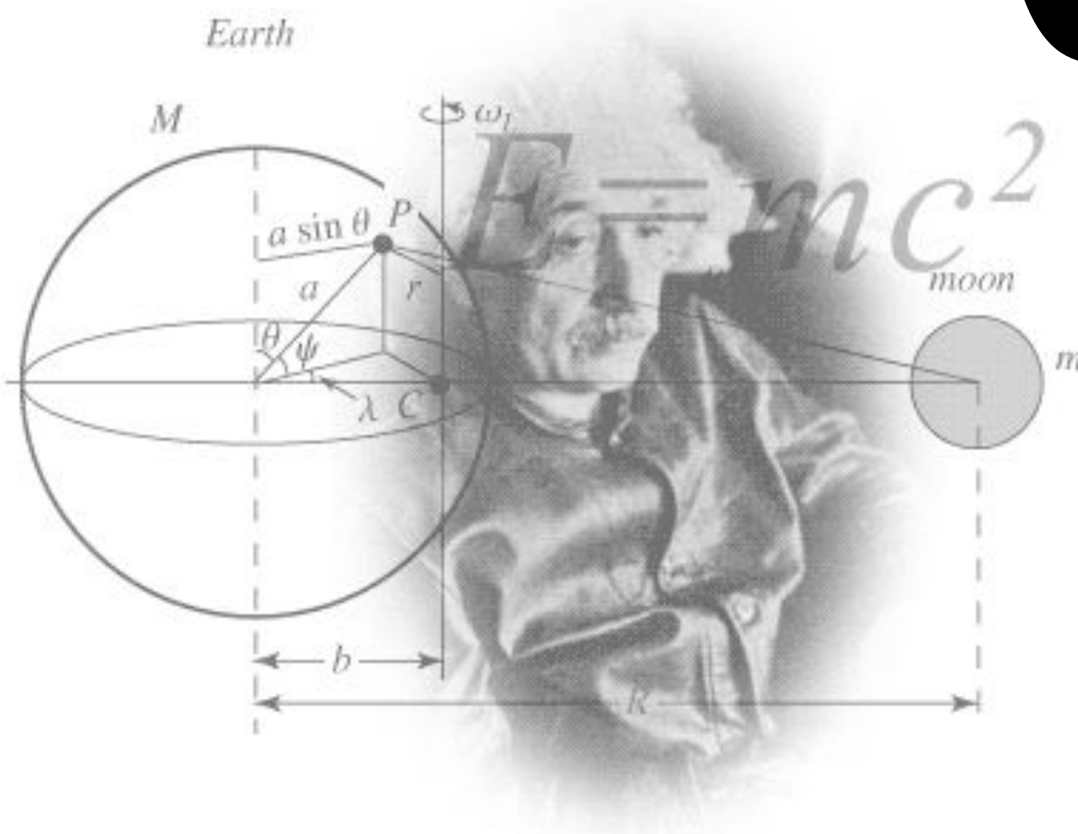
AP Physics C – Practice Workbook – Book 2

Electricity and Magnetism

C



2012-2013

Table of Contents

This book is a compilation of all the problems published by College Board in AP Physics C organized by topic.

The problems vary in level of difficulty and type and this book represents an invaluable resource for practice and review and should be used… often. Whether you are struggling or confident in a topic, you should be doing these problems as a reinforcement of ideas and concepts on a scale that could never be covered in the class time allotted.

The answers as presented are not the only method to solving many of these problems and physics teachers may present slightly different methods and/or different symbols and variables in each topic, but the underlying physics concepts are the same and we ask you read the solutions with an open mind and use these differences to expand your problem solving skills.

Finally, we *are* fallible and if you find any typographical errors, formatting errors or anything that strikes you as unclear or unreadable, please let us know so we can make the necessary announcements and corrections.

Table of Information and Equation Tables for AP Physics Exams

The accompanying Table of Information and Equation Tables will be provided to students when they take the AP Physics Exams. Therefore, students may NOT bring their own copies of these tables to the exam room, although they may use them throughout the year in their classes in order to become familiar with their content. **Check the Physics course home pages on AP Central for the latest versions of these tables (apcentral.collegeboard.com).**

Table of Information

For both the Physics B and Physics C Exams, the Table of Information is printed near the front cover of the multiple-choice section and on the green insert provided with the free-response section. The tables are identical for both exams except for one convention as noted.

Equation Tables

For both the Physics B and Physics C Exams, the equation tables for each exam are printed only on the green insert provided with the free-response section. The equation tables may be used by students when taking the free-response sections of both exams but NOT when taking the multiple-choice sections.

The equations in the tables express the relationships that are encountered most frequently in AP Physics courses and exams. However, the tables do not include all equations that might possibly be used. For example, they do not include many equations that can be derived by combining other equations in the tables. Nor do they include equations that are simply special cases of any that are in the tables. Students are responsible for understanding the physical principles that underlie each equation and for knowing the conditions for which each equation is applicable.

The equation tables are grouped in sections according to the major content category in which they appear. Within each section, the symbols used for the variables in that section are defined. However, in some cases the same symbol is used to represent different quantities in different tables. It should be noted that there is no uniform convention among textbooks for the symbols used in writing equations. The equation tables follow many common conventions, but in some cases consistency was sacrificed for the sake of clarity.

Some explanations about notation used in the equation tables:

1. The symbols used for physical constants are the same as those in the Table of Information and are defined in the Table of Information rather than in the right-hand columns of the tables.
2. Symbols in bold face represent vector quantities.
3. Subscripts on symbols in the equations are used to represent special cases of the variables defined in the right-hand columns.
4. The symbol Δ before a variable in an equation specifically indicates a change in the variable (i.e., final value minus initial value).
5. Several different symbols (e.g., d, r, s, h, ℓ) are used for linear dimensions such as length. The particular symbol used in an equation is one that is commonly used for that equation in textbooks.

TABLE OF INFORMATION DEVELOPED FOR 2012 (see note on cover page)

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^8$ m/s
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ m^3/kg•s^2
Universal gas constant,	$R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface,	$g = 9.8$ m/s^2
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K		

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}$ J•s $= 4.14 \times 10^{-15}$ eV•s
	$hc = 1.99 \times 10^{-25}$ J•m $= 1.24 \times 10^3$ eV•nm
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N•m^2
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^9$ N•m^2/C^2
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}$ (T•m)/A
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5 \text{ N/m}^2 = 1.0 \times 10^5$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

*IV. For mechanics and thermodynamics equations, *W* represents the work done on a system.

*Not on the Table of Information for Physics C, since Thermodynamics is not a Physics C topic.

ADVANCED PLACEMENT PHYSICS B EQUATIONS DEVELOPED FOR 2012

NEWTONIAN MECHANICS

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$F_{fric} \le \mu N$$

$$a_c = \frac{v^2}{r}$$

$$\tau = rF \sin\theta$$

$$\mathbf{p} = m\mathbf{v}$$

$$\mathbf{J} = \mathbf{F}\Delta t = \Delta\mathbf{p}$$

$$K = \frac{1}{2}mv^2$$

$$\Delta U_g = mgh$$

$$W = F\Delta r \cos\theta$$

$$P_{avg} = \frac{W}{\Delta t}$$

$$P = Fv\cos\theta$$

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2}kx^2$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T = \frac{1}{f}$$

$$F_G = -\frac{Gm_1m_2}{r^2}$$

$$U_G = -\frac{Gm_1m_2}{r}$$

a = acceleration
F = force
f = frequency
h = height
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque

ELECTRICITY AND MAGNETISM

$$F = \frac{kq_1q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$U_E = qV = \frac{kq_1q_2}{r}$$

$$E_{avg} = -\frac{V}{d}$$

$$V = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\right)$$

$$C = \frac{Q}{V}$$

$$C = \frac{\epsilon_0 A}{d}$$

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

$$R = \frac{\rho\ell}{A}$$

$$V = IR$$

$$P = IV$$

$$C_p = C_1 + C_2 + C_3 + \ldots$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

$$R_s = R_1 + R_2 + R_3 + \ldots$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

$$F_B = qvB\sin\theta$$

$$F_B = BI\ell\sin\theta$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$\phi_m = BA\cos\theta$$

$$\mathcal{E}_{avg} = -\frac{\Delta\phi_m}{\Delta t}$$

$$\mathcal{E} = B\ell v$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
ℓ = length
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential (stored) energy
V = electric potential or potential difference
v = velocity or speed
ρ = resistivity
θ = angle
ϕ_m = magnetic flux

ADVANCED PLACEMENT PHYSICS B EQUATIONS DEVELOPED FOR 2012

FLUID MECHANICS AND THERMAL PHYSICS

$\rho = m/V$

$P = P_0 + \rho g h$

$F_{buoy} = \rho V g$

$A_1 \upsilon_1 = A_2 \upsilon_2$

$P + \rho g y + \frac{1}{2}\rho \upsilon^2 = \text{const.}$

$\Delta \ell = \alpha \ell_0 \Delta T$

$H = \dfrac{kA\Delta T}{L}$

$P = \dfrac{F}{A}$

$PV = nRT = Nk_B T$

$K_{avg} = \frac{3}{2} k_B T$

$\upsilon_{rms} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3k_B T}{\mu}}$

$W = -P\Delta V$

$\Delta U = Q + W$

$e = \left|\dfrac{W}{Q_H}\right|$

$e_c = \dfrac{T_H - T_C}{T_H}$

A = area
e = efficiency
F = force
h = depth
H = rate of heat transfer
k = thermal conductivity
K_{avg} = average molecular kinetic energy
ℓ = length
L = thickness
m = mass
M = molar mass
n = number of moles
N = number of molecules
P = pressure
Q = heat transferred to a system
T = temperature
U = internal energy
V = volume
υ = velocity or speed
υ_{rms} = root-mean-square velocity
W = work done on a system
y = height
α = coefficient of linear expansion
μ = mass of molecule
ρ = density

ATOMIC AND NUCLEAR PHYSICS

$E = hf = pc$

$K_{\max} = hf - \phi$

$\lambda = \dfrac{h}{p}$

$\Delta E = (\Delta m)c^2$

E = energy
f = frequency
K = kinetic energy
m = mass
p = momentum
λ = wavelength
ϕ = work function

WAVES AND OPTICS

$\upsilon = f\lambda$

$n = \dfrac{c}{\upsilon}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sin\theta_c = \dfrac{n_2}{n_1}$

$\dfrac{1}{s_i} + \dfrac{1}{s_0} = \dfrac{1}{f}$

$M = \dfrac{h_i}{h_0} = -\dfrac{s_i}{s_0}$

$f = \dfrac{R}{2}$

$d\sin\theta = m\lambda$

$x_m \approx \dfrac{m\lambda L}{d}$

d = separation
f = frequency or focal length
h = height
L = distance
M = magnification
m = an integer
n = index of refraction
R = radius of curvature
s = distance
υ = speed
x = position
λ = wavelength
θ = angle

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

Rectangular Solid
$V = \ell w h$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

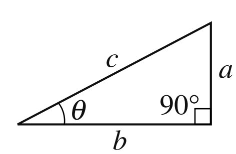
Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

Right Triangle
$a^2 + b^2 = c^2$

$\sin\theta = \dfrac{a}{c}$

$\cos\theta = \dfrac{b}{c}$

$\tan\theta = \dfrac{a}{b}$



A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

ADVANCED PLACEMENT PHYSICS C EQUATIONS DEVELOPED FOR 2012

MECHANICS

$\upsilon = \upsilon_0 + at$

$x = x_0 + \upsilon_0 t + \frac{1}{2}at^2$

$\upsilon^2 = \upsilon_0^{\,2} + 2a(x - x_0)$

$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_{fric} \le \mu N$

$W = \int \mathbf{F} \cdot d\mathbf{r}$

$K = \frac{1}{2}m\upsilon^2$

$P = \frac{dW}{dt}$

$P = \mathbf{F} \cdot \mathbf{v}$

$\Delta U_g = mgh$

$a_c = \frac{\upsilon^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2 dm = \sum mr^2$

$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$

$\upsilon = r\omega$

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
υ = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2}kx^2$

$x = x_{\max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa\epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$

$R = \frac{\rho\ell}{A}$

$\mathbf{E} = \rho\mathbf{J}$

$I = Nev_d A$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
υ = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$

$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$

$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 nI$

$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$

$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS DEVELOPED FOR 2012

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

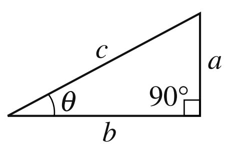
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

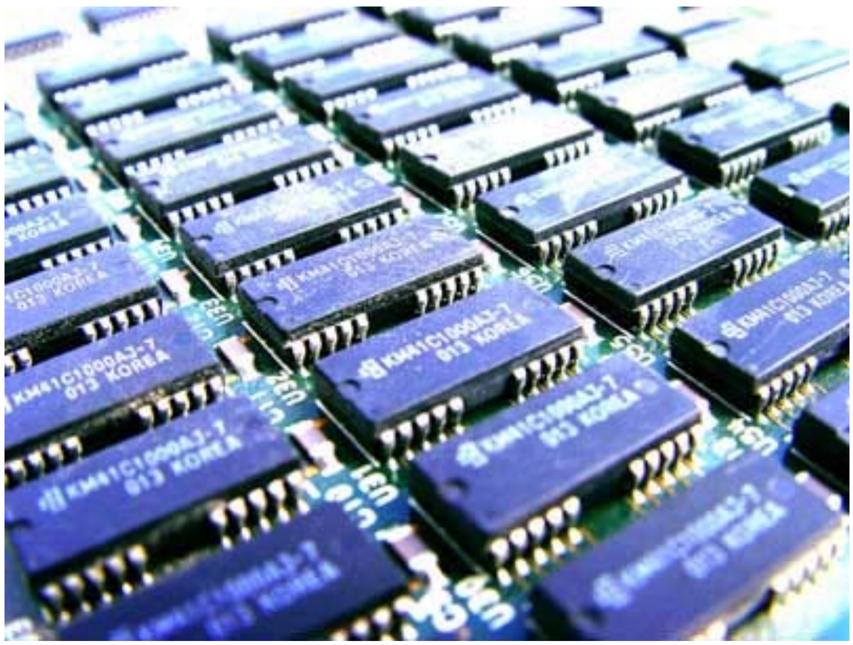
$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

Chapter 9

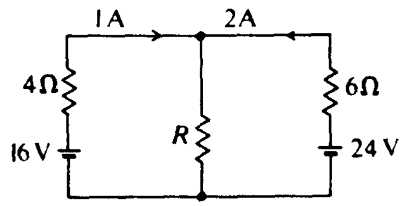
Circuits



AP Physics C Multiple Choice Practice – Circuits

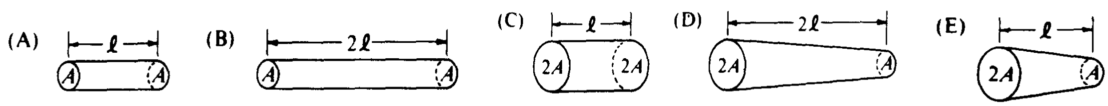
1. When lighted, a 100-watt light bulb operating on a 110-volt household circuit has a resistance closest to
(A) 10^{-2} Ω (B) 10^{-1} Ω (C) 1 Ω (D) 10 Ω (E) 100 Ω

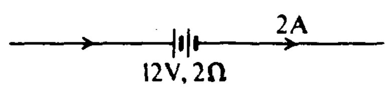
2. If i is current, t is time, E is electric field intensity, and x is distance, the ratio of $\int idt$ to $\int Edx$ may be expressed in
(A) coulombs (B) joules (C) newtons (D) farads (E) henrys



3. In the circuit shown above, what is the resistance R ?
(A) 3 Ω (B) 4 Ω (c) 6 Ω (D) 12 Ω (E) 18 Ω

4. The five resistors shown below have the lengths and cross-sectional areas indicated and are made of material with the same resistivity. Which has the greatest resistance?

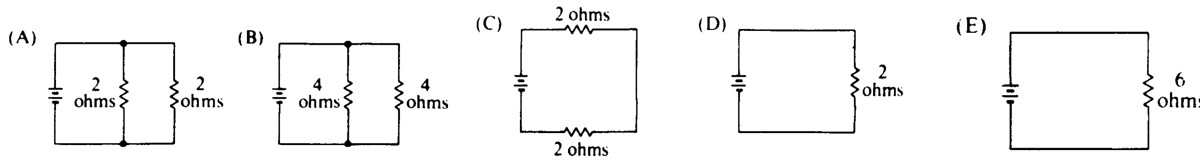




5. A 12-volt storage battery, with an internal resistance of 2Ω, is being charged by a current of 2 amperes as shown in the diagram above. Under these circumstances, a voltmeter connected across the terminals of the battery will read
(A) 4 V (B) 8 V (C) 10 V (D) 12 V (E) 16 V

6. A galvanometer has a resistance of 99 ohms and deflects full scale when a current of 10^{-3} ampere passes through it. In order to convert this galvanometer into an ammeter with a full-scale deflection of 0.1 ampere, one should connect a resistance of
(A) 1 Ω in series with it
(B) 901 Ω in series with it
(C) 9,900 Ω in series with it
(D) 1 Ω in parallel with it
(E) 9,900 Ω in parallel with it

Questions 7-9



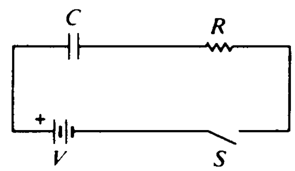
The batteries in each of the circuits shown above are identical and the wires have negligible resistance.

7. In which circuit is the current furnished by the battery the greatest?
   (A) (B) (C) (D) (E)

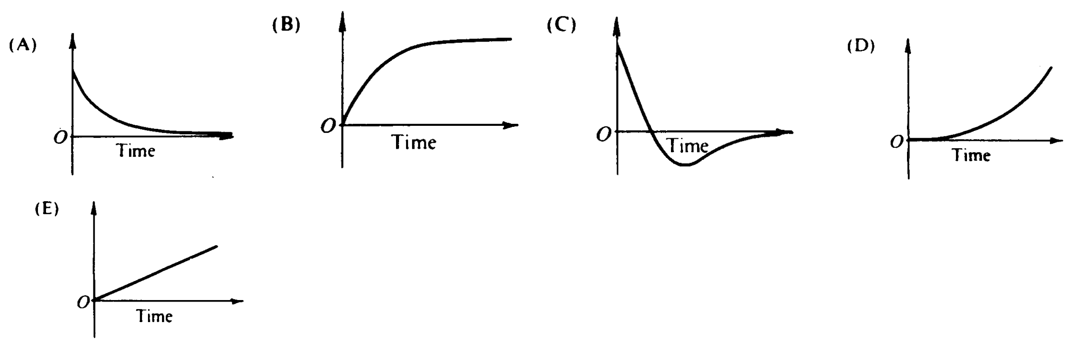
8. In which circuit is the equivalent resistance connected to the battery the greatest?
   (A) (B) (C) (D) (E)

9. Which circuit dissipates the least power?
   (A) (B) (C) (D) (E)

Questions 10-12 refer to the circuit shown below.



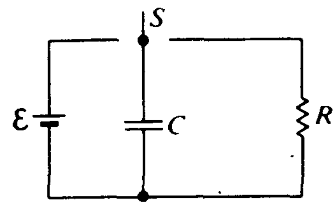
Assume the capacitor C is initially uncharged. The following graphs may represent different quantities related to the circuit as functions of time t after the switch S is closed



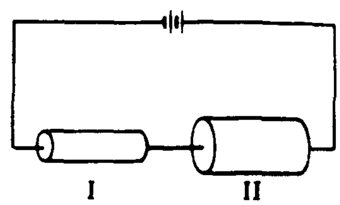
10. Which graph best represents the voltage versus time across the resistor R ?
    (A) (B) (C) (D) (E)

11. Which graph best represents the current versus time in the circuit?
    (A) (B) (C) (D) (E)

12. Which graph best represents the voltage across the capacitor versus time?
    (A) (B) (C) (D) (E)

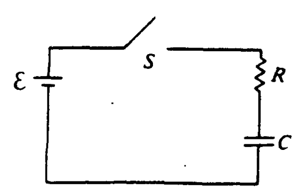


13. In the circuit shown above, the capacitor C is first charged by throwing switch S to the left, then discharged by throwing S to the right. The time constant for discharge could be increased by which of the following?
    (A) Placing another capacitor in parallel with C
    (B) Placing another capacitor in series with C
    (C) Placing another resistor in parallel with the resistor R
    (D) Increasing battery emf $\mathcal{E}$
    (E) Decreasing battery emf $\mathcal{E}$

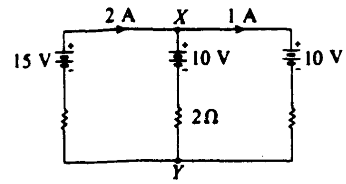


14. Two resistors of the same length, both made of the same material, are connected in a series to a battery as shown above. Resistor II has a greater cross sectional area than resistor I. Which of the following quantities has the same value for each resistor?
    (A) Potential difference between the two ends
    (B) Electric field strength within the resistor
    (C) Resistance
    (D) Current per unit area
    (E) Current

15. The emf of a battery is 12 volts. When the battery delivers a current of 0.5 ampere to a load, the potential difference between the terminals of the battery is 10 volts. The internal resistance of the battery is
    (A) 1 Ω (B) 2 Ω (C) 4 Ω (D) 20 Ω (E) 24 Ω



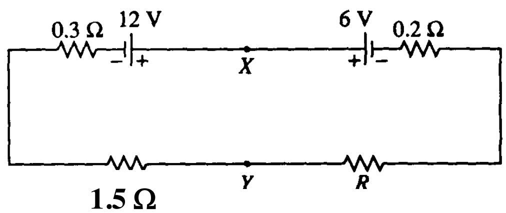
16. In the circuit shown above, the capacitor is initially uncharged. At time t = 0, switch S is closed. The natural logarithmic base is e. Which of the following is true at time t = RC?
    (A) The current is $\mathcal{E}$/eR .
    (B) The current is $\mathcal{E}$/R
    (C) The voltage across the capacitor is $\mathcal{E}$.
    (D) The voltage across the capacitor is $\mathcal{E}$/e .
    (E) The voltages across the capacitor and resistor are equal.



17. In the circuit shown above, the emf's of the batteries are given, as well as the currents in the outside branches and the resistance in the middle branch. What is the magnitude of the potential difference between X and Y ?
(A) 4 V (B) 8 V (C) 10 V (D) 12 V (E) 16 V

18. The power dissipated in a wire carrying a constant electric current I may be written as a function of I, the length l of the wire, the diameter d of the wire, and the resistivity ρ of the material in the wire. In this expression, the power dissipated is directly proportional to which of the following?
A) l only B) d only C) l and ρ only D) d and ρ only E) l, d, and ρ

Questions 19-21



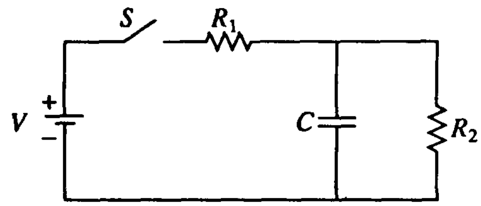
In the circuit above, the emf's and the resistances have the values shown. The current I in the circuit is 2 amperes.

19. The resistance R is
A) 1 Ω B) 2Ω C) 3 Ω D) 4 Ω E) 6 Ω

20. The potential difference between points X and Y is
A) 1.2 V B) 6.0 V C) 8.4 V D) 10.8 V E) 12.2 V

21. How much energy is dissipated by the 1.5-ohm resistor in 60 seconds?
A) 6 J B) 180 J C) 360 J D) 720 J E) 1,440 J

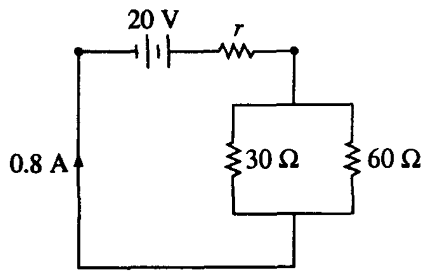
Questions 22-23



In the circuit shown above, the battery supplies a constant voltage V when the switch S is closed. The value of the capacitance is C, and the value of the resistances are R_1 and R_2.

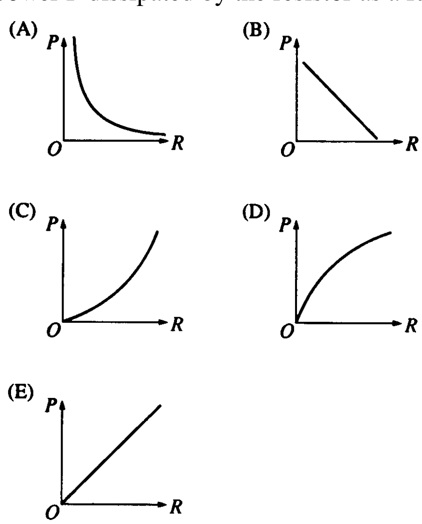
22. Immediately after the switch is closed, the current supplied by the battery is
A) $V/(R_1 + R_2)$ B) V/R_1 C) V/R_2 D) $V(R_1 + R_2)/R_1R_2$ E) zero

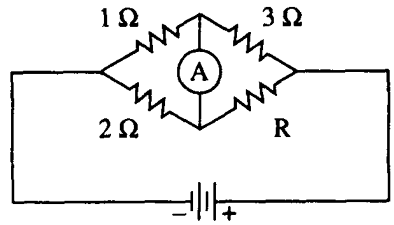
23. A long time after the switch has been closed, the current supplied by the battery is
A) $V/(R_1 + R_2)$ B) V/R_1 C) V/R_2 D) $V(R_1 + R_2)/R_1R_2$ E) zero



24. A 30-ohm resistor and a 60-ohm resistor are connected as shown above to a battery of emf 20 volts and internal resistance *r*. The current in the circuit is 0.8 ampere. What is the value of *r*?
A) 0.22 Ω B) 4.5 Ω C) 5 Ω D) 16Ω E) 70 Ω

25. A variable resistor is connected across a constant voltage source. Which of the following graphs represents the power P dissipated by the resistor as a function of its resistance R ?

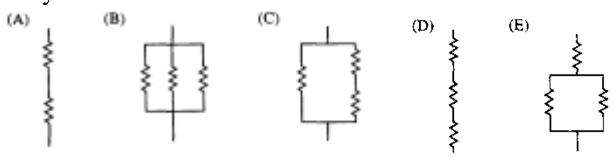




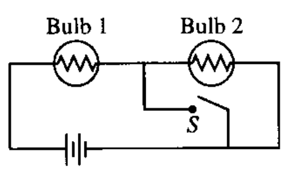
26. If the ammeter in the circuit above reads zero, what is the resistance R?
A) 1.5 Ω B) 2Ω C) 4 Ω D) 5 Ω E) 6Ω

27. A resistor R and a capacitor C are connected in series to a battery of terminal voltage V_0. Which of the following equations relating the current I in the circuit and the charge Q on the capacitor describes this circuit?
(A) $V_0 + QC - I^2R = 0$ (B) $V_0 - Q/C - IR = 0$ (C) $V_0^2 - Q^2/2C - I^2R = 0$
(D) $V_0 - C(dQ/dt) - I^2R = 0$ (E) $Q/C - IR = 0$

28. Which of the following combinations of 4Ω resistors would dissipate 24 W when connected to a 12 Volt battery?



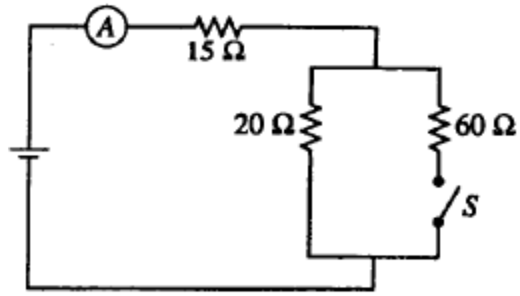
29. A wire of resistance *R* dissipates power *P* when a current *I* passes through it. The wire is replaced by another wire with resistance *3R*. The power dissipated by the new wire when the same current passes through it is
(A) P/9 (B) P/3 (C) P (D) 3P (E) 6P



30. The circuit in the figure above contains two identical lightbulbs in series with a battery. At first both bulbs glow with equal brightness. When switch S is closed, which of the following occurs to the bulbs?

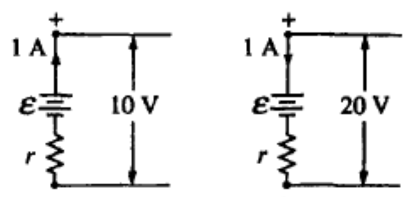
	Bulb 1	Bulb 2
(A)	Goes out	Gets brighter
(B)	Gets brighter	Goes out
(C)	Gets brighter	Gets slightly dimmer
(D)	Gets slightly dimmer	Gets brighter
(E)	Nothing	Goes out

31. A hair dryer is rated as 1200 W, 120 V. Its effective internal resistance is
(A) 0.1 Ω (B) 10 Ω (C) 12Ω (D) 120 Ω (E) 1440 Ω

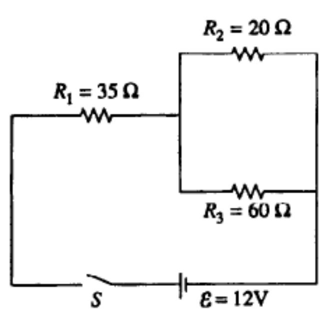


32. When the switch S is open in the circuit shown above, the reading on the ammeter A is 2.0 A. When the switch is closed, the reading on the ammeter is
(A) doubled
(B) increased slightly but not doubled
(C) the same
(D) decreased slightly but not halved
(E) halved

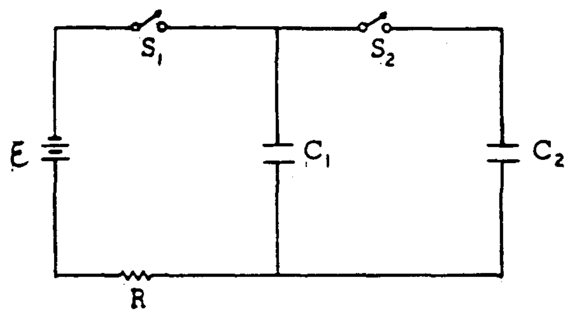
33. Two conducting cylindrical wires are made out of the same material. Wire X has twice the length and twice the diameter of wire Y. What is the ratio R_x/R_y of their resistances?
(A) 1/4 (B) ½ (C) 1 (D) 2 (E) 4



34. The figures above show parts of two circuits, each containing a battery of emf ε and internal resistance *r*. The current in each battery is 1 A, but the direction of the current in one battery is opposite to that in the other. If the potential differences across the batteries' terminals are 10 V and 20 V as shown, what are the values of $\mathcal{E}$ and *r*?
(A) $\mathcal{E}$ = 5 V, r = 15 Ω
(B) $\mathcal{E}$ =10 V, r = 100 Ω
(C) $\mathcal{E}$ = 15 V, r = 5 Ω
(D) $\mathcal{E}$ = 20 V, r = 10 Ω
(E) The values cannot be computed unless the complete circuits are shown.



35. In the circuit shown above, the equivalent resistance of the three resistors is
(A) 10.5 Ω (B) 15Ω (C) 20 Ω (D) 50 Ω (E) 115 Ω



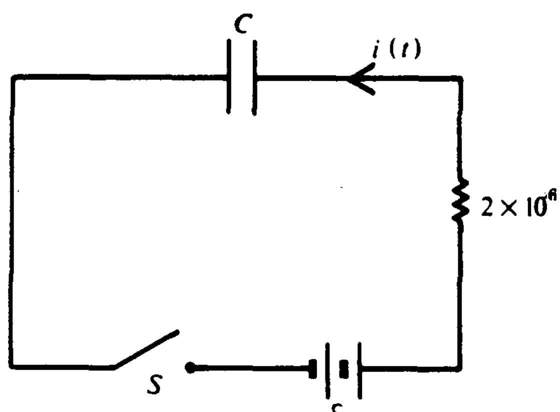
1975E2. In the diagram above, V = 100 volts; C_1 = 12 microfarads; C_2 = 24 microfarads; R = 10 ohms. Initially, C_1 and C_2 are uncharged, and all switches are open.

a. First, switch S_1 is closed. Determine the charge on C_1 when equilibrium is reached.

b. Next S_1 is opened and afterward S_2 is closed. Determine the charge on C_1 when equilibrium is again reached.

c. For the equilibrium condition of part (b), determine the voltage across C_1.

d. S_2 remains closed, and now S_1 is also closed. How much <u>additional</u> charge flows from the battery?

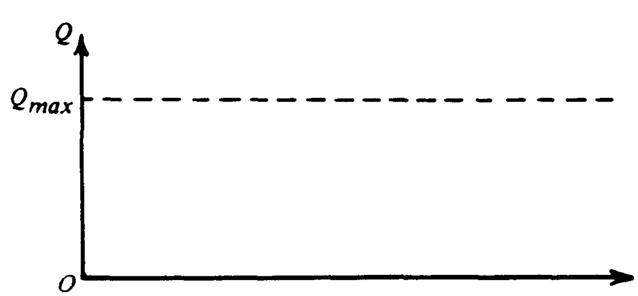


1983E2. The series circuit shown above contains a resistance $R = 2 \times 10^6$ ohms, a capacitor of unknown capacitance C, and a battery of unknown emf $\mathcal{E}$ and negligible internal resistance. Initially the capacitor is uncharged and the switch S is open. At time t = 0 the switch S is closed. For t > 0 the current in the circuit is described by the equation:

$i(t) = i_o e^{-t/6}$ where i_o = 10 microamperes and t is in seconds.

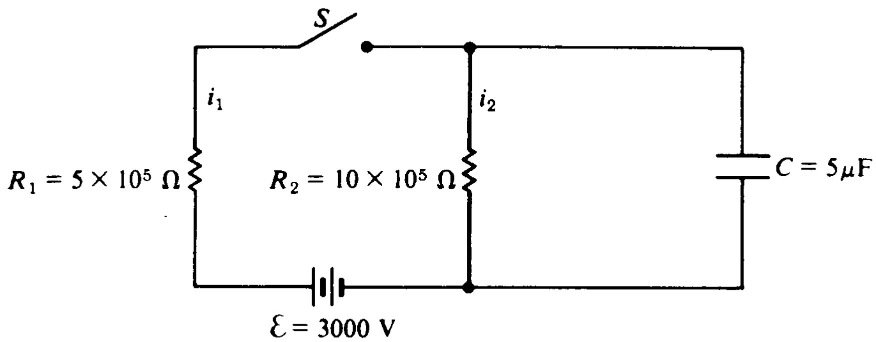
a. Determine the emf of the battery.

b. By evaluating an appropriate integral, develop an expression for the charge on the right-hand plate of the capacitor as a function of time for t > 0.



c. On the axes below sketch a graph of the charge Q on the capacitor as a function of time t.

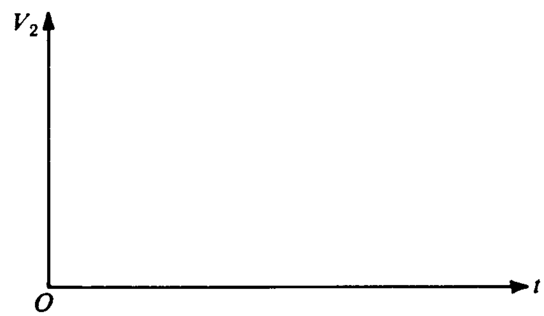
d. Determine the capacitance C.



1985E2. In the circuit shown above, i_1 and i_2 are the currents through resistors R_1 and R_2, respectively. V_1, V_2, and V_c are the potential differences across resistor R_1 resistor R_2, and capacitor C, respectively. Initially the capacitor is uncharged.

a. Calculate the current i_1 immediately after switch S is closed.

b. On the axes below, sketch the potential difference V_2 as a function of time t.



Assume switch S has been closed for a long time.

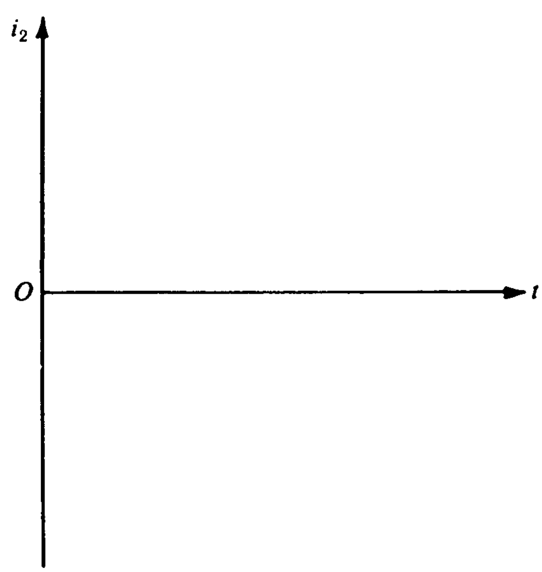
c. Calculate the current i_2.

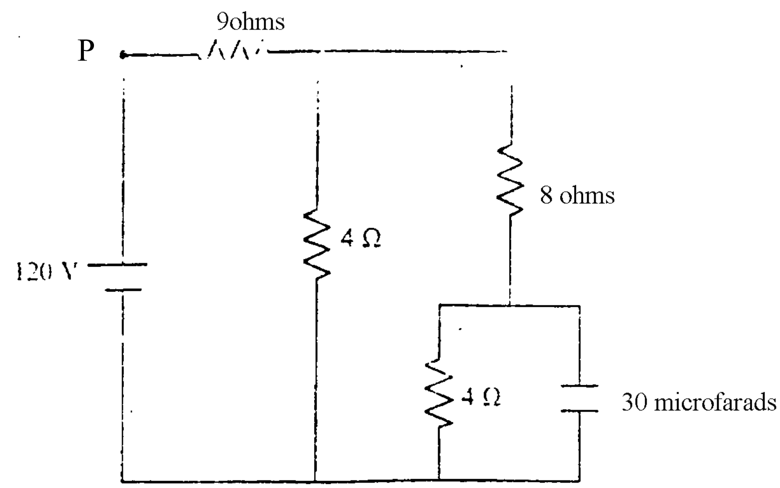
d. Calculate the charge Q on the capacitor.

e. Calculate the energy U stored in the capacitor.

Now the switch S is opened.

f. On the axes below, sketch the current i_2 as a function of time t and clearly indicate initial and final values.



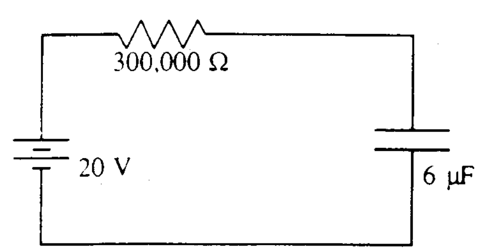


1988E2. In the circuit shown above. the battery has been connected for a long time so that the currents have steady values. Given these conditions, calculate each of the following

a. The current in the 9-ohm resistor.
b. The current in the 8-ohm resistor.
c. The potential difference across the 30-microfarad capacitor.
d. The energy stored in the 30-microfarad capacitor.

At some instant, the connection at point P fails, and the current in the 9-ohm resistor becomes zero.

e. Calculate the total amount of energy dissipated in the 8-ohm resistor after the connection fails.



1989E3. A battery with an emf of 20 volts is connected in series with a resistor of 300,000 ohms and an air-filled parallel-plate capacitor of capacitance 6 microfarads.

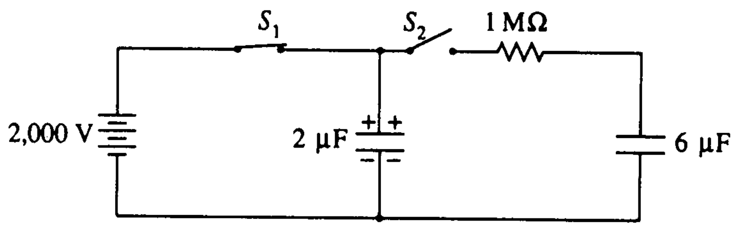
a. Determine the energy stored in the capacitor when it is fully charged.

The spacing between the capacitor plates is suddenly increased (in a time short compared to the time constant of the circuit) to four times its original value.

b. Determine the work that must be done in increasing the spacing in this fashion.
c. Determine the current in the resistor immediately after the spacing is increased.

After a long time. the circuit reaches a new static state.

d. Determine the total charge that has passed through the battery.
e. Determine the energy that has been added to the battery.



1992E2. The 2-microfarad (2×10^{-6} farad) capacitor shown in the circuit above is fully charged by closing switch S_1 and keeping switch S_2 open, thus connecting the capacitor to the 2,000-volt power supply.

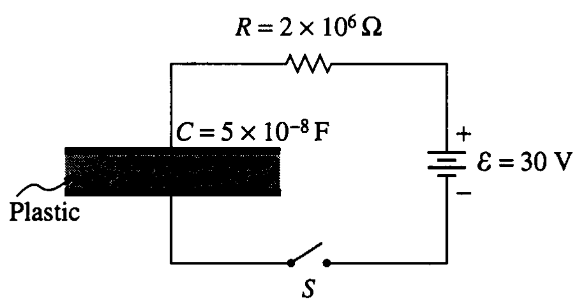
a. Determine each of the following for this fully charged capacitor.
   i. The magnitude of the charge on each plate of the capacitor.
   ii. The electrical energy stored in the capacitor.

   At a later time, switch S_1 is opened. Switch S_2 is then closed, connecting the charged 2-microfarad capacitor to a 1×10^6 Ω resistor and a 6-microfarad capacitor, which is initially uncharged.

b. Determine the initial current in the resistor the instant after switch S_2 is closed.

   Equilibrium is reached after a long period of time.

c. Determine the charge on the positive plate of each of the capacitors at equilibrium.
d. Determine the total electrical energy stored in the two capacitors at equilibrium. If the energy is greater than the energy determined in part (a) ii., where did the increase come from? If the energy is less than the energy determined in part (a) ii., where did the electrical energy go?

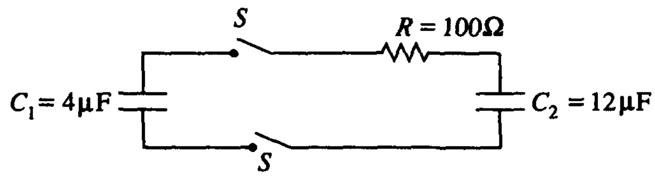


1995E2. A parallel-plate capacitor is made from two sheets of metal, each with an area of 1.0 square meter, separated by a sheet of plastic 1.0 millimeter (10^{-3} m) thick, as shown above. The capacitance is measured to be 0.05 microfarad (5×10^{-8} F).

a. What is the dielectric constant of the plastic?
b. The uncharged capacitor is connected in series with a resistor $R = 2 \times 10^6$ ohms, a 30-volt battery, and an open switch S, as shown above. The switch is then closed.
   i. What is the initial charging current when the switch S is closed?
   ii. What is the time constant for this circuit?
   iii. Determine the magnitude and sign of the final charge on the bottom plate of the fully charged capacitor.
   iv. How much electrical energy is stored in the fully charged capacitor?

   After the capacitor is fully charged, it is carefully disconnected, leaving the charged capacitor isolated in space. The plastic sheet is then removed from between the metal plates. The metal plates retain their original separation of 1.0 millimeter.

c. What is the new voltage across the plates?
d. If there is now more energy stored in the capacitor, where did it come from? If there is now less energy, what happened to it?



1996E2. Capacitors 1 and 2, of capacitance C_1 = 4μF and C_2 = 12μF, respectively, are connected in a circuit as shown above with a resistor of resistance R = 100Ω and two switches. Capacitor 1 is initially charged to a voltage V_o = 50 V, and capacitor 2 is initially uncharged. Both of the switches S are then closed at time t = 0.

a. What are the final charges on the positive plate of each of the capacitors 1 and 2 after equilibrium has been reached?

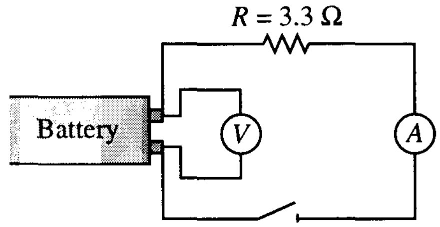
b. Determine the difference between the initial and the final stored energy of the system after equilibrium has been reached.

c. Write, but do not solve, an equation that, at any time after the switches are closed, relates the charge on capacitor C_1, its time derivative (which is the instantaneous current in the circuit), and the parameters V_o, R, C_1, and C_2.

The current in the resistor is given as a function of time by $I = I_o e^{-t/\tau}$, where I_o = 0.5A and $\tau = 3 \times 10^{-4}$s.

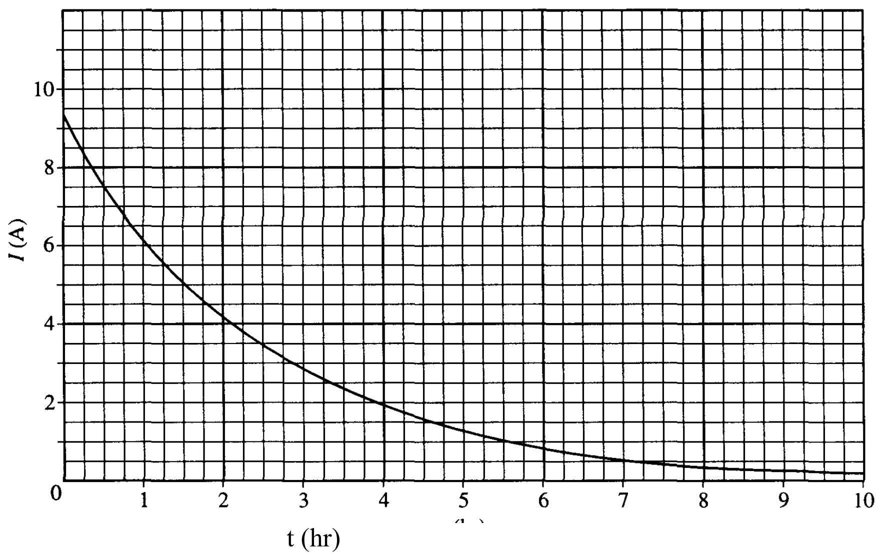
d. Determine the rate of energy dissipation in the resistor as an explicit function of time.

e. How much energy is dissipated in the resistor from the instant the switch is closed to when equilibrium is reached?



1997E1. A technician uses the circuit shown above to test prototypes of a new battery design. The switch is closed, and the technician records the current for a period of time. The curve that best fits the results is shown in the graph below.

The equation for this curve is $I = I_0 e^{-kt}$ where t is the time elapsed from the instant the switch is closed and I_o and k are constants.



a. i. Using the information in the graph, determine the potential difference V_o across the resistor immediately after the switch is closed.

ii. Would the open circuit voltage of the fresh battery have been less than, greater than, or equal to the value in part i? Justify your answer.

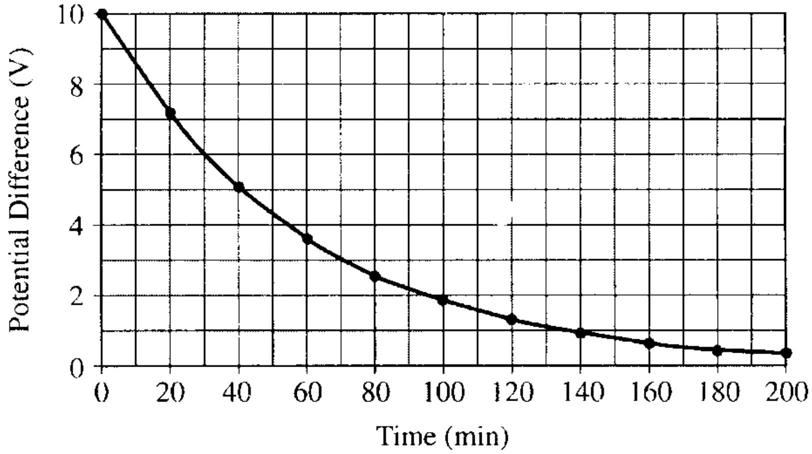
b. Determine the value of k from this best-fit curve. Show your work and be sure to include units in your answer.

c. Determine the following in terms of R, I_o, k, and t.

i. The power delivered to the resistor at time t = 0

ii. The power delivered to the resistor as a function of time t

iii. The total energy delivered to the resistor from t = 0 until the current is reduced to zero



2001E2. You have been hired to determine the internal resistance of 8.0 μF capacitors for an electronic component manufacturer. (Ideal capacitors have an infinite internal resistance - that is, the material between their plates is a perfect insulator. In practice, however, the material has a very small, but nonzero, conductivity.) You cannot simply connect the capacitors to an ohmmeter, because their resistance is too large for an ohmmeter to measure. Therefore you charge the capacitor to a potential difference of 10 V with a battery, disconnect it from the battery and measure the potential difference across the capacitor every 20 minutes with an ideal voltmeter, obtaining the graph shown above.

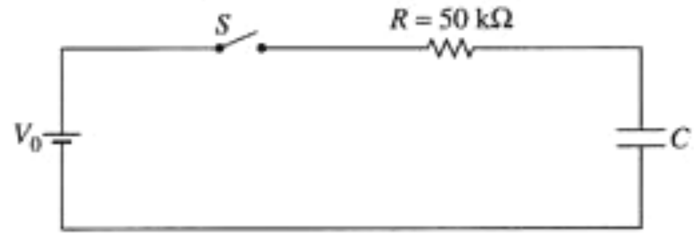
a. Determine the internal resistance of the capacitor.

The capacitor can be approximated as a parallel-plate capacitor separated by a 0.10 mm thick dielectric with $\kappa = 5.6$.

b. Determine the approximate surface area of one of the capacitor "plates."

c. Determine the resistivity of the dielectric.

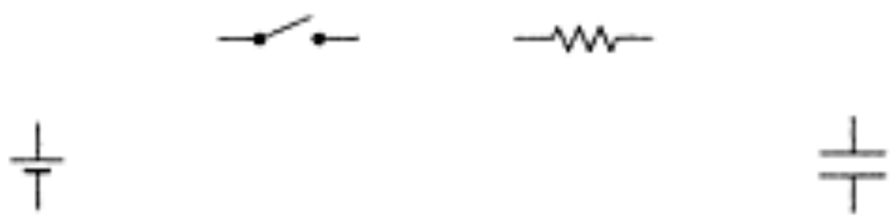
d. Determine the magnitude of the charge leaving the positive plate of the capacitor in the first 100 min.



2002E2. Your engineering firm has built the *RC* circuit shown above. The current is measured for the time t after the switch is closed at t = 0 and the best-fit curve is represented by the equation $I(t) = 5.20\ e^{-t/10}$, where I is in milliamperes and t is in seconds.

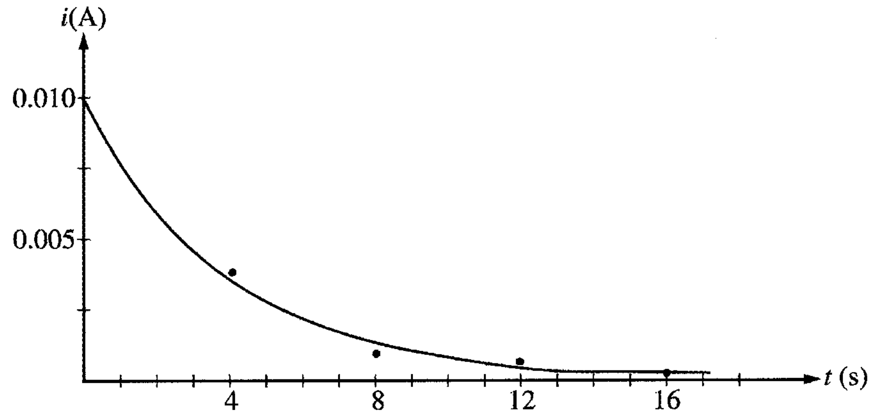
a. Determine the value of the charging voltage V_o predicted by the equation.
b. Determine the value of the capacitance C predicted by the equation.
c. The charging voltage is measured in the laboratory and found to be greater than predicted in part a.
i. Give one possible explanation for this finding.
ii. Explain the implications that your answer to part i has for the predicted value of the capacitance.
d. Your laboratory supervisor tells you that the charging time must be decreased. You may add resistors or capacitors to the original components and reconnect the *RC* circuit. In parts i and ii below, show how to reconnect the circuit, using either an additional resistor or a capacitor to decrease the charging time.

i. Indicate how a resistor may be added to decrease the charging time. Add the necessary resistor and connections to the following diagram.



ii. Instead of a resistor, use a capacitor. Indicate how the capacitor may be added to decrease the charging time. Add the necessary capacitor and connections to the following diagram.

2003E2. 1n the laboratory, you connect a resistor and a capacitor with unknown values in series with a battery of emf $\mathcal{E}$ = 12 V. You include a switch in the circuit. When the switch is closed at time $t = 0$, the circuit is completed, and you measure the current through the resistor as a function of time as plotted below.



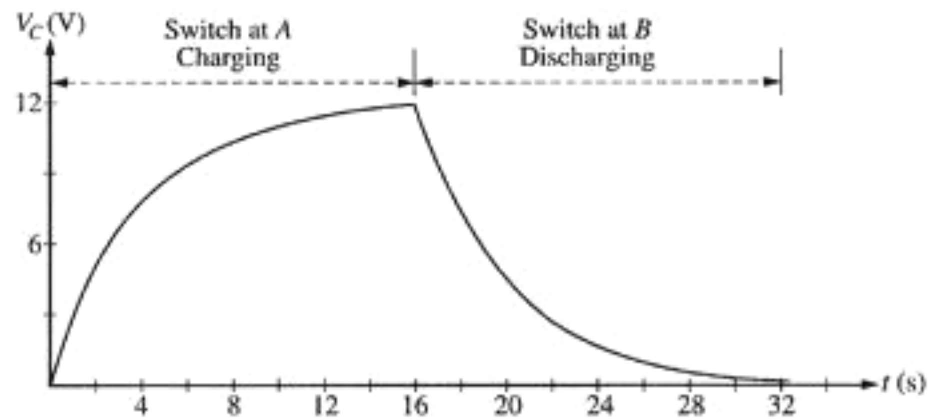
A data-fitting program finds that the current decays according to the equation $i(t) = \frac{\mathcal{E}}{R} e^{-t/4}$

a. Using common symbols for the battery, the resistor, the capacitor, and the switch, draw the circuit that you constructed. Show the circuit before the switch is closed and include whatever other devices you need to measure the current through the resistor to obtain the above plot. Label each component in your diagram.

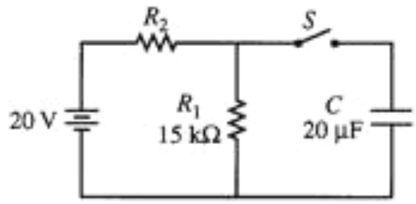
b. Having obtained the curve shown above, determine the value of the resistor that you placed in this circuit.

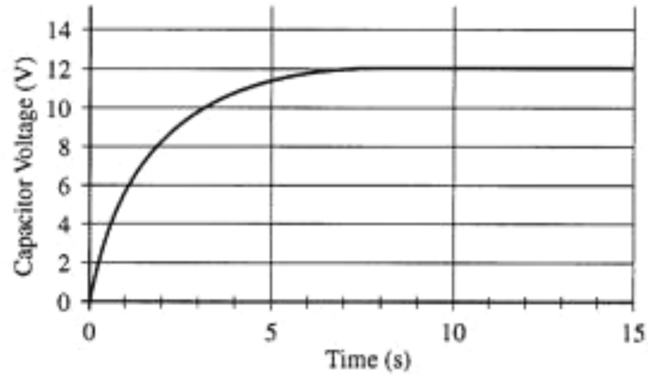
c. What capacitance did you insert in the circuit to give the result above?

You are now asked to reconnect the circuit with a new switch in such a way as to charge and discharge the capacitor. When the switch in the circuit is in position *A*, the capacitor is charging; and when the switch is in position *B*, the capacitor is discharging, as represented by the graph below of voltage V_C across the capacitor as a function of time



d. Draw a schematic diagram of the RC circuit that you constructed that would produce the graph above. Clearly indicate switch positions *A* and *B* on your circuit diagram and include whatever other devices you need to measure the voltage across the capacitor to obtain the above plot. Label each component in your diagram.

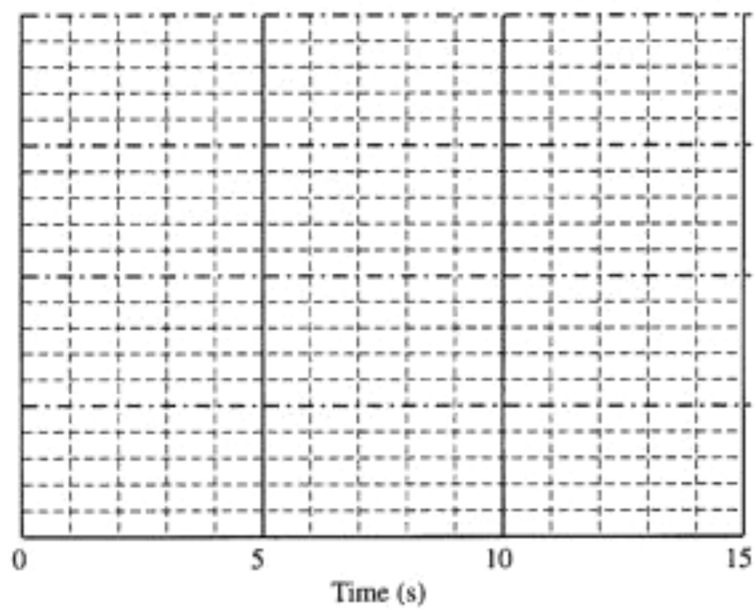




2004E2. In the circuit shown above left, the switch S is initially in the open position and the capacitor C is initially uncharged. A voltage probe and a computer (not shown) are used to measure the potential difference across the capacitor as a function of time after the switch is closed. The graph produced by the computer is shown above right. The battery has an emf of 20 V and negligible internal resistance. Resistor R_1 has a resistance of 15 kΩ and the capacitor C has a capacitance of 20 μF.

a. Determine the voltage across resistor R_2 immediately after the switch is closed.
b. Determine the voltage across resistor R_2 a long time after the switch is closed.
c. Calculate the value of the resistor R_2.
d. Calculate the energy stored in the capacitor a long time after the switch is closed.
e. On the axes below, graph the current in R_2 as a function of time from 0 to 15 s. Label the vertical axis with appropriate values.

Current in R_2

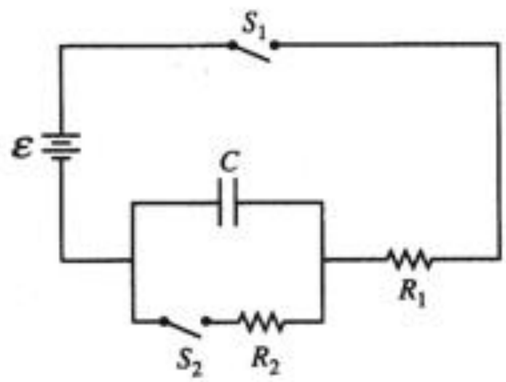


Resistor R_2 is removed and replaced with another resistor of lesser resistance. Switch S remains closed for a long time.

(f) Indicate below whether the energy stored in the capacitor is greater than, less than, or the same as it was with resistor R_2 in the circuit.

_____Greater than _____Less than _____The same as

Explain your reasoning.



2006E2. The circuit above contains a capacitor of capacitance C, a power supply of emf $\mathcal{E}$, two resistors of resistances R_1 and R_2, and two switches, S_1 and S_2. Initially, the capacitor is uncharged and both switches are open. Switch S_1 then gets closed at time t = 0.

a. Write a differential equation that can be solved to obtain the charge on the capacitor as a function of time t.

b. Solve the differential equation in part a. to determine the charge on the capacitor as a function of time.

Numerical values for the components are given as follows:

$\mathcal{E}$ =12V

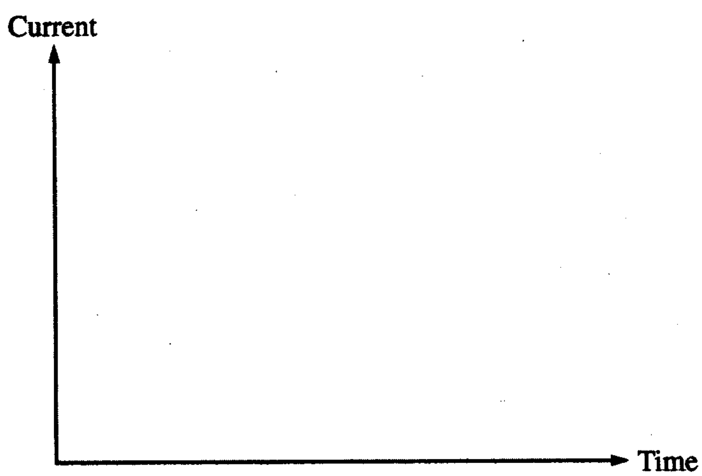
C = 0.060 F

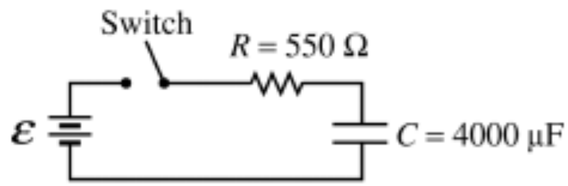
$R_1 = R_2 = 4700\ \Omega$

c. Determine the time at which the capacitor has a voltage 4.0 V across it.

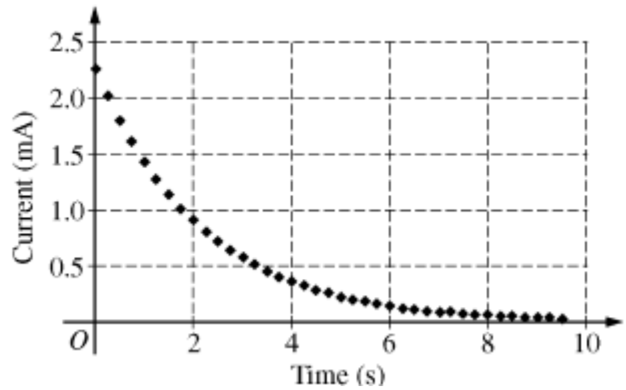
After switch S_1 has been closed for a long time, switch S_2 gets closed at a new time t = 0.

d. On the axes below, sketch graphs of the current I_1 in R_1 versus time and of the current I_2 in R_2 versus time, beginning when switch S_2 is closed at new time t = 0. Clearly label which graph is I_1 and which is I_2.

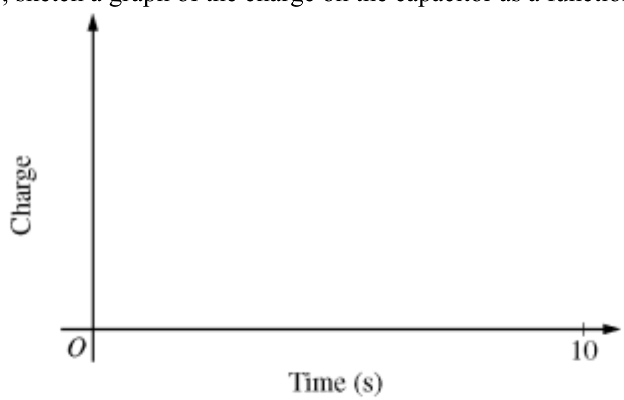




2007E1. A student sets up the circuit above in the lab. The values of the resistance and capacitance are as shown, but the constant voltage $\mathcal{E}$ delivered by the ideal battery is unknown. At time $t = 0$, the capacitor is uncharged and the student closes the switch. The current as a function of time is measured using a computer system, and the following graph is obtained.

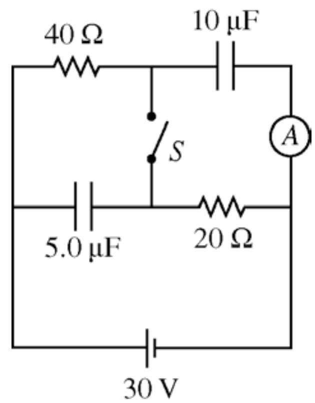


(a) Using the data above, calculate the battery voltage $\mathcal{E}$.
(b) Calculate the voltage across the capacitor at time $t = 4.0$ s.
(c) Calculate the charge on the capacitor at $t = 4.0$ s.
(d) On the axes below, sketch a graph of the charge on the capacitor as a function of time.



(e) Calculate the power being dissipated as heat in the resistor at t = 4.0 s.
(f) The capacitor is now discharged, its dielectric of constant $\kappa = 1$ is replaced by a dielectric of constant $\kappa = 3$, and the procedure is repeated. Is the amount of charge on one plate of the capacitor at $t = 4.0$ s now greater than, less than, or the same as before? Justify your answer.

________Greater than________Less than ________The same



2010E2. In the circuit illustrated above, switch *S* is initially open and the battery has been connected for a long time.

a. What is the steady-state current through the ammeter?
b. Calculate the charge on the 10 mF capacitor.
c. Calculate the energy stored in the 5.0 mF capacitor.

The switch is now closed, and the circuit comes to a new steady state.

d. Calculate the steady-state current through the battery.
e. Calculate the final charge on the 5.0 mF capacitor.
f. Calculate the energy dissipated as heat in the 40 Ω resistor in one minute once the circuit has reached steady state

ANSWERS - AP Physics C Multiple Choice Practice – Circuits

Solution	Answer
1. $P = V^2/R$	E
2. ∫idt is charge, ∫Edx is potential difference. Q/V is capacitance	D
3. The current through R is found using the junction rule at the top junction, where 1 A + 2 A enter giving I = 3 A. Now utilize Kirchhoff's loop rule through the left or right loops: (left side) + 16 V – (1 A)(4 Ω) – (3 A)R = 0 giving R = 4 Ω	B
4. R = ρL/A. Greatest resistance is the longest, narrowest resistor.	B
5. Summing the potential differences from left to right gives V_T = –12 V – (2 A)(2 Ω) = – 16 V. It is possible for $V_T > \mathcal{E}$.	E
6. When a current of 0.1 A passes through the circuit element connected to the galvanometer, we want 10^{-3} A though the "galvanometer" and $0.1–10^{-3}$ = 0.099 A to flow through, bypassing the coil. For full scale deflection we need a voltage of (99 Ω)(10^{-3}A) = .099 V so we need a resistance of (0.099 V)/(0.099 A) = 1 Ω in parallel	D
7. Current is greatest where resistance is least. The resistances are, in order, 1 Ω, 2 Ω, 4 Ω, 2 Ω and 6 Ω.	A
8. See above	E
9. Least power is for the greatest resistance ($P = \mathcal{E}^2/R$)	E
10. When the switch is closed, the circuit behaves as if the capacitor were just a wire and all the potential of the battery is across the resistor. As the capacitor charges, the voltage changes over to the capacitor over time, eventually making the current (and the potential difference across the resistor) zero and the potential difference across the capacitor equal to the emf of the battery.	A
11. See above	A
12. See above	B
13. The time constant is R×C. To increase it, we need to increase either R and/or C. In parallel, capacitors add their capacitances.	A
14. Since these resistors are in series, they must have the same current.	E
15. $V_T = \mathcal{E} – Ir$	C
16. When charging a capacitor, the voltage across the capacitor (and its charge) grows as $(1 – e^{-t})$ while the voltage across the resistor (and the current) decays as e^{-t}	A
17. Kirchhoff's junction rule applied at point X gives 2 A = I + 1 A, so the current in the middle wire is 1 A. Summing the potential differences through the middle wire from X to Y gives – 10 V – (1 A)(2 Ω) = –12 V	D
18. $P = I^2R$ and $R = \rho L/A$ giving $P \propto \rho L/d^2$	C
19. Utilizing Kirchhoff's loop rule starting at the upper left and moving clockwise: – (2 A)(0.3 Ω) + 12 V – 6 V – (2 A)(0.2 Ω) – (2A)(R) – (2A)(1.5 Ω) = 0	A
20. Summing the potential differences: – 6 V – (2 A)(0.2 Ω) – (2A)(1 Ω) = – 8.4 V	C
21. Energy = Pt = I^2Rt	C

22. When the switch is closed, the circuit behaves as if the capacitor were just a wire, shorting out the resistor on the right. B

23. When the capacitor is fully charged, the branch with the capacitor is "closed" to current, effectively removing it from the circuit for current analysis. A

24. Total resistance = $\mathcal{E}/I = 25\ \Omega$. Resistance of the 30 Ω and 60 Ω resistors in parallel = 20 Ω adding the internal resistance in series with the external circuit gives $R_{total} = 20\ \Omega + r = 25\ \Omega$ C

25. $P = V^2/R$ and if V is constant $P \propto 1/R$ A

26. For the ammeter to read zero means the junctions at the ends of the ammeter have the same potential. For this to be true, the potential drops across the 1 Ω and the 2 Ω resistor must be equal, which means the current through the 1 Ω resistor must be twice that of the 2 Ω resistor. This means the resistance of the upper branch (1 Ω and 3 Ω) must be ½ that of the lower branch (2 Ω and R) giving 1 Ω + 3 Ω = ½ (2 Ω + R) E

27. Kirchhoff's loop rule ($V = Q/C$ for a capacitor) B

28. To dissipate 24 W means $R = V^2/P = 6\ \Omega$. The resistances, in order, are: 8 Ω, 4/3 Ω, 8/3 Ω, 12 Ω and 6 Ω E

29. $P = I^2R$ D

30. Closing the switch short circuits Bulb 2 causing no current to flow to it. Since the bulbs were originally in series, this decreases the total resistance and increases the total current, making bulb 1 brighter. B

31. $P = V^2/R$ C

32. Closing the switch reduces the resistance in the right side from 20 Ω to 15 Ω, making the total circuit resistance decrease from 35 Ω to 30 Ω, a slight decrease, causing a slight increase in current. For the current to double, the total resistance must be cut in half. B

33. $R = \rho L/A \propto L/d^2$ where d is the diameter. $R_x/R_y = L_x/d_x^{\,2} \div L_y/d_y^{\,2} = (2L_y)d_y^{\,2}/[L_y(2d_y)^2] = ½$ B

34. Summing the potential differences from bottom to top: C
left circuit: $-(1\text{ A})r + \mathcal{E} = 10\text{ V}$
right circuit: $+(1\text{ A})r + \mathcal{E} = 20\text{ V}$, solve simultaneous equations

35. The equivalent resistance of the 20 Ω and the 60 Ω in parallel is 15 Ω, added to the 35 Ω resistor in series gives 15 Ω + 35 Ω = 50 Ω D

1975E2

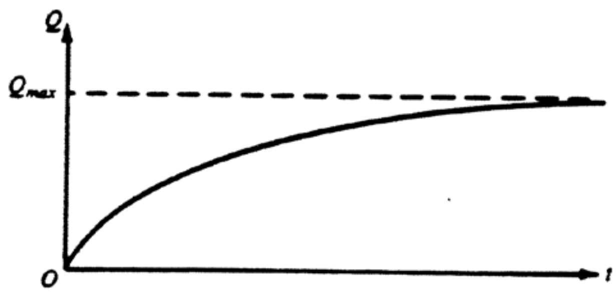
a. $Q = C\mathcal{E} = 12\ \mu F \times 100\ V = 1200\ \mu C$
b. Connecting the two capacitors puts them in parallel with the same voltage so $V_1 = V_2$ and $V = Q/C$ which gives $Q_1/C_1 = Q_2/C_2$ or $Q_1/12 = Q_2/24$ and $Q_2 = 2Q_1$. We also know the total charge is conserved so $Q_1 + Q_2 = 1200\ \mu C$ so we have $Q_1 + 2Q_1 = 1200\ \mu C$ so $Q_1 = 400\ \mu C$
c. $V = Q/C = 33.3\ V$
d. When the battery is reconnected, both capacitors charge to a potential difference of 100 V each. The total charge is then $Q = Q_1 + Q_2 = (C_1 + C_2)V = 3600\ \mu C$ making the *additional* charge from the battery 2400 μC.

1983E2

a. Initially there is no potential drop across the capacitor so $\mathcal{E}=i_0R = 20$ Volts
b.

$$Q = \int i\,dt = \int_0^t i_0 e^{-\frac{t}{6}}\,dt = -6i_0e^{-\frac{t}{6}} - (-6i_0e^0) = 6i_0(1 - e^{-\frac{t}{6}})$$

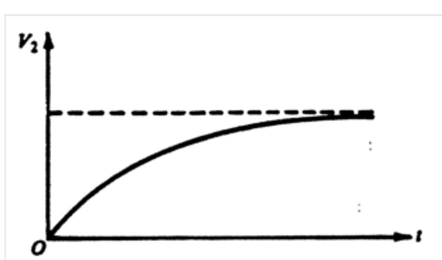
c.



d. For charging a capacitor, the time constant is RC. The numerical value of the time constant is 6 seconds so $C = 6/R = 3 \times 10^{-6}$ F

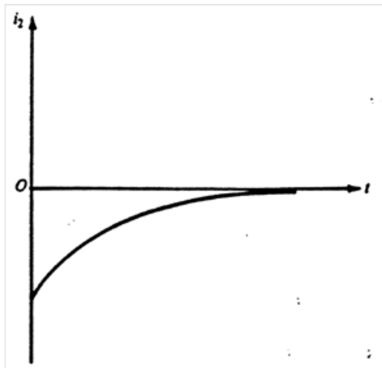
1985E2

a. Immediately after the switch is closed, the capacitor begins charging with current flowing to the capacitor as if it was just a wire. This short circuits R_2 making the total effective resistance of the circuit $5 \times 10^6\ \Omega$ and the total current $\mathcal{E}/R_{eff} = 0.006$ A
b.



c. When the capacitor is fully charged, no current flows through that branch and the circuit behaves as a simple series circuit with a total resistance of $15 \times 10^6\ \Omega$ and a total current of $\mathcal{E}/R = 0.002$ A
d. The voltage across the capacitor is equal to the voltage across the 10 MΩ resistor as they are in parallel. $V_C = V_{10M} = IR = 2000$ V and $Q = CV = 0.01$ C
e. $U_C = ½\ CV^2 = 10$ J

f.



<u>1988E2</u>

a. In their steady states, no current flows through the capacitor so the effective resistance of the branch on the right is 8 Ω + 4 Ω = 12 Ω. This is in parallel with the 4 Ω resistor making their effective resistance (12 × 4)/(12 + 4) = 3 Ω. Adding the 9 Ω resistor in the main branch gives a total circuit resistance of 12 Ω and a total current of $\mathcal{E}$/R = 10 A. This is the current in the 9 Ω resistor as it is in the main branch.
b. With 10 A across the 9 Ω resistor, the potential drop across it is 90 V, leaving 30 V across the two parallel branches on the right. With 30 V across the 12 Ω effective resistance in the right branch, we have a current through that branch (including the 8 Ω resistor) of V/R = 2.5 A
c. $V_C = V_4$ = IR = (2.5 A)(4 Ω) = 10 V
d. $U_C = ½ CV^2$ = 1500 μJ
e. Considering the capacitor as a battery, the equivalent circuit consists of a 4 Ω branch and a branch with a 4 Ω and 8 Ω resistor in series. The current from the discharging capacitor divides in the ratio of 1:3, with the 8 Ω resistor getting ¼ of the total and within the series branch, the 8 Ω resistor will receive 2/3 of the branch voltage so the fraction of the total energy dissipated in the 8 Ω resistor is $(1/4)(2/3)U_{total}$ = 250 μJ

<u>1989E3</u>

a. When charged, the potential difference across the capacitor is 20 V. $U_C = ½ CV^2$ = 1200 μJ
b. Given that the charge is initially unchanged, the work done is the change in the energy stored in the capacitor. Increasing the distance between plates to 4 times the initial value causes the capacitance to decrease to ¼ its initial value (C ∝ 1/d). Since $Q_i = Q_f$ we have $C_iV_i = C_fV_f$ so $V_f = 4V_i$
$W = \Delta U_C = ½ C_fV_f^2 – ½ C_iV_i^2 = ½ (¼ C(4V)^2) – ½ CV^2$ = 3600 μJ
c. After the spacing is increased, the capacitor acts as a battery with a voltage of 4V = 80 V with its emf opposite that of the 20 V battery making the effective voltage supplied to the circuit 80 V – 20 V = 60 V.
$I = \mathcal{E}_{eff}/R = 2 \times 10^{-4}$ A
d. The charge on the capacitor initially was Q = CV = 120 μC and after the plates have been separated and a new equilibrium is reach Q = (¼C)V = 30 μC so the charge that flowed back through the battery is 120 μC – 30 μC = 90 μC
e. For the battery $U = Q_{added}V$ = 1800 μJ

1992E2

a. i. $Q = CV = 4 \times 10^{-3}$ C
   ii. $U_C = \frac{1}{2} CV^2 = 4$ J
b. When the switch is closed, there is no charge on the 6 μF capacitor so the potential difference across the resistor equals that across the 2 μF capacitor, or 2000 V and $I = V/R = 2 \times 10^{-3}$ A
c. In equilibrium, charge is no longer moving so there is no potential difference across the resistor therefore the capacitors have the same potential difference. $V_2 = V_6$ gives $Q_2/C_2 = Q_6/C_6$ giving $Q_6 = 3Q_2$ and since total charge is conserved we have $Q_2 + Q_6 = Q_2 + 3Q_2 = 4Q_2 = 4 \times 10^{-3}$ C so $Q_2 = 1 \times 10^{-3}$ C and $Q_6 = 3 \times 10^{-3}$ C
d. $U_C = U_2 + U_6 = Q_2^2/2C_2 + Q_6^2/2C_6 = 1$ J. This is less than in part a. ii. Part of the energy was converted to heat in the resistor.

1995E2

a. $C = \kappa\varepsilon_0 A/d$ so $\kappa = Cd/\varepsilon_0 A = 5.65$
b. i. When the switch is closed, the voltage across the capacitor is zero thus all the voltage appears across the resistor and $I = \mathcal{E}/R = 1.5 \times 10^{-5}$ A
   ii. $\tau = RC = 0.1$ seconds
   iii. When fully charged, the current has stopped flowing and all the voltage now appears across the capacitor and $Q = CV = 1.5 \times 10^{-6}$ C and since the bottom plate is connected to the negative terminal of the battery the charge on that plate is also negative.
   iv. $U_C = \frac{1}{2} CV^2 = 2.25 \times 10^{-5}$ J
c. Since the capacitor is isolated, the charge on it remains the same. Removing the plastic reduces the capacitance to $C' = \varepsilon_0 A/d = C_{original}/\kappa$ and $V = Q/C' = 170$ V
d. $U' = Q^2/2C' = Q^2/2(C/\kappa) = \kappa(Q^2/2C) = \kappa U > U_{original}$. The increase came from the work that had to be done to remove the plastic from the capacitor.

1996E2

a. The initial charge on C_1 is $Q = CV_0 = 200\mu C$. In equilibrium, charge is no longer moving so there is no potential difference across the resistor therefore the capacitors have the same potential difference. $V_1 = V_2$ gives $Q_1/C_1 = Q_2/C_2$ giving $Q_2 = 3Q_1$ and since total charge is conserved we have $Q_1 + Q_2 = Q_1 + 3Q_1 = 4Q_1 = 200$ μC so $Q_1 = 50$ μC and $Q_2 = 150$ μC
b. $\Delta U = U_f - U_i = (Q_1^2/2C_1 + Q_2^2/2C_2) - \frac{1}{2} C_1 V_0^2 = -3750$ μJ
c. Kirchhoff's loop rule: $\mathcal{E} - IR - V_2 = 0$ with the following substitutions
   $\mathcal{E} = V_1 = Q_1/C_1$
   I = discharge rate of $C_1 = -dQ_1/dt$
   $V_2 = Q_2/C_2$ where $Q_2 = Q_0 - Q_1 = V_0 C_1 - Q_1$ all of which gives
   $Q_1/C_1 + (dQ_1/dt)R - (V_0 C_1 - Q_1)/C_2 = 0$
d.
$$P = I^2 R = \left(I_0 e^{-\frac{t}{\tau}}\right)^2 R = I_0{}^2 R e^{-\frac{2t}{\tau}} \; or \; 25 e^{-\frac{t}{1.5\times10^{-4}}}$$
e. The energy dissipated is equal to the difference in stored energy calculated in part (b): $\Delta U = 3750$ μJ
   (You could also integrate P dt)

1997E1

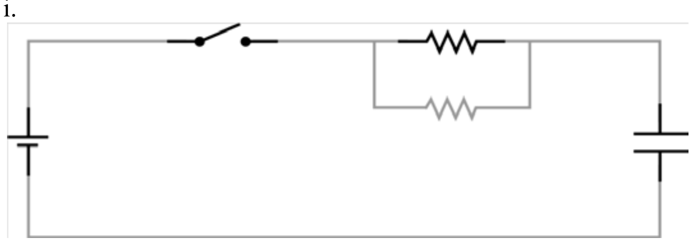
a. i. $V = IR$, from the graph $I_0 \approx 9.3$ A so $V = (9.3\text{ A})(3.3\ \Omega) \approx 31$ V
ii. The battery's open circuit voltage would be greater as a real battery has internal resistance so when it is connected in a circuit only part of the open circuit voltage appears across the external components
b. Taking the natural log of the expression for current gives $\ln(I) = \ln(I_0) - kt\ln(e)$, or $\ln(I/I_0) = -kt$
$k = \ln(I/I_0)/t$ and take a reading from the graph (for example 7.5 A at 0.5 h) gives $k = 0.4\text{ hr}^{-1}$
c. i. $P = I^2R$ at $t = 0$ gives $P = I_0^2R$
ii. substituting the expression for current gives $P = I_0^2Re^{-2kt}$
iii.
$$U = \int P\,dt = \int_0^{\infty} I_0^2Re^{-2kt}dt = I_0^2R\left(-\frac{1}{2k}\right)e^{-2kt}\Big|_0^{\infty} = -\frac{I_0^2R}{2k}(0-1) = \frac{I_0^2R}{2k}$$

2001E2

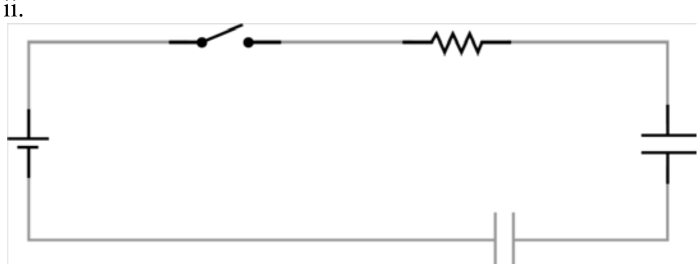
a. One method is to use the equation $V = V_0e^{-t/RC}$ and substitute values from the graph,
e.g. $V_0 = 10$ V, and $V = 2$ V at $t = 100$ min
$2 = 10e^{(-6000\text{ s})/R(8\mu\text{F})}$ giving $R = 4.7 \times 10^8\ \Omega$
b. $C = \kappa\varepsilon_0A/d$ so $A = Cd/\kappa\varepsilon_0 = 16\text{ m}^2$
c. $R = \rho L/A$ giving $\rho = RA/L = 7.2 \times 10^{13}\ \Omega$-m
d. One method is to use $\Delta Q = C\Delta V = (8 \times 10^{-6}\text{ F})(10\text{ V} - 2\text{ V}) = 64\ \mu$C
Another method is to integrate I dt

2002E2

a. $V_0 = I(0)R = (5.2 \times 10^{-3}\text{ A})(50 \times 10^3\ \Omega) = 260$ V
b. at $t = \infty$ the capacitor is fully charged so the total voltage drop occurs across it. We can integrate the current to find the total charge stored
$$Q = \int_0^{\infty} I\,dt = \int_0^{\infty} 5.2\,mA\,e^{-\frac{t}{10}}\,dt = -(10)(5.2\,mA)e^{-t/10}\Big|_0^{\infty} = 52\,mC$$
Now $C = Q/V_0 = 200\ \mu$F
Another method is to realize $\tau = 10$ seconds and $\tau = RC$
c. i. There is resistance in the connecting wires and within the power supply
ii. The predicted value of the capacitance is too high since the actual V_0 should be higher and $C = Q/V_0$
d. i.

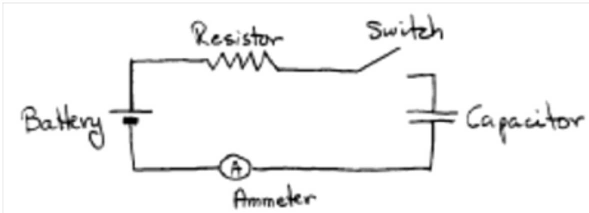


ii.



2003E2

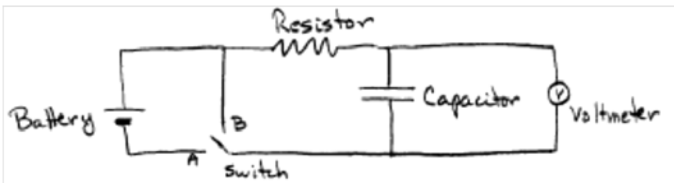
a.



b. $R = V/I = (12\text{ V})/(0.01\text{ A}) = 1200\ \Omega$

c. $\tau = RC$ so $C = \tau/R = (4\text{ s})/(1200\ \Omega) = 3.3 \times 10^{-3}$ F

d.



2004E2

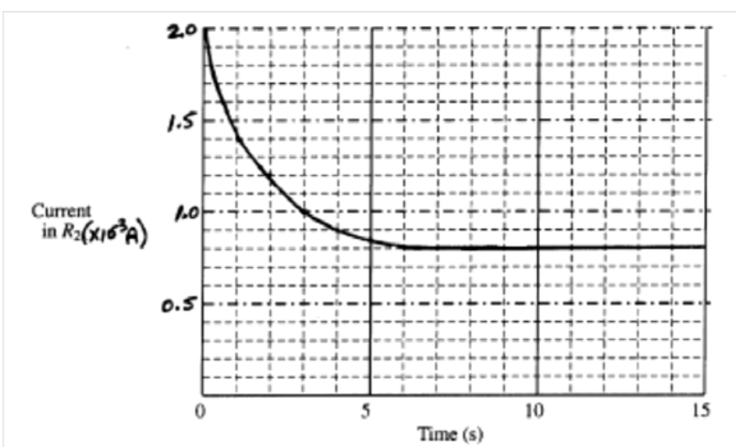
a. Once the switch is closed, $V_{R1} = V_C$ and $V_C = 0$ (initially uncharged) therefore all the voltage drop occurs across R_2 so $V_{R2} = 20$ V

b. From the graph, the maximum voltage across the capacitor (and thus also R_1) is 12 V, the remaining voltage drop occurs across R_2 so $V_{R2} = 8$ V

c. A long time after the switch is closed $I_1 = I_2$ and $I_1 = V_1/R_1 = (12\text{ V})/(15\text{ k}\Omega) = (8\text{ V})/R_2$ so $R_2 = 10\text{ k}\Omega$

d. $U = \frac{1}{2} CV^2 = \frac{1}{2}(20\ \mu\text{F})(12\text{ V})^2 = 1.44 \times 10^{-3}$ J

e.



f. The energy would be greater. Consider the circuit a long time after the switch is closed, when there is no current in the capacitor. If R_2 is replaced with a smaller resistance, then the total resistance decreases. This results in a larger current through the resistors. Therefore, the voltage across R_1, and thus across the capacitor, increases.

2006E2

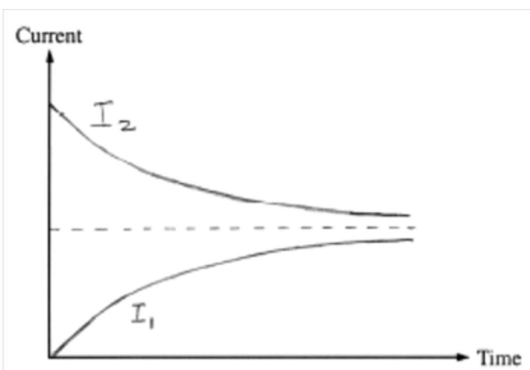
a. From Kirchhoff's loop rule $\mathcal{E} - V_R - V_C = 0$
$\mathcal{E} - IR_1 - q/C = 0$ where $I = dq/dt$
$\mathcal{E} - R_1 dq/dt - q/C = 0$

b.

$$\varepsilon - \frac{dq}{dt}R_1 - \frac{q}{C} = 0$$
$$\frac{dq}{\varepsilon C - q} = \frac{dt}{R_1 C}$$
$$\int_0^q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{R_1 C}$$
$$\ln(q - \varepsilon C)|_0^q = -\frac{t}{R_1 C}\bigg|_0^t$$
$$\ln(q - \varepsilon C) - \ln(-\varepsilon C) = \ln\frac{q - \varepsilon C}{-\varepsilon C} = -\frac{1}{R_1 C}$$
$$\frac{q - \varepsilon C}{-\varepsilon C} = e^{-\frac{t}{R_1 C}}$$
$$q - \varepsilon C = -\varepsilon C e^{-\frac{t}{R_1 C}}$$
$$q = \varepsilon C\left(1 - e^{-\frac{t}{R_1 C}}\right)$$

c. $q = CV$
$\mathcal{E}C(1 - e^{-t/RC}) = CV$ gives $t = R_1 C \ln(\mathcal{E}/(\mathcal{E} - V)) = (4700\ \Omega)(0.06\ \text{F}) \ln(12\ \text{V}/(12\ \text{V} - 4\text{V})) = 114$ seconds

d.



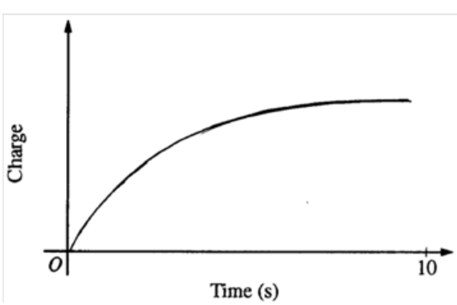
2007E1

a. Since $V_C = 0$ at $t = 0$, $\mathcal{E} = IR$. From the graph I is approximately 2.25 mA giving $\mathcal{E} = 1.24$ V

b. At t = 4 s, I = 0.35 mA and $V_C = \mathcal{E} - IR = 1.24\ \text{V} - (0.35\ \text{mA})(550\ \Omega) = 1.05$ V

c. $Q = CV = 4200\ \mu\text{C}$

d.



e. $P = I^2R = (0.35\text{ mA})^2(550\ \Omega) = 6.7 \times 10^{-5}$ W
f. Greater. C increases with a larger dielectric constant, but so does the time constant so we check
$Q(\kappa=3)/Q(\kappa=1) = \kappa C\mathcal{E}(1 - e^{-t/\kappa\tau})/\ C\mathcal{E}(1 - e^{-t/\kappa\tau}) = 3(1 - e^{-0.61})/\ (1 - e^{-1.82}) = 1.63$

2010E2

a. With a capacitor in each branch, the steady state current is zero.
b. Q = CV and each branch has 30 V (and the resistors have no potential drop with no current)
Q = (10 μF)(30 V) = 300 μC
c. $U = ½ CV^2 = ½ (5\ \mu\text{F})(30\text{ V})^2 = 2250\ \mu\text{J}$
d. With the switch closed, the capacitors are charged, but there is a steady current which winds its way through both resistors in series. R_T = 20 Ω + 40 Ω = 60 Ω and V = IR so I = 0.5 A
e. The voltage across the 5 μF capacitor is identical to the voltage across the 40 Ω resistor, which is (0.5 A)(40 Ω) = 20 V. So Q = CV = 100 μC
f. $P = I^2R = (0.5\text{ A})^2(40\ \Omega) = 10$ W
E = Pt = (10 W)(60 s) = 600 J
