the work done on a system (force over displacement) should equal the change in kinetic energy. In your experimental design, describe and compare two possible ways to calculate the work done.

Sample Response: One possible technique is to set up the motion detector at one end of the track, and have the computer record both displacement and velocity over time. Then attach the force sensor to one of the carts, and use this cart, through the force sensor, to push the second cart toward the motion detector. Calculate the difference between the final and initial kinetic energies (the kinetic energies after and before the push), and compare this to the area of a graph of force versus displacement for the duration of the push. They should be the same.

7.2 Kinetic Energy and the Work-Energy Theorem

Learning Objectives

By the end of this section, you will be able to:

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.E.1.1** The student is able to make predictions about the changes in kinetic energy of an object based on considerations of the direction of the net force on the object as the object moves. **(S.P. 6.4, 7.2)**
- **3.E.1.2** The student is able to use net force and velocity vectors to determine qualitatively whether kinetic energy of an object would increase, decrease, or remain unchanged. **(S.P. 1.4)**
- **3.E.1.3** The student is able to use force and velocity vectors to determine qualitatively or quantitatively the net force exerted on an object and qualitatively whether kinetic energy of that object would increase, decrease, or remain unchanged. **(S.P. 1.4, 2.2)**
- **3.E.1.4** The student is able to apply mathematical routines to determine the change in kinetic energy of an object given the forces on the object and the displacement of the object. **(S.P. 2.2)**
- **4.C.1.1** The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. **(S.P. 1.4, 2.1, 2.2)**
- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**
- **4.C.2.2** The student is able to apply the concepts of conservation of energy and the work-energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. **(S.P. 1.4, 2.2, 7.2)**
- **5.B.5.3** The student is able to predict and calculate from graphical data the energy transfer to or work done on an object or system from information about a force exerted on the object or system through a distance. **(S.P. 1.5, 2.2, 6.4)**

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in **Figure 7.2**(a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in **Figure 7.2**(d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in **Figure 7.2**(e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system.

Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in **Dynamics: Force and Newton's Laws of Motion** that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force $\mathbf{F}_{\text{net}}$. In equation form, this is

$W_{\text{net}} = F_{\text{net}} d \cos \theta$ where θ is the angle between the force vector and the displacement vector.

Figure 7.3(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $F \cos \theta$ vs. d graph. In this case, $F \cos \theta$ is constant. You can see that the area under the graph is $Fd \cos \theta$, or the work done. **Figure 7.3**(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $(F \cos \theta)_{i(\text{ave})}$. The work done is $(F \cos \theta)_{i(\text{ave})} d_i$ for each strip, and the total work done is the sum of the W_i. Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

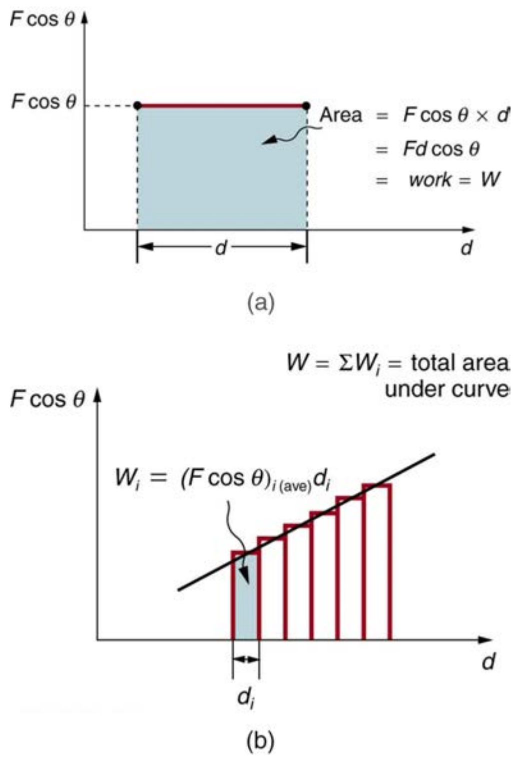


Figure 7.3 (a) A graph of $F \cos \theta$ vs. d, when $F \cos \theta$ is constant. The area under the curve represents the work done by the force. (b) A graph of $F \cos \theta$ vs. d in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Real World Connections: Work and Direction

Consider driving in a car. While moving, you have forward velocity and therefore kinetic energy. When you hit the brakes, they exert a force opposite to your direction of motion (acting through the wheels). The brakes do work on your car and reduce the kinetic energy. Similarly, when you accelerate, the engine (acting through the wheels) exerts a force in the direction of motion. The engine does work on your car, and increases the kinetic energy. Finally, if you go around a corner at a constant speed, you have the same kinetic energy both before and after the corner. The force exerted by the engine was perpendicular to the direction of motion, and therefore did no work and did not change the kinetic energy.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in **Figure 7.4**.

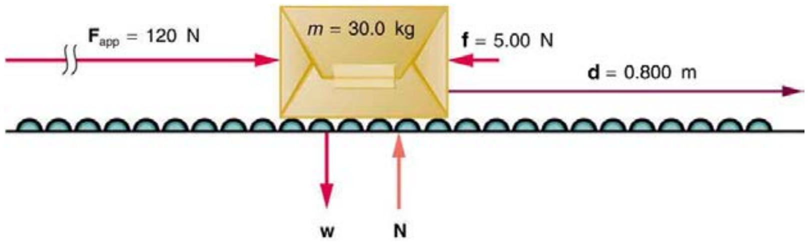


Figure 7.4 A package on a roller belt is pushed horizontally through a distance $\mathbf{d}$.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force $\mathbf{F}_{\text{app}}$ and the horizontal friction force $\mathbf{f}$. Thus, as expected, the net force is parallel to the displacement, so that $\theta = 0^{\circ}$ and $\cos\theta = 1$, and the net work is given by

$$W_{\text{net}} = F_{\text{net}}d. \tag{7.7}$$

The effect of the net force $\mathbf{F}_{\text{net}}$ is to accelerate the package from v_0 to v . The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See **Example 7.2**.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F_{\text{net}} = ma$ from Newton's second law gives

$$W_{\text{net}} = mad. \tag{7.8}$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in **Motion Equations for Constant Acceleration in One Dimension** for the change in speed over a distance d if the acceleration has the constant value a ; namely, $v^2 = v_0{}^2 + 2ad$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - v_0{}^2}{2d}$. When a is substituted into the preceding expression for W_{net} , we obtain

$$W_{\text{net}} = m\left(\frac{v^2 - v_0{}^2}{2d}\right)d. \tag{7.9}$$

The d cancels, and we rearrange this to obtain

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{7.10}$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

The Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \tag{7.11}$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v . (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$\text{KE} = \frac{1}{2}mv^2, \tag{7.12}$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in **Figure 7.4**, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Applying the Science Practices: Cars on a Hill

Assemble a ramp suitable for rolling some toy cars up or down. Then plan a series of experiments to determine how the direction of a force relative to the velocity of an object alters the kinetic energy of the object. Note that gravity will be pointing down in all cases. What happens if you start the car at the top? How about at the bottom, with an initial velocity that is increasing? If your ramp is wide enough, what happens if you send the toy car straight across? Does varying the surface of the ramp change your results?

Sample Response: When the toy car is going down the ramp, with a component of gravity in the same direction, the kinetic energy increases. Sending the car up the ramp decreases the kinetic energy, as gravity is opposing the motion. Sending the car sideways should result in little to no change. If you have a surface that generates more friction than a smooth surface (carpet), note that the friction always opposed the motion, and hence decreases the kinetic energy.

Example 7.2 Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in **Figure 7.4** is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\text{KE} = \frac{1}{2}mv^2$.

Solution

The kinetic energy is given by

$$\text{KE} = \frac{1}{2}mv^2. \tag{7.13}$$

Entering known values gives

$$\text{KE} = 0.5(30.0\text{ kg})(0.500\text{ m/s})^2, \tag{7.14}$$

which yields

$$\text{KE} = 3.75\text{ kg}\cdot\text{m}^2/\text{s}^2 = 3.75\text{ J}. \tag{7.15}$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Real World Connections: Center of Mass

Suppose we have two experimental carts, of equal mass, latched together on a track with a compressed spring between them. When the latch is released, the spring does 10 J of work on the carts (we'll see how in a couple of sections). The carts move relative to the spring, which is the center of mass of the system. However, the center of mass stays fixed. How can we consider the kinetic energy of this system?

By the work-energy theorem, the work done by the spring on the carts must turn into kinetic energy. So this system has 10 J of kinetic energy. The total kinetic energy of the system is the kinetic energy of the center of mass of the system relative to the fixed origin plus the kinetic energy of each cart relative to the center of mass. We know that the center of mass relative to the fixed origin does not move, and therefore all of the kinetic energy must be distributed among the carts relative to the center of mass. Since the carts have equal mass, they each receive an equal amount of kinetic energy, so each cart has 5.0 J of kinetic energy.

In our example, the forces between the spring and each cart are internal to the system. According to Newton's third law, these internal forces will cancel since they are equal and opposite in direction. However, this does not imply that these internal forces will not do work. Thus, the change in kinetic energy of the system is caused by work done by the force of the spring, and results in the motion of the two carts relative to the center of mass.

Example 7.3 Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in **Figure 7.4** with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See **Figure 7.4**.) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or $F_{\text{net}} = 120\text{ N} - 5.00\text{ N} = 115\text{ N}$. Thus the net work is

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}}d = (115\text{ N})(0.800\text{ m}) \\ &= 92.0\text{ N}\cdot\text{m} = 92.0\text{ J}. \end{aligned} \tag{7.16}$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

$$\begin{aligned} W_{\text{app}} &= F_{\text{app}}d\cos(0^\circ) = F_{\text{app}}d \\ &= (120\text{ N})(0.800\text{ m}) \\ &= 96.0\text{ J} \end{aligned} \tag{7.17}$$

The friction force and displacement are in opposite directions, so that $\theta = 180^\circ$, and the work done by friction is

$$\begin{aligned} W_{\text{fr}} &= F_{\text{fr}}d\cos(180^\circ) = -F_{\text{fr}}d \\ &= -(5.00\text{ N})(0.800\text{ m}) \\ &= -4.00\text{ J}. \end{aligned} \tag{7.18}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$\begin{aligned} W_{\text{gr}} &= 0, \\ W_{\text{N}} &= 0, \\ W_{\text{app}} &= 96.0\text{ J}, \\ W_{\text{fr}} &= -4.00\text{ J}. \end{aligned} \tag{7.19}$$

The total work done as the sum of the work done by each force is then seen to be

$$W_{\text{total}} = W_{\text{gr}} + W_{\text{N}} + W_{\text{app}} + W_{\text{fr}} = 92.0\text{ J}. \tag{7.20}$$

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

Example 7.4 Determining Speed from Work and Energy

Find the speed of the package in **Figure 7.4** at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net}, and the initial kinetic energy, $\frac{1}{2}mv_0^{\,2}$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed v.

Solution

The work-energy theorem in equation form is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\,2}. \tag{7.21}$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2 = W_{\text{net}} + \frac{1}{2}mv_0^{\,2}. \tag{7.22}$$

Thus,

$$\frac{1}{2}mv^2 = 92.0\text{ J}+3.75\text{ J} = 95.75\text{ J}. \tag{7.23}$$

Solving for the final speed as requested and entering known values gives

$$\begin{aligned} v &= \sqrt{\frac{2(95.75\text{ J})}{m}} = \sqrt{\frac{191.5\text{ kg}\cdot\text{m}^2/\text{s}^2}{30.0\text{ kg}}} \\ &= 2.53\text{ m/s}. \end{aligned} \tag{7.24}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Example 7.5 Work and Energy Can Reveal Distance, Too

How far does the package in **Figure 7.4** coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\theta = 180^\circ$. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{\text{fr}} = -95.75\text{ J}$. Furthermore, $W_{\text{fr}} = fd'\cos\theta = -fd'$, where d' is the distance it takes to stop. Thus,

$$d' = -\frac{W_{\text{fr}}}{f} = -\frac{-95.75\text{ J}}{5.00\text{ N}}, \tag{7.25}$$

and so

$$d' = 19.2\text{ m}. \tag{7.26}$$

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

7.3 Gravitational Potential Energy

Learning Objectives

By the end of this section, you will be able to:

- Explain gravitational potential energy in terms of work done against gravity.

- Show that the gravitational potential energy of an object of mass *m* at height *h* on Earth is given by *PE*g = *mgh*.
- Show how knowledge of potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.C.1.1** The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. **(S.P. 1.4, 2.1, 2.2)**
- **5.B.1.1** The student is able to set up a representation or model showing that a single object can only have kinetic energy and use information about that object to calculate its kinetic energy. **(S.P. 1.4, 2.2)**
- **5.B.1.2** The student is able to translate between a representation of a single object, which can only have kinetic energy, and a system that includes the object, which may have both kinetic and potential energies. **(S.P. 1.5)**

Work Done Against Gravity

Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in **Figure 7.5**. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then $W = Fd = mgh$. We define this to be the **gravitational potential energy** (PE_g) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the PE_g gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word "system"? Potential energy is a property of a system rather than of a single object—due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth's surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of PE_g to KE without explicitly considering the intermediate step of work. (See **Example 7.7**.) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.

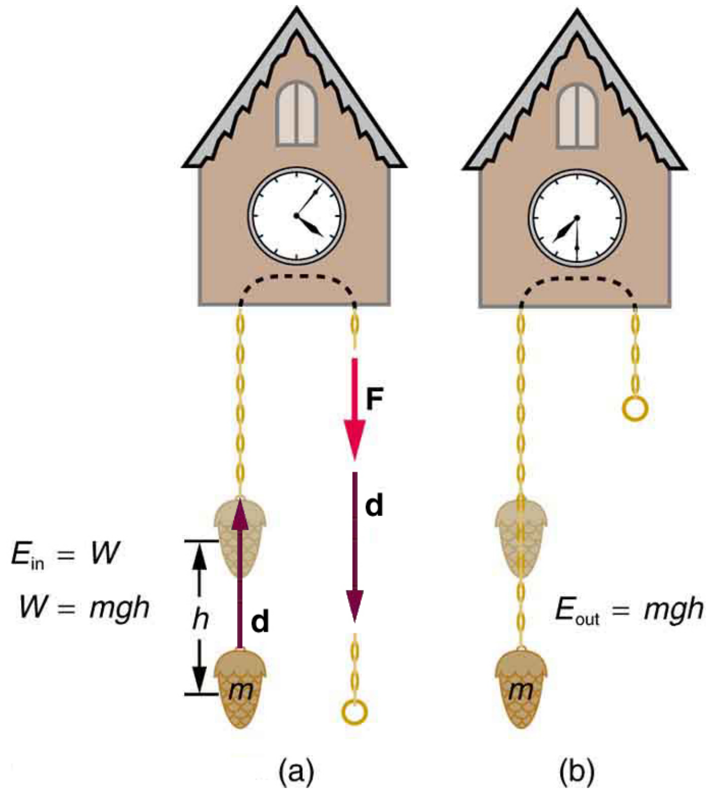


Figure 7.5 (a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy $\Delta \text{PE}_\text{g}$ to be

$$\Delta \text{PE}_\text{g} = mgh, \tag{7.27}$$

where, for simplicity, we denote the change in height by h rather than the usual Δh. Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

$$\begin{aligned} mgh &= (0.500 \text{ kg})(9.80 \text{ m/s}^2)(1.00 \text{ m}) \\ &= 4.90 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 4.90 \text{ J}. \end{aligned} \tag{7.28}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta \text{PE}_\text{g} = mgh$ applies for any path that has a change in height of h, not just when the mass is lifted straight up. (See **Figure 7.6**.) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.

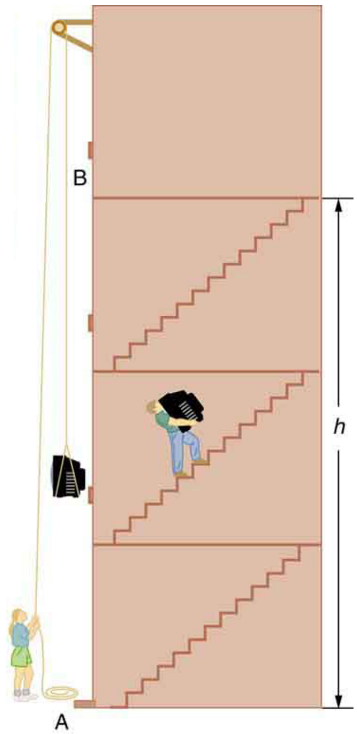


Figure 7.6 The change in gravitational potential energy $(\Delta \mathrm{PE_g})$ between points A and B is independent of the path. $\Delta \mathrm{PE_g} = mgh$ for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

Example 7.6 The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial $\mathrm{PE_g}$ is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

$$W = Fd\cos\theta = -Fd, \tag{7.29}$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions $(\cos\theta = \cos 180^\circ = -1)$. The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height h:

$$\mathrm{KE} = -\Delta \mathrm{PE_g} = -mgh, \tag{7.30}$$

The distance d that the person's knees bend is much smaller than the height h of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$W = -\mathrm{KE} = mgh. \tag{7.31}$$

Combining this equation with the expression for W gives

$$-Fd = mgh. \tag{7.32}$$

Recalling that h is negative because the person fell *down*, the force on the knee joints is given by

$$F = -\frac{mgh}{d} = -\frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(-3.00 \text{ m})}{5.00\times10^{-3} \text{ m}} = 3.53\times10^{5} \text{ N}. \tag{7.33}$$

Discussion

Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See **Figure 7.7**.)

Figure 7.7 The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

Example 7.7 Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in **Figure 7.8** if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?

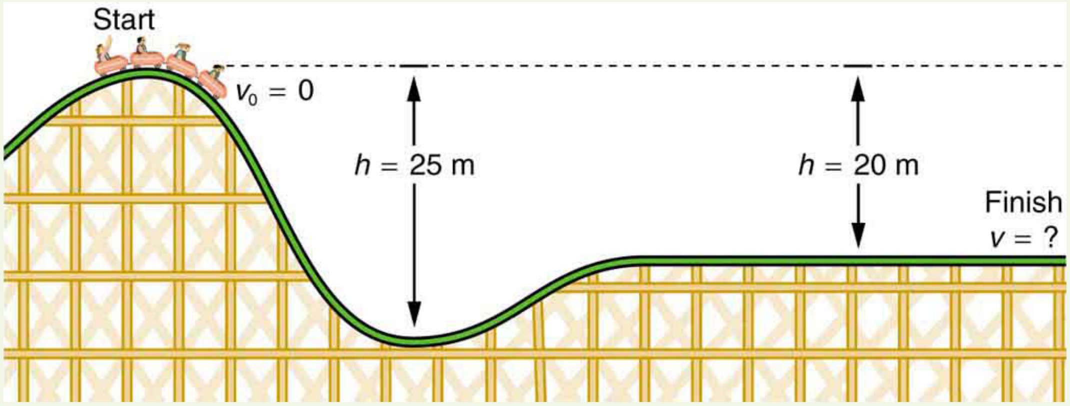


Figure 7.8 The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all ΔPE_g is converted to KE.

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta\text{PE}_\text{g} = \Delta\text{KE}$. Using the equations for PE_g and KE, we can solve for the final speed v, which is the desired quantity.

Solution for (a)

Here the initial kinetic energy is zero, so that $\Delta\text{KE} = \frac{1}{2}mv^2$. The equation for change in potential energy states that $\Delta\text{PE}_\text{g} = mgh$. Since h is negative in this case, we will rewrite this as $\Delta\text{PE}_\text{g} = -mg\mid h\mid$ to show the minus sign clearly. Thus,

$$-\Delta\text{PE}_\text{g} = \Delta\text{KE} \tag{7.34}$$

becomes

$$mg\mid h\mid = \frac{1}{2}mv^2. \tag{7.35}$$

Solving for v, we find that mass cancels and that

$$v = \sqrt{2g\mid h\mid}. \tag{7.36}$$

Substituting known values,

$$\begin{aligned} v &= \sqrt{2\left(9.80\text{ m/s}^2\right)(20.0\text{ m})} \\ &= 19.8\text{ m/s}. \end{aligned} \tag{7.37}$$

Solution for (b)

Again $-\Delta\text{PE}_\text{g} = \Delta\text{KE}$. In this case there is initial kinetic energy, so $\Delta\text{KE} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2$. Thus,

$$mg\mid h\mid = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2. \tag{7.38}$$

Rearranging gives

$$\frac{1}{2}mv^2 = mg\mid h\mid + \frac{1}{2}mv_0{}^2. \tag{7.39}$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v = \sqrt{2g\mid h\mid + v_0{}^2}. \tag{7.40}$$

This equation is very similar to the kinematics equation $v = \sqrt{v_0{}^2 + 2ad}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$\begin{aligned} v &= \sqrt{2(9.80\text{ m/s}^2)(20.0\text{ m}) + (5.00\text{ m/s})^2} \\ &= 20.4\text{ m/s}. \end{aligned} \tag{7.41}$$

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in **Falling Objects** that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see **Figure 7.9**). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.

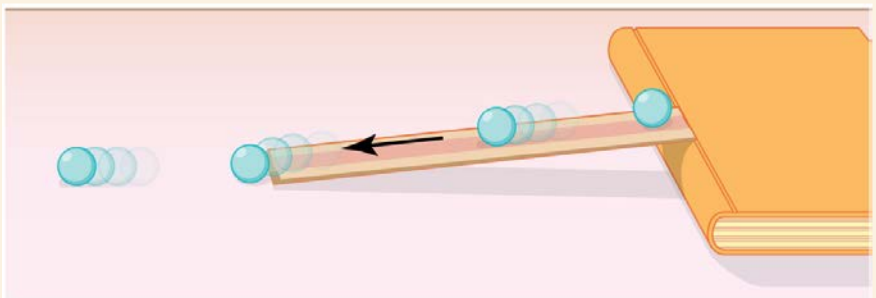


Figure 7.9 A marble rolls down a ruler, and its speed on the level surface is measured.

7.4 Conservative Forces and Potential Energy

Learning Objectives

By the end of this section, you will be able to:

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces leads to conservation of mechanical energy.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.C.1.1** The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. **(S.P. 1.4, 2.1, 2.2)**
- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**
- **5.B.1.1** The student is able to set up a representation or model showing that a single object can only have kinetic energy and use information about that object to calculate its kinetic energy. **(S.P. 1.4, 2.2)**
- **5.B.1.2** The student is able to translate between a representation of a single object, which can only have kinetic energy, and a system that includes the object, which may have both kinetic and potential energies. **(S.P. 1.5)**
- **5.B.3.1** The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. **(S.P. 2.2, 6.4, 7.2)**
- **5.B.3.2** The student is able to make quantitative calculations of the internal potential energy of a system from a description or diagram of that system. **(S.P. 1.4, 2.2)**
- **5.B.3.3** The student is able to apply mathematical reasoning to create a description of the internal potential energy of a system from a description or diagram of the objects and interactions in that system. **(S.P. 1.4, 2.2)**

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

Real World Connections: Energy of a Bowling Ball

How much energy does a bowling ball have? (Just think about it for a minute.)

If you are thinking that you need more information, you're right. If we can measure the ball's velocity, then determining its kinetic energy is simple. Note that this does require defining a reference frame in which to measure the velocity. Determining the ball's potential energy also requires more information. You need to know its height above the ground, which requires a reference frame of the ground. Without the ground—in other words, Earth—the ball does not classically have potential energy. Potential energy comes from the interaction between the ball and the ground. Another way of thinking about this is to compare the ball's potential energy on Earth and on the Moon. A bowling ball a certain height above Earth is going to have more potential energy than the same bowling ball the same height above the surface of the Moon, because Earth has greater mass than the Moon and therefore exerts more gravity on the ball. Thus, potential energy requires a system of at least two objects, or an object with an internal structure of at least two parts.

Potential Energy of a Spring

First, let us obtain an expression for the potential energy stored in a spring ($\mathrm{PE_s}$). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in **Elasticity: Stress and Strain**, and states that the magnitude of force F on the spring and the resulting deformation ΔL are proportional, $F = k\Delta L$.) (See **Figure 7.10**.) For our spring, we will replace ΔL (the amount of deformation produced by a force F) by the distance x that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude $F = kx$, where k is the spring's force constant. The force increases linearly from 0 at the start to kx in the fully stretched position. The average force is $kx/2$. Thus the work done in stretching or compressing the spring is $W_\mathrm{s} = Fd = \left(\frac{kx}{2}\right)x = \frac{1}{2}kx^2$. Alternatively, we noted in **Kinetic Energy and the Work-Energy Theorem** that the area under a graph of F vs. x is the work done by the force. In **Figure 7.10**(c) we see that this area is also $\frac{1}{2}kx^2$. We therefore define the **potential energy of a spring**, $\mathrm{PE_s}$, to be

$$\mathrm{PE_s} = \frac{1}{2}kx^2, \tag{7.42}$$

where k is the spring's force constant and x is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance x . The potential energy of the spring $\mathrm{PE_s}$ does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.

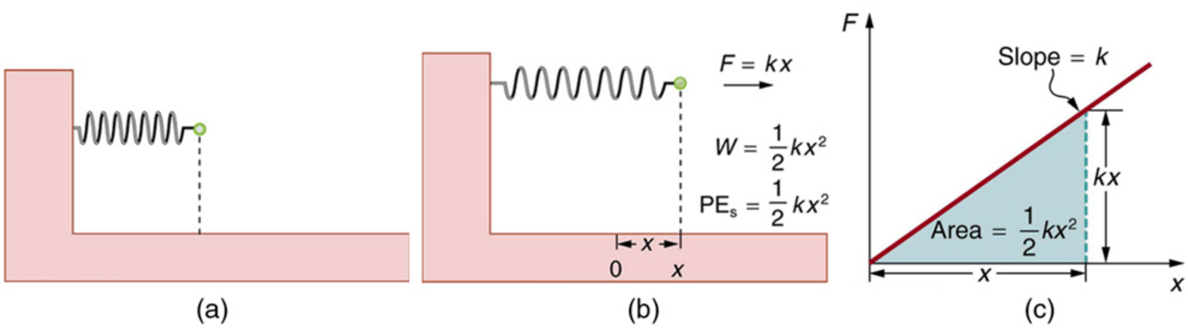


Figure 7.10 (a) An undeformed spring has no $\mathrm{PE_s}$ stored in it. (b) The force needed to stretch (or compress) the spring a distance x has a magnitude $F = kx$, and the work done to stretch (or compress) it is $\frac{1}{2}kx^2$. Because the force is conservative, this work is stored as potential energy $(\mathrm{PE_s})$ in the spring, and it can be fully recovered. (c) A graph of F vs. x has a slope of k , and the area under the graph is $\frac{1}{2}kx^2$. Thus the work done or potential energy stored is $\frac{1}{2}kx^2$.

The equation $\mathrm{PE_s} = \frac{1}{2}kx^2$ has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is $\mathrm{PE_s} = \frac{1}{2}kx^2$, where k is the force constant of the particular system and x is its deformation. Another example is seen in **Figure 7.11** for a guitar string.

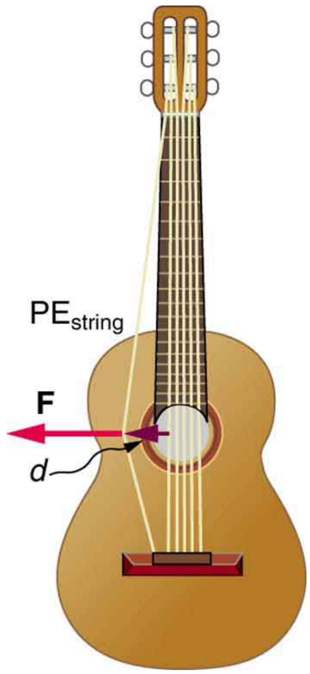


Figure 7.11 Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string.

Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\;2} = \Delta\mathrm{KE}. \tag{7.43}$$

If only conservative forces act, then

$$W_{\text{net}} = W_{\text{c}}, \tag{7.44}$$

where W_{c} is the total work done by all conservative forces. Thus,

$$W_{\text{c}} = \Delta\mathrm{KE}. \tag{7.45}$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is, $W_{\text{c}} = -\Delta\mathrm{PE}$. Therefore,

$$-\Delta\mathrm{PE} = \Delta\mathrm{KE} \tag{7.46}$$

or

$$\Delta\mathrm{KE} + \Delta\mathrm{PE} = 0. \tag{7.47}$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\left.\begin{array}{rc} & \mathrm{KE} + \mathrm{PE} = \text{constant} \\ \text{or} & \\ & \mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f} \end{array}\right\}\text{(conservative forces only)}, \tag{7.48}$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical**

energy, $(\mathrm{KE} + \mathrm{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

The internal energy of a system is the sum of the kinetic energies of all of its elements, plus the potential energy due to all of the interactions due to conservative forces between all of the elements.

Real World Connections

Consider a wind-up toy, such as a car. It uses a spring system to store energy. The amount of energy stored depends only on how many times it is wound, not how quickly or slowly the winding happens. Similarly, a dart gun using compressed air stores energy in its internal structure. In this case, the energy stored inside depends only on how many times it is pumped, not how quickly or slowly the pumping is done. The total energy put into the system, whether through winding or pumping, is equal to the total energy conserved in the system (minus any energy loss in the system due to interactions between its parts, such as air leaks in the dart gun). Since the internal energy of the system is conserved, you can calculate the amount of stored energy by measuring the kinetic energy of the system (the moving car or dart) when the potential energy is released.

Example 7.8 Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in **Figure 7.12**. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.

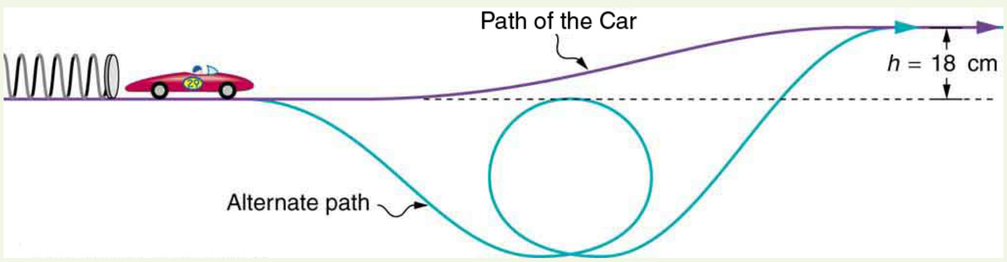


Figure 7.12 A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f} \tag{7.49}$$

or

$$\frac{1}{2}mv_{\mathrm{i}}{}^2 + mgh_{\mathrm{i}} + \frac{1}{2}kx_{\mathrm{i}}{}^2 = \frac{1}{2}mv_{\mathrm{f}}{}^2 + mgh_{\mathrm{f}} + \frac{1}{2}kx_{\mathrm{f}}{}^2, \tag{7.50}$$

where h is the height (vertical position) and x is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both h_{i} and h_{f} are zero. Furthermore, the initial speed v_{i} is zero and the final compression of the spring x_{f} is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\frac{1}{2}kx_{\mathrm{i}}{}^2 = \frac{1}{2}mv_{\mathrm{f}}{}^2. \tag{7.51}$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

$$\begin{aligned} v_\mathrm{f} &= \sqrt{\frac{k}{m}}x_\mathrm{i} \\ &= \sqrt{\frac{250.0\text{ N/m}}{0.100\text{ kg}}}(0.0400\text{ m}) \\ &= 2.00\text{ m/s.} \end{aligned} \tag{7.52}$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}kx_\mathrm{i}^{\;2} = \frac{1}{2}mv_\mathrm{f}^{\;2} + mgh_\mathrm{f}. \tag{7.53}$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_f and substituting known values gives

$$\begin{aligned} v_\mathrm{f} &= \sqrt{\frac{kx_\mathrm{i}^{\;2}}{m} - 2gh_\mathrm{f}} \\ &= \sqrt{\left(\frac{250.0\text{ N/m}}{0.100\text{ kg}}\right)(0.0400\text{ m})^2 - 2(9.80\text{ m/s}^2)(0.180\text{ m})} \\ &= 0.687\text{ m/s.} \end{aligned} \tag{7.54}$$

Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Applying the Science Practices: Potential Energy in a Spring

Suppose you are running an experiment in which two 250 g carts connected by a spring (with spring constant 120 N/m) are run into a solid block, and the compression of the spring is measured. In one run of this experiment, the spring was measured to compress from its rest length of 5.0 cm to a minimum length of 2.0 cm. What was the potential energy stored in this system?

Answer

Note that the change in length of the spring is 3.0 cm. Hence we can apply Equation 7.42 to find that the potential energy is PE = (1/2)(120 N/m)(0.030 m)^2 = 0.0541 J.

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in **Example 7.8**. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

PhET Explorations: Energy Skate Park

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

Figure 7.13 Energy Skate Park (http://cnx.org/content/m55076/1.4/energy-skate-park_en.jar)

7.5 Nonconservative Forces

Learning Objectives

By the end of this section, you will be able to:

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.C.1.2** The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. **(S.P. 6.4)**
- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**

Nonconservative Forces and Friction

Forces are either conservative or nonconservative. Conservative forces were discussed in **Conservative Forces and Potential Energy**. A **nonconservative force** is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in **Figure 7.14**, work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.

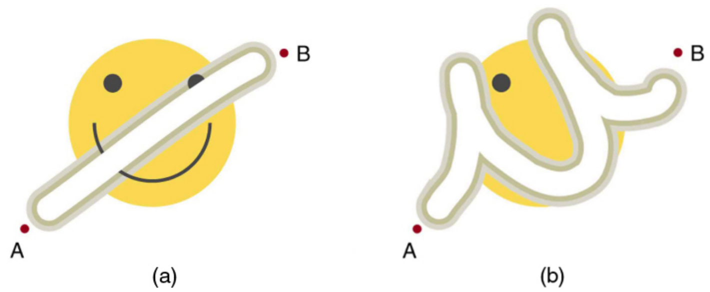


Figure 7.14 The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.

How Nonconservative Forces Affect Mechanical Energy

Mechanical energy *may* not be conserved when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. **Figure 7.15** compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in **Figure 7.15**(a) first before studying more complicated systems as in **Figure 7.15**(b).

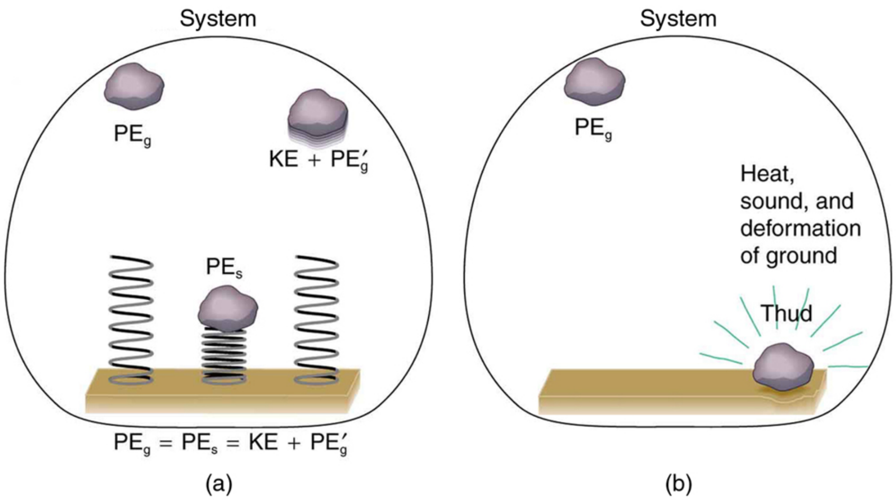


Figure 7.15 Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.

How the Work-Energy Theorem Applies

Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in **Kinetic Energy and the Work-Energy Theorem**, the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or $W_{\text{net}} = \Delta\text{KE}$. The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is,

$$W_{\text{net}} = W_{\text{nc}} + W_{\text{c}}, \tag{7.55}$$

so that

$$W_{\text{nc}} + W_{\text{c}} = \Delta\text{KE}, \tag{7.56}$$

where W_{nc} is the total work done by all nonconservative forces and W_{c} is the total work done by all conservative forces.

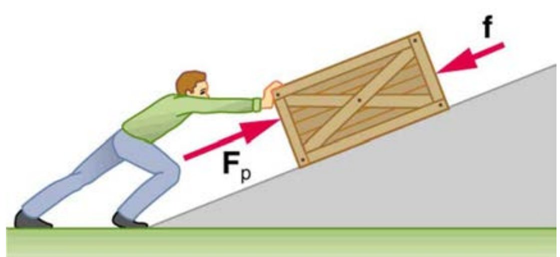


Figure 7.16 A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction.

Consider **Figure 7.16**, in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that $W_{\text{c}} = -\Delta\text{PE}$. Substituting this equation into the previous one and solving for W_{nc} gives

$$W_{\text{nc}} = \Delta\text{KE} + \Delta\text{PE}. \tag{7.57}$$

This equation means that the total mechanical energy $(\text{KE} + \text{PE})$ changes by exactly the amount of work done by nonconservative forces. In **Figure 7.16**, this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy.

We rearrange $W_{\text{nc}} = \Delta\text{KE} + \Delta\text{PE}$ to obtain

$$\text{KE}_\text{i} + \text{PE}_\text{i} + W_{\text{nc}} = \text{KE}_\text{f} + \text{PE}_\text{f}. \tag{7.58}$$

This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If W_{nc} is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in **Figure 7.16**. If W_{nc} is negative, then mechanical energy is decreased, such as when the rock hits the ground in **Figure 7.15**(b). If W_{nc} is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

Applying Energy Conservation with Nonconservative Forces

When no change in potential energy occurs, applying $\text{KE}_\text{i} + \text{PE}_\text{i} + W_{\text{nc}} = \text{KE}_\text{f} + \text{PE}_\text{f}$ amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation $\text{KE}_\text{i} + \text{PE}_\text{i} + W_{\text{nc}} = \text{KE}_\text{f} + \text{PE}_\text{f}$ says that you can start by finding the change in mechanical energy that would have resulted from just the conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

Example 7.9 Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in **Figure 7.17**, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.

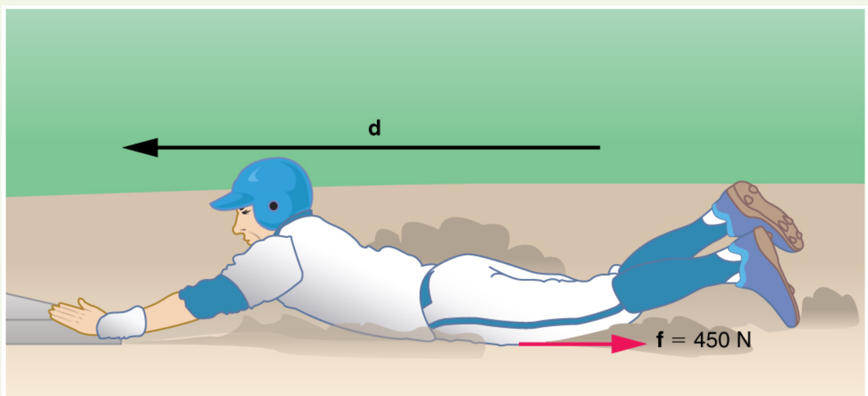


Figure 7.17 The baseball player slides to a stop in a distance d. In the process, friction removes the player's kinetic energy by doing an amount of work fd equal to the initial kinetic energy.

Strategy

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because $\mathbf{f}$ is in the opposite direction of the motion (that is, $\theta = 180^\text{o}$, and so $\cos\theta = -1$). Thus $W_{\text{nc}} = -fd$. The equation simplifies to

$$\frac{1}{2}m{v_\text{i}}^2 - fd = 0 \tag{7.59}$$

or

$$fd = \frac{1}{2}m{v_\text{i}}^2. \tag{7.60}$$

This equation can now be solved for the distance d.

Solution

Solving the previous equation for d and substituting known values yields

$$\begin{aligned} d &= \frac{mv_i^{\ 2}}{2f} \\ &= \frac{(65.0\text{ kg})(6.00\text{ m/s})^2}{(2)(450\text{ N})} \\ &= 2.60\text{ m.} \end{aligned} \tag{7.61}$$

Discussion

The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.

Example 7.10 Calculating Distance Traveled: Sliding Up an Incline

Suppose that the player from **Example 7.9** is running up a hill having a 5.00^o incline upward with a surface similar to that in the baseball stadium. The player slides with the same initial speed. Determine how far he slides.

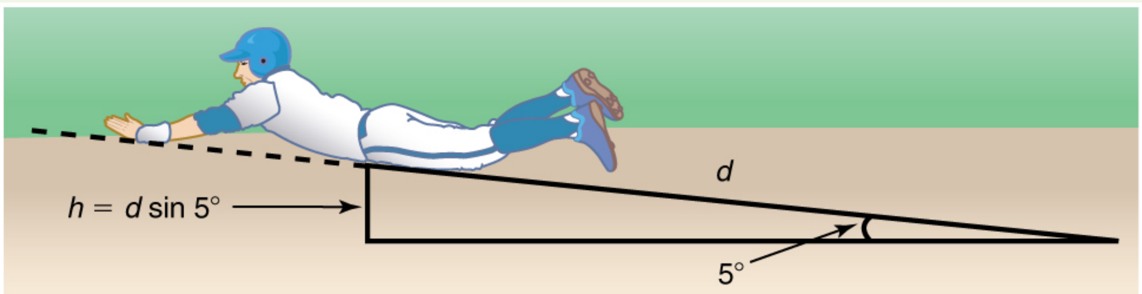


Figure 7.18 The same baseball player slides to a stop on a 5.00^o slope.

Strategy

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through distance d to reach height h along the hill, with $h = d\sin 5.00^\mathrm{o}$. This is expressed by the equation

$$\mathrm{KE} + \mathrm{PE}_\mathrm{i} + W_\mathrm{nc} = \mathrm{KE}_\mathrm{f} + \mathrm{PE}_\mathrm{f}. \tag{7.62}$$

Solution

The work done by friction is again $W_\mathrm{nc} = -fd$; initially the potential energy is $\mathrm{PE}_\mathrm{i} = mg\cdot 0 = 0$ and the kinetic energy is $\mathrm{KE}_\mathrm{i} = \frac{1}{2}mv_\mathrm{i}^{\ 2}$; the final energy contributions are $\mathrm{KE}_\mathrm{f} = 0$ for the kinetic energy and $\mathrm{PE}_\mathrm{f} = mgh = mgd\sin\theta$ for the potential energy.

Substituting these values gives

$$\frac{1}{2}mv_\mathrm{i}^{\ 2} + 0 + \left(-fd\right) = 0 + mgd\sin\theta. \tag{7.63}$$

Solve this for d to obtain

$$\begin{aligned} d &= \frac{\left(\frac{1}{2}\right)mv_\mathrm{i}^{\ 2}}{f + mg\sin\theta} \\ &= \frac{(0.5)(65.0\text{ kg})(6.00\text{ m/s})^2}{450\text{ N}+(65.0\text{ kg})(9.80\text{ m/s}^2)\sin(5.00^\mathrm{o})} \\ &= 2.31\text{ m.} \end{aligned} \tag{7.64}$$

Discussion

As might have been expected, the player slides a shorter distance by sliding uphill. Note that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance d that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy mgh, without combining and resolving force vectors. This simplifies the solution considerably.

Making Connections: Take-Home Investigation—Determining Friction from the Stopping Distance

This experiment involves the conversion of gravitational potential energy into thermal energy. Use the ruler, book, and marble from **Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy**. In addition, you will need a foam cup with a small hole in the side, as shown in **Figure 7.19**. From the 10-cm position on the ruler, let the marble roll into the cup positioned at the bottom of the ruler. Measure the distance d the cup moves before stopping. What forces caused it to stop? What happened to the kinetic energy of the marble at the bottom of the ruler? Next, place the marble at the 20-cm and the 30-cm positions and again measure the distance the cup moves after the marble enters it. Plot the distance the cup moves versus the initial marble position on the ruler. Is this relationship linear?

With some simple assumptions, you can use these data to find the coefficient of kinetic friction μ_{k} of the cup on the table. The force of friction f on the cup is $\mu_{\text{k}}N$, where the normal force N is just the weight of the cup plus the marble. The normal force and force of gravity do no work because they are perpendicular to the displacement of the cup, which moves horizontally. The work done by friction is fd. You will need the mass of the marble as well to calculate its initial kinetic energy.

It is interesting to do the above experiment also with a steel marble (or ball bearing). Releasing it from the same positions on the ruler as you did with the glass marble, is the velocity of this steel marble the same as the velocity of the marble at the bottom of the ruler? Is the distance the cup moves proportional to the mass of the steel and glass marbles?

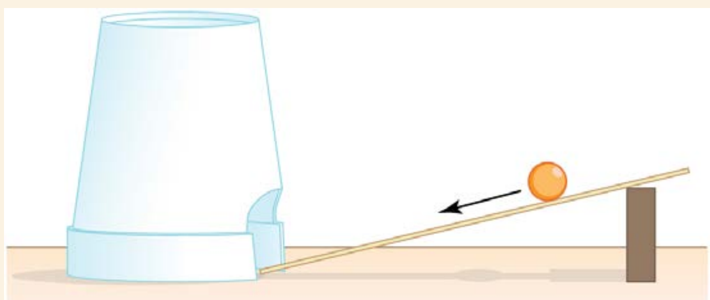


Figure 7.19 Rolling a marble down a ruler into a foam cup.

PhET Explorations: The Ramp

Explore forces, energy and work as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces acting on the file cabinet. Graphs show forces, energy and work.

PhET Interactive Simulation

Figure 7.20 The Ramp (http://cnx.org/content/m55047/1.4/the-ramp_en.jar)

7.6 Conservation of Energy

Learning Objectives

By the end of this section, you will be able to:

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.C.1.2** The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. **(S.P. 6.4)**

- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**
- **4.C.2.2** The student is able to apply the concepts of conservation of energy and the work-energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. **(S.P. 1.4, 2.2, 7.2)**
- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.B.5.4** The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). **(S.P. 6.4, 7.2)**
- **5.B.5.5** The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance. **(S.P. 2.2, 6.4)**

Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy $(\mathrm{KE} + \mathrm{PE})$ and energy transferred via work done by nonconservative forces (W_{nc}). But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called **other energy** (OE). Then we can state the conservation of energy in equation form as

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}. \tag{7.65}$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is KE, work done by a conservative force is represented by PE, work done by nonconservative forces is W_{nc}, and all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

Making Connections: Usefulness of the Energy Conservation Principle

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Real World Connections: Open or Closed System?

Consider whether the following systems are open or closed: a car, a spring-operated dart gun, and the system shown in **Figure 7.15**(a).

A car is not a closed system. You add energy in the form of more gas in the tank (or charging the batteries), and energy is lost due to air resistance and friction.

A spring-operated dart gun is not a closed system. You have to initially compress the spring. Once that has been done, however, the dart gun and dart can be treated as a closed system. All of the energy remains in the system consisting of these two objects.

Figure 7.15(a) is an example of a closed system, once it has been started. All of the energy in the system remains there; none is brought in from outside or leaves.

Table 7.1 gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

Problem-Solving Strategies for Energy

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}. \tag{7.66}$$

Step 4. If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

$$\mathrm{KE_i} + \mathrm{PE_i} + W_\mathrm{nc} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}. \tag{7.67}$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate W_c , the work done by conservative forces; it is already incorporated in the PE terms.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $h = 0$ at either the initial or final point, so that $\mathrm{PE_g}$ is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see **Figure 7.21**) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Figure 7.21 Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Table 7.1 Energy of Various Objects and Phenomena

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Energy released in a supernova	10^{44}
Fusion of all the hydrogen in Earth's oceans	10^{34}
Annual world energy use	4×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
1 kg hydrogen (fusion to helium)	6.4×10^{14}
1 kg uranium (nuclear fission)	8.0×10^{13}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
90,000-ton aircraft carrier at 30 knots	1.1×10^{10}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily home electricity use (developed countries)	7×10^{7}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
1 g fat (9.3 kcal)	3.9×10^{4}
ATP hydrolysis reaction	3.2×10^{4}
1 g carbohydrate (4.1 kcal)	1.7×10^{4}
1 g protein (4.1 kcal)	1.7×10^{4}
Tennis ball at 100 km/h	22
Mosquito $\left(10^{-2}\text{ g at }0.5\text{ m/s}\right)$	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency** of an energy conversion process is defined as

$$\text{Efficiency}(\quad) = \frac{\text{useful energy or work output}}{\text{total energy input}} = \frac{W_{\text{out}}}{E_{\text{in}}}. \tag{7.68}$$

Table 7.2 lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Table 7.2 Efficiency of the Human Body and Mechanical Devices

Activity/device	Efficiency (%)[1]
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

PhET Interactive Simulation

Figure 7.22 Masses and Springs (http://cnx.org/content/m55049/1.3/mass-spring-lab_en.jar)

7.7 Power

Learning Objectives

By the end of this section, you will be able to:

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in **Figure 7.23**.

1. Representative values

Figure 7.23 This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (P) as the rate at which work is done.

Power

Power is the rate at which work is done.

$$P = \frac{W}{t} \tag{7.69}$$

The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second $(1\ \text{W} = 1\ \text{J/s})$.

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example 7.11 Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See **Figure 7.24**.)

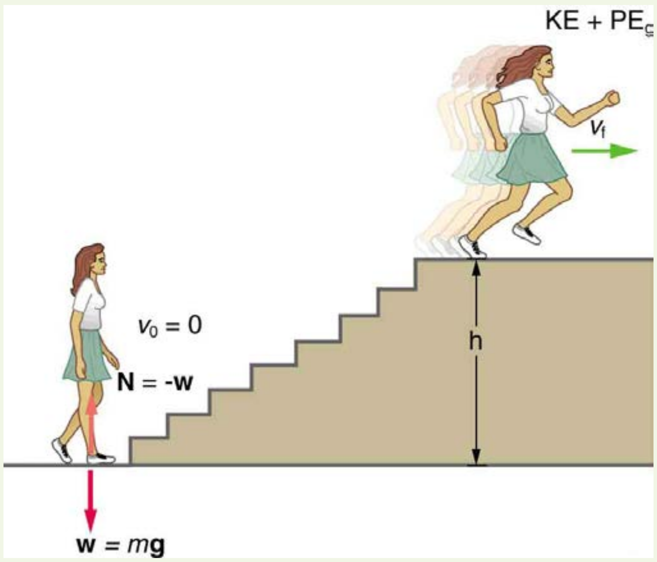


Figure 7.24 When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept

The work going into mechanical energy is $W = \text{KE} + \text{PE}$. At the bottom of the stairs, we take both KE and PE_g as initially zero; thus, $W = \text{KE}_\text{f} + \text{PE}_\text{g} = \frac{1}{2}mv_\text{f}^2 + mgh$, where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.

Solution

Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_\text{f}^2 + mgh}{t}. \tag{7.70}$$

Entering known values yields

$$\begin{aligned} P &= \frac{0.5(60.0\text{ kg})(2.00\text{ m/s})^2 + (60.0\text{ kg})\left(9.80\text{ m/s}^2\right)(3.00\text{ m})}{3.50\text{ s}} \\ &= \frac{120\text{ J} + 1764\text{ J}}{3.50\text{ s}} \\ &= 538\text{ W}. \end{aligned} \tag{7.71}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** $(1\text{ hp} = 746\text{ W})$! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

Making Connections: Take-Home Investigation—Measure Your Power Rating

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See **Table 7.3** for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter (kW/m^2). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is 10^6 W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See **Figure 7.25**.)

Figure 7.25 Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Table 7.3 Power Output or Consumption

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = W / t = E / t$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

$$E = Pt. \tag{7.72}$$

Electricity bills state the energy used in units of **kilowatt-hours** $(\mathrm{kW \cdot h})$, which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Example 7.12 Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is $0.120 per $\mathrm{kW \cdot h}$?

Strategy

Cost is based on energy consumed; thus, we must find E from $E = Pt$ and then calculate the cost. Because electrical energy is expressed in $\mathrm{kW \cdot h}$, at the start of a problem such as this it is convenient to convert the units into kW and hours.

Solution

The energy consumed in $\mathrm{kW \cdot h}$ is

$$\begin{aligned} E &= Pt = (0.200\ \mathrm{kW})(6.00\ \mathrm{h/d})(30.0\ \mathrm{d}) \\ &= 36.0\ \mathrm{kW \cdot h}, \end{aligned} \tag{7.73}$$

and the cost is simply given by

$$\text{cost} = (36.0\ \mathrm{kW \cdot h})(\$0.120\ \text{per}\ \mathrm{kW \cdot h}) = \$4.32\ \text{per month}. \tag{7.74}$$

Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in **Thermodynamics**, the potential for energy to produce useful work has been "degraded" in the energy transformation.

7.8 Work, Energy, and Power in Humans

Learning Objectives

By the end of this section, you will be able to:

- Explain the human body's consumption of energy when at rest versus when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See **Figure 7.26**.) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.

bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been "degraded" in the energy transformation. (This will be discussed in more detail in **Thermodynamics**.)

Glossary

basal metabolic rate: the total energy conversion rate of a person at rest

chemical energy: the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

conservation of mechanical energy: the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

conservative force: a force that does the same work for any given initial and final configuration, regardless of the path followed

efficiency: a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

electrical energy: the energy carried by a flow of charge

energy: the ability to do work

fossil fuels: oil, natural gas, and coal

friction: the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

gravitational potential energy: the energy an object has due to its position in a gravitational field

horsepower: an older non-SI unit of power, with $1\text{ hp} = 746\text{ W}$

joule: SI unit of work and energy, equal to one newton-meter

kilowatt-hour: $(\text{kW}\cdot\text{h})$ unit used primarily for electrical energy provided by electric utility companies

kinetic energy: the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

law of conservation of energy: the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

mechanical energy: the sum of kinetic energy and potential energy

metabolic rate: the rate at which the body uses food energy to sustain life and to do different activities

net work: work done by the net force, or vector sum of all the forces, acting on an object

nonconservative force: a force whose work depends on the path followed between the given initial and final configurations

nuclear energy: energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

potential energy: energy due to position, shape, or configuration

potential energy of a spring: the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression $\frac{1}{2}kx^2$ where x is the distance the spring is compressed or extended and k is the spring constant

power: the rate at which work is done

radiant energy: the energy carried by electromagnetic waves

renewable forms of energy: those sources that cannot be used up, such as water, wind, solar, and biomass

thermal energy: the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

useful work: work done on an external system

watt: (W) SI unit of power, with $1\ \text{W} = 1\ \text{J/s}$

work: the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

work-energy theorem: the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

Section Summary

7.1 Work: The Scientific Definition

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force $\mathbf{F}$ does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,

$$W = Fd\cos\theta.$$

- The SI unit for work and energy is the joule (J), where $1\ \text{J} = 1\ \text{N} \cdot \text{m} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

7.2 Kinetic Energy and the Work-Energy Theorem

- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\text{KE} = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2$.

7.3 Gravitational Potential Energy

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, ΔPE_g, is $\Delta\text{PE}_\text{g} = mgh$, with h being the increase in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, ΔPE_g, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta\text{KE} = -\Delta\text{PE}_\text{g}$.

7.4 Conservative Forces and Potential Energy

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined PE_g for the gravitational force.
- The potential energy of a spring is $\text{PE}_\text{s} = \frac{1}{2}kx^2$, where k is the spring's force constant and x is the displacement from its undeformed position.
- Mechanical energy is defined to be $\text{KE} + \text{PE}$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

$$\left.\begin{array}{c}\mathrm{KE}+\mathrm{PE}=\text{constant}\\ \text{or}\\ \mathrm{KE_i}+\mathrm{PE_i}=\mathrm{KE_f}+\mathrm{PE_f}\end{array}\right\}$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

7.5 Nonconservative Forces

- A nonconservative force is one for which work depends on the path.
- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work W_{nc} done by a nonconservative force changes the mechanical energy of a system. In equation form, $W_{\mathrm{nc}}=\Delta\mathrm{KE}+\Delta\mathrm{PE}$ or, equivalently, $\mathrm{KE_i}+\mathrm{PE_i}+W_{\mathrm{nc}}=\mathrm{KE_f}+\mathrm{PE_f}$.
- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws.

7.6 Conservation of Energy

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as $\mathrm{KE_i}+\mathrm{PE_i}+W_{\mathrm{nc}}+\mathrm{OE_i}=\mathrm{KE_f}+\mathrm{PE_f}+\mathrm{OE_f}$, where OE is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency of a machine or human is defined to be $=\dfrac{W_{\mathrm{out}}}{E_{\mathrm{in}}}$, where W_{out} is useful work output and E_{in} is the energy consumed.

7.7 Power

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t , $P=W/t$.
- The SI unit for power is the watt (W), where $1\ \mathrm{W}=1\ \mathrm{J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\ \mathrm{hp}=746\ \mathrm{W}$.

7.8 Work, Energy, and Power in Humans

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

7.9 World Energy Use

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

Conceptual Questions

7.1 Work: The Scientific Definition

1. Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

2. Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

3. Describe a situation in which a force is exerted for a long time but does no work. Explain.

7.2 Kinetic Energy and the Work-Energy Theorem

4. The person in **Figure 7.33** does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?

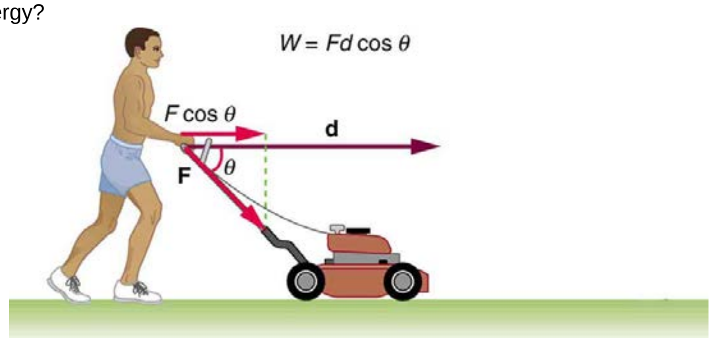


Figure 7.33

5. Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

6. When solving for speed in **Example 7.4**, we kept only the positive root. Why?

7.3 Gravitational Potential Energy

7. In **Example 7.7**, we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that it had the same final speed. Explain in terms of conservation of energy.

8. Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

7.4 Conservative Forces and Potential Energy

9. What is a conservative force?

10. The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

11. Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

12. What is the relationship of potential energy to conservative force?

7.6 Conservation of Energy

13. Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See **Figure 7.34**.)

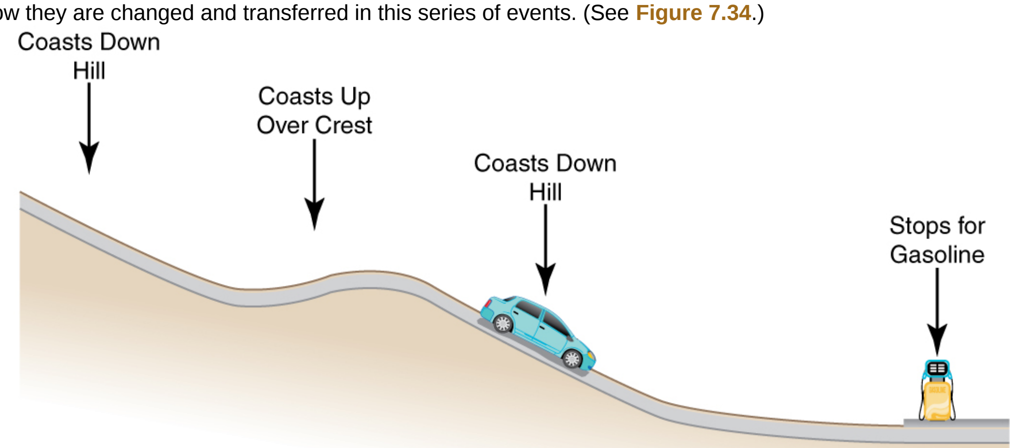


Figure 7.34 A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

14. Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

15. Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

16. List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

17. List the energy conversions that occur when riding a bicycle.

7.7 Power

18. Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

19. Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

20. A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

7.8 Work, Energy, and Power in Humans

21. Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?

22. Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?

23. Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?

24. Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal). Specifically, is it likely that exercise alone will be sufficient to lose weight? You may wish to consider that regular exercise may increase the metabolic rate, whereas protracted dieting may reduce it.

7.9 World Energy Use

25. What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.

26. If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?

Problems & Exercises

7.1 Work: The Scientific Definition

1. How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

2. A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

3. (a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?

4. Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See **Table 7.1** for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

5. Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See **Figure 7.35**.) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.

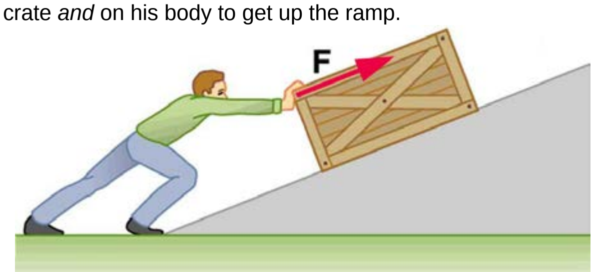


Figure 7.35 A man pushes a crate up a ramp.

6. How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in **Figure 7.36**? Assume no friction acts on the wagon.

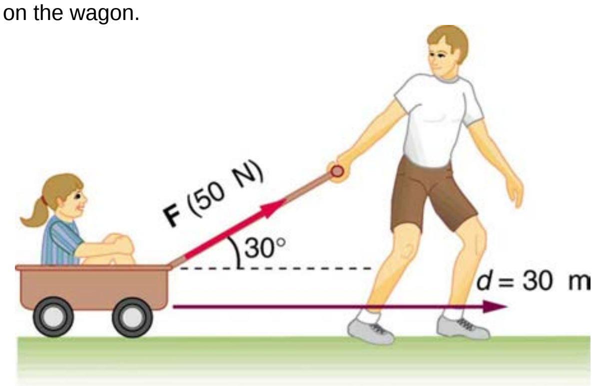


Figure 7.36 The boy does work on the system of the wagon and the child when he pulls them as shown.

7. A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

8. Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in **Figure 7.37**. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?

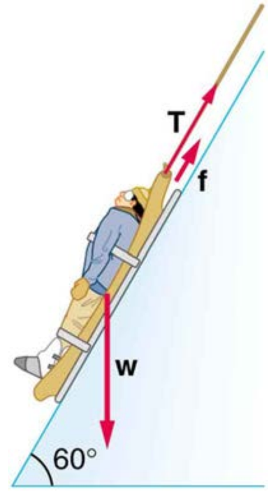


Figure 7.37 A rescue sled and victim are lowered down a steep slope.

7.2 Kinetic Energy and the Work-Energy Theorem

9. Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.

10. (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

11. Confirm the value given for the kinetic energy of an aircraft carrier in **Table 7.1**. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).

12. (a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

13. A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

14. Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

15. Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

7.3 Gravitational Potential Energy

16. A hydroelectric power facility (see **Figure 7.38**) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume $50.0\ \text{km}^3$ ($\text{mass} = 5.00 \times 10^{13}\ \text{kg})$, given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.

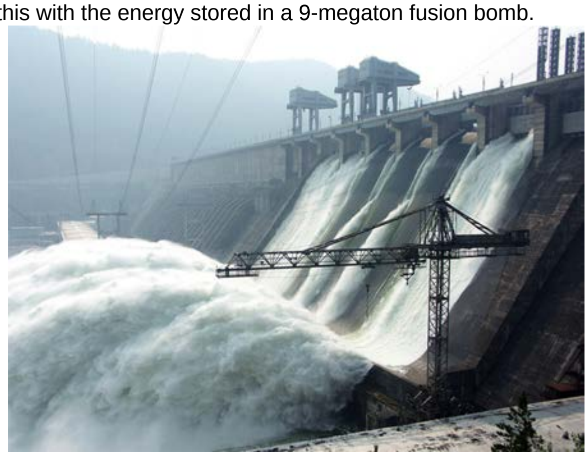


Figure 7.38 Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)

17. (a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about $7 \times 10^9\ \text{kg}$ and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?

18. Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

19. In **Example 7.7**, we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta\text{PE} >> \text{KE}_\text{i}$. Confirm this statement by taking the ratio of ΔPE to KE_i . (Note that mass cancels.)

20. A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in **Figure 7.39**. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.

Figure 7.39 A toy car moves up a sloped track. (credit: Leszek Leszczynski, Flickr)

21. In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30^o slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

7.4 Conservative Forces and Potential Energy

22. A $5.00 \times 10^5\text{-kg}$ subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring?

23. A pogo stick has a spring with a force constant of $2.50 \times 10^4\ \text{N/m}$, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Energy**.

7.5 Nonconservative Forces

24. A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in **Figure 7.40**. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)

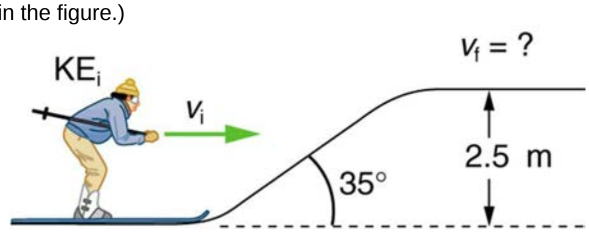


Figure 7.40 The skier's initial kinetic energy is partially used in coasting to the top of a rise.

25. (a) How high a hill can a car coast up (engine disengaged) if work done by friction is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the average force of friction if the hill has a slope 2.5^o above the horizontal?

7.6 Conservation of Energy

26. Using values from **Table 7.1**, how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

27. Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

28. If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from **Table 7.1**)? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.

29. (a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from **Table 7.1**. To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

7.7 Power

30. The Crab Nebula (see **Figure 7.41**) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from **Table 7.3**, calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.

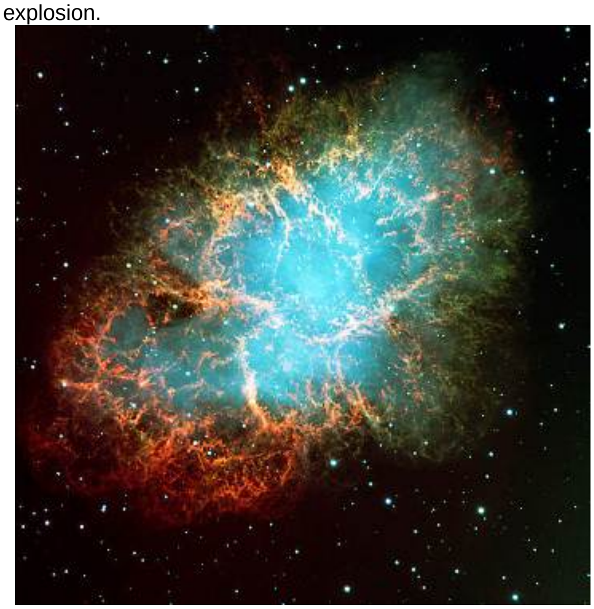


Figure 7.41 Crab Nebula (credit: ESO, via Wikimedia Commons)

31. Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from **Table 7.3**: (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10^{11} observable galaxies, the average brightness of which is somewhat less than our own galaxy.

32. A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

33. What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is $0.0900 per $\text{kW} \cdot \text{h}$?

34. A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is $0.110 per $\text{kW} \cdot \text{h}$?

35. (a) What is the average power consumption in watts of an appliance that uses $5.00\ \text{kW} \cdot \text{h}$ of energy per day? (b) How many joules of energy does this appliance consume in a year?

36. (a) What is the average useful power output of a person who does $6.00\times10^6\ \text{J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

37. A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

38. (a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

39. (a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is $0.0900 per $\text{kW} \cdot \text{h}$?

40. (a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply $8.00\times10^4\ \text{J}$ run a pocket calculator that consumes energy at the rate of $1.00\times10^{-3}\ \text{W}$?

41. (a) How long would it take a 1.50×10^5-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

42. Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Energy**.

43. (a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be $4.00 \times 10^{26}\ \text{W}$.) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of $1.30\ \text{kW/m}^2$ reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs $(1.05 \times 10^{20}\ \text{J})$? Australia's energy needs $(5.4 \times 10^{18}\ \text{J})$? China's energy needs $(6.3 \times 10^{19}\ \text{J})$? (These energy consumption values are from 2006.)

7.8 Work, Energy, and Power in Humans

44. (a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs?

45. (a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?

46. Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)

Figure 7.42 Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)

47. (a) What is the efficiency of an out-of-condition professor who does $2.10 \times 10^5\ \text{J}$ of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?

48. Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from **Table 7.5** for the energy consumption rates of these activities.

49. Using data from **Table 7.5**, calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)

50. What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See **Table 7.5**.)

51. Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she generate in the process?

52. Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.

53. Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.) (b) Compare this force with the weight of the jogger.

54. (a) Calculate the energy in kJ used by a 55.0-kg woman who does 50 deep knee bends in which her center of mass is lowered and raised 0.400 m. (She does work in both directions.) You may assume her efficiency is 20%. (b) What is the average power consumption rate in watts if she does this in 3.00 min?

55. Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the *Daedalus 88*, an aircraft powered by a bicycle-type drive mechanism (see **Figure 7.43**). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from **Table 7.2**, calculate the food energy in kilojoules he metabolized during the flight.

Figure 7.43 The Daedalus 88 in flight. (credit: NASA photo by Beasley)

56. The swimmer shown in **Figure 7.44** exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.

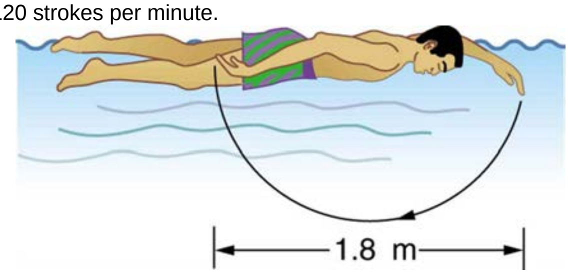


Figure 7.44

57. Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?

58. The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about $7\times10^{9}\ \text{kg}$. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see Figure 7.45), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)

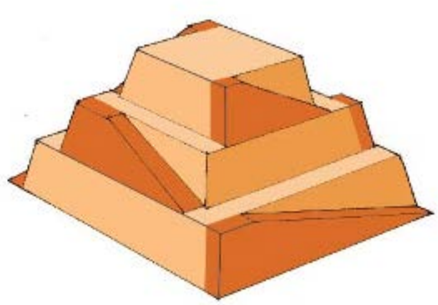


Figure 7.45 Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)

59. (a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

7.9 World Energy Use

60. Integrated Concepts

(a) Calculate the force the woman in Figure 7.46 exerts to do a push-up at constant speed, taking all data to be known to three digits. (b) How much work does she do if her center of mass rises 0.240 m? (c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in Work, Energy, and Power in Humans.

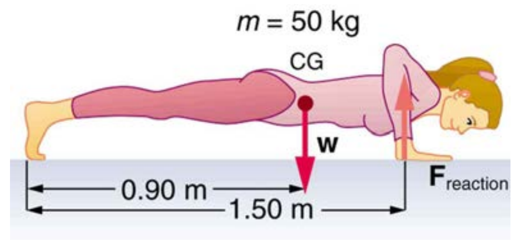


Figure 7.46 Forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity (CG).

61. Integrated Concepts

A 75.0-kg cross-country skier is climbing a 3.0° slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

62. Integrated Concepts

The 70.0-kg swimmer in Figure 7.44 starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke. (a) What is his initial acceleration if water resistance is 45.0 N? (b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s? (c) Discuss whether water resistance seems to increase linearly with velocity.

63. Integrated Concepts

A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible: (a) How much force is needed to compress the spring? (b) To what maximum height can the ball be shot? (c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun? (d) What is the gun's maximum range on level ground?

64. Integrated Concepts

(a) What force must be supplied by an elevator cable to produce an acceleration of $0.800\ \text{m/s}^2$ against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg? (b) How much work is done by the cable in lifting the elevator 20.0 m? (c) What is the final speed of the elevator if it starts from rest? (d) How much work went into thermal energy?

65. Unreasonable Results

A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures. (a) Calculate the car's efficiency. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

66. Unreasonable Results

Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can lose 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine. (a) How many kcal are supplied by the metabolization of 0.500 kg of fat? (b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

67. Construct Your Own Problem

Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)

68. Construct Your Own Problem

Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.

69. Integrated Concepts

A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, his feet leave the floor and his center of gravity rises 0.950 m above its normal standing erect position. (a) Using energy considerations, calculate his velocity when he leaves the floor. (b) What average force did he exert on the floor? (Do not neglect the force to support his weight as well as that to accelerate him.) (c) What was his power output during the acceleration phase?

Test Prep for AP® Courses

7.1 Work: The Scientific Definition

1. Given Table 7.7 about how much force does the rocket engine exert on the 3.0-kg payload?

Table 7.7

Distance traveled with rocket engine firing (m)	Payload final velocity (m/s)
500	310
490	300
1020	450
505	312

a. 150 N
b. 300 N
c. 450 N
d. 600 N

2. You have a cart track, a cart, several masses, and a position-sensing pulley. Design an experiment to examine how the force exerted on the cart does work as it moves through a distance.

3. Look at Figure 7.10(c). You compress a spring by *x*, and then release it. Next you compress the spring by 2*x*. How much more work did you do the second time than the first?

a. Half as much
b. The same
c. Twice as much
d. Four times as much

4. You have a cart track, two carts, several masses, a position-sensing pulley, and a piece of carpet (a rough surface) that will fit over the track. Design an experiment to examine how the force exerted on the cart does work as the cart moves through a distance.

5. A crane is lifting construction materials from the ground to an elevation of 60 m. Over the first 10 m, the motor linearly increases the force it exerts from 0 to 10 kN. It exerts that constant force for the next 40 m, and then winds down to 0 N again over the last 10 m, as shown in the figure. What is the total work done on the construction materials?

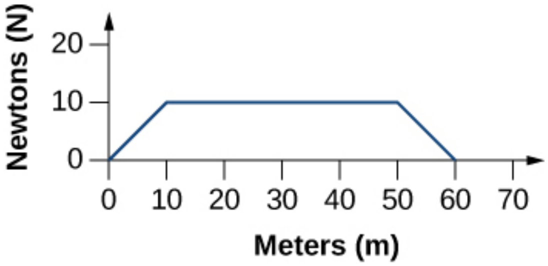


Figure 7.47

a. 500 kJ
b. 600 kJ
c. 300 kJ
d. 18 MJ

7.2 Kinetic Energy and the Work-Energy Theorem

6. A toy car is going around a loop-the-loop. Gravity ____ the kinetic energy on the upward side of the loop, ____ the kinetic energy at the top, and ____ the kinetic energy on the downward side of the loop.

a. increases, decreases, has no effect on
b. decreases, has no effect on, increases
c. increases, has no effect on, decreases
d. decreases, increases, has no effect on

7. A roller coaster is set up with a track in the form of a perfect cosine. Describe and graph what happens to the kinetic energy of a cart as it goes through the first full period of the track.

8. If wind is blowing horizontally toward a car with an angle of 30 degrees from the direction of travel, the kinetic energy will ____. If the wind is blowing at a car at 135 degrees from the direction of travel, the kinetic energy will ____.

a. increase, increase
b. increase, decrease
c. decrease, increase
d. decrease, decrease

9. In what direction relative to the direction of travel can a force act on a car (traveling on level ground), and not change the kinetic energy? Can you give examples of such forces?

10. A 2000-kg airplane is coming in for a landing, with a velocity 5 degrees below the horizontal and a drag force of 40 kN acting directly rearward. Kinetic energy will ____ due to the net force of ____.

a. increase, 20 kN
b. decrease, 40 kN
c. increase, 45 kN
d. decrease, 45 kN

11. You are participating in the Iditarod, and your sled dogs are pulling you across a frozen lake with a force of 1200 N while a 300 N wind is blowing at you at 135 degrees from your direction of travel. What is the net force, and will your kinetic energy increase or decrease?

12. A model drag car is being accelerated along its track from rest by a motor with a force of 75 N, but there is a drag force of 30 N due to the track. What is the kinetic energy after 2 m of travel?

a. 90 J
b. 150 J
c. 210 J
d. 60 J

13. You are launching a 2-kg potato out of a potato cannon. The cannon is 1.5 m long and is aimed 30 degrees above the horizontal. It exerts a 50 N force on the potato. What is the kinetic energy of the potato as it leaves the muzzle of the potato cannon?

14. When the force acting on an object is parallel to the direction of the motion of the center of mass, the mechanical energy ____. When the force acting on an object is antiparallel to the direction of the center of mass, the mechanical energy ____.

a. increases, increases
b. increases, decreases
c. decreases, increases
d. decreases, decreases

15. Describe a system in which the main forces acting are parallel or antiparallel to the center of mass, and justify your answer.

16. A child is pulling two red wagons, with the second one tied to the first by a (non-stretching) rope. Each wagon has a mass of 10 kg. If the child exerts a force of 30 N for 5.0 m, how much has the kinetic energy of the two-wagon system changed?

a. 300 J
b. 150 J
c. 75 J
d. 60 J

17. A child has two red wagons, with the rear one tied to the front by a (non-stretching) rope. If the child pushes on the rear wagon, what happens to the kinetic energy of each of the wagons, and the two-wagon system?

18. Draw a graph of the force parallel to displacement exerted on a stunt motorcycle going through a loop-the-loop versus the distance traveled around the loop. Explain the net change in energy.

7.3 Gravitational Potential Energy

19. A 1.0 kg baseball is flying at 10 m/s. How much kinetic energy does it have? Potential energy?

a. 10 J, 20 J
b. 50 J, 20 J
c. unknown, 50 J
d. 50 J, unknown

20. A 2.0-kg potato has been launched out of a potato cannon at 9.0 m/s. What is the kinetic energy? If you then learn that it is 4.0 m above the ground, what is the total mechanical energy relative to the ground?

a. 78 J, 3 J
b. 160 J, 81 J
c. 81 J, 160 J
d. 81 J, 3 J

21. You have a 120-g yo-yo that you are swinging at 0.9 m/s. How much energy does it have? How high can it get above the lowest point of the swing without your doing any additional work, on Earth? How high could it get on the Moon, where gravity is 1/6 Earth's?

7.4 Conservative Forces and Potential Energy

22. Two 4.0 kg masses are connected to each other by a spring with a force constant of 25 N/m and a rest length of 1.0 m. If the spring has been compressed to 0.80 m in length and the masses are traveling toward each other at 0.50 m/s (each), what is the total energy in the system?

a. 1.0 J
b. 1.5 J
c. 9.0 J
d. 8.0 J

23. A spring with a force constant of 5000 N/m and a rest length of 3.0 m is used in a catapult. When compressed to 1.0 m, it is used to launch a 50 kg rock. However, there is an error in the release mechanism, so the rock gets launched almost straight up. How high does it go, and how fast is it going when it hits the ground?

24. What information do you need to calculate the kinetic energy and potential energy of a spring? Potential energy due to gravity? How many objects do you need information about for each of these cases?

25. You are loading a toy dart gun, which has two settings, the more powerful with the spring compressed twice as far as the lower setting. If it takes 5.0 J of work to compress the dart gun to the lower setting, how much work does it take for the higher setting?

a. 20 J
b. 10 J
c. 2.5 J
d. 40 J

26. Describe a system you use daily with internal potential energy.

27. Old-fashioned pendulum clocks are powered by masses that need to be wound back to the top of the clock about once a week to counteract energy lost due to friction and to the chimes. One particular clock has three masses: 4.0 kg, 4.0 kg, and 6.0 kg. They can drop 1.3 meters. How much energy does the clock use in a week?

a. 51 J
b. 76 J
c. 127 J
d. 178 J

28. A water tower stores not only water, but (at least part of) the energy to move the water. How much? Make reasonable estimates for how much water is in the tower, and other quantities you need.

29. Old-fashioned pocket watches needed to be wound daily so they wouldn't run down and lose time, due to the friction in the internal components. This required a large number of turns of the winding key, but not much force per turn, and it was possible to overwind and break the watch. How was the energy stored?

a. A small mass raised a long distance
b. A large mass raised a short distance
c. A weak spring deformed a long way
d. A strong spring deformed a short way

30. Some of the very first clocks invented in China were powered by water. Describe how you think this was done.

7.5 Nonconservative Forces

31. You are in a room in a basement with a smooth concrete floor (friction force equals 40 N) and a nice rug (friction force equals 55 N) that is 3 m by 4 m. However, you have to push a very heavy box from one corner of the rug to the opposite corner of the rug. Will you do more work against friction going around the floor or across the rug, and how much extra?

a. Across the rug is 275 J extra
b. Around the floor is 5 J extra
c. Across the rug is 5 J extra
d. Around the floor is 280 J extra

32. In the Appalachians, along the interstate, there are ramps of loose gravel for semis that have had their brakes fail to drive into to stop. Design an experiment to measure how effective this would be.

7.6 Conservation of Energy

33. You do 30 J of work to load a toy dart gun. However, the dart is 10 cm long and feels a frictional force of 10 N while going through the dart gun's barrel. What is the kinetic energy of the fired dart?

a. 30 J
b. 29 J
c. 28 J
d. 27 J

34. When an object is lifted by a crane, it begins and ends its motion at rest. The same is true of an object pushed across a rough surface. Explain why this happens. What are the differences between these systems?

35. A child has two red wagons, with the rear one tied to the front by a stretchy rope (a spring). If the child pulls on the front wagon, the ____ increases.

a. kinetic energy of the wagons
b. potential energy stored in the spring
c. both A and B
d. not enough information

36. A child has two red wagons, with the rear one tied to the front by a stretchy rope (a spring). If the child pulls on the front wagon, the energy stored in the system increases. How do the relative amounts of potential and kinetic energy in this system change over time?

37. Which of the following are closed systems?

a. Earth
b. a car
c. a frictionless pendulum
d. a mass on a spring in a vacuum

38. Describe a real-world example of a closed system.

39. A 5.0-kg rock falls off of a 10 m cliff. If air resistance exerts a force of 10 N, what is the kinetic energy when the rock hits the ground?

a. 400 J
b. 12.6 m/s
c. 100 J
d. 500 J

40. Hydroelectricity is generated by storing water behind a dam, and then letting some of it run through generators in the dam to turn them. If the system is the water, what is the environment that is doing work on it? If a dam has water 100 m deep behind it, how much energy was generated if 10,000 kg of water exited the dam at 2.0 m/s?

41. Before railroads were invented, goods often traveled along canals, with mules pulling barges from the bank. If a mule is exerting a 1200 N force for 10 km, and the rope connecting the mule to the barge is at a 20 degree angle from the direction of travel, how much work did the mule do on the barge?

a. 12 MJ
b. 11 MJ
c. 4.1 MJ
d. 6 MJ

42. Describe an instance today in which you did work, by the scientific definition. Then calculate how much work you did in that instance, showing your work.

8 LINEAR MOMENTUM AND COLLISIONS

Figure 8.1 Each rugby player has great momentum, which will affect the outcome of their collisions with each other and the ground. (credit: ozzzie, Flickr)

Chapter Outline

Connection for AP® courses

In this chapter, you will learn about the concept of momentum and the relationship between momentum and force (both vector quantities) applied over a time interval. Have you ever considered why a glass dropped on a tile floor will often break, but a glass dropped on carpet will often remain intact? Both involve changes in momentum, but the actual collision with the floor is different in each case, just as an automobile collision without the benefit of an airbag can have a significantly different outcome than one with an airbag.

You will learn that the interaction of objects (like a glass and the floor or two automobiles) results in forces, which in turn result in changes in the momentum of each object. At the same time, you will see how the law of momentum conservation can be applied to a system to help determine the outcome of a collision.

The content in this chapter supports:

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.D A force exerted on an object can change the momentum of the object.

Essential Knowledge 3.D.2 The change in momentum of an object occurs over a time interval.

Big Idea 4: Interactions between systems can result in changes in those systems.

Enduring Understanding 4.B Interactions with other objects or systems can change the total linear momentum of a system.

Essential Knowledge 4.B.1 The change in linear momentum for a constant-mass system is the product of the mass of the system and the change in velocity of the center of mass.

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved.

Essential Knowledge 5.D.1 In a collision between objects, linear momentum is conserved. In an elastic collision, kinetic energy is the same before and after.

Essential Knowledge 5.D.2 In a collision between objects, linear momentum is conserved. In an inelastic collision, kinetic energy is not the same before and after the collision.

8.1 Linear Momentum and Force

Learning Objectives

By the end of this section, you will be able to:

- Define linear momentum.
- Explain the relationship between linear momentum and force.
- State Newton's second law of motion in terms of linear momentum.
- Calculate linear momentum given mass and velocity.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.D.1.1** The student is able to justify the selection of data needed to determine the relationship between the direction of the force acting on an object and the change in momentum caused by that force. **(S.P. 4.1)**

Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

$$\mathbf{p} = m\mathbf{v}. \tag{8.1}$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum $\mathbf{p}$ is a vector having the same direction as the velocity $\mathbf{v}$. The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.

Linear Momentum

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

$$\mathbf{p} = m\mathbf{v}. \tag{8.2}$$

Example 8.1 Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p. (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

$$p = mv \tag{8.3}$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

$$p_{\text{player}} = (110\text{ kg})(8.00\text{ m/s}) = 880\text{ kg} \cdot \text{m/s} \tag{8.4}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

$$p_{\text{ball}} = (0.410\text{ kg})(25.0\text{ m/s}) = 10.3\text{ kg} \cdot \text{m/s} \tag{8.5}$$

The ratio of the player's momentum to that of the ball is

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9. \quad (8.6)$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}, \quad (8.7)$$

where $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change in time.

Newton's Second Law of Motion in Terms of Momentum

The net external force equals the change in momentum of a system divided by the time over which it changes.

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} \quad (8.8)$$

Making Connections: Force and Momentum

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $\mathbf{F}_{\text{net}} = m\mathbf{a}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\Delta \mathbf{p}$ is given by

$$\Delta \mathbf{p} = \Delta(m\mathbf{v}). \quad (8.9)$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta \mathbf{v}. \quad (8.10)$$

So that for constant mass, Newton's second law of motion becomes

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t}. \quad (8.11)$$

Because $\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a}$, we get the familiar equation

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \quad (8.12)$$

when the mass of the system is constant.

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail**;** however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

Example 8.2 Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}. \tag{8.13}$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta p = m\Delta v = m(v_\text{f} - v_\text{i}). \tag{8.14}$$

In this example, the velocity just after impact and the change in time are given; thus, once Δp is calculated, $F_{\text{net}} = \frac{\Delta p}{\Delta t}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

$$\begin{aligned} \Delta p &= m(v_\text{f} - v_\text{i}) \\ &= (0.057 \text{ kg})(58 \text{ m/s} - 0 \text{ m/s}) \\ &= 3.306 \text{ kg} \cdot \text{m/s} \approx 3.3 \text{ kg} \cdot \text{m/s} \end{aligned} \tag{8.15}$$

Now the magnitude of the net external force can determined by using $F_{\text{net}} = \frac{\Delta p}{\Delta t}$:

$$\begin{aligned} F_{\text{net}} &= \frac{\Delta p}{\Delta t} = \frac{3.306 \text{ kg} \cdot \text{m/s}}{5.0\times10^{-3} \text{ s}} \\ &= 661 \text{ N} \approx 660 \text{ N}, \end{aligned} \tag{8.16}$$

where we have retained only two significant figures in the final step.

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $F_{\text{net}} = ma$, but one additional step would be required compared with the strategy used in this example.

Making Connections: Illustrative Example

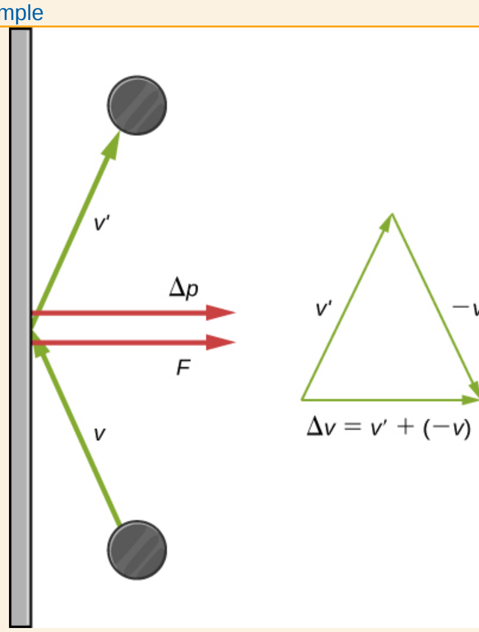


Figure 8.2 A puck has an elastic, glancing collision with the edge of an air hockey table.

In **Figure 8.2**, a puck is shown colliding with the edge of an air hockey table at a glancing angle. During the collision, the edge of the table exerts a force **F** on the puck, and the velocity of the puck changes as a result of the collision. The change in momentum is found by the equation:

$$\Delta p = m\Delta v = mv' - mv = m(v' + (-v)) \tag{8.17}$$

As shown, the direction of the change in velocity is the same as the direction of the change in momentum, which in turn is in the same direction as the force exerted by the edge of the table. Note that there is only a horizontal change in velocity. There is no difference in the vertical components of the initial and final velocity vectors; therefore, there is no vertical component to the change in velocity vector or the change in momentum vector. This is consistent with the fact that the force exerted by the edge of the table is purely in the horizontal direction.

8.2 Impulse

Learning Objectives

By the end of this section, you will be able to:

- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.D.2.1** The student is able to justify the selection of routines for the calculation of the relationships between changes in momentum of an object, average force, impulse, and time of interaction. **(S.P. 2.1)**
- **3.D.2.2** The student is able to predict the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted. **(S.P. 6.4)**
- **3.D.2.3** The student is able to analyze data to characterize the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted. **(S.P. 5.1)**
- **3.D.2.4** The student is able to design a plan for collecting data to investigate the relationship between changes in momentum and the average force exerted on an object over time. **(S.P. 4.1)**
- **4.B.2.1** The student is able to apply mathematical routines to calculate the change in momentum of a system by analyzing the average force exerted over a certain time on the system. **(S.P. 2.2)**
- **4.B.2.2** The student is able to perform analysis on data presented as a force-time graph and predict the change in momentum of a system. **(S.P. 5.1)**

The effect of a force on an object depends on how long it acts, as well as how great the force is. In **Example 8.1**, a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum $\Delta \mathbf{p}$.

By rearranging the equation $\mathbf{F}_{\text{net}} = \dfrac{\Delta \mathbf{p}}{\Delta t}$ to be

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}}\Delta t, \tag{8.18}$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $\mathbf{F}_{\text{net}}\Delta t$ is given the name **impulse**. Impulse is the same as the change in momentum.

Impulse: Change in Momentum

Change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}}\Delta t \tag{8.19}$$

The quantity $\mathbf{F}_{\text{net}}\Delta t$ is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

Making Connections: Illustrations of Force Exerted

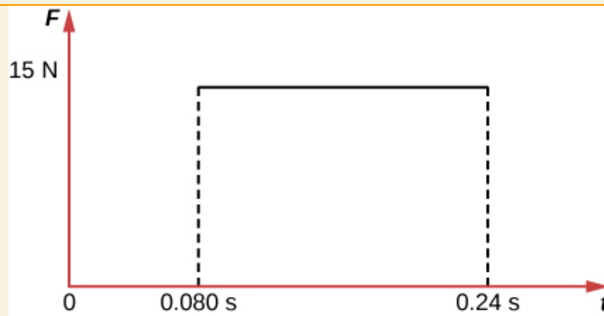


Figure 8.3 This is a graph showing the force exerted by a fixed barrier on a block versus time.

A 1.2-kg block slides across a horizontal, frictionless surface with a constant speed of 3.0 m/s before striking a fixed barrier and coming to a stop. In **Figure 8.3**, the force exerted by the barrier is assumed to be a constant 15 N during the 0.24-s collision. The impulse can be calculated using the area under the curve.

$$\Delta p = F\Delta t = (15 \quad \text{N})(0.24 \quad \text{s}) = \quad 3.6 \quad \text{kg}\bullet\text{m/s} \tag{8.20}$$

Note that the initial momentum of the block is:

$$p_{initial} = mv_{initial} = (1.2 \quad \text{kg})(-3.0 \quad \text{m/s}) = \quad -3.6\ \text{kg}\bullet\text{m/s} \tag{8.21}$$

We are assuming that the initial velocity is −3.0 m/s. We have established that the force exerted by the barrier is in the positive direction, so the initial velocity of the block must be in the negative direction. Since the final momentum of the block is zero, the impulse is equal to the change in momentum of the block.

Suppose that, instead of striking a fixed barrier, the block is instead stopped by a spring.Consider the force exerted by the spring over the time interval from the beginning of the collision until the block comes to rest.

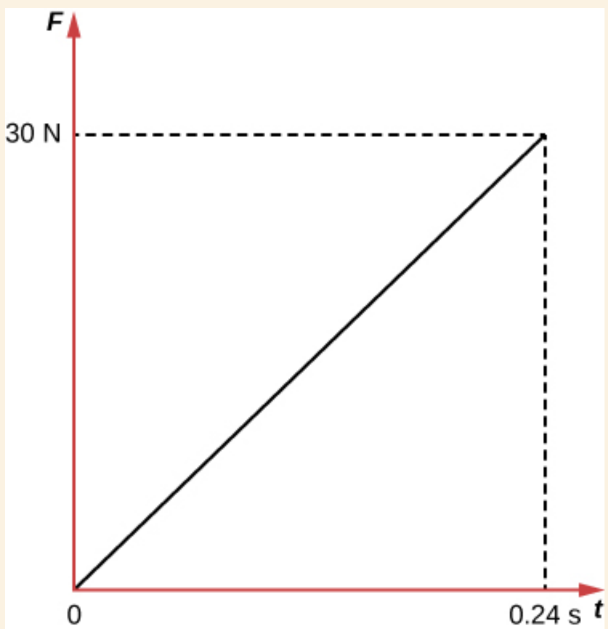


Figure 8.4 This is a graph showing the force exerted by a spring on a block versus time.

In this case, the impulse can be calculated again using the area under the curve (the area of a triangle):

$$\Delta p = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(0.24 \quad \text{s})(30 \quad \text{N}) = \quad 3.6 \quad \text{kg}\bullet\text{m/s} \tag{8.22}$$

Again, this is equal to the difference between the initial and final momentum of the block, so the impulse is equal to the change in momentum.

Example 8.3 Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of 30° from the perpendicular, and bounces off at an angle of 30° from perpendicular to the wall.

(a) Determine the direction of the force on the wall due to each ball.

(b) Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

Strategy for (a)

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to determine the direction. Assume the x-axis to be normal to the wall and to be positive in the initial direction of motion. Choose the y-axis to be along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

Solution for (a)

The first ball bounces directly into the wall and exerts a force on it in the $+x$ direction. Therefore the wall exerts a force on the ball in the $-x$ direction. The second ball continues with the same momentum component in the y direction, but reverses its x-component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the $-x$ direction, so the force of the wall on each ball is along the $-x$ direction.

Strategy for (b)

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

Solution for (b)

Let u be the speed of each ball before and after collision with the wall, and m the mass of each ball. Choose the x-axis and y-axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

$$p_{\text{xi}} = mu;\ p_{\text{yi}} = 0 \tag{8.23}$$

$$p_{\text{xf}} = -mu;\ p_{\text{yf}} = 0 \tag{8.24}$$

Impulse is the change in momentum vector. Therefore the x-component of impulse is equal to $-2mu$ and the y-component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

$$p_{\text{xi}} = mu\cos 30^\circ;\ p_{\text{yi}} = -mu\sin 30^\circ \tag{8.25}$$

$$p_{\text{xf}} = -mu\cos 30^\circ;\ p_{\text{yf}} = -mu\sin 30^\circ \tag{8.26}$$

It should be noted here that while p_{x} changes sign after the collision, p_{y} does not. Therefore the x-component of impulse is equal to $-2mu\cos 30^\circ$ and the y-component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

$$\frac{2mu}{2mu\cos 30^\circ} = \frac{2}{\sqrt{3}} = 1.155. \tag{8.27}$$

Discussion

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative x-direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive x-direction.

Our definition of impulse includes an assumption that the force is constant over the time interval Δt. *Forces are usually not constant*. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force. **Figure 8.5** shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times t_1 and t_2. That area is equal to the area inside the rectangle bounded by F_{eff}, t_1, and t_2. Thus the impulses and their effects are the same for both the actual and effective forces.

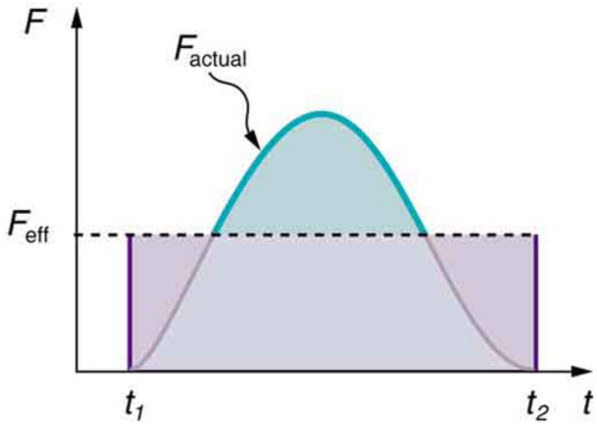


Figure 8.5 A graph of force versus time with time along the x -axis and force along the y -axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

Making Connections: Baseball

In most real-life collisions, the forces acting on an object are not constant. For example, when a bat strikes a baseball, the force is very small at the beginning of the collision since only a small portion of the ball is initially in contact with the bat. As the collision continues, the ball deforms so that a greater fraction of the ball is in contact with the bat, resulting in a greater force. As the ball begins to leave the bat, the force drops to zero, much like the force curve in **Figure 8.5**. Although the changing force is difficult to precisely calculate at each instant, the average force can be estimated very well in most cases.

Suppose that a 150-g baseball experiences an average force of 480 N in a direction opposite the initial 32 m/s speed of the baseball over a time interval of 0.017 s. What is the final velocity of the baseball after the collision?

$$\Delta p = F\Delta t = (480)(0.017) = \quad 8.16 \quad \text{kg}\bullet\text{m/s} \tag{8.28}$$

$$mv_f - mv_i = 8.16 \quad \text{kg} \bullet \text{m/s} \tag{8.29}$$

$$(0.150 \text{ kg})v_f - (0.150 \text{ kg})(-32 \text{ m/s}) = 8.16 \quad \text{kg} \bullet \text{m/s} \tag{8.30}$$

$$v_f = 22 \text{ m/s} \tag{8.31}$$

Note in the above example that the initial velocity of the baseball prior to the collision is negative, consistent with the assumption we initially made that the force exerted by the bat is positive and in the direction opposite the initial velocity of the baseball. In this case, even though the force acting on the baseball varies with time, the average force is a good approximation of the effective force acting on the ball for the purposes of calculating the impulse and the change in momentum.

Making Connections: Take-Home Investigation—Hand Movement and Impulse

Try catching a ball while “giving” with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

Making Connections: Constant Force and Constant Acceleration

The assumption of a constant force in the definition of impulse is analogous to the assumption of a constant acceleration in kinematics. In both cases, nature is adequately described without the use of calculus.

Applying the Science Practices: Verifying the Relationship between Force and Change in Linear Momentum

Design an experiment in order to experimentally verify the relationship between the impulse of a force and change in linear momentum. For simplicity, it would be best to ensure that frictional forces are very small or zero in your experiment so that the effect of friction can be neglected. As you design your experiment, consider the following:

- Would it be easier to analyze a one-dimensional collision or a two-dimensional collision?
- How will you measure the force?

- Should you have two objects in motion or one object bouncing off a rigid surface?
- How will you measure the duration of the collision?
- How will you measure the initial and final velocities of the object(s)?
- Would it be easier to analyze an elastic or inelastic collision?
- Should you verify the relationship mathematically or graphically?

8.3 Conservation of Momentum

Learning Objectives

By the end of this section, you will be able to:

- Describe the law of conservation of linear momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.
- Explain the law of conservation of momentum as it relates to atomic and subatomic particles.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.D.1.4** The student is able to design an experimental test of an application of the principle of the conservation of linear momentum, predict an outcome of the experiment using the principle, analyze data generated by that experiment whose uncertainties are expressed numerically, and evaluate the match between the prediction and the outcome. **(S.P. 4.2, 5.1, 5.3, 6.4)**
- **5.D.2.1** The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. **(S.P. 6.4, 7.2)**
- **5.D.2.2** The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically. **(S.P.4.1, 4.2, 5.1)**
- **5.D.3.1** The student is able to predict the velocity of the center of mass of a system when there is no interaction outside of the system but there is an interaction within the system (i.e., the student simply recognizes that interactions within a system do not affect the center of mass motion of the system and is able to determine that there is no external force). **(S.P. 6.4)**

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in **Impulse** and **Linear Momentum and Force**, where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils —conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in **Figure 8.6**. Both cars are coasting in the same direction when the lead car (labeled m_2) is bumped by the trailing car (labeled m_1). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.

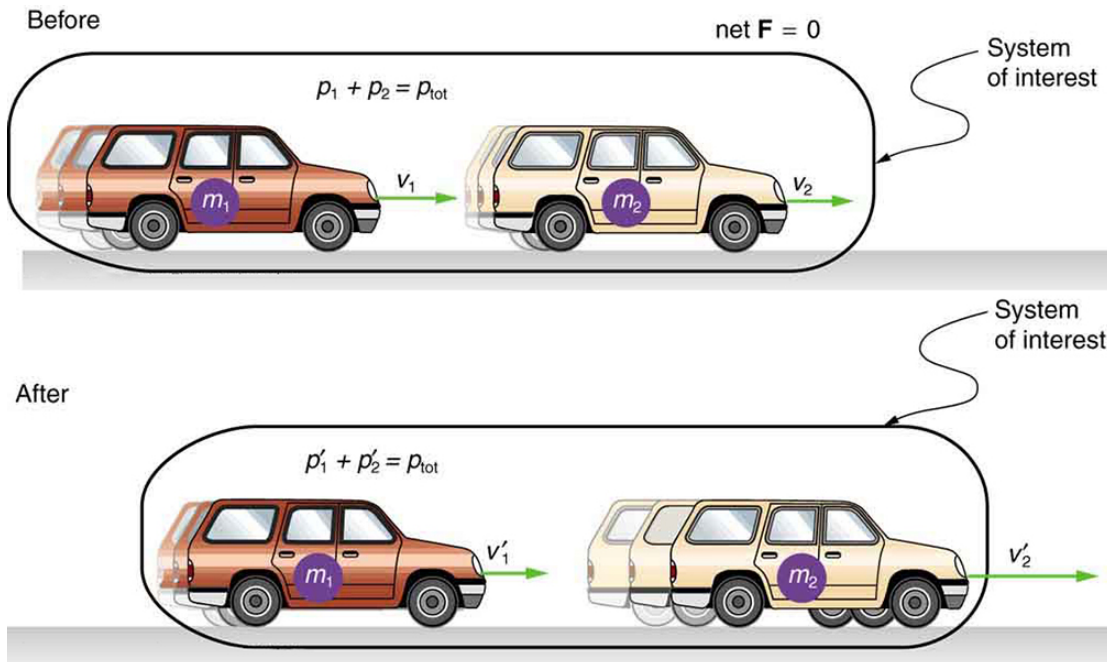


Figure 8.6 A car of mass m_1 moving with a velocity of v_1 bumps into another car of mass m_2 and velocity v_2 that it is following. As a result, the first car slows down to a velocity of v'_1 and the second speeds up to a velocity of v'_2. The momentum of each car is changed, but the total momentum p_{tot} of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

$$\Delta p_1 = F_1 \Delta t, \tag{8.32}$$

where F_1 is the force on car 1 due to car 2, and Δt is the time the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

$$\Delta p_2 = F_2 \Delta t, \tag{8.33}$$

where F_2 is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law that $F_2 = \ -F_1$, and so

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1. \tag{8.34}$$

Thus, the changes in momentum are equal and opposite, and

$$\Delta p_1 + \Delta p_2 = 0. \tag{8.35}$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$p_1 + p_2 = \text{constant}, \tag{8.36}$$

$$p_1 + p_2 = p'_1 + p'_2, \tag{8.37}$$

where p'_1 and p'_2 are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of objects in it. In equation form, the **conservation of momentum principle** for an isolated system is written

$$\mathbf{p}_{\text{tot}} = \text{constant}, \tag{8.38}$$

or

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}, \tag{8.39}$$

where $\mathbf{p}_{\text{tot}}$ is the total momentum (the sum of the momenta of the individual objects in the system) and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An **isolated system** is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.

Conservation of Momentum Principle

$$\begin{aligned} \mathbf{p}_{\text{tot}} &= \text{constant} \\ \mathbf{p}_{\text{tot}} &= \mathbf{p}'_{\text{tot}} \quad \text{(isolated system)} \end{aligned} \tag{8.40}$$

Isolated System

An isolated system is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.

Making Connections: Cart Collisions

Consider two air carts with equal mass *(m)* on a linear track. The first cart moves with a speed *v* towards the second cart, which is initially at rest. We will take the initial direction of motion of the first cart as the positive direction.

The momentum of the system will be conserved in the collision. If the collision is elastic, then the first cart will stop after the collision. Conservation of momentum therefore tells us that the second cart will have a final velocity *v* after the collision in the same direction as the initial velocity of the first cart.

The kinetic energy of the system will be conserved since the masses are equal and the final velocity of cart 2 is equal to the initial velocity of cart 1. What would a graph of total momentum vs. time look like in this case? What would a graph of total kinetic energy vs. time look like in this case?

Consider the center of mass of this system as the frame of reference. As cart 1 approaches cart 2, the center of mass remains exactly halfway between the two carts. The center of mass moves toward the stationary cart 2 at a speed $\frac{v}{2}$. After the collision, the center of mass continues moving in the same direction, away from (now stationary) cart 1 at a speed $\frac{v}{2}$.

How would a graph of center-of-mass velocity vs. time compare to a graph of momentum vs. time?

Suppose instead that the two carts move with equal speeds *v* in opposite directions towards the center of mass. Again, they have an elastic collision, so after the collision, they exchange velocities (each cart moving in the opposite direction of its initial motion with the same speed). As the two carts approach, the center of mass is exactly between the two carts, at the point where they will collide. In this case, how would a graph of center-of-mass velocity vs. time compare to a graph of the momentum of the system vs. time?

Let us return to the example where the first cart is moving with a speed *v* toward the second cart, initially at rest. Suppose the second cart has some putty on one end so that, when the collision occurs, the two carts stick together in an inelastic collision. In this case, conservation of momentum tells us that the final velocity of the two-cart system will be half the initial velocity of the first cart, in the same direction as the first cart's initial motion. Kinetic energy will not be conserved in this case, however. Compared to the moving cart before the collision, the overall moving mass after the collision is doubled, but the velocity is halved.

The initial kinetic energy of the system is:

$$k_i = \frac{1}{2}mv^2(\text{1st cart}){+}0(\text{2nd cart}){=}\frac{1}{2}mv^2 \tag{8.41}$$

The final kinetic energy of the two carts (2*m*) moving together (at speed *v*/2) is:

$$k_f = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2 \tag{8.42}$$

What would a graph of total momentum vs. time look like in this case? What would a graph of total kinetic energy vs. time look like in this case?

Consider the center of mass of this system. As cart 1 approaches cart 2, the center of mass remains exactly halfway between the two carts. The center of mass moves toward the stationary cart 2 at a speed $\frac{v}{2}$. After the collision, the two carts move together at a speed $\frac{v}{2}$. How would a graph of center-of-mass velocity vs. time compare to a graph of momentum vs. time?

Suppose instead that the two carts move with equal speeds *v* in opposite directions towards the center of mass. They have putty on the end of each cart so that they stick together after the collision. As the two carts approach, the center of mass is exactly between the two carts, at the point where they will collide. In this case, how would a graph of center-of-mass velocity vs. time compare to a graph of the momentum of the system vs. time?

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms of momentum, $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}_{\text{tot}}}{\Delta t}$. For an isolated system, $(\mathbf{F}_{\text{net}} = 0)$; thus, $\Delta \mathbf{p}_{\text{tot}} = 0$, and $\mathbf{p}_{\text{tot}}$ is constant.

We have noted that the three length dimensions in nature—x, y, and z—are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See **Figure 8.7**.) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.

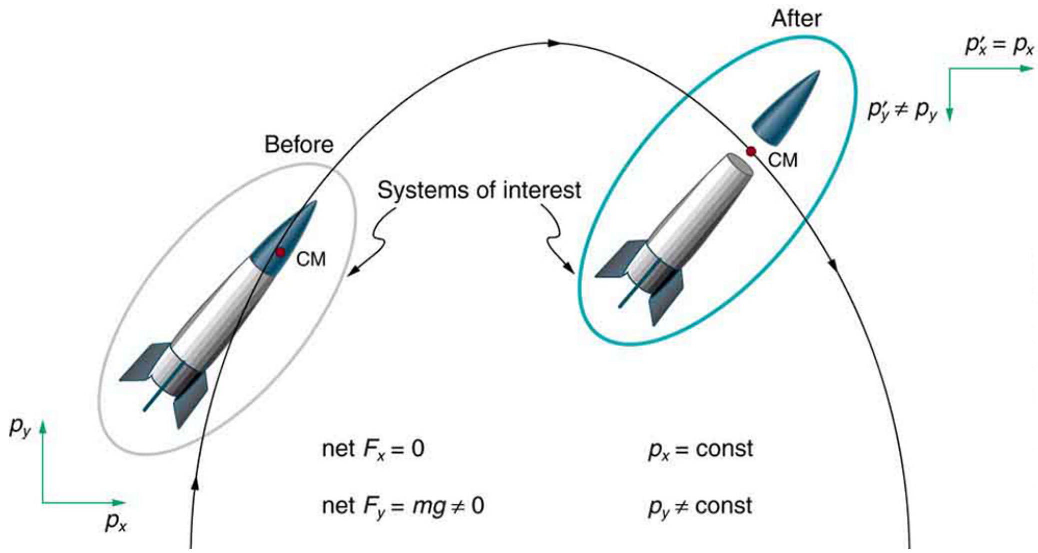


Figure 8.7 The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force $F_{x-\text{net}}$ is still zero. The vertical component of the momentum is not conserved, because the net vertical force $F_{y-\text{net}}$ is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

Making Connections: Take-Home Investigation—Drop of Tennis Ball and a Basketball

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

Making Connections: Take-Home Investigation—Two Tennis Balls in a Ballistic Trajectory

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

Applying Science Practices: Verifying the Conservation of Linear Momentum

Design an experiment to verify the conservation of linear momentum in a one-dimensional collision, both elastic and inelastic. For simplicity, try to ensure that friction is minimized so that it has a negligible effect on your experiment. As you consider your experiment, consider the following questions:

- Predict how the final momentum of the system will compare to the initial momentum of the system that you will measure. Justify your prediction.
- How will you measure the momentum of each object?
- Should you have two objects in motion or one object bouncing off a rigid surface?
- Should you verify the relationship mathematically or graphically?
- How will you estimate the uncertainty of your measurements? How will you express this uncertainty in your data?

When you have completed each experiment, compare the outcome to your prediction about the initial and final momentum of the system and evaluate your results.

Making Connections: Conservation of Momentum and Collision

Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

Subatomic Collisions and Momentum

The conservation of momentum principle not only applies to the macroscopic objects, it is also essential to our explorations of atomic and subatomic particles. Giant machines hurl subatomic particles at one another, and researchers evaluate the results by assuming conservation of momentum (among other things).

On the small scale, we find that particles and their properties are invisible to the naked eye but can be measured with our instruments, and models of these subatomic particles can be constructed to describe the results. Momentum is found to be a property of all subatomic particles including massless particles such as photons that compose light. Momentum being a property of particles hints that momentum may have an identity beyond the description of an object's mass multiplied by the object's velocity. Indeed, momentum relates to wave properties and plays a fundamental role in what measurements are taken and how we take these measurements. Furthermore, we find that the conservation of momentum principle is valid when considering systems of particles. We use this principle to analyze the masses and other properties of previously undetected particles, such as the nucleus of an atom and the existence of quarks that make up particles of nuclei. Figure 8.8 below illustrates how a particle scattering backward from another implies that its target is massive and dense. Experiments seeking evidence that **quarks** make up protons (one type of particle that makes up nuclei) scattered high-energy electrons off of protons (nuclei of hydrogen atoms). Electrons occasionally scattered straight backward in a manner that implied a very small and very dense particle makes up the proton—this observation is considered nearly direct evidence of quarks. The analysis was based partly on the same conservation of momentum principle that works so well on the large scale.

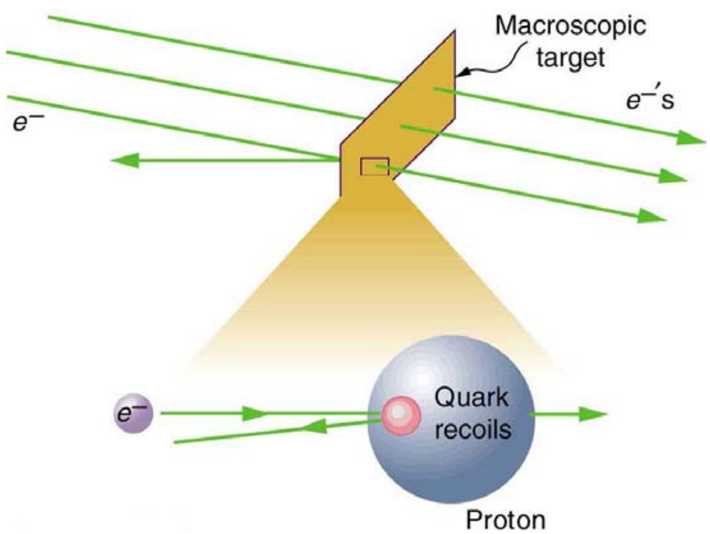


Figure 8.8 A subatomic particle scatters straight backward from a target particle. In experiments seeking evidence for quarks, electrons were observed to occasionally scatter straight backward from a proton.

8.4 Elastic Collisions in One Dimension

Learning Objectives

By the end of this section, you will be able to:

- Describe an elastic collision of two objects in one dimension.
- Define internal kinetic energy.
- Derive an expression for conservation of internal kinetic energy in a one-dimensional collision.
- Determine the final velocities in an elastic collision given masses and initial velocities.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.B.1.1** The student is able to calculate the change in linear momentum of a two-object system with constant mass in linear motion from a representation of the system (data, graphs, etc.). **(S.P. 1.4, 2.2)**
- **4.B.1.2** The student is able to analyze data to find the change in linear momentum for a constant-mass system using the product of the mass and the change in velocity of the center of mass. **(S.P. 5.1)**
- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.D.1.1** The student is able to make qualitative predictions about natural phenomena based on conservation of linear momentum and restoration of kinetic energy in elastic collisions. **(S.P. 6.4, 7.2)**
- **5.D.1.2** The student is able to apply the principles of conservation of momentum and restoration of kinetic energy to reconcile a situation that appears to be isolated and elastic, but in which data indicate that linear momentum and kinetic energy are not the same after the interaction, by refining a scientific question to identify interactions that have not been considered. Students will be expected to solve qualitatively and/or quantitatively for one-dimensional situations and only qualitatively in two-dimensional situations. **(S.P. 2.2, 3.2, 5.1, 5.3)**
- **5.D.1.5** The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. **(S.P. 2.1, 2.2)**
- **5.D.1.6** The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed. **(S.P. 6.4)**
- **5.D.1.7** The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. **(S.P. 2.1, 2.2)**
- **5.D.2.1** The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. **(S.P. 6.4, 7.2)**
- **5.D.2.2** The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically. **(S.P.4.1, 4.2, 5.1)**
- **5.D.3.2** The student is able to make predictions about the velocity of the center of mass for interactions within a defined one-dimensional system. **(S.P. 6.4)**

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves internal kinetic energy. **Internal kinetic energy** is the sum of the kinetic energies of the objects in the system. **Figure 8.9** illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

Elastic Collision

An **elastic collision** is one that conserves internal kinetic energy.

Internal Kinetic Energy

Internal kinetic energy is the sum of the kinetic energies of the objects in the system.

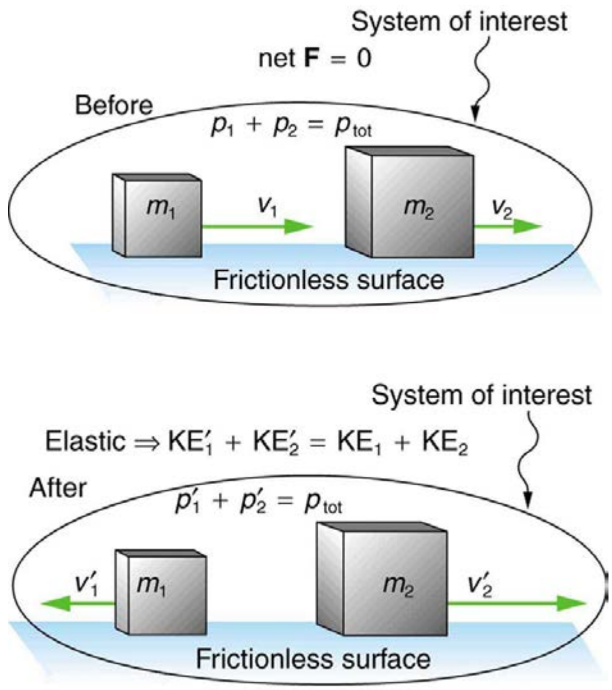


Figure 8.9 An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$p_1 + p_2 = p'_1 + p'_2 \quad (F_{\text{net}} = 0) \tag{8.43}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \quad (F_{\text{net}} = 0), \tag{8.44}$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

$$\frac{1}{2}m_1 v_1{}^2 + \frac{1}{2}m_2 v_2{}^2 = \frac{1}{2}m_1 v'_1{}^2 + \frac{1}{2}m_2 v'_2{}^2 \quad \text{(two-object elastic collision)} \tag{8.45}$$

expresses the equation for conservation of internal kinetic energy in a one-dimensional collision.

Making Connections: Collisions

Suppose data are collected on a collision between two masses sliding across a frictionless surface. Mass A (1.0 kg) moves with a velocity of +12 m/s, and mass B (2.0 kg) moves with a velocity of −12 m/s. The two masses collide and stick together after the collision. The table below shows the measured velocities of each mass at times before and after the collision:

Table 8.1

Time (s)	Velocity A (m/s)	Velocity B (m/s)
0	+12	−12
1.0 s	+12	−12
2.0 s	−4.0	−4.0
3.0 s	−4.0	−4.0

The total mass of the system is 3.0 kg. The velocity of the center of mass of this system can be determined from the conservation of momentum. Consider the system before the collision:

$$(m_A + m_B)v_{cm} = m_A v_A + m_B v_B \tag{8.46}$$

$$(3.0)v_{cm} = (1)(12) + (2)(-12) \tag{8.47}$$

$$v_{cm} = -4.0 \text{ m/s} \tag{8.48}$$

After the collision, the center-of-mass velocity is the same:

$$(m_A + m_B)v_{cm} = (m_A + m_B)v_{final} \tag{8.49}$$

$$(3.0)v_{cm} = (3)(-4.0) \tag{8.50}$$

$$v_{cm} = -4.0 \text{ m/s} \tag{8.51}$$

The total momentum of the system before the collision is:

$$m_A v_A + m_B v_B = (1)(12) + (2)(-12) = -12 \text{ kg} \bullet \text{m/s} \tag{8.52}$$

The total momentum of the system after the collision is:

$$(m_A + m_B)v_{final} = (3)(-4) = -12 \text{ kg} \bullet \text{m/s} \tag{8.53}$$

Thus, the change in momentum of the system is zero when measured this way. We get a similar result when we calculate the momentum using the center-of-mass velocity. Since the center-of-mass velocity is the same both before and after the collision, we calculate the same momentum for the system using this method both before and after the collision.

Example 8.4 Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

$$m_1 = 0.500 \text{ kg},\ m_2 = 3.50 \text{ kg},\ v_1 = 4.00 \text{ m/s, and } v_2 = 0. \tag{8.54}$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in **Figure 8.9** where both objects are initially moving. We are asked to find two unknowns (the final velocities v'_1 and v'_2). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2 = 0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p'_1 + p'_2 \tag{8.55}$$

or

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2. \tag{8.56}$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\frac{1}{2}m_1 v_1{}^2 = \frac{1}{2}m_1 v'_1{}^2 + \frac{1}{2}m_2 v'_2{}^2. \tag{8.57}$$

Solving the first equation (momentum equation) for v'_2, we obtain

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1). \tag{8.58}$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable v'_2, leaving only v'_1 as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v'_1 = 4.00 \text{ m/s} \tag{8.59}$$

and

$$v'_1 = -3.00 \text{ m/s}. \tag{8.60}$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution $(v'_1 = -3.00 \text{ m/s})$ is negative, meaning that the first object bounces backward. When this negative value of v'_1 is used to find the velocity of the second object after the collision, we get

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1) = \frac{0.500\text{ kg}}{3.50\text{ kg}}[4.00 - (-3.00)]\text{ m/s} \tag{8.61}$$

or

$$v'_2 = 1.00\text{ m/s}. \tag{8.62}$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

Making Connections: Take-Home Investigation—Ice Cubes and Elastic Collision

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

PhET Explorations: Collision Lab

Investigate collisions on an air hockey table. Set up your own experiments: vary the number of discs, masses and initial conditions. Is momentum conserved? Is kinetic energy conserved? Vary the elasticity and see what happens.

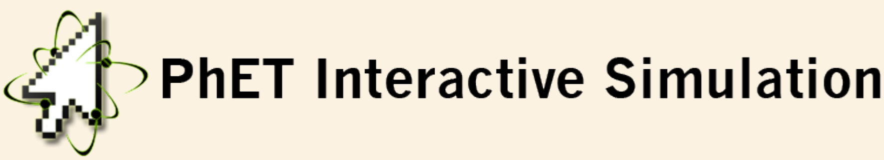


Figure 8.10 Collision Lab (http://cnx.org/content/m55171/1.3/collision-lab_en.jar)

8.5 Inelastic Collisions in One Dimension

Learning Objectives

By the end of this section, you will be able to:

- Define inelastic collision.
- Explain perfectly inelastic collisions.
- Apply an understanding of collisions to sports.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.B.1.1** The student is able to calculate the change in linear momentum of a two-object system with constant mass in linear motion from a representation of the system (data, graphs, etc.). **(S.P. 1.4, 2.2)**
- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.D.1.3** The student is able to apply mathematical routines appropriately to problems involving elastic collisions in one dimension and justify the selection of those mathematical routines based on conservation of momentum and restoration of kinetic energy. **(S.P. 2.1, 2.2)**
- **5.D.1.5** The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. **(S.P. 2.1, 2.2)**
- **5.D.2.1** The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. **(S.P. 6.4, 7.2)**
- **5.D.2.2** The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically. **(S.P.4.1, 4.2, 5.1)**
- **5.D.2.3** The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy. **(S.P. 6.4, 7.2)**

- **5.D.2.4** The student is able to analyze data that verify conservation of momentum in collisions with and without an external friction force. **(S.P. 4.1, 4.2, 4.4, 5.1, 5.3)**
- **5.D.2.5** The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum as the appropriate solution method for an inelastic collision, recognize that there is a common final velocity for the colliding objects in the totally inelastic case, solve for missing variables, and calculate their values. **(S.P. 2.1 2.2)**
- **5.D.2.6** The student is able to apply the conservation of linear momentum to an isolated system of objects involved in an inelastic collision to predict the change in kinetic energy. **(S.P. 6.4, 7.2)**

We have seen that in an elastic collision, internal kinetic energy is conserved. An **inelastic collision** is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

Inelastic Collision

An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

Figure 8.11 shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$. The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a **perfectly inelastic collision** because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

Perfectly Inelastic Collision

A collision in which the objects stick together is sometimes called "perfectly inelastic."

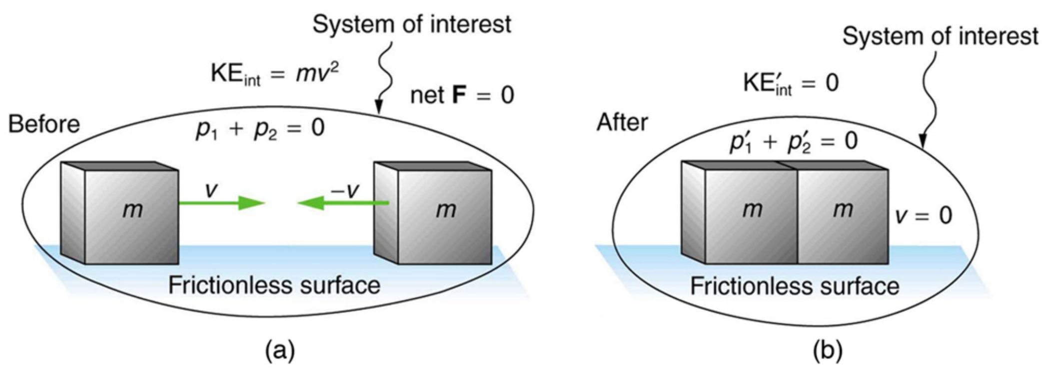


Figure 8.11 An inelastic one-dimensional two-object collision. Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.

Example 8.5 Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See **Figure 8.12**)

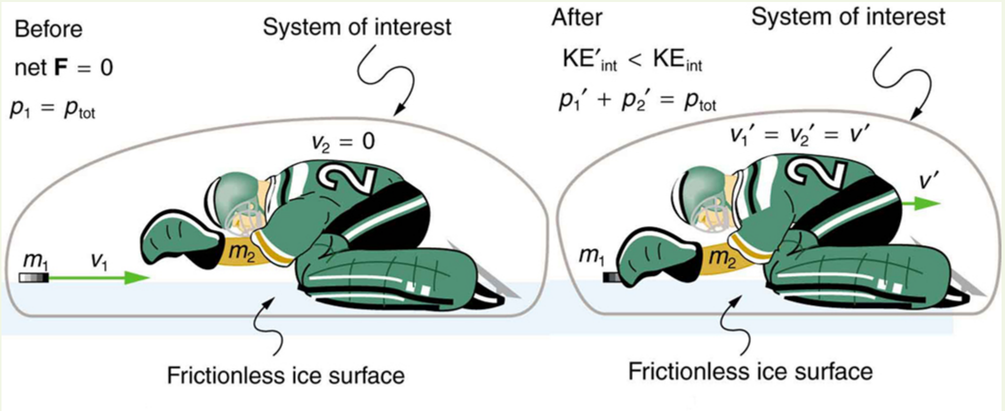


Figure 8.12 An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p'_1 + p'_2 \tag{8.63}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \tag{8.64}$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v'_1 = v'_2 = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1 v_1 = (m_1 + m_2)v'. \tag{8.65}$$

Solving for v' yields

$$v' = \frac{m_1}{m_1 + m_2} v_1. \tag{8.66}$$

Entering known values in this equation, we get

$$v' = \left(\frac{0.150 \text{ kg}}{70.0 \text{ kg} + 0.150 \text{ kg}}\right)(35.0 \text{ m/s}) = 7.48\times 10^{-2} \text{ m/s}. \tag{8.67}$$

Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

Solution for (b)

Before the collision, the internal kinetic energy KE_{int} of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, KE_{int} is initially

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(35.0 \text{ m/s})^2 \\ &= 91.9 \text{ J}. \end{aligned} \tag{8.68}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2}(m + M)v^2 = \frac{1}{2}(70.15 \text{ kg})\left(7.48\times 10^{-2} \text{ m/s}\right)^2 \\ &= 0.196 \text{ J}. \end{aligned} \tag{8.69}$$

The change in internal kinetic energy is thus

$$\begin{aligned}\mathrm{KE'}_{\mathrm{int}} - \mathrm{KE}_{\mathrm{int}} &= 0.196\ \mathrm{J} - 91.9\ \mathrm{J} \\ &= -91.7\ \mathrm{J}\end{aligned} \tag{8.70}$$

where the minus sign indicates that the energy was lost.

Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision. $\mathrm{KE}_{\mathrm{int}}$ is mostly converted to thermal energy and sound.

During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. **Figure 8.13** shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. **Example 8.6** deals with data from such a collision.

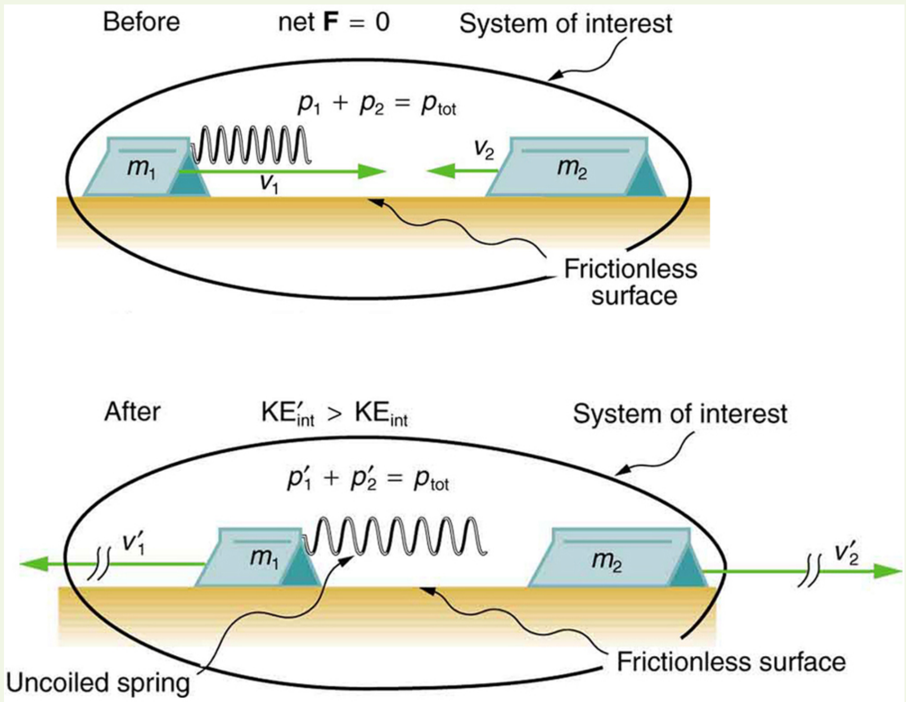


Figure 8.13 An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in **Example 8.6**, the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the "sweet spot" on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

Take-Home Experiment—Bouncing of Tennis Ball

1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure how high the ball bounces and observe what happens to your friend's hand during the collision. Explain your observations and measurements.
2. The coefficient of restitution (c) is a measure of the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a c of 1. For a ball bouncing off the floor (or a racquet on the floor), c can be shown to be $c = (h/H)^{1/2}$ where h is the height to which the ball bounces and H is the height from which the ball is dropped. Determine c for the cases in Part 1 and

for the case of a tennis ball bouncing off a concrete or wooden floor ($c = 0.85$ for new tennis balls used on a tennis court).

Example 8.6 Calculating Final Velocity and Energy Release: Two Carts Collide

In the collision pictured in **Figure 8.13**, two carts collide inelastically. Cart 1 (denoted m_1 carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s. Cart 2 (denoted m_2 in **Figure 8.13**) has a mass of 0.500 kg and an initial velocity of -0.500 m/s. After the collision, cart 1 is observed to recoil with a velocity of -4.00 m/s. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

Strategy

We can use conservation of momentum to find the final velocity of cart 2, because $F_{\text{net}} = 0$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \tag{8.71}$$

The only unknown in this equation is v'_2. Solving for v'_2 and substituting known values into the previous equation yields

$$\begin{aligned} v'_2 &= \frac{m_1 v_1 + m_2 v_2 - m_1 v'_1}{m_2} \\ &= \frac{(0.350\text{ kg})(2.00\text{ m/s}) + (0.500\text{ kg})(-0.500\text{ m/s})}{0.500\text{ kg}} - \frac{(0.350\text{ kg})(-4.00\text{ m/s})}{0.500\text{ kg}} \\ &= 3.70\text{ m/s}. \end{aligned} \tag{8.72}$$

Solution for (b)

The internal kinetic energy before the collision is

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \\ &= \frac{1}{2}(0.350\text{ kg})(2.00\text{ m/s})^2 + \frac{1}{2}(0.500\text{ kg})(-0.500\text{ m/s})^2 \\ &= 0.763\text{ J}. \end{aligned} \tag{8.73}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2}m_1 v'^2_1 + \frac{1}{2}m_2 v'^2_2 \\ &= \frac{1}{2}(0.350\text{ kg})(\text{-}4.00\text{ m/s})^2 + \frac{1}{2}(0.500\text{ kg})(3.70\text{ m/s})^2 \\ &= 6.22\text{ J}. \end{aligned} \tag{8.74}$$

The change in internal kinetic energy is thus

$$\begin{aligned} \text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 6.22\text{ J} - 0.763\text{ J} \\ &= 5.46\text{ J}. \end{aligned} \tag{8.75}$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

8.6 Collisions of Point Masses in Two Dimensions

Learning Objectives

By the end of this section, you will be able to:

- Discuss two-dimensional collisions as an extension of one-dimensional analysis.

- Define point masses.
- Derive an expression for conservation of momentum along the *x*-axis and *y*-axis.
- Describe elastic collisions of two objects with equal mass.
- Determine the magnitude and direction of the final velocity given initial velocity and scattering angle.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.D.1.2** The student is able to apply the principles of conservation of momentum and restoration of kinetic energy to reconcile a situation that appears to be isolated and elastic, but in which data indicate that linear momentum and kinetic energy are not the same after the interaction, by refining a scientific question to identify interactions that have not been considered. Students will be expected to solve qualitatively and/or quantitatively for one-dimensional situations and only qualitatively in two-dimensional situations.
- **5.D.3.3** The student is able to make predictions about the velocity of the center of mass for interactions within a defined two-dimensional system.

In the previous two sections, we considered only one-dimensional collisions; during such collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and we shall see that their study is an extension of the one-dimensional analysis already presented. The approach taken (similar to the approach in discussing two-dimensional kinematics and dynamics) is to choose a convenient coordinate system and resolve the motion into components along perpendicular axes. Resolving the motion yields a pair of one-dimensional problems to be solved simultaneously.

One complication arising in two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass by one another, they will spin in circles. We will not consider such rotation until later, and so for now we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of **point masses**—that is, structureless particles that cannot rotate or spin.

We start by assuming that $\mathbf{F}_{\text{net}} = 0$, so that momentum $\mathbf{p}$ is conserved. The simplest collision is one in which one of the particles is initially at rest. (See **Figure 8.14**.) The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in **Figure 8.14**. Because momentum is conserved, the components of momentum along the x- and y-axes $(p_x \text{ and } p_y)$ will also be conserved, but with the chosen coordinate system, p_y is initially zero and p_x is the momentum of the incoming particle. Both facts simplify the analysis. (Even with the simplifying assumptions of point masses, one particle initially at rest, and a convenient coordinate system, we still gain new insights into nature from the analysis of two-dimensional collisions.)

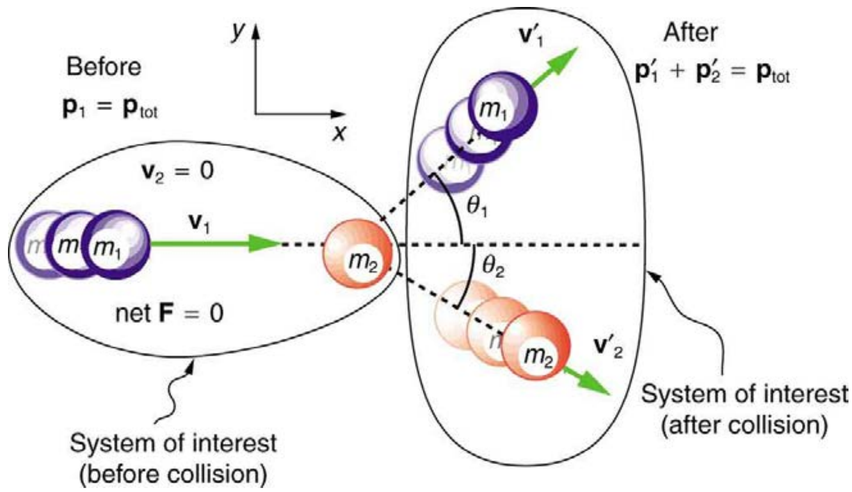


Figure 8.14 A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and v_1 is parallel to the x-axis. This coordinate system is sometimes called the laboratory coordinate system, because many scattering experiments have a target that is stationary in the laboratory, while particles are scattered from it to determine the particles that make-up the target and how they are bound together. The particles may not be observed directly, but their initial and final velocities are.

Along the x-axis, the equation for conservation of momentum is

$$p_{1x} + p_{2x} = p'_{1x} + p'_{2x}. \tag{8.76}$$

Where the subscripts denote the particles and axes and the primes denote the situation after the collision. In terms of masses and velocities, this equation is

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}. \tag{8.77}$$

But because particle 2 is initially at rest, this equation becomes

$$m_1 v_{1x} = m_1 v'_{1x} + m_2 v'_{2x}. \tag{8.78}$$

The components of the velocities along the x-axis have the form $v\cos\theta$. Because particle 1 initially moves along the x-axis, we find $v_{1x} = v_1$.

Conservation of momentum along the x-axis gives the following equation:

$$m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2, \tag{8.79}$$

where θ_1 and θ_2 are as shown in **Figure 8.14**.

Conservation of Momentum along the x-axis

$$m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2 \tag{8.80}$$

Along the y-axis, the equation for conservation of momentum is

$$p_{1y} + p_{2y} = p'_{1y} + p'_{2y} \tag{8.81}$$

or

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}. \tag{8.82}$$

But v_{1y} is zero, because particle 1 initially moves along the x-axis. Because particle 2 is initially at rest, v_{2y} is also zero. The equation for conservation of momentum along the y-axis becomes

$$0 = m_1 v'_{1y} + m_2 v'_{2y}. \tag{8.83}$$

The components of the velocities along the y-axis have the form $v\sin\theta$.

Thus, conservation of momentum along the y-axis gives the following equation:

$$0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2. \tag{8.84}$$

Conservation of Momentum along the y-axis

$$0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2 \tag{8.85}$$

The equations of conservation of momentum along the x-axis and y-axis are very useful in analyzing two-dimensional collisions of particles, where one is originally stationary (a common laboratory situation). But two equations can only be used to find two unknowns, and so other data may be necessary when collision experiments are used to explore nature at the subatomic level.

Making Connections: Real World Connections

We have seen, in one-dimensional collisions when momentum is conserved, that the center-of-mass velocity of the system remains unchanged as a result of the collision. If you calculate the momentum and center-of-mass velocity before the collision, you will get the same answer as if you calculate both quantities after the collision. This logic also works for two-dimensional collisions.

For example, consider two cars of equal mass. Car A is driving east (+*x*-direction) with a speed of 40 m/s. Car B is driving north (+*y*-direction) with a speed of 80 m/s. What is the velocity of the center-of-mass of this system before and after an inelastic collision, in which the cars move together as one mass after the collision?

Since both cars have equal mass, the center-of-mass velocity components are just the average of the components of the individual velocities before the collision. The *x*-component of the center of mass velocity is 20 m/s, and the y-component is 40 m/s.

Using momentum conservation for the collision in both the *x*-component and *y*-component yields similar answers:

$$m(40) + m(0) = (2m)v_{\text{final}}(x) \tag{8.86}$$

$$v_{\text{final}}(x) = 20\text{ m/s} \tag{8.87}$$

$$m(0) + m(80) = (2m)v_{\text{final}}(y) \tag{8.88}$$

$$v_{\text{final}}(y) = 40\text{ m/s} \tag{8.89}$$

Since the two masses move together after the collision, the velocity of this combined object is equal to the center-of-mass velocity. Thus, the center-of-mass velocity before and after the collision is identical, even in two-dimensional collisions, when momentum is conserved.

Example 8.7 Determining the Final Velocity of an Unseen Object from the Scattering of Another Object

Suppose the following experiment is performed. A 0.250-kg object (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object with mass of 0.400 kg (m_2). The 0.250-kg object emerges from the room at an angle of 45.0° with its incoming direction.

The speed of the 0.250-kg object is originally 2.00 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity $(v'_2$ and $\theta_2)$ of the 0.400-kg object after the collision.

Strategy

Momentum is conserved because the surface is frictionless. The coordinate system shown in **Figure 8.15** is one in which m_2 is originally at rest and the initial velocity is parallel to the x-axis, so that conservation of momentum along the x- and y-axes is applicable.

Everything is known in these equations except v'_2 and θ_2, which are precisely the quantities we wish to find. We can find two unknowns because we have two independent equations: the equations describing the conservation of momentum in the x- and y-directions.

Solution

Solving $m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$ for $v'_2 \cos\theta_2$ and $0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2$ for $v'_2 \sin\theta_2$ and taking the ratio yields an equation (in which θ2 is the only unknown quantity. Applying the identity $\left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$, we obtain:

$$\tan\theta_2 = \frac{v'_1 \sin\theta_1}{v'_1 \cos\theta_1 - v_1}. \tag{8.90}$$

Entering known values into the previous equation gives

$$\tan\theta_2 = \frac{(1.50\text{ m/s})(0.7071)}{(1.50\text{ m/s})(0.7071) - 2.00\text{ m/s}} = -1.129. \tag{8.91}$$

Thus,

$$\theta_2 = \tan^{-1}(-1.129) = 311.5^\circ \approx 312^\circ. \tag{8.92}$$

Angles are defined as positive in the counter clockwise direction, so this angle indicates that m_2 is scattered to the right in **Figure 8.15**, as expected (this angle is in the fourth quadrant). Either equation for the x- or y-axis can now be used to solve for v'_2, but the latter equation is easiest because it has fewer terms.

$$v'_2 = -\frac{m_1}{m_2} v'_1 \frac{\sin\theta_1}{\sin\theta_2} \tag{8.93}$$

Entering known values into this equation gives

$$v'_2 = -\left(\frac{0.250\text{ kg}}{0.400\text{ kg}}\right)(1.50\text{ m/s})\left(\frac{0.7071}{-0.7485}\right). \tag{8.94}$$

Thus,

$$v'_2 = 0.886\text{ m/s}. \tag{8.95}$$

Discussion

It is instructive to calculate the internal kinetic energy of this two-object system before and after the collision. (This calculation is left as an end-of-chapter problem.) If you do this calculation, you will find that the internal kinetic energy is less after the collision, and so the collision is inelastic. This type of result makes a physicist want to explore the system further.

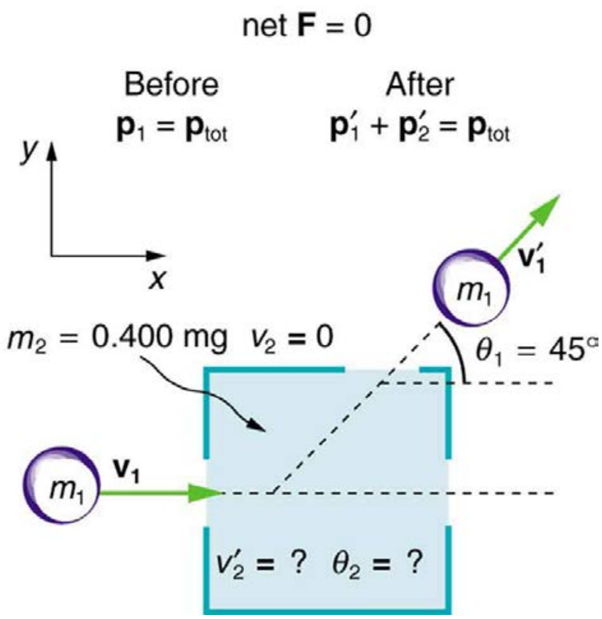


Figure 8.15 A collision taking place in a dark room is explored in **Example 8.7**. The incoming object m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Elastic Collisions of Two Objects with Equal Mass

Some interesting situations arise when the two colliding objects have equal mass and the collision is elastic. This situation is nearly the case with colliding billiard balls, and precisely the case with some subatomic particle collisions. We can thus get a mental image of a collision of subatomic particles by thinking about billiards (or pool). (Refer to **Figure 8.14** for masses and angles.) First, an elastic collision conserves internal kinetic energy. Again, let us assume object 2 (m_2) is initially at rest. Then, the internal kinetic energy before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1{}^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2. \tag{8.96}$$

Because the masses are equal, $m_1 = m_2 = m$. Algebraic manipulation (left to the reader) of conservation of momentum in the x- and y-directions can show that

$$\frac{1}{2}mv_1{}^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 + mv'_1 v'_2 \cos(\theta_1 - \theta_2). \tag{8.97}$$

(Remember that θ_2 is negative here.) The two preceding equations can both be true only if

$$mv'_1 v'_2 \cos(\theta_1 - \theta_2) = 0. \tag{8.98}$$

There are three ways that this term can be zero. They are

- $v'_1 = 0$: head-on collision; incoming ball stops
- $v'_2 = 0$: no collision; incoming ball continues unaffected
- $\cos(\theta_1 - \theta_2) = 0$: angle of separation $(\theta_1 - \theta_2)$ is 90° after the collision

All three of these ways are familiar occurrences in billiards and pool, although most of us try to avoid the second. If you play enough pool, you will notice that the angle between the balls is very close to 90° after the collision, although it will vary from this value if a great deal of spin is placed on the ball. (Large spin carries in extra energy and a quantity called *angular momentum*, which must also be conserved.) The assumption that the scattering of billiard balls is elastic is reasonable based on the correctness of the three results it produces. This assumption also implies that, to a good approximation, momentum is conserved for the two-ball system in billiards and pool. The problems below explore these and other characteristics of two-dimensional collisions.

Connections to Nuclear and Particle Physics

Two-dimensional collision experiments have revealed much of what we know about subatomic particles, as we shall see in **Medical Applications of Nuclear Physics** and **Particle Physics**. Ernest Rutherford, for example, discovered the nature of the atomic nucleus from such experiments.

8.7 Introduction to Rocket Propulsion

Learning Objectives

By the end of this section, you will be able to:

- State Newton's third law of motion.
- Explain the principle involved in propulsion of rockets and jet engines.
- Derive an expression for the acceleration of the rocket.
- Discuss the factors that affect the rocket's acceleration.
- Describe the function of a space shuttle.

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle—Newton's third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun's recoil or kick.

Making Connections: Take-Home Experiment—Propulsion of a Balloon

Hold a balloon and fill it with air. Then, let the balloon go. In which direction does the air come out of the balloon and in which direction does the balloon get propelled? If you fill the balloon with water and then let the balloon go, does the balloon's direction change? Explain your answer.

Figure 8.16 shows a rocket accelerating straight up. In part (a), the rocket has a mass m and a velocity v relative to Earth, and hence a momentum mv . In part (b), a time Δt has elapsed in which the rocket has ejected a mass Δm of hot gas at a velocity v_e relative to the rocket. The remainder of the mass $(m - \Delta m)$ now has a greater velocity $(v + \Delta v)$. The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time Δt , producing a negative impulse $\Delta p = -mg\Delta t$. (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum of the system.) So, the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket's thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.

By calculating the change in momentum for the entire system over Δt , and equating this change to the impulse, the following expression can be shown to be a good approximation for the acceleration of the rocket.

$$a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g \tag{8.99}$$

"The rocket" is that part of the system remaining after the gas is ejected, and g is the acceleration due to gravity.

Acceleration of a Rocket

Acceleration of a rocket is

$$a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g, \tag{8.100}$$

where a is the acceleration of the rocket, v_e is the escape velocity, m is the mass of the rocket, Δm is the mass of the ejected gas, and Δt is the time in which the gas is ejected.

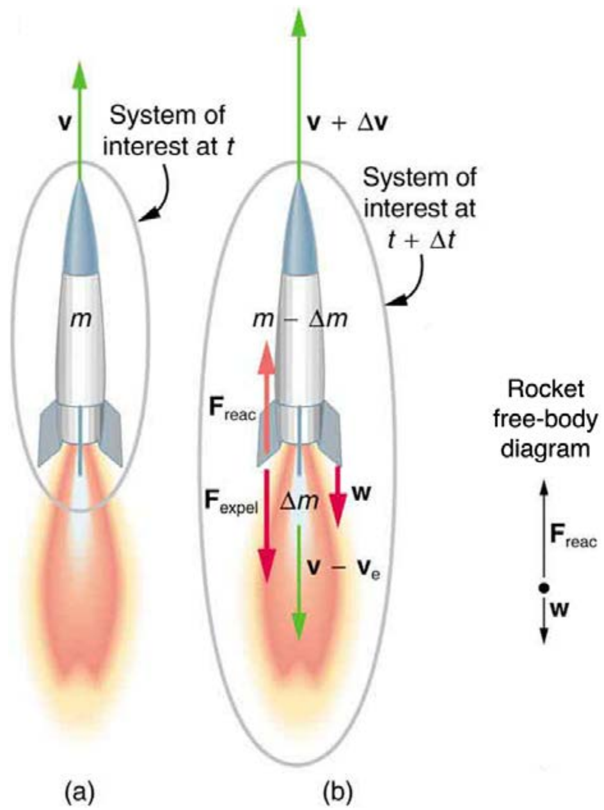


Figure 8.16 (a) This rocket has a mass m and an upward velocity v. The net external force on the system is $-mg$, if air resistance is neglected. (b) A time Δt later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket **.** First, the greater the exhaust velocity of the gases relative to the rocket, v_e, the greater the acceleration is. The practical limit for v_e is about $2.5\times10^3 \text{ m/s}$ for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor $\Delta m / \Delta t$ in the equation. The quantity $(\Delta m / \Delta t)v_e$, with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass m of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass m decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

Factors Affecting a Rocket's Acceleration

- The greater the exhaust velocity v_e of the gases relative to the rocket, the greater the acceleration.
- The faster the rocket burns its fuel, the greater its acceleration.
- The smaller the rocket's mass (all other factors being the same), the greater the acceleration.

Example 8.8 Calculating Acceleration: Initial Acceleration of a Moon Launch

A Saturn V's mass at liftoff was $2.80\times10^6 \text{ kg}$, its fuel-burn rate was $1.40\times10^4 \text{ kg/s}$, and the exhaust velocity was $2.40\times10^3 \text{ m/s}$. Calculate its initial acceleration.

Strategy

This problem is a straightforward application of the expression for acceleration because a is the unknown and all of the terms on the right side of the equation are given.

Solution

Substituting the given values into the equation for acceleration yields

$$\begin{aligned} a &= \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g \\ &= \frac{2.40\times10^3 \text{ m/s}}{2.80\times10^6 \text{ kg}}\left(1.40\times10^4 \text{ kg/s}\right) - 9.80 \text{ m/s}^2 \\ &= 2.20 \text{ m/s}^2. \end{aligned} \tag{8.101}$$

Discussion

This value is fairly small, even for an initial acceleration. The acceleration does increase steadily as the rocket burns fuel, because m decreases while v_e and $\frac{\Delta m}{\Delta t}$ remain constant. Knowing this acceleration and the mass of the rocket, you can show that the thrust of the engines was 3.36×10^7 N.

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$$v = v_e \ln\frac{m_0}{m_r}, \tag{8.102}$$

where $\ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass of the rocket (m_0) to what is left (m_r) after all of the fuel is exhausted. (Note that v is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.) For example, let us calculate the mass ratio needed to escape Earth's gravity starting from rest, given that the escape velocity from Earth is about 11.2×10^3 m/s, and assuming an exhaust velocity $v_e = 2.5\times10^3$ m/s.

$$\ln\frac{m_0}{m_r} = \frac{v}{v_e} = \frac{11.2\times10^3 \text{ m/s}}{2.5\times10^3 \text{ m/s}} = 4.48 \tag{8.103}$$

Solving for m_0/m_r gives

$$\frac{m_0}{m_r} = e^{4.48} = 88. \tag{8.104}$$

Thus, the mass of the rocket is

$$m_r = \frac{m_0}{88}. \tag{8.105}$$

This result means that only $1/88$ of the mass is left when the fuel is burnt, and $87/88$ of the initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%. Taking air resistance and gravitational force into account, the mass m_r remaining can only be about $m_0/180$. It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favorable, too.

The space shuttle was an attempt at an economical vehicle with some reusable parts, such as the solid fuel boosters and the craft itself. (See **Figure 8.17**) The shuttle's need to be operated by humans, however, made it at least as costly for launching satellites as expendable, unmanned rockets. Ideally, the shuttle would only have been used when human activities were required for the success of a mission, such as the repair of the Hubble space telescope. Rockets with satellites can also be launched from airplanes. Using airplanes has the double advantage that the initial velocity is significantly above zero and a rocket can avoid most of the atmosphere's resistance.

Figure 8.17 The space shuttle had a number of reusable parts. Solid fuel boosters on either side were recovered and refueled after each flight, and the entire orbiter returned to Earth for use in subsequent flights. The large liquid fuel tank was expended. The space shuttle was a complex assemblage of technologies, employing both solid and liquid fuel and pioneering ceramic tiles as reentry heat shields. As a result, it permitted multiple launches as opposed to single-use rockets. (credit: NASA)

PhET Explorations: Lunar Lander

Can you avoid the boulder field and land safely, just before your fuel runs out, as Neil Armstrong did in 1969? Our version of this classic video game accurately simulates the real motion of the lunar lander with the correct mass, thrust, fuel consumption rate, and lunar gravity. The real lunar lander is very hard to control.

Figure 8.18 Lunar Lander (http://cnx.org/content/m55174/1.2/lunar-lander_en.jar)

Glossary

change in momentum: the difference between the final and initial momentum; the mass times the change in velocity

conservation of momentum principle: when the net external force is zero, the total momentum of the system is conserved or constant

elastic collision: a collision that also conserves internal kinetic energy

impulse: the average net external force times the time it acts; equal to the change in momentum

inelastic collision: a collision in which internal kinetic energy is not conserved

internal kinetic energy: the sum of the kinetic energies of the objects in a system

isolated system: a system in which the net external force is zero

linear momentum: the product of mass and velocity

perfectly inelastic collision: a collision in which the colliding objects stick together

point masses: structureless particles with no rotation or spin

quark: fundamental constituent of matter and an elementary particle

second law of motion: physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

Section Summary

8.1 Linear Momentum and Force

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum $\mathbf{p}$ is defined to be

$$\mathbf{p} = m\mathbf{v},$$

where m is the mass of the system and $\mathbf{v}$ is its velocity.
- The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t},$$

$\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change time.

8.2 Impulse

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t.$$

- Forces are usually not constant over a period of time.

8.3 Conservation of Momentum

- The conservation of momentum principle is written

$$\mathbf{p}_{\text{tot}} = \text{constant}$$

or

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}} \quad \text{(isolated system)},$$

$\mathbf{p}_{\text{tot}}$ is the initial total momentum and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later.
- An isolated system is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.
- Conservation of momentum applies only when the net external force is zero.
- The conservation of momentum principle is valid when considering systems of particles.

8.4 Elastic Collisions in One Dimension

- An elastic collision is one that conserves internal kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

8.5 Inelastic Collisions in One Dimension

- An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces internal kinetic energy more than does any other type of inelastic collision.
- Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

8.6 Collisions of Point Masses in Two Dimensions

- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the x-axis parallel to the velocity of the incoming particle.
- Two-dimensional collisions of point masses where mass 2 is initially at rest conserve momentum along the initial direction of mass 1 (the x-axis), stated by $m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2$ and along the direction perpendicular to the initial direction (the y-axis) stated by $0 = m_1 v'_{1y} + m_2 v'_{2y}$.
- The internal kinetic before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv'^2_1 + \frac{1}{2}mv'^2_2 + mv'_1 v'_2 \cos(\theta_1 - \theta_2).$$

- Point masses are structureless particles that cannot spin.

8.7 Introduction to Rocket Propulsion

- Newton's third law of motion states that to every action, there is an equal and opposite reaction.
- Acceleration of a rocket is $a = \frac{v_\mathrm{e}}{m}\frac{\Delta m}{\Delta t} - g$.
- A rocket's acceleration depends on three main factors. They are
  1. The greater the exhaust velocity of the gases, the greater the acceleration.
  2. The faster the rocket burns its fuel, the greater its acceleration.
  3. The smaller the rocket's mass, the greater the acceleration.

Conceptual Questions

8.1 Linear Momentum and Force

1. An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

2. An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

3. Professional Application

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

4. How can a small force impart the same momentum to an object as a large force?

8.2 Impulse

5. Professional Application

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

6. While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

7. Professional Application

Tennis racquets have "sweet spots." If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

8.3 Conservation of Momentum

8. Professional Application

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

9. Under what circumstances is momentum conserved?

10. Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

11. Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

12. Professional Application

Explain in terms of momentum and Newton's laws how a car's air resistance is due in part to the fact that it pushes air in its direction of motion.

13. Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

14. Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

8.4 Elastic Collisions in One Dimension

15. What is an elastic collision?

8.5 Inelastic Collisions in One Dimension

16. What is an inelastic collision? What is a perfectly inelastic collision?

17. Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

18. A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

8.6 Collisions of Point Masses in Two Dimensions

19. Figure 8.19 shows a cube at rest and a small object heading toward it. (a) Describe the directions (angle θ_1) at which the small object can emerge after colliding elastically with the cube. How does θ_1 depend on b , the so-called impact parameter? Ignore any effects that might be due to rotation after the collision, and assume that the cube is much more massive than the small object. (b) Answer the same questions if the small object instead collides with a massive sphere.

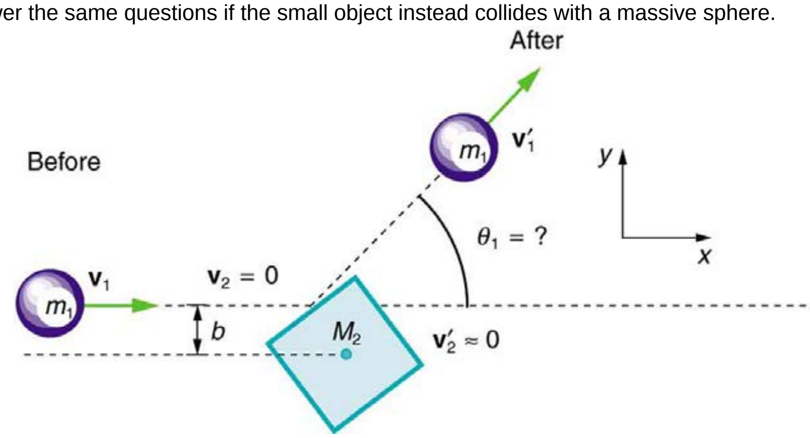


Figure 8.19 A small object approaches a collision with a much more massive cube, after which its velocity has the direction θ_1 . The angles at which the small object can be scattered are determined by the shape of the object it strikes and the impact parameter b .

8.7 Introduction to Rocket Propulsion

20. Professional Application

Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How is the motion of the center of mass affected by the explosion? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?

21. Professional Application

During a visit to the International Space Station, an astronaut was positioned motionless in the center of the station, out of reach of any solid object on which he could exert a force. Suggest a method by which he could move himself away from this position, and explain the physics involved.

22. Professional Application

It is possible for the velocity of a rocket to be greater than the exhaust velocity of the gases it ejects. When that is the case, the gas velocity and gas momentum are in the same direction as that of the rocket. How is the rocket still able to obtain thrust by ejecting the gases?

Problems & Exercises

8.1 Linear Momentum and Force

1. (a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s. (c) What is the momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant?

2. (a) What is the mass of a large ship that has a momentum of $1.60\times10^{9}\text{ kg}\cdot\text{m/s}$, when the ship is moving at a speed of 48.0 km/h? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of 1200 m/s.

3. (a) At what speed would a $2.00\times10^{4}\text{-kg}$ airplane have to fly to have a momentum of $1.60\times10^{9}\text{ kg}\cdot\text{m/s}$ (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of 60.0 m/s? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

4. (a) What is the momentum of a garbage truck that is $1.20\times10^{4}\text{ kg}$ and is moving at 10.0 m/s? (b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

5. A runaway train car that has a mass of 15,000 kg travels at a speed of 5.4 m/s down a track. Compute the time required for a force of 1500 N to bring the car to rest.

6. The mass of Earth is $5.972\times10^{24}\text{ kg}$ and its orbital radius is an average of $1.496\times10^{11}\text{ m}$. Calculate its linear momentum.

8.2 Impulse

7. A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

8. Professional Application

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

9. A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

10. Professional Application

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).

11. Professional Application

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

12. Professional Application

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of $4.00\times10^{3}\text{ m/s}$, given the collision lasts $6.00\times10^{-8}\text{ s}$.

13. Professional Application

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

14. Professional Application

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part. (a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun. (b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun? (c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

15. A cruise ship with a mass of $1.00\times10^{7}\text{ kg}$ strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

16. Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of $1.76\times10^{4}\text{ N}$ for $5.50\times10^{-2}\text{ s}$.

17. Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

18. A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?

19. Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

20. A ball with an initial velocity of 10 m/s moves at an angle 60° above the $+x$-direction. The ball hits a vertical wall and bounces off so that it is moving 60° above the $-x$-direction with the same speed. What is the impulse delivered by the wall?

21. When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

22. A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle 55° above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

8.3 Conservation of Momentum

23. Professional Application

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of -0.120 m/s. (The minus indicates direction of motion.) What is their final velocity?

24. Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

25. Professional Application

Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.* Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.

26. What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

27. A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

8.4 Elastic Collisions in One Dimension

28. Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

29. Professional Application

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00\times10^{3}\text{ kg}$, and the second a mass of $7.50\times10^{3}\text{ kg}$. If the two satellites collide elastically rather than dock, what is their final relative velocity?

30. A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

8.5 Inelastic Collisions in One Dimension

31. A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s. (a) Calculate the average force exerted on the ball by the bumper. (b) How much kinetic energy in joules is lost during the collision? (c) What percent of the original energy is left?

32. During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost?

33. Professional Application

Using mass and speed data from **Example 8.1** and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

34. A battleship that is $6.00\times10^{7}\text{ kg}$ and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

35. Professional Application

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00\times10^{3}\text{ kg}$, and the second a mass of $7.50\times10^{3}\text{ kg}$.

(a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest. (b) What is the loss of kinetic energy in this inelastic collision? (c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

36. Professional Application

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost?

37. Professional Application

Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

38. A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder.

(a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

39. Professional Application

One of the waste products of a nuclear reactor is plutonium-239 $\left(^{239}\text{Pu}\right)$. This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus $\left(^{4}\text{He} + {}^{235}\text{U}\right)$, the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is $8.40\times10^{-13}\text{ J}$ and is entirely converted to kinetic energy of the helium and uranium nuclei. The mass of the helium nucleus is $6.68\times10^{-27}\text{ kg}$, while that of the uranium is $3.92\times10^{-25}\text{ kg}$ (note that the ratio of the masses is 4 to 235). (a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest. (b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

40. Professional Application

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of $5.00\times10^{12}\text{ kg}$ (about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is $7.36\times10^{22}\text{ kg}$) ? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

41. Professional Application

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of –3.50 m/s. What is their velocity just after impact if they cling together?

42. What is the speed of a garbage truck that is $1.20\times10^{4}\text{ kg}$ and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

43. During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

44. (a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?

8.6 Collisions of Point Masses in Two Dimensions

45. Two identical pucks collide on an air hockey table. One puck was originally at rest. (a) If the incoming puck has a speed of 6.00 m/s and scatters to an angle of 30.0° ,what is the velocity (magnitude and direction) of the second puck? (You may use the result that $\theta_1 - \theta_2 = 90^\circ$ for elastic collisions of objects that have identical masses.) (b) Confirm that the collision is elastic.

46. Confirm that the results of the example Example 8.7 do conserve momentum in both the x - and y -directions.

47. A 3000-kg cannon is mounted so that it can recoil only in the horizontal direction. (a) Calculate its recoil velocity when it fires a 15.0-kg shell at 480 m/s at an angle of 20.0° above the horizontal. (b) What is the kinetic energy of the cannon? This energy is dissipated as heat transfer in shock absorbers that stop its recoil. (c) What happens to the vertical component of momentum that is imparted to the cannon when it is fired?

48. Professional Application

A 5.50-kg bowling ball moving at 9.00 m/s collides with a 0.850-kg bowling pin, which is scattered at an angle of 85.0° to the initial direction of the bowling ball and with a speed of 15.0 m/s. (a) Calculate the final velocity (magnitude and direction) of the bowling ball. (b) Is the collision elastic? (c) Linear kinetic energy is greater after the collision. Discuss how spin on the ball might be converted to linear kinetic energy in the collision.

49. Professional Application

Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei $\left(^{4}\text{He}\right)$ from gold-197 nuclei $\left(^{197}\text{Au}\right)$. The energy of the incoming helium nucleus was $8.00\times10^{-13}\text{ J}$, and the masses of the helium and gold nuclei were $6.68\times10^{-27}\text{ kg}$ and $3.29\times10^{-25}\text{ kg}$, respectively (note that their mass ratio is 4 to 197). (a) If a helium nucleus scatters to an angle of 120° during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus. (b) What is the final kinetic energy of the helium nucleus?

50. Professional Application

Two cars collide at an icy intersection and stick together afterward. The first car has a mass of 1200 kg and is approaching at 8.00 m/s due south. The second car has a mass of 850 kg and is approaching at 17.0 m/s due west. (a) Calculate the final velocity (magnitude and direction) of the cars. (b) How much kinetic energy is lost in the collision? (This energy goes into deformation of the cars.) Note that because both cars have an initial velocity, you cannot use the equations for conservation of momentum along the x-axis and y-axis; instead, you must look for other simplifying aspects.

51. Starting with equations $m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$ and $0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2$ for conservation of momentum in the x- and y-directions and assuming that one object is originally stationary, prove that for an elastic collision of two objects of equal masses,

$$\frac{1}{2}mv_1{}^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 + mv'_1 v'_2 \cos(\theta_1 - \theta_2)$$

as discussed in the text.

52. Integrated Concepts

A 90.0-kg ice hockey player hits a 0.150-kg puck, giving the puck a velocity of 45.0 m/s. If both are initially at rest and if the ice is frictionless, how far does the player recoil in the time it takes the puck to reach the goal 15.0 m away?

8.7 Introduction to Rocket Propulsion

53. Professional Application

Antiballistic missiles (ABMs) are designed to have very large accelerations so that they may intercept fast-moving incoming missiles in the short time available. What is the takeoff acceleration of a 10,000-kg ABM that expels 196 kg of gas per second at an exhaust velocity of $2.50\times10^3\ \text{m/s}$?

54. Professional Application

What is the acceleration of a 5000-kg rocket taking off from the Moon, where the acceleration due to gravity is only $1.6\ \text{m/s}^2$, if the rocket expels 8.00 kg of gas per second at an exhaust velocity of $2.20\times10^3\ \text{m/s}$?

55. Professional Application

Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of $2.00\times10^3\ \text{m/s}$. You may assume the gravitational force is negligible at the probe's location.

56. Professional Application

Ion-propulsion rockets have been proposed for use in space. They employ atomic ionization techniques and nuclear energy sources to produce extremely high exhaust velocities, perhaps as great as $8.00\times10^6\ \text{m/s}$. These techniques allow a much more favorable payload-to-fuel ratio. To illustrate this fact: (a) Calculate the increase in velocity of a 20,000-kg space probe that expels only 40.0-kg of its mass at the given exhaust velocity. (b) These engines are usually designed to produce a very small thrust for a very long time—the type of engine that might be useful on a trip to the outer planets, for example. Calculate the acceleration of such an engine if it expels $4.50\times10^{-6}\ \text{kg/s}$ at the given velocity, assuming the acceleration due to gravity is negligible.

57. Derive the equation for the vertical acceleration of a rocket.

58. Professional Application

(a) Calculate the maximum rate at which a rocket can expel gases if its acceleration cannot exceed seven times that of gravity. The mass of the rocket just as it runs out of fuel is 75,000-kg, and its exhaust velocity is $2.40\times10^3\ \text{m/s}$. Assume that the acceleration of gravity is the same as on Earth's surface $(9.80\ \text{m/s}^2)$. (b) Why might it be necessary to limit the acceleration of a rocket?

59. Given the following data for a fire extinguisher-toy wagon rocket experiment, calculate the average exhaust velocity of the gases expelled from the extinguisher. Starting from rest, the final velocity is 10.0 m/s. The total mass is initially 75.0 kg and is 70.0 kg after the extinguisher is fired.

60. How much of a single-stage rocket that is 100,000 kg can be anything but fuel if the rocket is to have a final speed of $8.00\ \text{km/s}$, given that it expels gases at an exhaust velocity of $2.20\times10^3\ \text{m/s}$?

61. Professional Application

(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement. (b) How much energy is lost to work done against friction?

62. Unreasonable Results

Squids have been reported to jump from the ocean and travel $30.0\ \text{m}$ (measured horizontally) before re-entering the water. (a) Calculate the initial speed of the squid if it leaves the water at an angle of 20.0°, assuming negligible lift from the air and negligible air resistance. (b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at $12.0\ \text{m/s}$; gravitational force and friction are neglected. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

63. Construct Your Own Problem

Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship. Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time. Among the things to be considered are the masses involved, the force she can exert on the packages through some distance, and the distance to the ship.

64. Construct Your Own Problem

Consider an artillery projectile striking armor plating. Construct a problem in which you find the force exerted by the projectile on the plate. Among the things to be considered are the mass and speed of the projectile and the distance over which its speed is reduced. Your instructor may also wish for you to consider the relative merits of depleted uranium versus lead projectiles based on the greater density of uranium.

Test Prep for AP® Courses

8.1 Linear Momentum and Force

1. A boy standing on a frictionless ice rink is initially at rest. He throws a snowball in the +*x*-direction, and it travels on a ballistic trajectory, hitting the ground some distance away. Which of the following is true about the boy while he is in the act of throwing the snowball?

a. He feels an upward force to compensate for the downward trajectory of the snowball.
b. He feels a backward force exerted by the snowball he is throwing.
c. He feels no net force.
d. He feels a forward force, the same force that propels the snowball.

2. A 150-g baseball is initially moving 80 mi/h in the –*x*-direction. After colliding with a baseball bat for 20 ms, the baseball moves 80 mi/h in the +*x*-direction. What is the magnitude and direction of the average force exerted by the bat on the baseball?

8.2 Impulse

3. A 1.0-kg ball of putty is released from rest and falls vertically 1.5 m until it strikes a hard floor, where it comes to rest in a 0.045-s time interval. What is the magnitude and direction of the average force exerted on the ball by the floor during the collision?

a. 33 N, up
b. 120 N, up
c. 120 N, down
d. 240 N, down

4. A 75-g ball is dropped from rest from a height of 2.2 m. It bounces off the floor and rebounds to a maximum height of 1.7 m. If the ball is in contact with the floor for 0.024 s, what is the magnitude and direction of the average force exerted on the ball by the floor during the collision?

5. A 2.4-kg ceramic bowl falls to the floor. During the 0.018-s impact, the bowl experiences an average force of 750 N from the floor. The bowl is at rest after the impact. From what initial height did the bowl fall?

a. 1.6 m
b. 2.8 m
c. 3.2 m
d. 5.6 m

6. Whether or not an object (such as a plate, glass, or bone) breaks upon impact depends on the average force exerted on that object by the surface. When a 1.2-kg glass figure hits the floor, it will break if it experiences an average force of 330 N. When it hits a tile floor, the glass comes to a stop in 0.015 s. From what minimum height must the glass fall to experience sufficient force to break? How would your answer change if the figure were falling to a padded or carpeted surface? Explain.

7. A 2.5-kg block slides across a frictionless table toward a horizontal spring.As the block bounces off the spring, a probe measures the velocity of the block (initially negative, moving away from the probe) over time as follows:

Table 8.2

Velocity (m/s)	Time (s)
−12.0	0
−10.0	0.10
−6.0	0.20
0	0.30
6.0	0.40
10.0	0.50
12.0	0.60

What is the average force exerted on the block by the spring over the entire 0.60-s time interval of the collision?

a. 50 N
b. 60 N
c. 100 N
d. 120 N

8. During an automobile crash test, the average force exerted by a solid wall on a 2500-kg car that hits the wall is measured to be 740,000 N over a 0.22-s time interval. What was the initial speed of the car prior to the collision, assuming the car is at rest at the end of the time interval?

9. A test car is driving toward a solid crash-test barrier with a speed of 45 mi/h. Two seconds prior to impact, the car begins to brake, but it is still moving when it hits the wall. After the collision with the wall, the car crumples somewhat and comes to a complete stop. In order to estimate the average force exerted by the wall on the car, what information would you need to collect?

a. The (negative) acceleration of the car before it hits the wall and the distance the car travels while braking.
b. The (negative) acceleration of the car before it hits the wall and the velocity of the car just before impact.
c. The velocity of the car just before impact and the duration of the collision with the wall.
d. The duration of the collision with the wall and the distance the car travels while braking.

10. Design an experiment to verify the relationship between the average force exerted on an object and the change in momentum of that object. As part of your explanation, list the equipment you would use and describe your experimental setup. What would you measure and how? How exactly would you verify the relationship? Explain.

11. A 22-g puck hits the wall of an air hockey table perpendicular to the wall with an initial speed of 14 m/s.The puck is in contact with the wall for 0.0055 s, and it rebounds from the wall with a speed of 14 m/s in the opposite direction.What is the magnitude of the average force exerted by the wall on the puck?

a. 0.308 N
b. 0.616 N
c. 56 N
d. 112 N

12. A 22-g puck hits the wall of an air hockey table perpendicular to the wall with an initial speed of 7 m/s. The puck is in contact with the wall for 0.011 s, and the wall exerts an average force of 28 N on the puck during that time. Calculate the magnitude and direction of the change in momentum of the puck.

13.

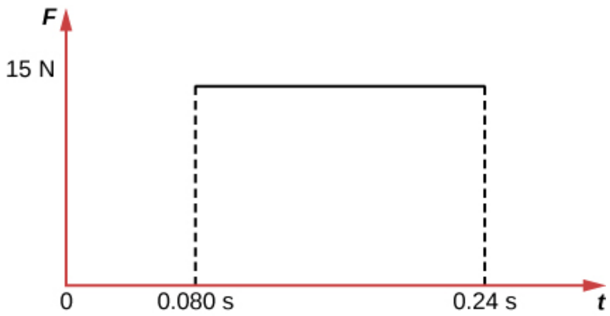


Figure 8.20 This is a graph showing the force exerted by a rigid wall versus time. The graph in **Figure 8.20** represents the force exerted on a particle during a collision. What is the magnitude of the change in momentum of the particle as a result of the collision?

a. 1.2 kg • m/s
b. 2.4 kg • m/s
c. 3.6 kg • m/s
d. 4.8 kg • m/s

14.

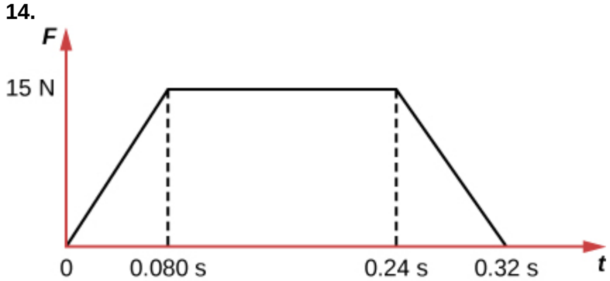


Figure 8.21 This is a graph showing the force exerted by a rigid wall versus time. The graph in **Figure 8.21** represents the force exerted on a particle during a collision. What is the magnitude of the change in momentum of the particle as a result of the collision?

8.3 Conservation of Momentum

15. Which of the following is an example of an open system?

a. Two air cars colliding on a track elastically.
b. Two air cars colliding on a track and sticking together.
c. A bullet being fired into a hanging wooden block and becoming embedded in the block, with the system then acting as a ballistic pendulum.
d. A bullet being fired into a hillside and becoming buried in the earth.

16. A 40-kg girl runs across a mat with a speed of 5.0 m/s and jumps onto a 120-kg hanging platform initially at rest, causing the girl and platform to swing back and forth like a pendulum together after her jump. What is the combined velocity of the girl and platform after the jump? What is the combined momentum of the girl and platform both before and after the collision?

A 50-kg boy runs across a mat with a speed of 6.0 m/s and collides with a soft barrier on the wall, rebounding off the wall and falling to the ground. The boy is at rest after the collision. What is the momentum of the boy before and after the collision? Is momentum conserved in this collision? Explain. Which of these is an example of an open system and which is an example of a closed system? Explain your answer.

17. A student sets up an experiment to measure the momentum of a system of two air cars, A and B, of equal mass, moving on a linear, frictionless track. Before the collision, car A has a certain speed, and car B is at rest. Which of the following will be true about the total momentum of the two cars?

a. It will be greater before the collision.
b. It will be equal before and after the collision.
c. It will be greater after the collision.
d. The answer depends on whether the collision is elastic or inelastic.

18. A group of students has two carts, *A* and *B*, with wheels that turn with negligible friction. The carts can travel along a straight horizontal track. Cart *A* has known mass *mA*. The students are asked to use a one-dimensional collision between the carts to determine the mass of cart *B*. Before the collision, cart *A* travels to the right and cart *B* is initially at rest. After the collision, the carts stick together.

a. Describe an experimental procedure to determine the velocities of the carts before and after a collision, including all the additional equipment you would need. You may include a labeled diagram of your setup to help in your description. Indicate what measurements you would take and how you would take them. Include enough detail so that another student could carry out your procedure.
b. There will be sources of error in the measurements taken in the experiment, both before and after the collision. For your experimental procedure, will the uncertainty in the calculated value of the mass of cart *B* be affected more by the error in the measurements taken before the collision or by those taken after the collision, or will it be equally affected by both sets of measurements? Justify your answer.

A group of students took measurements for one collision. A graph of the students' data is shown below.

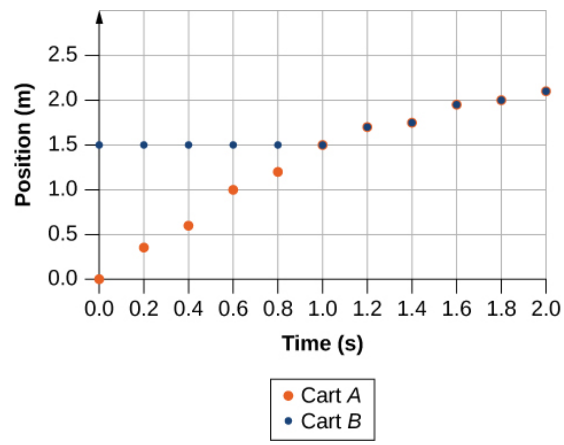


Figure 8.22 The image shows a graph with position in meters on the vertical axis and time in seconds on the horizontal axis.

c. Given m_A = 0.50 kg, use the graph to calculate the mass of cart *B*. Explicitly indicate the principles used in your calculations.
d. The students are now asked to Consider the kinetic energy changes in an inelastic collision, specifically whether the initial values of one of the physical quantities affect the fraction of mechanical energy dissipated in the collision. How could you modify the experiment to investigate this question? Be sure to explicitly describe the calculations you would make, specifying all equations you would use (but do not actually do any algebra or arithmetic).

19. Cart A is moving with an initial velocity +*v* (in the positive direction) toward cart B, initially at rest. Both carts have equal mass and are on a frictionless surface. Which of the following statements correctly characterizes the velocity of the center of mass of the system before and after the collision?

a. $\frac{+v}{2}$ before, $\frac{-v}{2}$ after
b. $\frac{+v}{2}$ before, 0 after
c. $\frac{+v}{2}$ before, $\frac{+v}{2}$ after
d. 0 before, 0 after

20. Cart A is moving with a velocity of +10 m/s toward cart B, which is moving with a velocity of +4 m/s. Both carts have equal mass and are moving on a frictionless surface. The two carts have an inelastic collision and stick together after the collision. Calculate the velocity of the center of mass of the system before and after the collision. If there were friction present in this problem, how would this external force affect the center-of-mass velocity both before and after the collision?

8.4 Elastic Collisions in One Dimension

21. Two cars (A and B) of mass 1.5 kg collide. Car A is initially moving at 12 m/s, and car B is initially moving in the same direction with a speed of 6 m/s. The two cars are moving along a straight line before and after the collision. What will be the change in momentum of this system after the collision?

a. −27 kg • m/s
b. zero
c. +27 kg • m/s
d. It depends on whether the collision is elastic or inelastic.

22. Two cars (A and B) of mass 1.5 kg collide. Car A is initially moving at 24 m/s, and car B is initially moving in the opposite direction with a speed of 12 m/s. The two cars are moving along a straight line before and after the collision. (a) If the two cars have an elastic collision, calculate the change in momentum of the two-car system. (b) If the two cars have a completely inelastic collision, calculate the change in momentum of the two-car system.

23. Puck A (200 g) slides across a frictionless surface to collide with puck B (800 g), initially at rest. The velocity of each puck is measured during the experiment as follows:

Table 8.3

Time	Velocity A	Velocity B
0	+8.0 m/s	0
1.0 s	+8.0 m/s	0
2.0 s	−2.0 m/s	+2.5 m/s
3.0 s	−2.0 m/s	+2.5 m/s

What is the change in momentum of the center of mass of the system as a result of the collision?

a. +1.6 kg•m/s
b. +0.8 kg•m/s
c. 0
d. −1.6 kg•m/s

24. For the table above, calculate the center-of-mass velocity of the system both before and after the collision, then calculate the center-of-mass momentum of the system both before and after the collision. From this, determine the change in the momentum of the system as a result of the collision.

25. Two cars (A and B) of equal mass have an elastic collision. Prior to the collision, car A is moving at 15 m/s in the +*x*-direction, and car B is moving at 10 m/s in the –*x*-direction. Assuming that both cars continue moving along the *x*-axis after the collision, what will be the velocity of car A after the collision?

a. same as the original 15 m/s speed, opposite direction
b. equal to car B's velocity prior to the collision
c. equal to the average of the two velocities, in its original direction
d. equal to the average of the two velocities, in the opposite direction

26. Two cars (A and B) of equal mass have an elastic collision. Prior to the collision, car A is moving at 20 m/s in the +*x*-direction, and car B is moving at 10 m/s in the –*x*-direction. Assuming that both cars continue moving along the *x*-axis after the collision, what will be the velocities of each car after the collision?

27. A rubber ball is dropped from rest at a fixed height. It bounces off a hard floor and rebounds upward, but it only reaches 90% of its original fixed height. What is the best way to explain the loss of kinetic energy of the ball during the collision?

a. Energy was required to deform the ball's shape during the collision with the floor.
b. Energy was lost due to work done by the ball pushing on the floor during the collision.
c. Energy was lost due to friction between the ball and the floor.
d. Energy was lost due to the work done by gravity during the motion.

28. A tennis ball strikes a wall with an initial speed of 15 m/s. The ball bounces off the wall but rebounds with slightly less speed (14 m/s) after the collision. Explain (a) what else changed its momentum in response to the ball's change in momentum so that overall momentum is conserved, and (b) how some of the ball's kinetic energy was lost.

29. Two objects, A and B, have equal mass. Prior to the collision, mass A is moving 10 m/s in the +*x*-direction, and mass B is moving 4 m/s in the +*x*-direction. Which of the following results represents an inelastic collision between A and B?

a. After the collision, mass A is at rest, and mass B moves 14 m/s in the +*x*-direction.
b. After the collision, mass A moves 4 m/s in the –*x*-direction, and mass B moves 18 m/s in the +*x*-direction.
c. After the collision, the two masses stick together and move 7 m/s in the +*x*-direction.
d. After the collision, mass A moves 4 m/s in the +*x*-direction, and mass B moves 10 m/s in the +*x*-direction.

30. Mass A is three times more massive than mass B. Mass A is initially moving 12 m/s in the +*x*-direction. Mass B is initially moving 12 m/s in the –*x*-direction. Assuming that the collision is elastic, calculate the final velocity of both masses after the collision. Show that your results are consistent with conservation of momentum and conservation of kinetic energy.

31. Two objects (A and B) of equal mass collide elastically. Mass A is initially moving 5.0 m/s in the +*x*-direction prior to the collision. Mass B is initially moving 3.0 m/s in the –*x*-direction prior to the collision. After the collision, mass A will be moving with a velocity of 3.0 m/s in the –*x*-direction. What will be the velocity of mass B after the collision?

a. 3.0 m/s in the +*x*-direction
b. 5.0 m/s in the +*x*-direction
c. 3.0 m/s in the –*x*-direction

d. 5.0 m/s in the –*x*-direction

32. Two objects (A and B) of equal mass collide elastically. Mass A is initially moving 4.0 m/s in the +*x*-direction prior to the collision. Mass B is initially moving 8.0 m/s in the –*x*-direction prior to the collision. After the collision, mass A will be moving with a velocity of 8.0 m/s in the –*x*-direction. (a) Use the principle of conservation of momentum to predict the velocity of mass B after the collision. (b) Use the fact that kinetic energy is conserved in elastic collisions to predict the velocity of mass B after the collision.

33. Two objects of equal mass collide. Object A is initially moving in the +*x*-direction with a speed of 12 m/s, and object B is initially at rest. After the collision, object A is at rest, and object B is moving away with some unknown velocity. There are no external forces acting on the system of two masses. What statement can we make about this collision?

a. Both momentum and kinetic energy are conserved.
b. Momentum is conserved, but kinetic energy is not conserved.
c. Neither momentum nor kinetic energy is conserved.
d. More information is needed in order to determine which is conserved.

34. Two objects of equal mass collide. Object A is initially moving with a velocity of 15 m/s in the +*x*-direction, and object B is initially at rest. After the collision, object A is at rest. There are no external forces acting on the system of two masses. (a) Use momentum conservation to deduce the velocity of object B after the collision. (b) Is this collision elastic? Justify your answer.

35. Which of the following statements is true about an inelastic collision?

a. Momentum is conserved, and kinetic energy is conserved.
b. Momentum is conserved, and kinetic energy is not conserved.
c. Momentum is not conserved, and kinetic energy is conserved.
d. Momentum is not conserved, and kinetic energy is not conserved.

36. Explain how the momentum and kinetic energy of a system of two colliding objects changes as a result of (a) an elastic collision and (b) an inelastic collision.

37. **Figure 8.9** shows the positions of two colliding objects measured before, during, and after a collision. Mass A is 1.0 kg. Mass B is 3.0 kg. Which of the following statements is true?

a. This is an elastic collision, with a total momentum of 0 kg • m/s.
b. This is an elastic collision, with a total momentum of 1.67 kg • m/s.
c. This is an inelastic collision, with a total momentum of 0 kg • m/s.
d. This is an inelastic collision, with a total momentum of 1.67 kg • m/s.

38. For the above graph, determine the initial and final momentum for both objects, assuming mass A is 1.0 kg and mass B is 3.0 kg. Also, determine the initial and final kinetic energies for both objects. Based on your results, explain whether momentum is conserved in this collision, and state whether the collision is elastic or inelastic.

39. Mass A (1.0 kg) slides across a frictionless surface with a velocity of 8 m/s in the positive direction. Mass B (3.0 kg) is initially at rest. The two objects collide and stick together. What will be the change in the center-of-mass velocity of the system as a result of the collision?

a. There will be no change in the center-of-mass velocity.
b. The center-of-mass velocity will decrease by 2 m/s.
c. The center-of-mass velocity will decrease by 6 m/s.
d. The center-of-mass velocity will decrease by 8 m/s.

40. Mass A (1.0 kg) slides across a frictionless surface with a velocity of 4 m/s in the positive direction. Mass B (1.0 kg) slides across the same surface in the opposite direction with a velocity of −8 m/s. The two objects collide and stick together after the collision. Predict how the center-of-mass velocity will change as a result of the collision, and explain your prediction. Calculate the center-of-mass velocity of the system both before and after the collision and explain why it remains the same or why it has changed.

8.5 Inelastic Collisions in One Dimension

41. Mass A (2.0 kg) has an initial velocity of 4 m/s in the +*x*-direction. Mass B (2.0 kg) has an initial velocity of 5 m/s in the –*x*-direction. If the two masses have an elastic collision, what will be the final velocities of the masses after the collision?

a. Both will move 0.5 m/s in the –*x*-direction.
b. Mass A will stop; mass B will move 9 m/s in the +*x*-direction.
c. Mass B will stop; mass A will move 9 m/s in the –*x*-direction.
d. Mass A will move 5 m/s in the –*x*-direction; mass B will move 4 m/s in the +*x*-direction.

42. Mass A has an initial velocity of 22 m/s in the +*x*-direction. Mass B is three times more massive than mass A and has an initial velocity of 22 m/s in the –*x*-direction. If the two masses have an elastic collision, what will be the final velocities of the masses after the collision?

43. Mass A (2.0 kg) is moving with an initial velocity of 15 m/s in the +*x*-direction, and it collides with mass B (5.0 kg), initially at rest. After the collision, the two objects stick together and move as one. What is the change in kinetic energy of the system as a result of the collision?

a. no change
b. decrease by 225 J
c. decrease by 161 J
d. decrease by 64 J

44. Mass A (2.0 kg) is moving with an initial velocity of 15 m/s in the +*x*-direction, and it collides with mass B (4.0 kg), initially moving 7.0 m/s in the +*x*-direction. After the collision, the two objects stick together and move as one. What is the change in kinetic energy of the system as a result of the collision?

45. Mass A slides across a rough table with an initial velocity of 12 m/s in the +*x*-direction. By the time mass A collides with mass B (a stationary object with equal mass), mass A has slowed to 10 m/s. After the collision, the two objects stick together and move as one. Immediately after the collision, the velocity of the system is measured to be 5 m/s in the +*x*-direction, and the system eventually slides to a stop. Which of the following statements is true about this motion?

a. Momentum is conserved during the collision, but it is not conserved during the motion before and after the collision.
b. Momentum is not conserved at any time during this analysis.
c. Momentum is conserved at all times during this analysis.
d. Momentum is not conserved during the collision, but it is conserved during the motion before and after the collision.

46. Mass A is initially moving with a velocity of 12 m/s in the +*x*-direction. Mass B is twice as massive as mass A and is initially at rest. After the two objects collide, the two masses move together as one with a velocity of 4 m/s in the +*x*-direction. Is momentum conserved in this collision?

47. Mass A is initially moving with a velocity of 24 m/s in the +*x*-direction. Mass B is twice as massive as mass A and is initially at rest. The two objects experience a totally inelastic collision. What is the final speed of both objects after the collision?

a. A is not moving; B is moving 24 m/s in the +*x*-direction.
b. Neither A nor B is moving.
c. A is moving 24 m/s in the –*x*-direction. B is not moving.
d. Both A and B are moving together 8 m/s in the +*x*-direction.

48. Mass A is initially moving with some unknown velocity in the +*x*-direction. Mass B is twice as massive as mass A and initially at rest. The two objects collide, and after the collision, they move together with a speed of 6 m/s in the +*x*-direction. (a) Is this collision elastic or inelastic? Explain. (b) Determine the initial velocity of mass A.

49. Mass A is initially moving with a velocity of 2 m/s in the +*x*-direction. Mass B is initially moving with a velocity of 6 m/s in the –*x*-direction. The two objects have equal masses. After they collide, mass A moves with a speed of 4 m/s in the –*x*-direction. What is the final velocity of mass B after the collision?

a. 6 m/s in the +*x*-direction
b. 4 m/s in the +*x*-direction
c. zero
d. 4 m/s in the –*x*-direction

50. Mass A is initially moving with a velocity of 15 m/s in the +*x*-direction. Mass B is twice as massive and is initially moving with a velocity of 10 m/s in the –*x*-direction. The two objects collide, and after the collision, mass A moves with a speed of 15 m/s in the –*x*-direction. (a) What is the final velocity of mass B after the collision? (b) Calculate the change in kinetic energy as a result of the collision, assuming mass A is 5.0 kg.

8.6 Collisions of Point Masses in Two Dimensions

51. Two cars of equal mass approach an intersection. Car A is moving east at a speed of 45 m/s. Car B is moving south at a speed of 35 m/s. They collide inelastically and stick together after the collision, moving as one object. Which of the following statements is true about the center-of-mass velocity of this system?

a. The center-of-mass velocity will decrease after the collision as a result of lost energy (but not drop to zero).
b. The center-of-mass velocity will remain the same after the collision since momentum is conserved.
c. The center-of-mass velocity will drop to zero since the two objects stick together.
d. The magnitude of the center-of-mass velocity will remain the same, but the direction of the velocity will change.

52. Car A has a mass of 2000 kg and approaches an intersection with a velocity of 38 m/s directed to the east. Car B has a mass of 3500 kg and approaches the intersection with a velocity of 53 m/s directed 63° north of east. The two cars collide and stick together after the collision. Will the center-of-mass velocity change as a result of the collision? Explain why or why not. Calculate the center-of-mass velocity before and after the collision.

9 STATICS AND TORQUE

Figure 9.1 On a short time scale, rocks like these in Australia's Kings Canyon are static, or motionless relative to the Earth. (credit: freeaussiestock.com)

Chapter Outline

Connection for AP® Courses

What might desks, bridges, buildings, trees, and mountains have in common? What do these objects have in common with a car moving at a constant velocity? While it may be apparent that the objects in the first group are all motionless relative to Earth, they also share something with the moving car and all objects moving at a constant velocity. All of these objects, stationary and moving, share an acceleration of zero. How can this be? Consider Newton's second law, $F = ma$. When acceleration is zero, as is the case for both stationary objects and objects moving at a constant velocity, the net external force must also be zero (Big Idea 3). Forces are acting on both stationary objects and on objects moving at a constant velocity, but the forces are balanced. That is, they are in *equilibrium*. In equilibrium, the net force is zero.

The first two sections of this chapter will focus on the two conditions necessary for equilibrium. They will not only help you to distinguish between stationary bridges and cars moving at constant velocity, but will introduce a second equilibrium condition, this time involving rotation. As you explore the second equilibrium condition, you will learn about torque, in support of both Enduring Understanding 3.F and Essential Knowledge 3.F.1. Much like a force, torque provides the capability for acceleration; however, with careful attention, torques may also be balanced and equilibrium can be reached.

The remainder of this chapter will discuss a variety of interesting equilibrium applications. From the art of balancing, to simple machines, to the muscles in your body, the world around you relies upon the principles of equilibrium to remain stable. This chapter will help you to see just how closely related these events truly are.

The content in this chapter supports:

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.F A force exerted on an object can cause a torque on that object.

Essential Knowledge 3.F.1 Only the force component perpendicular to the line connecting the axis of rotation and the point of application of the force results in a torque about that axis.

9.1 The First Condition for Equilibrium

Learning Objectives

By the end of this section, you will be able to:

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

$$\text{net } \mathbf{F} = 0 \tag{9.1}$$

Note that if net F is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical *x*- and *y*-axes are zero. This is written as

$$\text{net } F_x = 0 \text{ and } F_y = 0 \tag{9.2}$$

Figure 9.2 and **Figure 9.3** illustrate situations where net $F = 0$ for both **static equilibrium** (motionless), and **dynamic equilibrium** (constant velocity).

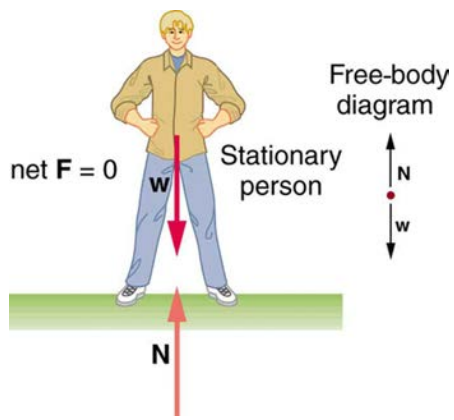


Figure 9.2 This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.

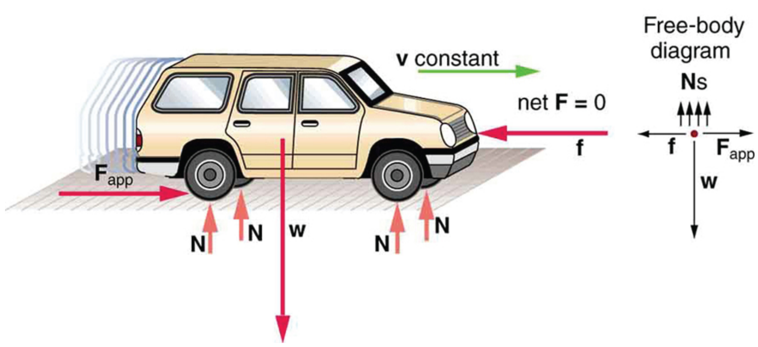


Figure 9.3 This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force F_{app} between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in **Figure 9.4** and **Figure 9.5** where forces are applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In **Figure 9.4**, the ice hockey stick remains motionless. But in **Figure 9.5**, with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.

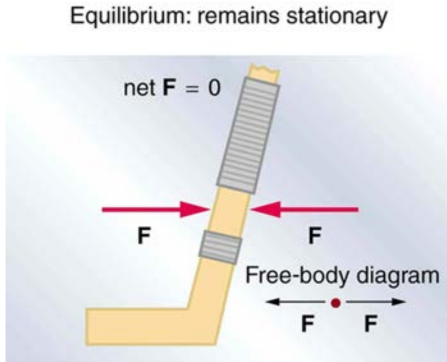


Figure 9.4 An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus, $\text{net } F = 0$. Equilibrium is achieved, which is static equilibrium in this case.

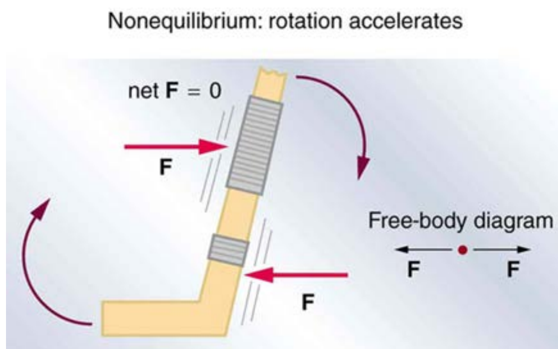


Figure 9.5 The same forces are applied at other points and the stick rotates—in fact, it experiences an accelerated rotation. Here $\text{net } F = 0$ but the system is *not* at equilibrium. Hence, the $\text{net } F = 0$ is a necessary—but not sufficient—condition for achieving equilibrium.

PhET Explorations: Torque

Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque.

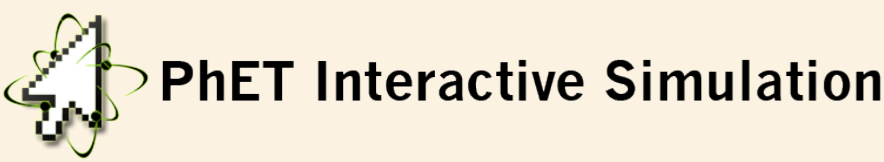


Figure 9.6 Torque (http://cnx.org/content/m55176/1.2/torque_en.jar)

9.2 The Second Condition for Equilibrium

Learning Objectives

By the end of this section, you will be able to:

- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.F.1.1** The student is able to use representations of the relationship between force and torque. **(S.P. 1.4)**
- **3.F.1.2** The student is able to compare the torques on an object caused by various forces. **(S.P. 1.4)**
- **3.F.1.3** The student is able to estimate the torque on an object caused by various forces in comparison to other situations. **(S.P. 2.3)**

Torque

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity. A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening the door. See **Figure 9.7**. First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.

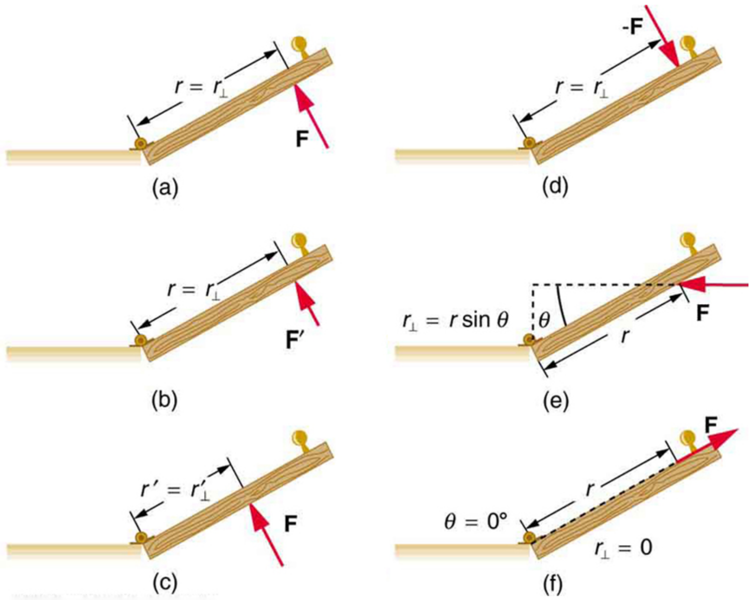


Figure 9.7 Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to $\mathbf{F}$. Note that $r_\perp$ is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force $\mathbf{F}'$ acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point but in a different direction. Here, θ is less than 90^o. (f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case, $\theta = 0^\text{o}$.

The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. **Torque** is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be

$$\tau = rF\sin\theta \tag{9.3}$$

where τ (the Greek letter tau) is the symbol for torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point, as seen in **Figure 9.7** and **Figure 9.8**. An alternative expression for torque is given in terms of the **perpendicular lever arm** $r_\perp$ as shown in **Figure 9.7** and **Figure 9.8**, which is defined as

$$r_\perp = r\sin\theta \tag{9.4}$$

so that

$$\tau = r_\perp F. \tag{9.5}$$

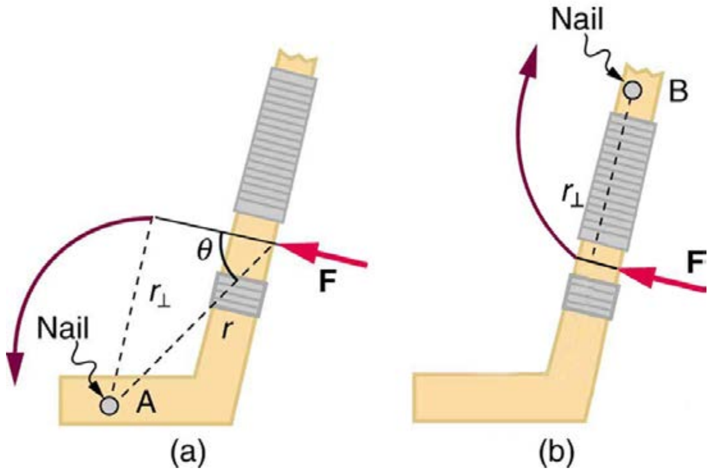


Figure 9.8 A force applied to an object can produce a torque, which depends on the location of the pivot point. (a) The three factors r , F , and θ for pivot point A on a body are shown here— r is the distance from the chosen pivot point to the point where the force F is applied, and θ is the angle between $\mathbf{F}$ and the vector directed from the point of application to the pivot point. If the object can rotate around point A, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot A. (b) In this case, point B is the pivot point. The torque from the applied force will cause a clockwise rotation around point B, and so it is a clockwise torque relative to B.

The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which $\mathbf{F}$ acts; it is shown as a dashed line in **Figure 9.7** and **Figure 9.8**. Note that the line segment that defines the distance $r_\perp$ is perpendicular to $\mathbf{F}$, as its name implies. It is sometimes easier to find or visualize $r_\perp$ than to find both r and θ . In such cases, it may be more convenient to use $\boldsymbol{\tau = r_\perp F}$ rather than $\tau = rF \sin\theta$ for torque, but both are equally valid.

The **SI unit of torque** is newtons times meters, usually written as $\text{N} \cdot \text{m}$. For example, if you push perpendicular to the door with a force of 40 N at a distance of 0.800 m from the hinges, you exert a torque of $32\ \text{N·m}(0.800\ \text{m}\times 40\ \text{N}\times \sin 90º)$ relative to the hinges. If you reduce the force to 20 N, the torque is reduced to $16\ \text{N·m}$, and so on.

The torque is always calculated with reference to some chosen pivot point. For the same applied force, a different choice for the location of the pivot will give you a different value for the torque, since both r and θ depend on the location of the pivot. Any point in any object can be chosen to calculate the torque about that point. The object may not actually pivot about the chosen "pivot point."

Note that for rotation in a plane, torque has two possible directions. Torque is either clockwise or counterclockwise relative to the chosen pivot point, as illustrated for points B and A, respectively, in **Figure 9.8**. If the object can rotate about point A, it will rotate counterclockwise, which means that the torque for the force is shown as counterclockwise relative to A. But if the object can rotate about point B, it will rotate clockwise, which means the torque for the force shown is clockwise relative to B. Also, the magnitude of the torque is greater when the lever arm is longer.

Making Connections: Pivoting Block

A solid block of length ***d*** is pinned to a wall on its right end. Three forces act on the block as shown below: $\mathbf{F}_A$, $\mathbf{F}_B$, and $\mathbf{F}_C$. While all three forces are of equal magnitude, and all three are equal distances away from the pivot point, all three forces will create a different torque upon the object.

$\mathbf{F}_A$ is vectored perpendicular to its distance from the pivot point; as a result, the magnitude of its torque can be found by the equation $\tau = F_A * d$. Vector $\mathbf{F}_B$ is parallel to the line connecting the point of application of force and the pivot point. As a result, it does not provide an ability to rotate the object and, subsequently, its torque is zero. $\mathbf{F}_C$, however, is directed at an angle Θ to the line connecting the point of application of force and the pivot point. In this instance, only the component perpendicular to this line is exerting a torque. This component, labeled $F_\perp$, can be found using the equation $F_\perp = F_C \sin\theta$. The component of the force parallel to this line, labeled $F_{/\!/}$, does not provide an ability to rotate the object and, as a result, does not provide a torque. Therefore, the resulting torque created by $\mathbf{F}_C$ is $\tau = F\perp * d$.

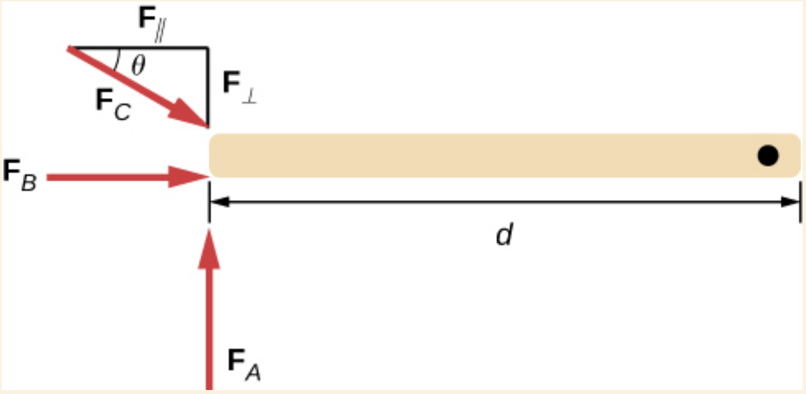


Figure 9.9 Forces on a block pinned to a wall. A solid block of length ***d*** is pinned to a wall on its right end. Three forces act on the block: F_A, F_B, and F_C.

Now, *the second condition necessary to achieve equilibrium* is that *the net external torque on a system must be zero*. An external torque is one that is created by an external force. You can choose the point around which the torque is calculated. The point can be the physical pivot point of a system or any other point in space—but it must be the same point for all torques. If the second condition (net external torque on a system is zero) is satisfied for one choice of pivot point, it will also hold true for any other choice of pivot point in or out of the system of interest. (This is true only in an inertial frame of reference.) The second condition necessary to achieve equilibrium is stated in equation form as

$$\text{net } \tau = 0 \tag{9.6}$$

where net means total. Torques, which are in opposite directions are assigned opposite signs. A common convention is to call counterclockwise (ccw) torques positive and clockwise (cw) torques negative.

When two children balance a seesaw as shown in **Figure 9.10**, they satisfy the two conditions for equilibrium. Most people have perfect intuition about seesaws, knowing that the lighter child must sit farther from the pivot and that a heavier child can keep a lighter one off the ground indefinitely.

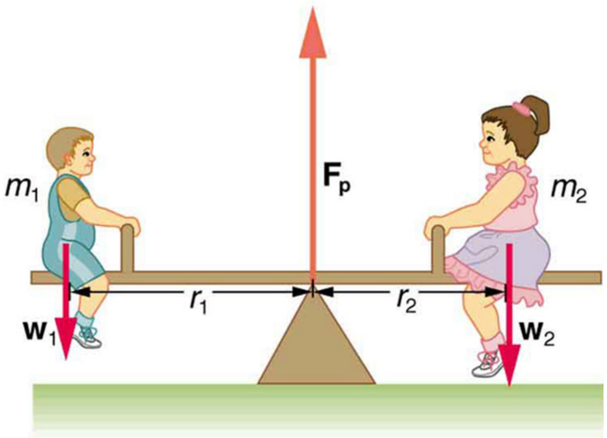


Figure 9.10 Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child.

Example 9.1 She Saw Torques On A Seesaw

The two children shown in **Figure 9.10** are balanced on a seesaw of negligible mass. (This assumption is made to keep the example simple—more involved examples will follow.) The first child has a mass of 26.0 kg and sits 1.60 m from the pivot.(a) If the second child has a mass of 32.0 kg, how far is she from the pivot? (b) What is F_{p}, the supporting force exerted by the pivot?

Strategy

Both conditions for equilibrium must be satisfied. In part (a), we are asked for a distance; thus, the second condition (regarding torques) must be used, since the first (regarding only forces) has no distances in it. To apply the second condition for equilibrium, we first identify the system of interest to be the seesaw plus the two children. We take the supporting pivot to be the point about which the torques are calculated. We then identify all external forces acting on the system.

Solution (a)

The three external forces acting on the system are the weights of the two children and the supporting force of the pivot. Let us examine the torque produced by each. Torque is defined to be

$$\tau = rF \sin \theta. \tag{9.7}$$

Here $\theta = 90^{\circ}$, so that $\sin \theta = 1$ for all three forces. That means $r_{\perp} = r$ for all three. The torques exerted by the three forces are first,

$$\tau_1 = r_1 w_1 \tag{9.8}$$

second,

$$\tau_2 = -r_2 w_2 \tag{9.9}$$

and third,

$$\begin{aligned} \tau_{\text{p}} &= r_{\text{p}} F_{\text{p}} \\ &= 0 \cdot F_{\text{p}} \\ &= 0. \end{aligned} \tag{9.10}$$

Note that a minus sign has been inserted into the second equation because this torque is clockwise and is therefore negative by convention. Since F_{p} acts directly on the pivot point, the distance r_{p} is zero. A force acting on the pivot cannot cause a rotation, just as pushing directly on the hinges of a door will not cause it to rotate. Now, the second condition for equilibrium is that the sum of the torques on both children is zero. Therefore

$$\tau_2 = -\tau_1, \tag{9.11}$$

or

$$r_2 w_2 = r_1 w_1. \tag{9.12}$$

Weight is mass times the acceleration due to gravity. Entering mg for w, we get

$$r_2 m_2 g = r_1 m_1 g. \tag{9.13}$$

Solve this for the unknown r_2:

$$r_2 = r_1 \frac{m_1}{m_2}. \tag{9.14}$$

The quantities on the right side of the equation are known; thus, r_2 is

$$r_2 = (1.60 \text{ m}) \frac{26.0 \text{ kg}}{32.0 \text{ kg}} = 1.30 \text{ m}. \tag{9.15}$$

As expected, the heavier child must sit closer to the pivot (1.30 m versus 1.60 m) to balance the seesaw.

Solution (b)

This part asks for a force F_{p}. The easiest way to find it is to use the first condition for equilibrium, which is

$$\text{net } \mathbf{F} = 0. \tag{9.16}$$

The forces are all vertical, so that we are dealing with a one-dimensional problem along the vertical axis; hence, the condition can be written as

$$\text{net } F_y = 0 \tag{9.17}$$

where we again call the vertical axis the *y*-axis. Choosing upward to be the positive direction, and using plus and minus signs to indicate the directions of the forces, we see that

$$F_{\text{p}} - w_1 - w_2 = 0. \tag{9.18}$$

This equation yields what might have been guessed at the beginning:

$$F_{\text{p}} = w_1 + w_2. \tag{9.19}$$

So, the pivot supplies a supporting force equal to the total weight of the system:

$$F_{\text{p}} = m_1 g + m_2 g. \tag{9.20}$$

Entering known values gives

$$\begin{aligned} F_\text{p} &= (26.0\text{ kg})\left(9.80\text{ m/s}^2\right) + (32.0\text{ kg})\left(9.80\text{ m/s}^2\right) \\ &= 568\text{ N}. \end{aligned} \tag{9.21}$$

Discussion

The two results make intuitive sense. The heavier child sits closer to the pivot. The pivot supports the weight of the two children. Part (b) can also be solved using the second condition for equilibrium, since both distances are known, but only if the pivot point is chosen to be somewhere other than the location of the seesaw's actual pivot!

Several aspects of the preceding example have broad implications. First, the choice of the pivot as the point around which torques are calculated simplified the problem. Since F_p is exerted on the pivot point, its lever arm is zero. Hence, the torque exerted by the supporting force F_p is zero relative to that pivot point. The second condition for equilibrium holds for any choice of pivot point, and so we choose the pivot point to simplify the solution of the problem.

Second, the acceleration due to gravity canceled in this problem, and we were left with a ratio of masses. *This will not always be the case*. Always enter the correct forces—do not jump ahead to enter some ratio of masses.

Third, the weight of each child is distributed over an area of the seesaw, yet we treated the weights as if each force were exerted at a single point. This is not an approximation—the distances r_1 and r_2 are the distances to points directly below the **center of gravity** of each child. As we shall see in the next section, the mass and weight of a system can act as if they are located at a single point.

Finally, note that the concept of torque has an importance beyond static equilibrium. *Torque plays the same role in rotational motion that force plays in linear motion.* We will examine this in the next chapter.

Take-Home Experiment

Take a piece of modeling clay and put it on a table, then mash a cylinder down into it so that a ruler can balance on the round side of the cylinder while everything remains still. Put a penny 8 cm away from the pivot. Where would you need to put two pennies to balance? Three pennies?

9.3 Stability

Learning Objectives

By the end of this section, you will be able to:

- State the types of equilibrium.
- Describe stable and unstable equilibriums.
- Describe neutral equilibrium.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.F.1.1** The student is able to use representations of the relationship between force and torque. **(S.P. 1.4)**
- **3.F.1.2** The student is able to compare the torques on an object caused by various forces. **(S.P. 1.4)**
- **3.F.1.3** The student is able to estimate the torque on an object caused by various forces in comparison to other situations. **(S.P. 2.3)**
- **3.F.1.4** The student is able to design an experiment and analyze data testing a question about torques in a balanced rigid system. **(S.P. 4.1, 4.2, 5.1)**
- **3.F.1.5** The student is able to calculate torques on a two-dimensional system in static equilibrium, by examining a representation or model (such as a diagram or physical construction). **(S.P. 1.4, 2.2)**

It is one thing to have a system in equilibrium; it is quite another for it to be stable. The toy doll perched on the man's hand in **Figure 9.11**, for example, is not in stable equilibrium. There are *three types of equilibrium*: *stable*, *unstable*, and *neutral*. Figures throughout this module illustrate various examples.

Figure 9.11 presents a balanced system, such as the toy doll on the man's hand, which has its center of gravity (cg) directly over the pivot, so that the torque of the total weight is zero. This is equivalent to having the torques of the individual parts balanced about the pivot point, in this case the hand. The cgs of the arms, legs, head, and torso are labeled with smaller type.

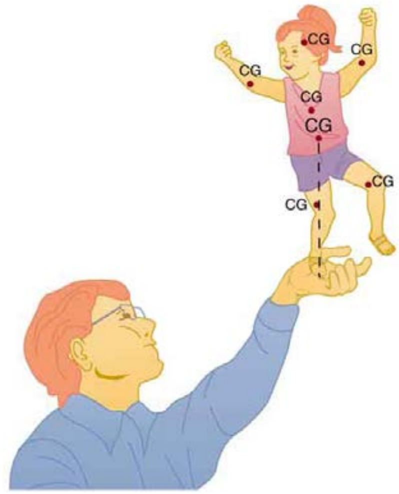


Figure 9.11 A man balances a toy doll on one hand.

A system is said to be in **stable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement. For example, a marble at the bottom of a bowl will experience a *restoring* force when displaced from its equilibrium position. This force moves it back toward the equilibrium position. Most systems are in stable equilibrium, especially for small displacements. For another example of stable equilibrium, see the pencil in **Figure 9.12**.

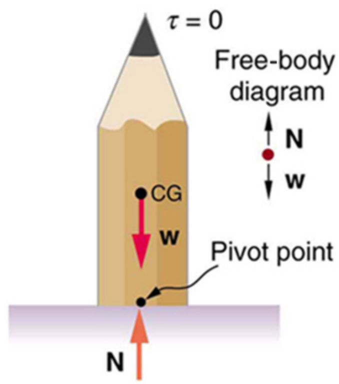


Figure 9.12 This pencil is in the condition of equilibrium. The net force on the pencil is zero and the total torque about any pivot is zero.

A system is in **unstable equilibrium** if, when displaced, it experiences a net force or torque in the *same* direction as the displacement from equilibrium. A system in unstable equilibrium accelerates away from its equilibrium position if displaced even slightly. An obvious example is a ball resting on top of a hill. Once displaced, it accelerates away from the crest. See the next several figures for examples of unstable equilibrium.

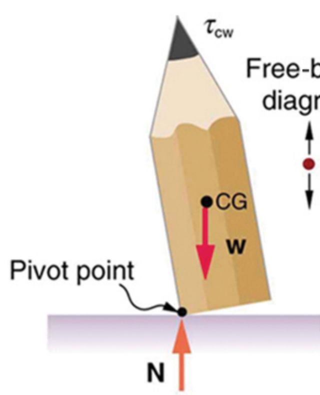


Figure 9.13 If the pencil is displaced slightly to the side (counterclockwise), it is no longer in equilibrium. Its weight produces a clockwise torque that returns the pencil to its equilibrium position.

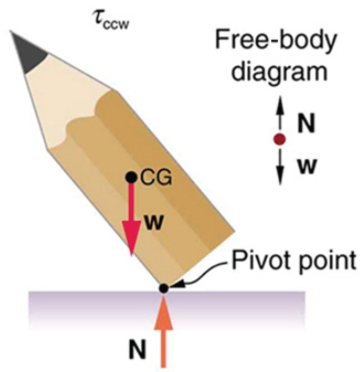


Figure 9.14 If the pencil is displaced too far, the torque caused by its weight changes direction to counterclockwise and causes the displacement to increase.

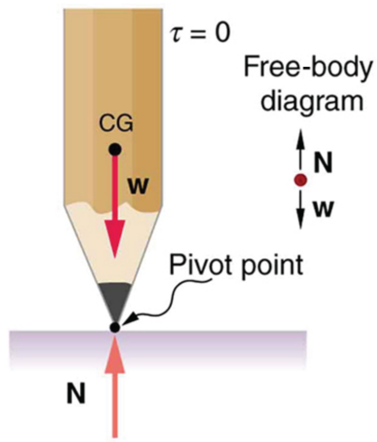


Figure 9.15 This figure shows unstable equilibrium, although both conditions for equilibrium are satisfied.

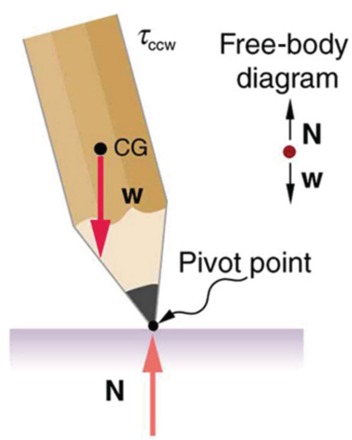


Figure 9.16 If the pencil is displaced even slightly, a torque is created by its weight that is in the same direction as the displacement, causing the displacement to increase.

A system is in **neutral equilibrium** if its equilibrium is independent of displacements from its original position. A marble on a flat horizontal surface is an example. Combinations of these situations are possible. For example, a marble on a saddle is stable for displacements toward the front or back of the saddle and unstable for displacements to the side. Figure 9.17 shows another example of neutral equilibrium.

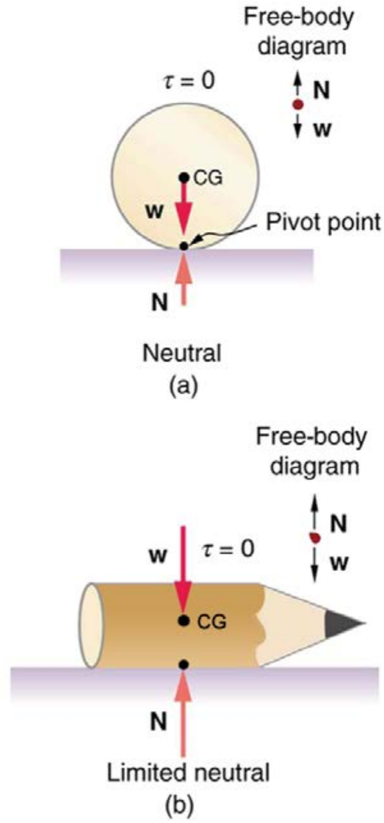


Figure 9.17 (a) Here we see neutral equilibrium. The cg of a sphere on a flat surface lies directly above the point of support, independent of the position on the surface. The sphere is therefore in equilibrium in any location, and if displaced, it will remain put. (b) Because it has a circular cross section, the pencil is in neutral equilibrium for displacements perpendicular to its length.

When we consider how far a system in stable equilibrium can be displaced before it becomes unstable, we find that some systems in stable equilibrium are more stable than others. The pencil in **Figure 9.12** and the person in **Figure 9.18**(a) are in stable equilibrium, but become unstable for relatively small displacements to the side. The critical point is reached when the cg is no longer *above* the base of support. Additionally, since the cg of a person's body is above the pivots in the hips, displacements must be quickly controlled. This control is a central nervous system function that is developed when we learn to hold our bodies erect as infants. For increased stability while standing, the feet should be spread apart, giving a larger base of support. Stability is also increased by lowering one's center of gravity by bending the knees, as when a football player prepares to receive a ball or braces themselves for a tackle. A cane, a crutch, or a walker increases the stability of the user, even more as the base of support widens. Usually, the cg of a female is lower (closer to the ground) than a male. Young children have their center of gravity between their shoulders, which increases the challenge of learning to walk.

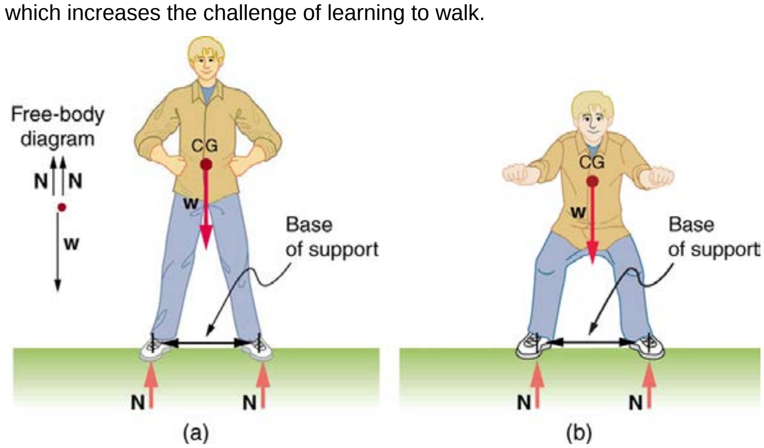


Figure 9.18 (a) The center of gravity of an adult is above the hip joints (one of the main pivots in the body) and lies between two narrowly-separated feet. Like a pencil standing on its eraser, this person is in stable equilibrium in relation to sideways displacements, but relatively small displacements take his cg outside the base of support and make him unstable. Humans are less stable relative to forward and backward displacements because the feet are not very long. Muscles are used extensively to balance the body in the front-to-back direction. (b) While bending in the manner shown, stability is increased by lowering the center of gravity. Stability is also increased if the base is expanded by placing the feet farther apart.

Animals such as chickens have easier systems to control. **Figure 9.19** shows that the cg of a chicken lies below its hip joints and between its widely separated and broad feet. Even relatively large displacements of the chicken's cg are stable and result in restoring forces and torques that return the cg to its equilibrium position with little effort on the chicken's part. Not all birds are like chickens, of course. Some birds, such as the flamingo, have balance systems that are almost as sophisticated as that of humans.

Figure 9.19 shows that the cg of a chicken is below the hip joints and lies above a broad base of support formed by widely-separated and large feet. Hence, the chicken is in very stable equilibrium, since a relatively large displacement is needed to render it unstable. The body of the chicken is supported from above by the hips and acts as a pendulum between the hips. Therefore, the chicken is stable for front-to-back displacements as well as for side-to-side displacements.

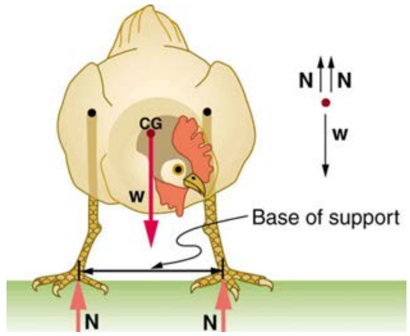


Figure 9.19 The center of gravity of a chicken is below the hip joints. The chicken is in stable equilibrium. The body of the chicken is supported from above by the hips and acts as a pendulum between them.

Engineers and architects strive to achieve extremely stable equilibriums for buildings and other systems that must withstand wind, earthquakes, and other forces that displace them from equilibrium. Although the examples in this section emphasize gravitational forces, the basic conditions for equilibrium are the same for all types of forces. The net external force must be zero, and the net torque must also be zero.

Take-Home Experiment

Stand straight with your heels, back, and head against a wall. Bend forward from your waist, keeping your heels and bottom against the wall, to touch your toes. Can you do this without toppling over? Explain why and what you need to do to be able to touch your toes without losing your balance. Is it easier for a woman to do this?

9.4 Applications of Statics, Including Problem-Solving Strategies

Learning Objectives

By the end of this section, you will be able to:

- Discuss the applications of statics in real life.
- State and discuss various problem-solving strategies in statics.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.F.1.1** The student is able to use representations of the relationship between force and torque. **(S.P. 1.4)**
- **3.F.1.2** The student is able to compare the torques on an object caused by various forces. **(S.P. 1.4)**
- **3.F.1.3** The student is able to estimate the torque on an object caused by various forces in comparison to other situations. **(S.P. 2.3)**
- **3.F.1.4** The student is able to design an experiment and analyze data testing a question about torques in a balanced rigid system. **(S.P. 4.1, 4.2, 5.1)**
- **3.F.1.5** The student is able to calculate torques on a two-dimensional system in static equilibrium, by examining a representation or model (such as a diagram or physical construction). **(S.P. 1.4, 2.2)**

Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We begin with a discussion of problem-solving strategies specifically used for statics. Since statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in **Problem-Solving Strategies**, still apply.

Problem-Solving Strategy: Static Equilibrium Situations

1. The first step is to determine whether or not the system is in **static equilibrium**. This condition is always the case when the *acceleration of the system is zero and accelerated rotation does not occur*.
2. It is particularly important to *draw a free body diagram for the system of interest*. Carefully label all forces, and note their relative magnitudes, directions, and points of application whenever these are known.
3. Solve the problem by applying either or both of the conditions for equilibrium (represented by the equations $\text{net } F = 0$ and $\text{net } \tau = 0$, depending on the list of known and unknown factors. If the second condition is involved,

choose the pivot point to simplify the solution. Any pivot point can be chosen, but the most useful ones cause torques by unknown forces to be zero. (Torque is zero if the force is applied at the pivot (then $r = 0$), or along a line through the pivot point (then $\theta = 0$)). Always choose a convenient coordinate system for projecting forces.

4. *Check the solution to see if it is reasonable* by examining the magnitude, direction, and units of the answer. The importance of this last step never diminishes, although in unfamiliar applications, it is usually more difficult to judge reasonableness. These judgments become progressively easier with experience.

Now let us apply this problem-solving strategy for the pole vaulter shown in the three figures below. The pole is uniform and has a mass of 5.00 kg. In **Figure 9.20**, the pole's cg lies halfway between the vaulter's hands. It seems reasonable that the force exerted by each hand is equal to half the weight of the pole, or 24.5 N. This obviously satisfies the first condition for equilibrium $(\text{net } F = 0)$. The second condition $(\text{net } \tau = 0)$ is also satisfied, as we can see by choosing the cg to be the pivot point. The weight exerts no torque about a pivot point located at the cg, since it is applied at that point and its lever arm is zero. The equal forces exerted by the hands are equidistant from the chosen pivot, and so they exert equal and opposite torques. Similar arguments hold for other systems where supporting forces are exerted symmetrically about the cg. For example, the four legs of a uniform table each support one-fourth of its weight.

In **Figure 9.20**, a pole vaulter holding a pole with its cg halfway between his hands is shown. Each hand exerts a force equal to half the weight of the pole, $F_R = F_L = w/2$. (b) The pole vaulter moves the pole to his left, and the forces that the hands exert are no longer equal. See **Figure 9.20**. If the pole is held with its cg to the left of the person, then he must push down with his right hand and up with his left. The forces he exerts are larger here because they are in opposite directions and the cg is at a long distance from either hand.

Similar observations can be made using a meter stick held at different locations along its length.

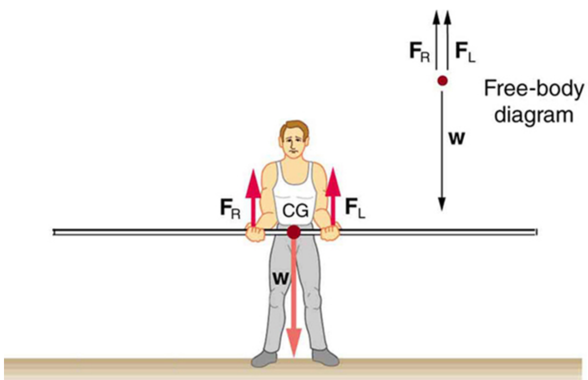


Figure 9.20 A pole vaulter holds a pole horizontally with both hands.

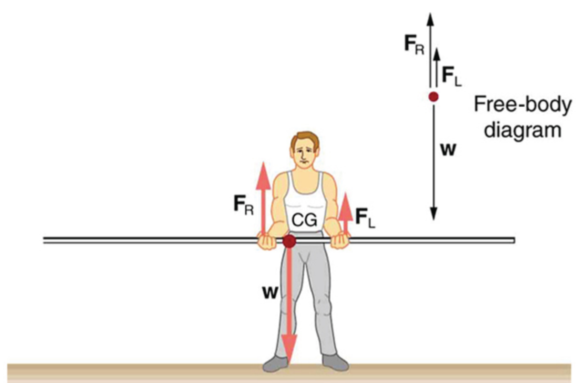


Figure 9.21 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is near his right hand.

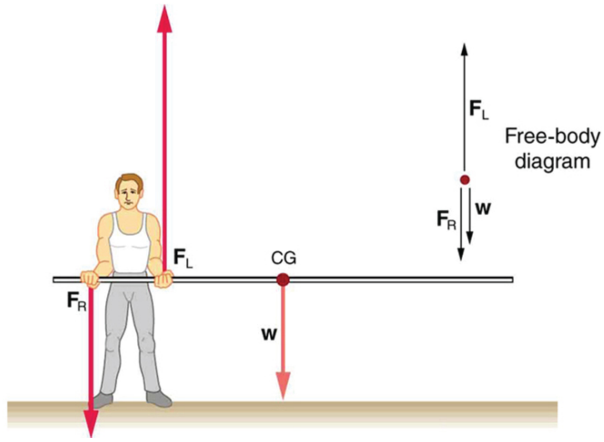


Figure 9.22 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter.

If the pole vaulter holds the pole as shown in **Figure 9.20**, the situation is not as simple. The total force he exerts is still equal to the weight of the pole, but it is not evenly divided between his hands. (If $F_L = F_R$, then the torques about the cg would not be equal since the lever arms are different.) Logically, the right hand should support more weight, since it is closer to the cg. In fact, if the right hand is moved directly under the cg, it will support all the weight. This situation is exactly analogous to two people carrying a load; the one closer to the cg carries more of its weight. Finding the forces F_L and F_R is straightforward, as the next example shows.

If the pole vaulter holds the pole from near the end of the pole (**Figure 9.22**), the direction of the force applied by the right hand of the vaulter reverses its direction.

Example 9.2 What Force Is Needed to Support a Weight Held Near Its CG?

For the situation shown in **Figure 9.20**, calculate: (a) F_R, the force exerted by the right hand, and (b) F_L, the force exerted by the left hand. The hands are 0.900 m apart, and the cg of the pole is 0.600 m from the left hand.

Strategy

Figure 9.20 includes a free body diagram for the pole, the system of interest. There is not enough information to use the first condition for equilibrium $(\text{net } F = 0)$, since two of the three forces are unknown and the hand forces cannot be assumed to be equal in this case. There is enough information to use the second condition for equilibrium $(\text{net } \tau = 0)$ if the pivot point is chosen to be at either hand, thereby making the torque from that hand zero. We choose to locate the pivot at the left hand in this part of the problem, to eliminate the torque from the left hand.

Solution for (a)

There are now only two nonzero torques, those from the gravitational force (τ_w) and from the push or pull of the right hand (τ_R). Stating the second condition in terms of clockwise and counterclockwise torques,

$$\text{net } \tau_\text{cw} = –\text{net } \tau_\text{ccw}. \tag{9.22}$$

or the algebraic sum of the torques is zero.

Here this is

$$\tau_R = –\tau_\text{w} \tag{9.23}$$

since the weight of the pole creates a counterclockwise torque and the right hand counters with a clockwise toque. Using the definition of torque, $\tau = rF \sin \theta$, noting that $\theta = 90^\text{o}$, and substituting known values, we obtain

$$(0.900 \text{ m})(F_R) = (0.600 \text{ m})(mg). \tag{9.24}$$

Thus,

$$\begin{aligned} F_R &= (0.667)(5.00 \text{ kg})\left(9.80 \text{ m/s}^2\right) \\ &= 32.7 \text{ N}. \end{aligned} \tag{9.25}$$

Solution for (b)

The first condition for equilibrium is based on the free body diagram in the figure. This implies that by Newton's second law:

$$F_L + F_R - mg = 0 \tag{9.26}$$

From this we can conclude:

$$F_L + F_R = w = mg \tag{9.27}$$

Solving for F_L, we obtain

$$\begin{aligned} F_L &= mg - F_R \\ &= mg - 32.7\text{ N} \\ &= (5.00\text{ kg})\left(9.80\text{ m/s}^2\right) - 32.7\text{ N} \\ &= 16.3\text{ N} \end{aligned} \tag{9.28}$$

Discussion

F_L is seen to be exactly half of F_R, as we might have guessed, since F_L is applied twice as far from the cg as F_R.

If the pole vaulter holds the pole as he might at the start of a run, shown in **Figure 9.22**, the forces change again. Both are considerably greater, and one force reverses direction.

Take-Home Experiment

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!

PhET Explorations: Balancing Act

Play with objects on a teeter totter to learn about balance. Test what you've learned by trying the Balance Challenge game.

PhET Interactive Simulation

Figure 9.23 Balancing Act (http://phet.colorado.edu/en/simulation/balancing-act)

9.5 Simple Machines

Learning Objectives

By the end of this section, you will be able to:

- Describe different simple machines.
- Calculate the mechanical advantage.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.F.1.1** The student is able to use representations of the relationship between force and torque. **(S.P. 1.4)**
- **3.F.1.2** The student is able to compare the torques on an object caused by various forces. **(S.P. 1.4)**
- **3.F.1.3** The student is able to estimate the torque on an object caused by various forces in comparison to other situations. **(S.P. 2.3)**
- **3.F.1.5** The student is able to calculate torques on a two-dimensional system in static equilibrium, by examining a representation or model (such as a diagram or physical construction). **(S.P. 1.4, 2.2)**

Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we apply the force. The word for “machine” comes from the Greek word meaning “to help make things easier.” Levers, gears, pulleys, wedges, and screws are some examples of machines. Energy is still conserved for these devices because a machine cannot do more work than the energy put into it. However, machines can reduce the input force that is needed to perform the job. The ratio of output to input force magnitudes for any simple machine is called its **mechanical advantage** (MA).

$$\text{MA} = \frac{F_\text{o}}{F_\text{i}} \tag{9.29}$$

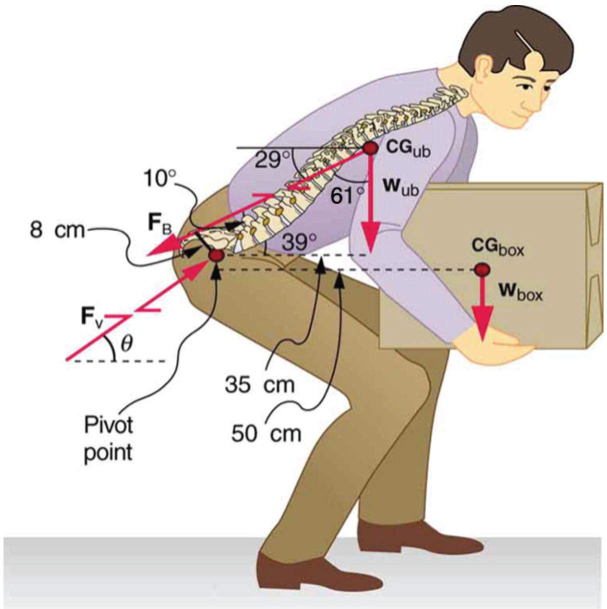


Figure 9.31 This figure shows that large forces are exerted by the back muscles and experienced in the vertebrae when a person lifts with their back, since these muscles have small effective perpendicular lever arms. The data shown here are analyzed in the preceding example, **Example 9.5**.

What are the benefits of having most skeletal muscles attached so close to joints? One advantage is speed because small muscle contractions can produce large movements of limbs in a short period of time. Other advantages are flexibility and agility, made possible by the large numbers of joints and the ranges over which they function. For example, it is difficult to imagine a system with biceps muscles attached at the wrist that would be capable of the broad range of movement we vertebrates possess.

There are some interesting complexities in real systems of muscles, bones, and joints. For instance, the pivot point in many joints changes location as the joint is flexed, so that the perpendicular lever arms and the mechanical advantage of the system change, too. Thus the force the biceps muscle must exert to hold up a book varies as the forearm is flexed. Similar mechanisms operate in the legs, which explain, for example, why there is less leg strain when a bicycle seat is set at the proper height. The methods employed in this section give a reasonable description of real systems provided enough is known about the dimensions of the system. There are many other interesting examples of force and torque in the body—a few of these are the subject of end-of-chapter problems.

Glossary

center of gravity: the point where the total weight of the body is assumed to be concentrated

dynamic equilibrium: a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

mechanical advantage: the ratio of output to input forces for any simple machine

neutral equilibrium: a state of equilibrium that is independent of a system's displacements from its original position

perpendicular lever arm: the shortest distance from the pivot point to the line along which $\mathbf{F}$ lies

SI units of torque: newton times meters, usually written as N·m

stable equilibrium: a system, when displaced, experiences a net force or torque in a direction opposite to the direction of the displacement

static equilibrium: a state of equilibrium in which the net external force and torque acting on a system is zero

static equilibrium: equilibrium in which the acceleration of the system is zero and accelerated rotation does not occur

torque: turning or twisting effectiveness of a force

unstable equilibrium: a system, when displaced, experiences a net force or torque in the same direction as the displacement from equilibrium

Section Summary

9.1 The First Condition for Equilibrium

- Statics is the study of forces in equilibrium.

- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that net $\mathbf{F} = 0$.

9.2 The Second Condition for Equilibrium

- The second condition assures those torques are also balanced. Torque is the rotational equivalent of a force in producing a rotation and is defined to be
$$\tau = rF\sin\theta$$
where τ is torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between $\mathbf{F}$ and the vector directed from the point where the force acts to the pivot point. The perpendicular lever arm $r_\perp$ is defined to be
$$r_\perp = r\sin\theta$$
so that
$$\tau = r_\perp F.$$
- The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which F acts. The SI unit for torque is newton-meter $(\mathrm{N{\cdot}m})$. The second condition necessary to achieve equilibrium is that the net external torque on a system must be zero:
$$\text{net } \tau = 0$$
By convention, counterclockwise torques are positive, and clockwise torques are negative.

9.3 Stability

- A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite the direction of the displacement.
- A system is in unstable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium.
- A system is in neutral equilibrium if its equilibrium is independent of displacements from its original position.

9.4 Applications of Statics, Including Problem-Solving Strategies

- Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We have discussed the problem-solving strategies specifically useful for statics. Statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in **Problem-Solving Strategies**, still apply.

9.5 Simple Machines

- Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we have to apply the force.
- The ratio of output to input forces for any simple machine is called its mechanical advantage
- A few simple machines are the lever, nail puller, wheelbarrow, crank, etc.

9.6 Forces and Torques in Muscles and Joints

- Statics plays an important part in understanding everyday strains in our muscles and bones.
- Many lever systems in the body have a mechanical advantage of significantly less than one, as many of our muscles are attached close to joints.
- Someone with good posture stands or sits in such as way that their center of gravity lies directly above the pivot point in their hips, thereby avoiding back strain and damage to disks.

Conceptual Questions

9.1 The First Condition for Equilibrium

1. What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.

2. Under what conditions can a rotating body be in equilibrium? Give an example.

9.2 The Second Condition for Equilibrium

3. What three factors affect the torque created by a force relative to a specific pivot point?

4. A wrecking ball is being used to knock down a building. One tall unsupported concrete wall remains standing. If the wrecking ball hits the wall near the top, is the wall more likely to fall over by rotating at its base or by falling straight down? Explain your answer. How is it most likely to fall if it is struck with the same force at its base? Note that this depends on how firmly the wall is attached at its base.

5. Mechanics sometimes put a length of pipe over the handle of a wrench when trying to remove a very tight bolt. How does this help? (It is also hazardous since it can break the bolt.)

9.3 Stability

6. A round pencil lying on its side as in **Figure 9.14** is in neutral equilibrium relative to displacements perpendicular to its length. What is its stability relative to displacements parallel to its length?

7. Explain the need for tall towers on a suspension bridge to ensure stable equilibrium.

9.4 Applications of Statics, Including Problem-Solving Strategies

8. When visiting some countries, you may see a person balancing a load on the head. Explain why the center of mass of the load needs to be directly above the person's neck vertebrae.

9.5 Simple Machines

9. Scissors are like a double-lever system. Which of the simple machines in **Figure 9.24** and **Figure 9.25** is most analogous to scissors?

10. Suppose you pull a nail at a constant rate using a nail puller as shown in **Figure 9.24**. Is the nail puller in equilibrium? What if you pull the nail with some acceleration – is the nail puller in equilibrium then? In which case is the force applied to the nail puller larger and why?

11. Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

12. Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces (see previous Question)?

9.6 Forces and Torques in Muscles and Joints

13. Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

14. Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces?

15. Certain types of dinosaurs were bipedal (walked on two legs). What is a good reason that these creatures invariably had long tails if they had long necks?

16. Swimmers and athletes during competition need to go through certain postures at the beginning of the race. Consider the balance of the person and why start-offs are so important for races.

17. If the maximum force the biceps muscle can exert is 1000 N, can we pick up an object that weighs 1000 N? Explain your answer.

18. Suppose the biceps muscle was attached through tendons to the upper arm close to the elbow and the forearm near the wrist. What would be the advantages and disadvantages of this type of construction for the motion of the arm?

19. Explain one of the reasons why pregnant women often suffer from back strain late in their pregnancy.

Problems & Exercises

9.2 The Second Condition for Equilibrium

1. (a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?

2. When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton × meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.

3. Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.

4. Use the second condition for equilibrium $(\text{net } \tau = 0)$ to calculate F_p in **Example 9.1**, employing any data given or solved for in part (a) of the example.

5. Repeat the seesaw problem in **Example 9.1** with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium.

9.3 Stability

6. Suppose a horse leans against a wall as in **Figure 9.32**. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal in magnitude and opposite in direction to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.

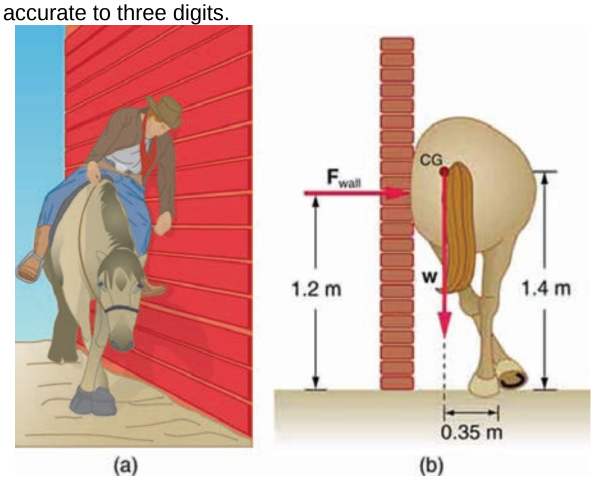


Figure 9.32

7. Two children of mass 20 kg and 30 kg sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3 m, at what distance from the pivot point is the small child sitting in order to maintain the balance?

8. (a) Calculate the magnitude and direction of the force on each foot of the horse in **Figure 9.32** (two are on the ground), assuming the center of mass of the horse is midway between the feet. The total mass of the horse and rider is 500kg. (b) What is the minimum coefficient of friction between the hooves and ground? Note that the force exerted by the wall is horizontal.

9. A person carries a plank of wood 2 m long with one hand pushing down on it at one end with a force F_1 and the other hand holding it up at 50 cm from the end of the plank with force F_2. If the plank has a mass of 20 kg and its center of gravity is at the middle of the plank, what are the magnitudes of the forces F_1 and F_2?

10. A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in **Figure 9.33**. The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.

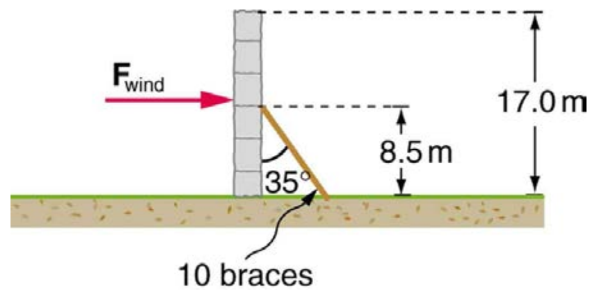


Figure 9.33

11. (a) What force must be exerted by the wind to support a 2.50-kg chicken in the position shown in **Figure 9.34**? (b) What is the ratio of this force to the chicken's weight? (c) Does this support the contention that the chicken has a relatively stable construction?

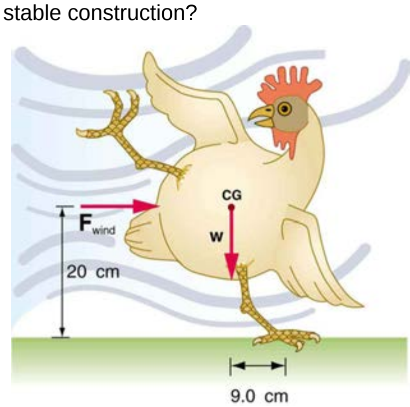


Figure 9.34

12. Suppose the weight of the drawbridge in **Figure 9.35** is supported entirely by its hinges and the opposite shore, so that its cables are slack. (a) What fraction of the weight is supported by the opposite shore if the point of support is directly beneath the cable attachments? (b) What is the direction and magnitude of the force the hinges exert on the bridge under these circumstances? The mass of the bridge is 2500 kg.

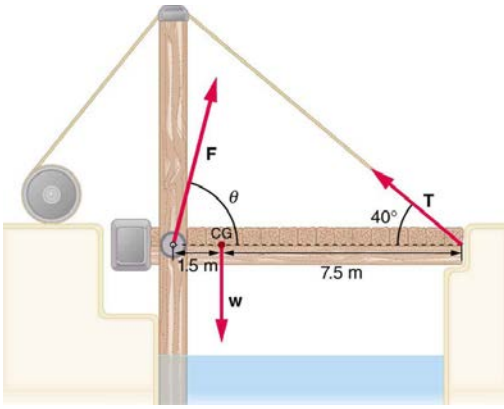


Figure 9.35 A small drawbridge, showing the forces on the hinges ($\mathbf{F}$), its weight ($\mathbf{w}$), and the tension in its wires ($\mathbf{T}$).

13. Suppose a 900-kg car is on the bridge in **Figure 9.35** with its center of mass halfway between the hinges and the cable attachments. (The bridge is supported by the cables and hinges only.) (a) Find the force in the cables. (b) Find the direction and magnitude of the force exerted by the hinges on the bridge.

14. A sandwich board advertising sign is constructed as shown in **Figure 9.36**. The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?

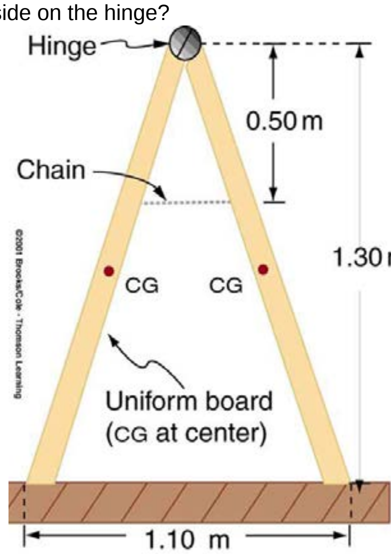


Figure 9.36 A sandwich board advertising sign demonstrates tension.

15. (a) What minimum coefficient of friction is needed between the legs and the ground to keep the sign in **Figure 9.36** in the position shown if the chain breaks? (b) What force is exerted by each side on the hinge?

16. A gymnast is attempting to perform splits. From the information given in **Figure 9.37**, calculate the magnitude and direction of the force exerted on each foot by the floor.

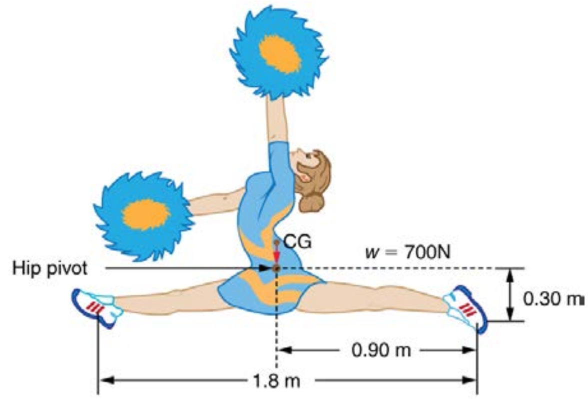


Figure 9.37 A gymnast performs full split. The center of gravity and the various distances from it are shown.

9.4 Applications of Statics, Including Problem-Solving Strategies

17. To get up on the roof, a person (mass 70.0 kg) places a 6.00-m aluminum ladder (mass 10.0 kg) against the house on a concrete pad with the base of the ladder 2.00 m from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 m from the bottom. The person is standing 3 m from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?

18. In **Figure 9.22**, the cg of the pole held by the pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by (a) his right hand and (b) his left hand. (c) If each hand supports half the weight of the pole in **Figure 9.20**, show that the second condition for equilibrium $(\text{net } \tau = 0)$ is satisfied for a pivot other than the one located at the center of gravity of the pole. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium described above.

9.5 Simple Machines

19. What is the mechanical advantage of a nail puller—similar to the one shown in **Figure 9.24** —where you exert a force 45 cm from the pivot and the nail is 1.8 cm on the other side? What minimum force must you exert to apply a force of 1250 N to the nail?

20. Suppose you needed to raise a 250-kg mower a distance of 6.0 cm above the ground to change a tire. If you had a 2.0-m long lever, where would you place the fulcrum if your force was limited to 300 N?

21. a) What is the mechanical advantage of a wheelbarrow, such as the one in **Figure 9.25**, if the center of gravity of the wheelbarrow and its load has a perpendicular lever arm of 5.50 cm, while the hands have a perpendicular lever arm of 1.02 m? (b) What upward force should you exert to support the wheelbarrow and its load if their combined mass is 55.0 kg? (c) What force does the wheel exert on the ground?

22. A typical car has an axle with 1.10 cm radius driving a tire with a radius of 27.5 cm . What is its mechanical advantage assuming the very simplified model in **Figure 9.26**(b)?

23. What force does the nail puller in **Exercise 9.19** exert on the supporting surface? The nail puller has a mass of 2.10 kg.

24. If you used an ideal pulley of the type shown in **Figure 9.27**(a) to support a car engine of mass 115 kg, (a) What would be the tension in the rope? (b) What force must the ceiling supply, assuming you pull straight down on the rope? Neglect the pulley system's mass.

25. Repeat **Exercise 9.24** for the pulley shown in **Figure 9.27**(c), assuming you pull straight up on the rope. The pulley system's mass is 7.00 kg.

9.6 Forces and Torques in Muscles and Joints

26. Verify that the force in the elbow joint in **Example 9.4** is 407 N, as stated in the text.

27. Two muscles in the back of the leg pull on the Achilles tendon as shown in **Figure 9.38**. What total force do they exert?

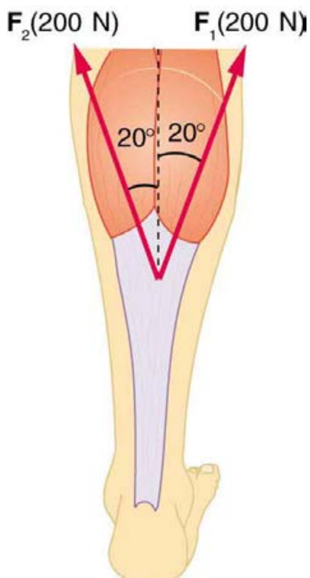


Figure 9.38 The Achilles tendon of the posterior leg serves to attach plantaris, gastrocnemius, and soleus muscles to calcaneus bone.

28. The upper leg muscle (quadriceps) exerts a force of 1250 N, which is carried by a tendon over the kneecap (the patella) at the angles shown in **Figure 9.39**. Find the direction and magnitude of the force exerted by the kneecap on the upper leg bone (the femur).

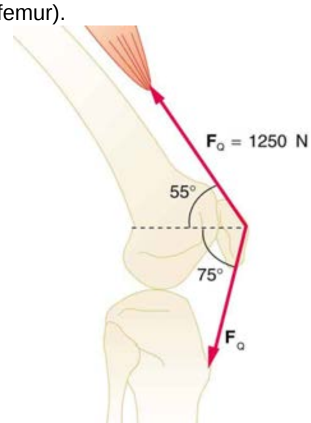


Figure 9.39 The knee joint works like a hinge to bend and straighten the lower leg. It permits a person to sit, stand, and pivot.

29. A device for exercising the upper leg muscle is shown in **Figure 9.40**, together with a schematic representation of an equivalent lever system. Calculate the force exerted by the upper leg muscle to lift the mass at a constant speed. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium in **Applications of Statistics, Including Problem-Solving Strategies**.

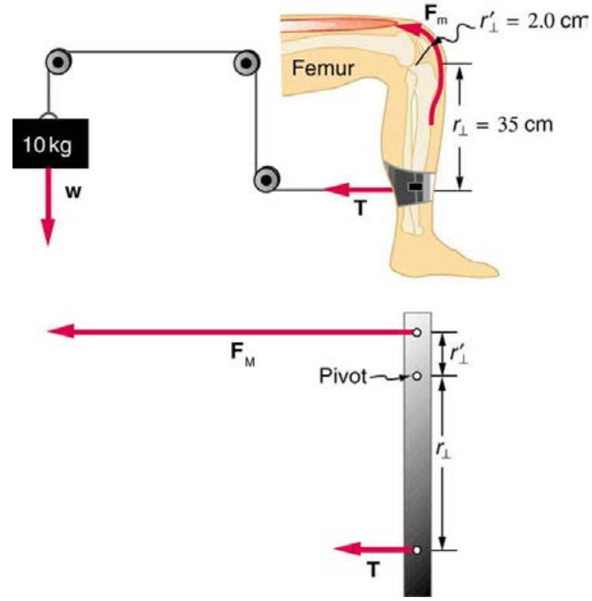


Figure 9.40 A mass is connected by pulleys and wires to the ankle in this exercise device.

30. A person working at a drafting board may hold her head as shown in **Figure 9.41**, requiring muscle action to support the head. The three major acting forces are shown. Calculate the direction and magnitude of the force supplied by the upper vertebrae $\mathbf{F}_\text{V}$ to hold the head stationary, assuming that this force acts along a line through the center of mass as do the weight and muscle force.

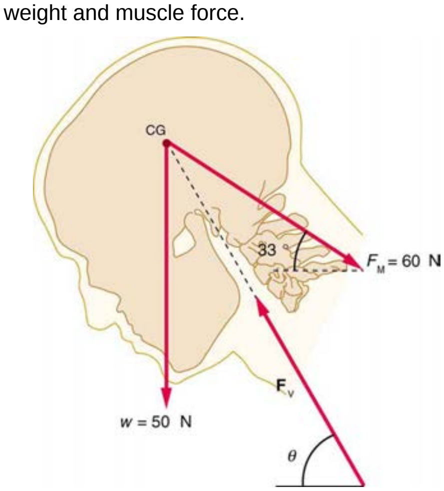


Figure 9.41

31. We analyzed the biceps muscle example with the angle between forearm and upper arm set at 90^o. Using the same numbers as in **Example 9.4**, find the force exerted by the biceps muscle when the angle is 120^o and the forearm is in a downward position.

32. Even when the head is held erect, as in **Figure 9.42**, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?

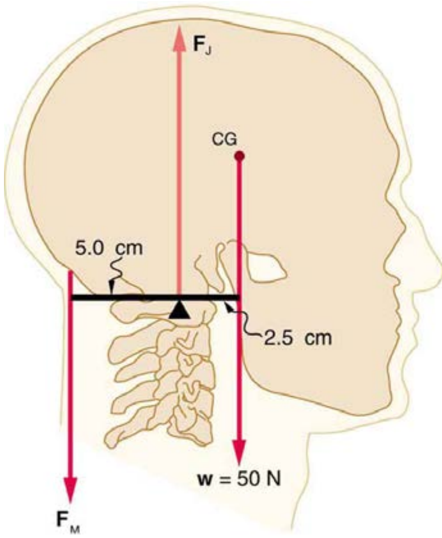


Figure 9.42 The center of mass of the head lies in front of its major point of support, requiring muscle action to hold the head erect. A simplified lever system is shown.

33. A 75-kg man stands on his toes by exerting an upward force through the Achilles tendon, as in **Figure 9.43**. (a) What is the force in the Achilles tendon if he stands on one foot? (b) Calculate the force at the pivot of the simplified lever system shown—that force is representative of forces in the ankle joint.

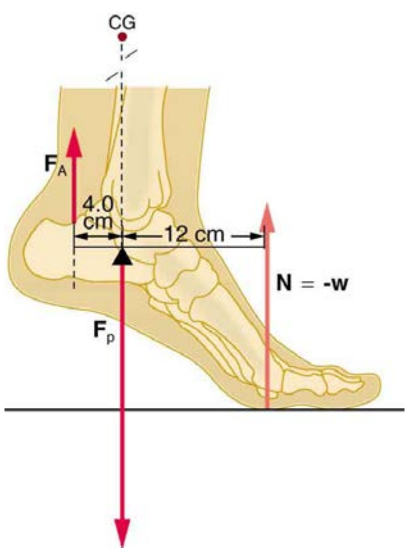


Figure 9.43 The muscles in the back of the leg pull the Achilles tendon when one stands on one's toes. A simplified lever system is shown.

34. A father lifts his child as shown in **Figure 9.44**. What force should the upper leg muscle exert to lift the child at a constant speed?

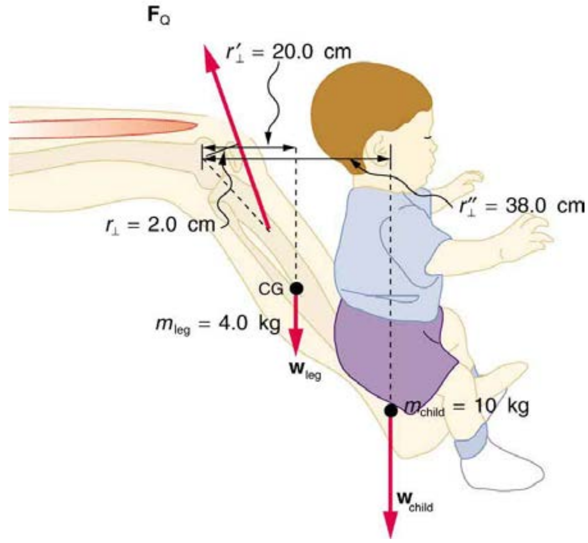


Figure 9.44 A child being lifted by a father's lower leg.

35. Unlike most of the other muscles in our bodies, the masseter muscle in the jaw, as illustrated in **Figure 9.45**, is attached relatively far from the joint, enabling large forces to be exerted by the back teeth. (a) Using the information in the figure, calculate the force exerted by the lower teeth on the bullet. (b) Calculate the force on the joint.

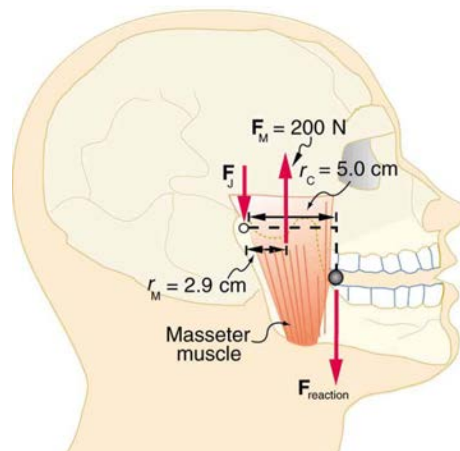


Figure 9.45 A person clenching a bullet between his teeth.

36. Integrated Concepts

Suppose we replace the 4.0-kg book in **Exercise 9.31** of the biceps muscle with an elastic exercise rope that obeys Hooke's Law. Assume its force constant $k = 600\ \text{N/m}$. (a) How much is the rope stretched (past equilibrium) to provide the same force F_B as in this example? Assume the rope is held in the hand at the same location as the book. (b) What force is on the biceps muscle if the exercise rope is pulled straight up so that the forearm makes an angle of 25^o with the horizontal? Assume the biceps muscle is still perpendicular to the forearm.

37. (a) What force should the woman in **Figure 9.46** exert on the floor with each hand to do a push-up? Assume that she moves up at a constant speed. (b) The triceps muscle at the back of her upper arm has an effective lever arm of 1.75 cm, and she exerts force on the floor at a horizontal distance of 20.0 cm from the elbow joint. Calculate the magnitude of the force in each triceps muscle, and compare it to her weight. (c)

How much work does she do if her center of mass rises 0.240 m? (d) What is her useful power output if she does 25 pushups in one minute?

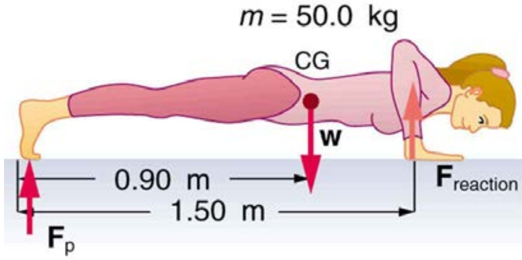


Figure 9.46 A woman doing pushups.

38. You have just planted a sturdy 2-m-tall palm tree in your front lawn for your mother's birthday. Your brother kicks a 500 g ball, which hits the top of the tree at a speed of 5 m/s and stays in contact with it for 10 ms. The ball falls to the ground near the base of the tree and the recoil of the tree is minimal. (a) What is the force on the tree? (b) The length of the sturdy section of the root is only 20 cm. Furthermore, the soil around the roots is loose and we can assume that an effective force is applied at the tip of the 20 cm length. What is the effective force exerted by the end of the tip of the root to keep the tree from toppling? Assume the tree will be uprooted rather than bend. (c) What could you have done to ensure that the tree does not uproot easily?

39. Unreasonable Results

Suppose two children are using a uniform seesaw that is 3.00 m long and has its center of mass over the pivot. The first child has a mass of 30.0 kg and sits 1.40 m from the pivot. (a) Calculate where the second 18.0 kg child must sit to balance the seesaw. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

40. Construct Your Own Problem

Consider a method for measuring the mass of a person's arm in anatomical studies. The subject lies on her back, extends her relaxed arm to the side and two scales are placed below the arm. One is placed under the elbow and the other under the back of her hand. Construct a problem in which you calculate the mass of the arm and find its center of mass based on the scale readings and the distances of the scales from the shoulder joint. You must include a free body diagram of the arm to direct the analysis. Consider changing the position of the scale under the hand to provide more information, if needed. You may wish to consult references to obtain reasonable mass values.

Test Prep for AP® Courses

9.2 The Second Condition for Equilibrium

1. Which of the following is not an example of an object undergoing a torque?

a. A car is rounding a bend at a constant speed.
b. A merry-go-round increases from rest to a constant rotational speed.
c. A pendulum swings back and forth.
d. A bowling ball rolls down a bowling alley.

2. Five forces of equal magnitude, labeled ***A***–***E***, are applied to the object shown below. If the object is anchored at point ***P***, which force will provide the greatest torque?

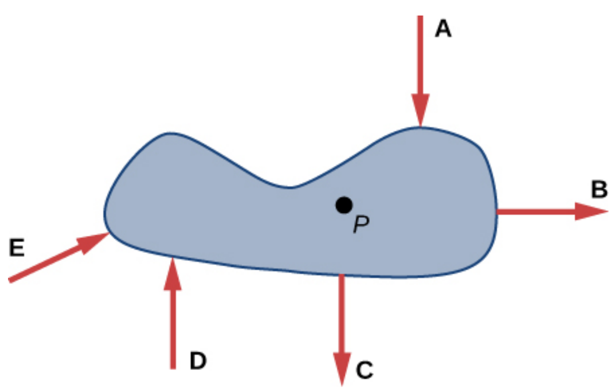


Figure 9.47 Five forces acting on an object.

a. Force ***A***

b. Force ***B***
c. Force ***C***
d. Force ***D***
e. Force ***E***

9.3 Stability

3. Using the concept of torque, explain why a traffic cone placed on its base is in stable equilibrium, while a traffic cone placed on its tip is in unstable equilibrium.

9.4 Applications of Statics, Including Problem-Solving Strategies

4. A child sits on the end of a playground see-saw. Which of the following values is the most appropriate estimate of the torque created by the child?

a. 6 N•m
b. 60 N•m
c. 600 N•m
d. 6000 N•m

5. A group of students is stacking a set of identical books, each one overhanging the one below it by 1 inch. They would like to estimate how many books they could place on top of each other before the stack tipped. What information below would they need to know to make this calculation?

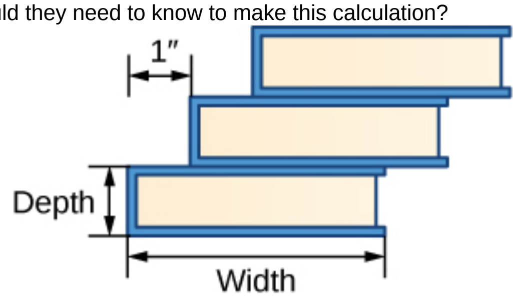


Figure 9.48 3 overlapping stacked books.

I. The mass of each book
II. The width of each book
III. The depth of each book

a. I only
b. I and II only
c. I and III only
d. II only
e. I, II, and III

6. A 10 N board of uniform density is 5 meters long. It is supported on the left by a string bearing a 3 N upward force. In order to prevent the string from breaking, a person must place an upward force of 7 N at a position along the bottom surface of the board. At what distance from its left edge would they need to place this force in order for the board to be in static equilibrium?

a. $\frac{3}{7}$ m
b. $\frac{5}{2}$ m
c. $\frac{25}{7}$ m
d. $\frac{30}{7}$ m
e. 5 m

7. A bridge is supported by two piers located 20 meters apart. Both the left and right piers provide an upward force on the bridge, labeled $\mathbf{F}_L$ and $\mathbf{F}_R$ respectively.

a. If a 1000 kg car comes to rest at a point 5 meters from the left pier, how much force will the bridge provide to the left and right piers?
b. How will $\mathbf{F}_L$ and $\mathbf{F}_R$ change as the car drives to the right side of the bridge?

8. An object of unknown mass is provided to a student. Without using a scale, design an experimental procedure detailing how the magnitude of this mass could be experimentally found. Your explanation must include the concept of torque and all steps should be provided in an orderly sequence. You may include a labeled diagram of your setup to help in your description. Include enough detail so that another student could carry out your procedure.

9.5 Simple Machines

9. As a young student, you likely learned that simple machines are capable of increasing the ability to lift and move objects. Now, as an educated AP Physics student, you are aware that this capability is governed by the relationship between force and torque.

In the space below, explain why torque is integral to the increase in force created by a simple machine. You may use an example or diagram to assist in your explanation. Be sure to cite the mechanical advantage in your explanation as well.

10. Figure 9.24(a) shows a wheelbarrow being lifted by an applied force $\mathbf{F}_i$. If the wheelbarrow is filled with twenty bricks massing 3 kg each, estimate the value of the applied force $\mathbf{F}_i$. Provide an explanation behind the total weight ***w*** and any reasoning toward your final answer. Additionally, provide a range of values over which you feel the force could exist.

9.6 Forces and Torques in Muscles and Joints

11. When you use your hand to raise a 20 lb dumbbell in a curling motion, the force on your bicep muscle is not equal to 20 lb.

a. Compare the size of the force placed on your bicep muscle to the force of the 20 lb dumbbell lifted by your hand. Using the concept of torque, which force is greater and explain why the two forces are not identical.
b. Does the force placed on your bicep muscle change as you curl the weight closer toward your body? (In other words, is the force on your muscle different when your forearm is 90° to your upper arm than when it is 45° to your upper arm?) Explain your answer using torque.

10 ROTATIONAL MOTION AND ANGULAR MOMENTUM

Figure 10.1 The mention of a tornado conjures up images of raw destructive power. Tornadoes blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw. They descend from clouds in funnel-like shapes that spin violently, particularly at the bottom where they are most narrow, producing winds as high as 500 km/h. (credit: Daphne Zaras, U.S. National Oceanic and Atmospheric Administration)

Chapter Outline

Connection for AP® Courses

Why do tornados spin? And why do tornados spin so rapidly? The answer is that the air masses that produce tornados are themselves rotating, and when the radii of the air masses decrease, their rate of rotation increases. An ice skater increases her spin in an exactly analogous manner, as seen in Figure 10.2. The skater starts her rotation with outstretched limbs and increases her rate of spin by pulling them in toward her body. The same physics describes the exhilarating spin of a skater and the wrenching force of a tornado. We will find that this is another example of the importance of conservation laws and their role in determining how changes happen in a system, supporting Big Idea 5. The idea that a change of a conserved quantity is always equal to the transfer of that quantity between interacting systems (Enduring Understanding 5.A) is presented for both energy and angular momentum (Enduring Understanding 5.E). The conservation of **angular momentum** in relation to the external net **torque** (Essential Knowledge 5.E.1) parallels that of linear momentum conservation in relation to the external net force. The concept of **rotational inertia** is introduced, a concept that takes into account not only the mass of an object or a system, but also the distribution of mass within the object or system. Therefore, changes in the rotational inertia of a system could lead to changes in the motion (Essential Knowledge 5.E.2) of the system. We shall see that all important aspects of rotational motion either have already been defined for linear motion or have exact analogues in linear motion.

Clearly, therefore, force, energy, and power are associated with rotational motion. This supports Big Idea 3, that interactions are described by forces. The ability of forces to cause torques (Enduring Understanding 3.F) is extended to the interactions between objects that result in nonzero net torque. This nonzero net torque in turn causes changes in the rotational motion of an object (Essential Knowledge 3.F.2) and results in changes of the angular momentum of an object (Essential Knowledge 3.F.3).

Similarly, Big Idea 4, that interactions between systems cause changes in those systems, is supported by the empirical observation that when torques are exerted on rigid bodies these torques cause changes in the angular momentum of the system (Enduring Understanding 4.D).

Again, there is a clear analogy between linear and rotational motion in this interaction. Both the angular kinematics variables (angular displacement, angular velocity, and **angular acceleration**) and the dynamics variables (torque and angular momentum) are vectors with direction depending on whether the rotation is clockwise or counterclockwise with respect to an axis of rotation (Essential Knowledge 4.D.1). The angular momentum of the system can change due to interactions (Essential Knowledge 4.D.2). This change is defined as the product of the average torque and the time interval during which torque is exerted (Essential Knowledge 4.D.3), analogous to the impulse-momentum theorem for linear motion.

The concepts in this chapter support:

Big Idea 3. The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.F. A force exerted on an object can cause a torque on that object.

Extended Knowledge 3.F.2. The presence of a net torque along any axis will cause a rigid system to change its rotational motion or an object to change its rotational motion about that axis.

Extended Knowledge 3.F.3. A torque exerted on an object can change the angular momentum of an object.

Big Idea 4. Interactions between systems can result in changes in those systems.

Enduring Understanding 4.D. A net torque exerted on a system by other objects or systems will change the angular momentum of the system.

Extended Knowledge 4.D.1. Torque, angular velocity, angular acceleration, and angular momentum are vectors and can be characterized as positive or negative depending upon whether they give rise to or correspond to counterclockwise or clockwise rotation with respect to an axis.

Extended Knowledge 4.D.2. The angular momentum of a system may change due to interactions with other objects or systems.

Extended Knowledge 4.D.3. The change in angular momentum is given by the product of the average torque and the time interval during which the torque is exerted.

Big Idea 5. Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A. Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Extended Knowledge 5.A.2. For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved.

Enduring Understanding 5.E. The angular momentum of a system is conserved.

Extended Knowledge 5.E.1. If the net external torque exerted on the system is zero, the angular momentum of the system does not change.

Extended Knowledge 5.E.2. The angular momentum of a system is determined by the locations and velocities of the objects that make up the system. The rotational inertia of an object or system depends upon the distribution of mass within the object or system. Changes in the radius of a system or in the distribution of mass within the system result in changes in the system's rotational inertia, and hence in its angular velocity and linear speed for a given angular momentum. Examples should include elliptical orbits in an Earth-satellite system. Mathematical expressions for the moments of inertia will be provided where needed. Students will not be expected to know the parallel axis theorem.

Figure 10.2 This figure skater increases her rate of spin by pulling her arms and her extended leg closer to her axis of rotation. (credit: Luu, Wikimedia Commons)

10.1 Angular Acceleration

Learning Objectives

By the end of this section, you will be able to:

- Describe uniform circular motion.
- Explain nonuniform circular motion.
- Calculate angular acceleration of an object.
- Observe the link between linear and angular acceleration.

Uniform Circular Motion and Gravitation discussed only uniform circular motion, which is motion in a circle at constant speed and, hence, constant angular velocity. Recall that angular velocity ω was defined as the time rate of change of angle θ:

$$\omega = \frac{\Delta\theta}{\Delta t}, \tag{10.1}$$

where θ is the angle of rotation as seen in **Figure 10.3**. The relationship between angular velocity ω and linear velocity v was also defined in **Rotation Angle and Angular Velocity** as

$$v = r\omega \tag{10.2}$$

or

$$\omega = \frac{v}{r}, \tag{10.3}$$

where r is the radius of curvature, also seen in **Figure 10.3**. According to the sign convention, the counter clockwise direction is considered as positive direction and clockwise direction as negative

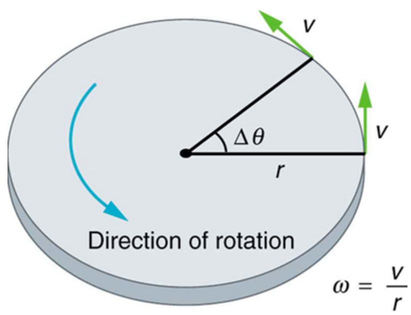


Figure 10.3 This figure shows uniform circular motion and some of its defined quantities.

Angular velocity is not constant when a skater pulls in her arms, when a child starts up a merry-go-round from rest, or when a computer's hard disk slows to a halt when switched off. In all these cases, there is an **angular acceleration**, in which ω changes. The faster the change occurs, the greater the angular acceleration. Angular acceleration α is defined as the rate of change of angular velocity. In equation form, angular acceleration is expressed as follows:

$$\alpha = \frac{\Delta\omega}{\Delta t}, \tag{10.4}$$

where $\Delta\omega$ is the **change in angular velocity** and Δt is the change in time. The units of angular acceleration are $(\text{rad/s})/\text{s}$, or rad/s^2. If ω increases, then α is positive. If ω decreases, then α is negative.

Example 10.1 Calculating the Angular Acceleration and Deceleration of a Bike Wheel

Suppose a teenager puts her bicycle on its back and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s. (a) Calculate the angular acceleration in rad/s^2. (b) If she now slams on the brakes, causing an angular acceleration of $-87.3\ \text{rad/s}^2$, how long does it take the wheel to stop?

Strategy for (a)

The angular acceleration can be found directly from its definition in $\alpha = \frac{\Delta\omega}{\Delta t}$ because the final angular velocity and time are given. We see that $\Delta\omega$ is 250 rpm and Δt is 5.00 s.

Solution for (a)

Entering known information into the definition of angular acceleration, we get

$$\begin{aligned}\alpha &= \frac{\Delta\omega}{\Delta t} \\ &= \frac{250 \text{ rpm}}{5.00 \text{ s}}.\end{aligned} \tag{10.5}$$

Because $\Delta\omega$ is in revolutions per minute (rpm) and we want the standard units of rad/s^2 for angular acceleration, we need to convert $\Delta\omega$ from rpm to rad/s:

$$\begin{aligned}\Delta\omega &= 250\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \\ &= 26.2\frac{\text{rad}}{\text{s}}.\end{aligned} \tag{10.6}$$

Entering this quantity into the expression for α, we get

$$\begin{aligned}\alpha &= \frac{\Delta\omega}{\Delta t} \\ &= \frac{26.2 \text{ rad/s}}{5.00 \text{ s}} \\ &= 5.24 \text{ rad/s}^2.\end{aligned} \tag{10.7}$$

Strategy for (b)

In this part, we know the angular acceleration and the initial angular velocity. We can find the stoppage time by using the definition of angular acceleration and solving for Δt, yielding

$$\Delta t = \frac{\Delta\omega}{\alpha}. \tag{10.8}$$

Solution for (b)

Here the angular velocity decreases from 26.2 rad/s (250 rpm) to zero, so that $\Delta\omega$ is -26.2 rad/s, and α is given to be -87.3 rad/s^2. Thus,

$$\begin{aligned}\Delta t &= \frac{-26.2 \text{ rad/s}}{-87.3 \text{ rad/s}^2} \\ &= 0.300 \text{ s}.\end{aligned} \tag{10.9}$$

Discussion

Note that the angular acceleration as the girl spins the wheel is small and positive; it takes 5 s to produce an appreciable angular velocity. When she hits the brake, the angular acceleration is large and negative. The angular velocity quickly goes to zero. In both cases, the relationships are analogous to what happens with linear motion. For example, there is a large deceleration when you crash into a brick wall—the velocity change is large in a short time interval.

If the bicycle in the preceding example had been on its wheels instead of upside-down, it would first have accelerated along the ground and then come to a stop. This connection between circular motion and linear motion needs to be explored. For example, it would be useful to know how linear and angular acceleration are related. In circular motion, linear acceleration is *tangent* to the circle at the point of interest, as seen in **Figure 10.4**. Thus, linear acceleration is called **tangential acceleration** a_t.

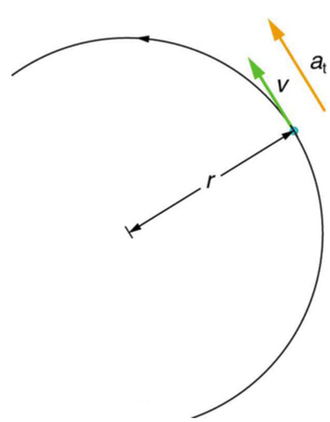


Figure 10.4 In circular motion, linear acceleration a, occurs as the magnitude of the velocity changes: a is tangent to the motion. In the context of circular motion, linear acceleration is also called tangential acceleration a_t.

Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction. We know from **Uniform Circular Motion and Gravitation** that in circular motion centripetal acceleration, a_c , refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration, as seen in **Figure 10.5**. Thus, a_t and a_c are perpendicular and independent of one another. Tangential acceleration a_t is directly related to the angular acceleration α and is linked to an increase or decrease in the velocity, but not its direction.

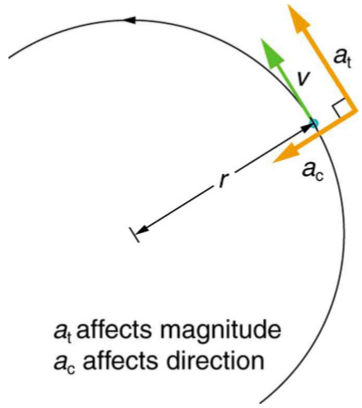


Figure 10.5 Centripetal acceleration a_c occurs as the direction of velocity changes; it is perpendicular to the circular motion. Centripetal and tangential acceleration are thus perpendicular to each other.

Now we can find the exact relationship between linear acceleration a_t and angular acceleration α . Because linear acceleration is proportional to a change in the magnitude of the velocity, it is defined (as it was in **One-Dimensional Kinematics**) to be

$$a_t = \frac{\Delta v}{\Delta t}. \tag{10.10}$$

For circular motion, note that $v = r\omega$, so that

$$a_t = \frac{\Delta(r\omega)}{\Delta t}. \tag{10.11}$$

The radius r is constant for circular motion, and so $\Delta(r\omega) = r(\Delta\omega)$. Thus,

$$a_t = r\frac{\Delta\omega}{\Delta t}. \tag{10.12}$$

By definition, $\alpha = \frac{\Delta\omega}{\Delta t}$. Thus,

$$a_t = r\alpha, \tag{10.13}$$

or

$$\alpha = \frac{a_t}{r}. \tag{10.14}$$

These equations mean that linear acceleration and angular acceleration are directly proportional. The greater the angular acceleration is, the larger the linear (tangential) acceleration is, and vice versa. For example, the greater the angular acceleration of a car's drive wheels, the greater the acceleration of the car. The radius also matters. For example, the smaller a wheel, the smaller its linear acceleration for a given angular acceleration α .

Example 10.2 Calculating the Angular Acceleration of a Motorcycle Wheel

A powerful motorcycle can accelerate from 0 to 30.0 m/s (about 108 km/h) in 4.20 s. What is the angular acceleration of its 0.320-m-radius wheels? (See **Figure 10.6**.)

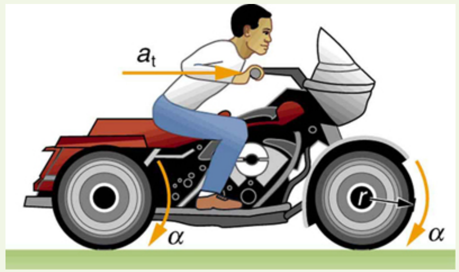


Figure 10.6 The linear acceleration of a motorcycle is accompanied by an angular acceleration of its wheels.

Strategy

We are given information about the linear velocities of the motorcycle. Thus, we can find its linear acceleration a_{t}. Then, the expression $\alpha = \frac{a_{\mathrm{t}}}{r}$ can be used to find the angular acceleration.

Solution

The linear acceleration is

$$\begin{aligned} a_{\mathrm{t}} &= \frac{\Delta v}{\Delta t} \\ &= \frac{30.0\text{ m/s}}{4.20\text{ s}} \\ &= 7.14\text{ m/s}^2. \end{aligned} \tag{10.15}$$

We also know the radius of the wheels. Entering the values for a_{t} and r into $\alpha = \frac{a_{\mathrm{t}}}{r}$, we get

$$\begin{aligned} \alpha &= \frac{a_{\mathrm{t}}}{r} \\ &= \frac{7.14\text{ m/s}^2}{0.320\text{ m}} \\ &= 22.3\text{ rad/s}^2. \end{aligned} \tag{10.16}$$

Discussion

Units of radians are dimensionless and appear in any relationship between angular and linear quantities.

So far, we have defined three rotational quantities— θ, ω, and α. These quantities are analogous to the translational quantities x, v, and a. **Table 10.1** displays rotational quantities, the analogous translational quantities, and the relationships between them.

Table 10.1 Rotational and Translational Quantities

Rotational	Translational	Relationship
θ	x	$\theta = \frac{x}{r}$
ω	v	$\omega = \frac{v}{r}$
α	a	$\alpha = \frac{a_t}{r}$

Making Connections: Take-Home Experiment

Sit down with your feet on the ground on a chair that rotates. Lift one of your legs such that it is unbent (straightened out). Using the other leg, begin to rotate yourself by pushing on the ground. Stop using your leg to push the ground but allow the chair to rotate. From the origin where you began, sketch the angle, angular velocity, and angular acceleration of your leg as a function of time in the form of three separate graphs. Estimate the magnitudes of these quantities.

Check Your Understanding

Angular acceleration is a vector, having both magnitude and direction. How do we denote its magnitude and direction? Illustrate with an example.

Solution

The magnitude of angular acceleration is α and its most common units are rad/s^2. The direction of angular acceleration along a fixed axis is denoted by a + or a – sign, just as the direction of linear acceleration in one dimension is denoted by a + or a – sign. For example, consider a gymnast doing a forward flip. Her angular momentum would be parallel to the mat and to her left. The magnitude of her angular acceleration would be proportional to her angular velocity (spin rate) and her moment of inertia about her spin axis.

PhET Explorations: Ladybug Revolution

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

PhET Interactive Simulation

Figure 10.7 Ladybug Revolution (http://cnx.org/content/m55183/1.2/rotation_en.jar)

10.2 Kinematics of Rotational Motion

Learning Objectives

By the end of this section, you will be able to:

- Observe the kinematics of rotational motion.
- Derive rotational kinematic equations.
- Evaluate problem solving strategies for rotational kinematics.

Just by using our intuition, we can begin to see how rotational quantities like θ, ω, and α are related to one another. For example, if a motorcycle wheel has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotates through many revolutions. In more technical terms, if the wheel's angular acceleration α is large for a long period of time t, then the final angular velocity ω and angle of rotation θ are large. The wheel's rotational motion is exactly analogous to the fact that the motorcycle's large translational acceleration produces a large final velocity, and the distance traveled will also be large.

Kinematics is the description of motion. The **kinematics of rotational motion** describes the relationships among rotation angle, angular velocity, angular acceleration, and time. Let us start by finding an equation relating ω, α, and t. To determine this equation, we recall a familiar kinematic equation for translational, or straight-line, motion:

$$v = v_0 + at \quad \text{(constant } a) \tag{10.17}$$

Note that in rotational motion $a = a_\text{t}$, and we shall use the symbol a for tangential or linear acceleration from now on. As in linear kinematics, we assume a is constant, which means that angular acceleration α is also a constant, because $a = r\alpha$. Now, let us substitute $v = r\omega$ and $a = r\alpha$ into the linear equation above:

$$r\omega = r\omega_0 + r\alpha t. \tag{10.18}$$

The radius r cancels in the equation, yielding

$$\omega = \omega_0 + at \quad \text{(constant } a), \tag{10.19}$$

where ω_0 is the initial angular velocity. This last equation is a *kinematic relationship* among ω, α, and t —that is, it describes their relationship without reference to forces or masses that may affect rotation. It is also precisely analogous in form to its translational counterpart.

Making Connections

Kinematics for rotational motion is completely analogous to translational kinematics, first presented in **One-Dimensional Kinematics**. Kinematics is concerned with the description of motion without regard to force or mass. We will find that translational kinematic quantities, such as displacement, velocity, and acceleration have direct analogs in rotational motion.

Starting with the four kinematic equations we developed in **One-Dimensional Kinematics**, we can derive the following four rotational kinematic equations (presented together with their translational counterparts):

Table 10.2 Rotational Kinematic Equations

Rotational	Translational	
$\theta = \bar{\omega} t$	$x = \bar{v} t$	
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	(constant α, a)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$	(constant α, a)
$\omega^2 = \omega_0{}^2 + 2\alpha\theta$	$v^2 = v_0{}^2 + 2ax$	(constant α, a)

In these equations, the subscript 0 denotes initial values (θ_0, x_0, and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity $\bar{v}$ are defined as follows:

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} \text{ and } \bar{v} = \frac{v_0 + v}{2}. \tag{10.20}$$

The equations given above in **Table 10.2** can be used to solve any rotational or translational kinematics problem in which a and α are constant.

Problem-Solving Strategy for Rotational Kinematics

1. *Examine the situation to determine that rotational kinematics (rotational motion) is involved*. Rotation must be involved, but without the need to consider forces or masses that affect the motion.
2. *Identify exactly what needs to be determined in the problem (identify the unknowns)*. A sketch of the situation is useful.
3. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns)*.
4. *Solve the appropriate equation or equations for the quantity to be determined (the unknown)*. It can be useful to think in terms of a translational analog because by now you are familiar with such motion.
5. *Substitute the known values along with their units into the appropriate equation, and obtain numerical solutions complete with units*. Be sure to use units of radians for angles.
6. *Check your answer to see if it is reasonable: Does your answer make sense*?

Example 10.3 Calculating the Acceleration of a Fishing Reel

A deep-sea fisherman hooks a big fish that swims away from the boat pulling the fishing line from his fishing reel. The whole system is initially at rest and the fishing line unwinds from the reel at a radius of 4.50 cm from its axis of rotation. The reel is given an angular acceleration of 110 rad/s^2 for 2.00 s as seen in **Figure 10.8**.

(a) What is the final angular velocity of the reel?

(b) At what speed is fishing line leaving the reel after 2.00 s elapses?

(c) How many revolutions does the reel make?

(d) How many meters of fishing line come off the reel in this time?

Strategy

In each part of this example, the strategy is the same as it was for solving problems in linear kinematics. In particular, known values are identified and a relationship is then sought that can be used to solve for the unknown.

Solution for (a)

Here α and t are given and ω needs to be determined. The most straightforward equation to use is $\omega = \omega_0 + \alpha t$ because the unknown is already on one side and all other terms are known. That equation states that

$$\omega = \omega_0 + \alpha t. \tag{10.21}$$

We are also given that $\omega_0 = 0$ (it starts from rest), so that

$$\omega = 0 + \left(110 \text{ rad/s}^2\right)(2.00\text{s}) = 220 \text{ rad/s}. \tag{10.22}$$

Solution for (b)

Now that ω is known, the speed v can most easily be found using the relationship

$$v = r\omega, \tag{10.23}$$

where the radius r of the reel is given to be 4.50 cm; thus,

$$v = (0.0450\ \text{m})(220\ \text{rad/s}) = 9.90\ \text{m/s}. \tag{10.24}$$

Note again that radians must always be used in any calculation relating linear and angular quantities. Also, because radians are dimensionless, we have $\text{m}\times\text{rad} = \text{m}$.

Solution for (c)

Here, we are asked to find the number of revolutions. Because $1\ \text{rev} = 2\pi\ \text{rad}$, we can find the number of revolutions by finding θ in radians. We are given α and t, and we know ω_0 is zero, so that θ can be obtained using

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$.

$$\begin{aligned}\theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ &= 0 + (0.500)\left(110\ \text{rad/s}^2\right)(2.00\ \text{s})^2 = 220\ \text{rad}.\end{aligned} \tag{10.25}$$

Converting radians to revolutions gives

$$\theta = (220\ \text{rad})\frac{1\ \text{rev}}{2\pi\ \text{rad}} = 35.0\ \text{rev}. \tag{10.26}$$

Solution for (d)

The number of meters of fishing line is x, which can be obtained through its relationship with θ:

$$x = r\theta = (0.0450\ \text{m})(220\ \text{rad}) = 9.90\ \text{m}. \tag{10.27}$$

Discussion

This example illustrates that relationships among rotational quantities are highly analogous to those among linear quantities. We also see in this example how linear and rotational quantities are connected. The answers to the questions are realistic. After unwinding for two seconds, the reel is found to spin at 220 rad/s, which is 2100 rpm. (No wonder reels sometimes make high-pitched sounds.) The amount of fishing line played out is 9.90 m, about right for when the big fish bites.

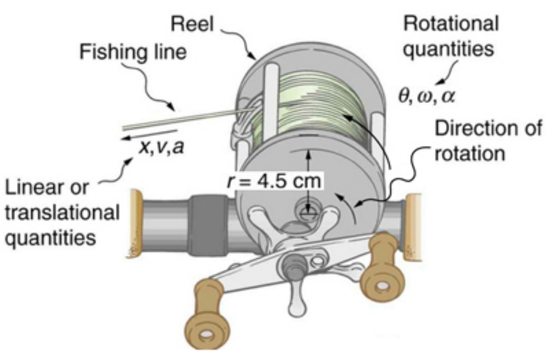


Figure 10.8 Fishing line coming off a rotating reel moves linearly. **Example 10.3** and **Example 10.4** consider relationships between rotational and linear quantities associated with a fishing reel.

Example 10.4 Calculating the Duration When the Fishing Reel Slows Down and Stops

Now let us consider what happens if the fisherman applies a brake to the spinning reel, achieving an angular acceleration of $-300\ \text{rad/s}^2$. How long does it take the reel to come to a stop?

Strategy

We are asked to find the time t for the reel to come to a stop. The initial and final conditions are different from those in the previous problem, which involved the same fishing reel. Now we see that the initial angular velocity is $\omega_0 = 220\ \text{rad/s}$ and the final angular velocity ω is zero. The angular acceleration is given to be $\alpha = -300\ \text{rad/s}^2$. Examining the available equations, we see all quantities but *t* are known in $\omega = \omega_0 + \alpha t,$ making it easiest to use this equation.

Solution

The equation states

$$\omega = \omega_0 + \alpha t. \tag{10.28}$$

We solve the equation algebraically for *t*, and then substitute the known values as usual, yielding

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 220 \text{ rad/s}}{-300 \text{ rad/s}^2} = 0.733 \text{ s}. \tag{10.29}$$

Discussion

Note that care must be taken with the signs that indicate the directions of various quantities. Also, note that the time to stop the reel is fairly small because the acceleration is rather large. Fishing lines sometimes snap because of the accelerations involved, and fishermen often let the fish swim for a while before applying brakes on the reel. A tired fish will be slower, requiring a smaller acceleration.

Example 10.5 Calculating the Slow Acceleration of Trains and Their Wheels

Large freight trains accelerate very slowly. Suppose one such train accelerates from rest, giving its 0.350-m-radius wheels an angular acceleration of 0.250 rad/s^2 . After the wheels have made 200 revolutions (assume no slippage): (a) How far has the train moved down the track? (b) What are the final angular velocity of the wheels and the linear velocity of the train?

Strategy

In part (a), we are asked to find x , and in (b) we are asked to find ω and v . We are given the number of revolutions θ , the radius of the wheels r , and the angular acceleration α .

Solution for (a)

The distance x is very easily found from the relationship between distance and rotation angle:

$$\theta = \frac{x}{r}. \tag{10.30}$$

Solving this equation for x yields

$$x = r\theta. \tag{10.31}$$

Before using this equation, we must convert the number of revolutions into radians, because we are dealing with a relationship between linear and rotational quantities:

$$\theta = (200 \text{ rev})\frac{2\pi \text{ rad}}{1 \text{ rev}} = 1257 \text{ rad}. \tag{10.32}$$

Now we can substitute the known values into $x = r\theta$ to find the distance the train moved down the track:

$$x = r\theta = (0.350 \text{ m})(1257 \text{ rad}) = 440 \text{ m}. \tag{10.33}$$

Solution for (b)

We cannot use any equation that incorporates t to find ω , because the equation would have at least two unknown values. The equation $\omega^2 = {\omega_0}^2 + 2\alpha\theta$ will work, because we know the values for all variables except ω :

$$\omega^2 = {\omega_0}^2 + 2\alpha\theta \tag{10.34}$$

Taking the square root of this equation and entering the known values gives

$$\begin{aligned} \omega &= \left[0 + 2(0.250 \text{ rad/s}^2)(1257 \text{ rad})\right]^{1/2} \\ &= 25.1 \text{ rad/s}. \end{aligned} \tag{10.35}$$

We can find the linear velocity of the train, v , through its relationship to ω :

$$v = r\omega = (0.350 \text{ m})(25.1 \text{ rad/s}) = 8.77 \text{ m/s}. \tag{10.36}$$

Discussion

The distance traveled is fairly large and the final velocity is fairly slow (just under 32 km/h).

There is translational motion even for something spinning in place, as the following example illustrates. **Figure 10.9** shows a fly on the edge of a rotating microwave oven plate. The example below calculates the total distance it travels.

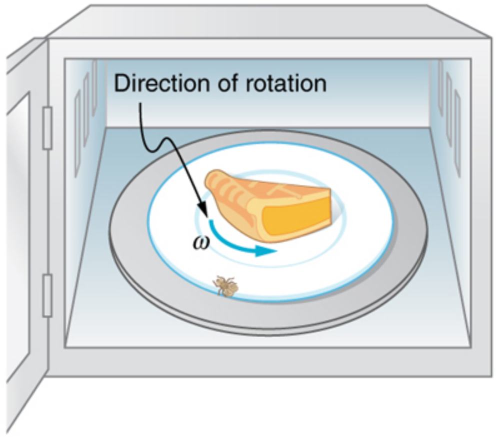


Figure 10.9 The image shows a microwave plate. The fly makes revolutions while the food is heated (along with the fly).

Example 10.6 Calculating the Distance Traveled by a Fly on the Edge of a Microwave Oven Plate

A person decides to use a microwave oven to reheat some lunch. In the process, a fly accidentally flies into the microwave and lands on the outer edge of the rotating plate and remains there. If the plate has a radius of 0.15 m and rotates at 6.0 rpm, calculate the total distance traveled by the fly during a 2.0-min cooking period. (Ignore the start-up and slow-down times.)

Strategy

First, find the total number of revolutions θ, and then the linear distance x traveled. $\theta = \bar{\omega} t$ can be used to find θ because $\bar{\omega}$ is given to be 6.0 rpm.

Solution

Entering known values into $\theta = \bar{\omega} t$ gives

$$\theta = \bar{\omega} t = (6.0 \text{ rpm})(2.0 \text{ min}) = 12 \text{ rev}. \tag{10.37}$$

As always, it is necessary to convert revolutions to radians before calculating a linear quantity like x from an angular quantity like θ:

$$\theta = (12 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 75.4 \text{ rad}. \tag{10.38}$$

Now, using the relationship between x and θ, we can determine the distance traveled:

$$x = r\theta = (0.15 \text{ m})(75.4 \text{ rad}) = 11 \text{ m}. \tag{10.39}$$

Discussion

Quite a trip (if it survives)! Note that this distance is the total distance traveled by the fly. Displacement is actually zero for complete revolutions because they bring the fly back to its original position. The distinction between total distance traveled and displacement was first noted in **One-Dimensional Kinematics**.

Check Your Understanding

Rotational kinematics has many useful relationships, often expressed in equation form. Are these relationships laws of physics or are they simply descriptive? (Hint: the same question applies to linear kinematics.)

Solution

Rotational kinematics (just like linear kinematics) is descriptive and does not represent laws of nature. With kinematics, we can describe many things to great precision but kinematics does not consider causes. For example, a large angular acceleration describes a very rapid change in angular velocity without any consideration of its cause.

10.3 Dynamics of Rotational Motion: Rotational Inertia

Learning Objectives

By the end of this section, you will be able to:

- Understand the relationship between force, mass, and acceleration.
- Study the turning effect of force.
- Study the analogy between force and torque, mass and moment of inertia, and linear acceleration and angular acceleration.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.D.1.1** The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system. **(S.P. 1.2, 1.4)**
- **4.D.1.2** The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, and angular momentum can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data. **(S.P. 3.2, 4.1, 5.1, 5.3)**
- **5.E.2.1** The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses. **(S.P. 2.2)**

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity as seen in **Figure 10.10**. In fact, your intuition is reliable in predicting many of the factors that are involved. For example, we know that a door opens slowly if we push too close to its hinges. Furthermore, we know that the more massive the door, the more slowly it opens. The first example implies that the farther the force is applied from the pivot, the greater the angular acceleration; another implication is that angular acceleration is inversely proportional to mass. These relationships should seem very similar to the familiar relationships among force, mass, and acceleration embodied in Newton's second law of motion. There are, in fact, precise rotational analogs to both force and mass.

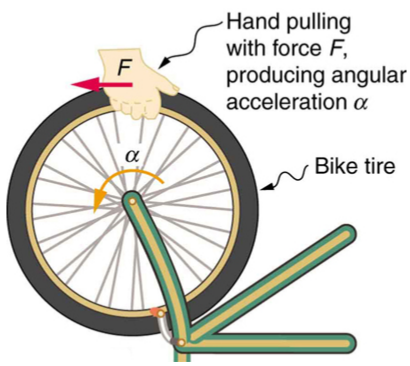


Figure 10.10 Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller.

To develop the precise relationship among force, mass, radius, and angular acceleration, consider what happens if we exert a force F on a point mass m that is at a distance r from a pivot point, as shown in **Figure 10.11**. Because the force is perpendicular to r, an acceleration $a = \frac{F}{m}$ is obtained in the direction of F. We can rearrange this equation such that $F = ma$ and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = ma$, yielding

$$F = mr\alpha. \tag{10.40}$$

Recall that **torque** is the turning effectiveness of a force. In this case, because $\mathbf{F}$ is perpendicular to r, torque is simply $\tau = Fr$. So, if we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,

$$rF = mr^2\alpha \tag{10.41}$$

or

$$\tau = mr^2\alpha. \tag{10.42}$$

This last equation is the rotational analog of Newton's second law ($F = ma$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia). The quantity mr^2 is called the **rotational inertia** or **moment of inertia** of a point mass m a distance r from the center of rotation.

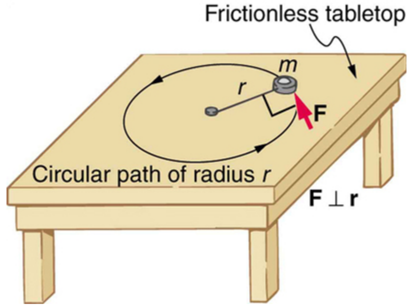


Figure 10.11 An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force F is applied to the object perpendicular to the radius r , causing it to accelerate about the pivot point. The force is kept perpendicular to r .

Making Connections: Rotational Motion Dynamics

Dynamics for rotational motion is completely analogous to linear or translational dynamics. Dynamics is concerned with force and mass and their effects on motion. For rotational motion, we will find direct analogs to force and mass that behave just as we would expect from our earlier experiences.

Rotational Inertia and Moment of Inertia

Before we can consider the rotation of anything other than a point mass like the one in **Figure 10.11**, we must extend the idea of rotational inertia to all types of objects. To expand our concept of rotational inertia, we define the **moment of inertia** I of an object to be the sum of mr^2 for all the point masses of which it is composed. That is, $I = \sum mr^2$. Here I is analogous to m in translational motion. Because of the distance r , the moment of inertia for any object depends on the chosen axis. Actually, calculating I is beyond the scope of this text except for one simple case—that of a hoop, which has all its mass at the same distance from its axis. A hoop's moment of inertia around its axis is therefore MR^2 , where M is its total mass and R its radius. (We use M and R for an entire object to distinguish them from m and r for point masses.) In all other cases, we must consult **Figure 10.12** (note that the table is piece of artwork that has shapes as well as formulae) for formulas for I that have been derived from integration over the continuous body. Note that I has units of mass multiplied by distance squared ($\text{kg} \cdot \text{m}^2$), as we might expect from its definition.

The general relationship among torque, moment of inertia, and angular acceleration is

$$\text{net } \tau = I\alpha \tag{10.43}$$

or

$$\alpha = \frac{\text{net } \tau}{I}, \tag{10.44}$$

where net τ is the total torque from all forces relative to a chosen axis. For simplicity, we will only consider torques exerted by forces in the plane of the rotation. Such torques are either positive or negative and add like ordinary numbers. The relationship in $\tau = I\alpha,\ \alpha = \frac{\text{net } \tau}{I}$ is the rotational analog to Newton's second law and is very generally applicable. This equation is actually valid for *any* torque, applied to *any* object, relative to *any* axis.

As we might expect, the larger the torque is, the larger the angular acceleration is. For example, the harder a child pushes on a merry-go-round, the faster it accelerates. Furthermore, the more massive a merry-go-round, the slower it accelerates for the same torque. The basic relationship between moment of inertia and angular acceleration is that the larger the moment of inertia, the smaller is the angular acceleration. But there is an additional twist. The moment of inertia depends not only on the mass of an object, but also on its *distribution* of mass relative to the axis around which it rotates. For example, it will be much easier to accelerate a merry-go-round full of children if they stand close to its axis than if they all stand at the outer edge. The mass is the same in both cases; but the moment of inertia is much larger when the children are at the edge.

Take-Home Experiment

Cut out a circle that has about a 10 cm radius from stiff cardboard. Near the edge of the circle, write numbers 1 to 12 like hours on a clock face. Position the circle so that it can rotate freely about a horizontal axis through its center, like a wheel. (You could loosely nail the circle to a wall.) Hold the circle stationary and with the number 12 positioned at the top, attach a lump of blue putty (sticky material used for fixing posters to walls) at the number 3. How large does the lump need to be to just rotate the circle? Describe how you can change the moment of inertia of the circle. How does this change affect the amount of blue putty needed at the number 3 to just rotate the circle? Change the circle's moment of inertia and then try rotating the circle by using different amounts of blue putty. Repeat this process several times.

In what direction did the circle rotate when you added putty at the number 3 (clockwise or counterclockwise)? In which of these directions was the resulting angular velocity? Was the angular velocity constant? What can we say about the direction (clockwise or counterclockwise) of the angular acceleration? How could you change the placement of the putty to create angular velocity in the opposite direction?

Problem-Solving Strategy for Rotational Dynamics

1. *Examine the situation to determine that torque and mass are involved in the rotation*. Draw a careful sketch of the situation.
2. *Determine the system of interest*.
3. *Draw a free body diagram*. That is, draw and label all external forces acting on the system of interest.
4. *Apply* $net\ \tau = I\alpha,\ \alpha = \frac{net\ \tau}{I}$ *, the rotational equivalent of Newton's second law, to solve the problem*. Care must be taken to use the correct moment of inertia and to consider the torque about the point of rotation.
5. *As always, check the solution to see if it is reasonable*.

Making Connections

In statics, the net torque is zero, and there is no angular acceleration. In rotational motion, net torque is the cause of angular acceleration, exactly as in Newton's second law of motion for rotation.

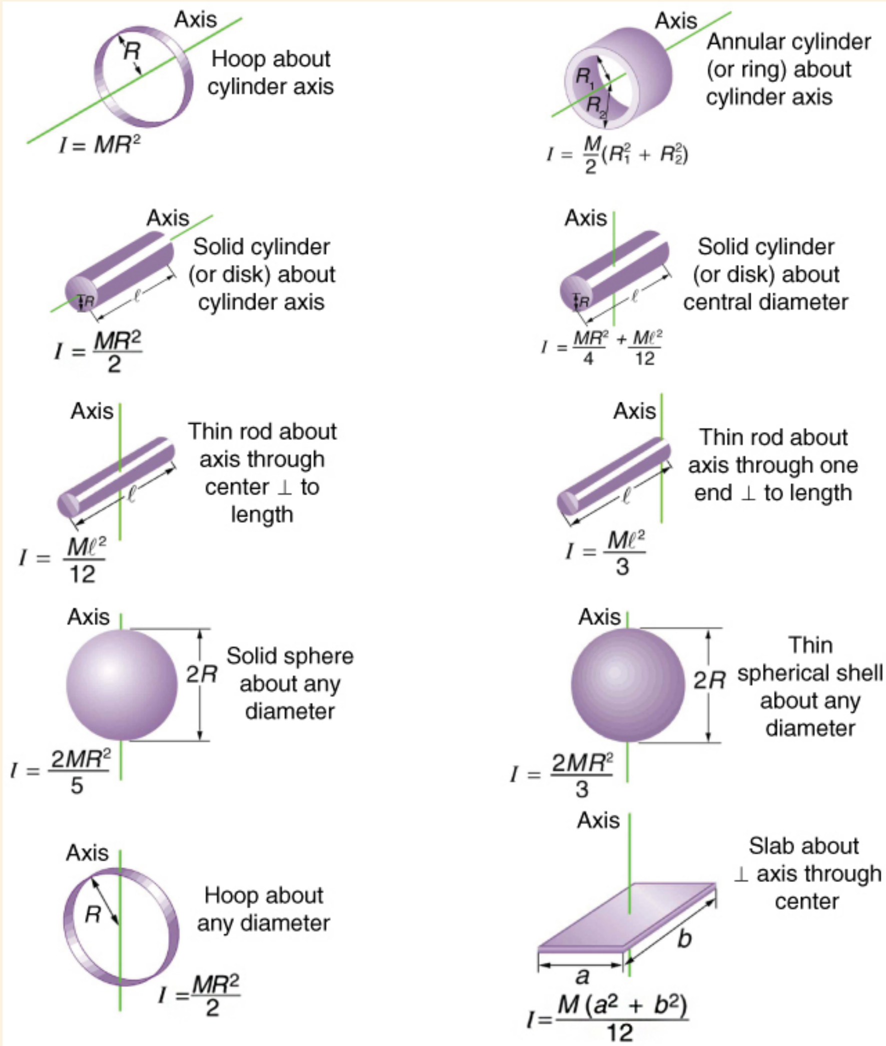


Figure 10.12 Some rotational inertias.

Example 10.7 Calculating the Effect of Mass Distribution on a Merry-Go-Round

Consider the father pushing a playground merry-go-round in **Figure 10.13**. He exerts a force of 250 N at the edge of the 50.0-kg merry-go-round, which has a 1.50 m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when an 18.0-kg child sits 1.25 m away from the center. Consider the merry-go-round itself to be a uniform disk with negligible retarding friction.

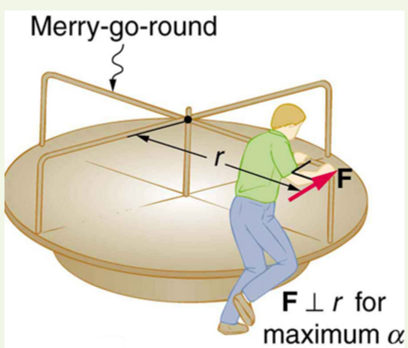


Figure 10.13 A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque.

Strategy

Angular acceleration is given directly by the expression $\alpha = \frac{\text{net } \tau}{I}$:

$$\alpha = \frac{\tau}{I}. \tag{10.45}$$

To solve for α, we must first calculate the torque τ (which is the same in both cases) and moment of inertia I (which is greater in the second case). To find the torque, we note that the applied force is perpendicular to the radius and friction is negligible, so that

$$\tau = rF\sin\theta = (1.50\text{ m})(250\text{ N}) = 375\text{ N}\cdot\text{m}. \tag{10.46}$$

Solution for (a)

The moment of inertia of a solid disk about this axis is given in **Figure 10.12** to be

$$\frac{1}{2}MR^2, \tag{10.47}$$

where $M = 50.0\text{ kg}$ and $R = 1.50\text{ m}$, so that

$$I = (0.500)(50.0\text{ kg})(1.50\text{ m})^2 = 56.25\text{ kg}\cdot\text{m}^2. \tag{10.48}$$

Now, after we substitute the known values, we find the angular acceleration to be

$$\alpha = \frac{\tau}{I} = \frac{375\text{ N}\cdot\text{m}}{56.25\text{ kg}\cdot\text{m}^2} = 6.67\frac{\text{rad}}{\text{s}^2}. \tag{10.49}$$

Solution for (b)

We expect the angular acceleration for the system to be less in this part, because the moment of inertia is greater when the child is on the merry-go-round. To find the total moment of inertia I, we first find the child's moment of inertia I_c by considering the child to be equivalent to a point mass at a distance of 1.25 m from the axis. Then,

$$I_\text{c} = MR^2 = (18.0\text{ kg})(1.25\text{ m})^2 = 28.13\text{ kg}\cdot\text{m}^2. \tag{10.50}$$

The total moment of inertia is the sum of moments of inertia of the merry-go-round and the child (about the same axis). To justify this sum to yourself, examine the definition of I:

$$I = 28.13\text{ kg}\cdot\text{m}^2 + 56.25\text{ kg}\cdot\text{m}^2 = 84.38\text{ kg}\cdot\text{m}^2. \tag{10.51}$$

Substituting known values into the equation for α gives

$$\alpha = \frac{\tau}{I} = \frac{375\text{ N}\cdot\text{m}}{84.38\text{ kg}\cdot\text{m}^2} = 4.44\frac{\text{rad}}{\text{s}^2}. \tag{10.52}$$

Discussion

The angular acceleration is less when the child is on the merry-go-round than when the merry-go-round is empty, as expected. The angular accelerations found are quite large, partly due to the fact that friction was considered to be negligible. If, for example, the father kept pushing perpendicularly for 2.00 s, he would give the merry-go-round an angular velocity of 13.3 rad/s when it is empty but only 8.89 rad/s when the child is on it. In terms of revolutions per second, these angular velocities are 2.12 rev/s and 1.41 rev/s, respectively. The father would end up running at about 50 km/h in the first case. Summer Olympics, here he comes! Confirmation of these numbers is left as an exercise for the reader.

Making Connections: Multiple Forces on One System

A large potter's wheel has a diameter of 60.0 cm and a mass of 8.0 kg. It is powered by a 20.0 N motor acting on the outer edge. There is also a brake capable of exerting a 15.0 N force at a radius of 12.0 cm from the axis of rotation, on the underside.

What is the angular acceleration when the motor is in use?

The torque is found by $\tau = rF\sin\theta = (0.300\,\text{m})(20.0\,\text{N}) = 6.00\,\text{N·m}$.

The moment of inertia is calculated as $I = \frac{1}{2}MR^2 = \frac{1}{2}(8.0\,\text{kg})(0.300\,\text{m})^2 = 0.36\,\text{kg}\cdot\text{m}^2$.

Thus, the angular acceleration would be $\alpha = \frac{\tau}{I} = \frac{6.00\,\text{N}\cdot\text{m}}{0.36\,\text{kg}\cdot\text{m}^2} = 17\,\text{rad/s}^2$.

Note that the friction is always acting in a direction opposite to the rotation that is currently happening in this system. If the potter makes a mistake and has both the brake and motor on simultaneously, the friction force of the brake will exert a torque opposite that of the motor.

The torque from the brake is $\tau = rF\sin\theta = (0.120\,\text{m})(15.0\,\text{N}) = 1.80\,\text{N·m}$.

Thus, the net torque is $6.00\ \text{N·m} - 1.80\,\text{N·m} = 4.20\,\text{N·m}$.

And the angular acceleration is $\alpha = \frac{\tau}{I} = \frac{4.20\,\text{N}\cdot\text{m}}{0.36\,\text{kg}\cdot\text{m}^2} = 12\,\text{rad/s}^2$.

Check Your Understanding

Torque is the analog of force and moment of inertia is the analog of mass. Force and mass are physical quantities that depend on only one factor. For example, mass is related solely to the numbers of atoms of various types in an object. Are torque and moment of inertia similarly simple?

Solution

No. Torque depends on three factors: force magnitude, force direction, and point of application. Moment of inertia depends on both mass and its distribution relative to the axis of rotation. So, while the analogies are precise, these rotational quantities depend on more factors.

10.4 Rotational Kinetic Energy: Work and Energy Revisited

Learning Objectives

By the end of this section, you will be able to:

- Derive the equation for rotational work.
- Calculate rotational kinetic energy.
- Demonstrate the law of conservation of energy.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.F.2.1** The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis. **(S.P. 6.4)**
- **3.F.2.2** The student is able to plan data collection and analysis strategies designed to test the relationship between a torque exerted on an object and the change in angular velocity of that object about an axis. **(S.P. 4.1, 4.2, 5.1)**

In this module, we will learn about work and energy associated with rotational motion. Figure 10.14 shows a worker using an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are created as layers of steel are pared from the pole. The stone continues to turn even after the motor is turned off, but it is eventually brought to a stop by friction. Clearly, the motor had to work to get the stone spinning. This work went into heat, light, sound, vibration, and considerable **rotational kinetic energy**.

Figure 10.14 The motor works in spinning the grindstone, giving it rotational kinetic energy. That energy is then converted to heat, light, sound, and vibration. (credit: U.S. Navy photo by Mass Communication Specialist Seaman Zachary David Bell)

Work must be done to rotate objects such as grindstones or merry-go-rounds. Work was defined in **Uniform Circular Motion and Gravitation** for translational motion, and we can build on that knowledge when considering work done in rotational motion. The simplest rotational situation is one in which the net force is exerted perpendicular to the radius of a disk (as shown in Figure 10.15) and remains perpendicular as the disk starts to rotate. The force is parallel to the displacement, and so the net work done is the product of the force times the arc length traveled:

$$\text{net } W = (\text{net } F)\Delta s. \quad (10.53)$$

To get torque and other rotational quantities into the equation, we multiply and divide the right-hand side of the equation by r, and gather terms:

$$\text{net } W = (r \text{ net } F)\frac{\Delta s}{r}. \tag{10.54}$$

We recognize that $r \text{ net } F = \text{net } \tau$ and $\Delta s / r = \theta$, so that

$$\text{net } W = (\text{net } \tau)\theta. \tag{10.55}$$

This equation is the expression for rotational work. It is very similar to the familiar definition of translational work as force multiplied by distance. Here, torque is analogous to force, and angle is analogous to distance. The equation $\text{net } W = (\text{net } \tau)\theta$ is valid in general, even though it was derived for a special case.

To get an expression for rotational kinetic energy, we must again perform some algebraic manipulations. The first step is to note that $\text{net } \tau = I\alpha$, so that

$$\text{net } W = I\alpha\theta. \tag{10.56}$$

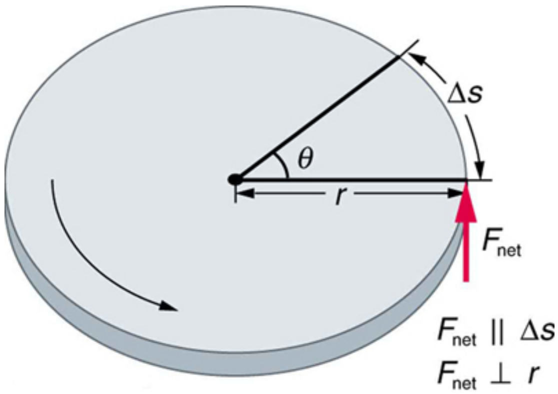


Figure 10.15 The net force on this disk is kept perpendicular to its radius as the force causes the disk to rotate. The net work done is thus $(\text{net } F)\Delta s$. The net work goes into rotational kinetic energy.

Making Connections

qWork and energy in rotational motion are completely analogous to work and energy in translational motion, first presented in **Uniform Circular Motion and Gravitation**.

Now, we solve one of the rotational kinematics equations for $\alpha\theta$. We start with the equation

$$\omega^2 = {\omega_0}^2 + 2\alpha\theta. \tag{10.57}$$

Next, we solve for $\alpha\theta$:

$$\alpha\theta = \frac{\omega^2 - {\omega_0}^2}{2}. \tag{10.58}$$

Substituting this into the equation for net W and gathering terms yields

$$\text{net } W = \frac{1}{2}I\omega^2 - \frac{1}{2}I{\omega_0}^2. \tag{10.59}$$

This equation is the **work-energy theorem** for rotational motion only. As you may recall, net work changes the kinetic energy of a system. Through an analogy with translational motion, we define the term $\left(\frac{1}{2}\right)I\omega^2$ to be **rotational kinetic energy** KE_{rot} for an object with a moment of inertia I and an angular velocity ω:

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2. \tag{10.60}$$

The expression for rotational kinetic energy is exactly analogous to translational kinetic energy, with I being analogous to m and ω to v. Rotational kinetic energy has important effects. Flywheels, for example, can be used to store large amounts of rotational kinetic energy in a vehicle, as seen in **Figure 10.16**.

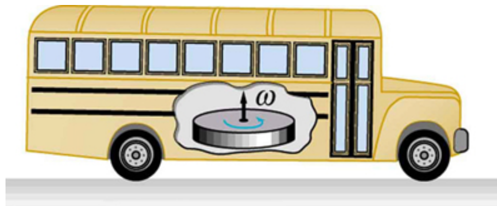


Figure 10.16 Experimental vehicles, such as this bus, have been constructed in which rotational kinetic energy is stored in a large flywheel. When the bus goes down a hill, its transmission converts its gravitational potential energy into KE_{rot} . It can also convert translational kinetic energy, when the bus stops, into KE_{rot} . The flywheel's energy can then be used to accelerate, to go up another hill, or to keep the bus from going against friction.

Example 10.8 Calculating the Work and Energy for Spinning a Grindstone

Consider a person who spins a large grindstone by placing her hand on its edge and exerting a force through part of a revolution as shown in **Figure 10.17**. In this example, we verify that the work done by the torque she exerts equals the change in rotational energy. (a) How much work is done if she exerts a force of 200 N through a rotation of $1.00\text{ rad}(57.3º)$? The force is kept perpendicular to the grindstone's 0.320-m radius at the point of application, and the effects of friction are negligible. (b) What is the final angular velocity if the grindstone has a mass of 85.0 kg? (c) What is the final rotational kinetic energy? (It should equal the work.)

Strategy

To find the work, we can use the equation $\text{net } W = (\text{net } \tau)\theta$. We have enough information to calculate the torque and are given the rotation angle. In the second part, we can find the final angular velocity using one of the kinematic relationships. In the last part, we can calculate the rotational kinetic energy from its expression in $\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2$.

Solution for (a)

The net work is expressed in the equation

$$\text{net } W = (\text{net } \tau)\theta, \tag{10.61}$$

where net τ is the applied force multiplied by the radius (rF) because there is no retarding friction, and the force is perpendicular to r . The angle θ is given. Substituting the given values in the equation above yields

$$\begin{aligned} \text{net } W &= rF\theta = (0.320\text{ m})(200\text{ N})(1.00\text{ rad}) \\ &= 64.0\text{ N}\cdot\text{m}. \end{aligned} \tag{10.62}$$

Noting that $1\text{ N}\cdot\text{m} = 1\text{ J}$,

$$\text{net } W = 64.0\text{ J}. \tag{10.63}$$

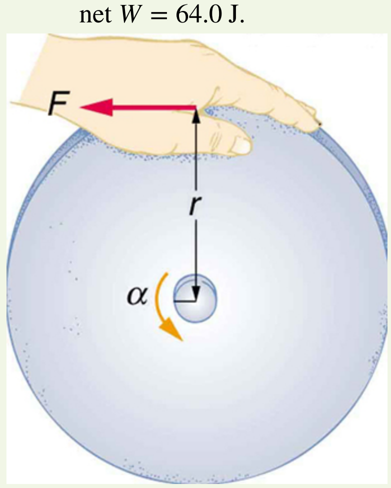


Figure 10.17 A large grindstone is given a spin by a person grasping its outer edge.

Solution for (b)

To find ω from the given information requires more than one step. We start with the kinematic relationship in the equation

$$\omega^2 = {\omega_0}^2 + 2\alpha\theta. \tag{10.64}$$

Note that $\omega_0 = 0$ because we start from rest. Taking the square root of the resulting equation gives

$$\omega = (2\alpha\theta)^{1/2}. \tag{10.65}$$

Now we need to find α. One possibility is

$$\alpha = \frac{\text{net } \tau}{I}, \tag{10.66}$$

where the torque is

$$\text{net } \tau = rF = (0.320 \text{ m})(200 \text{ N}) = 64.0 \text{ N} \cdot \text{m}. \tag{10.67}$$

The formula for the moment of inertia for a disk is found in **Figure 10.12**:

$$I = \frac{1}{2}MR^2 = 0.5(85.0 \text{ kg})(0.320 \text{ m})^2 = 4.352 \text{ kg} \cdot \text{m}^2. \tag{10.68}$$

Substituting the values of torque and moment of inertia into the expression for α, we obtain

$$\alpha = \frac{64.0 \text{ N} \cdot \text{m}}{4.352 \text{ kg} \cdot \text{m}^2} = 14.7\frac{\text{rad}}{\text{s}^2}. \tag{10.69}$$

Now, substitute this value and the given value for θ into the above expression for ω:

$$\omega = (2\alpha\theta)^{1/2} = \left[2\left(14.7\frac{\text{rad}}{\text{s}^2}\right)(1.00 \text{ rad})\right]^{1/2} = 5.42\frac{\text{rad}}{\text{s}}. \tag{10.70}$$

Solution for (c)

The final rotational kinetic energy is

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2. \tag{10.71}$$

Both I and ω were found above. Thus,

$$\text{KE}_{\text{rot}} = (0.5)\left(4.352 \text{ kg} \cdot \text{m}^2\right)(5.42 \text{ rad/s})^2 = 64.0 \text{ J}. \tag{10.72}$$

Discussion

The final rotational kinetic energy equals the work done by the torque, which confirms that the work done went into rotational kinetic energy. We could, in fact, have used an expression for energy instead of a kinematic relation to solve part (b). We will do this in later examples.

Helicopter pilots are quite familiar with rotational kinetic energy. They know, for example, that a point of no return will be reached if they allow their blades to slow below a critical angular velocity during flight. The blades lose lift, and it is impossible to immediately get the blades spinning fast enough to regain it. Rotational kinetic energy must be supplied to the blades to get them to rotate faster, and enough energy cannot be supplied in time to avoid a crash. Because of weight limitations, helicopter engines are too small to supply both the energy needed for lift and to replenish the rotational kinetic energy of the blades once they have slowed down. The rotational kinetic energy is put into them before takeoff and must not be allowed to drop below this crucial level. One possible way to avoid a crash is to use the gravitational potential energy of the helicopter to replenish the rotational kinetic energy of the blades by losing altitude and aligning the blades so that the helicopter is spun up in the descent. Of course, if the helicopter's altitude is too low, then there is insufficient time for the blade to regain lift before reaching the ground.

Take-Home Experiment

Rotational motion can be observed in wrenches, clocks, wheels or spools on axels, and seesaws. Choose an object or system that exhibits rotational motion and plan an experiment to test how torque affects angular velocity. How will you create and measure different amounts of torque? How will you measure angular velocity? Remember that $\text{net } \tau = I\alpha, \ I \propto mr^2, \ \text{and } \omega = \frac{v}{r}$.

Problem-Solving Strategy for Rotational Energy

1. *Determine that energy or work is involved in the rotation*.
2. *Determine the system of interest*. A sketch usually helps.
3. *Analyze the situation to determine the types of work and energy involved*.

4. *For closed systems, mechanical energy is conserved*. That is, $\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f}$. Note that KE_i and KE_f may each include translational and rotational contributions.
5. *For open systems*, mechanical energy may not be conserved, and other forms of energy (referred to previously as OE), such as heat transfer, may enter or leave the system. Determine what they are, and calculate them as necessary.
6. *Eliminate terms wherever possible to simplify the algebra*.
7. *Check the answer to see if it is reasonable*.

Example 10.9 Calculating Helicopter Energies

A typical small rescue helicopter, similar to the one in **Figure 10.18**, has four blades, each is 4.00 m long and has a mass of 50.0 kg. The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg. (a) Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm. (b) Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades. (c) To what height could the helicopter be raised if all of the rotational kinetic energy could be used to lift it?

Strategy

Rotational and translational kinetic energies can be calculated from their definitions. The last part of the problem relates to the idea that energy can change form, in this case from rotational kinetic energy to gravitational potential energy.

Solution for (a)

The rotational kinetic energy is

$$\text{KE}_\text{rot} = \frac{1}{2}I\omega^2. \tag{10.73}$$

We must convert the angular velocity to radians per second and calculate the moment of inertia before we can find KE_rot. The angular velocity ω is

$$\omega = \frac{300\text{ rev}}{1.00\text{ min}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} \cdot \frac{1.00\text{ min}}{60.0\text{ s}} = 31.4\frac{\text{rad}}{\text{s}}. \tag{10.74}$$

The moment of inertia of one blade will be that of a thin rod rotated about its end, found in **Figure 10.12**. The total I is four times this moment of inertia, because there are four blades. Thus,

$$I = 4\frac{M\ell^2}{3} = 4\times\frac{(50.0\text{ kg})(4.00\text{ m})^2}{3} = 1067\text{ kg}\cdot\text{m}^2. \tag{10.75}$$

Entering ω and I into the expression for rotational kinetic energy gives

$$\begin{aligned}\text{KE}_\text{rot} &= 0.5(1067\text{ kg}\cdot\text{m}^2)(31.4\text{ rad/s})^2 \\ &= 5.26\times10^5\text{ J}\end{aligned} \tag{10.76}$$

Solution for (b)

Translational kinetic energy was defined in **Uniform Circular Motion and Gravitation**. Entering the given values of mass and velocity, we obtain

$$\text{KE}_\text{trans} = \frac{1}{2}mv^2 = (0.5)(1000\text{ kg})(20.0\text{ m/s})^2 = 2.00\times10^5\text{ J}. \tag{10.77}$$

To compare kinetic energies, we take the ratio of translational kinetic energy to rotational kinetic energy. This ratio is

$$\frac{2.00\times10^5\text{ J}}{5.26\times10^5\text{ J}} = 0.380. \tag{10.78}$$

Solution for (c)

At the maximum height, all rotational kinetic energy will have been converted to gravitational energy. To find this height, we equate those two energies:

$$\text{KE}_\text{rot} = \text{PE}_\text{grav} \tag{10.79}$$

or

$$\frac{1}{2}I\omega^2 = mgh. \tag{10.80}$$

We now solve for h and substitute known values into the resulting equation

$$h = \frac{\frac{1}{2}I\omega^2}{mg} = \frac{5.26\times10^5\ \text{J}}{(1000\ \text{kg})\left(9.80\ \text{m/s}^2\right)} = 53.7\ \text{m}. \tag{10.81}$$

Discussion

The ratio of translational energy to rotational kinetic energy is only 0.380. This ratio tells us that most of the kinetic energy of the helicopter is in its spinning blades—something you probably would not suspect. The 53.7 m height to which the helicopter could be raised with the rotational kinetic energy is also impressive, again emphasizing the amount of rotational kinetic energy in the blades.

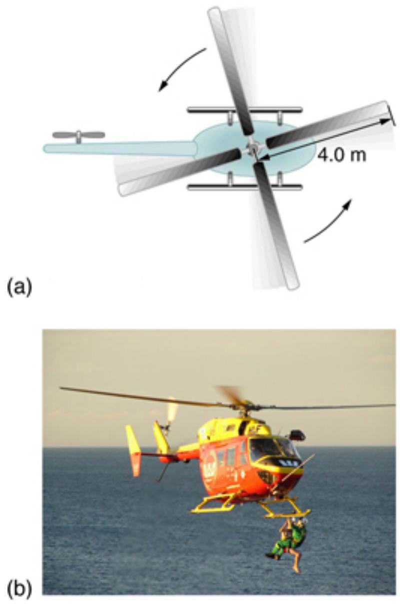


Figure 10.18 The first image shows how helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades. The second image shows a helicopter from the Auckland Westpac Rescue Helicopter Service. Over 50,000 lives have been saved since its operations beginning in 1973. Here, a water rescue operation is shown. (credit: 111 Emergency, Flickr)

Making Connections

Conservation of energy includes rotational motion, because rotational kinetic energy is another form of KE. **Uniform Circular Motion and Gravitation** has a detailed treatment of conservation of energy.

How Thick Is the Soup? Or Why Don't All Objects Roll Downhill at the Same Rate?

One of the quality controls in a tomato soup factory consists of rolling filled cans down a ramp. If they roll too fast, the soup is too thin. Why should cans of identical size and mass roll down an incline at different rates? And why should the thickest soup roll the slowest?

The easiest way to answer these questions is to consider energy. Suppose each can starts down the ramp from rest. Each can starting from rest means each starts with the same gravitational potential energy PE_{grav}, which is converted entirely to KE, provided each rolls without slipping. KE, however, can take the form of KE_{trans} or KE_{rot}, and total KE is the sum of the two. If a can rolls down a ramp, it puts part of its energy into rotation, leaving less for translation. Thus, the can goes slower than it would if it slid down. Furthermore, the thin soup does not rotate, whereas the thick soup does, because it sticks to the can. The thick soup thus puts more of the can's original gravitational potential energy into rotation than the thin soup, and the can rolls more slowly, as seen in **Figure 10.19**.

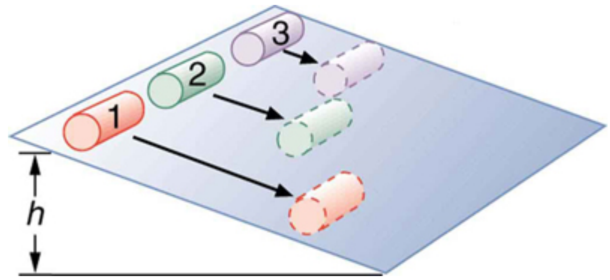


Figure 10.19 Three cans of soup with identical masses race down an incline. The first can has a low friction coating and does not roll but just slides down the incline. It wins because it converts its entire PE into translational KE. The second and third cans both roll down the incline without slipping. The second can contains thin soup and comes in second because part of its initial PE goes into rotating the can (but not the thin soup). The third can contains thick soup. It comes in third because the soup rotates along with the can, taking even more of the initial PE for rotational KE, leaving less for translational KE.

Assuming no losses due to friction, there is only one force doing work—gravity. Therefore the total work done is the change in kinetic energy. As the cans start moving, the potential energy is changing into kinetic energy. Conservation of energy gives

$$\mathrm{PE_i} = \mathrm{KE_f}. \tag{10.82}$$

More specifically,

$$\mathrm{PE_{grav}} = \mathrm{KE_{trans}} + \mathrm{KE_{rot}} \tag{10.83}$$

or

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \tag{10.84}$$

So, the initial mgh is divided between translational kinetic energy and rotational kinetic energy; and the greater I is, the less energy goes into translation. If the can slides down without friction, then $\omega = 0$ and all the energy goes into translation; thus, the can goes faster.

Take-Home Experiment

Locate several cans each containing different types of food. First, predict which can will win the race down an inclined plane and explain why. See if your prediction is correct. You could also do this experiment by collecting several empty cylindrical containers of the same size and filling them with different materials such as wet or dry sand.

Example 10.10 Calculating the Speed of a Cylinder Rolling Down an Incline

Calculate the final speed of a solid cylinder that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.

Strategy

We can solve for the final velocity using conservation of energy, but we must first express rotational quantities in terms of translational quantities to end up with v as the only unknown.

Solution

Conservation of energy for this situation is written as described above:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \tag{10.85}$$

Before we can solve for v , we must get an expression for I from **Figure 10.12**. Because v and ω are related (note here that the cylinder is rolling without slipping), we must also substitute the relationship $\omega = v / R$ into the expression. These substitutions yield

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right). \tag{10.86}$$

Interestingly, the cylinder's radius R and mass m cancel, yielding

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2. \tag{10.87}$$

Solving algebraically, the equation for the final velocity v gives

$$v = \left(\frac{4gh}{3}\right)^{1/2}. \tag{10.88}$$

Substituting known values into the resulting expression yields

$$v = \left[\frac{4\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m})}{3}\right]^{1/2} = 5.11\ \text{m/s}. \tag{10.89}$$

Discussion

Because m and R cancel, the result $v = \left(\frac{4}{3}gh\right)^{1/2}$ is valid for any solid cylinder, implying that all solid cylinders will roll down an incline at the same rate independent of their masses and sizes. (Rolling cylinders down inclines is what Galileo actually did to show that objects fall at the same rate independent of mass.) Note that if the cylinder slid without friction down the incline without rolling, then the entire gravitational potential energy would go into translational kinetic energy. Thus, $\frac{1}{2}mv^2 = mgh$ and $v = (2gh)^{1/2}$, which is 22% greater than $(4gh/3)^{1/2}$. That is, the cylinder would go faster at the bottom.

Check Your Understanding

Analogy of Rotational and Translational Kinetic Energy

Is rotational kinetic energy completely analogous to translational kinetic energy? What, if any, are their differences? Give an example of each type of kinetic energy.

Solution

Yes, rotational and translational kinetic energy are exact analogs. They both are the energy of motion involved with the coordinated (non-random) movement of mass relative to some reference frame. The only difference between rotational and translational kinetic energy is that translational is straight line motion while rotational is not. An example of both kinetic and translational kinetic energy is found in a bike tire while being ridden down a bike path. The rotational motion of the tire means it has rotational kinetic energy while the movement of the bike along the path means the tire also has translational kinetic energy. If you were to lift the front wheel of the bike and spin it while the bike is stationary, then the wheel would have only rotational kinetic energy relative to the Earth.

PhET Explorations: My Solar System

Build your own system of heavenly bodies and watch the gravitational ballet. With this orbit simulator, you can set initial positions, velocities, and masses of 2, 3, or 4 bodies, and then see them orbit each other.

PhET Interactive Simulation

Figure 10.20 My Solar System (http://cnx.org/content/m55188/1.4/my-solar-system_en.jar)

10.5 Angular Momentum and Its Conservation

Learning Objectives

By the end of this section, you will be able to:

- Understand the analogy between angular momentum and linear momentum.
- Observe the relationship between torque and angular momentum.
- Apply the law of conservation of angular momentum.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.D.2.1** The student is able to describe a model of a rotational system and use that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems. **(S.P. 1.2, 1.4)**
- **4.D.2.2** The student is able to plan a data collection and analysis strategy to determine the change in angular momentum of a system and relate it to interactions with other objects and systems. **(S.P. 2.2)**

- **4.D.3.1** The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum. **(S.P. 2.2)**
- **4.D.3.2** The student is able to plan a data collection strategy designed to test the relationship between the change in angular momentum of a system and the product of the average torque applied to the system and the time interval during which the torque is exerted. **(S.P. 4.1, 4.2)**
- **5.E.1.1** The student is able to make qualitative predictions about the angular momentum of a system for a situation in which there is no net external torque. **(S.P. 6.4, 7.2)**
- **5.E.1.2** The student is able to make calculations of quantities related to the angular momentum of a system when the net external torque on the system is zero. **(S.P. 2.1, 2.2)**
- **5.E.2.1** The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses. **(S.P. 2.2)**

Why does Earth keep on spinning? What started it spinning to begin with? And how does an ice skater manage to spin faster and faster simply by pulling her arms in? Why does she not have to exert a torque to spin faster? Questions like these have answers based in angular momentum, the rotational analog to linear momentum.

By now the pattern is clear—every rotational phenomenon has a direct translational analog. It seems quite reasonable, then, to define **angular momentum** L as

$$L = I\omega. \tag{10.90}$$

This equation is an analog to the definition of linear momentum as $p = mv$. Units for linear momentum are $\text{kg} \cdot \text{m/s}$ while units for angular momentum are $\text{kg} \cdot \text{m}^2\text{/s}$. As we would expect, an object that has a large moment of inertia I, such as Earth, has a very large angular momentum. An object that has a large angular velocity ω, such as a centrifuge, also has a rather large angular momentum.

Making Connections

Angular momentum is completely analogous to linear momentum, first presented in **Uniform Circular Motion and Gravitation**. It has the same implications in terms of carrying rotation forward, and it is conserved when the net external torque is zero. Angular momentum, like linear momentum, is also a property of the atoms and subatomic particles.

Example 10.11 Calculating Angular Momentum of the Earth

Strategy

No information is given in the statement of the problem; so we must look up pertinent data before we can calculate $L = I\omega$. First, according to **Figure 10.12**, the formula for the moment of inertia of a sphere is

$$I = \frac{2MR^2}{5} \tag{10.91}$$

so that

$$L = I\omega = \frac{2MR^2\omega}{5}. \tag{10.92}$$

Earth's mass M is $5.979\times10^{24}\ \text{kg}$ and its radius R is $6.376\times10^{6}\ \text{m}$. The Earth's angular velocity ω is, of course, exactly one revolution per day, but we must covert ω to radians per second to do the calculation in SI units.

Solution

Substituting known information into the expression for L and converting ω to radians per second gives

$$\begin{aligned} L &= 0.4\left(5.979\times10^{24}\ \text{kg}\right)\left(6.376\times10^{6}\ \text{m}\right)^2\left(\frac{1\ \text{rev}}{\text{d}}\right) \\ &= 9.72\times10^{37}\ \text{kg}\cdot\text{m}^2\cdot\text{rev/d}. \end{aligned} \tag{10.93}$$

Substituting 2π rad for 1 rev and $8.64\times10^{4}\ \text{s}$ for 1 day gives

$$\begin{aligned} L &= \left(9.72\times10^{37}\ \text{kg}\cdot\text{m}^2\right)\left(\frac{2\pi\ \text{rad/rev}}{8.64\times10^{4}\ \text{s/d}}\right)(1\ \text{rev/d}) \\ &= 7.07\times10^{33}\ \text{kg}\cdot\text{m}^2\text{/s}. \end{aligned} \tag{10.94}$$

Discussion

This number is large, demonstrating that Earth, as expected, has a tremendous angular momentum. The answer is approximate, because we have assumed a constant density for Earth in order to estimate its moment of inertia.

When you push a merry-go-round, spin a bike wheel, or open a door, you exert a torque. If the torque you exert is greater than opposing torques, then the rotation accelerates, and angular momentum increases. The greater the net torque, the more rapid the increase in L. The relationship between torque and angular momentum is

$$\text{net } \tau = \frac{\Delta L}{\Delta t}. \tag{10.95}$$

This expression is exactly analogous to the relationship between force and linear momentum, $F = \Delta p / \Delta t$. The equation $\text{net } \tau = \frac{\Delta L}{\Delta t}$ is very fundamental and broadly applicable. It is, in fact, the rotational form of Newton's second law.

Example 10.12 Calculating the Torque Putting Angular Momentum Into a Lazy Susan

Figure 10.21 shows a Lazy Susan food tray being rotated by a person in quest of sustenance. Suppose the person exerts a 2.50 N force perpendicular to the lazy Susan's 0.260-m radius for 0.150 s. (a) What is the final angular momentum of the lazy Susan if it starts from rest, assuming friction is negligible? (b) What is the final angular velocity of the lazy Susan, given that its mass is 4.00 kg and assuming its moment of inertia is that of a disk?

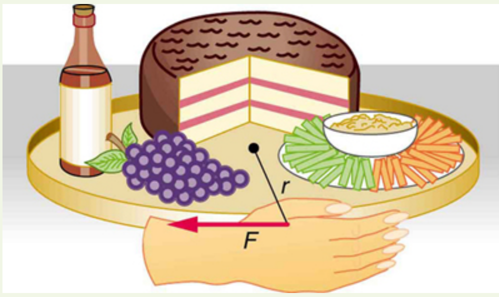


Figure 10.21 A partygoer exerts a torque on a lazy Susan to make it rotate. The equation $\text{net } \tau = \frac{\Delta L}{\Delta t}$ gives the relationship between torque and the angular momentum produced.

Strategy

We can find the angular momentum by solving $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for ΔL, and using the given information to calculate the torque. The final angular momentum equals the change in angular momentum, because the lazy Susan starts from rest. That is, $\Delta L = L$. To find the final velocity, we must calculate ω from the definition of L in $L = I\omega$.

Solution for (a)

Solving $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for ΔL gives

$$\Delta L = (\text{net } \tau)\Delta \text{t}. \tag{10.96}$$

Because the force is perpendicular to r, we see that $\text{net } \tau = rF$, so that

$$\begin{aligned} L &= \text{rF}\Delta t = (0.260 \text{ m})(2.50 \text{ N})(0.150 \text{ s}) \\ &= 9.75\times10^{-2} \text{ kg}\cdot\text{m}^2/\text{s}. \end{aligned} \tag{10.97}$$

Solution for (b)

The final angular velocity can be calculated from the definition of angular momentum,

$$L = I\omega. \tag{10.98}$$

Solving for ω and substituting the formula for the moment of inertia of a disk into the resulting equation gives

$$\omega = \frac{L}{I} = \frac{L}{\frac{1}{2}MR^2}. \tag{10.99}$$

And substituting known values into the preceding equation yields

$$\omega = \frac{9.75\times10^{-2}\ \text{kg}\cdot\text{m}^2/\text{s}}{(0.500)(4.00\ \text{kg})(0.260\ \text{m})} = 0.721\ \text{rad/s}. \tag{10.100}$$

Discussion

Note that the imparted angular momentum does not depend on any property of the object but only on torque and time. The final angular velocity is equivalent to one revolution in 8.71 s (determination of the time period is left as an exercise for the reader), which is about right for a lazy Susan.

Take-Home Experiment

Plan an experiment to analyze changes to a system's angular momentum. Choose a system capable of rotational motion such as a lazy Susan or a merry-go-round. Predict how the angular momentum of this system will change when you add an object to the lazy Susan or jump onto the merry-go-round. What variables can you control? What are you measuring? In other words, what are your independent and dependent variables? Are there any independent variables that it would be useful to keep constant (angular velocity, perhaps)? Collect data in order to calculate or estimate the angular momentum of your system when in motion. What do you observe? Collect data in order to calculate the change in angular momentum as a result of the interaction you performed.

Using your data, how does the angular momentum vary with the size and location of an object added to the rotating system?

Example 10.13 Calculating the Torque in a Kick

The person whose leg is shown in **Figure 10.22** kicks his leg by exerting a 2000-N force with his upper leg muscle. The effective perpendicular lever arm is 2.20 cm. Given the moment of inertia of the lower leg is $1.25\ \text{kg}\cdot\text{m}^2$, (a) find the angular acceleration of the leg. (b) Neglecting the gravitational force, what is the rotational kinetic energy of the leg after it has rotated through 57.3^o (1.00 rad)?

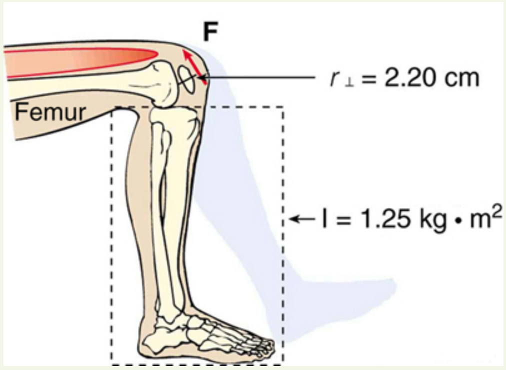


Figure 10.22 The muscle in the upper leg gives the lower leg an angular acceleration and imparts rotational kinetic energy to it by exerting a torque about the knee. $\mathbf{F}$ is a vector that is perpendicular to r. This example examines the situation.

Strategy

The angular acceleration can be found using the rotational analog to Newton's second law, or $\alpha = \text{net}\ \tau / I$. The moment of inertia I is given and the torque can be found easily from the given force and perpendicular lever arm. Once the angular acceleration α is known, the final angular velocity and rotational kinetic energy can be calculated.

Solution to (a)

From the rotational analog to Newton's second law, the angular acceleration α is

$$\alpha = \frac{\text{net}\ \tau}{I}. \tag{10.101}$$

Because the force and the perpendicular lever arm are given and the leg is vertical so that its weight does not create a torque, the net torque is thus

$$\begin{aligned} \text{net}\ \tau &= r_\perp\ F \\ &= (0.0220\ \text{m})(2000\ \text{N}) \\ &= 44.0\ \text{N}\cdot\text{m}. \end{aligned} \tag{10.102}$$

Substituting this value for the torque and the given value for the moment of inertia into the expression for α gives

$$\alpha = \frac{44.0\ \text{N}\cdot\text{m}}{1.25\ \text{kg}\cdot\text{m}^2} = 35.2\ \text{rad/s}^2. \tag{10.103}$$

Solution to (b)

The final angular velocity can be calculated from the kinematic expression

$$\omega^2 = {\omega_0}^2 + 2\alpha\theta \tag{10.104}$$

or

$$\omega^2 = 2\alpha\theta \tag{10.105}$$

because the initial angular velocity is zero. The kinetic energy of rotation is

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2 \tag{10.106}$$

so it is most convenient to use the value of ω^2 just found and the given value for the moment of inertia. The kinetic energy is then

$$\begin{aligned}\text{KE}_{\text{rot}} &= 0.5\left(1.25\ \text{kg}\cdot\text{m}^2\right)\left(70.4\ \text{rad}^2/\text{s}^2\right) \\ &= 44.0\ \text{J}\end{aligned}. \tag{10.107}$$

Discussion

These values are reasonable for a person kicking his leg starting from the position shown. The weight of the leg can be neglected in part (a) because it exerts no torque when the center of gravity of the lower leg is directly beneath the pivot in the knee. In part (b), the force exerted by the upper leg is so large that its torque is much greater than that created by the weight of the lower leg as it rotates. The rotational kinetic energy given to the lower leg is enough that it could give a ball a significant velocity by transferring some of this energy in a kick.

Making Connections: Conservation Laws

Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

Conservation of Angular Momentum

We can now understand why Earth keeps on spinning. As we saw in the previous example, $\Delta L = (\text{net }\tau)\Delta t$. This equation means that, to change angular momentum, a torque must act over some period of time. Because Earth has a large angular momentum, a large torque acting over a long time is needed to change its rate of spin. So what external torques are there? Tidal friction exerts torque that is slowing Earth's rotation, but tens of millions of years must pass before the change is very significant. Recent research indicates the length of the day was 18 h some 900 million years ago. Only the tides exert significant retarding torques on Earth, and so it will continue to spin, although ever more slowly, for many billions of years.

What we have here is, in fact, another conservation law. If the net torque is *zero*, then angular momentum is constant or *conserved*. We can see this rigorously by considering $\text{net }\tau = \frac{\Delta L}{\Delta t}$ for the situation in which the net torque is zero. In that case,

$$\text{net}\tau = 0 \tag{10.108}$$

implying that

$$\frac{\Delta L}{\Delta t} = 0. \tag{10.109}$$

If the change in angular momentum ΔL is zero, then the angular momentum is constant; thus,

$$L = \text{constant (net }\tau = 0) \tag{10.110}$$

or

$$L = L'(\text{net}\tau = 0). \tag{10.111}$$

These expressions are the **law of conservation of angular momentum**. Conservation laws are as scarce as they are important.

An example of conservation of angular momentum is seen in **Figure 10.23**, in which an ice skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice and because the friction is exerted very close to the pivot point. (Both F and r are small, and so τ is negligibly small.) Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that

$$L = L'. \tag{10.112}$$

Expressing this equation in terms of the moment of inertia,

$$I\omega = I'\omega', \tag{10.113}$$

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because I' is smaller, the angular velocity ω' must increase to keep the angular momentum constant. The change can be dramatic, as the following example shows.

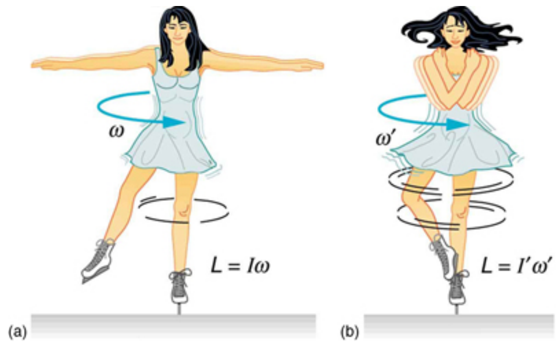


Figure 10.23 (a) An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small. In the next image, her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.

Example 10.14 Calculating the Angular Momentum of a Spinning Skater

Suppose an ice skater, such as the one in **Figure 10.23**, is spinning at 0.800 rev/ s with her arms extended. She has a moment of inertia of $2.34 \text{ kg} \cdot \text{m}^2$ with her arms extended and of $0.363 \text{ kg} \cdot \text{m}^2$ with her arms close to her body. (These moments of inertia are based on reasonable assumptions about a 60.0-kg skater.) (a) What is her angular velocity in revolutions per second after she pulls in her arms? (b) What is her rotational kinetic energy before and after she does this?

Strategy

In the first part of the problem, we are looking for the skater's angular velocity ω' after she has pulled in her arms. To find this quantity, we use the conservation of angular momentum and note that the moments of inertia and initial angular velocity are given. To find the initial and final kinetic energies, we use the definition of rotational kinetic energy given by

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2. \tag{10.114}$$

Solution for (a)

Because torque is negligible (as discussed above), the conservation of angular momentum given in $I\omega = I'\omega'$ is applicable. Thus,

$$L = L' \tag{10.115}$$

or

$$I\omega = I'\omega' \tag{10.116}$$

Solving for ω' and substituting known values into the resulting equation gives

$$\begin{aligned} \omega' &= \frac{I}{I'}\omega = \left(\frac{2.34 \text{ kg} \cdot \text{m}^2}{0.363 \text{ kg} \cdot \text{m}^2}\right)(0.800 \text{ rev/s}) \\ &= 5.16 \text{ rev/s}. \end{aligned} \tag{10.117}$$

Solution for (b)

Rotational kinetic energy is given by

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2. \tag{10.118}$$

The initial value is found by substituting known values into the equation and converting the angular velocity to rad/s:

$$\begin{aligned} \mathrm{KE_{rot}} &= (0.5)\left(2.34\ \mathrm{kg}\cdot\mathrm{m}^2\right)\left((0.800\ \mathrm{rev/s})(2\pi\ \mathrm{rad/rev})\right)^2 \\ &= 29.6\ \mathrm{J}. \end{aligned} \tag{10.119}$$

The final rotational kinetic energy is

$$\mathrm{KE_{rot}}' = \frac{1}{2}I'\omega'^2. \tag{10.120}$$

Substituting known values into this equation gives

$$\begin{aligned} KE_{\mathrm{rot}}' &= (0.5)\left(0.363\ \mathrm{kg}\cdot\mathrm{m}^2\right)\left[(5.16\ \mathrm{rev/s})(2\pi\ \mathrm{rad/rev})\right]^2 \\ &= 191\ \mathrm{J}. \end{aligned} \tag{10.121}$$

Discussion

In both parts, there is an impressive increase. First, the final angular velocity is large, although most world-class skaters can achieve spin rates about this great. Second, the final kinetic energy is much greater than the initial kinetic energy. The increase in rotational kinetic energy comes from work done by the skater in pulling in her arms. This work is internal work that depletes some of the skater's food energy.

There are several other examples of objects that increase their rate of spin because something reduced their moment of inertia. Tornadoes are one example. Storm systems that create tornadoes are slowly rotating. When the radius of rotation narrows, even in a local region, angular velocity increases, sometimes to the furious level of a tornado. Earth is another example. Our planet was born from a huge cloud of gas and dust, the rotation of which came from turbulence in an even larger cloud. Gravitational forces caused the cloud to contract, and the rotation rate increased as a result. (See **Figure 10.24**.)

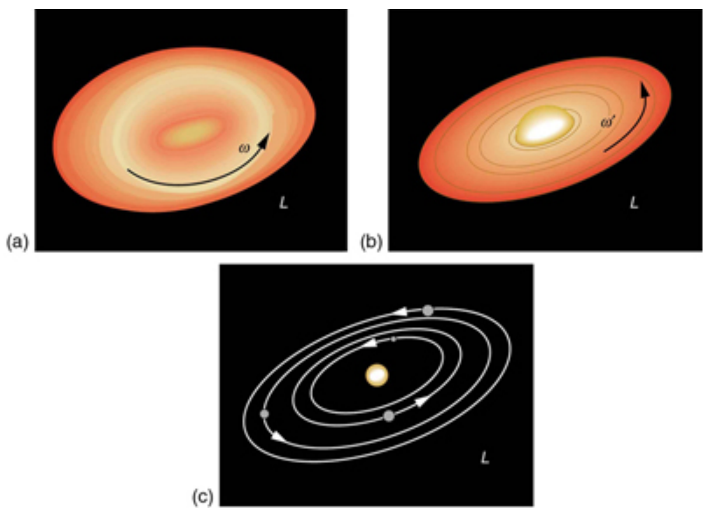


Figure 10.24 The Solar System coalesced from a cloud of gas and dust that was originally rotating. The orbital motions and spins of the planets are in the same direction as the original spin and conserve the angular momentum of the parent cloud.

In case of human motion, one would not expect angular momentum to be conserved when a body interacts with the environment as its foot pushes off the ground. Astronauts floating in space aboard the International Space Station have no angular momentum relative to the inside of the ship if they are motionless. Their bodies will continue to have this zero value no matter how they twist about as long as they do not give themselves a push off the side of the vessel.

Check Your Undestanding

Is angular momentum completely analogous to linear momentum? What, if any, are their differences?

Solution

Yes, angular and linear momentums are completely analogous. While they are exact analogs they have different units and are not directly inter-convertible like forms of energy are.

10.6 Collisions of Extended Bodies in Two Dimensions

Learning Objectives

By the end of this section, you will be able to:

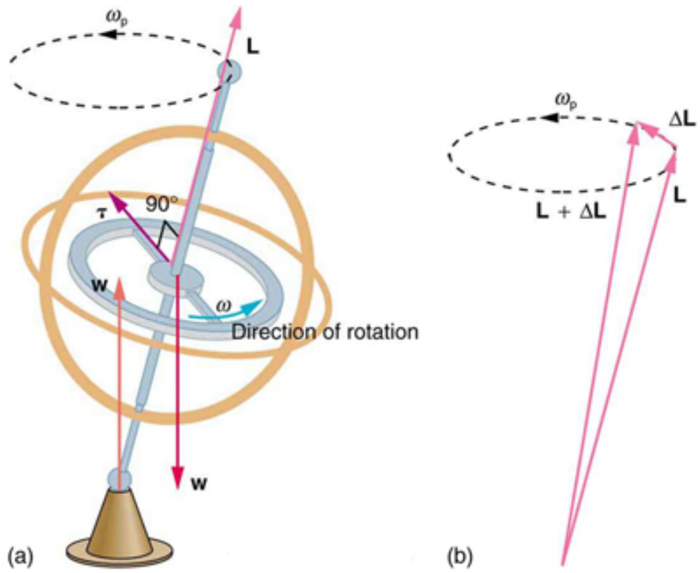


Figure 10.31 As seen in figure (a), the forces on a spinning gyroscope are its weight and the supporting force from the stand. These forces create a horizontal torque on the gyroscope, which create a change in angular momentum $\Delta\mathbf{L}$ that is also horizontal. In figure (b), $\Delta\mathbf{L}$ and $\mathbf{L}$ add to produce a new angular momentum with the same magnitude, but different direction, so that the gyroscope precesses in the direction shown instead of falling over.

Check Your Understanding

Rotational kinetic energy is associated with angular momentum? Does that mean that rotational kinetic energy is a vector?

Solution
No, energy is always a scalar whether motion is involved or not. No form of energy has a direction in space and you can see that rotational kinetic energy does not depend on the direction of motion just as linear kinetic energy is independent of the direction of motion.

Glossary

angular acceleration: the rate of change of angular velocity with time

angular momentum: the product of moment of inertia and angular velocity

change in angular velocity: the difference between final and initial values of angular velocity

kinematics of rotational motion: describes the relationships among rotation angle, angular velocity, angular acceleration, and time

law of conservation of angular momentum: angular momentum is conserved, i.e., the initial angular momentum is equal to the final angular momentum when no external torque is applied to the system

moment of inertia: mass times the square of perpendicular distance from the rotation axis; for a point mass, it is $I = mr^2$ and, because any object can be built up from a collection of point masses, this relationship is the basis for all other moments of inertia

right-hand rule: direction of angular velocity ω and angular momentum L in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation

rotational inertia: resistance to change of rotation. The more rotational inertia an object has, the harder it is to rotate

rotational kinetic energy: the kinetic energy due to the rotation of an object. This is part of its total kinetic energy

tangential acceleration: the acceleration in a direction tangent to the circle at the point of interest in circular motion

torque: the turning effectiveness of a force

work-energy theorem: if one or more external forces act upon a rigid object, causing its kinetic energy to change from KE_1 to KE_2, then the work W done by the net force is equal to the change in kinetic energy

Section Summary

10.1 Angular Acceleration

- Uniform circular motion is the motion with a constant angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$.
- In non-uniform circular motion, the velocity changes with time and the rate of change of angular velocity (i.e. angular acceleration) is $\alpha = \frac{\Delta\omega}{\Delta t}$.
- Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction, given as $a_{\mathrm{t}} = \frac{\Delta v}{\Delta t}$.
- For circular motion, note that $v = r\omega$, so that

$$a_{\mathrm{t}} = \frac{\Delta(r\omega)}{\Delta t}.$$

- The radius r is constant for circular motion, and so $\Delta(r\omega) = r\Delta\omega$. Thus,

$$a_{\mathrm{t}} = r\frac{\Delta\omega}{\Delta t}.$$

- By definition, $\Delta\omega / \Delta t = \alpha$. Thus,

$$a_{\mathrm{t}} = r\alpha$$

or

$$\alpha = \frac{a_{\mathrm{t}}}{r}.$$

10.2 Kinematics of Rotational Motion

- Kinematics is the description of motion.
- The kinematics of rotational motion describes the relationships among rotation angle, angular velocity, angular acceleration, and time.
- Starting with the four kinematic equations we developed in the **One-Dimensional Kinematics**, we can derive the four rotational kinematic equations (presented together with their translational counterparts) seen in **Table 10.2**.
- In these equations, the subscript 0 denotes initial values (x_0 and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity $\bar{v}$ are defined as follows:

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} \text{ and } \bar{v} = \frac{v_0 + v}{2}.$$

10.3 Dynamics of Rotational Motion: Rotational Inertia

- The farther the force is applied from the pivot, the greater is the angular acceleration; angular acceleration is inversely proportional to mass.
- If we exert a force F on a point mass m that is at a distance r from a pivot point and because the force is perpendicular to r, an acceleration $a = F/m$ is obtained in the direction of F. We can rearrange this equation such that

$$F = ma,$$

and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F{=}ma$, yielding

$$F{=}mr\alpha$$

- Torque is the turning effectiveness of a force. In this case, because F is perpendicular to r, torque is simply $\tau = rF$. If we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,

$$rF = mr^2\alpha$$

or

$$\tau = mr^2\alpha.$$

- The moment of inertia I of an object is the sum of MR^2 for all the point masses of which it is composed. That is,

$$I = \sum mr^2.$$

- The general relationship among torque, moment of inertia, and angular acceleration is

$$\tau = I\alpha$$

or

$$\alpha = \frac{\text{net } \tau}{I}.$$

10.4 Rotational Kinetic Energy: Work and Energy Revisited

- The rotational kinetic energy $\mathrm{KE_{rot}}$ for an object with a moment of inertia I and an angular velocity ω is given by

$$\mathrm{KE_{rot}} = \frac{1}{2}I\omega^2.$$

- Helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades.
- Work and energy in rotational motion are completely analogous to work and energy in translational motion.
- The equation for the **work-energy theorem** for rotational motion is,

$$\text{net } W = \frac{1}{2}I\omega^2 - \frac{1}{2}I{\omega_0}^2.$$

10.5 Angular Momentum and Its Conservation

- Every rotational phenomenon has a direct translational analog , likewise angular momentum L can be defined as $L = I\omega.$
- This equation is an analog to the definition of linear momentum as $p = mv$. The relationship between torque and angular momentum is $\text{net } \tau = \frac{\Delta L}{\Delta t}.$
- Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

10.6 Collisions of Extended Bodies in Two Dimensions

- Angular momentum L is analogous to linear momentum and is given by $L = I\omega$.
- Angular momentum is changed by torque, following the relationship $\text{net } \tau = \frac{\Delta L}{\Delta t}.$
- Angular momentum is conserved if the net torque is zero $L = \text{constant } (\text{net } \tau = 0)$ or $L = L' \ (\text{net } \tau = 0)$. This equation is known as the law of conservation of angular momentum, which may be conserved in collisions.

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

- Torque is perpendicular to the plane formed by r and $\mathbf{F}$ and is the direction your right thumb would point if you curled the fingers of your right hand in the direction of $\mathbf{F}$. The direction of the torque is thus the same as that of the angular momentum it produces.
- The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to $\mathbf{L}$. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque ($\mathbf{L} = \Delta\mathbf{L}$), and it rotates about a horizontal axis, falling over just as we would expect.
- Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star.

Conceptual Questions

10.1 Angular Acceleration

1. Analogies exist between rotational and translational physical quantities. Identify the rotational term analogous to each of the following: acceleration, force, mass, work, translational kinetic energy, linear momentum, impulse.

2. Explain why centripetal acceleration changes the direction of velocity in circular motion but not its magnitude.

3. In circular motion, a tangential acceleration can change the magnitude of the velocity but not its direction. Explain your answer.

4. Suppose a piece of food is on the edge of a rotating microwave oven plate. Does it experience nonzero tangential acceleration, centripetal acceleration, or both when: (a) The plate starts to spin? (b) The plate rotates at constant angular velocity? (c) The plate slows to a halt?

10.3 Dynamics of Rotational Motion: Rotational Inertia

5. The moment of inertia of a long rod spun around an axis through one end perpendicular to its length is $ML^2/3$. Why is this moment of inertia greater than it would be if you spun a point mass M at the location of the center of mass of the rod (at $L/2$)? (That would be $ML^2/4$.)

6. Why is the moment of inertia of a hoop that has a mass M and a radius R greater than the moment of inertia of a disk that has the same mass and radius? Why is the moment of inertia of a spherical shell that has a mass M and a radius R greater than that of a solid sphere that has the same mass and radius?

7. Give an example in which a small force exerts a large torque. Give another example in which a large force exerts a small torque.

8. While reducing the mass of a racing bike, the greatest benefit is realized from reducing the mass of the tires and wheel rims. Why does this allow a racer to achieve greater accelerations than would an identical reduction in the mass of the bicycle's frame?

Figure 10.32 The image shows a side view of a racing bicycle. Can you see evidence in the design of the wheels on this racing bicycle that their moment of inertia has been purposely reduced? (credit: Jesús Rodriguez)

9. A ball slides up a frictionless ramp. It is then rolled without slipping and with the same initial velocity up another frictionless ramp (with the same slope angle). In which case does it reach a greater height, and why?

10.4 Rotational Kinetic Energy: Work and Energy Revisited

10. Describe the energy transformations involved when a yo-yo is thrown downward and then climbs back up its string to be caught in the user's hand.

11. What energy transformations are involved when a dragster engine is revved, its clutch let out rapidly, its tires spun, and it starts to accelerate forward? Describe the source and transformation of energy at each step.

12. The Earth has more rotational kinetic energy now than did the cloud of gas and dust from which it formed. Where did this energy come from?

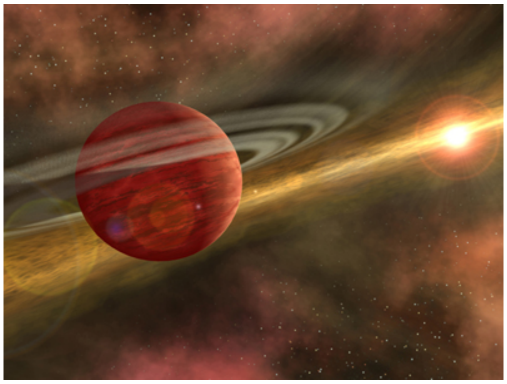


Figure 10.33 An immense cloud of rotating gas and dust contracted under the influence of gravity to form the Earth and in the process rotational kinetic energy increased. (credit: NASA)

10.5 Angular Momentum and Its Conservation

13. When you start the engine of your car with the transmission in neutral, you notice that the car rocks in the opposite sense of the engine's rotation. Explain in terms of conservation of angular momentum. Is the angular momentum of the car conserved for long (for more than a few seconds)?

14. Suppose a child walks from the outer edge of a rotating merry-go round to the inside. Does the angular velocity of the merry-go-round increase, decrease, or remain the same? Explain your answer.

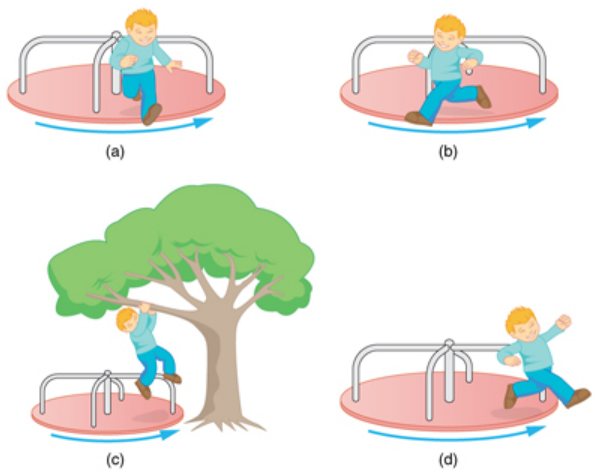


Figure 10.34 A child may jump off a merry-go-round in a variety of directions.

15. Suppose a child gets off a rotating merry-go-round. Does the angular velocity of the merry-go-round increase, decrease, or remain the same if: (a) He jumps off radially? (b) He jumps backward to land motionless? (c) He jumps straight up and hangs onto an overhead tree branch? (d) He jumps off forward, tangential to the edge? Explain your answers. (Refer to **Figure 10.34**).

16. Helicopters have a small propeller on their tail to keep them from rotating in the opposite direction of their main lifting blades. Explain in terms of Newton's third law why the helicopter body rotates in the opposite direction to the blades.

17. Whenever a helicopter has two sets of lifting blades, they rotate in opposite directions (and there will be no tail propeller). Explain why it is best to have the blades rotate in opposite directions.

18. Describe how work is done by a skater pulling in her arms during a spin. In particular, identify the force she exerts on each arm to pull it in and the distance each moves, noting that a component of the force is in the direction moved. Why is angular momentum not increased by this action?

19. When there is a global heating trend on Earth, the atmosphere expands and the length of the day increases very slightly. Explain why the length of a day increases.

20. Nearly all conventional piston engines have flywheels on them to smooth out engine vibrations caused by the thrust of individual piston firings. Why does the flywheel have this effect?

21. Jet turbines spin rapidly. They are designed to fly apart if something makes them seize suddenly, rather than transfer angular momentum to the plane's wing, possibly tearing it off. Explain how flying apart conserves angular momentum without transferring it to the wing.

22. An astronaut tightens a bolt on a satellite in orbit. He rotates in a direction opposite to that of the bolt, and the satellite rotates in the same direction as the bolt. Explain why. If a handhold is available on the satellite, can this counter-rotation be prevented? Explain your answer.

23. Competitive divers pull their limbs in and curl up their bodies when they do flips. Just before entering the water, they fully extend their limbs to enter straight down. Explain the effect of both actions on their angular velocities. Also explain the effect on their angular momenta.

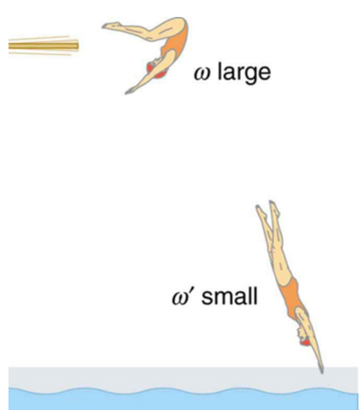


Figure 10.35 The diver spins rapidly when curled up and slows when she extends her limbs before entering the water.

24. Draw a free body diagram to show how a diver gains angular momentum when leaving the diving board.

25. In terms of angular momentum, what is the advantage of giving a football or a rifle bullet a spin when throwing or releasing it?

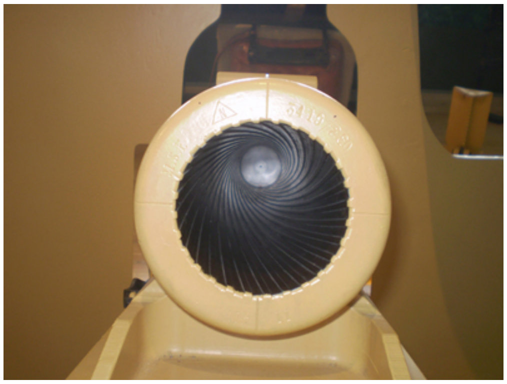


Figure 10.36 The image shows a view down the barrel of a cannon, emphasizing its rifling. Rifling in the barrel of a canon causes the projectile to spin just as is the case for rifles (hence the name for the grooves in the barrel). (credit: Elsie esq., Flickr)

10.6 Collisions of Extended Bodies in Two Dimensions

26. Describe two different collisions—one in which angular momentum is conserved, and the other in which it is not. Which condition determines whether or not angular momentum is conserved in a collision?

27. Suppose an ice hockey puck strikes a hockey stick that lies flat on the ice and is free to move in any direction. Which quantities are likely to be conserved: angular momentum, linear momentum, or kinetic energy (assuming the puck and stick are very resilient)?

28. While driving his motorcycle at highway speed, a physics student notices that pulling back lightly on the right handlebar tips the cycle to the left and produces a left turn. Explain why this happens.

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

29. While driving his motorcycle at highway speed, a physics student notices that pulling back lightly on the right handlebar tips the cycle to the left and produces a left turn. Explain why this happens.

30. Gyroscopes used in guidance systems to indicate directions in space must have an angular momentum that does not change in direction. Yet they are often subjected to large forces and accelerations. How can the direction of their angular momentum be constant when they are accelerated?

Problems & Exercises

10.1 Angular Acceleration

1. At its peak, a tornado is 60.0 m in diameter and carries 500 km/h winds. What is its angular velocity in revolutions per second?

2. Integrated Concepts

An ultracentrifuge accelerates from rest to 100,000 rpm in 2.00 min. (a) What is its angular acceleration in rad/s^2 ? (b) What is the tangential acceleration of a point 9.50 cm from the axis of rotation? (c) What is the radial acceleration in m/s^2 and multiples of g of this point at full rpm?

3. Integrated Concepts

You have a grindstone (a disk) that is 90.0 kg, has a 0.340-m radius, and is turning at 90.0 rpm, and you press a steel axe against it with a radial force of 20.0 N. (a) Assuming the kinetic coefficient of friction between steel and stone is 0.20, calculate the angular acceleration of the grindstone. (b) How many turns will the stone make before coming to rest?

4. Unreasonable Results

You are told that a basketball player spins the ball with an angular acceleration of $100\ \text{rad/s}^2$. (a) What is the ball's final angular velocity if the ball starts from rest and the acceleration lasts 2.00 s? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

10.2 Kinematics of Rotational Motion

5. With the aid of a string, a gyroscope is accelerated from rest to 32 rad/s in 0.40 s.

(a) What is its angular acceleration in rad/s^2?

(b) How many revolutions does it go through in the process?

6. Suppose a piece of dust finds itself on a CD. If the spin rate of the CD is 500 rpm, and the piece of dust is 4.3 cm from the center, what is the total distance traveled by the dust in 3 minutes? (Ignore accelerations due to getting the CD rotating.)

7. A gyroscope slows from an initial rate of 32.0 rad/s at a rate of $0.700\ \text{rad/s}^2$.

(a) How long does it take to come to rest?

(b) How many revolutions does it make before stopping?

8. During a very quick stop, a car decelerates at $7.00\ \text{m/s}^2$.

(a) What is the angular acceleration of its 0.280-m-radius tires, assuming they do not slip on the pavement?

(b) How many revolutions do the tires make before coming to rest, given their initial angular velocity is $95.0\ \text{rad/s}$?

(c) How long does the car take to stop completely?

(d) What distance does the car travel in this time?

(e) What was the car's initial velocity?

(f) Do the values obtained seem reasonable, considering that this stop happens very quickly?

Figure 10.37 Yo-yos are amusing toys that display significant physics and are engineered to enhance performance based on physical laws. (credit: Beyond Neon, Flickr)

9. Everyday application: Suppose a yo-yo has a center shaft that has a 0.250 cm radius and that its string is being pulled.

(a) If the string is stationary and the yo-yo accelerates away from it at a rate of $1.50\ \text{m/s}^2$, what is the angular acceleration of the yo-yo?

(b) What is the angular velocity after 0.750 s if it starts from rest?

(c) The outside radius of the yo-yo is 3.50 cm. What is the tangential acceleration of a point on its edge?

10.3 Dynamics of Rotational Motion: Rotational Inertia

10. This problem considers additional aspects of example **Calculating the Effect of Mass Distribution on a Merry-Go-Round**. (a) How long does it take the father to give the merry-go-round an angular velocity of 1.50 rad/s? (b) How many revolutions must he go through to generate this velocity? (c) If he exerts a slowing force of 300 N at a radius of 1.35 m, how long would it take him to stop them?

11. Calculate the moment of inertia of a skater given the following information. (a) The 60.0-kg skater is approximated as a cylinder that has a 0.110-m radius. (b) The skater with arms extended is approximately a cylinder that is 52.5 kg, has a 0.110-m radius, and has two 0.900-m-long arms which are 3.75 kg each and extend straight out from the cylinder like rods rotated about their ends.

12. The triceps muscle in the back of the upper arm extends the forearm. This muscle in a professional boxer exerts a force of $2.00\times10^3\ \text{N}$ with an effective perpendicular lever arm of 3.00 cm, producing an angular acceleration of the forearm of $120\ \text{rad/s}^2$. What is the moment of inertia of the boxer's forearm?

13. A soccer player extends her lower leg in a kicking motion by exerting a force with the muscle above the knee in the front of her leg. She produces an angular acceleration of $30.00\ \text{rad/s}^2$ and her lower leg has a moment of inertia of $0.750\ \text{kg}\cdot\text{m}^2$. What is the force exerted by the muscle if its effective perpendicular lever arm is 1.90 cm?

14. Suppose you exert a force of 180 N tangential to a 0.280-m-radius 75.0-kg grindstone (a solid disk).

(a)What torque is exerted? (b) What is the angular acceleration assuming negligible opposing friction? (c) What is the angular acceleration if there is an opposing frictional force of 20.0 N exerted 1.50 cm from the axis?

15. Consider the 12.0 kg motorcycle wheel shown in **Figure 10.38**. Assume it to be approximately an annular ring with an inner radius of 0.280 m and an outer radius of 0.330 m. The motorcycle is on its center stand, so that the wheel can spin freely. (a) If the drive chain exerts a force of 2200 N at a radius of 5.00 cm, what is the angular acceleration of the wheel? (b) What is the tangential acceleration of a point on the outer edge of the tire? (c) How long, starting from rest, does it take to reach an angular velocity of 80.0 rad/s?

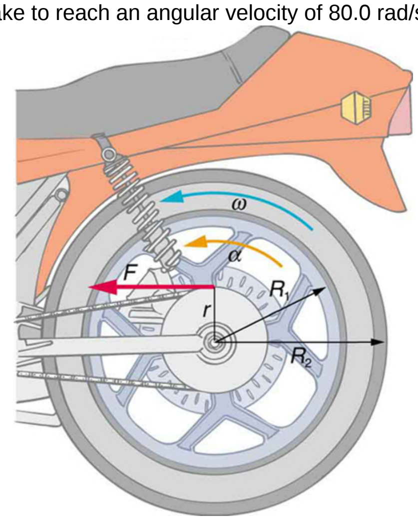


Figure 10.38 A motorcycle wheel has a moment of inertia approximately that of an annular ring.

16. Zorch, an archenemy of Superman, decides to slow Earth's rotation to once per 28.0 h by exerting an opposing force at and parallel to the equator. Superman is not immediately concerned, because he knows Zorch can only exert a force of $4.00 \times 10^7 \text{ N}$ (a little greater than a Saturn V rocket's thrust). How long must Zorch push with this force to accomplish his goal? (This period gives Superman time to devote to other villains.) Explicitly show how you follow the steps found in **Problem-Solving Strategy for Rotational Dynamics**.

17. An automobile engine can produce 200 N · m of torque. Calculate the angular acceleration produced if 95.0% of this torque is applied to the drive shaft, axle, and rear wheels of a car, given the following information. The car is suspended so that the wheels can turn freely. Each wheel acts like a 15.0 kg disk that has a 0.180 m radius. The walls of each tire act like a 2.00-kg annular ring that has inside radius of 0.180 m and outside radius of 0.320 m. The tread of each tire acts like a 10.0-kg hoop of radius 0.330 m. The 14.0-kg axle acts like a rod that has a 2.00-cm radius. The 30.0-kg drive shaft acts like a rod that has a 3.20-cm radius.

18. Starting with the formula for the moment of inertia of a rod rotated around an axis through one end perpendicular to its length $\left(I = M\ell^2/3\right)$, prove that the moment of inertia of a rod rotated about an axis through its center perpendicular to its length is $I = M\ell^2/12$. You will find the graphics in **Figure 10.12** useful in visualizing these rotations.

19. Unreasonable Results

A gymnast doing a forward flip lands on the mat and exerts a 500-N · m torque to slow and then reverse her angular velocity. Her initial angular velocity is 10.0 rad/s, and her moment of inertia is $0.050 \text{ kg} \cdot \text{m}^2$. (a) What time is required for her to exactly reverse her spin? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

20. Unreasonable Results

An advertisement claims that an 800-kg car is aided by its 20.0-kg flywheel, which can accelerate the car from rest to a speed of 30.0 m/s. The flywheel is a disk with a 0.150-m radius. (a) Calculate the angular velocity the flywheel must have if 95.0% of its rotational energy is used to get the car up to speed. (b) What is unreasonable about the result? (c) Which premise is unreasonable or which premises are inconsistent?

10.4 Rotational Kinetic Energy: Work and Energy Revisited

21. This problem considers energy and work aspects of **Example 10.7**—use data from that example as needed. (a) Calculate the rotational kinetic energy in the merry-go-round plus child when they have an angular velocity of 20.0 rpm. (b) Using energy considerations, find the number of revolutions the father will have to push to achieve this angular velocity starting from rest. (c) Again, using energy considerations, calculate the force the father must exert to stop the merry-go-round in two revolutions

22. What is the final velocity of a hoop that rolls without slipping down a 5.00-m-high hill, starting from rest?

23. (a) Calculate the rotational kinetic energy of Earth on its axis. (b) What is the rotational kinetic energy of Earth in its orbit around the Sun?

24. Calculate the rotational kinetic energy in the motorcycle wheel (**Figure 10.38**) if its angular velocity is 120 rad/s. Assume M = 12.0 kg, R_1 = 0.280 m, and R_2 = 0.330 m.

25. A baseball pitcher throws the ball in a motion where there is rotation of the forearm about the elbow joint as well as other movements. If the linear velocity of the ball relative to the elbow joint is 20.0 m/s at a distance of 0.480 m from the joint and the moment of inertia of the forearm is $0.500 \text{ kg} \cdot \text{m}^2$, what is the rotational kinetic energy of the forearm?

26. While punting a football, a kicker rotates his leg about the hip joint. The moment of inertia of the leg is $3.75 \text{ kg} \cdot \text{m}^2$ and its rotational kinetic energy is 175 J. (a) What is the angular velocity of the leg? (b) What is the velocity of tip of the punter's shoe if it is 1.05 m from the hip joint? (c) Explain how the football can be given a velocity greater than the tip of the shoe (necessary for a decent kick distance).

27. A bus contains a 1500 kg flywheel (a disk that has a 0.600 m radius) and has a total mass of 10,000 kg. (a) Calculate the angular velocity the flywheel must have to contain enough energy to take the bus from rest to a speed of 20.0 m/s, assuming 90.0% of the rotational kinetic energy can be transformed into translational energy. (b) How high a hill can the bus climb with this stored energy and still have a speed of 3.00 m/s at the top of the hill? Explicitly show how you follow the steps in the **Problem-Solving Strategy for Rotational Energy**.

28. A ball with an initial velocity of 8.00 m/s rolls up a hill without slipping. Treating the ball as a spherical shell, calculate the vertical height it reaches. (b) Repeat the calculation for the same ball if it slides up the hill without rolling.

29. While exercising in a fitness center, a man lies face down on a bench and lifts a weight with one lower leg by contacting the muscles in the back of the upper leg. (a) Find the angular acceleration produced given the mass lifted is 10.0 kg at a distance of 28.0 cm from the knee joint, the moment of inertia of the lower leg is $0.900\text{ kg}\cdot\text{m}^2$, the muscle force is 1500 N, and its effective perpendicular lever arm is 3.00 cm. (b) How much work is done if the leg rotates through an angle of 20.0^o with a constant force exerted by the muscle?

30. To develop muscle tone, a woman lifts a 2.00-kg weight held in her hand. She uses her biceps muscle to flex the lower arm through an angle of 60.0^o. (a) What is the angular acceleration if the weight is 24.0 cm from the elbow joint, her forearm has a moment of inertia of $0.250\text{ kg}\cdot\text{m}^2$, and the net force she exerts is 750 N at an effective perpendicular lever arm of 2.00 cm? (b) How much work does she do?

31. Consider two cylinders that start down identical inclines from rest except that one is frictionless. Thus one cylinder rolls without slipping, while the other slides frictionlessly without rolling. They both travel a short distance at the bottom and then start up another incline. (a) Show that they both reach the same height on the other incline, and that this height is equal to their original height. (b) Find the ratio of the time the rolling cylinder takes to reach the height on the second incline to the time the sliding cylinder takes to reach the height on the second incline. (c) Explain why the time for the rolling motion is greater than that for the sliding motion.

32. What is the moment of inertia of an object that rolls without slipping down a 2.00-m-high incline starting from rest, and has a final velocity of 6.00 m/s? Express the moment of inertia as a multiple of MR^2, where M is the mass of the object and R is its radius.

33. Suppose a 200-kg motorcycle has two wheels like, the one described in **Example 10.15** and is heading toward a hill at a speed of 30.0 m/s. (a) How high can it coast up the hill, if you neglect friction? (b) How much energy is lost to friction if the motorcycle only gains an altitude of 35.0 m before coming to rest?

34. In softball, the pitcher throws with the arm fully extended (straight at the elbow). In a fast pitch the ball leaves the hand with a speed of 139 km/h. (a) Find the rotational kinetic energy of the pitcher's arm given its moment of inertia is $0.720\text{ kg}\cdot\text{m}^2$ and the ball leaves the hand at a distance of 0.600 m from the pivot at the shoulder. (b) What force did the muscles exert to cause the arm to rotate if their effective perpendicular lever arm is 4.00 cm and the ball is 0.156 kg?

35. Construct Your Own Problem

Consider the work done by a spinning skater pulling her arms in to increase her rate of spin. Construct a problem in which you calculate the work done with a “force multiplied by distance” calculation and compare it to the skater's increase in kinetic energy.

10.5 Angular Momentum and Its Conservation

36. (a) Calculate the angular momentum of the Earth in its orbit around the Sun.

(b) Compare this angular momentum with the angular momentum of Earth on its axis.

37. (a) What is the angular momentum of the Moon in its orbit around Earth?

(b) How does this angular momentum compare with the angular momentum of the Moon on its axis? Remember that the Moon keeps one side toward Earth at all times.

(c) Discuss whether the values found in parts (a) and (b) seem consistent with the fact that tidal effects with Earth have caused the Moon to rotate with one side always facing Earth.

38. Suppose you start an antique car by exerting a force of 300 N on its crank for 0.250 s. What angular momentum is given to the engine if the handle of the crank is 0.300 m from the pivot and the force is exerted to create maximum torque the entire time?

39. A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s. What is its angular velocity after a 22.0-kg child gets onto it by grabbing its outer edge? The child is initially at rest.

40. Three children are riding on the edge of a merry-go-round that is 100 kg, has a 1.60-m radius, and is spinning at 20.0 rpm. The children have masses of 22.0, 28.0, and 33.0 kg. If the child who has a mass of 28.0 kg moves to the center of the merry-go-round, what is the new angular velocity in rpm?

41. (a) Calculate the angular momentum of an ice skater spinning at 6.00 rev/s given his moment of inertia is $0.400\text{ kg}\cdot\text{m}^2$. (b) He reduces his rate of spin (his angular velocity) by extending his arms and increasing his moment of inertia. Find the value of his moment of inertia if his angular velocity decreases to 1.25 rev/s. (c) Suppose instead he keeps his arms in and allows friction of the ice to slow him to 3.00 rev/s. What average torque was exerted if this takes 15.0 s?

42. Consider the Earth-Moon system. Construct a problem in which you calculate the total angular momentum of the system including the spins of the Earth and the Moon on their axes and the orbital angular momentum of the Earth-Moon system in its nearly monthly rotation. Calculate what happens to the Moon's orbital radius if the Earth's rotation decreases due to tidal drag. Among the things to be considered are the amount by which the Earth's rotation slows and the fact that the Moon will continue to have one side always facing the Earth.

10.6 Collisions of Extended Bodies in Two Dimensions

43. Repeat **Example 10.15** in which the disk strikes and adheres to the stick 0.100 m from the nail.

44. Repeat **Example 10.15** in which the disk originally spins clockwise at 1000 rpm and has a radius of 1.50 cm.

45. Twin skaters approach one another as shown in **Figure 10.39** and lock hands. (a) Calculate their final angular velocity, given each had an initial speed of 2.50 m/s relative to the ice. Each has a mass of 70.0 kg, and each has a center of mass located 0.800 m from their locked hands. You may approximate their moments of inertia to be that of point masses at this radius. (b) Compare the initial kinetic energy and final kinetic energy.

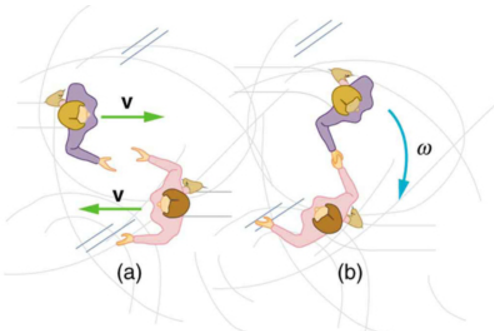


Figure 10.39 Twin skaters approach each other with identical speeds. Then, the skaters lock hands and spin.

46. Suppose a 0.250-kg ball is thrown at 15.0 m/s to a motionless person standing on ice who catches it with an outstretched arm as shown in **Figure 10.40**.

(a) Calculate the final linear velocity of the person, given his mass is 70.0 kg.

(b) What is his angular velocity if each arm is 5.00 kg? You may treat the ball as a point mass and treat the person's arms as uniform rods (each has a length of 0.900 m) and the rest of his body as a uniform cylinder of radius 0.180 m. Neglect the effect of the ball on his center of mass so that his center of mass remains in his geometrical center.

(c) Compare the initial and final total kinetic energies.

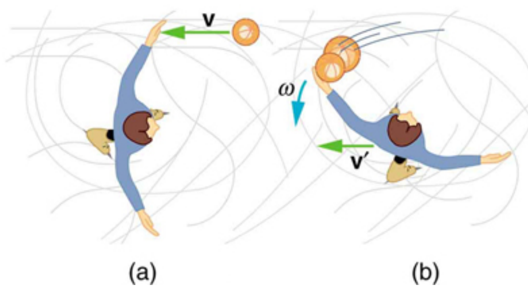


Figure 10.40 The figure shows the overhead view of a person standing motionless on ice about to catch a ball. Both arms are outstretched. After catching the ball, the skater recoils and rotates.

47. Repeat **Example 10.15** in which the stick is free to have translational motion as well as rotational motion.

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

48. Integrated Concepts

The axis of Earth makes a 23.5° angle with a direction perpendicular to the plane of Earth's orbit. As shown in **Figure 10.41**, this axis precesses, making one complete rotation in 25,780 y.

(a) Calculate the change in angular momentum in half this time.

(b) What is the average torque producing this change in angular momentum?

(c) If this torque were created by a single force (it is not) acting at the most effective point on the equator, what would its magnitude be?

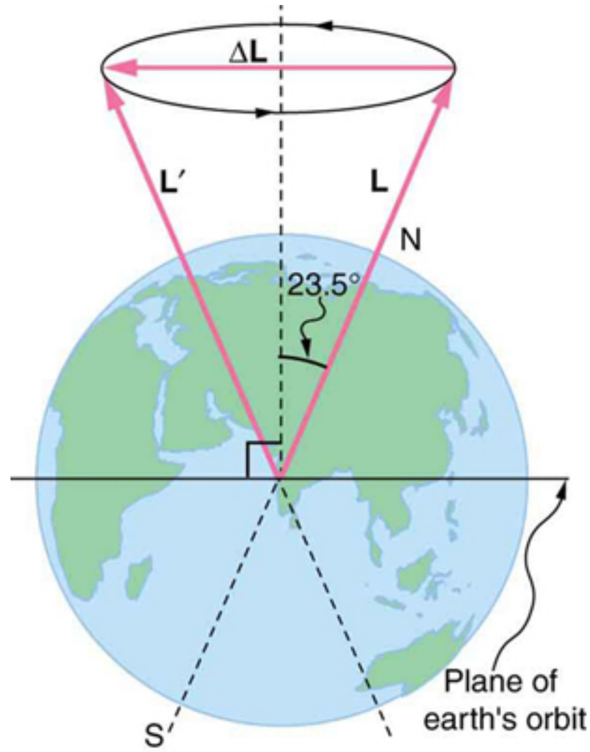


Figure 10.41 The Earth's axis slowly precesses, always making an angle of 23.5° with the direction perpendicular to the plane of Earth's orbit. The change in angular momentum for the two shown positions is quite large, although the magnitude $\mathbf{L}$ is unchanged.

Test Prep for AP® Courses

10.3 Dynamics of Rotational Motion: Rotational Inertia

1. A piece of wood can be carved by spinning it on a motorized lathe and holding a sharp chisel to the edge of the wood as it spins. How does the angular velocity of a piece of wood with a radius of 0.2 m spinning on a lathe change when a chisel is held to the wood's edge with a force of 50 N?

a. It increases by 0.1 N•m multiplied by the moment of inertia of the wood.
b. It decreases by 0.1 N•m divided by the moment of inertia of the wood-and-lathe system.
c. It decreases by 0.1 N•m multiplied by the moment of inertia of the wood.
d. It decreases by 0.1 m/s^2.

2. A Ferris wheel is loaded with people in the chairs at the following positions: 4 o'clock, 1 o'clock, 9 o'clock, and 6 o'clock. As the wheel begins to turn, what forces are acting on the system? How will each force affect the angular velocity and angular momentum?

3. A lever is placed on a fulcrum. A rock is placed on the left end of the lever and a downward (clockwise) force is applied to the right end of the lever. What measurements would be most effective to help you determine the angular momentum of the system? (Assume the lever itself has negligible mass.)

a. the angular velocity and mass of the rock
b. the angular velocity and mass of the rock, and the radius of the lever
c. the velocity of the force, the radius of the lever, and the mass of the rock
d. the mass of the rock, the length of the lever on both sides of the fulcrum, and the force applied on the right side of the lever

4. You can use the following setup to determine angular acceleration and angular momentum: A lever is placed on a fulcrum. A rock is placed on the left end of the lever and a known downward (clockwise) force is applied to the right end of the lever. What calculations would you perform? How would you account for gravity in your calculations?

5. Consider two sizes of disk, both of mass *M*. One size of disk has radius *R*; the other has radius 2*R*. System A consists of two of the larger disks rigidly connected to each other with a common axis of rotation. System B consists of one of the larger disks and a number of the smaller disks rigidly connected with a common axis of rotation. If the moment of inertia for system A equals the moment of inertia for system B, how many of the smaller disks are in system B?

a. 1
b. 2
c. 3
d. 4

6. How do you arrange these objects so that the resulting system has the maximum possible moment of inertia? What is that moment of inertia?

10.4 Rotational Kinetic Energy: Work and Energy Revisited

7. Gear A, which turns clockwise, meshes with gear B, which turns counterclockwise. When more force is applied through gear A, torque is created. How does the angular velocity of gear B change as a result?

a. It increases in magnitude.
b. It decreases in magnitude.
c. It changes direction.
d. It stays the same.

8. Which will cause a greater increase in the angular velocity of a disk: doubling the torque applied or halving the radius at which the torque is applied? Explain.

9. Which measure would not be useful to help you determine the change in angular velocity when the torque on a fishing reel is increased?

a. the radius of the reel
b. the amount of line that unspools
c. the angular momentum of the fishing line
d. the time it takes the line to unspool

10. What data could you collect to study the change in angular velocity when two people push a merry-go-round instead of one, providing twice as much torque? How would you use the data you collect?

10.5 Angular Momentum and Its Conservation

11. Which rotational system would be best to use as a model to measure how angular momentum changes when forces on the system are changed?

a. a fishing reel
b. a planet and its moon
c. a figure skater spinning
d. a person's lower leg

12. You are collecting data to study changes in the angular momentum of a bicycle wheel when a force is applied to it. Which of the following measurements would be least helpful to you?

a. the time for which the force is applied
b. the radius at which the force is applied
c. the angular velocity of the wheel when the force is applied
d. the direction of the force

13. Which torque applied to a disk with radius 7.0 cm for 3.5 s will produce an angular momentum of 25 N•m•s?

a. 7.1 N•m
b. 357.1 N•m
c. 3.6 N•m
d. 612.5 N•m

14. Which of the following would be the best way to produce measurable amounts of torque on a system to test the relationship between the angular momentum of the system, the average torque applied to the system, and the time for which the torque is applied?

a. having different numbers of people push on a merry-go-round
b. placing known masses on one end of a seesaw
c. touching the outer edge of a bicycle wheel to a treadmill that is moving at different speeds
d. hanging known masses from a string that is wound around a spool suspended horizontally on an axle

15.

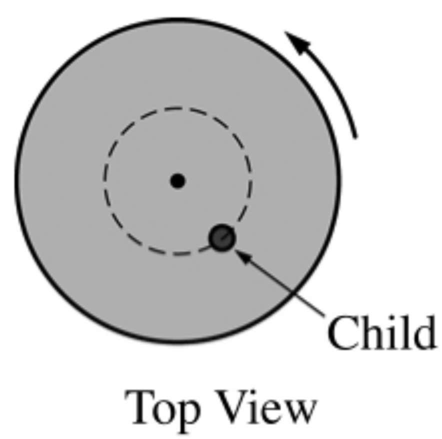


Figure 10.42 A curved arrow lies at the side of a gray disk. There is a point at the center of the disk, and around the point there is a dashed circle. There is a point labeled “Child” on the dashed circle. Below the disc is a label saying “Top View”. The diagram above shows a top view of a child of mass *M* on a circular platform of mass 2*M* that is rotating counterclockwise. Assume the platform rotates without friction. Which of the following describes an action by the child that will increase the angular speed of the platform-child system and why?

a. The child moves toward the center of the platform, increasing the total angular momentum of the system.
b. The child moves toward the center of the platform, decreasing the rotational inertia of the system.
c. The child moves away from the center of the platform, increasing the total angular momentum of the system.
d. The child moves away from the center of the platform, decreasing the rotational inertia of the system.

16.

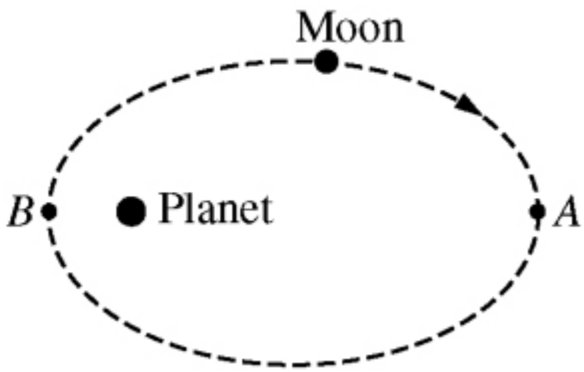


Figure 10.43 A point labeled “Moon” lies on a dashed ellipse. Two other points, labeled “A” and “B”, lie at opposite ends of the ellipse. A point labeled “Planet” lies inside the ellipse. A moon is in an elliptical orbit about a planet as shown above. At point *A* the moon has speed *uA* and is at distance *RA* from the planet. At point *B* the moon has speed *uB*. Has the moon's angular momentum changed? Explain your answer.

17. A hamster sits 0.10 m from the center of a lazy Susan of negligible mass. The wheel has an angular velocity of 1.0 rev/s. How will the angular velocity of the lazy Susan change if the hamster walks to 0.30 m from the center of rotation? Assume zero friction and no external torque.

a. It will speed up to 2.0 rev/s.
b. It will speed up to 9.0 rev/s.
c. It will slow to 0.01 rev/s.
d. It will slow to 0.02 rev/s.

18. Earth has a mass of 6.0×10^{24} kg, a radius of 6.4×10^{6} m, and an angular velocity of 1.2×10^{-5} rev/s. How would the planet's angular velocity change if a layer of Earth with mass 1.0×10^{23} kg broke off of the Earth, decreasing Earth's radius by 0.2×10^{6} m? Assume no friction.

19. Consider system A, consisting of two disks of radius *R*, with both rotating clockwise. Now consider system B, consisting of one disk of radius *R* rotating counterclockwise and another disk of radius 2*R* rotating clockwise. All of the disks have the same mass, and all have the same magnitude of angular velocity.

Which system has the greatest angular momentum?

a. A
b. B
c. They're equal.
d. Not enough information

20. Assume that a baseball bat being swung at 3π rad/s by a batting machine is equivalent to a 1.1 m thin rod with a mass of 1.0 kg. How fast would a 0.15 kg baseball that squarely hits the very tip of the bat have to be going for the net angular momentum of the bat-ball system to be zero?

10.6 Collisions of Extended Bodies in Two Dimensions

21. A box with a mass of 2.0 kg rests on one end of a seesaw. The seesaw is 6.0 m long, and we can assume it has negligible mass. Approximately what angular momentum will the box have if someone with a mass of 65 kg sits on the other end of the seesaw quickly, with a velocity of 1.2 m/s?

a. 702 kg•m^2/s
b. 39 kg•m^2/s
c. 18 kg•m^2/s
d. 1.2 kg•m^2/s

22. A spinner in a board game can be thought of as a thin rod that spins about an axis at its center. The spinner in a certain game is 12 cm long and has a mass of 10 g. How will its angular velocity change when it is flicked at one end with a force equivalent to 15 g travelling at 5.0 m/s if all the energy of the collision is transferred to the spinner? (You can use the table in **Figure 10.12** to estimate the rotational inertia of the spinner.)

23. A cyclist pedals to exert a torque on the rear wheel of the bicycle. When the cyclist changes to a higher gear, the torque increases. Which of the following would be the most effective strategy to help you determine the change in angular momentum of the bicycle wheel?

a. multiplying the ratio between the two torques by the mass of the bicycle and rider
b. adding the two torques together, and multiplying by the time for which both torques are applied
c. multiplying the difference in the two torques by the time for which the new torque is applied
d. multiplying both torques by the mass of the bicycle and rider

24. An electric screwdriver has two speeds, each of which exerts a different torque on a screw. Describe what calculations you could use to help you compare the angular momentum of a screw at each speed. What measurements would you need to make in order to calculate this?

25. Why is it important to consider the shape of an object when determining the object's angular momentum?

a. The shape determines the location of the center of mass. The location of the center of mass in turn determines the angular velocity of the object.
b. The shape helps you determine the location of the object's outer edge, where rotational velocity will be greatest.
c. The shape helps you determine the location of the center of rotation.
d. The shape determines the location of the center of mass. The location of the center of mass contributes to the object's rotational inertia, which contributes to its angular momentum.

26. How could you collect and analyze data to test the difference between the torques provided by two speeds on a tabletop fan?

27. Describe a rotational system you could use to demonstrate the effect on the system's angular momentum of applying different amounts of external torque.

28. How could you use simple equipment such as balls and string to study the changes in angular momentum of a system when it interacts with another system?

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

29. A globe (model of the Earth) is a hollow sphere with a radius of 16 cm. By wrapping a cord around the equator of a globe and pulling on it, a person exerts a torque on the globe of 120 N • m for 1.2 s. What angular momentum does the globe have after 1.2 s?

30. How could you use a fishing reel to test the relationship between the torque applied to a system, the time for which the torque was applied, and the resulting angular momentum of the system? How would you measure angular momentum?

16 OSCILLATORY MOTION AND WAVES

Figure 16.1 There are at least four types of waves in this picture—only the water waves are evident. There are also sound waves, light waves, and waves on the guitar strings. (credit: John Norton)

Chapter Outline

Connection for AP® Courses

In this chapter, students are introduced to oscillation, the regular variation in the position of a system about a central point accompanied by transfer of energy and momentum, and to **waves**. A child's swing, a pendulum, a spring, and a vibrating string are all examples of oscillations. This chapter will address simple harmonic motion and periods of vibration, aspects of oscillation that produce waves, a common phenomenon in everyday life. Waves carry energy from one place to another." This chapter will show how harmonic oscillations produce waves that transport energy across space and through time. The information and examples presented support Big Ideas 1, 2, and 3 of the AP® Physics Curriculum Framework.

The chapter opens by discussing the forces that govern oscillations and waves. It goes on to discuss important concepts such as **simple harmonic motion**, uniform harmonic motion, and damped harmonic motion. You will also learn about energy in simple harmonic motion and how it changes from kinetic to potential, and how the total sum, which would be the mechanical energy of the oscillator, remains constant or conserved at all times. The chapter also discusses characteristics of waves, such as their frequency, period of oscillation, and the forms in which they can exist, i.e., transverse or longitudinal. The chapter ends by discussing what happens when two or more waves overlap and how the **amplitude** of the resultant wave changes, leading to the phenomena of **superposition** and **interference**.

The concepts in this chapter support:

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.B Classically, the acceleration of an object interacting with other objects can be predicted by using $\vec{a} = \sum \frac{\vec{F}}{m}$.

Essential Knowledge 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion. Examples should include gravitational force exerted by the Earth on a simple pendulum and a mass-spring oscillator.

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding 4.C Interactions with other objects or systems can change the total energy of a system.

Essential Knowledge 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy.

Essential Knowledge 4.C.2 Mechanical energy (the sum of kinetic and potential energy) is transferred into or out of a system when an external force is exerted on a system such that a component of the force is parallel to its displacement. The process through which the energy is transferred is called work.

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.B The energy of a system is conserved.

Essential Knowledge 5.B.2 A system with internal structure can have internal energy, and changes in a system's internal structure can result in changes in internal energy. [Physics 1: includes mass-spring oscillators and simple pendulums. Physics 2: includes charged object in electric fields and examining changes in internal energy with changes in configuration.]

Big Idea 6 Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.

Enduring Understanding 6.A A wave is a traveling disturbance that transfers energy and momentum.

Essential Knowledge 6.A.1 Waves can propagate via different oscillation modes such as transverse and longitudinal.

Essential Knowledge 6.A.2 For propagation, mechanical waves require a medium, while electromagnetic waves do not require a physical medium. Examples should include light traveling through a vacuum and sound not traveling through a vacuum.

Essential Knowledge 6.A.3 The amplitude is the maximum displacement of a wave from its equilibrium value.

Essential Knowledge 6.A.4 Classically, the energy carried by a wave depends on and increases with amplitude. Examples should include sound waves.

Enduring Understanding 6.B A periodic wave is one that repeats as a function of both time and position and can be described by its amplitude, frequency, wavelength, speed, and energy.

Essential Knowledge 6.B.1 The period is the repeat time of the wave. The frequency is the number of repetitions over a period of time.

Essential Knowledge 6.B.2 The wavelength is the repeat distance of the wave.

Essential Knowledge 6.B.3 A simple wave can be described by an equation involving one sine or cosine function involving the wavelength, amplitude, and frequency of the wave.

Essential Knowledge 6.B.4 The wavelength is the ratio of speed over frequency.

Enduring Understanding 6.C Only waves exhibit interference and diffraction.

Essential Knowledge 6.C.1 When two waves cross, they travel through each other; they do not bounce off each other. Where the waves overlap, the resulting displacement can be determined by adding the displacements of the two waves. This is called superposition.

Enduring Understanding 6.D Interference and superposition lead to standing waves and beats.

Essential Knowledge 6.D.1 Two or more wave pulses can interact in such a way as to produce amplitude variations in the resultant wave. When two pulses cross, they travel through each other; they do not bounce off each other. Where the pulses overlap, the resulting displacement can be determined by adding the displacements of the two pulses. This is called superposition.

Essential Knowledge 6.D.2 Two or more traveling waves can interact in such a way as to produce amplitude variations in the resultant wave.

Essential Knowledge 6.D.3 Standing waves are the result of the addition of incident and reflected waves that are confined to a region and have nodes and antinodes. Examples should include waves on a fixed length of string, and sound waves in both closed and open tubes.

Essential Knowledge 6.D.4 The possible wavelengths of a standing wave are determined by the size of the region to which it is confined.

Essential Knowledge 6.D.5 Beats arise from the addition of waves of slightly different frequency.

16.1 Hooke's Law: Stress and Strain Revisited

Learning Objectives

By the end of this section, you will be able to:

- Explain Newton's third law of motion with respect to stress and deformation.
- Describe the restoring force and displacement.
- Use Hooke's law of deformation, and calculate stored energy in a spring.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.B.3.3** The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. **(S.P. 2.2, 5.1)**
- **3.B.3.4** The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force. **(S.P. 2.2, 6.2)**

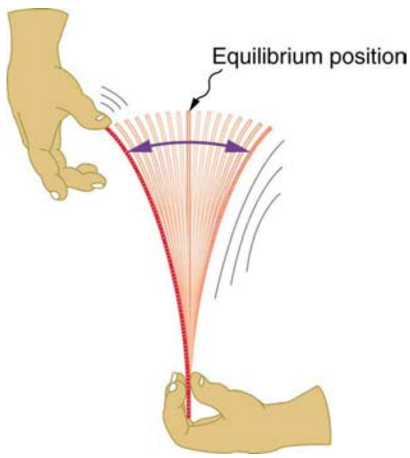


Figure 16.2 When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement. When the ruler is on the left, there is a force to the right, and vice versa.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in **Figure 16.2**. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.

The simplest oscillations occur when the restoring force is directly proportional to displacement. When stress and strain were covered in **Newton's Third Law of Motion**, the name was given to this relationship between force and displacement was Hooke's law:

$$F = -kx. \tag{16.1}$$

Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.

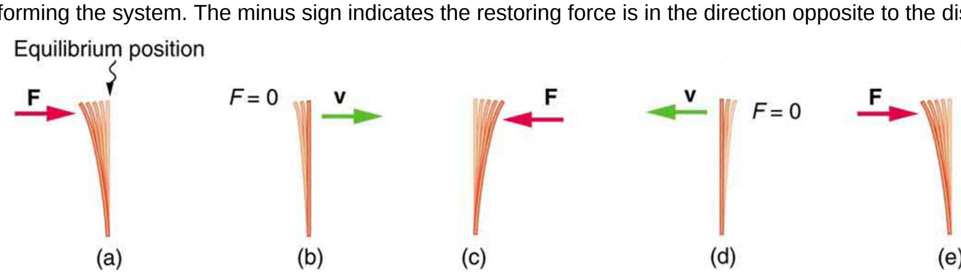
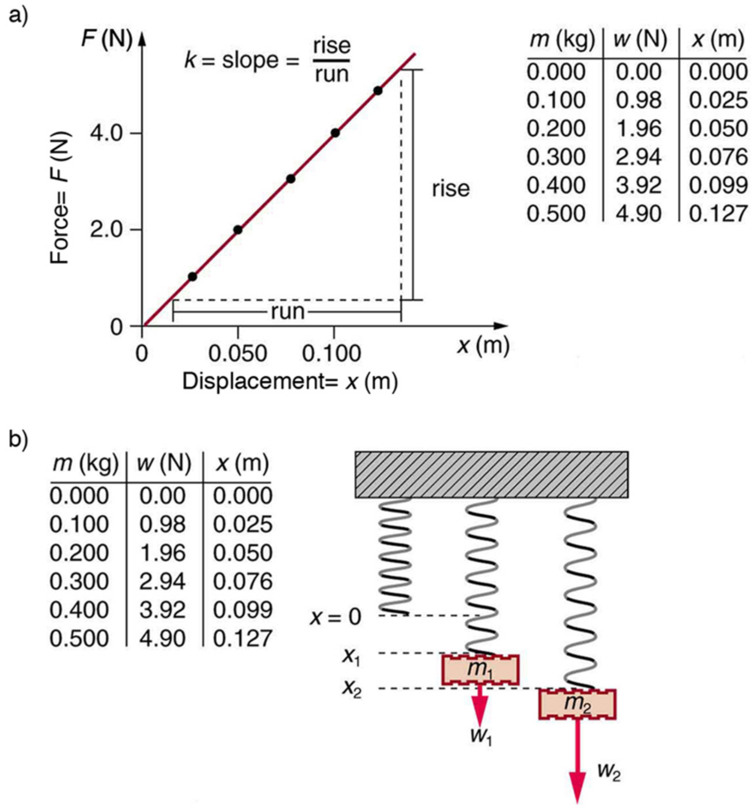


Figure 16.3 (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

The **force constant** k is related to the rigidity (or stiffness) of a system—the larger the force constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). For example, k is directly related to Young's modulus when we stretch a string. **Figure 16.4** shows a graph of the absolute value of the restoring force versus the displacement for a system that can be described by Hooke's law—a simple spring in this case. The slope of the graph equals the force constant k in newtons per meter. A common physics laboratory exercise is to measure restoring forces created by springs, determine if they follow Hooke's law, and calculate their force constants if they do.



m (kg)	w (N)	x (m)
0.000	0.00	0.000
0.100	0.98	0.025
0.200	1.96	0.050
0.300	2.94	0.076
0.400	3.92	0.099
0.500	4.90	0.127

m (kg)	w (N)	x (m)
0.000	0.00	0.000
0.100	0.98	0.025
0.200	1.96	0.050
0.300	2.94	0.076
0.400	3.92	0.099
0.500	4.90	0.127

Figure 16.4 (a) A graph of absolute value of the restoring force versus displacement is displayed. The fact that the graph is a straight line means that the system obeys Hooke's law. The slope of the graph is the force constant k. (b) The data in the graph were generated by measuring the displacement of a spring from equilibrium while supporting various weights. The restoring force equals the weight supported, if the mass is stationary.

Example 16.1 How Stiff Are Car Springs?

Figure 16.5 The mass of a car increases due to the introduction of a passenger. This affects the displacement of the car on its suspension system. (credit: exfordy on Flickr)

What is the force constant for the suspension system of a car that settles 1.20 cm when an 80.0-kg person gets in?

Strategy

Consider the car to be in its equilibrium position $x = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $x = -1.20\times10^{-2}\text{ m}$. At that point, the springs supply a restoring force F equal to the person's weight $w = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$. We take this force to be F in Hooke's law. Knowing F and x, we can then solve the force constant k.

Solution

1. Solve Hooke's law, $F = -kx$, for k:

$$k = -\frac{F}{x}. \tag{16.2}$$

Substitute known values and solve k:

$$\begin{aligned} k &= -\frac{784\text{ N}}{-1.20\times10^{-2}\text{ m}} \\ &= 6.53\times10^{4}\text{ N/m}. \end{aligned} \tag{16.3}$$

Discussion

Note that F and x have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in if it were not for damping (due to frictional forces) provided by shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Energy in Hooke's Law of Deformation

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\text{PE}_{\text{el}} = \frac{1}{2}kx^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke's law. Hence,

$$\text{PE}_{\text{el}} = \frac{1}{2}kx^2, \tag{16.4}$$

where PE_{el} is the **elastic potential energy** stored in any deformed system that obeys Hooke's law and has a displacement x from equilibrium and a force constant k.

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app}. The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{\text{app}} = kx$. Figure 16.6 shows a graph of the applied force versus deformation x for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)kx^2$ (Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx, so that the average force is $(1/2)kx$, the distance moved is x, and thus $W = F_{\text{app}}d = [(1/2)kx](x) = (1/2)kx^2$ (Method B in the figure).

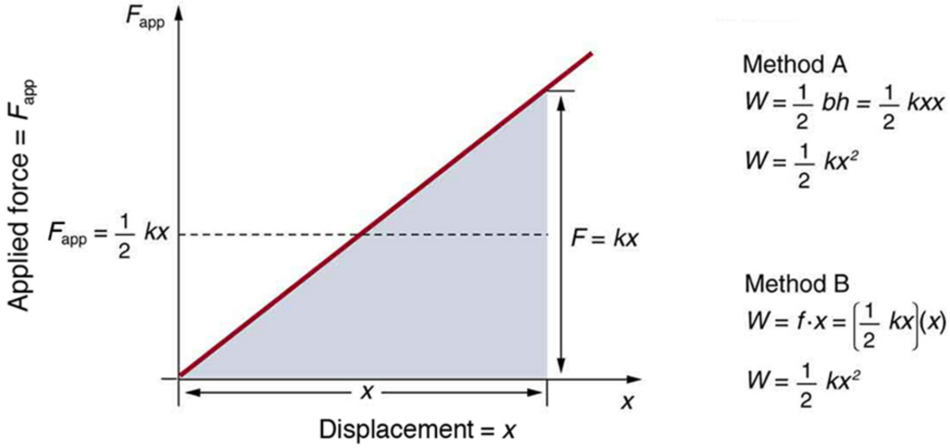


Figure 16.6 A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W = (1/2)kx^2$.

Example 16.2 Calculating Stored Energy: A Tranquilizer Gun Spring

We can use a toy gun's spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?

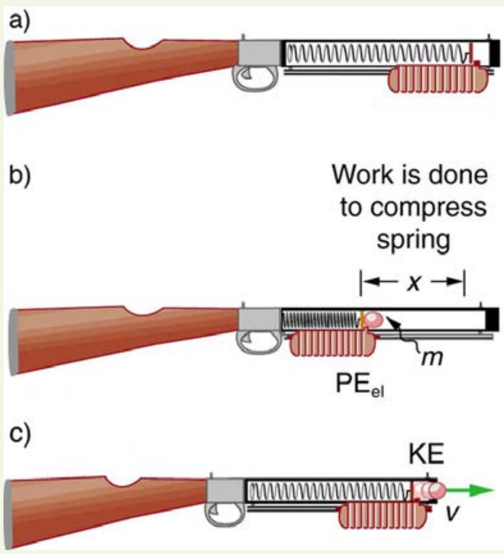


Figure 16.7 (a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x, and the projectile is in place. (c) When released, the spring converts elastic potential energy PE_{el} into kinetic energy.

Strategy for a

(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

$$\begin{aligned} PE_{el} &= \frac{1}{2}kx^2 = \frac{1}{2}(50.0\ \text{N/m})(0.150\ \text{m})^2 = 0.563\ \text{N} \cdot \text{m} \\ &= 0.563\ \text{J} \end{aligned} \tag{16.5}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:

$$\text{KE}_\text{f} = \text{PE}_\text{el} \text{ or } 1/2mv^2 = (1/2)kx^2 = \text{PE}_\text{el} = 0.563 \text{ J} \tag{16.6}$$

2. Solve for v:

$$v = \left[\frac{2\text{PE}_\text{el}}{m}\right]^{1/2} = \left[\frac{2(0.563 \text{ J})}{0.002 \text{ kg}}\right]^{1/2} = 23.7(\text{J/kg})^{1/2} \tag{16.7}$$

3. Convert units: 23.7 m/s

Discussion

(a) and (b): This projectile speed is impressive for a tranquilizer gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

Check your Understanding

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Solution

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

Check your Understanding

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Solution

It was stored in the object as potential energy.

16.2 Period and Frequency in Oscillations

Learning Objectives

By the end of this section, you will be able to:

- Relate recurring mechanical vibrations to the frequency and period of harmonic motion, such as the motion of a guitar string.
- Compute the frequency and period of an oscillation.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.B.3.3** The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. **(S.P. 2.2, 5.1)**

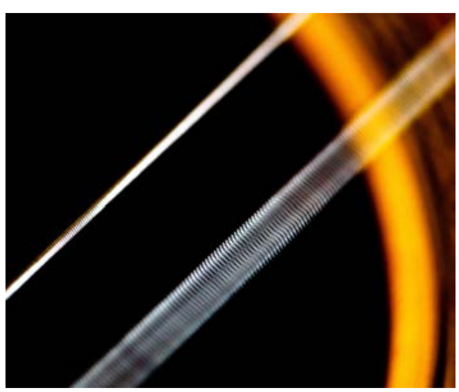


Figure 16.8 The strings on this guitar vibrate at regular time intervals. (credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T. Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

$$f = \frac{1}{T}. \tag{16.8}$$

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

$$1 \text{ Hz} = 1\frac{\text{cycle}}{\text{sec}} \text{or } 1 \text{ Hz} = \frac{1}{\text{s}} \tag{16.9}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

Example 16.3 Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let’s try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f. In question (b), the frequency f is given and we are asked to find the period T.

Solution a

1. Substitute $0.400\ \mu\text{s}$ for T in $f = \frac{1}{T}$:

$$f = \frac{1}{T} = \frac{1}{0.400\times 10^{-6}\ \text{s}}. \tag{16.10}$$

Solve to find

$$f = 2.50\times 10^{6}\ \text{Hz}. \tag{16.11}$$

Discussion a

The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.

Solution b

1. Identify the known values:
The time for one complete oscillation is the period T:

$$f = \frac{1}{T}. \tag{16.12}$$

2. Solve for T:

$$T = \frac{1}{f}. \tag{16.13}$$

3. Substitute the given value for the frequency into the resulting expression:

$$T = \frac{1}{f} = \frac{1}{264\ \text{Hz}} = \frac{1}{264\ \text{cycles/s}} = 3.79\times 10^{-3}\ \text{s} = 3.79\ \text{ms}. \tag{16.14}$$

Discussion

The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

Check your Understanding

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Solution

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

16.3 Simple Harmonic Motion: A Special Periodic Motion

Learning Objectives

By the end of this section, you will be able to:

- Describe a simple harmonic oscillator.
- Relate physical characteristics of a vibrating system to aspects of simple harmonic motion and any resulting waves.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.B.3.1** The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. **(S.P. 6.4, 7.2)**
- **3.B.3.4** The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force. **(S.P. 2.2, 6.2)**
- **6.A.3.1** The student is able to use graphical representation of a periodic mechanical wave to determine the amplitude of the wave. **(S.P. 1.4)**
- **6.B.1.1** The student is able to use a graphical representation of a periodic mechanical wave (position versus time) to determine the period and frequency of the wave and describe how a change in the frequency would modify features of the representation. **(S.P. 1.4, 2.2)**

The oscillations of a system in which the net force can be described by Hooke’s law are of special importance, because they are very common. They are also the simplest oscillatory systems. **Simple Harmonic Motion** (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke’s law, and such a system is called a **simple harmonic oscillator**. If the net force can be described by Hooke’s law and there is no *damping* (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in **Figure 16.9**. The maximum displacement from equilibrium is called the **amplitude** X. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation.

Take-Home Experiment: SHM and the Marble

Find a bowl or basin that is shaped like a hemisphere on the inside. Place a marble inside the bowl and tilt the bowl periodically so the marble rolls from the bottom of the bowl to equally high points on the sides of the bowl. Get a feel for the force required to maintain this periodic motion. What is the restoring force and what role does the force you apply play in the simple harmonic motion (SHM) of the marble?

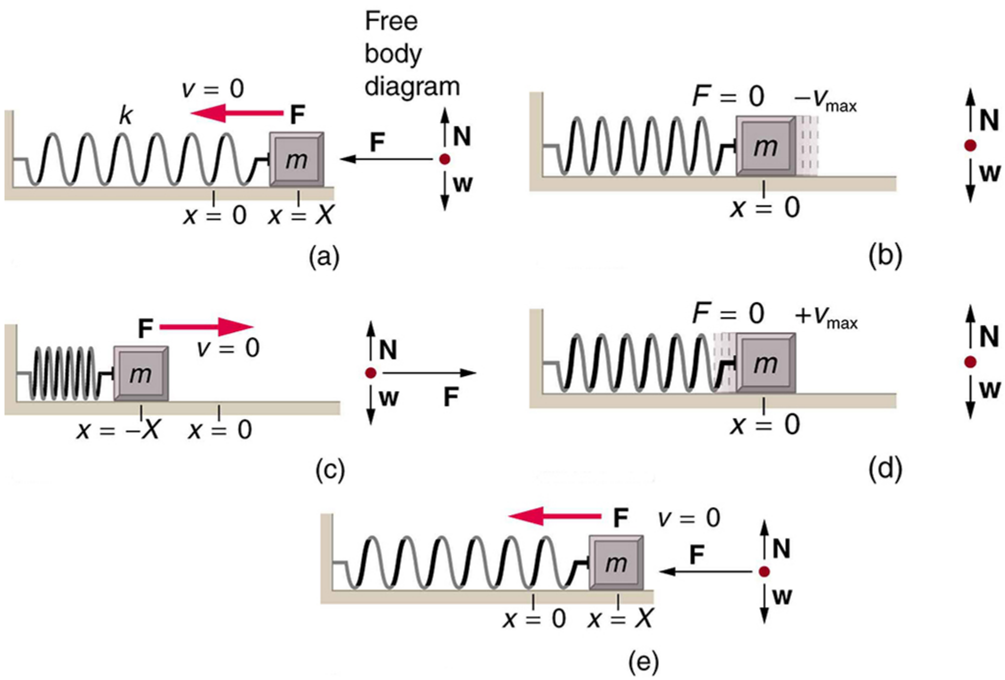


Figure 16.9 An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T.

What is so significant about simple harmonic motion? One special thing is that the period T and frequency f of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, will oscillate with the same frequency whether plucked gently or hard. Because the period is constant, a simple harmonic oscillator can be used as a clock.

Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large force constant k, which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness—the stiffer it is, the faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one.

In fact, the mass m and the force constant k are the *only* factors that affect the period and frequency of simple harmonic motion.

Period of Simple Harmonic Oscillator

The *period of a simple harmonic oscillator* is given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{16.15}$$

and, because $f = 1/T$, the *frequency of a simple harmonic oscillator* is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{16.16}$$

Note that neither T nor f has any dependence on amplitude.

Example 16.4 Mechanical Waves

What do sound waves, water waves, and seismic waves have in common? They are all governed by Newton's laws and they can exist only when traveling in a medium, such as air, water, or rocks. Waves that require a medium to travel are collectively known as "mechanical waves."

Take-Home Experiment: Mass and Ruler Oscillations

Find two identical wooden or plastic rulers. Tape one end of each ruler firmly to the edge of a table so that the length of each ruler that protrudes from the table is the same. On the free end of one ruler tape a heavy object such as a few large coins. Pluck the ends of the rulers at the same time and observe which one undergoes more cycles in a time period, and measure the period of oscillation of each of the rulers.

Example 16.5 Calculate the Frequency and Period of Oscillations: Bad Shock Absorbers in a Car

If the shock absorbers in a car go bad, then the car will oscillate at the least provocation, such as when going over bumps in the road and after stopping (See **Figure 16.10**). Calculate the frequency and period of these oscillations for such a car if the car's mass (including its load) is 900 kg and the force constant (k) of the suspension system is $6.53\times10^4\ \text{N/m}$.

Strategy

The frequency of the car's oscillations will be that of a simple harmonic oscillator as given in the equation $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The mass and the force constant are both given.

Solution

1. Enter the known values of *k* and *m*:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{6.53\times10^4\ \text{N/m}}{900\ \text{kg}}}. \tag{16.17}$$

2. Calculate the frequency:

$$\frac{1}{2\pi}\sqrt{72.6\,/\,\text{s}^{-2}} = 1.3656\,/\,\text{s}^{-1} \approx 1.36\,/\,\text{s}^{-1} = 1.36\ \text{Hz}. \tag{16.18}$$

3. You could use $T = 2\pi\sqrt{\frac{m}{k}}$ to calculate the period, but it is simpler to use the relationship $T = 1\,/\,f$ and substitute the value just found for f :

$$T = \frac{1}{f} = \frac{1}{1.356\ \text{Hz}} = 0.738\ \text{s}. \tag{16.19}$$

Discussion

The values of T and f both seem about right for a bouncing car. You can observe these oscillations if you push down hard on the end of a car and let go.

The Link between Simple Harmonic Motion and Waves

If a time-exposure photograph of the bouncing car were taken as it drove by, the headlight would make a wavelike streak, as shown in **Figure 16.10**. Similarly, **Figure 16.11** shows an object bouncing on a spring as it leaves a wavelike "trace of its position on a moving strip of paper. Both waves are sine functions. All simple harmonic motion is intimately related to sine and cosine waves.

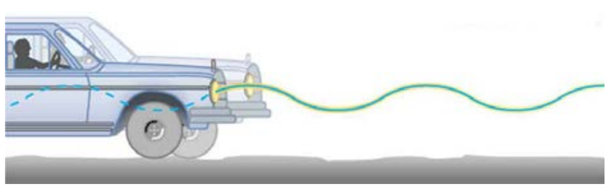


Figure 16.10 The bouncing car makes a wavelike motion. If the restoring force in the suspension system can be described only by Hooke's law, then the wave is a sine function. (The wave is the trace produced by the headlight as the car moves to the right.)

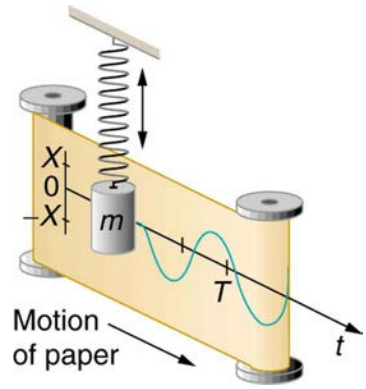


Figure 16.11 The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave.

The displacement as a function of time *t* in any simple harmonic motion—that is, one in which the net restoring force can be described by Hooke's law, is given by

$$x(t) = X \cos\frac{2\pi t}{T}, \tag{16.20}$$

where X is amplitude. At $t = 0$, the initial position is $x_0 = X$, and the displacement oscillates back and forth with a period T. (When $t = T$, we get $x = X$ again because $\cos 2\pi = 1$.). Furthermore, from this expression for x, the velocity v as a function of time is given by:

$$v(t) = -v_{\text{max}} \sin\left(\frac{2\pi t}{T}\right), \tag{16.21}$$

where $v_{\text{max}} = 2\pi X / T = X\sqrt{k / m}$. The object has zero velocity at maximum displacement—for example, $v = 0$ when $t = 0$, and at that time $x = X$. The minus sign in the first equation for $v(t)$ gives the correct direction for the velocity. Just after the start of the motion, for instance, the velocity is negative because the system is moving back toward the equilibrium point. Finally, we can get an expression for acceleration using Newton's second law. [Then we have $x(t),\ v(t),\ t,$ and $a(t)$, the quantities needed for kinematics and a description of simple harmonic motion.] According to Newton's second law, the acceleration is $a = F / m = kx / m$. So, $a(t)$ is also a cosine function:

$$a(t) = -\frac{kX}{m}\cos\frac{2\pi t}{T}. \tag{16.22}$$

Hence, $a(t)$ is directly proportional to and in the opposite direction to $x(t)$.

Figure 16.12 shows the simple harmonic motion of an object on a spring and presents graphs of $x(t), v(t),$ and $a(t)$ versus time.

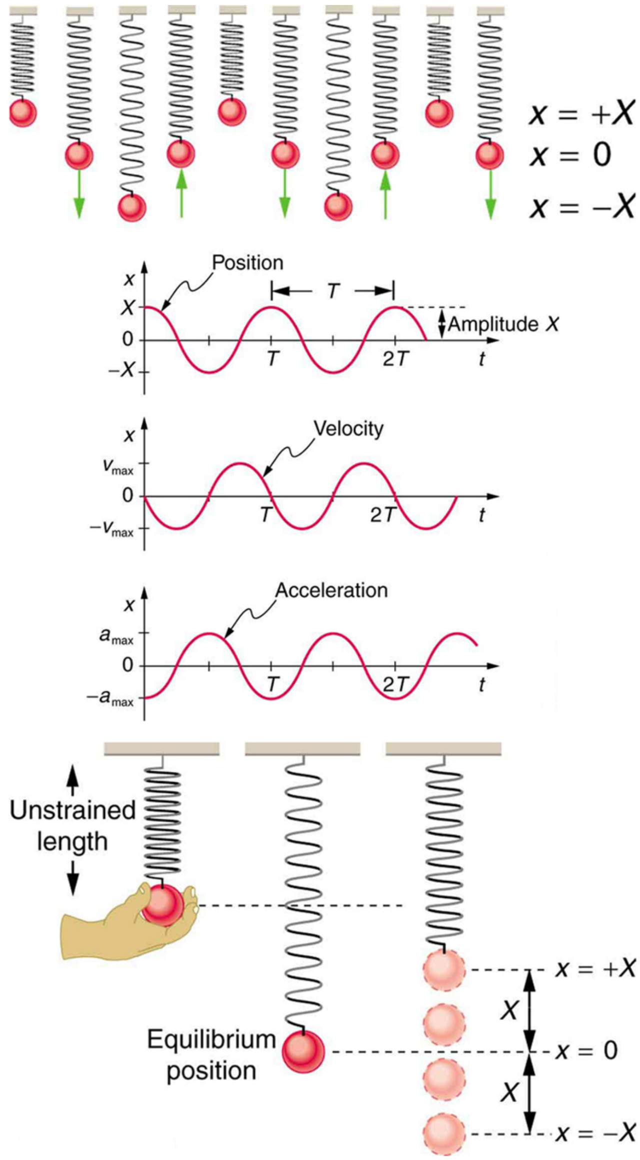


Figure 16.12 Graphs of $x(t)$, $v(t)$, and $a(t)$ versus t for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value X; v is initially zero and then negative as the object moves down; and the initial acceleration is negative, back toward the equilibrium position and becomes zero at that point.

The most important point here is that these equations are mathematically straightforward and are valid for all simple harmonic motion. They are very useful in visualizing waves associated with simple harmonic motion, including visualizing how waves add with one another.

Check Your Understanding

Suppose you pluck a banjo string. You hear a single note that starts out loud and slowly quiets over time. Describe what happens to the sound waves in terms of period, frequency and amplitude as the sound decreases in volume.

Solution

Frequency and period remain essentially unchanged. Only amplitude decreases as volume decreases.

Check Your Understanding

A babysitter is pushing a child on a swing. At the point where the swing reaches x, where would the corresponding point on a wave of this motion be located?

Solution

x is the maximum deformation, which corresponds to the amplitude of the wave. The point on the wave would either be at the very top or the very bottom of the curve.

PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

PhET Interactive Simulation

Figure 16.13 Masses and Springs (http://cnx.org/content/m55273/1.2/mass-spring-lab_en.jar)

16.4 The Simple Pendulum

Learning Objectives

By the end of this section, you will be able to:

- Determine the period of oscillation of a hanging pendulum.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.B.3.1** The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. **(S.P. 6.4, 7.2)**
- **3.B.3.2** The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force. **(S.P. 4.2)**
- **3.B.3.3** The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. **(S.P. 2.2, 5.1)**
- **3.B.3.4** The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force. **(S.P. 2.2, 6.2)**

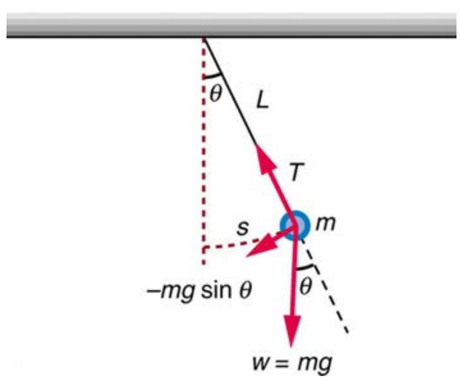


Figure 16.14 A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg\ \sin\theta$ toward the equilibrium position—that is, a restoring force.

Pendulums are in common usage. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing; and some are just there, such as the sinker on a fishing line. For small displacements, a pendulum is a simple harmonic oscillator. A **simple pendulum** is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string, such as shown in **Figure 16.14**. Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.

We begin by defining the displacement to be the arc length s. We see from **Figure 16.14** that the net force on the bob is tangent to the arc and equals $-mg\ \sin\ \theta$. (The weight mg has components $mg\ \cos\ \theta$ along the string and $mg\ \sin\ \theta$ tangent to the arc.) Tension in the string exactly cancels the component $mg\ \cos\ \theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position at $\theta = 0$.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about $15º$), $\sin\theta \approx \theta$ ($\sin\theta$ and θ differ by about 1% or less at smaller angles). Thus, for angles less than about $15º$, the restoring force F is

$$F \approx -mg\theta. \tag{16.23}$$

The displacement s is directly proportional to θ. When θ is expressed in radians, the arc length in a circle is related to its radius (L in this instance) by:

$$s = L\theta, \tag{16.24}$$

so that

$$\theta = \frac{s}{L}. \tag{16.25}$$

For small angles, then, the expression for the restoring force is:

$$F \approx -\frac{mg}{L}s \tag{16.26}$$

This expression is of the form:

$$F = -kx, \tag{16.27}$$

where the force constant is given by $k = mg/L$ and the displacement is given by $x = s$. For angles less than about $15º$, the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about $15º$. For the simple pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}}. \tag{16.28}$$

Thus,

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{16.29}$$

for the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. As with simple harmonic oscillators, the period T for a pendulum is nearly independent of amplitude, especially if θ is less than about $15º$. Even simple pendulum clocks can be finely adjusted and accurate.

Note the dependence of T on g. If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. Consider the following example.

Example 16.6 Measuring Acceleration due to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find g given the period T and the length L of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{g}}$ for g, assuming only that the angle of deflection is less than $15º$.

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for g:

$$g = 4\pi^2\frac{L}{T^2}. \tag{16.30}$$

2. Substitute known values into the new equation:

$$g = 4\pi^2\frac{0.75000\text{ m}}{(1.7357\text{ s})^2}. \tag{16.31}$$

3. Calculate to find g:

$$g = 9.8281\text{ m/s}^2. \tag{16.32}$$

Discussion

This method for determining g can be very accurate. This is why length and period are given to five digits in this example. For the precision of the approximation $\sin\theta \approx \theta$ to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about 0.5°.

Making Career Connections

Knowing g can be important in geological exploration; for example, a map of g over large geographical regions aids the study of plate tectonics and helps in the search for oil fields and large mineral deposits.

Take Home Experiment: Determining g

Use a simple pendulum to determine the acceleration due to gravity g in your own locale. Cut a piece of a string or dental floss so that it is about 1 m long. Attach a small object of high density to the end of the string (for example, a metal nut or a car key). Starting at an angle of less than 10°, allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch. Calculate g. How accurate is this measurement? How might it be improved?

Check Your Understanding

An engineer builds two simple pendula. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10 kg. Pendulum 2 has a bob with a mass of 100 kg. Describe how the motion of the pendula will differ if the bobs are both displaced by 12°.

Solution

The movement of the pendula will not differ at all because the mass of the bob has no effect on the motion of a simple pendulum. The pendula are only affected by the period (which is related to the pendulum's length) and by the acceleration due to gravity.

PhET Explorations: Pendulum Lab

Play with one or two pendulums and discover how the period of a simple pendulum depends on the length of the string, the mass of the pendulum bob, and the amplitude of the swing. It's easy to measure the period using the photogate timer. You can vary friction and the strength of gravity. Use the pendulum to find the value of g on planet X. Notice the anharmonic behavior at large amplitude.

Figure 16.15 Pendulum Lab (http://cnx.org/content/m55274/1.2/pendulum-lab_en.jar)

16.5 Energy and the Simple Harmonic Oscillator

Learning Objectives

By the end of this section, you will be able to:

- Describe the changes in energy that occur while a system undergoes simple harmonic motion.

To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have We know from **Hooke's Law: Stress and Strain Revisited** that the energy stored in the deformation of a simple harmonic oscillator is a form of potential energy given by:

$$\text{PE}_{\text{el}} = \frac{1}{2}kx^2. \tag{16.33}$$

Because a simple harmonic oscillator has no dissipative forces, the other important form of energy is kinetic energy KE. Conservation of energy for these two forms is:

$$\text{KE} + \text{PE}_{\text{el}} = \text{constant} \tag{16.34}$$

or

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}. \tag{16.35}$$

This statement of conservation of energy is valid for *all* simple harmonic oscillators, including ones where the gravitational force plays a role

Namely, for a simple pendulum we replace the velocity with $v = L\omega$, the spring constant with $k = mg / L$, and the displacement term with $x = L\theta$. Thus

$$\frac{1}{2}mL^2\omega^2 + \frac{1}{2}mgL\theta^2 = \text{constant}. \tag{16.36}$$

In the case of undamped simple harmonic motion, the energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates. So for the simple example of an object on a frictionless surface attached to a spring, as shown again in **Figure 16.16**, the motion starts with all of the energy stored in the spring. As the object starts to move, the elastic potential energy is converted to kinetic energy, becoming entirely kinetic energy at the equilibrium position. It is then converted back into elastic potential energy by the spring, the velocity becomes zero when the kinetic energy is completely converted, and so on. This concept provides extra insight here and in later applications of simple harmonic motion, such as alternating current circuits.

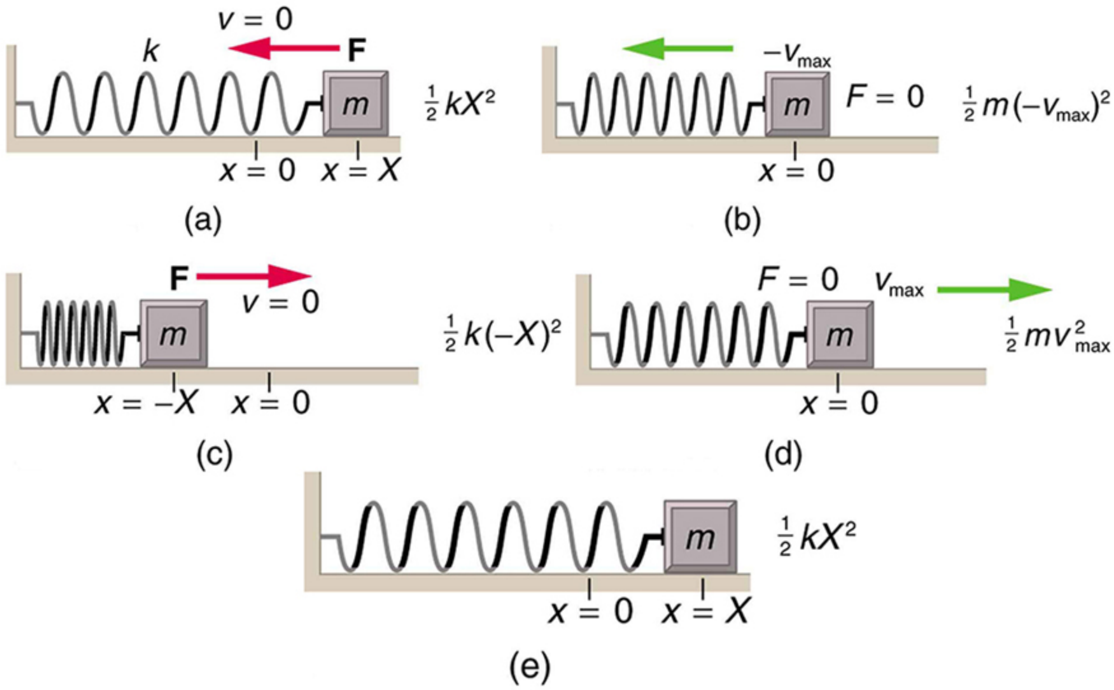


Figure 16.16 The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface.

The conservation of energy principle can be used to derive an expression for velocity v. If we start our simple harmonic motion with zero velocity and maximum displacement ($x = X$), then the total energy is

$$\frac{1}{2}kX^2. \tag{16.37}$$

This total energy is constant and is shifted back and forth between kinetic energy and potential energy, at most times being shared by each. The conservation of energy for this system in equation form is thus:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kX^2. \tag{16.38}$$

Solving this equation for v yields:

$$v = \pm\sqrt{\frac{k}{m}\left(X^2 - x^2\right)}. \tag{16.39}$$

Manipulating this expression algebraically gives:

$$v = \pm\sqrt{\frac{k}{m}}X\sqrt{1 - \frac{x^2}{X^2}} \tag{16.40}$$

and so

$$v = \pm v_{\text{max}}\sqrt{1 - \frac{x^2}{X^2}}, \tag{16.41}$$

where

$$v_{\text{max}} = \sqrt{\frac{k}{m}}X. \tag{16.42}$$

From this expression, we see that the velocity is a maximum (v_{max}) at $x = 0$, as stated earlier in $v(t) = -v_{\text{max}}\ \sin\frac{2\pi t}{T}$.

Notice that the maximum velocity depends on three factors. Maximum velocity is directly proportional to amplitude. As you might guess, the greater the maximum displacement the greater the maximum velocity. Maximum velocity is also greater for stiffer systems, because they exert greater force for the same displacement. This observation is seen in the expression for $v_{\text{max}};$ it is proportional to the square root of the force constant k . Finally, the maximum velocity is smaller for objects that have larger masses, because the maximum velocity is inversely proportional to the square root of m . For a given force, objects that have large masses accelerate more slowly.

A similar calculation for the simple pendulum produces a similar result, namely:

$$\omega_{\text{max}} = \sqrt{\frac{g}{L}}\theta_{\text{max}}. \tag{16.43}$$

Making Connections: Mass Attached to a Spring

Consider a mass *m* attached to a spring, with spring constant *k*, fixed to a wall. When the mass is displaced from its equilibrium position and released, the mass undergoes simple harmonic motion. The spring exerts a force $F = -kv$ on the mass. The potential energy of the system is stored in the spring. It will be zero when the spring is in the equilibrium position. All the internal energy exists in the form of kinetic energy, given by $KE = \frac{1}{2}mv^2$. As the system oscillates, which means that the spring compresses and expands, there is a change in the structure of the system and a corresponding change in its internal energy. Its kinetic energy is converted to potential energy and vice versa. This occurs at an equal rate, which means that a loss of kinetic energy yields a gain in potential energy, thus preserving the work-energy theorem and the law of conservation of energy.

Example 16.7 Determine the Maximum Speed of an Oscillating System: A Bumpy Road

Suppose that a car is 900 kg and has a suspension system that has a force constant $k = 6.53\times10^4\ \text{N/m}$. The car hits a bump and bounces with an amplitude of 0.100 m. What is its maximum vertical velocity if you assume no damping occurs?

Strategy

We can use the expression for v_{max} given in $v_{\text{max}} = \sqrt{\frac{k}{m}}X$ to determine the maximum vertical velocity. The variables m and k are given in the problem statement, and the maximum displacement X is 0.100 m.

Solution

1. Identify known.
2. Substitute known values into $v_{\text{max}} = \sqrt{\frac{k}{m}}X$:

$$v_{\text{max}} = \sqrt{\frac{6.53\times10^4\ \text{N/m}}{900\ \text{kg}}}(0.100\ \text{m}). \tag{16.44}$$

3. Calculate to find $v_{\text{max}} = 0.852\ \text{m/s}.$

Discussion

This answer seems reasonable for a bouncing car. There are other ways to use conservation of energy to find v_{max} . We could use it directly, as was done in the example featured in **Hooke's Law: Stress and Strain Revisited**.

The small vertical displacement y of an oscillating simple pendulum, starting from its equilibrium position, is given as

$$y(t) = a \sin \omega t, \tag{16.45}$$

where a is the amplitude, ω is the angular velocity and t is the time taken. Substituting $\omega = \frac{2\pi}{T}$, we have

$$yt = a \sin\left(\frac{2\pi t}{T}\right). \tag{16.46}$$

Thus, the displacement of pendulum is a function of time as shown above.

Also the velocity of the pendulum is given by

$$v(t) = \frac{2a\pi}{T} \cos\left(\frac{2\pi t}{T}\right), \tag{16.47}$$

so the motion of the pendulum is a function of time.

Check Your Understanding

Why does it hurt more if your hand is snapped with a ruler than with a loose spring, even if the displacement of each system is equal?

Solution

The ruler is a stiffer system, which carries greater force for the same amount of displacement. The ruler snaps your hand with greater force, which hurts more.

Check Your Understanding

You are observing a simple harmonic oscillator. Identify one way you could decrease the maximum velocity of the system.

Solution

You could increase the mass of the object that is oscillating.

16.6 Uniform Circular Motion and Simple Harmonic Motion

Learning Objectives

By the end of this section, you will be able to:

- Compare simple harmonic motion with uniform circular motion.

Figure 16.17 The horses on this merry-go-round exhibit uniform circular motion. (credit: Wonderlane, Flickr)

There is an easy way to produce simple harmonic motion by using uniform circular motion. Figure 16.18 shows one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected on the floor as shown. The shadow undergoes simple harmonic motion. Hooke’s law usually describes uniform circular motions (ω constant) rather than systems that have large visible displacements. So observing the projection of uniform circular motion, as in Figure 16.18, is often easier than observing a precise large-scale simple harmonic oscillator. If studied in sufficient depth, simple harmonic motion produced in this manner can give considerable insight into many aspects of oscillations and waves and is very useful mathematically. In our brief treatment, we shall indicate some of the major features of this relationship and how they might be useful.

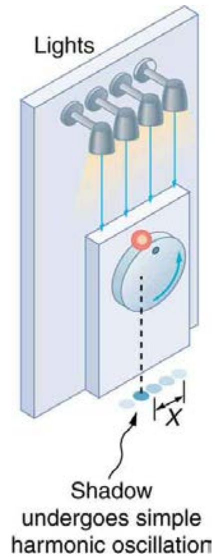


Figure 16.18 The shadow of a ball rotating at constant angular velocity ω on a turntable goes back and forth in precise simple harmonic motion.

Figure 16.19 shows the basic relationship between uniform circular motion and simple harmonic motion. The point P travels around the circle at constant angular velocity ω . The point P is analogous to an object on the merry-go-round. The projection of the position of P onto a fixed axis undergoes simple harmonic motion and is analogous to the shadow of the object. At the time shown in the figure, the projection has position x and moves to the left with velocity v . The velocity of the point P around the circle equals $\bar{v}_{\text{max}}$ .The projection of $\bar{v}_{\text{max}}$ on the x -axis is the velocity v of the simple harmonic motion along the x -axis.

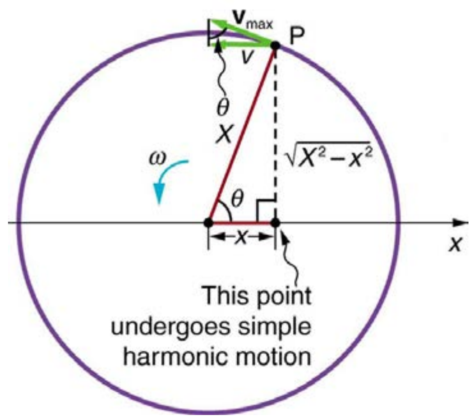


Figure 16.19 A point P moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion. Its projection on the x-axis undergoes simple harmonic motion. Also shown is the velocity of this point around the circle, $\bar{v}_{\text{max}}$, and its projection, which is v . Note that these velocities form a similar triangle to the displacement triangle.

To see that the projection undergoes simple harmonic motion, note that its position x is given by

$$x = X \cos \theta, \tag{16.48}$$

where $\theta = \omega t$, ω is the constant angular velocity, and X is the radius of the circular path. Thus,

$$x = X \cos \omega t. \tag{16.49}$$

The angular velocity ω is in radians per unit time; in this case 2π radians is the time for one revolution T . That is, $\omega = 2\pi / T$. Substituting this expression for ω , we see that the position x is given by:

$$x(t) = \cos\left(\frac{2\pi t}{T}\right). \tag{16.50}$$

This expression is the same one we had for the position of a simple harmonic oscillator in **Simple Harmonic Motion: A Special Periodic Motion**. If we make a graph of position versus time as in **Figure 16.20**, we see again the wavelike character (typical of simple harmonic motion) of the projection of uniform circular motion onto the x -axis.

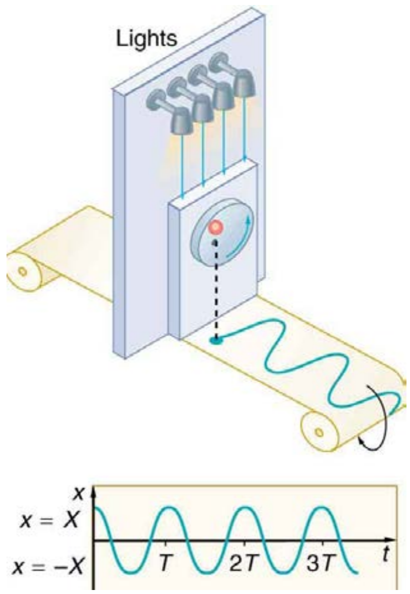


Figure 16.20 The position of the projection of uniform circular motion performs simple harmonic motion, as this wavelike graph of x versus t indicates.

Now let us use **Figure 16.19** to do some further analysis of uniform circular motion as it relates to simple harmonic motion. The triangle formed by the velocities in the figure and the triangle formed by the displacements ($X, x,$ and $\sqrt{X^2 - x^2}$) are similar right triangles. Taking ratios of similar sides, we see that

$$\frac{v}{v_{\text{max}}} = \frac{\sqrt{X^2 - x^2}}{X} = \sqrt{1 - \frac{x^2}{X^2}}. \tag{16.51}$$

We can solve this equation for the speed v or

$$v = v_{\text{max}}\sqrt{1 - \frac{x^2}{X^2}}. \tag{16.52}$$

This expression for the speed of a simple harmonic oscillator is exactly the same as the equation obtained from conservation of energy considerations in **Energy and the Simple Harmonic Oscillator**. You can begin to see that it is possible to get all of the characteristics of simple harmonic motion from an analysis of the projection of uniform circular motion.

Finally, let us consider the period T of the motion of the projection. This period is the time it takes the point P to complete one revolution. That time is the circumference of the circle $2\pi X$ divided by the velocity around the circle, v_{max} . Thus, the period T is

$$T = \frac{2\pi X}{v_{\text{max}}}. \tag{16.53}$$

We know from conservation of energy considerations that

$$v_{\text{max}} = \sqrt{\frac{k}{m}}X. \tag{16.54}$$

Solving this equation for X / v_{max} gives

$$\frac{X}{v_{\text{max}}} = \sqrt{\frac{m}{k}}. \tag{16.55}$$

Substituting this expression into the equation for T yields

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{16.56}$$

Thus, the period of the motion is the same as for a simple harmonic oscillator. We have determined the period for any simple harmonic oscillator using the relationship between uniform circular motion and simple harmonic motion.

Some modules occasionally refer to the connection between uniform circular motion and simple harmonic motion. Moreover, if you carry your study of physics and its applications to greater depths, you will find this relationship useful. It can, for example, help to analyze how waves add when they are superimposed.

Check Your Understanding

A famous magic trick involves a performer singing a note toward a crystal glass until the glass shatters. Explain why the trick works in terms of resonance and natural frequency.

Solution
The performer must be singing a note that corresponds to the natural frequency of the glass. As the sound wave is directed at the glass, the glass responds by resonating at the same frequency as the sound wave. With enough energy introduced into the system, the glass begins to vibrate and eventually shatters.

16.9 Waves

Learning Objectives

By the end of this section, you will be able to:

- Describe various characteristics associated with a wave.
- Differentiate between transverse and longitudinal waves.

Figure 16.29 Waves in the ocean behave similarly to all other types of waves. (credit: Steve Jurveston, Flickr)

What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a **wave** is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances, including disturbance of Earth's surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in **Figure 16.30**. The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period T. The wave's frequency is $f = 1 / T$, as usual. The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define **wave velocity** v_w to be the speed at which the disturbance moves. Wave velocity is sometimes also called the *propagation velocity or propagation speed,* because the disturbance propagates from one location to another.

Misconception Alert

Many people think that water waves push water from one direction to another. In fact, the particles of water tend to stay in one location, save for moving up and down due to the energy in the wave. The energy moves forward through the water, but the water stays in one place. If you feel yourself pushed in an ocean, what you feel is the energy of the wave, not a rush of water.

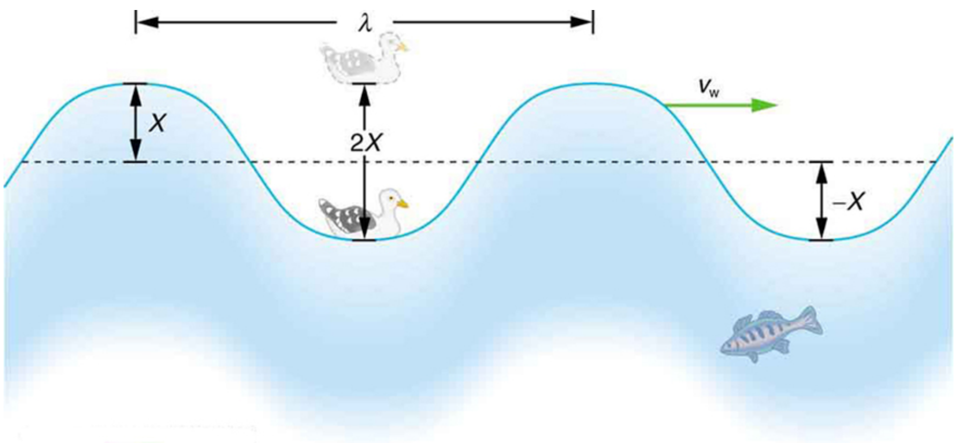


Figure 16.30 An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed v_{w}.

The water wave in the figure also has a length associated with it, called its **wavelength** λ, the distance between adjacent identical parts of a wave. (λ is the distance parallel to the direction of propagation.) The speed of propagation v_{w} is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

$$v_{\text{w}} = \frac{\lambda}{T} \tag{16.66}$$

or

$$v_{\text{w}} = f\lambda. \tag{16.67}$$

This fundamental relationship holds for all types of waves. For water waves, v_{w} is the speed of a surface wave; for sound, v_{w} is the speed of sound; and for visible light, v_{w} is the speed of light, for example.

Applying the Science Practices: Different Types of Waves

Consider a spring fixed to a wall with a mass connected to its end. This fixed point on the wall exerts a force on the complete spring-and-mass system, and this implies that the momentum of the complete system is not conserved. Now, consider energy. Since the system is fixed to a point on the wall, it does not do any work; hence, the total work done is conserved, which means that the energy is conserved. Consequently, we have an oscillator in which energy is conserved but momentum is not. Now, consider a system of two masses connected to each other by a spring. This type of system also forms an oscillator. Since there is no fixed point, momentum is conserved as the forces acting on the two masses are equal and opposite. Energy for such a system will be conserved, because there are no external forces acting on the spring-two-masses system. It is clear from above that, for momentum to be conserved, momentum needs to be carried by waves. This is a typical example of a mechanical oscillator producing mechanical waves that need a medium in which to propagate. Sound waves are also examples of mechanical waves. There are some waves that can travel in the absence of a medium of propagation. Such waves are called “electromagnetic waves.” Light waves are examples of electromagnetic waves. Electromagnetic waves are created by the vibration of electric charge. This vibration creates a wave with both electric and magnetic field components.

Take-Home Experiment: Waves in a Bowl

Fill a large bowl or basin with water and wait for the water to settle so there are no ripples. Gently drop a cork into the middle of the bowl. Estimate the wavelength and period of oscillation of the water wave that propagates away from the cork. Remove the cork from the bowl and wait for the water to settle again. Gently drop the cork at a height that is different from the first drop. Does the wavelength depend upon how high above the water the cork is dropped?

Example 16.9 Calculate the Velocity of Wave Propagation: Gull in the Ocean

Calculate the wave velocity of the ocean wave in Figure 16.30 if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.

Strategy

We are asked to find v_{w} . The given information tells us that $\lambda = 10.0\text{ m}$ and $T = 5.00\text{ s}$. Therefore, we can use $v_{\text{w}} = \frac{\lambda}{T}$ to find the wave velocity.

Solution

1. Enter the known values into $v_{\text{w}} = \frac{\lambda}{T}$:

$$v_{\text{w}} = \frac{10.0\text{ m}}{5.00\text{ s}}. \tag{16.68}$$

2. Solve for v_{w} to find $v_{\text{w}} = 2.00\text{ m/s}$.

Discussion

This slow speed seems reasonable for an ocean wave. Note that the wave moves to the right in the figure at this speed, not the varying speed at which the sea gull moves up and down.

Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in **Figure 16.31** propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a **transverse wave** or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a **longitudinal wave** or compressional wave, the disturbance is parallel to the direction of propagation. **Figure 16.32** shows an example of a longitudinal wave. The size of the disturbance is its amplitude *X* and is completely independent of the speed of propagation v_{w} .

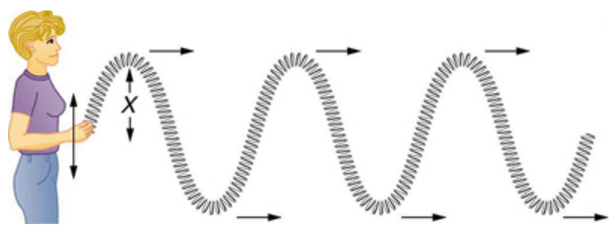


Figure 16.31 In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction.

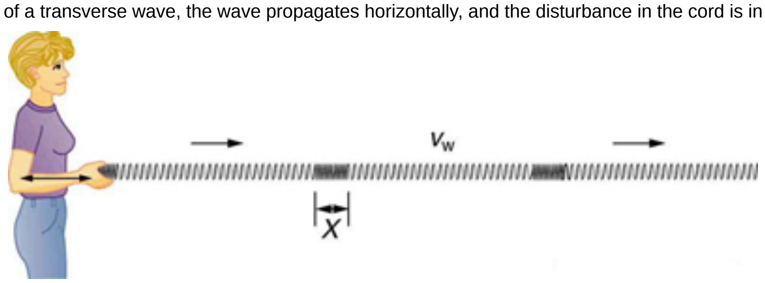


Figure 16.32 In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction.

Waves may be transverse, longitudinal, or *a combination of the two*. (Water waves are actually a combination of transverse and longitudinal. The simplified water wave illustrated in **Figure 16.30** shows no longitudinal motion of the bird.) The waves on the strings of musical instruments are transverse—so are electromagnetic waves, such as visible light.

Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.

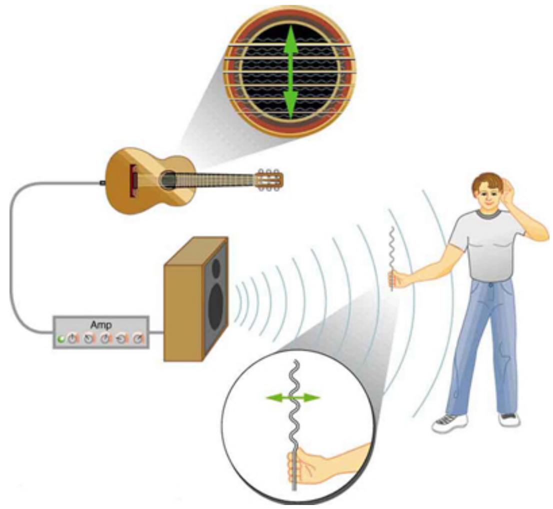


Figure 16.33 The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal.

Earthquake waves under Earth's surface also have both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). These components have important individual characteristics—they propagate at different speeds, for example. Earthquakes also have surface waves that are similar to surface waves on water.

Applying the Science Practices: Electricity in Your Home

The source of electricity is of a sinusoidal nature. If we appropriately probe using an oscilloscope (an instrument used to display and analyze electronic signals), we can precisely determine the frequency and wavelength of the waveform. Inquire about the maximum voltage current that you get in your house and plot a sinusoidal waveform representing the frequency, wavelength, and period for it.

Check Your Understanding

Why is it important to differentiate between longitudinal and transverse waves?

Solution
In the different types of waves, energy can propagate in a different direction relative to the motion of the wave. This is important to understand how different types of waves affect the materials around them.

PhET Explorations: Wave on a String

Watch a string vibrate in slow motion. Wiggle the end of the string and make waves, or adjust the frequency and amplitude of an oscillator. Adjust the damping and tension. The end can be fixed, loose, or open.

Figure 16.34 Wave on a String (http://cnx.org/content/m55281/1.2/wave-on-a-string_en.jar)

16.10 Superposition and Interference

Learning Objectives

By the end of this section, you will be able to:

- Determine the resultant waveform when two waves act in superposition relative to each other.
- Explain standing waves.
- Describe the mathematical representation of overtones and beat frequency.

Figure 16.35 These waves result from the superposition of several waves from different sources, producing a complex pattern. (credit: waterborough, Wikimedia Commons)

Most waves do not look very simple. They look more like the waves in Figure 16.35 than like the simple water wave considered in Waves. (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add. Figure 16.36 and Figure 16.37 illustrate superposition in two special cases, both of which produce simple results.

Figure 16.36 shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure **constructive interference**. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

Figure 16.37 shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.

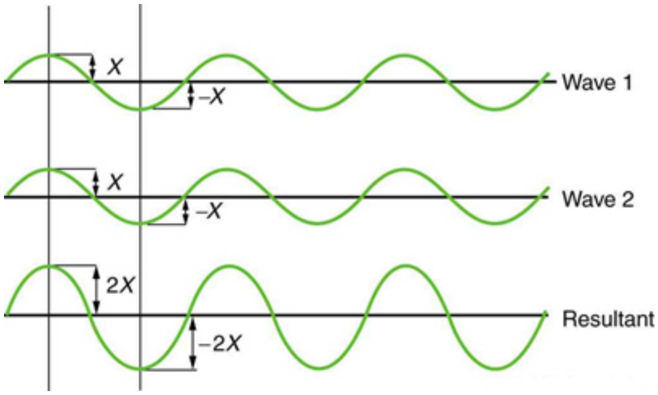


Figure 16.36 Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.

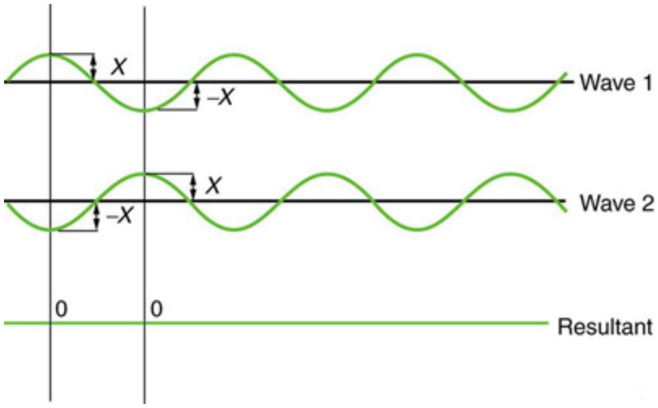


Figure 16.37 Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in **Figure 16.38**. Here again, the disturbances add and subtract, producing a more complicated looking wave.

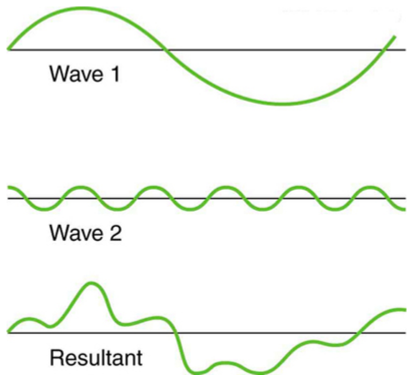


Figure 16.38 Superposition of non-identical waves exhibits both constructive and destructive interference.

Standing Waves

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in **Figure 16.39** for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a **standing wave**. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.

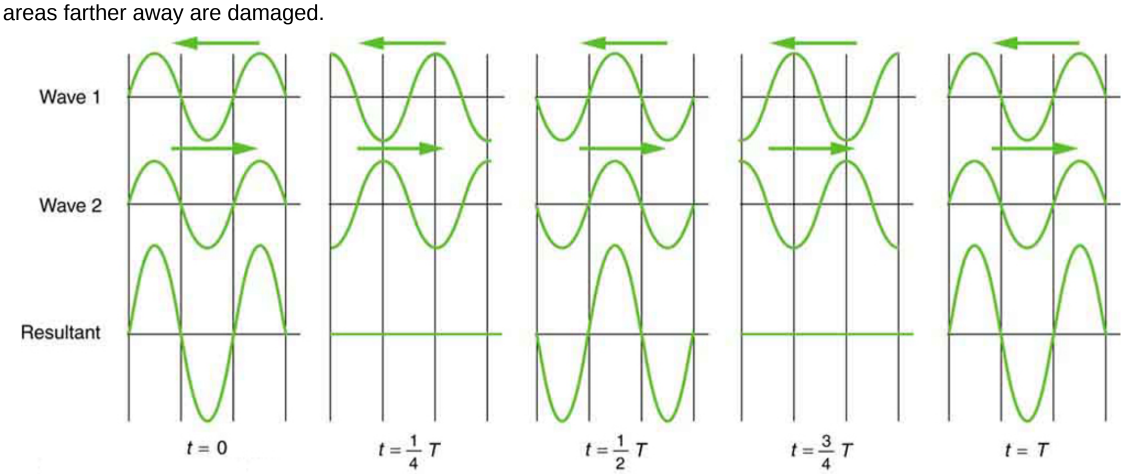


Figure 16.39 Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference.

Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. **Figure 16.40** and **Figure 16.41** show three standing waves that can be created on a string that is fixed at both ends. **Nodes** are the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word **antinode** is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the **fundamental frequency**, is thus for the longest wavelength, which is seen to be $\lambda_1 = 2L$. Therefore, the fundamental frequency is $f_1 = v_\mathrm{w} / \lambda_1 = v_\mathrm{w} / 2L$. In this case, the **overtones** or harmonics are multiples of the fundamental frequency. As seen in **Figure 16.41**, the first harmonic can easily be calculated since $\lambda_2 = L$. Thus, $f_2 = v_\mathrm{w} / \lambda_2 = v_\mathrm{w} / 2L = 2f_1$. Similarly, $f_3 = 3f_1$, and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater v_w is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.

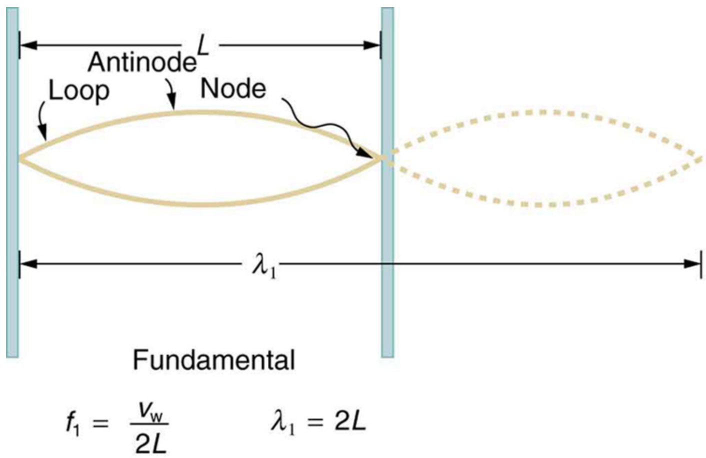


Figure 16.40 The figure shows a string oscillating at its fundamental frequency.

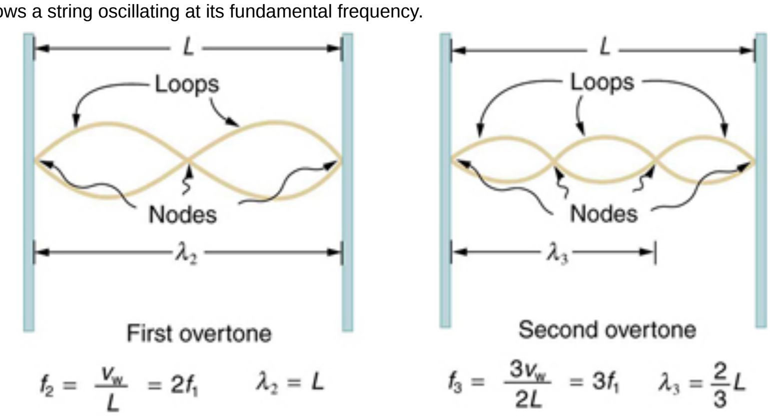


Figure 16.41 First and second harmonic frequencies are shown.

Beats

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. **Figure 16.42** illustrates this graphically.

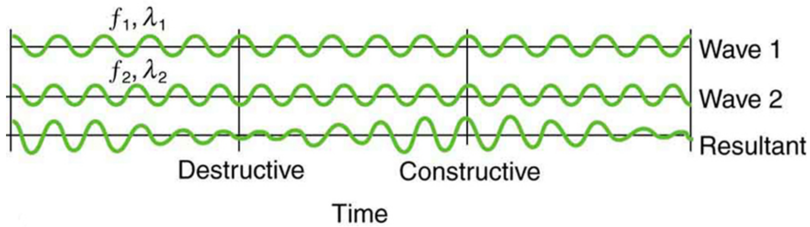


Figure 16.42 Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or *beats*, with a frequency called the **beat frequency**. We can determine the beat frequency by adding two waves together mathematically. Note that a wave can be represented at one point in space as

$$x = X\cos\left(\frac{2\pi\, t}{T}\right) = X\cos(2\pi\, ft), \tag{16.69}$$

where $f = 1/T$ is the frequency of the wave. Adding two waves that have different frequencies but identical amplitudes produces a resultant

$$x = x_1 + x_2. \tag{16.70}$$

More specifically,

$$x = X\cos(2\pi\, f_1\, t) + X\cos(2\pi\, f_2\, t). \tag{16.71}$$

Using a trigonometric identity, it can be shown that

$$x = 2X\cos(\pi\, f_{\rm B}\, t)\cos(2\pi\, f_{\rm ave}\, t), \tag{16.72}$$

where

$$f_{\rm B} = |f_1 - f_2| \tag{16.73}$$

is the beat frequency, and $f_{\rm ave}$ is the average of f_1 and f_2. These results mean that the resultant wave has twice the amplitude and the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency $f_{\rm B}$. The first cosine term in the expression effectively causes the amplitude to go up and down. The second cosine term is the wave with frequency $f_{\rm ave}$. This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.

Real World Connections: Tuning Forks

The **MIT physics demo (http://openstaxcollege.org/l/31tuningforks/)** entitled “Tuning Forks: Resonance and Beat Frequency” provides a qualitative picture of how wave interference produces beats.

Description: Two identical forks and sounding boxes are placed next to each other. Striking one tuning fork will cause the other to resonate at the same frequency. When a weight is attached to one tuning fork, they are no longer identical. Thus, one will not cause the other to resonate. When two different forks are struck at the same time, the interference of their pitches produces beats.

Real World Connections: Jump Rop

This is a fun activity with which to learn about interference and superposition. Take a jump rope and hold it at the two ends with one of your friends. While each of you is holding the rope, snap your hands to produce a wave from each side. Record your observations and see if they match with the following:

a. One wave starts from the right end and travels to the left end of the rope.
b. Another wave starts at the left end and travels to the right end of the rope.
c. The waves travel at the same speed.
d. The shape of the waves depends on the way the person snaps his or her hands.
e. There is a region of overlap.
f. The shapes of the waves are identical to their original shapes after they overlap.

Now, snap the rope up and down and ask your friend to snap his or her end of the rope sideways. The resultant that one sees here is the vector sum of two individual displacements.

This activity illustrates superposition and interference. When two or more waves interact with each other at a point, the disturbance at that point is given by the sum of the disturbances each wave will produce in the absence of the other. This is the principle of superposition. Interference is a result of superposition of two or more waves to form a resultant wave of greater or lower amplitude.

While beats may sometimes be annoying in audible sounds, we will find that beats have many applications. Observing beats is a very useful way to compare similar frequencies. There are applications of beats as apparently disparate as in ultrasonic imaging and radar speed traps.

Check Your Understanding

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

Solution
The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium with no amplitude at all. The wavelengths will result in both constructive and destructive interference

Check Your Understanding

Define nodes and antinodes.

Solution
Nodes are areas of wave interference where there is no motion. Antinodes are areas of wave interference where the motion is at its maximum point.

Check Your Understanding

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

Solution
With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

PhET Explorations: Wave Interference

Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

PhET Interactive Simulation

Figure 16.43 Wave Interference (http://cnx.org/content/m55282/1.2/wave-interference_en.jar)

16.11 Energy in Waves: Intensity

Learning Objectives

By the end of this section, you will be able to:

- Calculate the intensity and the power of rays and waves.

frequency: number of events per unit of time

fundamental frequency: the lowest frequency of a periodic waveform

intensity: power per unit area

longitudinal wave: a wave in which the disturbance is parallel to the direction of propagation

natural frequency: the frequency at which a system would oscillate if there were no driving and no damping forces

nodes: the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

oscillate: moving back and forth regularly between two points

over damping: the condition in which damping of an oscillator causes it to return to equilibrium without oscillating; oscillator moves more slowly toward equilibrium than in the critically damped system

overtones: multiples of the fundamental frequency of a sound

period: time it takes to complete one oscillation

periodic motion: motion that repeats itself at regular time intervals

resonance: the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate: a system being driven at its natural frequency

restoring force: force acting in opposition to the force caused by a deformation

simple harmonic motion: the oscillatory motion in a system where the net force can be described by Hooke’s law

simple harmonic oscillator: a device that implements Hooke’s law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support such as a wall

simple pendulum: an object with a small mass suspended from a light wire or string

superposition: the phenomenon that occurs when two or more waves arrive at the same point

transverse wave: a wave in which the disturbance is perpendicular to the direction of propagation

under damping: the condition in which damping of an oscillator causes it to return to equilibrium with the amplitude gradually decreasing to zero; system returns to equilibrium faster but overshoots and crosses the equilibrium position one or more times

wave: a disturbance that moves from its source and carries energy

wave velocity: the speed at which the disturbance moves. Also called the propagation velocity or propagation speed

wavelength: the distance between adjacent identical parts of a wave

Section Summary

16.1 Hooke’s Law: Stress and Strain Revisited

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations and waves are related to systems that can be described by Hooke’s law:
$$F = -kx,$$
where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.
- Elastic potential energy PE_{el} stored in the deformation of a system that can be described by Hooke’s law is given by
$$\text{PE}_{\text{el}} = (1/2)kx^2.$$

16.2 Period and Frequency in Oscillations

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency f.
- These quantities are related by

$$f = \frac{1}{T}.$$

16.3 Simple Harmonic Motion: A Special Periodic Motion

- Simple harmonic motion is oscillatory motion for a system that can be described only by Hooke's law. Such a system is also called a simple harmonic oscillator.
- Maximum displacement is the amplitude X. The period T and frequency f of a simple harmonic oscillator are given by $T = 2\pi\sqrt{\frac{m}{k}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where m is the mass of the system.
- Displacement in simple harmonic motion as a function of time is given by $x(t) = X \cos\frac{2\pi t}{T}$.
- The velocity is given by $v(t) = -v_{\text{max}} \sin\frac{2\pi t}{T}$, where $v_{\text{max}} = \sqrt{k/m}X$.
- The acceleration is found to be $a(t) = -\frac{kX}{m}\cos\frac{2\pi t}{T}$.

16.4 The Simple Pendulum

- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about 15°.
  The period of a simple pendulum is
  $$T = 2\pi\sqrt{\frac{L}{g}},$$
  where L is the length of the string and g is the acceleration due to gravity.

16.5 Energy and the Simple Harmonic Oscillator

- Energy in the simple harmonic oscillator is shared between elastic potential energy and kinetic energy, with the total being constant:
  $$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}.$$
- Maximum velocity depends on three factors: it is directly proportional to amplitude, it is greater for stiffer systems, and it is smaller for objects that have larger masses:
  $$v_{\text{max}} = \sqrt{\frac{k}{m}}X.$$

16.6 Uniform Circular Motion and Simple Harmonic Motion

A projection of uniform circular motion undergoes simple harmonic oscillation.

16.7 Damped Harmonic Motion

- Damped harmonic oscillators have non-conservative forces that dissipate their energy.
- Critical damping returns the system to equilibrium as fast as possible without overshooting.
- An underdamped system will oscillate through the equilibrium position.
- An overdamped system moves more slowly toward equilibrium than one that is critically damped.

16.8 Forced Oscillations and Resonance

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- The less damping a system has, the higher the amplitude of the forced oscillations near resonance. The more damping a system has, the broader response it has to varying driving frequencies.

16.9 Waves

- A wave is a disturbance that moves from the point of creation with a wave velocity v_{w}.
- A wave has a wavelength λ, which is the distance between adjacent identical parts of the wave.
- Wave velocity and wavelength are related to the wave's frequency and period by $v_{\text{w}} = \frac{\lambda}{T}$ or $v_{\text{w}} = f\lambda$.
- A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

16.10 Superposition and Interference

- Superposition is the combination of two waves at the same location.

- Constructive interference occurs when two identical waves are superimposed in phase.
- Destructive interference occurs when two identical waves are superimposed exactly out of phase.
- A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.
- Nodes are points of no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Waves on a string are resonant standing waves with a fundamental frequency and can occur at higher multiples of the fundamental, called overtones or harmonics.
- Beats occur when waves of similar frequencies f_1 and f_2 are superimposed. The resulting amplitude oscillates with a beat frequency given by

$$f_B = |f_1 - f_2|.$$

16.11 Energy in Waves: Intensity

Intensity is defined to be the power per unit area:

$I = \frac{P}{A}$ and has units of W/m^2.

Conceptual Questions

16.1 Hooke's Law: Stress and Strain Revisited

1. Describe a system in which elastic potential energy is stored.

16.3 Simple Harmonic Motion: A Special Periodic Motion

2. What conditions must be met to produce simple harmonic motion?

3. (a) If frequency is not constant for some oscillation, can the oscillation be simple harmonic motion?

(b) Can you think of any examples of harmonic motion where the frequency may depend on the amplitude?

4. Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude.

5. Explain why you expect an object made of a stiff material to vibrate at a higher frequency than a similar object made of a spongy material.

6. As you pass a freight truck with a trailer on a highway, you notice that its trailer is bouncing up and down slowly. Is it more likely that the trailer is heavily loaded or nearly empty? Explain your answer.

7. Some people modify cars to be much closer to the ground than when manufactured. Should they install stiffer springs? Explain your answer.

16.4 The Simple Pendulum

8. Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you, will you have to lengthen or shorten the pendulum to keep the correct time, other factors remaining constant? Explain your answer.

16.5 Energy and the Simple Harmonic Oscillator

9. Explain in terms of energy how dissipative forces such as friction reduce the amplitude of a harmonic oscillator. Also explain how a driving mechanism can compensate. (A pendulum clock is such a system.)

16.7 Damped Harmonic Motion

10. Give an example of a damped harmonic oscillator. (They are more common than undamped or simple harmonic oscillators.)

11. How would a car bounce after a bump under each of these conditions?

- overdamping
- underdamping
- critical damping

12. Most harmonic oscillators are damped and, if undriven, eventually come to a stop. How is this observation related to the second law of thermodynamics?

16.8 Forced Oscillations and Resonance

13. Why are soldiers in general ordered to "route step" (walk out of step) across a bridge?

16.9 Waves

14. Give one example of a transverse wave and another of a longitudinal wave, being careful to note the relative directions of the disturbance and wave propagation in each.

15. What is the difference between propagation speed and the frequency of a wave? Does one or both affect wavelength? If so, how?

16.10 Superposition and Interference

16. Speakers in stereo systems have two color-coded terminals to indicate how to hook up the wires. If the wires are reversed, the speaker moves in a direction opposite that of a properly connected speaker. Explain why it is important to have both speakers connected the same way.

16.11 Energy in Waves: Intensity

17. Two identical waves undergo pure constructive interference. Is the resultant intensity twice that of the individual waves? Explain your answer.

18. Circular water waves decrease in amplitude as they move away from where a rock is dropped. Explain why.

Problems & Exercises

16.1 Hooke's Law: Stress and Strain Revisited

1. Fish are hung on a spring scale to determine their mass (most fishermen feel no obligation to truthfully report the mass).

(a) What is the force constant of the spring in such a scale if it the spring stretches 8.00 cm for a 10.0 kg load?

(b) What is the mass of a fish that stretches the spring 5.50 cm?

(c) How far apart are the half-kilogram marks on the scale?

2. It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke's law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring's effective spring constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85 kg team?

3. One type of BB gun uses a spring-driven plunger to blow the BB from its barrel. (a) Calculate the force constant of its plunger's spring if you must compress it 0.150 m to drive the 0.0500-kg plunger to a top speed of 20.0 m/s. (b) What force must be exerted to compress the spring?

4. (a) The springs of a pickup truck act like a single spring with a force constant of $1.30\times10^{5}\ \text{N/m}$. By how much will the truck be depressed by its maximum load of 1000 kg?

(b) If the pickup truck has four identical springs, what is the force constant of each?

5. When an 80.0-kg man stands on a pogo stick, the spring is compressed 0.120 m.

(a) What is the force constant of the spring? (b) Will the spring be compressed more when he hops down the road?

6. A spring has a length of 0.200 m when a 0.300-kg mass hangs from it, and a length of 0.750 m when a 1.95-kg mass hangs from it. (a) What is the force constant of the spring? (b) What is the unloaded length of the spring?

16.2 Period and Frequency in Oscillations

7. What is the period of $60.0\ \text{Hz}$ electrical power?

8. If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?

9. Find the frequency of a tuning fork that takes $2.50\times10^{-3}\ \text{s}$ to complete one oscillation.

10. A stroboscope is set to flash every $8.00\times10^{-5}\ \text{s}$. What is the frequency of the flashes?

11. A tire has a tread pattern with a crevice every 2.00 cm. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 m/s?

12. Engineering Application

Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

16.3 Simple Harmonic Motion: A Special Periodic Motion

13. A type of cuckoo clock keeps time by having a mass bouncing on a spring, usually something cute like a cherub in a chair. What force constant is needed to produce a period of 0.500 s for a 0.0150-kg mass?

14. If the spring constant of a simple harmonic oscillator is doubled, by what factor will the mass of the system need to change in order for the frequency of the motion to remain the same?

15. A 0.500-kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s?

16. By how much leeway (both percentage and mass) would you have in the selection of the mass of the object in the previous problem if you did not wish the new period to be greater than 2.01 s or less than 1.99 s?

17. Suppose you attach the object with mass m to a vertical spring originally at rest, and let it bounce up and down. You release the object from rest at the spring's original rest length. (a) Show that the spring exerts an upward force of $2.00\ mg$ on the object at its lowest point. (b) If the spring has a force constant of $10.0\ \text{N/m}$ and a 0.25-kg-mass object is set in motion as described, find the amplitude of the oscillations. (c) Find the maximum velocity.

18. A diver on a diving board is undergoing simple harmonic motion. Her mass is 55.0 kg and the period of her motion is 0.800 s. The next diver is a male whose period of simple harmonic oscillation is 1.05 s. What is his mass if the mass of the board is negligible?

19. Suppose a diving board with no one on it bounces up and down in a simple harmonic motion with a frequency of 4.00 Hz. The board has an effective mass of 10.0 kg. What is the frequency of the simple harmonic motion of a 75.0-kg diver on the board?

20.

Figure 16.46 This child's toy relies on springs to keep infants entertained. (credit: By Humboldthead, Flickr)

The device pictured in **Figure 16.46** entertains infants while keeping them from wandering. The child bounces in a harness suspended from a door frame by a spring constant.

(a) If the spring stretches 0.250 m while supporting an 8.0-kg child, what is its spring constant?

(b) What is the time for one complete bounce of this child? (c) What is the child's maximum velocity if the amplitude of her bounce is 0.200 m?

21. A 90.0-kg skydiver hanging from a parachute bounces up and down with a period of 1.50 s. What is the new period of oscillation when a second skydiver, whose mass is 60.0 kg, hangs from the legs of the first, as seen in **Figure 16.47**.

Figure 16.47 The oscillations of one skydiver are about to be affected by a second skydiver. (credit: U.S. Army, www.army.mil)

16.4 The Simple Pendulum

As usual, the acceleration due to gravity in these problems is taken to be $g = 9.80\text{ m}/\text{s}^2$, unless otherwise specified.

22. What is the length of a pendulum that has a period of 0.500 s?

23. Some people think a pendulum with a period of 1.00 s can be driven with "mental energy" or psycho kinetically, because its period is the same as an average heartbeat. True or not, what is the length of such a pendulum?

24. What is the period of a 1.00-m-long pendulum?

25. How long does it take a child on a swing to complete one swing if her center of gravity is 4.00 m below the pivot?

26. The pendulum on a cuckoo clock is 5.00 cm long. What is its frequency?

27. Two parakeets sit on a swing with their combined center of mass 10.0 cm below the pivot. At what frequency do they swing?

28. (a) A pendulum that has a period of 3.00000 s and that is located where the acceleration due to gravity is 9.79 m/s^2 is moved to a location where it the acceleration due to gravity is 9.82 m/s^2. What is its new period? (b) Explain why so many digits are needed in the value for the period, based on the relation between the period and the acceleration due to gravity.

29. A pendulum with a period of 2.00000 s in one location $\left(g = 9.80\text{ m/s}^2\right)$ is moved to a new location where the period is now 1.99796 s. What is the acceleration due to gravity at its new location?

30. (a) What is the effect on the period of a pendulum if you double its length?

(b) What is the effect on the period of a pendulum if you decrease its length by 5.00%?

31. Find the ratio of the new/old periods of a pendulum if the pendulum were transported from Earth to the Moon, where the acceleration due to gravity is 1.63 m/s^2.

32. At what rate will a pendulum clock run on the Moon, where the acceleration due to gravity is 1.63 m/s^2, if it keeps time accurately on Earth? That is, find the time (in hours) it takes the clock's hour hand to make one revolution on the Moon.

33. Suppose the length of a clock's pendulum is changed by 1.000%, exactly at noon one day. What time will it read 24.00 hours later, assuming it the pendulum has kept perfect time before the change? Note that there are two answers, and perform the calculation to four-digit precision.

34. If a pendulum-driven clock gains 5.00 s/day, what fractional change in pendulum length must be made for it to keep perfect time?

16.5 Energy and the Simple Harmonic Oscillator

35. The length of nylon rope from which a mountain climber is suspended has a force constant of $1.40\times10^4\text{ N/m}$.

(a) What is the frequency at which he bounces, given his mass plus and the mass of his equipment are 90.0 kg?

(b) How much would this rope stretch to break the climber's fall if he free-falls 2.00 m before the rope runs out of slack? Hint: Use conservation of energy.

(c) Repeat both parts of this problem in the situation where twice this length of nylon rope is used.

36. Engineering Application

Near the top of the Citigroup Center building in New York City, there is an object with mass of $4.00\times10^5\text{ kg}$ on springs that have adjustable force constants. Its function is to dampen wind-driven oscillations of the building by oscillating at the same frequency as the building is being driven—the driving force is transferred to the object, which oscillates instead of the entire building. (a) What effective force constant should the springs have to make the object oscillate with a period of 2.00 s? (b) What energy is stored in the springs for a 2.00-m displacement from equilibrium?

16.6 Uniform Circular Motion and Simple Harmonic Motion

37. (a)What is the maximum velocity of an 85.0-kg person bouncing on a bathroom scale having a force constant of $1.50\times10^6\text{ N/m}$, if the amplitude of the bounce is 0.200 cm? (b)What is the maximum energy stored in the spring?

38. A novelty clock has a 0.0100-kg mass object bouncing on a spring that has a force constant of 1.25 N/m. What is the maximum velocity of the object if the object bounces 3.00 cm above and below its equilibrium position? (b) How many joules of kinetic energy does the object have at its maximum velocity?

39. At what positions is the speed of a simple harmonic oscillator half its maximum? That is, what values of x/X give $v = \pm v_{\text{max}}/2$, where X is the amplitude of the motion?

40. A ladybug sits 12.0 cm from the center of a Beatles music album spinning at 33.33 rpm. What is the maximum velocity of its shadow on the wall behind the turntable, if illuminated parallel to the record by the parallel rays of the setting Sun?

16.7 Damped Harmonic Motion

41. The amplitude of a lightly damped oscillator decreases by 3.0% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

16.8 Forced Oscillations and Resonance

42. How much energy must the shock absorbers of a 1200-kg car dissipate in order to damp a bounce that initially has a velocity of 0.800 m/s at the equilibrium position? Assume the car returns to its original vertical position.

43. If a car has a suspension system with a force constant of $5.00 \times 10^{4}\ \text{N/m}$, how much energy must the car's shocks remove to dampen an oscillation starting with a maximum displacement of 0.0750 m?

44. (a) How much will a spring that has a force constant of 40.0 N/m be stretched by an object with a mass of 0.500 kg when hung motionless from the spring? (b) Calculate the decrease in gravitational potential energy of the 0.500-kg object when it descends this distance. (c) Part of this gravitational energy goes into the spring. Calculate the energy stored in the spring by this stretch, and compare it with the gravitational potential energy. Explain where the rest of the energy might go.

45. Suppose you have a 0.750-kg object on a horizontal surface connected to a spring that has a force constant of 150 N/m. There is simple friction between the object and surface with a static coefficient of friction $\mu_{\text{s}} = 0.100$. (a) How far can the spring be stretched without moving the mass? (b) If the object is set into oscillation with an amplitude twice the distance found in part (a), and the kinetic coefficient of friction is $\mu_{\text{k}} = 0.0850$, what total distance does it travel before stopping? Assume it starts at the maximum amplitude.

46. Engineering Application: A suspension bridge oscillates with an effective force constant of $1.00 \times 10^{8}\ \text{N/m}$. (a) How much energy is needed to make it oscillate with an amplitude of 0.100 m? (b) If soldiers march across the bridge with a cadence equal to the bridge's natural frequency and impart $1.00 \times 10^{4}\ \text{J}$ of energy each second, how long does it take for the bridge's oscillations to go from 0.100 m to 0.500 m amplitude?

16.9 Waves

47. Storms in the South Pacific can create waves that travel all the way to the California coast, which are 12,000 km away. How long does it take them if they travel at 15.0 m/s?

48. Waves on a swimming pool propagate at 0.750 m/s. You splash the water at one end of the pool and observe the wave go to the opposite end, reflect, and return in 30.0 s. How far away is the other end of the pool?

49. Wind gusts create ripples on the ocean that have a wavelength of 5.00 cm and propagate at 2.00 m/s. What is their frequency?

50. How many times a minute does a boat bob up and down on ocean waves that have a wavelength of 40.0 m and a propagation speed of 5.00 m/s?

51. Scouts at a camp shake the rope bridge they have just crossed and observe the wave crests to be 8.00 m apart. If they shake it the bridge twice per second, what is the propagation speed of the waves?

52. What is the wavelength of the waves you create in a swimming pool if you splash your hand at a rate of 2.00 Hz and the waves propagate at 0.800 m/s?

53. What is the wavelength of an earthquake that shakes you with a frequency of 10.0 Hz and gets to another city 84.0 km away in 12.0 s?

54. Radio waves transmitted through space at $3.00 \times 10^{8}\ \text{m/s}$ by the Voyager spacecraft have a wavelength of 0.120 m. What is their frequency?

55. Your ear is capable of differentiating sounds that arrive at the ear just 1.00 ms apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different times on a day when the speed of sound is 340 m/s?

56. (a) Seismographs measure the arrival times of earthquakes with a precision of 0.100 s. To get the distance to the epicenter of the quake, they compare the arrival times of S- and P-waves, which travel at different speeds. **Figure 16.48**) If S- and P-waves travel at 4.00 and 7.20 km/s, respectively, in the region considered, how precisely can the distance to the source of the earthquake be determined? (b) Seismic waves from underground detonations of nuclear bombs can be used to locate the test site and detect violations of test bans. Discuss whether your answer to (a) implies a serious limit to such detection. (Note also that the uncertainty is greater if there is an uncertainty in the propagation speeds of the S- and P-waves.)

Figure 16.48 A seismograph as described in above problem.(credit: Oleg Alexandrov)

16.10 Superposition and Interference

57. A car has two horns, one emitting a frequency of 199 Hz and the other emitting a frequency of 203 Hz. What beat frequency do they produce?

58. The middle-C hammer of a piano hits two strings, producing beats of 1.50 Hz. One of the strings is tuned to 260.00 Hz. What frequencies could the other string have?

59. Two tuning forks having frequencies of 460 and 464 Hz are struck simultaneously. What average frequency will you hear, and what will the beat frequency be?

60. Twin jet engines on an airplane are producing an average sound frequency of 4100 Hz with a beat frequency of 0.500 Hz. What are their individual frequencies?

61. A wave traveling on a Slinky® that is stretched to 4 m takes 2.4 s to travel the length of the Slinky and back again. (a) What is the speed of the wave? (b) Using the same Slinky stretched to the same length, a standing wave is created which consists of three antinodes and four nodes. At what frequency must the Slinky be oscillating?

62. Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?

16.11 Energy in Waves: Intensity

63. Medical Application

Ultrasound of intensity $1.50\times10^2\ \text{W/m}^2$ is produced by the rectangular head of a medical imaging device measuring 3.00 by 5.00 cm. What is its power output?

64. The low-frequency speaker of a stereo set has a surface area of $0.05\ \text{m}^2$ and produces 1W of acoustical power. What is the intensity at the speaker? If the speaker projects sound uniformly in all directions, at what distance from the speaker is the intensity $0.1\ \text{W/m}^2$?

65. To increase intensity of a wave by a factor of 50, by what factor should the amplitude be increased?

66. Engineering Application

A device called an insolation meter is used to measure the intensity of sunlight has an area of 100 cm^2 and registers 6.50 W. What is the intensity in W/m^2?

67. Astronomy Application

Energy from the Sun arrives at the top of the Earth’s atmosphere with an intensity of $1.30\ \text{kW/m}^2.$ How long does it take for $1.8\times10^9\ \text{J}$ to arrive on an area of $1.00\ \text{m}^2$?

68. Suppose you have a device that extracts energy from ocean breakers in direct proportion to their intensity. If the device produces 10.0 kW of power on a day when the breakers are 1.20 m high, how much will it produce when they are 0.600 m high?

69. Engineering Application

(a) A photovoltaic array of (solar cells) is 10.0% efficient in gathering solar energy and converting it to electricity. If the average intensity of sunlight on one day is $700\ \text{W/m}^2,$ what area should your array have to gather energy at the rate of 100 W? (b) What is the maximum cost of the array if it must pay for itself in two years of operation averaging 10.0 hours per day? Assume that it earns money at the rate of 9.00 ¢ per kilowatt-hour.

70. A microphone receiving a pure sound tone feeds an oscilloscope, producing a wave on its screen. If the sound intensity is originally $2.00\times10^{-5}\ \text{W/m}^2,$ but is turned up until the amplitude increases by 30.0%, what is the new intensity?

71. Medical Application

(a) What is the intensity in W/m^2 of a laser beam used to burn away cancerous tissue that, when 90.0% absorbed, puts 500 J of energy into a circular spot 2.00 mm in diameter in 4.00 s? (b) Discuss how this intensity compares to the average intensity of sunlight (about $700\ \text{W/m}^2$) and the implications that would have if the laser beam entered your eye. Note how your answer depends on the time duration of the exposure.

Test Prep for AP® Courses

16.1 Hooke's Law: Stress and Strain Revisited

1. Which of the following represents the distance (how much ground the particle covers) moved by a particle in a simple harmonic motion in one time period? (Here, *A* represents the amplitude of the oscillation.)

a. 0 cm
b. *A* cm
c. 2*A* cm
d. 4*A* cm

2. A spring has a spring constant of 80 N·m^{-1}. What is the force required to (a) compress the spring by 5 cm and (b) expand the spring by 15 cm?

3. In the formula $F = -kx$, what does the minus sign indicate?

a. It indicates that the restoring force is in the direction of the displacement.
b. It indicates that the restoring force is in the direction opposite the displacement.
c. It indicates that mechanical energy in the system decreases when a system undergoes oscillation.
d. None of the above

4. The splashing of a liquid resembles an oscillation. The restoring force in this scenario will be due to which of the following?

a. Potential energy
b. Kinetic energy
c. Gravity
d. Mechanical energy

16.2 Period and Frequency in Oscillations

5. A mass attached to a spring oscillates and completes 50 full cycles in 30 s. What is the time period and frequency of this system?

16.3 Simple Harmonic Motion: A Special Periodic Motion

6. Use these figures to answer the following questions.

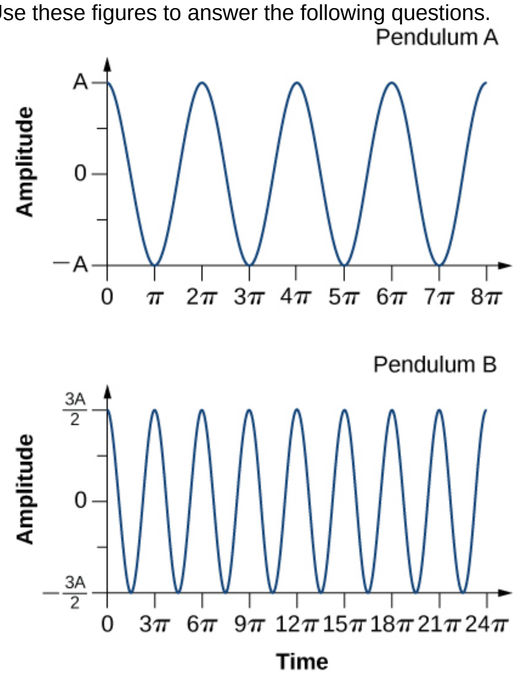


Figure 16.49

a. Which of the two pendulums oscillates with larger amplitude?
b. Which of the two pendulums oscillates at a higher frequency?

7. A particle of mass 100 g undergoes a simple harmonic motion. The restoring force is provided by a spring with a spring constant of 40 N·m^{-1}. What is the period of oscillation?

a. 10 s
b. 0.5 s
c. 0.1 s
d. 1

8. The graph shows the simple harmonic motion of a mass *m* attached to a spring with spring constant *k*.

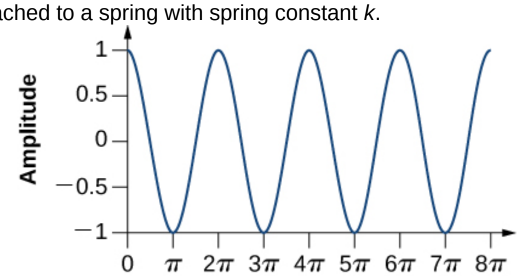


Figure 16.50

What is the displacement at time 8π?

a. 1 m
b. 0 m
c. Not defined
d. −1 m

9. A pendulum of mass 200 g undergoes simple harmonic motion when acted upon by a force of 15 N. The pendulum crosses the point of equilibrium at a speed of 5 m·s^{-1}. What is the energy of the pendulum at the center of the oscillation?

16.4 The Simple Pendulum

10. A ball is attached to a string of length 4 m to make a pendulum. The pendulum is placed at a location that is away from the Earth's surface by twice the radius of the Earth. What is the acceleration due to gravity at that height and what is the period of the oscillations?

11. Which of the following gives the correct relation between the acceleration due to gravity and period of a pendulum?

a. $g = \frac{2\pi L}{T^2}$
b. $g = \frac{4\pi^2 L}{T^2}$
c. $g = \frac{2\pi L}{T}$
d. $g = \frac{2\pi^2 L}{T}$

12. Tom has two pendulums with him. Pendulum 1 has a ball of mass 0.1 kg attached to it and has a length of 5 m. Pendulum 2 has a ball of mass 0.5 kg attached to a string of length 1 m. How does mass of the ball affect the frequency of the pendulum? Which pendulum will have a higher frequency and why?

16.5 Energy and the Simple Harmonic Oscillator

13. A mass of 1 kg undergoes simple harmonic motion with amplitude of 1 m. If the period of the oscillation is 1 s, calculate the internal energy of the system.

16.6 Uniform Circular Motion and Simple Harmonic Motion

14. In the equation $x = A \sin wt,$ what values can the position x take?

a. −1 to +1
b. –*A* to +*A*
c. 0
d. –*t* to *t*

16.7 Damped Harmonic Motion

15. The non-conservative damping force removes energy from a system in which form?

a. Mechanical energy
b. Electrical energy
c. Thermal energy
d. None of the above

16. The time rate of change of mechanical energy for a damped oscillator is always:

a. 0
b. Negative
c. Positive
d. Undefined

17. A 0.5-kg object is connected to a spring that undergoes oscillatory motion. There is friction between the object and the surface it is kept on given by coefficient of friction $\mu_k = 0.06$. If the object is released 0.2 m from equilibrium, what is the distance that the object travels? Given that the force constant of the spring is 50 N m^{-1} and the frictional force between the objects is 0.294 N.

16.8 Forced Oscillations and Resonance

18. How is constant amplitude sustained in forced oscillations?

16.9 Waves

19. What is the difference between the waves coming from a tuning fork and electromagnetic waves?

20. Represent longitudinal and transverse waves in a graphical form.

21. Why is the sound produced by a tambourine different from that produced by drums?

22. A transverse wave is traveling left to right. Which of the following is correct about the motion of particles in the wave?

a. The particles move up and down when the wave travels in a vacuum.
b. The particles move left and right when the wave travels in a medium.
c. The particles move up and down when the wave travels in a medium.
d. The particles move right and left when the wave travels in a vacuum.

23.

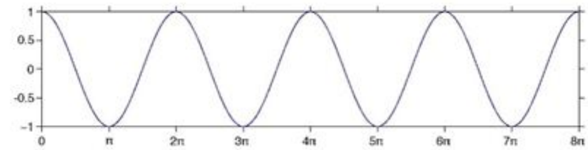


Figure 16.51 The graph shows propagation of a mechanical wave. What is the wavelength of this wave?

16.10 Superposition and Interference

24. A guitar string has a number of frequencies at which it vibrates naturally. Which of the following is true in this context?

a. The resonant frequencies of the string are integer multiples of fundamental frequencies.
b. The resonant frequencies of the string are not integer multiples of fundamental frequencies.
c. They have harmonic overtones.
d. None of the above

25. Explain the principle of superposition with figures that show the changes in the wave amplitude.

26. In this figure which points represent the points of constructive interference?

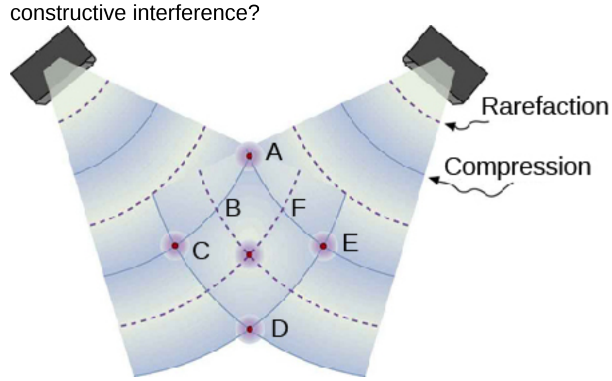


Figure 16.52

a. A, B, F
b. A, B, C, D, E, F
c. A, C, D, E
d. A, B, D

27. A string is fixed on both sides. It is snapped from both ends at the same time by applying an equal force. What happens to the shape of the waves generated in the string? Also, will you observe an overlap of waves?

28. In the preceding question, what would happen to the amplitude of the waves generated in this way? Also, consider another scenario where the string is snapped up from one end and down from the other end. What will happen in this situation?

29. Two sine waves travel in the same direction in a medium. The amplitude of each wave is *A*, and the phase difference between the two is 180°. What is the resultant amplitude?

a. 2*A*
b. 3*A*
c. 0
d. 9*A*

30. Standing wave patterns consist of nodes and antinodes formed by repeated interference between two waves of the same frequency traveling in opposite directions. What are nodes and antinodes and how are they produced?

17 PHYSICS OF HEARING

Figure 17.1 This tree fell some time ago. When it fell, atoms in the air were disturbed. Physicists would call this disturbance sound whether someone was around to hear it or not. (credit: B.A. Bowen Photography)

Chapter Outline

Connection for AP® Courses

In this chapter, the concept of waves is specifically applied to the phenomena of sound. As such, Big Idea 6 continues to be supported, as sound waves carry energy and momentum from one location to another without the permanent transfer of mass. This energy is carried through vibrations caused by disturbances in air pressure (Enduring Understanding 6.A). As air pressure increases, amplitudes of vibration and energy transfer do as well. This idea (Enduring Understanding 6.A.4) explains why a very loud sound can break glass.

The chapter continues the fundamental analysis of waves addressed in Chapter 16. Sound waves are periodic, and can therefore be expressed as a function of position and time. Furthermore, sound waves are described by amplitude, frequency, wavelength, and speed (Enduring Understanding 6.B). The relationship between speed and frequency is analyzed further in **Section 17.4**, as the frequency of sound depends upon the relative motion between the source and observer. This concept, known as the Doppler effect, supports Essential Knowledge 6.B.5.

Like all other waves, sound waves can overlap. When they do so, their interaction will produce an amplitude variation within the resultant wave. This amplitude can be determined by adding the displacement of the two pulses, through a process called superposition. This process, covered in **Section 17.5**, reinforces the content in Enduring Understanding 6.D.1.

In situations where the interfering waves are confined, such as on a fixed length of string or in a tube, standing waves can result. These waves are the result of interference between the incident and reflecting wave. Standing waves are described using nodes and antinodes, and their wavelengths are determined by the size of the region to which they are confined. This chapter's description of both standing waves and the concept of beats strongly support Enduring Understanding 6.D, as well as Essential Knowledge 6.D.1, 6.D.3, and 6.D.4.

The concepts in this chapter support:

Big Idea 6 Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.

Enduring Understanding 6.B A periodic wave is one that repeats as a function of both time and position and can be described by its amplitude, frequency, wavelength, speed, and energy.

Essential Knowledge 6.B.5 The observed frequency of a wave depends on the relative motion of the source and the observer. This is a qualitative measurement only.

Enduring Understanding 6.D Interference and superposition lead to standing waves and beats.

Essential Knowledge 6.D.1 Two or more wave pulses can interact in such a way as to produce amplitude variations in the resultant wave. When two pulses cross, they travel through each other; they do not bounce off each other. Where the pulses overlap, the resulting displacement can be determined by adding the displacements of the two pulses. This is called superposition.

Essential Knowledge 6.D.3 Standing waves are the result of the addition of incident and reflected waves that are confined to a region and have nodes and antinodes. Examples should include waves on a fixed length of string, and sound waves in both closed and open tubes.

Essential Knowledge 6.D.4 The possible wavelengths of a standing wave are determined by the size of the region in which it is confined.

17.1 Sound

Learning Objectives

By the end of this section, you will be able to:

- Define sound and hearing.
- Describe sound as a longitudinal wave.

Figure 17.2 This glass has been shattered by a high-intensity sound wave of the same frequency as the resonant frequency of the glass. While the sound is not visible, the effects of the sound prove its existence. (credit: ||read||, Flickr)

Sound can be used as a familiar illustration of waves. Because hearing is one of our most important senses, it is interesting to see how the physical properties of sound correspond to our perceptions of it. **Hearing** is the perception of sound, just as vision is the perception of visible light. But sound has important applications beyond hearing. Ultrasound, for example, is not heard but can be employed to form medical images and is also used in treatment.

The physical phenomenon of **sound** is defined to be a disturbance of matter that is transmitted from its source outward. Sound is a wave. On the atomic scale, it is a disturbance of atoms that is far more ordered than their thermal motions. In many instances, sound is a periodic wave, and the atoms undergo simple harmonic motion. In this text, we shall explore such periodic sound waves.

A vibrating string produces a sound wave as illustrated in **Figure 17.3**, **Figure 17.4**, and **Figure 17.5**. As the string oscillates back and forth, it transfers energy to the air, mostly as thermal energy created by turbulence. But a small part of the string's energy goes into compressing and expanding the surrounding air, creating slightly higher and lower local pressures. These compressions (high pressure regions) and rarefactions (low pressure regions) move out as longitudinal pressure waves having the same frequency as the string—they are the disturbance that is a sound wave. (Sound waves in air and most fluids are longitudinal, because fluids have almost no shear strength. In solids, sound waves can be both transverse and longitudinal.) **Figure 17.5** shows a graph of gauge pressure versus distance from the vibrating string.

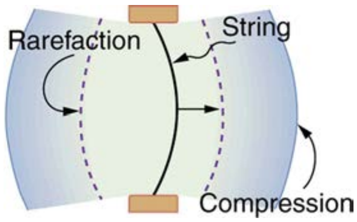


Figure 17.3 A vibrating string moving to the right compresses the air in front of it and expands the air behind it.

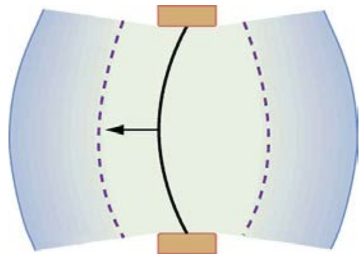


Figure 17.4 As the string moves to the left, it creates another compression and rarefaction as the ones on the right move away from the string.

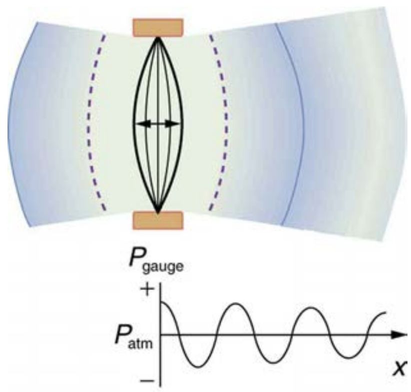


Figure 17.5 After many vibrations, there are a series of compressions and rarefactions moving out from the string as a sound wave. The graph shows gauge pressure versus distance from the source. Pressures vary only slightly from atmospheric for ordinary sounds.

The amplitude of a sound wave decreases with distance from its source, because the energy of the wave is spread over a larger and larger area. But it is also absorbed by objects, such as the eardrum in **Figure 17.6**, and converted to thermal energy by the viscosity of air. In addition, during each compression a little heat transfers to the air and during each rarefaction even less heat transfers from the air, so that the heat transfer reduces the organized disturbance into random thermal motions. (These processes can be viewed as a manifestation of the second law of thermodynamics presented in **Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency**.) Whether the heat transfer from compression to rarefaction is significant depends on how far apart they are—that is, it depends on wavelength. Wavelength, frequency, amplitude, and speed of propagation are important for sound, as they are for all waves.

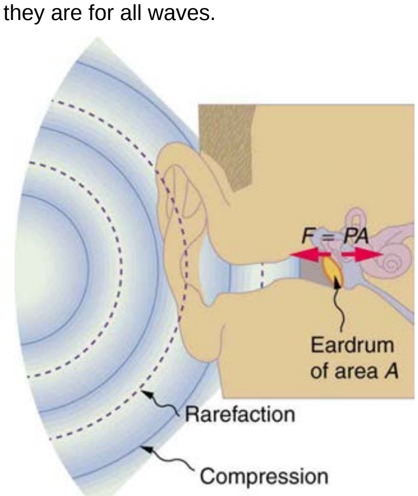


Figure 17.6 Sound wave compressions and rarefactions travel up the ear canal and force the eardrum to vibrate. There is a net force on the eardrum, since the sound wave pressures differ from the atmospheric pressure found behind the eardrum. A complicated mechanism converts the vibrations to nerve impulses, which are perceived by the person.

PhET Explorations: Wave Interference

Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

PhET Interactive Simulation

Figure 17.7 Wave Interference (http://cnx.org/content/m55288/1.2/wave-interference_en.jar)

17.2 Speed of Sound, Frequency, and Wavelength

Learning Objectives

By the end of this section, you will be able to:

- Define pitch.
- Describe the relationship between the speed of sound, its frequency, and its wavelength.
- Describe the effects on the speed of sound as it travels through various media.
- Describe the effects of temperature on the speed of sound.

The information presented in this section supports the following AP® learning objectives and science practices:

- **6.B.4.1** The student is able to design an experiment to determine the relationship between periodic wave speed, wavelength, and frequency, and relate these concepts to everyday examples. **(S.P. 4.2, 5.1, 7.2)**

Figure 17.8 When a firework explodes, the light energy is perceived before the sound energy. Sound travels more slowly than light does. (credit: Dominic Alves, Flickr)

Sound, like all waves, travels at a certain speed and has the properties of frequency and wavelength. You can observe direct evidence of the speed of sound while watching a fireworks display. The flash of an explosion is seen well before its sound is heard, implying both that sound travels at a finite speed and that it is much slower than light. You can also directly sense the frequency of a sound. Perception of frequency is called **pitch**. The wavelength of sound is not directly sensed, but indirect evidence is found in the correlation of the size of musical instruments with their pitch. Small instruments, such as a piccolo, typically make high-pitch sounds, while large instruments, such as a tuba, typically make low-pitch sounds. High pitch means small wavelength, and the size of a musical instrument is directly related to the wavelengths of sound it produces. So a small instrument creates short-wavelength sounds. Similar arguments hold that a large instrument creates long-wavelength sounds.

The relationship of the speed of sound, its frequency, and wavelength is the same as for all waves:

$$v_\mathrm{w} = f\lambda, \tag{17.1}$$

where v_w is the speed of sound, f is its frequency, and λ is its wavelength. The wavelength of a sound is the distance between adjacent identical parts of a wave—for example, between adjacent compressions as illustrated in Figure 17.9. The frequency is the same as that of the source and is the number of waves that pass a point per unit time.

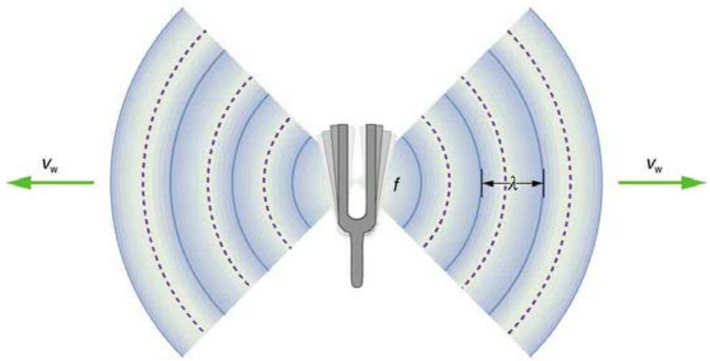


Figure 17.9 A sound wave emanates from a source vibrating at a frequency f, propagates at v_w, and has a wavelength λ.

Table 17.4 makes it apparent that the speed of sound varies greatly in different media. The speed of sound in a medium is determined by a combination of the medium's rigidity (or compressibility in gases) and its density. The more rigid (or less compressible) the medium, the faster the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is directly proportional to the stiffness of the oscillating object. The greater the density of a medium, the slower the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is inversely proportional to the mass of the oscillating object. The speed of sound in air is low, because air is compressible. Because liquids and solids are relatively rigid and very difficult to compress, the speed of sound in such media is generally greater than in gases.

Applying the Science Practices: Bottle Music

When liquid is poured into a small-necked container like a soda bottle, it can make for a fun musical experience! Find a small-necked bottle and pour water into it. When you blow across the surface of the bottle, a musical pitch should be created. This pitch, which corresponds to the resonant frequency of the air remaining in the bottle, can be determined using **Equation 17.1**. Your task is to design an experiment and collect data to confirm this relationship between the frequency created by blowing into the bottle and the depth of air remaining.

1. Use the explanation above to design an experiment that will yield data on depth of air column and frequency of pitch. Use the data table below to record your data.

Table 17.1

Depth of air column (λ)	Frequency of pitch generated (*f*)

2. Construct a graph using the information collected above. The graph should include all five data points and should display frequency on the dependent axis.
3. What type of relationship is displayed on your graph? (direct, inverse, quadratic, etc.)
4. Does your graph align with equation 17.1, given earlier in this section? Explain.

Note: For an explanation of why a frequency is created when you blow across a small-necked container, explore **Section 17.5** later in this chapter.

Answer

1. As the depth of the air column increases, the frequency values must decrease. A sample set of data is displayed below.

Table 17.2

Depth of air column (λ)	Frequency of pitch generated (*f*)
24 cm	689.6 Hz
22 cm	752.3 Hz
20 cm	827.5 Hz
18 cm	919.4 Hz
16 cm	1034.4 Hz

2. The graph drawn should have frequency on the vertical axis, contain five data points, and trend downward and to the right. A graph using the sample data from above is displayed below.

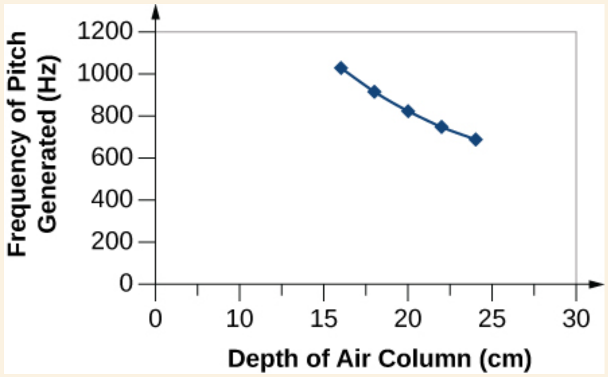


Figure 17.10 A graph of the depth of air column versus the frequency of pitch generated.

3. Inverse relationship.

Table 17.3

Depth of air column (λ)	Frequency of pitch generated (*f*)	Product of wavelength and frequency
24 cm	689.6 Hz	165.5
22 cm	752.3 Hz	165.5
20 cm	827.5 Hz	165.5
18 cm	919.4 Hz	165.5
16 cm	1034.4 Hz	165.5

4. The graph does align with the equation $v = f\lambda$. As the wavelength decreases, the frequency of the pitch generated increases. This relationship is validated by both the sample data table and the sample graph. Additionally, as **Table 17.1** demonstrates, the product of λ and *f* is constant across all five data points.
In addition to these explanations, the student may use the formula as given in the problem statement to show that the product *f* × *air column height* is consistently 165.5.

Table 17.4 Speed of Sound in Various Media

Medium	v_{w}(m/s)
Gases at $0^{\circ}C$	
Air	331
Carbon dioxide	259
Oxygen	316
Helium	965
Hydrogen	1290
Liquids at $20^{\circ}C$	
Ethanol	1160
Mercury	1450
Water, fresh	1480
Sea water	1540
Human tissue	1540
Solids (longitudinal or bulk)	
Vulcanized rubber	54
Polyethylene	920
Marble	3810
Glass, Pyrex	5640
Lead	1960
Aluminum	5120
Steel	5960

Earthquakes, essentially sound waves in Earth's crust, are an interesting example of how the speed of sound depends on the rigidity of the medium. Earthquakes have both longitudinal and transverse components, and these travel at different speeds. The bulk modulus of granite is greater than its shear modulus. For that reason, the speed of longitudinal or pressure waves (P-waves) in earthquakes in granite is significantly higher than the speed of transverse or shear waves (S-waves). Both components of earthquakes travel slower in less rigid material, such as sediments. P-waves have speeds of 4 to 7 km/s, and S-waves correspondingly range in speed from 2 to 5 km/s, both being faster in more rigid material. The P-wave gets progressively farther ahead of the S-wave as they travel through Earth's crust. The time between the P- and S-waves is routinely used to determine the distance to their source, the epicenter of the earthquake.

The speed of sound is affected by temperature in a given medium. For air at sea level, the speed of sound is given by

$$v_{\text{w}} = (331\ \text{m/s})\sqrt{\frac{T}{273\ \text{K}}}, \tag{17.2}$$

where the temperature (denoted as T) is in units of kelvin. The speed of sound in gases is related to the average speed of particles in the gas, v_{rms} , and that

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}, \tag{17.3}$$

where k is the Boltzmann constant ($1.38\times10^{-23}\ \text{J/K}$) and m is the mass of each (identical) particle in the gas. So, it is reasonable that the speed of sound in air and other gases should depend on the square root of temperature. While not negligible, this is not a strong dependence. At 0°C , the speed of sound is 331 m/s, whereas at 20.0°C it is 343 m/s, less than a 4% increase. **Figure 17.11** shows a use of the speed of sound by a bat to sense distances. Echoes are also used in medical imaging.

Figure 17.11 A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance.

One of the more important properties of sound is that its speed is nearly independent of frequency. This independence is certainly true in open air for sounds in the audible range of 20 to 20,000 Hz. If this independence were not true, you would certainly notice it for music played by a marching band in a football stadium, for example. Suppose that high-frequency sounds traveled faster—then the farther you were from the band, the more the sound from the low-pitch instruments would lag that from the high-pitch ones. But the music from all instruments arrives in cadence independent of distance, and so all frequencies must travel at nearly the same speed. Recall that

$$v_{\mathrm{w}} = f\lambda. \tag{17.4}$$

In a given medium under fixed conditions, v_{w} is constant, so that there is a relationship between f and λ; the higher the frequency, the smaller the wavelength. See **Figure 17.12** and consider the following example.

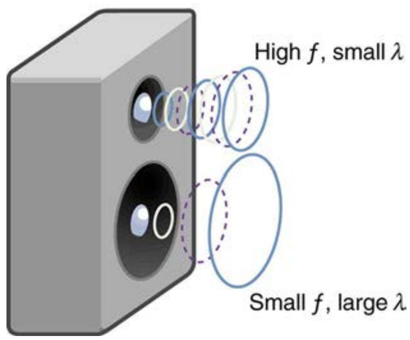


Figure 17.12 Because they travel at the same speed in a given medium, low-frequency sounds must have a greater wavelength than high-frequency sounds. Here, the lower-frequency sounds are emitted by the large speaker, called a woofer, while the higher-frequency sounds are emitted by the small speaker, called a tweeter.

Example 17.1 Calculating Wavelengths: What Are the Wavelengths of Audible Sounds?

Calculate the wavelengths of sounds at the extremes of the audible range, 20 and 20,000 Hz, in $30.0^{\circ}\mathrm{C}$ air. (Assume that the frequency values are accurate to two significant figures.)

Strategy

To find wavelength from frequency, we can use $v_{\mathrm{w}} = f\lambda$.

Solution

1. Identify knowns. The value for v_{w}, is given by

$$v_{\mathrm{w}} = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}}. \tag{17.5}$$

2. Convert the temperature into kelvin and then enter the temperature into the equation

$$v_{\mathrm{w}} = (331\text{ m/s})\sqrt{\frac{303\text{ K}}{273\text{ K}}} = 348.7\text{ m/s}. \tag{17.6}$$

3. Solve the relationship between speed and wavelength for λ:

$$\lambda = \frac{v_{\mathrm{w}}}{f}. \tag{17.7}$$

4. Enter the speed and the minimum frequency to give the maximum wavelength:

$$\lambda_{\max} = \frac{348.7\text{ m/s}}{20\text{ Hz}} = 17\text{ m}. \tag{17.8}$$

5. Enter the speed and the maximum frequency to give the minimum wavelength:

$$\lambda_{\text{min}} = \frac{348.7\text{ m/s}}{20,000\text{ Hz}} = 0.017\text{ m} = 1.7\text{ cm.} \tag{17.9}$$

Discussion

Because the product of f multiplied by λ equals a constant, the smaller f is, the larger λ must be, and vice versa.

The speed of sound can change when sound travels from one medium to another. However, the frequency usually remains the same because it is like a driven oscillation and has the frequency of the original source. If v_{w} changes and f remains the same, then the wavelength λ must change. That is, because $v_{\text{w}} = f\lambda$, the higher the speed of a sound, the greater its wavelength for a given frequency.

Making Connections: Take-Home Investigation—Voice as a Sound Wave

Suspend a sheet of paper so that the top edge of the paper is fixed and the bottom edge is free to move. You could tape the top edge of the paper to the edge of a table. Gently blow near the edge of the bottom of the sheet and note how the sheet moves. Speak softly and then louder such that the sounds hit the edge of the bottom of the paper, and note how the sheet moves. Explain the effects.

Check Your Understanding

Imagine you observe two fireworks explode. You hear the explosion of one as soon as you see it. However, you see the other firework for several milliseconds before you hear the explosion. Explain why this is so.

Solution

Sound and light both travel at definite speeds. The speed of sound is slower than the speed of light. The first firework is probably very close by, so the speed difference is not noticeable. The second firework is farther away, so the light arrives at your eyes noticeably sooner than the sound wave arrives at your ears.

Check Your Understanding

You observe two musical instruments that you cannot identify. One plays high-pitch sounds and the other plays low-pitch sounds. How could you determine which is which without hearing either of them play?

Solution

Compare their sizes. High-pitch instruments are generally smaller than low-pitch instruments because they generate a smaller wavelength.

17.3 Sound Intensity and Sound Level

Learning Objectives

By the end of this section, you will be able to:

- Define intensity, sound intensity, and sound pressure level.
- Calculate sound intensity levels in decibels (dB).

The information presented in this section supports the following AP® learning objectives and science practices:

- **6.A.4.1** The student is able to explain and/or predict qualitatively how the energy carried by a sound wave relates to the amplitude of the wave, and/or apply this concept to a real-world example. **(S.P. 6.4)**

Figure 17.13 Noise on crowded roadways like this one in Delhi makes it hard to hear others unless they shout. (credit: Lingaraj G J, Flickr)

In a quiet forest, you can sometimes hear a single leaf fall to the ground. After settling into bed, you may hear your blood pulsing through your ears. But when a passing motorist has his stereo turned up, you cannot even hear what the person next to you in your car is saying. We are all very familiar with the loudness of sounds and aware that they are related to how energetically the source is vibrating. In cartoons depicting a screaming person (or an animal making a loud noise), the cartoonist often shows an open mouth with a vibrating uvula, the hanging tissue at the back of the mouth, to suggest a loud sound coming from the throat **Figure 17.14**. High noise exposure is hazardous to hearing, and it is common for musicians to have hearing losses that are sufficiently severe that they interfere with the musicians' abilities to perform. The relevant physical quantity is sound intensity, a concept that is valid for all sounds whether or not they are in the audible range.

Intensity is defined to be the power per unit area carried by a wave. Power is the rate at which energy is transferred by the wave. In equation form, **intensity** I is

$$I = \frac{P}{A}, \tag{17.10}$$

where P is the power through an area A. The SI unit for I is $\mathrm{W/m^2}$. The intensity of a sound wave is related to its amplitude squared by the following relationship:

$$I = \frac{(\Delta p)^2}{2\rho v_\mathrm{w}}. \tag{17.11}$$

Here Δp is the pressure variation or pressure amplitude (half the difference between the maximum and minimum pressure in the sound wave) in units of pascals (Pa) or $\mathrm{N/m^2}$. (We are using a lower case p for pressure to distinguish it from power, denoted by P above.) The energy (as kinetic energy $\frac{mv^2}{2}$) of an oscillating element of air due to a traveling sound wave is proportional to its amplitude squared. In this equation, ρ is the density of the material in which the sound wave travels, in units of $\mathrm{kg/m^3}$, and v_w is the speed of sound in the medium, in units of m/s. The pressure variation is proportional to the amplitude of the oscillation, and so I varies as $(\Delta p)^2$ (**Figure 17.14**). This relationship is consistent with the fact that the sound wave is produced by some vibration; the greater its pressure amplitude, the more the air is compressed in the sound it creates.

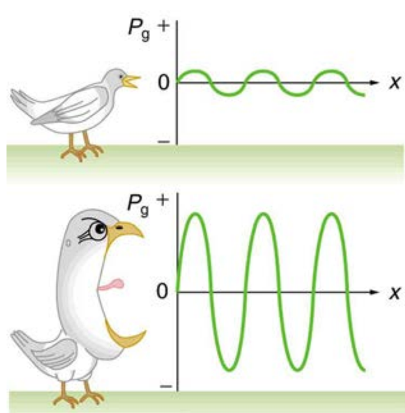


Figure 17.14 Graphs of the gauge pressures in two sound waves of different intensities. The more intense sound is produced by a source that has larger-amplitude oscillations and has greater pressure maxima and minima. Because pressures are higher in the greater-intensity sound, it can exert larger forces on the objects it encounters.

Sound intensity levels are quoted in decibels (dB) much more often than sound intensities in watts per meter squared. Decibels are the unit of choice in the scientific literature as well as in the popular media. The reasons for this choice of units are related to how we perceive sounds. How our ears perceive sound can be more accurately described by the logarithm of the intensity rather than directly to the intensity. The **sound intensity level** β in decibels of a sound having an intensity I in watts per meter squared is defined to be

$$\beta\,(\text{dB}) = 10\,\log_{10}\left(\frac{I}{I_0}\right), \tag{17.12}$$

where $I_0 = 10^{-12}\ \text{W/m}^2$ is a reference intensity. In particular, I_0 is the lowest or threshold intensity of sound a person with normal hearing can perceive at a frequency of 1000 Hz. Sound intensity level is not the same as intensity. Because β is defined in terms of a ratio, it is a unitless quantity telling you the *level* of the sound relative to a fixed standard ($10^{-12}\ \text{W/m}^2$, in this case). The units of decibels (dB) are used to indicate this ratio is multiplied by 10 in its definition. The bel, upon which the decibel is based, is named for Alexander Graham Bell, the inventor of the telephone.

Table 17.5 Sound Intensity Levels and Intensities

Sound intensity level β (dB)	Intensity I(W/m^2)	Example/effect
0	1×10^{-12}	Threshold of hearing at 1000 Hz
10	1×10^{-11}	Rustle of leaves
20	1×10^{-10}	Whisper at 1 m distance
30	1×10^{-9}	Quiet home
40	1×10^{-8}	Average home
50	1×10^{-7}	Average office, soft music
60	1×10^{-6}	Normal conversation
70	1×10^{-5}	Noisy office, busy traffic
80	1×10^{-4}	Loud radio, classroom lecture
90	1×10^{-3}	Inside a heavy truck; damage from prolonged exposure[1]
100	1×10^{-2}	Noisy factory, siren at 30 m; damage from 8 h per day exposure
110	1×10^{-1}	Damage from 30 min per day exposure
120	1	Loud rock concert, pneumatic chipper at 2 m; threshold of pain
140	1×10^{2}	Jet airplane at 30 m; severe pain, damage in seconds
160	1×10^{4}	Bursting of eardrums

The decibel level of a sound having the threshold intensity of $10^{-12}\ \text{W/m}^2$ is $\beta = 0\ \text{dB}$, because $\log_{10} 1 = 0$. That is, the threshold of hearing is 0 decibels. **Table 17.5** gives levels in decibels and intensities in watts per meter squared for some familiar sounds.

One of the more striking things about the intensities in **Table 17.5** is that the intensity in watts per meter squared is quite small for most sounds. The ear is sensitive to as little as a trillionth of a watt per meter squared—even more impressive when you realize that the area of the eardrum is only about $1\ \text{cm}^2$, so that only 10^{-16} W falls on it at the threshold of hearing! Air molecules in a sound wave of this intensity vibrate over a distance of less than one molecular diameter, and the gauge pressures involved are less than 10^{-9} atm.

1. Several government agencies and health-related professional associations recommend that 85 dB not be exceeded for 8-hour daily exposures in the absence of hearing protection.

Another impressive feature of the sounds in **Table 17.5** is their numerical range. Sound intensity varies by a factor of 10^{12} from threshold to a sound that causes damage in seconds. You are unaware of this tremendous range in sound intensity because how your ears respond can be described approximately as the logarithm of intensity. Thus, sound intensity levels in decibels fit your experience better than intensities in watts per meter squared. The decibel scale is also easier to relate to because most people are more accustomed to dealing with numbers such as 0, 53, or 120 than numbers such as 1.00×10^{-11}.

One more observation readily verified by examining **Table 17.5** or using $I = \frac{(\Delta p)^2}{2\rho v_\mathrm{w}}$ is that each factor of 10 in intensity corresponds to 10 dB. For example, a 90 dB sound compared with a 60 dB sound is 30 dB greater, or three factors of 10 (that is, 10^3 times) as intense. Another example is that if one sound is 10^7 as intense as another, it is 70 dB higher. See **Table 17.6**.

Table 17.6 Ratios of Intensities and Corresponding Differences in Sound Intensity Levels

I_2 / I_1	$\beta_2 - \beta_1$
2.0	3.0 dB
5.0	7.0 dB
10.0	10.0 dB

Example 17.2 Calculating Sound Intensity Levels: Sound Waves

Calculate the sound intensity level in decibels for a sound wave traveling in air at $0^\circ\mathrm{C}$ and having a pressure amplitude of 0.656 Pa.

Strategy

We are given Δp, so we can calculate I using the equation $I = (\Delta p)^2 / (2pv_\mathrm{w})^2$. Using I, we can calculate β straight from its definition in $\beta\,(\mathrm{dB}) = 10\,\log_{10}(I / I_0)$.

Solution

(1) Identify knowns:

Sound travels at 331 m/s in air at $0^\circ\mathrm{C}$.

Air has a density of $1.29\ \mathrm{kg/m^3}$ at atmospheric pressure and $0^\circ\mathrm{C}$.

(2) Enter these values and the pressure amplitude into $I = (\Delta p)^2 / (2\rho v_\mathrm{w})$:

$$I = \frac{(\Delta p)^2}{2\rho v_\mathrm{w}} = \frac{(0.656\ \mathrm{Pa})^2}{2\left(1.29\ \mathrm{kg/m^3}\right)(331\ \mathrm{m/s})} = 5.04\times10^{-4}\ \mathrm{W/m^2}. \tag{17.13}$$

(3) Enter the value for I and the known value for I_0 into $\beta\,(\mathrm{dB}) = 10\,\log_{10}(I / I_0)$. Calculate to find the sound intensity level in decibels:

$$10\,\log_{10}\left(5.04\times10^8\right) = 10\,(8.70)\ \mathrm{dB} = 87\ \mathrm{dB}. \tag{17.14}$$

Discussion

This 87 dB sound has an intensity five times as great as an 80 dB sound. So a factor of five in intensity corresponds to a difference of 7 dB in sound intensity level. This value is true for any intensities differing by a factor of five.

Example 17.3 Change Intensity Levels of a Sound: What Happens to the Decibel Level?

Show that if one sound is twice as intense as another, it has a sound level about 3 dB higher.

Strategy

You are given that the ratio of two intensities is 2 to 1, and are then asked to find the difference in their sound levels in decibels. You can solve this problem using of the properties of logarithms.

Solution

(1) Identify knowns:

The ratio of the two intensities is 2 to 1, or:

$$\frac{I_2}{I_1} = 2.00. \tag{17.15}$$

We wish to show that the difference in sound levels is about 3 dB. That is, we want to show:

$$\beta_2 - \beta_1 = 3 \text{ dB}. \tag{17.16}$$

Note that:

$$\log_{10} b - \log_{10} a = \log_{10}\left(\frac{b}{a}\right). \tag{17.17}$$

(2) Use the definition of β to get:

$$\beta_2 - \beta_1 = 10 \log_{10}\left(\frac{I_2}{I_1}\right) = 10 \log_{10} 2.00 = 10\ (0.301) \text{ dB}. \tag{17.18}$$

Thus,

$$\beta_2 - \beta_1 = 3.01 \text{ dB}. \tag{17.19}$$

Discussion

This means that the two sound intensity levels differ by 3.01 dB, or about 3 dB, as advertised. Note that because only the ratio I_2/I_1 is given (and not the actual intensities), this result is true for any intensities that differ by a factor of two. For example, a 56.0 dB sound is twice as intense as a 53.0 dB sound, a 97.0 dB sound is half as intense as a 100 dB sound, and so on.

It should be noted at this point that there is another decibel scale in use, called the **sound pressure level**, based on the ratio of the pressure amplitude to a reference pressure. This scale is used particularly in applications where sound travels in water. It is beyond the scope of most introductory texts to treat this scale because it is not commonly used for sounds in air, but it is important to note that very different decibel levels may be encountered when sound pressure levels are quoted. For example, ocean noise pollution produced by ships may be as great as 200 dB expressed in the sound pressure level, where the more familiar sound intensity level we use here would be something under 140 dB for the same sound.

Take-Home Investigation: Feeling Sound

Find a CD player and a CD that has rock music. Place the player on a light table, insert the CD into the player, and start playing the CD. Place your hand gently on the table next to the speakers. Increase the volume and note the level when the table just begins to vibrate as the rock music plays. Increase the reading on the volume control until it doubles. What has happened to the vibrations?

Check Your Understanding

Describe how amplitude is related to the loudness of a sound.

Solution

Amplitude is directly proportional to the experience of loudness. As amplitude increases, loudness increases.

Check Your Understanding

Identify common sounds at the levels of 10 dB, 50 dB, and 100 dB.

Solution

10 dB: Running fingers through your hair.

50 dB: Inside a quiet home with no television or radio.

100 dB: Take-off of a jet plane.

17.4 Doppler Effect and Sonic Booms

Learning Objectives

By the end of this section, you will be able to:

- Define Doppler effect, Doppler shift, and sonic boom.
- Calculate the frequency of a sound heard by someone observing Doppler shift.
- Describe the sounds produced by objects moving faster than the speed of sound.

The information presented in this section supports the following AP® learning objectives and science practices:

- **6.B.5.1** The student is able to create or use a wave front diagram to demonstrate or interpret qualitatively the observed frequency of a wave, dependent upon relative motions of source and observer. **(S.P. 1.4)**

The characteristic sound of a motorcycle buzzing by is an example of the **Doppler effect**. The high-pitch scream shifts dramatically to a lower-pitch roar as the motorcycle passes by a stationary observer. The closer the motorcycle brushes by, the more abrupt the shift. The faster the motorcycle moves, the greater the shift. We also hear this characteristic shift in frequency for passing race cars, airplanes, and trains. It is so familiar that it is used to imply motion and children often mimic it in play.

The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer. Although less familiar, this effect is easily noticed for a stationary source and moving observer. For example, if you ride a train past a stationary warning bell, you will hear the bell's frequency shift from high to low as you pass by. The actual change in frequency due to relative motion of source and observer is called a **Doppler shift**. The Doppler effect and Doppler shift are named for the Austrian physicist and mathematician Christian Johann Doppler (1803–1853), who did experiments with both moving sources and moving observers. Doppler, for example, had musicians play on a moving open train car and also play standing next to the train tracks as a train passed by. Their music was observed both on and off the train, and changes in frequency were measured.

What causes the Doppler shift? **Figure 17.15**, **Figure 17.16**, and **Figure 17.17** compare sound waves emitted by stationary and moving sources in a stationary air mass. Each disturbance spreads out spherically from the point where the sound was emitted. If the source is stationary, then all of the spheres representing the air compressions in the sound wave centered on the same point, and the stationary observers on either side see the same wavelength and frequency as emitted by the source, as in **Figure 17.15**. If the source is moving, as in **Figure 17.16**, then the situation is different. Each compression of the air moves out in a sphere from the point where it was emitted, but the point of emission moves. This moving emission point causes the air compressions to be closer together on one side and farther apart on the other. Thus, the wavelength is shorter in the direction the source is moving (on the right in **Figure 17.16**), and longer in the opposite direction (on the left in **Figure 17.16**). Finally, if the observers move, as in **Figure 17.17**, the frequency at which they receive the compressions changes. The observer moving toward the source receives them at a higher frequency, and the person moving away from the source receives them at a lower frequency.

Figure 17.15 Sounds emitted by a source spread out in spherical waves. Because the source, observers, and air are stationary, the wavelength and frequency are the same in all directions and to all observers.

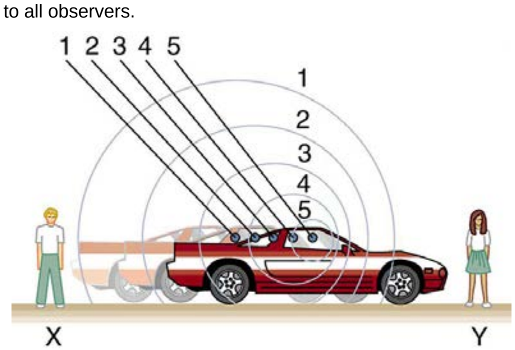


Figure 17.16 Sounds emitted by a source moving to the right spread out from the points at which they were emitted. The wavelength is reduced and, consequently, the frequency is increased in the direction of motion, so that the observer on the right hears a higher-pitch sound. The opposite is true for the observer on the left, where the wavelength is increased and the frequency is reduced.

Figure 17.17 The same effect is produced when the observers move relative to the source. Motion toward the source increases frequency as the observer on the right passes through more wave crests than she would if stationary. Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests than he would if stationary.

We know that wavelength and frequency are related by $v_{\text{w}} = f\lambda$, where v_{w} is the fixed speed of sound. The sound moves in a medium and has the same speed v_{w} in that medium whether the source is moving or not. Thus f multiplied by λ is a constant. Because the observer on the right in **Figure 17.16** receives a shorter wavelength, the frequency she receives must be higher. Similarly, the observer on the left receives a longer wavelength, and hence he hears a lower frequency. The same thing happens in **Figure 17.17**. A higher frequency is received by the observer moving toward the source, and a lower frequency is received by an observer moving away from the source. In general, then, relative motion of source and observer toward one another increases the received frequency. Relative motion apart decreases frequency. The greater the relative speed is, the greater the effect.

The Doppler Effect

The Doppler effect occurs not only for sound but for any wave when there is relative motion between the observer and the source. There are Doppler shifts in the frequency of sound, light, and water waves, for example. Doppler shifts can be used to determine velocity, such as when ultrasound is reflected from blood in a medical diagnostic. The recession of galaxies is determined by the shift in the frequencies of light received from them and has implied much about the origins of the universe. Modern physics has been profoundly affected by observations of Doppler shifts.

For a stationary observer and a moving source, the frequency f_{obs} received by the observer can be shown to be

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right), \tag{17.20}$$

where f_{s} is the frequency of the source, v_{s} is the speed of the source along a line joining the source and observer, and v_{w} is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away from the observer, producing the appropriate shifts up and down in frequency. Note that the greater the speed of the source, the greater the effect. Similarly, for a stationary source and moving observer, the frequency received by the observer f_{obs} is given by

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}} \pm v_{\text{obs}}}{v_{\text{w}}}\right), \tag{17.21}$$

where v_{obs} is the speed of the observer along a line joining the source and observer. Here the plus sign is for motion toward the source, and the minus is for motion away from the source.

Example 17.4 Calculate Doppler Shift: A Train Horn

Suppose a train that has a 150-Hz horn is moving at 35.0 m/s in still air on a day when the speed of sound is 340 m/s.

(a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes?

(b) What frequency is observed by the train's engineer traveling on the train?

Strategy

To find the observed frequency in (a), $f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right)$, must be used because the source is moving. The minus sign is used for the approaching train, and the plus sign for the receding train. In (b), there are two Doppler shifts—one for a moving source and the other for a moving observer.

Solution for (a)

(1) Enter known values into $f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} - v_{\text{s}}}\right)$.

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} - v_{\text{s}}}\right) = (150\text{ Hz})\left(\frac{340\text{ m/s}}{340\text{ m/s} - 35.0\text{ m/s}}\right) \tag{17.22}$$

(2) Calculate the frequency observed by a stationary person as the train approaches.

$$f_{\text{obs}} = (150\text{ Hz})(1.11) = 167\text{ Hz} \tag{17.23}$$

(3) Use the same equation with the plus sign to find the frequency heard by a stationary person as the train recedes.

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} + v_{\text{s}}}\right) = (150\text{ Hz})\left(\frac{340\text{ m/s}}{340\text{ m/s} + 35.0\text{ m/s}}\right) \tag{17.24}$$

(4) Calculate the second frequency.

$$f_{\text{obs}} = (150\text{ Hz})(0.907) = 136\text{ Hz} \tag{17.25}$$

Discussion on (a)

The numbers calculated are valid when the train is far enough away that the motion is nearly along the line joining train and observer. In both cases, the shift is significant and easily noticed. Note that the shift is 17.0 Hz for motion toward and 14.0 Hz for motion away. The shifts are not symmetric.

Solution for (b)

(1) Identify knowns:

- It seems reasonable that the engineer would receive the same frequency as emitted by the horn, because the relative velocity between them is zero.
- Relative to the medium (air), the speeds are $v_{\text{s}} = v_{\text{obs}} = 35.0\text{ m/s}$.
- The first Doppler shift is for the moving observer; the second is for the moving source.

(2) Use the following equation:

$$f_{\text{obs}} = \left[f_{\text{s}}\left(\frac{v_{\text{w}} \pm v_{\text{obs}}}{v_{\text{w}}}\right)\right]\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right). \tag{17.26}$$

The quantity in the square brackets is the Doppler-shifted frequency due to a moving observer. The factor on the right is the effect of the moving source.

(3) Because the train engineer is moving in the direction toward the horn, we must use the plus sign for v_{obs}; however, because the horn is also moving in the direction away from the engineer, we also use the plus sign for v_{s}. But the train is carrying both the engineer and the horn at the same velocity, so $v_{\text{s}} = v_{\text{obs}}$. As a result, everything but f_{s} cancels, yielding

$$f_{\text{obs}} = f_{\text{s}}. \tag{17.27}$$

Discussion for (b)

We may expect that there is no change in frequency when source and observer move together because it fits your experience. For example, there is no Doppler shift in the frequency of conversations between driver and passenger on a motorcycle. People talking when a wind moves the air between them also observe no Doppler shift in their conversation. The crucial point is that source and observer are not moving relative to each other.

Sonic Booms to Bow Wakes

What happens to the sound produced by a moving source, such as a jet airplane, that approaches or even exceeds the speed of sound? The answer to this question applies not only to sound but to all other waves as well.

Suppose a jet airplane is coming nearly straight at you, emitting a sound of frequency f_{s}. The greater the plane's speed v_{s}, the greater the Doppler shift and the greater the value observed for f_{obs}. Now, as v_{s} approaches the speed of sound, f_{obs} approaches infinity, because the denominator in $f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right)$ approaches zero. At the speed of sound, this result means that in front of the source, each successive wave is superimposed on the previous one because the source moves forward at the speed of sound. The observer gets them all at the same instant, and so the frequency is infinite. (Before airplanes exceeded the speed of sound, some people argued it would be impossible because such constructive superposition would produce pressures great enough to destroy the airplane.) If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the approaching source are mixed with those from it when receding. This mixing appears messy, but something interesting happens—a sonic boom is created. (See **Figure 17.18**.)

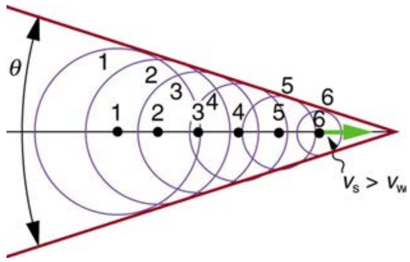


Figure 17.18 Sound waves from a source that moves faster than the speed of sound spread spherically from the point where they are emitted, but the source moves ahead of each. Constructive interference along the lines shown (actually a cone in three dimensions) creates a shock wave called a sonic boom. The faster the speed of the source, the smaller the angle θ.

There is constructive interference along the lines shown (a cone in three dimensions) from similar sound waves arriving there simultaneously. This superposition forms a disturbance called a **sonic boom**, a constructive interference of sound created by an object moving faster than sound. Inside the cone, the interference is mostly destructive, and so the sound intensity there is much less than on the shock wave. An aircraft creates two sonic booms, one from its nose and one from its tail. (See **Figure 17.19**.) During television coverage of space shuttle landings, two distinct booms could often be heard. These were separated by exactly the time it would take the shuttle to pass by a point. Observers on the ground often do not see the aircraft creating the sonic boom, because it has passed by before the shock wave reaches them, as seen in **Figure 17.19**. If the aircraft flies close by at low altitude, pressures in the sonic boom can be destructive and break windows as well as rattle nerves. Because of how destructive sonic booms can be, supersonic flights are banned over populated areas of the United States.

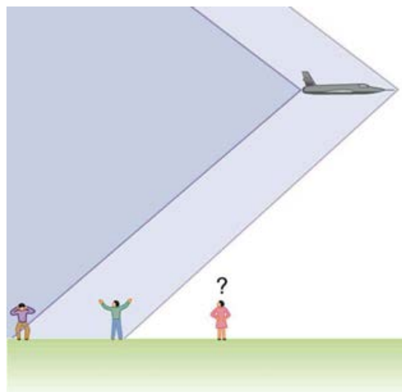


Figure 17.19 Two sonic booms, created by the nose and tail of an aircraft, are observed on the ground after the plane has passed by.

Sonic booms are one example of a broader phenomenon called bow wakes. A **bow wake**, such as the one in **Figure 17.20**, is created when the wave source moves faster than the wave propagation speed. Water waves spread out in circles from the point where created, and the bow wake is the familiar V-shaped wake trailing the source. A more exotic bow wake is created when a subatomic particle travels through a medium faster than the speed of light travels in that medium. (In a vacuum, the maximum speed of light will be $c = 3.00\times10^8\text{ m/s}$; in the medium of water, the speed of light is closer to $0.75c$. If the particle creates light in its passage, that light spreads on a cone with an angle indicative of the speed of the particle, as illustrated in **Figure 17.21**. Such a bow wake is called Cerenkov radiation and is commonly observed in particle physics.

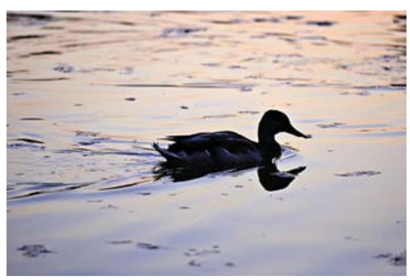


Figure 17.20 Bow wake created by a duck. Constructive interference produces the rather structured wake, while there is relatively little wave action inside the wake, where interference is mostly destructive. (credit: Horia Varlan, Flickr)

Figure 17.21 The blue glow in this research reactor pool is Cerenkov radiation caused by subatomic particles traveling faster than the speed of light in water. (credit: U.S. Nuclear Regulatory Commission)

Doppler shifts and sonic booms are interesting sound phenomena that occur in all types of waves. They can be of considerable use. For example, the Doppler shift in ultrasound can be used to measure blood velocity, while police use the Doppler shift in radar (a microwave) to measure car velocities. In meteorology, the Doppler shift is used to track the motion of storm clouds; such “Doppler Radar” can give velocity and direction and rain or snow potential of imposing weather fronts. In astronomy, we can examine the light emitted from distant galaxies and determine their speed relative to ours. As galaxies move away from us, their light is shifted to a lower frequency, and so to a longer wavelength—the so-called red shift. Such information from galaxies far, far away has allowed us to estimate the age of the universe (from the Big Bang) as about 14 billion years.

Check Your Understanding

Why did scientist Christian Doppler observe musicians both on a moving train and also from a stationary point not on the train?

Solution

Doppler needed to compare the perception of sound when the observer is stationary and the sound source moves, as well as when the sound source and the observer are both in motion.

Check Your Understanding

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Solution

If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and let the ambulance through.

17.5 Sound Interference and Resonance: Standing Waves in Air Columns

Learning Objectives

By the end of this section, you will be able to:

- Define antinode, node, fundamental, overtones, and harmonics.
- Identify instances of sound interference in everyday situations.
- Describe how sound interference occurring inside open and closed tubes changes the characteristics of the sound, and how this applies to sounds produced by musical instruments.
- Calculate the length of a tube using sound wave measurements.

The information presented in this section supports the following AP® learning objectives and science practices:

- **6.D.1.1** The student is able to use representations of individual pulses and construct representations to model the interaction of two wave pulses to analyze the superposition of two pulses. **(S.P. 1.1, 1.4)**
- **6.D.1.2** The student is able to design a suitable experiment and analyze data illustrating the superposition of mechanical waves (only for wave pulses or standing waves). **(S.P. 4.2, 5.1)**
- **6.D.1.3** The student is able to design a plan for collecting data to quantify the amplitude variations when two or more traveling waves or wave pulses interact in a given medium. **(S.P. 4.2)**

- **6.D.3.1** The student is able to refine a scientific question related to standing waves and design a detailed plan for the experiment that can be conducted to examine the phenomenon qualitatively or quantitatively. **(S.P. 2.1, 2.2, 4.2)**
- **6.D.3.2** The student is able to predict properties of standing waves that result from the addition of incident and reflected waves that are confined to a region and have nodes and antinodes. **(S.P. 6.4)**
- **6.D.3.3** The student is able to plan data collection strategies, predict the outcome based on the relationship under test, perform data analysis, evaluate evidence compared to the prediction, explain any discrepancy and, if necessary, revise the relationship among variables responsible for establishing standing waves on a string or in a column of air. **(S.P. 3.2, 4.1, 5.1, 5.2, 5.3)**
- **6.D.3.4** The student is able to describe representations and models of situations in which standing waves result from the addition of incident and reflected waves confined to a region. **(S.P. 1.2)**
- **6.D.4.2** The student is able to calculate wavelengths and frequencies (if given wave speed) of standing waves based on boundary conditions and length of region within which the wave is confined, and calculate numerical values of wavelengths and frequencies. Examples should include musical instruments. **(S.P. 2.2)**

Figure 17.22 Some types of headphones use the phenomena of constructive and destructive interference to cancel out outside noises. (credit: JVC America, Flickr)

Interference is the hallmark of waves, all of which exhibit constructive and destructive interference exactly analogous to that seen for water waves. In fact, one way to prove something "is a wave" is to observe interference effects. So, sound being a wave, we expect it to exhibit interference; we have already mentioned a few such effects, such as the beats from two similar notes played simultaneously.

Figure 17.23 shows a clever use of sound interference to cancel noise. Larger-scale applications of active noise reduction by destructive interference are contemplated for entire passenger compartments in commercial aircraft. To obtain destructive interference, a fast electronic analysis is performed, and a second sound is introduced with its maxima and minima exactly reversed from the incoming noise. Sound waves in fluids are pressure waves and consistent with Pascal's principle; pressures from two different sources add and subtract like simple numbers; that is, positive and negative gauge pressures add to a much smaller pressure, producing a lower-intensity sound. Although completely destructive interference is possible only under the simplest conditions, it is possible to reduce noise levels by 30 dB or more using this technique.

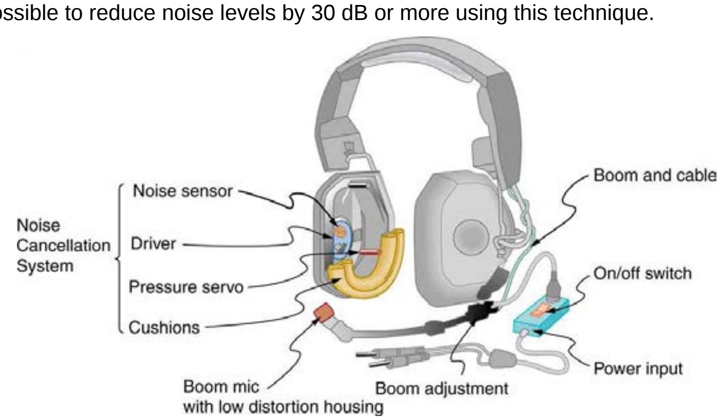


Figure 17.23 Headphones designed to cancel noise with destructive interference create a sound wave exactly opposite to the incoming sound. These headphones can be more effective than the simple passive attenuation used in most ear protection. Such headphones were used on the record-setting, around the world nonstop flight of the Voyager aircraft to protect the pilots' hearing from engine noise.

Where else can we observe sound interference? All sound resonances, such as in musical instruments, are due to constructive and destructive interference. Only the resonant frequencies interfere constructively to form standing waves, while others interfere destructively and are absent. From the toot made by blowing over a bottle, to the characteristic flavor of a violin's sounding box, to the recognizability of a great singer's voice, resonance and standing waves play a vital role.

Interference

Interference is such a fundamental aspect of waves that observing interference is proof that something is a wave. The wave nature of light was established by experiments showing interference. Similarly, when electrons scattered from crystals exhibited interference, their wave nature was confirmed to be exactly as predicted by symmetry with certain wave characteristics of light.

Applying the Science Practices: Standing Wave

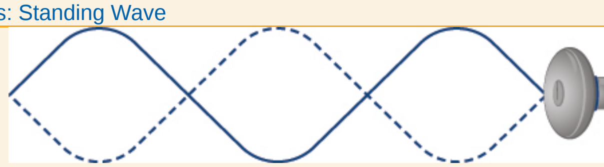


Figure 17.24 The standing wave pattern of a rubber tube attached to a doorknob.

Tie one end of a strip of long rubber tubing to a stable object (doorknob, fence post, etc.) and shake the other end up and down until a standing wave pattern is achieved. Devise a method to determine the frequency and wavelength generated by your arm shaking. Do your results align with the equation? Do you find that the velocity of the wave generated is consistent for each trial? If not, explain why this is the case.

Answer

This task will likely require two people. The frequency of the wave pattern can be found by timing how long it takes the student shaking the rubber tubing to move his or her hand up and down one full time. (It may be beneficial to time how long it takes the student to do this ten times, and then divide by ten to reduce error.) The wavelength of the standing wave can be measured with a meter stick by measuring the distance between two nodes and multiplying by two. This information should be gathered for standing wave patterns of multiple different wavelengths. As students collect their data, they can use the equation to determine if the wave velocity is consistent. There will likely be some error in the experiment yielding velocities of slightly different value. This error is probably due to an inaccuracy in the wavelength and/or frequency measurements.

Suppose we hold a tuning fork near the end of a tube that is closed at the other end, as shown in **Figure 17.25**, **Figure 17.26**, **Figure 17.27**, and **Figure 17.28**. If the tuning fork has just the right frequency, the air column in the tube resonates loudly, but at most frequencies it vibrates very little. This observation just means that the air column has only certain natural frequencies. The figures show how a resonance at the lowest of these natural frequencies is formed. A disturbance travels down the tube at the speed of sound and bounces off the closed end. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly half a cycle later, and it interferes constructively with the continuing sound produced by the tuning fork. The incoming and reflected sounds form a standing wave in the tube as shown.

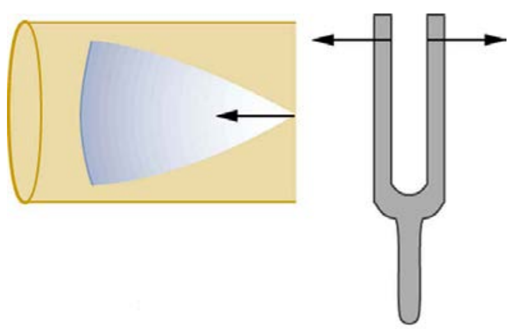


Figure 17.25 Resonance of air in a tube closed at one end, caused by a tuning fork. A disturbance moves down the tube.

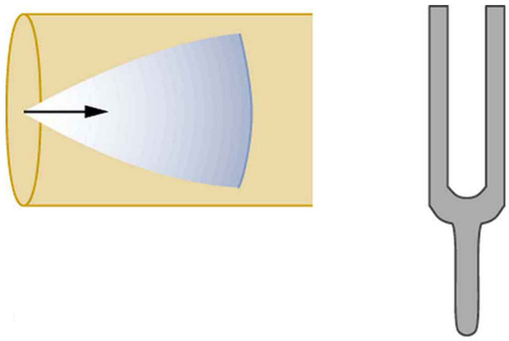


Figure 17.26 Resonance of air in a tube closed at one end, caused by a tuning fork. The disturbance reflects from the closed end of the tube.

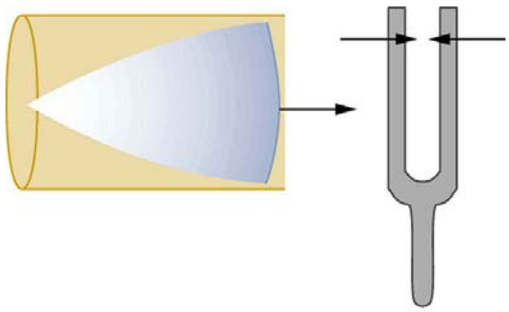


Figure 17.27 Resonance of air in a tube closed at one end, caused by a tuning fork. If the length of the tube L is just right, the disturbance gets back to the tuning fork half a cycle later and interferes constructively with the continuing sound from the tuning fork. This interference forms a standing wave, and the air column resonates.

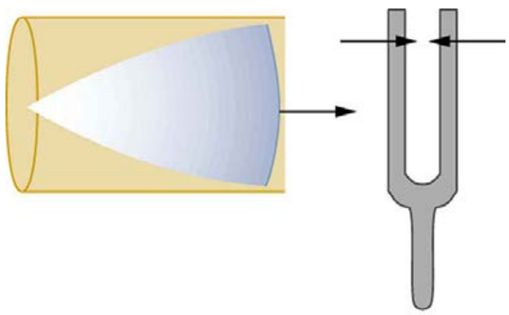


Figure 17.28 Resonance of air in a tube closed at one end, caused by a tuning fork. A graph of air displacement along the length of the tube shows none at the closed end, where the motion is constrained, and a maximum at the open end. This standing wave has one-fourth of its wavelength in the tube, so that $\lambda = 4L$.

The standing wave formed in the tube has its maximum air displacement (an **antinode**) at the open end, where motion is unconstrained, and no displacement (a **node**) at the closed end, where air movement is halted. The distance from a node to an antinode is one-fourth of a wavelength, and this equals the length of the tube; thus, $\lambda = 4L$. This same resonance can be produced by a vibration introduced at or near the closed end of the tube, as shown in Figure 17.29. It is best to consider this a natural vibration of the air column independently of how it is induced.

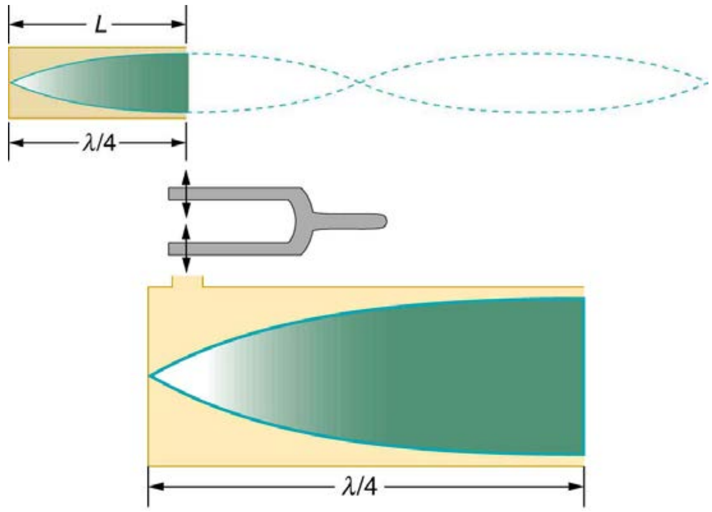


Figure 17.29 The same standing wave is created in the tube by a vibration introduced near its closed end.

Given that maximum air displacements are possible at the open end and none at the closed end, there are other, shorter wavelengths that can resonate in the tube, such as the one shown in **Figure 17.30**. Here the standing wave has three-fourths of its wavelength in the tube, or $L = (3/4)\lambda'$, so that $\lambda' = 4L/3$. Continuing this process reveals a whole series of shorter-wavelength and higher-frequency sounds that resonate in the tube. We use specific terms for the resonances in any system. The lowest resonant frequency is called the **fundamental**, while all higher resonant frequencies are called **overtones**. All resonant frequencies are integral multiples of the fundamental, and they are collectively called **harmonics**. The fundamental is the first harmonic, the first overtone is the second harmonic, and so on. **Figure 17.31** shows the fundamental and the first three overtones (the first four harmonics) in a tube closed at one end.

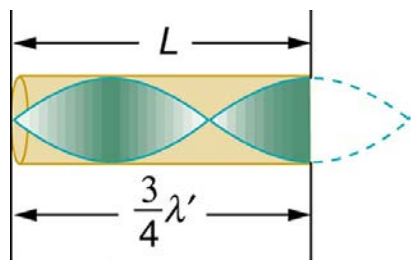


Figure 17.30 Another resonance for a tube closed at one end. This has maximum air displacements at the open end, and none at the closed end. The wavelength is shorter, with three-fourths λ' equaling the length of the tube, so that $\lambda' = 4L/3$. This higher-frequency vibration is the first overtone.

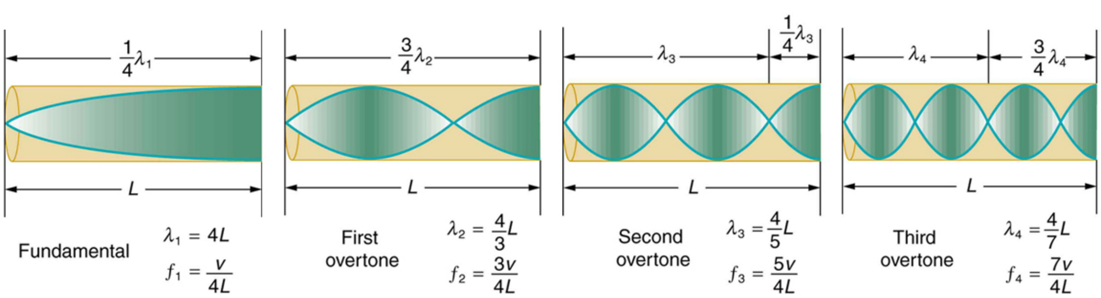


Figure 17.31 The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

The fundamental and overtones can be present simultaneously in a variety of combinations. For example, middle C on a trumpet has a sound distinctively different from middle C on a clarinet, both instruments being modified versions of a tube closed at one end. The fundamental frequency is the same (and usually the most intense), but the overtones and their mix of intensities are different and subject to shading by the musician. This mix is what gives various musical instruments (and human voices) their distinctive characteristics, whether they have air columns, strings, sounding boxes, or drumheads. In fact, much of our speech is determined by shaping the cavity formed by the throat and mouth and positioning the tongue to adjust the fundamental and combination of overtones. Simple resonant cavities can be made to resonate with the sound of the vowels, for example. (See **Figure 17.32**.) In boys, at puberty, the larynx grows and the shape of the resonant cavity changes giving rise to the difference in predominant frequencies in speech between men and women.

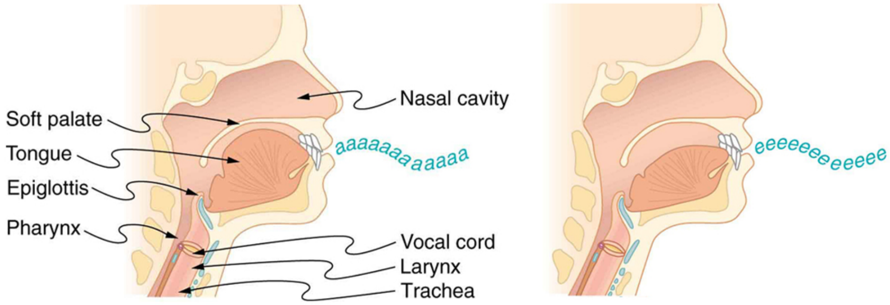


Figure 17.32 The throat and mouth form an air column closed at one end that resonates in response to vibrations in the voice box. The spectrum of overtones and their intensities vary with mouth shaping and tongue position to form different sounds. The voice box can be replaced with a mechanical vibrator, and understandable speech is still possible. Variations in basic shapes make different voices recognizable.

Now let us look for a pattern in the resonant frequencies for a simple tube that is closed at one end. The fundamental has $\lambda = 4L$, and frequency is related to wavelength and the speed of sound as given by:

$$v_{\text{w}} = f\lambda. \tag{17.28}$$

Solving for f in this equation gives

$$f = \frac{v_{\text{w}}}{\lambda} = \frac{v_{\text{w}}}{4L}, \tag{17.29}$$

where v_{w} is the speed of sound in air. Similarly, the first overtone has $\lambda' = 4L/3$ (see **Figure 17.31**), so that

$$f' = 3\frac{v_{\text{w}}}{4L} = 3f. \tag{17.30}$$

Because $f' = 3f$, we call the first overtone the third harmonic. Continuing this process, we see a pattern that can be generalized in a single expression. The resonant frequencies of a tube closed at one end are

$$f_n = n\frac{v_{\text{w}}}{4L},\ n = 1,3,5, \tag{17.31}$$

where f_1 is the fundamental, f_3 is the first overtone, and so on. It is interesting that the resonant frequencies depend on the speed of sound and, hence, on temperature. This dependence poses a noticeable problem for organs in old unheated cathedrals, and it is also the reason why musicians commonly bring their wind instruments to room temperature before playing them.

Example 17.5 Find the Length of a Tube with a 128 Hz Fundamental

(a) What length should a tube closed at one end have on a day when the air temperature, is 22.0°C , if its fundamental frequency is to be 128 Hz (C below middle C)?

(b) What is the frequency of its fourth overtone?

Strategy

The length L can be found from the relationship in $f_n = n\frac{v_{\text{w}}}{4L}$, but we will first need to find the speed of sound v_{w} .

Solution for (a)

(1) Identify knowns:

- the fundamental frequency is 128 Hz
- the air temperature is 22.0°C

(2) Use $f_n = n\frac{v_{\text{w}}}{4L}$ to find the fundamental frequency ($n = 1$).

$$f_1 = \frac{v_{\text{w}}}{4L} \tag{17.32}$$

(3) Solve this equation for length.

$$L = \frac{v_{\text{w}}}{4f_1} \tag{17.33}$$

(4) Find the speed of sound using $v_{\mathrm{w}} = (331\ \mathrm{m/s})\sqrt{\dfrac{T}{273\ \mathrm{K}}}$.

$$v_{\mathrm{w}} = (331\ \mathrm{m/s})\sqrt{\frac{295\ \mathrm{K}}{273\ \mathrm{K}}} = 344\ \mathrm{m/s} \tag{17.34}$$

(5) Enter the values of the speed of sound and frequency into the expression for L.

$$L = \frac{v_{\mathrm{w}}}{4f_1} = \frac{344\ \mathrm{m/s}}{4(128\ \mathrm{Hz})} = 0.672\ \mathrm{m} \tag{17.35}$$

Discussion on (a)

Many wind instruments are modified tubes that have finger holes, valves, and other devices for changing the length of the resonating air column and hence, the frequency of the note played. Horns producing very low frequencies, such as tubas, require tubes so long that they are coiled into loops.

Solution for (b)

(1) Identify knowns:

- the first overtone has $n = 3$
- the second overtone has $n = 5$
- the third overtone has $n = 7$
- the fourth overtone has $n = 9$

(2) Enter the value for the fourth overtone into $f_n = n\dfrac{v_{\mathrm{w}}}{4L}$.

$$f_9 = 9\frac{v_{\mathrm{w}}}{4L} = 9f_1 = 1.15\ \mathrm{kHz} \tag{17.36}$$

Discussion on (b)

Whether this overtone occurs in a simple tube or a musical instrument depends on how it is stimulated to vibrate and the details of its shape. The trombone, for example, does not produce its fundamental frequency and only makes overtones.

Another type of tube is one that is *open* at both ends. Examples are some organ pipes, flutes, and oboes. The resonances of tubes open at both ends can be analyzed in a very similar fashion to those for tubes closed at one end. The air columns in tubes open at both ends have maximum air displacements at both ends, as illustrated in **Figure 17.33**. Standing waves form as shown.

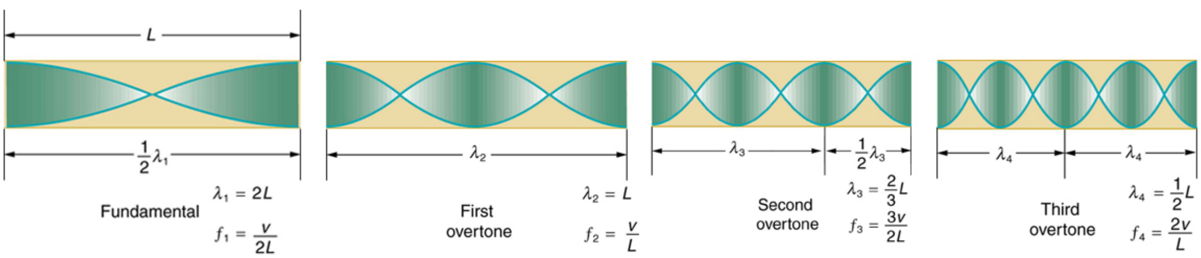


Figure 17.33 The resonant frequencies of a tube open at both ends are shown, including the fundamental and the first three overtones. In all cases the maximum air displacements occur at both ends of the tube, giving it different natural frequencies than a tube closed at one end.

Based on the fact that a tube open at both ends has maximum air displacements at both ends, and using **Figure 17.33** as a guide, we can see that the resonant frequencies of a tube open at both ends are:

$$f_n = n\frac{v_{\mathrm{w}}}{2L},\ \ n = 1, 2, 3..., \tag{17.37}$$

where f_1 is the fundamental, f_2 is the first overtone, f_3 is the second overtone, and so on. Note that a tube open at both ends has a fundamental frequency twice what it would have if closed at one end. It also has a different spectrum of overtones than a tube closed at one end. So if you had two tubes with the same fundamental frequency but one was open at both ends and the other was closed at one end, they would sound different when played because they have different overtones. Middle C, for example, would sound richer played on an open tube, because it has even multiples of the fundamental as well as odd. A closed tube has only odd multiples.

Applying the Science Practices: Closed- and Open-Ended Tubes

Strike an open-ended length of plastic pipe while holding it in the air. Now place one end of the pipe on a hard surface, sealing one opening, and strike it again. How does the sound change? Further investigate the sound created by the pipe by striking pipes of different lengths and composition.

Answer

When the pipe is placed on the ground, the standing wave within the pipe changes from being open on both ends to being closed on one end. As a result, the fundamental frequency will change from $f = \frac{v}{2L}$ to $f = \frac{v}{4L}$. This decrease in frequency results in a decrease in observed pitch.

Real-World Applications: Resonance in Everyday Systems

Resonance occurs in many different systems, including strings, air columns, and atoms. Resonance is the driven or forced oscillation of a system at its natural frequency. At resonance, energy is transferred rapidly to the oscillating system, and the amplitude of its oscillations grows until the system can no longer be described by Hooke's law. An example of this is the distorted sound intentionally produced in certain types of rock music.

Wind instruments use resonance in air columns to amplify tones made by lips or vibrating reeds. Other instruments also use air resonance in clever ways to amplify sound. **Figure 17.34** shows a violin and a guitar, both of which have sounding boxes but with different shapes, resulting in different overtone structures. The vibrating string creates a sound that resonates in the sounding box, greatly amplifying the sound and creating overtones that give the instrument its characteristic flavor. The more complex the shape of the sounding box, the greater its ability to resonate over a wide range of frequencies. The marimba, like the one shown in **Figure 17.35** uses pots or gourds below the wooden slats to amplify their tones. The resonance of the pot can be adjusted by adding water.

Figure 17.34 String instruments such as violins and guitars use resonance in their sounding boxes to amplify and enrich the sound created by their vibrating strings. The bridge and supports couple the string vibrations to the sounding boxes and air within. (credits: guitar, Feliciano Guimares, Fotopedia; violin, Steve Snodgrass, Flickr)

Figure 17.35 Resonance has been used in musical instruments since prehistoric times. This marimba uses gourds as resonance chambers to amplify its sound. (credit: APC Events, Flickr)

We have emphasized sound applications in our discussions of resonance and standing waves, but these ideas apply to any system that has wave characteristics. Vibrating strings, for example, are actually resonating and have fundamentals and overtones similar to those for air columns. More subtle are the resonances in atoms due to the wave character of their electrons. Their orbitals can be viewed as standing waves, which have a fundamental (ground state) and overtones (excited states). It is fascinating that wave characteristics apply to such a wide range of physical systems.

Check Your Understanding

Describe how noise-canceling headphones differ from standard headphones used to block outside sounds.

Solution

Regular headphones only block sound waves with a physical barrier. Noise-canceling headphones use destructive interference to reduce the loudness of outside sounds.

Check Your Understanding

How is it possible to use a standing wave's node and antinode to determine the length of a closed-end tube?

Solution

When the tube resonates at its natural frequency, the wave's node is located at the closed end of the tube, and the antinode is located at the open end. The length of the tube is equal to one-fourth of the wavelength of this wave. Thus, if we know the wavelength of the wave, we can determine the length of the tube.

PhET Explorations: Sound

This simulation lets you see sound waves. Adjust the frequency or volume and you can see and hear how the wave changes. Move the listener around and hear what she hears.

PhET Interactive Simulation

Figure 17.36 Sound (http://cnx.org/content/m55293/1.2/sound_en.jar)

Applying the Science Practices: Variables Affecting Superposition

In the PhET Interactive Simulation above, select the tab titled ‘Two Source Interference.’ Within this tab, manipulate the variables present (frequency, amplitude, and speaker separation) to investigate the relationship the variables have with the superposition pattern constructed on the screen. Record all observations.

(3) Calculate to find the frequency returning to the source: 2,500,649 Hz.

Solution for (c)

(1) Identify knowns:

- The beat frequency is simply the absolute value of the difference between f_s and f_{obs} , as stated in:

$$f_B = |f_{obs} - f_s|. \quad (17.47)$$

(2) Substitute known values:

$$|2,500,649\text{ Hz} - 2,500,000\text{ Hz}| \quad (17.48)$$

(3) Calculate to find the beat frequency: 649 Hz.

Discussion

The Doppler shifts are quite small compared with the original frequency of 2.50 MHz. It is far easier to measure the beat frequency than it is to measure the echo frequency with an accuracy great enough to see shifts of a few hundred hertz out of a couple of megahertz. Furthermore, variations in the source frequency do not greatly affect the beat frequency, because both f_s and f_{obs} would increase or decrease. Those changes subtract out in $f_B = |f_{obs} - f_s|$.

Industrial and Other Applications of Ultrasound

Industrial, retail, and research applications of ultrasound are common. A few are discussed here. Ultrasonic cleaners have many uses. Jewelry, machined parts, and other objects that have odd shapes and crevices are immersed in a cleaning fluid that is agitated with ultrasound typically about 40 kHz in frequency. The intensity is great enough to cause cavitation, which is responsible for most of the cleansing action. Because cavitation-produced shock pressures are large and well transmitted in a fluid, they reach into small crevices where even a low-surface-tension cleaning fluid might not penetrate.

Sonar is a familiar application of ultrasound. Sonar typically employs ultrasonic frequencies in the range from 30.0 to 100 kHz. Bats, dolphins, submarines, and even some birds use ultrasonic sonar. Echoes are analyzed to give distance and size information both for guidance and finding prey. In most sonar applications, the sound reflects quite well because the objects of interest have significantly different density than the medium in which they travel. When the Doppler shift is observed, velocity information can also be obtained. Submarine sonar can be used to obtain such information, and there is evidence that some bats also sense velocity from their echoes.

Similarly, there are a range of relatively inexpensive devices that measure distance by timing ultrasonic echoes. Many cameras, for example, use such information to focus automatically. Some doors open when their ultrasonic ranging devices detect a nearby object, and certain home security lights turn on when their ultrasonic rangers observe motion. Ultrasonic “measuring tapes” also exist to measure such things as room dimensions. Sinks in public restrooms are sometimes automated with ultrasound devices to turn faucets on and off when people wash their hands. These devices reduce the spread of germs and can conserve water.

Ultrasound is used for nondestructive testing in industry and by the military. Because ultrasound reflects well from any large change in density, it can reveal cracks and voids in solids, such as aircraft wings, that are too small to be seen with x-rays. For similar reasons, ultrasound is also good for measuring the thickness of coatings, particularly where there are several layers involved.

Basic research in solid state physics employs ultrasound. Its attenuation is related to a number of physical characteristics, making it a useful probe. Among these characteristics are structural changes such as those found in liquid crystals, the transition of a material to a superconducting phase, as well as density and other properties.

These examples of the uses of ultrasound are meant to whet the appetites of the curious, as well as to illustrate the underlying physics of ultrasound. There are many more applications, as you can easily discover for yourself.

Check Your Understanding

Why is it possible to use ultrasound both to observe a fetus in the womb and also to destroy cancerous tumors in the body?

Solution

Ultrasound can be used medically at different intensities. Lower intensities do not cause damage and are used for medical imaging. Higher intensities can pulverize and destroy targeted substances in the body, such as tumors.

Glossary

acoustic impedance: property of medium that makes the propagation of sound waves more difficult

antinode: point of maximum displacement

bow wake: V-shaped disturbance created when the wave source moves faster than the wave propagation speed

Doppler effect: an alteration in the observed frequency of a sound due to motion of either the source or the observer

Doppler shift: the actual change in frequency due to relative motion of source and observer

Doppler-shifted ultrasound: a medical technique to detect motion and determine velocity through the Doppler shift of an echo

fundamental: the lowest-frequency resonance

harmonics: the term used to refer collectively to the fundamental and its overtones

hearing: the perception of sound

infrasound: sounds below 20 Hz

intensity: the power per unit area carried by a wave

intensity reflection coefficient: a measure of the ratio of the intensity of the wave reflected off a boundary between two media relative to the intensity of the incident wave

loudness: the perception of sound intensity

node: point of zero displacement

note: basic unit of music with specific names, combined to generate tunes

overtones: all resonant frequencies higher than the fundamental

phon: the numerical unit of loudness

pitch: the perception of the frequency of a sound

sonic boom: a constructive interference of sound created by an object moving faster than sound

sound: a disturbance of matter that is transmitted from its source outward

sound intensity level: a unitless quantity telling you the level of the sound relative to a fixed standard

sound pressure level: the ratio of the pressure amplitude to a reference pressure

timbre: number and relative intensity of multiple sound frequencies

tone: number and relative intensity of multiple sound frequencies

ultrasound: sounds above 20,000 Hz

Section Summary

17.1 Sound

- Sound is a disturbance of matter that is transmitted from its source outward.
- Sound is one type of wave.
- Hearing is the perception of sound.

17.2 Speed of Sound, Frequency, and Wavelength

The relationship of the speed of sound v_{w} , its frequency f , and its wavelength λ is given by

$$v_{\text{w}} = f\lambda,$$

which is the same relationship given for all waves.

In air, the speed of sound is related to air temperature T by

$$v_{\text{w}} = (331\ \text{m/s})\sqrt{\frac{T}{273\ \text{K}}}.$$

v_{w} is the same for all frequencies and wavelengths.

17.3 Sound Intensity and Sound Level

- Intensity is the same for a sound wave as was defined for all waves; it is

$$I = \frac{P}{A},$$

where P is the power crossing area A. The SI unit for I is watts per meter squared. The intensity of a sound wave is also related to the pressure amplitude Δp

$$I = \frac{(\Delta p)^2}{2\rho v_{\text{w}}},$$

where ρ is the density of the medium in which the sound wave travels and v_{w} is the speed of sound in the medium.

- Sound intensity level in units of decibels (dB) is

$$\beta\,(\text{dB}) = 10\,\log_{10}\left(\frac{I}{I_0}\right),$$

where $I_0 = 10^{-12}\ \text{W/m}^2$ is the threshold intensity of hearing.

17.4 Doppler Effect and Sonic Booms

- The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer.
- The actual change in frequency is called the Doppler shift.
- A sonic boom is constructive interference of sound created by an object moving faster than sound.
- A sonic boom is a type of bow wake created when any wave source moves faster than the wave propagation speed.
- For a stationary observer and a moving source, the observed frequency f_{obs} is:

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right),$$

where f_{s} is the frequency of the source, v_{s} is the speed of the source, and v_{w} is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away.

- For a stationary source and moving observer, the observed frequency is:

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}} \pm v_{\text{obs}}}{v_{\text{w}}}\right),$$

where v_{obs} is the speed of the observer.

17.5 Sound Interference and Resonance: Standing Waves in Air Columns

- Sound interference and resonance have the same properties as defined for all waves.
- In air columns, the lowest-frequency resonance is called the fundamental, whereas all higher resonant frequencies are called overtones. Collectively, they are called harmonics.
- The resonant frequencies of a tube closed at one end are:

$$f_n = n\frac{v_{\text{w}}}{4L},\ n = 1, 3, 5...,$$

f_1 is the fundamental and L is the length of the tube.

- The resonant frequencies of a tube open at both ends are:

$$f_n = n\frac{v_{\text{w}}}{2L},\ n = 1, 2, 3...$$

17.6 Hearing

- The range of audible frequencies is 20 to 20,000 Hz.
- Those sounds above 20,000 Hz are ultrasound, whereas those below 20 Hz are infrasound.
- The perception of frequency is pitch.
- The perception of intensity is loudness.
- Loudness has units of phons.

17.7 Ultrasound

- The acoustic impedance is defined as:

$$Z = \rho v,$$

ρ is the density of a medium through which the sound travels and v is the speed of sound through that medium.

- The intensity reflection coefficient a, a measure of the ratio of the intensity of the wave reflected off a boundary between two media relative to the intensity of the incident wave, is given by

$$a = \frac{(Z_2 - Z_1)^2}{(Z_1 + Z_2)^2}.$$

- The intensity reflection coefficient is a unitless quantity.

Conceptual Questions

17.2 Speed of Sound, Frequency, and Wavelength

1. How do sound vibrations of atoms differ from thermal motion?

2. When sound passes from one medium to another where its propagation speed is different, does its frequency or wavelength change? Explain your answer briefly.

17.3 Sound Intensity and Sound Level

3. Six members of a synchronized swim team wear earplugs to protect themselves against water pressure at depths, but they can still hear the music and perform the combinations in the water perfectly. One day, they were asked to leave the pool so the dive team could practice a few dives, and they tried to practice on a mat, but seemed to have a lot more difficulty. Why might this be?

4. A community is concerned about a plan to bring train service to their downtown from the town's outskirts. The current sound intensity level, even though the rail yard is blocks away, is 70 dB downtown. The mayor assures the public that there will be a difference of only 30 dB in sound in the downtown area. Should the townspeople be concerned? Why?

17.4 Doppler Effect and Sonic Booms

5. Is the Doppler shift real or just a sensory illusion?

6. Due to efficiency considerations related to its bow wake, the supersonic transport aircraft must maintain a cruising speed that is a constant ratio to the speed of sound (a constant Mach number). If the aircraft flies from warm air into colder air, should it increase or decrease its speed? Explain your answer.

7. When you hear a sonic boom, you often cannot see the plane that made it. Why is that?

17.5 Sound Interference and Resonance: Standing Waves in Air Columns

8. How does an unamplified guitar produce sounds so much more intense than those of a plucked string held taut by a simple stick?

9. You are given two wind instruments of identical length. One is open at both ends, whereas the other is closed at one end. Which is able to produce the lowest frequency?

10. What is the difference between an overtone and a harmonic? Are all harmonics overtones? Are all overtones harmonics?

17.6 Hearing

11. Why can a hearing test show that your threshold of hearing is 0 dB at 250 Hz, when **Figure 17.39** implies that no one can hear such a frequency at less than 20 dB?

17.7 Ultrasound

12. If audible sound follows a rule of thumb similar to that for ultrasound, in terms of its absorption, would you expect the high or low frequencies from your neighbor's stereo to penetrate into your house? How does this expectation compare with your experience?

13. Elephants and whales are known to use infrasound to communicate over very large distances. What are the advantages of infrasound for long distance communication?

14. It is more difficult to obtain a high-resolution ultrasound image in the abdominal region of someone who is overweight than for someone who has a slight build. Explain why this statement is accurate.

15. Suppose you read that 210-dB ultrasound is being used to pulverize cancerous tumors. You calculate the intensity in watts per centimeter squared and find it is unreasonably high ($10^5\ \text{W/cm}^2$). What is a possible explanation?

Problems & Exercises

17.2 Speed of Sound, Frequency, and Wavelength

1. When poked by a spear, an operatic soprano lets out a 1200-Hz shriek. What is its wavelength if the speed of sound is 345 m/s?

2. What frequency sound has a 0.10-m wavelength when the speed of sound is 340 m/s?

3. Calculate the speed of sound on a day when a 1500 Hz frequency has a wavelength of 0.221 m.

4. (a) What is the speed of sound in a medium where a 100-kHz frequency produces a 5.96-cm wavelength? (b) Which substance in **Table 17.4** is this likely to be?

5. Show that the speed of sound in 20.0°C air is 343 m/s, as claimed in the text.

6. Air temperature in the Sahara Desert can reach 56.0°C (about 134°F). What is the speed of sound in air at that temperature?

7. Dolphins make sounds in air and water. What is the ratio of the wavelength of a sound in air to its wavelength in seawater? Assume air temperature is 20.0°C .

8. A sonar echo returns to a submarine 1.20 s after being emitted. What is the distance to the object creating the echo? (Assume that the submarine is in the ocean, not in fresh water.)

9. (a) If a submarine's sonar can measure echo times with a precision of 0.0100 s, what is the smallest difference in distances it can detect? (Assume that the submarine is in the ocean, not in fresh water.)

(b) Discuss the limits this time resolution imposes on the ability of the sonar system to detect the size and shape of the object creating the echo.

10. A physicist at a fireworks display times the lag between seeing an explosion and hearing its sound, and finds it to be 0.400 s. (a) How far away is the explosion if air temperature is 24.0°C and if you neglect the time taken for light to reach the physicist? (b) Calculate the distance to the explosion taking the speed of light into account. Note that this distance is negligibly greater.

11. Suppose a bat uses sound echoes to locate its insect prey, 3.00 m away. (See **Figure 17.11**.) (a) Calculate the echo times for temperatures of 5.00°C and 35.0°C . (b) What percent uncertainty does this cause for the bat in locating the insect? (c) Discuss the significance of this uncertainty and whether it could cause difficulties for the bat. (In practice, the bat continues to use sound as it closes in, eliminating most of any difficulties imposed by this and other effects, such as motion of the prey.)

17.3 Sound Intensity and Sound Level

12. What is the intensity in watts per meter squared of 85.0-dB sound?

13. The warning tag on a lawn mower states that it produces noise at a level of 91.0 dB. What is this in watts per meter squared?

14. A sound wave traveling in 20°C air has a pressure amplitude of 0.5 Pa. What is the intensity of the wave?

15. What intensity level does the sound in the preceding problem correspond to?

16. What sound intensity level in dB is produced by earphones that create an intensity of $4.00\times10^{-2}\ \text{W/m}^2$?

17. Show that an intensity of $10^{-12}\ \text{W/m}^2$ is the same as $10^{-16}\ \text{W/cm}^2$.

18. (a) What is the decibel level of a sound that is twice as intense as a 90.0-dB sound? (b) What is the decibel level of a sound that is one-fifth as intense as a 90.0-dB sound?

19. (a) What is the intensity of a sound that has a level 7.00 dB lower than a $4.00\times10^{-9}\ \text{W/m}^2$ sound? (b) What is the intensity of a sound that is 3.00 dB higher than a $4.00\times10^{-9}\ \text{W/m}^2$ sound?

20. (a) How much more intense is a sound that has a level 17.0 dB higher than another? (b) If one sound has a level 23.0 dB less than another, what is the ratio of their intensities?

21. People with good hearing can perceive sounds as low in level as $-8.00\ \text{dB}$ at a frequency of 3000 Hz. What is the intensity of this sound in watts per meter squared?

22. If a large housefly 3.0 m away from you makes a noise of 40.0 dB, what is the noise level of 1000 flies at that distance, assuming interference has a negligible effect?

23. Ten cars in a circle at a boom box competition produce a 120-dB sound intensity level at the center of the circle. What is the average sound intensity level produced there by each stereo, assuming interference effects can be neglected?

24. The amplitude of a sound wave is measured in terms of its maximum gauge pressure. By what factor does the amplitude of a sound wave increase if the sound intensity level goes up by 40.0 dB?

25. If a sound intensity level of 0 dB at 1000 Hz corresponds to a maximum gauge pressure (sound amplitude) of $10^{-9}\ \text{atm}$, what is the maximum gauge pressure in a 60-dB sound? What is the maximum gauge pressure in a 120-dB sound?

26. An 8-hour exposure to a sound intensity level of 90.0 dB may cause hearing damage. What energy in joules falls on a 0.800-cm-diameter eardrum so exposed?

27. (a) Ear trumpets were never very common, but they did aid people with hearing losses by gathering sound over a large area and concentrating it on the smaller area of the eardrum. What decibel increase does an ear trumpet produce if its sound gathering area is $900\ \text{cm}^2$ and the area of the eardrum is $0.500\ \text{cm}^2$, but the trumpet only has an efficiency of 5.00% in transmitting the sound to the eardrum? (b) Comment on the usefulness of the decibel increase found in part (a).

28. Sound is more effectively transmitted into a stethoscope by direct contact than through the air, and it is further intensified by being concentrated on the smaller area of the eardrum. It is reasonable to assume that sound is transmitted into a stethoscope 100 times as effectively compared with transmission though the air. What, then, is the gain in decibels produced by a stethoscope that has a sound gathering area of $15.0\ \text{cm}^2$, and concentrates the sound onto two eardrums with a total area of $0.900\ \text{cm}^2$ with an efficiency of 40.0%?

29. Loudspeakers can produce intense sounds with surprisingly small energy input in spite of their low efficiencies. Calculate the power input needed to produce a 90.0-dB sound intensity level for a 12.0-cm-diameter speaker that has an efficiency of 1.00%. (This value is the sound intensity level right at the speaker.)

17.4 Doppler Effect and Sonic Booms

30. (a) What frequency is received by a person watching an oncoming ambulance moving at 110 km/h and emitting a steady 800-Hz sound from its siren? The speed of sound on this day is 345 m/s. (b) What frequency does she receive after the ambulance has passed?

31. (a) At an air show a jet flies directly toward the stands at a speed of 1200 km/h, emitting a frequency of 3500 Hz, on a day when the speed of sound is 342 m/s. What frequency is received by the observers? (b) What frequency do they receive as the plane flies directly away from them?

32. What frequency is received by a mouse just before being dispatched by a hawk flying at it at 25.0 m/s and emitting a screech of frequency 3500 Hz? Take the speed of sound to be 331 m/s.

33. A spectator at a parade receives an 888-Hz tone from an oncoming trumpeter who is playing an 880-Hz note. At what speed is the musician approaching if the speed of sound is 338 m/s?

34. A commuter train blows its 200-Hz horn as it approaches a crossing. The speed of sound is 335 m/s. (a) An observer waiting at the crossing receives a frequency of 208 Hz. What is the speed of the train? (b) What frequency does the observer receive as the train moves away?

35. Can you perceive the shift in frequency produced when you pull a tuning fork toward you at 10.0 m/s on a day when the speed of sound is 344 m/s? To answer this question, calculate the factor by which the frequency shifts and see if it is greater than 0.300%.

36. Two eagles fly directly toward one another, the first at 15.0 m/s and the second at 20.0 m/s. Both screech, the first one emitting a frequency of 3200 Hz and the second one emitting a frequency of 3800 Hz. What frequencies do they receive if the speed of sound is 330 m/s?

37. What is the minimum speed at which a source must travel toward you for you to be able to hear that its frequency is Doppler shifted? That is, what speed produces a shift of 0.300% on a day when the speed of sound is 331 m/s?

17.5 Sound Interference and Resonance: Standing Waves in Air Columns

38. A “showy” custom-built car has two brass horns that are supposed to produce the same frequency but actually emit 263.8 and 264.5 Hz. What beat frequency is produced?

39. What beat frequencies will be present: (a) If the musical notes A and C are played together (frequencies of 220 and 264 Hz)? (b) If D and F are played together (frequencies of 297 and 352 Hz)? (c) If all four are played together?

40. What beat frequencies result if a piano hammer hits three strings that emit frequencies of 127.8, 128.1, and 128.3 Hz?

41. A piano tuner hears a beat every 2.00 s when listening to a 264.0-Hz tuning fork and a single piano string. What are the two possible frequencies of the string?

42. (a) What is the fundamental frequency of a 0.672-m-long tube, open at both ends, on a day when the speed of sound is 344 m/s? (b) What is the frequency of its second harmonic?

43. If a wind instrument, such as a tuba, has a fundamental frequency of 32.0 Hz, what are its first three overtones? It is closed at one end. (The overtones of a real tuba are more complex than this example, because it is a tapered tube.)

44. What are the first three overtones of a bassoon that has a fundamental frequency of 90.0 Hz? It is open at both ends. (The overtones of a real bassoon are more complex than this example, because its double reed makes it act more like a tube closed at one end.)

45. How long must a flute be in order to have a fundamental frequency of 262 Hz (this frequency corresponds to middle C on the evenly tempered chromatic scale) on a day when air temperature is 20.0°C ? It is open at both ends.

46. What length should an oboe have to produce a fundamental frequency of 110 Hz on a day when the speed of sound is 343 m/s? It is open at both ends.

47. What is the length of a tube that has a fundamental frequency of 176 Hz and a first overtone of 352 Hz if the speed of sound is 343 m/s?

48. (a) Find the length of an organ pipe closed at one end that produces a fundamental frequency of 256 Hz when air temperature is 18.0°C . (b) What is its fundamental frequency at 25.0°C ?

49. By what fraction will the frequencies produced by a wind instrument change when air temperature goes from 10.0°C to 30.0°C ? That is, find the ratio of the frequencies at those temperatures.

50. The ear canal resonates like a tube closed at one end. (See **Figure 17.41**.) If ear canals range in length from 1.80 to 2.60 cm in an average population, what is the range of fundamental resonant frequencies? Take air temperature to be 37.0°C , which is the same as body temperature. How does this result correlate with the intensity versus frequency graph (**Figure 17.39** of the human ear?

51. Calculate the first overtone in an ear canal, which resonates like a 2.40-cm-long tube closed at one end, by taking air temperature to be 37.0°C . Is the ear particularly sensitive to such a frequency? (The resonances of the ear canal are complicated by its nonuniform shape, which we shall ignore.)

52. A crude approximation of voice production is to consider the breathing passages and mouth to be a resonating tube closed at one end. (See **Figure 17.32**.) (a) What is the fundamental frequency if the tube is 0.240-m long, by taking air temperature to be 37.0°C ? (b) What would this frequency become if the person replaced the air with helium? Assume the same temperature dependence for helium as for air.

53. (a) Students in a physics lab are asked to find the length of an air column in a tube closed at one end that has a fundamental frequency of 256 Hz. They hold the tube vertically and fill it with water to the top, then lower the water while a 256-Hz tuning fork is rung and listen for the first resonance. What is the air temperature if the resonance occurs for a length of 0.336 m? (b) At what length will they observe the second resonance (first overtone)?

54. What frequencies will a 1.80-m-long tube produce in the audible range at 20.0°C if: (a) The tube is closed at one end? (b) It is open at both ends?

17.6 Hearing

55. The factor of 10^{-12} in the range of intensities to which the ear can respond, from threshold to that causing damage after brief exposure, is truly remarkable. If you could measure distances over the same range with a single instrument and the smallest distance you could measure was 1 mm, what would the largest be?

56. The frequencies to which the ear responds vary by a factor of 10^3. Suppose the speedometer on your car measured speeds differing by the same factor of 10^3, and the greatest speed it reads is 90.0 mi/h. What would be the slowest nonzero speed it could read?

57. What are the closest frequencies to 500 Hz that an average person can clearly distinguish as being different in frequency from 500 Hz? The sounds are not present simultaneously.

58. Can the average person tell that a 2002-Hz sound has a different frequency than a 1999-Hz sound without playing them simultaneously?

59. If your radio is producing an average sound intensity level of 85 dB, what is the next lowest sound intensity level that is clearly less intense?

60. Can you tell that your roommate turned up the sound on the TV if its average sound intensity level goes from 70 to 73 dB?

61. Based on the graph in **Figure 17.38**, what is the threshold of hearing in decibels for frequencies of 60, 400, 1000, 4000, and 15,000 Hz? Note that many AC electrical appliances produce 60 Hz, music is commonly 400 Hz, a reference frequency is 1000 Hz, your maximum sensitivity is near 4000 Hz, and many older TVs produce a 15,750 Hz whine.

62. What sound intensity levels must sounds of frequencies 60, 3000, and 8000 Hz have in order to have the same loudness as a 40-dB sound of frequency 1000 Hz (that is, to have a loudness of 40 phons)?

63. What is the approximate sound intensity level in decibels of a 600-Hz tone if it has a loudness of 20 phons? If it has a loudness of 70 phons?

64. (a) What are the loudnesses in phons of sounds having frequencies of 200, 1000, 5000, and 10,000 Hz, if they are all at the same 60.0-dB sound intensity level? (b) If they are all at 110 dB? (c) If they are all at 20.0 dB?

65. Suppose a person has a 50-dB hearing loss at all frequencies. By how many factors of 10 will low-intensity sounds need to be amplified to seem normal to this person? Note that smaller amplification is appropriate for more intense sounds to avoid further hearing damage.

66. If a woman needs an amplification of 5.0×10^{12} times the threshold intensity to enable her to hear at all frequencies, what is her overall hearing loss in dB? Note that smaller amplification is appropriate for more intense sounds to avoid further damage to her hearing from levels above 90 dB.

67. (a) What is the intensity in watts per meter squared of a just barely audible 200-Hz sound? (b) What is the intensity in watts per meter squared of a barely audible 4000-Hz sound?

68. (a) Find the intensity in watts per meter squared of a 60.0-Hz sound having a loudness of 60 phons. (b) Find the intensity in watts per meter squared of a 10,000-Hz sound having a loudness of 60 phons.

69. A person has a hearing threshold 10 dB above normal at 100 Hz and 50 dB above normal at 4000 Hz. How much more intense must a 100-Hz tone be than a 4000-Hz tone if they are both barely audible to this person?

70. A child has a hearing loss of 60 dB near 5000 Hz, due to noise exposure, and normal hearing elsewhere. How much more intense is a 5000-Hz tone than a 400-Hz tone if they are both barely audible to the child?

71. What is the ratio of intensities of two sounds of identical frequency if the first is just barely discernible as louder to a person than the second?

17.7 Ultrasound

Unless otherwise indicated, for problems in this section, assume that the speed of sound through human tissues is 1540 m/s.

72. What is the sound intensity level in decibels of ultrasound of intensity $10^5\ \text{W/m}^2$, used to pulverize tissue during surgery?

73. Is 155-dB ultrasound in the range of intensities used for deep heating? Calculate the intensity of this ultrasound and compare this intensity with values quoted in the text.

74. Find the sound intensity level in decibels of $2.00\times10^{-2}\ \text{W/m}^2$ ultrasound used in medical diagnostics.

75. The time delay between transmission and the arrival of the reflected wave of a signal using ultrasound traveling through a piece of fat tissue was 0.13 ms. At what depth did this reflection occur?

76. In the clinical use of ultrasound, transducers are always coupled to the skin by a thin layer of gel or oil, replacing the air that would otherwise exist between the transducer and the skin. (a) Using the values of acoustic impedance given in **Table 17.8** calculate the intensity reflection coefficient between transducer material and air. (b) Calculate the intensity reflection coefficient between transducer material and gel (assuming for this problem that its acoustic impedance is identical to that of water). (c) Based on the results of your calculations, explain why the gel is used.

77. (a) Calculate the minimum frequency of ultrasound that will allow you to see details as small as 0.250 mm in human tissue. (b) What is the effective depth to which this sound is effective as a diagnostic probe?

78. (a) Find the size of the smallest detail observable in human tissue with 20.0-MHz ultrasound. (b) Is its effective penetration depth great enough to examine the entire eye (about 3.00 cm is needed)? (c) What is the wavelength of such ultrasound in 0°C air?

79. (a) Echo times are measured by diagnostic ultrasound scanners to determine distances to reflecting surfaces in a patient. What is the difference in echo times for tissues that are 3.50 and 3.60 cm beneath the surface? (This difference is the minimum resolving time for the scanner to see details as small as 0.100 cm, or 1.00 mm. Discrimination of smaller time differences is needed to see smaller details.) (b) Discuss whether the period T of this ultrasound must be smaller than the minimum time resolution. If so, what is the minimum frequency of the ultrasound and is that out of the normal range for diagnostic ultrasound?

80. (a) How far apart are two layers of tissue that produce echoes having round-trip times (used to measure distances) that differ by $0.750\ \mu s$? (b) What minimum frequency must the ultrasound have to see detail this small?

81. (a) A bat uses ultrasound to find its way among trees. If this bat can detect echoes 1.00 ms apart, what minimum distance between objects can it detect? (b) Could this distance explain the difficulty that bats have finding an open door when they accidentally get into a house?

82. A dolphin is able to tell in the dark that the ultrasound echoes received from two sharks come from two different objects only if the sharks are separated by 3.50 m, one being that much farther away than the other. (a) If the ultrasound has a frequency of 100 kHz, show this ability is not limited by its wavelength. (b) If this ability is due to the dolphin's ability to detect the arrival times of echoes, what is the minimum time difference the dolphin can perceive?

83. A diagnostic ultrasound echo is reflected from moving blood and returns with a frequency 500 Hz higher than its original 2.00 MHz. What is the velocity of the blood? (Assume that the frequency of 2.00 MHz is accurate to seven significant figures and 500 Hz is accurate to three significant figures.)

84. Ultrasound reflected from an oncoming bloodstream that is moving at 30.0 cm/s is mixed with the original frequency of 2.50 MHz to produce beats. What is the beat frequency? (Assume that the frequency of 2.50 MHz is accurate to seven significant figures.)

Test Prep for AP® Courses

17.2 Speed of Sound, Frequency, and Wavelength

1. A teacher wants to demonstrate that the speed of sound is not a constant value. Considering her regular classroom voice as the control, which of the following will increase the speed of sound leaving her mouth?

I. Submerge her mouth underwater and speak at the same volume.
II. Increase the temperature of the room and speak at the same volume.
III. Increase the pitch of her voice and speak at the same volume.

a. I only
b. I and II only
c. I, II and III
d. II and III
e. III only

2. All members of an orchestra begin tuning their instruments at the same time. While some woodwind instruments play high frequency notes, other stringed instruments play notes of lower frequency. Yet an audience member will hear all notes simultaneously, in apparent contrast to the equation.

Explain how a student could demonstrate the flaw in the above logic, using a slinky, stopwatch, and meter stick. Make sure to explain what relationship is truly demonstrated in the above equation, in addition to what would be necessary to get the speed of the slinky to actually change. You may include diagrams and equations as part of your explanation.

17.3 Sound Intensity and Sound Level

3. In order to waken a sleeping child, the volume on an alarm clock is tripled. Under this new scenario, how much more energy will be striking the child's ear drums each second?

a. twice as much
b. three times as much
c. approximately 4.8 times as much
d. six times as much
e. nine times as much

4. A musician strikes the strings of a guitar such that they vibrate with twice the amplitude.

a. Explain why this requires an energy input greater than twice the original value.
b. Explain why the sound leaving the string will not result in a decibel level that is twice as great.

17.4 Doppler Effect and Sonic Booms

5. A baggage handler stands on the edge of a runway as a landing plane approaches. Compared to the pitch of the plane as heard by the plane's pilot, which of the following correctly describes the sensation experienced by the handler?

a. The frequency of the plane will be lower pitched according to the baggage handler and will become even lower pitched as the plane slows to a stop.
b. The frequency of the plane will be lower pitched according to the baggage handler but will increase in pitch as the plane slows to a stop.
c. The frequency of the plane will be higher pitched according to the baggage handler but will decrease in pitch as the plane slows to a stop.
d. The frequency of the plane will be higher pitched according to the baggage handler and will further increase in pitch as the plane slows to a stop.

6. The following graph represents the perceived frequency of a car as it passes a student.

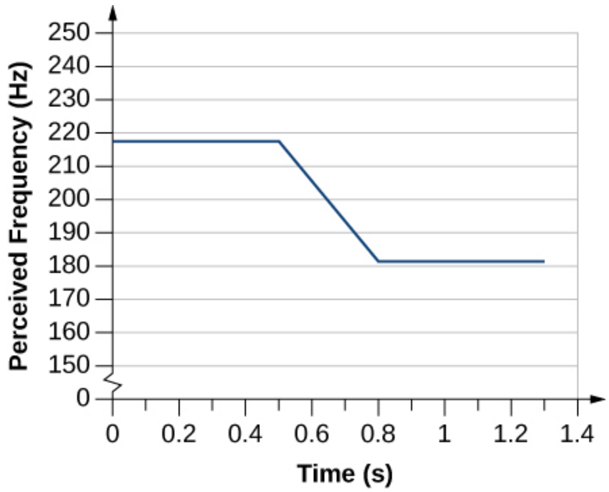


Figure 17.51 Plot of time versus perceived frequency to illustrate the Doppler effect.

a. If the true frequency of the car’s horn is 200 Hz, how fast was the car traveling?
b. On the graph above, draw a line demonstrating the perceived frequency for a car traveling twice as fast. Label all intercepts, maximums, and minimums on the graph.

17.5 Sound Interference and Resonance: Standing Waves in Air Columns

7. A common misconception is that two wave pulses traveling in opposite directions will reflect off each other. Outline a procedure that you would use to convince someone that the two wave pulses do not reflect off each other, but instead travel through each other. You may use sketches to represent your understanding. Be sure to provide evidence to not only refute the original claim, but to support yours as well.

8. Two wave pulses are traveling toward each other on a string, as shown below. Which of the following representations correctly shows the string as the two pulses overlap?

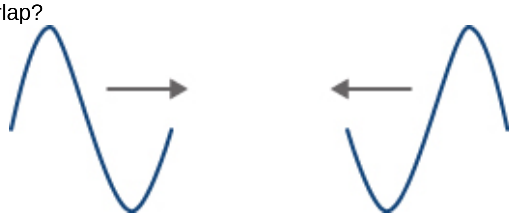


Figure 17.52

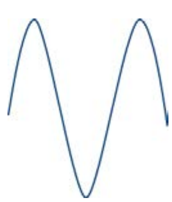
a. 

Figure 17.53

b. 

Figure 17.54

c.

Figure 17.55

d. 

Figure 17.56

9. A student sends a transverse wave pulse of amplitude *A* along a rope attached at one end. As the pulse returns to the student, a second pulse of amplitude 3*A* is sent along the opposite side of the rope. What is the resulting amplitude when the two pulses interact?

a. 4*A*
b. *A*
c. 2*A*, on the side of the original wave pulse
d. 2*A*, on the side of the second wave pulse

10. A student would like to demonstrate destructive interference using two sound sources. Explain how the student could set up this demonstration and what restrictions they would need to place upon their sources. Be sure to consider both the layout of space and the sounds created in your explanation.

11. A student is shaking a flexible string attached to a wooden board in a rhythmic manner. Which of the following choices will decrease the wavelength within the rope?

I. The student could shake her hand back and forth with greater frequency.
II. The student could shake her hand back in forth with a greater amplitude.
III. The student could increase the tension within the rope by stepping backwards from the board.

a. I only
b. I and II
c. I and III
d. II and III
e. I, II, and III

12. A ripple tank has two locations (L1 and L2) that vibrate in tandem as shown below. Both L1 and L2 vibrate in a plane perpendicular to the page, creating a two-dimensional interference pattern.

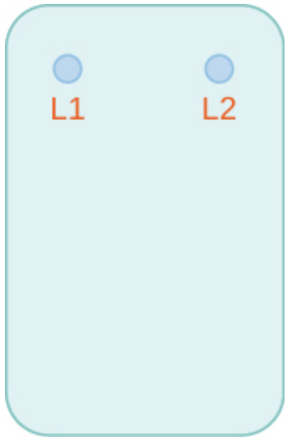


Figure 17.57

Describe an experimental procedure to determine the speed of the waves created within the water, including all additional equipment that you would need. You may use the diagram below to help your description, or you may create one of your own. Include enough detail so that another student could carry out your experiment.

13. A string is vibrating between two posts as shown above. Students are to determine the speed of the wave within this string. They have already measured the amount of time necessary for the wave to oscillate up and down. The students must also take what other measurements to determine the speed of the wave?

a. The distance between the two posts.
b. The amplitude of the wave
c. The tension in the string
d. The amplitude of the wave and the tension in the string
e. The distance between the two posts, the amplitude of the wave, and the tension in the string

14. The accepted speed of sound in room temperature air is 346 m/s. Knowing that their school is colder than usual, a group of students is asked to determine the speed of sound in their room. They are permitted to use any materials necessary; however, their lab procedure must utilize standing wave patterns. The students collect the information **Table 17.9**.

Table 17.9

Trial Number	Wavelength (m)	Frequency (Hz)	
1	3.45	95	
2	2.32	135	
3	1.70	190	
4	1.45	240	
5	1.08	305	

a. Describe an experimental procedure the group of students could have used to obtain this data. Include diagrams of the experimental setup and any equipment used in the process.
b. Select a set of data points from the table and plot those points on a graph to determine the speed of sound within the classroom. Fill in the blank column in the table for any quantities you graph other than the given data. Label the axes and indicate the scale for each. Draw a best-fit line or curve through your data points.
c. Using information from the graph, determine the speed of sound within the student's classroom, and explain what characteristic of the graph provides this evidence.
d. Determine the temperature of the classroom.

15. A tube is open at one end. If the fundamental frequency f is created by a wavelength λ, then which of the following describes the frequency and wavelength associated with the tube's fourth overtone?

	f	λ
(a)	$4f$	$\lambda/4$
(b)	$4f$	λ
(c)	$9f$	$\lambda/9$
(d)	$9f$	λ

16. A group of students were tasked with collecting information about standing waves. **Table 17.10** a series of their data, showing the length of an air column and a resonant frequency present when the column is struck.

Table 17.10

Length (m)	Resonant Frequency (Hz)
1	85.75
2	43
3	29
4	21.5

a. From their data, determine whether the air column was open or closed on each end.
b. Predict the resonant frequency of the column at a length of 2.5 meters.

17. When a student blows across a glass half-full of water, a resonant frequency is created within the air column remaining in the glass. Which of the following can the student do to increase this resonant frequency?

I. Add more water to the glass.
II. Replace the water with a more dense fluid.
III. Increase the temperature of the room.

a. I only
b. I and III
c. II and III
d. all of the above

18. A wooden ruler rests on a desk with half of its length protruding off the desk edge. A student holds one end in place and strikes the protruding end with his other hand, creating a musical sound.

a. Explain, without using a sound meter, how the student could experimentally determine the speed of sound that travels within the ruler.
b. A sound meter is then used to measure the true frequency of the ruler. It is found that the experimental result is lower than the true value. Explain a factor that may have caused this difference. Also explain what affect this result has on the calculated speed of sound.

19. A musician stands outside in a field and plucks a string on an acoustic guitar. Standing waves will most likely occur in which of the following media? Select two answers.

a. The guitar string
b. The air inside the guitar
c. The air surrounding the guitar

d. The ground beneath the musician

20.

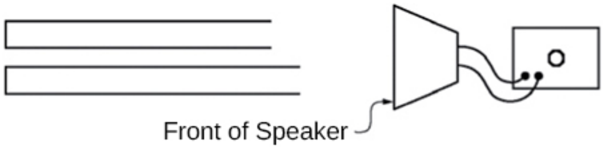


Figure 17.58 This figure shows two tubes that are identical except for their slightly different lengths. Both tubes have one open end and one closed end. A speaker connected to a variable frequency generator is placed in front of the tubes, as shown. The speaker is set to produce a note of very low frequency when turned on. The frequency is then slowly increased to produce resonances in the tubes. Students observe that at first only one of the tubes resonates at a time. Later, as the frequency gets very high, there are times when both tubes resonate.

In a clear, coherent, paragraph-length answer, explain why there are some high frequencies, but no low frequencies, at which both tubes resonate. You may include diagrams and/or equations as part of your explanation.

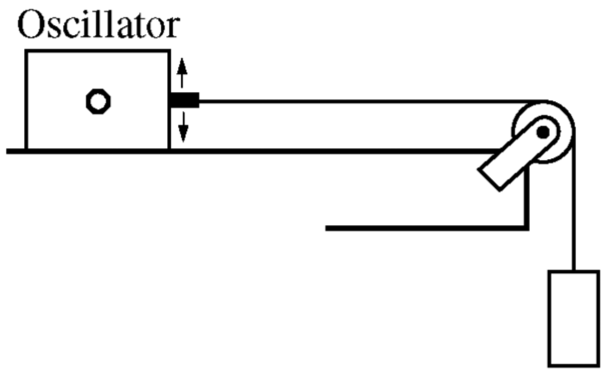


Figure 17.59

21. A student connects one end of a string with negligible mass to an oscillator. The other end of the string is passed over a pulley and attached to a suspended weight, as shown above. The student finds that a standing wave with one antinode is formed on the string when the frequency of the oscillator is f_0. The student then moves the oscillator to shorten the horizontal segment of string to half its original length. At what frequency will a standing wave with one antinode now be formed on the string?

a. $f_0/2$
b. f_0
c. $2f_0$
d. There is no frequency at which a standing wave will be formed.

22. A guitar string of length L is bound at both ends. Table 17.11 shows the string's harmonic frequencies when struck.

Table 17.11

Harmonic Number	Frequency
1	$225/L$
2	$450/L$
3	$675/L$
4	$900/L$

a. Based on the information above, what is the speed of the wave within the string?
b. The guitarist then slides her finger along the neck of the guitar, changing the string length as a result. Calculate the fundamental frequency of the string and wave speed present if the string length is reduced to 2/3 L.

18 ELECTRIC CHARGE AND ELECTRIC FIELD

Figure 18.1 Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)

Chapter Outline

Connection for AP® Courses

The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm (shown in Figure 18.2) is familiar to every schoolchild. In this experiment, Franklin demonstrated a connection between lightning and static electricity. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. Much has been written about Franklin. His experiments were only part of the life of a man who was a scientist, inventor, revolutionary, statesman, and writer. Franklin's experiments were not performed in isolation, nor were they the only ones to reveal connections.

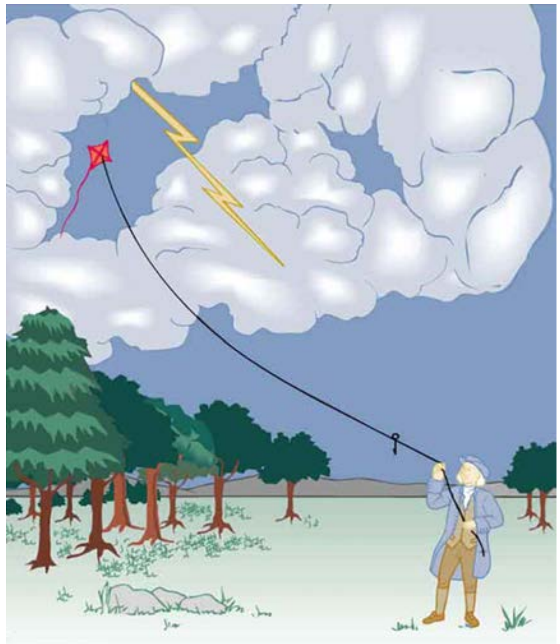


Figure 18.2 Benjamin Franklin, his kite, and electricity.

When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces (except gravitational force) are manifestations of the electromagnetic force. For example, the Italian scientist Luigi Galvani (1737-1798) performed a series of experiments in which static electricity was used to stimulate contractions of leg muscles of dead frogs, an effect already known in humans subjected to static discharges. But Galvani also found that if he joined one end of two metal wires (say copper and zinc) and touched the other ends of the wires to muscles; he produced the same effect in frogs as static discharge. Alessandro Volta (1745-1827), partly inspired by Galvani's work, experimented with various combinations of metals and developed the battery.

During the same era, other scientists made progress in discovering fundamental connections. The periodic table was developed as systematic properties of the elements were discovered. This influenced the development and refinement of the concept of atoms as the basis of matter. Such submicroscopic descriptions of matter also help explain a great deal more. Atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the electromagnetic force.

Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism. All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

This chapter begins the study of electromagnetic phenomena at a fundamental level. The next several chapters will cover static electricity, moving electricity, and magnetism – collectively known as electromagnetism. In this chapter, we begin with the study of electric phenomena due to charges that are at least temporarily stationary, called electrostatics, or static electricity.

The chapter introduces several very important concepts of charge, electric force, and electric field, as well as defining the relationships between these concepts. The charge is defined as a property of a system (Big Idea 1) that can affect its interaction with other charged systems (Enduring Understanding 1.B). The law of conservation of electric charge is also discussed (Essential Knowledge 1.B.1). The two kinds of electric charge are defined as positive and negative, providing an explanation for having positively charged, negatively charged, or neutral objects (containing equal quantities of positive and negative charges) (Essential Knowledge 1.B.2). The discrete nature of the electric charge is introduced in this chapter by defining the elementary charge as the smallest observed unit of charge that can be isolated, which is the electron charge (Essential Knowledge 1.B.3). The concepts of a system (having internal structure) and of an object (having no internal structure) are implicitly introduced to explain charges carried by the electron and proton (Enduring Understanding 1.A, Essential Knowledge 1.A.1).

An electric field is caused by the presence of charged objects (Enduring Understanding 2.C) and can be used to explain interactions between electrically charged objects (Big Idea 2). The electric force represents the effect of an electric field on a charge placed in the field. The magnitude and direction of the electric force are defined by the magnitude and direction of the electric field and magnitude and sign of the charge (Essential Knowledge 2.C.1). The magnitude of the electric field is proportional to the net charge of the objects that created that field (Essential Knowledge 2.C.2). For the special case of a spherically symmetric charged object, the electric field outside the object is radial, and its magnitude varies as the inverse square of the radial distance from the center of that object (Essential Knowledge 2.C.3). The chapter provides examples of vector field maps for various charged systems, including point charges, spherically symmetric charge distributions, and uniformly charged parallel plates (Essential Knowledge 2.C.1, Essential Knowledge 2.C.2). For multiple point charges, the chapter explains how to find the vector field map by adding the electric field vectors of each individual object, including the special case of two equal charges having opposite signs, known as an electric dipole (Essential Knowledge 2.C.4). The special case of two oppositely charged parallel plates with uniformly distributed electric charge when the electric field is perpendicular to the plates and is constant in both magnitude and direction is described in detail, providing many opportunities for problem solving and applications (Essential Knowledge 2.C.5).

The idea that interactions can be described by forces is also reinforced in this chapter (Big Idea 3). Like all other forces that you have learned about so far, electric force is a vector that affects the motion according to Newton's laws (Enduring Understanding 3.A). It is clearly stated in the chapter that electric force appears as a result of interactions between two charged objects (Essential Knowledge 3.A.3, Essential Knowledge 3.C.2). At the macroscopic level the electric force is a long-range force (Enduring Understanding 3.C); however, at the microscopic level many contact forces, such as friction, can be explained by interatomic electric forces (Essential Knowledge 3.C.4). This understanding of friction is helpful when considering properties of conductors and insulators and the transfer of charge by conduction.

Interactions between systems can result in changes in those systems (Big Idea 4). In the case of charged systems, such interactions can lead to changes of electric properties (Enduring Understanding 4.E), such as charge distribution (Essential Knowledge 4.E.3). Any changes are governed by conservation laws (Big Idea 5). Depending on whether the system is closed or open, certain quantities of the system remain the same or changes in those quantities are equal to the amount of transfer of this quantity from or to the system (Enduring Understanding 5.A). The electric charge is one of these quantities (Essential Knowledge 5.A.2). Therefore, the electric charge of a system is conserved (Enduring Understanding 5.C) and the exchange of electric charge between objects in a system does not change the total electric charge of the system (Essential Knowledge 5.C.2).

Big Idea 1 Objects and systems have properties such as mass and charge. Systems may have internal structure.

Enduring Understanding 1.A The internal structure of a system determines many properties of the system.

Essential Knowledge 1.A.1 A system is an object or a collection of objects. Objects are treated as having no internal structure.

Enduring Understanding 1.B Electric charge is a property of an object or system that affects its interactions with other objects or systems containing charge.

Essential Knowledge 1.B.1 Electric charge is conserved. The net charge of a system is equal to the sum of the charges of all the objects in the system.

Essential Knowledge 1.B.2 There are only two kinds of electric charge. Neutral objects or systems contain equal quantities of positive and negative charge, with the exception of some fundamental particles that have no electric charge.

Essential Knowledge 1.B.3 The smallest observed unit of charge that can be isolated is the electron charge, also known as the elementary charge.

Big Idea 2 Fields existing in space can be used to explain interactions.

Enduring Understanding 2.C An electric field is caused by an object with electric charge.

Essential Knowledge 2.C.1 The magnitude of the electric force *F* exerted on an object with electric charge *q* by an electric field ($\vec{E}$ is $\vec{F} = q\vec{E}$. The direction of the force is determined by the direction of the field and the sign of the charge, with positively charged objects accelerating in the direction of the field and negatively charged objects accelerating in the direction opposite the field. This should include a vector field map for positive point charges, negative point charges, spherically symmetric charge distribution, and uniformly charged parallel plates.

Essential Knowledge 2.C.2 The magnitude of the electric field vector is proportional to the net electric charge of the object(s) creating that field. This includes positive point charges, negative point charges, spherically symmetric charge distributions, and uniformly charged parallel plates.

Essential Knowledge 2.C.3 The electric field outside a spherically symmetric charged object is radial, and its magnitude varies as the inverse square of the radial distance from the center of that object. Electric field lines are not in the curriculum. Students will be expected to rely only on the rough intuitive sense underlying field lines, wherein the field is viewed as analogous to something emanating uniformly from a source.

Essential Knowledge 2.C.4 The electric field around dipoles and other systems of electrically charged objects (that can be modeled as point objects) is found by vector addition of the field of each individual object. Electric dipoles are treated qualitatively in this course as a teaching analogy to facilitate student understanding of magnetic dipoles.

Essential Knowledge 2.C.5 Between two oppositely charged parallel plates with uniformly distributed electric charge, at points far from the edges of the plates, the electric field is perpendicular to the plates and is constant in both magnitude and direction.

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A All forces share certain common characteristics when considered by observers in inertial reference frames.

Essential Knowledge 3.A.3 A force exerted on an object is always due to the interaction of that object with another object.

Enduring Understanding 3.C At the macroscopic level, forces can be categorized as either long-range (action-at-a-distance) forces or contact forces.

Essential Knowledge 3.C.2 Electric force results from the interaction of one object that has an electric charge with another object that has an electric charge.

Essential Knowledge 3.C.4 Contact forces result from the interaction of one object touching another object, and they arise from interatomic electric forces. These forces include tension, friction, normal, spring (Physics 1), and buoyant (Physics 2).

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding 4.E The electric and magnetic properties of a system can change in response to the presence of, or changes in, other objects or systems.

Essential Knowledge 4.E.3 The charge distribution in a system can be altered by the effects of electric forces produced by a charged object.

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved.

Enduring Understanding 5.C The electric charge of a system is conserved.

Essential Knowledge 5.C.2 The exchange of electric charges among a set of objects in a system conserves electric charge.

18.1 Static Electricity and Charge: Conservation of Charge

Learning Objectives

By the end of this section, you will be able to:

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.

The information presented in this section supports the following AP® learning objectives and science practices:

- **1.B.1.1** The student is able to make claims about natural phenomena based on conservation of electric charge. **(S.P. 6.4)**
- **1.B.1.2** The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits. **(S.P. 6.4, 7.2)**
- **1.B.2.1** The student is able to construct an explanation of the two-charge model of electric charge based on evidence produced through scientific practices. **(S.P. 6.4)**
- **1.B.3.1** The student is able to challenge the claim that an electric charge smaller than the elementary charge has been isolated. **(S.P. 1.5, 6.1, 7.2)**
- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.C.2.1** The student is able to predict electric charges on objects within a system by application of the principle of charge conservation within a system. **(S.P. 6.4)**
- **5.C.2.2** The student is able to design a plan to collect data on the electrical charging of objects and electric charge induction on neutral objects and qualitatively analyze that data. **(S.P. 4.2, 5.1)**
- **5.C.2.3** The student is able to justify the selection of data relevant to an investigation of the electrical charging of objects and electric charge induction on neutral objects. **(S.P. 4.1)**

Figure 18.3 Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge. At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it to attract bits of straw (see **Figure 18.3**). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge “positive”, and the other type “negative.” For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. **Figure 18.4** shows how these simple materials can be used to explore the nature of the force between charges.

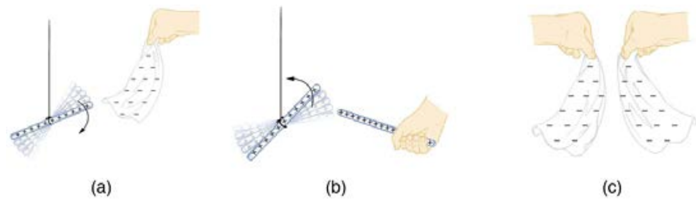


Figure 18.4 A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged. (a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

Figure 18.5 shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.

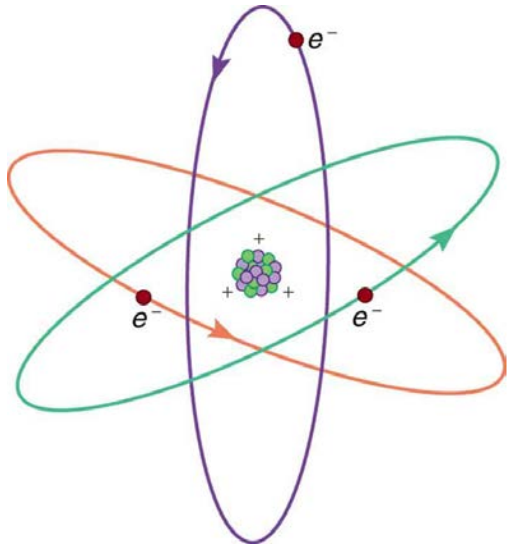


Figure 18.5 This simplified (and not to scale) view of an atom is called the planetary model of the atom. Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual negative and positive charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

$$|q_e| = 1.60 \times 10^{-19}\ \text{C}. \tag{18.1}$$

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

$$1.00\ \text{C} \times \frac{1\ \text{proton}}{1.60 \times 10^{-19}\ \text{C}} = 6.25 \times 10^{18}\ \text{protons}. \tag{18.2}$$

Similarly, 6.25×10^{18} electrons have a combined charge of −1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $|q_e|$ (see **Things Great and Small: The Submicroscopic Origin of Charge**), and all observed charges are integral multiples of $|q_e|$.

Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See **Figure 18.6**.) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

Figure 18.6 shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.

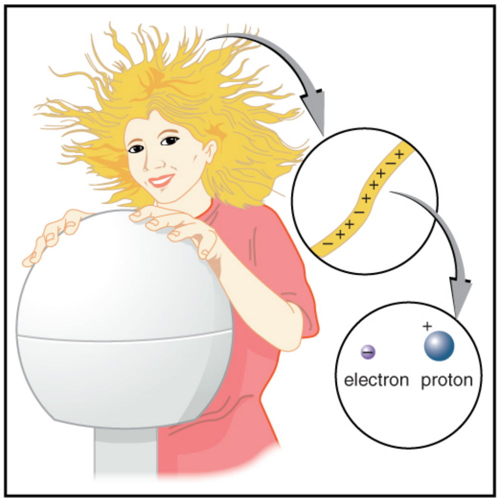


Figure 18.6 When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in **Figure 18.7**. Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$. There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.

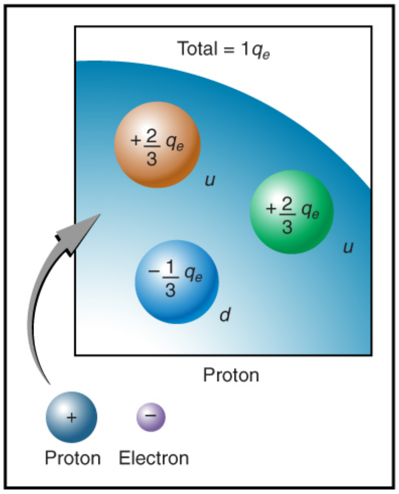


Figure 18.7 Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton: $-\frac{1}{3}q_e + \frac{2}{3}q_e + \frac{2}{3}q_e = +1q_e$.

Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See **Figure 18.8**.) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.

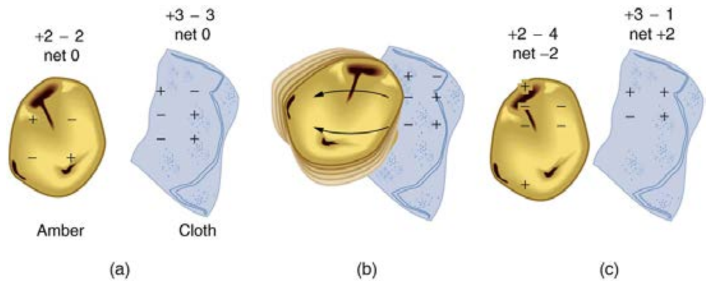


Figure 18.8 When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

Law of Conservation of Charge

Total charge is constant in any process.

Making Connections: Net Charge

Hence if a closed system is neutral, it will remain neutral. Similarly, if a closed system has a charge, say, −10*e*, it will always have that charge. The only way to change the charge of a system is to transfer charge outside, either by bringing in charge or removing charge. If it is possible to transfer charge outside, the system is no longer closed/isolated and is known as an open system. However, charge is always conserved, for both open and closed systems. Consequently, the charge transferred to/from an open system is equal to the change in the system's charge.

For example, each of the two materials (amber and cloth) discussed in **Figure 18.8** have no net charge initially. The only way to change their charge is to transfer charge from outside each object. When they are rubbed together, negative charge is transferred to the amber and the final charge of the amber is the sum of the initial charge and the charge transferred to it. On the other hand, the final charge on the cloth is equal to its initial charge minus the charge transferred out.

Similarly when glass is rubbed with silk, the net charge on the silk is its initial charge plus the incoming charge and the charge on the glass is the initial charge minus the outgoing charge. Also the charge gained by the silk will be equal to the charge lost by the glass, which means that if the silk gains –5*e* charge, the glass would have lost −5*e* charge.

In more exotic situations, such as in particle accelerators, mass, Δm , can be created from energy in the amount $\Delta m = \dfrac{E}{c^2}$.

Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are “matter-antimatter” counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See **Figure 18.9**.) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy *E*, again obeying the relationship $\Delta m = \dfrac{E}{c^2}$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

Making Connections: Conservation Laws

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.

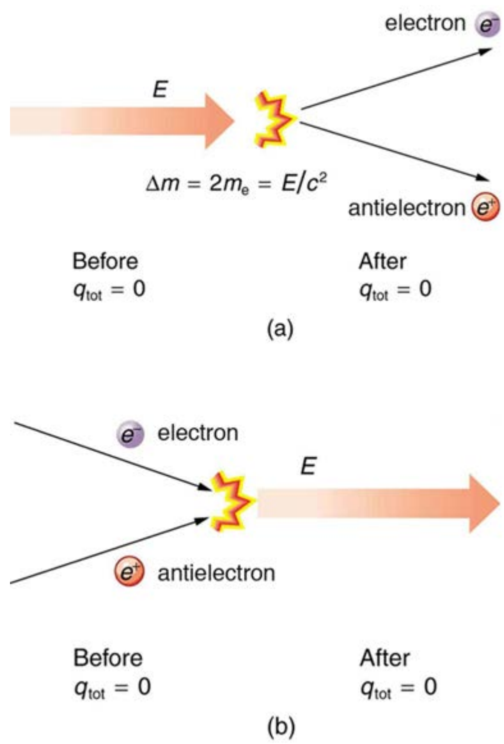


Figure 18.9 (a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. (m_e is the electron's mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

PhET Explorations: Balloons and Static Electricity

Why does a balloon stick to your sweater? Rub a balloon on a sweater, then let go of the balloon and it flies over and sticks to the sweater. View the charges in the sweater, balloons, and the wall.

PhET Interactive Simulation

Figure 18.10 Balloons and Static Electricity (http://cnx.org/content/m55300/1.2/balloons_en.jar)

Applying the Science Practices: Electrical Charging

Design an experiment to demonstrate the electrical charging of objects, by using a glass rod, a balloon, small bits of paper, and different pieces of cloth (like silk, wool, or nylon). Also show that like charges repel each other whereas unlike charges attract each other.

18.2 Conductors and Insulators

Learning Objectives

By the end of this section, you will be able to:

- Define conductor and insulator, explain the difference, and give examples of each.
- Describe three methods for charging an object.
- Explain what happens to an electric force as you move farther from the source.
- Define polarization.

The information presented in this section supports the following AP® learning objectives and science practices:

- **1.B.2.2** The student is able to make a qualitative prediction about the distribution of positive and negative electric charges within neutral systems as they undergo various processes. **(S.P. 6.4, 7.2)**
- **1.B.2.3** The student is able to challenge claims that polarization of electric charge or separation of charge must result in a net charge on the object. **(S.P. 6.1)**
- **4.E.3.1** The student is able to make predictions about the redistribution of charge during charging by friction, conduction, and induction. **(S.P. 6.4)**
- **4.E.3.2** The student is able to make predictions about the redistribution of charge caused by the electric field due to other systems, resulting in charged or polarized objects. **(S.P. 6.4, 7.2)**
- **4.E.3.3** The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors. **(S.P. 1.1, 1.4, 6.4)**
- **4.E.3.4** The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors that predicts charge distribution in processes involving induction or conduction. **(S.P. 1.1, 1.4, 6.4)**
- **4.E.3.5** The student is able to plan and/or analyze the results of experiments in which electric charge rearrangement occurs by electrostatic induction, or is able to refine a scientific question relating to such an experiment by identifying anomalies in a data set or procedure. **(S.P. 3.2, 4.1, 4.2, 5.1, 5.3)**

Figure 18.11 This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: Evan-Amos, Wikimedia Commons)

Some substances, such as metals and salty water, allow charges to move through them with relative ease. Some of the electrons in metals and similar conductors are not bound to individual atoms or sites in the material. These **free electrons** can move through the material much as air moves through loose sand. Any substance that has free electrons and allows charge to move relatively freely through it is called a **conductor**. The moving electrons may collide with fixed atoms and molecules, losing some energy, but they can move in a conductor. Superconductors allow the movement of charge without any loss of energy. Salty water and other similar conducting materials contain free ions that can move through them. An ion is an atom or molecule having a positive or negative (nonzero) total charge. In other words, the total number of electrons is not equal to the total number of protons.

Other substances, such as glass, do not allow charges to move through them. These are called **insulators**. Electrons and ions in insulators are bound in the structure and cannot move easily—as much as 10^{23} times more slowly than in conductors. Pure water and dry table salt are insulators, for example, whereas molten salt and salty water are conductors.

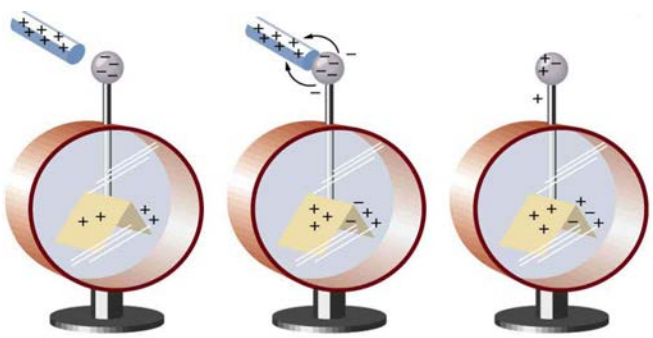


Figure 18.12 An electroscope is a favorite instrument in physics demonstrations and student laboratories. It is typically made with gold foil leaves hung from a (conducting) metal stem and is insulated from the room air in a glass-walled container. (a) A positively charged glass rod is brought near the tip of the electroscope, attracting electrons to the top and leaving a net positive charge on the leaves. Like charges in the light flexible gold leaves repel, separating them. (b) When the rod is touched against the ball, electrons are attracted and transferred, reducing the net charge on the glass rod but leaving the electroscope positively charged. (c) The excess charges are evenly distributed in the stem and leaves of the electroscope once the glass rod is removed.

Charging by Contact

Figure 18.12 shows an electroscope being charged by touching it with a positively charged glass rod. Because the glass rod is an insulator, it must actually touch the electroscope to transfer charge to or from it. (Note that the extra positive charges reside on the surface of the glass rod as a result of rubbing it with silk before starting the experiment.) Since only electrons move in metals, we see that they are attracted to the top of the electroscope. There, some are transferred to the positive rod by touch, leaving the electroscope with a net positive charge.

Electrostatic repulsion in the leaves of the charged electroscope separates them. The electrostatic force has a horizontal component that results in the leaves moving apart as well as a vertical component that is balanced by the gravitational force. Similarly, the electroscope can be negatively charged by contact with a negatively charged object.

Charging by Induction

It is not necessary to transfer excess charge directly to an object in order to charge it. **Figure 18.13** shows a method of **induction** wherein a charge is created in a nearby object, without direct contact. Here we see two neutral metal spheres in contact with one another but insulated from the rest of the world. A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

This is an example of induced **polarization** of neutral objects. Polarization is the separation of charges in an object that remains neutral. If the spheres are now separated (before the rod is pulled away), each sphere will have a net charge. Note that the object closest to the charged rod receives an opposite charge when charged by induction. Note also that no charge is removed from the charged rod, so that this process can be repeated without depleting the supply of excess charge.

Another method of charging by induction is shown in **Figure 18.14**. The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since the earth is large and most ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction and the charged rod loses none of its excess charge.

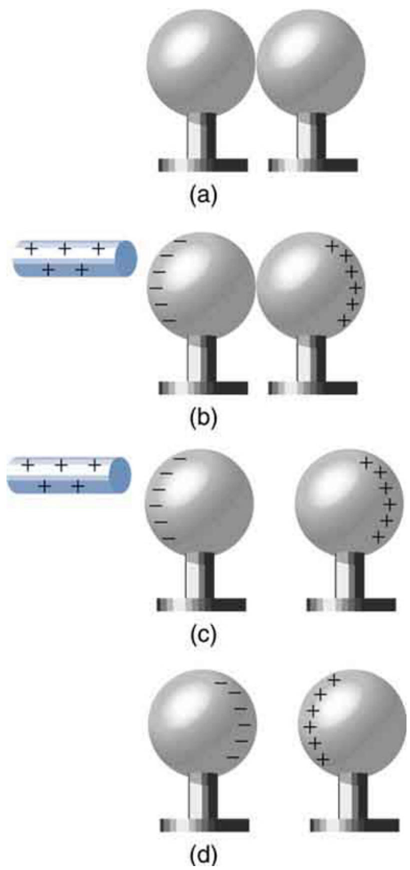


Figure 18.13 Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The spheres are separated before the rod is removed, thus separating negative and positive charge. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.

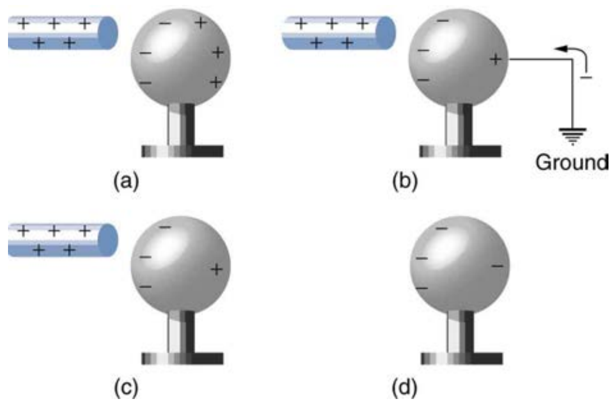


Figure 18.14 Charging by induction, using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from the earth's ample supply. (c) The ground connection is broken. (d) The positive rod is removed, leaving the sphere with an induced negative charge.

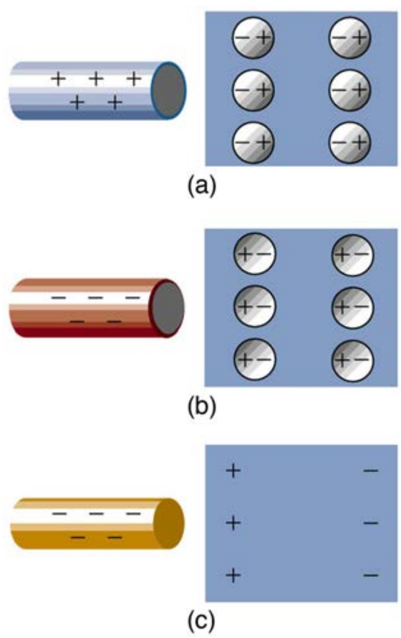


Figure 18.15 Both positive and negative objects attract a neutral object by polarizing its molecules. (a) A positive object brought near a neutral insulator polarizes its molecules. There is a slight shift in the distribution of the electrons orbiting the molecule, with unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. **Figure 18.15** shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

Check Your Understanding

Can you explain the attraction of water to the charged rod in the figure below?

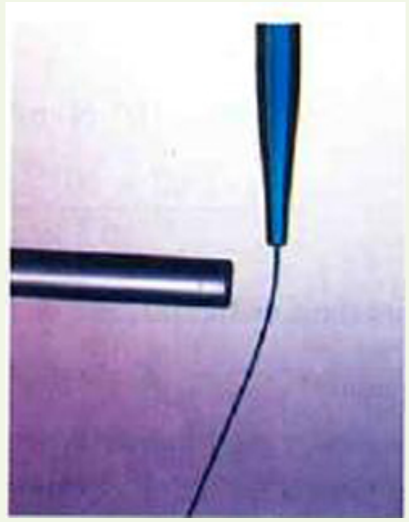


Figure 18.16

Solution

Water molecules are polarized, giving them slightly positive and slightly negative sides. This makes water even more susceptible to a charged rod's attraction. As the water flows downward, due to the force of gravity, the charged conductor exerts a net attraction to the opposite charges in the stream of water, pulling it closer.

Applying the Science Practices: Electrostatic Induction

Plan an experiment to demonstrate electrostatic induction using household items, like balloons, woolen cloth, aluminum drink cans, or foam cups. Explain the process of induction in your experiment by discussing details of (and making diagrams relating to) the movement and alignment of charges.

PhET Explorations: John Travoltage

Make sparks fly with John Travoltage. Wiggle Johnnie's foot and he picks up charges from the carpet. Bring his hand close to the door knob and get rid of the excess charge.

PhET Interactive Simulation

Figure 18.17 John Travoltage (http://cnx.org/content/m55301/1.2/travoltage_en.jar)

18.3 Conductors and Electric Fields in Static Equilibrium

Learning Objectives

By the end of this section, you will be able to:

- List the three properties of a conductor in electrostatic equilibrium.
- Explain the effect of an electric field on free charges in a conductor.
- Explain why no electric field may exist inside a conductor.
- Describe the electric field surrounding Earth.
- Explain what happens to an electric field applied to an irregular conductor.
- Describe how a lightning rod works.
- Explain how a metal car may protect passengers inside from the dangerous electric fields caused by a downed line touching the car.

The information presented in this section supports the following AP learning objectives:

- **2.C.3.1** The student is able to explain the inverse square dependence of the electric field surrounding a spherically symmetric electrically charged object.

- **2.C.5.1** The student is able to create representations of the magnitude and direction of the electric field at various distances (small compared to plate size) from two electrically charged plates of equal magnitude and opposite signs and is able to recognize that the assumption of uniform field is not appropriate near edges of plates.

Conductors contain **free charges** that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called **electrostatic equilibrium**.

Figure 18.18 shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor's surface. There can be no component of the field parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge. A positive free charge is shown, but free charges can be either positive or negative and are, in fact, negative in metals. The motion of a positive charge is equivalent to the motion of a negative charge in the opposite direction.

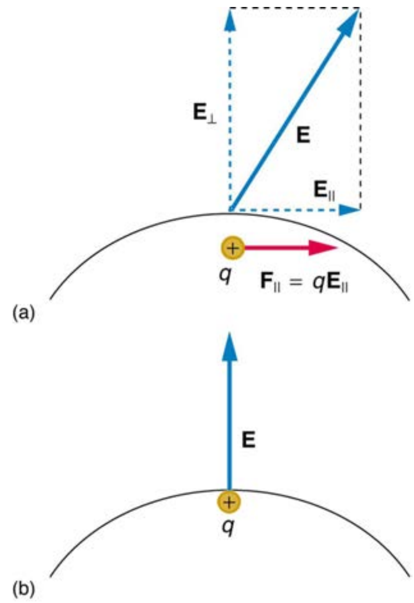


Figure 18.18 When an electric field $\mathbf{E}$ is applied to a conductor, free charges inside the conductor move until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component ($\mathbf{E}_{||}$) exerts a force ($\mathbf{F}_{||}$) on the free charge q , which moves the charge until $\mathbf{F}_{||} = 0$. (b) The resulting field is perpendicular to the surface. The free charge has been brought to the conductor's surface, leaving electrostatic forces in equilibrium.

A conductor placed in an **electric field** will be **polarized**. **Figure 18.19** shows the result of placing a neutral conductor in an originally uniform electric field. The field becomes stronger near the conductor but entirely disappears inside it.

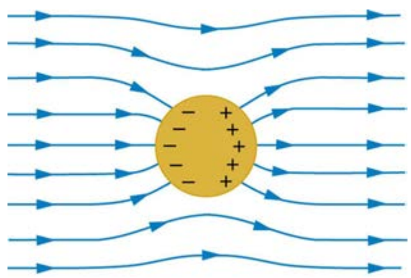


Figure 18.19 This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin again on excess positive charge on the opposite side. No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized.

Misconception Alert: Electric Field inside a Conductor

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in **Figure 18.20**. Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its center equal to the excess charge.

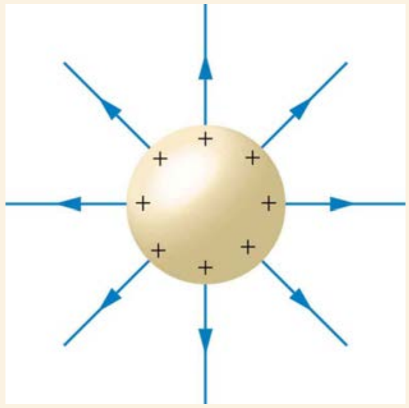


Figure 18.20 The mutual repulsion of excess positive charges on a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the conductor, the field is identical to that of a point charge at the center equal to the excess charge.

Properties of a Conductor in Electrostatic Equilibrium

1. The electric field is zero inside a conductor.
2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in **Figure 18.21**. The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges, the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat). The edge effects are less important when the plates are close together.

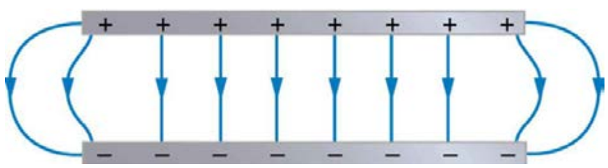


Figure 18.21 Two metal plates with equal, but opposite, excess charges. The field between them is uniform in strength and direction except near the edges. One use of such a field is to produce uniform acceleration of charges between the plates, such as in the electron gun of a TV tube.

Earth's Electric Field

A near uniform electric field of approximately 150 N/C, directed downward, surrounds Earth, with the magnitude increasing slightly as we get closer to the surface. What causes the electric field? At around 100 km above the surface of Earth we have a layer of charged particles, called the **ionosphere**. The ionosphere is responsible for a range of phenomena including the electric field surrounding Earth. In fair weather the ionosphere is positive and the Earth largely negative, maintaining the electric field (**Figure 18.22**(a)).

In storm conditions clouds form and localized electric fields can be larger and reversed in direction (**Figure 18.22**(b)). The exact charge distributions depend on the local conditions, and variations of **Figure 18.22**(b) are possible.

If the electric field is sufficiently large, the insulating properties of the surrounding material break down and it becomes conducting. For air this occurs at around 3×10^6 N/C. Air ionizes ions and electrons recombine, and we get discharge in the form of lightning sparks and corona discharge.

Figure 18.22 Earth's electric field. (a) Fair weather field. Earth and the ionosphere (a layer of charged particles) are both conductors. They produce a uniform electric field of about 150 N/C. (credit: D. H. Parks) (b) Storm fields. In the presence of storm clouds, the local electric fields can be larger. At very high fields, the insulating properties of the air break down and lightning can occur. (credit: Jan-Joost Verhoef)

Electric Fields on Uneven Surfaces

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points.

To see how and why this happens, consider the charged conductor in **Figure 18.23**. The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. This is because the forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surfaces are different. The component parallel to the surface is greatest on the flattest surface and, hence, more effective in moving the charge.

The same effect is produced on a conductor by an externally applied electric field, as seen in **Figure 18.23** (c). Since the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.

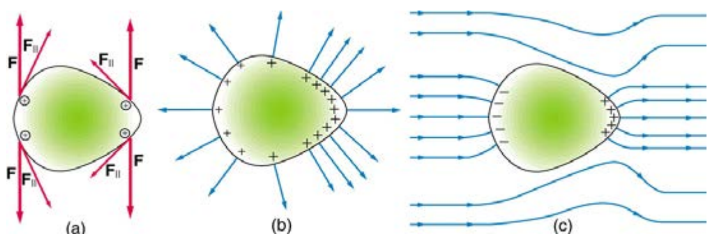


Figure 18.23 Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature. (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is $\mathbf{F}_{||}$ that moves the charges apart once they have reached the surface. (b) $\mathbf{F}_{||}$ is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c) An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.

Applications of Conductors

On a very sharply curved surface, such as shown in **Figure 18.24**, the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible. (See **Figure 18.25**.) Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a **Faraday cage**. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active (“hot”) electrical wire was broken (in a storm or an accident) and fell on your car.

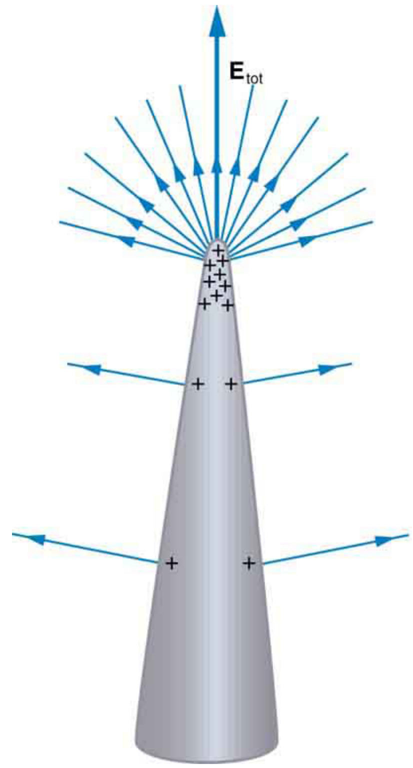


Figure 18.24 A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor. Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.

Figure 18.25 (a) A lightning rod is pointed to facilitate the transfer of charge. (credit: Romaine, Wikimedia Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person's hair while touching the metal sphere. (credit: Jon 'ShakataGaNai' Davis/Wikimedia Commons).

18.4 Coulomb's Law

Learning Objectives

By the end of this section, you will be able to:

- State Coulomb's law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two point charges, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.A.3.4** The student is able to make claims about the force on an object due to the presence of other objects with the same property: mass, electric charge. **(S.P. 6.1, 6.4)**
- **3.C.2.1** The student is able to use Coulomb's law qualitatively and quantitatively to make predictions about the interaction between two electric point charges (interactions between collections of electric point charges are not covered in Physics 1 and instead are restricted to Physics 2). **(S.P. 2.2, 6.4)**
- **3.C.2.2** The student is able to connect the concepts of gravitational force and electric force to compare similarities and differences between the forces. **(S.P. 7.2)**

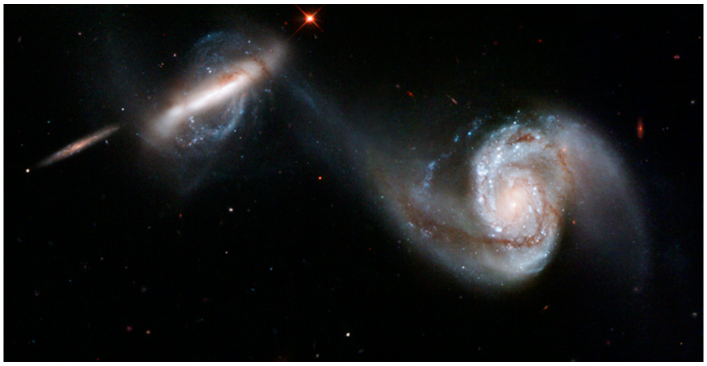


Figure 18.26 This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Through the work of scientists in the late 18th century, the main features of the **electrostatic force**—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called **Coulomb's law** after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Coulomb's Law

$$F = k\frac{|q_1 q_2|}{r^2}. \tag{18.3}$$

Coulomb's law calculates the magnitude of the force F between two point charges, q_1 and q_2, separated by a distance r. In SI units, the constant k is equal to

$$k = 8.988\times10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \approx 8.99\times10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}. \tag{18.4}$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See **Figure 18.27**.)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared $\left(F \propto 1/r^2\right)$ to an accuracy of 1 part in 10^{16}. No exceptions have ever been found, even at the small distances within the atom.

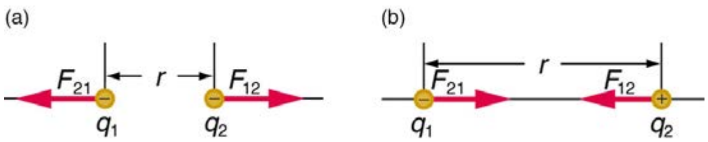


Figure 18.27 The magnitude of the electrostatic force F between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2. (a) Like charges. (b) Unlike charges.

Making Connections: Comparing Gravitational and Electrostatic Forces

Recall that the gravitational force (Newton's law of gravitation) quantifies force as $F_s = G\frac{mM}{r^2}$.

The comparison between the two forces—gravitational and electrostatic—shows some similarities and differences. Gravitational force is proportional to the masses of interacting objects, and the electrostatic force is proportional to the magnitudes of the charges of interacting objects. Hence both forces are proportional to a property that represents the strength of interaction for a given field. In addition, both forces are inversely proportional to the square of the distances between them. It may seem that the two forces are related but that is not the case. In fact, there are huge variations in the magnitudes of the two forces as they depend on different parameters and different mechanisms. For electrons (or protons), electrostatic force is dominant and is much greater than the gravitational force. On the other hand, gravitational force is generally dominant for objects with large masses. Another major difference between the two forces is that gravitational force can only be attractive, whereas electrostatic could be attractive or repulsive (depending on the sign of charges; unlike charges attract and like charges repel).

Example 18.1 How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by $0.530\times10^{-10}\text{ m}$ with the gravitational force between them. This distance is their average separation in a hydrogen atom.

Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law, $F = k\frac{|q_1 q_2|}{r^2}$. We then calculate the gravitational force using Newton's universal law of gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

Solution

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

$$F = k\frac{|q_1 q_2|}{r^2} \tag{18.5}$$

$$= \left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\times\frac{(1.60\times10^{-19}\text{ C})(1.60\times10^{-19}\text{ C})}{(0.530\times10^{-10}\text{ m})^2} \tag{18.6}$$

Thus the Coulomb force is

$$F = 8.19\times10^{-8}\text{ N}. \tag{18.7}$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of $8.99\times10^{22}\text{ m/s}^2$ (verification is left as an end-of-section problem).The gravitational force is given by Newton's law of gravitation as:

$$F_G = G\frac{mM}{r^2}, \tag{18.8}$$

where $G = 6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2$. Here m and M represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

$$F_G = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\times\frac{(9.11\times10^{-31}\text{ kg})(1.67\times10^{-27}\text{ kg})}{(0.530\times10^{-10}\text{ m})^2} = 3.61\times10^{-47}\text{ N} \tag{18.9}$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

$$\frac{F}{F_G} = 2.27\times10^{39}. \tag{18.10}$$

Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive **Coulomb forces** nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. **Figure 18.38** shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

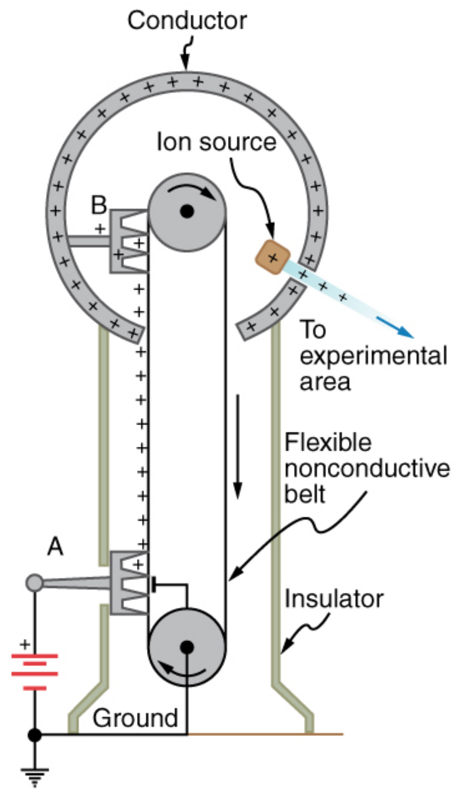


Figure 18.38 Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Take-Home Experiment: Electrostatics and Humidity

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in **Figure 18.39**.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is **grounded** so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull electric field is given to be upward, the electric force is upward. We thus have a one-dimensional (vertical direction) problem, and we can state Newton's second law as

$$a = \frac{F_{\text{net}}}{m}. \tag{18.24}$$

where $F_{\text{net}} = F - w$. Entering this and the known values into the expression for Newton's second law yields

$$\begin{aligned} a &= \frac{F - w}{m} \\ &= \frac{9.60\times10^{-14}\text{ N} - 3.92\times10^{-14}\text{ N}}{4.00\times10^{-15}\text{ kg}} \\ &= 14.2\text{ m/s}^2. \end{aligned} \tag{18.25}$$

Discussion for (c)

This is an upward acceleration great enough to carry the drop to places where you might not wish to have gasoline.

This worked example illustrates how to apply problem-solving strategies to situations that include topics in different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Unreasonable Results

The Unreasonable Results exercises for this module have results that are unreasonable because some premise is unreasonable or because certain of the premises are inconsistent with one another. Physical principles applied correctly then produce unreasonable results. The purpose of these problems is to give practice in assessing whether nature is being accurately described, and if it is not to trace the source of difficulty.

Problem-Solving Strategy

To determine if an answer is reasonable, and to determine the cause if it is not, do the following.

1. Solve the problem using strategies as outlined above. Use the format followed in the worked examples in the text to solve the problem as usual.
2. Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, and so on?
3. If the answer is unreasonable, look for what specifically could cause the identified difficulty. Usually, the manner in which the answer is unreasonable is an indication of the difficulty. For example, an extremely large Coulomb force could be due to the assumption of an excessively large separated charge.

Glossary

conductor: a material that allows electrons to move separately from their atomic orbits

conductor: an object with properties that allow charges to move about freely within it

Coulomb force: another term for the electrostatic force

Coulomb interaction: the interaction between two charged particles generated by the Coulomb forces they exert on one another

Coulomb's law: the mathematical equation calculating the electrostatic force vector between two charged particles

dipole: a molecule's lack of symmetrical charge distribution, causing one side to be more positive and another to be more negative

electric charge: a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

electric field: a three-dimensional map of the electric force extended out into space from a point charge

electric field lines: a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

electromagnetic force: one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

electron: a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

electrostatic equilibrium: an electrostatically balanced state in which all free electrical charges have stopped moving about

electrostatic force: the amount and direction of attraction or repulsion between two charged bodies

electrostatic precipitators: filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

electrostatic repulsion: the phenomenon of two objects with like charges repelling each other

electrostatics: the study of electric forces that are static or slow-moving

Faraday cage: a metal shield which prevents electric charge from penetrating its surface

field: a map of the amount and direction of a force acting on other objects, extending out into space

free charge: an electrical charge (either positive or negative) which can move about separately from its base molecule

free electron: an electron that is free to move away from its atomic orbit

grounded: when a conductor is connected to the Earth, allowing charge to freely flow to and from Earth's unlimited reservoir

grounded: connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

induction: the process by which an electrically charged object brought near a neutral object creates a charge in that object

ink-jet printer: small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

insulator: a material that holds electrons securely within their atomic orbits

ionosphere: a layer of charged particles located around 100 km above the surface of Earth, which is responsible for a range of phenomena including the electric field surrounding Earth

laser printer: uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

law of conservation of charge: states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

photoconductor: a substance that is an insulator until it is exposed to light, when it becomes a conductor

point charge: A charged particle, designated Q, generating an electric field

polar molecule: a molecule with an asymmetrical distribution of positive and negative charge

polarization: slight shifting of positive and negative charges to opposite sides of an atom or molecule

polarized: a state in which the positive and negative charges within an object have collected in separate locations

proton: a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

screening: the dilution or blocking of an electrostatic force on a charged object by the presence of other charges nearby

static electricity: a buildup of electric charge on the surface of an object

test charge: A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

Van de Graaff generator: a machine that produces a large amount of excess charge, used for experiments with high voltage

vector: a quantity with both magnitude and direction

vector addition: mathematical combination of two or more vectors, including their magnitudes, directions, and positions

xerography: a dry copying process based on electrostatics

Section Summary

18.1 Static Electricity and Charge: Conservation of Charge

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $|q_e|$ is

$$|q_e| = 1.60\times10^{-19}\text{ C}.$$

- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.
- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

18.2 Conductors and Insulators

- Polarization is the separation of positive and negative charges in a neutral object.
- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge within its atomic structure.
- Objects with like charges repel each other, while those with unlike charges attract each other.
- A conducting object is said to be grounded if it is connected to the Earth through a conductor. Grounding allows transfer of charge to and from the earth's large reservoir.
- Objects can be charged by contact with another charged object and obtain the same sign charge.
- If an object is temporarily grounded, it can be charged by induction, and obtains the opposite sign charge.
- Polarized objects have their positive and negative charges concentrated in different areas, giving them a non-symmetrical charge.
- Polar molecules have an inherent separation of charge.

18.3 Conductors and Electric Fields in Static Equilibrium

- A conductor allows free charges to move about within it.
- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.
- Electrical storms result when the electrical field of Earth's surface in certain locations becomes more strongly charged, due to changes in the insulating effect of the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.

18.4 Coulomb’s Law

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is

$$F = k\frac{|q_1 q_2|}{r^2},$$

  where q_1 and q_2 are two point charges separated by a distance r, and $k \approx 8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$
- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

18.5 Electric Field: Concept of a Field Revisited

- The electrostatic force field surrounding a charged object extends out into space in all directions.

- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the distance between the two.
- The electric field $\mathbf{E}$ is defined to be

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge q . $\mathbf{E}$ has units of N/C.
- The magnitude of the electric field $\mathbf{E}$ created by a point charge Q is

$$\mathbf{E} = k\frac{|Q|}{r^2}.$$

where r is the distance from Q . The electric field $\mathbf{E}$ is a vector and fields due to multiple charges add like vectors.

18.6 Electric Field Lines: Multiple Charges

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

18.7 Electric Forces in Biology

- Many molecules in living organisms, such as DNA, carry a charge.
- An uneven distribution of the positive and negative charges within a polar molecule produces a dipole.
- The effect of a Coulomb field generated by a charged object may be reduced or blocked by other nearby charged objects.
- Biological systems contain water, and because water molecules are polar, they have a strong effect on other molecules in living systems.

18.8 Applications of Electrostatics

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

Conceptual Questions

18.1 Static Electricity and Charge: Conservation of Charge

1. There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

2. Why do most objects tend to contain nearly equal numbers of positive and negative charges?

18.2 Conductors and Insulators

3. An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on himself, his clothes fly off. Explain.

4. If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near its knob?

5. When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

6. Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

7. Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?

8. What is grounding? What effect does it have on a charged conductor? On a charged insulator?

18.3 Conductors and Electric Fields in Static Equilibrium

9. Is the object in a conductor or an insulator? Justify your answer.

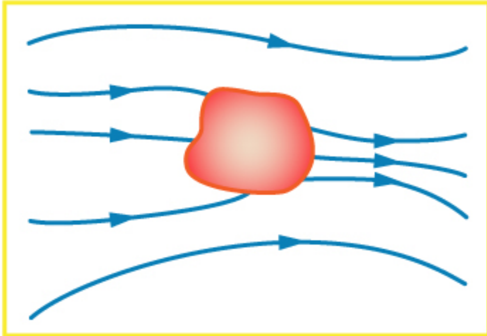


Figure 18.43

10. If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.

11. The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?

12. Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor by the application of a similar externally created electric field? (The answers to both questions have implications for charge transfer utilizing points.)

13. Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?

14. Can the belt of a Van de Graaff accelerator be a conductor? Explain.

15. Are you relatively safe from lightning inside an automobile? Give two reasons.

16. Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.

17. Using the symmetry of the arrangement, show that the net Coulomb force on the charge q at the center of the square below (**Figure 18.44**) is zero if the charges on the four corners are exactly equal.

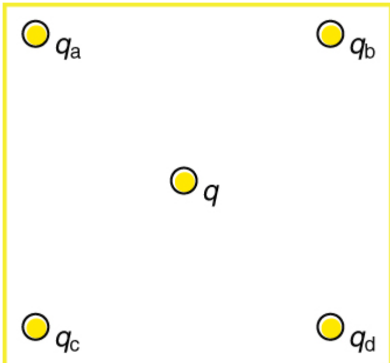


Figure 18.44 Four point charges q_a, q_b, q_c, and q_d lie on the corners of a square and q is located at its center.

18. (a) Using the symmetry of the arrangement, show that the electric field at the center of the square in **Figure 18.44** is zero if the charges on the four corners are exactly equal. (b) Show that this is also true for any combination of charges in which $q_a = q_b$ and $q_b = q_c$

19. (a) What is the direction of the total Coulomb force on q in **Figure 18.44** if q is negative, $q_a = q_c$ and both are negative, and $q_b = q_c$ and both are positive? (b) What is the direction of the electric field at the center of the square in this situation?

20. Considering **Figure 18.44**, suppose that $q_a = q_d$ and $q_b = q_c$. First show that q is in static equilibrium. (You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of q from the center of the square.

21. If $q_a = 0$ in **Figure 18.44**, under what conditions will there be no net Coulomb force on q?

22. In regions of low humidity, one develops a special “grip” when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one's fingers. Discuss the induced charge and explain why this is done.

23. Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?

24. Suppose a woman carries an excess charge. To maintain her charged status can she be standing on ground wearing just any pair of shoes? How would you discharge her? What are the consequences if she simply walks away?

18.4 Coulomb's Law

25. Figure 18.45 shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.

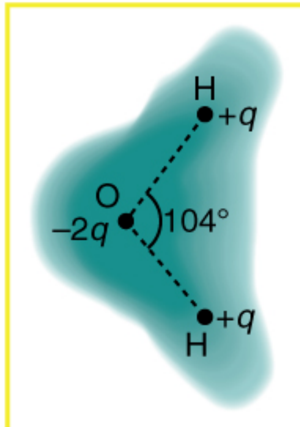


Figure 18.45 Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar molecule*. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

26. Using Figure 18.45, explain, in terms of Coulomb's law, why a polar molecule (such as in Figure 18.45) is attracted by both positive and negative charges.

27. Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

18.5 Electric Field: Concept of a Field Revisited

28. Why must the test charge q in the definition of the electric field be vanishingly small?

29. Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

18.6 Electric Field Lines: Multiple Charges

30. Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

31. Figure 18.46 shows an electric field extending over three regions, labeled I, II, and III. Answer the following questions. (a) Are there any isolated charges? If so, in what region and what are their signs? (b) Where is the field strongest? (c) Where is it weakest? (d) Where is the field the most uniform?

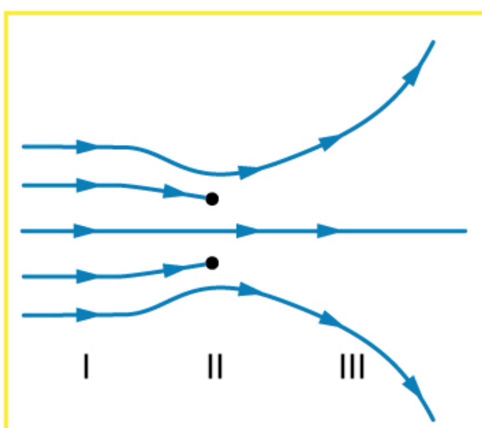


Figure 18.46

18.7 Electric Forces in Biology

32. A cell membrane is a thin layer enveloping a cell. The thickness of the membrane is much less than the size of the cell. In a static situation the membrane has a charge distribution of -2.5×10^{-6} C/m ^{2} on its inner surface and $+2.5\times10^{-6}$ C/m^2 on its outer surface. Draw a diagram of the cell and the surrounding cell membrane. Include on this diagram the charge distribution and the corresponding electric field. Is there any electric field inside the cell? Is there any electric field outside the cell?

Problems & Exercises

18.1 Static Electricity and Charge: Conservation of Charge

1. Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of -2.00 nC (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500\ \mu\text{C}$?

2. If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

3. To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

4. A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $|q_e|$ is this?

18.2 Conductors and Insulators

5. Suppose a speck of dust in an electrostatic precipitator has 1.0000×10^{12} protons in it and has a net charge of –5.00 nC (a very large charge for a small speck). How many electrons does it have?

6. An amoeba has 1.00×10^{16} protons and a net charge of 0.300 pC. (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?

7. A 50.0 g ball of copper has a net charge of $2.00\ \mu\text{C}$. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

8. What net charge would you place on a 100 g piece of sulfur if you put an extra electron on 1 in 10^{12} of its atoms? (Sulfur has an atomic mass of 32.1.)

9. How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

18.3 Conductors and Electric Fields in Static Equilibrium

10. Sketch the electric field lines in the vicinity of the conductor in **Figure 18.47** given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?

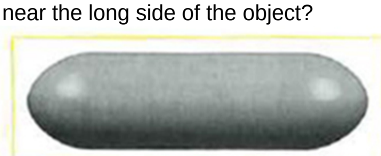


Figure 18.47

11. Sketch the electric field lines in the vicinity of the conductor in **Figure 18.48** given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?

Figure 18.48

12. Sketch the electric field between the two conducting plates shown in **Figure 18.49**, given the top plate is positive and an equal amount of negative charge is on the bottom plate. Be certain to indicate the distribution of charge on the plates.

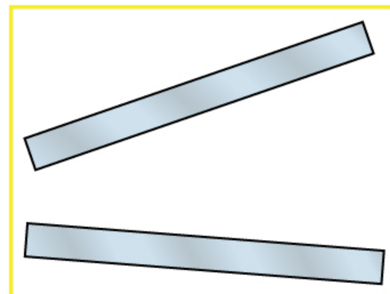


Figure 18.49

13. Sketch the electric field lines in the vicinity of the charged insulator in **Figure 18.50** noting its nonuniform charge distribution.

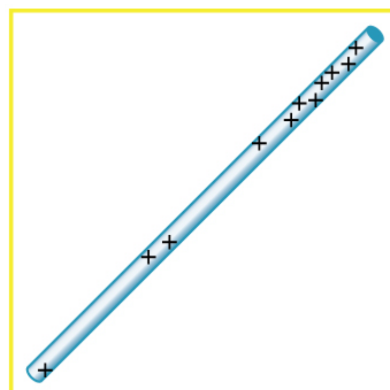


Figure 18.50 A charged insulating rod such as might be used in a classroom demonstration.

14. What is the force on the charge located at $x = 8.00\text{ cm}$ in **Figure 18.51**(a) given that $q = 1.00\ \mu\text{C}$?

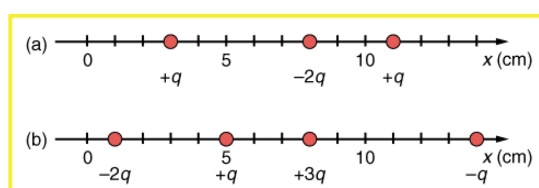


Figure 18.51 (a) Point charges located at 3.00, 8.00, and 11.0 cm along the *x*-axis. (b) Point charges located at 1.00, 5.00, 8.00, and 14.0 cm along the *x*-axis.

15. (a) Find the total electric field at $x = 1.00\text{ cm}$ in **Figure 18.51**(b) given that $q = 5.00\text{ nC}$. (b) Find the total electric field at $x = 11.00\text{ cm}$ in **Figure 18.51**(b). (c) If the charges are allowed to move and eventually be brought to rest by friction, what will the final charge configuration be? (That is, will there be a single charge, double charge, etc., and what will its value(s) be?)

16. (a) Find the electric field at $x = 5.00\text{ cm}$ in **Figure 18.51**(a), given that $q = 1.00\ \mu\text{C}$. (b) At what position between 3.00 and 8.00 cm is the total electric field the same as that for $-2q$ alone? (c) Can the electric field be zero anywhere between 0.00 and 8.00 cm? (d) At very large positive or negative values of *x*, the electric field approaches zero in both (a) and (b). In which does it most rapidly approach zero and why? (e) At what position to the right of 11.0 cm is the total electric field zero, other than at infinity? (Hint: A graphing calculator can yield considerable insight in this problem.)

17. (a) Find the total Coulomb force on a charge of 2.00 nC located at $x = 4.00\text{ cm}$ in **Figure 18.51** (b), given that $q = 1.00\ \mu\text{C}$. (b) Find the *x*-position at which the electric field is zero in **Figure 18.51** (b).

18. Using the symmetry of the arrangement, determine the direction of the force on q in the figure below, given that $q_a = q_b = +7.50\ \mu\text{C}$ and $q_c = q_d = -7.50\ \mu\text{C}$. (b) Calculate the magnitude of the force on the charge q, given that the square is 10.0 cm on a side and $q = 2.00\ \mu\text{C}$.

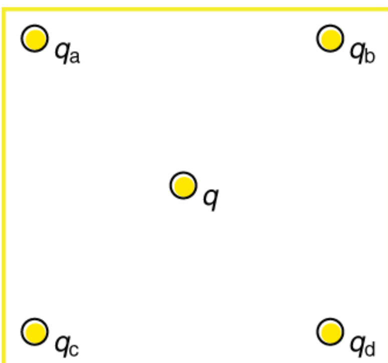


Figure 18.52

19. (a) Using the symmetry of the arrangement, determine the direction of the electric field at the center of the square in **Figure 18.52**, given that $q_a = q_b = -1.00\ \mu\text{C}$ and $q_c = q_d = +1.00\ \mu\text{C}$. (b) Calculate the magnitude of the electric field at the location of q, given that the square is 5.00 cm on a side.

20. Find the electric field at the location of q_a in **Figure 18.52** given that $q_b = q_c = q_d = +2.00\text{ nC}$, $q = -1.00\text{ nC}$, and the square is 20.0 cm on a side.

21. Find the total Coulomb force on the charge q in **Figure 18.52**, given that $q = 1.00\ \mu\text{C}$, $q_a = 2.00\ \mu\text{C}$, $q_b = -3.00\ \mu\text{C}$, $q_c = -4.00\ \mu\text{C}$, and $q_d = +1.00\ \mu\text{C}$. The square is 50.0 cm on a side.

22. (a) Find the electric field at the location of q_a in **Figure 18.53**, given that $q_\text{b} = +10.00\ \mu\text{C}$ and $q_\text{c} = -5.00\ \mu\text{C}$. (b) What is the force on q_a, given that $q_\text{a} = +1.50\text{ nC}$?

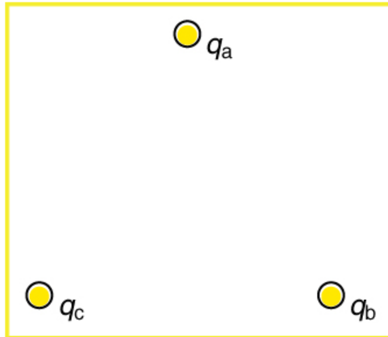


Figure 18.53 Point charges located at the corners of an equilateral triangle 25.0 cm on a side.

23. (a) Find the electric field at the center of the triangular configuration of charges in **Figure 18.53**, given that $q_a = +2.50\text{ nC}$, $q_b = -8.00\text{ nC}$, and $q_c = +1.50\text{ nC}$. (b) Is there any combination of charges, other than $q_a = q_b = q_c$, that will produce a zero strength electric field at the center of the triangular configuration?

18.4 Coulomb's Law

24. What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of – 30.0 nC?

25. (a) How strong is the attractive force between a glass rod with a $0.700\ \mu\text{C}$ charge and a silk cloth with a $-0.600\ \mu\text{C}$ charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

26. Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

27. Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

28. How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

29. If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

30. A test charge of $+2\ \mu\text{C}$ is placed halfway between a charge of $+6\ \mu\text{C}$ and another of $+4\ \mu\text{C}$ separated by 10 cm. (a) What is the magnitude of the force on the test charge? (b) What is the direction of this force (away from or toward the $+6\ \mu\text{C}$ charge)?

31. Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

32. (a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

33. Suppose you have a total charge q_{tot} that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

34. (a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

35. (a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

36. At what distance is the electrostatic force between two protons equal to the weight of one proton?

37. A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

38. (a) Two point charges totaling $8.00\ \mu\text{C}$ exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?

39. Point charges of $5.00\ \mu\text{C}$ and $-3.00\ \mu\text{C}$ are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?

40. Two point charges q_1 and q_2 are $3.00\ \text{m}$ apart, and their total charge is $20\ \mu\text{C}$. (a) If the force of repulsion between them is 0.075N, what are magnitudes of the two charges? (b) If one charge attracts the other with a force of 0.525N, what are the magnitudes of the two charges? Note that you may need to solve a quadratic equation to reach your answer.

18.5 Electric Field: Concept of a Field Revisited

41. What is the magnitude and direction of an electric field that exerts a $2.00\times10^{-5}\ \text{N}$ upward force on a $-1.75\ \mu\text{C}$ charge?

42. What is the magnitude and direction of the force exerted on a $3.50\ \mu\text{C}$ charge by a 250 N/C electric field that points due east?

43. Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

44. (a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?

45. Calculate the initial (from rest) acceleration of a proton in a $5.00\times10^{6}\ \text{N/C}$ electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

46. (a) Find the direction and magnitude of an electric field that exerts a $4.80\times10^{-17}\ \text{N}$ westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

18.6 Electric Field Lines: Multiple Charges

47. (a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$.

48. Sketch the electric field lines a long distance from the charge distributions shown in Figure 18.34 (a) and (b)

49. Figure 18.54 shows the electric field lines near two charges q_1 and q_2. What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.

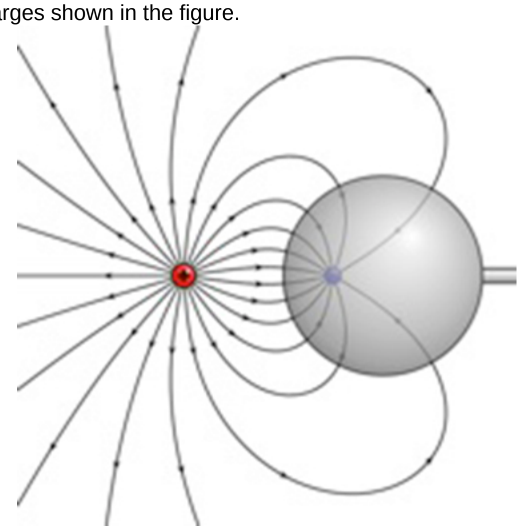


Figure 18.54 The electric field near two charges.

50. Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See Figure 18.54 for a similar situation).

18.8 Applications of Electrostatics

51. (a) What is the electric field 5.00 m from the center of the terminal of a Van de Graaff with a 3.00 mC charge, noting that the field is equivalent to that of a point charge at the center of the terminal? (b) At this distance, what force does the field exert on a $2.00\ \mu\text{C}$ charge on the Van de Graaff's belt?

52. (a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth? (b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and electrostatic forces.

53. A simple and common technique for accelerating electrons is shown in Figure 18.55, where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving. (a) Calculate the acceleration of the electron if the field strength is $2.50\times10^{4}\ \text{N/C}$. (b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.

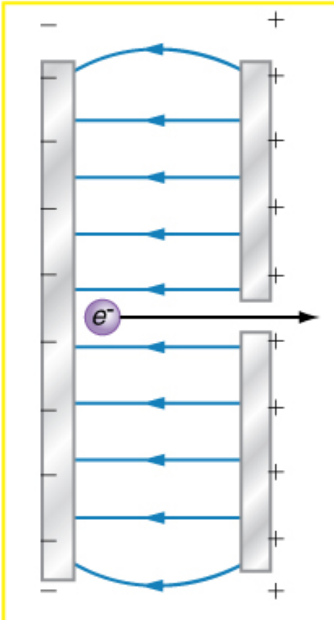


Figure 18.55 Parallel conducting plates with opposite charges on them create a relatively uniform electric field used to accelerate electrons to the right. Those that go through the hole can be used to make a TV or computer screen glow or to produce X-rays.

54. Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface. (a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its center? (b) What acceleration will the field produce on a free electron near Earth's surface? (c) What mass object with a single extra electron will have its weight supported by this field?

55. Point charges of $25.0\ \mu\text{C}$ and $45.0\ \mu\text{C}$ are placed 0.500 m apart. (a) At what point along the line between them is the electric field zero? (b) What is the electric field halfway between them?

56. What can you say about two charges q_1 and q_2, if the electric field one-fourth of the way from q_1 to q_2 is zero?

57. Integrated Concepts

Calculate the angular velocity ω of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is $0.530\times10^{-10}\ \text{m}$. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.

58. Integrated Concepts

An electron has an initial velocity of $5.00\times10^{6}\ \text{m/s}$ in a uniform $2.00\times10^{5}\ \text{N/C}$ strength electric field. The field accelerates the electron in the direction opposite to its initial velocity. (a) What is the direction of the electric field? (b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?

59. Integrated Concepts

The practical limit to an electric field in air is about $3.00\times10^{6}\ \text{N/C}$. Above this strength, sparking takes place because air begins to ionize and charges flow, reducing the field. (a) Calculate the distance a free proton must travel in this field to reach 3.00% of the speed of light, starting from rest. (b) Is this practical in air, or must it occur in a vacuum?

60. Integrated Concepts

A 5.00 g charged insulating ball hangs on a 30.0 cm long string in a uniform horizontal electric field as shown in **Figure 18.56**. Given the charge on the ball is $1.00\ \mu\text{C}$, find the strength of the field.

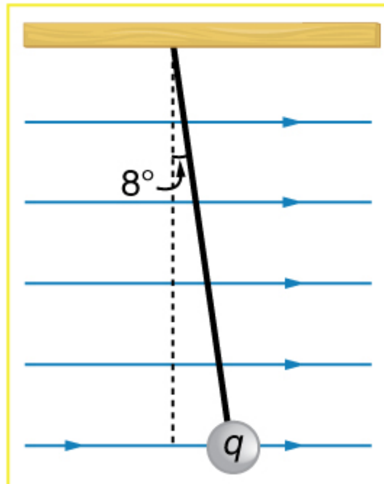


Figure 18.56 A horizontal electric field causes the charged ball to hang at an angle of 8.00°.

61. Integrated Concepts

Figure 18.57 shows an electron passing between two charged metal plates that create an 100 N/C vertical electric field perpendicular to the electron's original horizontal velocity. (These can be used to change the electron's direction, such as in an oscilloscope.) The initial speed of the electron is $3.00\times10^{6}\ \text{m/s}$, and the horizontal distance it travels in the uniform field is 4.00 cm. (a) What is its vertical deflection? (b) What is the vertical component of its final velocity? (c) At what angle does it exit? Neglect any edge effects.

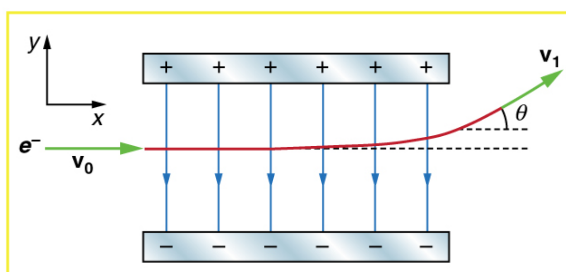


Figure 18.57

62. Integrated Concepts

The classic Millikan oil drop experiment was the first to obtain an accurate measurement of the charge on an electron. In it, oil drops were suspended against the gravitational force by a vertical electric field. (See **Figure 18.58**.) Given the oil drop to be $1.00\ \mu\text{m}$ in radius and have a density of $920\ \text{kg/m}^3$:

(a) Find the weight of the drop. (b) If the drop has a single excess electron, find the electric field strength needed to balance its weight.

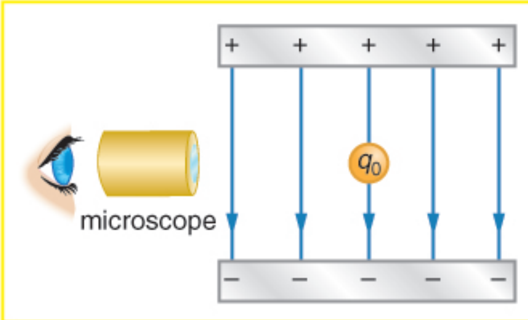


Figure 18.58 In the Millikan oil drop experiment, small drops can be suspended in an electric field by the force exerted on a single excess electron. Classically, this experiment was used to determine the electron charge q_e by measuring the electric field and mass of the drop.

63. Integrated Concepts

(a) In **Figure 18.59**, four equal charges q lie on the corners of a square. A fifth charge Q is on a mass m directly above the center of the square, at a height equal to the length d of one side of the square. Determine the magnitude of q in terms of Q, m, and d, if the Coulomb force is to equal the weight of m. (b) Is this equilibrium stable or unstable? Discuss.

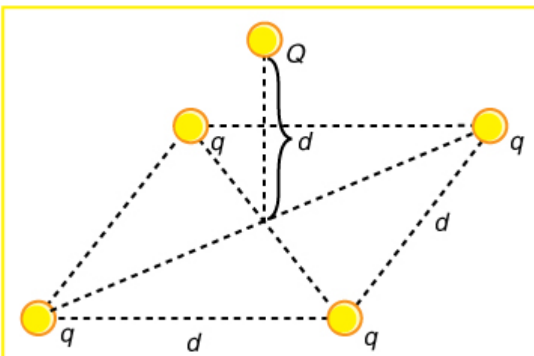


Figure 18.59 Four equal charges on the corners of a horizontal square support the weight of a fifth charge located directly above the center of the square.

64. Unreasonable Results

(a) Calculate the electric field strength near a 10.0 cm diameter conducting sphere that has 1.00 C of excess charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

65. Unreasonable Results

(a) Two 0.500 g raindrops in a thunderhead are 1.00 cm apart when they each acquire 1.00 mC charges. Find their acceleration. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

66. Unreasonable Results

A wrecking yard inventor wants to pick up cars by charging a 0.400 m diameter ball and inducing an equal and opposite charge on the car. If a car has a 1000 kg mass and the ball is to be able to lift it from a distance of 1.00 m: (a) What minimum charge must be used? (b) What is the electric field near the surface of the ball? (c) Why are these results unreasonable? (d) Which premise or assumption is responsible?

67. Construct Your Own Problem

Consider two insulating balls with evenly distributed equal and opposite charges on their surfaces, held with a certain distance between the centers of the balls. Construct a problem in which you calculate the electric field (magnitude and direction) due to the balls at various points along a line running through the centers of the balls and extending to infinity on either side. Choose interesting points and comment on the meaning of the field at those points. For example, at what points might the field be just that due to one ball and where does the field become negligibly small? Among the things to be considered are the magnitudes of the charges and the distance between the centers of the balls. Your instructor may wish for you to consider the electric field off axis or for a more complex array of charges, such as those in a water molecule.

68. Construct Your Own Problem

Consider identical spherical conducting space ships in deep space where gravitational fields from other bodies are negligible compared to the gravitational attraction between the ships. Construct a problem in which you place identical excess charges on the space ships to exactly counter their gravitational attraction. Calculate the amount of excess charge needed. Examine whether that charge depends on the distance between the centers of the ships, the masses of the ships, or any other factors. Discuss whether this would be an easy, difficult, or even impossible thing to do in practice.

Test Prep for AP® Courses

18.1 Static Electricity and Charge: Conservation of Charge

1. When a glass rod is rubbed against silk, which of the following statements is true?
a. Electrons are removed from the silk.
b. Electrons are removed from the rod.
c. Protons are removed from the silk.
d. Protons are removed from the rod.

2. In an experiment, three microscopic latex spheres are sprayed into a chamber and become charged with $+3e$, $+5e$, and $-3e$, respectively. Later, all three spheres collide simultaneously and then separate. Which of the following are possible values for the final charges on the spheres? Select *two* answers.

	X	Y	Z
(a)	$+4e$	$-4e$	$+5e$
(b)	$-4e$	$+4.5e$	$+5.5e$
(c)	$+5e$	$-8e$	$+7e$
(d)	$+6e$	$+6e$	$-7e$

3. If objects X and Y attract each other, which of the following will be false?
a. X has positive charge and Y has negative charge.
b. X has negative charge and Y has positive charge.
c. X and Y both have positive charge.
d. X is neutral and Y has a charge.

4. Suppose a positively charged object A is brought in contact with an uncharged object B in a closed system. What type of charge will be left on object B?
a. negative
b. positive
c. neutral
d. cannot be determined

5. What will be the net charge on an object which attracts neutral pieces of paper but repels a negatively charged balloon?
a. negative
b. positive
c. neutral
d. cannot be determined

6. When two neutral objects are rubbed against each other, the first one gains a net charge of $3e$. Which of the following statements is true?
a. The second object gains $3e$ and is negatively charged.
b. The second object loses $3e$ and is negatively charged.
c. The second object gains $3e$ and is positively charged.
d. The second object loses $3e$ and is positively charged.

7. In an experiment, a student runs a comb through his hair several times and brings it close to small pieces of paper. Which of the following will he observe?
a. Pieces of paper repel the comb.
b. Pieces of paper are attracted to the comb.
c. Some pieces of paper are attracted and some repel the comb.
d. There is no attraction or repulsion between the pieces of paper and the comb.

8. In an experiment a negatively charged balloon (balloon X) is repelled by another charged balloon Y. However, an object Z is attracted to balloon Y. Which of the following can be the charge on Z? Select *two* answers.
a. negative
b. positive
c. neutral
d. cannot be determined

9. Suppose an object has a charge of 1 C and gains 6.88×10^{18} electrons.
a. What will be the net charge of the object?
b. If the object has gained electrons from a neutral object, what will be the charge on the neutral object?
c. Find and explain the relationship between the total charges of the two objects before and after the transfer.
d. When a third object is brought in contact with the first object (after it gains the electrons), the resulting charge on the third object is 0.4 C. What was its initial charge?

10. The charges on two identical metal spheres (placed in a closed system) are -2.4×10^{-17} C and -4.8×10^{-17} C.
a. How many electrons will be equivalent to the charge on each sphere?
b. If the two spheres are brought in contact and then separated, find the charge on each sphere.
c. Calculate the number of electrons that would be equivalent to the resulting charge on each sphere.

11. In an experiment the following observations are made by a student for four charged objects W, X, Y, and Z:
- A glass rod rubbed with silk attracts W.
- W attracts Z but repels X.
- X attracts Z but repels Y.
- Y attracts W and Z.

Estimate whether the charges on each of the four objects are positive, negative, or neutral.

18.2 Conductors and Insulators

12. Some students experimenting with an uncharged metal sphere want to give the sphere a net charge using a charged aluminum pie plate. Which of the following steps would give the sphere a net charge of the same sign as the pie plate?
a. bringing the pie plate close to, but not touching, the metal sphere, then moving the pie plate away.
b. bringing the pie plate close to, but not touching, the metal sphere, then momentarily touching a grounding wire to the metal sphere.
c. bringing the pie plate close to, but not touching, the metal sphere, then momentarily touching a grounding wire to the pie plate.
d. touching the pie plate to the metal sphere.

13.

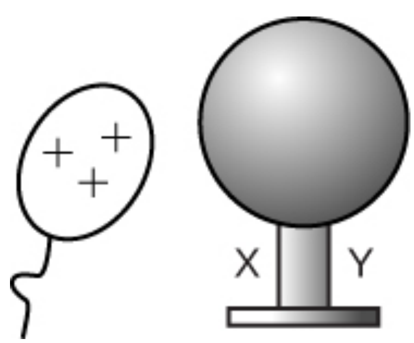


Figure 18.60 Balloon and sphere.

When the balloon is brought closer to the sphere, there will be a redistribution of charges. What is this phenomenon called?
a. electrostatic repulsion

b. conduction
c. polarization
d. none of the above

14. What will be the charge at Y (i.e., the part of the sphere furthest from the balloon)?
a. positive
b. negative
c. zero
d. It can be positive or negative depending on the material.

15. What will be the net charge on the sphere?
a. positive
b. negative
c. zero
d. It can be positive or negative depending on the material.

16. If Y is grounded while the balloon is still close to X, which of the following will be true?
a. Electrons will flow from the sphere to the ground.
b. Electrons will flow from the ground to the sphere.
c. Protons will flow from the sphere to the ground.
d. Protons will flow from the ground to the sphere.

17. If the balloon is moved away after grounding, what will be the net charge on the sphere?
a. positive
b. negative
c. zero
d. It can be positive or negative depending on the material.

18. A positively charged rod is used to charge a sphere by induction. Which of the following is true?
a. The sphere must be a conductor.
b. The sphere must be an insulator.
c. The sphere can be a conductor or insulator but must be connected to ground.
d. The sphere can be a conductor or insulator but must be already charged.

19.

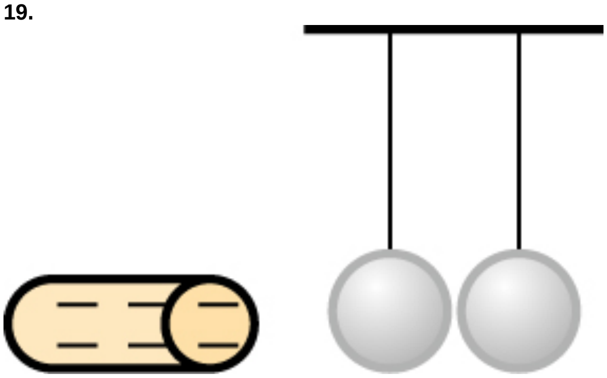


Figure 18.61 Rod and metal balls.

As shown in the figure above, two metal balls are suspended and a negatively charged rod is brought close to them.

a. If the two balls are in contact with each other what will be the charges on each ball?
b. Explain how the balls get these charges.
c. What will happen to the charge on the second ball (i.e., the ball further away from the rod) if it is momentarily grounded while the rod is still there?
d. If (instead of grounding) the second ball is moved away and then the rod is removed from the first ball, will the two balls have induced charges? If yes, what will be the charges? If no, why not?

20. Two experiments are performed using positively charged glass rods and neutral electroscopes. In the first experiment the rod is brought in contact with the electroscope. In the second experiment the rod is only brought close to the electroscope but not in contact. However, while the rod is close, the electroscope is momentarily grounded and then the rod is removed. In both experiments the needles of the electroscopes deflect, which indicates the presence of charges.
a. What is the charging method in each of the two experiments?
b. What is the net charge on the electroscope in the first experiment? Explain how the electroscope obtains that charge.
c. Is the net charge on the electroscope in the second experiment different from that of the first experiment? Explain why.

18.3 Conductors and Electric Fields in Static Equilibrium

21.

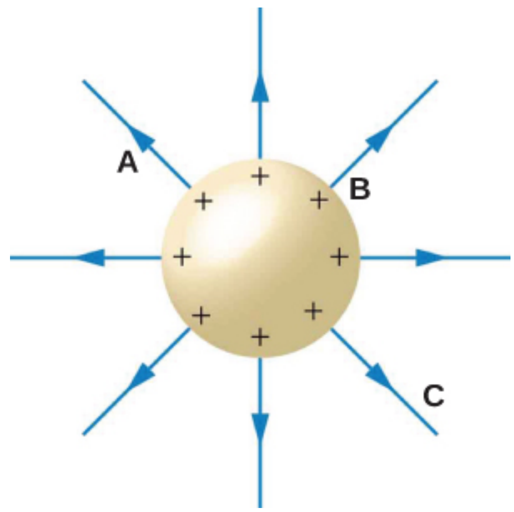


Figure 18.62 A sphere conductor.

An electric field due to a positively charged spherical conductor is shown above. Where will the electric field be weakest?

a. Point A
b. Point B
c. Point C
d. Same at all points

22.

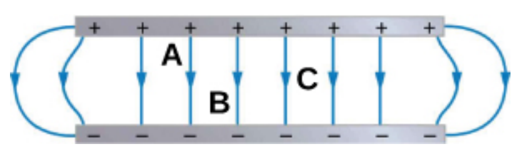


Figure 18.63 Electric field between two parallel metal plates.

The electric field created by two parallel metal plates is shown above. Where will the electric field be strongest?

a. Point A
b. Point B
c. Point C
d. Same at all points

23. Suppose that the electric field experienced due to a positively charged small spherical conductor at a certain distance is *E*. What will be the percentage change in electric field experienced at thrice the distance if the charge on the conductor is doubled?

24.

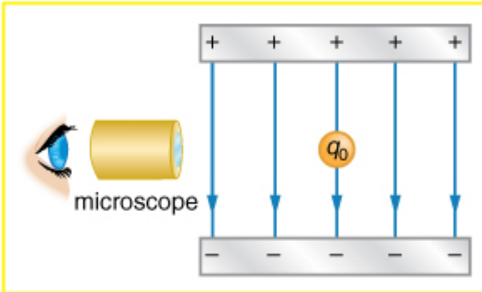


Figure 18.64 Millikan oil drop experiment.

The classic Millikan oil drop experiment setup is shown above. In this experiment oil drops are suspended in a vertical electric field against the gravitational force to measure their charge. If the mass of a negatively charged drop suspended in an electric field of 1.18×10^{-4} N/C strength is 3.85×10^{-21} g, find the number of excess electrons in the drop.

18.4 Coulomb's Law

25. For questions 25–27, suppose that the electrostatics force between two charges is *F*.

What will be the force if the distance between them is halved?

a. 4*F*
b. 2*F*
c. *F*/4
d. *F*/2

26. Which of the following is false?
a. If the charge of one of the particles is doubled and that of the second is unchanged, the force will become 2*F*.
b. If the charge of one of the particles is doubled and that of the second is halved, the force will remain *F*.
c. If the charge of both the particles is doubled, the force will become 4*F*.
d. None of the above.

27. Which of the following is true about the gravitational force between the particles?
a. It will be 3.25×10^{-38} *F*.
b. It will be 3.25×10^{38} *F*.
c. It will be equal to *F*.
d. It is not possible to determine the gravitational force as the masses of the particles are not given.

28. Two massive, positively charged particles are initially held a fixed distance apart. When they are moved farther apart, the magnitude of their mutual gravitational force changes by a factor of *n*. Which of the following indicates the factor by which the magnitude of their mutual electrostatic force changes?
a. $1/n^2$
b. $1/n$
c. n
d. n^2

29.
a. What is the electrostatic force between two charges of 1 C each, separated by a distance of 0.5 m?
b. How will this force change if the distance is increased to 1 m?

30.
a. Find the ratio of the electrostatic force to the gravitational force between two electrons.
b. Will this ratio change if the two electrons are replaced by protons? If yes, find the new ratio.

18.5 Electric Field: Concept of a Field Revisited

31. Two particles with charges +2*q* and +*q* are separated by a distance *r*. The +2*q* particle has an electric field *E* at distance *r* and exerts a force *F* on the +*q* particle. Use this information to answer questions 31–32.

What is the electric field of the +*q* particle at the same distance and what force does it exert on the +2*q* particle?

a. *E*/2, *F*/2
b. *E*, *F*/2
c. *E*/2, *F*
d. *E*, *F*

32. When the +*q* particle is replaced by a +3*q* particle, what will be the electric field and force from the +2*q* particle experienced by the +3*q* particle?
a. *E*/3, 3*F*
b. *E*, 3*F*
c. *E*/3, *F*
d. *E*, *F*

33. The direction of the electric field of a negative charge is
a. inward for both positive and negative charges.
b. outward for both positive and negative charges.
c. inward for other positive charges and outward for other negative charges.
d. outward for other positive charges and inward for other negative charges.

34. The force responsible for holding an atom together is
a. frictional
b. electric
c. gravitational
d. magnetic

35. When a positively charged particle exerts an inward force on another particle *P*, what will be the charge of *P*?
a. positive
b. negative
c. neutral
d. cannot be determined

36. Find the force exerted due to a particle having a charge of 3.2×10^{-19} C on another identical particle 5 cm away.

37. Suppose that the force exerted on an electron is 5.6×10^{-17} N, directed to the east.
a. Find the magnitude of the electric field that exerts the force.
b. What will be the direction of the electric field?
c. If the electron is replaced by a proton, what will be the magnitude of force exerted?
d. What will be the direction of force on the proton?

18.6 Electric Field Lines: Multiple Charges

38.

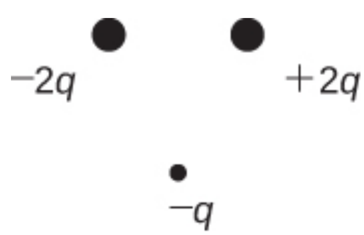


Figure 18.65 An electric dipole (with +2*q* and –2*q* as the two charges) is shown in the figure above. A third charge, −*q* is

placed equidistant from the dipole charges. What will be the direction of the net force on the third charge?

a. →
b. ←
c. ↓
d. ↑

39.

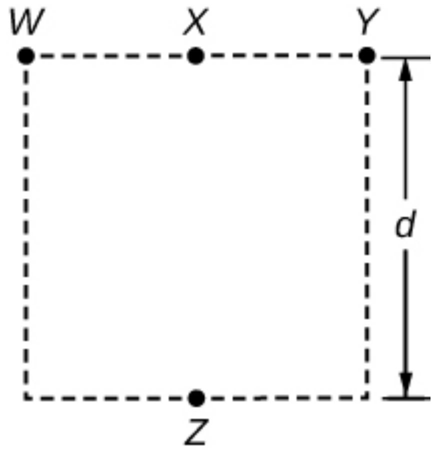


Figure 18.66

Four objects, each with charge +*q*, are held fixed on a square with sides of length *d*, as shown in the figure. Objects *X* and *Z* are at the midpoints of the sides of the square. The electrostatic force exerted by object *W* on object *X* is *F*. Use this information to answer questions 39–40.

What is the magnitude of force exerted by object *W* on *Z*?

a. *F*/7
b. *F*/5
c. *F*/3
d. *F*/2

40. What is the magnitude of the net force exerted on object *X* by objects *W*, *Y*, and *Z*?

a. *F*/4
b. *F*/2
c. 9*F*/4
d. 3*F*

41.

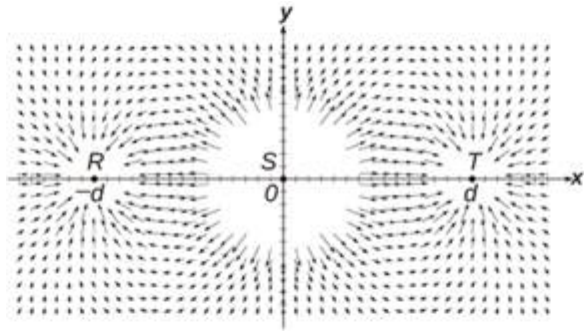


Figure 18.67 Electric field with three charged objects.

The figure above represents the electric field in the vicinity of three small charged objects, *R*, *S*, and *T*. The objects have charges −*q*, +2*q*, and −*q*, respectively, and are located on the *x*-axis at −*d*, 0, and *d*. Field vectors of very large magnitude are omitted for clarity.

(a) i) Briefly describe the characteristics of the field diagram that indicate that the sign of the charges of objects *R* and *T* is negative and that the sign of the charge of object *S* is positive.

ii) Briefly describe the characteristics of the field diagram that indicate that the magnitudes of the charges of objects *R* and *T* are equal and that the magnitude of the charge of object *S* is about twice that of objects *R* and *T*.

For the following parts, an electric field directed to the right is defined to be positive.

(b) On the axes below, sketch a graph of the electric field *E* along the *x*-axis as a function of position *x*.

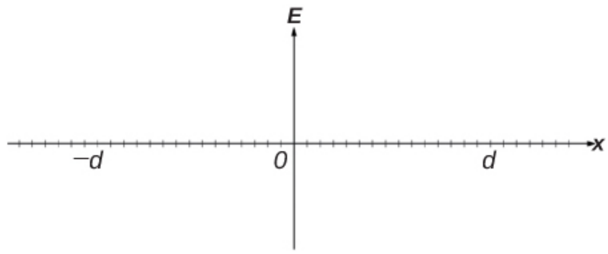


Figure 18.68 An Electric field (E) axis and Position (x) axis.

(c) Write an expression for the electric field *E* along the *x*-axis as a function of position *x* in the region between objects *S* and *T* in terms of *q*, *d*, and fundamental constants, as appropriate.

(d) Your classmate tells you there is a point between *S* and *T* where the electric field is zero. Determine whether this statement is true, and explain your reasoning using two of the representations from parts (a), (b), or (c).

20 ELECTRIC CURRENT, RESISTANCE, AND OHM'S LAW

Figure 20.1 Electric energy in massive quantities is transmitted from this hydroelectric facility, the **Srisailam power station located along the Krishna River in India (http://en.wikipedia.org/wiki/Srisailam_Dam)** , by the movement of charge—that is, by electric current. (credit: Chintohere, Wikimedia Commons)

Chapter Outline

Connection for AP® Courses

In our daily lives, we see and experience many examples of electricity which involve electric current, the movement of charge. These include the flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling its load over a mountain pass, and a hydroelectric plant sending energy to metropolitan and rural users.

Humankind has indeed harnessed electricity, the basis of technology, to improve the quality of life. While the previous two chapters concentrated on static electricity and the fundamental force underlying its behavior, the next few chapters will be devoted to electric and magnetic phenomena involving electric current. In addition to exploring applications of electricity, we shall gain new insights into its nature – in particular, the fact that all magnetism results from electric current.

This chapter supports learning objectives covered under Big Ideas 1, 4, and 5 of the AP Physics Curriculum Framework. Electric charge is a property of a system (Big Idea 1) that affects its interaction with other charged systems (Enduring Understanding 1.B), whereas electric current is fundamentally the movement of charge through a conductor and is based on the fact that electric charge is conserved within a system (Essential Knowledge 1.B.1). The conservation of charge also leads to the concept of an electric circuit as a closed loop of electrical current. In addition, this chapter discusses examples showing that the current in a circuit is resisted by the elements of the circuit and the strength of the resistance depends on the material of the elements. The macroscopic properties of materials, including resistivity, depend on their molecular and atomic structure (Enduring Understanding 1.E). In addition, resistivity depends on the temperature of the material (Essential Knowledge 1.E.2).

The chapter also describes how the interaction of systems of objects can result in changes in those systems (Big Idea 4). For example, electric properties of a system of charged objects can change in response to the presence of, or changes in, other charged objects or systems (Enduring Understanding 4.E). A simple circuit with a resistor and an energy source is an example of such a system. The current through the resistor in the circuit is equal to the difference of potentials across the resistor divided by its resistance (Essential Knowledge 4.E.4).

The unifying theme of the physics curriculum is that any changes in the systems due to interactions are governed by laws of conservation (Big Idea 5). This chapter applies the idea of energy conservation (Enduring Understanding 5.B) to electric circuits and connects concepts of electric energy and electric power as rates of energy use (Essential Knowledge 5.B.5). While the laws of conservation of energy in electric circuits are fully described by Kirchoff's rules, which are introduced in the next chapter (Essential Knowledge 5.B.9), the specific definition of power (based on Essential Knowledge 5.B.9) is that it is the rate at which energy is transferred from a resistor as the product of the electric potential difference across the resistor and the current through the resistor.

Big Idea 1 Objects and systems have properties such as mass and charge. Systems may have internal structure.

Enduring Understanding 1.B Electric charge is a property of an object or system that affects its interactions with other objects or systems containing charge.

Essential Knowledge 1.B.1 Electric charge is conserved. The net charge of a system is equal to the sum of the charges of all the objects in the system.

Enduring Understanding 1.E Materials have many macroscopic properties that result from the arrangement and interactions of the atoms and molecules that make up the material.

Essential Knowledge 1.E.2 Matter has a property called resistivity.

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding 4.E The electric and magnetic properties of a system can change in response to the presence of, or changes in, other objects or systems.

Essential Knowledge 4.E.4 The resistance of a resistor, and the capacitance of a capacitor, can be understood from the basic properties of electric fields and forces, as well as the properties of materials and their geometry.

Big Idea 5: Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.B The energy of a system is conserved.

Essential Knowledge 5.B.5 Energy can be transferred by an external force exerted on an object or system that moves the object or system through a distance; this energy transfer is called work. Energy transfer in mechanical or electrical systems may occur at different rates. Power is defined as the rate of energy transfer into, out of, or within a system. [A piston filled with gas getting compressed or expanded is treated in Physics 2 as a part of thermodynamics.]

Essential Knowledge 5.B.9 Kirchhoff's loop rule describes conservation of energy in electrical circuits. [The application of Kirchhoff's laws to circuits is introduced in Physics 1 and further developed in Physics 2 in the context of more complex circuits, including those with capacitors.]

20.1 Current

Learning Objectives

By the end of this section, you will be able to:

- Define electric current, ampere, and drift velocity.
- Describe the direction of charge flow in conventional current.
- Use drift velocity to calculate current and vice versa.

The information presented in this section supports the following AP® learning objectives and science practices:

- **1.B.1.1** The student is able to make claims about natural phenomena based on conservation of electric charge. **(S.P. 6.4)**
- **1.B.1.2** The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits. **(S.P. 6.4, 7.2)**

Electric Current

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** I is defined to be

$$I = \frac{\Delta Q}{\Delta t}, \tag{20.1}$$

where ΔQ is the amount of charge passing through a given area in time Δt. (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t = t$.) (See **Figure 20.2**.) The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \Delta Q / \Delta t$, we see that an ampere is one coulomb per second:

$$1 \text{ A} = 1 \text{ C/s} \tag{20.2}$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

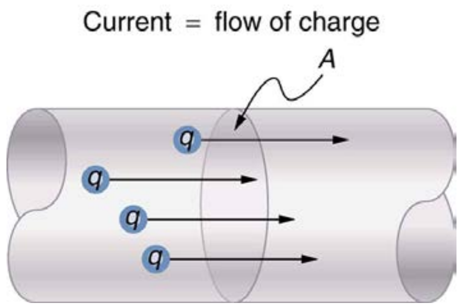


Figure 20.2 The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Example 20.1 Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Strategy

We can use the definition of current in the equation $I = \Delta Q / \Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

Solution for (a)

Entering the given values for charge and time into the definition of current gives

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = \frac{720 \text{ C}}{4.00 \text{ s}} = 180 \text{ C/s} \\ &= 180 \text{ A}. \end{aligned} \tag{20.3}$$

Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large because large frictional forces need to be overcome when setting something in motion.

Solution for (b)

Solving the relationship $I = \Delta Q / \Delta t$ for time Δt, and entering the known values for charge and current gives

$$\begin{aligned} \Delta t &= \frac{\Delta Q}{I} = \frac{1.00 \text{ C}}{0.300 \times 10^{-3} \text{ C/s}} \\ &= 3.33 \times 10^{3} \text{ s}. \end{aligned} \tag{20.4}$$

Discussion for (b)

This time is slightly less than an hour. The small current used by the hand-held calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It's because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

Figure 20.3 shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in **Figure 20.3** (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.

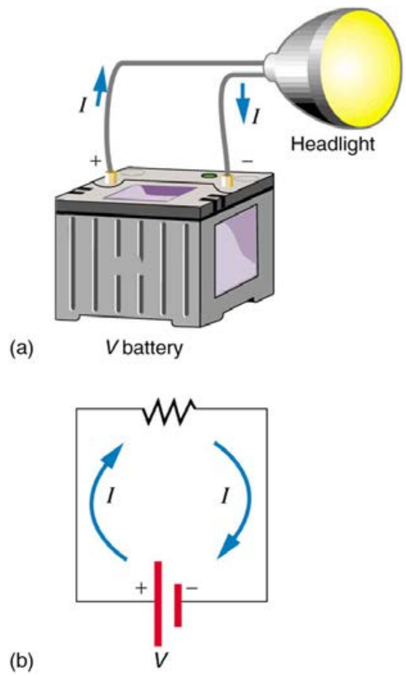


Figure 20.3 (a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide variety of similar circuits.

Note that the direction of current in **Figure 20.3** is from positive to negative. *The direction of conventional current is the direction that positive charge would flow*. In a single loop circuit (as shown in **Figure 20.3**), the value for current at all points of the circuit should be the same if there are no losses. This is because current is the flow of charge and charge is conserved, i.e., the charge flowing out from the battery will be the same as the charge flowing into the battery. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. **Figure 20.4** illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in **Figure 20.4**. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.

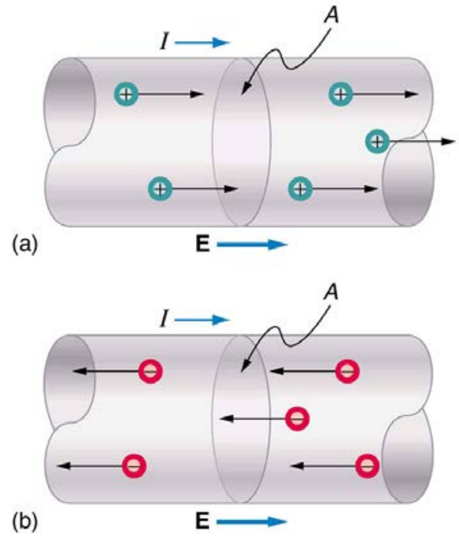


Figure 20.4 Current I is the rate at which charge moves through an area A, such as the cross-section of a wire. Conventional current is defined to move in the direction of the electric field. (a) Positive charges move in the direction of the electric field and the same direction as conventional current. (b) Negative charges move in the direction opposite to the electric field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

Example 20.2 Calculating the Number of Electrons that Move through a Calculator

If the 0.300-mA current through the calculator mentioned in the **Example 20.1** example is carried by electrons, how many electrons per second pass through it?

Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{\text{electrons}} = -0.300\times10^{-3}\text{ C/s}$ .Since each electron (e^-) has a charge of $-1.60\times10^{-19}\text{ C}$, we can convert the current in coulombs per second to electrons per second.

Solution

Starting with the definition of current, we have

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300\times10^{-3}\text{ C}}{\text{s}}. \tag{20.5}$$

We divide this by the charge per electron, so that

$$\begin{aligned} \frac{e^-}{\text{s}} &= \frac{-0.300\times10^{-3}\text{ C}}{\text{s}}\times\frac{1\ e^-}{-1.60\times10^{-19}\text{ C}} \\ &= 1.88\times10^{15}\ \frac{e^-}{\text{s}}. \end{aligned} \tag{20.6}$$

Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

Drift Velocity

Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of 10^8 m/s , a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move *much* more slowly on average, typically drifting at speeds on the order of 10^{-4} m/s . How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in **Figure 20.5**, the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The density of charge in a system cannot easily be increased, and so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in electric field.

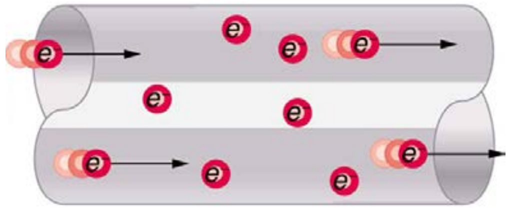


Figure 20.5 When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. **Figure 20.6** shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** v_d is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.

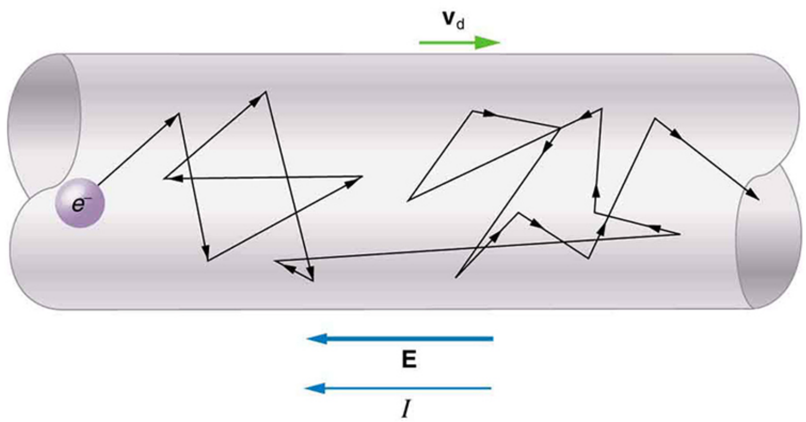


Figure 20.6 Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity, v_d, and it is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Conduction of Electricity and Heat

Good electrical conductors are often good heat conductors, too. This is because large numbers of free electrons can carry electrical current and can transport thermal energy.

The free-electron collisions transfer energy to the atoms of the conductor. The electric field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed, therefore) of the electrons. The work is transferred to the conductor's atoms, possibly increasing temperature. Thus a continuous power input is required to maintain current. An exception, of course, is found in superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings. In contrast, the supply of energy can be useful, such as in a lightbulb filament. The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

Making Connections: Take-Home Investigation—Filament Observations

Find a lightbulb with a filament. Look carefully at the filament and describe its structure. To what points is the filament connected?

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in **Figure 20.7**. *The number of free charges per unit volume* is given the symbol n and depends on the material. The shaded segment has a volume Ax, so that the number of free charges in it is nAx. The charge ΔQ in this segment is thus $qnAx$, where q is the amount of charge on each carrier. (Recall that for electrons, q is

$-1.60\times10^{-19}\ \text{C}$.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time Δt , the current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{qnAx}{\Delta t}. \tag{20.7}$$

Note that $x / \Delta t$ is the magnitude of the drift velocity, v_{d} , since the charges move an average distance x in a time Δt . Rearranging terms gives

$$I = nqAv_{\text{d}}, \tag{20.8}$$

where I is the current through a wire of cross-sectional area A made of a material with a free charge density n . The carriers of the current each have charge q and move with a drift velocity of magnitude v_{d} .

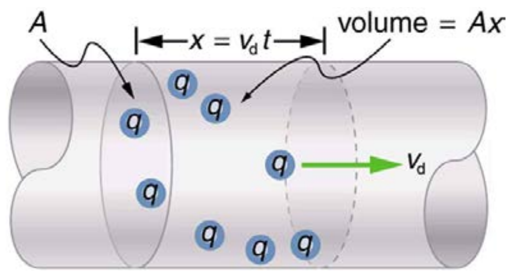


Figure 20.7 All the charges in the shaded volume of this wire move out in a time t , having a drift velocity of magnitude $v_{\text{d}} = x / t$. See text for further discussion.

Note that simple drift velocity is not the entire story. The speed of an electron is much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons? Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as much as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a “sea” of electrons. These free electrons respond by accelerating when an electric field is applied. Of course as they move they collide with the atoms in the lattice and other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

Example 20.3 Calculating Drift Velocity in a Common Wire

Calculate the drift velocity of electrons in a 12-gauge copper wire (which has a diameter of 2.053 mm) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20 A.) The density of copper is $8.80\times10^{3}\ \text{kg/m}^{3}$.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_{\text{d}}$. The current $I = 20.0\ \text{A}$ is given, and $q = -1.60\times10^{-19}\,\text{C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^{2}$, where r is one-half the given diameter, 2.053 mm. We are given the density of copper, $8.80\times10^{3}\ \text{kg/m}^{3}$, and the periodic table shows that the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, $6.02\times10^{23}\ \text{atoms/mol}$, to determine n, the number of free electrons per cubic meter.

Solution

First, calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, is the same as the number of copper atoms per m^{3} . We can now find n as follows:

$$\begin{aligned} n &= \frac{1\ e^{-}}{\text{atom}}\times\frac{6.02\times10^{23}\ \text{atoms}}{\text{mol}}\times\frac{1\ \text{mol}}{63.54\ \text{g}}\times\frac{1000\ \text{g}}{\text{kg}}\times\frac{8.80\times10^{3}\ \text{kg}}{1\ \text{m}^{3}} \\ &= 8.342\times10^{28}\ e^{-}/\text{m}^{3}. \end{aligned} \tag{20.9}$$

The cross-sectional area of the wire is

$$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(\frac{2.053\times10^{-3}\text{ m}}{2}\right)^2 \\ &= 3.310\times10^{-6}\text{ m}^2. \end{aligned} \tag{20.10}$$

Rearranging $I = nqAv_\text{d}$ to isolate drift velocity gives

$$v_\text{d} = \frac{I}{nqA} \tag{20.11}$$

$$= \frac{20.0\text{ A}}{(8.342\times10^{28}/\text{m}^3)(-1.60\times10^{-19}\text{ C})(3.310\times10^{-6}\text{ m}^2)}$$

$$= -4.53\times10^{-4}\text{ m/s}.$$

Discussion

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of 10^{-4} m/s) confirms that the signal moves on the order of 10^{12} times faster (about 10^8 m/s) than the charges that carry it.

20.2 Ohm's Law: Resistance and Simple Circuits

Learning Objectives

By the end of this section, you will be able to:

- Explain the origin of Ohm's law.
- Calculate voltages, currents, and resistances with Ohm's law.
- Explain the difference between ohmic and non-ohmic materials.
- Describe a simple circuit.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.E.4.1** The student is able to make predictions about the properties of resistors and/or capacitors when placed in a simple circuit based on the geometry of the circuit element and supported by scientific theories and mathematical relationships. **(S.P. 2.2, 6.4)**

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm's Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V. \tag{20.12}$$

This important relationship is known as **Ohm's law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** R. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

$$I \propto \frac{1}{R}. \tag{20.13}$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

$$I = \frac{V}{R}. \tag{20.14}$$

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I. An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol Ω (upper case Greek omega). Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\ \Omega = 1\frac{V}{A}. \tag{20.15}$$

Figure 20.8 shows the schematic for a simple circuit. A **simple circuit** has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R.

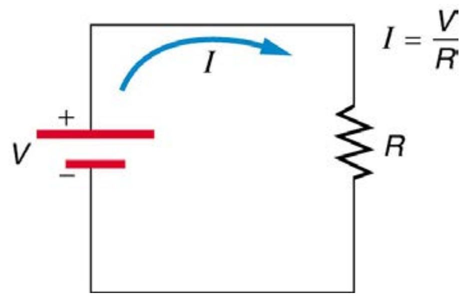


Figure 20.8 A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

Making Connections: Real World Connections

Ohm's law ($V = IR$) is a fundamental relationship that could be presented by a linear function with the slope of the line being the resistance. The resistance represents the voltage that needs to be applied to the resistor to create a current of 1 A through the circuit. The graph (in the figure below) shows this representation for two simple circuits with resistors that have different resistances and thus different slopes.

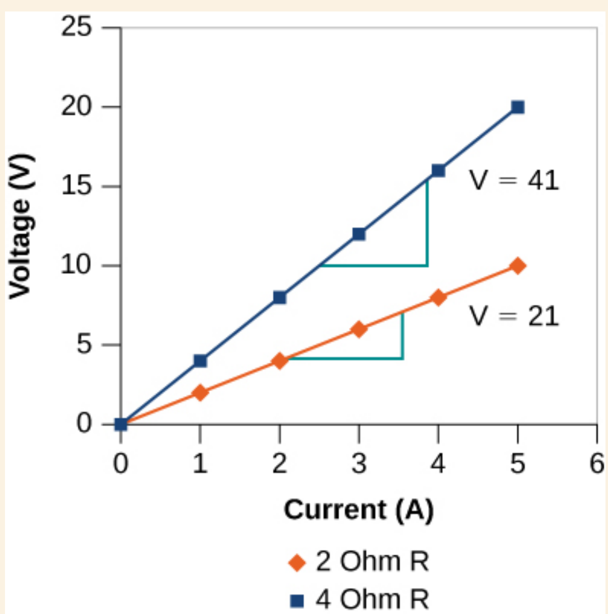


Figure 20.9 The figure illustrates the relationship between current and voltage for two different resistors. The slope of the graph represents the resistance value, which is 2Ω and 4Ω for the two lines shown.

Making Connections: Real World Connections

The materials which follow Ohm's law by having a linear relationship between voltage and current are known as ohmic materials. On the other hand, some materials exhibit a nonlinear voltage-current relationship and hence are known as non-ohmic materials. The figure below shows current voltage relationships for the two types of materials.

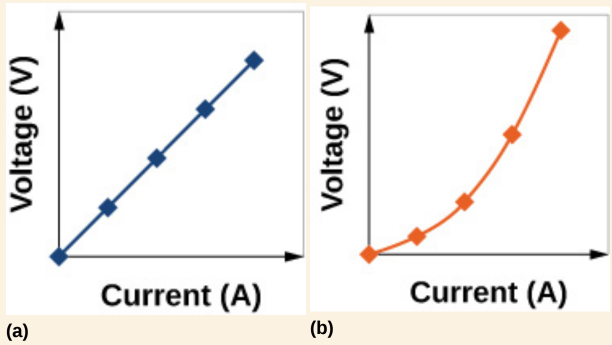


Figure 20.10 The relationship between voltage and current for ohmic and non-ohmic materials are shown.

Clearly the resistance of an ohmic material (shown in (a)) remains constant and can be calculated by finding the slope of the graph but that is not true for a non-ohmic material (shown in (b)).

Example 20.4 Calculating Resistance: An Automobile Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy

We can rearrange Ohm's law as stated by $I = V/R$ and use it to find the resistance.

Solution

Rearranging $I = V/R$ and substituting known values gives

$$R = \frac{V}{I} = \frac{12.0\text{ V}}{2.50\text{ A}} = 4.80\ \Omega. \tag{20.16}$$

Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. As we shall see in **Resistance and Resistivity**, resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^{5}\ \Omega$, whereas the resistance of the human heart is about $10^{3}\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed, as will be seen in **Resistance and Resistivity**.

Additional insight is gained by solving $I = V/R$ for $V,$ yielding

$$V = IR. \tag{20.17}$$

This expression for V can be interpreted as the *voltage drop across a resistor produced by the current* I. The phrase IR *drop* is often used for this voltage. For instance, the headlight in **Example 20.4** has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $\text{PE} = q\Delta V$, and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See **Figure 20.11**.)

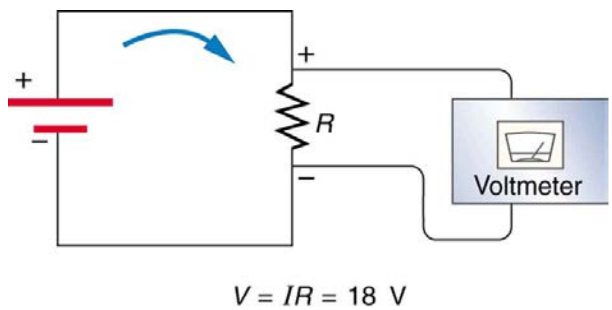


Figure 20.11 The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

Making Connections: Conservation of Energy

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

PhET Explorations: Ohm's Law

See how the equation form of Ohm's law relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

PhET Interactive Simulation

Figure 20.12 Ohm's Law (http://cnx.org/content/m55356/1.2/ohms-law_en.jar)

20.3 Resistance and Resistivity

Learning Objectives

By the end of this section, you will be able to:

- Explain the concept of resistivity.
- Use resistivity to calculate the resistance of specified configurations of material.
- Use the thermal coefficient of resistivity to calculate the change of resistance with temperature.

The information presented in this section supports the following AP® learning objectives and science practices:

- **1.E.2.1** The student is able to choose and justify the selection of data needed to determine resistivity for a given material. **(S.P. 4.1)**
- **4.E.4.2** The student is able to design a plan for the collection of data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors. **(S.P. 4.1, 4.2)**
- **4.E.4.3** The student is able to analyze data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors. **(S.P. 5.1)**

Material and Shape Dependence of Resistance

The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor in **Figure 20.13** is easy to analyze, and, by so doing, we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance R is directly proportional to its length L, similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again similar to the flow of fluid through a pipe). In fact, R is inversely proportional to the cylinder's cross-sectional area A.

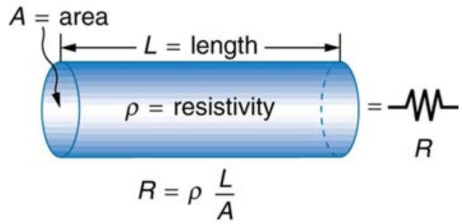


Figure 20.13 A uniform cylinder of length L and cross-sectional area A. Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its resistance. The larger its cross-sectional area A, the smaller its resistance.

For a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the **resistivity** ρ of a substance so that the **resistance** R of an object is directly proportional to ρ. Resistivity ρ is an *intrinsic* property of a material, independent of its shape or size. The resistance R of a uniform cylinder of length L, of cross-sectional area A, and made of a material with resistivity ρ, is

$$R = \frac{\rho L}{A}. \tag{20.18}$$

Table 20.1 gives representative values of ρ. The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivities. Conductors have the smallest resistivities, and insulators have the largest; semiconductors have intermediate resistivities. Conductors have varying but large free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as will be explored in later chapters.

Table 20.1 Resistivities ρ of Various materials at 20°C

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
Conductors	
Silver	1.59×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.65×10^{-8}
Tungsten	5.6×10^{-8}
Iron	9.71×10^{-8}
Platinum	10.6×10^{-8}
Steel	20×10^{-8}
Lead	22×10^{-8}
Manganin (Cu, Mn, Ni alloy)	44×10^{-8}
Constantan (Cu, Ni alloy)	49×10^{-8}
Mercury	96×10^{-8}
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}
Semiconductors[1]	
Carbon (pure)	3.5×10^{5}
Carbon	$(3.5 - 60) \times 10^{5}$
Germanium (pure)	600×10^{-3}
Germanium	$(1 - 600) \times 10^{-3}$
Silicon (pure)	2300
Silicon	0.1–2300
Insulators	
Amber	5×10^{14}
Glass	$10^{9} - 10^{14}$
Lucite	$>10^{13}$
Mica	$10^{11} - 10^{15}$
Quartz (fused)	75×10^{16}
Rubber (hard)	$10^{13} - 10^{16}$
Sulfur	10^{15}
Teflon	$>10^{13}$
Wood	$10^{8} - 10^{14}$

Example 20.5 Calculating Resistor Diameter: A Headlight Filament

A car headlight filament is made of tungsten and has a cold resistance of $0.350 \ \Omega$. If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter?

Strategy

We can rearrange the equation $R = \frac{\rho L}{A}$ to find the cross-sectional area A of the filament from the given information. Then its diameter can be found by assuming it has a circular cross-section.

Solution

The cross-sectional area, found by rearranging the expression for the resistance of a cylinder given in $R = \frac{\rho L}{A}$, is

$$A = \frac{\rho L}{R}. \tag{20.19}$$

Substituting the given values, and taking ρ from **Table 20.1**, yields

$$\begin{aligned} A &= \frac{(5.6\times10^{-8} \ \Omega \cdot \text{m})(4.00\times10^{-2} \ \text{m})}{0.350 \ \Omega} \\ &= 6.40\times10^{-9} \ \text{m}^2. \end{aligned} \tag{20.20}$$

The area of a circle is related to its diameter D by

$$A = \frac{\pi D^2}{4}. \tag{20.21}$$

Solving for the diameter D, and substituting the value found for A, gives

$$\begin{aligned} D &= 2\left(\frac{A}{p}\right)^{\frac{1}{2}} = 2\left(\frac{6.40\times10^{-9} \ \text{m}^2}{3.14}\right)^{\frac{1}{2}} \\ &= 9.0\times10^{-5} \ \text{m}. \end{aligned} \tag{20.22}$$

Discussion

The diameter is just under a tenth of a millimeter. It is quoted to only two digits, because ρ is known to only two digits.

Temperature Variation of Resistance

The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures. (See **Figure 20.14**.) Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about 100°C or less), resistivity ρ varies with temperature change ΔT as expressed in the following equation

$$\rho = \rho_0(1 + \alpha \Delta T), \tag{20.23}$$

where ρ_0 is the original resistivity and α is the **temperature coefficient of resistivity**. (See the values of α in **Table 20.2** below.) For larger temperature changes, α may vary or a nonlinear equation may be needed to find ρ. Note that α is positive for metals, meaning their resistivity increases with temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (which is made of copper, manganese and nickel), for example, has α close to zero (to three digits on the scale in **Table 20.2**), and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.

1. Values depend strongly on amounts and types of impurities

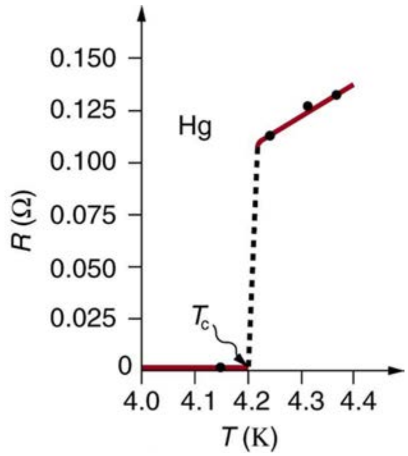


Figure 20.14 The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Table 20.2 Temperature Coefficients of Resistivity α

Material	Coefficient α (1/°C)[2]
Conductors	
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.93×10^{-3}
Lead	4.3×10^{-3}
Manganin (Cu, Mn, Ni alloy)	0.000×10^{-3}
Constantan (Cu, Ni alloy)	0.002×10^{-3}
Mercury	0.89×10^{-3}
Nichrome (Ni, Fe, Cr alloy)	0.4×10^{-3}
Semiconductors	
Carbon (pure)	-0.5×10^{-3}
Germanium (pure)	-50×10^{-3}
Silicon (pure)	-70×10^{-3}

Note also that α is negative for the semiconductors listed in **Table 20.2**, meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

2. Values at 20°C.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ. For a cylinder we know $R = \rho L/A$, and so, if L and A do not change greatly with temperature, R will have the same temperature dependence as ρ. (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on L and A is about two orders of magnitude less than on ρ.) Thus,

$$R = R_0(1 + \alpha\Delta T) \tag{20.24}$$

is the temperature dependence of the resistance of an object, where R_0 is the original resistance and R is the resistance after a temperature change ΔT. Numerous thermometers are based on the effect of temperature on resistance. (See **Figure 20.15**.) One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.

Figure 20.15 These familiar thermometers are based on the automated measurement of a thermistor's temperature-dependent resistance. (credit: Biol, Wikimedia Commons)

Example 20.6 Calculating Resistance: Hot-Filament Resistance

Although caution must be used in applying $\rho = \rho_0(1 + \alpha\Delta T)$ and $R = R_0(1 + \alpha\Delta T)$ for temperature changes greater than 100°C, for tungsten the equations work reasonably well for very large temperature changes. What, then, is the resistance of the tungsten filament in the previous example if its temperature is increased from room temperature (20°C) to a typical operating temperature of 2850°C?

Strategy

This is a straightforward application of $R = R_0(1 + \alpha\Delta T)$, since the original resistance of the filament was given to be $R_0 = 0.350\ \Omega$, and the temperature change is $\Delta T = 2830^\circ\text{C}$.

Solution

The hot resistance R is obtained by entering known values into the above equation:

$$\begin{aligned} R &= R_0(1 + \alpha\Delta T) \\ &= (0.350\ \Omega)[1 + (4.5\times10^{-3}/^\circ\text{C})(2830^\circ\text{C})] \\ &= 4.8\ \Omega. \end{aligned} \tag{20.25}$$

Discussion

This value is consistent with the headlight resistance example in **Ohm's Law: Resistance and Simple Circuits**.

PhET Explorations: Resistance in a Wire

Learn about the physics of resistance in a wire. Change its resistivity, length, and area to see how they affect the wire's resistance. The sizes of the symbols in the equation change along with the diagram of a wire.

PhET Interactive Simulation

Figure 20.16 Resistance in a Wire (http://cnx.org/content/m55357/1.2/resistance-in-a-wire_en.jar)

Applying the Science Practices: Examining Resistance

Using the PhET Simulation "Resistance in a Wire", design an experiment to determine how different variables – resistivity, length, and area – affect the resistance of a resistor. For each variable, you should record your results in a table and then create a graph to determine the relationship.

20.4 Electric Power and Energy

Learning Objectives

By the end of this section, you will be able to:

- Calculate the power dissipated by a resistor and the power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.B.9.8** The student is able to translate between graphical and symbolic representations of experimental data describing relationships among power, current, and potential difference across a resistor. **(S.P. 1.5)**

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for **electric power**? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See **Figure 20.17**(a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?

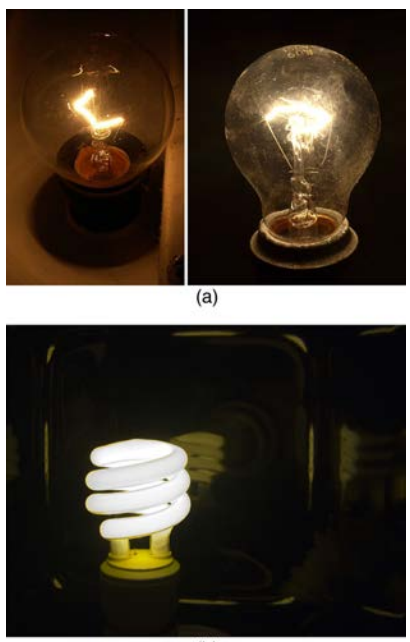


Figure 20.17 (a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $\text{PE} = qV$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the charge moves through). Power is the rate at which energy is moved, and so electric power is

$$P = \frac{PE}{t} = \frac{qV}{t}. \tag{20.26}$$

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

$$P = IV. \tag{20.27}$$

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1\ \text{A} \cdot \text{V} = 1\ \text{W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P = IV = (20\ \text{A})(12\ \text{V}) = 240\ \text{W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1\ \text{kA} \cdot \text{V} = 1\ \text{kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P = IV$. Substituting $I = V/R$ gives $P = (V / R)V = V^2/R$. Similarly, substituting $V = IR$ gives $P = I(IR) = I^2R$. Three expressions for electric power are listed together here for convenience:

$$P = IV \tag{20.28}$$

$$P = \frac{V^2}{R} \tag{20.29}$$

$$P = I^2R. \tag{20.30}$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Making Connections: Using Graphs to Calculate Resistance

As $p \propto I^2$ and $p \propto V^2$, the graph for power versus current or voltage is quadratic. An example is shown in the figure below.

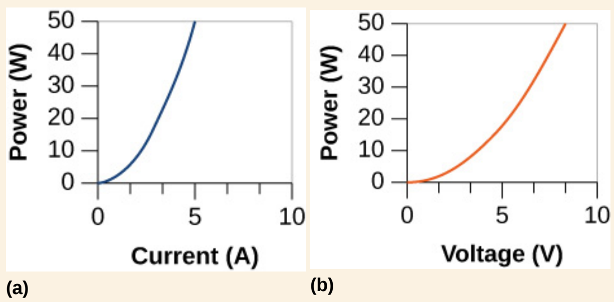


Figure 20.18 The figure shows (a) power versus current and (b) power versus voltage relationships for simple resistor circuits.

Using equations (20.29) and (20.30), we can calculate the resistance in each case. In graph (a), the power is 50 W when current is 5 A; hence, the resistance can be calculated as $R = P \big/ I^2 = 50 \big/ 5^2 = 2\ \Omega$. Similarly, the resistance value can be calculated in graph (b) as $R = V^2 \big/ P = 10^2 \big/ 50 = 2\ \Omega$

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2 / R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2 / R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

Example 20.7 Calculating Power Dissipation and Current: Hot and Cold Power

(a) Consider the examples given in **Ohm's Law: Resistance and Simple Circuits** and **Resistance and Resistivity**. Then find the power dissipated by the car headlight in these examples, both when it is hot and when it is cold. (b) What current does it draw when cold?

Strategy for (a)

For the hot headlight, we know voltage and current, so we can use $P = IV$ to find the power. For the cold headlight, we know the voltage and resistance, so we can use $P = V^2 / R$ to find the power.

Solution for (a)

Entering the known values of current and voltage for the hot headlight, we obtain

$$P = IV = (2.50\text{ A})(12.0\text{ V}) = 30.0\text{ W}. \tag{20.31}$$

The cold resistance was $0.350\ \Omega$, and so the power it uses when first switched on is

$$P = \frac{V^2}{R} = \frac{(12.0\text{ V})^2}{0.350\ \Omega} = 411\text{ W}. \tag{20.32}$$

Discussion for (a)

The 30 W dissipated by the hot headlight is typical. But the 411 W when cold is surprisingly higher. The initial power quickly decreases as the bulb's temperature increases and its resistance increases.

Strategy and Solution for (b)

The current when the bulb is cold can be found several different ways. We rearrange one of the power equations, $P = I^2R$, and enter known values, obtaining

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{411\text{ W}}{0.350\ \Omega}} = 34.3\text{ A}. \tag{20.33}$$

Discussion for (b)

The cold current is remarkably higher than the steady-state value of 2.50 A, but the current will quickly decline to that value as the bulb's temperature increases. Most fuses and circuit breakers (used to limit the current in a circuit) are designed to tolerate very high currents briefly as a device comes on. In some cases, such as with electric motors, the current remains high for several seconds, necessitating special “slow blow” fuses.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E/t$, we see that

$$E = Pt \tag{20.34}$$

is the energy used by a device using power P for a time interval t . For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW}\cdot\text{h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $1\text{ kW}\cdot\text{h} = 3.6{\times}10^6\text{ J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See Figure 20.17(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

Making Connections: Energy, Power, and Time

The relationship $E = Pt$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

Example 20.8 Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs $1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

$$E = Pt = (60\text{ W})(1000\text{ h}) = 60{,}000\text{ W}\cdot\text{h}. \tag{20.35}$$

In kilowatt-hours, this is

$$E = 60.0\text{ kW}\cdot\text{h}. \tag{20.36}$$

Now the electricity cost is

$$\text{cost} = (60.0\text{ kW}\cdot\text{h})(\$0.12/\text{kW}\cdot\text{h}) = \$7.20. \tag{20.37}$$

The total cost will be $7.20 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $7.20/4 = $1.80. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or 0.1($1.50) = $0.15. Therefore, the total cost will be $1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P = IV$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

20.5 Alternating Current versus Direct Current

Learning Objectives

By the end of this section, you will be able to:

- Explain the differences and similarities between AC and DC current.
- Calculate *rms* voltage, current, and average power.
- Explain why AC current is used for power transmission.

Alternating Current

Most of the examples dealt with so far, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. **Direct current** (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Most well-known applications, however, use a time-varying voltage source. **Alternating current** (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. **Figure 20.19** shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world.

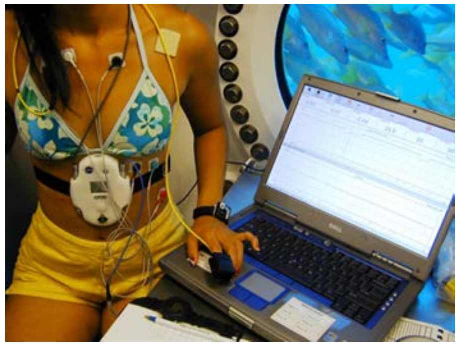


Figure 20.38 This NASA scientist and NEEMO 5 aquanaut's heart rate and other vital signs are being recorded by a portable device while living in an underwater habitat. (credit: NASA, Life Sciences Data Archive at Johnson Space Center, Houston, Texas)

PhET Explorations: Neuron

PhET Interactive Simulation

Figure 20.39 Neuron (http://cnx.org/content/m55361/1.2/neuron_en.jar)

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

Glossary

AC current: current that fluctuates sinusoidally with time, expressed as $I = I_0 \sin 2\pi ft$, where I is the current at time t, I_0 is the peak current, and f is the frequency in hertz

AC voltage: voltage that fluctuates sinusoidally with time, expressed as $V = V_0 \sin 2\pi ft$, where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz

alternating current: (AC) the flow of electric charge that periodically reverses direction

ampere: (amp) the SI unit for current; 1 A = 1 C/s

bioelectricity: electrical effects in and created by biological systems

direct current: (DC) the flow of electric charge in only one direction

drift velocity: the average velocity at which free charges flow in response to an electric field

electric current: the rate at which charge flows, $I = \Delta Q/\Delta t$

electric power: the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

electrocardiogram (ECG): usually abbreviated ECG, a record of voltages created by depolarization and repolarization, especially in the heart

microshock sensitive: a condition in which a person's skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

nerve conduction: the transport of electrical signals by nerve cells

ohm: the unit of resistance, given by $1\Omega = 1$ V/A

Ohm's law: an empirical relation stating that the current I is proportional to the potential difference V, $\propto V$; it is often written as $I = V/R$, where R is the resistance

ohmic: a type of a material for which Ohm's law is valid

resistance: the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$

resistivity: an intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by ρ

rms current: the root mean square of the current, $I_{\text{rms}} = I_0 / \sqrt{2}$, where I_0 is the peak current, in an AC system

rms voltage: the root mean square of the voltage, $V_{\text{rms}} = V_0 / \sqrt{2}$, where V_0 is the peak voltage, in an AC system

semipermeable: property of a membrane that allows only certain types of ions to cross it

shock hazard: when electric current passes through a person

short circuit: also known as a "short," a low-resistance path between terminals of a voltage source

simple circuit: a circuit with a single voltage source and a single resistor

temperature coefficient of resistivity: an empirical quantity, denoted by α, which describes the change in resistance or resistivity of a material with temperature

thermal hazard: a hazard in which electric current causes undesired thermal effects

Section Summary

20.1 Current

- Electric current I is the rate at which charge flows, given by

$$I = \frac{\Delta Q}{\Delta t},$$

  where ΔQ is the amount of charge passing through an area in time Δt .
- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1\ \text{A} = 1\ \text{C/s}.$
- Current is the flow of free charges, such as electrons and ions.
- Drift velocity v_d is the average speed at which these charges move.
- Current I is proportional to drift velocity v_d , as expressed in the relationship $I = nqAv_\text{d}$. Here, I is the current through a wire of cross-sectional area A . The wire's material has a free-charge density n , and each carrier has charge q and a drift velocity v_d .
- Electrical signals travel at speeds about 10^{12} times greater than the drift velocity of free electrons.

20.2 Ohm's Law: Resistance and Simple Circuits

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I , voltage V , and resistance R in a simple circuit to be $I = \frac{V}{R}.$
- Resistance has units of ohms (Ω), related to volts and amperes by $1\ \Omega = 1\ \text{V/A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V = IR$.

20.3 Resistance and Resistivity

- The resistance R of a cylinder of length L and cross-sectional area A is $R = \frac{\rho L}{A}$, where ρ is the resistivity of the material.
- Values of ρ in **Table 20.1** show that materials fall into three groups—*conductors, semiconductors, and insulators*.
- Temperature affects resistivity; for relatively small temperature changes ΔT , resistivity is $\rho = \rho_0(1 + \alpha \Delta T)$, where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- **Table 20.2** gives values for α , the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature: $R = R_0(1 + \alpha \Delta T)$, where R_0 is the original resistance, and R is the resistance after the temperature change.

20.4 Electric Power and Energy

- Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device.

- Three expressions for electrical power are
$$P = IV,$$
$$P = \frac{V^2}{R},$$
and
$$P = I^2R.$$
- The energy used by a device with a power P over a time t is $E = Pt$.

20.5 Alternating Current versus Direct Current

- Direct current (DC) is the flow of electric current in only one direction. It refers to systems where the source voltage is constant.
- The voltage source of an alternating current (AC) system puts out $V = V_0 \sin 2\pi ft$, where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz.
- In a simple circuit, $I = V/R$ and AC current is $I = I_0 \sin 2\pi ft$, where I is the current at time t, and $I_0 = V_0/R$ is the peak current.
- The average AC power is $P_{\text{ave}} = \frac{1}{2}I_0 V_0$.
- Average (rms) current I_{rms} and average (rms) voltage V_{rms} are $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$ and $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$, where rms stands for root mean square.
- Thus, $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$.
- Ohm's law for AC is $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$.
- Expressions for the average power of an AC circuit are $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$, $P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R}$, and $P_{\text{ave}} = I_{\text{rms}}^2 R$, analogous to the expressions for DC circuits.

20.6 Electric Hazards and the Human Body

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- **Table 20.3** lists shock hazards as a function of current.
- **Figure 20.28** graphs the threshold current for two hazards as a function of frequency.

20.7 Nerve Conduction–Electrocardiograms

- Electric potentials in neurons and other cells are created by ionic concentration differences across semipermeable membranes.
- Stimuli change the permeability and create action potentials that propagate along neurons.
- Myelin sheaths speed this process and reduce the needed energy input.
- This process in the heart can be measured with an electrocardiogram (ECG).

Conceptual Questions

20.1 Current

1. Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

2. Car batteries are rated in ampere-hours ($\text{A} \cdot \text{h}$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

3. If two different wires having identical cross-sectional areas carry the same current, will the drift velocity be higher or lower in the better conductor? Explain in terms of the equation $v_{\text{d}} = \frac{I}{nqA}$, by considering how the density of charge carriers n relates to whether or not a material is a good conductor.

4. Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

5. In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

6. Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

20.2 Ohm’s Law: Resistance and Simple Circuits

7. The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

8. How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

20.3 Resistance and Resistivity

9. In which of the three semiconducting materials listed in **Table 20.1** do impurities supply free charges? (Hint: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)

10. Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width? (See **Figure 20.40**.)

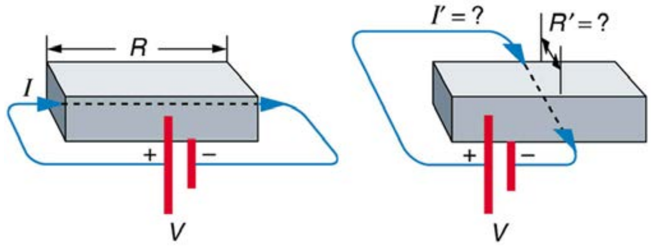


Figure 20.40 Does current taking two different paths through the same object encounter different resistance?

11. If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?

12. Explain why $R = R_0(1 + \alpha\Delta T)$ for the temperature variation of the resistance R of an object is not as accurate as $\rho = \rho_0(1 + \alpha\Delta T)$, which gives the temperature variation of resistivity ρ.

20.4 Electric Power and Energy

13. Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

14. The power dissipated in a resistor is given by $P = V^2 / R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2R$, which means power increases if resistance increases. Explain why there is no contradiction here.

20.5 Alternating Current versus Direct Current

15. Give an example of a use of AC power other than in the household. Similarly, give an example of a use of DC power other than that supplied by batteries.

16. Why do voltage, current, and power go through zero 120 times per second for 60-Hz AC electricity?

17. You are riding in a train, gazing into the distance through its window. As close objects streak by, you notice that the nearby fluorescent lights make *dashed* streaks. Explain.

20.6 Electric Hazards and the Human Body

18. Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

19. What are the two major hazards of electricity?

20. Why isn't a short circuit a shock hazard?

21. What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

22. An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

23. Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

24. Some devices often used in bathrooms, such as hairdryers, often have safety messages saying “Do not use when the bathtub or basin is full of water.” Why is this so?

25. We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

26. Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

27. Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

28. Could a person on intravenous infusion (an IV) be microshock sensitive?

29. In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

20.7 Nerve Conduction–Electrocardiograms

30. Note that in **Figure 20.31**, both the concentration gradient and the Coulomb force tend to move Na^+ ions into the cell. What prevents this?

31. Define depolarization, repolarization, and the action potential.

32. Explain the properties of myelinated nerves in terms of the insulating properties of myelin.

Problems & Exercises

20.1 Current

1. What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

2. A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

3. What is the current when a typical static charge of $0.250\ \mu\text{C}$ moves from your finger to a metal doorknob in $1.00\ \mu\text{s}$?

4. Find the current when 2.00 nC jumps between your comb and hair over a $0.500\text{-}\mu\text{s}$ time interval.

5. A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

6. The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

7. (a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in the figure below. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $P = I^2R$.)

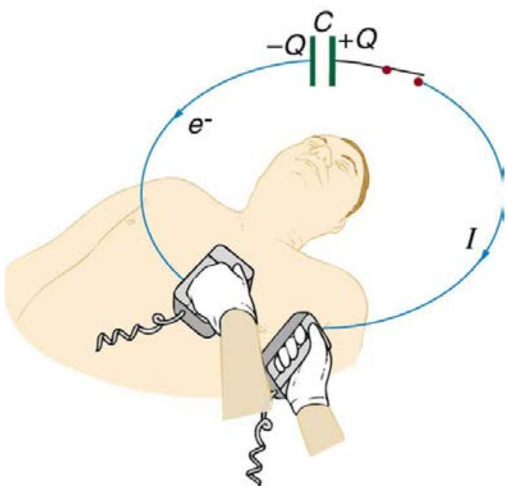


Figure 20.41 The capacitor in a defibrillation unit drives a current through the heart of a patient.

8. During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is $500\ \Omega$ and a 10.0-mA current is needed. What voltage should be applied?

9. (a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

10. A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

11. The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?

12. Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

13. A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA. (a) How many He^{++} nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of He^{++} nuclei strike the target?

14. Repeat the above example on **Example 20.3**, but for a wire made of silver and given there is one free electron per silver atom.

15. Using the results of the above example on **Example 20.3**, find the drift velocity in a copper wire of twice the diameter and carrying 20.0 A.

16. A 14-gauge copper wire has a diameter of 1.628 mm. What magnitude current flows when the drift velocity is 1.00 mm/s? (See above example on **Example 20.3** for useful information.)

17. SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator (closed in 2009), has a 20.0-A circulating beam of electrons that are moving at nearly the speed of light. (See **Figure 20.42**.) How many electrons are in the beam?

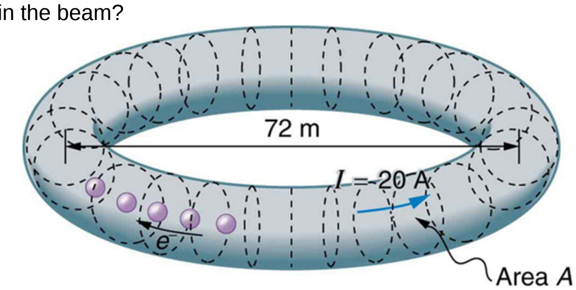


Figure 20.42 Electrons circulating in the storage ring called SPEAR constitute a 20.0-A current. Because they travel close to the speed of light, each electron completes many orbits in each second.

20.2 Ohm's Law: Resistance and Simple Circuits

18. What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

19. Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

20. What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

21. How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

22. (a) Find the voltage drop in an extension cord having a $0.0600\text{-}\Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of $0.300\ \Omega$. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

23. A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00\times10^{9}\ \Omega$. What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

20.3 Resistance and Resistivity

24. What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

25. The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.

26. If the 0.100-mm diameter tungsten filament in a light bulb is to have a resistance of $0.200\ \Omega$ at 20.0°C, how long should it be?

27. Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).

28. What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when $1.00 \times 10^{3}\ \text{V}$ is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)

29. (a) To what temperature must you raise a copper wire, originally at 20.0°C, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

30. A resistor made of Nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at 20.0°C. Over what temperature range can it be used?

31. Of what material is a resistor made if its resistance is 40.0% greater at 100°C than at 20.0°C?

32. An electronic device designed to operate at any temperature in the range from -10.0°C to 55.0°C contains pure carbon resistors. By what factor does their resistance increase over this range?

33. (a) Of what material is a wire made, if it is 25.0 m long with a 0.100 mm diameter and has a resistance of $77.7\ \Omega$ at 20.0°C? (b) What is its resistance at 150°C?

34. Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at 20.0°C?

35. A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?

36. A copper wire has a resistance of $0.500\ \Omega$ at 20.0°C, and an iron wire has a resistance of $0.525\ \Omega$ at the same temperature. At what temperature are their resistances equal?

37. (a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.0600/^\circ\text{C}$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at 37.0°C (normal body temperature)? (b) The negative value for α may not be maintained for very low temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)

38. Integrated Concepts

(a) Redo **Exercise 20.25** taking into account the thermal expansion of the tungsten filament. You may assume a thermal expansion coefficient of $12\times10^{-6}/^\circ\text{C}$. (b) By what percentage does your answer differ from that in the example?

39. Unreasonable Results

(a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?

20.4 Electric Power and Energy

40. What is the power of a $1.00\times10^{2}\ \text{MV}$ lightning bolt having a current of $2.00 \times 10^{4}\ \text{A}$?

41. What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

42. A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See **Figure 20.43**.)

Figure 20.43 The strip of solar cells just above the keys of this calculator convert light to electricity to supply its energy needs. (credit: Evan-Amos, Wikimedia Commons)

43. How many watts does a flashlight that has $6.00\times10^{2}\ \text{C}$ pass through it in 0.500 h use if its voltage is 3.00 V?

44. Find the power dissipated in each of these extension cords: (a) an extension cord having a $0.0600\text{-}\Omega$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of $0.300\ \Omega$.

45. Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.

46. Show that the units $1\ \text{V}^2/\Omega = 1\text{W}$, as implied by the equation $P = V^2/R$.

47. Show that the units $1\ \text{A}^2 \cdot \Omega = 1\ \text{W}$, as implied by the equation $P = I^2R$.

48. Verify the energy unit equivalence that $1\ \text{kW} \cdot \text{h} = 3.60\times10^{6}\ \text{J}$.

49. Electrons in an X-ray tube are accelerated through $1.00\times10^{2}\ \text{kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.

50. An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs $12.0\ \text{cents/kW} \cdot \text{h}$? See **Figure 20.44**.

Figure 20.44 On-demand electric hot water heater. Heat is supplied to water only when needed. (credit: aviddavid, Flickr)

51. With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At $9.0\ \text{cents/kW} \cdot \text{h}$, how much does this cost?

52. What would be the maximum cost of a CFL such that the total cost (investment plus operating) would be the same for both CFL and incandescent 60-W bulbs? Assume the cost of the incandescent bulb is 25 cents and that electricity costs $10\ \text{cents/kWh}$. Calculate the cost for 1000 hours, as in the cost effectiveness of CFL example.

53. Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

54. Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at $1.00\ \text{A} \cdot \text{h}$ and 1.58 V keep a 1.00-W flashlight bulb burning?

55. A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

56. The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages $12.0\ \text{cents/kW} \cdot \text{h}$.

57. An old lightbulb draws only 50.0 W, rather than its original 60.0 W, due to evaporative thinning of its filament. By what factor is its diameter reduced, assuming uniform thinning along its length? Neglect any effects caused by temperature differences.

58. 00-gauge copper wire has a diameter of 9.266 mm. Calculate the power loss in a kilometer of such wire when it carries $1.00\times10^{2}\ \text{A}$.

59. Integrated Concepts

Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See **Figure 20.45**.)

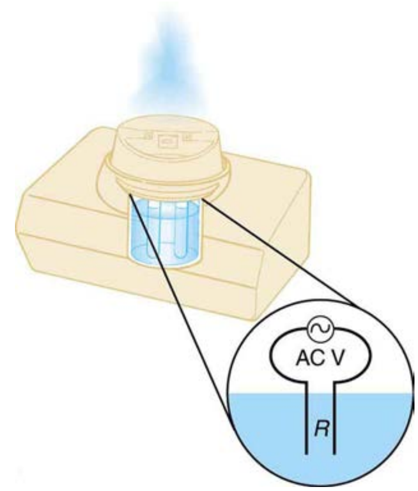


Figure 20.45 This cold vaporizer passes current directly through water, vaporizing it directly with relatively little temperature increase.

60. Integrated Concepts

(a) What energy is dissipated by a lightning bolt having a 20,000-A current, a voltage of $1.00\times10^{2}\ \text{MV}$, and a length of 1.00 ms? (b) What mass of tree sap could be raised from 18.0°C to its boiling point and then evaporated by this energy, assuming sap has the same thermal characteristics as water?

61. Integrated Concepts

What current must be produced by a 12.0-V battery-operated bottle warmer in order to heat 75.0 g of glass, 250 g of baby formula, and $3.00\times10^{2}\ \text{g}$ of aluminum from 20.0°C to 90.0°C in 5.00 min?

62. Integrated Concepts

How much time is needed for a surgical cauterizer to raise the temperature of 1.00 g of tissue from 37.0°C to 100°C and then boil away 0.500 g of water, if it puts out 2.00 mA at 15.0 kV? Ignore heat transfer to the surroundings.

63. Integrated Concepts

Hydroelectric generators (see **Figure 20.46**) at Hoover Dam produce a maximum current of $8.00\times10^{3}\ \text{A}$ at 250 kV. (a) What is the power output? (b) The water that powers the generators enters and leaves the system at low speed (thus its kinetic energy does not change) but loses 160 m in altitude. How many cubic meters per second are needed, assuming 85.0% efficiency?

Figure 20.46 Hydroelectric generators at the Hoover dam. (credit: Jon Sullivan)

64. Integrated Concepts

(a) Assuming 95.0% efficiency for the conversion of electrical power by the motor, what current must the 12.0-V batteries of a 750-kg electric car be able to supply: (a) To accelerate from rest to 25.0 m/s in 1.00 min? (b) To climb a $2.00\times10^{2}\text{-m}$ -high hill in 2.00 min at a constant 25.0-m/s speed while exerting $5.00\times10^{2}\text{ N}$ of force to overcome air resistance and friction? (c) To travel at a constant 25.0-m/s speed, exerting a $5.00\times10^{2}\text{ N}$ force to overcome air resistance and friction? See **Figure 20.47**.

Figure 20.47 This REVAi, an electric car, gets recharged on a street in London. (credit: Frank Hebbert)

65. Integrated Concepts

A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is $5.30\times10^{4}\text{ kg}$, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.

66. Integrated Concepts

(a) An aluminum power transmission line has a resistance of $0.0580\ \Omega/\text{km}$. What is its mass per kilometer? (b) What is the mass per kilometer of a copper line having the same resistance? A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

67. Integrated Concepts

(a) An immersion heater utilizing 120 V can raise the temperature of a $1.00\times10^{2}\text{-g}$ aluminum cup containing 350 g of water from 20.0°C to 95.0°C in 2.00 min. Find its resistance, assuming it is constant during the process. (b) A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

68. Integrated Concepts

(a) What is the cost of heating a hot tub containing 1500 kg of water from 10.0°C to 40.0°C, assuming 75.0% efficiency to account for heat transfer to the surroundings? The cost of electricity is $9\text{ cents/kW}\cdot\text{h}$. (b) What current was used by the 220-V AC electric heater, if this took 4.00 h?

69. Unreasonable Results

(a) What current is needed to transmit $1.00\times10^{2}\text{ MW}$ of power at 480 V? (b) What power is dissipated by the transmission lines if they have a $1.00\text{-}\Omega$ resistance? (c) What is unreasonable about this result? (d) Which assumptions are unreasonable, or which premises are inconsistent?

70. Unreasonable Results

(a) What current is needed to transmit $1.00\times10^{2}\text{ MW}$ of power at 10.0 kV? (b) Find the resistance of 1.00 km of wire that would cause a 0.0100% power loss. (c) What is the diameter of a 1.00-km-long copper wire having this resistance? (d) What is unreasonable about these results? (e) Which assumptions are unreasonable, or which premises are inconsistent?

71. Construct Your Own Problem

Consider an electric immersion heater used to heat a cup of water to make tea. Construct a problem in which you calculate the needed resistance of the heater so that it increases the temperature of the water and cup in a reasonable amount of time. Also calculate the cost of the electrical energy used in your process. Among the things to be considered are the voltage used, the masses and heat capacities involved, heat losses, and the time over which the heating takes place. Your instructor may wish for you to consider a thermal safety switch (perhaps bimetallic) that will halt the process before damaging temperatures are reached in the immersion unit.

20.5 Alternating Current versus Direct Current

72. (a) What is the hot resistance of a 25-W light bulb that runs on 120-V AC? (b) If the bulb's operating temperature is 2700°C, what is its resistance at 2600°C?

73. Certain heavy industrial equipment uses AC power that has a peak voltage of 679 V. What is the rms voltage?

74. A certain circuit breaker trips when the rms current is 15.0 A. What is the corresponding peak current?

75. Military aircraft use 400-Hz AC power, because it is possible to design lighter-weight equipment at this higher frequency. What is the time for one complete cycle of this power?

76. A North American tourist takes his 25.0-W, 120-V AC razor to Europe, finds a special adapter, and plugs it into 240 V AC. Assuming constant resistance, what power does the razor consume as it is ruined?

77. In this problem, you will verify statements made at the end of the power losses for **Example 20.10**. (a) What current is needed to transmit 100 MW of power at a voltage of 25.0 kV? (b) Find the power loss in a $1.00\text{-}\ \Omega$ transmission line. (c) What percent loss does this represent?

78. A small office-building air conditioner operates on 408-V AC and consumes 50.0 kW. (a) What is its effective resistance? (b) What is the cost of running the air conditioner during a hot summer month when it is on 8.00 h per day for 30 days and electricity costs $9.00\text{ cents/kW}\cdot\text{h}$?

79. What is the peak power consumption of a 120-V AC microwave oven that draws 10.0 A?

80. What is the peak current through a 500-W room heater that operates on 120-V AC power?

81. Two different electrical devices have the same power consumption, but one is meant to be operated on 120-V AC and the other on 240-V AC. (a) What is the ratio of their resistances? (b) What is the ratio of their currents? (c) Assuming its resistance is unaffected, by what factor will the power increase if a 120-V AC device is connected to 240-V AC?

82. Nichrome wire is used in some radiative heaters. (a) Find the resistance needed if the average power output is to be 1.00 kW utilizing 120-V AC. (b) What length of Nichrome wire, having a cross-sectional area of 5.00mm^2, is needed if the operating temperature is 500° C? (c) What power will it draw when first switched on?

83. Find the time after $t = 0$ when the instantaneous voltage of 60-Hz AC first reaches the following values: (a) $V_0 / 2$ (b) V_0 (c) 0.

84. (a) At what two times in the first period following $t = 0$ does the instantaneous voltage in 60-Hz AC equal V_{rms}? (b) $-V_{\text{rms}}$?

20.6 Electric Hazards and the Human Body

85. (a) How much power is dissipated in a short circuit of 240-V AC through a resistance of $0.250\ \Omega$? (b) What current flows?

86. What voltage is involved in a 1.44-kW short circuit through a $0.100\text{-}\ \Omega$ resistance?

87. Find the current through a person and identify the likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of $300\text{ k}\Omega$; (b) if she is standing barefoot on wet grass and has a resistance of only $4000\text{ k}\Omega$.

88. While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of $4000\ \Omega$. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

89. Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

90. (a) During surgery, a current as small as $20.0\ \mu\text{A}$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $300\ \Omega$, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

91. (a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?

92. A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

93. Integrated Concepts

A short circuit in a 120-V appliance cord has a $0.500\text{-}\ \Omega$ resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200\text{ cal/g}\cdot^\circ\text{C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

94. Construct Your Own Problem

Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, …), and acceptable currents (safe but sensed, safe and unfelt, …).

20.7 Nerve Conduction–Electrocardiograms

95. Integrated Concepts

Use the ECG in **Figure 20.37** to determine the heart rate in beats per minute assuming a constant time between beats.

96. Integrated Concepts

(a) Referring to **Figure 20.37**, find the time systolic pressure lags behind the middle of the QRS complex. (b) Discuss the reasons for the time lag.

Test Prep for AP® Courses

20.1 Current

1. Which of the following can be explained on the basis of conservation of charge in a closed circuit consisting of a battery, resistor, and metal wires?

a. The number of electrons leaving the battery will be equal to the number of electrons entering the battery.
b. The number of electrons leaving the battery will be less than the number of electrons entering the battery.
c. The number of protons leaving the battery will be equal to the number of protons entering the battery.
d. The number of protons leaving the battery will be less than the number of protons entering the battery.

2. When a battery is connected to a bulb, there is 2.5 A of current in the circuit. What amount of charge will flow though the circuit in a time of 0.5 s?

a. 0.5 C
b. 1 C
c. 1.25 C
d. 1.5 C

3. If 0.625×10^{20} electrons flow through a circuit each second, what is the current in the circuit?

4. Two students calculate the charge flowing through a circuit. The first student concludes that 300 C of charge flows in 1 minute. The second student concludes that 3.125×10^{19} electrons flow per second. If the current measured in the circuit is 5 A, which of the two students (if any) have performed the calculations correctly?

20.2 Ohm's Law: Resistance and Simple Circuits

5. If the voltage across a fixed resistance is doubled, what happens to the current?

a. It doubles.
b. It halves.
c. It stays the same.
d. The current cannot be determined.

6. The table below gives the voltages and currents recorded across a resistor.

Table 20.4

Voltage (V)	2.50	5.00	7.50	10.00	12.50
Current (A)	0.69	1.38	2.09	2.76	3.49

a. Plot the graph and comment on the shape.
b. Calculate the value of the resistor.

7. What is the resistance of a bulb if the current in it is 1.25 A when a 4 V voltage supply is connected to it? If the voltage supply is increased to 7 V, what will be the current in the bulb?

20.3 Resistance and Resistivity

8. Which of the following affect the resistivity of a wire?

a. length
b. area of cross section
c. material
d. all of the above

9. The lengths and diameters of four wires are given as shown.

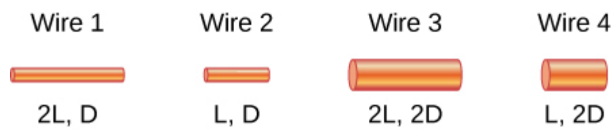


Figure 20.48

If the four wires are made from the same material, which of the following is true? Select *two* answers.

a. Resistance of Wire 3 > Resistance of Wire 2
b. Resistance of Wire 1 > Resistance of Wire 2
c. Resistance of Wire 1 < Resistance of Wire 4
d. Resistance of Wire 4 < Resistance of Wire 3

10. Suppose the resistance of a wire is *R* Ω. What will be the resistance of another wire of the same material having the same length but double the diameter?

a. *R*/2
b. 2*R*
c. *R*/4
d. 4*R*

11. The resistances of two wires having the same lengths and cross section areas are 3 Ω and 11 Ω. If the resistivity of the 3 Ω wire is 2.65×10^{-8} Ω·m, find the resistivity of the 1 Ω wire.

12. The lengths and diameters of three wires are given below. If they all have the same resistance, find the ratio of their resistivities.

Table 20.5

Wire	*Length*	*Diameter*
Wire 1	2 m	1 cm
Wire 2	1 m	0.5 cm
Wire 3	1 m	1 cm

13. Suppose the resistance of a wire is 2 Ω. If the wire is stretched to three times its length, what will be its resistance? Assume that the volume does not change.

20.4 Electric Power and Energy

14.

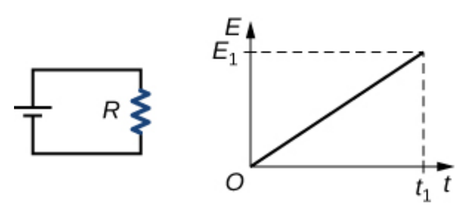
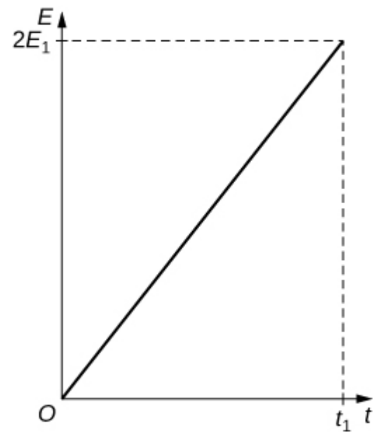
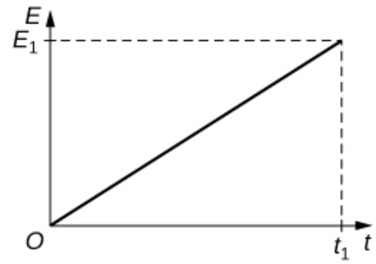


Figure 20.49 The circuit shown contains a resistor *R* connected to a voltage supply. The graph shows the total energy *E* dissipated by the resistance as a function of time. Which of the following shows the corresponding graph for double resistance, i.e., if *R* is replaced by 2*R*?



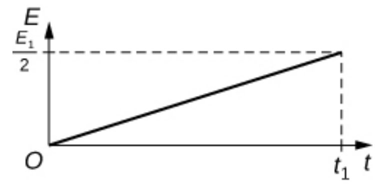
a.

Figure 20.50



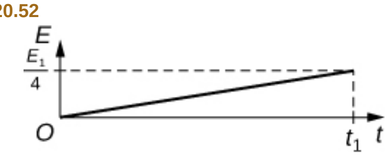
b.

Figure 20.51



c.

Figure 20.52



d.

Figure 20.53

15. What will be the ratio of the resistance of a 120 W, 220 V lamp to that of a 100 W, 110 V lamp?

21 CIRCUITS, BIOELECTRICITY, AND DC INSTRUMENTS

Figure 21.1 Electric circuits in a computer allow large amounts of data to be quickly and accurately analyzed.. (credit: Airman 1st Class Mike Meares, United States Air Force)

Chapter Outline

Connection for AP® Courses

Electric circuits are commonplace in our everyday lives. Some circuits are simple, such as those in flashlights while others are extremely complex, such as those used in supercomputers. This chapter takes the topic of electric circuits a step beyond simple circuits by addressing both changes that result from interactions between systems (Big Idea 4) and constraints on such changes due to laws of conservation (Big Idea 5). When the circuit is purely resistive, everything in this chapter applies to both DC and AC. However, matters become more complex when capacitance is involved. We do consider what happens when capacitors are connected to DC voltage sources, but the interaction of capacitors (and other nonresistive devices) with AC sources is left for a later chapter. In addition, a number of important DC instruments, such as meters that measure voltage and current, are covered in this chapter.

Information and examples presented in the chapter examine cause-effect relationships inherent in interactions involving electrical systems. The electrical properties of an electric circuit can change due to other systems (Enduring Understanding 4.E). More specifically, values of currents and potential differences in electric circuits depend on arrangements of individual circuit components (Essential Knowledge 4.E.5). In this chapter several series and parallel combinations of resistors are discussed and their effects on currents and potential differences are analyzed.

In electric circuits the total energy (Enduring Understanding 5.B) and the total electric charge (Enduring Understanding 5.C) are conserved. Kirchoff's rules describe both, energy conservation (Essential Knowledge 5.B.9) and charge conservation (Essential

Knowledge 5.C.3). Energy conservation is discussed in terms of the loop rule which specifies that the potential around any closed circuit path must be zero. Charge conservation is applied as conservation of current by equating the sum of all currents entering a junction to the sum of all currents leaving the junction (also known as the junction rule). Kirchoff's rules are used to calculate currents and potential differences in circuits that combine resistors in series and parallel, and resistors and capacitors.

The concepts in this chapter support:

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding 4.E The electric and magnetic properties of a system can change in response to the presence of, or changes in, other objects or systems.

Essential Knowledge 4.E.5 The values of currents and electric potential differences in an electric circuit are determined by the properties and arrangement of the individual circuit elements such as sources of emf, resistors, and capacitors.

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.B The energy of a system is conserved.

Essential Knowledge 5.B.9 Kirchhoff's loop rule describes conservation of energy in electrical circuits.

Enduring Understanding 5.C The electric charge of a system is conserved.

Essential Knowledge 5.C.3 Kirchhoff's junction rule describes the conservation of electric charge in electrical circuits. Since charge is conserved, current must be conserved at each junction in the circuit. Examples should include circuits that combine resistors in series and parallel.

21.1 Resistors in Series and Parallel

Learning Objectives

By the end of this section, you will be able to:

- Draw a circuit with resistors in parallel and in series.
- Use Ohm's law to calculate the voltage drop across a resistor when current passes through it.
- Contrast the way total resistance is calculated for resistors in series and in parallel.
- Explain why total resistance of a parallel circuit is less than the smallest resistance of any of the resistors in that circuit.
- Calculate total resistance of a circuit that contains a mixture of resistors connected in series and in parallel.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.E.5.1** The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. **(S.P. 2.2, 6.4)**
- **4.E.5.2** The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. **(S.P. 6.1, 6.4)**
- **4.E.5.3** The student is able to plan data collection strategies and perform data analysis to examine the values of currents and potential differences in an electric circuit that is modified by changing or rearranging circuit elements, including sources of emf, resistors, and capacitors. **(S.P. 2.2, 4.2, 5.1)**
- **5.B.9.3** The student is able to apply conservation of energy (Kirchhoff's loop rule) in calculations involving the total electric potential difference for complete circuit loops with only a single battery and resistors in series and/or in, at most, one parallel branch. **(S.P. 2.2, 6.4, 7.2)**

Most circuits have more than one component, called a **resistor** that limits the flow of charge in the circuit. A measure of this limit on charge flow is called **resistance**. The simplest combinations of resistors are the series and parallel connections illustrated in **Figure 21.2**. The total resistance of a combination of resistors depends on both their individual values and how they are connected.

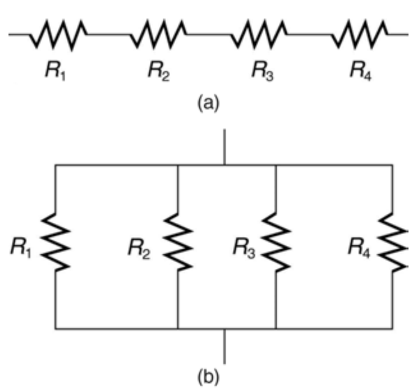


Figure 21.2 (a) A series connection of resistors. (b) A parallel connection of resistors.

Resistors in Series

When are resistors in **series**? Resistors are in series whenever the flow of charge, called the **current**, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then R_1 in **Figure 21.2**(a) could be the resistance of the screwdriver's shaft, R_2 the resistance of its handle, R_3 the person's body resistance, and R_4 the resistance of her shoes.

Figure 21.3 shows resistors in series connected to a **voltage** source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubber-soled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)

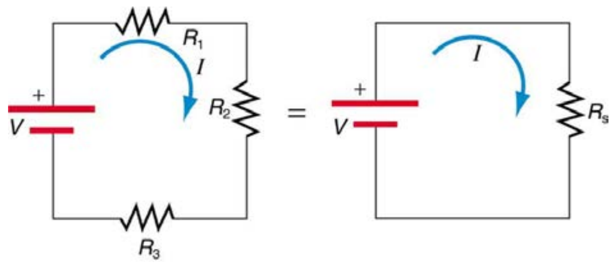


Figure 21.3 Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a **voltage drop**, in each resistor in **Figure 21.3**.

According to **Ohm's law**, the voltage drop, V , across a resistor when a current flows through it is calculated using the equation $V = IR$, where I equals the current in amps (A) and R is the resistance in ohms (Ω) . Another way to think of this is that V is the voltage necessary to make a current I flow through a resistance R .

So the voltage drop across R_1 is $V_1 = IR_1$, that across R_2 is $V_2 = IR_2$, and that across R_3 is $V_3 = IR_3$. The sum of these voltages equals the voltage output of the source; that is,

$$V = V_1 + V_2 + V_3. \tag{21.1}$$

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation $PE = qV$, where q is the electric charge and V is the voltage. Thus the energy supplied by the source is qV , while that dissipated by the resistors is

$$qV_1 + qV_2 + qV_3. \tag{21.2}$$

Connections: Conservation Laws

The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus, $qV = qV_1 + qV_2 + qV_3$. The charge q cancels, yielding $V = V_1 + V_2 + V_3$, as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)

Now substituting the values for the individual voltages gives

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3). \tag{21.3}$$

Note that for the equivalent single series resistance R_s , we have

$$V = IR_s. \tag{21.4}$$

This implies that the total or equivalent series resistance R_s of three resistors is $R_s = R_1 + R_2 + R_3$.

This logic is valid in general for any number of resistors in series; thus, the total resistance R_s of a series connection is

$$R_s = R_1 + R_2 + R_3 + ..., \tag{21.5}$$

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

Example 21.1 Calculating Resistance, Current, Voltage Drop, and Power Dissipation: Analysis of a Series Circuit

Suppose the voltage output of the battery in Figure 21.3 is 12.0 V, and the resistances are $R_1 = 1.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 13.0\ \Omega$. (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance is simply the sum of the individual resistances, as given by this equation:

$$\begin{aligned} R_\text{s} &= R_1 + R_2 + R_3 \\ &= 1.00\ \Omega + 6.00\ \Omega + 13.0\ \Omega \\ &= 20.0\ \Omega. \end{aligned} \tag{21.6}$$

Strategy and Solution for (b)

The current is found using Ohm's law, $V = IR$. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

$$I = \frac{V}{R_\text{s}} = \frac{12.0\text{ V}}{20.0\ \ \Omega} = 0.600\text{ A}. \tag{21.7}$$

Strategy and Solution for (c)

The voltage—or IR drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

$$V_1 = IR_1 = (0.600\text{ A})(1.0\ \ \Omega\,) = 0.600\text{ V}. \tag{21.8}$$

Similarly,

$$V_2 = IR_2 = (0.600\text{ A})(6.0\ \ \Omega\,) = 3.60\text{ V} \tag{21.9}$$

and

$$V_3 = IR_3 = (0.600\text{ A})(13.0\ \ \Omega\,) = 7.80\text{ V}. \tag{21.10}$$

Discussion for (c)

The three IR drops add to 12.0 V, as predicted:

$$V_1 + V_2 + V_3 = (0.600 + 3.60 + 7.80)\text{ V} = 12.0\text{ V}. \tag{21.11}$$

Strategy and Solution for (d)

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use **Joule's law**, $P = IV$, where P is electric power. In this case, each resistor has the same full current flowing through it. By substituting Ohm's law $V = IR$ into Joule's law, we get the power dissipated by the first resistor as

$$P_1 = I^2R_1 = (0.600\text{ A})^2(1.00\ \ \Omega\,) = 0.360\text{ W}. \tag{21.12}$$

Similarly,

$$P_2 = I^2R_2 = (0.600\text{ A})^2(6.00\ \ \Omega\,) = 2.16\text{ W} \tag{21.13}$$

and

$$P_3 = I^2R_3 = (0.600\text{ A})^2(13.0\ \ \Omega\,) = 4.68\text{ W}. \tag{21.14}$$

Discussion for (d)

Power can also be calculated using either $P = IV$ or $P = \frac{V^2}{R}$, where V is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

Strategy and Solution for (e)

The easiest way to calculate power output of the source is to use $P = IV$, where V is the source voltage. This gives

$$P = (0.600\ \text{A})(12.0\ \text{V}) = 7.20\ \text{W}. \tag{21.15}$$

Discussion for (e)

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

$$P_1 + P_2 + P_3 = (0.360 + 2.16 + 4.68)\ \text{W} = 7.20\ \text{W}. \tag{21.16}$$

Power is energy per unit time (watts), and so conservation of energy requires the power output of the source to be equal to the total power dissipated by the resistors.

Major Features of Resistors in Series

1. Series resistances add: $R_\text{s} = R_1 + R_2 + R_3 + ...$
2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it.

Resistors in Parallel

Figure 21.4 shows resistors in **parallel**, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile’s headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the source and can operate completely independently. The same is true in your house, or any building. (See **Figure 21.4**(b).)

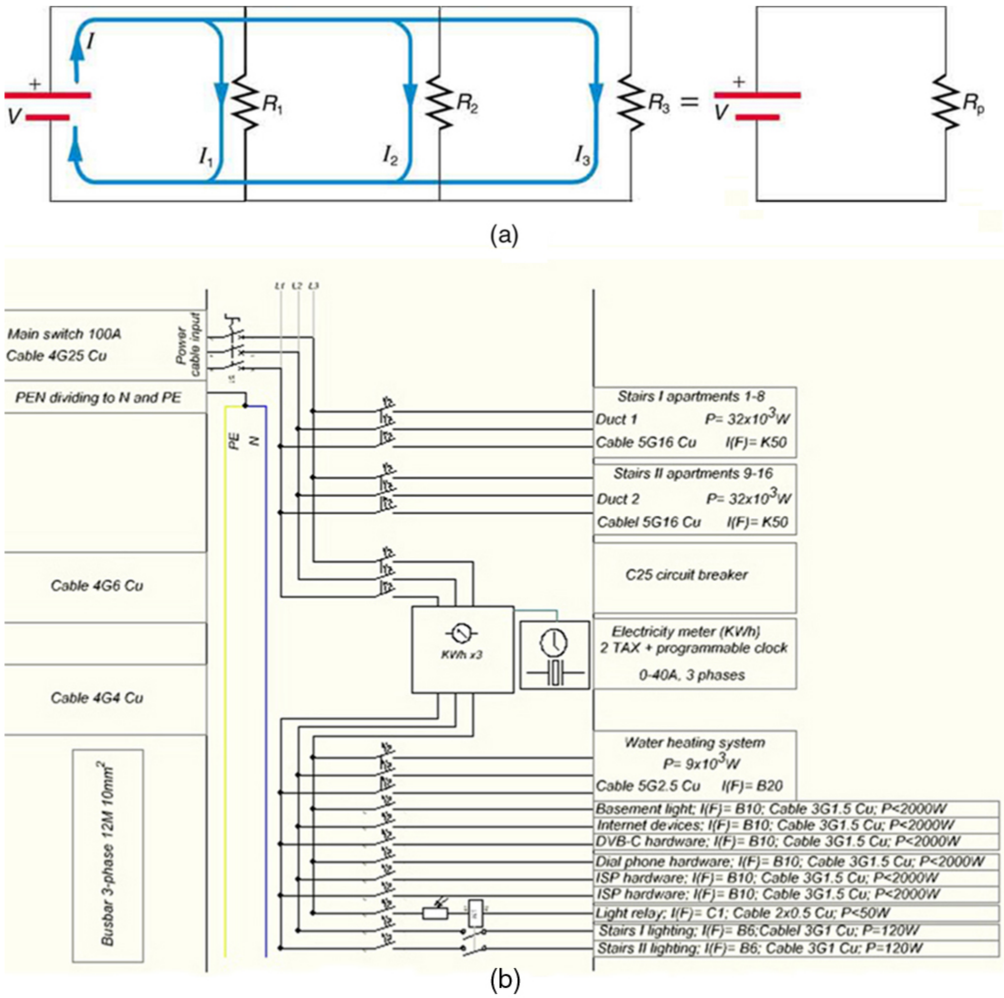


Figure 21.4 (a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. (credit: Dmitry G, Wikimedia Commons)

To find an expression for the equivalent parallel resistance R_p, let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are $I_1 = \dfrac{V}{R_1}$, $I_2 = \dfrac{V}{R_2}$, and $I_3 = \dfrac{V}{R_3}$. Conservation of charge implies that the total current I produced by the source is the sum of these currents:

$$I = I_1 + I_2 + I_3. \tag{21.17}$$

Substituting the expressions for the individual currents gives

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right). \tag{21.18}$$

Note that Ohm’s law for the equivalent single resistance gives

$$I = \frac{V}{R_\text{p}} = V\left(\frac{1}{R_\text{p}}\right). \tag{21.19}$$

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance R_p of a parallel connection is related to the individual resistances by

$$\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{.3}} + \text{....} \tag{21.20}$$

This relationship results in a total resistance R_p that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

Example 21.2 Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in **Figure 21.4** be the same as the previously considered series connection: $V = 12.0\text{ V}$, $R_1 = 1.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 13.0\ \Omega$. (a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

$$\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00\ \Omega} + \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega}. \tag{21.21}$$

Thus,

$$\frac{1}{R_\text{p}} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega} = \frac{1.2436}{\Omega}. \tag{21.22}$$

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance R_p. This yields

$$R_\text{p} = \frac{1}{1.2436}\ \Omega = 0.8041\ \Omega. \tag{21.23}$$

The total resistance with the correct number of significant digits is $R_\text{p} = 0.804\ \Omega$.

Discussion for (a)

R_p is, as predicted, less than the smallest individual resistance.

Strategy and Solution for (b)

The total current can be found from Ohm's law, substituting R_p for the total resistance. This gives

$$I = \frac{V}{R_\text{p}} = \frac{12.0\text{ V}}{0.8041\ \Omega} = 14.92\text{ A}. \tag{21.24}$$

Discussion for (b)

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

Strategy and Solution for (c)

The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

$$I_1 = \frac{V}{R_1} = \frac{12.0\text{ V}}{1.00\ \Omega} = 12.0\text{ A}. \tag{21.25}$$

Similarly,

$$I_2 = \frac{V}{R_2} = \frac{12.0\text{ V}}{6.00\ \Omega} = 2.00\text{ A} \tag{21.26}$$

and

$$I_3 = \frac{V}{R_3} = \frac{12.0\text{ V}}{13.0\ \Omega} = 0.92\text{ A}. \tag{21.27}$$

Discussion for (c)

The total current is the sum of the individual currents:

$$I_1 + I_2 + I_3 = 14.92\text{ A}. \tag{21.28}$$

This is consistent with conservation of charge.

Strategy and Solution for (d)

The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P = \frac{V^2}{R}$, since each resistor gets full voltage. Thus,

$$P_1 = \frac{V^2}{R_1} = \frac{(12.0\text{ V})^2}{1.00\ \ \Omega} = 144\text{ W}. \tag{21.29}$$

Similarly,

$$P_2 = \frac{V^2}{R_2} = \frac{(12.0\text{ V})^2}{6.00\ \ \Omega} = 24.0\text{ W} \tag{21.30}$$

and

$$P_3 = \frac{V^2}{R_3} = \frac{(12.0\text{ V})^2}{13.0\ \ \Omega} = 11.1\text{ W}. \tag{21.31}$$

Discussion for (d)

The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.

Strategy and Solution for (e)

The total power can also be calculated in several ways. Choosing $P = IV$, and entering the total current, yields

$$P = IV = (14.92\text{ A})(12.0\text{ V}) = 179\text{ W}. \tag{21.32}$$

Discussion for (e)

Total power dissipated by the resistors is also 179 W:

$$P_1 + P_2 + P_3 = 144\text{ W} + 24.0\text{ W} + 11.1\text{ W} = 179\text{ W}. \tag{21.33}$$

This is consistent with the law of conservation of energy.

Overall Discussion

Note that both the currents and powers in parallel connections are greater than for the same devices in series.

Major Features of Resistors in Parallel

1. Parallel resistance is found from $\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$, and it is smaller than any individual resistance in the combination.
2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
3. Parallel resistors do not each get the total current; they divide it.

Combinations of Series and Parallel

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in **Figure 21.5**. Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.

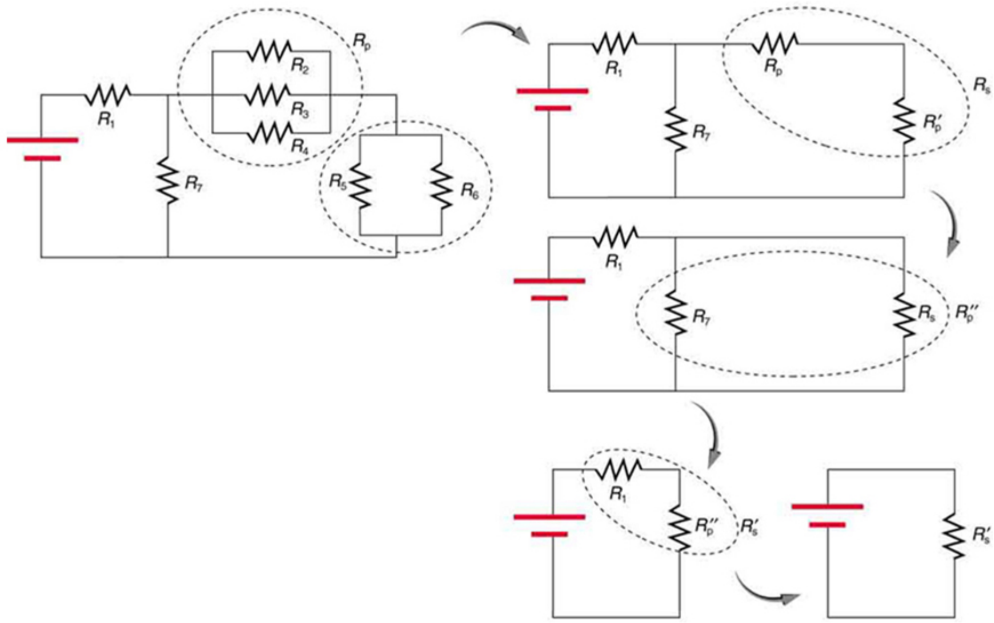


Figure 21.5 This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

The simplest combination of series and parallel resistance, shown in Figure 21.6, is also the most instructive, since it is found in many applications. For example, R_1 could be the resistance of wires from a car battery to its electrical devices, which are in parallel. R_2 and R_3 could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

Example 21.3 Calculating Resistance, IR Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits

Figure 21.6 shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3. (a) Find the total resistance. (b) What is the IR drop in R_1? (c) Find the current I_2 through R_2. (d) What power is dissipated by R_2?

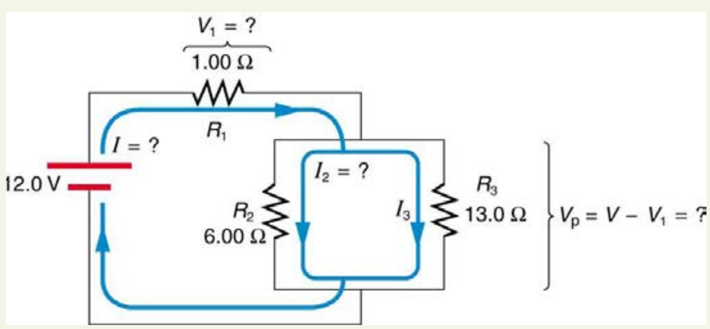


Figure 21.6 These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1.

Strategy and Solution for (a)

To find the total resistance, we note that R_2 and R_3 are in parallel and their combination R_p is in series with R_1. Thus the total (equivalent) resistance of this combination is

$$R_\text{tot} = R_1 + R_\text{p}. \tag{21.34}$$

First, we find R_p using the equation for resistors in parallel and entering known values:

$$\frac{1}{R_\text{p}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega} = \frac{0.2436}{\Omega}. \tag{21.35}$$

Inverting gives

$$R_p = \frac{1}{0.2436}\ \Omega = 4.11\ \Omega. \tag{21.36}$$

So the total resistance is

$$R_{tot} = R_1 + R_p = 1.00\ \Omega + 4.11\ \Omega = 5.11\ \Omega. \tag{21.37}$$

Discussion for (a)

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0\ \Omega$ and $0.804\ \Omega$, respectively) found for the same resistors in the two previous examples.

Strategy and Solution for (b)

To find the IR drop in R_1, we note that the full current I flows through R_1. Thus its IR drop is

$$V_1 = IR_1. \tag{21.38}$$

We must find I before we can calculate V_1. The total current I is found using Ohm's law for the circuit. That is,

$$I = \frac{V}{R_{tot}} = \frac{12.0\ \text{V}}{5.11\ \Omega} = 2.35\ \text{A}. \tag{21.39}$$

Entering this into the expression above, we get

$$V_1 = IR_1 = (2.35\ \text{A})(1.00\ \Omega) = 2.35\ \text{V}. \tag{21.40}$$

Discussion for (b)

The voltage applied to R_2 and R_3 is less than the total voltage by an amount V_1. When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3.

Strategy and Solution for (c)

To find the current through R_2, we must first find the voltage applied to it. We call this voltage V_p, because it is applied to a parallel combination of resistors. The voltage applied to both R_2 and R_3 is reduced by the amount V_1, and so it is

$$V_p = V - V_1 = 12.0\ \text{V} - 2.35\ \text{V} = 9.65\ \text{V}. \tag{21.41}$$

Now the current I_2 through resistance R_2 is found using Ohm's law:

$$I_2 = \frac{V_p}{R_2} = \frac{9.65\ \text{V}}{6.00\ \Omega} = 1.61\ \text{A}. \tag{21.42}$$

Discussion for (c)

The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.

Strategy and Solution for (d)

The power dissipated by R_2 is given by

$$P_2 = (I_2)^2 R_2 = (1.61\ \text{A})^2 (6.00\ \Omega) = 15.5\ \text{W}. \tag{21.43}$$

Discussion for (d)

The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

Applying the Science Practices: Circuit Construction Kit (DC only)

Plan an experiment to analyze the effect on currents and potential differences due to rearrangement of resistors and variations in voltage sources. Your experimental investigation should include data collection for at least five different scenarios of rearranged resistors (i.e., several combinations of series and parallel) and three scenarios of different voltage sources.

Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in **Figure 21.7**. The device represented by R_3 has a very low resistance, and so when it is switched on, a large current flows. This increased current causes a larger IR drop in the wires represented by R_1, reducing the voltage across the light bulb (which is R_2), which then dims noticeably.

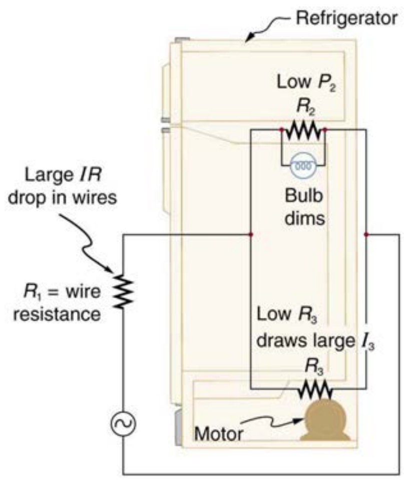


Figure 21.7 Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant IR drop in the wires and reduces the voltage across the light.

Check Your Understanding

Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

Solution

No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions. In such cases Kirchhoff's rules, to be introduced in **Kirchhoff's Rules**, will allow you to analyze the circuit.

Problem-Solving Strategies for Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding R, the reciprocal must be taken with care.
5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance should be greater, whereas total parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

21.2 Electromotive Force: Terminal Voltage

Learning Objectives

By the end of this section, you will be able to:

- Compare and contrast the voltage and the electromagnetic force of an electric power source.
- Describe what happens to the terminal voltage, current, and power delivered to a load as internal resistance of the voltage source increases.
- Explain why it is beneficial to use more than one voltage source connected in parallel.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.B.9.7** The student is able to refine and analyze a scientific question for an experiment using Kirchhoff's loop rule for circuits that includes determination of internal resistance of the battery and analysis of a nonohmic resistor. **(S.P. 4.1, 4.2, 5.1, 5.3)**

When you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they simply blink off when the battery's energy is gone? Their gradual dimming implies that battery output voltage decreases as the battery is depleted.

Furthermore, if you connect an excessive number of 12-V lights in parallel to a car battery, they will be dim even when the battery is fresh and even if the wires to the lights have very low resistance. This implies that the battery's output voltage is reduced by the overload.

The reason for the decrease in output voltage for depleted or overloaded batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an **internal resistance**. Let us examine both.

Electromotive Force

You can think of many different types of voltage sources. Batteries themselves come in many varieties. There are many types of mechanical/electrical generators, driven by many different energy sources, ranging from nuclear to wind. Solar cells create voltages directly from light, while thermoelectric devices create voltage from temperature differences.

A few voltage sources are shown in **Figure 21.8**. All such devices create a **potential difference** and can supply current if connected to a resistance. On the small scale, the potential difference creates an electric field that exerts force on charges, causing current. We thus use the name **electromotive force**, abbreviated emf.

Emf is not a force at all; it is a special type of potential difference. To be precise, the electromotive force (emf) is the potential difference of a source when no current is flowing. Units of emf are volts.

Figure 21.8 A variety of voltage sources (clockwise from top left): the Brazos Wind Farm in Fluvanna, Texas (credit: Leaflet, Wikimedia Commons); the Krasnoyarsk Dam in Russia (credit: Alex Polezhaev); a solar farm (credit: U.S. Department of Energy); and a group of nickel metal hydride batteries (credit: Tiaa Monto). The voltage output of each depends on its construction and load, and equals emf only if there is no load.

Electromotive force is directly related to the source of potential difference, such as the particular combination of chemicals in a battery. However, emf differs from the voltage output of the device when current flows. The voltage across the terminals of a battery, for example, is less than the emf when the battery supplies current, and it declines further as the battery is depleted or loaded down. However, if the device's output voltage can be measured without drawing current, then output voltage will equal emf (even for a very depleted battery).

Internal Resistance

As noted before, a 12-V truck battery is physically larger, contains more charge and energy, and can deliver a larger current than a 12-V motorcycle battery. Both are lead-acid batteries with identical emf, but, because of its size, the truck battery has a smaller internal resistance r. Internal resistance is the inherent resistance to the flow of current within the source itself.

Figure 21.9 is a schematic representation of the two fundamental parts of any voltage source. The emf (represented by a script E in the figure) and internal resistance r are in series. The smaller the internal resistance for a given emf, the more current and the more power the source can supply.

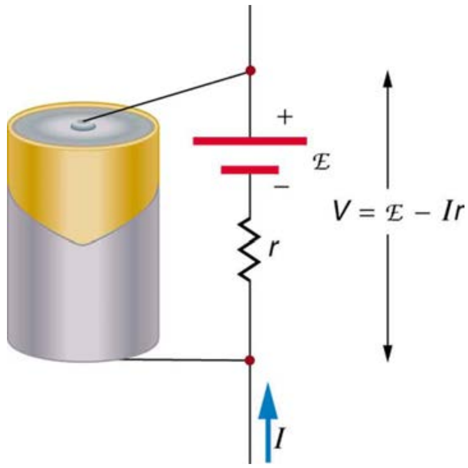


Figure 21.9 Any voltage source (in this case, a carbon-zinc dry cell) has an emf related to its source of potential difference, and an internal resistance r related to its construction. (Note that the script E stands for emf.). Also shown are the output terminals across which the terminal voltage V is measured. Since $V = \text{emf} - Ir$, terminal voltage equals emf only if there is no current flowing.

The internal resistance r can behave in complex ways. As noted, r increases as a battery is depleted. But internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted.

Things Great and Small: The Submicroscopic Origin of Battery Potential

Various types of batteries are available, with emfs determined by the combination of chemicals involved. We can view this as a molecular reaction (what much of chemistry is about) that separates charge.

The lead-acid battery used in cars and other vehicles is one of the most common types. A single cell (one of six) of this battery is seen in **Figure 21.10**. The cathode (positive) terminal of the cell is connected to a lead oxide plate, while the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

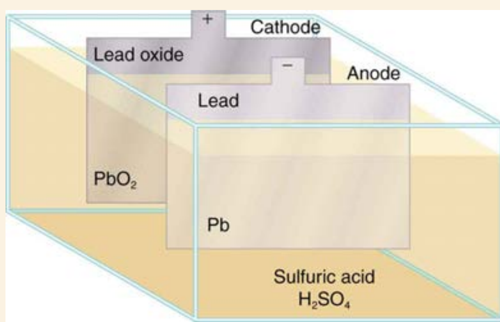


Figure 21.10 Artist’s conception of a lead-acid cell. Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge as well as participating in the chemical reaction.

The details of the chemical reaction are left to the reader to pursue in a chemistry text, but their results at the molecular level help explain the potential created by the battery. **Figure 21.11** shows the result of a single chemical reaction. Two electrons are placed on the anode, making it negative, provided that the cathode supplied two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction will not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.

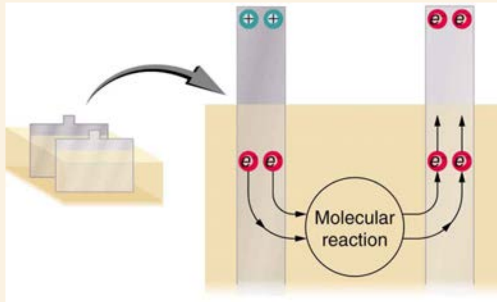


Figure 21.11 Artist's conception of two electrons being forced onto the anode of a cell and two electrons being removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a closed circuit to proceed, since the two electrons must be supplied to the cathode.

Why are the chemicals able to produce a unique potential difference? Quantum mechanical descriptions of molecules, which take into account the types of atoms and numbers of electrons in them, are able to predict the energy states they can have and the energies of reactions between them.

In the case of a lead-acid battery, an energy of 2 eV is given to each electron sent to the anode. Voltage is defined as the electrical potential energy divided by charge: $V = \frac{P_\mathrm{E}}{q}$. An electron volt is the energy given to a single electron by a voltage of 1 V. So the voltage here is 2 V, since 2 eV is given to each electron. It is the energy produced in each molecular reaction that produces the voltage. A different reaction produces a different energy and, hence, a different voltage.

Terminal Voltage

The voltage output of a device is measured across its terminals and, thus, is called its **terminal voltage** V. Terminal voltage is given by

$$V = \mathrm{emf} - Ir, \tag{21.44}$$

where r is the internal resistance and I is the current flowing at the time of the measurement.

I is positive if current flows away from the positive terminal, as shown in **Figure 21.9**. You can see that the larger the current, the smaller the terminal voltage. And it is likewise true that the larger the internal resistance, the smaller the terminal voltage.

Suppose a load resistance R_load is connected to a voltage source, as in **Figure 21.12**. Since the resistances are in series, the total resistance in the circuit is $R_\mathrm{load} + r$. Thus the current is given by Ohm's law to be

$$I = \frac{\mathrm{emf}}{R_\mathrm{load} + r}. \tag{21.45}$$

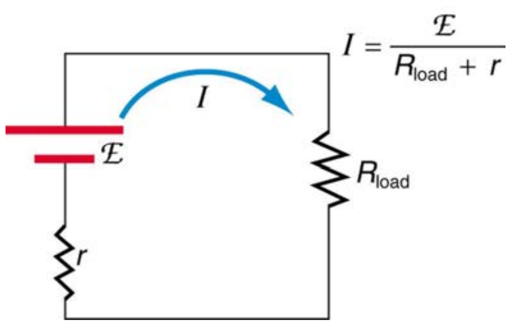


Figure 21.12 Schematic of a voltage source and its load R_load. Since the internal resistance r is in series with the load, it can significantly affect the terminal voltage and current delivered to the load. (Note that the script E stands for emf.)

We see from this expression that the smaller the internal resistance r, the greater the current the voltage source supplies to its load R_load. As batteries are depleted, r increases. If r becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

Example 21.4 Calculating Terminal Voltage, Power Dissipation, Current, and Resistance: Terminal Voltage and Load

A certain battery has a 12.0-V emf and an internal resistance of $0.100\ \Omega$. (a) Calculate its terminal voltage when connected to a $10.0\text{-}\ \Omega$ load. (b) What is the terminal voltage when connected to a $0.500\text{-}\ \Omega$ load? (c) What power does the $0.500\text{-}\ \Omega$ load dissipate? (d) If the internal resistance grows to $0.500\ \Omega$, find the current, terminal voltage, and power dissipated by a $0.500\text{-}\ \Omega$ load.

Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated using the equation $V = \text{emf} - Ir$. Once current is found, the power dissipated by a resistor can also be found.

Solution for (a)

Entering the given values for the emf, load resistance, and internal resistance into the expression above yields

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\text{ V}}{10.1\ \Omega} = 1.188\text{ A}. \tag{21.46}$$

Enter the known values into the equation $V = \text{emf} - Ir$ to get the terminal voltage:

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\text{ V} - (1.188\text{ A})(0.100\ \Omega) \\ &= 11.9\text{ V}. \end{aligned} \tag{21.47}$$

Discussion for (a)

The terminal voltage here is only slightly lower than the emf, implying that $10.0\ \Omega$ is a light load for this particular battery.

Solution for (b)

Similarly, with $R_{\text{load}} = 0.500\ \Omega$, the current is

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\text{ V}}{0.600\ \Omega} = 20.0\text{ A}. \tag{21.48}$$

The terminal voltage is now

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\text{ V} - (20.0\text{ A})(0.100\ \Omega) \\ &= 10.0\text{ V}. \end{aligned} \tag{21.49}$$

Discussion for (b)

This terminal voltage exhibits a more significant reduction compared with emf, implying $0.500\ \Omega$ is a heavy load for this battery.

Solution for (c)

The power dissipated by the $0.500\text{ - }\Omega$ load can be found using the formula $P = I^2R$. Entering the known values gives

$$P_{\text{load}} = I^2R_{\text{load}} = (20.0\text{ A})^2(0.500\ \Omega) = 2.00\times10^2\text{ W}. \tag{21.50}$$

Discussion for (c)

Note that this power can also be obtained using the expressions $\frac{V^2}{R}$ or IV , where V is the terminal voltage (10.0 V in this case).

Solution for (d)

Here the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\text{ V}}{1.00\ \Omega} = 12.0\text{ A}. \tag{21.51}$$

Now the terminal voltage is

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\text{ V} - (12.0\text{ A})(0.500\ \Omega) \\ &= 6.00\text{ V}, \end{aligned} \tag{21.52}$$

and the power dissipated by the load is

$$P_{\text{load}} = I^2 R_{\text{load}} = (12.0\text{ A})^2 (0.500\ \ \Omega\ ) = 72.0\text{ W}. \tag{21.53}$$

Discussion for (d)

We see that the increased internal resistance has significantly decreased terminal voltage, current, and power delivered to a load.

Applying the Science Practices: Internal Resistance

The internal resistance of a battery can be estimated using a simple activity. The circuit shown in the figure below includes a resistor R in series with a battery along with an ammeter and voltmeter to measure the current and voltage respectively.

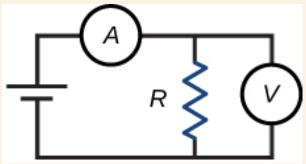


Figure 21.13

The currents and voltages measured for several *R* values are shown in the table below. Using the data given in the table and applying graphical analysis, determine the emf and internal resistance of the battery. Your response should clearly explain the method used to obtain the result.

Table 21.1

Resistance	Current (A)	Voltage (V)
R_1	3.53	4.24
R_2	2.07	4.97
R_3	1.46	5.27
R_4	1.13	5.43

Answer

Plot the measured currents and voltages on a graph. The terminal voltage of a battery is equal to the emf of the battery minus the voltage drop across the internal resistance of the battery or $V = emf - Ir$. Using this linear relationship and the plotted graph, the internal resistance and emf of the battery can be found. The graph for this case is shown below. The equation is $V = -0.50I + 6.0$ and hence the internal resistance will be equal to 0.5 Ω and emf will be equal to 6 V.

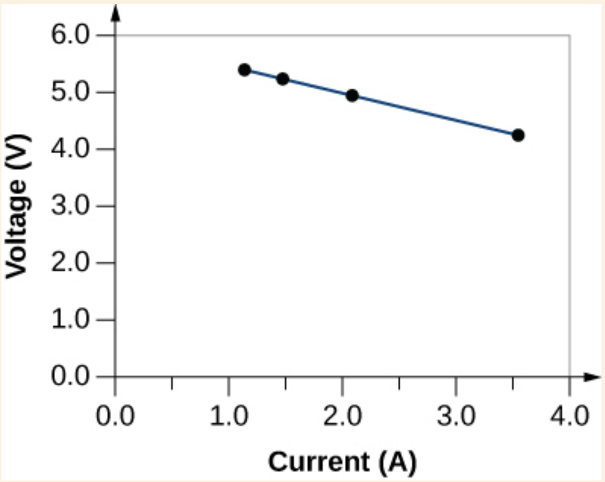


Figure 21.14

Battery testers, such as those in **Figure 21.15**, use small load resistors to intentionally draw current to determine whether the terminal voltage drops below an acceptable level. They really test the internal resistance of the battery. If internal resistance is high, the battery is weak, as evidenced by its low terminal voltage.

Figure 21.15 These two battery testers measure terminal voltage under a load to determine the condition of a battery. The large device is being used by a U.S. Navy electronics technician to test large batteries aboard the aircraft carrier USS *Nimitz* and has a small resistance that can dissipate large amounts of power. (credit: U.S. Navy photo by Photographer's Mate Airman Jason A. Johnston) The small device is used on small batteries and has a digital display to indicate the acceptability of their terminal voltage. (credit: Keith Williamson)

Some batteries can be recharged by passing a current through them in the direction opposite to the current they supply to a resistance. This is done routinely in cars and batteries for small electrical appliances and electronic devices, and is represented pictorially in Figure 21.16. The voltage output of the battery charger must be greater than the emf of the battery to reverse current through it. This will cause the terminal voltage of the battery to be greater than the emf, since $V = \text{emf} - Ir$, and I is now negative.

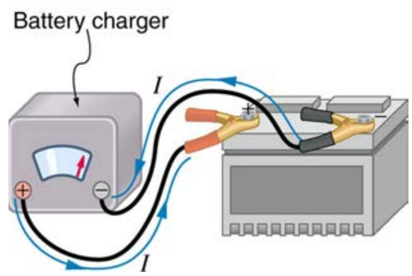


Figure 21.16 A car battery charger reverses the normal direction of current through a battery, reversing its chemical reaction and replenishing its chemical potential.

Multiple Voltage Sources

There are two voltage sources when a battery charger is used. Voltage sources connected in series are relatively simple. When voltage sources are in series, their internal resistances add and their emfs add algebraically. (See Figure 21.17.) Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf.

But if the cells oppose one another, such as when one is put into an appliance backward, the total emf is less, since it is the algebraic sum of the individual emfs.

A battery is a multiple connection of voltaic cells, as shown in Figure 21.18. The disadvantage of series connections of cells is that their internal resistances add. One of the authors once owned a 1957 MGA that had two 6-V batteries in series, rather than a single 12-V battery. This arrangement produced a large internal resistance that caused him many problems in starting the engine.

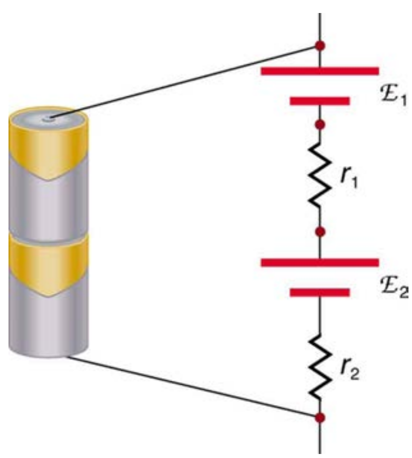


Figure 21.17 A series connection of two voltage sources. The emfs (each labeled with a script E) and internal resistances add, giving a total emf of $\text{emf}_1 + \text{emf}_2$ and a total internal resistance of $r_1 + r_2$.

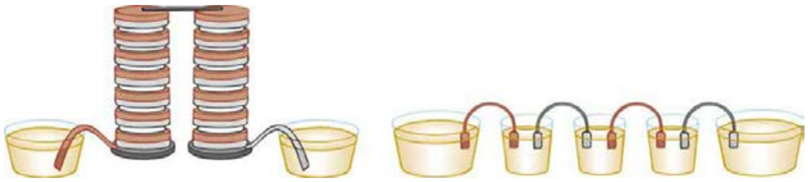


Figure 21.18 Batteries are multiple connections of individual cells, as shown in this modern rendition of an old print. Single cells, such as AA or C cells, are commonly called batteries, although this is technically incorrect.

If the *series* connection of two voltage sources is made into a complete circuit with the emfs in opposition, then a current of magnitude $I = \dfrac{(\text{emf}_1 - \text{emf}_2)}{r_1 + r_2}$ flows. See **Figure 21.19**, for example, which shows a circuit exactly analogous to the battery charger discussed above. If two voltage sources in series with emfs in the same sense are connected to a load R_{load}, as in **Figure 21.20**, then $I = \dfrac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$ flows.

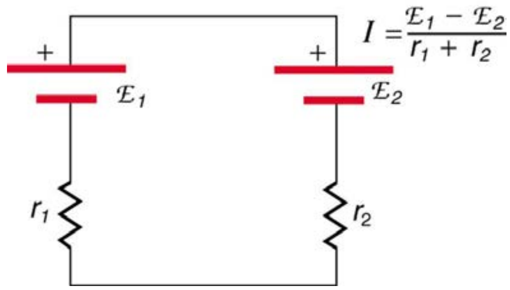


Figure 21.19 These two voltage sources are connected in series with their emfs in opposition. Current flows in the direction of the greater emf and is limited to $I = \dfrac{(\text{emf}_1 - \text{emf}_2)}{r_1 + r_2}$ by the sum of the internal resistances. (Note that each emf is represented by script E in the figure.) A battery charger connected to a battery is an example of such a connection. The charger must have a larger emf than the battery to reverse current through it.

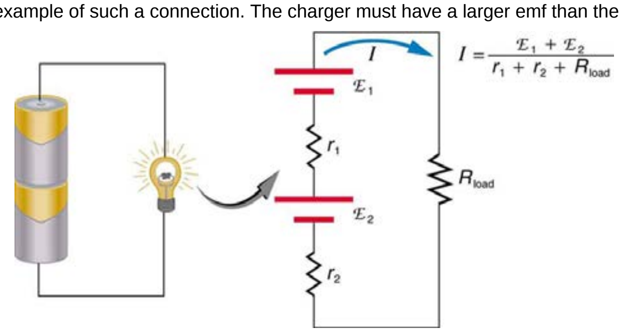


Figure 21.20 This schematic represents a flashlight with two cells (voltage sources) and a single bulb (load resistance) in series. The current that flows is $I = \dfrac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$. (Note that each emf is represented by script E in the figure.)

Take-Home Experiment: Flashlight Batteries

Find a flashlight that uses several batteries and find new and old batteries. Based on the discussions in this module, predict the brightness of the flashlight when different combinations of batteries are used. Do your predictions match what you observe? Now place new batteries in the flashlight and leave the flashlight switched on for several hours. Is the flashlight still quite bright? Do the same with the old batteries. Is the flashlight as bright when left on for the same length of time with old and new batteries? What does this say for the case when you are limited in the number of available new batteries?

Figure 21.21 shows two voltage sources with identical emfs in parallel and connected to a load resistance. In this simple case, the total emf is the same as the individual emfs. But the total internal resistance is reduced, since the internal resistances are in parallel. The parallel connection thus can produce a larger current.

Here, $I = \dfrac{\text{emf}}{(r_{\text{tot}} + R_{\text{load}})}$ flows through the load, and r_{tot} is less than those of the individual batteries. For example, some diesel-powered cars use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.

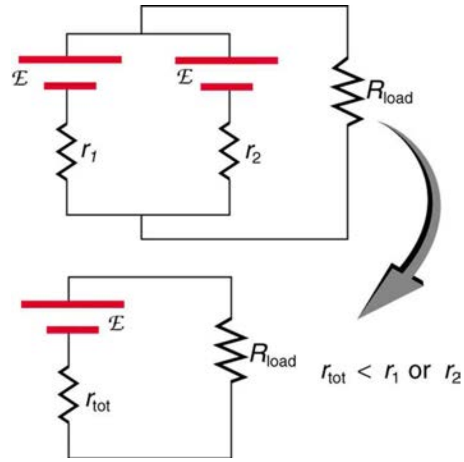


Figure 21.21 Two voltage sources with identical emfs (each labeled by script E) connected in parallel produce the same emf but have a smaller total internal resistance than the individual sources. Parallel combinations are often used to deliver more current. Here $I = \dfrac{\text{emf}}{\left(r_{\text{tot}} + R_{\text{load}}\right)}$ flows through the load.

Animals as Electrical Detectors

A number of animals both produce and detect electrical signals. Fish, sharks, platypuses, and echidnas (spiny anteaters) all detect electric fields generated by nerve activity in prey. Electric eels produce their own emf through biological cells (electric organs) called electroplaques, which are arranged in both series and parallel as a set of batteries.

Electroplaques are flat, disk-like cells; those of the electric eel have a voltage of 0.15 V across each one. These cells are usually located toward the head or tail of the animal, although in the case of the electric eel, they are found along the entire body. The electroplaques in the South American eel are arranged in 140 rows, with each row stretching horizontally along the body and containing 5,000 electroplaques. This can yield an emf of approximately 600 V, and a current of 1 A—deadly.

The mechanism for detection of external electric fields is similar to that for producing nerve signals in the cell through depolarization and repolarization—the movement of ions across the cell membrane. Within the fish, weak electric fields in the water produce a current in a gel-filled canal that runs from the skin to sensing cells, producing a nerve signal. The Australian platypus, one of the very few mammals that lay eggs, can detect fields of 30 $\frac{\text{mV}}{\text{m}}$, while sharks have been found to be able to sense a field in their snouts as small as 100 $\frac{\text{mV}}{\text{m}}$ (**Figure 21.22**). Electric eels use their own electric fields produced by the electroplaques to stun their prey or enemies.

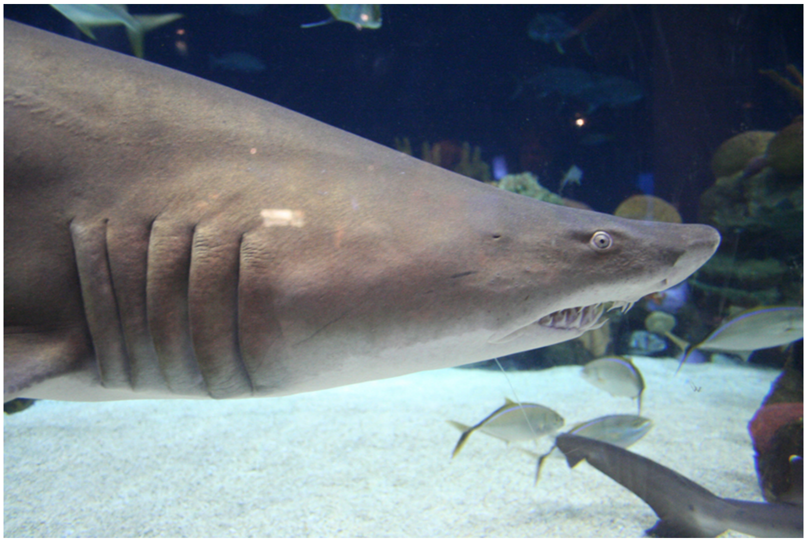


Figure 21.22 Sand tiger sharks (*Carcharias taurus*), like this one at the Minnesota Zoo, use electroreceptors in their snouts to locate prey. (credit: Jim Winstead, Flickr)

Solar Cell Arrays

Another example dealing with multiple voltage sources is that of combinations of solar cells—wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation (PV), the conversion of sunlight directly into electricity, is based upon the photoelectric effect, in which photons hitting the surface of a solar cell create an electric current in the cell.

Most solar cells are made from pure silicon—either as single-crystal silicon, or as a thin film of silicon deposited upon a glass or metal backing. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the amount of sunlight upon the cell (the incident solar radiation—the insolation). Under bright noon sunlight, a current of about 100 mA/cm^2 of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical-energy needs. They can be wired together in series or in parallel—connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

The output of the solar cells is direct current. For most uses in a home, AC is required, so a device called an inverter must be used to convert the DC to AC. Any extra output can then be passed on to the outside electrical grid for sale to the utility.

Take-Home Experiment: Virtual Solar Cells

One can assemble a "virtual" solar cell array by using playing cards, or business or index cards, to represent a solar cell. Combinations of these cards in series and/or parallel can model the required array output. Assume each card has an output of 0.5 V and a current (under bright light) of 2 A. Using your cards, how would you arrange them to produce an output of 6 A at 3 V (18 W)?

Suppose you were told that you needed only 18 W (but no required voltage). Would you need more cards to make this arrangement?

21.3 Kirchhoff's Rules

Learning Objectives

By the end of this section, you will be able to:

- Analyze a complex circuit using Kirchhoff's rules, applying the conventions for determining the correct signs of various terms.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.B.9.1** The student is able to construct or interpret a graph of the energy changes within an electrical circuit with only a single battery and resistors in series and/or in, at most, one parallel branch as an application of the conservation of energy (Kirchhoff's loop rule). **(S.P. 1.1, 1.4)**
- **5.B.9.2** The student is able to apply conservation of energy concepts to the design of an experiment that will demonstrate the validity of Kirchhoff's loop rule in a circuit with only a battery and resistors either in series or in, at most, one pair of parallel branches. **(S.P. 4.2, 6.4, 7.2)**
- **5.B.9.3** The student is able to apply conservation of energy (Kirchhoff's loop rule) in calculations involving the total electric potential difference for complete circuit loops with only a single battery and resistors in series and/or in, at most, one parallel branch. **(S.P. 2.2, 6.4, 7.2)**
- **5.B.9.4** The student is able to analyze experimental data including an analysis of experimental uncertainty that will demonstrate the validity of Kirchhoff's loop rule. **(S.P. 5.1)**
- **5.B.9.5** The student is able to use conservation of energy principles (Kirchhoff's loop rule) to describe and make predictions regarding electrical potential difference, charge, and current in steady-state circuits composed of various combinations of resistors and capacitors. **(S.P. 6.4)**
- **5.C.3.1** The student is able to apply conservation of electric charge (Kirchhoff's junction rule) to the comparison of electric current in various segments of an electrical circuit with a single battery and resistors in series and in, at most, one parallel branch and predict how those values would change if configurations of the circuit are changed. **(S.P. 6.4, 7.2)**
- **5.C.3.2** The student is able to design an investigation of an electrical circuit with one or more resistors in which evidence of conservation of electric charge can be collected and analyzed. **(S.P. 4.1, 4.2, 5.1)**
- **5.C.3.3** The student is able to use a description or schematic diagram of an electrical circuit to calculate unknown values of current in various segments or branches of the circuit. **(S.P. 1.4, 2.2)**
- **5.C.3.4** The student is able to predict or explain current values in series and parallel arrangements of resistors and other branching circuits using Kirchhoff's junction rule and relate the rule to the law of charge conservation. **(S.P. 6.4, 7.2)**
- **5.C.3.5** The student is able to determine missing values and direction of electric current in branches of a circuit with resistors and NO capacitors from values and directions of current in other branches of the circuit through appropriate selection of nodes and application of the junction rule. **(S.P. 1.4, 2.2)**

Many complex circuits, such as the one in **Figure 21.23**, cannot be analyzed with the series-parallel techniques developed in **Resistors in Series and Parallel** and **Electromotive Force: Terminal Voltage**. There are, however, two circuit analysis rules that can be used to analyze any circuit, simple or complex. These rules are special cases of the laws of conservation of charge and conservation of energy. The rules are known as **Kirchhoff's rules**, after their inventor Gustav Kirchhoff (1824–1887).

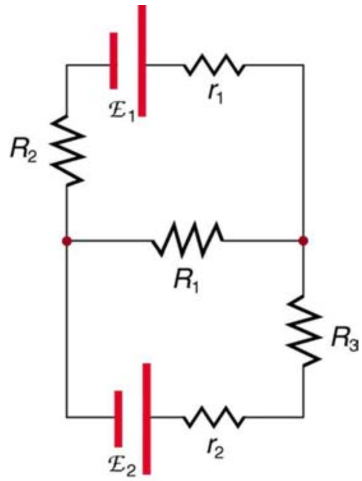


Figure 21.23 This circuit cannot be reduced to a combination of series and parallel connections. Kirchhoff's rules, special applications of the laws of conservation of charge and energy, can be used to analyze it. (Note: The script E in the figure represents electromotive force, emf.)

Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.

Explanations of the two rules will now be given, followed by problem-solving hints for applying Kirchhoff's rules, and a worked example that uses them.

Kirchhoff's First Rule

Kirchhoff's first rule (the **junction rule**) is an application of the conservation of charge to a junction; it is illustrated in **Figure 21.24**. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out. Kirchhoff's first rule requires that $I_1 = I_2 + I_3$ (see figure). Equations like this can and will be used to analyze circuits and to solve circuit problems.

Making Connections: Conservation Laws

Kirchhoff's rules for circuit analysis are applications of **conservation laws** to circuits. The first rule is the application of conservation of charge, while the second rule is the application of conservation of energy. Conservation laws, even used in a specific application, such as circuit analysis, are so basic as to form the foundation of that application.

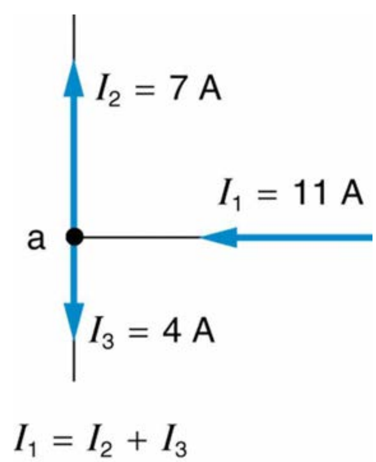


Figure 21.24 The junction rule. The diagram shows an example of Kirchhoff's first rule where the sum of the currents into a junction equals the sum of the currents out of a junction. In this case, the current going into the junction splits and comes out as two currents, so that $I_1 = I_2 + I_3$. Here I_1 must be 11 A, since I_2 is 7 A and I_3 is 4 A.

Kirchhoff’s Second Rule

Kirchhoff’s second rule (the **loop rule**) is an application of conservation of energy. The loop rule is stated in terms of potential, V , rather than potential energy, but the two are related since $\text{PE}_{\text{elec}} = qV$. Recall that **emf** is the potential difference of a source when no current is flowing. In a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. **Figure 21.25** illustrates the changes in potential in a simple series circuit loop.

Kirchhoff’s second rule requires $\text{emf} - Ir - IR_1 - IR_2 = 0$. Rearranged, this is $\text{emf} = Ir + IR_1 + IR_2$, which means the emf equals the sum of the IR (voltage) drops in the loop.

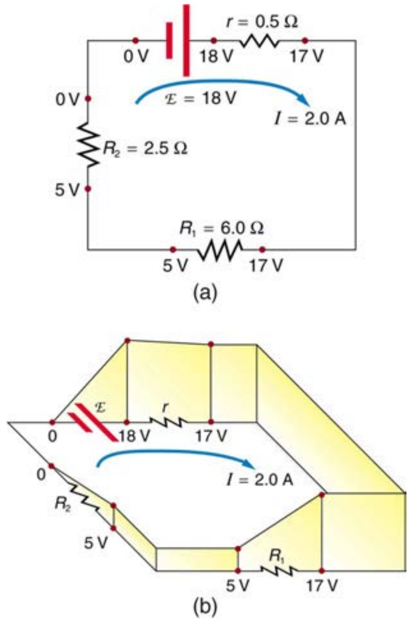


Figure 21.25 The loop rule. An example of Kirchhoff’s second rule where the sum of the changes in potential around a closed loop must be zero. (a) In this standard schematic of a simple series circuit, the emf supplies 18 V, which is reduced to zero by the resistances, with 1 V across the internal resistance, and 12 V and 5 V across the two load resistances, for a total of 18 V. (b) This perspective view represents the potential as something like a roller coaster, where charge is raised in potential by the emf and lowered by the resistances. (Note that the script E stands for emf.)

Applying Kirchhoff’s Rules

By applying Kirchhoff’s rules, we generate equations that allow us to find the unknowns in circuits. The unknowns may be currents, emfs, or resistances. Each time a rule is applied, an equation is produced. If there are as many independent equations as unknowns, then the problem can be solved. There are two decisions you must make when applying Kirchhoff’s rules. These decisions determine the signs of various quantities in the equations you obtain from applying the rules.

1. When applying Kirchhoff’s first rule, the junction rule, you must label the current in each branch and decide in what direction it is going. For example, in **Figure 21.23**, **Figure 21.24**, and **Figure 21.25**, currents are labeled I_1 , I_2 , I_3 , and I , and arrows indicate their directions. There is no risk here, for if you choose the wrong direction, the current will be of the correct magnitude but negative.
2. When applying Kirchhoff’s second rule, the loop rule, you must identify a closed loop and decide in which direction to go around it, clockwise or counterclockwise. For example, in **Figure 21.25** the loop was traversed in the same direction as the current (clockwise). Again, there is no risk; going around the circuit in the opposite direction reverses the sign of every term in the equation, which is like multiplying both sides of the equation by $-1.$

Figure 21.26 and the following points will help you get the plus or minus signs right when applying the loop rule. Note that the resistors and emfs are traversed by going from a to b. In many circuits, it will be necessary to construct more than one loop. In traversing each loop, one needs to be consistent for the sign of the change in potential. (See **Example 21.5**.)

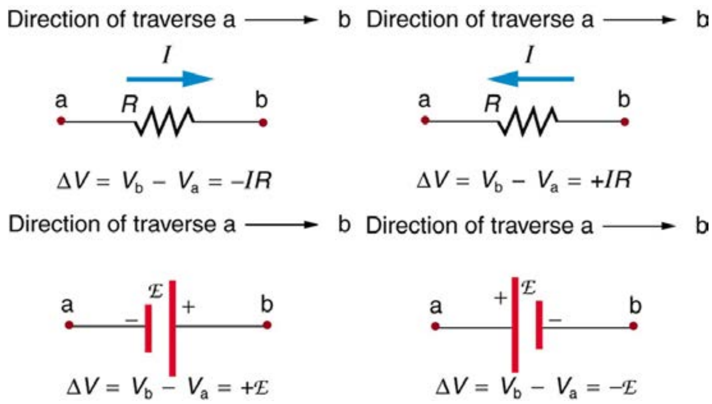


Figure 21.26 Each of these resistors and voltage sources is traversed from a to b. The potential changes are shown beneath each element and are explained in the text. (Note that the script E stands for emf.)

- When a resistor is traversed in the same direction as the current, the change in potential is $-IR$. (See **Figure 21.26**.)
- When a resistor is traversed in the direction opposite to the current, the change in potential is $+IR$. (See **Figure 21.26**.)
- When an emf is traversed from $-$ to + (the same direction it moves positive charge), the change in potential is +emf. (See **Figure 21.26**.)
- When an emf is traversed from + to $-$ (opposite to the direction it moves positive charge), the change in potential is $-$ emf. (See **Figure 21.26**.)

Example 21.5 Calculating Current: Using Kirchhoff's Rules

Find the currents flowing in the circuit in **Figure 21.27**.

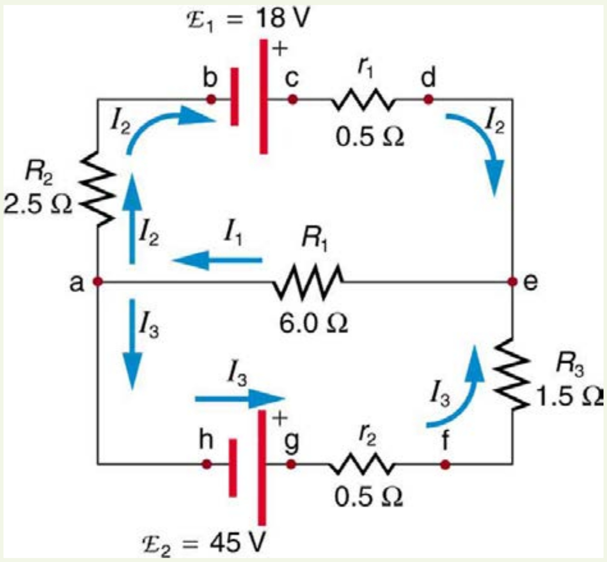


Figure 21.27 This circuit is similar to that in **Figure 21.23**, but the resistances and emfs are specified. (Each emf is denoted by script E.) The currents in each branch are labeled and assumed to move in the directions shown. This example uses Kirchhoff's rules to find the currents.

Strategy

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled I_1 , I_2 , and I_3 in the figure and assumptions have been made about their directions. Locations on the diagram have been labeled with letters a through h. In the solution we will apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

Solution

We begin by applying Kirchhoff's first or junction rule at point a. This gives

$$I_1 = I_2 + I_3, \tag{21.54}$$

since I_1 flows into the junction, while I_2 and I_3 flow out. Applying the junction rule at e produces exactly the same equation, so that no new information is obtained. This is a single equation with three unknowns—three independent equations are needed, and so the loop rule must be applied.

Now we consider the loop abcdea. Going from a to b, we traverse R_2 in the same (assumed) direction of the current I_2, and so the change in potential is $-I_2R_2$. Then going from b to c, we go from $-$ to +, so that the change in potential is $+\text{emf}_1$. Traversing the internal resistance r_1 from c to d gives $-I_2r_1$. Completing the loop by going from d to a again traverses a resistor in the same direction as its current, giving a change in potential of $-I_1R_1$.

The loop rule states that the changes in potential sum to zero. Thus,

$$-I_2R_2 + \text{emf}_1 - I_2r_1 - I_1R_1 = -I_2(R_2 + r_1) + \text{emf}_1 - I_1R_1 = 0. \tag{21.55}$$

Substituting values from the circuit diagram for the resistances and emf, and canceling the ampere unit gives

$$-3I_2 + 18 - 6I_1 = 0. \tag{21.56}$$

Now applying the loop rule to aefgha (we could have chosen abcdefgha as well) similarly gives

$$+ I_1R_1 + I_3R_3 + I_3r_2 - \text{emf}_2 = +I_1 R_1 + I_3\big(R_3 + r_2\big) - \text{emf}_2 = 0. \tag{21.57}$$

Note that the signs are reversed compared with the other loop, because elements are traversed in the opposite direction. With values entered, this becomes

$$+ 6I_1 + 2I_3 - 45 = 0. \tag{21.58}$$

These three equations are sufficient to solve for the three unknown currents. First, solve the second equation for I_2:

$$I_2 = 6 - 2I_1. \tag{21.59}$$

Now solve the third equation for I_3:

$$I_3 = 22.5 - 3I_1. \tag{21.60}$$

Substituting these two new equations into the first one allows us to find a value for I_1:

$$I_1 = I_2 + I_3 = (6 - 2I_1) + (22.5 - 3I_1) = 28.5 - 5I_1. \tag{21.61}$$

Combining terms gives

$$6I_1 = 28.5, \text{ and} \tag{21.62}$$

$$I_1 = 4.75 \text{ A}. \tag{21.63}$$

Substituting this value for I_1 back into the fourth equation gives

$$I_2 = 6 - 2I_1 = 6 - 9.50 \tag{21.64}$$

$$I_2 = -3.50 \text{ A}. \tag{21.65}$$

The minus sign means I_2 flows in the direction opposite to that assumed in **Figure 21.27**.

Finally, substituting the value for I_1 into the fifth equation gives

$$I_3 = 22.5 - 3I_1 = 22.5 - 14.25 \tag{21.66}$$

$$I_3 = 8.25 \text{ A}. \tag{21.67}$$

Discussion

Just as a check, we note that indeed $I_1 = I_2 + I_3$. The results could also have been checked by entering all of the values into the equation for the abcdefgha loop.

Problem-Solving Strategies for Kirchhoff's Rules

1. Make certain there is a clear circuit diagram on which you can label all known and unknown resistances, emfs, and currents. If a current is unknown, you must assign it a direction. This is necessary for determining the signs of potential changes. If you assign the direction incorrectly, the current will be found to have a negative value—no harm done.
2. Apply the junction rule to any junction in the circuit. Each time the junction rule is applied, you should get an equation with a current that does not appear in a previous application—if not, then the equation is redundant.
3. Apply the loop rule to as many loops as needed to solve for the unknowns in the problem. (There must be as many independent equations as unknowns.) To apply the loop rule, you must choose a direction to go around the loop. Then

carefully and consistently determine the signs of the potential changes for each element using the four bulleted points discussed above in conjunction with **Figure 21.26**.

4. Solve the simultaneous equations for the unknowns. This may involve many algebraic steps, requiring careful checking and rechecking.
5. Check to see whether the answers are reasonable and consistent. The numbers should be of the correct order of magnitude, neither exceedingly large nor vanishingly small. The signs should be reasonable—for example, no resistance should be negative. Check to see that the values obtained satisfy the various equations obtained from applying the rules. The currents should satisfy the junction rule, for example.

The material in this section is correct in theory. We should be able to verify it by making measurements of current and voltage. In fact, some of the devices used to make such measurements are straightforward applications of the principles covered so far and are explored in the next modules. As we shall see, a very basic, even profound, fact results—making a measurement alters the quantity being measured.

Check Your Understanding

Can Kirchhoff's rules be applied to simple series and parallel circuits or are they restricted for use in more complicated circuits that are not combinations of series and parallel?

Solution

Kirchhoff's rules can be applied to any circuit since they are applications to circuits of two conservation laws. Conservation laws are the most broadly applicable principles in physics. It is usually mathematically simpler to use the rules for series and parallel in simpler circuits so we emphasize Kirchhoff's rules for use in more complicated situations. But the rules for series and parallel can be derived from Kirchhoff's rules. Moreover, Kirchhoff's rules can be expanded to devices other than resistors and emfs, such as capacitors, and are one of the basic analysis devices in circuit analysis.

Making Connections: Parallel Resistors

A simple circuit shown below – with two parallel resistors and a voltage source – is implemented in a laboratory experiment with $\varepsilon = 6.00 \pm 0.02$ V and $R_1 = 4.8 \pm 0.1\ \Omega$ and $R_2 = 9.6 \pm 0.1\ \Omega$. The values include an allowance for experimental uncertainties as they cannot be measured with perfect certainty. For example if you measure the value for a resistor a few times, you may get slightly different results. Hence values are expressed with some level of uncertainty.

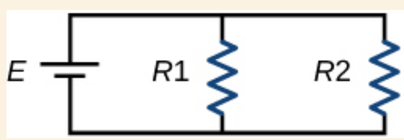


Figure 21.28

In the laboratory experiment the currents measured in the two resistors are $I_1 = 1.27$ A and $I_2 = 0.62$ A respectively. Let us examine these values using Kirchhoff's laws.

For the two loops,

$E - I_1R_1 = 0$ or $I_1 = E/R_1$

$E - I_2R_2 = 0$ or $I_2 = E/R_2$

Converting the given uncertainties for voltage and resistances into percentages, we get

$E = 6.00$ V ± 0.33%

$R_1 = 4.8\ \Omega$ ± 2.08%

$R_2 = 9.6\ \Omega$ ± 1.04%

We now find the currents for the two loops. While the voltage is divided by the resistance to find the current, uncertainties in voltage and resistance are directly added to find the uncertainty in the current value.

$I_1 = (6.00/4.8) \pm (0.33\%+2.08\%)$

$= 1.25 \pm 2.4\%$

$= 1.25 \pm 0.03$ A

$I_2 = (6.00/9.6) \pm (0.33\%+1.04\%)$

$= 0.63 \pm 1.4\%$

$= 0.63 \pm 0.01$ A

Finally you can check that the two measured values in this case are within the uncertainty ranges found for the currents. However there can also be additional experimental uncertainty in the measurements of currents.

21.4 DC Voltmeters and Ammeters

Learning Objectives

By the end of this section, you will be able to:

- Explain why a voltmeter must be connected in parallel with the circuit.
- Draw a diagram showing an ammeter correctly connected in a circuit.
- Describe how a galvanometer can be used as either a voltmeter or an ammeter.
- Find the resistance that must be placed in series with a galvanometer to allow it to be used as a voltmeter with a given reading.
- Explain why measuring the voltage or current in a circuit can never be exact.

Voltmeters measure voltage, whereas **ammeters** measure current. Some of the meters in automobile dashboards, digital cameras, cell phones, and tuner-amplifiers are voltmeters or ammeters. (See **Figure 21.29**.) The internal construction of the simplest of these meters and how they are connected to the system they monitor give further insight into applications of series and parallel connections.

Figure 21.29 The fuel and temperature gauges (far right and far left, respectively) in this 1996 Volkswagen are voltmeters that register the voltage output of “sender” units, which are hopefully proportional to the amount of gasoline in the tank and the engine temperature. (credit: Christian Giersing)

Voltmeters are connected in parallel with whatever device’s voltage is to be measured. A parallel connection is used because objects in parallel experience the same potential difference. (See **Figure 21.30**, where the voltmeter is represented by the symbol V.)

Ammeters are connected in series with whatever device’s current is to be measured. A series connection is used because objects in series have the same current passing through them. (See **Figure 21.31**, where the ammeter is represented by the symbol A.)

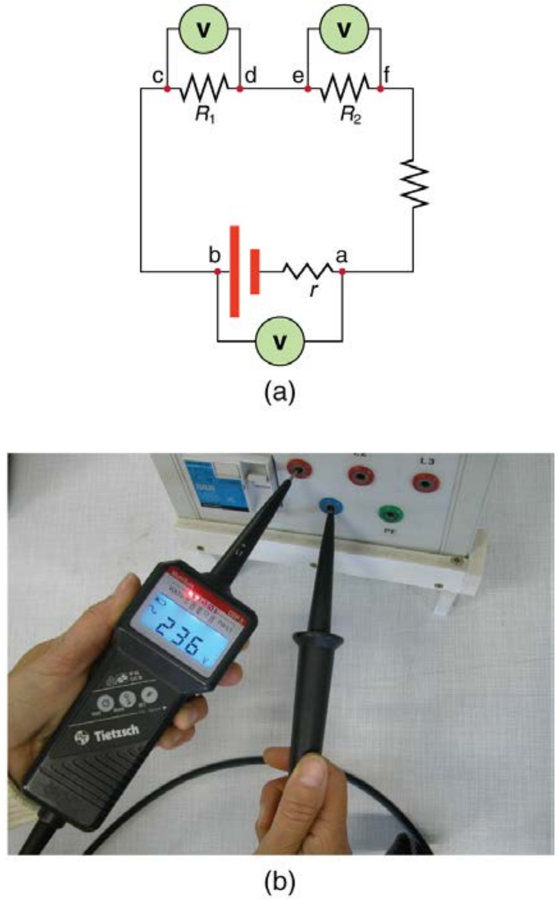


Figure 21.30 (a) To measure potential differences in this series circuit, the voltmeter (V) is placed in parallel with the voltage source or either of the resistors. Note that terminal voltage is measured between points a and b. It is not possible to connect the voltmeter directly across the emf without including its internal resistance, r . (b) A digital voltmeter in use. (credit: Messtechniker, Wikimedia Commons)

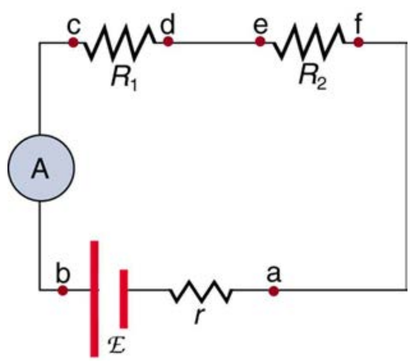


Figure 21.31 An ammeter (A) is placed in series to measure current. All of the current in this circuit flows through the meter. The ammeter would have the same reading if located between points d and e or between points f and a as it does in the position shown. (Note that the script capital E stands for emf, and r stands for the internal resistance of the source of potential difference.)

Analog Meters: Galvanometers

Analog meters have a needle that swivels to point at numbers on a scale, as opposed to **digital meters**, which have numerical readouts similar to a hand-held calculator. The heart of most analog meters is a device called a **galvanometer**, denoted by G. Current flow through a galvanometer, I_{G} , produces a proportional needle deflection. (This deflection is due to the force of a magnetic field upon a current-carrying wire.)

The two crucial characteristics of a given galvanometer are its resistance and current sensitivity. **Current sensitivity** is the current that gives a **full-scale deflection** of the galvanometer's needle, the maximum current that the instrument can measure. For example, a galvanometer with a current sensitivity of $50\ \mu\text{A}$ has a maximum deflection of its needle when $50\ \mu\text{A}$ flows through it, reads half-scale when $25\ \mu\text{A}$ flows through it, and so on.

If such a galvanometer has a $25\text{-}\ \Omega$ resistance, then a voltage of only $V = IR = (50\ \mu\text{A})(25\ \Omega) = 1.25\ \text{mV}$ produces a full-scale reading. By connecting resistors to this galvanometer in different ways, you can use it as either a voltmeter or ammeter that can measure a broad range of voltages or currents.

Galvanometer as Voltmeter

Figure 21.32 shows how a galvanometer can be used as a voltmeter by connecting it in series with a large resistance, R. The value of the resistance R is determined by the maximum voltage to be measured. Suppose you want 10 V to produce a full-scale deflection of a voltmeter containing a $25\text{-}\Omega$ galvanometer with a $50\text{-}\mu\text{A}$ sensitivity. Then 10 V applied to the meter must produce a current of $50\ \mu\text{A}$. The total resistance must be

$$R_{\text{tot}} = R + r = \frac{V}{I} = \frac{10\ \text{V}}{50\ \mu\text{A}} = 200\ \text{k}\Omega,\ \text{or} \tag{21.68}$$

$$R = R_{\text{tot}} - r = 200\ \text{k}\Omega - 25\ \ \Omega\ \approx 200\ \text{k}\ \Omega\,. \tag{21.69}$$

(R is so large that the galvanometer resistance, r, is nearly negligible.) Note that 5 V applied to this voltmeter produces a half-scale deflection by producing a $25\text{-}\mu\text{A}$ current through the meter, and so the voltmeter's reading is proportional to voltage as desired.

This voltmeter would not be useful for voltages less than about half a volt, because the meter deflection would be small and difficult to read accurately. For other voltage ranges, other resistances are placed in series with the galvanometer. Many meters have a choice of scales. That choice involves switching an appropriate resistance into series with the galvanometer.

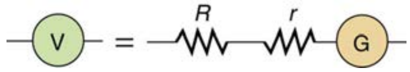


Figure 21.32 A large resistance R placed in series with a galvanometer G produces a voltmeter, the full-scale deflection of which depends on the choice of R. The larger the voltage to be measured, the larger R must be. (Note that r represents the internal resistance of the galvanometer.)

Galvanometer as Ammeter

The same galvanometer can also be made into an ammeter by placing it in parallel with a small resistance R, often called the **shunt resistance**, as shown in **Figure 21.33**. Since the shunt resistance is small, most of the current passes through it, allowing an ammeter to measure currents much greater than those producing a full-scale deflection of the galvanometer.

Suppose, for example, an ammeter is needed that gives a full-scale deflection for 1.0 A, and contains the same $25\text{-}\ \Omega$ galvanometer with its $50\text{-}\mu\text{A}$ sensitivity. Since R and r are in parallel, the voltage across them is the same.

These IR drops are $IR = I_{\text{G}}r$ so that $IR = \frac{I_{\text{G}}}{I} = \frac{R}{r}$. Solving for R, and noting that I_{G} is $50\ \mu\text{A}$ and I is 0.999950 A, we have

$$R = r\frac{I_{\text{G}}}{I} = (25\ \ \Omega\,)\frac{50\ \mu\text{A}}{0.999950\ \text{A}} = 1.25\times10^{-3}\ \ \Omega\,. \tag{21.70}$$

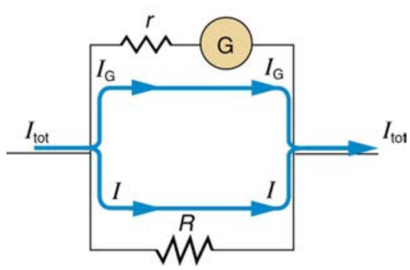


Figure 21.33 A small shunt resistance R placed in parallel with a galvanometer G produces an ammeter, the full-scale deflection of which depends on the choice of R. The larger the current to be measured, the smaller R must be. Most of the current (I) flowing through the meter is shunted through R to protect the galvanometer. (Note that r represents the internal resistance of the galvanometer.) Ammeters may also have multiple scales for greater flexibility in application. The various scales are achieved by switching various shunt resistances in parallel with the galvanometer—the greater the maximum current to be measured, the smaller the shunt resistance must be.

Taking Measurements Alters the Circuit

When you use a voltmeter or ammeter, you are connecting another resistor to an existing circuit and, thus, altering the circuit. Ideally, voltmeters and ammeters do not appreciably affect the circuit, but it is instructive to examine the circumstances under which they do or do not interfere.

First, consider the voltmeter, which is always placed in parallel with the device being measured. Very little current flows through the voltmeter if its resistance is a few orders of magnitude greater than the device, and so the circuit is not appreciably affected. (See Figure 21.34(a).) (A large resistance in parallel with a small one has a combined resistance essentially equal to the small one.) If, however, the voltmeter's resistance is comparable to that of the device being measured, then the two in parallel have a smaller resistance, appreciably affecting the circuit. (See Figure 21.34(b).) The voltage across the device is not the same as when the voltmeter is out of the circuit.

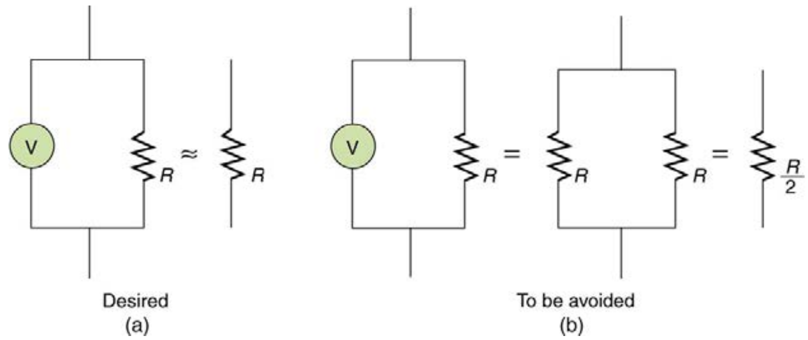


Figure 21.34 (a) A voltmeter having a resistance much larger than the device ($R_{\text{Voltmeter}} >> R$) with which it is in parallel produces a parallel resistance essentially the same as the device and does not appreciably affect the circuit being measured. (b) Here the voltmeter has the same resistance as the device ($R_{\text{Voltmeter}} \cong R$), so that the parallel resistance is half of what it is when the voltmeter is not connected. This is an example of a significant alteration of the circuit and is to be avoided.

An ammeter is placed in series in the branch of the circuit being measured, so that its resistance adds to that branch. Normally, the ammeter's resistance is very small compared with the resistances of the devices in the circuit, and so the extra resistance is negligible. (See Figure 21.35(a).) However, if very small load resistances are involved, or if the ammeter is not as low in resistance as it should be, then the total series resistance is significantly greater, and the current in the branch being measured is reduced. (See Figure 21.35(b).)

A practical problem can occur if the ammeter is connected incorrectly. If it was put in parallel with the resistor to measure the current in it, you could possibly damage the meter; the low resistance of the ammeter would allow most of the current in the circuit to go through the galvanometer, and this current would be larger since the effective resistance is smaller.

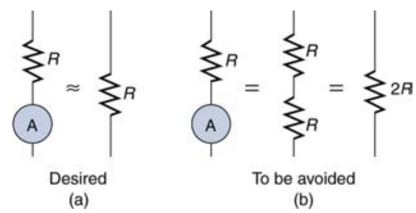


Figure 21.35 (a) An ammeter normally has such a small resistance that the total series resistance in the branch being measured is not appreciably increased. The circuit is essentially unaltered compared with when the ammeter is absent. (b) Here the ammeter's resistance is the same as that of the branch, so that the total resistance is doubled and the current is half what it is without the ammeter. This significant alteration of the circuit is to be avoided.

One solution to the problem of voltmeters and ammeters interfering with the circuits being measured is to use galvanometers with greater sensitivity. This allows construction of voltmeters with greater resistance and ammeters with smaller resistance than when less sensitive galvanometers are used.

There are practical limits to galvanometer sensitivity, but it is possible to get analog meters that make measurements accurate to a few percent. Note that the inaccuracy comes from altering the circuit, not from a fault in the meter.

Connections: Limits to Knowledge

Making a measurement alters the system being measured in a manner that produces uncertainty in the measurement. For macroscopic systems, such as the circuits discussed in this module, the alteration can usually be made negligibly small, but it cannot be eliminated entirely. For submicroscopic systems, such as atoms, nuclei, and smaller particles, measurement alters the system in a manner that cannot be made arbitrarily small. This actually limits knowledge of the system—even limiting what nature can know about itself. We shall see profound implications of this when the Heisenberg uncertainty principle is discussed in the modules on quantum mechanics.

There is another measurement technique based on drawing no current at all and, hence, not altering the circuit at all. These are called null measurements and are the topic of Null Measurements. Digital meters that employ solid-state electronics and null measurements can attain accuracies of one part in 10^6.

heart defibrillator is slightly more complex than the one in **Figure 21.42**, to compensate for magnetic and AC effects that will be covered in **Magnetism**.

Check Your Understanding

When is the potential difference across a capacitor an emf?

Solution
Only when the current being drawn from or put into the capacitor is zero. Capacitors, like batteries, have internal resistance, so their output voltage is not an emf unless current is zero. This is difficult to measure in practice so we refer to a capacitor's voltage rather than its emf. But the source of potential difference in a capacitor is fundamental and it is an emf.

PhET Explorations: Circuit Construction Kit (DC only)

An electronics kit in your computer! Build circuits with resistors, light bulbs, batteries, and switches. Take measurements with the realistic ammeter and voltmeter. View the circuit as a schematic diagram, or switch to a life-like view.

PhET Interactive Simulation

Figure 21.45 Circuit Construction Kit (DC only) (http://cnx.org/content/m55370/1.3/circuit-construction-kit-dc_en.jar)

Glossary

ammeter: an instrument that measures current

analog meter: a measuring instrument that gives a readout in the form of a needle movement over a marked gauge

bridge device: a device that forms a bridge between two branches of a circuit; some bridge devices are used to make null measurements in circuits

capacitance: the maximum amount of electric potential energy that can be stored (or separated) for a given electric potential

capacitor: an electrical component used to store energy by separating electric charge on two opposing plates

conservation laws: require that energy and charge be conserved in a system

current: the flow of charge through an electric circuit past a given point of measurement

current sensitivity: the maximum current that a galvanometer can read

digital meter: a measuring instrument that gives a readout in a digital form

electromotive force (emf): the potential difference of a source of electricity when no current is flowing; measured in volts

full-scale deflection: the maximum deflection of a galvanometer needle, also known as current sensitivity; a galvanometer with a full-scale deflection of $50\ \mu\text{A}$ has a maximum deflection of its needle when $50\ \mu\text{A}$ flows through it

galvanometer: an analog measuring device, denoted by G, that measures current flow using a needle deflection caused by a magnetic field force acting upon a current-carrying wire

internal resistance: the amount of resistance within the voltage source

Joule's law: the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by: $P_e = IV$

junction rule: Kirchhoff's first rule, which applies the conservation of charge to a junction; current is the flow of charge; thus, whatever charge flows into the junction must flow out; the rule can be stated $I_1 = I_2 + I_3$

Kirchhoff's rules: a set of two rules, based on conservation of charge and energy, governing current and changes in potential in an electric circuit

loop rule: Kirchhoff's second rule, which states that in a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of

the circuit. Thus, the emf equals the sum of the IR (voltage) drops in the loop and can be stated:
$$\text{emf} = Ir + IR_1 + IR_2$$

null measurements: methods of measuring current and voltage more accurately by balancing the circuit so that no current flows through the measurement device

ohmmeter: an instrument that applies a voltage to a resistance, measures the current, calculates the resistance using Ohm's law, and provides a readout of this calculated resistance

Ohm's law: the relationship between current, voltage, and resistance within an electrical circuit: $V = IR$

parallel: the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

potential difference: the difference in electric potential between two points in an electric circuit, measured in volts

potentiometer: a null measurement device for measuring potentials (voltages)

RC circuit: a circuit that contains both a resistor and a capacitor

resistance: causing a loss of electrical power in a circuit

resistor: a component that provides resistance to the current flowing through an electrical circuit

series: a sequence of resistors or other components wired into a circuit one after the other

shunt resistance: a small resistance R placed in parallel with a galvanometer G to produce an ammeter; the larger the current to be measured, the smaller R must be; most of the current flowing through the meter is shunted through R to protect the galvanometer

terminal voltage: the voltage measured across the terminals of a source of potential difference

voltage: the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery

voltage drop: the loss of electrical power as a current travels through a resistor, wire or other component

voltmeter: an instrument that measures voltage

Wheatstone bridge: a null measurement device for calculating resistance by balancing potential drops in a circuit

Section Summary

21.1 Resistors in Series and Parallel

- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances: $R_s = R_1 + R_2 + R_3 + ...$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula:
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

21.2 Electromotive Force: Terminal Voltage

- All voltage sources have two fundamental parts—a source of electrical energy that has a characteristic electromotive force (emf), and an internal resistance r.
- The emf is the potential difference of a source when no current is flowing.
- The numerical value of the emf depends on the source of potential difference.
- The internal resistance r of a voltage source affects the output voltage when a current flows.
- The voltage output of a device is called its terminal voltage V and is given by $V = \text{emf} - Ir$, where I is the electric current and is positive when flowing away from the positive terminal of the voltage source.

- When multiple voltage sources are in series, their internal resistances add and their emfs add algebraically.
- Solar cells can be wired in series or parallel to provide increased voltage or current, respectively.

21.3 Kirchhoff's Rules

- Kirchhoff's rules can be used to analyze any circuit, simple or complex.
- Kirchhoff's first rule—the junction rule: The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule: The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.
- The two rules are based, respectively, on the laws of conservation of charge and energy.
- When calculating potential and current using Kirchhoff's rules, a set of conventions must be followed for determining the correct signs of various terms.
- The simpler series and parallel rules are special cases of Kirchhoff's rules.

21.4 DC Voltmeters and Ammeters

- Voltmeters measure voltage, and ammeters measure current.
- A voltmeter is placed in parallel with the voltage source to receive full voltage and must have a large resistance to limit its effect on the circuit.
- An ammeter is placed in series to get the full current flowing through a branch and must have a small resistance to limit its effect on the circuit.
- Both can be based on the combination of a resistor and a galvanometer, a device that gives an analog reading of current.
- Standard voltmeters and ammeters alter the circuit being measured and are thus limited in accuracy.

21.5 Null Measurements

- Null measurement techniques achieve greater accuracy by balancing a circuit so that no current flows through the measuring device.
- One such device, for determining voltage, is a potentiometer.
- Another null measurement device, for determining resistance, is the Wheatstone bridge.
- Other physical quantities can also be measured with null measurement techniques.

21.6 DC Circuits Containing Resistors and Capacitors

- An RC circuit is one that has both a resistor and a capacitor.
- The time constant τ for an RC circuit is $\tau = RC$.
- When an initially uncharged ($V_0 = 0$ at $t = 0$) capacitor in series with a resistor is charged by a DC voltage source, the voltage rises, asymptotically approaching the emf of the voltage source; as a function of time,
$$V = \text{emf}(1 - e^{-t/RC})(\text{charging}).$$
- Within the span of each time constant τ , the voltage rises by 0.632 of the remaining value, approaching the final voltage asymptotically.
- If a capacitor with an initial voltage V_0 is discharged through a resistor starting at $t = 0$, then its voltage decreases exponentially as given by
$$V = V_0 e^{-t/RC}\,(\text{discharging}).$$
- In each time constant τ , the voltage falls by 0.368 of its remaining initial value, approaching zero asymptotically.

Conceptual Questions

21.1 Resistors in Series and Parallel

1. A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in **Figure 21.46** has on current when open and when closed.

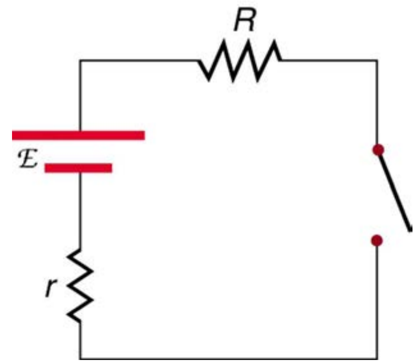


Figure 21.46 A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

2. What is the voltage across the open switch in **Figure 21.46**?

3. There is a voltage across an open switch, such as in **Figure 21.46**. Why, then, is the power dissipated by the open switch small?

4. Why is the power dissipated by a closed switch, such as in **Figure 21.46**, small?

5. A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in **Figure 21.47**. Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)

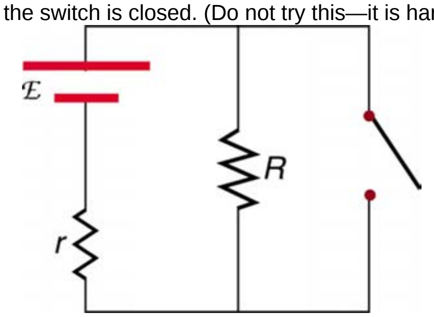


Figure 21.47 A wiring mistake put this switch in parallel with the device represented by R . (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

6. Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

7. Would your headlights dim when you start your car's engine if the wires in your automobile were superconductors? (Do not neglect the battery's internal resistance.) Explain.

8. Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

9. If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

10. Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

11. Before World War II, some radios got power through a "resistance cord" that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio's tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

12. Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

21.2 Electromotive Force: Terminal Voltage

13. Is every emf a potential difference? Is every potential difference an emf? Explain.

14. Explain which battery is doing the charging and which is being charged in **Figure 21.48**.

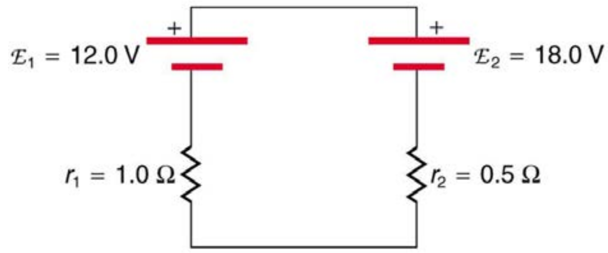


Figure 21.48

15. Given a battery, an assortment of resistors, and a variety of voltage and current measuring devices, describe how you would determine the internal resistance of the battery.

16. Two different 12-V automobile batteries on a store shelf are rated at 600 and 850 "cold cranking amps." Which has the smallest internal resistance?

17. What are the advantages and disadvantages of connecting batteries in series? In parallel?

18. Semitractor trucks use four large 12-V batteries. The starter system requires 24 V, while normal operation of the truck's other electrical components utilizes 12 V. How could the four batteries be connected to produce 24 V? To produce 12 V? Why is 24 V better than 12 V for starting the truck's engine (a very heavy load)?

21.3 Kirchhoff's Rules

19. Can all of the currents going into the junction in **Figure 21.49** be positive? Explain.

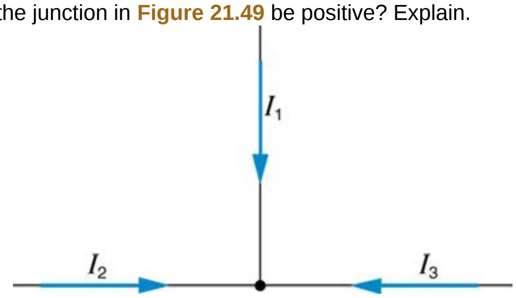


Figure 21.49

20. Apply the junction rule to junction b in **Figure 21.50**. Is any new information gained by applying the junction rule at e? (In the figure, each emf is represented by script E.)

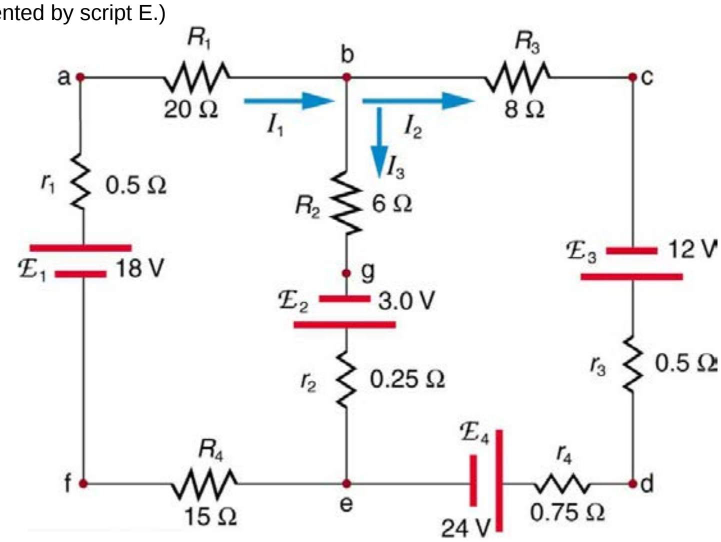


Figure 21.50

21. (a) What is the potential difference going from point a to point b in **Figure 21.50**? (b) What is the potential difference going from c to b? (c) From e to g? (d) From e to d?

22. Apply the loop rule to loop afedcba in **Figure 21.50**.

23. Apply the loop rule to loops abgefa and cbgedc in **Figure 21.50**.

21.4 DC Voltmeters and Ammeters

24. Why should you not connect an ammeter directly across a voltage source as shown in **Figure 21.51**? (Note that script E in the figure stands for emf.)

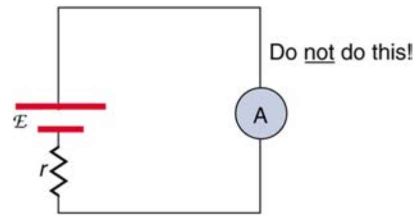


Figure 21.51

25. Suppose you are using a multimeter (one designed to measure a range of voltages, currents, and resistances) to measure current in a circuit and you inadvertently leave it in a voltmeter mode. What effect will the meter have on the circuit? What would happen if you were measuring voltage but accidentally put the meter in the ammeter mode?

26. Specify the points to which you could connect a voltmeter to measure the following potential differences in **Figure 21.52**: (a) the potential difference of the voltage source; (b) the potential difference across R_1 ; (c) across R_2 ; (d) across R_3 ; (e) across R_2 and R_3 . Note that there may be more than one answer to each part.

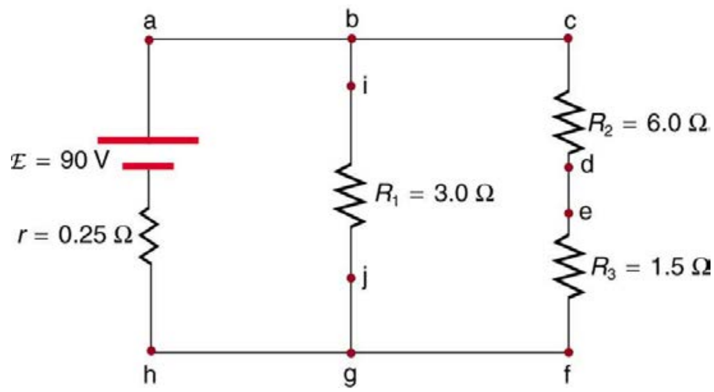


Figure 21.52

27. To measure currents in **Figure 21.52**, you would replace a wire between two points with an ammeter. Specify the points between which you would place an ammeter to measure the following: (a) the total current; (b) the current flowing through R_1 ; (c) through R_2 ; (d) through R_3 . Note that there may be more than one answer to each part.

21.5 Null Measurements

28. Why can a null measurement be more accurate than one using standard voltmeters and ammeters? What factors limit the accuracy of null measurements?

29. If a potentiometer is used to measure cell emfs on the order of a few volts, why is it most accurate for the standard emf_s to be the same order of magnitude and the resistances to be in the range of a few ohms?

21.6 DC Circuits Containing Resistors and Capacitors

30. Regarding the units involved in the relationship $\tau = RC$, verify that the units of resistance times capacitance are time, that is, $\Omega \cdot \text{F} = \text{s}$.

31. The RC time constant in heart defibrillation is crucial to limiting the time the current flows. If the capacitance in the defibrillation unit is fixed, how would you manipulate resistance in the circuit to adjust the RC constant τ ? Would an adjustment of the applied voltage also be needed to ensure that the current delivered has an appropriate value?

32. When making an ECG measurement, it is important to measure voltage variations over small time intervals. The time is limited by the RC constant of the circuit—it is not possible to measure time variations shorter than RC . How would you manipulate R and C in the circuit to allow the necessary measurements?

33. Draw two graphs of charge versus time on a capacitor. Draw one for charging an initially uncharged capacitor in series with a resistor, as in the circuit in **Figure 21.41**, starting from $\text{t} = 0$. Draw the other for discharging a capacitor through a resistor, as in the circuit in **Figure 21.42**, starting at $\text{t} = 0$, with an initial charge Q_0 . Show at least two intervals of τ .

34. When charging a capacitor, as discussed in conjunction with **Figure 21.41**, how long does it take for the voltage on the capacitor to reach emf? Is this a problem?

35. When discharging a capacitor, as discussed in conjunction with **Figure 21.42**, how long does it take for the voltage on the capacitor to reach zero? Is this a problem?

36. Referring to **Figure 21.41**, draw a graph of potential difference across the resistor versus time, showing at least two intervals of τ . Also draw a graph of current versus time for this situation.

37. A long, inexpensive extension cord is connected from inside the house to a refrigerator outside. The refrigerator doesn't run as it should. What might be the problem?

38. In **Figure 21.44**, does the graph indicate the time constant is shorter for discharging than for charging? Would you expect ionized gas to have low resistance? How would you adjust R to get a longer time between flashes? Would adjusting R affect the discharge time?

39. An electronic apparatus may have large capacitors at high voltage in the power supply section, presenting a shock hazard even when the apparatus is switched off. A "bleeder resistor" is therefore placed across such a capacitor, as shown schematically in **Figure 21.53**, to bleed the charge from it after the apparatus is off. Why must the bleeder resistance be much greater than the effective resistance of the rest of the circuit? How does this affect the time constant for discharging the capacitor?

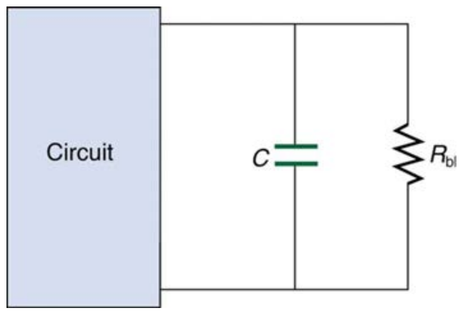


Figure 21.53 A bleeder resistor R_{bl} discharges the capacitor in this electronic device once it is switched off.

Problems & Exercises

21.1 Resistors in Series and Parallel

Note: Data taken from figures can be assumed to be accurate to three significant digits.

1. (a) What is the resistance of ten $275\text{-}\Omega$ resistors connected in series? (b) In parallel?

2. (a) What is the resistance of a $1.00\times10^{2}\text{-}\Omega$, a $2.50\text{-k}\Omega$, and a $4.00\text{-k}\ \Omega$ resistor connected in series? (b) In parallel?

3. What are the largest and smallest resistances you can obtain by connecting a $36.0\text{-}\Omega$, a $50.0\text{-}\Omega$, and a $700\text{-}\Omega$ resistor together?

4. An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A, 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

5. Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

6. (a) Given a 48.0-V battery and $24.0\text{-}\Omega$ and $96.0\text{-}\Omega$ resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

7. Referring to the example combining series and parallel circuits and **Figure 21.6**, calculate I_3 in the following two different ways: (a) from the known values of I and I_2; (b) using Ohm's law for R_3. In both parts explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

8. Referring to **Figure 21.6**: (a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

9. Refer to **Figure 21.7** and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.400\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

10. A 240-kV power transmission line carrying $5.00\times10^{2}\ \text{A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00\times10^{9}\text{-}\Omega$ resistance. **Figure 21.54**. (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

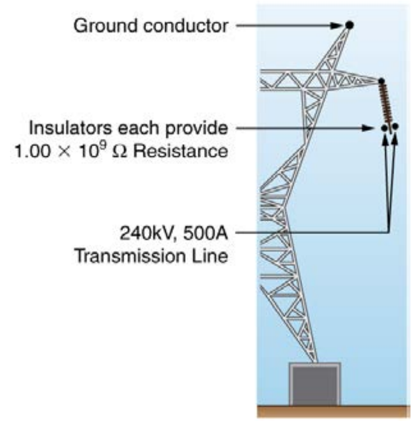


Figure 21.54 High-voltage (240-kV) transmission line carrying $5.00\times10^{2}\ \text{A}$ is hung from a grounded metal transmission tower. The row of ceramic insulators provide $1.00\times10^{9}\ \Omega$ of resistance each.

11. Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 >> R_2$): (a) Their series resistance is very nearly equal to the greater resistance R_1. (b) Their parallel resistance is very nearly equal to smaller resistance R_2.

12. Unreasonable Results

Two resistors, one having a resistance of $145\ \Omega$, are connected in parallel to produce a total resistance of $150\ \Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

13. Unreasonable Results

Two resistors, one having a resistance of $900\ \text{k}\Omega$, are connected in series to produce a total resistance of $0.500\ \text{M}\Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

21.2 Electromotive Force: Terminal Voltage

14. Standard automobile batteries have six lead-acid cells in series, creating a total emf of 12.0 V. What is the emf of an individual lead-acid cell?

15. Carbon-zinc dry cells (sometimes referred to as non-alkaline cells) have an emf of 1.54 V, and they are produced as single cells or in various combinations to form other voltages. (a) How many 1.54-V cells are needed to make the common 9-V battery used in many small electronic devices? (b) What is the actual emf of the approximately 9-V battery? (c) Discuss how internal resistance in the series connection of cells will affect the terminal voltage of this approximately 9-V battery.

16. What is the output voltage of a 3.0000-V lithium cell in a digital wristwatch that draws 0.300 mA, if the cell's internal resistance is $2.00\ \Omega$?

17. (a) What is the terminal voltage of a large 1.54-V carbon-zinc dry cell used in a physics lab to supply 2.00 A to a circuit, if the cell's internal resistance is $0.100\ \Omega$? (b) How much electrical power does the cell produce? (c) What power goes to its load?

18. What is the internal resistance of an automobile battery that has an emf of 12.0 V and a terminal voltage of 15.0 V while a current of 8.00 A is charging it?

19. (a) Find the terminal voltage of a 12.0-V motorcycle battery having a $0.600\text{-}\Omega$ internal resistance, if it is being charged by a current of 10.0 A. (b) What is the output voltage of the battery charger?

20. A car battery with a 12-V emf and an internal resistance of $0.050\ \Omega$ is being charged with a current of 60 A. Note that in this process the battery is being charged. (a) What is the potential difference across its terminals? (b) At what rate is thermal energy being dissipated in the battery? (c) At what rate is electric energy being converted to chemical energy? (d) What are the answers to (a) and (b) when the battery is used to supply 60 A to the starter motor?

21. The hot resistance of a flashlight bulb is $2.30\ \Omega$, and it is run by a 1.58-V alkaline cell having a $0.100\text{-}\Omega$ internal resistance. (a) What current flows? (b) Calculate the power supplied to the bulb using $I^2 R_{\text{bulb}}$. (c) Is this power the same as calculated using $\dfrac{V^2}{R_{\text{bulb}}}$?

22. The label on a portable radio recommends the use of rechargeable nickel-cadmium cells (nicads), although they have a 1.25-V emf while alkaline cells have a 1.58-V emf. The radio has a $3.20\text{-}\Omega$ resistance. (a) Draw a circuit diagram of the radio and its batteries. Now, calculate the power delivered to the radio. (b) When using Nicad cells each having an internal resistance of $0.0400\ \Omega$. (c) When using alkaline cells each having an internal resistance of $0.200\ \Omega$. (d) Does this difference seem significant, considering that the radio's effective resistance is lowered when its volume is turned up?

23. An automobile starter motor has an equivalent resistance of $0.0500\ \Omega$ and is supplied by a 12.0-V battery with a $0.0100\text{-}\Omega$ internal resistance. (a) What is the current to the motor? (b) What voltage is applied to it? (c) What power is supplied to the motor? (d) Repeat these calculations for when the battery connections are corroded and add $0.0900\ \Omega$ to the circuit. (Significant problems are caused by even small amounts of unwanted resistance in low-voltage, high-current applications.)

24. A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of $0.0200\ \Omega$ in series with a 1.53-V carbon-zinc dry cell having a $0.100\text{-}\Omega$ internal resistance. The load resistance is $10.0\ \Omega$. (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?

25. (a) What is the internal resistance of a voltage source if its terminal voltage drops by 2.00 V when the current supplied increases by 5.00 A? (b) Can the emf of the voltage source be found with the information supplied?

26. A person with body resistance between his hands of $10.0\ \text{k}\ \Omega$ accidentally grasps the terminals of a 20.0-kV power supply. (Do NOT do this!) (a) Draw a circuit diagram to represent the situation. (b) If the internal resistance of the power supply is $2000\ \Omega$, what is the current through his body? (c) What is the power dissipated in his body? (d) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in this situation to be 1.00 mA or less? (e) Will this modification compromise the effectiveness of the power supply for driving low-resistance devices? Explain your reasoning.

27. Electric fish generate current with biological cells called electroplaques, which are physiological emf devices. The electroplaques in the South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. Each electroplaque has an emf of 0.15 V and internal resistance of $0.25\ \Omega$. If the water surrounding the fish has resistance of $800\ \Omega$, how much current can the eel produce in water from near its head to near its tail?

28. Integrated Concepts

A 12.0-V emf automobile battery has a terminal voltage of 16.0 V when being charged by a current of 10.0 A. (a) What is the battery's internal resistance? (b) What power is dissipated inside the battery? (c) At what rate (in $^{\circ}\text{C/min}$) will its temperature increase if its mass is 20.0 kg and it has a specific heat of $0.300\ \text{kcal/kg}\cdot{}^{\circ}\text{C}$, assuming no heat escapes?

29. Unreasonable Results

A 1.58-V alkaline cell with a $0.200\text{-}\Omega$ internal resistance is supplying 8.50 A to a load. (a) What is its terminal voltage? (b) What is the value of the load resistance? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable or inconsistent?

30. Unreasonable Results

(a) What is the internal resistance of a 1.54-V dry cell that supplies 1.00 W of power to a $15.0\text{-}\Omega$ bulb? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

21.3 Kirchhoff's Rules

31. Apply the loop rule to loop abcdefgha in **Figure 21.27**.

32. Apply the loop rule to loop aedcba in **Figure 21.27**.

33. Verify the second equation in **Example 21.5** by substituting the values found for the currents I_1 and I_2.

34. Verify the third equation in **Example 21.5** by substituting the values found for the currents I_1 and I_3.

35. Apply the junction rule at point a in **Figure 21.55**.

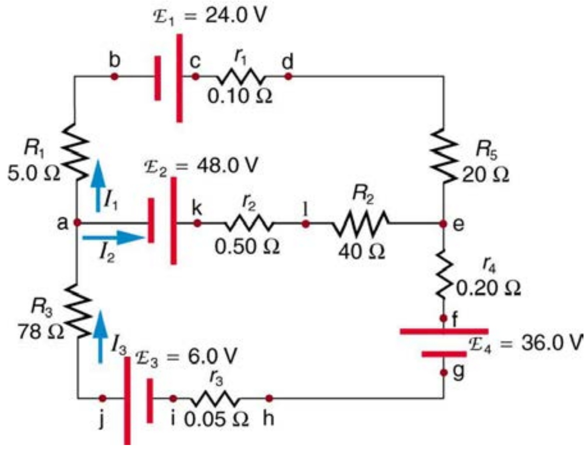


Figure 21.55

36. Apply the loop rule to loop abcdefghija in Figure 21.55.

37. Apply the loop rule to loop akledcba in Figure 21.55.

38. Find the currents flowing in the circuit in Figure 21.55. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

39. Solve **Example 21.5**, but use loop abcdefgha instead of loop akledcba. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

40. Find the currents flowing in the circuit in Figure 21.50.

41. Unreasonable Results

Consider the circuit in Figure 21.56, and suppose that the emfs are unknown and the currents are given to be $I_1 = 5.00\text{ A}$, $I_2 = 3.0\text{ A}$, and $I_3 = -2.00\text{ A}$. (a) Could you find the emfs? (b) What is wrong with the assumptions?

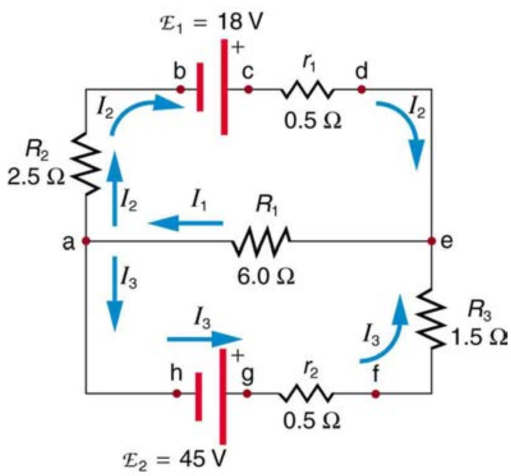


Figure 21.56

21.4 DC Voltmeters and Ammeters

42. What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $1.00\text{-M}\ \Omega$ resistance on its 30.0-V scale?

43. What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $25.0\text{-k}\ \Omega$ resistance on its 100-V scale?

44. Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 0.100-V full-scale reading.

45. Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 3000-V full-scale reading. Include a circuit diagram with your solution.

46. Find the resistance that must be placed in parallel with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 10.0-A full-scale reading. Include a circuit diagram with your solution.

47. Find the resistance that must be placed in parallel with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 300-mA full-scale reading.

48. Find the resistance that must be placed in series with a $10.0\text{-}\Omega$ galvanometer having a $100\text{-}\mu\text{A}$ sensitivity to allow it to be used as a voltmeter with: (a) a 300-V full-scale reading, and (b) a 0.300-V full-scale reading.

49. Find the resistance that must be placed in parallel with a $10.0\text{-}\Omega$ galvanometer having a $100\text{-}\mu\text{A}$ sensitivity to allow it to be used as an ammeter with: (a) a 20.0-A full-scale reading, and (b) a 100-mA full-scale reading.

50. Suppose you measure the terminal voltage of a 1.585-V alkaline cell having an internal resistance of $0.100\ \Omega$ by placing a $1.00\text{-k}\ \Omega$ voltmeter across its terminals. (See Figure 21.57.) (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.

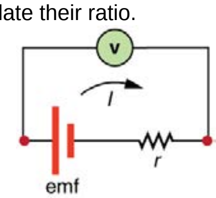


Figure 21.57

51. Suppose you measure the terminal voltage of a 3.200-V lithium cell having an internal resistance of $5.00\ \Omega$ by placing a $1.00\text{-k}\ \Omega$ voltmeter across its terminals. (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.

52. A certain ammeter has a resistance of $5.00\times10^{-5}\ \Omega$ on its 3.00-A scale and contains a $10.0\text{-}\Omega$ galvanometer. What is the sensitivity of the galvanometer?

53. A $1.00\text{-M}\Omega$ voltmeter is placed in parallel with a $75.0\text{-k}\ \Omega$ resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) What is the resistance of the combination? (c) If the voltage across the combination is kept the same as it was across the $75.0\text{-k}\ \Omega$ resistor alone, what is the percent increase in current? (d) If the current through the combination is kept the same as it was through the $75.0\text{-k}\ \Omega$ resistor alone, what is the percentage decrease in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

54. A $0.0200\text{-}\Omega$ ammeter is placed in series with a $10.00\text{-}\Omega$ resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) Calculate the resistance of the combination. (c) If the voltage is kept the same across the combination as it was through the $10.00\text{-}\Omega$ resistor alone, what is the percent decrease in current? (d) If the current is kept the same through the combination as it was through the $10.00\text{-}\Omega$ resistor alone, what is the percent increase in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

55. Unreasonable Results

Suppose you have a $40.0\text{-}\Omega$ galvanometer with a $25.0\text{-}\mu\text{A}$ sensitivity. (a) What resistance would you put in series with it to allow it to be used as a voltmeter that has a full-scale deflection for 0.500 mV? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

56. Unreasonable Results

(a) What resistance would you put in parallel with a $40.0\text{-}\Omega$ galvanometer having a $25.0\text{-}\mu\text{A}$ sensitivity to allow it to be used as an ammeter that has a full-scale deflection for $10.0\text{-}\mu\text{A}$? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

21.5 Null Measurements

57. What is the emf_x of a cell being measured in a potentiometer, if the standard cell's emf is 12.0 V and the potentiometer balances for $R_\text{x} = 5.000\ \Omega$ and $R_\text{s} = 2.500\ \Omega$?

58. Calculate the emf_x of a dry cell for which a potentiometer is balanced when $R_\text{x} = 1.200\ \Omega$, while an alkaline standard cell with an emf of 1.600 V requires $R_\text{s} = 1.247\ \Omega$ to balance the potentiometer.

59. When an unknown resistance R_x is placed in a Wheatstone bridge, it is possible to balance the bridge by adjusting R_3 to be $2500\ \Omega$. What is R_x if $\dfrac{R_2}{R_1} = 0.625$?

60. To what value must you adjust R_3 to balance a Wheatstone bridge, if the unknown resistance R_x is $100\ \Omega$, R_1 is $50.0\ \Omega$, and R_2 is $175\ \Omega$?

61. (a) What is the unknown emf_x in a potentiometer that balances when R_x is $10.0\ \Omega$, and balances when R_s is $15.0\ \Omega$ for a standard 3.000-V emf? (b) The same emf_x is placed in the same potentiometer, which now balances when R_s is $15.0\ \Omega$ for a standard emf of 3.100 V. At what resistance R_x will the potentiometer balance?

62. Suppose you want to measure resistances in the range from $10.0\ \Omega$ to $10.0\ \text{k}\Omega$ using a Wheatstone bridge that has $\dfrac{R_2}{R_1} = 2.000$. Over what range should R_3 be adjustable?

21.6 DC Circuits Containing Resistors and Capacitors

63. The timing device in an automobile's intermittent wiper system is based on an RC time constant and utilizes a $0.500\text{-}\mu\text{F}$ capacitor and a variable resistor. Over what range must R be made to vary to achieve time constants from 2.00 to 15.0 s?

64. A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

65. The duration of a photographic flash is related to an RC time constant, which is $0.100\ \mu\text{s}$ for a certain camera. (a) If the resistance of the flash lamp is $0.0400\ \Omega$ during discharge, what is the size of the capacitor supplying its energy? (b) What is the time constant for charging the capacitor, if the charging resistance is $800\ \text{k}\Omega$?

66. A 2.00- and a $7.50\text{-}\mu\text{F}$ capacitor can be connected in series or parallel, as can a 25.0- and a $100\text{-k}\Omega$ resistor. Calculate the four RC time constants possible from connecting the resulting capacitance and resistance in series.

67. After two time constants, what percentage of the final voltage, emf, is on an initially uncharged capacitor C , charged through a resistance R ?

68. A $500\text{-}\Omega$ resistor, an uncharged $1.50\text{-}\mu\text{F}$ capacitor, and a 6.16-V emf are connected in series. (a) What is the initial current? (b) What is the RC time constant? (c) What is the current after one time constant? (d) What is the voltage on the capacitor after one time constant?

69. A heart defibrillator being used on a patient has an RC time constant of 10.0 ms due to the resistance of the patient and the capacitance of the defibrillator. (a) If the defibrillator has an $8.00\text{-}\mu\text{F}$ capacitance, what is the resistance of the path through the patient? (You may neglect the capacitance of the patient and the resistance of the defibrillator.) (b) If the initial voltage is 12.0 kV, how long does it take to decline to $6.00\times10^{2}\ \text{V}$?

70. An ECG monitor must have an RC time constant less than $1.00\times10^{2}\ \mu\text{s}$ to be able to measure variations in voltage over small time intervals. (a) If the resistance of the circuit (due mostly to that of the patient's chest) is $1.00\ \text{k}\Omega$, what is the maximum capacitance of the circuit? (b) Would it be difficult in practice to limit the capacitance to less than the value found in (a)?

71. Figure 21.58 shows how a bleeder resistor is used to discharge a capacitor after an electronic device is shut off, allowing a person to work on the electronics with less risk of shock. (a) What is the time constant? (b) How long will it take to reduce the voltage on the capacitor to 0.250% (5% of 5%) of its full value once discharge begins? (c) If the capacitor is charged to a voltage V_0 through a $100\text{-}\Omega$ resistance, calculate the time it takes to rise to $0.865V_0$ (This is about two time constants.)

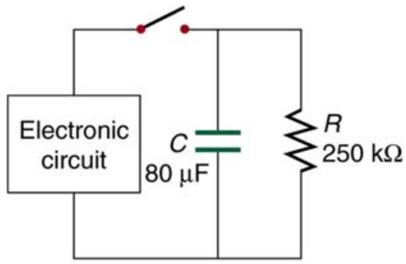


Figure 21.58

72. Using the exact exponential treatment, find how much time is required to discharge a $250\text{-}\mu\text{F}$ capacitor through a $500\text{-}\Omega$ resistor down to 1.00% of its original voltage.

73. Using the exact exponential treatment, find how much time is required to charge an initially uncharged 100-pF capacitor through a $75.0\text{-M}\ \Omega$ resistor to 90.0% of its final voltage.

74. Integrated Concepts

If you wish to take a picture of a bullet traveling at 500 m/s, then a very brief flash of light produced by an RC discharge through a flash tube can limit blurring. Assuming 1.00 mm of motion during one RC constant is acceptable, and given that the flash is driven by a $600\text{-}\mu\text{F}$ capacitor, what is the resistance in the flash tube?

75. Integrated Concepts

A flashing lamp in a Christmas earring is based on an RC discharge of a capacitor through its resistance. The effective duration of the flash is 0.250 s, during which it produces an average 0.500 W from an average 3.00 V. (a) What energy does it dissipate? (b) How much charge moves through the lamp? (c) Find the capacitance. (d) What is the resistance of the lamp?

76. Integrated Concepts

A $160\text{-}\mu\text{F}$ capacitor charged to 450 V is discharged through a $31.2\text{-k}\ \Omega$ resistor. (a) Find the time constant. (b) Calculate the temperature increase of the resistor, given that its mass is 2.50 g and its specific heat is $1.67\frac{\text{kJ}}{\text{kg}\cdot{}^{\circ}\text{C}}$, noting that most of the thermal energy is retained in the short time of the discharge. (c) Calculate the new resistance, assuming it is pure carbon. (d) Does this change in resistance seem significant?

77. Unreasonable Results

(a) Calculate the capacitance needed to get an RC time constant of $1.00\times10^{3}\ \text{s}$ with a $0.100\text{-}\Omega$ resistor. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

78. Construct Your Own Problem

Consider a camera's flash unit. Construct a problem in which you calculate the size of the capacitor that stores energy for the flash lamp. Among the things to be considered are the voltage applied to the capacitor, the energy needed in the flash and the associated charge needed on the capacitor, the resistance of the flash lamp during discharge, and the desired RC time constant.

79. Construct Your Own Problem

Consider a rechargeable lithium cell that is to be used to power a camcorder. Construct a problem in which you calculate the internal resistance of the cell during normal operation. Also, calculate the minimum voltage output of a battery charger to be used to recharge your lithium cell. Among the things to be considered are the emf and useful terminal voltage of a lithium cell and the current it should be able to supply to a camcorder.

Test Prep for AP® Courses

21.1 Resistors in Series and Parallel

1.

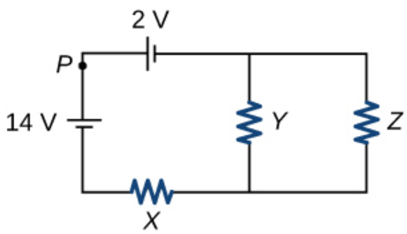


Figure 21.59 The figure above shows a circuit containing two batteries and three identical resistors with resistance *R*. Which of the following changes to the circuit will result in an increase in the current at point *P*? Select *two* answers.

a. Reversing the connections to the 14 V battery.
b. Removing the 2 V battery and connecting the wires to close the left loop.
c. Rearranging the resistors so all three are in series.
d. Removing the branch containing resistor *Z*.

2. In a circuit, a parallel combination of six 1.6-kΩ resistors is connected in series with a parallel combination of four 2.4-kΩ resistors. If the source voltage is 24 V, what will be the percentage of total current in one of the 2.4-kΩ resistors?

a. 10%
b. 12%
c. 20%
d. 25%

3. If the circuit in the previous question is modified by removing some of the 1.6 kΩ resistors, the total current in the circuit is 24 mA. How many resistors were removed?

a. 1
b. 2
c. 3
d. 4

4.

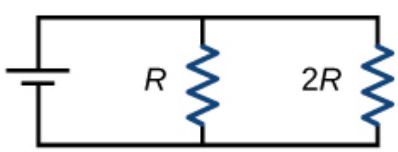


Figure 21.60 Two resistors, with resistances *R* and 2*R* are connected to a voltage source as shown in this figure. If the power dissipated in *R* is 10 W, what is the power dissipated in 2*R*?

a. 1 W
b. 2.5 W
c. 5 W
d. 10 W

5. In a circuit, a parallel combination of two 20-Ω and one 10-Ω resistors is connected in series with a 4-Ω resistor. The source voltage is 36 V.

a. Find the resistor(s) with the maximum current.
b. Find the resistor(s) with the maximum voltage drop.
c. Find the power dissipated in each resistor and hence the total power dissipated in all the resistors. Also find the power output of the source. Are they equal or not? Justify your answer.
d. Will the answers for questions (a) and (b) differ if a 3 Ω resistor is added in series to the 4 Ω resistor? If yes, repeat the question(s) for the new resistor combination.
e. If the values of all the resistors and the source voltage are doubled, what will be the effect on the current?

21.2 Electromotive Force: Terminal Voltage

6. Suppose there are two voltage sources – Sources A and B – with the same emfs but different internal resistances, i.e., the internal resistance of Source A is lower than Source B. If they both supply the same current in their circuits, which of the following statements is true?

a. External resistance in Source A's circuit is more than Source B's circuit.
b. External resistance in Source A's circuit is less than Source B's circuit.
c. External resistance in Source A's circuit is the same as Source B's circuit.
d. The relationship between external resistances in the two circuits can't be determined.

7. Calculate the internal resistance of a voltage source if the terminal voltage of the source increases by 1 V when the current supplied decreases by 4 A? Suppose this source is connected in series (in the same direction) to another source with a different voltage but same internal resistance. What will be the total internal resistance? How will the total internal resistance change if the sources are connected in the opposite direction?

21.3 Kirchhoff's Rules

8. An experiment was set up with the circuit diagram shown. Assume $R_1 = 10\ \Omega$, $R_2 = R_3 = 5\ \Omega$, $r = 0\ \Omega$ and $E = 6$ V.

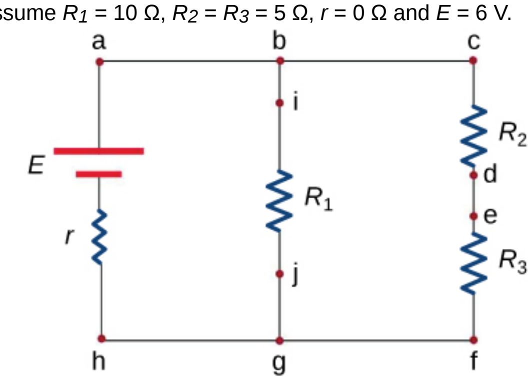


Figure 21.61

a. One of the steps to examine the set-up is to test points with the same potential. Which of the following points can be tested?
   a. Points *b*, *c* and *d*.
   b. Points *d*, *e* and *f*.
   c. Points *f*, *h* and *j*.
   d. Points *a*, *h* and *i*.
b. At which three points should the currents be measured so that Kirchhoff's junction rule can be directly confirmed?
   a. Points *b*, *c* and *d*.
   b. Points *d*, *e* and *f*.
   c. Points *f*, *h* and *j*.
   d. Points *a*, *h* and *i*.
c. If the current in the branch with the voltage source is upward and currents in the other two branches are downward, i.e. $I_a = I_i + I_c$, identify which of the following can be true? Select *two* answers.
   a. $I_i = I_j - I_f$
   b. $I_e = I_h - I_i$
   c. $I_c = I_j - I_a$
   d. $I_d = I_h - I_j$

d. The measurements reveal that the current through R_1 is 0.5 A and R_3 is 0.6 A. Based on your knowledge of Kirchoff’s laws, confirm which of the following statements are true.
   a. The measured current for R_1 is correct but for R_3 is incorrect.
   b. The measured current for R_3 is correct but for R_1 is incorrect.
   c. Both the measured currents are correct.
   d. Both the measured currents are incorrect.
e. The graph shown in the following figure is the energy dissipated at R_1 as a function of time.

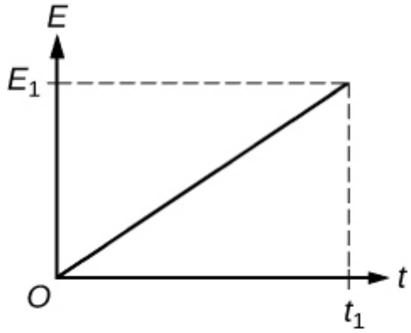
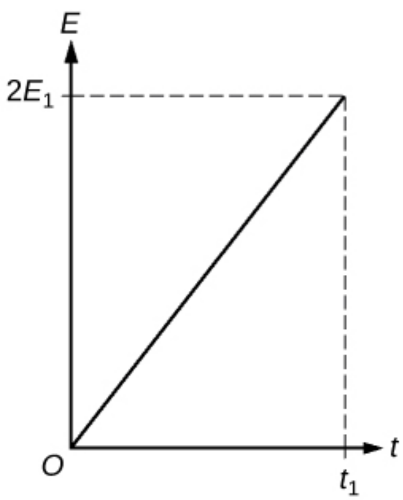


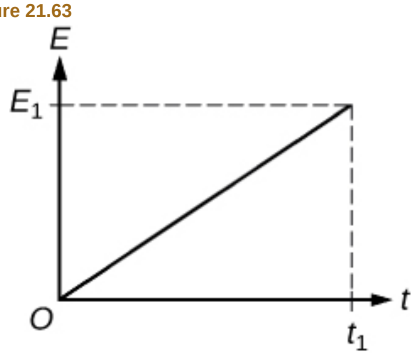
Figure 21.62

Which of the following shows the graph for energy dissipated at R_2 as a function of time?



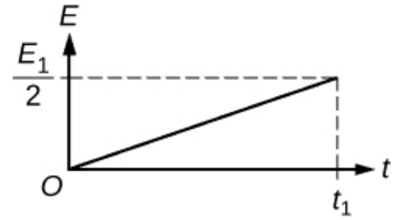
a.

Figure 21.63



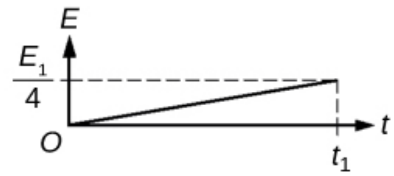
b.

Figure 21.64



c.

Figure 21.65



d.

Figure 21.66

9. For this question, consider the circuit shown in the following figure.

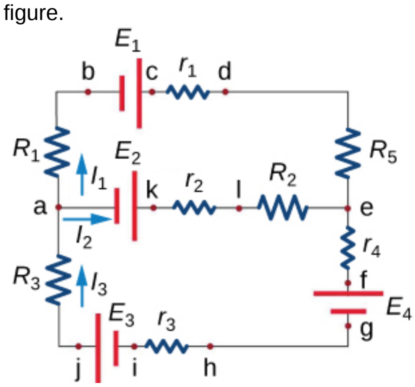


Figure 21.67

a. Assuming that none of the three currents (I_1, I_2, and I_3) are equal to zero, which of the following statements is false?
   a. $I_3 = I_1 + I_2$ at point *a*.
   b. $I_2 = I_3 - I_1$ at point *e*.
   c. The current through R_3 is equal to the current through R_5.
   d. The current through R_1 is equal to the current through R_5.
b. Which of the following statements is true?
   a. $E_1 + E_2 + I_1R_1 - I_2R_2 + I_1r_1 - I_2r_2 + I_1R_5 = 0$
   b. $- E_1 + E_2 + I_1R_1 - I_2R_2 + I_1r_1 - I_2r_2 - I_1R_5 = 0$
   c. $E_1 - E_2 - I_1R_1 + I_2R_2 - I_1r_1 + I_2r_2 - I_1R_5 = 0$
   d. $E_1 + E_2 - I_1R_1 + I_2R_2 - I_1r_1 + I_2r_2 + I_1R_5 = 0$
c. If I_1 = 5 A and I_3 = -2 A, which of the following statements is false?
   a. The current through R_1 will flow from *a* to *b* and will be equal to 5 A.
   b. The current through R_3 will flow from *a* to *j* and will be equal to 2 A.
   c. The current through R_5 will flow from *d* to *e* and will be equal to 5 A.
   d. None of the above.
d. If I_1 = 5 A and I_3 = -2 A, I_2 will be equal to
   a. 3 A
   b. -3 A
   c. 7 A
   d. -7 A

10.

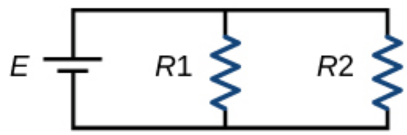


Figure 21.68 In an experiment this circuit is set up. Three ammeters are used to record the currents in the three vertical branches (with R_1, R_2, and E). The readings of the ammeters in the resistor branches (i.e. currents in R_1 and R_2) are 2 A and 3 A respectively.

a. Find the equation obtained by applying Kirchhoff's loop rule in the loop involving R_1 and R_2.
b. What will be the reading of the third ammeter (i.e. the branch with E)? If E were replaced by $3E$, how would this reading change?
c. If the original circuit is modified by adding another voltage source (as shown in the following circuit), find the readings of the three ammeters.

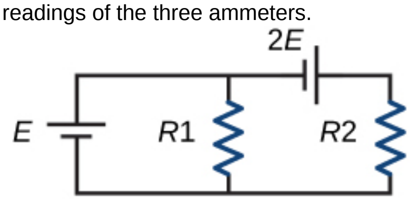


Figure 21.69

11.

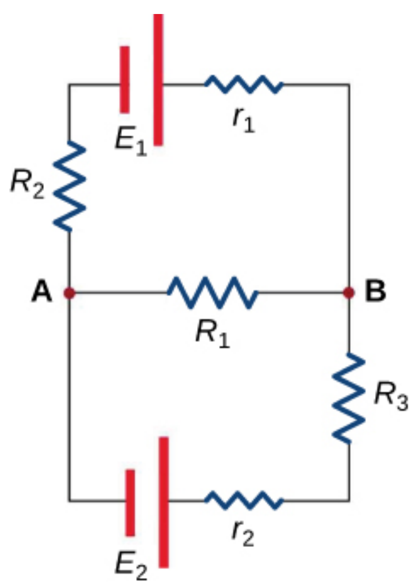


Figure 21.70 In this circuit, assume the currents through R_1, R_2 and R_3 are I_1, I_2 and I_3 respectively and all are flowing in the clockwise direction.

a. Find the equation obtained by applying Kirchhoff's junction rule at point A.
b. Find the equations obtained by applying Kirchhoff's loop rule in the upper and lower loops.
c. Assume $R_1 = R_2 = 6\ \Omega$, $R_3 = 12\ \Omega$, $r_1 = r_2 = 0\ \Omega$, $E_1 = 6$ V and $E_2 = 4$ V. Calculate I_1, I_2 and I_3.
d. For the situation in which E_2 is replaced by a closed switch, repeat parts (a) and (b). Using the values for R_1, R_2, R_3, r_1 and E_1 from part (c) calculate the currents through the three resistors.
e. For the circuit in part (d) calculate the output power of the voltage source and across all the resistors. Examine if energy is conserved in the circuit.
f. A student implemented the circuit of part (d) in the lab and measured the current though one of the resistors as 0.19 A. According to the results calculated in part (d) identify the resistor(s). Justify any difference in measured and calculated value.

21.6 DC Circuits Containing Resistors and Capacitors

12. A battery is connected to a resistor and an uncharged capacitor. The switch for the circuit is closed at $t = 0$ s.

a. While the capacitor is being charged, which of the following is true?
    a. Current through and voltage across the resistor increase.
    b. Current through and voltage across the resistor decrease.
    c. Current through and voltage across the resistor first increase and then decrease.
    d. Current through and voltage across the resistor first decrease and then increase.
b. When the capacitor is fully charged, which of the following is NOT zero?
    a. Current in the resistor.
    b. Voltage across the resistor.
    c. Current in the capacitor.
    d. None of the above.

13. An uncharged capacitor C is connected in series (with a switch) to a resistor R_1 and a voltage source E. Assume $E = 24$ V, $R_1 = 1.2$ kΩ and $C = 1$ mF.

a. What will be the current through the circuit as the switch is closed? Draw a circuit diagram and show the direction of current after the switch is closed. How long will it take for the capacitor to be 99% charged?
b. After full charging, this capacitor is connected in series to another resistor, $R_2 = 1$ kΩ. What will be the current in the circuit as soon as it's connected? Draw a circuit diagram and show the direction of current. How long will it take for the capacitor voltage to reach 3.24 V?

A ATOMIC MASSES

Table A1 Atomic Masses

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
0	neutron	1	n	1.008 665	β^-	10.37 min
1	Hydrogen	1	^1H	1.007 825	99.985%	
	Deuterium	2	^2H or D	2.014 102	0.015%	
	Tritium	3	^3H or T	3.016 050	β^-	12.33 y
2	Helium	3	^3He	3.016 030	$1.38\times10^{-4}\%$	
		4	^4He	4.002 603	$\approx 100\%$	
3	Lithium	6	^6Li	6.015 121	7.5%	
		7	^7Li	7.016 003	92.5%	
4	Beryllium	7	^7Be	7.016 928	EC	53.29 d
		9	^9Be	9.012 182	100%	
5	Boron	10	^{10}B	10.012 937	19.9%	
		11	^{11}B	11.009 305	80.1%	
6	Carbon	11	^{11}C	11.011 432	EC, β^+	
		12	^{12}C	12.000 000	98.90%	
		13	^{13}C	13.003 355	1.10%	
		14	^{14}C	14.003 241	β^-	5730 y
7	Nitrogen	13	^{13}N	13.005 738	β^+	9.96 min
		14	^{14}N	14.003 074	99.63%	
		15	^{15}N	15.000 108	0.37%	
8	Oxygen	15	^{15}O	15.003 065	EC, β^+	122 s
		16	^{16}O	15.994 915	99.76%	
		18	^{18}O	17.999 160	0.200%	
9	Fluorine	18	^{18}F	18.000 937	EC, β^+	1.83 h
		19	^{19}F	18.998 403	100%	
10	Neon	20	^{20}Ne	19.992 435	90.51%	
		22	^{22}Ne	21.991 383	9.22%	
11	Sodium	22	^{22}Na	21.994 434	β^+	2.602 y

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		23	^{23}Na	22.989 767	100%	
		24	^{24}Na	23.990 961	β^-	14.96 h
12	Magnesium	24	^{24}Mg	23.985 042	78.99%	
13	Aluminum	27	^{27}Al	26.981 539	100%	
14	Silicon	28	^{28}Si	27.976 927	92.23%	2.62h
		31	^{31}Si	30.975 362	β^-	
15	Phosphorus	31	^{31}P	30.973 762	100%	
		32	^{32}P	31.973 907	β^-	14.28 d
16	Sulfur	32	^{32}S	31.972 070	95.02%	
		35	^{35}S	34.969 031	β^-	87.4 d
17	Chlorine	35	^{35}Cl	34.968 852	75.77%	
		37	^{37}Cl	36.965 903	24.23%	
18	Argon	40	^{40}Ar	39.962 384	99.60%	
19	Potassium	39	^{39}K	38.963 707	93.26%	
		40	^{40}K	39.963 999	0.0117%, EC, β^-	1.28×10^9 y
20	Calcium	40	^{40}Ca	39.962 591	96.94%	
21	Scandium	45	^{45}Sc	44.955 910	100%	
22	Titanium	48	^{48}Ti	47.947 947	73.8%	
23	Vanadium	51	^{51}V	50.943 962	99.75%	
24	Chromium	52	^{52}Cr	51.940 509	83.79%	
25	Manganese	55	^{55}Mn	54.938 047	100%	
26	Iron	56	^{56}Fe	55.934 939	91.72%	
27	Cobalt	59	^{59}Co	58.933 198	100%	
		60	^{60}Co	59.933 819	β^-	5.271 y
28	Nickel	58	^{58}Ni	57.935 346	68.27%	
		60	^{60}Ni	59.930 788	26.10%	
29	Copper	63	^{63}Cu	62.939 598	69.17%	
		65	^{65}Cu	64.927 793	30.83%	
30	Zinc	64	^{64}Zn	63.929 145	48.6%	
		66	^{66}Zn	65.926 034	27.9%	
31	Gallium	69	^{69}Ga	68.925 580	60.1%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
32	Germanium	72	$^{72}\mathrm{Ge}$	71.922 079	27.4%	
		74	$^{74}\mathrm{Ge}$	73.921 177	36.5%	
33	Arsenic	75	$^{75}\mathrm{As}$	74.921 594	100%	
34	Selenium	80	$^{80}\mathrm{Se}$	79.916 520	49.7%	
35	Bromine	79	$^{79}\mathrm{Br}$	78.918 336	50.69%	
36	Krypton	84	$^{84}\mathrm{Kr}$	83.911 507	57.0%	
37	Rubidium	85	$^{85}\mathrm{Rb}$	84.911 794	72.17%	
38	Strontium	86	$^{86}\mathrm{Sr}$	85.909 267	9.86%	
		88	$^{88}\mathrm{Sr}$	87.905 619	82.58%	
		90	$^{90}\mathrm{Sr}$	89.907 738	β^-	28.8 y
39	Yttrium	89	$^{89}\mathrm{Y}$	88.905 849	100%	
		90	$^{90}\mathrm{Y}$	89.907 152	β^-	64.1 h
40	Zirconium	90	$^{90}\mathrm{Zr}$	89.904 703	51.45%	
41	Niobium	93	$^{93}\mathrm{Nb}$	92.906 377	100%	
42	Molybdenum	98	$^{98}\mathrm{Mo}$	97.905 406	24.13%	
43	Technetium	98	$^{98}\mathrm{Tc}$	97.907 215	β^-	$4.2\times10^{6}\mathrm{y}$
44	Ruthenium	102	$^{102}\mathrm{Ru}$	101.904 348	31.6%	
45	Rhodium	103	$^{103}\mathrm{Rh}$	102.905 500	100%	
46	Palladium	106	$^{106}\mathrm{Pd}$	105.903 478	27.33%	
47	Silver	107	$^{107}\mathrm{Ag}$	106.905 092	51.84%	
		109	$^{109}\mathrm{Ag}$	108.904 757	48.16%	
48	Cadmium	114	$^{114}\mathrm{Cd}$	113.903 357	28.73%	
49	Indium	115	$^{115}\mathrm{In}$	114.903 880	95.7%, β^-	$4.4\times10^{14}\mathrm{y}$
50	Tin	120	$^{120}\mathrm{Sn}$	119.902 200	32.59%	
51	Antimony	121	$^{121}\mathrm{Sb}$	120.903 821	57.3%	
52	Tellurium	130	$^{130}\mathrm{Te}$	129.906 229	33.8%, β^-	$2.5\times10^{21}\mathrm{y}$
53	Iodine	127	$^{127}\mathrm{I}$	126.904 473	100%	
		131	$^{131}\mathrm{I}$	130.906 114	β^-	8.040 d
54	Xenon	132	$^{132}\mathrm{Xe}$	131.904 144	26.9%	
		136	$^{136}\mathrm{Xe}$	135.907 214	8.9%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
55	Cesium	133	$^{133}\mathrm{Cs}$	132.905 429	100%	
		134	$^{134}\mathrm{Cs}$	133.906 696	EC, β^-	2.06 y
56	Barium	137	$^{137}\mathrm{Ba}$	136.905 812	11.23%	
		138	$^{138}\mathrm{Ba}$	137.905 232	71.70%	
57	Lanthanum	139	$^{139}\mathrm{La}$	138.906 346	99.91%	
58	Cerium	140	$^{140}\mathrm{Ce}$	139.905 433	88.48%	
59	Praseodymium	141	$^{141}\mathrm{Pr}$	140.907 647	100%	
60	Neodymium	142	$^{142}\mathrm{Nd}$	141.907 719	27.13%	
61	Promethium	145	$^{145}\mathrm{Pm}$	144.912 743	EC, α	17.7 y
62	Samarium	152	$^{152}\mathrm{Sm}$	151.919 729	26.7%	
63	Europium	153	$^{153}\mathrm{Eu}$	152.921 225	52.2%	
64	Gadolinium	158	$^{158}\mathrm{Gd}$	157.924 099	24.84%	
65	Terbium	159	$^{159}\mathrm{Tb}$	158.925 342	100%	
66	Dysprosium	164	$^{164}\mathrm{Dy}$	163.929 171	28.2%	
67	Holmium	165	$^{165}\mathrm{Ho}$	164.930 319	100%	
68	Erbium	166	$^{166}\mathrm{Er}$	165.930 290	33.6%	
69	Thulium	169	$^{169}\mathrm{Tm}$	168.934 212	100%	
70	Ytterbium	174	$^{174}\mathrm{Yb}$	173.938 859	31.8%	
71	Lutecium	175	$^{175}\mathrm{Lu}$	174.940 770	97.41%	
72	Hafnium	180	$^{180}\mathrm{Hf}$	179.946 545	35.10%	
73	Tantalum	181	$^{181}\mathrm{Ta}$	180.947 992	99.98%	
74	Tungsten	184	$^{184}\mathrm{W}$	183.950 928	30.67%	
75	Rhenium	187	$^{187}\mathrm{Re}$	186.955 744	62.6%, β^-	$4.6\times10^{10}\mathrm{y}$
76	Osmium	191	$^{191}\mathrm{Os}$	190.960 920	β^-	15.4 d
		192	$^{192}\mathrm{Os}$	191.961 467	41.0%	
77	Iridium	191	$^{191}\mathrm{Ir}$	190.960 584	37.3%	
		193	$^{193}\mathrm{Ir}$	192.962 917	62.7%	
78	Platinum	195	$^{195}\mathrm{Pt}$	194.964 766	33.8%	
79	Gold	197	$^{197}\mathrm{Au}$	196.966 543	100%	
		198	$^{198}\mathrm{Au}$	197.968 217	β^-	2.696 d
80	Mercury	199	$^{199}\mathrm{Hg}$	198.968 253	16.87%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		202	^{202}Hg	201.970 617	29.86%	
81	Thallium	205	^{205}Tl	204.974 401	70.48%	
82	Lead	206	^{206}Pb	205.974 440	24.1%	
		207	^{207}Pb	206.975 872	22.1%	
		208	^{208}Pb	207.976 627	52.4%	
		210	^{210}Pb	209.984 163	α, β^-	22.3 y
		211	^{211}Pb	210.988 735	β^-	36.1 min
		212	^{212}Pb	211.991 871	β^-	10.64 h
83	Bismuth	209	^{209}Bi	208.980 374	100%	
		211	^{211}Bi	210.987 255	α, β^-	2.14 min
84	Polonium	210	^{210}Po	209.982 848	α	138.38 d
85	Astatine	218	^{218}At	218.008 684	α, β^-	1.6 s
86	Radon	222	^{222}Rn	222.017 570	α	3.82 d
87	Francium	223	^{223}Fr	223.019 733	α, β^-	21.8 min
88	Radium	226	^{226}Ra	226.025 402	α	$1.60 \times 10^3 y$
89	Actinium	227	^{227}Ac	227.027 750	α, β^-	21.8 y
90	Thorium	228	^{228}Th	228.028 715	α	1.91 y
		232	^{232}Th	232.038 054	100%, α	$1.41 \times 10^{10} y$
91	Protactinium	231	^{231}Pa	231.035 880	α	$3.28 \times 10^4 y$
92	Uranium	233	^{233}U	233.039 628	α	$1.59 \times 10^3 y$
		235	^{235}U	235.043 924	0.720%, α	$7.04 \times 10^8 y$
		236	^{236}U	236.045 562	α	$2.34 \times 10^7 y$
		238	^{238}U	238.050 784	99.2745%, α	$4.47 \times 10^9 y$
		239	^{239}U	239.054 289	β^-	23.5 min
93	Neptunium	239	^{239}Np	239.052 933	β^-	2.355 d
94	Plutonium	239	^{239}Pu	239.052 157	α	$2.41 \times 10^4 y$
95	Americium	243	^{243}Am	243.061 375	α, fission	$7.37 \times 10^3 y$
96	Curium	245	^{245}Cm	245.065 483	α	$8.50 \times 10^3 y$
97	Berkelium	247	^{247}Bk	247.070 300	α	$1.38 \times 10^3 y$

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
98	Californium	249	^{249}Cf	249.074 844	α	351 y
99	Einsteinium	254	^{254}Es	254.088 019	α, β^-	276 d
100	Fermium	253	^{253}Fm	253.085 173	EC, α	3.00 d
101	Mendelevium	255	^{255}Md	255.091 081	EC, α	27 min
102	Nobelium	255	^{255}No	255.093 260	EC, α	3.1 min
103	Lawrencium	257	^{257}Lr	257.099 480	EC, α	0.646 s
104	Rutherfordium	261	^{261}Rf	261.108 690	α	1.08 min
105	Dubnium	262	^{262}Db	262.113 760	α, fission	34 s
106	Seaborgium	263	^{263}Sg	263.11 86	α, fission	0.8 s
107	Bohrium	262	^{262}Bh	262.123 1	α	0.102 s
108	Hassium	264	^{264}Hs	264.128 5	α	0.08 ms
109	Meitnerium	266	^{266}Mt	266.137 8	α	3.4 ms

B SELECTED RADIOACTIVE ISOTOPES

Decay modes are α, β^-, β^+, electron capture (EC) and isomeric transition (IT). EC results in the same daughter nucleus as would β^+ decay. IT is a transition from a metastable excited state. Energies for $\beta^{\pm}$ decays are the maxima; average energies are roughly one-half the maxima.

Table B1 Selected Radioactive Isotopes

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)	Percent
^{3}H	12.33 y	β^-	0.0186	100%			
^{14}C	5730 y	β^-	0.156	100%			
^{13}N	9.96 min	β^+	1.20	100%			
^{22}Na	2.602 y	β^+	0.55	90%	γ	1.27	100%
^{32}P	14.28 d	β^-	1.71	100%			
^{35}S	87.4 d	β^-	0.167	100%			
^{36}Cl	3.00×10^{5}y	β^-	0.710	100%			
^{40}K	1.28×10^{9}y	β^-	1.31	89%			
^{43}K	22.3 h	β^-	0.827	87%	γs	0.373	87%
						0.618	87%
^{45}Ca	165 d	β^-	0.257	100%			
^{51}Cr	27.70 d	EC			γ	0.320	10%
^{52}Mn	5.59d	β^+	3.69	28%	γs	1.33	28%
						1.43	28%
^{52}Fe	8.27 h	β^+	1.80	43%		0.169	43%
						0.378	43%
^{59}Fe	44.6 d	β^-s	0.273	45%	γs	1.10	57%
			0.466	55%		1.29	43%
^{60}Co	5.271 y	β^-	0.318	100%	γs	1.17	100%
						1.33	100%
^{65}Zn	244.1 d	EC			γ	1.12	51%
^{67}Ga	78.3 h	EC			γs	0.0933	70%
						0.185	35%
						0.300	19%
						others	
^{75}Se	118.5 d	EC			γs	0.121	20%
						0.136	65%
						0.265	68%

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)	Percent
						0.280	20%
						others	
^{86}Rb	18.8 d	β^- s	0.69	9%	γ	1.08	9%
			1.77	91%			
^{85}Sr	64.8 d	EC			γ	0.514	100%
^{90}Sr	28.8 y	β^-	0.546	100%			
^{90}Y	64.1 h	β^-	2.28	100%			
^{99m}Tc	6.02 h	IT			γ	0.142	100%
^{113m}In	99.5 min	IT			γ	0.392	100%
^{123}I	13.0 h	EC			γ	0.159	$\approx 100\%$
^{131}I	8.040 d	β^- s	0.248	7%	γ s	0.364	85%
			0.607	93%		others	
			others				
^{129}Cs	32.3 h	EC			γ s	0.0400	35%
						0.372	32%
						0.411	25%
						others	
^{137}Cs	30.17 y	β^- s	0.511	95%	γ	0.662	95%
			1.17	5%			
^{140}Ba	12.79 d	β^-	1.035	$\approx 100\%$	γ s	0.030	25%
						0.044	65%
						0.537	24%
						others	
^{198}Au	2.696 d	β^-	1.161	$\approx 100\%$	γ	0.412	$\approx 100\%$
^{197}Hg	64.1 h	EC			γ	0.0733	100%
^{210}Po	138.38 d	α	5.41	100%			
^{226}Ra	1.60×10^{3}y	αs	4.68	5%	γ	0.186	100%
			4.87	95%			
^{235}U	7.038×10^{8}y	α	4.68	$\approx 100\%$	γ s	numerous	<0.400%
^{238}U	4.468×10^{9}y	αs	4.22	23%	γ	0.050	23%
			4.27	77%			
^{237}Np	2.14×10^{6}y	αs	numerous		γ s	numerous	<0.250%
			4.96 (max.)				
^{239}Pu	2.41×10^{4}y	αs	5.19	11%	γ s	7.5×10^{-5}	73%
			5.23	15%		0.013	15%
			5.24	73%		0.052	10%

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)	Percent
						others	
^{243}Am	$7.37\times10^3 y$	αs	Max. 5.44		γs	0.075	
			5.37	88%		others	
			5.32	11%			
			others				

C USEFUL INFORMATION

This appendix is broken into several tables.

- **Table C1**, Important Constants
- **Table C2**, Submicroscopic Masses
- **Table C3**, Solar System Data
- **Table C4**, Metric Prefixes for Powers of Ten and Their Symbols
- **Table C5**, The Greek Alphabet
- **Table C6**, SI units
- **Table C7**, Selected British Units
- **Table C8**, Other Units
- **Table C9**, Useful Formulae

Table C1 Important Constants[1]

Symbol	Meaning	Best Value	Approximate Value
c	Speed of light in vacuum	$2.99792458 \times 10^8\,\text{m/s}$	$3.00 \times 10^8\,\text{m/s}$
G	Gravitational constant	$6.67384(80) \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$	$6.67 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$
N_A	Avogadro's number	$6.02214129(27) \times 10^{23}$	6.02×10^{23}
k	Boltzmann's constant	$1.3806488(13) \times 10^{-23}\,\text{J/K}$	$1.38 \times 10^{-23}\,\text{J/K}$
R	Gas constant	$8.3144621(75)\,\text{J/mol} \cdot \text{K}$	$8.31\,\text{J/mol} \cdot \text{K} = 1.99\,\text{cal/mol} \cdot \text{K} = 0.0821\text{atm} \cdot \text{L/mol} \cdot \text{K}$
σ	Stefan-Boltzmann constant	$5.670373(21) \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}$	$5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}$
k	Coulomb force constant	$8.987551788... \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2$	$8.99 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2$
q_e	Charge on electron	$-1.602176565(35) \times 10^{-19}\,\text{C}$	$-1.60 \times 10^{-19}\,\text{C}$
ε_0	Permittivity of free space	$8.854187817... \times 10^{-12}\,\text{C}^2/\text{N} \cdot \text{m}^2$	$8.85 \times 10^{-12}\,\text{C}^2/\text{N} \cdot \text{m}^2$
μ_0	Permeability of free space	$4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A}$	$1.26 \times 10^{-6}\,\text{T} \cdot \text{m/A}$
h	Planck's constant	$6.62606957(29) \times 10^{-34}\,\text{J} \cdot \text{s}$	$6.63 \times 10^{-34}\,\text{J} \cdot \text{s}$

Table C2 Submicroscopic Masses[2]

Symbol	Meaning	Best Value	Approximate Value
m_e	Electron mass	$9.10938291(40)\times10^{-31}\text{kg}$	$9.11\times10^{-31}\text{kg}$
m_p	Proton mass	$1.672621777(74)\times10^{-27}\text{kg}$	$1.6726\times10^{-27}\text{kg}$

1. Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (http://www.physics.nist.gov/cuu) (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.
2. Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (http://www.physics.nist.gov/cuu) (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Symbol	Meaning	Best Value	Approximate Value
m_n	Neutron mass	$1.674927351(74) \times 10^{-27} \text{kg}$	$1.6749 \times 10^{-27} \text{kg}$
u	Atomic mass unit	$1.660538921(73) \times 10^{-27} \text{kg}$	$1.6605 \times 10^{-27} \text{kg}$

Table C3 Solar System Data

Sun	mass	$1.99 \times 10^{30} \text{kg}$
	average radius	$6.96 \times 10^{8} \text{m}$
	Earth-sun distance (average)	$1.496 \times 10^{11} \text{m}$
Earth	mass	$5.9736 \times 10^{24} \text{kg}$
	average radius	$6.376 \times 10^{6} \text{m}$
	orbital period	$3.16 \times 10^{7} \text{s}$
Moon	mass	$7.35 \times 10^{22} \text{kg}$
	average radius	$1.74 \times 10^{6} \text{m}$
	orbital period (average)	$2.36 \times 10^{6} \text{s}$
	Earth-moon distance (average)	$3.84 \times 10^{8} \text{m}$

Table C4 Metric Prefixes for Powers of Ten and Their Symbols

Prefix	Symbol	Value	Prefix	Symbol	Value
tera	T	10^{12}	deci	d	10^{-1}
giga	G	10^{9}	centi	c	10^{-2}
mega	M	10^{6}	milli	m	10^{-3}
kilo	k	10^{3}	micro	μ	10^{-6}
hecto	h	10^{2}	nano	n	10^{-9}
deka	da	10^{1}	pico	p	10^{-12}
—	—	$10^{0}(=1)$	femto	f	10^{-15}

Table C5 The Greek Alphabet

Alpha	A	α	Eta	H	η	Nu	N	ν	Tau	T	τ
Beta	B	β	Theta	Θ	θ	Xi	Ξ	ξ	Upsilon	Υ	υ
Gamma	Γ	γ	Iota	I	ι	Omicron	O	o	Phi	Φ	ϕ
Delta	Δ	δ	Kappa	K	κ	Pi	Π	π	Chi	X	χ
Epsilon	E	ε	Lambda	Λ	λ	Rho	P	ρ	Psi	Ψ	ψ
Zeta	Z	ζ	Mu	M	μ	Sigma	Σ	σ	Omega	Ω	ω

Table C6 SI Units

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$\text{N} = \text{kg} \cdot \text{m}/\text{s}^2$	newton
	Energy	$\text{J} = \text{kg} \cdot \text{m}^2/\text{s}^2$	joule
	Power	$\text{W} = \text{J}/\text{s}$	watt
	Pressure	$\text{Pa} = \text{N}/\text{m}^2$	pascal
	Frequency	$\text{Hz} = 1/\text{s}$	hertz
	Electronic potential	$\text{V} = \text{J}/\text{C}$	volt
	Capacitance	$\text{F} = \text{C}/\text{V}$	farad
	Charge	$\text{C} = \text{s} \cdot \text{A}$	coulomb
	Resistance	$\Omega = \text{V}/\text{A}$	ohm
	Magnetic field	$\text{T} = \text{N}/(\text{A} \cdot \text{m})$	tesla
	Nuclear decay rate	$\text{Bq} = 1/\text{s}$	becquerel

Table C7 Selected British Units

Length	$1 \text{ inch (in.)} = 2.54 \text{ cm (exactly)}$
	$1 \text{ foot (ft)} = 0.3048 \text{ m}$
	$1 \text{ mile (mi)} = 1.609 \text{ km}$
Force	$1 \text{ pound (lb)} = 4.448 \text{ N}$
Energy	$1 \text{ British thermal unit (Btu)} = 1.055 \times 10^3 \text{ J}$
Power	$1 \text{ horsepower (hp)} = 746 \text{ W}$
Pressure	$1 \text{ lb}/\text{in}^2 = 6.895 \times 10^3 \text{ Pa}$

Table C8 Other Units

Length	$1 \text{ light year (ly)} = 9.46 \times 10^{15} \text{ m}$
	$1 \text{ astronomical unit (au)} = 1.50 \times 10^{11} \text{ m}$
	$1 \text{ nautical mile} = 1.852 \text{ km}$
	$1 \text{ angstrom(Å)} = 10^{-10} \text{ m}$
Area	$1 \text{ acre (ac)} = 4.05 \times 10^3 \text{ m}^2$
	$1 \text{ square foot (ft}^2) = 9.29 \times 10^{-2} \text{ m}^2$
	$1 \text{ barn } (b) = 10^{-28} \text{ m}^2$
Volume	$1 \text{ liter } (L) = 10^{-3} \text{ m}^3$

	1 U.S. gallon (gal) $= 3.785 \times 10^{-3}\ \text{m}^3$
Mass	1 solar mass $= 1.99 \times 10^{30}$ kg
	1 metric ton $= 10^3$ kg
	1 atomic mass unit (u) $= 1.6605 \times 10^{-27}$ kg
Time	1 year (y) $= 3.16 \times 10^7$ s
	1 day (d) $= 86{,}400$ s
Speed	1 mile per hour (mph) $= 1.609$ km / h
	1 nautical mile per hour (naut) $= 1.852$ km / h
Angle	1 degree (°) $= 1.745 \times 10^{-2}$ rad
	1 minute of arc ($'$) $= 1 / 60$ degree
	1 second of arc ($''$) $= 1 / 60$ minute of arc
	1 grad $= 1.571 \times 10^{-2}$ rad
Energy	1 kiloton TNT (kT) $= 4.2 \times 10^{12}$ J
	1 kilowatt hour ($\text{kW} \cdot h$) $= 3.60 \times 10^6$ J
	1 food calorie (kcal) $= 4186$ J
	1 calorie (cal) $= 4.186$ J
	1 electron volt (eV) $= 1.60 \times 10^{-19}$ J
Pressure	1 atmosphere (atm) $= 1.013 \times 10^5$ Pa
	1 millimeter of mercury (mm Hg) $= 133.3$ Pa
	1 torricelli (torr) $= 1$ mm Hg $= 133.3$ Pa
Nuclear decay rate	1 curie (Ci) $= 3.70 \times 10^{10}$ Bq

Table C9 Useful Formulae

Circumference of a circle with radius r or diameter d	$C = 2\pi r = \pi d$
Area of a circle with radius r or diameter d	$A = \pi r^2 = \pi d^2 / 4$
Area of a sphere with radius r	$A = 4\pi r^2$
Volume of a sphere with radius r	$V = (4/3)\left(\pi r^3\right)$

D GLOSSARY OF KEY SYMBOLS AND NOTATION

In this glossary, key symbols and notation are briefly defined.

Table D1

Symbol	Definition
$\overline{\text{any symbol}}$	average (indicated by a bar over a symbol—e.g., $\bar{v}$ is average velocity)
$°\text{C}$	Celsius degree
$°\text{F}$	Fahrenheit degree
$//$	parallel
$\perp$	perpendicular
$\propto$	proportional to
$\pm$	plus or minus
$_0$	zero as a subscript denotes an initial value
α	alpha rays
α	angular acceleration
α	temperature coefficient(s) of resistivity
β	beta rays
β	sound level
β	volume coefficient of expansion
β^-	electron emitted in nuclear beta decay
β^+	positron decay
γ	gamma rays
γ	surface tension
$\gamma = 1/\sqrt{1 - v^2/c^2}$	a constant used in relativity
Δ	change in whatever quantity follows
δ	uncertainty in whatever quantity follows
ΔE	change in energy between the initial and final orbits of an electron in an atom
ΔE	uncertainty in energy
Δm	difference in mass between initial and final products
ΔN	number of decays that occur
Δp	change in momentum
Δp	uncertainty in momentum
$\Delta \text{PE}_\text{g}$	change in gravitational potential energy

Symbol	Definition
$\Delta\theta$	rotation angle
Δs	distance traveled along a circular path
Δt	uncertainty in time
Δt_0	proper time as measured by an observer at rest relative to the process
ΔV	potential difference
Δx	uncertainty in position
ε_0	permittivity of free space
η	viscosity
θ	angle between the force vector and the displacement vector
θ	angle between two lines
θ	contact angle
θ	direction of the resultant
θ_b	Brewster's angle
θ_c	critical angle
κ	dielectric constant
λ	decay constant of a nuclide
λ	wavelength
λ_n	wavelength in a medium
μ_0	permeability of free space
μ_k	coefficient of kinetic friction
μ_s	coefficient of static friction
v_e	electron neutrino
π^+	positive pion
π^-	negative pion
π^0	neutral pion
ρ	density
ρ_c	critical density, the density needed to just halt universal expansion
ρ_fl	fluid density
$\bar{\rho}_\text{obj}$	average density of an object
ρ / ρ_w	specific gravity
τ	characteristic time constant for a resistance and inductance (RL) or resistance and capacitance (RC) circuit
τ	characteristic time for a resistor and capacitor (RC) circuit
τ	torque
Υ	upsilon meson

Symbol	Definition
Φ	magnetic flux
ϕ	phase angle
Ω	ohm (unit)
ω	angular velocity
A	ampere (current unit)
A	area
A	cross-sectional area
A	total number of nucleons
a	acceleration
a_{B}	Bohr radius
a_{c}	centripetal acceleration
a_{t}	tangential acceleration
AC	alternating current
AM	amplitude modulation
atm	atmosphere
B	baryon number
B	blue quark color
$\bar{B}$	antiblue (yellow) antiquark color
b	quark flavor bottom or beauty
B	bulk modulus
B	magnetic field strength
$\mathrm{B}_{\mathrm{int}}$	electron’s intrinsic magnetic field
$\mathrm{B}_{\mathrm{orb}}$	orbital magnetic field
BE	binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons
BE/A	binding energy per nucleon
Bq	becquerel—one decay per second
C	capacitance (amount of charge stored per volt)
C	coulomb (a fundamental SI unit of charge)
C_{p}	total capacitance in parallel
C_{s}	total capacitance in series
CG	center of gravity
CM	center of mass
c	quark flavor charm
c	specific heat
c	speed of light

Symbol	Definition
Cal	kilocalorie
cal	calorie
COP_{hp}	heat pump’s coefficient of performance
COP_{ref}	coefficient of performance for refrigerators and air conditioners
$\cos\theta$	cosine
$\cot\theta$	cotangent
$\csc\theta$	cosecant
D	diffusion constant
d	displacement
d	quark flavor down
dB	decibel
d_{i}	distance of an image from the center of a lens
d_{o}	distance of an object from the center of a lens
DC	direct current
E	electric field strength
ε	emf (voltage) or Hall electromotive force
emf	electromotive force
E	energy of a single photon
E	nuclear reaction energy
E	relativistic total energy
E	total energy
E_0	ground state energy for hydrogen
E_0	rest energy
EC	electron capture
E_{cap}	energy stored in a capacitor
	efficiency—the useful work output divided by the energy input
$_C$	Carnot efficiency
E_{in}	energy consumed (food digested in humans)
E_{ind}	energy stored in an inductor
E_{out}	energy output
e	emissivity of an object
e^+	antielectron or positron
eV	electron volt
F	farad (unit of capacitance, a coulomb per volt)
F	focal point of a lens

Symbol	Definition
$\mathbf{F}$	force
F	magnitude of a force
F	restoring force
F_B	buoyant force
F_c	centripetal force
F_i	force input
$\mathbf{F}_{net}$	net force
F_o	force output
FM	frequency modulation
f	focal length
f	frequency
f_0	resonant frequency of a resistance, inductance, and capacitance (RLC) series circuit
f_0	threshold frequency for a particular material (photoelectric effect)
f_1	fundamental
f_2	first overtone
f_3	second overtone
f_B	beat frequency
f_k	magnitude of kinetic friction
f_s	magnitude of static friction
G	gravitational constant
G	green quark color
$\bar{G}$	antigreen (magenta) antiquark color
g	acceleration due to gravity
g	gluons (carrier particles for strong nuclear force)
h	change in vertical position
h	height above some reference point
h	maximum height of a projectile
h	Planck's constant
hf	photon energy
h_i	height of the image
h_o	height of the object
I	electric current
I	intensity
I	intensity of a transmitted wave

Symbol	Definition
I	moment of inertia (also called rotational inertia)
I_0	intensity of a polarized wave before passing through a filter
I_{ave}	average intensity for a continuous sinusoidal electromagnetic wave
I_{rms}	average current
J	joule
J/Ψ	Joules/psi meson
K	kelvin
k	Boltzmann constant
k	force constant of a spring
K_α	x rays created when an electron falls into an $n = 1$ shell vacancy from the $n = 3$ shell
K_β	x rays created when an electron falls into an $n = 2$ shell vacancy from the $n = 3$ shell
kcal	kilocalorie
KE	translational kinetic energy
$\text{KE} + \text{PE}$	mechanical energy
KE_e	kinetic energy of an ejected electron
KE_{rel}	relativistic kinetic energy
KE_{rot}	rotational kinetic energy
$\overline{\text{KE}}$	thermal energy
kg	kilogram (a fundamental SI unit of mass)
L	angular momentum
L	liter
L	magnitude of angular momentum
L	self-inductance
ℓ	angular momentum quantum number
L_α	x rays created when an electron falls into an $n = 2$ shell from the $n = 3$ shell
L_e	electron total family number
L_μ	muon family total number
L_τ	tau family total number
L_{f}	heat of fusion
L_{f} and L_{v}	latent heat coefficients
L_{orb}	orbital angular momentum
L_{s}	heat of sublimation
L_{v}	heat of vaporization
L_z	z - component of the angular momentum

Symbol	Definition
M	angular magnification
M	mutual inductance
m	indicates metastable state
m	magnification
m	mass
m	mass of an object as measured by a person at rest relative to the object
m	meter (a fundamental SI unit of length)
m	order of interference
m	overall magnification (product of the individual magnifications)
$m\left({}^{A}\text{X}\right)$	atomic mass of a nuclide
MA	mechanical advantage
m_{e}	magnification of the eyepiece
m_e	mass of the electron
m_{ℓ}	angular momentum projection quantum number
m_n	mass of a neutron
m_{o}	magnification of the objective lens
mol	mole
m_p	mass of a proton
m_{s}	spin projection quantum number
N	magnitude of the normal force
N	newton
$\mathbf{N}$	normal force
N	number of neutrons
n	index of refraction
n	number of free charges per unit volume
N_{A}	Avogadro's number
N_{r}	Reynolds number
$\text{N} \cdot \text{m}$	newton-meter (work-energy unit)
$\text{N} \cdot \text{m}$	newtons times meters (SI unit of torque)
OE	other energy
P	power
P	power of a lens
P	pressure
$\mathbf{p}$	momentum
p	momentum magnitude
p	relativistic momentum

Symbol	Definition
$\mathbf{p}_{\text{tot}}$	total momentum
$\mathbf{p}'_{\text{tot}}$	total momentum some time later
P_{abs}	absolute pressure
P_{atm}	atmospheric pressure
P_{atm}	standard atmospheric pressure
PE	potential energy
PE_{el}	elastic potential energy
PE_{elec}	electric potential energy
PE_{s}	potential energy of a spring
P_{g}	gauge pressure
P_{in}	power consumption or input
P_{out}	useful power output going into useful work or a desired, form of energy
Q	latent heat
Q	net heat transferred into a system
Q	flow rate—volume per unit time flowing past a point
$+Q$	positive charge
$-Q$	negative charge
q	electron charge
q_p	charge of a proton
q	test charge
QF	quality factor
R	activity, the rate of decay
R	radius of curvature of a spherical mirror
R	red quark color
$\bar{R}$	antired (cyan) quark color
R	resistance
R	resultant or total displacement
R	Rydberg constant
R	universal gas constant
r	distance from pivot point to the point where a force is applied
r	internal resistance
$r_{\perp}$	perpendicular lever arm
r	radius of a nucleus
r	radius of curvature

Symbol	Definition
r	resistivity
r or rad	radiation dose unit
rem	roentgen equivalent man
rad	radian
RBE	relative biological effectiveness
RC	resistor and capacitor circuit
rms	root mean square
r_n	radius of the *n*th H-atom orbit
R_p	total resistance of a parallel connection
R_s	total resistance of a series connection
R_s	Schwarzschild radius
S	entropy
$\mathbf{S}$	intrinsic spin (intrinsic angular momentum)
S	magnitude of the intrinsic (internal) spin angular momentum
S	shear modulus
S	strangeness quantum number
s	quark flavor strange
s	second (fundamental SI unit of time)
s	spin quantum number
$\mathbf{s}$	total displacement
$\sec\theta$	secant
$\sin\theta$	sine
s_z	*z*-component of spin angular momentum
T	period—time to complete one oscillation
T	temperature
T_c	critical temperature—temperature below which a material becomes a superconductor
T	tension
T	tesla (magnetic field strength *B*)
t	quark flavor top or truth
t	time
$t_{1/2}$	half-life—the time in which half of the original nuclei decay
$\tan\theta$	tangent
U	internal energy
u	quark flavor up
u	unified atomic mass unit
$\mathbf{u}$	velocity of an object relative to an observer

Symbol	Definition
$\mathbf{u}'$	velocity relative to another observer
V	electric potential
V	terminal voltage
V	volt (unit)
V	volume
$\mathbf{v}$	relative velocity between two observers
v	speed of light in a material
$\mathbf{v}$	velocity
$\bar{\mathbf{v}}$	average fluid velocity
$V_\mathrm{B} - V_\mathrm{A}$	change in potential
$\mathbf{v}_\mathrm{d}$	drift velocity
V_p	transformer input voltage
V_rms	rms voltage
V_s	transformer output voltage
$\mathbf{v}_\mathrm{tot}$	total velocity
v_w	propagation speed of sound or other wave
$\mathbf{v}_\mathrm{w}$	wave velocity
W	work
W	net work done by a system
W	watt
w	weight
w_fl	weight of the fluid displaced by an object
W_c	total work done by all conservative forces
W_nc	total work done by all nonconservative forces
W_out	useful work output
X	amplitude
X	symbol for an element
${}^{Z}X_{N}$	notation for a particular nuclide
x	deformation or displacement from equilibrium
x	displacement of a spring from its undeformed position
x	horizontal axis
X_C	capacitive reactance
X_L	inductive reactance
x_rms	root mean square diffusion distance
y	vertical axis

Symbol	Definition
Y	elastic modulus or Young's modulus
Z	atomic number (number of protons in a nucleus)
Z	impedance

ANSWER KEY

Chapter 1

Problems & Exercises

1

a. 27.8 m/s

b. 62.1 mph

3

$$\frac{1.0 \text{ m}}{\text{s}} = \frac{1.0 \text{ m}}{\text{s}} \times \frac{3600 \text{ s}}{1 \text{ hr}} \times \frac{1 \text{ km}}{1000 \text{ m}}$$

$= 3.6 \text{ km/h}$.

5

length: 377 ft; $4.53\times10^3 \text{ in}$. width: 280 ft; $3.3\times10^3 \text{ in}$.

7

8.847 km

9

(a) $1.3\times10^{-9} \text{ m}$

(b) 40 km/My

11

2 kg

13

a. $85.5 \text{ to } 94.5 \text{ km/h}$

b. $53.1 \text{ to } 58.7 \text{ mi/h}$

15

(a) $7.6\times10^7 \text{ beats}$

(b) $7.57\times10^7 \text{ beats}$

(c) $7.57\times10^7 \text{ beats}$

17

a. 3

b. 3

c. 3

19

a) 2.2%

(b) $59 \text{ to } 61 \text{ km/h}$

21

$80 \pm 3 \text{ beats/min}$

23

2.8 h

25

$11 \pm 1 \text{ cm}^3$

27

$12.06 \pm 0.04 \text{ m}^2$

29

Sample answer: 2×10^{9} heartbeats

31

Sample answer: 2×10^{31} if an average human lifetime is taken to be about 70 years.

33

Sample answer: 50 atoms

35

Sample answers:

(a) 10^{12} cells/hummingbird

(b) 10^{16} cells/human

Chapter 2

Problems & Exercises

1

(a) 7 m

(b) 7 m

(c) $+7\text{ m}$

3

(a) 13 m

(b) 9 m

(c) $+9\text{ m}$

5

(a) $3.0\times10^{4}\text{ m/s}$

(b) 0 m/s

7

2×10^{7} years

9

$34.689\text{ m/s} = 124.88\text{ km/h}$

11

(a) 40.0 km/h

(b) 34.3 km/h, 25° S of E.

(c) average speed $= 3.20\text{ km/h}$, $\bar{v} = 0$.

13

384,000 km

15

(a) $6.61\times10^{15}\text{ rev/s}$

(b) 0 m/s

16

4.29 m/s^2

18

(a) 1.43 s

(b) -2.50 m/s^2

20

(a) 10.8 m/s

(b)

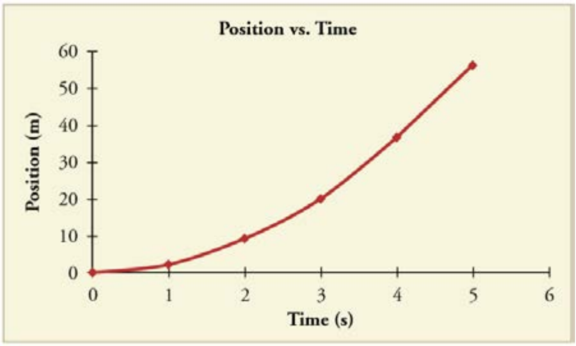


Figure 2.48.

21
38.9 m/s (about 87 miles per hour)

23

(a) 16.5 s

(b) 13.5 s

(c) -2.68 m/s^2

25

(a) 20.0 m

(b) -1.00 m/s

(c) This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at 2.00 m/s^2, then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

27
0.799 m

29

(a) 28.0 m/s

(b) 50.9 s

(c) 7.68 km to accelerate and 713 m to decelerate

31

(a) 51.4 m

(b) 17.1 s

33

(a) -80.4 m/s^2

(b) $9.33\times10^{-2}\text{ s}$

35

(a) 7.7 m/s

(b) $-15\times10^2\text{ m/s}^2$. This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

37

(a) 32.6 m/s^2

(b) 162 m/s

(c) $v > v_{\text{max}}$, because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at 32.6 m/s^2 during the last few meters, but substantially less, and the final velocity would be less than 162 m/s.

39
104 s
40

(a) $v = 12.2\text{ m/s}$; $a = 4.07\text{ m/s}^2$

(b) $v = 11.2\text{ m/s}$

41

(a) $y_1 = 6.28\text{ m}$; $v_1 = 10.1\text{ m/s}$

(b) $y_2 = 10.1\text{ m}$; $v_2 = 5.20\text{ m/s}$

(c) $y_3 = 11.5\text{ m}$; $v_3 = 0.300\text{ m/s}$

(d) $y_4 = 10.4\text{ m}$; $v_4 = -4.60\text{ m/s}$

43
$v_0 = 4.95\text{ m/s}$
45

(a) $a = -9.80\text{ m/s}^2$; $v_0 = 13.0\text{ m/s}$; $y_0 = 0\text{ m}$

(b) $v = 0\text{m/s}$. Unknown is distance y to top of trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a(y - y_0)$ because it contains all known values except for y , so we can solve for y . Solving for y gives

$$\begin{aligned} v^2 - v_0^2 &= 2a(y - y_0) \\ \frac{v^2 - v_0^2}{2a} &= y - y_0 \\ y &= y_0 + \frac{v^2 - v_0^2}{2a} = 0\text{ m} + \frac{(0\text{ m/s})^2 - (13.0\text{ m/s})^2}{2\left(-9.80\text{ m/s}^2\right)} = 8.62\text{ m} \end{aligned} \tag{2.100}$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.

(c) 2.65 s

47

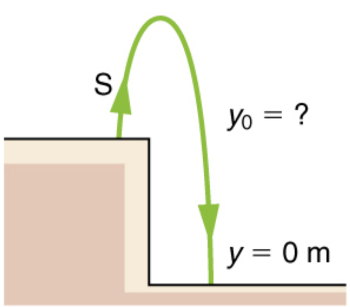


Figure 2.57.

(a) 8.26 m

(b) 0.717 s

49
1.91 s
51

(a) 94.0 m

(b) 3.13 s

53

(a) -70.0 m/s (downward)

(b) 6.10 s

55

(a) 19.6 m

(b) 18.5 m

57

(a) 305 m

(b) 262 m, -29.2 m/s

(c) 8.91 s

59

(a) 115 m/s

(b) 5.0 m/s^2

61

$$v = \frac{(11.7 - 6.95)\times 10^3 \text{ m}}{(40.0 - 20.0) \text{ s}} = 238 \text{ m/s} \tag{2.114}$$

63

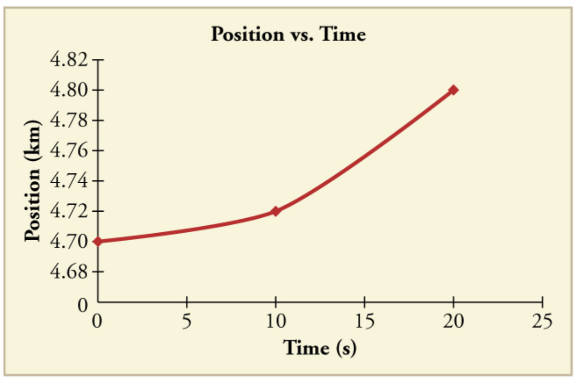


Figure 2.63.

65

(a) 6 m/s

(b) 12 m/s

(c) 3 m/s^2

(d) 10 s

Test Prep for AP® Courses

1

(a)

3

a. Use tape to mark off two distances on the track — one for cart *A* before the collision and one for the combined carts after the collision. Push cart *A* to give it an initial speed. Use a stopwatch to measure the time it takes for the cart(s) to cross the marked distances. The speeds are the distances divided by the times.

b. If the measurement errors are of the same magnitude, they will have a greater effect after the collision. The speed of the combined carts will be less than the initial speed of cart *A*. As a result, these errors will be a greater percentage of the actual velocity value after the collision occurs. (Note: Other arguments could properly be made for ‘more error before the collision' and error that ‘equally affects both sets of measurement.')

5

The position vs. time graph should be represented with a positively sloped line whose slope steadily decreases to zero. The *y*-intercept of the graph may be any value. The line on the velocity vs. time graph should have a positive *y*-intercept and a negative slope. Because the final velocity of the book is zero, the line should finish on the *x*-axis.

The position vs. time graph should be represented with a negatively sloped line whose slope steadily decreases to zero. The *y*-intercept of the graph may be any value. The line on the velocity vs. time graph should have a negative *y*-intercept and a positive slope. Because the final velocity of the book is zero, the line should finish on the *x*-axis.]

7
(c)

Chapter 3

Problems & Exercises

1

(a) 480 m

(b) 379 m, $18.4°$ east of north

3
north component 3.21 km, east component 3.83 km
5
19.5 m, $4.65°$ south of west
7

(a) 26.6 m, $65.1°$ north of east

(b) 26.6 m, $65.1°$ south of west

9
52.9 m, $90.1°$ with respect to the *x*-axis.
11

x-component 4.41 m/s

y-component 5.07 m/s

13

(a) 1.56 km

(b) 120 m east

15
North-component 87.0 km, east-component 87.0 km
17
30.8 m, 35.8 west of north
19

(a) 30.8 m, $54.2°$ south of west

(b) 30.8 m, $54.2°$ north of east

21
18.4 km south, then 26.2 km west(b) 31.5 km at $45.0°$ south of west, then 5.56 km at $45.0°$ west of north
23
7.34 km, $63.5°$ south of east
25

$$\begin{aligned} x &= 1.30 \text{ m} \times 10^2 \\ y &= 30.9 \text{ m.} \end{aligned}$$

27

(a) 3.50 s

(b) 28.6 m/s (c) 34.3 m/s

(d) 44.7 m/s, $50.2°$ below horizontal

29

(a) $18.4°$

(b) The arrow will go over the branch.

31

$$R = \frac{v_0}{\sin 2\theta_0 g}$$

$$\text{For } \theta = 45°, \quad R = \frac{v_0}{g}$$

$R = 91.8 \text{ m}$ for $v_0 = 30 \text{ m/s}$; $R = 163 \text{ m}$ for $v_0 = 40 \text{ m/s}$; $R = 255 \text{ m}$ for $v_0 = 50 \text{ m/s}$.

33

(a) 560 m/s

(b) $8.00 \times 10^3 \text{ m}$

(c) 80.0 m. This error is not significant because it is only 1% of the answer in part (b).

35
1.50 m, assuming launch angle of $45°$

37

$\theta = 6.1°$

yes, the ball lands at 5.3 m from the net

39

(a) −0.486 m

(b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

41
4.23 m. No, the owl is not lucky; he misses the nest.

43
No, the maximum range (neglecting air resistance) is about 92 m.

45
15.0 m/s

47

(a) 24.2 m/s

(b) The ball travels a total of 57.4 m with the brief gust of wind.

49

$$y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta)t - \frac{1}{2}gt^2,$$

so that $t = \dfrac{2(v_0 \sin\theta)}{g}$

$x - x_0 = v_{0x}t = (v_0 \cos\theta)t = R,$ and substituting for t gives:

$$R = v_0 \cos\theta\left(\frac{2v_0 \sin\theta}{g}\right) = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

since $2\sin\theta\cos\theta = \sin 2\theta,$ the range is:

$$R = \frac{v_0{}^2 \sin 2\theta}{g}.$$

52

(a) 35.8 km, $45°$ south of east

(b) 5.53 m/s, $45°$ south of east

(c) 56.1 km , 45^{o} south of east

54

(a) 0.70 m/s faster

(b) Second runner wins

(c) 4.17 m

56

17.0 m/s , 22.1^{o}

58

(a) 230 m/s , 8.0^{o} south of west

(b) The wind should make the plane travel slower and more to the south, which is what was calculated.

60

(a) 63.5 m/s

(b) 29.6 m/s

62

6.68 m/s , 53.3^{o} south of west

64

(a) $H_{\text{average}} = 14.9\frac{\text{km/s}}{\text{Mly}}$

(b) 20.2 billion years

66

1.72 m/s , 42.3^{o} north of east

Test Prep for AP® Courses

1

(d)

3

We would need to know the horizontal and vertical positions of each ball at several times. From that data, we could deduce the velocities over several time intervals and also the accelerations (both horizontal and vertical) for each ball over several time intervals.

5

The graph of the ball's vertical velocity over time should begin at 4.90 m/s during the time interval 0 - 0.1 sec (there should be a data point at t = 0.05 sec, v = 4.90 m/s). It should then have a slope of -9.8 m/s^2, crossing through v = 0 at t = 0.55 sec and ending at v = -0.98 m/s at t = 0.65 sec.

The graph of the ball's horizontal velocity would be a constant positive value, a flat horizontal line at some positive velocity from t = 0 until t = 0.7 sec.

Chapter 4

Problems & Exercises

1

265 N

3

13.3 m/s^2

7

(a) 12 m/s^2 .

(b) The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.

9

(a) The system is the child in the wagon plus the wagon.

(b

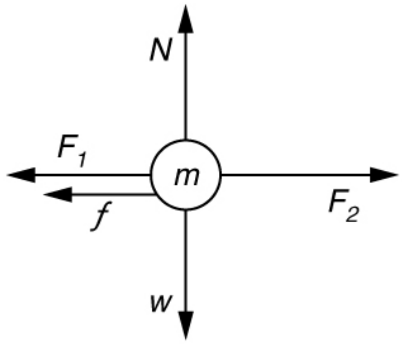


Figure 4.10.

(c) $a = 0.130 \text{ m/s}^2$ in the direction of the second child's push.

(d) $a = 0.00 \text{ m/s}^2$

11

(a) $3.68 \times 10^3 \text{ N}$. This force is 5.00 times greater than his weight.

(b) 3750 N; 11.3° above horizontal

13

$1.5 \times 10^3 \text{ N}, 150 \text{ kg}, 150 \text{ kg}$

15

Force on shell: $2.64 \times 10^7 \text{ N}$

Force exerted on ship = $-2.64 \times 10^7 \text{ N}$, by Newton's third law

17

a. 0.11 m/s^2

b. $1.2 \times 10^4 \text{ N}$

19

(a) $7.84 \times 10^{-4} \text{ N}$

(b) $1.89 \times 10^{-3} \text{ N}$. This is 2.41 times the tension in the vertical strand.

21

Newton's second law applied in vertical direction gives

$$F_y = F - 2T \sin\theta = 0 \quad ()$$

$$F = 2T \sin\theta \quad ()$$

$$T = \frac{F}{2 \sin\theta}. \quad ()$$

23

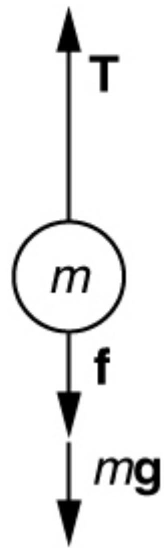


Figure 4.26.

Using the free-body diagram:

$F_{\text{net}} = T - f - mg = ma$,

so that

$$a = \frac{T - f - mg}{m} = \frac{1.250 \times 10^7 \text{ N} - 4.50 \times 10^6 \, N - (5.00 \times 10^5 \text{ kg})(9.80 \text{ m/s}^2)}{5.00 \times 10^5 \text{ kg}} = 6.20 \text{ m/s}^2 .$$

25

1. Use Newton's laws of motion.

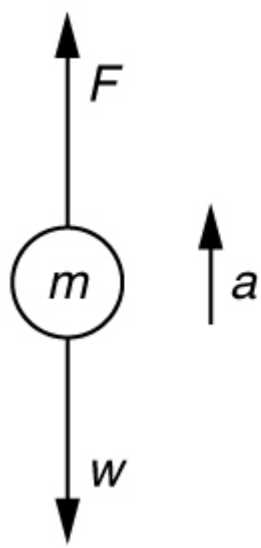


Figure 4.26.

2. Given : $a = 4.00g = (4.00)(9.80 \text{ m/s}^2) = 39.2 \text{ m/s}^2$; $m = 70.0 \text{ kg}$,
   Find: F .
3. $\sum F = +F - w = ma,$ so that $F = ma + w = ma + mg = m(a + g)$.
   $F = (70.0 \text{ kg})[(39.2 \text{ m/s}^2) + (9.80 \text{ m/s}^2)] = 3.43 \times 10^3 \text{N}$. The force exerted by the high-jumper is actually down on the ground, but F is up from the ground and makes him jump.
4. This result is reasonable, since it is quite possible for a person to exert a force of the magnitude of 10^3 N .

27

(a) $4.41\times10^5\ \mathrm{N}$

(b) $1.50\times10^5\ \mathrm{N}$

29

(a) $910\ \mathrm{N}$

(b) $1.11\times10^3\ \mathrm{N}$

31

$a = 0.139\ \mathrm{m/s}$, $\theta = 12.4°$ north of east

33

1. Use Newton’s laws since we are looking for forces.
2. Draw a free-body diagram:

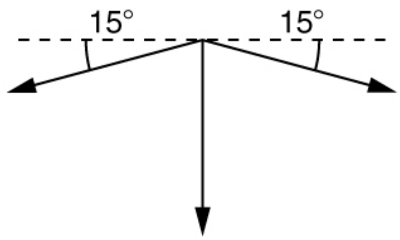


Figure 4.29.

3. The tension is given as $T = 25.0\ \mathrm{N}$. Find F_{app}. Using Newton’s laws gives: $\Sigma F_y = 0,$ so that applied force is due to the *y*-components of the two tensions: $F_{app} = 2\ T\ \sin\theta = 2(25.0\ \mathrm{N})\sin(15°) = 12.9\ \mathrm{N}$

   The *x*-components of the tension cancel. $\sum F_x = 0$.
4. This seems reasonable, since the applied tensions should be greater than the force applied to the tooth.

40

$10.2\ \mathrm{m/s^2}$, $4.67°$ from vertical

42

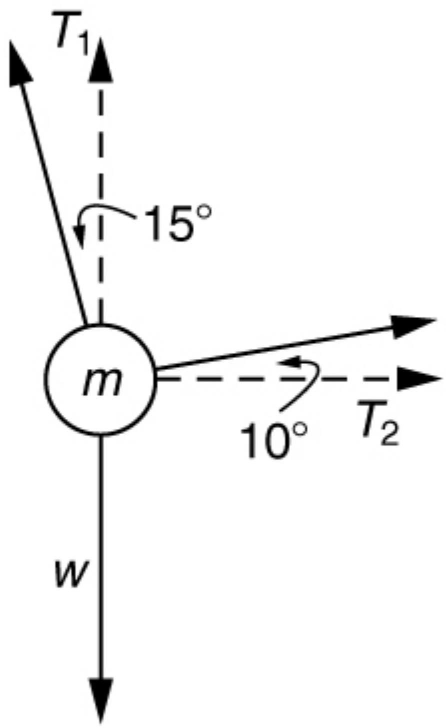


Figure 4.35.

$T_1 = 736\text{ N}$

$T_2 = 194\text{ N}$

44

(a) 7.43 m/s

(b) 2.97 m

46

(a) 4.20 m/s

(b) 29.4 m/s^2

(c) $4.31\times10^3\text{ N}$

48

(a) 47.1 m/s

(b) $2.47\times10^3\text{ m/s}^2$

(c) $6.18\times10^3\text{ N}$. The average force is 252 times the shell’s weight.

52

(a) 1×10^{-13}

(b) 1×10^{-11}

54

10^2

Test Prep for AP® Courses

1

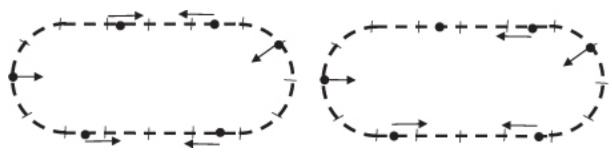


Figure 4.4. Car *X* is shown on the left, and Car *Y* is shown on the right.

i.

Car *X* takes longer to accelerate and does not spend any time traveling at top speed. Car *Y* accelerates over a shorter time and spends time going at top speed. So Car *Y* must cover the straightaways in a shorter time. Curves take the same time, so Car *Y* must overall take a shorter time.

ii.

The only difference in the calculations for the time of one segment of linear acceleration is the difference in distances. That shows that Car *X* takes longer to accelerate. The equation $\frac{d}{4v_c} = t_c$ corresponds to Car *Y* traveling for a time at top speed.

Substituting $a = \frac{v_c}{t_1}$ into the displacement equation in part (b) ii gives $D = \frac{3}{2}v_c t_1$. This shows that a car takes less time to reach its maximum speed when it accelerates over a shorter distance. Therefore, Car *Y* reaches its maximum speed more quickly, and spends more time at its maximum speed than Car *X* does, as argued in part (b) i.

3

A body cannot exert a force on itself. The hawk may accelerate as a result of several forces. The hawk may accelerate toward Earth as a result of the force due to gravity. The hawk may accelerate as a result of the additional force exerted on it by wind. The hawk may accelerate as a result of orienting its body to create less air resistance, thus increasing the net force forward.

5

(a) A soccer player, gravity, air, and friction commonly exert forces on a soccer ball being kicked.

(b) Gravity and the surrounding water commonly exert forces on a dolphin jumping. (The dolphin moves its muscles to exert a force on the water. The water exerts an equal force on the dolphin, resulting in the dolphin's motion.)

(c) Gravity and air exert forces on a parachutist drifting to Earth.

7

(c)

9

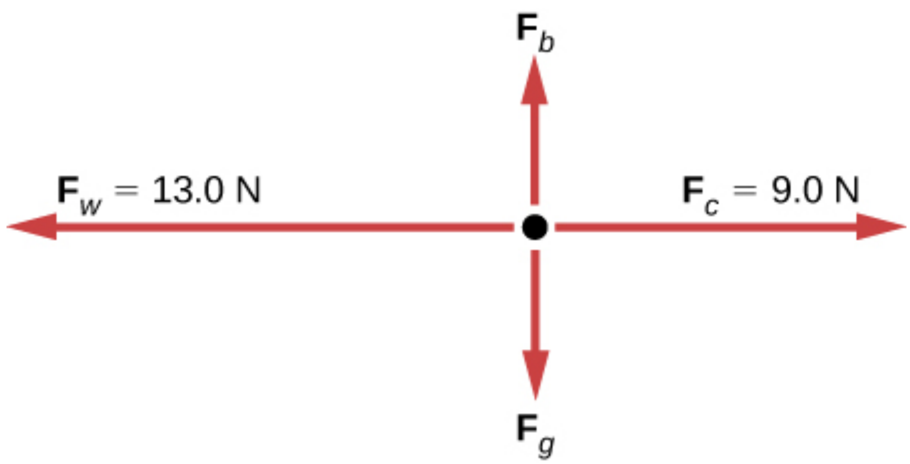


Figure 4.14.

The diagram consists of a black dot in the center and two small red arrows pointing up (Fb) and down (Fg) and two long red arrows pointing right (Fc = 9.0 N) and left (Fw=13.0 N).

In the diagram, F_g represents the force due to gravity on the balloon, and F_b represents the buoyant force. These two forces are equal in magnitude and opposite in direction. F_c represents the force of the current. F_w represents the force of the wind. The net force on the balloon will be $F_w - F_c = 4.0$ N and the balloon will accelerate in the direction the wind is blowing.

11

Since $m = F / a$, the parachutist has a mass of $539\,\text{N}/9.8\,\text{km/s}^2 = 55\,\text{kg}$.

For the first 2 s, the parachutist accelerates at 9.8 m/s^2.

$$\begin{aligned} v &= at \\ &= 9.8\frac{\text{m}}{\text{s}^2} \bullet 2\text{s} \\ &= 17.6\frac{\text{m}}{\text{s}} \end{aligned}$$

Her speed after 2 s is 19.6 m/s.

From 2 s to 10 s, the net force on the parachutist is 539 N – 615 N, or 76 N upward.

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{-76\,\text{N}}{55\,\text{kg}} \\ &= -1.4\,\frac{\text{m}}{\text{s}^2} \end{aligned}$$

Since $v = v_0 + at$, $v = 17.6\,\text{m/s}^2 + (-1.4\,\text{m/s}^2)(8\text{s}) = 6.5\,\text{m/s}^2$.

At 10 s, the parachutist is falling to Earth at 8.4 m/s.

13

The system includes the gardener and the wheelbarrow with its contents. The following forces are important to include: the weight of the wheelbarrow, the weight of the gardener, the normal force for the wheelbarrow and the gardener, the force of the gardener pushing against the ground and the equal force of the ground pushing back against the gardener, and any friction in the wheelbarrow's wheels.

15

The system undergoing acceleration is the two figure skaters together.

Net force = $120\,\text{N} - 5.0\,\text{N} = 115\,\text{N}$.

Total mass = $40\,\text{kg} + 50\,\text{kg} = 90\,\text{kg}$.

Using Newton's second law, we have that

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{115\,\text{N}}{90\,\text{kg}} \\ &= 1.28\,\frac{\text{m}}{\text{s}^2} \end{aligned}$$

The pair accelerates forward at 1.28 m/s^2.

17

The force of tension must equal the force of gravity plus the force necessary to accelerate the mass. $F = mg$ can be used to calculate the first, and $F = ma$ can be used to calculate the second.

For gravity:

$$\begin{aligned} F &= mg \\ &= (120.0\,\text{kg})(9.8\,\text{m/s}^2) \\ &= 1205.4\,\text{N} \end{aligned}$$

For acceleration:

$$\begin{aligned} F &= ma \\ &= (120.0\,\text{kg})(1.3\,\text{m/s}^2) \\ &= 159.9\,\text{N} \end{aligned}$$

The total force of tension in the cable is 1176 N + 156 N = 1332 N.

19

(b)

21

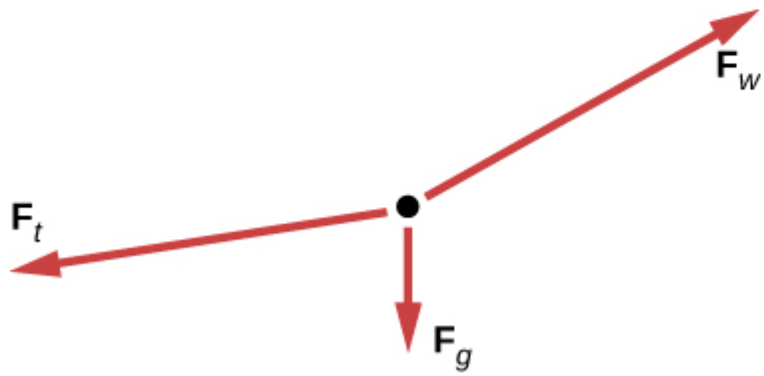


Figure 4.24.

The diagram has a black dot and three solid red arrows pointing away from the dot. Arrow Ft is long and pointing to the left and slightly down. Arrow Fw is also long and is a bit below a diagonal line halfway between pointing up and pointing to the right. A short arrow Fg is pointing down.

$\mathbf{F}_g$ is the force on the kite due to gravity.

$\mathbf{F}_w$ is the force exerted on the kite by the wind.

$\mathbf{F}_t$ is the force of tension in the string holding the kite. It must balance the vector sum of the other two forces for the kite to float stationary in the air.

23
(b)
25
(d)
27

A free-body diagram would show a northward force of 64 N and a westward force of 38 N. The net force is equal to the sum of the two applied forces. It can be found using the Pythagorean theorem:

$$\begin{aligned} F_{\text{net}} &= \sqrt{F_x{}^2 + F_y{}^w} \\ &= \sqrt{(38\,\text{N})^2 + (64\,\text{N})^2} \\ &= 74.4\,\text{N} \end{aligned}$$

Since $a = \frac{F}{m}$,

$$\begin{aligned} a &= \frac{74.4\,\text{N}}{825\,\text{kg}} \\ &= 0.09\,\text{m/s}^2 \end{aligned}$$

The boulder will accelerate at 0.09 m/s^2.

29
(b)
31
(b)
33
(d)

Chapter 5

Problems & Exercises

1
$5.00\,\text{N}$
4

(a) 588 N

(b) $1.96\,\text{m/s}^2$

6

(a) 3.29 m/s^2

(b) 3.52 m/s^2

(c) 980 N; 945 N

10

$1.83\ \text{m/s}^2$

14

(a) $4.20\ \text{m/s}^2$

(b) $2.74\ \text{m/s}^2$

(c) $–0.195\ \text{m/s}^2$

16

(a) $1.03{\times}10^6\ \text{N}$

(b) $3.48{\times}10^5\ \text{N}$

18

(a) $51.0\ \text{N}$

(b) $0.720\ \text{m/s}^2$

20

$115\ \text{m/s; }414\ \text{km/hr}$

22

$25\ \text{m/s; }9.9\ \text{m/s}$

24

2.9

26

$$[\eta] = \frac{[F_\text{s}]}{[r][v]} = \frac{\text{kg}\cdot\text{m/s}^2}{\text{m}\cdot\text{m/s}} = \frac{\text{kg}}{\text{m}\cdot\text{s}} \tag{5.30}$$

28

$0.76\ \text{kg/m}\cdot\text{s}$

29

$$1.90{\times}10^{-3}\ \text{cm} \tag{5.58}$$

31

(a)1 mm

(b) This does seem reasonable, since the lead does seem to shrink a little when you push on it.

33

(a)9 cm

(b)This seems reasonable for nylon climbing rope, since it is not supposed to stretch that much.

35

8.59 mm

37

$$1.49{\times}10^{-7}\ \text{m} \tag{5.59}$$

39

(a) $3.99{\times}10^{-7}\ \text{m}$

(b) $9.67{\times}10^{-8}\ \text{m}$

41

$4{\times}10^6\ \text{N/m}^2$. This is about 36 atm, greater than a typical jar can withstand.

43

1.4 cm

Test Prep for AP® Courses

1
(b)
3
(c)

Chapter 6

Problems & Exercises

1
723 km
3
5×10^7 rotations
5
117 rad/s
7

76.2 rad/s

728 rpm

8

(a) 33.3 rad/s

(b) 500 N

(c) 40.8 m

10
12.9 rev/min
12
4×10^{21} m
14

a) 3.47×10^4 m / s^2, 3.55×10^3 g

b) 51.1 m / s

16

a) 31.4 rad/s

b) 118 m/s

c) 384 m/s

d)The centripetal acceleration felt by Olympic skaters is 12 times larger than the acceleration due to gravity. That's quite a lot of acceleration in itself. The centripetal acceleration felt by Button's nose was 39.2 times larger than the acceleration due to gravity. It is no wonder that he ruptured small blood vessels in his spins.

18

a) 0.524 km/s

b) 29.7 km/s

20

(a) 1.35×10^3 rpm

(b) 8.47×10^3 m/s^2

(c) 8.47×10^{-12} N

(d) 865

21

(a) 16.6 m/s

(b) $19.6\ \text{m/s}^2$

(c)

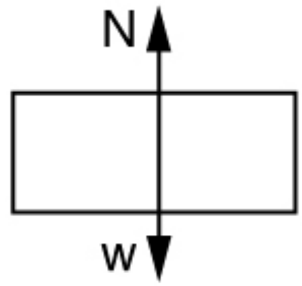


Figure 6.10.

(d) $1.76\times10^3\ \text{N or } 3.00\ w$, that is, the normal force (upward) is three times her weight.

(e) This answer seems reasonable, since she feels like she's being forced into the chair MUCH stronger than just by gravity.

22

a) $40.5\ \text{m/s}^2$

b) 905 N

c) The force in part (b) is very large. The acceleration in part (a) is too much, about 4 g.

d) The speed of the swing is too large. At the given velocity at the bottom of the swing, there is enough kinetic energy to send the child all the way over the top, ignoring friction.

23

a) 483 N

b) 17.4 N

c) 2.24 times her weight, 0.0807 times her weight

25

4.14°

27

a) 24.6 m

b) $36.6\ \text{m/s}^2$

c) $a_\text{c} = 3.73\ g$. This does not seem too large, but it is clear that bobsledders feel a lot of force on them going through sharply banked turns.

29

a) 2.56 rad/s

b) 5.71°

30

a) 16.2 m/s

b) 0.234

32

a) 1.84

b) A coefficient of friction this much greater than 1 is unreasonable .

c) The assumed speed is too great for the tight curve.

33

a) $5.979\times10^{24}\ \text{kg}$

b) This is identical to the best value to three significant figures.

35

a) $1.62\ \ \mathrm{m/s}^2$

b) $3.75\ \ \mathrm{m/s}^2$

37

a) $3.42\times10^{-5}\ \mathrm{m/s}^2$

b) $3.34\times10^{-5}\ \mathrm{m/s}^2$

The values are nearly identical. One would expect the gravitational force to be the same as the centripetal force at the core of the system.

39

a) $7.01\times10^{-7}\ \mathrm{N}$

b) $1.35\times10^{-6}\ \mathrm{N}\ ,\ 0.521$

41

a) $1.66\times10^{-10}\ \mathrm{m/s}^2$

b) $2.17\times10^{5}\ \mathrm{m/s}$

42

a) $2.94\times10^{17}\ \mathrm{kg}$

b) 4.92×10^{-8}

of the Earth's mass.

c) The mass of the mountain and its fraction of the Earth's mass are too great.

d) The gravitational force assumed to be exerted by the mountain is too great.

44

$1.98\times10^{30}\ \mathrm{kg}$

46

$\dfrac{M_J}{M_E} = 316$

48

a) $7.4\times10^{3}\ \mathrm{m/s}$

b) $1.05\times10^{3}\ \mathrm{m/s}$

c) $2.86\times10^{-7}\ \mathrm{s}$

d) $1.84\times10^{7}\ \mathrm{N}$

e) $2.76\times10^{4}\ \mathrm{J}$

49

a) $5.08\times10^{3}\ \mathrm{km}$

b) This radius is unreasonable because it is less than the radius of earth.

c) The premise of a one-hour orbit is inconsistent with the known radius of the earth.

Test Prep for AP® Courses

1

(a)

3

(b)

5
(b)

Chapter 7

Problems & Exercises

1

$$3.00 \text{ J} = 7.17\times10^{-4} \text{ kcal} \tag{7.8}$$

3

(a) 5.92×10^{5} J

(b) -5.88×10^{5} J

(c) The net force is zero.

5

$$3.14\times10^{3} \text{ J} \tag{7.9}$$

7

(a) -700 J

(b) 0

(c) 700 J

(d) 38.6 N

(e) 0

9
$1/250$

11
1.1×10^{10} J

13
$\mathbf{2.8\times10^{3}\ N}$

15
102 N

16

(a) 1.96×10^{16} J

(b) The ratio of gravitational potential energy in the lake to the energy stored in the bomb is 0.52. That is, the energy stored in the lake is approximately half that in a 9-megaton fusion bomb.

18

(a) 1.8 J

(b) 8.6 J

20

$$v_f = \sqrt{2gh + v_0{}^2} = \sqrt{2(9.80 \text{ m/s}^2)(-0.180 \text{ m}) + (2.00 \text{ m/s})^2} = 0.687 \text{ m/s} \tag{7.45}$$

22

$$7.81\times10^{5} \text{ N/m} \tag{7.60}$$

24
9.46 m/s

26
4×10^{4} molecules

27

Equating ΔPE_g and ΔKE, we obtain $v = \sqrt{2gh + v_0{}^2} = \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m}) + (15.0 \text{ m/s})^2} = 24.8 \text{ m/s}$

29

(a) 25×10^{6} years

(b) This is much, much longer than human time scales.

30

$$2 \times 10^{-10} \tag{7.81}$$

32

(a) 40

(b) 8 million

34

$149

36

(a) 208 W

(b) 141 s

38

(a) 3.20 s

(b) 4.04 s

40

(a) $9.46 \times 10^{7} \text{ J}$

(b) 2.54 y

42

Identify knowns: $m = 950 \text{ kg}$, slope angle $\theta = 2.00^{\circ}$, $v = 3.00 \text{ m/s}$, $f = 600 \text{ N}$

Identify unknowns: power P of the car, force F that car applies to road

Solve for unknown:

$$P = \frac{W}{t} = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv,$$

where F is parallel to the incline and must oppose the resistive forces and the force of gravity:

$$F = f + w = 600 \text{ N} + mg \sin \theta$$

Insert this into the expression for power and solve:

$$\begin{aligned} P &= (f + mg \sin \theta)v \\ &= \left[600 \text{ N} + (950 \text{ kg})\left(9.80 \text{ m/s}^2\right)\sin 2^{\circ}\right](30.0 \text{ m/s}) \\ &= 2.77 \times 10^{4} \text{ W} \end{aligned}$$

About 28 kW (or about 37 hp) is reasonable for a car to climb a gentle incline.

44

(a) 9.5 min

(b) 69 flights of stairs

46

641 W, 0.860 hp

48

31 g

50

14.3%

52

(a) $3.21 \times 10^{4} \text{ N}$

(b) $2.35 \times 10^{3} \text{ N}$

(c) Ratio of net force to weight of person is 41.0 in part (a); 3.00 in part (b)

54

(a) 108 kJ

(b) 599 W

56

(a) 144 J

(b) 288 W

58

(a) 2.50×10^{12} J

(b) 2.52%

(c) 1.4×10^{4} kg (14 metric tons)

60

(a) 294 N

(b) 118 J

(c) 49.0 W

62

(a) 0.500 m/s^2

(b) 62.5 N

(c) Assuming the acceleration of the swimmer decreases linearly with time over the 5.00 s interval, the frictional force must therefore be increasing linearly with time, since $f = F - ma$. If the acceleration decreases linearly with time, the velocity will contain a term dependent on time squared (t^2). Therefore, the water resistance will not depend linearly on the velocity.

64

(a) 16.1×10^{3} N

(b) 3.22×10^{5} J

(c) 5.66 m/s

(d) 4.00 kJ

66

(a) 4.65×10^{3} kcal

(b) 38.8 kcal/min

(c) This power output is higher than the highest value on **Table 7.5**, which is about 35 kcal/min (corresponding to 2415 watts) for sprinting.

(d) It would be impossible to maintain this power output for 2 hours (imagine sprinting for 2 hours!).

69

(a) 4.32 m/s

(b) 3.47×10^{3} N

(c) 8.93 kW

Test Prep for AP® Courses

1
(b)
3
(d)
5
(a)
7

The kinetic energy should change in the form of –cos, with an initial value of 0 or slightly above, and ending at the same level.

9

Any force acting perpendicular will have no effect on kinetic energy. Obvious examples are gravity and the normal force, but others include wind directly from the side and rain or other precipitation falling straight down.

11

Note that the wind is pushing from behind and one side, so your KE will increase. The net force has components of 1400 N in the direction of travel and 212 N perpendicular to the direction of travel. So the net force is 1420 N at 8.5 degrees from the direction of travel.

13

Gravity has a component perpendicular to the cannon (and to displacement, so it is irrelevant) and has a component parallel to the cannon. The latter is equal to 9.8 N. Thus the net force in the direction of the displacement is 50 N − 9.8 N, and the kinetic energy is 60 J.

15

The potato cannon (and many other projectile launchers) above is an option, with a force launching the projectile, friction, potentially gravity depending on the direction it is pointed, etc. A drag (or other) car accelerating is another possibility.

17

The kinetic energy of the rear wagon increases. The front wagon does not, until the rear wagon collides with it. The total system may be treated by its center of mass, halfway between the wagons, and its energy increases by the same amount as the sum of the two individual wagons.

19

(d)

21

0.049 J; 0.041 m, 0.25 m

23

20 m high, 20 m/s.

25

(a)

27

(d)

29

(c)

31

(b)

33

(c)

35

(c)

37

(c), (d)

39

(a)

41

(b)

Chapter 8

Problems & Exercises

1

(a) $1.50\times10^{4}\ \text{kg}\cdot\text{m/s}$

(b) 625 to 1

(c) $6.66\times10^{2}\ \text{kg}\cdot\text{m/s}$

3

(a) $8.00\times10^{4}\ \text{m/s}$

(b) $1.20\times10^{6}\ \text{kg}\cdot\text{m/s}$

(c) Because the momentum of the airplane is 3 orders of magnitude smaller than of the ship, the ship will not recoil very much. The recoil would be $-0.0100\ \text{m/s}$, which is probably not noticeable.

5

54 s

7

$9.00\times10^{3}\ \text{N}$

9

a) $2.40\times10^{3}\ \text{N}$ toward the leg

b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the change in momentum and the time interval are the same.

11

a) $800\ \text{kg}\cdot\text{m/s}$ away from the wall

b) $1.20\ \text{m/s}$ away from the wall

13

(a) $1.50\times10^{6}\ \text{N}$ away from the dashboard

(b) $1.00\times10^{5}\ \text{N}$ away from the dashboard

15

$4.69\times10^{5}\ \text{N}$ in the boat's original direction of motion

17

$2.10\times10^{3}\ \text{N}$ away from the wall

19

$$\begin{aligned}&\mathbf{p} = m\mathbf{v} \Rightarrow p^2 = m^2v^2 \Rightarrow \frac{p^2}{m} = mv^2\\ &\Rightarrow \frac{p^2}{2m} = \frac{1}{2}mv^2 = KE\\ &KE = \frac{p^2}{2m}\end{aligned} \tag{8.35}$$

21

60.0 g

23

0.122 m/s

25

In a collision with an identical car, momentum is conserved. Afterwards $v_\text{f} = 0$ for both cars. The change in momentum will be the same as in the crash with the tree. However, the force on the body is not determined since the time is not known. A padded stop will reduce injurious force on body.

27

22.4 m/s in the same direction as the original motion

29

0.250 m/s

31

(a) 86.4 N perpendicularly away from the bumper

(b) 0.389 J

(c) 64.0%

33

(a) 8.06 m/s

(b) -56.0 J

(c)(i) 7.88 m/s; (ii) -223 J

35

(a) 0.163 m/s in the direction of motion of the more massive satellite

(b) 81.6 J

(c) 8.70×10^{-2} m/s in the direction of motion of the less massive satellite, 81.5 J. Because there are no external forces, the velocity of the center of mass of the two-satellite system is unchanged by the collision. The two velocities calculated above are the velocity of the center of mass in each of the two different individual reference frames. The loss in KE is the same in both reference frames because the KE lost to internal forces (heat, friction, etc.) is the same regardless of the coordinate system chosen.

37

0.704 m/s

–2.25 m/s

38

(a) 4.58 m/s away from the bullet

(b) 31.5 J

(c) –0.491 m/s

(d) 3.38 J

40

(a) 1.02×10^{-6} m/s

(b) $\mathbf{5.63 \times 10^{20}\ J}$ (almost all KE lost)

(c) Recoil speed is 6.79×10^{-17} m/s , energy lost is 6.25×10^{9} J . The plume will not affect the momentum result because the plume is still part of the Moon system. The plume may affect the kinetic energy result because a significant part of the initial kinetic energy may be transferred to the kinetic energy of the plume particles.

42

24.8 m/s

44

(a) 4.00 kg

(b) 210 J

(c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

45

(a) 3.00 m/s, 60° below x -axis

(b) Find speed of first puck after collision: $0 = mv'_1 \sin 30^\circ - mv'_2 \sin 60^\circ \Rightarrow v'_1 = v'_2 \dfrac{\sin 60^\circ}{\sin 30^\circ} = 5.196 \text{ m/s}$

Verify that ratio of initial to final KE equals one: $\left.\begin{aligned} \text{KE} &= \tfrac{1}{2}mv_1^{\,2} = 18m \text{ J} \\ \text{KE} &= \tfrac{1}{2}mv'_1{}^2 + \tfrac{1}{2}mv'_2{}^2 = 18m \text{ J} \end{aligned}\right\} \dfrac{\text{KE}}{\text{KE}'} = 1.00$

47

(a) -2.26 m/s

(b) 7.63×10^{3} J

(c) The ground will exert a normal force to oppose recoil of the cannon in the vertical direction. The momentum in the vertical direction is transferred to the earth. The energy is transferred into the ground, making a dent where the cannon is. After long barrages, cannon have erratic aim because the ground is full of divots.

49

(a) 5.36×10^{5} m/s at -29.5°

(b) 7.52×10^{-13} J

51

We are given that $m_1 = m_2 \equiv m$. The given equations then become:

$$v_1 = v_1 \cos\theta_1 + v_2 \cos\theta_2 \tag{8.107}$$

and

$$0 = v'_1 \sin\theta_1 + v'_2 \sin\theta_2. \tag{8.108}$$

Square each equation to get

$$\begin{aligned} v_1{}^2 &= v'_1{}^2 \cos^2\theta_1 + v'_2{}^2 \cos^2\theta_2 + 2v'_1 v'_2 \cos\theta_1 \cos\theta_2 \\ 0 &= v'_1{}^2 \sin^2\theta_1 + v'_2{}^2 \sin^2\theta_2 + 2v'_1 v'_2 \sin\theta_1 \sin\theta_2 . \end{aligned} \tag{8.109}$$

Add these two equations and simplify:

$$\begin{aligned} v_1{}^2 &= v'_1{}^2 + v'_2{}^2 + 2v'_1 v'_2(\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2) \\ &= v'_1{}^2 + v'_2{}^2 + 2v'_1 v'_2\left[\frac{1}{2}\cos(\theta_1 - \theta_2) + \frac{1}{2}\cos(\theta_1 + \theta_2) + \frac{1}{2}\cos(\theta_1 - \theta_2) - \frac{1}{2}\cos(\theta_1 + \theta_2)\right] \\ &= v'_1{}^2 + v'_2{}^2 + 2v'_1 v'_2 \cos(\theta_1 - \theta_2). \end{aligned} \tag{8.110}$$

Multiply the entire equation by $\frac{1}{2}m$ to recover the kinetic energy:

$$\frac{1}{2}mv_1{}^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 + mv'_1 v'_2 \cos(\theta_1 - \theta_2) \tag{8.111}$$

53

39.2 m/s^2

55

$4.16\times10^3 \text{ m/s}$

57

The force needed to give a small mass Δm an acceleration $a_{\Delta m}$ is $F = \Delta m a_{\Delta m}$. To accelerate this mass in the small time interval Δt at a speed v_e requires $v_e = a_{\Delta m}\Delta t$, so $F = v_e\frac{\Delta m}{\Delta t}$. By Newton's third law, this force is equal in magnitude to the thrust force acting on the rocket, so $F_{\text{thrust}} = v_e\frac{\Delta m}{\Delta t}$, where all quantities are positive. Applying Newton's second law to the rocket gives $F_{\text{thrust}} - mg = ma \Rightarrow a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g$, where m is the mass of the rocket and unburnt fuel.

60

$2.63\times10^3 \text{ kg}$

61

(a) 0.421 m/s away from the ejected fluid.

(b) 0.237 J.

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1

(b)

3

(b)

5

(a)

7

(c) (based on calculation of $F = \frac{m\Delta v}{\Delta t}$)

9

(c)

11

(d)

13

(b)

15
(d)
17
(b)
19
(c)
21
(b)
23
(c)
25
(b)
27
(a)
29
(c)
31
(b)
33
(a)
35
(b)
37
(a)
39
(a)
41
(d)
43
(c). Because of conservation of momentum, the final velocity of the combined mass must be 4.286 m/s. The initial kinetic energy is $(0.5)(2.0)(15)^2 = 225\ \text{J}$. The final kinetic energy is $(0.5)(7.0)(4.286)^2 = 64\ \text{J}$, so the difference is −161 J.
45
(a)
47
(d)
49
(c)
51
(b)

Chapter 9

Problems & Exercises

1

a) $46.8\ \text{N·m}$

b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's application from the hinges. (Children don't have a tougher time opening a door because they push lower than adults, they have a tougher time because they don't push far enough from the hinges.)

3
23.3 N
5

Given:

$$\begin{aligned} m_1 &= 26.0\ \text{kg}, m_2 = 32.0\ \text{kg}, m_\text{s} = 12.0\ \text{kg}, \\ r_1 &= 1.60\ \text{m}, r_\text{s} = 0.160\ \text{m, find (a) } r_{2,}\ \text{(b) } F_\text{p} \end{aligned} \tag{9.26}$$

a) Since children are balancing:

$$\text{net } \tau_{\text{cw}} = -\text{net } \tau_{\text{ccw}}$$
$$\Rightarrow w_1 r_1 + m_s g r_s = w_2 r_2 \tag{9.27}$$

So, solving for r_2 gives:

$$\begin{aligned} r_2 &= \frac{w_1 r_1 + m_s g r_s}{w_2} = \frac{m_1 g r_1 + m_s g r_s}{m_2 g} = \frac{m_1 r_1 + m_s r_s}{m_2} \\ &= \frac{(26.0 \text{ kg})(1.60 \text{ m}) + (12.0 \text{ kg})(0.160 \text{ m})}{32.0 \text{ kg}} \\ &= 1.36 \text{ m} \end{aligned} \tag{9.28}$$

b) Since the children are not moving:

$$\text{net } F = 0 = F_p - w_1 - w_2 - w_s$$
$$\Rightarrow F_p = w_1 + w_2 + w_s \tag{9.29}$$

So that

$$\begin{aligned} F_p &= (26.0 \text{ kg} + 32.0 \text{ kg} + 12.0 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 686 \text{ N} \end{aligned} \tag{9.30}$$

6

$F_{\text{wall}} = 1.43 \times 10^3 \text{ N}$

8

a) 2.55×10^3 N, 16.3º to the left of vertical (i.e., toward the wall)

b) 0.292

10

$F_B = 2.12 \times 10^4 \text{ N}$

12

a) 0.167, or about one-sixth of the weight is supported by the opposite shore.

b) $F = 2.0 \times 10^4 \text{ N}$, straight up.

14

a) 21.6 N

b) 21.6 N

16

350 N directly upwards

19

25

50 N

21

a) $\text{MA} = 18.5$

b) $F_i = 29.1 \text{ N}$

c) 510 N downward

23

$1.3 \times 10^3 \text{ N}$

25

a) $T = 299 \text{ N}$

b) 897 N upward

26

$$\begin{aligned}
F_B &= 470\text{ N};\ r_1 = 4.00\text{ cm};\ w_a = 2.50\text{ kg};\ r_2 = 16.0\text{ cm}; w_b = 4.00\text{ kg};\ r_3 = 38.0\text{ cm} \\
F_E &= w_a\left(\frac{r_2}{r_1} - 1\right) + w_b\left(\frac{r_3}{r_1} - 1\right) \\
&= (2.50\text{ kg})(9.80\text{ m/s}^2)\left(\frac{16.0\text{ cm}}{4.0\text{ cm}} - 1\right) \\
&\quad +(4.00\text{ kg})(9.80\text{ m/s}^2)\left(\frac{38.0\text{ cm}}{4.00\text{ cm}} - 1\right) \\
&= 407\text{ N}
\end{aligned}$$

28

$1.1\times10^3\text{ N}$

$\theta = 190^\circ$ ccw from positive x axis

30

$F_V = 97\text{ N}, \theta = 59^\circ$

32

(a) 25 N downward

(b) 75 N upward

33

(a) $F_A = 2.21\times10^3\text{ N}$ upward

(b) $F_B = 2.94\times10^3\text{ N}$ downward

35

(a) $F_{\text{teeth on bullet}} = 1.2\times10^2\text{ N}$ upward

(b) $F_J = 84\text{ N}$ downward

37

(a) 147 N downward

(b) 1680 N, 3.4 times her weight

(c) 118 J

(d) 49.0 W

39

a) $\bar{x}_2 = 2.33\text{ m}$

b) The seesaw is 3.0 m long, and hence, there is only 1.50 m of board on the other side of the pivot. The second child is off the board.

c) The position of the first child must be shortened, i.e. brought closer to the pivot.

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1

(a)

3

Both objects are in equilibrium. However, they will respond differently if a force is applied to their sides. If the cone placed on its base is displaced to the side, its center of gravity will remain over its base and it will return to its original position. When the traffic cone placed on its tip is displaced to the side, its center of gravity will drift from its base, causing a torque that will accelerate it to the ground.

5

(d)

7

a. $\mathbf{F}_L$ = 7350 N, $\mathbf{F}_R$ = 2450 N

b. As the car moves to the right side of the bridge, $\mathbf{F}_L$ will decrease and $\mathbf{F}_R$ will increase. (At exactly halfway across the bridge, $\mathbf{F}_L$ and $\mathbf{F}_R$ will both be 4900 N.)

9

The student should mention that the guiding principle behind simple machines is the second condition of equilibrium. Though the torque leaving a machine must be equivalent to torque entering a machine, the same requirement does not exist for forces. As a result, by decreasing the lever arm to the existing force, the size of the existing force will be increased. The mechanical advantage will be equivalent to the ratio of the forces exiting and entering the machine.

11

a. The force placed on your bicep muscle will be greater than the force placed on the dumbbell. The bicep muscle is closer to your elbow than the downward force placed on your hand from the dumbbell. Because the elbow is the pivot point of the system, this results in a decreased lever arm for the bicep. As a result, the force on the bicep must be greater than that placed on the dumbbell. (How much greater? The ratio between the bicep and dumbbell forces is equal to the inverted ratio of their distances from the elbow. If the dumbbell is ten times further from the elbow than the bicep, the force on the bicep will be 200 pounds!)

b. The force placed on your bicep muscle will decrease. As the forearm lifts the dumbbell, it will get closer to the elbow. As a result, the torque placed on the arm from the weight will decrease and the countering torque created by the bicep muscle will do so as well.

Chapter 10

Problems & Exercises

1

$\omega = 0.737\text{ rev/s}$

3

(a) -0.26 rad/s^2

(b) 27 rev

5

(a) 80 rad/s^2

(b) 1.0 rev

7

(a) 45.7 s

(b) 116 rev

9

a) 600 rad/s^2

b) 450 rad/s

c) 21.0 m/s

10

(a) 0.338 s

(b) 0.0403 rev

(c) 0.313 s

12

$0.50\text{ kg}\cdot\text{m}^2$

14

(a) $50.4\text{ N}\cdot\text{m}$

(b) 17.1 rad/s^2

(c) 17.0 rad/s^2

16

$3.96\times10^{18}\text{ s}$

or $1.26\times10^{11}\text{ y}$

18

$$I_{end} = I_{center} + m\left(\frac{l}{2}\right)^2$$

Thus, $I_{center} = I_{end} - \frac{1}{4}ml^2 = \frac{1}{3}ml^2 - \frac{1}{4}ml^2 = \frac{1}{12}ml^2$

19

(a) 2.0 ms

(b) The time interval is too short.

(c) The moment of inertia is much too small, by one to two orders of magnitude. A torque of $500 \text{ N} \cdot \text{m}$ is reasonable.

20

(a) 17,500 rpm

(b) This angular velocity is very high for a disk of this size and mass. The radial acceleration at the edge of the disk is > 50,000 gs.

(c) Flywheel mass and radius should both be much greater, allowing for a lower spin rate (angular velocity).

21

(a) 185 J

(b) 0.0785 rev

(c) $W = 9.81 \text{ N}$

23

(a) $2.57 \times 10^{29} \text{ J}$

(b) $\text{KE}_{\text{rot}} = 2.65 \times 10^{33} \text{ J}$

25

$$\text{KE}_{\text{rot}} = 434 \text{ J} \tag{10.104}$$

27

(a) 128 rad/s

(b) 19.9 m

29

(a) 10.4 rad/s^2

(b) net $W = 6.11 \text{ J}$

34

(a) 1.49 kJ

(b) $2.52 \times 10^4 \text{ N}$

36

(a) $2.66 \times 10^{40} \text{ kg} \cdot \text{m}^2/\text{s}$

(b) $7.07 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}$

The angular momentum of the Earth in its orbit around the Sun is 3.77×10^6 times larger than the angular momentum of the Earth around its axis.

38

$22.5 \text{ kg} \cdot \text{m}^2/\text{s}$

40

25.3 rpm

43

(a) $0.156\ \text{rad/s}$

(b) $1.17\times10^{-2}\ \text{J}$

(c) $0.188\ \text{kg}\cdot\text{m/s}$

45

(a) 3.13 rad/s

(b) Initial KE = 438 J, final KE = 438 J

47

(a) 1.70 rad/s

(b) Initial KE = 22.5 J, final KE = 2.04 J

(c) $1.50\ \text{kg}\cdot\text{m/s}$

48

(a) $5.64\times10^{33}\ \text{kg}\cdot\text{m}^2\text{/s}$

(b) $1.39\times10^{22}\ \text{N}\cdot\text{m}$

(c) $2.17\times10^{15}\ \text{N}$

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1
(b)
3
(d)
5

(d)

You are given a thin rod of length 1.0 m and mass 2.0 kg, a small lead weight of 0.50 kg, and a not-so-small lead weight of 1.0 kg. The rod has three holes, one in each end and one through the middle, which may either hold a pivot point or one of the small lead weights.

7
(a)
9
(c)
11
(a)
13
(a)
15
(b)
17
(c)
19
(b)
21
(b)
23
(c)
25
(d)
27
A door on hinges is a rotational system. When you push or pull on the door handle, the angular momentum of the system changes. If a weight is hung on the door handle, then pushing on the door with the same force will cause a different increase in angular momentum. If you push or pull near the hinges with the same force, the resulting angular momentum of the system will also be different.
29

Since the globe is stationary to start with,

$\tau = \frac{\Delta L}{\Delta t}$

$\tau \cdot \Delta t = \Delta L$

By substituting,

120 N•m • 1.2 s = 144 N•m•s.

The angular momentum of the globe after 1.2 s is 144 N•m•s.

Chapter 11

Problems & Exercises

1

$1.610\ \text{cm}^3$

3

(a) 2.58 g

(b) The volume of your body increases by the volume of air you inhale. The average density of your body decreases when you take a deep breath, because the density of air is substantially smaller than the average density of the body before you took the deep breath.

4

$2.70\ \text{g/cm}^3$

6

(a) 0.163 m

(b) Equivalent to 19.4 gallons, which is reasonable

8

$7.9\times10^2\ \text{kg/m}^3$

9

$15.6\ \text{g/cm}^3$

10

(a) $10^{18}\ \text{kg/m}^3$

(b) $2\times10^4\ \text{m}$

11

$3.59\times10^6\ \text{Pa}$; or $521\ \text{lb/in}^2$

13

$2.36\times10^3\ \text{N}$

14

0.760 m

16

$$\begin{aligned}(h\rho g)_{\text{units}} &= (\text{m})\left(\text{kg/m}^3\right)\left(\text{m/s}^2\right) = \left(\text{kg}\cdot\text{m}^2\right)/\left(\text{m}^3\cdot\text{s}^2\right) \\ &= \left(\text{kg}\cdot\text{m/s}^2\right)\left(1/\text{m}^2\right) \\ &= \text{N/m}^2\end{aligned} \tag{11.30}$$

18

(a) 20.5 mm Hg

(b) The range of pressures in the eye is 12–24 mm Hg, so the result in part (a) is within that range

20

$1.09\times10^3\ \text{N/m}^2$

22

24.0 N

24

$2.55\times10^{7}\ \text{Pa}$; or 251 atm

26

$5.76\times10^{3}\ \text{N}$ extra force

28

(a) $V = d_{\text{i}}A_{\text{i}} = d_{\text{o}}A_{\text{o}} \Rightarrow d_{\text{o}} = d_{\text{i}}\left(\frac{A_{\text{i}}}{A_{\text{o}}}\right)$.

Now, using equation:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_{\text{o}} = F_{\text{i}}\left(\frac{A_{\text{o}}}{A_{\text{i}}}\right). \tag{11.32}$$

Finally,

$$W_{\text{o}} = F_{\text{o}}d_{\text{o}} = \left(\frac{F_{\text{i}}A_{\text{o}}}{A_{\text{i}}}\right)\left(\frac{d_{\text{i}}A_{\text{i}}}{A_{\text{o}}}\right) = F_{\text{i}}d_{\text{i}} = W_{\text{i}}. \tag{11.33}$$

In other words, the work output equals the work input.

(b) If the system is not moving, friction would not play a role. With friction, we know there are losses, so that $W_{\text{out}} = W_{\text{in}} - W_{\text{f}}$; therefore, the work output is less than the work input. In other words, with friction, you need to push harder on the input piston than was calculated for the nonfriction case.

29

Balloon:

$$\begin{aligned} P_{\text{g}} &= 5.00\ \text{cm H}_2\text{O}, \\ P_{\text{abs}} &= 1.035\times10^{3}\ \text{cm H}_2\text{O}. \end{aligned}$$

Jar:

$$\begin{aligned} P_{\text{g}} &= -50.0\ \text{mm Hg}, \\ P_{\text{abs}} &= 710\ \text{mm Hg}. \end{aligned}$$

31

4.08 m

33

$$\Delta P = 38.7\ \text{mm Hg},$$
$$\text{Leg blood pressure} = \frac{159}{119}.$$

35

$22.4\ \text{cm}^2$

36

91.7%

38

$815\ \text{kg/m}^3$

40

(a) 41.4 g

(b) $41.4\ \text{cm}^3$

(c) $1.09\ \text{g/cm}^3$

42

(a) 39.5 g

(b) $50\ \text{cm}^3$

(c) $0.79\ \text{g/cm}^3$

It is ethyl alcohol.

44

8.21 N

46

(a) 960 kg/m^3

(b) 6.34%

She indeed floats more in seawater.

48

(a) 0.24

(b) 0.68

(c) Yes, the cork will float because $\rho_{\text{obj}} < \rho_{\text{ethyl alcohol}} (0.678 \text{ g/cm}^3 < 0.79 \text{ g/cm}^3)$

50

The difference is 0.006%.

52

$$F_{\text{net}} = F_2 - F_1 = P_2 A - P_1 A = (P_2 - P_1)A$$
$$= (h_2 \rho_{\text{fl}} g - h_1 \rho_{\text{fl}} g)A$$
$$= (h_2 - h_1)\rho_{\text{fl}} g A$$

where ρ_{fl} = density of fluid. Therefore,

$$F_{\text{net}} = (h_2 - h_1)A\rho_{\text{fl}} g = V_{\text{fl}}\rho_{\text{fl}} g = m_{\text{fl}} g = w_{\text{fl}}$$

where is w_{fl} the weight of the fluid displaced.

54

592 N/m^2

56

$2.23 \times 10^{-2} \text{ mm Hg}$

58

(a) $1.65 \times 10^{-3} \text{ m}$

(b) $3.71 \times 10^{-4} \text{ m}$

60

$6.32 \times 10^{-2} \text{ N/m}$

Based on the values in table, the fluid is probably glycerin.

62

$$\begin{aligned} P_{\text{w}} &= 14.6 \text{ N/m}^2, \\ P_{\text{a}} &= 4.46 \text{ N/m}^2, \\ P_{\text{sw}} &= 7.40 \text{ N/m}^2. \end{aligned}$$

Alcohol forms the most stable bubble, since the absolute pressure inside is closest to atmospheric pressure.

64

5.1°

This is near the value of $\theta = 0^{\circ}$ for most organic liquids.

66

-2.78

The ratio is negative because water is raised whereas mercury is lowered.

68
479 N
70
1.96 N
71
$-63.0 \text{ cm } H_2O$
73

(a) $3.81\times10^3 \text{ N/m}^2$

(b) 28.7 mm Hg, which is sufficient to trigger micturition reflex

75

(a) 13.6 m water

(b) 76.5 cm water

77

(a) $3.98\times10^6 \text{ Pa}$

(b) $2.1\times10^{-3} \text{ cm}$

79

(a) 2.97 cm

(b) $3.39\times10^{-6} \text{ J}$

(c) Work is done by the surface tension force through an effective distance $h/2$ to raise the column of water.

81

(a) $2.01\times10^4 \text{ N}$

(b) $1.17\times10^{-3} \text{ m}$

(c) $2.56\times10^{10} \text{ N/m}^2$

83

(a) $1.38\times10^4 \text{ N}$

(b) $2.81\times10^7 \text{ N/m}^2$

(c) 283 N

85

(a) 867 N

(b) This is too much force to exert with a hand pump.

(c) The assumed radius of the pump is too large; it would be nearly two inches in diameter—too large for a pump or even a master cylinder. The pressure is reasonable for bicycle tires.

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1
(e)
3
(a) 100 kg/m^3 (b) 60% (c) yes; yes (76% will be submerged) (d) answers vary
5
(d)

Chapter 12

Problems & Exercises

1

$2.78 \text{ cm}^3/\text{s}$

3

27 cm/s

5

(a) 0.75 m/s

(b) 0.13 m/s

7

(a) 40.0 cm^2

(b) 5.09×10^7

9

(a) 22 h

(b) 0.016 s

11

(a) 12.6 m/s

(b) $0.0800 \text{ m}^3/\text{s}$

(c) No, independent of density.

13

(a) 0.402 L/s

(b) 0.584 cm

15

(a) $127 \text{ cm}^3/\text{s}$

(b) 0.890 cm

17

$$\begin{aligned} P &= \frac{\text{Force}}{\text{Area}}, \\ (P)_{\text{units}} &= \text{N/m}^2 = \text{N} \cdot \text{m/m}^3 = \text{J/m}^3 \\ &= \text{energy/volume} \end{aligned}$$

19

184 mm Hg

21

$2.54 \times 10^5 \text{ N}$

23

(a) $1.58 \times 10^6 \text{ N/m}^2$

(b) 163 m

25

(a) $9.56 \times 10^8 \text{ W}$

(b) 1.4

27

1.26 W

29

(a) $3.02 \times 10^{-3} \text{ N}$

(b) 1.03×10^{-3}

31

$1.60 \text{ cm}^3/\text{min}$

33
$8.7\times10^{-11}\ \mathrm{m^3/s}$
35
0.316
37
(a) 1.52

(b) Turbulence will decrease the flow rate of the blood, which would require an even larger increase in the pressure difference, leading to higher blood pressure.

39
$$225\ \mathrm{mPa\cdot s} \tag{12.98}$$
41
$$0.138\ \mathrm{Pa\cdot s}, \tag{12.99}$$

or

Olive oil.

43

(a) $1.62\times10^{4}\ \mathrm{N/m^2}$

(b) $0.111\ \mathrm{cm^3/s}$

(c)10.6 cm

45
1.59
47
$2.95\times10^{6}\ \mathrm{N/m^2}$ (gauge pressure)
51
$N_\mathrm{R}=1.99\times10^{2}<2000$
53

(a) nozzle: 1.27×10^{5} , not laminar

(b) hose: 3.51×10^{4} , not laminar.

55
2.54 << 2000, laminar.
57

1.02 m/s

$1.28\times10^{-2}\ \mathrm{L/s}$

59

(a) $\geq 13.0\ \mathrm{m}$

(b) $2.68\times10^{-6}\ \mathrm{N/m^2}$

61

(a) 23.7 atm or $344\ \mathrm{lb/in^2}$

(b) The pressure is much too high.

(c) The assumed flow rate is very high for a garden hose.

(d) 5.27×10^{6} > > 3000, turbulent, contrary to the assumption of laminar flow when using this equation.

62
$1.41\times10^{-3}\ \mathrm{m}$
64
$1.3\times10^{2}\ \mathrm{s}$
66

0.391 s

Test Prep for AP® Courses

1
(c)
3
(a)
5
(a)
7
(a)
9
(d)

Chapter 13

Problems & Exercises

1
102°F

3
20.0°C and 25.6°C

5
9890°F

7

(a) 22.2°C

(b)
$$\begin{aligned}\Delta T(^\circ\text{F}) &= T_2(^\circ\text{F}) - T_1(^\circ\text{F}) \\ &= \frac{9}{5}T_2(^\circ\text{C}) + 32.0^\circ - \left(\frac{9}{5}T_1(^\circ\text{C}) + 32.0^\circ\right) \\ &= \frac{9}{5}(T_2(^\circ\text{C}) - T_1(^\circ\text{C})) = \frac{9}{5}\Delta T(^\circ\text{C})\end{aligned}$$

9
169.98 m
11
$5.4\times10^{-6}\ \text{m}$
13
Because the area gets smaller, the price of the land DECREASES by $\sim\$17,000$.
15
$$\begin{aligned}V &= V_0 + \Delta V = V_0(1+\beta\Delta T) \\ &= (60.00\ \text{L})\left[1+\left(950\times10^{-6}/^\circ\text{C}\right)(35.0^\circ\text{C} - 15.0^\circ\text{C})\right] \\ &= 61.1\ \text{L}\end{aligned} \tag{13.25}$$
17

(a) 9.35 mL

(b) 7.56 mL

19
0.832 mm
21

We know how the length changes with temperature: $\Delta L = \alpha L_0 \Delta T$. Also we know that the volume of a cube is related to its length by $V = L^3$, so the final volume is then $V = V_0 + \Delta V = (L_0 + \Delta L)^3$. Substituting for ΔL gives

$$V = (L_0 + \alpha L_0 \Delta T)^3 = L_0^3(1+\alpha\Delta T)^3 . \tag{13.26}$$

Now, because $\alpha\Delta T$ is small, we can use the binomial expansion:

$$V \approx L_0^3(1 + 3\alpha\Delta \text{T}) = L_0^3 + 3\alpha L_0^3 \Delta T. \tag{13.27}$$

So writing the length terms in terms of volumes gives $V = V_0 + \Delta V \approx V_0 + 3\alpha V_0 \Delta T,$ and so

$$\Delta V = \beta V_0 \Delta T \approx 3\alpha V_0 \Delta T, \text{ or } \beta \approx 3\alpha. \quad (13.28)$$

22
1.62 atm
24

(a) 0.136 atm

(b) 0.135 atm. The difference between this value and the value from part (a) is negligible.

26

(a) $nRT = (\text{mol})(\text{J/mol} \cdot \text{K})(\text{K}) = \text{J}$

(b) $nRT = (\text{mol})(\text{cal/mol} \cdot \text{K})(\text{K}) = \text{cal}$

(c) $$\begin{aligned} nRT &= (\text{mol})(\text{L} \cdot \text{atm/mol} \cdot \text{K})(\text{K}) \\ &= \text{L} \cdot \text{atm} = (\text{m}^3)(\text{N/m}^2) \\ &= \text{N} \cdot \text{m} = \text{J} \end{aligned}$$

28
7.86×10^{-2} mol
30

(a) $6.02 \times 10^5\ \text{km}^3$

(b) 6.02×10^8 km

32
-73.9ºC
34

(a) $9.14 \times 10^6\ \text{N/m}^2$

(b) $8.23 \times 10^6\ \text{N/m}^2$

(c) 2.16 K

(d) No. The final temperature needed is much too low to be easily achieved for a large object.

36
41 km
38

(a) 3.7×10^{-17} Pa

(b) $6.0 \times 10^{17}\ \text{m}^3$

(c) 8.4×10^2 km

39
1.25×10^3 m/s
41

(a) 1.20×10^{-19} J

(b) 1.24×10^{-17} J

43
458 K
45
1.95×10^7 K
47
6.09×10^5 m/s
49

$7.89 \times 10^4 \text{ Pa}$

51

(a) $1.99 \times 10^5 \text{ Pa}$

(b) 0.97 atm

53

$3.12 \times 10^4 \text{ Pa}$

55

78.3%

57

(a) $2.12 \times 10^4 \text{ Pa}$

(b) $1.06\ \%$

59

(a) $8.80 \times 10^{-2} \text{ g}$

(b) $6.30 \times 10^3 \text{ Pa}$; the two values are nearly identical.

61

82.3%

63

4.77°C

65

38.3 m

67

$\dfrac{(F_\text{B} / w_\text{Cu})}{(F_\text{B} / w_\text{Cu})'} = 1.02$. The buoyant force supports nearly the exact same amount of force on the copper block in both circumstances.

69

(a) $4.41 \times 10^{10} \text{ mol/m}^3$

(b) It's unreasonably large.

(c) At high pressures such as these, the ideal gas law can no longer be applied. As a result, unreasonable answers come up when it is used.

71

(a) $7.03 \times 10^8 \text{ m/s}$

(b) The velocity is too high—it's greater than the speed of light.

(c) The assumption that hydrogen inside a supernova behaves as an idea gas is responsible, because of the great temperature and density in the core of a star. Furthermore, when a velocity greater than the speed of light is obtained, classical physics must be replaced by relativity, a subject not yet covered.

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1

(a), (c)

3

(d)

5

(b)

7

(a) 7.29 × 10^{-21} J; (b) 352K or 79ºC

Chapter 14

Problems & Exercises

1

$$5.02\times10^{8}\ \text{J} \tag{14.18}$$

3

$$3.07\times10^{3}\ \text{J} \tag{14.19}$$

5

$$0.171^\circ\text{C} \tag{14.20}$$

7
10.8
9
617 W
11
35.9 kcal
13

(a) 591 kcal

(b) $4.94\times10^{3}\ \text{s}$

15
13.5 W
17

(a) 148 kcal

(b) 0.418 s, 3.34 s, 4.19 s, 22.6 s, 0.456 s

19
33.0 g
20

(a) 9.67 L

(b) Crude oil is less dense than water, so it floats on top of the water, thereby exposing it to the oxygen in the air, which it uses to burn. Also, if the water is under the oil, it is less efficient in absorbing the heat generated by the oil.

22

a) 319 kcal

b) 2.00°C

24
20.6°C
26
4.38 kg
28

(a) $1.57\times10^{4}\ \text{kcal}$

(b) $18.3\ \text{kW}\cdot\text{h}$

(c) $1.29\times10^{4}\ \text{kcal}$

30

(a) 1.01×10^{3} W

(b) One

32
84.0 W
34
2.59 kg
36

(a) 39.7 W

(b) 820 kcal

38
35 to 1, window to wall

40

$1.05\times10^3\ \mathrm{K}$

42

(a) 83 W

(b) 24 times that of a double pane window.

44

20.0 W, 17.2% of 2400 kcal per day

45

10 m/s

47

85.7ºC

49

1.48 kg

51

$2\times10^4\ \mathrm{MW}$

53

(a) 97.2 J

(b) 29.2 W

(c) 9.49 W

(d) The total rate of heat loss would be $29.2\ \mathrm{W} + 9.49\ \mathrm{W} = 38.7\ \mathrm{W}$. While sleeping, our body consumes 83 W of power, while sitting it consumes 120 to 210 W. Therefore, the total rate of heat loss from breathing will not be a major form of heat loss for this person.

55

$-21.7\ \mathrm{kW}$

Note that the negative answer implies heat loss to the surroundings.

57

$-266\ \mathrm{kW}$

59

$-36.0\ \mathrm{W}$

61

(a) 1.31%

(b) 20.5%

63

(a) $-15.0\ \mathrm{kW}$

(b) 4.2 cm

65

(a) 48.5ºC

(b) A pure white object reflects more of the radiant energy that hits it, so a white tent would prevent more of the sunlight from heating up the inside of the tent, and the white tunic would prevent that heat which entered the tent from heating the rider. Therefore, with a white tent, the temperature would be lower than 48.5ºC, and the rate of radiant heat transferred to the rider would be less than 20.0 W.

67

(a) $3\times10^{17}\ \mathrm{J}$

(b) $1\times10^{13}\ \mathrm{kg}$

(c) When a large meteor hits the ocean, it causes great tidal waves, dissipating large amount of its energy in the form of kinetic energy of the water.

69

(a) $3.44\times10^5\ \mathrm{m^3/s}$

(b) This is equivalent to 12 million cubic feet of air per second. That is tremendous. This is too large to be dissipated by heating the air by only 5°C . Many of these cooling towers use the circulation of cooler air over warmer water to increase the rate of evaporation. This would allow much smaller amounts of air necessary to remove such a large amount of heat because evaporation removes larger quantities of heat than was considered in part (a).

71
20.9 min
73

(a) 3.96×10^{-2} g

(b) 96.2 J

(c) 16.0 W

75

(a) 1.102

(b) $2.79\times10^4\ \text{J}$

(c) 12.6 J. This will not cause a significant cooling of the air because it is much less than the energy found in part (b), which is the energy required to warm the air from 20.0°C to 50.0°C .

76

(a) 36°C

(b) Any temperature increase greater than about 3°C would be unreasonably large. In this case the final temperature of the person would rise to $73^\circ\text{C}\ (163^\circ\text{F})$.

(c) The assumption of 95% heat retention is unreasonable.

78

(a) 1.46 kW

(b) Very high power loss through a window. An electric heater of this power can keep an entire room warm.

(c) The surface temperatures of the window do not differ by as great an amount as assumed. The inner surface will be warmer, and the outer surface will be cooler.

Test Prep for AP® Courses

1
(c)
3
(a)
5
(b)
7
(a)
9
(d)

Chapter 15

Problems & Exercises

1
$1.6\times10^9\ \text{J}$
3
$-9.30\times10^8\ \text{J}$
5

(a) $-1.0\times10^4\ \text{J}$, or $-2.39\ \text{kcal}$

(b) 5.00%

7

(a) 122 W

(b) $2.10 \times 10^6\ \text{J}$

(c) Work done by the motor is $1.61 \times 10^7\ \text{J}$;thus the motor produces 7.67 times the work done by the man

9

(a) 492 kJ

(b) This amount of heat is consistent with the fact that you warm quickly when exercising. Since the body is inefficient, the excess heat produced must be dissipated through sweating, breathing, etc.

10

$6.77 \times 10^3\ \text{J}$

12

(a) $W = P\Delta V = 1.76 \times 10^5\ \text{J}$

(b) $W = Fd = 1.76 \times 10^5\ \text{J}$. Yes, the answer is the same.

14

$W = 4.5 \times 10^3\ \text{J}$

16

W is not equal to the difference between the heat input and the heat output.

20

(a) $18.5\ \text{kJ}$

(b) 54.1%

22

(a) $1.32 \times 10^9\ \text{J}$

(b) $4.68 \times 10^9\ \text{J}$

24

(a) $3.80 \times 10^9\ \text{J}$

(b) 0.667 barrels

26

(a) $8.30 \times 10^{12}\ \text{J}$, which is 3.32% of $2.50 \times 10^{14}\ \text{J}$.

(b) $-8.30 \times 10^{12}\ \text{J}$, where the negative sign indicates a reduction in heat transfer to the environment.

28

403°C

30

(a) 244°C

(b) 477°C

(c)Yes, since automobiles engines cannot get too hot without overheating, their efficiency is limited.

32

(a) ${}_1 = 1 - \frac{T_{\text{c},1}}{T_{\text{h},1}} = 1 - \frac{543\ \text{K}}{723\ \text{K}} = 0.249 \text{ or } 24.9\%$

(b) ${}_2 = 1 - \frac{423\ \text{K}}{543\ \text{K}} = 0.221 \text{ or } 22.1\%$

(c) $_1 = 1 - \frac{T_{c,1}}{T_{h,1}} \Rightarrow T_{c,1} = T_{h,1}(1, \ -, \quad _1)$ similarly, $T_{c,2} = T_{h,2}(1 - \quad _2)$

using $T_{h,2} = T_{c,1}$ in above equation gives

$$T_{c,2} = T_{h,1}(1 - \quad _1)(1 - \quad _2) \equiv T_{h,1}(1 - \quad _{\text{overall}})$$
$$?(1 - \quad _{\text{overall}}) = (1 - \quad _1)(1 - \quad _2)$$
$$_{\text{overall}} = 1 - (1 - 0.249)(1 - 0.221) = 41.5\%$$

(d) $_{\text{overall}} = 1 - \frac{423\ \text{K}}{723\ \text{K}} = 0.415 \text{ or } 41.5\%$

34

The heat transfer to the cold reservoir is $Q_c = Q_h - W = 25\ \text{kJ} - 12\ \text{kJ} = 13\ \text{kJ}$, so the efficiency is $= 1 - \frac{Q_c}{Q_h} = 1 - \frac{13\ \text{kJ}}{25\ \text{kJ}} = 0.48$. The Carnot efficiency is $_C = 1 - \frac{T_c}{T_h} = 1 - \frac{300\ \text{K}}{600\ \text{K}} = 0.50$. The actual efficiency is 96% of the Carnot efficiency, which is much higher than the best-ever achieved of about 70%, so her scheme is likely to be fraudulent.

36

(a) -56.3ºC

(b) The temperature is too cold for the output of a steam engine (the local environment). It is below the freezing point of water.

(c) The assumed efficiency is too high.

37

4.82

39

0.311

41

(a) 4.61

(b) $1.66 \times 10^8\ \text{J or } 3.97 \times 10^4\ \text{kcal}$

(c) To transfer $1.66 \times 10^8\ \text{J}$, heat pump costs $1.00, natural gas costs $1.34.

43

27.6ºC

45

(a) $1.44 \times 10^7\ \text{J}$

(b) 40 cents

(c) This cost seems quite realistic; it says that running an air conditioner all day would cost $9.59 (if it ran continuously).

47

(a) $9.78 \times 10^4\ \text{J/K}$

(b) In order to gain more energy, we must generate it from things within the house, like a heat pump, human bodies, and other appliances. As you know, we use a lot of energy to keep our houses warm in the winter because of the loss of heat to the outside.

49

$8.01 \times 10^5\ \text{J}$

51

(a) $1.04 \times 10^{31}\ \text{J/K}$

(b) $3.28 \times 10^{31}\ \text{J}$

53

199 J/K

55

(a) $2.47\times10^{14}\text{ J}$

(b) $1.60\times10^{14}\text{ J}$

(c) $2.85\times10^{10}\text{ J/K}$

(d) $8.29\times10^{12}\text{ J}$

57

It should happen twice in every $1.27\times10^{30}\text{ s}$ or once in every $6.35\times10^{29}\text{ s}$

$$\left(6.35\times10^{29}\text{ s}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1\text{ d}}{24\text{ h}}\right)\left(\frac{1\text{ y}}{365.25\text{ d}}\right)$$

$$= 2.0\times10^{22}\text{ y}$$

59

(a) 3.0×10^{29}

(b) 24%

61

(a) $-2.38\times10^{-23}\text{ J/K}$

(b) 5.6 times more likely

(c) If you were betting on two heads and 8 tails, the odds of breaking even are 252 to 45, so on average you would break even. So, no, you wouldn't bet on odds of 252 to 45.

Test Prep for AP® Courses

1
(d)
3
(a)
5
(b)
7
(c)
9
(d)
11
(a)
13
(c)
15
(b)

Chapter 16

Problems & Exercises

1

(a) $1.23\times10^{3}\text{ N/m}$

(b) 6.88 kg

(c) 4.00 mm

3

(a) 889 N/m

(b) 133 N

5

(a) $6.53\times10^{3}\text{ N/m}$

(b) Yes

7
16.7 ms
8
$0.400\ \text{s / beats}$
9
400 Hz
10
12,500 Hz
11
1.50 kHz
12

(a) 93.8 m/s

(b) $11.3 \times 10^3\ \text{rev/min}$

13
$2.37\ \text{N/m}$
15
0.389 kg
18
94.7 kg
21
1.94 s
22
6.21 cm
24
2.01 s
26
2.23 Hz
28

(a) 2.99541 s

(b) Since the period is related to the square root of the acceleration of gravity, when the acceleration changes by 1% the period changes by $(0.01)^2 = 0.01\%$ so it is necessary to have at least 4 digits after the decimal to see the changes.

30

(a) Period increases by a factor of 1.41 ($\sqrt{2}$)

(b) Period decreases to 97.5% of old period

32
Slow by a factor of 2.45
34
length must increase by 0.0116%.
35

(a) $1.99\ \text{Hz}$

(b) 50.2 cm

(c) 1.41 Hz, 0.710 m

36

(a) $3.95 \times 10^6\ \text{N/m}$

(b) $7.90 \times 10^6\ \text{J}$

37

a). 0.266 m/s

b). 3.00 J

39

$\pm\frac{\sqrt{3}}{2}$

42
384 J
44

(a). 0.123 m

(b). −0.600 J

(c). 0.300 J. The rest of the energy may go into heat caused by friction and other damping forces.

46

(a) 5.00×10^{5} J

(b) 1.20×10^{3} s

47

$$t = 9.26\text{ d} \tag{16.75}$$

49

$$f = 40.0\text{ Hz} \tag{16.76}$$

51

$$v_{\text{w}} = 16.0\text{ m/s} \tag{16.77}$$

53

$$\lambda = 700\text{ m} \tag{16.78}$$

55

$$d = 34.0\text{ cm} \tag{16.79}$$

57
$f = 4\text{ Hz}$
59

462 Hz,

4 Hz

61

(a) 3.33 m/s

(b) 1.25 Hz

63
0.225 W
65
7.07
67
16.0 d
68
2.50 kW
70
$3.38\times10^{-5}\text{ W/m}^{2}$

Test Prep for AP® Courses

1
(d)
3
(b)
5

The frequency is given by

$$f = \frac{1}{T} = \frac{50\,cycles}{30s} = 1.66\,Hz$$

Time period is:

$$T = \frac{1}{f} = \frac{1}{1.66} = 0.6\,\mathrm{s}$$

7
(c)
9

The energy of the particle at the center of the oscillation is given by

$$E = \frac{1}{2}mv^2 \quad = \frac{1}{2}\times 0.2\,\mathrm{kg}\times(5\ \mathrm{m{\cdot}s^{-1}})^2 \quad = 2.5\,\mathrm{J}$$

11
(b)
13
19.7 J
15
(c)
17

$$d = \frac{k}{2\mu_K mg}\left(X^2 - \frac{\mu_K mg}{k}\right)^2 \quad \mathit{where}\, k = 50\,\mathrm{N\cdot m^{-1}} \quad \mu_k = 0.06 \quad m = 0.5\mathrm{kg}$$

$$d = \frac{50\mathrm{N\cdot m^{-1}}}{2\times 0.06\times 9.8\mathrm{m\cdot s^{-2}}}\left((0.2)^2 - \left(\frac{\left(0.06\times 0.5\mathrm{kg}\times 9.8\mathrm{m\cdot s^{-2}}\right)^2}{(50\mathrm{N\cdot m^{-1}})^2}\right)\right) = 1.698\,\mathrm{m}$$

19
The waves coming from a tuning fork are mechanical waves that are longitudinal in nature, whereas electromagnetic waves are transverse in nature.
21
The sound energy coming out of an instrument depends on its size. The sound waves produced are relative to the size of the musical instrument. A smaller instrument such as a tambourine will produce a high-pitched sound (higher frequency, shorter wavelength), whereas a larger instrument such as a drum will produce a deeper sound (lower frequency, longer wavelength).
23
$2\pi\,\mathrm{m}$
25
The student explains the principle of superposition and then shows two waves adding up to form a bigger wave when a crest adds with a crest and a trough with another trough. Also the student shows a wave getting cancelled out when a crest meets a trough and vice versa.
27
The student must note that the shape of the wave remains the same and there is first an overlap and then receding of the waves.
29
(c)

Chapter 17

Problems & Exercises

1
0.288 m
3
332 m/s
5

$$\begin{aligned} v_\mathrm{w} &= (331\ \mathrm{m/s})\sqrt{\frac{T}{273\ \mathrm{K}}} = (331\ \mathrm{m/s})\sqrt{\frac{293\ \mathrm{K}}{273\ \mathrm{K}}} \\ &= 343\ \mathrm{m/s} \end{aligned} \quad (17.12)$$

7
0.223
9

(a) 7.70 m

(b) This means that sonar is good for spotting and locating large objects, but it isn't able to resolve smaller objects, or detect the detailed shapes of objects. Objects like ships or large pieces of airplanes can be found by sonar, while smaller pieces must be found by other means.

11

(a) 18.0 ms, 17.1 ms

(b) 5.00%

(c) This uncertainty could definitely cause difficulties for the bat, if it didn't continue to use sound as it closed in on its prey. A 5% uncertainty could be the difference between catching the prey around the neck or around the chest, which means that it could miss grabbing its prey.

12

$$3.16\times10^{-4}\ \mathrm{W/m^2} \tag{17.23}$$

14

$$3.04\times10^{-4}\ \mathrm{W/m^2} \tag{17.24}$$

16

106 dB

18

(a) 93 dB

(b) 83 dB

20

(a) 50.1

(b) 5.01×10^{-3} or $\frac{1}{200}$

22

70.0 dB

24

100

26

$$1.45\times10^{-3}\ \mathrm{J} \tag{17.25}$$

28

28.2 dB

30

(a) 878 Hz

(b) 735 Hz

32

$$3.79\times10^{3}\ \mathrm{Hz} \tag{17.36}$$

34

(a) 12.9 m/s

(b) 193 Hz

36

First eagle hears $4.23\times10^{3}\ \mathrm{Hz}$

Second eagle hears $3.56\times10^{3}\ \mathrm{Hz}$

38

0.7 Hz

40

0.3 Hz, 0.2 Hz, 0.5 Hz

42

(a) 256 Hz

(b) 512 Hz

44

180 Hz, 270 Hz, 360 Hz

46
1.56 m
48

(a) 0.334 m

(b) 259 Hz

50
3.39 to 4.90 kHz
52

(a) 367 Hz

(b) 1.07 kHz

54

(a) $f_n = n(47.6\text{ Hz}),\ \ n = 1, 3, 5,..., 419$

(b) $f_n = n(95.3\text{ Hz}),\ \ n = 1, 2, 3,..., 210$

55

$$1{\times}10^6\text{ km} \tag{17.49}$$

57
498.5 or 501.5 Hz
59
82 dB
61
approximately 48, 9, 0, –7, and 20 dB, respectively
63

(a) 23 dB

(b) 70 dB

65
Five factors of 10
67

(a) $2{\times}10^{-10}\text{ W/m}^2$

(b) $2{\times}10^{-13}\text{ W/m}^2$

69
2.5
71
1.26
72
170 dB
74
103 dB
76

(a) 1.00

(b) 0.823

(c) Gel is used to facilitate the transmission of the ultrasound between the transducer and the patient’s body.

78

(a) $77.0\ \mu\text{m}$

(b) Effective penetration depth = 3.85 cm, which is enough to examine the eye.

(c) $16.6\ \mu\text{m}$

80

(a) $5.78{\times}10^{-4}\text{ m}$

(b) $2.67{\times}10^{6}\text{ Hz}$

82

(a) $v_{\text{w}} = 1540 \text{ m/s} = f\lambda \quad \lambda = \dfrac{1540 \text{ m/s}}{100\times10^{3} \text{ Hz}} = 0.0154 \text{ m} < 3.50 \text{ m}$. Because the wavelength is much shorter than the distance in question, the wavelength is not the limiting factor.

(b) 4.55 ms

84

974 Hz

(Note: extra digits were retained in order to show the difference.)

Test Prep for AP® Courses

1
(b)
3
(e)
5
(c)
7
Answers vary. Students could include a sketch showing an increased amplitude when two waves occupy the same location. Students could also cite conceptual evidence such as sound waves passing through each other.
9
(d)
11
(c)
13
(a)
15
(c)
17
(b)
19
(a), (b)
21
(c)

Chapter 18

Problems & Exercises

1

(a) 1.25×10^{10}

(b) 3.13×10^{12}

3
-600 C
5
1.03×10^{12}
7
9.09×10^{-13}
9
$1.48\times10^{8} \text{ C}$
15

(a) $E_{x = 1.00 \text{ cm}} = -\infty$

(b) $2.12\times10^{5} \text{ N/C}$

(c) one charge of $+q$

17

(a) 0.252 N to the left

(b) $x = 6.07 \text{ cm}$

19

(a)The electric field at the center of the square will be straight up, since q_a and q_b are positive and q_c and q_d are negative and all have the same magnitude.

(b) $2.04 \times 10^7 \text{ N/C (upward)}$

21

$0.102 \text{ N},$ in the $-y$ direction

23

(a) $\vec{E} = 4.36 \times 10^3 \text{ N/C}, 35.0°$, below the horizontal.

(b) No

25

(a) 0.263 N

(b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

27

The separation decreased by a factor of 5.

31

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{r^2} = ma \quad a = \frac{kq^2}{mr^2} \Rightarrow \\ &= \frac{\left(9.00 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right)\left(1.60 \times 10^{-19} \text{ m}\right)^2}{\left(1.67 \times 10^{-27} \text{ kg}\right)\left(2.00 \times 10^{-9} \text{ m}\right)^2} \\ &= 3.45 \times 10^{16} \text{ m/s}^2 \end{aligned}$$

32

(a) 3.2

(b) If the distance increases by 3.2, then the force will decrease by a factor of 10 ; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

34

(a) 1.04×10^{-9} C

(b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

37

1.02×10^{-11}

39

a. 0.859 m beyond negative charge on line connecting two charges
b. 0.109 m from lesser charge on line connecting two charges

42

8.75×10^{-4} N

44

(a) $6.94 \times 10^{-8} \text{ C}$

(b) 6.25 N/C

46

(a) 300 N/C (east)

(b) 4.80×10^{-17} N (east)

52

(a) 5.58×10^{-11} N/C

(b)the coulomb force is extraordinarily stronger than gravity

54

(a) -6.76×10^{5} C

(b) 2.63×10^{13} m/s^2 (upward)

(c) 2.45×10^{-18} kg

56

The charge q_2 is 9 times greater than q_1.

Test Prep for AP® Courses

1
(b)
3
(c)
5
(a)
7
(b)
9
(a) -0.1 C, (b) 1.1 C, (c) Both charges will be equal to 1 C, law of conservation of charge, (d) 0.9 C
11
W is negative, X is positive, Y is negative, Z is neutral.
13
(c)
15
(c)
17
(b)
19
a) Ball 1 will have positive charge and Ball 2 will have negative charge. b) The negatively charged rod attracts positive charge of Ball 1. The electrons of Ball 1 are transferred to Ball 2, making it negatively charged. c) If Ball 2 is grounded while the rod is still there, it will lose its negative charge to the ground. d) Yes, Ball 1 will be positively charged and Ball 2 will be negatively charge.
21
(c)
23
decrease by 77.78%.
25
(a)
27
(d)
29
(a) 3.60×10^{10} N, (b) It will become 1/4 of the original value; hence it will be equal to 8.99×10^{9} N
31
(c)
33
(a)
35
(b)
37
(a) 350 N/C, (b) west, (c) 5.6×10−17 N, (d) west.
39
(b)

41

(a) i) Field vectors near objects point toward negatively charged objects and away from positively charged objects.

(a) ii) The vectors closest to *R* and *T* are about the same length and start at about the same distance. We have that $q_R \big/ d^2 = q_T \big/ d^2$, so the charge on *R* is about the same as the charge on *T*. The closest vectors around *S* are about the same length as those around *R* and *T*. The vectors near *S* start at about 6 units away, while vectors near *R* and *T* start at about 4 units. We have that $q_R \big/ d^2 = q_S \big/ D^2$, so $q_S \big/ q_R = D^2 \big/ d^2 = 36 \big/ 16 = 2.25$, and so the charge on *S* is about twice that on *R* and *T*.

(b)

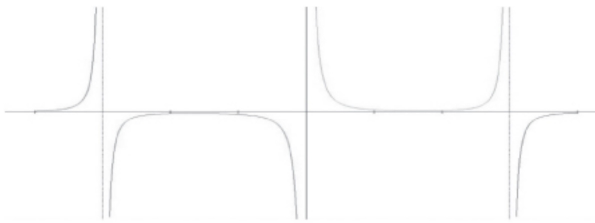

Figure 18.35. A vector diagram.

(c)

$$E = k\left[-\frac{q}{(d+x)^2} + \frac{2q}{(x)^2} + \frac{q}{(d-x)^2}\right]$$

(d) The statement is not true. The vector diagram shows field vectors in this region with nonzero length, and the vectors not shown have even greater lengths. The equation in part (c) shows that, when $0 < x < d$, the denominator of the negative term is always greater than the denominator of the third term, but the numerator is the same. So the negative term always has a smaller magnitude than the third term and since the second term is positive the sum of the terms is always positive.

Chapter 19

Problems & Exercises

1
42.8
4
1.00×10^5 K
6

(a) 4×10^4 W

(b) A defibrillator does not cause serious burns because the skin conducts electricity well at high voltages, like those used in defibrillators. The gel used aids in the transfer of energy to the body, and the skin doesn't absorb the energy, but rather lets it pass through to the heart.

8

(a) 7.40×10^3 C

(b) 1.54×10^{20} electrons per second

9
3.89×10^6 C
11

(a) 1.44×10^{12} V

(b) This voltage is very high. A 10.0 cm diameter sphere could never maintain this voltage; it would discharge.

(c) An 8.00 C charge is more charge than can reasonably be accumulated on a sphere of that size.

15

(a) $3.00\ \text{kV}$

(b) $750\ \text{V}$

17

(a) No. The electric field strength between the plates is $2.5\times10^{6}\ \text{V/m}$, which is lower than the breakdown strength for air ($3.0\times10^{6}\ \text{V/m}$).

(b) 1.7 mm

19

44.0 mV

21

$15\ \text{kV}$

23

(a) $800\ \text{KeV}$

(b) $25.0\ \text{km}$

24

144 V

26

(a) 1.80 km

(b) A charge of 1 C is a very large amount of charge; a sphere of radius 1.80 km is not practical.

28

$-2.22\times10^{-13}\ \text{C}$

30

(a) $3.31\times10^{6}\ \text{V}$

(b) 152 MeV

32

(a) $2.78\times10^{-7}\ \text{C}$

(b) $2.00\times10^{-10}\ \text{C}$

35

(a) $2.96\times10^{9}\ \text{m/s}$

(b) This velocity is far too great. It is faster than the speed of light.

(c) The assumption that the speed of the electron is far less than that of light and that the problem does not require a relativistic treatment produces an answer greater than the speed of light.

46

$21.6\ \text{mC}$

48

$80.0\ \text{mC}$

50

20.0 kV

52

$667\ \text{pF}$

54

(a) $4.4\ \mu\text{F}$

(b) $4.0\times10^{-5}\ \text{C}$

56

(a) 14.2 kV

(b) The voltage is unreasonably large, more than 100 times the breakdown voltage of nylon.

(c) The assumed charge is unreasonably large and cannot be stored in a capacitor of these dimensions.

57
$0.293\ \mu\text{F}$

59
$3.08\ \mu\text{F}$ in series combination, $13.0\ \mu\text{F}$ in parallel combination

60
$2.79\ \mu\text{F}$

62

(a) $–3.00\ \mu\text{F}$

(b) You cannot have a negative value of capacitance.

(c) The assumption that the capacitors were hooked up in parallel, rather than in series, was incorrect. A parallel connection always produces a greater capacitance, while here a smaller capacitance was assumed. This could happen only if the capacitors are connected in series.

63

(a) $405\ \text{J}$

(b) $90.0\ \text{mC}$

64

(a) 3.16 kV

(b) 25.3 mC

66

(a) $1.42\times10^{-5}\ \text{C}$, $6.38\times10^{-5}\ \text{J}$

(b) $8.46\times10^{-5}\ \text{C}$, $3.81\times10^{-4}\ \text{J}$

67

(a) $4.43\times10^{-12}\ \text{F}$

(b) $452\ \text{V}$

(c) $4.52\times10^{-7}\ \text{J}$

70

(a) $133\ \text{F}$

(b) Such a capacitor would be too large to carry with a truck. The size of the capacitor would be enormous.

(c) It is unreasonable to assume that a capacitor can store the amount of energy needed.

Test Prep for AP® Courses

1
(a)
3
(b)
5
(c)
7
(a)
9
(b)
11
(b)
13
(a)
15
(c)
17

(b)
19
(a)
21
(d)
23
(d)
25
(a)
27
(b)
29
(c)
31
(d)
33
(a)
35
(c)
37
(c)
39
(b)
41
(a)
43
(d)

Chapter 20

Problems & Exercises

1
0.278 mA
3
0.250 A
5
1.50ms
7

(a) $1.67\text{k } \Omega$

(b) If a 50 times larger resistance existed, keeping the current about the same, the power would be increased by a factor of about 50 (based on the equation $P = I^2R$), causing much more energy to be transferred to the skin, which could cause serious burns. The gel used reduces the resistance, and therefore reduces the power transferred to the skin.

9

(a) 0.120 C

(b) $7.50\times10^{17}\,\text{electrons}$

11
96.3 s
13

(a) $7.81 \times 10^{14}\ \text{He}^{++}\ \text{nuclei/s}$

(b) $4.00 \times 10^3\ \text{s}$

(c) $7.71 \times 10^8\ \text{s}$

15
$-1.13\times10^{-4}\,\text{m/s}$

17

$9.42\times10^{13}\text{ electrons}$

18

0.833 A

20

$7.33\times10^{-2}\ \ \Omega$

22

(a) 0.300 V

(b) 1.50 V

(c) The voltage supplied to whatever appliance is being used is reduced because the total voltage drop from the wall to the final output of the appliance is fixed. Thus, if the voltage drop across the extension cord is large, the voltage drop across the appliance is significantly decreased, so the power output by the appliance can be significantly decreased, reducing the ability of the appliance to work properly.

24

$0.104\ \Omega$

26

$2.8\times10^{-2}\text{ m}$

28

$1.10\times10^{-3}\text{ A}$

30

$-5^\circ\text{C to }45^\circ\text{C}$

32

1.03

34

0.06%

36

-17°C

38

(a) $4.7\ \ \Omega$ (total)

(b) 3.0% decrease

40

$2.00\times10^{12}\text{ W}$

44

(a) 1.50 W

(b) 7.50 W

46

$$\frac{\text{V}^2}{\Omega}=\frac{\text{V}^2}{\text{V/A}}=\text{AV}=\left(\frac{\text{C}}{\text{s}}\right)\left(\frac{\text{J}}{\text{C}}\right)=\frac{\text{J}}{\text{s}}=1\text{ W}$$

48

$$1\text{ kW}\cdot\text{h}=\left(\frac{1\times10^3\text{ J}}{1\text{ s}}\right)(1\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right)=3.60\times10^6\text{ J}$$

50

$438/y

52

$6.25

54

1.58 h

56

$3.94 billion/year

58

25.5 W

60

(a) $2.00\times10^{9}\text{ J}$

(b) 769 kg

62
45.0 s
64

(a) 343 A

(b) 2.17×10^3 A

(c) 1.10×10^3 A

66

(a) 1.23×10^3 kg

(b) 2.64×10^3 kg

69

(a) 2.08×10^5 A

(b) 4.33×10^4 MW

(c) The transmission lines dissipate more power than they are supposed to transmit.

(d) A voltage of 480 V is unreasonably low for a transmission voltage. Long-distance transmission lines are kept at much higher voltages (often hundreds of kilovolts) to reduce power losses.

73
480 V
75
2.50 ms
77

(a) 4.00 kA

(b) 16.0 MW

(c) 16.0%

79
2.40 kW
81

(a) 4.0

(b) 0.50

(c) 4.0

83

(a) 1.39 ms

(b) 4.17 ms

(c) 8.33 ms

85

(a) 230 kW

(b) 960 A

87

(a) 0.400 mA, no effect

(b) 26.7 mA, muscular contraction for duration of the shock (can't let go)

89
$1.20 \times 10^5\ \Omega$
91

(a) $1.00\ \Omega$

(b) 14.4 kW

93

Temperature increases 860° C. It is very likely to be damaging.

95

80 beats/minute

Test Prep for AP® Courses

1
(a)
3
10 A
5
(a)
7
3.2 Ω, 2.19 A
9
(b), (d)
11
9.72×10^{-8} Ω·m
13
18 Ω
15
10:3 or 3.33

Chapter 21

Problems & Exercises

1

(a) $2.75\text{ k }\Omega$

(b) $27.5\ \ \Omega$

3

(a) $786\ \ \Omega$

(b) $20.3\ \ \Omega$

5

29.6 W

7

(a) 0.74 A

(b) 0.742 A

9

(a) 60.8 W

(b) 3.18 kW

11

(a) $$\begin{aligned} R_\text{s} &= R_1 + R_2 \\ \Rightarrow R_\text{s} &\approx R_1 (R_1 >> R_2) \end{aligned}$$

(b) $\dfrac{1}{R_\text{p}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{R_1 + R_2}{R_1 R_2}$,

so that

$$R_\text{p} = \frac{R_1 R_2}{R_1 + R_2} \approx \frac{R_1 R_2}{R_1} = R_2 (R_1 >> R_2).$$

13

(a) $-400\ \mathrm{k}\ \Omega$

(b) Resistance cannot be negative.

(c) Series resistance is said to be less than one of the resistors, but it must be greater than any of the resistors.

14
2.00 V
16
2.9994 V
18
$0.375\ \ \Omega$
21

(a) 0.658 A

(b) 0.997 W

(c) 0.997 W; yes

23

(a) 200 A

(b) 10.0 V

(c) 2.00 kW

(d) $0.1000\ \ \Omega$; 80.0 A, 4.0 V, 320 W

25

(a) $0.400\ \Omega$

(b) No, there is only one independent equation, so only r can be found.

29

(a) –0.120 V

(b) $-1.41\times10^{-2}\ \ \Omega$

(c) Negative terminal voltage; negative load resistance.

(d) The assumption that such a cell could provide 8.50 A is inconsistent with its internal resistance.

31

$$-I_2R_2+\mathrm{emf}_1-I_2r_1+I_3R_3+I_3r_2-\mathrm{emf}_2=0 \tag{21.69}$$

35

$$I_3=I_1+I_2 \tag{21.70}$$

37

$$\mathrm{emf}_2-I_2r_2-I_2R_2+I_1R_5+I_1r_1-\mathrm{emf}_1+I_1R_1=0 \tag{21.71}$$

39

(a) $\mathrm{I}_1=4.75\ \mathrm{A}$

(b) $\mathrm{I}_2=-3.5\ \mathrm{A}$

(c) $\mathrm{I}_3=8.25\ \mathrm{A}$

41

(a) No, you would get inconsistent equations to solve.

(b) $I_1\neq I_2+I_3$. The assumed currents violate the junction rule.

42
$30\ \mu A$
44
$1.98\ \mathrm{k}\ \Omega$
46

$$1.25\times10^{-4}\ \ \Omega \tag{21.75}$$

48

(a) $3.00\text{ M }\Omega$

(b) $2.99\text{ k }\Omega$

50

(a) 1.58 mA

(b) 1.5848 V (need four digits to see the difference)

(c) 0.99990 (need five digits to see the difference from unity)

52

$15.0\ \mu\text{A}$

54

(a)

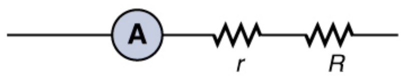

Figure 21.39.

(b) $10.02\ \ \Omega$

(c) 0.9980, or a 2.0×10^{-1} percent decrease

(d) 1.002, or a 2.0×10^{-1} percent increase

(e) Not significant.

56

(a) $-66.7\ \ \Omega$

(b) You can't have negative resistance.

(c) It is unreasonable that I_{G} is greater than I_{tot} (see **Figure 21.36**). You cannot achieve a full-scale deflection using a current less than the sensitivity of the galvanometer.

57

24.0 V

59

$1.56\text{ k }\Omega$

61

(a) 2.00 V

(b) $9.68\ \ \Omega$

62

$$\text{Range} = 5.00\ \ \Omega\ \text{ to } 5.00\text{ k }\Omega \tag{21.82}$$

63

range $4.00\text{ to }30.0\text{ M }\Omega$

65

(a) $2.50\ \mu\text{F}$

(b) 2.00 s

67

86.5%

69

(a) $1.25\text{ k }\Omega$

(b) 30.0 ms

71

(a) 20.0 s

(b) 120 s

(c) 16.0 ms

73

1.73×10^{-2} s

74

$3.33\times10^{-3}\ \Omega$

76

(a) 4.99 s

(b) $3.87^{\circ}\mathrm{C}$

(c) $31.1\ \mathrm{k}\ \Omega$

(d) No

Test Prep for AP® Courses

1

(a), (b)

3

(b)

5

(a) 4-Ω resistor; (b) combination of 20-Ω, 20-Ω, and 10-Ω resistors; (c) 20 W in each 20-Ω resistor, 40 W in 10-Ω resistor, 64 W in 4-Ω resistor, total 144W total in resistors, output power is 144 W, yes they are equal (law of conservation of energy); (d) 4 Ω and 3 Ω for part (a) and no change for part (b); (e) no effect, it will remain the same.

7

0.25 Ω, 0.50 Ω, no change

9

a. (c)

b. (c)

c. (d)

d. (d)

11

a. $I_1 + I_3 = I_2$

b. $E_1 - I_1R_1 - I_2R_2 - I_1r_1 = 0$; $-E_2 + I_1R_1 - I_3R_3 - I_3r_2 = 0$

c. $I_1 = 8/15$ A, $I_2 = 7/15$ A and $I_3 = -1/15$ A

d. $I_1 = 2/5$ A, $I_2 = 3/5$ A and $I_3 = 1/5$ A

e. $P_{E1} = 18/5$ W and $P_{R1} = 24/25$ W, $P_{R2} = 54/25$ W, $P_{R3} = 12/25$ W. Yes, $P_{E1} = P_{R1} + P_{R2} + P_{R3}$

f. R_3, losses in the circuit

13

(a) 20 mA, **Figure 21.44**, 5.5 s; (b) 24 mA, **Figure 21.35**, 2 s

Chapter 22

Problems & Exercises

1

(a) Left (West)

(b) Into the page

(c) Up (North)

(d) No force

(e) Right (East)

(f) Down (South)

3

(a) East (right)

(b) Into page

(c) South (down)

5

(a) Into page

(b) West (left)

(c) Out of page

7

$7.50\times10^{-7}\text{ N}$ perpendicular to both the magnetic field lines and the velocity

9

(a) $3.01\times10^{-5}\text{ T}$

(b) This is slightly less then the magnetic field strength of $5\times10^{-5}\text{ T}$ at the surface of the Earth, so it is consistent.

11

(a) $6.67\times10^{-10}\text{ C}$ (taking the Earth's field to be $5.00\times10^{-5}\text{ T}$)

(b) Less than typical static, therefore difficult

12

4.27 m

14

(a) 0.261 T

(b) This strength is definitely obtainable with today's technology. Magnetic field strengths of 0.500 T are obtainable with permanent magnets.

16

$4.36\times10^{-4}\text{ m}$

18

(a) 3.00 kV/m

(b) 30.0 V

20

0.173 m

22

$7.50\times10^{-4}\text{ V}$

24

(a) 1.18×10^{3} m/s

(b) Once established, the Hall emf pushes charges one direction and the magnetic force acts in the opposite direction resulting in no net force on the charges. Therefore, no current flows in the direction of the Hall emf. This is the same as in a current-carrying conductor—current does not flow in the direction of the Hall emf.

26

11.3 mV

28

$1.16\ \mu\text{V}$

30

2.00 T

31

(a) west (left)

(b) into page

(c) north (up)

(d) no force

(e) east (right)

(f) south (down)

33

(a) into page

(b) west (left)

(c) out of page

35

(a) 2.50 N

(b) This is about half a pound of force per 100 m of wire, which is much less than the weight of the wire itself. Therefore, it does not cause any special concerns.

37

1.80 T

39

(a) 30°

(b) 4.80 N

41

(a) τ decreases by 5.00% if B decreases by 5.00%

(b) 5.26% increase

43

10.0 A

45

$\mathrm{A \cdot m^2 \cdot T = A \cdot m^2}\left(\frac{\mathrm{N}}{\mathrm{A \cdot m}}\right) = \mathrm{N \cdot m}$.

47

$3.48 \times 10^{-26}\ \mathrm{N \cdot m}$

49

(a) $0.666\ \mathrm{N \cdot m}$ west

(b) This is not a very significant torque, so practical use would be limited. Also, the current would need to be alternated to make the loop rotate (otherwise it would oscillate).

50

(a) 8.53 N, repulsive

(b) This force is repulsive and therefore there is never a risk that the two wires will touch and short circuit.

52

400 A in the opposite direction

54

(a) $1.67 \times 10^{-3}\ \mathrm{N/m}$

(b) $3.33 \times 10^{-3}\ \mathrm{N/m}$

(c) Repulsive

(d) No, these are very small forces

56

(a) Top wire: $2.65 \times 10^{-4}\ \mathrm{N/m}$ s, 10.9° to left of up

(b) Lower left wire: $3.61 \times 10^{-4}\ \mathrm{N/m}$, 13.9° down from right

(c) Lower right wire: $3.46 \times 10^{-4}\ \mathrm{N/m}$, 30.0° down from left

58

(a) right-into page, left-out of page

(b) right-out of page, left-into page

(c) right-out of page, left-into page

60

(a) clockwise

(b) clockwise as seen from the left

(c) clockwise as seen from the right

61

1.01×10^{13} T

63

(a) 4.80×10^{-4} T

(b) Zero

(c) If the wires are not paired, the field is about 10 times stronger than Earth's magnetic field and so could severely disrupt the use of a compass.

65

39.8 A

67

(a) 3.14×10^{-5} T

(b) 0.314 T

69

7.55×10^{-5} T, 23.4°

71

10.0 A

73

(a) 9.09×10^{-7} N upward

(b) 3.03×10^{-5} m/s^2

75

60.2 cm

77

(a) 1.02×10^{3} N/m^2

(b) Not a significant fraction of an atmosphere

79

$17.0 \times 10^{-4}\%/^{\circ}\mathrm{C}$

81

18.3 MHz

83

(a) Straight up

(b) 6.00×10^{-4} N/m

(c) $94.1\ \mu\mathrm{m}$

(d)2.47 Ω/m, 49.4 V/m

85

(a) 571 C

(b) Impossible to have such a large separated charge on such a small object.

(c) The 1.00-N force is much too great to be realistic in the Earth's field.

87

(a) 2.40×10^{6} m/s

(b) The speed is too high to be practical $\leq$ 1% speed of light

(c) The assumption that you could reasonably generate such a voltage with a single wire in the Earth's field is unreasonable

89

(a) 25.0 kA

(b) This current is unreasonably high. It implies a total power delivery in the line of 50.0x10^9 W, which is much too high for standard transmission lines.

(c)100 meters is a long distance to obtain the required field strength. Also coaxial cables are used for transmission lines so that there is virtually no field for DC power lines, because of cancellation from opposing currents. The surveyor's concerns are not a problem for his magnetic field measurements.

Test Prep for AP® Courses

1
(a)
3
(b)
5
(b)
7
(a)
9
(b)
11
(e)
13
(c)
15
(c)

Chapter 23

Problems & Exercises

1
Zero
3

(a) CCW

(b) CW

(c) No current induced

5

(a) 1 CCW, 2 CCW, 3 CW

(b) 1, 2, and 3 no current induced

(c) 1 CW, 2 CW, 3 CCW

9
(a) 3.04 mV
(b) As a lower limit on the ring, estimate R = 1.00 mΩ. The heat transferred will be 2.31 mJ. This is not a significant amount of heat.
11
0.157 V
13

proportional to $\frac{1}{r}$

17

(a) 0.630 V

(b) No, this is a very small emf.

19
2.22 m/s
25

(a) 10.0 N

(b) 2.81×10^{8} J

(c) 0.36 m/s

(d) For a week-long mission (168 hours), the change in velocity will be 60 m/s, or approximately 1%. In general, a decrease in velocity would cause the orbit to start spiraling inward because the velocity would no longer be sufficient to keep the circular orbit. The long-term consequences are that the shuttle would require a little more fuel to maintain the desired speed, otherwise the orbit would spiral slightly inward.

28
474 V
30
0.247 V
32

(a) 50

(b) yes

34

(a) 0.477 T

(b) This field strength is small enough that it can be obtained using either a permanent magnet or an electromagnet.

36

(a) 5.89 V

(b) At t=0

(c) 0.393 s

(d) 0.785 s

38

(a) 1.92×10^{6} rad/s

(b) This angular velocity is unreasonably high, higher than can be obtained for any mechanical system.

(c) The assumption that a voltage as great as 12.0 kV could be obtained is unreasonable.

39

(a) $12.00\ \Omega$

(b) 1.67 A

41
72.0 V
43
$0.100\ \Omega$
44

(a) 30.0

(b) 9.75×10^{-2} A

46

(a) 20.0 mA

(b) 2.40 W

(c) Yes, this amount of power is quite reasonable for a small appliance.

48

(a) 0.063 A

(b) Greater input current needed.

50

(a) 2.2

(b) 0.45

(c) 0.20, or 20.0%

52

(a) 335 MV

(b) way too high, well beyond the breakdown voltage of air over reasonable distances

(c) input voltage is too high

54

(a) 15.0 V

(b) 75.0 A

(c) yes

55

1.80 mH

57

3.60 V

61

(a) 31.3 kV

(b) 125 kJ

(c) 1.56 MW

(d) No, it is not surprising since this power is very high.

63

(a) 1.39 mH

(b) 3.33 V

(c) Zero

65

60.0 mH

67

(a) 200 H

(b) $5.00°\text{C}$

69

500 H

71

$50.0\ \Omega$

73

$1.00\times10^{-18}\ \text{s}$ to 0.100 s

75

95.0%

77

(a) 24.6 ms

(b) 26.7 ms

(c) 9% difference, which is greater than the inherent uncertainty in the given parameters.

79

531 Hz

81

1.33 nF

83

(a) 2.55 A

(b) 1.53 mA

85

$63.7\ \mu\text{H}$

87

(a) 21.2 mH

(b) $8.00\ \Omega$

89

(a) 3.18 mF

(b) $16.7\ \Omega$

92

(a) $40.02\ \Omega$ at 60.0 Hz, $193\ \Omega$ at 10.0 kHz

(b) At 60 Hz, with a capacitor, $Z=531\ \Omega$, over 13 times as high as without the capacitor. The capacitor makes a large difference at low frequencies. At 10 kHz, with a capacitor $Z=190\ \Omega$, about the same as without the capacitor. The capacitor has a smaller effect at high frequencies.

94

(a) $529\ \Omega$ at 60.0 Hz, $185\ \Omega$ at 10.0 kHz

(b) These values are close to those obtained in **Example 23.12** because at low frequency the capacitor dominates and at high frequency the inductor dominates. So in both cases the resistor makes little contribution to the total impedance.

96
9.30 nF to 101 nF
98
3.17 pF
100

(a) $1.31\ \mu\text{H}$

(b) 1.66 pF

102

(a) $12.8\ \text{k}\Omega$

(b) $1.31\ \text{k}\Omega$

(c) 31.9 mA at 500 Hz, 312 mA at 7.50 kHz

(d) 82.2 kHz

(e) 0.408 A

104

(a) 0.159

(b) 80.9°

(c) 26.4 W

(d) 166 W

106
16.0 W

Test Prep for AP® Courses

1

(c)

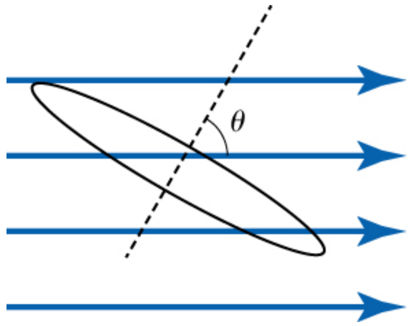

Figure 23.6.

3
(c)
5
(a), (d)
7
(c)

Chapter 24

Problems & Exercises

3
150 kV/m
6

(a) 33.3 cm (900 MHz) 11.7 cm (2560 MHz)

(b) The microwave oven with the smaller wavelength would produce smaller hot spots in foods, corresponding to the one with the frequency 2560 MHz.

8
26.96 MHz
10
5.0×10^{14} Hz
12

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{1.20\times10^{15}\text{ Hz}} = 2.50\times10^{-7}\text{ m} \qquad ()$$

14
0.600 m
16

(a) $f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{1\times10^{-10}\text{ m}} = 3\times10^{18}\text{ Hz}$

(b) X-rays

19

(a) 6.00×10^{6} m

(b) 4.33×10^{-5} T

21
(a) 1.50 × 10^{6} Hz, AM band
(b) The resonance of currents on an antenna that is 1/4 their wavelength is analogous to the fundamental resonant mode of an air column closed at one end, since the tube also has a length equal to 1/4 the wavelength of the fundamental oscillation.
23

(a) 1.55×10^{15} Hz

(b) The shortest wavelength of visible light is 380 nm, so that

$$\begin{aligned}&\frac{\lambda_{\text{visible}}}{\lambda_{\text{UV}}}\\ &= \frac{380 \text{ nm}}{193 \text{ nm}}\\ &= 1.97.\end{aligned} \quad ()$$

In other words, the UV radiation is 97% more accurate than the shortest wavelength of visible light, or almost twice as accurate!

25

3.90×10^{8} m

27

(a) 1.50×10^{11} m

(b) $0.500\ \mu$s

(c) 66.7 ns

29

(a) $-3.5 \times 10^{2}\ \text{W/m}^2$

(b) 88%

(c) $1.7\ \mu$T

30

$$\begin{aligned} I &= \frac{c\varepsilon_0 E_0^2}{2} \\ &= \frac{\left(3.00 \times 10^{8}\ \text{m/s}\right)\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(125\ \text{V/m})^2}{2} \\ &= 20.7\ \text{W/m}^2 \end{aligned} \quad ()$$

32

(a) $I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{0.250 \times 10^{-3}\ \text{W}}{\pi\left(0.500 \times 10^{-3}\ \text{m}\right)^2} = 318\ \text{W/m}^2$

(b) $$\begin{aligned} I_{\text{ave}} &= \frac{cB_0^2}{2\mu_0} \Rightarrow B_0 = \left(\frac{2\mu_0 I}{c}\right)^{1/2} \\ &= \left(\frac{2\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(318.3\ \text{W/m}^2\right)}{3.00 \times 10^{8}\ \text{m/s}}\right)^{1/2} \\ &= 1.63 \times 10^{-6}\ \text{T} \end{aligned}$$

(c) $$\begin{aligned} E_0 &= cB_0 = \left(3.00 \times 10^{8}\ \text{m/s}\right)\left(1.633 \times 10^{-6}\ \text{T}\right) \\ &= 4.90 \times 10^{2}\ \text{V/m} \end{aligned}$$

34

(a) 89.2 cm

(b) 27.4 V/m

36

(a) 333 T

(b) $1.33 \times 10^{19}\ \text{W/m}^2$

(c) 13.3 kJ

38

(a) $I = \frac{P}{A} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$

(b) $I \propto E_0^2, B_0^2 \Rightarrow E_0^2, B_0^2 \propto \frac{1}{r^2} \Rightarrow E_0, B_0 \propto \frac{1}{r}$

40
13.5 pF
42

(a) 4.07 kW/m^2

(b) 1.75 kV/m

(c) $5.84\ \mu\text{T}$

(d) 2 min 19 s

44

(a) $5.00 \times 10^3 \text{ W/m}^2$

(b) $3.88 \times 10^{-6} \text{ N}$

(c) $5.18 \times 10^{-12} \text{ N}$

46

(a) $t = 0$

(b) $7.50 \times 10^{-10} \text{ s}$

(c) $1.00 \times 10^{-9} \text{ s}$

48

(a) $1.01 \times 10^6 \text{ W/m}^2$

(b) Much too great for an oven.

(c) The assumed magnetic field is unreasonably large.

50

(a) $2.53 \times 10^{-20} \text{ H}$

(b) L is much too small.

(c) The wavelength is unreasonably small.

Test Prep for AP® Courses

1
(b)
3
(a)
5
(d)
7
(d)
9
(d)
11
(a)

Chapter 25

Problems & Exercises

1

Top 1.715 m from floor, bottom 0.825 m from floor. Height of mirror is 0.890 m, or precisely one-half the height of the person.

5

$2.25\times10^{8}\text{ m/s}$ in water

$2.04\times10^{8}\text{ m/s}$ in glycerine

7

1.490, polystyrene

9

1.28 s

11

1.03 ns

13

$n = 1.46$, fused quartz

17

(a) 0.898

(b) Can't have $n < 1.00$ since this would imply a speed greater than c.

(c) Refracted angle is too big relative to the angle of incidence.

19

(a) $\frac{c}{5.00}$

(b) Speed of light too slow, since index is much greater than that of diamond.

(c) Angle of refraction is unreasonable relative to the angle of incidence.

22

66.3°

24

> 1.414

26

1.50, benzene

29

$46.5^{\circ}\text{, red; }46.0^{\circ}\text{, violet}$

31

(a) 0.043°

(b) 1.33 m

33

71.3°

35

$53.5^{\circ}\text{, red; }\ 55.2^{\circ}\text{, violet}$

37

$5.00\text{ to }12.5\text{ D}$

39

-0.222 m

41

(a) 3.43 m

(b) 0.800 by 1.20 m

42

(a) -1.35 m (on the object side of the lens).

(b) $+10.0$

(c) 5.00 cm

43
44.4 cm
45

(a) 6.60 cm

(b) –0.333

47

(a) $+7.50 \text{ cm}$

(b) 13.3 D

(c) Much greater

49

(a) +6.67

(b) +20.0

(c) The magnification increases without limit (to infinity) as the object distance increases to the limit of the focal distance.

51
-0.933 mm

53
+0.667 m
55

(a) $-1.5\times10^{-2} \text{ m}$

(b) -66.7 D

57
+0.360 m (concave)
59

(a) +0.111

(b) -0.334 cm (behind “mirror”)

(c) 0.752cm

61

$$m = \frac{h_{\text{i}}}{h_{\text{o}}} = -\frac{d_{\text{i}}}{d_{\text{o}}} = -\frac{-d_{\text{o}}}{d_{\text{o}}} = \frac{d_{\text{o}}}{d_{\text{o}}} = 1 \Rightarrow h_{\text{i}} = h_{\text{o}} \tag{25.61}$$

63
6.82 kW/m^2

Test Prep for AP® Courses

1
(c)
3
(c)
5
(a)
7
Since light bends toward the normal upon entering a medium with a higher index of refraction, the upper path is a more accurate representation of a light ray moving from A to B.
9

First, measure the angle of incidence and the angle of refraction for light entering the plastic from air. Since the two angles can be measured and the index of refraction of air is known, the student can solve for the index of refraction of the plastic.

Next, measure the angle of incidence and the angle of refraction for light entering the gas from the plastic. Since the two angles can be measured and the index of refraction of the plastic is known, the student can solve for the index of refraction of the gas.

11

The speed of light in a medium is simply *c*/*n*, so the speed of light in water is 2.25×10^8 m/s. From Snell's law, the angle of incidence is 44°.

13
(d)
15
(a)
17
(a)
19
(b)

Chapter 26

Problems & Exercises

1
52.0 D

3

(a) -0.233 mm

(b) The size of the rods and the cones is smaller than the image height, so we can distinguish letters on a page.

5

(a) $+62.5\text{ D}$

(b) -0.250 mm

(c) -0.0800 mm

6
2.00 m
8

(a) $\pm 0.45\text{ D}$

(b) The person was nearsighted because the patient was myopic and the power was reduced.

10
0.143 m
12
1.00 m
14
20.0 cm
16
-5.00 D
18
25.0 cm
20
-0.198 D
22
30.8 cm
24
-0.444 D
26

(a) 4.00

(b) 1600

28

(a) 0.501 cm

(b) Eyepiece should be 204 cm behind the objective lens.

30

(a) +18.3 cm (on the eyepiece side of the objective lens)

(b) -60.0

(c) -11.3 cm (on the objective side of the eyepiece)

(d) +6.67

(e) -400

33
-40.0

35
-1.67

37
$+10.0 \text{ cm}$

39

(a) 0.251 µm

(b) Yes, this thickness implies that the shape of the cornea can be very finely controlled, producing normal distant vision in more than 90% of patients.

Test Prep for AP® Courses

1
(a)
3
(c)
5
(a)
7
(b)
9
(d)
11
(c)

Chapter 27

Problems & Exercises

1
$1 / 1.333 = 0.750$
3
1.49, Polystyrene
5
0.877 glass to water
6
0.516°
8
$1.22\times10^{-6} \text{ m}$
10
600 nm
12
2.06°
14
1200 nm (not visible)
16

(a) 760 nm

(b) 1520 nm

18

For small angles $\sin\theta - \tan\theta \approx \theta \text{ (in radians)}$.

For two adjacent fringes we have,

$$d \ \sin \theta_{\mathrm{m}} = m\lambda \tag{27.11}$$

and

$$d \ \sin \theta_{\mathrm{m}+1} = (m+1)\lambda \tag{27.12}$$

Subtracting these equations gives

$$\begin{gathered} d(\sin \theta_{\mathrm{m}+1} - \sin \theta_{\mathrm{m}}) = [(m+1) - m]\lambda \\ d(\theta_{\mathrm{m}+1} - \theta_{\mathrm{m}}) = \lambda \\ \tan \theta_{\mathrm{m}} = \frac{y_{\mathrm{m}}}{x} \approx \theta_{\mathrm{m}} \Rightarrow d\left(\frac{y_{\mathrm{m}+1}}{x} - \frac{y_{\mathrm{m}}}{x}\right) = \lambda \\ d\frac{\Delta y}{x} = \lambda \Rightarrow \Delta y = \frac{x\lambda}{d} \end{gathered} \tag{27.13}$$

20
450 nm
21
$5.97°$
23
8.99×10^{3}
25
707 nm
27
(a) $11.8°, \ 12.5°, \ 14.1°, \ 19.2°$

(b) $24.2°, \ 25.7°, \ 29.1°, \ 41.0°$

(c) Decreasing the number of lines per centimeter by a factor of x means that the angle for the x-order maximum is the same as the original angle for the first- order maximum.

29
589.1 nm and 589.6 nm
31
$28.7°$
33
$43.2°$
35
$90.0°$
37
(a) The longest wavelength is 333.3 nm, which is not visible.

(b) 333 nm (UV)

(c) 6.58×10^{3} cm

39
1.13×10^{-2} m
41
(a) 42.3 nm

(b) Not a visible wavelength

The number of slits in this diffraction grating is too large. Etching in integrated circuits can be done to a resolution of 50 nm, so slit separations of 400 nm are at the limit of what we can do today. This line spacing is too small to produce diffraction of light.

43
(a) $33.4°$

(b) No

45
(a) 1.35×10^{-6} m

(b) 69.9°

47
750 nm
49

(a) 9.04°

(b) 12

51

(a) 0.0150°

(b) 0.262 mm

(c) This distance is not easily measured by human eye, but under a microscope or magnifying glass it is quite easily measurable.

53

(a) 30.1°

(b) 48.7°

(c) No

(d) $2\theta_1 = (2)(14.5^\circ) = 29^\circ, \theta_2 - \theta_1 = 30.05^\circ - 14.5^\circ = 15.56^\circ$. Thus, $29^\circ \approx (2)(15.56^\circ) = 31.1^\circ$.

55
23.6° and 53.1°
57

(a) 1.63×10^{-4} rad

(b) 326 ly

59
1.46×10^{-5} rad
61

(a) 3.04×10^{-7} rad

(b) Diameter of 235 m

63
5.15 cm
65

(a) Yes. Should easily be able to discern.

(b) The fact that it is just barely possible to discern that these are separate bodies indicates the severity of atmospheric aberrations.

70
532 nm (green)
72
83.9 nm
74
620 nm (orange)
76
380 nm
78
33.9 nm
80
4.42×10^{-5} m
82
The oil film will appear black, since the reflected light is not in the visible part of the spectrum.
84
45.0°
86

45.7 mW/m^2

88

90.0%

90

I_0

92

48.8°

94

41.2°

96

(a) 1.92, not diamond (Zircon)

(b) 55.2°

98

$B_2 = 0.707\ B_1$

100

(a) 2.07×10^{-2} °C/s

(b) Yes, the polarizing filters get hot because they absorb some of the lost energy from the sunlight.

Test Prep for AP® Courses

1

(b)

3

(b) and (c)

5

(b)

7

(b)

9

(b)

11

(d)

13

(b)

15

(d)

17

(b)

Chapter 28

Problems & Exercises

1

(a) 1.0328

(b) 1.15

3

$5.96\times10^{-8}\text{ s}$

5

$0.800c$

7

$0.140c$

9

(a) $0.745c$

(b) $0.99995c$ (to five digits to show effect)

11

(a) 0.996

(b) γ cannot be less than 1.

(c) Assumption that time is longer in moving ship is unreasonable.

12

48.6 m

14

(a) 1.387 km = 1.39 km

(b) 0.433 km

(c) $=$

Thus, the distances in parts (a) and (b) are related when $\gamma = 3.20$.

16

(a) 4.303 y (to four digits to show any effect)

(b) 0.1434 y

(c) $\Delta t = \gamma \Delta t_0 \Rightarrow \gamma = \frac{\Delta t}{\Delta t_0} = \frac{4.303 \text{ y}}{0.1434 \text{ y}} = 30.0$

Thus, the two times are related when γ=30.00.

18

(a) 0.250

(b) γ must be ≥1

(c) The Earth-bound observer must measure a shorter length, so it is unreasonable to assume a longer length.

20

(a) $0.909c$

(b) $0.400c$

22

$0.198c$

24

a) 658 nm

b) red

c) $v/c = 9.92 \times 10^{-5}$ (negligible)

26

$0.991c$

28

$-0.696c$

30

$0.01324c$

32

$u' = c$, so

$$\begin{aligned} u &= \frac{v+u'}{1+(vu'/c^2)} = \frac{v+c}{1+(vc/c^2)} = \frac{v+c}{1+(v/c)} \\ &= \frac{c(v+c)}{c+v} = c \end{aligned}$$

34

a) $0.99947c$

b) 1.2064×10^{11} y

c) 1.2058×10^{11} y (all to sufficient digits to show effects)

35

4.09×10^{-19} kg · m/s

37

(a) $3.000000015 \times 10^{13}$ kg · m/s .

(b) Ratio of relativistic to classical momenta equals 1.000000005 (extra digits to show small effects)

39

2.9957×10^{8} m/s

41

(a) 1.121×10^{-8} m/s

(b) The small speed tells us that the mass of a proton is substantially smaller than that of even a tiny amount of macroscopic matter!

43

8.20×10^{-14} J

0.512 MeV

45

2.3×10^{-30} kg

47

(a) 1.11×10^{27} kg

(b) 5.56×10^{-5}

49

7.1×10^{-3} kg

7.1×10^{-3}

The ratio is greater for hydrogen.

51

208

$0.999988c$

53

6.92×10^{5} J

1.54

55

(a) $0.914c$

(b) The rest mass energy of an electron is 0.511 MeV, so the kinetic energy is approximately 150% of the rest mass energy. The electron should be traveling close to the speed of light.

57

90.0 MeV

59

(a)
$E^2 = p^2c^2 + m^2c^4 = \gamma^2 m^2 c^4$, so that
$p^2 c^2 = (\gamma^2 - 1)m^2 c^4$, and therefore
$\frac{(pc)^2}{(mc^2)^2} = \gamma^2 - 1$

(b) yes

61

1.07×10^3

63

6.56×10^{-8} kg

4.37×10^{-10}

65

$0.314c$

$0.99995c$

67

(a) 1.00 kg

(b) This much mass would be measurable, but probably not observable just by looking because it is 0.01% of the total mass.

69

(a) 6.3×10^{11} kg/s

(b) 4.5×10^{10} y

(c) 4.44×10^9 kg

(d) 0.32%

Test Prep for AP® Courses

1

(a)

3

The relativistic Doppler effect takes into account the special relativity concept of time dilation and also does not require a medium of propagation to be used as a point of reference (light does not require a medium for propagation).

5

Relativistic kinetic energy is given as $\text{KE}_{\text{rel}} = (\gamma - 1)mc^2$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Classical kinetic energy is given as $\text{KE}_{\text{class}} = \frac{1}{2}mv^2$

At low velocities $v = 0$, a binomial expansion and subsequent approximation of γ gives:

$\gamma = 1 + \frac{1v^2}{2c^2}$ or $\gamma - 1 = \frac{1v^2}{2c^2}$

Substituting $\gamma - 1$ in the expression for KE_{rel} gives

$$\text{KE}_{\text{rel}} = \left[\frac{1v^2}{2c^2}\right]mc^2 = \frac{1}{2}mv^2 = \text{KE}_{\text{class}}$$

Hence, relativistic kinetic energy becomes classical kinetic energy when $v \ll c$.

Chapter 29

Problems & Exercises

1

(a) 0.070 eV

(b) 14

3

(a) 2.21×10^{34} J

(b) 2.26×10^{34}

(c) No

4

263 nm

6

3.69 eV

8

0.483 eV

10

2.25 eV

12

(a) 264 nm

(b) Ultraviolet

14

1.95×10^{6} m/s

16

(a) 4.02×10^{15} /s

(b) 0.256 mW

18

(a) -1.90 eV

(b) Negative kinetic energy

(c) That the electrons would be knocked free.

20

6.34×10^{-9} eV, 1.01×10^{-27} J

22

2.42×10^{20} Hz

24

$$\begin{aligned} hc &= \left(6.62607\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.99792\times10^{8}\text{ m/s}\right)\left(\frac{10^{9}\text{ nm}}{1\text{ m}}\right)\left(\frac{1.00000\text{ eV}}{1.60218\times10^{-19}\text{ J}}\right) \\ &= 1239.84\text{ eV}\cdot\text{nm} \\ &\approx 1240\text{ eV}\cdot\text{nm} \end{aligned} \tag{29.22}$$

26

(a) 0.0829 eV

(b) 121

(c) 1.24 MeV

(d) 1.24×10^{5}

28

(a) 25.0×10^{3} eV

(b) 6.04×10^{18} Hz

30

(a) 2.69

(b) 0.371

32

(a) 1.25×10^{13} photons/s

(b) 997 km

34

8.33×10^{13} photons/s

36

181 km

38

(a) 1.66×10^{-32} kg · m/s

(b) The wavelength of microwave photons is large, so the momentum they carry is very small.

40

(a) 13.3 μm

(b) 9.38×10^{-2} eV

42

(a) 2.65×10^{-28} kg · m/s

(b) 291 m/s

(c) electron 3.86×10^{-26} J, photon 7.96×10^{-20} J, ratio 2.06×10^{6}

44

(a) 1.32×10^{-13} m

(b) 9.39 MeV

(c) 4.70×10^{-2} MeV

46

$E=\gamma mc^2$ and $P=\gamma mu$, so

$$\frac{E}{P}=\frac{\gamma mc^2}{\gamma mu}=\frac{c^2}{u}. \tag{29.35}$$

As the mass of particle approaches zero, its velocity u will approach c, so that the ratio of energy to momentum in this limit is

$$\lim_{m\to 0}\frac{E}{P}=\frac{c^2}{c}=c \tag{29.36}$$

which is consistent with the equation for photon energy.

48

(a) 3.00×10^{6} W

(b) Headlights are way too bright.

(c) Force is too large.

49

7.28×10^{-4} m

51

6.62×10^{7} m/s

53

1.32×10^{-13} m

55

(a) 6.62×10^{7} m/s

(b) 22.9 MeV

57

$$15.1 \text{ keV} \tag{29.42}$$

59

(a) 5.29 fm

(b) 4.70×10^{-12} J

(c) 29.4 MV

61

(a) 7.28×10^{12} m/s

(b) This is thousands of times the speed of light (an impossibility).

(c) The assumption that the electron is non-relativistic is unreasonable at this wavelength.

62

(a) 57.9 m/s

(b) 9.55×10^{-9} eV

(c) From **Table 29.1**, we see that typical molecular binding energies range from about 1eV to 10 eV, therefore the result in part (b) is approximately 9 orders of magnitude smaller than typical molecular binding energies.

64

29 nm,

290 times greater

66

1.10×10^{-13} eV

68

3.3×10^{-22} s

70

2.66×10^{-46} kg

72

0.395 nm

74

(a) 1.3×10^{-19} J

(b) 2.1×10^{23}

(c) 1.4×10^{2} s

76

(a) 3.35×10^{5} J

(b) 1.12×10^{-3} kg · m/s

(c) 1.12×10^{-3} m/s

(d) 6.23×10^{-7} J

78

(a) 1.06×10^{3}

(b) $5.33\times10^{-16}\ \text{kg}\cdot\text{m/s}$

(c) $1.24\times10^{-18}\ \text{m}$

80

(a) $1.62\times10^{3}\ \text{m/s}$

(b) $4.42\times10^{-19}\ \text{J}$ for photon, $1.19\times10^{-24}\ \text{J}$ for electron, photon energy is 3.71×10^{5} times greater

(c) The light is easier to make because 450-nm light is blue light and therefore easy to make. Creating electrons with $7.43\ \mu\text{eV}$ of energy would not be difficult, but would require a vacuum.

81

(a) $2.30\times10^{-6}\ \text{m}$

(b) $3.20\times10^{-12}\ \text{m}$

83

$3.69\times10^{-4}\ ^\circ\text{C}$

85

(a) 2.00 kJ

(b) $1.33\times10^{-5}\ \text{kg}\cdot\text{m/s}$

(c) $1.33\times10^{-5}\ \text{N}$

(d) yes

Test Prep for AP® Courses

1
(b)
3
(c)
5
(b)
7
(c)
9
(c)
11
(a)
13
(a)
15
(c)
17
(d)
19
(d)

Chapter 30

Problems & Exercises

1

1.84×10^{3}

3
50 km

4

$6 \times 10^{20}\ \text{kg/m}^3$

6

(a) $10.0\ \mu\text{m}$

(b) It isn't hard to make one of approximately this size. It would be harder to make it exactly $10.0\ \mu\text{m}$.

7

$\frac{1}{\lambda} = R\left(\frac{1}{n_{\text{f}}^2} - \frac{1}{n_{\text{i}}^2}\right) \Rightarrow \lambda = \frac{1}{R}\left[\frac{(n_{\text{i}} \cdot n_{\text{f}})^2}{n_{\text{i}}^2 - n_{\text{f}}^2}\right]; n_{\text{i}} = 2, n_{\text{f}} = 1,$ so that

$\lambda = \left(\frac{\text{m}}{1.097 \times 10^7}\right)\left[\frac{(2 \times 1)^2}{2^2 - 1^2}\right] = 1.22 \times 10^{-7}\ \text{m} = 122\ \text{nm}$, which is UV radiation.

9

$$a_{\text{B}} = \frac{h^2}{4\pi^2 m_e kZq_e^2} = \frac{(6.626 \times 10^{-34}\ \text{J·s})^2}{4\pi^2(9.109 \times 10^{-31}\ \text{kg})(8.988 \times 10^9\ \text{N·m}^2/\text{C}^2)(1)(1.602 \times 10^{-19}\ \text{C})^2} = 0.529 \times 10^{-10}\ \text{m}$$

11

0.850 eV

13

$2.12 \times 10^{-10}\ \text{m}$

15

365 nm

It is in the ultraviolet.

17

No overlap

365 nm

122 nm

19

7

21

(a) 2

(b) 54.4 eV

23

$\frac{kZq_e^2}{r_n^2} = \frac{m_e V^2}{r_n}$, so that $r_n = \frac{kZq_e^2}{m_e V^2} = \frac{kZq_e^2}{m_e}\frac{1}{V^2}$. From the equation $m_e v r_n = n\frac{h}{2\pi}$, we can substitute for the velocity, giving: $r_n = \frac{kZq_e^2}{m_e} \cdot \frac{4\pi^2 m_e^2 r_n^2}{n^2 h^2}$ so that $r_n = \frac{n^2}{Z}\frac{h^2}{4\pi^2 m_e kq_e^2} = \frac{n^2}{Z}a_{\text{B}}$, where $a_{\text{B}} = \frac{h^2}{4\pi^2 m_e kq_e^2}$.

25

(a) $0.248 \times 10^{-10}\ \text{m}$

(b) 50.0 keV

(c) The photon energy is simply the applied voltage times the electron charge, so the value of the voltage in volts is the same as the value of the energy in electron volts.

27

(a) $100 \times 10^3\ \text{eV}$, $1.60 \times 10^{-14}\ \text{J}$

(b) $0.124 \times 10^{-10}\ \text{m}$

29

(a) 8.00 keV

(b) 9.48 keV

30

(a) 1.96 eV

(b) $(1240 \text{ eV}\cdot\text{nm})/(1.96 \text{ eV}) = 633 \text{ nm}$

(c) 60.0 nm

32

693 nm

34

(a) 590 nm

(b) $(1240 \text{ eV}\cdot\text{nm})/(1.17 \text{ eV}) = 1.06 \text{ μm}$

35

$l = 4, 3$ are possible since $l < n$ and $|m_l| \le l$.

37

$n = 4 \Rightarrow l = 3, 2, 1, 0 \Rightarrow m_l = \pm 3, \pm 2, \pm 1, 0$ are possible.

39

(a) $1.49 \times 10^{-34} \text{ J}\cdot\text{s}$

(b) $1.06 \times 10^{-34} \text{ J}\cdot\text{s}$

41

(a) $3.66 \times 10^{-34} \text{ J}\cdot\text{s}$

(b) $s = 9.13 \times 10^{-35} \text{ J}\cdot\text{s}$

(c) $\frac{L}{S} = \frac{\sqrt{12}}{\sqrt{3/4}} = 4$

43

$\theta = 54.7°, 125.3°$

44

(a) 32. (b) $2 \text{ in } s, 6 \text{ in } p, 10 \text{ in } d,$ and 14 in f, for a total of 32.

46

(a) 2

(b) $3d^9$

48

(b) $n \ge l$ is violated,

(c) cannot have 3 electrons in s subshell since $3 > (2l + 1) = 2$

(d) cannot have 7 electrons in p subshell since $7 > (2l + 1) = 2(2 + 1) = 6$

50

(a) The number of different values of m_l is $\pm l, \pm(l - 1), ...,0$ for each $l > 0$ and one for $l = 0 \Rightarrow (2l + 1)$. Also an overall factor of 2 since each m_l can have m_s equal to either $+1/2$ or $-1/2 \Rightarrow 2(2l + 1)$.

(b) for each value of l, you get $2(2l + 1)$

$= 0, 1, 2, ...,(n-1) \Rightarrow 2\{[(2)(0) + 1] + [(2)(1) + 1] + ... + [(2)(n - 1) + 1]\} = 2[\underbrace{1 + 3 + ... + (2n - 3) + (2n - 1)}_{n \text{ terms}}]$ to see that the expression in the box is $= n^2,$ imagine taking $(n - 1)$ from the last term and adding it to first term

$= 2[1 + (n–1) + 3 + ... + (2n - 3) + (2n - 1)–(n - 1)] = 2[n + 3 + ... + (2n - 3) + n]$. Now take $(n - 3)$ from penultimate term and add to the second term $2[\underbrace{n + n + ... + n + n}_{n \text{ terms}}] = 2n^2$.

52
The electric force on the electron is up (toward the positively charged plate). The magnetic force is down (by the RHR).
54
401 nm
56
(a) 6.54×10^{-16} kg

(b) 5.54×10^{-7} m

58
1.76×10^{11} C/kg , which agrees with the known value of 1.759×10^{11} C/kg to within the precision of the measurement
60
(a) 2.78 fm

(b) 0.37 of the nuclear radius.

62
(a) 1.34×10^{23}

(b) 2.52 MW

64
(a) 6.42 eV

(b) 7.27×10^{-20} J/molecule

(c) 0.454 eV, 14.1 times less than a single UV photon. Therefore, each photon will evaporate approximately 14 molecules of tissue. This gives the surgeon a rather precise method of removing corneal tissue from the surface of the eye.

66
91.18 nm to 91.22 nm
68
(a) 1.24×10^{11} V

(b) The voltage is extremely large compared with any practical value.

(c) The assumption of such a short wavelength by this method is unreasonable.

Test Prep for AP® Courses

1
(a), (d)
3
(a)
5
(a)
7
(b)
9
(a)
11
(d)
13
(d)
15
(a), (c)

Chapter 31

Problems & Exercises

1

1.67×10^4

5

$$m = \rho V = \rho d^3 \;\; \Rightarrow \;\; a = \left(\frac{m}{\rho}\right)^{1/3} = \left(\frac{2.3\times10^{17}\ \text{kg}}{1000\ \text{kg/m}^3}\right)^{\frac{1}{3}} \\ = \;\; 61\times10^3\ \text{m} = 61\ \text{km} \tag{31.17}$$

7

$1.9\ \text{fm}$

9

(a) $4.6\ \text{fm}$

(b) $0.61\ \text{to}\ 1$

11

$85.4\ \text{to}\ 1$

13

$12.4\ \text{GeV}$

15

19.3 to 1

17

$${}^{3}_{1}\text{H}_2 \rightarrow {}^{3}_{2}\text{He}_1 + \beta^- + \bar{\nu}_e \tag{31.47}$$

19

$${}^{50}_{25}M_{25} \rightarrow {}^{50}_{24}\text{Cr}_{26} + \beta^+ + \nu_e \tag{31.48}$$

21

$${}^{7}_{4}\text{Be}_3 + e^- \rightarrow {}^{7}_{3}\text{Li}_4 + \nu_e \tag{31.49}$$

23

$${}^{210}_{84}\text{Po}_{126} \rightarrow {}^{206}_{82}\text{Pb}_{124} + {}^{4}_{2}\text{He}_2 \tag{31.50}$$

25

$${}^{137}_{55}\text{Cs}_{82} \rightarrow {}^{137}_{56}\text{Ba}_{81} + \beta^- + \bar{\nu}_e \tag{31.51}$$

27

$${}^{232}_{90}\text{Th}_{142} \rightarrow {}^{228}_{88}\text{Ra}_{140} + {}^{4}_{2}\text{He}_2 \tag{31.52}$$

29

(a) $\text{charge:}(+1) + (-1) = 0;\quad \text{electron family number: } (+1) + (-1) = 0;\quad A\text{: } 0 + 0 = 0$

(b) 0.511 MeV

(c) The two γ rays must travel in exactly opposite directions in order to conserve momentum, since initially there is zero momentum if the center of mass is initially at rest.

31

$$Z = (Z + 1) - 1;\quad A = A;\quad \text{efn} : 0 = (+1) + (-1) \tag{31.53}$$

33

$$Z - 1 = Z - 1;\quad A = A;\quad \text{efn} :(+1) = (+1) \tag{31.54}$$

35

(a) ${}^{226}_{88}\text{Ra}_{138} \rightarrow {}^{222}_{86}\text{Rn}_{136} + {}^{4}_{2}\text{He}_2$

(b) 4.87 MeV

37

(a) $\mathrm{n} \rightarrow \mathrm{p} + \beta^{-} + \bar{\nu}_e$

(b)) 0.783 MeV

39
1.82 MeV
41

(a) 4.274 MeV

(b) 1.927×10^{-5}

(c) Since U-238 is a slowly decaying substance, only a very small number of nuclei decay on human timescales; therefore, although those nuclei that decay lose a noticeable fraction of their mass, the change in the total mass of the sample is not detectable for a macroscopic sample.

43

(a) ${}^{15}_{8}\mathrm{O}_7 + e^{-} \rightarrow {}^{15}_{7}\mathrm{N}_8 + \nu_e$

(b) 2.754 MeV

44
57,300 y
46

(a) 0.988 Ci

(b) The half-life of ${}^{226}\mathrm{Ra}$ is now better known.

48
$1.22 \times 10^{3}\ \mathrm{Bq}$

50

(a) 16.0 mg

(b) 0.0114%

52
$1.48 \times 10^{17}\ \mathrm{y}$

54
$5.6 \times 10^{4}\ \mathrm{y}$

56
2.71 y
58

(a) 1.56 mg

(b) 11.3 Ci

60

(a) 1.23×10^{-3}

(b) Only part of the emitted radiation goes in the direction of the detector. Only a fraction of that causes a response in the detector. Some of the emitted radiation (mostly α particles) is observed within the source. Some is absorbed within the source, some is absorbed by the detector, and some does not penetrate the detector.

62

(a) $1.68 \times 10^{-5}\ \mathrm{Ci}$

(b) $8.65 \times 10^{10}\ \mathrm{J}$

(c) $\$\,2.9 \times 10^{3}$

64

(a) $6.97 \times 10^{15}\ \mathrm{Bq}$

(b) 6.24 kW

(c) 5.67 kW

68

(a) 84.5 Ci

(b) An extremely large activity, many orders of magnitude greater than permitted for home use.

(c) The assumption of $1.00\ \mu A$ is unreasonably large. Other methods can detect much smaller decay rates.

69

1.112 MeV, consistent with graph

71

7.848 MeV, consistent with graph

73

(a) 7.680 MeV, consistent with graph

(b) 7.520 MeV, consistent with graph. Not significantly different from value for ^{12}C, but sufficiently lower to allow decay into another nuclide that is more tightly bound.

75

(a) 1.46×10^{-8} u vs. 1.007825 u for ^{1}H

(b) 0.000549 u

(c) 2.66×10^{-5}

76

(a) $-9.315\ \text{MeV}$

(b) The negative binding energy implies an unbound system.

(c) This assumption that it is two bound neutrons is incorrect.

78

22.8 cm

79

(a) ${}^{235}_{92}\text{U}_{143} \rightarrow {}^{231}_{90}\text{Th}_{141} + {}^{4}_{2}\text{He}_{2}$

(b) 4.679 MeV

(c) 4.599 MeV

81

a) 2.4×10^{8} u

(b) The greatest known atomic masses are about 260. This result found in (a) is extremely large.

(c) The assumed radius is much too large to be reasonable.

82

(a) $-1.805\ \text{MeV}$

(b) Negative energy implies energy input is necessary and the reaction cannot be spontaneous.

(c) Although all conversation laws are obeyed, energy must be supplied, so the assumption of spontaneous decay is incorrect.

Test Prep for AP® Courses

1

(c)

3

(a)

5

When ${}^{241}_{95}\text{Am}$ undergoes α decay, it loses 2 neutrons and 2 protons. The resulting nucleus is therefore ${}^{237}_{93}\text{Np}$.

7

During this process, the nucleus emits a particle with -1 charge. In order for the overall charge of the system to remain constant, the charge of the nucleus must therefore increase by +1.

9

a. No. Nucleon number is conserved (238 = 234 + 4), but the atomic number or charge is NOT conserved (92 ≠ 88+2).

b. Yes. Nucleon number is conserved (223 = 209 + 14), and atomic number is conserved (88 = 82 + 6).

c. Yes. Nucleon number is conserved (14 = 14), and charge is conserved if the electron's charge is properly counted (6 = 7 + (-1)).

d. No. Nucleon number is not conserved (24 ≠ 23). The positron released counts as a charge to conserve charge, but it doesn't count as a nucleon.

11

This must be alpha decay since 4 nucleons (2 positive charges) are lost from the parent nucleus. The number remaining is found from:

$$\mathrm{N}(t) = \mathrm{N}_0 e^{\left(\frac{-0.693t}{t_{\frac{1}{2}}}\right)} = 3.4\times10^{17} e^{\left(\frac{-(0.693)(0.035)}{0.00173}\right)}$$

$$\mathrm{N}(t) = 4.1\times10^{11} \text{ nuclei}$$

Chapter 32

Problems & Exercises

1

5.701 MeV

3

$${}^{99}_{42}\mathrm{Mo}_{57} \rightarrow {}^{99}_{43}\mathrm{Tc}_{56} + \beta^- + \bar{v}_e$$

5

1.43×10^{-9} g

7

(a) 6.958 MeV

(b) 5.7×10^{-10} g

8

(a) 100 mSv

(b) 80 mSv

(c) ~30 mSv

10

~2 Gy

12

1.69 mm

14

1.24 MeV

16

7.44×10^{8}

18

4.92×10^{-4} Sv

20

4.43 g

22

0.010 g

24

95%

26

(a) $A=1+1=2$, $Z=1+1=1+1$, $\text{efn} = 0 = -1 + 1$

(b) $A=1+2=3$, $Z=1+1=2$, $\text{efn}=0=0$

(c) $A=3+3=4+1+1$, $Z=2+2=2+1+1$, $\text{efn}=0=0$

28

$$\begin{aligned} E &= (m_\text{i} - m_\text{f})c^2 \\ &= \left[4m\left({}^1\text{H}\right) - m\left({}^4\text{He}\right)\right]c^2 \\ &= [4(1.007825) - 4.002603](931.5\text{ MeV}) \\ &= 26.73\text{ MeV} \end{aligned}$$

30

$3.12\times10^5\text{ kg}$ (about 200 tons)

32

$$\begin{aligned} E &= (m_\text{i} - m_\text{f})c^2 \\ E_1 &= (1.008665 + 3.016030 - 4.002603)(931.5\text{ MeV}) \\ &= 20.58\text{ MeV} \\ E_2 &= (1.008665 + 1.007825 - 2.014102)(931.5\text{ MeV}) \\ &= 2.224\text{ MeV} \end{aligned}$$

${}^4\text{He}$ is more tightly bound, since this reaction gives off more energy per nucleon.

34

$1.19\times10^4\text{ kg}$

36

$2e^- + 4\,{}^1\text{H} \rightarrow {}^4\text{He} + 7\gamma + 2v_e$

38

(a) $A=12+1=13$, $Z=6+1=7$, $\text{efn} = 0 = 0$

(b) $A=13=13$, $Z=7=6+1$, $\text{efn} = 0 = -1 + 1$

(c) $A=13 + 1=14$, $Z=6+1=7$, $\text{efn} = 0 = 0$

(d) $A=14 + 1=15$, $Z=7+1=8$, $\text{efn} = 0 = 0$

(e) $A=15=15$, $Z=8=7+1$, $\text{efn} = 0 = -1 + 1$

(f) $A=15 + 1=12 + 4$, $Z=7+1=6 + 2$, $\text{efn} = 0 = 0$

40

$E_\gamma = 20.6\text{ MeV}$

$E_{{}^4\text{He}} = 5.68\times10^{-2}\text{ MeV}$

42

(a) 3×10^9 y

(b) This is approximately half the lifetime of the Earth.

43

(a) 177.1 MeV

(b) Because the gain of an external neutron yields about 6 MeV, which is the average BE/A for heavy nuclei.

(c) $A = 1 + 238 = 96 + 140 + 1 + 1 + 1, Z = 92 = 38 + 53, \text{efn} = 0 = 0$

45

(a) 180.6 MeV

(b) $A = 1 + 239 = 96 + 140 + 1 + 1 + 1 + 1, Z = 94 = 38 + 56, \text{efn} = 0 = 0$

47

$^{238}\text{U} + n \rightarrow {}^{239}\text{U} + \gamma$ 4.81 MeV

$^{239}\text{U} \rightarrow {}^{239}\text{Np} + \beta^- + v_e$ 0.753 MeV

$\text{Np} \rightarrow \text{Pu} + \beta^- + v_e$ 0.211 MeV

49

(a) 2.57×10^3 MW

(b) 8.03×10^{19} fission/s

(c) 991 kg

51
0.56 g
53
4.781 MeV
55

(a) Blast yields 2.1×10^{12} J to 8.4×10^{11} J , or 2.5 to 1, conventional to radiation enhanced.

(b) Prompt radiation yields 6.3×10^{11} J to 2.1×10^{11} J , or 3 to 1, radiation enhanced to conventional.

57

(a) 1.1×10^{25} fissions , 4.4 kg

(b) 3.2×10^{26} fusions , 2.7 kg

(c) The nuclear fuel totals only 6 kg, so it is quite reasonable that some missiles carry 10 overheads. The mass of the fuel would only be 60 kg and therefore the mass of the 10 warheads, weighing about 10 times the nuclear fuel, would be only 1500 lbs. If the fuel for the missiles weighs 5 times the total weight of the warheads, the missile would weigh about 9000 lbs or 4.5 tons. This is not an unreasonable weight for a missile.

59
7×10^4 g
61

(a) 4.86×10^9 W

(b) 11.0 y

Test Prep for AP® Courses

1
(b)
3
(c)
5
(d)
7
(d)
9
(b)

Chapter 33

Problems & Exercises

1
3×10^{-39} s
3

1.99×10^{-16} m (0.2 fm)

4

(a) 10^{-11} to 1, weak to EM

(b) 1 to 1

6

(a) 2.09×10^{-5} s

(b) 4.77×10^{4} Hz

8

78.0 cm

10

1.40×10^{6}

12

100 GeV

13

67.5 MeV

15

(a) 1×10^{14}

(b) 2×10^{17}

17

(a) 1671 MeV

(b) $Q = 1, Q' = 1 + 0 + 0 = 1. L_\tau = -1; L'\tau = -1; L\mu = 0; L'\mu = -1 + 1 = 0$

(c) $\tau^- \rightarrow \mu^- + \nu_\mu + \bar{\nu}_\tau$
$\Rightarrow \mu^-$ antiparticle of μ^+; ν_μ of $\bar{\nu}_\mu$; $\bar{\nu}_\tau$ of ν_τ

19

(a) 3.9 eV

(b) 2.9×10^{-8}

21

(a) The uud composition is the same as for a proton.

(b) 3.3×10^{-24} s

(c) Strong (short lifetime)

23

a) $\Delta^{++}(uuu); B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

b)

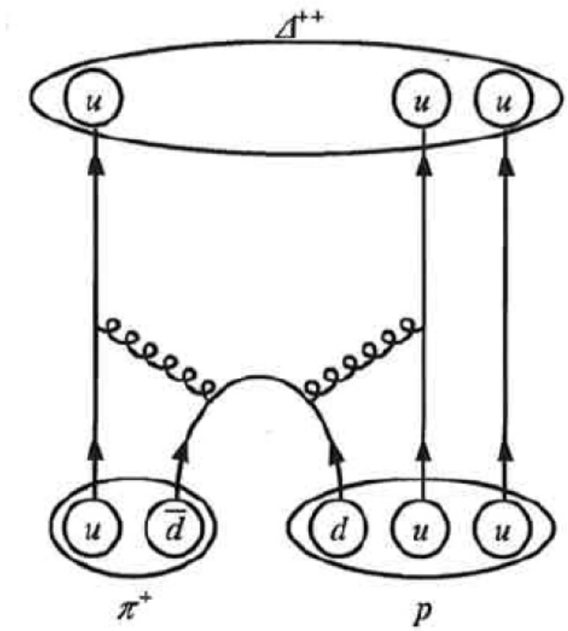

Figure 33.20.

25

(a) $+1$

(b) $B = 1 = 1 + 0, Z = \ = 0 + (- 1)$, all lepton numbers are 0 before and after

(c) $(sss) \to (uds) + (\bar{u}\,s)$

27

(a) $(u\,\bar{u}\, + d\,\bar{d}\,) \to (u\,\bar{u}\, + d\,\bar{d}\,) + (u\,\bar{u}\, + d\,\bar{d}\,)$

(b) 277.9 MeV

(c) 547.9 MeV

29

No. $\text{Charge} = -1$ is conserved. $L_{e_\text{i}} = 0 \neq L_{e_\text{f}} = 2$ is not conserved. $L_\mu = 1$ is conserved.

31

(a)Yes. $Z = -1 = 0 + (- 1)$, $B = 1 = 1 + 0$, all lepton family numbers are 0 before and after, spontaneous since mass greater before reaction.

(b) $dds \to udd + \bar{u}\,d$

33

(a) 216

(b) There are more baryons observed because we have the 6 antiquarks and various mixtures of quarks (as for the π-meson) as well.

35

$$\Omega^+ \ (\bar{s}\ \bar{s}\ \bar{s})$$
$$B = -\frac{1}{3} - \frac{1}{3} - \frac{1}{3} = -1,$$
$$L_e,\ \mu,\ \tau = 0 + 0 + 0 = 0,$$
$$Q = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1,$$
$$S = 1 + 1 + 1 = 3.$$

37

(a)803 MeV

(b) 938.8 MeV

(c) The annihilation energy of an extra electron is included in the total energy.

39

$c\bar{d}$

41

a)The antiproton

b) $\bar{p} \rightarrow \pi^0 + e^-$

43

(a) 5×10^{10}

(b) $5 \times 10^4 \text{ particles/m}^2$

45

$2.5 \times 10^{-17} \text{ m}$

47

(a) 33.9 MeV

(b) Muon antineutrino 29.8 MeV, muon 4.1 MeV (kinetic energy)

49

(a) $7.2 \times 10^5 \text{ kg}$

(b) $7.2 \times 10^2 \text{ m}^3$

(c) 100 months

Test Prep for AP® Courses

1
(d)
3
(d)
5
(b)
7
(a)
9
(c), though this comes from Einstein's special relativity
11
(a)
13
(d)
15
(b)
17
(b)

Chapter 34

Problems & Exercises

1

$3 \times 10^{41} \text{ kg}$

3

(a) $3 \times 10^{52} \text{ kg}$

(b) 2×10^{79}

(c) 4×10^{88}

5
0.30 Gly
7

(a) $2.0 \times 10^{5}\ \text{km/s}$

(b) $0.67c$

9
$2.7 \times 10^{5}\ \text{m/s}$
11
6×10^{-11} (an overestimate, since some of the light from Andromeda is blocked by gas and dust within that galaxy)
13

(a) $2 \times 10^{-8}\ \text{kg}$

(b) 1×10^{19}

15

(a) $30\text{km/s} \cdot \text{Mly}$

(b) $15\text{km/s} \cdot \text{Mly}$

17
960 rev/s
19
89.999773° (many digits are used to show the difference between 90°)
22
23.6 km
24

(a) $2.95 \times 10^{12}\ \text{m}$

(b) $3.12 \times 10^{-4}\ \text{ly}$

26

(a) 1×10^{20}

(b) 10 times greater

27
$$1.5 \times 10^{15} \tag{34.6}$$
29
$$0.6\ \text{m}^{-3} \tag{34.7}$$
31
$$0.30\ \Omega \tag{34.8}$$

Index

Symbols

M

N

O

P

Q

R

S