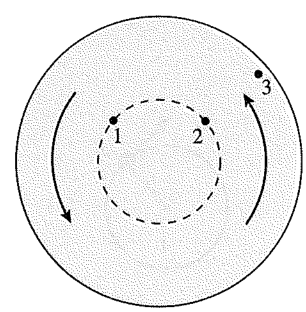
6 Circular Motion, Orbits, and Gravity

6.1 Uniform Circular Motion

1. a. The crankshaft in your car rotates at 3000 rpm. What is the frequency in revolutions per second?

   b. A record turntable rotates at 33.3 rpm. What is the rotation period in seconds?

2. The figure shows three points on a steadily rotating wheel.
   a. Draw the velocity vectors at each of the three points.
   b. Rank in order, from largest to smallest, the speeds v_1, v_2, and v_3 of these points.

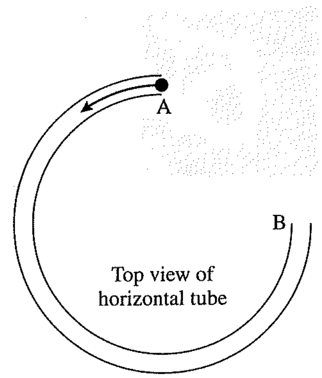


Order:

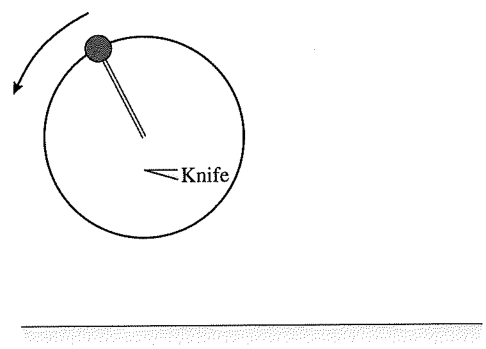
Explanation:

6.2 Dynamics of Uniform Circular Motion

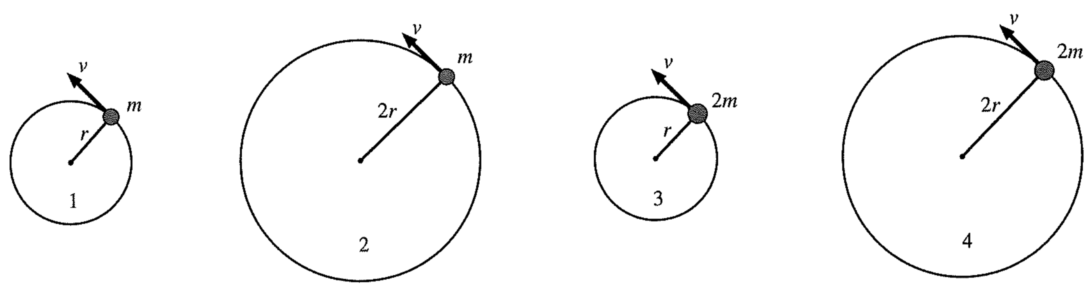
3. The figure shows a *top view* of a plastic tube that is fixed on a horizontal table top. A marble is shot into the tube at A. Sketch the marble's trajectory after it leaves the tube at B. Explain.



4. A ball swings in a *vertical* circle on a string. During one revolution, a very sharp knife is used to cut the string at the instant when the ball is at its lowest point. Sketch the subsequent trajectory of the ball until it hits the ground. Explain.



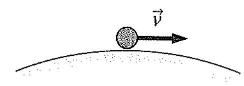
5. The figures are a bird's-eye view of particles moving in horizontal circles on a table top. All are moving at the same speed. Rank in order, from largest to smallest, the tensions T_1 to T_4.

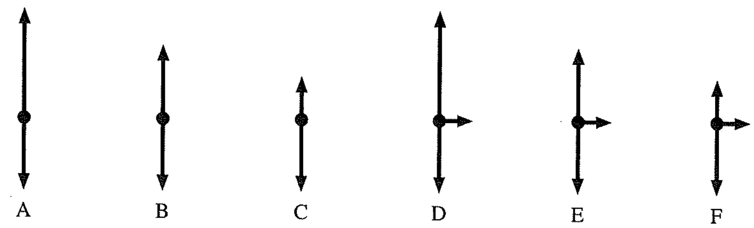


Order:

Explanation:

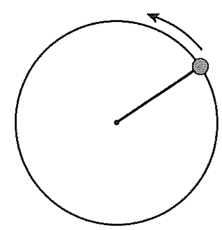
6. A ball rolls over the top of a circular hill. Rolling friction is negligible. Circle the letter of the ball's free-body diagram at the very top of the hill.



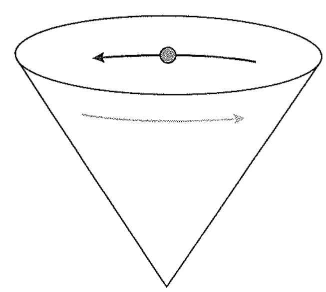


Explanation:

7. A ball on a string moves in a vertical circle. When the ball is at its lowest point, is the tension in the string greater than, less than, or equal to the ball's weight? Explain. (You should include a free-body diagram as part of your explanation.)



8. A marble rolls around the inside of a cone. Draw a free-body diagram of the marble when it is on the left side of the cone, coming toward you.



9. A coin of mass m is placed at distance r from the center of a turntable. The coefficient of static friction between the coin and the turntable is μ_s. Starting from rest, the turntable is gradually rotated faster and faster. At what rotation frequency does the coin slip and "fly off"?

PSS 6.1

a. Begin with a visual overview. Draw the turntable both as seen from above and as an edge view with the coin on the left side coming toward you. Label radius r, make a table of known information, and indicate what you're trying to find.

b. What direction does $\vec{f}_s$ point?
Explain.

c. What condition describes the situation just as the coin starts to slip? Write this condition as a mathematical statement.

d. Now draw a free-body diagram of the coin. Following Problem Solving Strategy 6.1, draw the free-body diagram with the circle viewed edge on, the x-axis pointing toward the center of the circle, and the y-axis perpendicular to the plane of the circle. Your free-body diagram should have three forces on it.

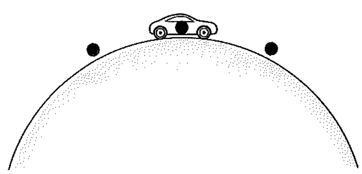
e. Referring to Problem Solving Strategy 6.1, write Newton's second law for the x- and y-components of the forces. One sum should equal 0, the other mv^2/r.

f. The two equations of part e are valid for any speed up to the point of slipping. If you combine these with your statement of part c, you can solve for the speed v_{max} at which the coin slips. Do so.

g. Finally, use the relationship between v and f to find the frequency at which the coin slips.

6.3 Apparent Forces in Circular Motion

10. The drawing is a partial motion diagram for a car rolling at constant speed over the top of a circular hill.
    a. Complete the motion diagram by adding the car's velocity vectors. Then use the velocity vectors to determine *and show* the car's acceleration $\vec{a}$ at the top of the hill.
    b. To the right, draw a free-body diagram of the car at the top of the hill. Next to the free-body diagram, indicate the direction of the net force on the car or, if appropriate, write $\vec{F}_{net} = \vec{0}$.



    c. Does the net force point of your free-body diagram point in the same direction you showed for the car's acceleration?
    If not, you may want to reconsider your work thus far because Newton's second law requires $\vec{F}_{net}$ and $\vec{a}$ to point the same way.
    d. Is there a maximum speed at which the car can travel over the top of the hill and not lose contact with the ground? If so, show how the free-body diagram would look at that speed. If not, why not?

11. A stunt plane does a series of vertical loop-the-loops. At what point in the circle does the pilot feel the heaviest? Explain. Include a free-body diagram with your explanation.

12. A roller-coaster car goes around the inside of a loop-the-loop. One of the following statements is true at the highest point in the loop, and one is true at the lowest point. Check the true statements.

	Highest	Lowest
The car's apparent weight w_{app} is always less than w		
The car's apparent weight w_{app} is always equal to w		
The car's apparent weight w_{app} is always greater than w		
w_{app} could be less than, equal to, or greater than w		

13. You can swing a ball on a string in a *vertical* circle if you swing it fast enough.
    a. Draw two free-body diagrams of the ball at the top of the circle. On the left, show the ball when it is going around the circle very fast. On the right, show the ball as it goes around the circle more slowly.

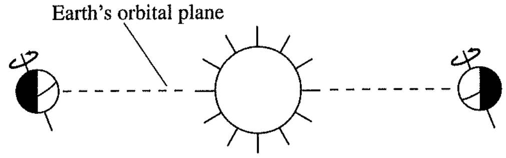
Very fast

Slower

    b. Suppose the ball has the smallest possible frequency that allows it to go all the way around the circle. What is the tension in the string when the ball is at the highest point? Explain.

6.4 Circular Orbits and Weightlessness

14. The earth has seasons because the axis of the earth's rotation is tilted 23° away from a line perpendicular to the plane of the earth's orbit. You can see this in the figure, which shows the edge of the earth's orbit around the sun. For both positions of the earth, draw a force vector to show the net force acting on the earth or, if appropriate, write $\vec{F} = \vec{0}$.



15. A small projectile is launched parallel to the ground at height $h = 1$ m with sufficient speed to orbit a completely smooth, airless planet. A bug rides in a small hole inside the projectile. Is the bug weightless? Explain.

16. It's been proposed that future space stations create "artificial gravity" by rotating around an axis. (The space station would have to be much larger than the present space station for this to be feasible.)

a. How would this work? Explain.

b. Would the artificial gravity be equally effective throughout the space station? If not, where in the space station would the residents want to live and work?

6.5 Newton's Law of Gravity

17. Is the earth's gravitational force on the sun larger than, smaller than, or equal to the sun's gravitational force on the earth? Explain.

18. Star A is twice as massive as star B.

a. Draw gravitational force vectors on both stars. The length of each vector should be proportional to the size of the force.

b. Is the acceleration of star A toward B larger than, smaller than, or equal to the acceleration of star B toward A? Explain.

19. The quantity y is inversely proportional to the square of x, and $y = 4$ when $x = 5$.

a. Write an equation to represent this inverse-square relationship for all y and x.

b. Find y if $x = 2$. ______

c. Find x if $y = 100$. ______

20. The quantity y is inversely proportional to the square of x. For one value of x, $y = 12$.

a. What is the value of y if x is doubled? ______

b. What is the value of y if the original value of x is halved? ______

21. How far away from the earth does an orbiting spacecraft have to be in order for the astronauts inside to be "weightless"?

22. The acceleration due to gravity at the surface of Planet X is 20 m/s^2. The radius and the mass of Planet Z are twice those of Planet X. What is g on Planet Z?

6.6 Gravity and Orbits

23. Planet X orbits the star Omega with a "year" that is 200 earth days long. Planet Y circles Omega at four times the distance of Planet X. How long is a year on Planet Y?

24. The mass of Jupiter is $M_{Jupiter} = 300M_{earth}$. Jupiter orbits around the sun with $T_{Jupiter} = 11.9$ years in an orbit with $r_{Jupiter} = 5.2r_{earth}$. Suppose the earth could be moved to the distance of Jupiter and placed in a circular orbit around the sun. The new period of the earth's orbit would be

a. 1 year.

b. 11.9 years.

c. Between 1 year and 11.9 years.

d. More than 11.9 years.

e. It could be anything, depending on the speed the earth is given.

f. It is impossible for a planet of earth's mass to orbit at the distance of Jupiter.

Circle the letter of the true statement. Then explain your choice.

25. The gravitational force of a star on orbiting planet 1 is F_1. Planet 2, which is twice as massive as Planet 1 and orbits at twice the distance from the star, experiences gravitational force F_2.

a. What is the ratio F_2/F_1?

b. Planet 1 orbits the star with period T_1 and Planet 2 with period T_2. What is the ratio T_2/T_1?

26. Satellite A orbits a planet with a speed of 10,000 m/s. Satellite B, orbiting at the same distance from the center of the planet, is twice as massive as Satellite A. What is the speed of Satellite B?

You Write the Problem!

Exercises 27–30: You are given the equation that is used to solve a problem. For each of these:

a. Write a realistic physics problem for which this is the correct equation. Look at worked examples and end-of-chapter problems in the textbook to see what realistic physics problems are like. Be sure that the problem you write, and the answer you ask for, is consistent with the information given in the equation.
b. If appropriate, draw the free-body diagram and pictorial representation for your problem.
c. Finish the solution of the problem.

27. $60\text{ N} = \dfrac{(0.30\text{ kg})\,v^2}{0.50\text{ m}}$

28. $(1500\text{ kg})(9.8\text{ m/s}^2) - 11{,}760\text{ N} = \dfrac{(1500\text{ kg})\,v^2}{200\text{ m}}$

29. $\dfrac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.90 \times 10^{27}\text{ kg})}{R^2} = \dfrac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}{6.37 \times 10^{6}\text{ m}}$

30. $\dfrac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})(1000\text{ kg})}{r^2} = \dfrac{(1000\text{ kg})(1997\text{ m/s})^2}{r}$