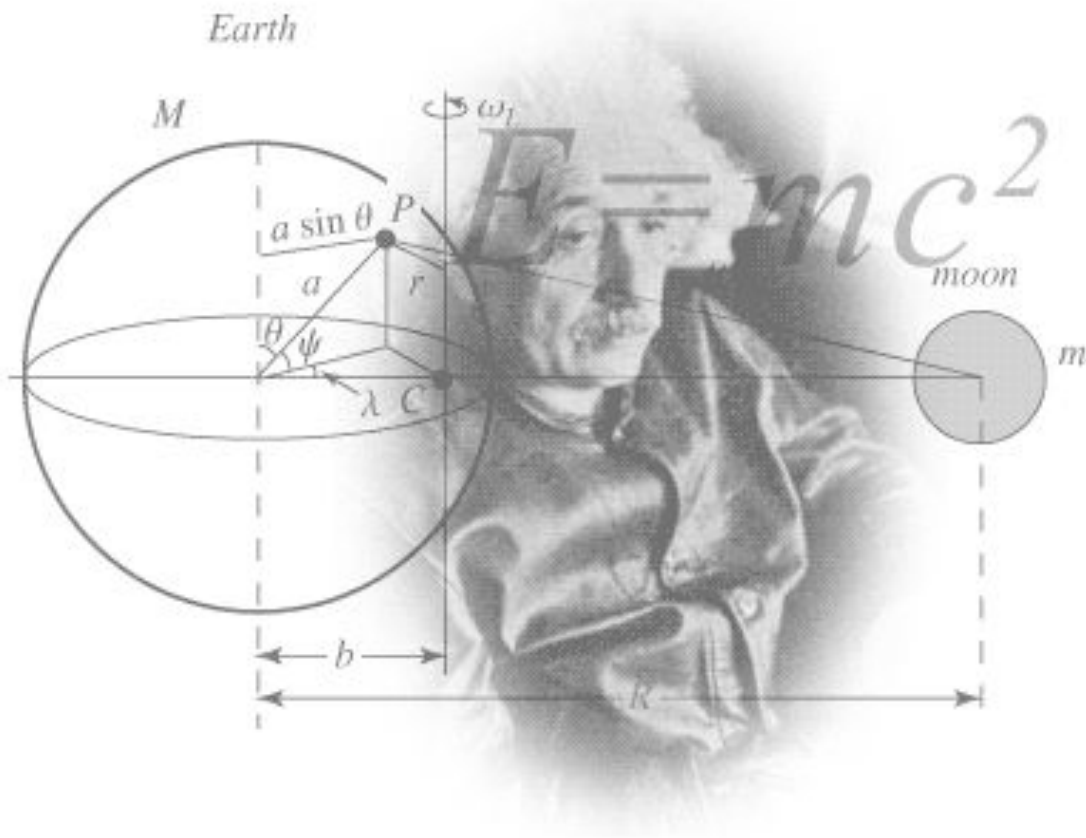


# AP Physics 2 – Practice Workbook – Book 2

## Electricity, Magnetism, Fluids, Thermodynamics, Optics and Modern Physics



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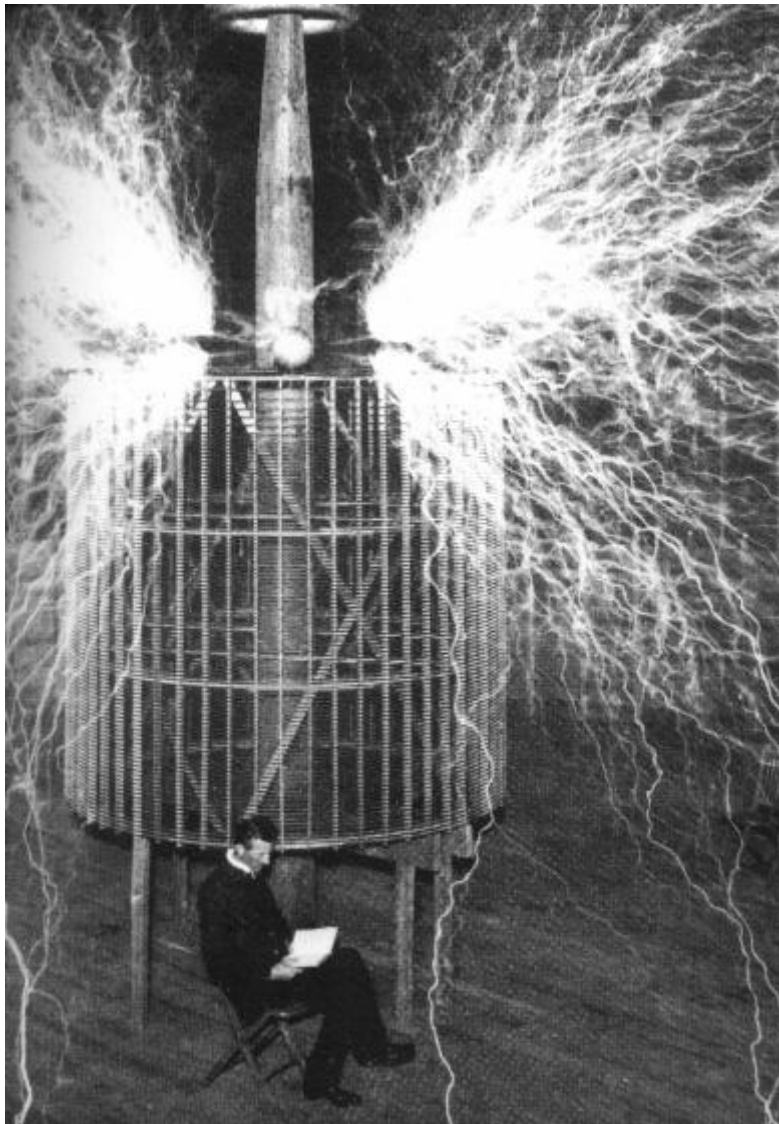
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# Chapter 12

## Electrostatics

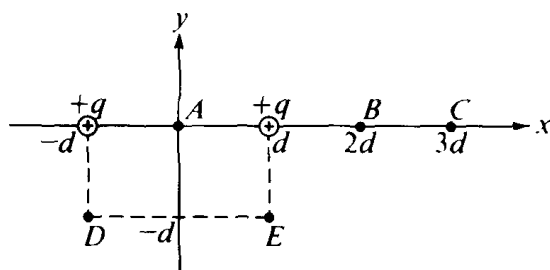




AP Physics Multiple Choice Practice – Electrostatics

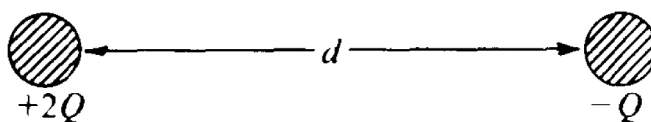
1. A solid conducting sphere is given a positive charge  $Q$ . How is the charge  $Q$  distributed in or on the sphere?  
 (A) It is concentrated at the center of the sphere.  
 (B) It is uniformly distributed throughout the sphere.  
 (C) Its density decreases radially outward from the center.  
 (D) It is uniformly distributed on the surface of the sphere only.
2. A parallel-plate capacitor is charged by connection to a battery. If the battery is disconnected and the separation between the plates is increased, what will happen to the charge on the capacitor and the voltage across it?  
 (A) Both remain fixed. (B) Both increase. (C) The charge increases and the voltage decreases. (D) The charge remains fixed and the voltage increases.
3. One joule of work is needed to move one coulomb of charge from one point to another with no change in velocity. Which of the following is true between the two points?  
 (A) The current is one ampere.  
 (B) The potential difference is one volt. (C) The electric field strength is one newton per coulomb.  
 (D) The electric field strength is one joule per electron.

Questions 4-5



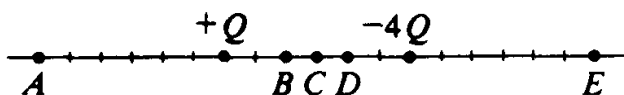
Two positive charges of magnitude  $q$  are each a distance  $d$  from the origin  $A$  of a coordinate system as shown above.

4. At which of the following points is the electric field least in magnitude?  
 (A) A (B) B (C) C (D) D
5. At which of the following points is the electric potential greatest in magnitude?  
 (A) A (B) B (C) C (D) D
6. A parallel-plate capacitor has a capacitance  $C_0$ . A second parallel-plate capacitor has plates with twice the area and twice the separation. The capacitance of the second capacitor is most nearly  
 (A)  $\frac{1}{4}C_0$  (B)  $C_0$  (C)  $2C_0$  (D)  $4C_0$

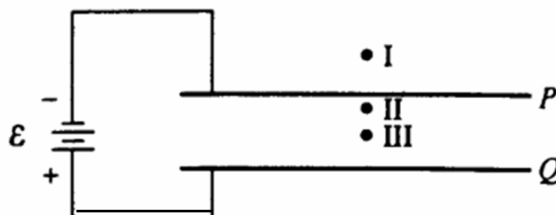


7. Two identical conducting spheres are charged to  $+2Q$  and  $-Q$ , respectively, and are separated by a distance  $d$  (much greater than the radii of the spheres) as shown above. The magnitude of the force of attraction on the left sphere is  $F_1$ . After the two spheres are made to touch and then are re-separated by distance  $d$ , the magnitude of the force on the left sphere is  $F_2$ . Which of the following relationships is correct?  
 (A)  $2F_1 = F_2$  (B)  $F_1 = F_2$  (C)  $F_1 = 2F_2$  (D)  $F_1 = 8F_2$

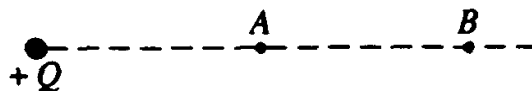
8. The capacitance of a parallel-plate capacitor can be increased by increasing which of the following?  
 (A) The distance between the plates (B) The charge on each plate (C) The area of the plates  
 (D) The potential difference across the plates
9. A hollow metal sphere of radius  $R$  is positively charged. Of the following distances from the center of the sphere, which location will have the greatest electric field strength?  
 (A) 0 (center of the sphere) (B)  $3R/2$  (C)  $2R$   
 (D) None of the above because the field is of constant strength
10. Two isolated charges,  $+q$  and  $-2q$ , are 2 centimeters apart. If  $F$  is the magnitude of the force acting on charge  $-2q$ , what are the magnitude and direction of the force acting on charge  $+q$ ?
- | <u>Magnitude</u> | <u>Direction</u>       |
|------------------|------------------------|
| (A) $(1/2) F$    | Toward charge $-2q$    |
| (B) $2 F$        | Away from charge $-2q$ |
| (C) $F$          | Toward charge $-2q$    |
| (D) $F$          | Away from charge $-2q$ |



11. Charges  $+Q$  and  $-4Q$  are situated as shown above. The net electric field is zero nearest which point?  
 (A) A (B) C (C) D (D) E
12. A positive charge of  $10^{-6}$  coulomb is placed on an insulated solid conducting sphere. Which of the following is true?  
 (A) The charge resides uniformly throughout the sphere.  
 (B) The electric field in the region surrounding the sphere increases with increasing distance from the sphere.  
 (C) An insulated metal object acquires a net positive charge when brought near to, but not in contact with, the sphere.  
 (D) When a second conducting sphere is connected by a conducting wire to the first sphere, charge is transferred until the electric potentials of the two spheres are equal.



13. Two large parallel conducting plates  $P$  and  $Q$  are connected to a battery of emf  $\mathcal{E}$ , as shown above. A test charge is placed successively at points I, II, and III. If edge effects are negligible, the force on the charge when it is at point III is  
 (A) of equal magnitude and in the same direction as the force on the charge when it is at point I  
 (B) of equal magnitude and in the same direction as the force on the charge when it is at point II  
 (C) much greater in magnitude than the force on the charge when it is at point II, but in the same direction  
 (D) much less in magnitude than the force on the charge when it is at point II, but in the same direction

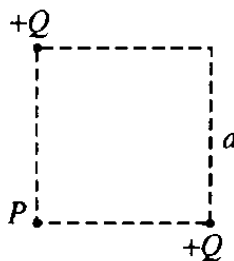


14. The diagram above shows an isolated, positive charge  $Q$ . Point (B) is twice as far away from  $Q$  as point A. The ratio of the electric field strength at point A to the electric field strength at point B is  
 (A) 8 to 1      (B) 4 to 1      (C) 2 to 1      (D) 1 to 2
15. Which of the following is true about the net force on an uncharged conducting sphere in a uniform electric field?  
 (A) It is zero.    (B) It is in the direction of the field.    (C) It is in the direction opposite to the field.  
 (D) It causes the sphere to oscillate about an equilibrium position.


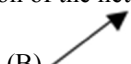
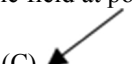



16. Two conducting spheres of different radii, as shown above, each have charge  $-Q$ . Which of the following occurs when the two spheres are connected with a conducting wire?  
 (A) No charge flows.  
 (B) Negative charge flows from the larger sphere to the smaller sphere until the electric potential of each sphere is the same.  
 (C) Negative charge flows from the smaller sphere to the larger sphere until the electric field at the surface of each sphere is the same.  
 (D) Negative charge flows from the smaller sphere to the larger sphere until the electric potential of each sphere is the same.
17. Two parallel conducting plates are connected to a constant voltage source. The magnitude of the electric field between the plates is  $2,000 \text{ N/C}$ . If the voltage is doubled and the distance between the plates is reduced to  $1/5$  the original distance, the magnitude of the new electric field is  
 (A)  $800 \text{ N/C}$       (B)  $1,600 \text{ N/C}$       (C)  $2,400 \text{ N/C}$       (D)  $20,000 \text{ N/C}$

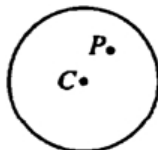
Questions 18-19



The figure above shows two particles, each with a charge of  $+Q$ , that are located at the opposite corners of a square of side  $d$ .

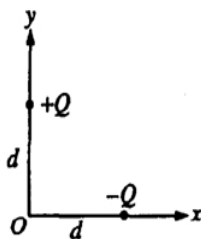
18. What is the direction of the net electric field at point P ?  
 (A)     (B)     (C)     (D) 
19. What is the potential energy of a particle of charge  $+q$  that is held at point P ?  
 (A) Zero    (B)  $\frac{\sqrt{2}}{4\pi\epsilon_0} \frac{qQ}{d}$     (C)  $\frac{2}{4\pi\epsilon_0} \frac{qQ}{d}$     (D)  $\frac{2\sqrt{2}}{4\pi\epsilon_0} \frac{qQ}{d}$

20. Two parallel conducting plates, separated by a distance  $d$ , are connected to a battery of emf  $\mathcal{E}$ . Which of the following is correct if the plate separation is doubled while the battery remains connected?
- (A) The electric charge on the plates is doubled.  
 (B) The electric charge on the plates is halved.  
 (C) The potential difference between the plates is doubled.  
 (E) The capacitance is unchanged.



21. The hollow metal sphere shown above is positively charged. Point C is the center of the sphere and point P is any other point within the sphere. Which of the following is true of the electric field at these points?
- (A) It is zero at both points.  
 (B) It is zero at C, but at P it is not zero and is directed inward.  
 (C) It is zero at C, but at P it is not zero and is directed outward.  
 (D) It is not zero at either point.

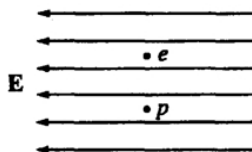
Questions 22-23



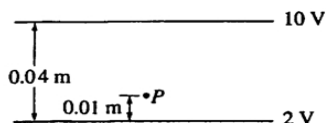
Charges  $-Q$  and  $+Q$  are located on the x- and y-axes, respectively, each at a distance  $d$  from the origin  $O$ , as shown above.

22. What is the direction of the electric field at the origin  $O$ ?

- (A) (B) (C) (D)



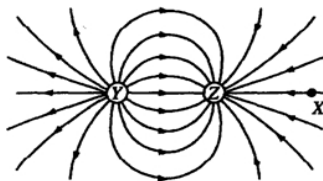
23. An electron  $e$  and a proton  $p$  are simultaneously released from rest in a uniform electric field  $E$ , as shown above. Assume that the particles are sufficiently far apart so that the only force acting on each particle after it is released is that due to the electric field. At a later time when the particles are still in the field, the electron and the proton will have the same
- (A) direction of motion (B) speed (C) magnitude of acceleration  
 (D) magnitude of force acting on them



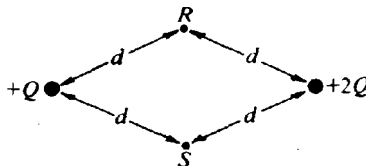
24. Two large, flat, parallel, conducting plates are 0.04 m apart, as shown above. The lower plate is at a potential of 2 V with respect to ground. The upper plate is at a potential of 10 V with respect to ground. Point P is located 0.01 m above the lower plate. The electric potential at point P is
- (A) 10 V (B) 8 V (C) 6 V (D) 4 V

25. A particle of charge  $Q$  and mass  $m$  is accelerated from rest through a potential difference  $V$ , attaining a kinetic energy  $K$ . What is the kinetic energy of a particle of charge  $2Q$  and mass  $m/2$  that is accelerated from rest through the same potential difference?

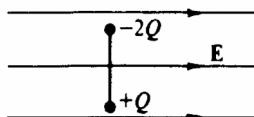
(A)  $\frac{1}{2}K$       (B)  $K$       (C)  $2K$       (D)  $4K$



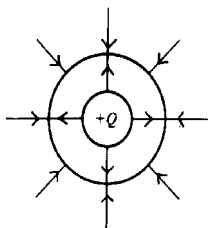
26. The diagram above shows electric field lines in an isolated region of space containing two small charged spheres,  $Y$  and  $Z$ . Which of the following statements is true?
- (A) The charge on  $Y$  is negative and the charge on  $Z$  is positive.  
 (B) The strength of the electric field is the same everywhere.  
 (C) The electric field is strongest midway between  $Y$  and  $Z$ .  
 (D) A small negatively charged object placed at point  $X$  would tend to move toward the right.
27. A parallel-plate capacitor has a capacitance  $C_0$ . A second parallel-plate capacitor has plates with twice the area and twice the separation. The capacitance of the second capacitor is most nearly
- (A)  $\frac{1}{4}C_0$       (B)  $\frac{1}{2}C_0$       (C)  $C_0$       (D)  $2C_0$
28. The electric field  $E$  just outside the surface of a charged conductor is
- (A) directed perpendicular to the surface    (B) directed parallel to the surface  
 (C) zero    (D) infinite



29. Points  $R$  and  $S$  are each the same distance  $d$  from two unequal charges,  $+Q$  and  $+2Q$ , as shown above. The work required to move a charge  $-Q$  from point  $R$  to point  $S$  is
- (A) dependent on the path taken from  $R$  to  $S$   
 (B) positive  
 (C) zero  
 (D) negative



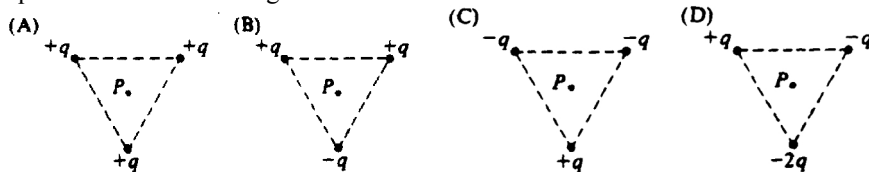
30. A rigid insulated rod, with two unequal charges attached to its ends, is placed in a uniform electric field  $E$  as shown above. The rod experiences a
- (A) net force to the left and a clockwise rotation  
 (B) net force to the left and a counterclockwise rotation  
 (C) net force to the right and a clockwise rotation  
 (D) net force to the right and a counterclockwise rotation



31. The electric field of two long coaxial cylinders is represented by lines of force as shown above. The charge on the inner cylinder is  $+Q$ . The charge on the outer cylinder is  
 (A)  $+3Q$  (B)  $+Q$  (C)  $-Q$  (D)  $-3Q$
32. An isolated capacitor with air between its plates has a potential difference  $V_0$  and a charge  $Q_0$ . After the space between the plates is filled with oil, the difference in potential is  $V$  and the charge is  $Q$ . Which of the following pairs of relationships is correct?  
 (A)  $Q = Q_0$  and  $V > V_0$  (B)  $Q = Q_0$  and  $V < V_0$  (C)  $Q > Q_0$  and  $V = V_0$  (D)  $Q < Q_0$  and  $V < V_0$
33. Two small spheres have equal charges  $q$  and are separated by a distance  $d$ . The force exerted on each sphere by the other has magnitude  $F$ . If the charge on each sphere is doubled and  $d$  is halved, the force on each sphere has magnitude  
 (A)  $F$  (B)  $4F$  (C)  $8F$  (D)  $16F$
34. Which of the following statements about conductors under electrostatic conditions is true?  
 (A) Positive work is required to move a positive charge over the surface of a conductor.  
 (B) Charge that is placed on the surface of a conductor always spreads evenly over the surface.  
 (C) The electric potential inside a conductor is always zero.  
 (D) The surface of a conductor is always an equipotential surface.
35. A charged particle traveling with a velocity  $v$  in an electric field  $E$  experiences a force  $F$  that must be  
 (A) parallel to  $v$  (B) perpendicular to  $v$  (C) parallel to  $E$  (D) perpendicular to  $E$
36. A positive charge of  $3.0 \times 10^{-8}$  coulomb is placed in an upward directed uniform electric field of  $4.0 \times 10^4$  N/C. When the charge is moved 0.5 meter upward, the work done by the electric force on the charge is  
 (A)  $6 \times 10^{-4}$  J (B)  $12 \times 10^{-4}$  J (C)  $2 \times 10^4$  J (D)  $8 \times 10^4$  J

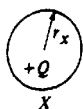
#### Questions 37-38

The following configurations of electric charges are located at the vertices of an equilateral triangle. Point P is equidistant from the charges.



37. In which configuration is the electric field at P equal to zero?  
 (A) A (B) B (C) C (D) D
38. In which configuration is the electric field at P pointed at the midpoint between two of the charges?  
 (A) A (B) B (C) C (D) D

39. A sheet of mica is inserted between the plates of an isolated charged parallel-plate capacitor. Which of the following statements is true?
- (A) The capacitance decreases.
  - (B) The potential difference across the capacitor decreases.
  - (C) The charge on the capacitor plates decreases
  - (D) The electric field between the capacitor plates increases.



40. Two conducting spheres, X and Y have the same positive charge  $+Q$ , but different radii ( $r_x > r_y$ ) as shown above. The spheres are separated so that the distance between them is large compared with either radius. If a wire is connected between them, in which direction will electrons be directed in the wire?
- (A) From X to Y
  - (B) From Y to X
  - (C) There will be no flow of charge in the wire.
  - (D) It cannot be determined without knowing the magnitude of  $Q$ .

Questions 41-42

A sphere of radius  $R$  has positive charge  $Q$  uniformly distributed on its surface

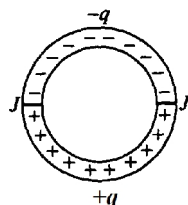
41. Which of the following represents the magnitude of the electric field  $E$  and the potential  $V$  as functions of  $r$ , the distance from the center of the sphere, when  $r < R$  ?

$E$	$V$
(A) 0	$kQ/R$
(B) 0	$kQ/r$
(C) $kQ/r^2$	0
(D) $kQ/R^2$	0

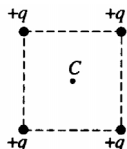
42. Which of the following represents the magnitude, of the electric field  $E$  and the potential  $V$  as functions of  $r$ , the distance from the center of sphere, when  $r > R$  ?

$E$	$V$
(A) $kQ/R^2$	$kQ/R$
(B) $kQ/R$	$kQ/r$
(C) $kQ/r^2$	$kQ/r$
(D) $kQ/r^2$	$kQ/r^2$

43. From the electric field vector at a point, one can determine which of the following?
- I. The direction of the electrostatic force on a test charge of known sign at that point
  - II. The magnitude of the electrostatic force exerted per unit charge on a test charge at that point
  - III. The electrostatic charge at that point
- (A) I only   (B) III only   (C) I and II only   (D) II and III only
44. A conducting sphere of radius  $R$  carries a charge  $Q$ . Another conducting sphere has a radius  $R/2$ , but carries the same charge. The spheres are far apart. The ratio of the electric field near the surface of the smaller sphere to the field near the surface of the larger sphere is most nearly
- (A)  $1/4$    (B)  $1/2$    (C) 2   (D) 4

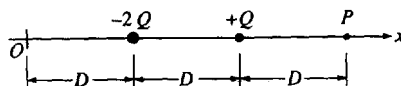


45. A circular ring made of an insulating material is cut in half. One half is given a charge  $-q$  uniformly distributed along its arc. The other half is given a charge  $+q$  also uniformly distributed along its arc. The two halves are then rejoined with insulation at the junctions J, as shown above. If there is no change in the charge distributions, what is the direction of the net electrostatic force on an electron located at the center of the circle?
- (A) Toward the top of the page (B) Toward the bottom of the page (C) To the right (D) To the left

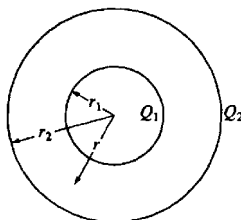


46. Four positive charges of magnitude  $q$  are arranged at the corners of a square, as shown above. At the center C of the square, the potential due to one charge alone is  $V_0$  and the electric field due to one charge alone has magnitude  $E_0$ . Which of the following correctly gives the electric potential and the magnitude of the electric field at the center of the square due to all four charges?

	Electric Potential	Electric Field
(A)	Zero	Zero
(B)	Zero	$2E_0$
(C)	$4V_0$	Zero
(D)	$4V_0$	$2E_0$



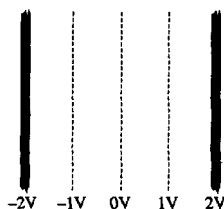
47. Two charges,  $-2Q$  and  $+Q$ , are located on the  $x$ -axis, as shown above. Point P, at a distance of  $3D$  from the origin O, is one of two points on the positive  $x$ -axis at which the electric potential is zero. How far from the origin O is the other point?
- (A)  $2/3 D$  (B)  $3/2 D$  (C)  $5/3 D$  (D)  $2D$



48. Two concentric, spherical conducting shells have radii  $r_1$  and  $r_2$  and charges  $Q_1$  and  $Q_2$ , as shown above. Let  $r$  be the distance from the center of the spheres and consider the region  $r_1 < r < r_2$ . In this region the electric field is proportional to
- (A)  $Q_1/r^2$  (B)  $(Q_1 + Q_2)/r^2$  (C)  $(Q_1 + Q_2)/r$  (D)  $Q_1/r + Q_2/r_2$

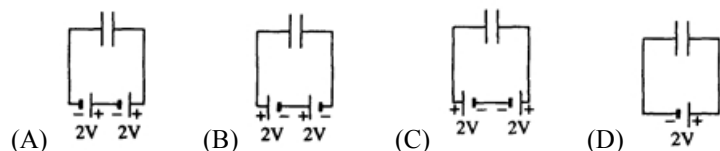


Questions 49-50

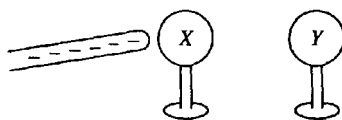


A battery or batteries connected to two parallel plates produce the equipotential lines between the plates shown above.

49. Which of the following configurations is most likely to produce these equipotential lines?



50. The force on an electron located on the 0 volt potential line is  
 (A) 0 N (B) 1 N, directed to the right (C) 1 N, directed to the left  
 (D) to the right, but its magnitude cannot be determined without knowing the distance between the lines

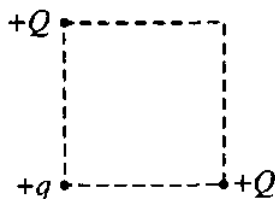


51. Two metal spheres that are initially uncharged are mounted on insulating stands, as shown above. A negatively charged rubber rod is brought close to, but does not make contact with, sphere X. Sphere Y is then brought close to X on the side opposite to the rubber rod. Y is allowed to touch X and then is removed some distance away. The rubber rod is then moved far away from X and Y. What are the final charges on the spheres?

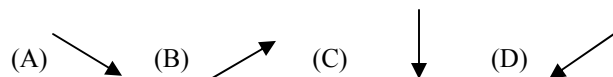
<u>Sphere X</u>	<u>Sphere Y</u>
A) Zero	Zero
B) Negative	Positive
C) Positive	Negative
D) Positive	Positive

52. Two initially uncharged conductors, 1 and 2, are mounted on insulating stands and are in contact, as shown above. A negatively charged rod is brought near but does not touch them. With the rod held in place, conductor 2 is moved to the right by pushing its stand, so that the conductors are separated. Which of the following is now true of conductor 2?  
 (A) It is uncharged. (B) It is positively charged. (C) It is negatively charged.  
 (D) It is charged, but its sign cannot be predicted.

Questions 53-54



53. As shown above, two particles, each of charge  $+Q$ , are fixed at opposite corners of a square that lies in the plane of the page. A positive test charge  $+q$  is placed at a third corner. What is the direction of the force on the test charge due to the two other charges?

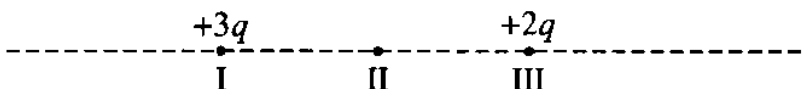


54. If  $F$  is the magnitude of the force on the test charge due to only one of the other charges, what is the magnitude of the net force acting on the test charge due to both of these charges?

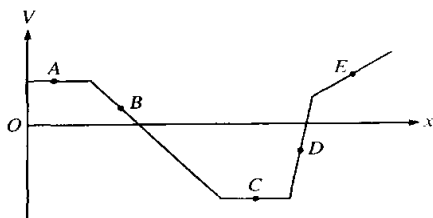
- (A) Zero (B)  $\frac{F}{\sqrt{2}}$  (C)  $\sqrt{2}F$  (D)  $2F$  (E)  $2$

Questions 55-56

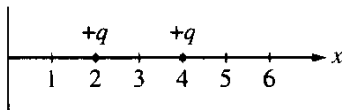
Two charges are located on the line shown in the figure below, in which the charge at point I is  $+3q$  and the charge at point III is  $+2q$ . Point II is halfway between points I and III.



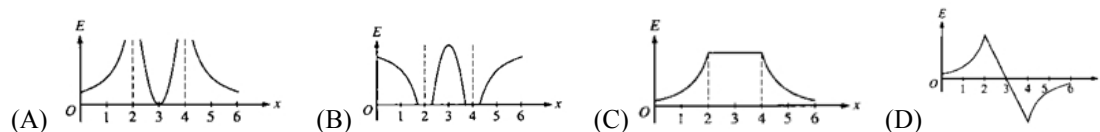
55. Other than at infinity, the electric field strength is zero at a point on the line in which of the following ranges?  
 (A) To the left of I (B) Between I and II (C) Between II and III (D) To the right of III
56. The electric potential is negative at some points on the line in which of the following ranges?  
 (A) To the left of I (B) Between I and II (C) Between II and III  
 (D) None; this potential is never negative.



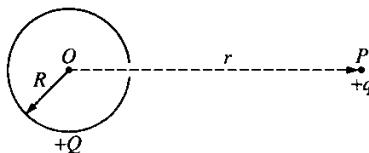
57. The graph above shows the electric potential  $V$  in a region of space as a function of position along the  $x$ -axis. At which point would a charged particle experience the force of greatest magnitude?  
 (A) A (B) B (C) D (D) E
58. If the only force acting on an electron is due to a uniform electric field, the electron moves with constant  
 (A) acceleration in a direction opposite to that of the field  
 (B) acceleration in the direction of the field  
 (C) speed in a direction opposite to that of the field  
 (D) speed in the direction of the field



59. Two charged particles, each with a charge of  $+q$ , are located along the  $x$ -axis at  $x = 2$  and  $x = 4$ , as shown above. Which of the following shows the graph of the magnitude of the electric field along the  $x$ -axis from the origin to  $x = 6$ ?



60. A positive electric charge is moved at a constant speed between two locations in an electric field, with no work done by or against the field at any time during the motion. This situation can occur only if the  
 (A) charge is moved in the direction of the field  
 (B) charge is moved opposite to the direction of the field  
 (C) charge is moved perpendicular to an equipotential line  
 (D) charge is moved along an equipotential line

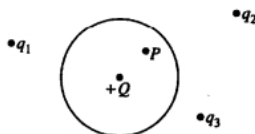


61. The nonconducting hollow sphere of radius  $R$  shown above carries a large charge  $+Q$ , which is uniformly distributed on its surface. There is a small hole in the sphere. A small charge  $+q$  is initially located at point  $P$ , a distance  $r$  from the center of the sphere. If  $k = 1/4\pi\epsilon_0$ , what is the work that must be done by an external agent in moving the charge  $+q$  from  $P$  through the hole to the center  $O$  of the sphere?  
 (A)  $kqQ/r$  (B)  $kqQ/R$  (C)  $kq(Q - q)/r$  (D)  $kqQ(1/R - 1/r)$

#### Questions 62-63

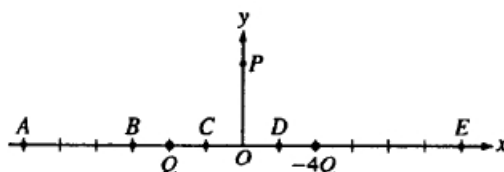
A capacitor is constructed of two identical conducting plates parallel to each other and separated by a distance  $d$ . The capacitor is charged to a potential difference of  $V_0$  by a battery, which is then disconnected.

62. If any edge effects are negligible, what is the magnitude of the electric field between the plates?  
 (A)  $V_0 d$  (B)  $V_0/d$  (C)  $V_0/d^2$  (D)  $V_0^2/d$
63. A sheet of insulating plastic material is inserted between the plates without otherwise disturbing the system. What effect does this have on the capacitance?  
 (A) It causes the capacitance to increase.  
 (B) It causes the capacitance to decrease.  
 (C) None; the capacitance does not change.  
 (E) Nothing can be said about the effect without knowing the thickness of the sheet.



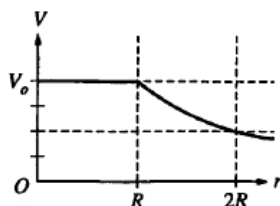
64. A point charge  $+Q$  is inside an uncharged conducting spherical shell that in turn is near several isolated point charges, as shown above. The electric field at point P inside the shell depends on the magnitude of  
 (A)  $Q$  only  
 (B) the charge distribution on the sphere only  
 (C)  $Q$  and the charge distribution on the sphere  
 (D) all of the point charges
65. A potential difference  $V$  is maintained between two large, parallel conducting plates. An electron starts from rest on the surface of one plate and accelerates toward the other. Its speed as it reaches the second plate is proportional to  
 (A)  $\frac{1}{\sqrt{V}}$  (B)  $\sqrt{V}$  (C)  $V$  (D)  $V^2$

Questions 66-67

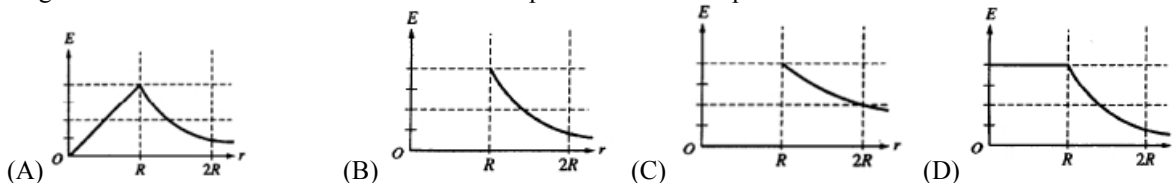


Particles of charge  $Q$  and  $-4Q$  are located on the  $x$ -axis as shown in the figure above. Assume the particles are isolated from all other charges.

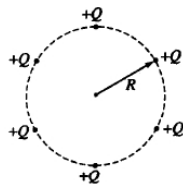
66. Which of the following describes the direction of the electric field at point P?  
 (A)  $+y$  (B)  $-y$  (C) Components in both the  $-x$  and  $+y$  directions  
 (D) Components in both the  $+x$  and  $-y$  directions
67. At which of the labeled points on the  $x$ -axis is the electric field zero?  
 (A) A (B) B (C) C (D) E



68. A solid metallic sphere of radius  $R$  has charge  $Q$  uniformly distributed on its outer surface. A graph of electric potential  $V$  as a function of position  $r$  is shown above. Which of the following graphs best represents the magnitude of the electric field  $E$  as a function of position  $r$  for this sphere?



Questions 69-70



As shown in the figure above, six particles, each with charge  $+Q$ , are held fixed and are equally spaced around the circumference of a circle of radius  $R$ .

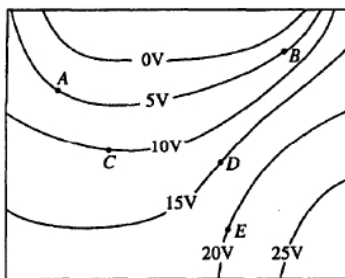
69. What is the magnitude of the resultant electric field at the center of the circle?

(A) 0      (B)  $\frac{\sqrt{6}}{4\pi\epsilon_0} \frac{Q}{R^2}$       (C)  $\frac{3\sqrt{2}}{4\pi\epsilon_0} \frac{Q}{R^2}$       (D)  $\frac{3}{2\pi\epsilon_0} \frac{Q}{R^2}$

70. With the six particles held fixed, how much work would be required to bring a seventh particle of charge  $+Q$  from very far away and place it at the center of the circle?

(A) 0      (B)  $\frac{\sqrt{6}}{4\pi\epsilon_0} \frac{Q}{R}$       (C)  $\frac{3}{2\pi\epsilon_0} \frac{Q^2}{R^2}$       (D)  $\frac{3}{2\pi\epsilon_0} \frac{Q^2}{R}$

Questions 71-73



The diagram above shows equipotential lines produced by an unknown charge distribution. A, B, C, D, and E are points in the plane.

71. Which vector below best describes the direction of the electric field at point A ?



72. At which point does the electric field have the greatest magnitude?

(A) A      (B) B      (C) C      (D) E

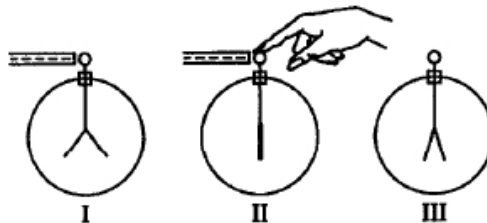
73. How much net work must be done by an external force to move a  $-1 \mu\text{C}$  point charge from rest at point C to rest at point E ?

(A)  $-20 \mu\text{J}$       (B)  $-10 \mu\text{J}$       (C)  $10 \mu\text{J}$       (D)  $20 \mu\text{J}$

74. A physics problem starts: "A solid sphere has charge distributed uniformly throughout. . ." It may be correctly concluded that the

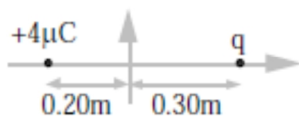
(A) electric field is zero everywhere inside the sphere  
 (B) electric potential on the surface of the sphere is not constant  
 (C) electric potential in the center of the sphere is zero  
 (D) sphere is not made of metal

75. A uniform spherical charge distribution has radius  $R$ . Which of the following is true of the electric field strength due to this charge distribution at a distance  $r$  from the center of the charge?
- (A) It is greatest when  $r = 0$ .  
 (B) It is directly proportional to  $r$  when  $r > R$ .  
 (C) It is directly proportional to  $r$  when  $r < R$ .  
 (D) It is directly proportional to  $r^2$ .



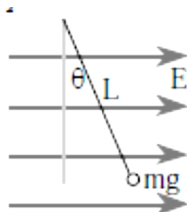
76. When a negatively charged rod is brought near, but does not touch, the initially uncharged electroscope shown above, the leaves spring apart (I). When the electroscope is then touched with a finger, the leaves collapse (II). When next the finger and finally the rod are removed, the leaves spring apart a second time (III). The charge on the leaves is
- (A) positive in both I and III  
 (B) negative in both I and III  
 (C) positive in I, negative in III  
 (D) negative in I, positive in III
77. A positively charged conductor attracts a second object. Which of the following statements *could* be true?
- I. The second object is a conductor with negative net charge.  
 II. The second object is a conductor with zero net charge.  
 III. The second object is an insulator with zero net charge..
- (A) I only (B) II only (C) III only (D) I, II & III

#### Questions 78-79



A point charge of  $+4.0 \mu\text{C}$  is placed on the negative  $x$ -axis  $0.20 \text{ m}$  to the left of the origin, as shown in the accompanying figure. A second point charge  $q$  is placed on the positive  $x$ -axis  $0.30 \text{ m}$  to the right of the origin.

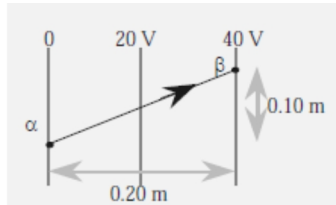
78. If the net electric field at the origin is zero. What is  $q$ ?
- (A)  $+9.0 \mu\text{C}$  (B)  $+6.0 \mu\text{C}$  (C)  $-6.0 \mu\text{C}$  (D)  $-9.0 \mu\text{C}$
79. If the net electric potential at the origin is zero, what is  $q$ ?
- (A)  $+9.0 \mu\text{C}$  (B)  $+6.0 \mu\text{C}$  (C)  $-6.0 \mu\text{C}$  (D)  $-9.0 \mu\text{C}$



80. A small object with charge  $q$  and weight  $mg$  is attached to one end of a string of length  $L$ . The other end is attached to a stationary support. The system is placed in a uniform horizontal electric field  $E$ , as shown in the accompanying figure. In the presence of the field, the string makes a constant angle  $q$  with the vertical. What is the sign and magnitude of  $q$ ?

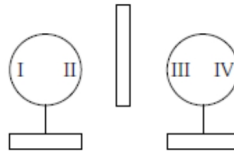
- (A) positive with magnitude  $\frac{mg}{E}$   
 (B) negative with magnitude  $\frac{mg}{E}$   
 (C) positive with magnitude  $\frac{mg}{E} \tan \theta$   
 (D) negative with magnitude  $\frac{mg}{E} \tan \theta$

81. Two large parallel plates a distance  $d$  apart are charged by connecting them to a battery of potential difference  $V$ . The battery is disconnected, and the plates are slowly moved apart. As the distance between plates increases:  
 (A) the charge on the plates decreases.  
 (B) the electric field intensity between the plates increases.  
 (C) the potential difference between the plates decreases.  
 (D) the potential difference between the plates increases.



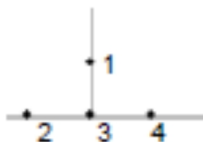
82. In the figure above, equipotential lines are drawn at 0, 20.0 V, and 40.0 V. The total work done in moving a point charge of  $+3.00 \text{ mC}$  from position a to position b is:  
 (A) 4.00 mJ (B) 8.00 mJ (C) 12.0 mJ (D) 120 mJ
83. Two positive point charges repel each other with force 0.36 N when their separation is 1.5 m. What force do they exert on each other when their separation is 1.0 m?  
 (A) 0.81 N (B) 0.54 N (C) 0.24 N (D) 0.16 N
84. Two isolated conducting spheres ( $S_1$  of radius 0.030 m and initial charge  $+6.0 \text{ nC}$  and  $S_2$  of radius 0.040 m and initial charge  $+2.0 \text{ nC}$ ) are connected by a conducting wire. Charge will flow in the wire until:  
 (A) both spheres are equally charged.  
 (B) the force of repulsion between the two spheres becomes equal.  
 (C) both spheres have the same surface charge density.  
 (D) both spheres are at the same potential.
85. A point charge  $+q$  is placed midway between two point charges  $+3q$  and  $-q$  separated by a distance  $2d$ . If Coulomb's constant is  $k$ , the magnitude of the force on the charge  $+q$  is:

- (A)  $2 \frac{kq^2}{d^2}$  (B)  $4 \frac{kq^2}{d^2}$  (C)  $6 \frac{kq^2}{d^2}$  (D)  $8 \frac{kq^2}{d^2}$



86. A charged rod is placed between two insulated conducting spheres as shown. The spheres have no net charge. Region II has the same polarity as Region  
 (A) I only (B) III only (C) IV only (D) I & IV only

87. Two large oppositely charged insulated plates have a uniform electric field between them. The distance between the plates is increased. Which of the following statements is true?
- The field strength decreases.
  - The field strength increases.
  - The potential difference between the plates increases.
- (A) I only (B) II only (C) III only (D) I and III only
88. When two charged point-like objects are separated by a distance  $R$ , the force between them is  $F$ . If the distance between them is quadrupled, the force between them is
- (A)  $16 F$  (B)  $4 F$  (C)  $F/4$  (D)  $F/16$
89. An electroscope is given a positive charge, causing its foil leaves to separate. When an object is brought near the top plate of the electroscope, the foils separate even further. We could conclude
- that the object is positively charged.
  - that the object is negatively charged.
  - only that the object is charged.
  - only that the object is uncharged.



90. Four positive point charges are arranged as shown in the accompanying diagram. The force between charges 1 and 3 is  $6.0 \text{ N}$ ; the force between charges 2 and 3 is  $5.0 \text{ N}$ ; and the force between charges 3 and 4 is  $3.0 \text{ N}$ . The magnitude of the total force on charge 3 is most nearly
- (A)  $6.3 \text{ N}$  (B)  $8.0 \text{ N}$  (C)  $11 \text{ N}$  (D)  $14 \text{ N}$
91. Two isolated parallel plates are separated by a distance  $d$ . They carry opposite charges  $Q$  and each has surface area  $A$ . Which of the following would increase the strength of the electric field between the plates?
- Increasing  $Q$
  - Increasing  $A$
  - Increasing  $d$
- (A) I only (B) II only (C) III only (D) II & III only
92. When a positive electrically charged glass rod is brought near a neutral hollow metal sphere suspended by an insulating string, the sphere will be attracted to the rod because:
- the rod removes electron from the sphere
  - the electric charge produces a magnetic field to attract the sphere
  - the charge on the rod causes a separation of charge in the sphere
  - some of the protons from the rod have been given to the sphere



$\alpha^{2+}$   $\text{P}^+$



93. An alpha particle and a proton are placed equal distance between two large charged metal plates as shown. Which of the following would best describe the motion of the two particles if they were free to move?
- The alpha particle will travel upwards with twice the velocity of the proton.
  - Both particles will travel upwards with the same velocity.
  - Both particles will accelerate upwards with the same acceleration.
  - The alpha particle will accelerate upwards with half the acceleration of the proton.



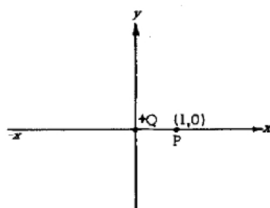
94. Two parallel metal plates carry opposite electrical charges each with a magnitude of  $Q$ . The plates are separated by a distance  $d$  and each plate has an area  $A$ . Consider the following:
- I. increasing  $Q$
  - II. increasing  $d$
  - III. increasing  $A$

Which of the following would have the effect of reducing the potential difference between the plates?

(A) I only (B) II only (C) III only (D) II and III

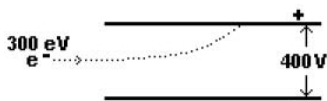
95. A positive point charge of  $+q$  and a negative point charge of  $-q$  are separated by a distance  $d$ . What would be the magnitude of the electric field midway between the two charges?

(A)  $E = \frac{kq}{d^2}$  (B)  $E = \frac{2kq}{d^2}$  (C)  $E = \frac{4kq}{d}$  (D)  $E = \frac{8kq}{d^2}$



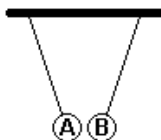
96. A positive charge  $+Q$  located at the origin produces an electric field  $E_0$  at point P ( $x = +1$ ,  $y = 0$ ). A negative charge  $-2Q$  is placed at such a point as to produce a net field of zero at point P. The second charge will be placed on the

(A) x-axis where  $x > 1$  (B) x-axis where  $x < 0$  (C) y-axis where  $y > 0$  (D) y-axis where  $y < 0$



97. A 300 eV electron is aimed midway between two parallel metal plates with a potential difference of 400 V. The electron is deflected upwards and strikes the upper plate as shown. What would be the kinetic energy of the electron just before striking the metal plate?

(A) 360 eV (B) 400 eV (C) 500 eV (D) 700 eV



98. Two small hollow metal spheres hung on insulating threads attract one another as shown. It is known that a positively charged rod will attract ball A.

- I. Ball A has a positive charge
- II. Ball B has a negative charge
- III. Ball A and Ball B have opposite charges

Which of the above can be correctly concluded about the charge on the balls?

(A) I only (B) II only (C) III only (D) none of these

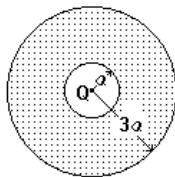
99. A  $5 \times 10^{-6}$  coulomb electric charge is placed midway between two parallel metal plates connected to a 9-volt battery. If the electric charge experiences a force of  $1.5 \times 10^{-4}$  newtons, what is the separation of the metal plates?

(A)  $6.75 \times 10^{-9}$  m (B)  $2.7 \times 10^{-4}$  m (C)  $3.7 \times 10^{-3}$  m (D) 0.30 m

100. A parallel-plate capacitor is connected with wires of negligible resistance to a battery having emf  $\mathcal{E}$  until the capacitor is fully charged. The battery is then disconnected from the circuit and the plates of the capacitor are moved to half of their original separation using insulated gloves. Let  $V_{\text{new}}$  be the potential difference across the capacitor plates when the plates are moved together. Let  $V_{\text{old}}$  be the potential difference across the capacitor

plates when connected to the battery.  $\frac{V_{\text{new}}}{V_{\text{old}}} =$

- (A)  $\frac{1}{2}$  (B) 1 (C) 2 (D) 4

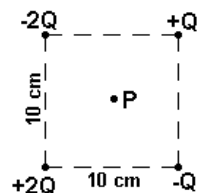


101. A solid, uncharged conducting sphere of radius  $3a$  contains a hollowed spherical region of radius  $a$ . A point charge  $+Q$  is placed at the common center of the spheres. Taking  $V = 0$  as  $r$  approaches infinity, the potential at position  $r = 2a$  from the center of the spheres is:

- (A) 0 (B)  $\frac{2kQ}{3a}$  (C)  $\frac{kQ}{3a}$  (D)  $\frac{kQ}{a}$

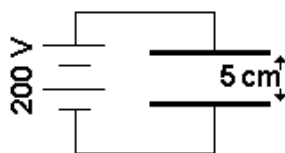
102. Two identical electrical point charges  $Q$ , separated by a distance  $d$  produce an electrical force of  $F$  on one another. If the distance is decreased to a distance of  $0.40d$ , what is the strength of the resulting force?

- (A)  $6.3F$  (B)  $2.5F$  (C)  $0.40F$  (D)  $0.16F$



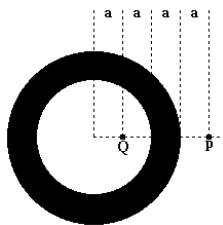
103. Four electrical charges are arranged on the corners of a 10 cm square as shown. What would be the direction of the resulting electric field at the center point P?

- (A)  $\rightarrow$  (B)  $\uparrow$  (C)  $\downarrow$  (D)  $\nearrow$



104. A proton is released between the two parallel plates of the fully charged capacitor shown above. What would be the resulting acceleration of the proton?

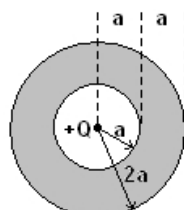
- (A)  $7.3 \times 10^{13} \text{ m/s}^2$  (B)  $9.6 \times 10^8 \text{ m/s}^2$  (C)  $6.3 \times 10^{19} \text{ m/s}^2$  (D)  $3.8 \times 10^{11} \text{ m/s}^2$



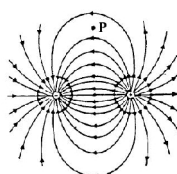
105. A solid uncharged conducting sphere has radius  $3a$  contains a hollowed spherical region of radius  $2a$ . A point charge  $+Q$  is placed at a position a distance  $a$  from the common center of the spheres. What is the magnitude of the electric field at the position  $r = 4a$  from the center of the spheres as marked in the figure by P?

- (A) 0 (B)  $\frac{kQ}{16a^2}$  (C)  $\frac{3kQ}{16a^2}$  (D)  $\frac{kQ}{9a^2}$

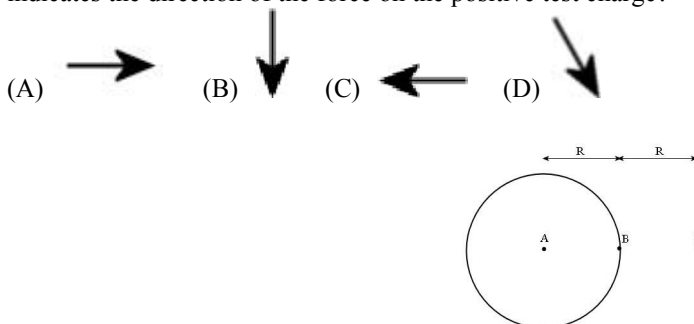
106. A positively charged object is brought near but not in contact with the top of an uncharged gold leaf electroscope. The experimenter then briefly touches the electroscope with a finger. The finger is removed, followed by the removal of the positively charged object. What happens to the leaves of the electroscope when a negative charge is now brought near but not in contact with the top of the electroscope?
- (A) they remain uncharged  
 (B) they move farther apart  
 (C) they move closer together  
 (D) they remain negatively charged but unmoved



107. A solid spherical conducting shell has inner radius  $a$  and outer radius  $2a$ . At the center of the shell is located a point charge  $+Q$ . What must the excess charge of the shell be in order for the charge density on the inner and outer surfaces of the shell to be exactly equal?
- (A)  $-5Q$  (B)  $+3Q$  (C)  $-4Q$  (D)  $+4Q$



108. A small positive test charge is placed at point P in the region near two charges. Which of the following arrows indicates the direction of the force on the positive test charge?

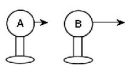
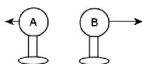
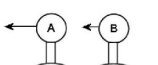
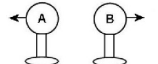


109. A spherical conducting shell has a net charge  $+Q$  placed on it. Which of the following is the correct relationship for the electric potential at the points labeled A, B, and C? Point A is at the center of the sphere, point B is at the surface of the shell, a distance  $R$  from point A, and point C is a distance  $R$  from point B outside the sphere. As  $r$  goes to infinity,  $V = 0$ .
- (A)  $V_C < V_B < V_A$  (B)  $V_A < V_B < V_C$  (C)  $V_C = V_B = V_A$  (D)  $V_C < V_B = V_A$
110. Which statement about a system of point charges that are fixed in space is necessarily true?
- (A) If the potential energy of the system is negative, net positive work by an external agent is required to take the charges in the system back to infinity.  
 (B) If the potential energy of the system is positive, net positive work is required to bring any new charge not part of the system in from infinity to its final resting location.  
 (C) If the potential energy of the system is zero, no negative charges are in the configuration.  
 (D) If the potential energy of the system is negative, net positive work by an external agent was required to assemble the system of charges.

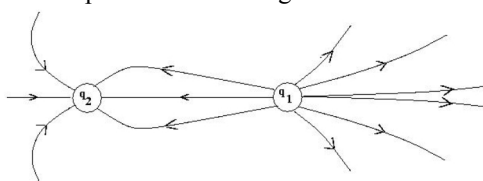
111. A positive point charge exerts a force of magnitude  $F$  on a negative point charge placed a distance  $x$  away. The negative point charge is replaced with a positive point charge and the distance between the two point charges is halved. What is the magnitude of the new force that the positive point charge exerts on the negative point charge?

- (A)  $4F$  (B)  $2F$  (C)  $F$  (D)  $F/2$

112. Two uniformly charged non-conducting spheres on insulating bases are placed on an air table. Sphere A has a charge  $+3Q$  coulombs and sphere B has a charge  $+Q$  coulombs. Which of the following correctly illustrates the magnitude and direction of the electrostatic force between the spheres when they are released?

- (A) 
- (B) 
- (C) 
- (D) 

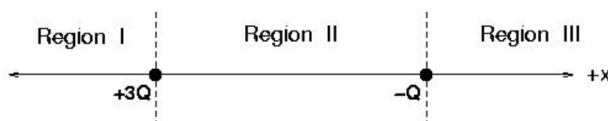
113. For the diagram shown below, what is the ratio of the charges  $q_2/q_1$  where the diagram shown has a representation of the field lines in the space near the charges?



- (A)  $-3/2$  (B)  $-2/3$  (C)  $2/3$  (D)  $3/2$

#### Questions 114-115

Two point charges are fixed on the  $x$ -axis in otherwise empty space as shown below.



114. In which Region(s) is there a place on the  $x$ -axis (aside from infinity) at which the electric potential is equal to zero?

- (A) Only in Region II (C) In both Regions I and III  
(B) In both Regions I and II (D) In both Regions II and III

115. In which Region(s) is there a place on the  $x$ -axis (aside from infinity) at which the electric field is equal to zero?

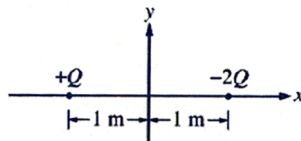
- (A) Only in Region I (C) Only in Region III  
(B) In both Regions I and II (D) In both Regions II and III

116. A parallel-plate capacitor is connected to a battery. Without disconnecting the capacitor, a student pulls the capacitor's plates apart so that the plate separation doubles. As a result of this action, what happens to the voltage across the capacitor and the energy stored by the capacitor?

- (A) the voltage doubles; the energy stays the same  
(B) the voltage doubles; the energy halves  
(C) the voltage stays the same; the energy halves  
(D) the voltage stays the same; the energy doubles

117. A person rubs a neutral comb through their hair and the comb becomes negatively charged. Which of the following is the best explanation for this phenomenon?

- (A) The hair gains protons from the comb.
- (B) The hair gains protons from the comb while giving electrons to the comb.
- (C) The hair loses electrons to the comb.
- (D) The comb loses protons to the person's hand holding the comb.



118. A charge of  $+Q$  is located on the  $x$ -axis at  $x = -1$  meter and a charge of  $-2Q$  is held at  $x = +1$  meter, as shown in the diagram above. At what position on the  $x$ -axis will a test charge of  $+q$  experience a zero net electrostatic force?

- (A)  $-(3 + \sqrt{8})$  m
- (B)  $-1/3$  m
- (C)  $1/3$  m
- (D)  $(3 + \sqrt{8})$  m

119. The two plates of a parallel-plate capacitor are a distance  $d$  apart and are mounted on insulating supports. A battery is connected across the capacitor to charge it and is then disconnected. The distance between the insulated plates is then increased to  $2d$ . If fringing of the field is still negligible, which of the following quantities is doubled?

- (A) The capacitance of the capacitor
- (B) The surface density of the charge on the plates of the capacitor
- (C) The energy stored in the capacitor
- (D) The intensity of the electric field between the plates of the capacitor

120. Two point objects each carrying charge  $10Q$  are separated by a distance  $d$ . The force between them is  $F$ . If half the charge on one object is transferred to the other object while at the same time the distance between them is doubled, what is the new force between the two objects?

- (A)  $0.19 F$
- (B)  $0.25 F$
- (C)  $0.75 F$
- (D)  $4.0 F$

121. Two identical spheres carry identical electric charges. If the spheres are set a distance  $d$  apart they repel one another with a force  $F$ . A third sphere, identical to the other two but initially uncharged is then touched to one sphere and then to the other before being removed. What would be the resulting force between the original two spheres?

- (A)  $\frac{3}{4} F$
- (B)  $\frac{5}{8} F$
- (C)  $\frac{1}{2} F$
- (D)  $\frac{3}{8} F$

122. An alpha particle is accelerated to a velocity  $v$  in a particle accelerator by a potential difference of 1200 V. Which of the following potential differences would be needed to give the alpha particle twice the velocity?

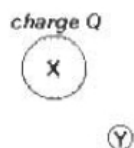
- (A) 4800 V
- (B) 4100 V
- (C) 2400 V
- (D) 1700 V

123. An electrical charge  $Q$  is placed at one vertex of an equilateral triangle. When an identical charge is placed at another vertex, each charge feels a force of 15 N. When a third charge identical to the first two, is placed at the third vertex, what would be the magnitude of the force on each charge?

- (A) 15 N
- (B) 26 N
- (C) 30 N
- (D) 42 N

124. Two conducting spheres with the same charge  $Q$  are separated by an infinite distance. Sphere A has a radius of 10 cm while sphere B has a radius of 20 cm. At what distance from the centers of the spheres would the magnitude of the electric field be the same?

- (A) 15 cm from A and 15 cm from B
- (C) 20 cm from A and 40 cm from B
- (B) 30 cm from A and 40 cm from B
- (D) 40 cm from A and 40 cm from B

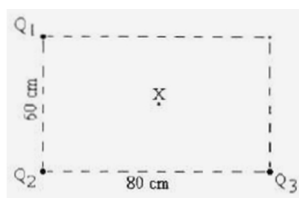


125. A large conducting sphere labeled X contains an electrical charge  $Q$ . Sphere X is connected by a metal wire to a small uncharged conducting sphere labeled Y. The wire is then removed. How does the electrical field ( $E_y$ ) at the surface of sphere Y compare to the electrical field ( $E_x$ ) at the surface of sphere X?  
 (A)  $E_x = E_y = 0$  (B)  $E_y = E_x \neq 0$  (C)  $E_y < E_x$  (D)  $E_y > E_x$
126. What voltage would be required across a 8.9 nF capacitor to accumulate  $1.5 \times 10^{12}$  excess electrons on one plate of the capacitor?  
 (A) 3.7 V (B) 5.9 V (C) 14 V (D) 27 V



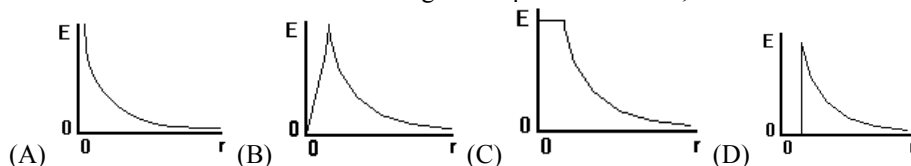
127. A hollow metal sphere is uniformly charged with positive charge. Points K and L are inside the sphere and points M and N are outside the sphere as shown in the diagram. At which pair of points would the field be the smallest?  
 (A) points K and N (B) points L and M (C) points K and L (D) points M and N

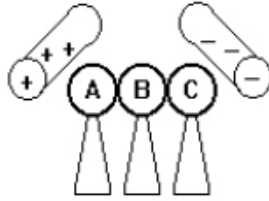
#### Questions 128-129



Three electric charges ( $Q_1$ ,  $Q_2$ , and  $Q_3$ ) are arranged at three corners of a rectangle as shown in the diagram and each has a charge of  $-40$  nC.

128. What is the magnitude of the net force on  $Q_2$ ?  
 (A)  $1.4 \times 10^{-5}$  N (B)  $1.7 \times 10^{-5}$  N (C)  $4.2 \times 10^{-5}$  N (D)  $4.6 \times 10^{-5}$  N
129. What would be the magnitude of the total electric field at center point X?  
 (A) 1440 N/C (B) 720 N/C (C) 360 N/C (D) 180 N/C
130. Which of the following graphs would best represent the electric field of a hollow sphere as a function of distance from its center when it is charged to a potential of 400,000 volts?

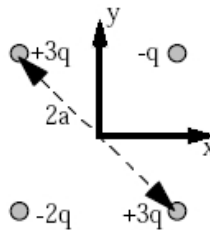
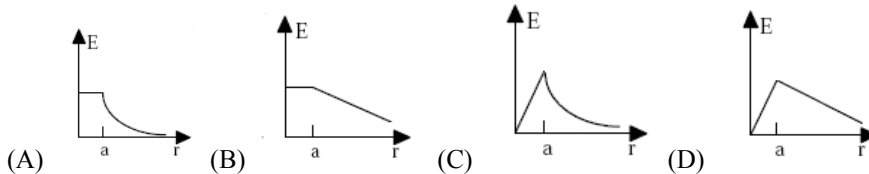




131. Three metal spheres A, B, and C are mounted on insulating stands. The spheres are touching one another, as shown in the diagram below. A strong positively charged object is brought near sphere A and a strong negative charge is brought near sphere C. While the charged objects remain near spheres A and C, sphere B is removed by means of its insulating stand. After the charged objects are removed, sphere B is first touched to sphere A and then to sphere C. The resulting charge on B would be of what relative amount and sign?

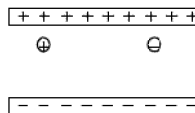
(A) the same sign but  $1/2$  the magnitude as originally on sphere A  
 (B) the opposite sign but  $1/2$  the magnitude as originally on sphere A  
 (C) the opposite sign but  $1/4$  the magnitude as originally on sphere A  
 (D) the same sign but  $1/2$  the magnitude as originally on sphere C

132. A charge is uniformly distributed through a volume of radius  $a$ . Which of the graphs below best represents the magnitude of the electric field as a function of distance from the center of the sphere?



133. Four point charges are placed at the corners of a square with diagonal  $2a$  as shown in the diagram. What is the total electric field at the center of the square?

(A)  $kq/a^2$  at an angle  $45^\circ$  above the  $+x$  axis.  
 (B)  $kq/a^2$  at an angle  $45^\circ$  below the  $-x$  axis.  
 (C)  $3kq/a^2$  at an angle  $45^\circ$  above the  $-x$  axis.  
 (D)  $3kq/a^2$  at an angle  $45^\circ$  below the  $+x$  axis.

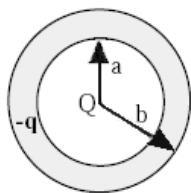


134. A free electron and a free proton are placed between two oppositely charged parallel plates. Both are closer to the positive plate than the negative plate. See the diagram below. Which of the following statements is true?

I. The force on the proton is greater than the force on the electron.  
 II. The potential energy of the proton is greater than that of the electron.  
 III. The potential energy of the proton and the electron is the same.

(A) II only (B) III only (C) I & II only (D) I & III only

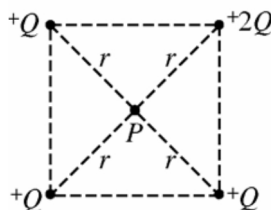
Questions 135-136



A spherical shell with an inner surface of radius  $a$  and an outer surface of radius  $b$  is made of conducting material. A charge  $+Q$  is placed at the center of the spherical shell and a total charge  $-q$  is placed on the shell.

135. How is the charge  $-q$  distributed after it has reached equilibrium?
- (A)  $+Q$  on the inner surface,  $-q - Q$  on the outer surface.
  - (B) The charge  $-q$  is spread uniformly between the inner and outer surface.
  - (C)  $-Q$  on the inner surface,  $-q + Q$  on the outer surface.
  - (D)  $-Q$  on the inner surface,  $-q$  on the outer surface.
136. What is the electrostatic potential at a distance  $R$  from the center of the shell, where  $b < R < a$ ?
- (A)  $kQ/a$  (B)  $kQ/R$  (C)  $k(Q - q)/R$  (D)  $k(Q - q)/b$
137. Conducting sphere  $X$  is initially uncharged. Conducting sphere  $Y$  has twice the diameter of sphere  $X$  and initially has charge  $q$ . If the spheres are connected by a long thin wire, which of the following is true once equilibrium has been reached?
- (A) Sphere  $Y$  has half the potential of sphere  $X$ .
  - (B) Spheres  $X$  and  $Y$  have the same potential.
  - (C) Sphere  $Y$  has half the charge of sphere  $X$ .
  - (D) Spheres  $X$  and  $Y$  have the same charge.

Questions 138-139



Four positive charges are fixed at the corners of a square, as shown above. Three of the charges have magnitude  $Q$ , and the fourth charge has a magnitude  $2Q$ . Point  $P$  is at the center of the square at a distance  $r$  from each charge.

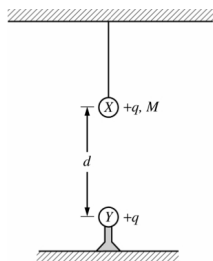
138. What is the electric potential at point  $P$ ?
- (A) Zero (B)  $2kQ/r$  (C)  $4kQ/r$  (D)  $5kQ/r$
139. What is the magnitude of the electric field at point  $P$ ?
- (A)  $kQ/r^2$  (B)  $2kQ/r^2$  (C)  $4kQ/r^2$  (D)  $5kQ/r^2$



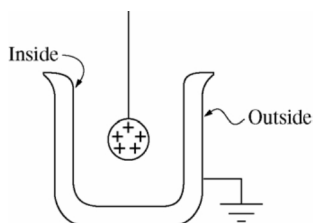
140. The two charged metal spheres  $X$  and  $Y$  shown above are far apart, and each is isolated from all other charges. The radius of sphere  $X$  is greater than that of sphere  $Y$ , and the magnitudes of the electric fields just outside their surfaces are the same. How does the charge on sphere  $X$  compare with that on sphere  $Y$ ?
- (A) It is greater.
  - (B) It is less.
  - (C) It is the same.
  - (D) It cannot be determined without knowing the actual value of the electric field just outside the spheres.



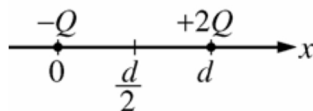
141. Two negative point charges are a distance  $x$  apart and have potential energy  $U$ . If the distance between the point charges increases to  $3x$ , what is their new potential energy?  
 (A)  $9U$  (B)  $3U$  (C)  $1/3 U$  (D)  $1/9 U$



142. Sphere  $X$  of mass  $M$  and charge  $+q$  hangs from a string as shown above. Sphere  $Y$  has an equal charge  $+q$  and is fixed in place a distance  $d$  directly below sphere  $X$ . If sphere  $X$  is in equilibrium, the tension in the string is most nearly  
 (A)  $Mg + kq/d$  (B)  $Mg - kq/d$  (C)  $Mg + kq^2/d^2$  (D)  $Mg - kq^2/d^2$

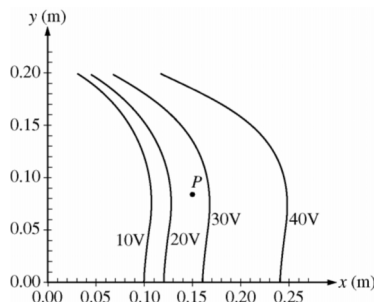


143. A small positively charged sphere is lowered by a nonconducting thread into a grounded metal cup without touching the inside surface of the cup, as shown above. The grounding wire attached to the outside surface is disconnected and the charged sphere is then removed from the cup. Which of the following best describes the subsequent distribution of excess charge on the surface of the cup?  
 (A) Negative charge resides on the inside surface, and no charge resides on the outside surface.  
 (B) Negative charge resides on the outside surface, and no charge resides on the inside surface.  
 (C) Positive charge resides on the inside surface, and no charge resides on the outside surface.  
 (D) Positive charge resides on the outside surface, and no charge resides on the inside surface.
144. A helium nucleus (charge  $+2q$  and mass  $4m$ ) and a lithium nucleus (charge  $+3q$  and mass  $7m$ ) are accelerated through the same electric potential difference,  $V_0$ . What is the ratio of their resultant kinetic energies,  $\frac{K_{\text{lithium}}}{K_{\text{Helium}}}$ ?  
 (A)  $2/3$  (B)  $6/7$  (C)  $7/6$  (D)  $3/2$



145. A point charge  $-Q$  is located at the origin, while a second point charge  $+2Q$  is located at  $x = d$  on the  $x$ -axis, as shown above. A point on the  $x$ -axis where the net electric field is zero is located in which of the following regions?  
 (A)  $-\infty < x < 0$  (B)  $0 < x < d/2$  (C)  $d/2 < x < d$  (D)  $d < x < \infty$

Questions 146-147



A fixed charge distribution produces the equipotential lines shown in the figure above.

146. Which of the following expressions best represents the magnitude of the electric field at point  $P$  ?

- (A)  $10 \text{ V}/0.14 \text{ m}$  (B)  $10 \text{ V}/0.04 \text{ m}$  (C)  $25 \text{ V}/0.14 \text{ m}$  (D)  $25 \text{ V}/0.04 \text{ m}$

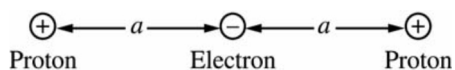
147. The direction of the electric field at point  $P$  is most nearly

- (A) toward the left  
(B) toward the right  
(C) toward the bottom of the page  
(D) toward the top of the page

148. A cloud contains spherical drops of water of radius  $R$  and charge  $Q$ . Assume the drops are far apart.

If two droplets happen to combine into a single larger droplet, the new potential  $V$  at the surface of the larger droplet is most nearly equal to

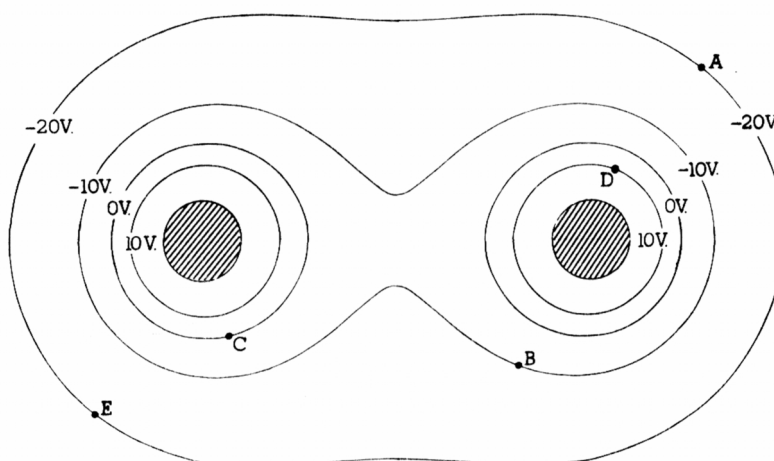
- (A)  $2V_0$  (B)  $\frac{2}{\sqrt{2}}V_0$  (C)  $\sqrt[3]{2}V_0$  (D)  $V_0$



149. Two protons and an electron are assembled along a line, as shown above. The distance between the electron and each proton is  $a$ . What is the work done by an external force in assembling this configuration of charges?

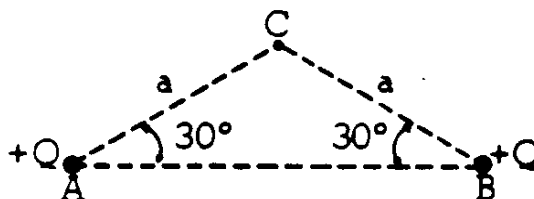
- (A)  $-2ke^2/a$  (B)  $-3ke^2/2a$  (C)  $ke^2/2a$  (D)  $3ke^2/a$

AP Physics Free Response Practice – Electrostatics



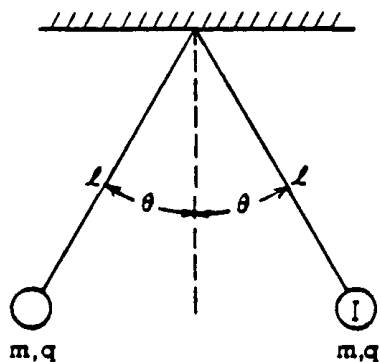
1974B5. The diagram above shows some of the equipotentials in a plane perpendicular to two parallel charged metal cylinders. The potential of each line is labeled.

- The left cylinder is charged positively. What is the sign of the charge on the other cylinder?
- On the diagram above, sketch lines to describe the electric field produced by the charged cylinders.
- Determine the potential difference,  $V_A - V_B$ , between points A and B.
- How much work is done by the field if a charge of 0.50 coulomb is moved along a path from point A to point E and then to point D?



1975B2. Two identical electric charges  $+Q$  are located at two corners A and B of an isosceles triangle as shown above.

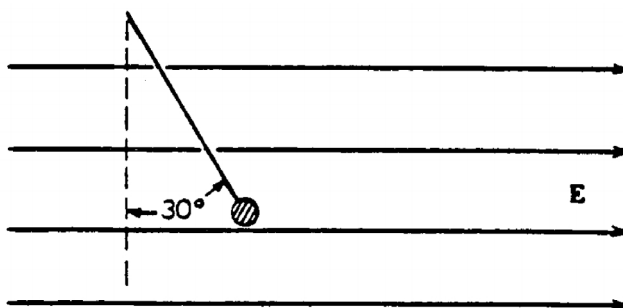
- How much work does the electric field do on a small test charge  $+q$  as the charge moves from point C to infinity,
- In terms of the given quantities, determine where a third charge  $+2Q$  should be placed so that the electric field at point C is zero. Indicate the location of this charge on the diagram above.



1979B7. Two small spheres, each of mass  $m$  and positive charge  $q$ , hang from light threads of lengths  $l$ .

Each thread makes an angle  $\theta$  with the vertical as shown above.

- On the diagram draw and label all forces on sphere I.
- Develop an expression for the charge  $q$  in terms of  $m$ ,  $l$ ,  $\theta$ ,  $g$ , and the Coulomb's law constant.



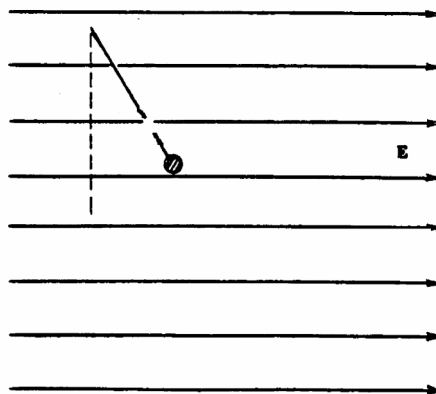
1981B3. A small conducting sphere of mass  $5 \times 10^{-3}$  kilogram, attached to a string of length 0.2 meter, is at rest in a uniform electric field  $E$ , directed horizontally to the right as shown above. There is a charge of  $5 \times 10^{-6}$  coulomb on the sphere. The string makes an angle of  $30^\circ$  with the vertical.

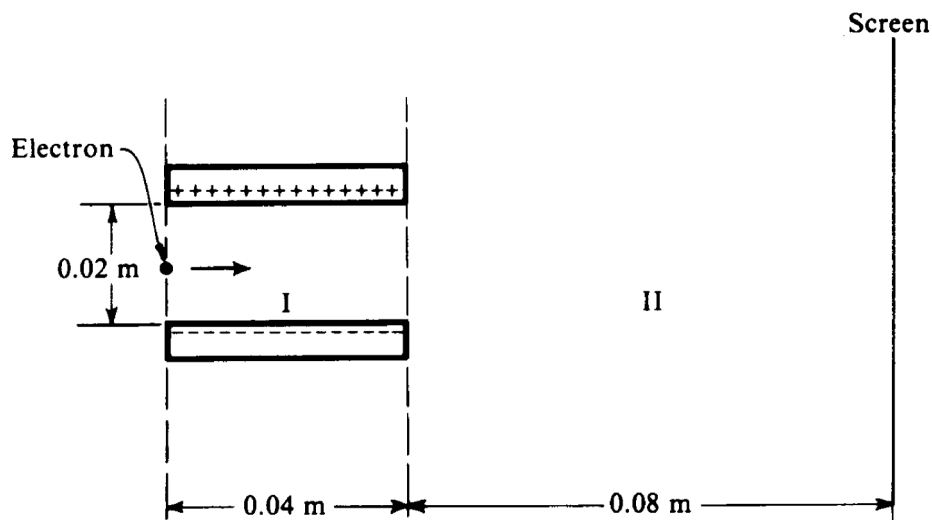
Assume  $g = 10$  meters per second squared.

- In the space below, draw and label all the forces acting on the sphere.



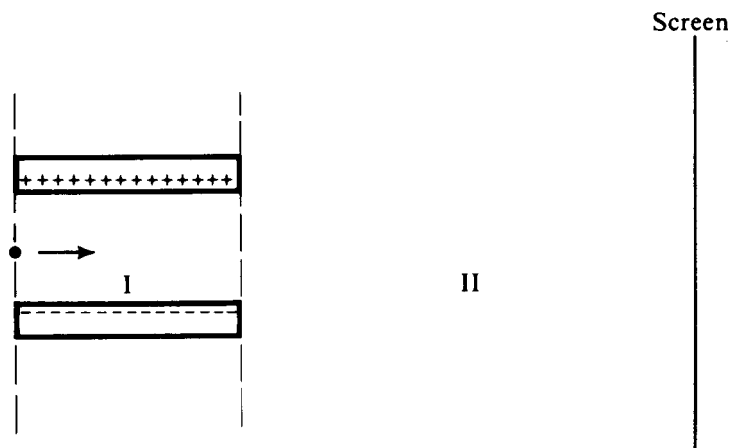
- Calculate the tension in the string and the magnitude of the electric field.
- The string now breaks. Describe the subsequent motion of the sphere and sketch on the following diagram the path of the sphere while in the electric field.

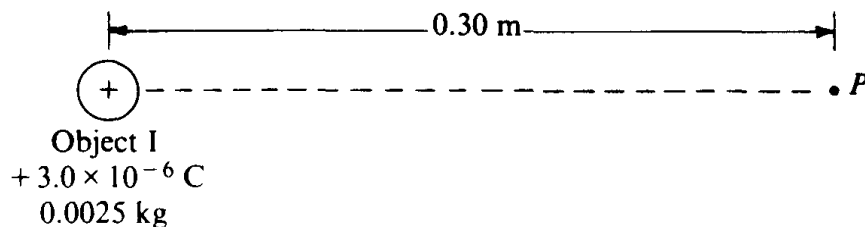




1985B3. An electron initially moves in a horizontal direction and has a kinetic energy of  $2.0 \times 10^3$  electron-volts when it is in the position shown above. It passes through a uniform electric field between two oppositely charged horizontal plates (region I) and a field-free region (region II) before eventually striking a screen at a distance of 0.08 meter from the edge of the plates. The plates are 0.04 meter long and are separated from each other by a distance of 0.02 meter. The potential difference across the plates is 250 volts. Gravity is negligible.

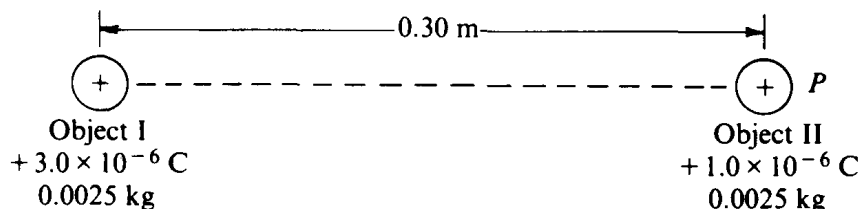
- Calculate the initial speed of the electron as it enters region I.
- Calculate the magnitude of the electric field  $E$  between the plates, and indicate its direction on the diagram above.
- Calculate the magnitude of the electric force  $F$  acting on the electron while it is in region I.
- On the diagram below, sketch the path of the electron in regions I and II. For each region describe the shape of the path.





1987B2. Object I, shown above, has a charge of  $+ 3 \times 10^{-6}$  coulomb and a mass of 0.0025 kilogram.

- a. What is the electric potential at point P, 0.30 meter from object I?

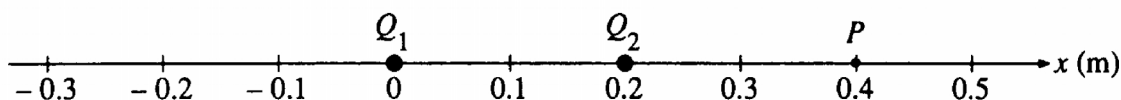


Object II, of the same mass as object I, but having a charge of  $+ 1 \times 10^{-6}$  coulomb, is brought from infinity to point P, as shown above.

- b. How much work must be done to bring the object II from infinity to point P?  
 c. What is the magnitude of the electric force between the two objects when they are 0.30 meter apart?  
 d. What are the magnitude and direction of the electric field at the point midway between the two objects?

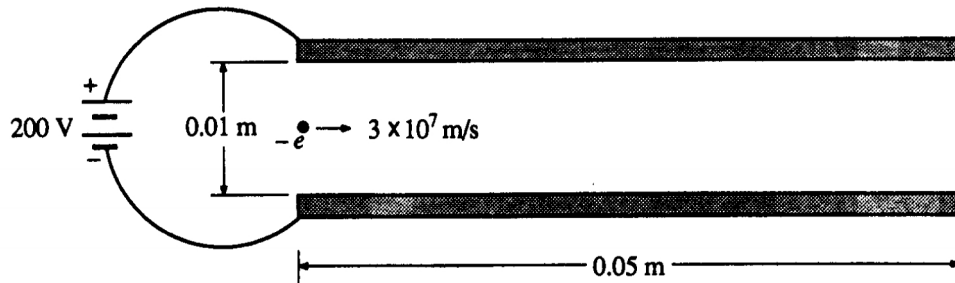
The two objects are then released simultaneously and move apart due to the electric force between them. No other forces act on the objects.

- e. What is the speed of object I when the objects are very far apart?

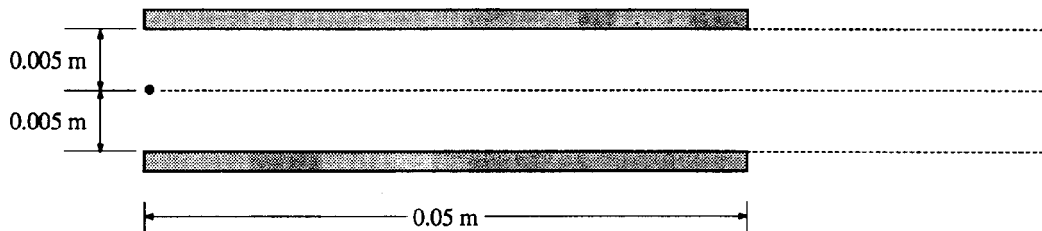


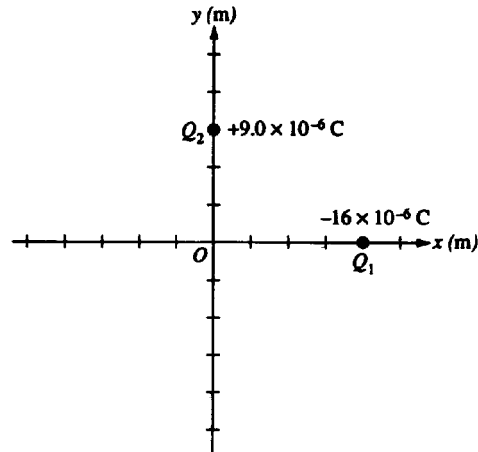
1989B2. Two point charges,  $Q_1$  and  $Q_2$ , are located a distance 0.20 meter apart, as shown above. Charge  $Q_1 = +8.0 \mu\text{C}$ . The net electric field is zero at point P, located 0.40 meter from  $Q_1$  and 0.20 meter from  $Q_2$ .

- a. Determine the magnitude and sign of charge  $Q_2$ .  
 b. Determine the magnitude and direction of the net force on charge  $Q_1$ .  
 c. Calculate the electrostatic potential energy of the system.  
 d. Determine the coordinate of the point R on the x-axis between the two charges at which the electric potential is zero.  
 e. How much work is needed to bring an electron from infinity to point R, which was determined in the previous part?



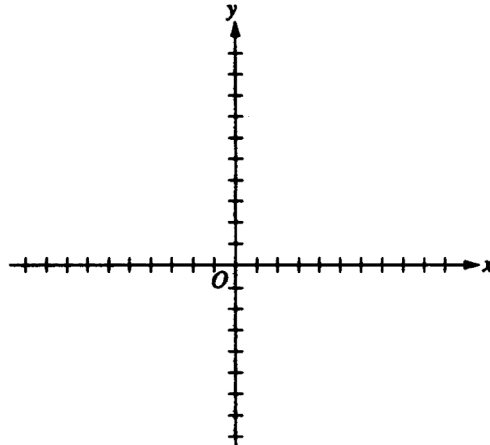
- 1990B2 (modified) A pair of square parallel conducting plates, having sides of length 0.05 meter, are 0.01 meter apart and are connected to a 200-volt power supply, as shown above. An electron is moving horizontally with a speed of  $3 \times 10^7$  meters per second when it enters the region between the plates. Neglect gravitation and the distortion of the electric field around the edges of the plates.
- Determine the magnitude of the electric field in the region between the plates and indicate its direction on the figure above.
  - Determine the magnitude and direction of the acceleration of the electron in the region between the plates.
  - Determine the magnitude of the vertical displacement of the electron for the time interval during which it moves through the region between the plates.
  - On the diagram below, sketch the path of the electron as it moves through and after it emerges from the region between the plates. The dashed lines in the diagram have been added for reference only.



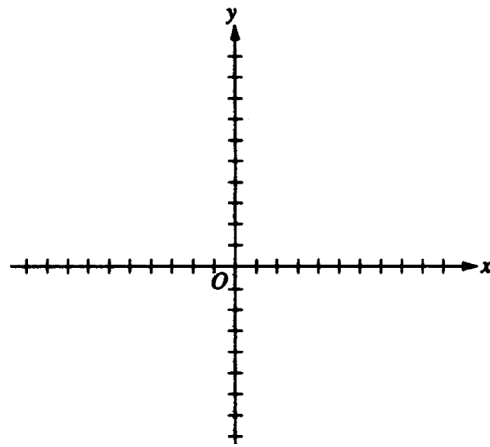


1993B2. A charge  $Q_1 = -1.6 \times 10^{-6}$  coulomb is fixed on the  $x$ -axis at +4.0 meters, and a charge  $Q_2 = +9 \times 10^{-6}$  coulomb is fixed on the  $y$ -axis at +3.0 meters, as shown on the diagram above.

- a.
  - i. Calculate the magnitude of the electric field  $E_1$  at the origin O due to charge  $Q_1$
  - ii. Calculate the magnitude of the electric field  $E_2$  at the origin O due to charge  $Q_2$ .
  - iii. On the axes below, draw and label vectors to show the electric fields  $E_1$  and  $E_2$  due to each charge, and also indicate the resultant electric field  $E$  at the origin.

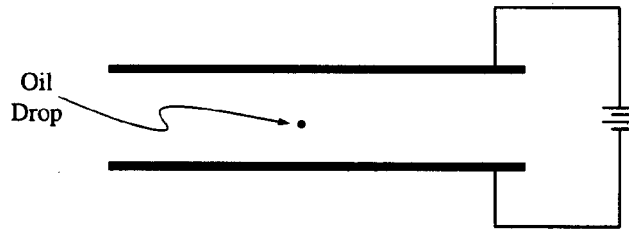


- b. Calculate the electric potential  $V$  at the origin.  
A charge  $Q_3 = -4 \times 10^{-6}$  coulomb is brought from a very distant point by an external force and placed at the origin.
- c. On the axes below, indicate the direction of the force on  $Q_3$  at the origin.



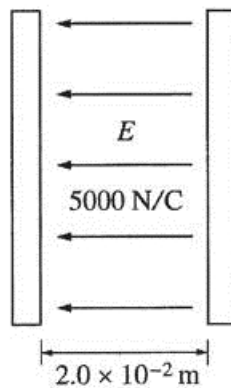
- d. Calculate the work that had to be done by the external force to bring  $Q_3$  to the origin from the distant point.





1996B6 Robert Millikan received a Nobel Prize for determining the charge on the electron. To do this, he set up a potential difference between two horizontal parallel metal plates. He then sprayed drops of oil between the plates and adjusted the potential difference until drops of a certain size remained suspended at rest between the plates, as shown above. Suppose that when the potential difference between the plates is adjusted until the electric field is  $10,000 \text{ N/C}$  downward, a certain drop with a mass of  $3.27 \times 10^{-16} \text{ kg}$  remains suspended.

- What is the magnitude of the charge on this drop?
- The electric field is downward, but the electric force on the drop is upward. Explain why.
- If the distance between the plates is  $0.01 \text{ m}$ , what is the potential difference between the plates?
- The oil in the drop slowly evaporates while the drop is being observed, but the charge on the drop remains the same. Indicate whether the drop remains at rest, moves upward, or moves downward. Explain briefly.



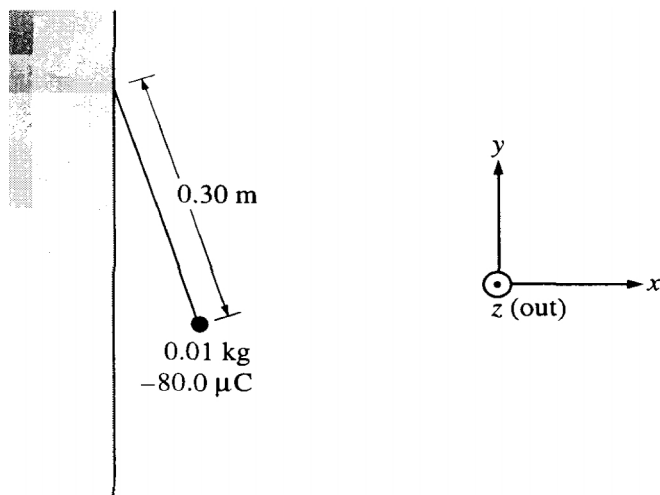
Note: Figure not drawn to scale.

2002B5B. Two parallel conducting plates, each of area  $0.30 \text{ m}^2$ , are separated by a distance of  $2.0 \times 10^{-2} \text{ m}$  of air. One plate has charge  $+Q$ ; the other has charge  $-Q$ . An electric field of  $5000 \text{ N/C}$  is directed to the left in the space between the plates, as shown in the diagram above.

- Indicate on the diagram which plate is positive (+) and which is negative (-).
- Determine the potential difference between the plates.
- Determine the capacitance of this arrangement of plates.

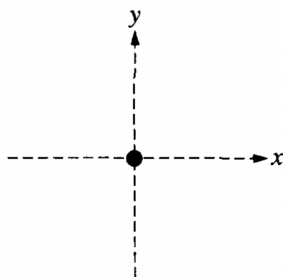
An electron is initially located at a point midway between the plates.

- Determine the magnitude of the electrostatic force on the electron at this location and state its direction.
- If the electron is released from rest at this location midway between the plates, determine its speed just before striking one of the plates. Assume that gravitational effects are negligible.



1998B2. A wall has a negative charge distribution producing a uniform horizontal electric field. A small plastic ball of mass  $0.01 \text{ kg}$ , carrying a charge of  $-80.0 \mu\text{C}$ , is suspended by an uncharged, nonconducting thread  $0.30 \text{ m}$  long. The thread is attached to the wall and the ball hangs in equilibrium, as shown above, in the electric and gravitational fields. The electric force on the ball has a magnitude of  $0.032 \text{ N}$ .

- a. On the diagram below, draw and label the forces acting on the ball.

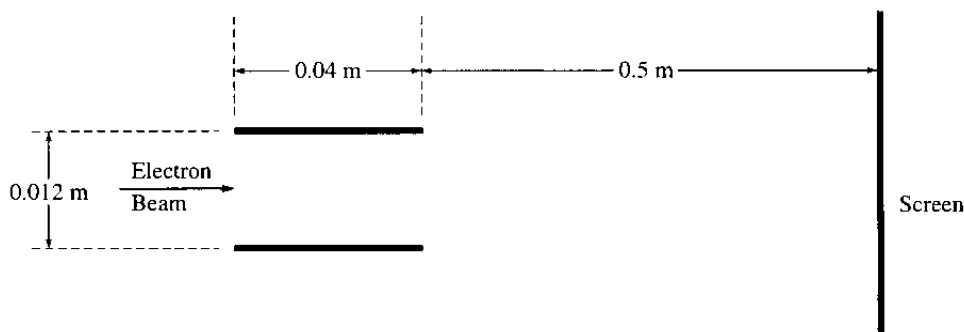


- b. Calculate the magnitude of the electric field at the ball's location due to the charged wall, and state its direction relative to the coordinate axes shown.
- c. Determine the perpendicular distance from the wall to the center of the ball.
- d. The string is now cut.
- Calculate the magnitude of the resulting acceleration of the ball, and state its direction relative to the coordinate axes shown.
  - Describe the resulting path of the ball.

1999B2. In a television set, electrons are first accelerated from rest through a potential difference in an electron gun. They then pass through deflecting plates before striking the screen.

- a. Determine the potential difference through which the electrons must be accelerated in the electron gun in order to have a speed of  $6.0 \times 10^7$  m/s when they enter the deflecting plates.

The pair of horizontal plates shown below is used to deflect electrons up or down in the television set by placing a potential difference across them. The plates have length 0.04 m and separation 0.012 m, and the right edge of the plates is 0.50 m from the screen. A potential difference of 200 V is applied across the plates, and the electrons are deflected toward the top of the screen. Assume that the electrons enter horizontally midway between the plates with a speed of  $6.0 \times 10^7$  m/s and that fringing effects at the edges of the plates and gravity are negligible.



Note: Figure not drawn to scale.

- b. Which plate in the pair must be at the higher potential for the electrons to be deflected upward? Check the appropriate box below.

☐

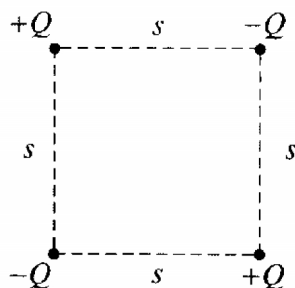
Upper plate

☐

Lower plate

Justify your answer.

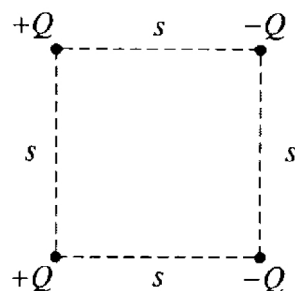
- c. Considering only an electron's motion as it moves through the space between the plates, compute the following.
- The time required for the electron to move through the plates
  - The vertical displacement of the electron while it is between the plates
- d. Show why it is a reasonable assumption to neglect gravity in part c.
- e. Still neglecting gravity, describe the path of the electrons from the time they leave the plates until they strike the screen. State a reason for your answer.



Arrangement 1

2001B3. Four charged particles are held fixed at the corners of a square of side  $s$ . All the charges have the same magnitude  $Q$ , but two are positive and two are negative. In Arrangement 1, shown above, charges of the same sign are at opposite corners. Express your answers to parts a. and b. in terms of the given quantities and fundamental constants.

- a. For Arrangement 1, determine the following.
  - i. The electrostatic potential at the center of the square
  - ii. The magnitude of the electric field at the center of the square



Arrangement 2

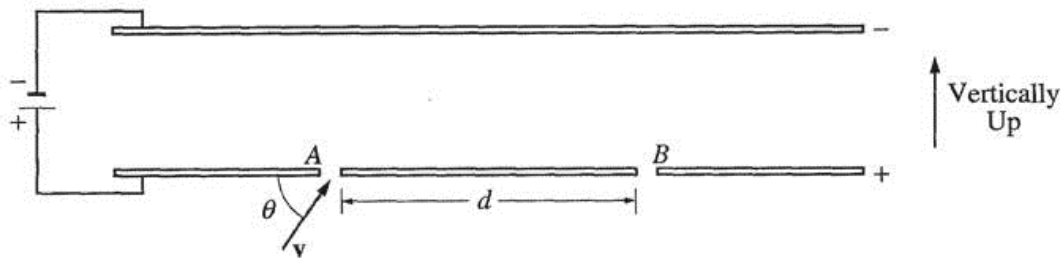
The bottom two charged particles are now switched to form Arrangement 2, shown above, in which the positively charged particles are on the left and the negatively charged particles are on the right.

- b. For Arrangement 2, determine the following.
  - i. The electrostatic potential at the center of the square
  - ii. The magnitude of the electric field at the center of the square
- c. In which of the two arrangements would more work be required to remove the particle at the upper right corner from its present position to a distance a long way away from the arrangement?

\_\_\_\_\_ Arrangement 1

\_\_\_\_\_ Arrangement 2

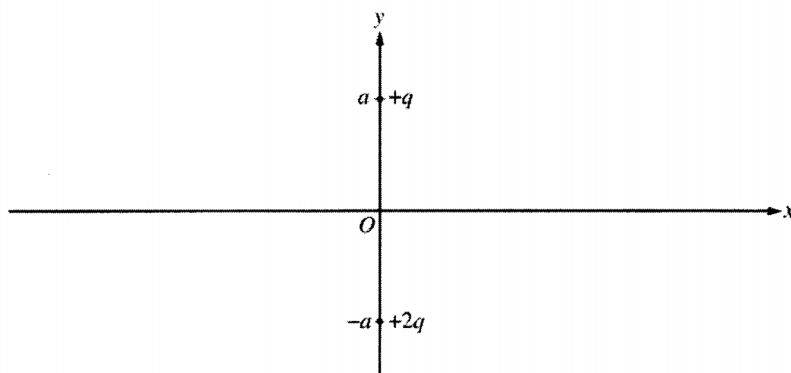
Justify your answer



2003Bb4. An electric field  $E$  exists in the region between the two electrically charged parallel plates shown above. A beam of electrons of mass  $m$ , charge  $q$ , and velocity  $v$  enters the region through a small hole at position A. The electrons exit the region between the plates through a small hole at position B.

Express your answers to the following questions in terms of the quantities  $m$ ,  $q$ ,  $E$ ,  $\theta$ , and  $v$ . Ignore the effects of gravity.

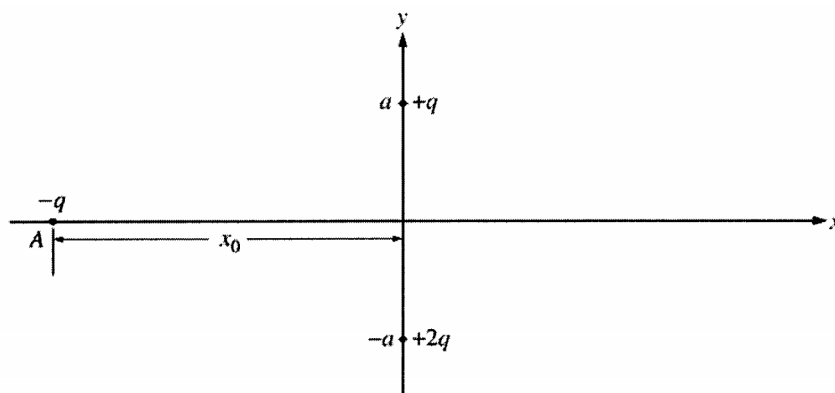
- a.
    - i. On the diagram of the parallel plates above, draw and label a vector to show the direction of the electric field  $E$  between the plates.
    - ii. On the following diagram, show the direction of the force(s) acting on an electron after it enters the region between the plates.
- 
- iii. On the diagram of the parallel plates above, show the trajectory of an electron that will exit through the small hole at position B.
  - b. Determine the magnitude of the acceleration of an electron after it has entered the region between the parallel plates.
  - c. Determine the total time that it takes the electrons to go from position A to position B.
  - d. Determine the distance  $d$  between positions A and B.
  - e. Now assume that the effects of gravity cannot be ignored in this problem. How would the distance where the electron exits the region between the plates change for an electron entering the region at A? Explain your reasoning.



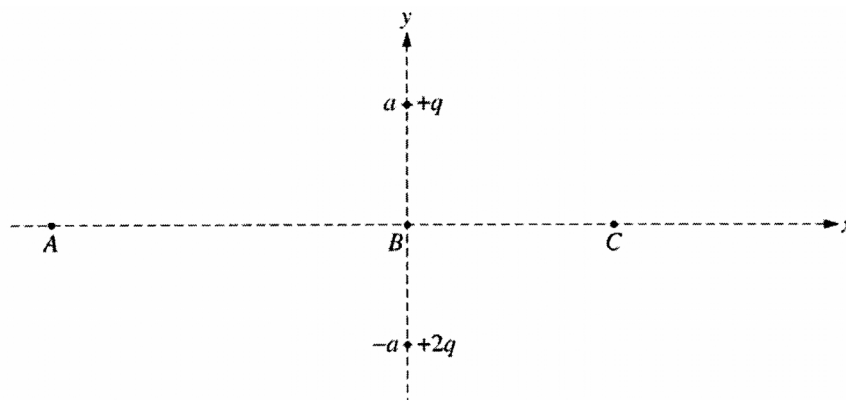
2005B3 Two point charges are fixed on the  $y$ -axis at the locations shown in the figure above. A charge of  $+q$  is located at  $y = +a$  and a charge of  $+2q$  is located at  $y = -a$ . Express your answers to parts a. and b. in terms of  $q$ ,  $a$ , and fundamental constants.

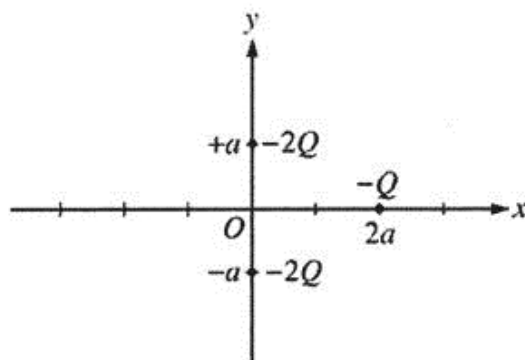
- Determine the magnitude and direction of the electric field at the origin.
- Determine the electric potential at the origin.

A third charge of  $-q$  is first placed at an arbitrary point A ( $x = -x_0$ ) on the  $x$ -axis as shown in the figure below.



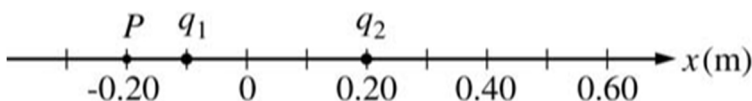
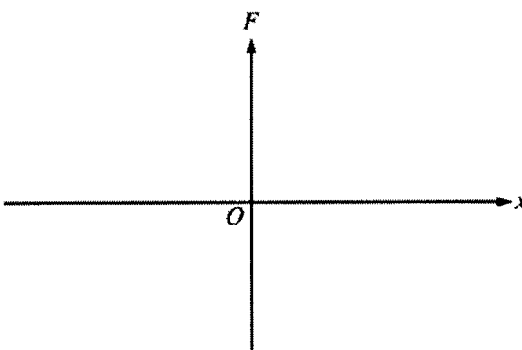
- Write expressions in terms of  $q$ ,  $a$ ,  $x_0$ , and fundamental constants for the magnitudes of the forces on the  $-q$  charge at point A caused by each of the following.
  - The  $+q$  charge
  - The  $+2q$  charge
- The  $-q$  charge can also be placed at other points on the  $x$ -axis. At each of the labeled points (A, B, and C) in the following diagram, draw a vector to represent the direction of the net force on the  $-q$  charge due to the other two charges when it is at those points.





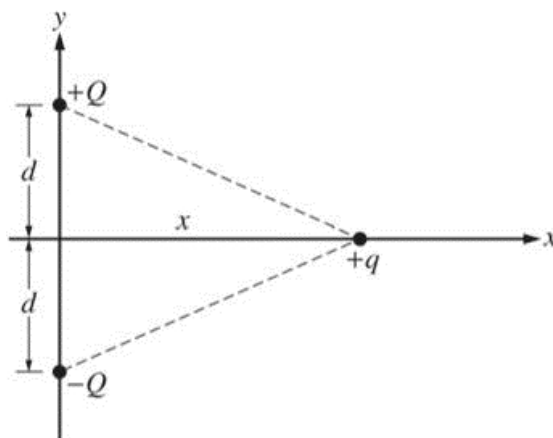
2005Bb3 The figure above shows two point charges, each of charge  $-2Q$ , fixed on the  $y$ -axis at  $y = +a$  and at  $y = -a$ . A third point charge of charge  $-Q$  is placed on the  $x$ -axis at  $x = 2a$ . Express all algebraic answers in terms of  $Q$ ,  $a$ , and fundamental constants.

- Derive an expression for the magnitude of the net force on the charge  $-Q$  due to the other two charges, and state its direction.
- Derive an expression for the magnitude of the net electric field at the origin due to all three charges, and state its direction.
- Derive an expression for the electrical potential at the origin due to all three charges.
- On the axes below, sketch a graph of the force  $F$  on the  $-Q$  charge caused by the other two charges as it is moved along the  $x$ -axis from a large positive position to a large negative position. Let the force be positive when it acts to the right and negative when it acts to the left.



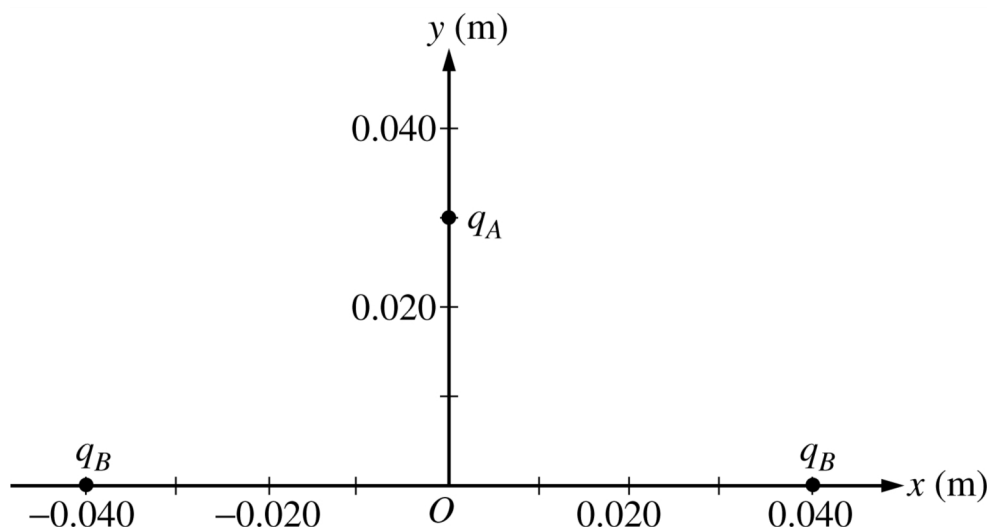
2006B3. Two point charges,  $q_1$  and  $q_2$ , are placed 0.30 m apart on the  $x$ -axis, as shown in the figure above. Charge  $q_1$  has a value of  $-3.0 \times 10^{-9}$  C. The net electric field at point P is zero.

- What is the sign of charge  $q_2$ ?  
 \_\_\_\_\_ Positive      \_\_\_\_\_ Negative  
 Justify your answer.
- Calculate the magnitude of charge  $q_2$ .
- Calculate the magnitude of the electric force on  $q_2$  and indicate its direction.
- Determine the  $x$ -coordinate of the point on the line between the two charges at which the electric potential is zero.
- How much work must be done by an external force to bring an electron from infinity to the point at which the electric potential is zero? Explain your reasoning.



2006Bb3. Three electric charges are arranged on an  $x$ - $y$  coordinate system, as shown above. Express all algebraic answers to the following parts in terms of  $Q$ ,  $q$ ,  $x$ ,  $d$ , and fundamental constants.

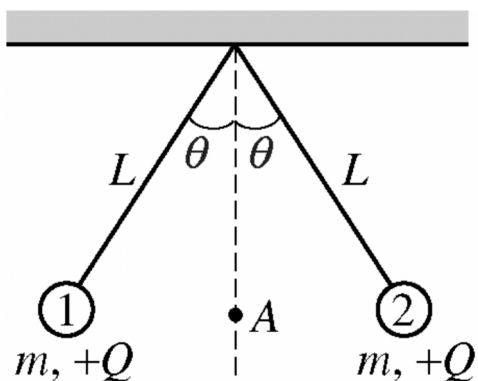
- On the diagram, draw vectors representing the forces  $F_1$  and  $F_2$  exerted on the  $+q$  charge by the  $+Q$  and  $-Q$  charges, respectively.
- Determine the magnitude and direction of the total electric force on the  $+q$  charge.
- Determine the electric field (magnitude and direction) at the position of the  $+q$  charge due to the other two charges.
- Calculate the electric potential at the position of the  $+q$  charge due to the other two charges.
- Charge  $+q$  is now moved along the positive  $x$ -axis to a very large distance from the other two charges. The magnitude of the force on the  $+q$  charge at this large distance now varies as  $1/x^3$ . Explain why this happens.



2009B2B.(modified) Three particles are arranged on coordinate axes as shown above. Particle A has charge  $q_A = -0.20 \text{ nC}$ , and is initially on the  $y$ -axis at  $y = 0.030 \text{ m}$ . The other two particles each have charge  $q_B = +0.30 \text{ nC}$  and are held fixed on the  $x$ -axis at  $x = -0.040 \text{ m}$  and  $x = +0.040 \text{ m}$ , respectively.

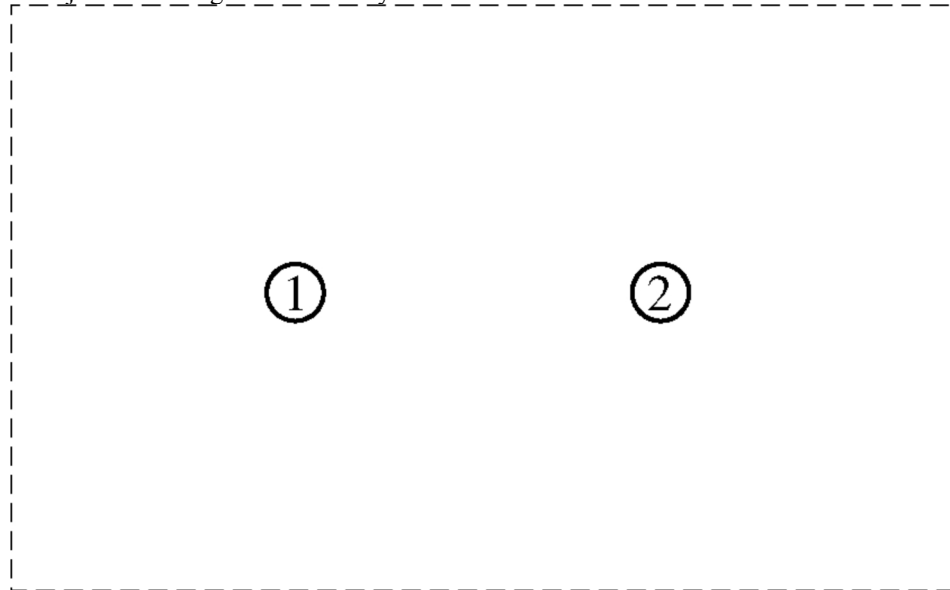
- Calculate the magnitude of the net electric force on particle A when it is at  $y = 0.030 \text{ m}$ , and state its direction.
- Particle A is then released from rest. Qualitatively describe its motion over a long time.



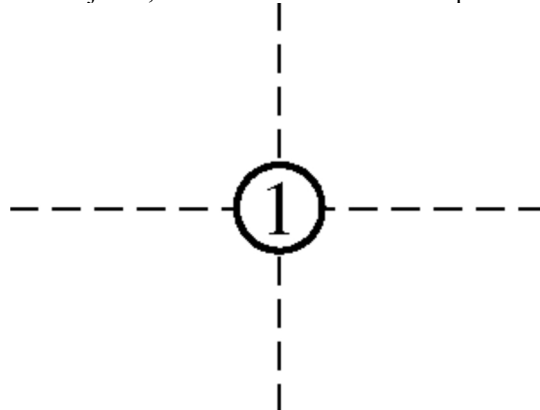


2009B2. Two small objects, labeled 1 and 2 in the diagram above, are suspended in equilibrium from strings of length  $L$ . Each object has mass  $m$  and charge  $+Q$ . Assume that the strings have negligible mass and are insulating and electrically neutral. Express all algebraic answers in terms of  $m$ ,  $L$ ,  $Q$ ,  $q$ , and fundamental constants.

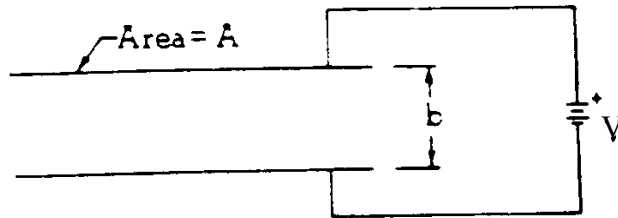
- a. On the following diagram, sketch lines to illustrate a 2-dimensional view of the net electric field due to the two objects in the region enclosed by the dashed lines.



- b. Derive an expression for the electric potential at point A, shown in the diagram at the top of the page, which is midway between the charged objects.
- c. On the following diagram of object 1, draw and label vectors to represent the forces on the object.



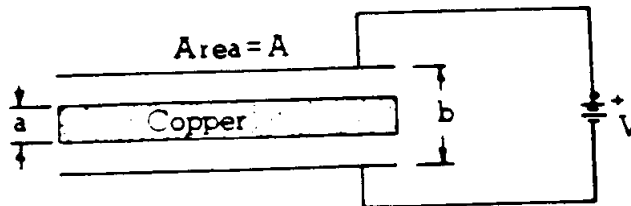
- d. Using the conditions of equilibrium, write—but do not solve—two equations that could, together, be solved for  $q$  and the tension  $T$  in the left-hand string.



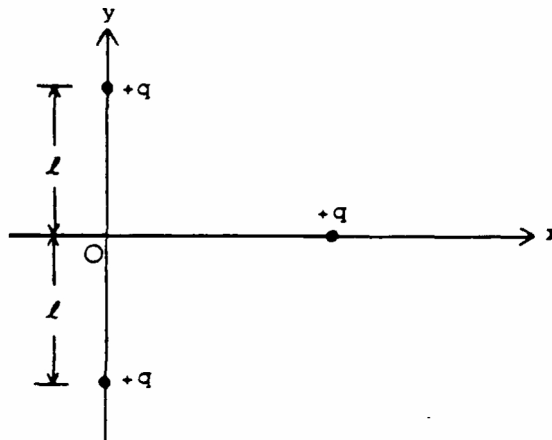
\*1974E2. A parallel-plate capacitor with spacing  $b$  and area  $A$  is connected to a battery of voltage  $V$  as shown above. Initially the space between the plates is empty. Make the following determinations in terms of the given symbols.

- Determine the electric field between the plates.
- Determine the charge stored on each capacitor plate.

A copper slab of thickness  $a$  is now inserted midway between the plates as shown below.

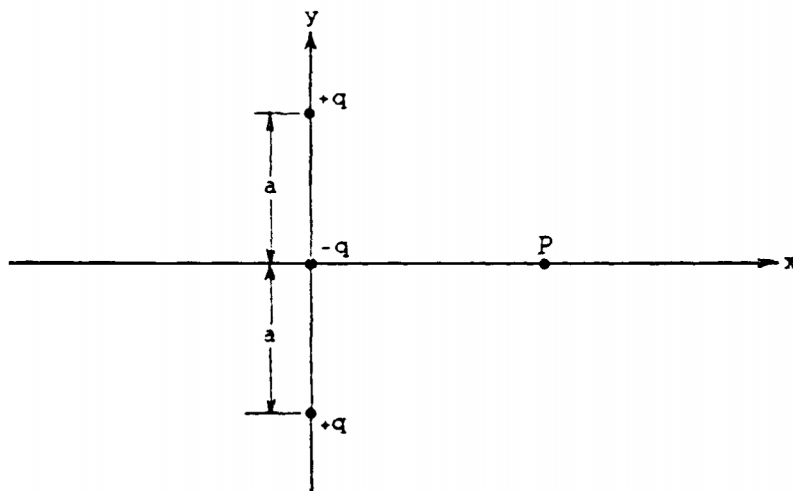


- Determine the electric field in the spaces above and below the slab.
- Determine the ratio of capacitances  $\frac{C_{\text{with copper}}}{C_{\text{original}}}$  when the slab is inserted



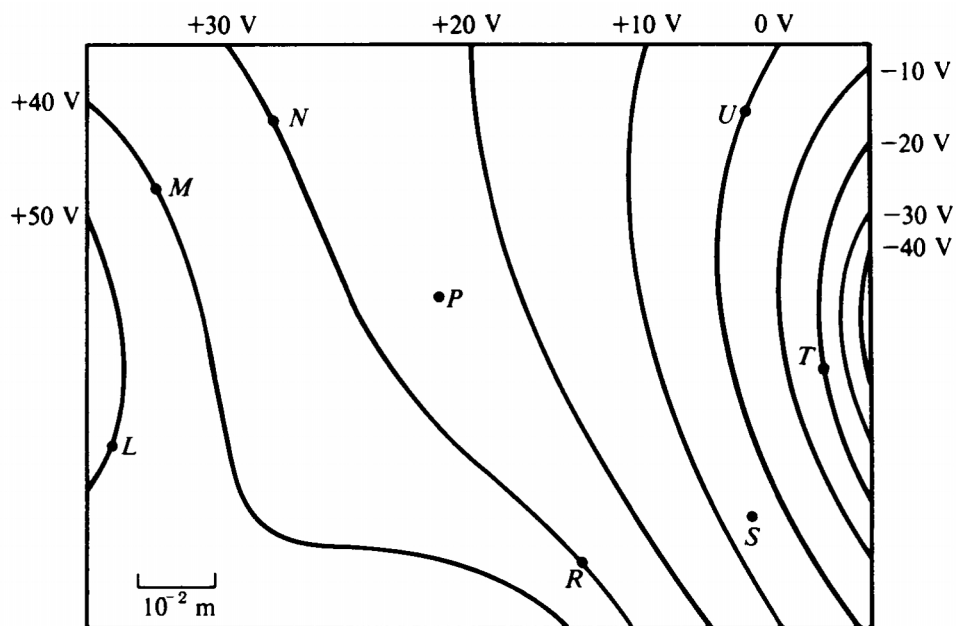
1975E1. Two stationary point charges  $+q$  are located on the  $y$ -axis as shown above. A third charge  $+q$  is brought in from infinity along the  $x$ -axis.

- Express the potential energy of the movable charge as a function of its position on the  $x$ -axis.
- Determine the magnitude and direction of the force acting on the movable charge when it is located at the position  $x = l$
- Determine the work done by the electric field as the charge moves from infinity to the origin.



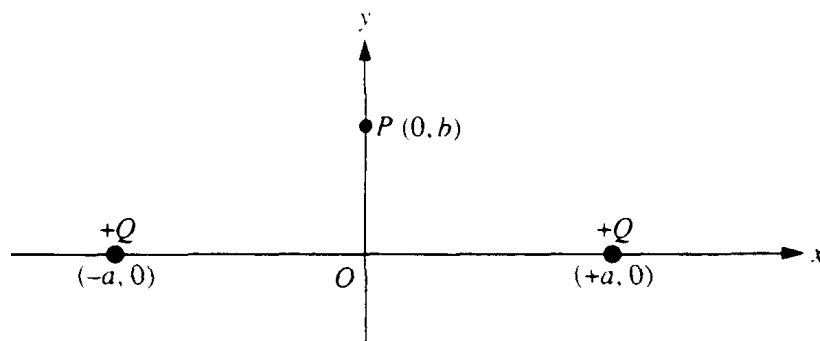
1982E1 (modified) Three point charges are arranged on the  $y$ -axis as shown above. The charges are  $+q$  at  $(0, a)$ ,  $-q$  at  $(0, 0)$ , and  $+q$  at  $(0, -a)$ . Any other charge or material is infinitely far away.

- Determine the point(s) on the  $x$ -axis where the electric potential due to this system of charges is zero.
- Determine the  $x$  and  $y$  components of the electric field at a point  $P$  on the  $x$ -axis at a distance  $x$  from the origin.



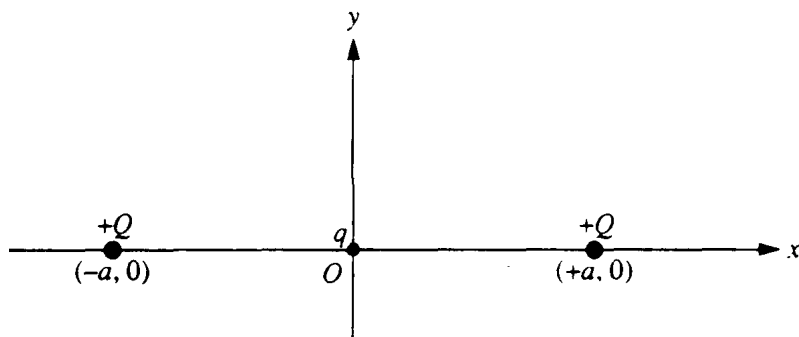
\*1986E1. Three point charges produce the electric equipotential lines shown on the diagram above.

- Draw arrows at points  $L$ ,  $N$ , and  $U$  on the diagram to indicate the direction of the electric field at these points.
- At which of the lettered points is the electric field  $E$  greatest in magnitude? Explain your reasoning.
- Compute an approximate value for the magnitude of the electric field  $E$  at point  $P$ .
- Compute an approximate value for the potential difference,  $V_M - V_S$ , between points  $M$  and  $S$ .
- Determine the work done by the field if a charge of  $+5 \times 10^{-12}$  coulomb is moved from point  $M$  to point  $R$ .
- If the charge of  $+5 \times 10^{-12}$  coulomb were moved from point  $M$  first to point  $S$ , and then to point  $R$ , would the answer to  $e$ . be different, and if so, how?



1991E1. Two equal positive charges  $Q$  are fixed on the  $x$ -axis, one at  $+a$  and the other at  $-a$ , as shown above. Point  $P$  is a point on the  $y$ -axis with coordinates  $(0, b)$ . Determine each of the following in terms of the given quantities and fundamental constants.

- The electric field  $E$  at the origin  $O$ .
- The electric potential  $V$  at the origin  $O$ .
- The magnitude of the electric field  $E$  at point  $P$ .



A small particle of charge  $q$  ( $q \ll Q$ ) and mass  $m$  is placed at the origin, displaced slightly, and then released. Assume that the only subsequent forces acting are the electric forces from the two fixed charges  $Q$ , at  $x = +a$  and  $x = -a$ , and that the particle moves only in the  $xy$ -plane. In each of the following cases, describe briefly the motion of the charged particle after it is released. Write an expression for its speed when far away if the resulting force pushes it away from the origin.

- $q$  is positive and is displaced in the  $+x$  direction.
- $q$  is positive and is displaced in the  $+y$  direction.
- $q$  is negative and is displaced in the  $+y$  direction.

2000E2 (modified) Three particles, A, B, and C, have equal positive charges  $Q$  and are held in place at the vertices of an equilateral triangle with sides of length  $l$ , as shown in the figures below. The dotted lines represent the bisectors for each side. The base of the triangle lies on the  $x$ -axis, and the altitude of the triangle lies on the  $y$ -axis.

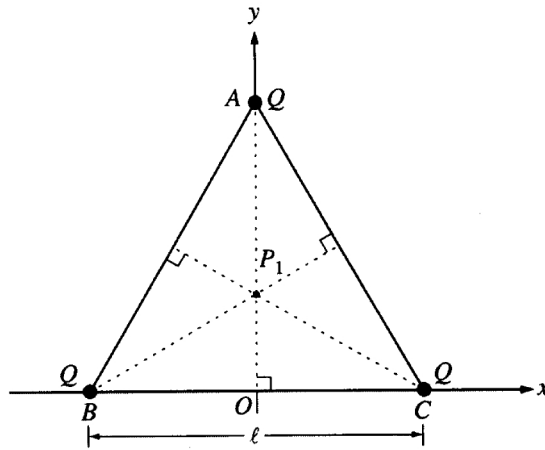


Figure 1

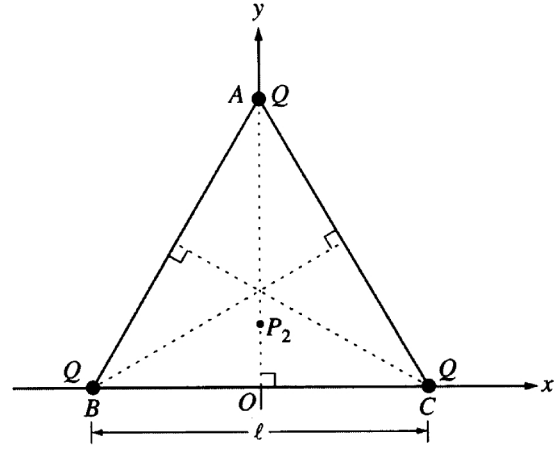
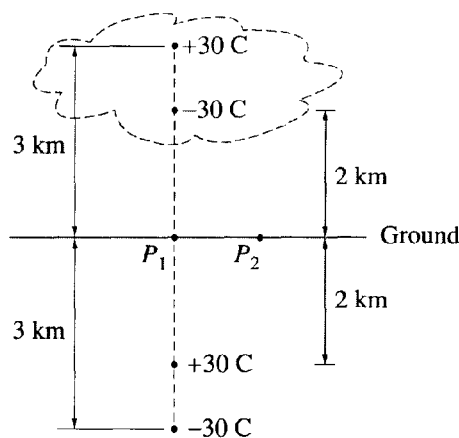
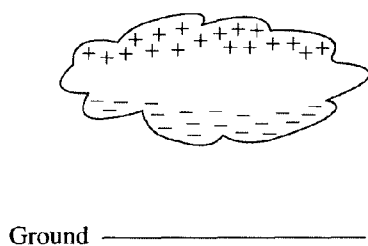


Figure 2

- i. Point  $P_1$ , the intersection of the three bisectors, locates the geometric center of the triangle and is one point where the electric field is zero. On Figure 1 above, draw the electric field vectors  $E_A$ ,  $E_B$ , and  $E_C$  at  $P_1$  due to each of the three charges. Be sure your arrows are drawn to reflect the relative magnitude of the fields.
- ii. Another point where the electric field is zero is point  $P_2$  at  $(0, y_2)$ . On Figure 2 above, draw electric field vectors  $E_A$ ,  $E_B$ , and  $E_C$  at  $P_2$  due to each of the three point charges. Indicate below whether the magnitude of each of these vectors is greater than, less than, or the same as for point  $P_1$ .

	Greater than at $P_1$	Less than at $P_1$	The same as at $P_1$
$E_A$			
$E_B$			
$E_C$			

- b. Explain why the  $x$ -component of the total electric field is zero at any point on the  $y$ -axis.
- c. Write a general expression for the electric potential  $V$  at any point on the  $y$ -axis inside the triangle in terms of  $Q$ ,  $l$ , and  $y$ .



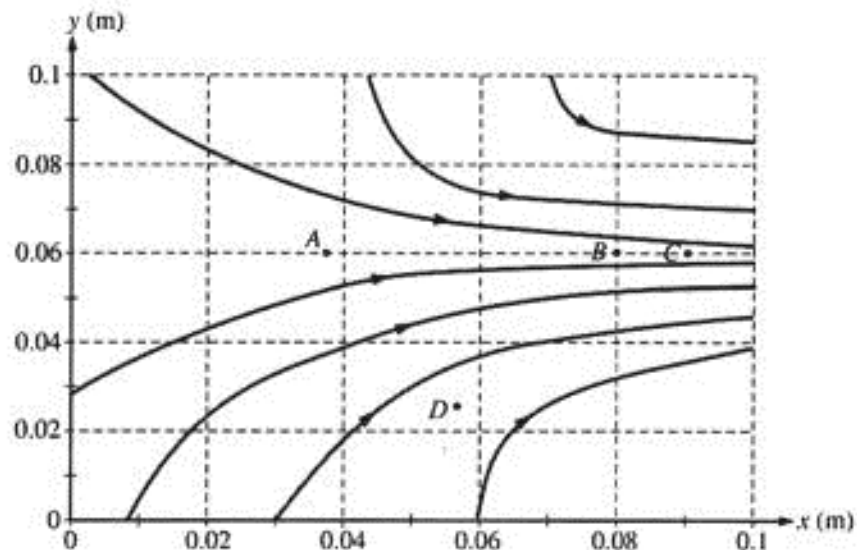
Note: Figures not drawn to scale.

2001E1. A thundercloud has the charge distribution illustrated above left. Treat this distribution as two point charges, a negative charge of  $-30\text{ C}$  at a height of  $2\text{ km}$  above ground and a positive charge of  $+30\text{ C}$  at a height of  $3\text{ km}$ . The presence of these charges induces charges on the ground. Assuming the ground is a conductor, it can be shown that the induced charges can be treated as a charge of  $+30\text{ C}$  at a depth of  $2\text{ km}$  below ground and a charge of  $-30\text{ C}$  at a depth of  $3\text{ km}$ , as shown above right. Consider point  $P_1$ , which is just above the ground directly below the thundercloud, and point  $P_2$ , which is  $1\text{ km}$  horizontally away from  $P_1$ .

- Determine the direction and magnitude of the electric field at point  $P_1$ .
- On the diagram, clearly indicate the direction of the electric field at point  $P_2$
  - How does the magnitude of the field at this point compare with the magnitude at point  $P_1$ ? Justify your answer:

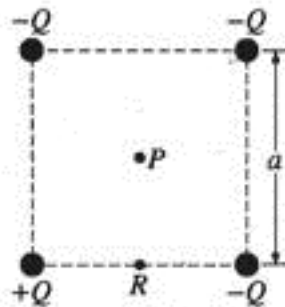
\_\_\_\_\_ Greater      \_\_\_\_\_ Equal      \_\_\_\_\_ Less

- Letting the zero of potential be at infinity, determine the potential at these points.
  - Point  $P_1$
  - Point  $P_2$
- Determine the electric potential at an altitude of  $1\text{ km}$  directly above point  $P_1$ .
- Determine the total electric potential energy of this arrangement of charges.



\*2005E1. Consider the electric field diagram above.

- Points A, B, and C are all located at  $y = 0.06$  m.
    - At which of these three points is the magnitude of the electric field the greatest? Justify your answer.
    - At which of these three points is the electric potential the greatest? Justify your answer.
  - An electron is released from rest at point B.
    - Qualitatively describe the electron's motion in terms of direction, speed, and acceleration.
    - Calculate the electron's speed after it has moved through a potential difference of 10 V.
  - Points B and C are separated by a potential difference of 20 V. Estimate the magnitude of the electric field midway between them and state any assumptions that you make.
  - On the diagram, draw an equipotential line that passes through point D and intersects at least three electric field lines.
-



2006E1. The square of side  $a$  above contains a positive point charge  $+Q$  fixed at the lower left corner and negative point charges  $-Q$  fixed at the other three corners of the square. Point  $P$  is located at the center of the square.

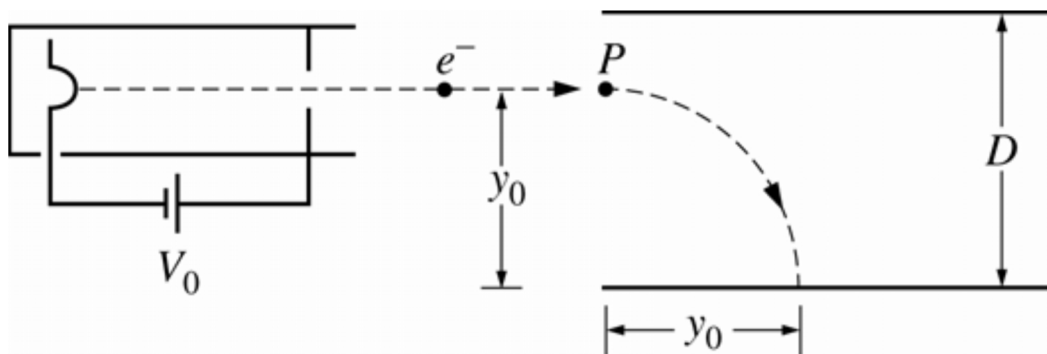
- On the diagram, indicate with an arrow the direction of the net electric field at point  $P$ .
- Derive expressions for each of the following in terms of the given quantities and fundamental constants.
  - The magnitude of the electric field at point  $P$
  - The electric potential at point  $P$
- A positive charge is placed at point  $P$ . It is then moved from point  $P$  to point  $R$ , which is at the midpoint of the bottom side of the square. As the charge is moved, is the work done on it by the electric field positive, negative, or zero?

\_\_\_\_ Positive      \_\_\_\_ Negative      \_\_\_\_ Zero

Explain your reasoning.

- Describe one way to replace a single charge in this configuration that would make the electric field at the center of the square equal to zero. Justify your answer.
    - Describe one way to replace a single charge in this configuration such that the electric potential at the center of the square is zero but the electric field is not zero. Justify your answer.
-





2009E2 (modified) Electrons created at the filament at the left end of the tube represented above are accelerated through a voltage  $V_0$  and exit the tube. The electrons then move with constant speed to the right, as shown, before entering a region in which there is a uniform electric field between two parallel plates separated by a distance  $D$ . The electrons enter the field at point  $P$ , which is a distance  $y_0$  from the bottom plate, and are deflected toward that plate. Express your answers to the following in terms of  $V_0$ ,  $D$ ,  $y_0$ , and fundamental constants.

- Calculate the speed of the electrons as they exit the tube.
- Calculate the magnitude of the electric field required to cause the electrons to land the distance  $y_0$  from the edge of the plate.
  - Indicate the direction of the electric field.

\_\_\_\_ To the left

\_\_\_\_ To the right

\_\_\_\_ Toward the top of the page

\_\_\_\_ Toward the bottom of the page

\_\_\_\_ Into the page

\_\_\_\_ Out of the page

Justify your answer.

- Calculate the potential difference between the two plates required to produce the electric field determined in part b.

# ANSWERS - AP Physics Multiple Choice Practice – Electrostatics

<u>Solution</u>	<u>Answer</u>
1. Since charge is free to move around on/in a conductor, excess charges will repel each other to the outer surface	D
2. When the battery is disconnected, Q remains constant. Since C decreases when d increases and $Q = CV$ , V will increase	D
3. $W = qV$	B
4. Since both charges are positive, the electric field vectors point in opposite directions at points between the two. At point A, the magnitudes of the electric field vectors are equal and therefore cancel out, making $E = 0$ at point A	A
5. $V = \Sigma kQ/r$ and since both charges are positive, the largest potential is at the closest point to the two charges (it is more mathematically complex than that, but this reasoning works for the choices given)	A
6. $C = \epsilon_0 A/d$ ; if $A \times 2$ , $C \times 2$ and if $d \times 2$ , $C \div 2$ so the net effect is C is unchanged	B
7. The net charge on the two spheres is +Q so when they touch and separate, the charge on each sphere (divided equally) is $\frac{1}{2} Q$ . $F \propto Q_1 Q_2$ so before contact $F \propto (2Q)(Q) = 2Q^2$ and after contact $F \propto (\frac{1}{2} Q)(\frac{1}{2} Q) = \frac{1}{4} Q^2$ or 1/8 of the original force	D
8. $C = \epsilon_0 A/d$ and changing Q or V has no effect on the capacitance	C
9. Inside the metal sphere $E = 0$ . Once outside the sphere E decreases as you move away so the strongest field will be the closest point to the <i>outside</i> of the sphere	B
10. Newton's third law	C
11. Where E is zero must be closer to the smaller charge to make up for the weaker field. The vectors point in opposite directions when outside the two opposite charges. These two criteria eliminate 4 of the choices.	A
12. Charges flow when there is a difference in potential. Analyzing the other choices: A is wrong because the charge resides on the surface. For B, $E = 0$ in a charged conducting sphere. $E = kQ/r^2$ eliminates choice C. And for D, charge separation will occur, but the object will not acquire any charge.	D
13. E is uniform between charged parallel plates therefore the force on a charge is also uniform between the plates	B
14. $E \propto 1/r^2$ so if $r \times 2$ , $E \div 4$	B
15. While the charges may separate, the forces on the opposite charges are in opposite directions, canceling out	A
16. $V = kQ/r$ so the smaller sphere is at the lower potential (more negative = lower) Negative charge flows from low to high potential so the charge will flow from the smaller sphere to the larger. The flow of charge ceases when there is no difference in potential.	D
17. $E = V/d$ so if $V \times 2$ , $E \times 2$ and if $d \div 5$ , $E \times 5$ so the net effect is $E \times 10$	D
18. The electric field vectors from the two charges point down and to the left (away from the charges) so the resultant field points down and left	C
19. The potential energy of a particle at a location is the potential at that location times the charge. In this case, the potential is $kQ/d + kQ/d = (2kQ/d)$	C

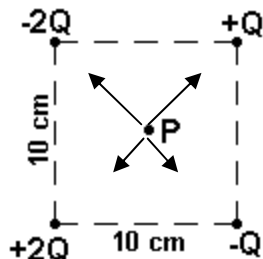
20. If the battery remains connected, the potential remains constant.  $C$  decreases as the separation increases so the charge  $Q = CV$  will also decrease B
21. In metals under electrostatic conditions, the electric field is zero everywhere inside. A
22. The electric field vector from the  $+Q$  charge points down and from the  $-Q$  charge points to the right so the resultant field points down and right C
23. The two vectors, each of magnitude  $E = kQ/d^2$ , point at right angles to each other so the resultant field is  $\sqrt{2}E$  D
24. The electric field between charged parallel plates is uniform, which means the potential changes uniformly with distance. For a change of 8 V over 4 cm means the change of potential with position (and the electric field strength) is 2 V/cm, which gives the potential 1 cm away from the 2 V plate as 4 V D
25.  $W = \Delta K = QV$  (mass doesn't have an effect on the kinetic energy, just on the speed in this case) C
26. The field lines point away from Y and toward Z making Y positive and Z negative. D
27.  $C = \epsilon_0 A/d$ ;  $\epsilon_0(2A)/(2d) = \epsilon_0 A/d$  C
28. Charges arrange themselves on conductors so there is no electric field inside, and no electric field component along the surface A
29. By symmetry  $V_R = V_S$  so  $\Delta V_{RS} = 0$  and  $W = q\Delta V$  C
30. The force on the upper charge is to the left and twice the magnitude of the force on the bottom charge, which is to the right. This makes the net force to the left and the torque on the rod to be counterclockwise. B
31. Compared to the  $+Q$  charge at the center, the charge on the outer surface of the outer cylinder has twice the magnitude and is of opposite sign (so it is  $-2Q$ ). There is also an equal and opposite charge induced on the inner surface of the outer cylinder making the total charge on the outer cylinder  $-2Q + -Q$  D
32. Since the capacitor is isolated,  $Q$  remains constant. Filling the place with oil (a dielectric) will increase the capacitance, causing the potential ( $V = Q/C$ ) to decrease. B
33.  $F_E \propto q_1 q_2 / r^2$ ; if  $q_1$  and  $q_2 \times 2$ ;  $F \times 4$  and if  $r \div 2$ ,  $F \times 4$  making the net effect  $F \times 4 \times 4$  D
34. Since there is no component of the electric field along a conducting surface under electrostatic conditions, no work is done moving the charge around the surface, meaning no differences in potential D
35. Regardless of velocity, the force on a charge in an electric field is parallel to the field ( $\mathbf{F} = q\mathbf{E}$ ) C
36.  $W = Fd = qEd$  A
37.  $E$  points away from  $+$  charges and toward  $-$  charges. Use symmetry. A
38. D is not symmetric so the field will not point at the midpoint of any side. The field in choice B points *at* the bottom charge. C
39. Since the capacitor is isolated,  $Q$  remains constant. Filling the place with oil (a dielectric) will increase the capacitance, causing the potential ( $V = Q/C$ ) to decrease. B
40.  $V = kQ/r$  so the smaller sphere (Y) is at the higher potential. Negative charge flows from low to high potential so the charge will flow from X to Y. A
41. Once inside a uniform sphere of charge, the electric field is zero. Since  $E = 0$  the potential does not change within the sphere (meaning it is the same value as the surface) A

42. Outside a uniform sphere of charge, it behaves as a point charge. C
43.  $\mathbf{E} = \mathbf{F}/q$ . The vector nature of the equation allows one to find the direction of  $\mathbf{F}$  and the equation itself allows one to find the ratio  $F/q$ , but not  $q$  specifically C
44. Outside a uniform sphere of charge, it behaves as a point charge.  $E = kQ/r^2$  D
45. By symmetry, the force on an electron at the center from the top half will be straight down and the force from the bottom half will also be straight down B
46.  $\mathbf{E}$  is a vector so all the individual  $\mathbf{E}$  field vectors from the four charges will cancel.  $V$  is a scalar and will add since they are all positive charges. C
47. The points where  $V = 0$  must lie closer to the smaller charge. Unlike electric field vectors which also require the individual vectors point in opposite directions, there are a locus of points (in this case in a ring surrounding the  $+Q$  charge) where  $V = 0$  as the two charges are opposite in sign and  $V$  is a scalar. So the other point on the  $x$  axis is between the two charges, but closer to the  $+Q$  charge. This must be a value between 1.5 D and 2 D C
48. When inside a uniform shell of charge, there is an electric field due to the shell. When outside a uniform shell of charge, the electric field is as if the shell was a point charge. A
49. The potential difference between the plates is 4 V and the right side is the positive plate. We need the batteries pointing in the same direction with the positive terminal on the right. A
50. The electron experiences a force toward the positive plate of magnitude  $F = Eq$ .  $E = V/d$  and cannot be calculated without knowing  $d$ . D
51. While spheres X and Y are in contact, electrons will repel away from the rod out of sphere X into sphere Y. C
52. While spheres 1 and 2 are in contact, electrons will repel away from the rod out of sphere 1 into sphere 2. C
53. The force vectors from the two  $+Q$  charges point down and to the left (away from the charges) so the resultant force points down and left D
54. The two vectors, each of magnitude  $F$ , point at right angles to each other so the resultant field is  $\sqrt{2}F$  D
55. Where  $E$  is zero must be closer to the smaller charge to make up for the weaker field. The vectors point in opposite directions when between the two like charges. These two criteria eliminate 4 of the choices C
56. Since both charges are positive and  $V$  is a scalar equal to  $\Sigma kQ/r$ , the potential will never be zero in the vicinity of these two charges. D
57. The electric field (and hence, the electric force on a charge) is greatest where the potential changes most rapidly with position (the greatest gradient) since  $E = V/d$ . On this graph, this would be the point where the slope is the greatest C
58. If  $F$  is constant and  $F = ma$ , the acceleration is also constant. Negative charges experience forces opposite in direction to electric field lines. A
59. By symmetry,  $E = 0$  at the midpoint and goes to infinity near each charge ( $E = kQ/r^2$ ) A
60. If no work is done by the field and there is a field present, the motion must be perpendicular to the field, along an equipotential line, making the force perpendicular to the displacement of the charge (a requirement for zero work). Along an equipotential line,  $\Delta V = 0$  and  $W = q\Delta V$ . D
61. Inside the sphere,  $E = 0$  which means the potential does not change with position and is the same value as the surface, which is  $kQ/R$ . At point P, the potential is  $kQ/r$ .  $W = q\Delta V = q(kQ/R - kQ/r)$  D

62.  $E = V/d$  B
63. If the battery is disconnected,  $Q$  remains constant. If a dielectric is inserted between the plates, the capacitance increases and since  $Q = CV$ , the potential difference decreases. A
64. Conductors under electrostatic conditions will arrange their charges so no electric field exists inside (other than those created by charges placed inside the empty cavity). Fields from external charges will not penetrate into conducting enclosures. A
65.  $W = K = q\Delta V$  and  $K = \frac{1}{2}mv^2$  B
66. At point P, the field due to charge  $Q$  points up and to the right and the field due to charge  $-4Q$  is larger in magnitude and points down and to the right. Due to the asymmetry, no components will cancel. D
67. Where  $E$  is zero must be closer to the smaller charge to make up for the weaker field. The vectors point in opposite directions outside the two opposite charges. These two criteria eliminate 3 of the choices. For the magnitudes of the electric fields to be zero the ratios  $Q/r^2$  must be equal giving (in units along the x axis)  $Q/r^2 = 4Q/(r + 4 \text{ units})^2$  giving  $r = 4$  units A
68. Since  $E = \Delta V/d$ ,  $E$  represents the slope of the line on the graph which could be choice C or D. since  $V \propto 1/r$  the slope is proportional to  $\Delta V/r = (1/r)/r = 1/r^2$  which is choice C B
69. By symmetry, all the vectors cancel A
70.  $W = q\Delta V = +Q(V_{\text{center}} - V_{\infty}) = +QV_{\text{center}}$  where  $V_{\text{center}} = \Sigma V = \Sigma kQ/r = 6kQ/R$  D
71.  $E$  points from high potential to low potential, perpendicular to equipotential lines (the direction of the force on a positive charge) A
72.  $E$  is greatest in magnitude where  $V$  changes most rapidly with position (the largest gradient) which is where the lines are closest together. B
73.  $\Delta V_{CE} = V_E - V_C = 10 \text{ V}$ . The amount of work,  $W = q\Delta V = 1 \mu\text{C} \times 10 \text{ V} = 10 \mu\text{J}$ . Since the external force must push against the negative charge to keep it from accelerating and bring it to rest at point E, the work done by the external force must be negative. B
74. For charge to be distributed *throughout* a material, it must be non-conducting D
75. Like gravity inside a uniform sphere of mass, the field is directly proportional to  $r$  when inside the sphere (and proportional to  $1/r^2$  when outside) C
76. In I, charge separation occurs (negative charges repel to the leaves). The whole process describes charging by induction, where the electrons leave the electroscope to ground (the finger) and once contact with ground is broken, the electroscope is left with a positive charge (III) B
77. Charged objects attract object with an opposite charge, but also neutral objects by separation of charges. D
78. If  $E = 0$ , the field vectors point in opposite directions, making  $q$  positive. In magnitude we can find  $q$  by  $(+4 \mu\text{C})/(0.2 \text{ m})^2 = q/(0.3 \text{ m})^2$  A
79. If  $V = 0$  and  $V = \Sigma kQ/r$  then  $q$  must be negative and  $(+4 \mu\text{C})/(0.2 \text{ m}) = q/(0.3 \text{ m})$  C
80. Since the electrostatic force pushes the charge to the right, with the field line, it is a positive charge.  $\Sigma F_y = 0$  gives  $T \cos \theta = mg$  and  $\Sigma F_x = 0$  gives  $T \sin \theta = F_E = qE$ . Divide the two expressions to eliminate  $T$ . C
81. If the battery is disconnected, the charge on the plates remains constant. If the separation increases,  $C$  decreases ( $C = \epsilon_0 A/d$ ). Since  $Q = CV$ ,  $V$  must increase. D
82.  $W = q\Delta V$  (motion along an equipotential line requires no work so only  $\Delta V$  matters, not the path) D

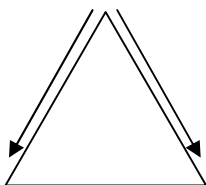
83.  $F \propto 1/r^2$  A
84. Charges flow when there is a difference in potential. D
85. The distance between the +q charge and each charge is d. The force on the +q charge from each charge is in the same direction, making the net force  $kq^2/d^2 + k(3q^2)/d^2$  B
86. The rod will attract the same charge from each sphere to the side closer to the rod. B
87. Since the plates are insulated, the charge remains constant. If the distance is increased, the capacitance will decrease ( $C \propto A/d$ ) and since  $Q = CV$ , the potential difference must increase by the same factor that the distance increases. This means  $E = V/d$  remains the same. C
88.  $F \propto 1/r^2$ ; if  $r \times 4$ ,  $F \div 16$  D
89. If the leaves are positive, further separation means they are becoming more positive, which implies electrons are leaving the leaves, attracted to the top plate of the electroscope. This will occur if the object is positively charged. A
90. Vector addition. Since all the charges are positive, the forces due to charges 2 and 4 point in opposite directions, making the magnitude of the net force along the x axis 2 N. Combine this with a net force along the y axis of 6 N using the Pythagorean theorem A
91.  $Q = CV$  and  $V = Ed$  and using  $C = \epsilon_0 A/d$  gives  $E = Q/\epsilon_0 A$  A
92. Charged objects attract neutral objects by separating the charges within the neutral object. C
93. An alpha particle has twice the charge and four times the mass of a proton. Twice the charge means twice the electric force. This, combined with four times the mass gives half the acceleration. D
94.  $Q = CV$  and  $C = \epsilon_0 A/d$  which gives  $V = Qd/\epsilon_0 A$  C
95. At a point midway between the charges  $E = kq/(d/2)^2$  from each charge. Since they are opposite charges, the field vectors between the charges point in the same direction. D
96. For the E field vectors to point in opposite directions, point P must lie outside the two charges. For the magnitudes of E due to each charge to cancel, Point P must be closer to the smaller charge. B
97. The extra kinetic energy gained by the electron is  $W = K = q\Delta V$ , where  $\Delta V$  is the potential difference between the midway line and the upper plate, which is 200 V. This makes the additional kinetic energy 200 eV. Kinetic energy is a scalar so the total KE of the electron is now 300 eV + 200 eV C
98. If a positive rod attracts ball A, it is either negative or neutral. For ball B to also attract ball A means ball B can be charged positive or negative (if ball A is neutral) or neutral (if ball A is positive) D
99.  $F = Eq$  and  $E = V/d$  giving  $d = qV/F$  D
100. Since the battery is removed, the charge remains constant. If the distance is decreased, the capacitance will increase ( $C \propto A/d$ ) and since  $Q = CV$ , the potential difference must decrease by the same factor that the distance decreases. A
101. Since the spherical shell is conducting, a charge of  $-Q$  is induced on the inner surface. This gives a charge of  $+Q$  on the outer surface since the spherical shell is neutral. As  $E = 0$  inside the conducting shell, the potential inside is constant and the same as on the surface, which is  $kQ/r$  C
102.  $F \propto 1/r^2$ ; if  $r \times 0.4$  then  $F \div 0.4^2$  A

103. The force vectors from each charge and their relative magnitude are drawn below B



104. Using  $F = ma = qE$  and  $E = V/d$  gives  $a = qV/md$  D
105. Since the spherical shell is conducting, a charge of  $-Q$  is induced on the inner surface. This gives a charge of  $+Q$  on the outer surface since the spherical shell is neutral and the field outside the shell is as if the shell was a point charge. B
106. The process described is charging by induction which gives the electroscope in this case a net negative charge. Bringing a negative charge near the top of the electroscope will cause electrons to repel to the leaves. Since the leaves are already negative, this will cause them to separate further. B
107. The charge density is  $Q/\text{area}$  which is  $Q/4\pi r^2$  so for the inner surface it is  $Q_{\text{inner}}/4\pi a^2$  and for the outer surface it is  $Q_{\text{outer}}/16\pi a^2$ . For these to be equal  $Q_{\text{outer}}$  must be  $4Q_{\text{inner}}$ . Because of the  $+Q$  charge inside, there is a charge of  $-Q$  induced on the inner surface, which means the outer surface must have charge  $-4Q$ . Thus the *total* charge on the shell is  $-5Q$  A
108. The force on a positive charge is in the direction of the electric field at that location. C
109. Once inside a uniform sphere of charge, the electric field is zero. Since  $E = 0$  the potential does not change within the sphere (meaning it is the same value as the surface).  $V \propto 1/r$  outside the sphere. D
110. Negative potential energy means the system is bound. This means energy input is required to break the system apart. A
111.  $F \propto 1/r^2$  A
112. Newton's third law requires the forces be equal and opposite. This eliminates choices A, B and C. Since they both positive, the force is repulsive. D
113.  $q_2/q_1 = \text{lines on } q_2/\text{lines on } q_1$  and since the lines point toward  $q_2$  and away from  $q_1$  they are oppositely charged, making the ratio negative. B
114. The points where  $V = 0$  must lie closer to the smaller charge. Unlike electric field vectors which also require the individual vectors point in opposite directions, there are a locus of points (in this case in a ring surrounding the  $-Q$  charge) where  $V = 0$  as the two charges are opposite in sign and  $V$  is a scalar. D
115. For  $E$  to be zero, the electric field vectors from each charge must point in opposite directions and must therefore occur at a point outside the charges. For the electric field vectors from each charge to be equal in magnitude so they can cancel, it must also occur at a point closer to the smaller charge to make up for the weaker field. C
116. Since the battery remains connected,  $V$  remains constant.  $C$  decreases as  $d$  increases ( $C \propto 1/d$ ) and  $U_C = \frac{1}{2} CV^2$  C
117. Only electrons are transferred in static charging processes. C

118. Any charge will experience a net force of zero where the electric field is zero. This must be where the fields from each charge point in opposite directions and also closer to the smaller charge, which is to the left of the  $+Q$  charge (the answer will be to the left of  $-1$  m). Let the distances to the  $+Q$  and the  $-2Q$  charge be  $x$  and  $(x + 2)$ , respectively. This gives  $E_1 = E_2$  and  $kQ/x^2 = k(2Q)/(x + 2)^2$ . Solve for  $x$  and add the extra  $1$  m to the origin. A
119. Since the battery is removed, the charge remains constant. If the distance is increased, the capacitance will decrease ( $C \propto A/d$ ) and since  $Q = CV$ , the potential difference must increase by the same factor that the distance increases and  $U_C = \frac{1}{2} QV$  C
120.  $F \propto q_1q_2/r^2$ ; the original force  $F \propto 100Q^2/d^2$ . The new charges are  $15Q$  and  $5Q$  making the new force  $F \propto 75Q^2/(2d)^2 = 19Q^2/d^2$  A
121. When one sphere is touched, the charge divides equally ( $\frac{1}{2} Q$  each). When this sphere is then touched to the second sphere, the net charge ( $\frac{3}{2} Q$ ) is divided equally ( $\frac{3}{4} Q$  each). Since  $F \propto q_1q_2$ , the original force is proportional to  $Q^2$  and the new force is then proportional to  $(\frac{1}{2} Q)(\frac{3}{4} Q) = \frac{3}{8} Q^2$  D
122.  $W = K = q\Delta V$  so  $\Delta V \propto v^2$  and for  $v$  to double,  $\Delta V$  must increase by  $4$  A
123. Adding the force vectors shown (each  $15$  N) with  $x$  components that cancel and  $y$  components that equal  $15 \text{ N} \cos 30^\circ$  gives  $F = 2 \times 15 \text{ N} \cos 30^\circ = 26 \text{ N}$  B



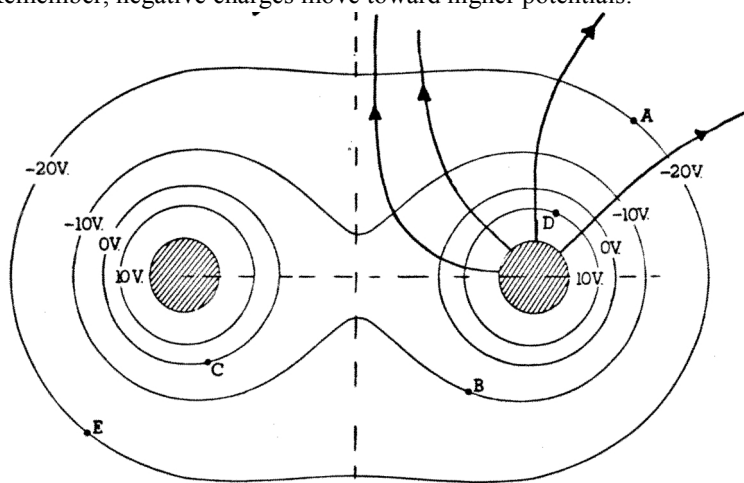
124. Once outside the spheres, they act as point charges and their difference in size is irrelevant D
125. When connected, the potentials become equal. This gives  $kQ_X/r_X = kQ_Y/r_Y$  and since  $E = kQ/r^2$ , dividing the potentials by their respective radii gives  $kQ_X/(r_X)^2 < kQ_Y/(r_Y)^2$  D
126.  $1.5 \times 10^{12}$  excess electrons is a charge of magnitude  $(1.5 \times 10^{12}) \times (1.6 \times 10^{-19}) = 2.4 \times 10^{-7} \text{ C}$ . Use  $Q = CV$  D
127. The field is zero everywhere inside a metal sphere. C
128. The force on  $Q_2$  from  $Q_1$  points downward and the force from  $Q_3$  points at right angles to the left. Compute each force using  $F = kq_1q_2/r^2$  and use the Pythagorean theorem. D
129. The field at the center due to  $Q_1$  and  $Q_3$  cancels. The only contribution to the field then is that due to  $Q_2$ .  $E = kQ/r^2$  where  $r^2 = 0.3^2 + 0.4^2$  A
130.  $E$  inside  $= 0$  and outside  $E \propto 1/r^2$  D
131. Initially, when B is removed, A and C are equally and oppositely charged and B is neutral. Touching B to A gives B  $\frac{1}{2}$  the charge of A (split equally). The charge on B is then  $\frac{1}{2}$  that of C and oppositely charged. When B and C touch, the total charge between them is  $\frac{1}{2}$  the charge of C and the same sign as C. Each sphere then has  $\frac{1}{4}$  of the charge of C after contact is made. This makes the end result that the charge on sphere B is  $\frac{1}{4}$  the original charge of A and the same sign as sphere C, which is opposite that of A C
132. Advanced question (not exactly in the B curriculum, but interesting). Like gravity inside a uniform sphere of mass, the field is directly proportional to  $r$  when inside the sphere (and proportional to  $1/r^2$  when outside) C



133. The field due to the two  $+3q$  charges cancel. The  $-q$  in the upper right counters  $-q$  from the lower left, leaving the net contribution to the field a  $-q$  from the lower left. B
134. With equal charge, the forces are the same. The potential energy of the charges is equal in magnitude, but positive for the proton and negative for the electron. For scalars, positive numbers are higher than negative numbers. A
135. The charge  $Q$  in the middle will induce a charge  $-Q$  on the inner surface of the shell. For the net charge of the shell to be  $-q$ , the outer surface must have the rest of the charge such that  $q_{\text{outer}} + q_{\text{inner}} = -q$  so  $q_{\text{outer}} = -q - q_{\text{inner}} = -q - (-Q) = Q - q$  C
136. The potential inside the shell is the same as the potential at the surface of the shell since  $E = 0$  inside the shell.  $V = kq_{\text{outer}}/b$  D
137. Charges flow when there is a difference in potential. B
138.  $V = \Sigma kQ/r$  D
139. The electric field cancels from symmetry all but  $+Q$  remaining in the upper right corner and  $E = kQ/r^2$  A
140. If the  $E$  fields are the same, that means  $kQ_X/r_X^2 = kQ_Y/r_Y^2$ , or  $Q_X/Q_Y = r_X^2/r_Y^2$  A
141.  $U_E \propto 1/r$  C
142.  $\Sigma F = 0$  so we have  $T + k(q)(q)/d^2 - Mg = 0$  giving  $T = Mg - kq^2/d^2$  D
143. When lowered inside, the charged sphere induces a negative charge on the inner surface of the cup. The outer surface remains neutral since it is grounded. When the grounding wire is removed, the cup has a net negative charge, which when the sphere is removed, will move to the outer surface of the cup. B
144.  $K = q\Delta V$  so  $K_1/K_2 = q_1/q_2$  D
145. The field is zero where the fields from each charge point in opposite directions and also closer to the smaller charge, which is to the left of the  $-Q$  charge A
146.  $E = \Delta V/d$  B
147.  $E$  fields point from high potential to low potential, perpendicular to the equipotential lines. A
148. Combining two droplets doubles the charge. The volume is doubled, which means the radius is multiplied by  $\sqrt[3]{2}$ . This gives  $V = kQ/r = k(2Q)/(\sqrt[3]{2}r)$  B
149. The work to assemble the charges is the potential energy of the system, which is the sum of the potential energies of each pair of charges  $U_E = -ke^2/a - ke^2/a + ke^2/2a$  B

## 1974B5

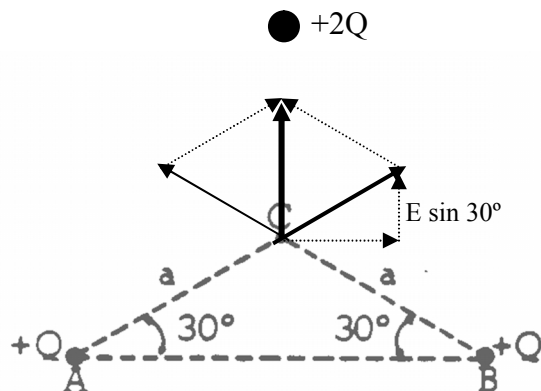
- a. Since the potential increases as you near the cylinder on the right, it must also have a positive charge. Remember, negative charges move toward higher potentials.
- b.



- c.  $V_A - V_B = (-20 \text{ V}) - (-10 \text{ V}) = -10 \text{ V}$
- d.  $W_{AED} = W_{AD} = -q\Delta V = -(0.5 \text{ C})(30 \text{ V}) = -15 \text{ J}$

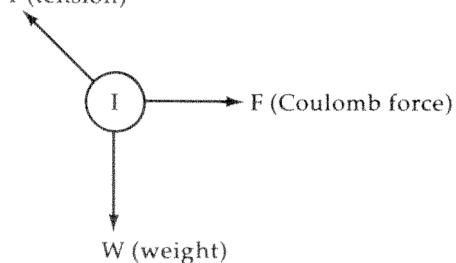
## 1975B2

- a.  $V_C = kQ/a + kQ/a = 2kQ/a$ ;  $W = -q\Delta V = -(+q)(V_\infty - V_C) = -q(0 - 2kQ/a) = 2kQq/a$
- b. Looking at the diagram below, the fields due to the two point charges cancel their x components and add their y components, each of which has a value  $(kQ/a^2) \sin 30^\circ = \frac{1}{2} kQ/a^2$  making the net E field (shown by the arrow pointing upward)  $2 \times \frac{1}{2} kQ/a^2 = kQ/a^2$ . For this field to be cancelled, we need a field of the same magnitude pointing downward. This means the positive charge  $+2Q$  must be placed directly above point C at a distance calculated by  $k(2Q)/d^2 = kQ/a^2$  giving  $d = \sqrt{2}a$



1979B7

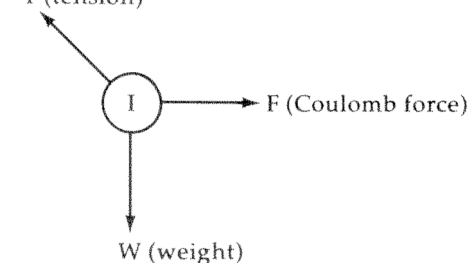
a. T (tension)



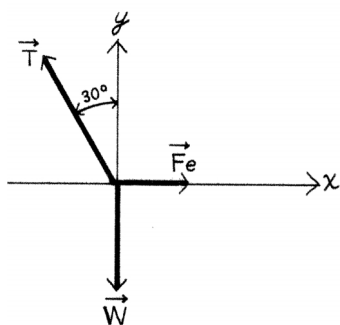
- b. Resolving the tension into components we have  $T \cos \theta = W$  and  $T \sin \theta = F$  where  $W = mg$  and  $F = kq^2/r^2$  and  $r = 2l \sin \theta$  giving  $F = kq^2/(4l^2 \sin^2 \theta)$  Dividing the two expressions we get  $\tan \theta = F/mg = kq^2/(4l^2 \sin^2 \theta mg)$  solving yields  $q^2 = 4mgl^2 (\sin^2 \theta)(\tan \theta)/k$

1981 B3

a. T (tension)

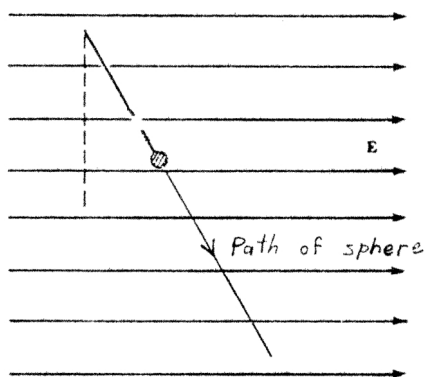


b.



$$T \cos 30^\circ = mg \text{ so } T = 0.058 \text{ N}$$

$$T \sin \theta = F_E = Eq \text{ gives } E = 5.8 \times 10^3 \text{ N/C}$$

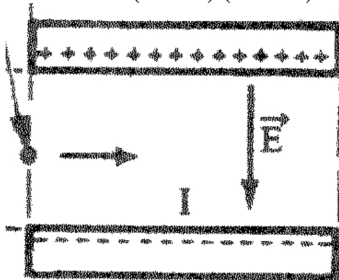


- c. After the string is cut, the only forces are gravity, which acts down, and the electrical force which acts to the right. The resultant of these two forces causes a constant acceleration along the line of the string. The path is therefore down and to the right, along the direction of the string as shown above.

1985B3

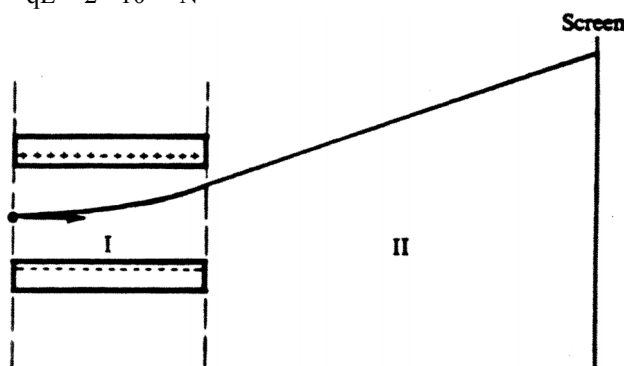
a.  $K = (2 \times 10^3 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = 3.2 \times 10^{-16} \text{ J}$   
 $K = \frac{1}{2} mv^2$  gives  $v = 2.7 \times 10^7 \text{ m/s}$

b.  $E = \Delta V/d = (250 \text{ V})/(0.02 \text{ m}) = 1.25 \times 10^4 \text{ V/m}$



c.  $F = qE = 2 \times 10^{-15} \text{ N}$

d.



Path curves parabolically toward the upper plate in region I and moves in a straight line in region II.

1987B2

a.  $V = kQ/r = 9 \times 10^4 \text{ V}$

b.  $W = q\Delta V$  (where  $V$  at infinity is zero)  $= 0.09 \text{ J}$

c.  $F = kqQ/r^2 = 0.3 \text{ N}$

d. Between the two charges, the fields from each charge point in opposite directions, making the resultant field the difference between the magnitudes of the individual fields.

$E = kQ/r^2$  gives  $E_I = 1.2 \times 10^6 \text{ N/C}$  to the right and  $E_{II} = 0.4 \times 10^6 \text{ N/C}$  to the left

The resultant field is therefore  $E = E_I - E_{II} = 8 \times 10^5 \text{ N/C}$  to the right

e. From conservation of momentum  $m_I v_I = m_{II} v_{II}$  and since the masses are equal we have  $v_I = v_{II}$ .

Conservation of energy gives  $U = K = 2(\frac{1}{2} mv^2) = 0.09 \text{ J}$  giving  $v = 6 \text{ m/s}$

1989B2

a.  $E = kQ/r^2$  and since the field is zero  $E_1 + E_2 = 0$  giving  $k(Q_1/r_1^2 + Q_2/r_2^2) = 0$

This gives the magnitude of  $Q_2 = Q_1(r_2^2/r_1^2) = 2\mu\text{C}$  and since the fields must point in opposite directions from each charge at point P,  $Q_2$  must be negative.

b.  $F = kQ_1Q_2/r^2 = 3.6 \text{ N}$  to the right (they attract)

c.  $U = kQ_1Q_2/r = -0.72 \text{ J}$

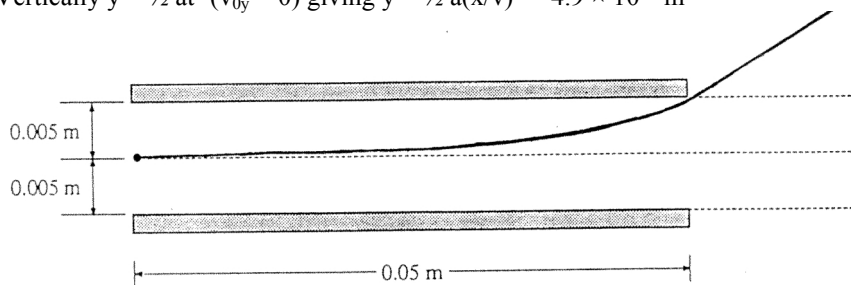
d. between the charges we have a distance from  $Q_1$  of  $x$  and from  $Q_2$  of  $(0.2 \text{ m} - x)$

$V = kQ_1/x + kQ_2/(0.2 \text{ m} - x) = 0$ , solving for  $x$  gives  $x = 0.16 \text{ m}$

e.  $W = q\Delta V$  where  $\Delta V = V_\infty - V_R = 0$  so  $W = 0$

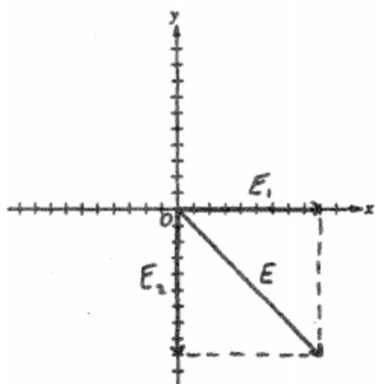
1990B2

- $E = V/d = 2 \times 10^4 \text{ V/m}$
- $F = Eq = ma$  gives  $a = qE/m = 3.5 \times 10^{15} \text{ m/s}^2$
- Horizontally:  $x = vt$  giving  $t = x/v$   
Vertically  $y = \frac{1}{2} at^2$  ( $v_{0y} = 0$ ) giving  $y = \frac{1}{2} a(x/v)^2 = 4.9 \times 10^{-3} \text{ m}$
- 

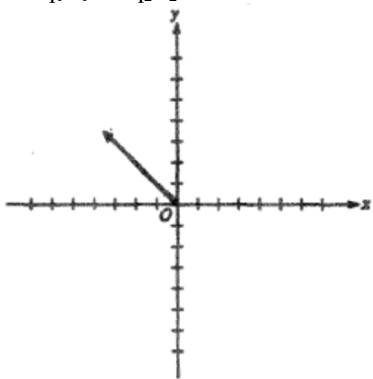


1993B2

- $E = kq/r^2 = 9000 \text{ N/C}$
  - $E = kq/r^2 = 9000 \text{ N/C}$
  -



- $V = kq_1/r_1 + kq_2/r_2 = -9000 \text{ V}$
- 



Since the charge is negative, the force acts opposite the direction of the net  $E$  field.

- $W = q\Delta V = 0.036 \text{ J}$

1996B6

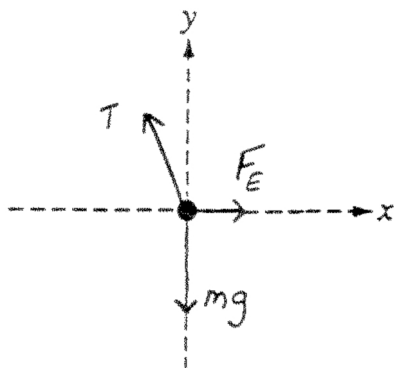
- $\Sigma F = 0$  gives  $qE = mg$  and  $q = mg/E = 3.27 \times 10^{-19} \text{ C}$
- The drop must have a net negative charge. The electric force on a negative charge acts opposite the direction of the electric field.
- $V = Ed = 100 \text{ V}$
- The drop moves upward. The reduced mass decreases the downward force of gravity on the drop while if the charge remains the same, the upward electric force is unchanged.

2002B5B

- Electric field lines point away from positive charges and toward negative charges. The plate on the left is negative and the plate on the right is positive.
- $V = Ed = 100 \text{ V}$
- $C = \epsilon_0 A/d = 1.3 \times 10^{-10} \text{ F}$
- $F = qE = 8 \times 10^{-16} \text{ N}$  to the right (opposite the direction of the electric field)
- The potential difference between the center and one of the plates is  $50 \text{ V}$ .  
 $W = qV = \frac{1}{2} mv^2$  gives  $v = 4.2 \times 10^6 \text{ m/s}$

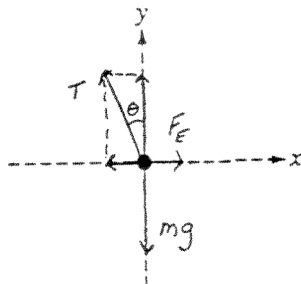
1998B2

a.



b.  $E = F/q = 400 \text{ N/C}$

c.



$T \sin \theta = F_E$  and  $T \cos \theta = mg$ . Dividing gives  $\tan \theta = F/mg$  and  $\theta = 18^\circ$ .

From the diagram  $\sin \theta = x/(0.30 \text{ m})$  giving  $x = 0.09 \text{ m}$

- $a_x = F/m = 3.2 \text{ m/s}^2$ ;  $a_y = 9.8 \text{ m/s}^2$   
 $a = \sqrt{a_x^2 + a_y^2} = 10.3 \text{ m/s}^2$ ;  $\tan \theta = (9.8 \text{ m/s}^2)/(3.2 \text{ m/s}^2) = 72^\circ$  below the x axis  
 (or  $18^\circ$  to the right of the y axis, the same as the angle of the string)
  - The ball moves in a straight line down and to the right

1999B2

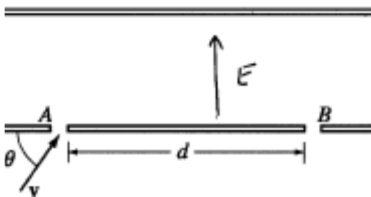

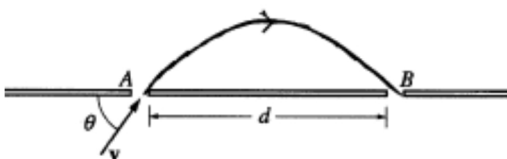
- $W = qV = \frac{1}{2}mv^2$  gives  $V = mv^2/2q = 1.0 \times 10^4$  V
- Electrons travel toward higher potential making the upper plate at the higher potential.
- $x = v_x t$  gives  $t = 6.7 \times 10^{-10}$  s
  - $F = ma = qE$  and  $E = V/d$  gives  $a = qV/md$  and  $y = \frac{1}{2}at^2$  ( $v_{0y} = 0$ ) gives  $y = qVt^2/2md = 6.5 \times 10^{-4}$  m
- $F_g$  is on the order of  $10^{-30}$  N (mg) and  $F_E = qE = qV/d$  is around  $10^{-14}$  N so  $F_E \gg F_g$
- Since there is no more electric force, the path is a straight line.

2001B3

- $V = \Sigma kQ/r = k(-Q/r + -Q/r + Q/r + Q/r) = 0$
  - The fields from the charges on opposing corners cancels which gives  $E = 0$
- $V = \Sigma kQ/r = k(-Q/r + -Q/r + Q/r + Q/r) = 0$
  - The field from each individual charge points along a diagonal, with an  $x$ -component to the right. The vertical components cancel in pairs, and the  $x$ -components are equal in magnitude. Each  $x$  component being  $E = kQ/r^2 \cos 45^\circ$  and the distance from a corner to the center of  $r^2 = s^2/2$  gives  

$$E = 4E_x = 4 \frac{kQ}{s^2/2} \frac{\sqrt{2}}{2} = 4\sqrt{2}kQ/s^2$$
- Arrangement 1. The force of attraction on the upper right charge is greater in arrangement 1 because the two closest charges are both positive, whereas in arrangement 2 one is positive and one is negative.

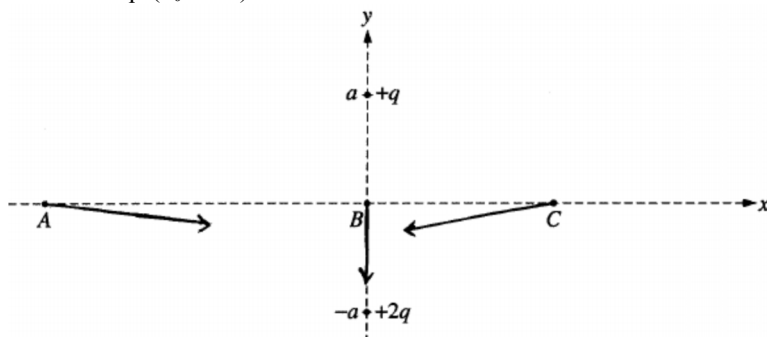
2003B4B

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  - 

- $F = ma = qE$  gives  $a = qE/m$
- The acceleration is downward and at the top of the path,  $v_y = v_{0y} - at = 0$  and  $v_{0y} = v \sin \theta$  which gives  $t_{top} = v \sin \theta / a$  or  $t_{total} = 2t_{top} = 2v \sin \theta / a$  and substituting  $a$  from part b gives  $t = (2mv \sin \theta) / qE$
- $d = x_x t$  where  $v_x = v \cos \theta$  giving  $d = (2mv^2 \sin \theta \cos \theta) / qE$
- The distance would be less because gravity, acting downward, will increase the electron's downward acceleration, decreasing the time spent in the field.

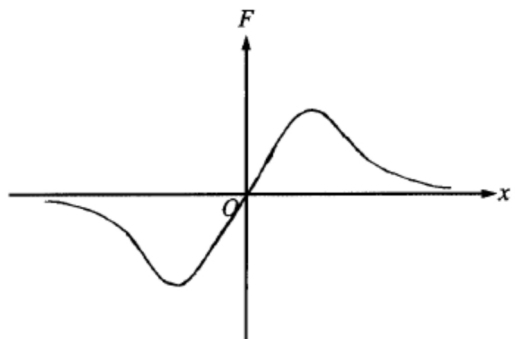
2005B3

- $E = kq/r^2$  and the field from each charge points in opposite directions, with the larger field contribution pointing upward.  $E_O = k(2q)/a^2 - kq/a^2 = kq/a^2$  upward (+y)
- $V_O = \sum kq/r = k(2q)/a + kq/a = 3kq/a$
- $F = kq_1q_2/r^2$  where in this case  $r^2 = x_0^2 + a^2$ 
  - $F = kq^2/(x_0^2 + a^2)$
  - $F = 2kq^2/(x_0^2 + a^2)$
- 



2005B3B

- The distance between the charges is  $r = \sqrt{a^2 + (2a)^2} = \sqrt{5}a$ . The y components of the forces due to the two  $-2Q$  charges cancel so the magnitude of the net force equals the sum of the x components, where  $F_x = F \cos \theta$  and  $\cos \theta = 2a/r = 2/\sqrt{5}$ .  
Putting this all together gives  $F_x = 2 \times (kQ(2Q)/r^2) \cos \theta = 8kQ^2/5\sqrt{5}a^2$  to the right (+x)
- The contribution to the field from the  $-2Q$  charges cancel. This gives  $E = kQ/(2a)^2 = kQ/4a^2$  to the right (+x)
- $V = \sum kQ/r = k(-2Q)/a + k(-2Q)/a + k(-Q)/2a = -9kQ/2a$
- At the origin the force is zero (they cancel). As the charge moves away from the origin, the force first increases as the x components grow, then decrease as the distance grows larger.



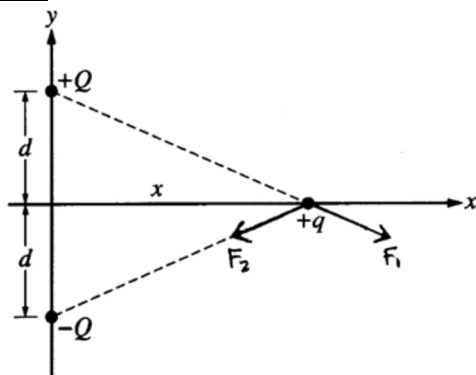
2006B3

- Positive. The electric field due to  $q_1$  points to the right since  $q_1$  is negative. For the electric field to be zero at point P, the field from  $q_2$  must point to the left, away from  $q_2$  making  $q_2$  positive.
- $E_1 + E_2 = 0$  so setting the fields from each charge equal in magnitude gives  $kq_1/d_1^2 = kq_2/d_2^2$ , or  $q_2 = q_1(d_2^2/d_1^2) = 4.8 \times 10^{-8} \text{ C}$
- $F = kq_1q_2/r^2 = 1.4 \times 10^{-5} \text{ N}$  to the left
- $V_1 + V_2 = 0 = kq_1/r_1 + kq_2/r_2$  and let  $r_2 = d$  and  $r_1 = (0.3 \text{ m} - d)$   
solving yields  $d = 0.28 \text{ m}$  to the left of  $q_2$  which is at  $x = 0.20 \text{ m} - 0.28 \text{ m} = -0.08 \text{ m}$
- $W = q\Delta V$  and since  $\Delta V = 0$ ,  $W = 0$



2006B3B

a.



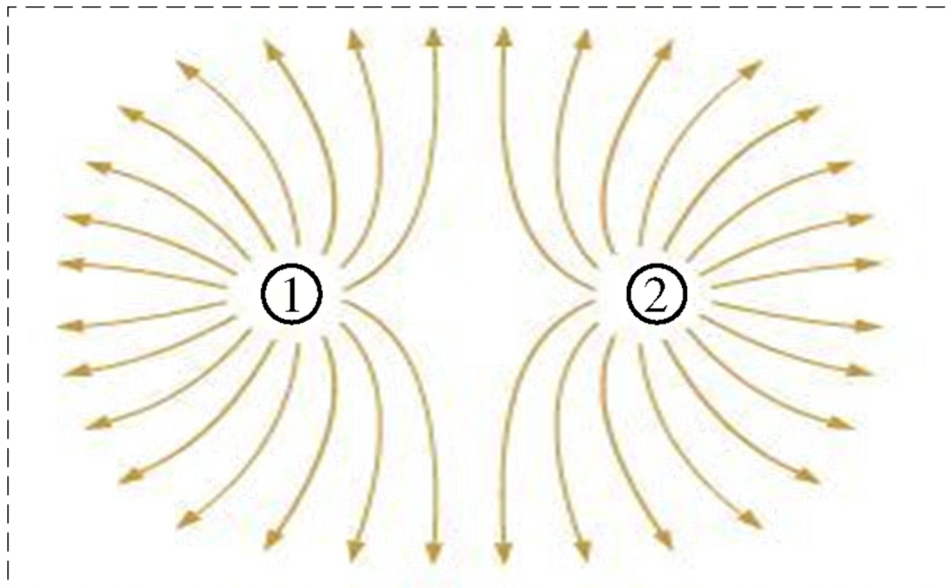
- b. The x components of the forces cancel so the net force is the sum of the y components, which are equal in magnitude and direction.  $F_{\text{net}} = 2 \times F \cos \theta$  where  $\theta$  is the angle between the y axis and the dashed line in the diagram above.  $\cos \theta = d/r = d/\sqrt{x^2 + d^2}$   
 This gives  $F_{\text{net}} = 2 \times kqQ/r^2 \times \cos \theta = 2kqQd/(x^2 + d^2)^{3/2}$
- c.  $E = F/q$  at the point where  $q_1$  lies.  $E = 2kQd/(x^2 + d^2)^{3/2}$
- d. Since the charges  $Q$  and  $-Q$  are equidistant from the point and  $V = \Sigma kQ/r$ , the potential  $V = 0$
- e. As  $x$  gets large, the distance to the charges  $r$  and the value of  $x$  become similar, that is  $\sqrt{x^2 + d^2} \approx x$ . Substituting this into the answer to b. yields  $F = 2kqQd/x^3$

2009B2B

- a. The x components of the forces due to the charges  $q_B$  cancel making the net force equal to the sum of the y components which are equal in magnitude and both point downward. The distance between  $q_A$  and either  $q_B$  is found by the Pythagorean theorem to be 0.05 m.  $F_y = F \sin \theta$  where  $\theta$  is the angle between the line joining  $q_A$  and  $q_B$  and the x axis, giving  $\sin \theta = 3/5$ .  
 This gives  $F_{\text{net}} = 2 \times F_y = 2 (kq_Aq_B/r^2) \times \sin \theta = 2.6 \times 10^{-7}$  N down ( $-y$ )
- b. Particle A will accelerate downward, but as the particle approaches the origin, the force and the acceleration will decrease to zero at the origin. It will then pass through the origin, with a net force now pointing upward, where it will eventually slow down and reverse direction, repeating the process. The short answer is the particle will oscillate vertically about the origin.

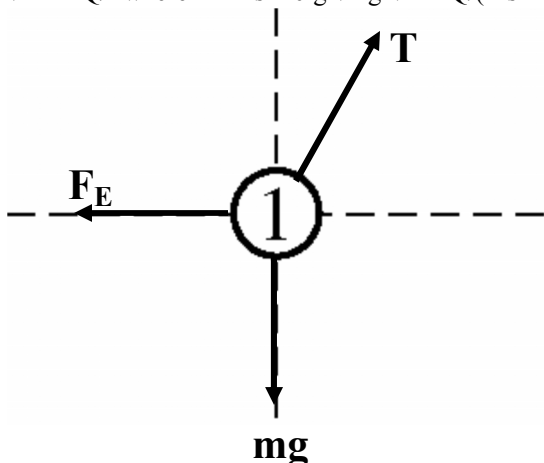
2009B2

a.



b.  $V = \Sigma kQ/r$  where  $r = L \sin \theta$  giving  $V = kQ/(L \sin \theta) + kQ/(L \sin \theta) = 2kQ/(L \sin \theta)$

c.



d.  $\Sigma F_y = 0; T \cos \theta = mg$   
 $\Sigma F_x = 0; T \sin \theta = F_E = kQ^2/(2L \sin \theta)^2$

#### 1974E2

- $E = V/d = V/b$
- $C = \epsilon_0 A/d = \epsilon_0 A/b; Q = CV = \epsilon_0 AV/b$
- This arrangement acts as two capacitors in series, which each have a potential difference  $\frac{1}{2} V$ . Using  $E = V/d$  where  $d = \frac{1}{2}(b - a)$  for each of the spaces above and below. This gives  $E = V/d = (\frac{1}{2} V)/(\frac{1}{2}(b - a)) = V/(b - a)$
- With the copper inserted, we have two capacitors in series, each with a spacing  $\frac{1}{2}(b - a)$ . The capacitance of each is then  $\epsilon_0 A/(\frac{1}{2}(b - a))$  and in series, two equal capacitors have an equivalent capacitance of  $\frac{1}{2} C$  making the total capacitance with the copper inserted  $\frac{1}{2} \epsilon_0 A/(\frac{1}{2}(b - a)) = \epsilon_0 A/(b - a)$  making the ratio  $b/(b - a)$ . Notice the final capacitance is effectively a new single capacitor with an air gap of  $(b - a)$ . Imagine sliding the copper slab up to touch the top plate, this is the same result. This is why adding capacitors in series decreases the capacitance as if the gap between the plates was increased.

#### 1975E1

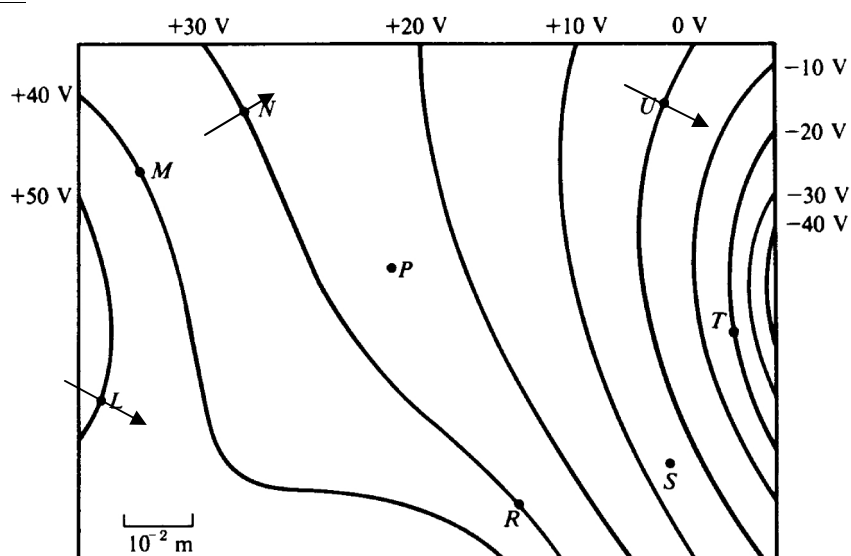
- To find  $V$  along the  $x$  axis we use  $V = \Sigma kq/r$  where  $r = \sqrt{l^2 + x^2}$  giving  $V = 2kq/\sqrt{l^2 + x^2}$  and  $U_E = qV$  so as a function of  $x$  we have  $U_E = 2kq^2/\sqrt{l^2 + x^2}$
- Along the  $x$  axis, the  $y$  components of the forces cancel and the net force is then the sum of the  $x$  components of the forces. Since  $x = l$  in this case, the forces make an angle of  $45^\circ$  to the  $x$  axis and we have  $F = 2 \times F_x = 2 \times F \times \cos 45^\circ = 2 \times kq^2/(\sqrt{l^2 + l^2})^2 \times \cos 45^\circ = kq^2/\sqrt{2}l^2$
- At the origin, the potential is  $V = kq/l + kq/l = 2kq/l$  and with  $V_\infty = 0$  we have  $W = -q\Delta V = -2kq^2/l$

#### 1982E1

- $V = \Sigma kq/r = -kq/x + 2kq/\sqrt{a^2 + x^2} = 0$  which gives  $1/x = 2/\sqrt{a^2 + x^2}$  cross multiplying and squaring gives  $4x^2 = a^2 + x^2$  yielding  $x = \pm a/\sqrt{3}$
- $E = kq/r^2$  and by symmetry, the  $y$  components cancel. The  $x$  components of the electric field from the positive charges points to the right and has magnitude  $(kq/r^2) \cos \theta$  where  $\cos \theta = x/r = x/\sqrt{x^2 + a^2}$  and the  $x$  component of the electric field from the  $-q$  charge points to the left with magnitude  $kq/x^2$  making the net field  $E = 2kqx/(x^2 + a^2)^{3/2} - kq/x^2$

1986E1

a.



The field lines point perpendicular to the equipotential lines from high to low potential.

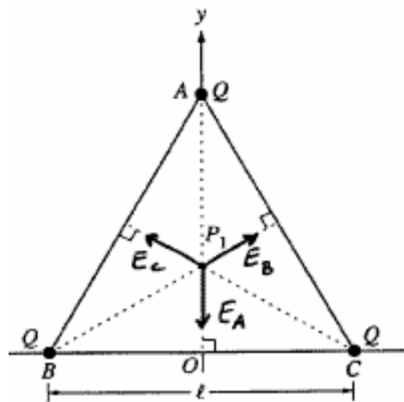
- The magnitude of the field is greatest at point T because the equipotential lines are closest together, meaning  $\Delta V$  has the largest gradient, which is related to the strength of the electric field.
- $E = \Delta V/d = (10 \text{ V})/(0.02 \text{ m}) = 500 \text{ V/m}$
- $V_M - V_S = 40 \text{ V} - 5 \text{ V} = 35 \text{ V}$
- $W = -q\Delta V$  and  $\Delta V = -10 \text{ V}$  which gives  $W = 5 \times 10^{-11} \text{ J}$
- The work done is independent of the path so the answer would be the same.

1991E1

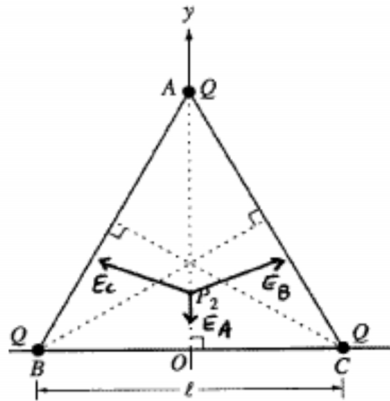
- $E = kQ/a^2$  for each charge, but each vector points in the opposite direction so  $E = 0$
- $V = kQ/a + kQ/a = 2kQ/a$
- the distance to point P from either charge is  $r = \sqrt{a^2 + b^2}$  and the magnitude of E is  $kQ/r^2 = kQ/(a^2 + b^2)$   
The x components cancel so we have only the y components which are  $E \sin \theta$  where  $\sin \theta = b/\sqrt{a^2 + b^2}$  and adding the 2 y components from the two charges gives  $E_{\text{net}} = 2kQb/(a^2 + b^2)^{3/2}$
- The particle will be pushed back toward the origin and oscillate left and right about the origin.
- The particle will accelerate away from the origin.  
The potential of at the center is  $2kQ/a$  and far away  $V_{\infty} = 0$ . To find the speed when far away we use  $W = q\Delta V$   
 $= K = \frac{1}{2}mv^2$  which gives  $v = 2 \sqrt{\frac{kQq}{ma}}$
- The particle will be pulled back toward the origin and oscillate up and down around the origin.

2000E2

a. i.



ii.



	Greater than at $P_1$	Less than at $P_1$	The same as at $P_1$
$E_A$		✓	
$E_B$	✓		
$E_C$	✓		

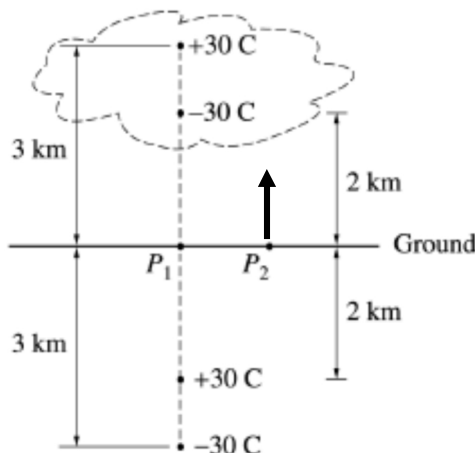
b. The x components cancel due to the symmetry about the y axis.

c.  $V = \sum kQ/r = kQ_A/r_A + kQ_B/r_B + kQ_C/r_C$  where the terms for B and C are equal so we have  $V = kQ_A/r_A + 2Q/r_B$

and using the proper geometry for the distances gives  $V = k \left[ \frac{Q}{\frac{\sqrt{3}l}{2} - y} + \frac{2Q}{\sqrt{\frac{l^2}{4} + y^2}} \right]$

2001E1

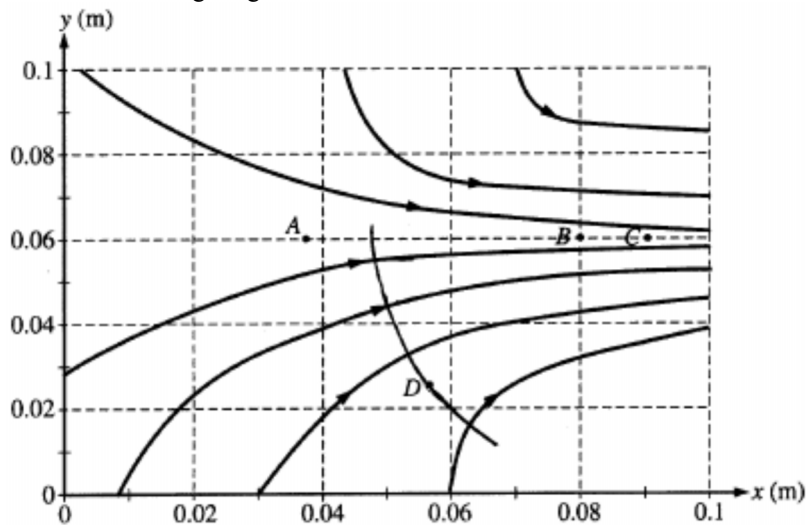
- a.  $E$  is the vector sum of  $kQ/r^2$ . Let fields directed upward be positive and fields directed downward be negative. This gives  $E = k[-30 \text{ C}/(3000 \text{ m})^2 + 30 \text{ C}/(2000 \text{ m})^2 + 30 \text{ C}/(2000 \text{ m})^2 - 30 \text{ C}/(3000 \text{ m})^2] = 75,000 \text{ N/C}$  upward
- b. i.



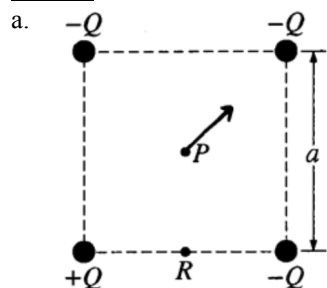
- ii. Because it is a larger distance from the charges, the magnitude is less.
- c. i. By symmetry, the potentials cancel and  $V = 0$
- ii. By symmetry, the potentials cancel and  $V = 0$
- d.  $V = \Sigma kQ/r = k[30 \text{ C}/(2000 \text{ m}) - 30 \text{ C}/(1000 \text{ m}) + 30 \text{ C}/(3000 \text{ m}) - 30 \text{ C}/(4000 \text{ m})] = -1.12 \times 10^8 \text{ V}$
- e.  $U = kq_1q_2/r$  for each pair of charges  
 $= k[(30)(-30)/1000 + (30)(30)/5000 + (30)(-30)/6000 + -30(30)/4000 + -30(-30)/5000 + 30(-30)/1000] = -1.6 \times 10^{10} \text{ J}$

2005E1

- a. i. The magnitude of the field is greatest at point C because this is where the field lines are closest together.
- ii. The potential is greatest at point A. Electric field lines point from high to low potential.
- b. i. The electron moves to the left, against the field lines. As the field gets weaker the electron's acceleration to the left decreases in magnitude, all the while gaining speed to the left.
- ii.  $W = q\Delta V = \frac{1}{2}mv^2$  gives  $v = 1.9 \times 10^6 \text{ m/s}$
- c. If we assume the field is nearly uniform between B and C we can use  $E = \Delta V/d$  where the distance between B and C  $d = 0.01 \text{ m}$  giving  $E = 20 \text{ V}/0.01 \text{ m} = 2000 \text{ V/m}$
- d.



2006E1



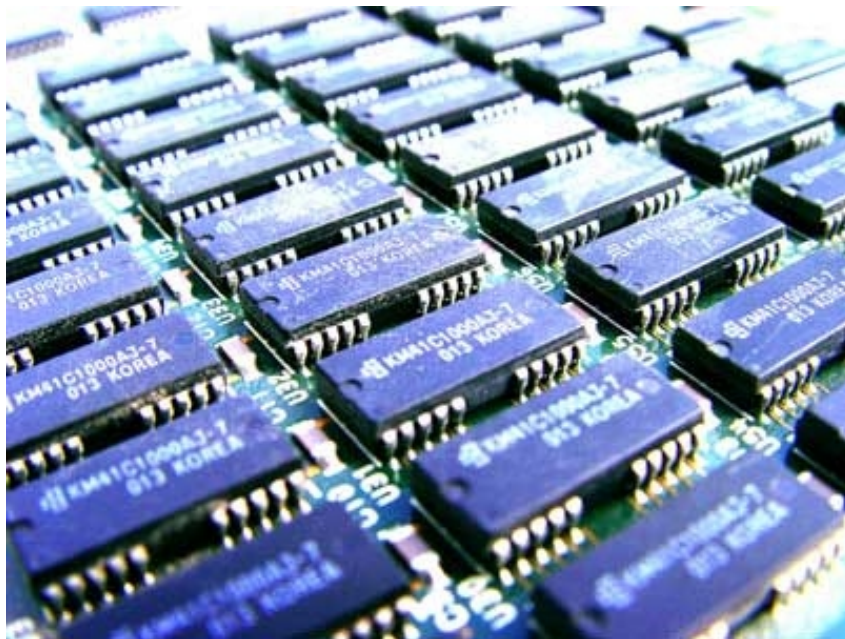
- b. i. The fields at point  $P$  due to the upper left and lower right negative charges are equal in magnitude and opposite in direction so they sum to zero. The fields at point  $P$  due to the other two charges are equal in magnitude and in the same direction so they add.  
Using  $r^2 = a^2/2$  we have  $E = 2 \times kQ/r^2 = 4kQ/a^2$
- ii.  $V = \Sigma kQ/r = k(-Q - Q - Q + Q)/r = -2kQ/r$  with  $r = a/\sqrt{2}$  giving  $V = -2\sqrt{2}kQ/a$
- c. Negative. The field is directed generally from  $R$  to  $P$  and the charge moves in the opposite direction. Thus, the field does negative work on the charge.
- d. i. Replace the top right negative charge with a positive charge OR replace the bottom left positive charge with a negative charge. The vector fields/forces all cancel from oppositely located same charge pairs.
- ii. Replace the top left negative charge with a positive charge OR replace the bottom right negative charge with a positive charge. The scalar potentials all cancel from equidistant located opposite charge pairs. The field vectors in these cases will not cancel.

2009E2

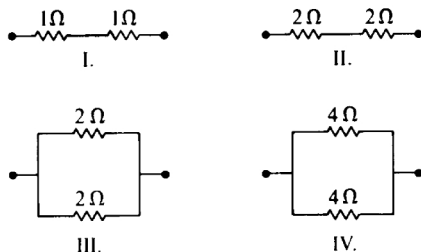
- a.  $W = qV_0 = \frac{1}{2}mv^2$  giving  $v = \sqrt{\frac{2eV_0}{m}}$
- b. i. The time to travel horizontally a distance  $y_0$  is found from  $v = d/t$  giving  $t = d/v = y_0 / \sqrt{\frac{2eV_0}{m}}$
- The downward acceleration of the electron is found from  $F = qE = ma$  giving  $a = eE/m$  and using  $y = \frac{1}{2}at^2$  and substituting the values found earlier we have  $y = y_0 = \frac{1}{2}(eE/m)(y_0^2)/(2eV_0/m)$  which yields  $E = 4V_0/y_0$
- ii. For the electron to accelerate downward requires the electric field to point upward, toward the top of the page since negative charges experience forces opposite electric field lines.
- c.  $\Delta V = ED = (4D/y_0)V_0$

# Chapter 13

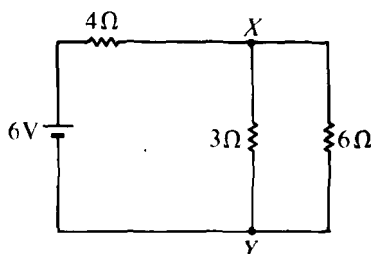
## Circuits



AP Physics Multiple Choice Practice – Circuits



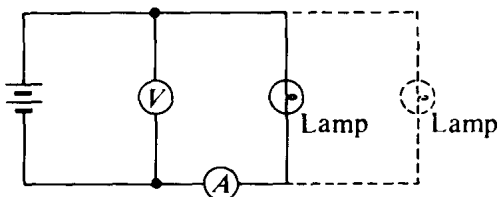
1. **Multiple Correct.** Which arrangements of resistors shown above have the same resistance between the terminals? Select two answers
- (A) I  
(B) II  
(C) III  
(D) IV



2. In the circuit shown above, what is the value of the potential difference between points X and Y if the  $6\text{-volt}$  battery has no internal resistance?
- (A)  $2\text{ V}$     (B)  $3\text{ V}$     (C)  $4\text{ V}$     (D)  $6\text{ V}$

Questions 3-4:

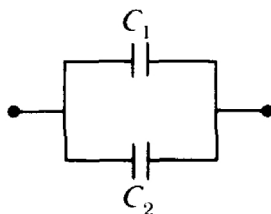
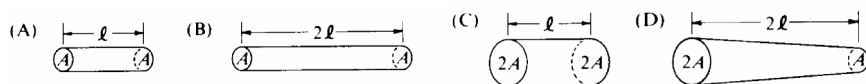
A lamp, a voltmeter V, an ammeter A, and a battery with zero internal resistance are connected as shown.



3. How would the ammeter reading change when another lamp is connected in parallel with the first lamp as shown by the dashed lines?
- (A) increases, because the current through the ammeter splits to feed both branches  
(B) remains the same, because the ammeter measures the current provided by the battery  
(C) decreases, because the resistance of the circuit is increased  
(D) remains the same, because energy is conserved in the circuit
4. How would the voltmeter reading change when another lamp is connected in parallel with the first lamp as shown by the dashed lines?
- (A) decreases, because the current is split between the two branches  
(B) remains the same, because charge is conserved in the circuit  
(C) increases, because the resistance of the circuit is increased  
(D) remains the same, because energy is conserved in the circuit



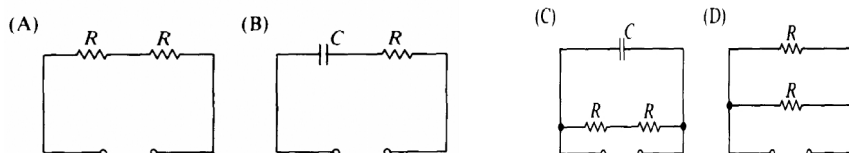
5. The four resistors shown below have the lengths and cross-sectional areas indicated and are made of material with the same resistivity. Which has the greatest resistance?



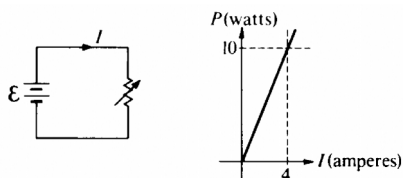
6. Two capacitors are connected in parallel as shown above. A voltage  $V$  is applied to the pair. What is the ratio of charge stored on  $C_1$  to the charge stored on  $C_2$ , when  $C_1 = 1.5C_2$ ?
- (A)  $2/3$     (B)  $1$     (C)  $3/2$     (D)  $9/4$

#### Questions 7-8

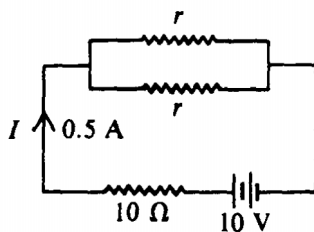
The five incomplete circuits below are composed of resistors  $R$ , all of equal resistance, and capacitors  $C$ , all of equal capacitance. A battery that can be used to complete any of the circuits is available.



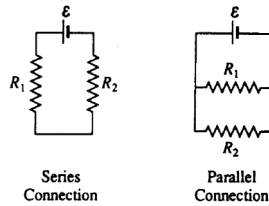
7. Into which circuit should the battery be connected to obtain the greatest steady power dissipation?
- (A) A    (B) B    (C) C    (D) D
8. Which circuit will retain stored energy if the battery is connected to it and then disconnected?
- (A) A    (B) B    (C) C    (D) D



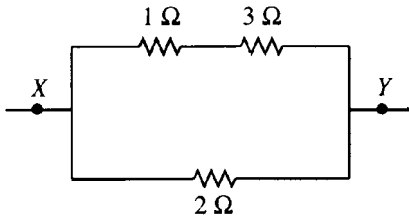
9. The circuit shown above left is made up of a variable resistor and a battery with negligible internal resistance. A graph of the power  $P$  dissipated in the resistor as a function of the current  $I$  supplied by the battery is given above right. What is the emf of the battery?
- (A)  $0.025 \text{ V}$     (B)  $2.5 \text{ V}$     (C)  $6.25 \text{ V}$     (D)  $40 \text{ V}$



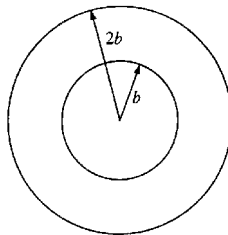
10. In the circuit shown above, the value of  $r$  for which the current  $I$  is  $0.5$  ampere is
- (A)  $1 \Omega$     (B)  $5 \Omega$     (C)  $10 \Omega$     (D)  $20 \Omega$



11. In the diagrams above, resistors  $R_1$  and  $R_2$  are shown in two different connections to the same source of emf  $\varepsilon$  that has no internal resistance. How does the power dissipated by the resistors in these two cases compare?
- (A) It is greater for the series connection, because the current is not split.  
 (B) It is greater for the series connection, because the equivalent resistance is greater.  
 (C) It is greater for the parallel connection, because the total current is greater.  
 (D) It is greater for the parallel connection, because both resistors have the same voltage.

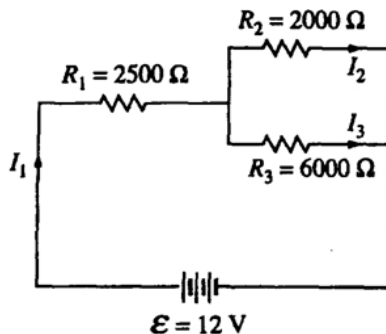


12. The diagram above shows part of a closed electrical circuit. When there is a steady current in the circuit, the amount of charge passing a point per unit of time is
- (A) greater in the  $1\ \Omega$  resistor than in the  $3\ \Omega$  resistor  
 (B) greater in the  $1\ \Omega$  resistor than in the  $2\ \Omega$  resistor  
 (C) greater in the  $2\ \Omega$  resistor than in the  $3\ \Omega$  resistor  
 (D) greater at point X than at point Y

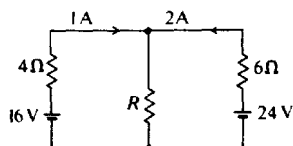


13. Two concentric circular loops of radii  $b$  and  $2b$ , made of the same type of wire, lie in the plane of the page, as shown above. The total resistance of the wire loop of radius  $b$  is  $R$ . What is the resistance of the wire loop of radius  $2b$ ?
- (A)  $R/2$       (B)  $R$       (C)  $2R$       (D)  $4R$
14. The total capacitance of several capacitors in parallel is the sum of the individual capacitances for which of the following reasons?
- (A) The charge on each capacitor depends on its capacitance, but the potential difference across each is the same.  
 (B) The charge is the same on each capacitor, but the potential difference across each capacitor depends on its capacitance.  
 (C) Capacitors in a circuit always combine like resistors in series.  
 (D) The parallel combination increases the effective separation of the plates.
15. A wire of length  $L$  and radius  $r$  has a resistance  $R$ . What is the resistance of a second wire made from the same material that has a length  $L/2$  and a radius  $r/2$ ?
- (A)  $4R$       (B)  $2R$       (C)  $R$       (D)  $R/2$

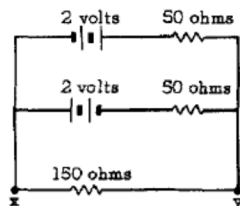
Questions 16-18



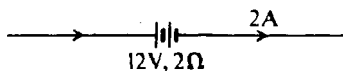
16. Which current is greater  $I_1$  or  $I_2$ ?
- (A)  $I_1$  is greater, because it has more resistance.  
 (B)  $I_2$  is greater, because it has less resistance.  
 (C)  $I_1$  is greater, because of charge conservation.  
 (D)  $I_2$  is greater, because of energy conservation.
17. What is the current  $I_1$ ?
- (A) 1 mA      (B) 3 mA      (C) 4 mA      (D) 12 mA
18. Which of the following changes would increase the value of  $I_1$ ?
- (A) Remove  $R_3$  and the branch containing it.  
 (B) Replace  $R_2$  with another 6000 Ohm resistor.  
 (C) Add an 8000 Ohm resistor in parallel with  $R_2$  and  $R_3$ .  
 (D) Rewire the circuit, putting all three resistors in series.
19. A 60-W incandescent bulb and a 13.3-Watt compact fluorescent light are both plugged into a 110-volt household circuit and lit. Which bulb has the greater resistance?
- (A) Neither, since they are connected across the same potential difference.  
 (B) The 60-W bulb, because it draws more current.  
 (C) The 13.3-W bulb, because it uses less power  
 (D) The 13.3-W bulb, because it is more efficient.



20. In the circuit shown above, what is the resistance  $R$ ?
- (A) 3  $\Omega$     (B) 4  $\Omega$     (C) 6  $\Omega$     (D) 12  $\Omega$

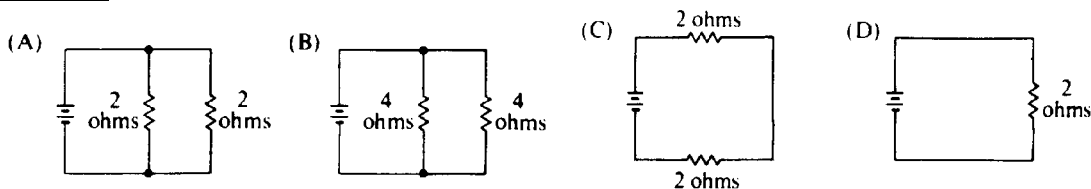


21. In the circuit shown above, the current in each battery is 0.04 ampere. What is the potential difference between the points  $x$  and  $y$ ?
- (A) 8 V    (B) 2 V    (C) 0 V    (D) 4 V



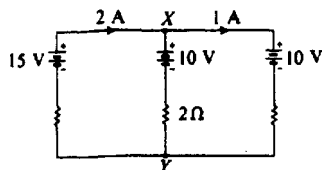
22. A 12-volt storage battery, with an internal resistance of  $2\Omega$ , is being charged by a current of 2 amperes as shown in the diagram above. Under these circumstances, a voltmeter connected across the terminals of the battery will read  
 (A) 8 V (B) 10 V (C) 12 V (D) 16 V

Questions 23-25



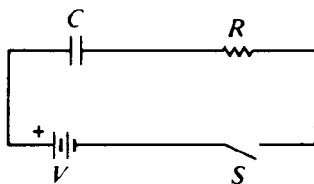
The batteries in each of the circuits shown above are identical and the wires have negligible resistance.

23. In which circuit is the current furnished by the battery the greatest?  
 (A) A (B) B (C) C (D) D
24. In which circuit is the equivalent resistance connected to the battery the greatest?  
 (A) A (B) B (C) C (D) D
25. Which circuit dissipates the least power?  
 (A) A (B) B (C) C (D) D
26. When two identical parallel-plate capacitors are connected in series, which of the following is true of the equivalent capacitance?  
 (A) It depends on the charge on each capacitor.  
 (B) It depends on the potential difference across both capacitors.  
 (C) It is larger than the capacitance of each capacitor.  
 (D) It is smaller than the capacitance of each capacitor.
27. The emf of a battery is 12 volts. When the battery delivers a steady current of 0.5 ampere to a load, the potential difference between the terminals of the battery is 10 volts. What terminal voltage would you expect if the battery were connected to a load with less resistance?  
 (A) 12 V (B) 10 V (C) Between 10 and 12 V (D) Less than 10 V

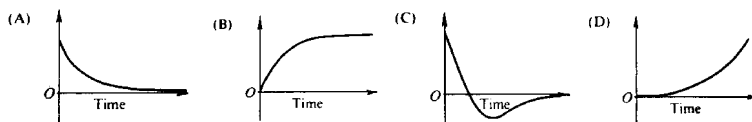


28. In the circuit shown above, the emf's of the batteries are given, as well as the currents in the outside branches and the resistance in the middle branch. What is the magnitude of the potential difference between X and Y?  
 (A) 4 V (B) 8 V (C) 10 V (D) 12 V

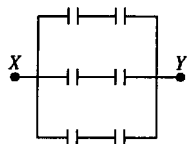
Questions 29-31



Assume the capacitor  $C$  is initially uncharged. The following graphs may represent different quantities related to the circuit as functions of time  $t$  after the switch  $S$  is closed

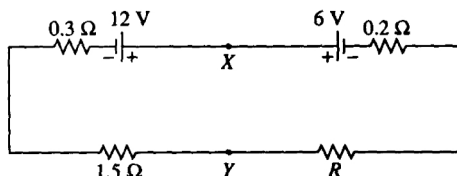


29. Which graph best represents the voltage versus time across the resistor  $R$ ?  
 (A) A (B) B (C) C (D) D
30. Which graph best represents the current versus time in the circuit?  
 (A) A (B) B (C) C (D) D
31. Which graph best represents the voltage across the capacitor versus time?  
 (A) A (B) B (C) C (D) D
32. A wire of resistance  $R$  dissipates power  $P$  when a current  $I$  passes through it. The wire is replaced by another wire with resistance  $3R$ . The power dissipated by the new wire when the same current passes through it is  
 (A)  $P/9$  (B)  $P/3$  (C)  $P$  (D)  $3P$



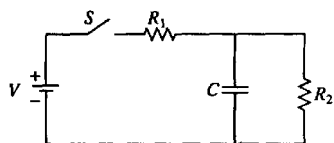
33. The diagram above represents a circuit of six 2-microfarad capacitors. What potential difference must be applied between points  $X$  and  $Y$  so that the charge on each plate of each capacitor will have magnitude 6 microcoulombs?  
 (A) 3 V (B) 6 V (C) 9 V (D) 18 V

Questions 34-35



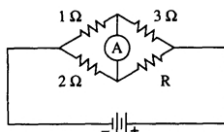
In the circuit above, the emf's and the resistances have the values shown ( $0.3 \Omega$ ,  $0.2 \Omega$  and  $1.5 \Omega$ ). The current  $I$  in the circuit is 2 amperes.

34. The resistance  $R$  is  
 (A)  $1 \Omega$  (B)  $2 \Omega$  (C)  $3 \Omega$  (D)  $4 \Omega$
35. The potential difference between points  $X$  and  $Y$  is  
 (A) 1.2 V (B) 6.0 V (C) 8.4 V (D) 10.8 V

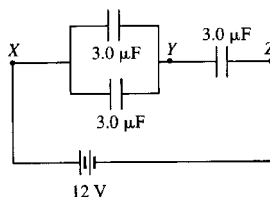
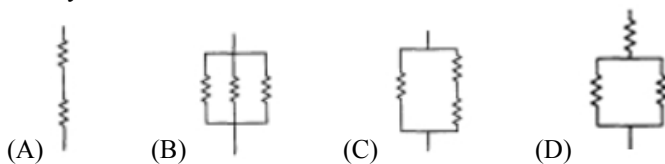


36. In the circuit shown above, the battery supplies a constant voltage  $V$  when the switch  $S$  is closed. The value of the capacitance is  $C$ , and the value of the resistances are  $R_1$  and  $R_2$ . In the time after the switch is closed, the current supplied by the battery is
- (A) constant, because batteries always provide constant current
  - (B) constant, because the capacitor is an open circuit and doesn't affect the two resistors in series
  - (C) decreasing, because current initially flows to the capacitor
  - (D) decreasing, because it becomes zero when the capacitor is full

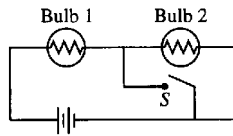
Questions 37-38:



37. If the ammeter in the circuit above reads zero, what is the resistance  $R$  ?
- (A)  $1.5 \Omega$
  - (B)  $2 \Omega$
  - (C)  $4 \Omega$
  - (D)  $6 \Omega$
38. If the current through the ammeter is to flow toward the bottom of the page, how must the resistance  $R$  compare to the value found in #37?
- (A) it must be larger, so that the voltage across  $R$  increases
  - (B) it must be larger, so that the equivalent resistance of the circuit increases
  - (C) it must be smaller, so that the voltage across  $R$  decreases
  - (D) it must be smaller, so that the equivalent resistance of the circuit decreases
39. A resistor  $R$  and a capacitor  $C$  are connected in series to a battery of terminal voltage  $V_0$ . Which of the following equations relating the current  $I$  in the circuit and the charge  $Q$  on the capacitor describes this circuit?
- (A)  $V_0 + QC - I^2R = 0$
  - (B)  $V_0 - Q/C - IR = 0$
  - (C)  $V_0^2 - Q^2/2C - I^2R = 0$
  - (D)  $V_0 - CI - I^2R = 0$
40. Which of the following combinations of  $4\Omega$  resistors would dissipate  $24 \text{ W}$  when connected to a  $12 \text{ Volt}$  battery?



41. Three identical capacitors, each of capacitance  $3.0 \mu\text{F}$ , are connected in a circuit with a  $12 \text{ V}$  battery as shown above. The potential difference between points  $Y$  and  $Z$  is
- (A)  $6 \text{ V}$ , because all capacitors will have the same potential difference
  - (B)  $8 \text{ V}$ , because charge is conserved in the set of wires containing point  $Y$ .
  - (C)  $6 \text{ V}$ , because charge is conserved in the set of wires containing point  $Y$ .
  - (D)  $8 \text{ V}$ , because all capacitors have the same charge.

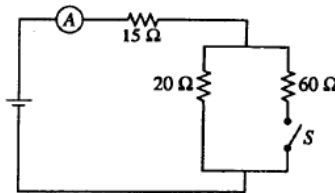


42. The circuit in the figure above contains two identical lightbulbs in series with a battery. At first both bulbs glow with equal brightness. When switch S is closed, which of the following occurs to the bulbs?

Bulb 1

Bulb 2

- |                          |                      |
|--------------------------|----------------------|
| (A) Goes out             | Gets brighter        |
| (B) Gets brighter        | Goes out             |
| (C) Gets brighter        | Gets slightly dimmer |
| (D) Gets slightly dimmer | Gets brighter        |

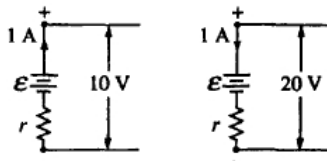


43. When the switch S is open in the circuit shown above, the reading on the ammeter A is 2.0 A. When the switch is closed, the reading on the ammeter is

- (A) doubled  
(B) increased slightly but not doubled  
(C) the same  
(D) decreased slightly

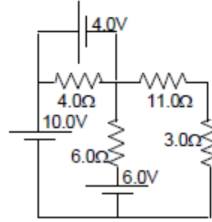
44. Two conducting cylindrical wires are made out of the same material. Wire X has twice as much resistance than wire Y. Which of the following could be true?

- (A) Wire X is twice the diameter of wire Y.  
(B) Wire X is twice as long and twice the diameter of wire Y.  
(C) Wire Y is twice as long and twice the diameter of wire X.  
(D) Wire Y is twice as long as wire X.



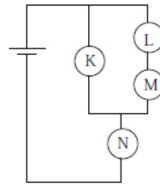
45. The figures above show parts of two circuits, each containing a battery of emf  $\epsilon$  and internal resistance  $r$ . The current in each battery is 1 A, but the direction of the current in one battery is opposite to that in the other. If the potential differences across the batteries' terminals are 10 V and 20 V as shown, what are the values of  $\epsilon$  and  $r$ ?

- (A)  $\epsilon = 5 \text{ V}$ ,  $r = 15 \Omega$   
(B)  $\epsilon = 10 \text{ V}$ ,  $r = 100 \Omega$   
(C)  $\epsilon = 15 \text{ V}$ ,  $r = 5 \Omega$   
(D)  $\epsilon = 20 \text{ V}$ ,  $r = 10 \Omega$



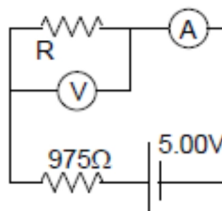
46. What is the current through the  $6.0\ \Omega$  resistor shown in the accompanying circuit diagram? Assume all batteries have negligible resistance.  
 (A) 0 A (B) 0.40 A (C) 1.3 A (D) 1.5 A

Questions 47-49



Four identical light bulbs K, L, M, and N are connected in the electrical circuit shown above.

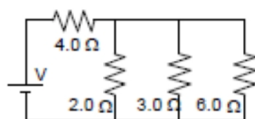
47. Rank the current through the bulbs.  
 (A)  $K > L > M > N$   
 (B)  $L = M > K = N$   
 (C)  $L > M > K > N$   
 (D)  $N > K > L = M$
48. Bulb K burns out. Which of the following statements is true?  
 (A) Only bulb N goes out.  
 (B) Bulb N becomes brighter.  
 (C) The brightness of bulb N remains the same.  
 (D) Bulb N becomes dimmer but does not go out.
49. Bulb M burns out. Which of the following statements is true?  
 (A) Only bulb M goes out.  
 (B) Bulb N goes out but at least one other bulb remains lit.  
 (C) The brightness of bulb N remains the same.  
 (D) Bulb N becomes dimmer but does not go out.



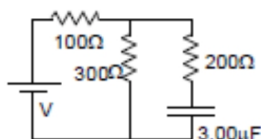
50. The voltmeter in the accompanying circuit diagram has internal resistance  $10.0\ \text{k}\Omega$  and the ammeter has internal resistance  $25.0\ \Omega$ . The ammeter reading is  $1.00\ \text{mA}$ . The voltmeter reading is most nearly:  
 (A) 1.0 V (B) 3.0 V (C) 4.0 V (D) 5.0 V
51. When two resistors, having resistance  $R_1$  and  $R_2$ , are connected in parallel, the equivalent resistance of the combination is  $5\ \Omega$ . Which of the following statements about the resistances is correct?  
 (A) Both  $R_1$  and  $R_2$  are greater than  $5\ \Omega$ .  
 (B) Both  $R_1$  and  $R_2$  are equal to  $5\ \Omega$ .  
 (C) Both  $R_1$  and  $R_2$  are less than  $5\ \Omega$ .  
 (D) The sum of  $R_1$  and  $R_2$  is  $5\ \Omega$ .



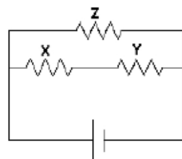
52. When a single resistor is connected to a battery, a total power  $P$  is dissipated in the circuit. How much total power is dissipated in a circuit if  $n$  identical resistors are connected in series using the same battery? Assume the internal resistance of the battery is zero.  
 (A)  $n^2P$  (B)  $nP$  (C)  $P/n$  (D)  $P/n^2$



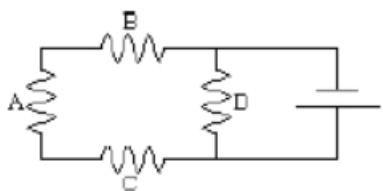
53. In the accompanying circuit diagram, the current through the  $6.0\text{-}\Omega$  resistor is  $1.0\text{ A}$ . What is the power supply voltage  $V$ ?  
 (A)  $10\text{ V}$  (B)  $18\text{ V}$  (C)  $24\text{ V}$  (D)  $30\text{ V}$



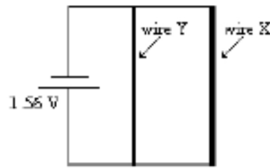
54. In the circuit diagrammed above, the  $3.00\text{-}\mu\text{F}$  capacitor is fully charged at  $18.0\text{ }\mu\text{C}$ . What is the value of the power supply voltage  $V$ ?  
 (A)  $4.40\text{ V}$  (B)  $6.00\text{ V}$  (C)  $8.00\text{ V}$  (D)  $10.4\text{ V}$



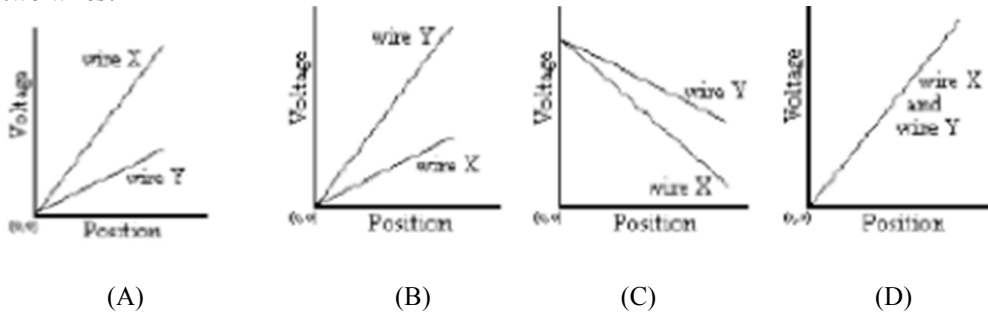
55. Given the simple electrical circuit above, if the current in all three resistors is equal, which of the following statements must be true?  
 (A) X, Y, and Z all have equal resistance  
 (B) X and Y have equal resistance  
 (C) X and Y added together have the same resistance as Z  
 (D) X and Y each have more resistance than Z



56. If all of the resistors in the above simple circuit have the same resistance, which would dissipate the greatest power?  
 (A) resistor A  
 (B) resistor B  
 (C) resistor C  
 (D) resistor D

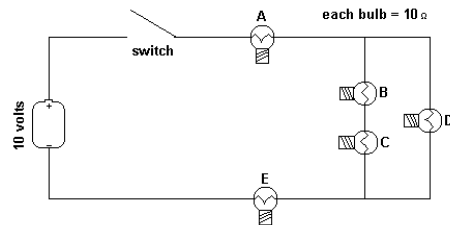


57. The following diagram represents an electrical circuit containing two uniform resistance wires connected to a single flashlight cell. Both wires have the same length, but the thickness of wire X is twice that of wire Y. Which of the following would best represent the dependence of electric potential on position along the length of the two wires?

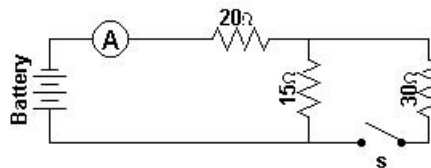


58. Five identical light bulbs, each with a resistance of 10 ohms, are connected in a simple electrical circuit with a switch and a 10 volt battery as shown in the diagram below. Which bulb (or bulbs) could burn out without causing other bulbs in the circuit to also go out?

- (A) only bulb D (C) only bulbs C or D  
(B) only bulb E (D) bulbs B, C, or D



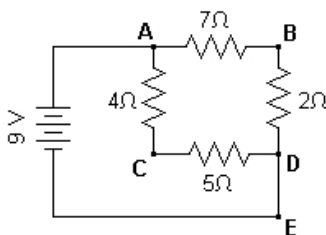
#### Questions 59-61



An ideal battery, an ideal ammeter, a switch and three resistors are connected as shown. With the switch open as shown in the diagram the ammeter reads 2.0 amperes.

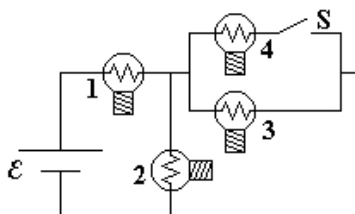
59. With the switch open, what would be the potential difference across the 15 ohm resistor?  
(A) 30 V (B) 40 V (C) 60 V (D) 70 V
60. With the switch open, what must be the voltage supplied by the battery?  
(A) 30 V (B) 40 V (C) 60 V (D) 70 V
61. When the switch is closed, what would be the current in the circuit?  
(A) 1.7 A (B) 2.0 A (C) 2.3 A (D) 3.0 A

Questions 62-63



A 9-volt battery is connected to four resistors to form a simple circuit as shown above.

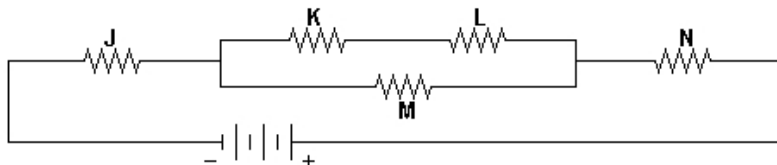
62. How would the current through the 2 ohm resistor compare to the current through the 4 ohm resistor?  
 (A) twice as large (B) one-half as large (C) equally as large (D) four times as large
63. What would be the potential at point B with respect to point C in the above circuit?  
 (A) +7 V (B) +3 V (C) 0 V (D) -3 V



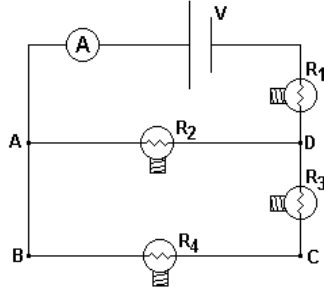
64. A circuit is connected as shown. All light bulbs are identical. When the switch in the circuit is closed illuminating bulb #4, which other bulb(s) also become brighter?  
 (A) Bulb #1 only (B) Bulb #2 only (C) Bulbs #2 and #3 only (D) Bulbs #1, #2, and #3

Questions 65-66

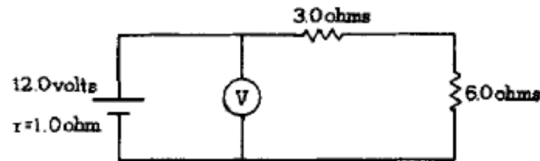
The diagram below shows five identical resistors connected in a combination series and parallel circuit to a voltage source.



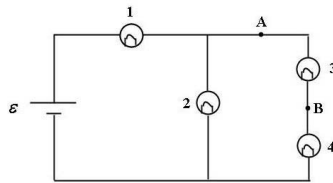
65. Through which resistor(s) would there be the greatest current?  
 (A) M only (B) N only (C) J and N only (D) K and L only
66. Which resistor(s) have the greatest rate of energy dissipation?  
 (A) M only (B) N only (C) J and N only (D) K and L only



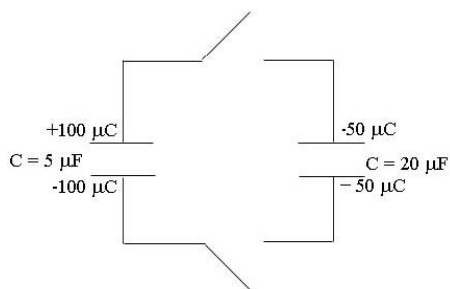
67. The circuit shown has an ideal ammeter with zero resistance and four identical resistance light bulbs which are initially illuminated. A person removes the bulb  $R_4$  from its socket thereby permanently breaking the electrical circuit at that point. Which statement is true of the circuit after removing the bulb?
- (A) The voltage from  $B \rightarrow C$  increases.
  - (B) The power supplied by the battery increases
  - (C) The voltage across  $R_1$  increases.
  - (D) The ammeter reading is unchanged.
68. A current through the thin filament wire of a light bulb causes the filament to become white hot, while the larger wires connected to the light bulb remain much cooler. This happens because
- (A) the larger connecting wires have more resistance than the filament.
  - (B) the thin filament has more resistance than the larger connecting wires.
  - (C) the filament wire is not insulated.
  - (D) the current in the filament is greater than that through the connecting wires.



69. In the circuit above the voltmeter  $V$  draws negligible current and the internal resistance of the battery is  $1.0$  ohm. The reading of the voltmeter is
- (A)  $10.0$  V
  - (B)  $10.5$  V
  - (C)  $10.8$  V
  - (D)  $11.6$  V

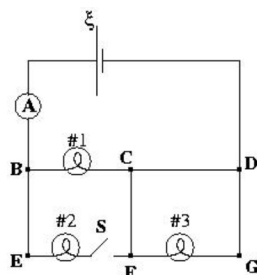


70. For the circuit shown, a shorting wire of negligible resistance is added to the circuit between points  $A$  and  $B$ . When this shorting wire is added, bulb #3 goes out. Which bulbs (all identical) in the circuit brighten?
- (A) Only Bulb 4
  - (B) Only Bulbs 1 and 4
  - (C) Only Bulbs 2 and 4
  - (D) Bulbs 1, 2 and 4

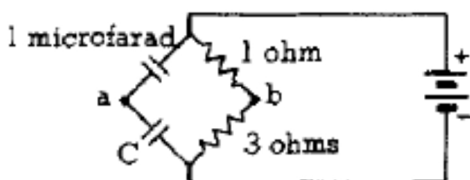


71. For the configuration of capacitors shown, both switches are closed simultaneously. After equilibrium is established, what is the charge on the top plate of the  $5\ \mu\text{F}$  capacitor?  
 (A)  $50\ \mu\text{C}$  (B)  $30\ \mu\text{C}$  (C)  $25\ \mu\text{C}$  (D)  $10\ \mu\text{C}$

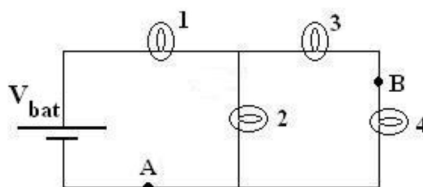
72. A student wants to make a brighter light bulb. He decides to modify the filament. How should the filament of a light bulb be modified in order to make the light bulb produce more light at a given voltage?  
 (A) Increase the resistivity only.  
 (B) Increase the diameter only.  
 (C) Decrease the diameter only.  
 (D) Decrease the diameter and increase the resistivity.



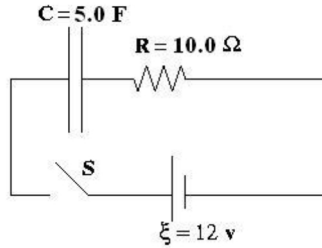
73. For the circuit shown, the ammeter reading is initially  $I$ . The switch in the circuit then is closed. Consequently:  
 (A) The ammeter reading decreases.  
 (B) The potential difference between  $E$  and  $F$  increases.  
 (C) Bulb #3 lights up more brightly.  
 (D) The power supplied by the battery decreases.



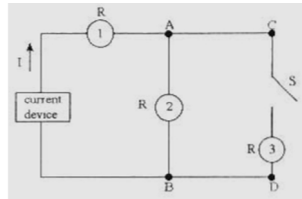
74. In the circuit shown above, the potential difference between points  $a$  and  $b$  is zero for a value of capacitance  $C$  of  
 (A)  $1/3$  microfarad (B)  $2/3$  microfarad (C)  $2$  microfarads (D)  $3$  microfarads



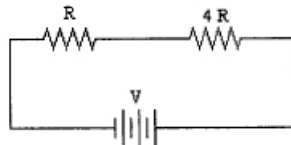
75. For the circuit shown, when a shorting wire (no resistance) connects the points labeled  $A$  and  $B$ , which of the numbered light bulbs become brighter? Assume that all four bulbs are identical and have resistance  $R$ .  
 (A) Bulb 1 only (B) Bulb 2 only (C) Bulb 3 only (D) Bulbs 1 and 3 only



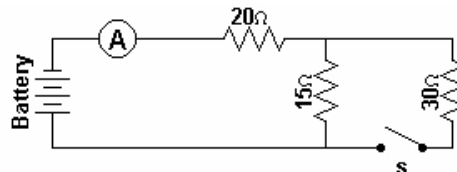
76. For the RC circuit shown, the resistance is  $R = 10.0 \, \Omega$ , the capacitance is  $C = 5.0 \, \text{F}$  and the battery has voltage  $\xi = 12 \, \text{volts}$ . The capacitor is initially uncharged when the switch  $S$  is closed at time  $t = 0$ . At some time later, the current in the circuit is  $0.50 \, \text{A}$ . What is the magnitude of the voltage across the capacitor at that moment?  
 (A) 5 volts (B) 6 volts (C) 7 volts (D) 12 volts



77. In the circuit shown above, a constant current device is connected to some identical light bulbs. After the switch  $S$  in the circuit is closed, which statement is correct about the circuit?  
 (A) Bulb #2 becomes brighter. (B) Bulb #1 becomes dimmer.  
 (C) All three bulbs become equally brighter. (D) The voltage between points  $C$  and  $D$  is decreased.



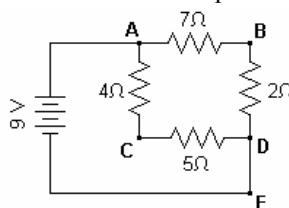
78. Two resistors, one with resistance  $R$  and the second with resistance  $4R$  are placed in a circuit with a voltage  $V$ . If resistance  $R$  dissipates power  $P$ , what would be the power dissipated by the  $4R$  resistance?  
 (A)  $4P$  (B)  $P$  (C)  $1/2 P$  (D)  $1/4 P$



79. A battery, an ammeter, three resistors, and a switch are connected to form the simple circuit shown above. When the switch is closed what would happen to the potential difference across the  $15 \, \text{ohm}$  resistor?  
 (A) it would equal the potential difference across the  $20 \, \text{ohm}$  resistor  
 (B) it would be twice the potential difference across the  $30 \, \text{ohm}$  resistor  
 (C) it would equal the potential difference across the  $30 \, \text{ohm}$  resistor  
 (D) it would be half the potential difference across the  $30 \, \text{ohm}$  resistor

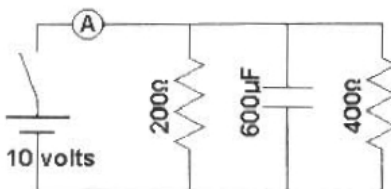
#### Questions 80-81

A 9-volt battery is connected to four resistors to form a simple circuit as shown below.

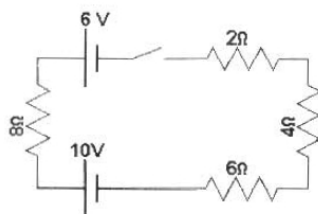


80. What would be the current at point  $E$  in the circuit?  
 (A) 2 amp (B) 4 amp (C) 5 amp (D) 7 amp

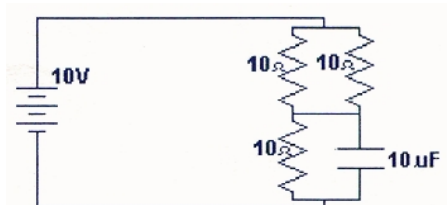
81. What would be the potential at point B with respect to point D?  
 (A) +2 V (B) +4 V (C) +5 V (D) +7 V



82. Two resistors and a capacitor are connected with a 10 volt battery, a switch and an ideal ammeter to form the simple electrical circuit shown. After the switch is closed and the current in the circuit reaches a constant value, what is the reading on the ammeter in the circuit?  
 (A)  $8.1 \times 10^{-2} \text{ A}$  (B)  $7.5 \times 10^{-2} \text{ A}$  (C)  $6.9 \times 10^{-2} \text{ A}$  (D) zero

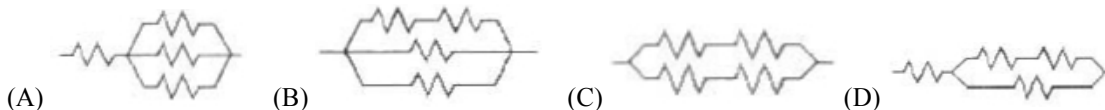


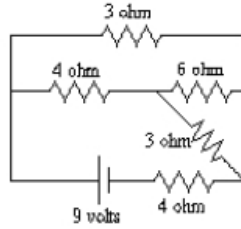
83. When the switch is closed, what would be the current in the circuit shown in the diagram above if the two batteries are opposing one another?  
 (A) 0.75 A (B) 0.5 A (C) 0.3 A (D) 0.2 A



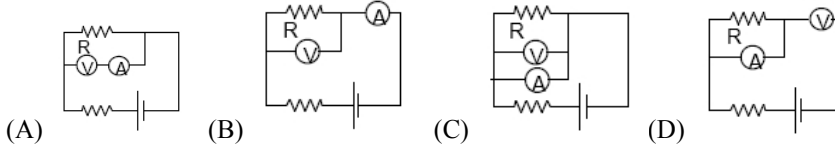
84. The diagram above shows an electrical circuit composed of 3 resistors and 1 capacitor. If each resistor has a resistance of  $10 \Omega$  and the capacitor has a value of  $10 \mu\text{F}$ , what would be the charge stored in the capacitor when an EMF of 10 V is maintained in the circuit for a sufficient time to fully charge the capacitor?  
 (A)  $23 \mu\text{C}$  (B)  $40 \mu\text{C}$  (C)  $67 \mu\text{C}$  (D)  $100 \mu\text{C}$

85. Given 4 identical resistors of resistance  $R$ , which of the following configurations would have an equivalent resistance of  $\frac{4}{3} R$ ?

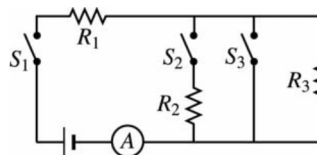




86. What would be the total current being supplied by the battery in the circuit shown above?  
 (A) 3.0 amperes (B) 2.0 (C) 1.5 amperes (D) 1.0 amperes
87. Which of the following wiring diagrams could be used to experimentally determine  $R$  using Ohm's Law? Assume an ideal voltmeter and an ideal ammeter.



Questions 88-90

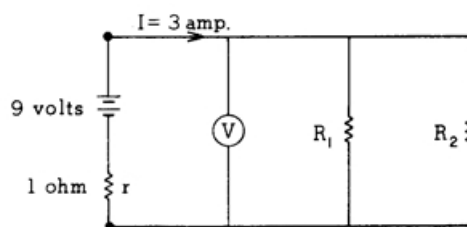


In the circuit above, the resistors all have the same resistance. The battery, wires, and ammeter have negligible resistance. A closed switch also has negligible resistance.

88. Closing which of the switches will produce the greatest power dissipation in  $R_2$ ?  
 (A)  $S_1$  only (B)  $S_2$  only (C)  $S_1$  and  $S_2$  only (D)  $S_1$  and  $S_3$  only
89. Closing which of the switches will produce the greatest reading on the ammeter?  
 (A)  $S_1$  only (B)  $S_2$  only (C)  $S_1$  and  $S_2$  (D)  $S_1$  and  $S_3$
90. Closing which of the switches will produce the greatest voltage across  $R_3$ ?  
 (A)  $S_1$  only (B)  $S_2$  only (C)  $S_1$  and  $S_2$  only (D)  $S_1$  and  $S_3$  only

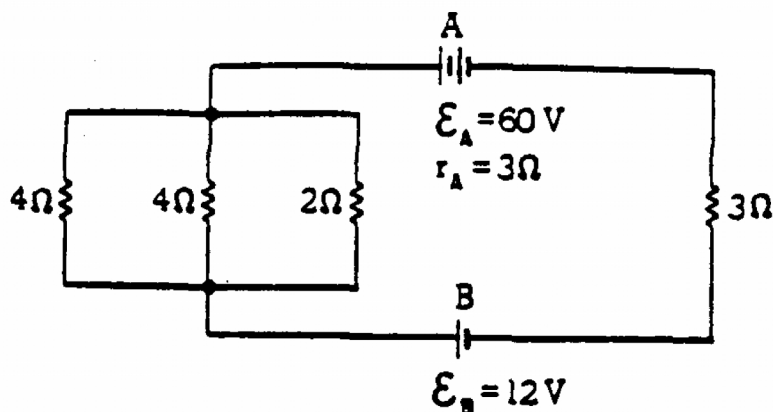


AP Physics Free Response Practice – Circuits



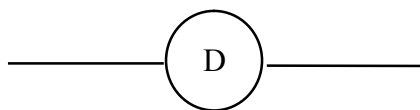
1976B3. In the circuit shown above, the current delivered by the 9-volt battery of internal resistance 1 ohm is 3 amperes. The power dissipated in  $R_2$  is 12 watts.

- Determine the reading of voltmeter V in the diagram.
- Determine the resistance of  $R_2$ .
- Determine the resistance of  $R_1$ .



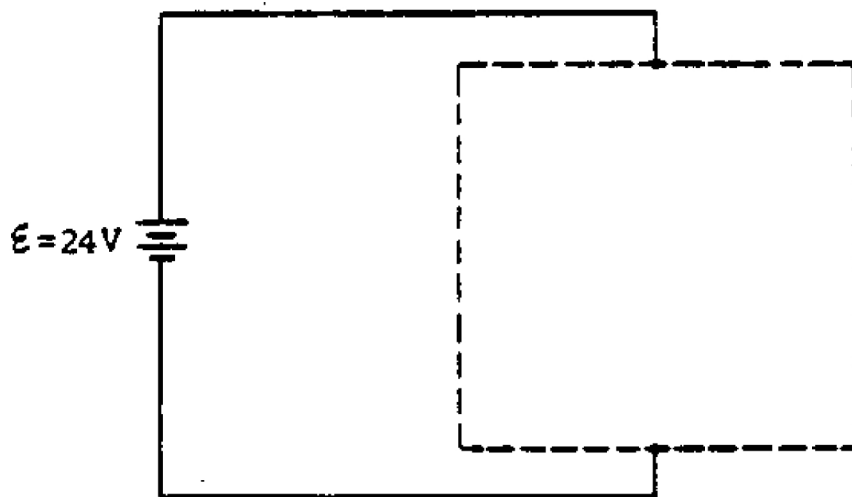
1981B4. A circuit consists of battery A of emf  $\mathcal{E}_A = 60$  volts and internal resistance  $r_A = 3$  ohms; battery B of emf  $\mathcal{E}_B = 12$  volts and internal resistance  $r_B = 1$  ohm; and four resistors connected as shown in the diagram above.

- Calculate the current in the 2-ohm resistor.
- Calculate the power dissipated in the 3-ohm resistor.
- Calculate the terminal voltage of battery B.

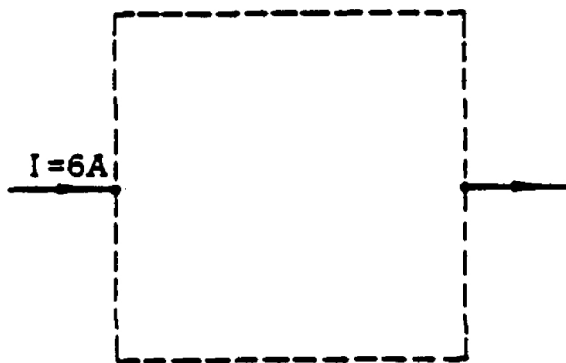


1980B2. The electrical device whose symbol is shown above requires a terminal voltage of 12 volts and a current of 2 amperes for proper operation.

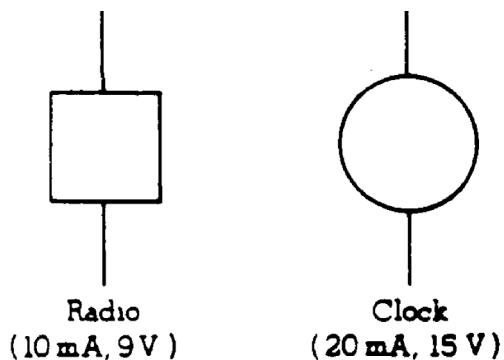
- a. Using only this device and one or more 3-ohm resistors design a circuit so that the device will operate properly when the circuit is connected across a battery of emf 24 volts and negligible internal resistance. Within the dashed-line box in the diagram below, draw the circuit using the symbol for the device and the appropriate symbol for each 3-ohm resistor.



- b. Using only this device and one or more 3-ohm resistors, design a circuit so that the device will operate properly when connected to a source that supplies a fixed current of 6 amperes. Within the dashed-line box in the diagram below, draw the circuit using the symbol for the device and the appropriate symbol.

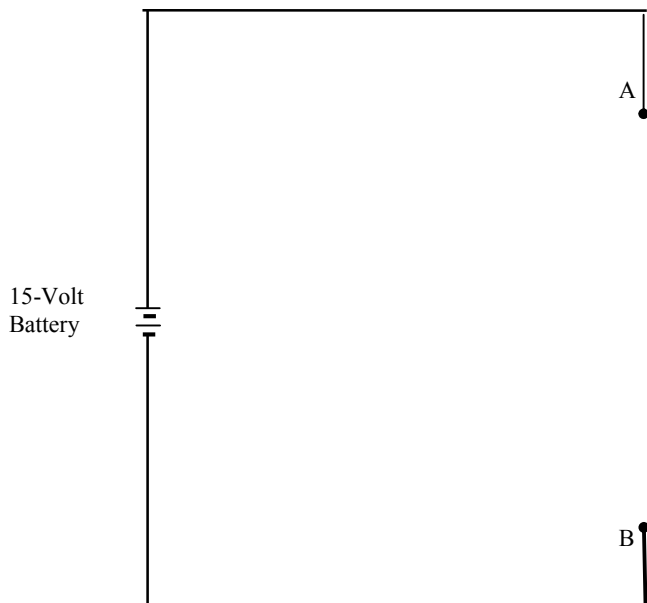


- c. Calculate the power dissipation in each 3-ohm resistor used in the circuit in part b.

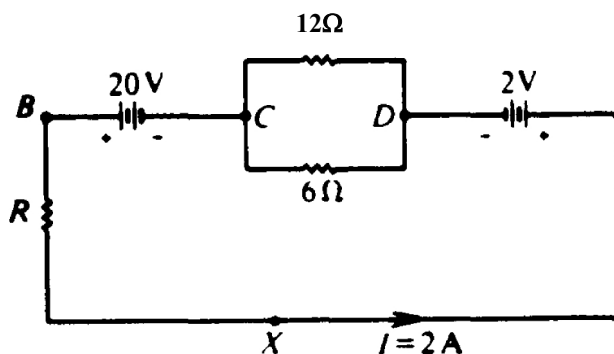


1982B4. A cabin contains only two small electrical appliances: a radio that requires 10 milliamperes of current at 9 volts, and a clock that requires 20 milliamperes at 15 volts. A 15-volt battery with negligible internal resistance supplies the electrical energy to operate the radio and the clock.

- a. Complete the diagram below to show how the radio, the clock, and a single resistor  $R$  can be connected between points A and B so that the correct potential difference is applied across each appliance. Use the symbols in the diagram above to indicate the radio and the clock.

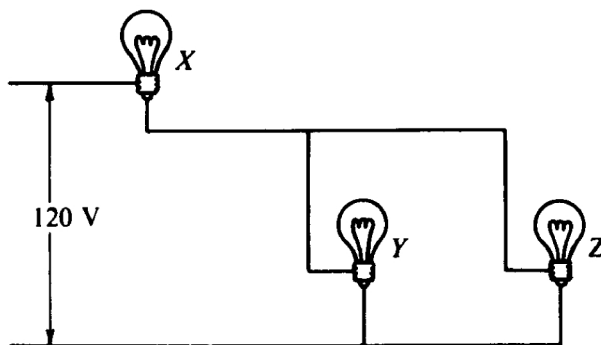


- b. Calculate the resistance of  $R$ .
- c. Calculate the electrical energy that must be supplied by the battery to operate the circuits for 1 minute.



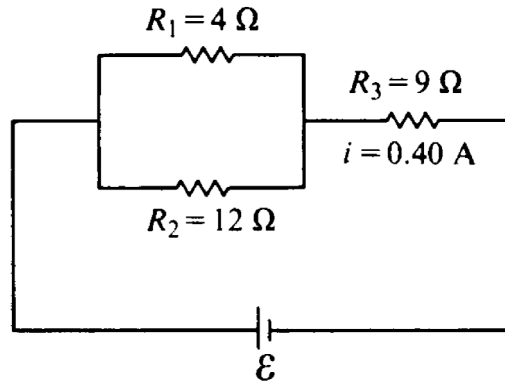
1983B3. The circuit shown above is constructed with two batteries and three resistors. The connecting wires may be considered to have negligible resistance. The current  $I$  is 2 amperes.

- Calculate the resistance  $R$ .
- Calculate the current in the
  - 6-ohm resistor
  - 12-ohm resistor
- The potential at point  $X$  is 0 volts. Calculate the electric potential at points  $B$ ,  $C$ , and  $D$  in the circuit.
- Calculate the power supplied by the 20-volt battery.



1986B3. In the circuit shown above,  $X$ ,  $Y$ , and  $Z$  represent three light bulbs, each rated at 60 watts, 120 volts. Assume that the resistances of the bulbs are constant and do not depend on the current.

- What is the resistance of each bulb?
- What is the equivalent resistance of the three light bulbs when arranged as shown?
- What is the total power dissipation of this combination when connected to a 120-volt source as shown?
- What is the current in bulb  $X$ ?
- What is the potential difference across bulb  $X$ ?
- What is the potential difference across bulb  $Z$ ?

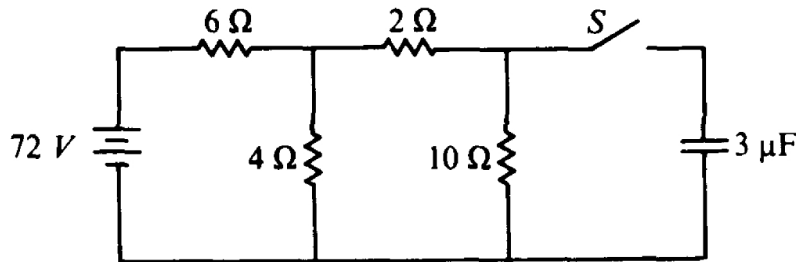


1987B4. Three resistors are arranged in a circuit as shown above. The battery has an unknown but constant emf  $\mathcal{E}$  and a negligible internal resistance.

- a. Determine the equivalent resistance of the three resistors.

The current  $I$  in resistor  $R_3$  is 0.40 ampere.

- b. Determine the emf  $\mathcal{E}$  (Voltage) of the battery.  
 c. Determine the potential difference across resistor  $R_1$   
 d. Determine the power dissipated in resistor  $R_1$   
 e. Determine the amount of charge that passes through resistor  $R_3$  in one minute.



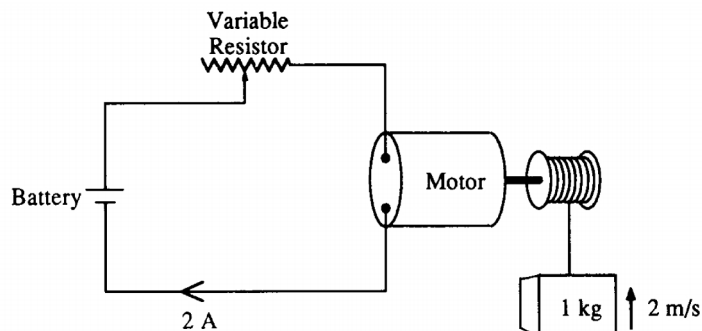
1988B3. The circuit shown above includes a switch  $S$ , which can be closed to connect the 3-microfarad capacitor in parallel with the 10-ohm resistor or opened to disconnect the capacitor from the circuit.

Case I: Switch  $S$  is open. The capacitor is not connected. Under these conditions determine:

- a. the current in the battery  
 b. the current in the 10-ohm resistor  
 c. the potential difference across the 10-ohm resistor

Case II: Switch  $S$  is closed. The capacitor is connected. After some time, the currents reach constant values. Under these conditions determine:

- d. the charge on the capacitor  
 e. the energy stored in the capacitor

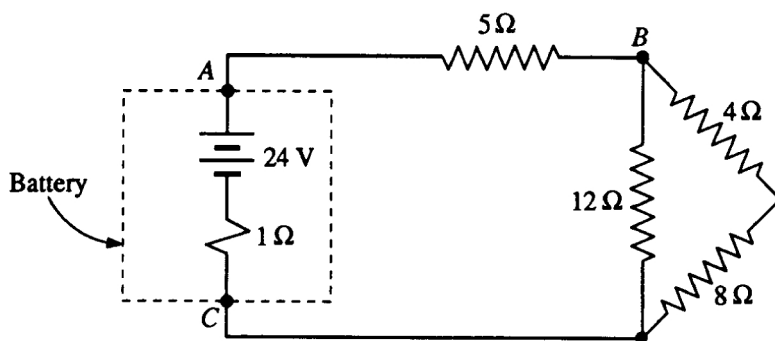


1989B3. A series circuit consists of a battery of negligible internal resistance, a variable resistor, and an electric motor of negligible resistance. The current in the circuit is 2 amperes when the resistance in the circuit is adjusted to 10 ohms. Under these conditions the motor lifts a 1-kilogram mass vertically at a constant speed of 2 meters per second.

- Determine the electrical power that is
  - dissipated in the resistor
  - used by the motor in lifting the mass
  - supplied by the battery
- Determine the potential difference across
  - the resistor
  - the motor
  - the battery

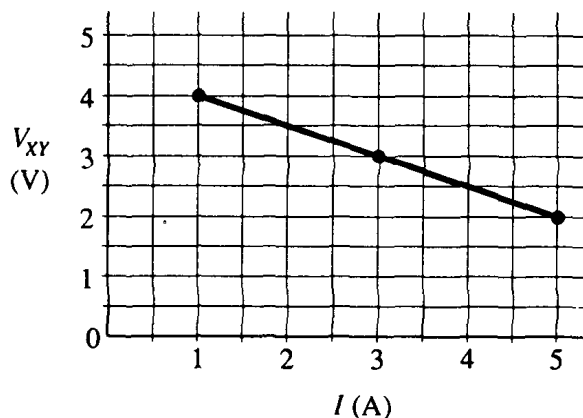
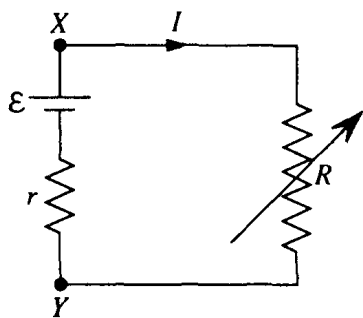
The resistor is now adjusted until the mass rises vertically at a constant speed of 3 meters per second. The voltage drop across the motor is proportional to the speed of the motor, and the current remains constant.

- Determine the voltage drop across the motor.
- Determine the new resistance in the circuit.





1990B3. A battery with an emf of 24 volts and an internal resistance of 1 ohm is connected to an external circuit as shown above. Determine each of the following:

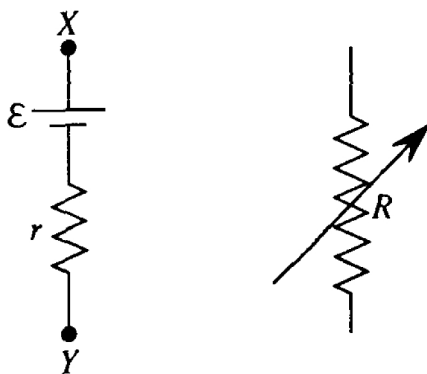
- the equivalent resistance of the combination of the 4-ohm, 8-ohm, and 12-ohm resistors
- the current in the 5-ohm resistor
- the terminal voltage,  $V_{AC}$  of the battery
- the rate at which energy is dissipated in the 12-ohm resistor
- the magnitude of the potential difference  $V_{BC}$
- the power delivered by the battery to the external circuit



1991B4. A battery with emf  $\mathcal{E}$  and internal resistance  $r$  is connected to a variable resistance  $R$  at points  $X$  and  $Y$ , as shown above on the left. Varying  $R$  changes both the current  $I$  and the terminal voltage  $V_{XY}$ . The quantities  $I$  and  $V_{XY}$  are measured for several values of  $R$  and the data are plotted in a graph, as shown above on the right.

- Determine the emf  $\mathcal{E}$  of the battery.
- Determine the internal resistance  $r$  of the battery.
- Determine the value of the resistance  $R$  that will produce a current  $I$  of 3 amperes.
- Determine the maximum current that the battery can produce.
- The current and voltage measurements were made with an ammeter and a voltmeter. On the diagram

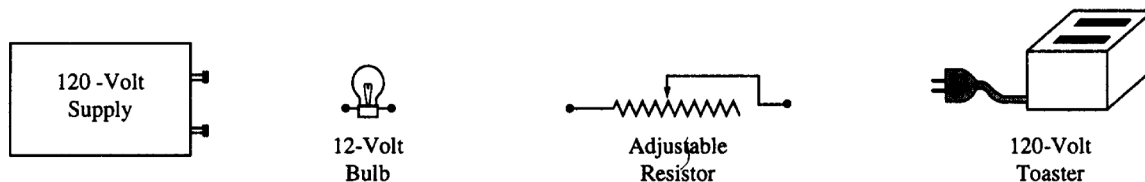
below, show a proper circuit for performing these measurements. Use  to represent the ammeter and  to represent the voltmeter.



- 1995B2. A certain light bulb is designed to dissipate 6 watts when it is connected to a 12-volt source.
- Calculate the resistance of the light bulb.
  - If the light bulb functions as designed and is lit continuously for 30 days, how much energy is used? Be sure to indicate the units in your answer.

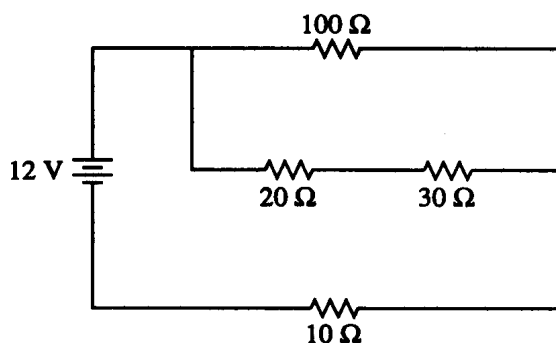
The 6-watt, 12-volt bulb is connected in a circuit with a 1,500-watt, 120-volt toaster; an adjustable resistor; and a 120-volt power supply. The circuit is designed such that the bulb and the toaster operate at the given values and, if the light bulb fails, the toaster will still function at these values.

- On the diagram below, draw in wires connecting the components shown to make a complete circuit that will function as described above.



- Determine the value of the adjustable resistor that must be used in order for the circuit to work as designed.
- If the resistance of the adjustable resistor is increased, what will happen to the following?
  - The brightness of the bulb. Briefly explain your reasoning.
  - The power dissipated by the toaster. Briefly explain your reasoning.

- 1996B4. A student is provided with a 12.0-V battery of negligible internal resistance and four resistors with the following resistances:  $100\ \Omega$ ,  $30\ \Omega$ ,  $20\ \Omega$ , and  $10\ \Omega$ . The student also has plenty of wire of negligible resistance available to make connections as desired.
- Using all of these components, draw a circuit diagram in which each resistor has nonzero current flowing through it, but in which the current from the battery is as small as possible.
  - Using all of these components, draw a circuit diagram in which each resistor has nonzero current flowing through it, but in which the current from the battery is as large as possible (without short circuiting the battery).








The battery and resistors are now connected in the circuit shown above.

- Determine the following for this circuit.
  - The current in the  $10\text{-}\Omega$  resistor
  - The total power consumption of the circuit
- Assuming that the current remains constant, how long will it take to provide a total of 10 kJ of electrical energy to the circuit?

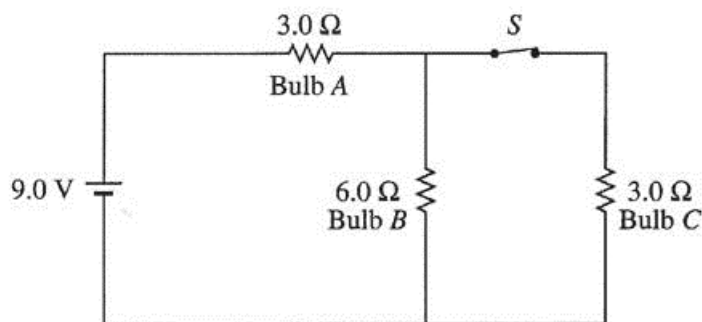
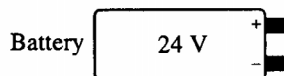


1997B4 (modified) Three identical resistors, each of resistance  $30\ \Omega$  are connected in a circuit to heat water in a glass beaker.  $24\text{ V}$  battery with negligible internal resistance provides the power. The three resistors may be connected in series or in parallel.

- If they are connected in series, what power is developed in the circuit?
  - If they are connected in parallel, what power is developed in the circuit?
- Using the battery and one or more of the resistors, design a circuit that will heat the water at the fastest rate when the resistor(s) are placed in the water. Include an ammeter to measure the current in the circuit and a voltmeter to measure the total potential difference of the circuit. Assume the wires are insulated and have no resistance. Draw a diagram of the circuit in the box below, using the following symbols to represent the components in your diagram.

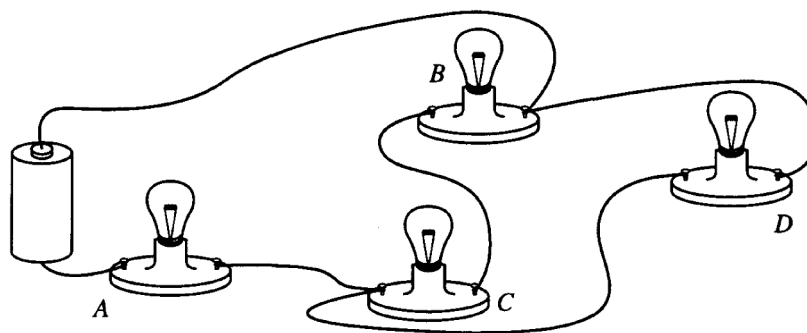
Symbols to be Used:  Resistors   Ammeter  Voltmeter 

Draw your diagram in this box only.



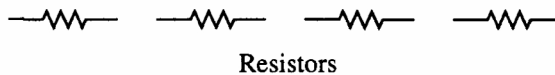
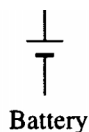
2002B3B. Lightbulbs of fixed resistance  $3.0\ \Omega$  and  $6.0\ \Omega$ , a  $9.0\text{ V}$  battery, and a switch  $S$  are connected as shown in the schematic diagram above. The switch  $S$  is closed.

- Calculate the current in bulb A.
- Which lightbulb is brightest? Justify your answer.
- Switch  $S$  is then opened. By checking the appropriate spaces below, indicate whether the brightness of each lightbulb increases, decreases, or remains the same. Explain your reasoning for each lightbulb.
  - Bulb A: The brightness    \_\_\_ increases    \_\_\_ decreases    \_\_\_ remains the same  
Explanation:
  - Bulb B: The brightness    \_\_\_ increases    \_\_\_ decreases    \_\_\_ remains the same  
Explanation:
  - Bulb C: The brightness    \_\_\_ increases    \_\_\_ decreases    \_\_\_ remains the same  
Explanation:



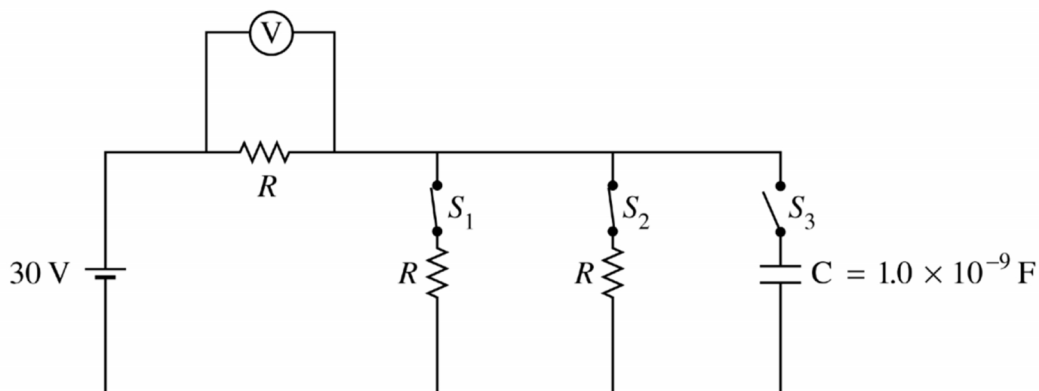
1998B4 In the circuit shown above, A, B, C, and D are identical lightbulbs. Assume that the battery maintains a constant potential difference between its terminals (i.e., the internal resistance of the battery is assumed to be negligible) and the resistance of each lightbulb remains constant.

- a. Draw a diagram of the circuit in the box below, using the following symbols to represent the components in your diagram. Label the resistors A, B, C, and D to refer to the corresponding lightbulbs.



Draw your diagram in this box only.

- b. List the bulbs in order of their brightnesses, from brightest to least bright. If any two or more bulbs have the same brightness, state which ones. Justify your answer.
- c. Bulb D is then removed from its socket.
- Describe the change in the brightness, if any, of bulb A when bulb D is removed from its socket. Justify your answer.
  - Describe the change in the brightness, if any, of bulb B when bulb D is removed from its socket. Justify your answer.



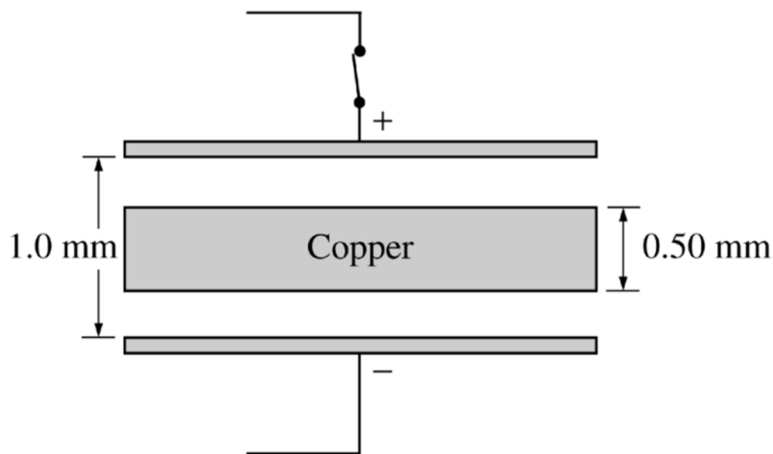
2000B3. Three identical resistors, each with resistance  $R$ , and a capacitor of  $1.0 \times 10^{-9} \text{ F}$  are connected to a 30 V battery with negligible internal resistance, as shown in the circuit diagram above. Switches  $S_1$  and  $S_2$  are initially closed, and switch  $S_3$  is initially open. A voltmeter is connected as shown.

- a. Determine the reading on the voltmeter.

Switches  $S_1$  and  $S_2$  are now opened, and then switch  $S_3$  is closed.

- b. Determine the charge  $Q$  on the capacitor after  $S_3$  has been closed for a very long time.

After the capacitor is fully charged, switches  $S_1$  and  $S_2$  remain open, switch  $S_3$  remains closed, the plates are held fixed, and a conducting copper block is inserted midway between the plates, as shown below. The plates of the capacitor are separated by a distance of 1.0 mm, and the copper block has a thickness of 0.5 mm.



- c. i. What is the potential difference between the plates?  
 ii. What is the electric field inside the copper block?  
 iii. On the diagram above, draw arrows to clearly indicate the direction of the electric field between the plates.  
 iv. Determine the magnitude of the electric field in each of the spaces between the plates and the copper block.

2002B3 Two lightbulbs, one rated 30 W at 120 V and another rated 40 W at 120 V, are arranged in two different circuits.

- The two bulbs are first connected in parallel to a 120 V source.
  - Determine the resistance of the bulb rated 30 W and the current in it when it is connected in this circuit.
  - Determine the resistance of the bulb rated 40 W and the current in it when it is connected in this circuit.
- The bulbs are now connected in series with each other and a 120 V source.
  - Determine the resistance of the bulb rated 30 W and the current in it when it is connected in this circuit.
  - Determine the resistance of the bulb rated 40 W and the current in it when it is connected in this circuit.
- In the spaces below, number the bulbs in each situation described, in order of their brightness. (1 = brightest, 4 = dimmest)

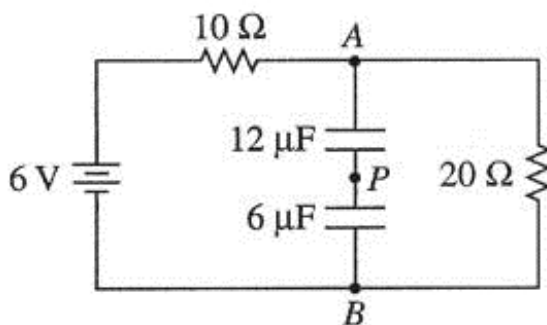
\_\_\_\_ 30 W bulb in the parallel circuit

\_\_\_\_ 40 W bulb in the parallel circuit

\_\_\_\_ 30 W bulb in the series circuit

\_\_\_\_ 40 W bulb in the series circuit

- Calculate the total power dissipated by the two bulbs in each of the following cases.
  - The parallel circuit
  - The series circuit



2003B2 A circuit contains two resistors ( $10\ \Omega$  and  $20\ \Omega$ ) and two capacitors ( $12\ \mu\text{F}$  and  $6\ \mu\text{F}$ ) connected to a 6 V battery, as shown in the diagram above. The circuit has been connected for a long time.

- Calculate the total capacitance of the circuit.
- Calculate the current in the  $10\ \Omega$  resistor.
- Calculate the potential difference between points A and B.
- Calculate the charge stored on one plate of the  $6\ \mu\text{F}$  capacitor.
- The wire is cut at point P. Will the potential difference between points A and B increase, decrease, or remain the same?

\_\_\_\_ increase

\_\_\_\_ decrease

\_\_\_\_ remain the same

Justify your answer.



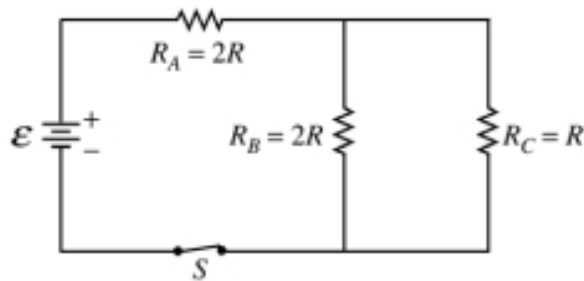
2003Bb2. A student is asked to design a circuit to supply an electric motor with 1.0 mA of current at 3.0 V potential difference.

- Determine the power to be supplied to the motor.
- Determine the electrical energy to be supplied to the motor in 60 s.
- Operating as designed above, the motor can lift a 0.012 kg mass a distance of 1.0 m in 60 s at constant velocity. Determine the efficiency of the motor.

To operate the motor, the student has available only a 9.0 V battery to use as the power source and the following five resistors.



- In the space below, complete a schematic diagram of a circuit that shows how one or more of these resistors can be connected to the battery and motor so that 1.0 mA of current and 3.0 V of potential difference are supplied to the motor. Be sure to label each resistor in the circuit with the correct value of its resistance.



2007B3. The circuit above contains a battery with negligible internal resistance, a closed switch S, and three resistors, each with a resistance of  $R$  or  $2R$ .

- a. i. Rank the currents in the three resistors from greatest to least, with number 1 being greatest. If two resistors have the same current, give them the same ranking.

\_\_\_\_\_  $I_A$       \_\_\_\_\_  $I_B$       \_\_\_\_\_  $I_C$

ii. Justify your answers.

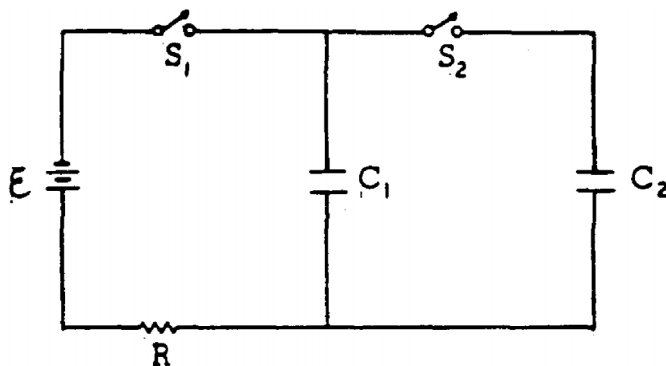
- b. i. Rank the voltages across the three resistors from greatest to least, with number 1 being greatest. If two resistors have the same voltage across them, give them the same ranking.

\_\_\_\_\_  $V_A$       \_\_\_\_\_  $V_B$       \_\_\_\_\_  $V_C$

ii. Justify your answers.

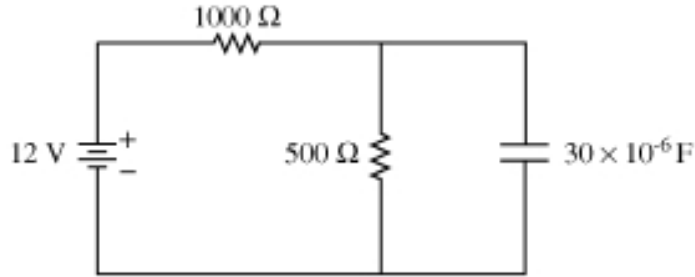
For parts c. through e., use  $\mathcal{E} = 12 \text{ V}$  and  $R = 200 \, \Omega$ .

- c. Calculate the equivalent resistance of the circuit.  
d. Calculate the current in resistor  $R_C$ .  
e. The switch S is opened, resistor  $R_B$  is removed and replaced by a capacitor of capacitance  $2.0 \times 10^{-6} \text{ F}$ , and the switch S is again closed. Calculate the charge on the capacitor after all the currents have reached their final steady-state values.



1975E2. In the diagram above,  $V = 100 \text{ volts}$ ;  $C_1 = 12 \text{ microfarads}$ ;  $C_2 = 24 \text{ microfarads}$ ;  $R = 10 \text{ ohms}$ . Initially,  $C_1$  and  $C_2$  are uncharged, and all switches are open.

- a. First, switch  $S_1$  is closed. Determine the charge on  $C_1$  when equilibrium is reached.  
b. Next  $S_1$  is opened and afterward  $S_2$  is closed. Determine the charge on  $C_1$  when equilibrium is again reached.  
c. For the equilibrium condition of part b., determine the voltage across  $C_1$ .  
d.  $S_2$  remains closed, and now  $S_1$  is also closed. How much additional charge flows from the battery?



B2007b3. In the circuit above, a 12.0 V battery is connected to two resistors, one of resistance 1000  $\Omega$  and the other of resistance 500  $\Omega$ . A capacitor with a capacitance of  $30 \times 10^{-6}$  F is connected in parallel with the 500  $\Omega$  resistor. The circuit has been connected for a long time, and all currents have reached their steady states.

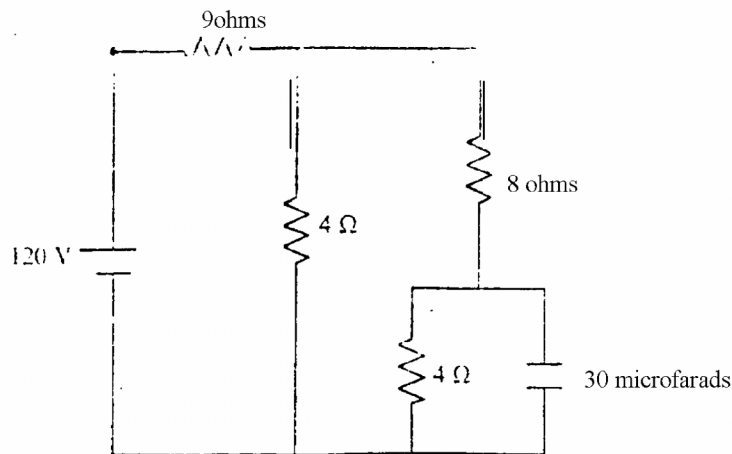
- Calculate the current in the 500  $\Omega$  resistor.
- Draw an ammeter in the circuit above in a location such that it could measure the current in the 500  $\Omega$  resistor. Use the symbol  $\textcircled{A}$  to indicate the ammeter.
  - Draw a voltmeter in the circuit above in a location such that it could measure the voltage across the 1000  $\Omega$  resistor. Use the symbol  $\textcircled{V}$  to indicate the voltmeter.
- Calculate the charge stored on the capacitor.
- Calculate the power dissipated in the 1000  $\Omega$  resistor.
- The capacitor is now discharged, and the 500  $\Omega$  resistor is removed and replaced by a resistor of greater resistance. The circuit is reconnected, and currents are again allowed to come to their steady-state values. Is the charge now stored on the capacitor larger, smaller, or the same as it was in part c.?

\_\_\_\_\_ Larger

\_\_\_\_\_ Smaller

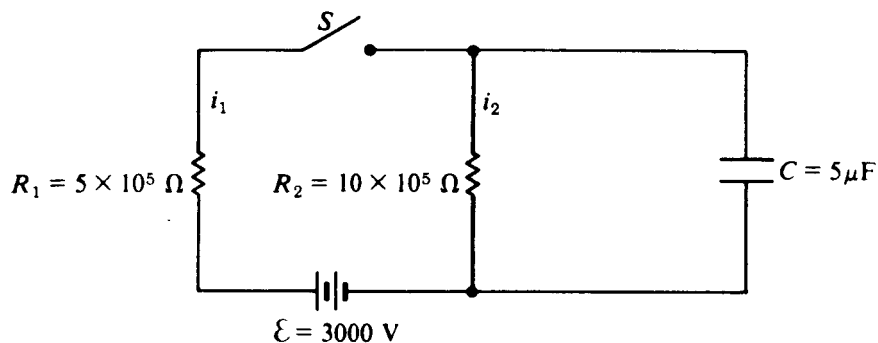
\_\_\_\_\_ The same as

Justify your answer.



1988E2. In the circuit shown above, the battery has been connected for a long time so that the currents have steady values. Given these conditions, calculate each of the following

- The current in the 9-ohm resistor.
- The current in the 8-ohm resistor.
- The potential difference across the 30-microfarad capacitor.
- The energy stored in the 30-microfarad capacitor.

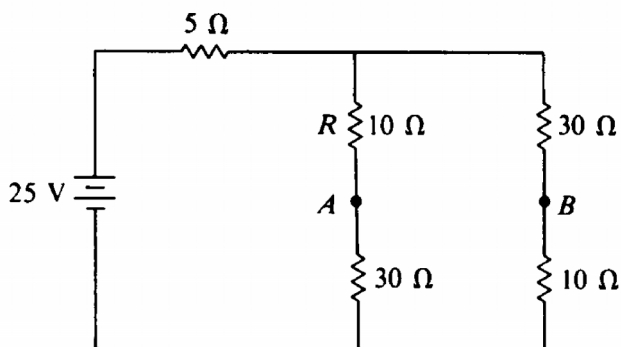


1985E2 (modified) In the circuit shown above,  $i_1$  and  $i_2$  are the currents through resistors  $R_1$  and  $R_2$ , respectively.  $V_1$ ,  $V_2$ , and  $V_c$  are the potential differences across resistor  $R_1$ , resistor  $R_2$ , and capacitor  $C$ , respectively. Initially the capacitor is uncharged.

- Calculate the current  $i_1$  immediately after switch  $S$  is closed.

Assume switch  $S$  has been closed for a long time.

- Calculate the current  $i_2$ .
- Calculate the charge  $Q$  on the capacitor.
- Calculate the energy  $U$  stored in the capacitor.



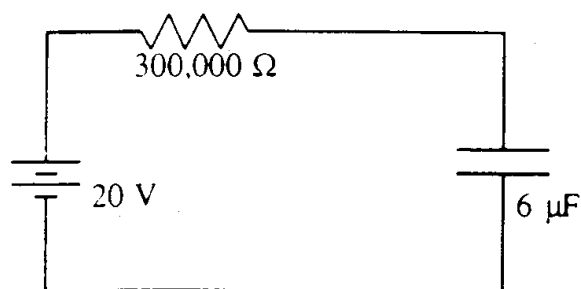
1986E2 (modified) Five resistors are connected as shown above to a 25-volt source of emf with zero internal resistance.

- Determine the current in the resistor labeled  $R$ .

A 10-microfarad capacitor is connected between points  $A$  and  $B$ . The currents in the circuit and the charge on the capacitor soon reach constant values. Determine the constant value for each of the following.

- The current in the resistor  $R$
- The charge on the capacitor





1989E3. A battery with an emf of 20 volts is connected in series with a resistor of 300,000 ohms and an air-filled parallel-plate capacitor of capacitance 6 microfarads.

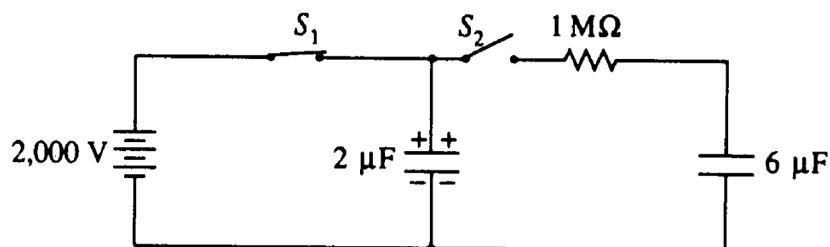
- a. Determine the energy stored in the capacitor when it is fully charged.

The spacing between the capacitor plates is suddenly increased (in a time short enough so the charge does not have time to readjust) to four times its original value.

- b. Determine the work that must be done in increasing the spacing in this fashion.  
c. Determine the current in the resistor immediately after the spacing is increased.

After a long time, the circuit reaches a new static state.

- d. Determine the total charge that has passed through the battery.  
e. Determine the energy that has been added to the battery.



1992E2. The 2-microfarad ( $2 \times 10^{-6}$  farad) capacitor shown in the circuit above is fully charged by closing switch  $S_1$  and keeping switch  $S_2$  open, thus connecting the capacitor to the 2,000-volt power supply.

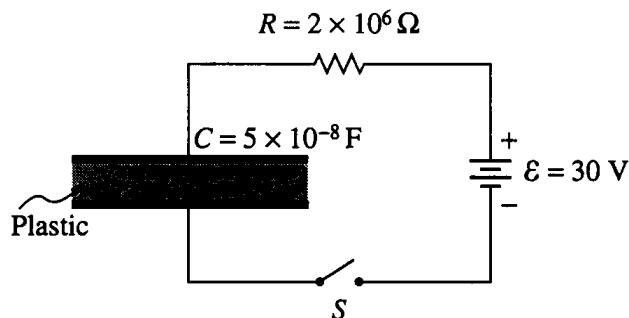
- a. Determine each of the following for this fully charged capacitor.  
i. The magnitude of the charge on each plate of the capacitor.  
ii. The electrical energy stored in the capacitor.

At a later time, switch  $S_1$  is opened. Switch  $S_2$  is then closed, connecting the charged 2-microfarad capacitor to a 1-megohm ( $1 \times 10^6 \Omega$ ) resistor and a 6-microfarad capacitor, which is initially uncharged.

- b. Determine the initial current in the resistor the instant after switch  $S_2$  is closed.

Equilibrium is reached after a long period of time.

- c. Determine the charge on the positive plate of each of the capacitors at equilibrium.  
d. Determine the total electrical energy stored in the two capacitors at equilibrium. If the energy is greater than the energy determined in part a. ii., where did the increase come from? If the energy is less than the energy determined in part a. ii., where did the electrical energy go?



1995E2 (modified) A parallel-plate capacitor is made from two sheets of metal, each with an area of 1.0 square meter, separated by a sheet of plastic 1.0 millimeter ( $10^{-3}$  m) thick, as shown above. The capacitance is measured to be 0.05 microfarad ( $5 \times 10^{-8}$  F).

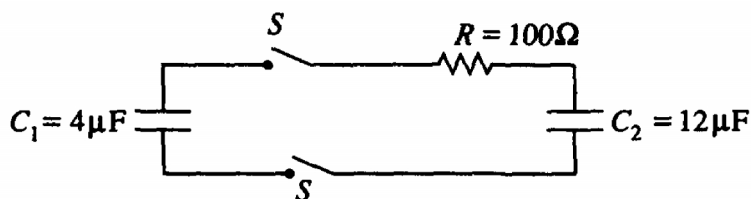
- a. What is the dielectric constant of the plastic?

The uncharged capacitor is connected in series with a resistor  $R = 2 \times 10^6$  ohms, a 30-volt battery, and an open switch  $S$ , as shown above. The switch is then closed.

- b. What is the initial charging current when the switch  $S$  is closed?  
 c. Determine the magnitude and sign of the final charge on the bottom plate of the fully charged capacitor.  
 d. How much electrical energy is stored in the fully charged capacitor?

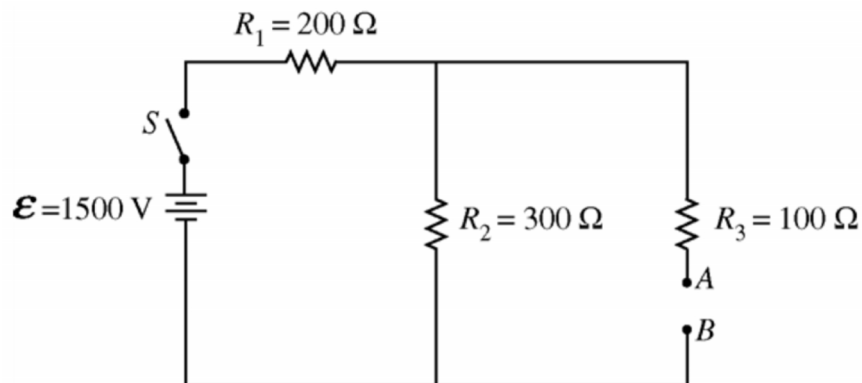
After the capacitor is fully charged, it is carefully disconnected, leaving the charged capacitor isolated in space. The plastic sheet is then removed from between the metal plates. The metal plates retain their original separation of 1.0 millimeter.

- e. What is the new voltage across the plates?  
 f. If there is now more energy stored in the capacitor, where did it come from? If there is now less energy, what happened to it?

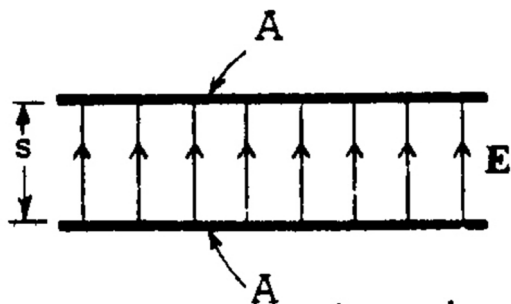


1996E2 (modified) Capacitors 1 and 2, of capacitance  $C_1 = 4\mu\text{F}$  and  $C_2 = 12\mu\text{F}$ , respectively, are connected in a circuit as shown above with a resistor of resistance  $R = 100\ \Omega$  and two switches. Capacitor 1 is initially charged to a voltage  $V_0 = 50$  V and capacitor 2 is initially uncharged. Both of the switches  $S$  are then closed at time  $t = 0$ .

- a. What are the final charges on the positive plate of each of the capacitors 1 and 2 after equilibrium has been reached?  
 b. Determine the difference between the initial and the final stored energy of the system after equilibrium has been reached.



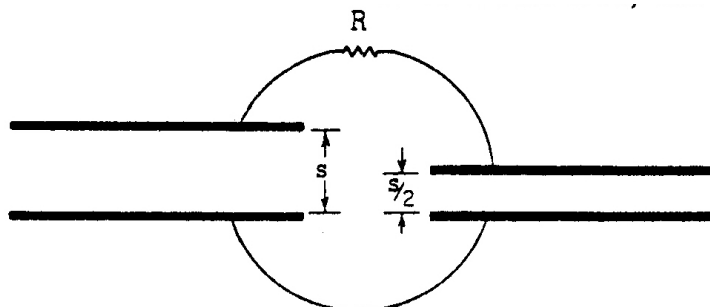
- 2008E2 (modified) In the circuit shown above, A and B are terminals to which different circuit components can be connected.
- Calculate the potential difference across  $R_2$  immediately after the switch S is closed in each of the following cases.
    - A  $50 \Omega$  resistor connects A and B.
    - An initially uncharged  $0.80 \mu\text{F}$  capacitor connects A and B.



- 1978B3. A uniform electric field  $E$  is established between two capacitor plates, each of area  $A$ , which are separated by a distance  $s$  as shown above.
- What is the electric potential difference  $V$  between the plates?
  - Specify the sign of the charge on each plate.

The capacitor above is then connected electrically through a resistor to a second parallel-plate capacitor, initially uncharged, whose plates have the same area  $A$  but a separation of only  $s/2$ .

- Indicate on the diagram below the direction of the current in each wire, and explain why the current will eventually cease.



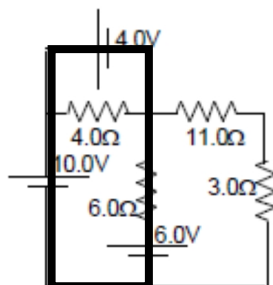
- After the current has ceased, which capacitor has the greater charge? Explain your reasoning.
- The total energy stored in the two capacitors after the current has ceased is less than the initial stored energy. Explain qualitatively what has become of this “lost” energy.

# ANSWERS - AP Physics Multiple Choice Practice – Circuits

<u>Solution</u>	<u>Answer</u>
1. The resistances are as follows: I: $2\ \Omega$ , II: $4\ \Omega$ , III: $1\ \Omega$ , IV: $2\ \Omega$	A,D
2. The total resistance of the $3\ \Omega$ and $6\ \Omega$ in parallel is $2\ \Omega$ making the total circuit resistance $6\ \Omega$ and the total current $\mathcal{E}/R = 1\ \text{A}$ . This $1\ \text{A}$ will divide in the ratio of 2:1 through the $3\ \Omega$ and $6\ \Omega$ respectively so the $3\ \Omega$ resistor receives $2/3\ \text{A}$ making the potential difference $IR = (2/3\ \text{A})(3\ \Omega) = 2\ \text{V}$ .	A
3. The voltage across each branch does not change (the battery is a constant voltage device and energy is conserved on each loop), neither does the current through each branch. The ammeter measures the current feeding both branches, according to the junction rule.	A
4. The battery is a constant voltage device and energy is conserved on each loop.	D
5. $R = \rho L/A$ . Greatest resistance is the longest, narrowest resistor.	B
6. In parallel $V_1 = V_2$ . $Q_1 = C_1 V_1$ and $Q_2 = C_2 V_2$ so $Q_1/Q_2 = C_1/C_2 = 1.5$	C
7. For steady power dissipation, the circuit must allow current to flow indefinitely. For the greatest power, the total resistance should be the smallest value. These criteria are met with the resistors in parallel.	D
8. To retain energy, there must be a capacitor that will not discharge through a resistor. Capacitors in circuits C and E will discharge through the resistors in parallel with them.	B
9. $P = I\mathcal{E}$	B
10. The resistance of the two resistors in parallel is $r/2$ . The total circuit resistance is then $10\ \Omega + \frac{1}{2}r$ , which is equivalent to $\mathcal{E}/I = (10\ \text{V})/(0.5\ \text{A}) = 20\ \Omega = 10\ \Omega + r/2$	D
11. With more current drawn from the battery for the parallel connection, more power is dissipated in this connection. While the resistors in series share the voltage of the battery, the resistors in parallel have the full potential difference of the battery across them.	C
12. The upper branch, with twice the resistance of the lower branch, will have $\frac{1}{2}$ the current of the lower branch.	C
13. The larger loop, with twice the radius, has twice the circumference (length) and $R = \rho L/A$	C
14. By process of elimination, A is the only possible true statement.	A
15. $R = \rho L/A$ . If $L \div 2$ , $R \div 2$ and if $r \div 2$ then $A \div 4$ and $R \times 4$ making the net effect $R \div 2 \times 4$	B
16. Because of the junction rule, $I_1$ must equal the sum of $I_2$ and $I_3$ . Resistance is only relevant to the comparison of currents if the potential difference is the same.	C
17. Resistance of the $2000\ \Omega$ and $6000\ \Omega$ in parallel $= 1500\ \Omega$ , adding the $2500\ \Omega$ in series gives a total circuit resistance of $4000\ \Omega$ . $I_{\text{total}} = I_1 = \mathcal{E}/R_{\text{total}}$	B
18. Only C would decrease the total resistance of the circuit, by adding a new parallel branch	C
19. The lower-Watt bulb across the same potential difference draws less current. It therefore has a greater resistance.	B
20. The current through R is found using the junction rule at the top junction, where $1\ \text{A} + 2\ \text{A}$ enter giving $I = 3\ \text{A}$ . Now utilize Kirchhoff's loop rule through the left or right loops: (left side) $+16\ \text{V} - (1\ \text{A})(4\ \Omega) - (3\ \text{A})R = 0$ giving $R = 4\ \Omega$	B

21. Utilizing Kirchhoff's loop rule with any loop including the lower branch gives 0 V since the resistance next to each battery drops the 2 V of each battery leaving the lower branch with no current. You can also think of the junction rule where there is 0.04 A going into each junction and 0.04 A leaving to the other battery, with no current for the lower branch. C
22. Summing the potential differences from left to right gives  $V_T = -12 \text{ V} - (2 \text{ A})(2 \Omega) = -16 \text{ V}$ . It is possible for  $V_T > \mathcal{E}$ . D
23. Current is greatest where resistance is least. The resistances are, in order, 1  $\Omega$ , 2  $\Omega$ , 4  $\Omega$ , and 2  $\Omega$ . A
24. See above C
25. Least power is for the greatest resistance ( $P = \mathcal{E}^2/R$ ) C
26. In series, the equivalent capacitance is calculated using reciprocals, like resistors in parallel. This results in an equivalent capacitance smaller than the smallest capacitor. C
27. With a load of less resistance, a greater current flows, and there is therefore a greater potential difference over the internal resistance. D
28. Kirchhoff's junction rule applied at point X gives  $2 \text{ A} = I + 1 \text{ A}$ , so the current in the middle wire is 1 A. Summing the potential differences through the middle wire from X to Y gives  $-10 \text{ V} - (1 \text{ A})(2 \Omega) = -12 \text{ V}$  D
29. When the switch is closed, the circuit behaves as if the capacitor were just a wire and all the potential of the battery is across the resistor. As the capacitor charges, the voltage changes over to the capacitor over time, eventually making the current (and the potential difference across the resistor) zero and the potential difference across the capacitor equal to the emf of the battery. A
30. See above A
31. See above B
32.  $P = I^2 R$  D
33. For each capacitor to have 6  $\mu\text{C}$ , each *branch* will have 6  $\mu\text{C}$  since the two capacitors in series in each branch has the same charge. The total charge for the three branches is then 18  $\mu\text{C}$ .  $Q = CV$  gives  $18 \mu\text{C} = (3 \mu\text{F})V$  B
34. Utilizing Kirchhoff's loop rule starting at the upper left and moving clockwise:  $-(2 \text{ A})(0.3 \Omega) + 12 \text{ V} - 6 \text{ V} - (2 \text{ A})(0.2 \Omega) - (2 \text{ A})(R) - (2 \text{ A})(1.5 \Omega) = 0$  A
35. Summing the potential differences:  $-6 \text{ V} - (2 \text{ A})(0.2 \Omega) - (2 \text{ A})(1 \Omega) = -8.4 \text{ V}$  C
36. As the capacitor becomes charged, more current flows through R2, thus increasing the equivalent resistance of the circuit and decreasing the current flow – but it never becomes zero. C
37. For the ammeter to read zero means the junctions at the ends of the ammeter have the same potential. For this to be true, the potential drops across the 1  $\Omega$  and the 2  $\Omega$  resistor must be equal, which means the current through the 1  $\Omega$  resistor must be twice that of the 2  $\Omega$  resistor. This means the resistance of the upper branch (1  $\Omega$  and 3  $\Omega$ ) must be  $\frac{1}{2}$  that of the lower branch (2  $\Omega$  and R) giving  $1 \Omega + 3 \Omega = \frac{1}{2} (2 \Omega + R)$  D
38. The current through the ammeter will flow toward the bottom of the page if that flow is from a higher to a lower potential. To accomplish this,  $R/2\Omega$  must be greater than  $3\Omega / 1\Omega$ . A
39. Kirchhoff's loop rule ( $V = Q/C$  for a capacitor) B
40. To dissipate 24 W means  $R = V^2/P = 6 \Omega$ . The resistances, in order, are: 8  $\Omega$ ,  $4/3 \Omega$ ,  $8/3 \Omega$ , 12  $\Omega$  and 6  $\Omega$  D

41. The equivalent capacitance between X and Y is twice the capacitance between Y and Z. This means the voltage between X and Y is  $\frac{1}{2}$  the voltage between Y and Z. For a total of 12 V, this gives 4 V between X and Y and 8 V between Y and Z. B
42. Closing the switch short circuits Bulb 2 causing no current to flow to it. Since the bulbs were originally in series, this decreases the total resistance and increases the total current, making bulb 1 brighter. B
43. Closing the switch reduces the resistance in the right side from  $20\ \Omega$  to  $15\ \Omega$ , making the total circuit resistance decrease from  $35\ \Omega$  to  $30\ \Omega$ , a slight decrease, causing a slight increase in current. For the current to double, the total resistance must be cut in half. B
44.  $R = \rho L/A \propto L/d^2$  where d is the diameter.  $R_x/R_y = L_x/d_x^2 \div L_y/d_y^2 = (2L_y)d_y^2/[L_y(2d_y)^2] = \frac{1}{2}$  C
45. Summing the potential differences from bottom to top:  
left circuit:  $-(1\text{ A})r + \mathcal{E} = 10\text{ V}$   
right circuit:  $+(1\text{ A})r + \mathcal{E} = 20\text{ V}$ , solve simultaneous equations C
46. If you perform Kirchhoff's loop rule for the highlighted loop, you get a current of 0 A through the  $6\ \Omega$  resistor. A



47. N is in the main branch, with the most current. The current then divides into the two branches, with K receiving twice the current as L and M. The L/M branch has twice the resistance of the K branch. L and M in series have the same current. D
48. If K burns out, the circuit becomes a series circuit with the three resistors, N, M and L all in series, reducing the current through bulb N. D
49. If M burns out, the circuit becomes a series circuit with the two resistors, N and K in series, with bulb L going out as well since it is in series with bulb M. D
50. Using Kirchhoff's loop rule around the circuit going through either V or R since they are in parallel and will have the same potential drop gives:  $-V - (1.00\text{ mA})(25\ \Omega) + 5.00\text{ V} - (1.00\text{ mA})(975\ \Omega) = 0$  C
51. The equivalent resistance in parallel is smaller than the smallest resistance. A
52.  $P = \mathcal{E}^2/R$ . Total resistance of n resistors in series is nR making the power  $P = \mathcal{E}^2/nR = P/n$  C
53. If the current in the  $6\ \Omega$  resistor is 1 A, then by ratios, the currents in the  $2\ \Omega$  and  $3\ \Omega$  resistor are 3 A and 2 A respectively (since they have  $\frac{1}{3}$  and  $\frac{1}{2}$  the resistance). This makes the total current 6 A and the potential drop across the  $4\ \Omega$  resistor 24 V. Now use Kirchhoff's loop rule for any branch. D
54. The voltage across the capacitor is 6 V ( $Q = CV$ ) and since the capacitor is in parallel with the  $300\ \Omega$  resistor, the voltage across the  $300\ \Omega$  resistor is also 6 V. The  $200\ \Omega$  resistor is not considered since the capacitor is charged and no current flows through that branch. The  $100\ \Omega$  resistor in series with the  $300\ \Omega$  resistor has  $\frac{1}{3}$  the voltage (2 V) since it is  $\frac{1}{3}$  the resistance. Kirchhoff's loop rule for the left loop gives  $\mathcal{E} = 8\text{ V}$ . C

55. For the currents in the branches to be equal, each branch must have the same resistance. C
56. Resistor D is in a branch by itself while resistors A, B and C are in series, drawing less current than resistor D. D
57. Even though the wires have different resistances and currents, the potential drop across each is 1.56 V and will vary by the same gradient, dropping all 1.56 V along the same length. D
58. A and E failing in the main branch would cause the entire circuit to fail. B and C would affect each other. A
59.  $V = IR$  A
60.  $\mathcal{E} = IR_{\text{total}}$  where  $R_{\text{total}} = 35 \Omega$  D
61. With the switch closed, the resistance of the  $15 \Omega$  and the  $30 \Omega$  in parallel is  $10 \Omega$ , making the total circuit resistance  $30 \Omega$  and  $\mathcal{E} = IR$  C
62. The equivalent resistance through path ACD is equal to the equivalent resistance through path ABD, making the current through the two branches equal C
63. The resistance in each of the two paths is  $9 \Omega$ , making the current in each branch 1 A. From point A, the potential drop across the  $7 \Omega$  resistor is then 7 V and across the  $4 \Omega$  resistor is 4 V, making point B 3 V lower than point C D
64. Closing the switch reduces the total resistance of the circuit, increasing the current in the main branch containing bulb 1 A
65. Resistors J and N are in the main branch and therefore receive the largest current. C
66.  $P = I^2R$  C
67. Breaking the circuit in the lower branch lowers the total current in the circuit, decreasing the voltage across  $R_1$ . Looking at the upper loop, this means  $R_2$  now has a larger share of the battery voltage and the voltage across AD is the same as the voltage across BC A
68. In series circuits, larger resistors develop more power B
69. With a total resistance of  $10 \Omega$ , the total current is 1.2 A. The terminal voltage  $V_T = \mathcal{E} - Ir$  C
70. Shorting bulb 3 decreases the resistance in the right branch, increasing the current through bulb 4 and decreasing the total circuit resistance. This increases the total current in the main branch containing bulb 1. B
71. The total charge to be distributed is  $+100 \mu\text{C} - 50 \mu\text{C} = +50 \mu\text{C}$ . In parallel, the capacitors must have the same voltage so the  $20 \mu\text{F}$  capacitor has four times the charge of the  $5 \mu\text{F}$  capacitor. This gives  $Q_{20} = 4Q_5$  and  $Q_{20} + Q_5 = 4Q_5 + Q_5 = 5Q_5 = 50 \mu\text{C}$ , or  $Q_5 = 10 \mu\text{C}$  D
72. For more light at a given voltage, more current is required, which requires less resistance.  $R = \rho L/A$  B
73. Wire CD shorts out bulb #3 so it will never light. Closing the switch merely adds bulb #2 in parallel to bulb #1, which does not change the potential difference across bulb #1. C
74. For points a and b to be at the same potential, the potential drop across the  $3 \Omega$  resistor must be equal to the potential drop across capacitor C. The potential drop across the  $3 \Omega$  resistor is three times the drop across the  $1 \Omega$  resistor. For the potential drop across capacitor C to be three times the drop across the  $1 \mu\text{F}$  capacitor, C must be  $1/3$  the capacitance, or  $1/3 \mu\text{F}$  A
75. Shorting bulb 4 decreases the resistance in the right branch, increasing the current through bulb 3 and in the main branch containing bulb 1. D

76. When the current is 0.5 A, the voltage across the resistor is  $V = IR = 5 \text{ V}$ . According to the loop rule, the remaining 7 V must be across the capacitor. C
77. Since there is constant current, bulb 1 remains unchanged and bulbs 2 and three must now split the current. With half the current through bulb 2, the potential difference between A and B is also halved. D
78. The voltmeter is essentially another resistor. The voltmeter in parallel with the  $100 \Omega$  resistor acts as a  $500 \Omega$  resistor, which will half  $\frac{1}{2}$  the voltage of the  $100 \Omega$  resistor on the left. Thus the 120 V will split into 80 V for the  $1000 \Omega$  resistor and 40 V for the voltmeter combination. A
79. The  $15 \Omega$  resistor would be in parallel with the  $30 \Omega$  resistor when the switch is closed. C
80.  $ACD = 9 \Omega$ ,  $ABD = 9 \Omega$  so the total resistance is  $4.5 \Omega$  making the total current  $\mathcal{E}/R = 2 \text{ A}$ . A
81. The 2 A will divide equally between the two branches with 1 A going through each branch. From B to D we have  $-(1 \text{ A})(2 \Omega) = -2 \text{ V}$ , with B at the higher potential A
82. When the capacitor is charged, the branch is effectively removed from the circuit, making it a simple parallel circuit. The total resistance is  $133.3 \Omega$  and  $V = IR$  B
83. In a simple series circuit with two batteries opposing one another the voltages subtract from one another. The total effective voltage for this circuit is then 4 V. With a total resistance of  $20 \Omega$  the total current is  $(4 \text{ V})/(20 \Omega)$  D
84. When the capacitor is charged, the branch is effectively removed from the circuit, making the circuit a  $10 \Omega$  resistor in series with two  $10 \Omega$  resistors in parallel. The lone  $10 \Omega$  resistor has twice the voltage of the two  $10 \Omega$  resistors in parallel with an effective resistance of  $5 \Omega$ . The 10 volts will then divide with 3.3 V going to the parallel combination and 6.7 V going to the single  $10 \Omega$  resistor. The capacitor is in parallel with the single  $10 \Omega$  resistor.  $Q = CV$  C
85. The resistances are, respectively,  $\frac{4}{3} R$ ,  $\frac{2}{5} R$ ,  $R$ , and  $\frac{5}{3} R$  A
86. Closing the switch adds another parallel branch, increasing the total current delivered by the battery. Bulb 3 will get brighter. Bulb 2, in its own loop with bulb 3 and the battery will then lose some of its share of the potential difference from the battery and will get dimmer. C
87. Voltmeters must be placed in parallel and ammeters must be placed in series. B
88.  $S_1$  must be closed to have any current. Closing  $S_2$  will allow current in  $R_2$  but closing  $R_3$  would short circuit  $R_2$ . C
89.  $S_1$  must be closed to have any current. Closing  $S_3$  will short circuit  $R_3$ , leaving only resistor  $R_1$ , which is the lowest possible resistance. D
90.  $S_1$  must be closed to have any current. The greatest voltage will occur with the greatest current through  $R_3$  but closing  $S_2$  or  $S_3$  will draw current away from  $R_3$ . A



AP Physics Free Response Practice – Circuits – ANSWERS

1976B3

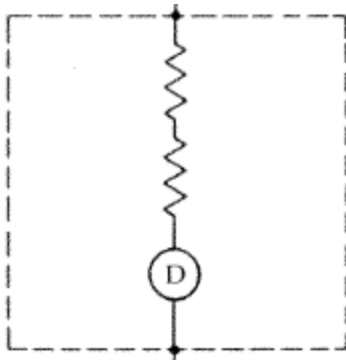
- $V_T = E - Ir = 6 \text{ V}$
- In parallel, each resistor gets 6 V and  $P = V^2/R$  gives  $R = 3 \Omega$
- For the  $3 \Omega$  resistor we have  $I = V/R = 2 \text{ A}$  leaving 1 A for the branch with  $R_1$ .  $R = V/I = 6 \Omega$

1981B4

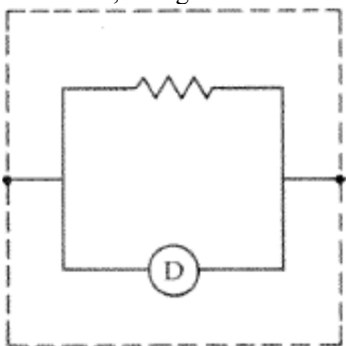
- The two batteries are connected with opposing emfs so the total emf in the circuit is  $\mathcal{E} = 60 \text{ V} - 12 \text{ V} = 48 \text{ V}$ . The resistance of the parallel combination of resistors is  $(\frac{1}{4} + \frac{1}{4} + \frac{1}{2})^{-1} = 1 \Omega$  combining with the rest of the resistors in series gives a total circuit resistance of  $8 \Omega$ . The total current is then  $\mathcal{E}/R = 6 \text{ A}$ . The voltage across the parallel combination of resistors is  $V_p = IR_p = 6 \text{ V}$  so the current through the  $2 \Omega$  resistor is  $I = V/R = 3 \text{ A}$
- $P = I^2 R = 108 \text{ W}$
- The current is forced through battery B from the positive to the negative terminal, charging the battery. This makes the equation for the terminal voltage  $V_T = \mathcal{E} + Ir = 18 \text{ V}$

1980B2

- The resistance of the device is found from  $R = V/I = 6 \Omega$ . With a 24 volt source, to provide a current of 2 A requires a total resistance of  $12 \Omega$ . For the additional  $6 \Omega$  resistance, place two  $3 \Omega$  resistors in series with the device.



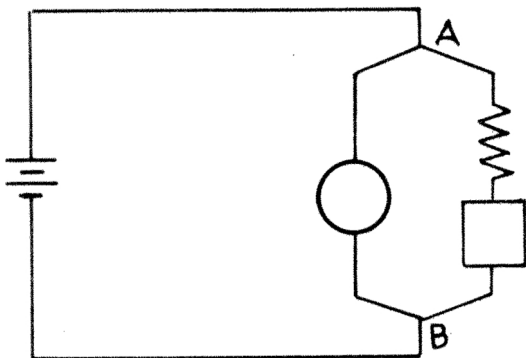
- Since the device requires 2 A, a resistor in parallel with the device must carry a current of  $6 \text{ A} - 2 \text{ A} = 4 \text{ A}$ . In parallel with the device, the resistor will have a potential difference of 12 V so must have a resistance of  $V/I = 3 \Omega$ . Thus, a single  $3 \Omega$  resistor in parallel will suffice.



- $P = I^2 R = 48 \text{ W}$

1982B4

- a. Since the clock requires 15 V it must be directly connected between A and B. Since the radio requires less than 15 V, there must be a resistor in series with it.



- b. The current through the radio (and R) is 10 mA. The voltage across the radio is 9 V, which leaves 6 V across the resistor giving  $R = V/I = 600 \, \Omega$
- c.  $P = IV$  where  $V = 15 \, \text{V}$  and  $I = 10 \, \text{mA} + 20 \, \text{mA} = 30 \, \text{mA}$  so  $P = 0.45 \, \text{W}$  and energy =  $Pt = 27 \, \text{J}$

1983B3

- a. The two batteries are connected with opposing emfs so the total emf in the circuit is  $\mathcal{E} = 20 \, \text{V} - 2 \, \text{V} = 18 \, \text{V}$ . The equivalent resistance of the two parallel resistors is  $(6 \times 12)/(6 + 12) = 4 \, \Omega$  and since R is in series with the pair, the total circuit resistance is  $(4 + R) \, \Omega = \mathcal{E}/I = 9 \, \Omega$  giving  $R = 5 \, \Omega$
- b. Because the voltages of the two resistors in parallel are equal we have  $6I_1 = 12I_2$  and  $I_1 + I_2 = 2 \, \text{A}$  giving
- $4/3 \, \text{A}$
  - $2/3 \, \text{A}$
- c. Summing the potential differences from point X gives  $V_X + IR = 0 + (2 \, \text{A})(5 \, \Omega) = V_B = 10 \, \text{V}$ . Continuing along gives  $V_B - 20 \, \text{V} = V_C = -10 \, \text{V}$ . And  $V_C + (2/3 \, \text{A})(12 \, \Omega) = V_D = -2 \, \text{V}$
- d.  $P = \mathcal{E}I = 40 \, \text{W}$

1986B3

- a.  $P = V^2/R$  gives  $R = 240 \, \Omega$
- b. Bulbs Y and Z in parallel have an equivalent resistance of  $120 \, \Omega$ . Adding bulb X in series with the pair gives  $R = 360 \, \Omega$
- c.  $P_T = \mathcal{E}^2/R_T = 40 \, \text{W}$
- d.  $I = \mathcal{E}/R = 1/3 \, \text{A}$
- e.  $V_X = IR_X = 80 \, \text{V}$
- f. The current splits equally through Y and Z.  $V_Z = I_Z R_Z = (1/6 \, \text{A})(240 \, \Omega) = 40 \, \text{V}$

1987B4

- a. The equivalent resistance of  $R_1$  and  $R_2$  is  $(12 \times 4)/(12 + 4) = 3 \, \Omega$ . Adding  $R_3$  in series with the pair gives  $R = 12 \, \Omega$
- b.  $\mathcal{E} = IR_T = 4.8 \, \text{V}$
- c. The voltage across resistor 1 (equal to the voltage across  $R_2$ ) is the emf of the battery minus the drop across  $R_3$  which is  $4.8 \, \text{V} - (0.4 \, \text{A})(9 \, \Omega) = 1.2 \, \text{V}$
- d.  $P = V^2/R = 0.36 \, \text{W}$
- e.  $Q = It = (0.4 \, \text{C/s})(60 \, \text{s}) = 24 \, \text{C}$

1988B3

- On the right we have two resistors in series:  $10\ \Omega + 2\ \Omega = 12\ \Omega$ . This is in parallel with the  $4\ \Omega$  resistor which is an equivalent resistance of  $3\ \Omega$  and adding the remaining main branch resistor in series gives a total circuit resistance of  $9\ \Omega$ . The current is then  $I = \mathcal{E}/R_T = 8\ \text{A}$
- The voltage remaining for the parallel branches on the right is the emf of the battery minus the potential dropped across the  $6\ \Omega$  resistor which is  $72\ \text{V} - (8\ \text{A})(6\ \Omega) = 24\ \text{V}$ . Thus the current in the  $10\ \Omega$  resistor is the current through the whole  $12\ \Omega$  branch which is  $I = V/R = (24\ \text{V})/(12\ \Omega) = 2\ \text{A}$
- $V_{10} = I_{10}R_{10} = 20\ \text{V}$
- When charged, the capacitor is in parallel with the  $10\ \Omega$  resistor so  $V_C = V_{10} = 20\ \text{V}$  and  $Q = CV = 60\ \mu\text{C}$
- $U_C = \frac{1}{2} CV^2 = 6 \times 10^{-4}\ \text{J}$

1989B3

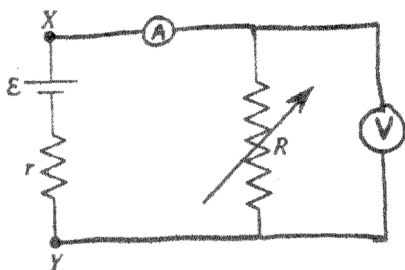
- $P = I^2 R = (2\ \text{A})^2 (10\ \Omega) = 40\ \text{W}$
  - $P = Fv = mgv = 20\ \text{W}$  (using  $g = 10\ \text{m/s}^2$ )
  - $P_B = P_R + P_M = 40\ \text{W} + 20\ \text{W} = 60\ \text{W}$
- $V = IR = 20\ \text{V}$
  - $V = P/I = (20\ \text{W})/(2\ \text{A}) = 10\ \text{V}$
  - $\mathcal{E} = V_R + V_M = 30\ \text{V}$
- Since the speed is increased by  $3/2$ , the voltage drop increases by the same value and is now  $(3/2)(10\ \text{V}) = 15\ \text{V}$
- The new voltage across the resistor is found from  $V_R = \mathcal{E} - V_M = 15\ \text{V}$  and  $I = V_R/R = (15\ \text{V})/(2\ \Omega) = 7.5\ \text{A}$

1990B3

- The  $4\ \Omega$  and  $8\ \Omega$  are in series so their equivalent resistance is  $12\ \Omega$ . Another  $12\ \Omega$  resistor in parallel makes the equivalent resistance  $(12 \times 12)/(12 + 12) = 6\ \Omega$
- Adding the remaining resistors in series throughout the circuit gives a total circuit resistance of  $12\ \Omega$  and the total current (which is also the current in the  $5\ \Omega$  resistor) is  $\mathcal{E}/R = 2\ \text{A}$
- $V_{AC} = \mathcal{E} - Ir = 22\ \text{V}$
- The current divides equally between the two branches on the right so  $P_{12} = I^2 R = (1\ \text{A})^2 (12\ \Omega) = 12\ \text{W}$
- From B to C you only have to pass through the  $12\ \Omega$  resistor which gives  $V = (1\ \text{A})(12\ \Omega) = 12\ \text{V}$
- $P_B = V_{AC}^2 / R_{\text{external}} = (22\ \text{V})^2 / 11\ \Omega = 44\ \text{W}$

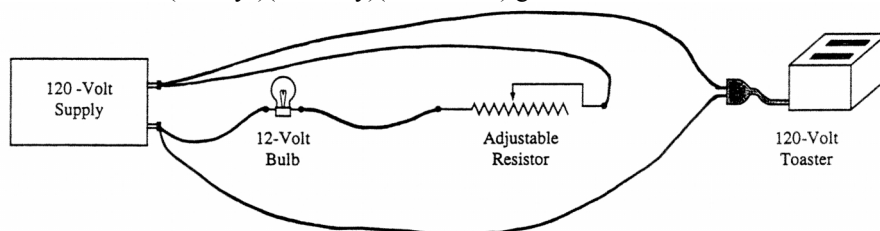
1991B4

- $V_{XY} = \mathcal{E} - Ir$  and using data from the graph we can find two equations to solve simultaneously  
 $4\ \text{V} = \mathcal{E} - (1\ \text{A})r$  and  $3\ \text{V} = \mathcal{E} - (3\ \text{A})r$  will yield the solutions  $\mathcal{E} = 4.5\ \text{V}$  and  $r = 0.5\ \Omega$
  - $V_{XY} = IR$  which gives  $3\ \text{V} = (3\ \text{A})R$  and  $R = 1\ \Omega$
  - $I_{\text{max}}$  occurs for  $R = 0$  and  $V_{XY} = 0$  which gives  $\mathcal{E} = I_{\text{max}}r$  and  $I_{\text{max}} = 9\ \text{A}$  (this is the x intercept of the graph)



1995B2

- $P = V^2/R$  gives  $R = 24 \Omega$
- $E = Pt$  where  $t = (30 \text{ days})(24 \text{ h/day})(3600 \text{ sec/h})$  gives  $E = 1.6 \times 10^7 \text{ J}$
- 

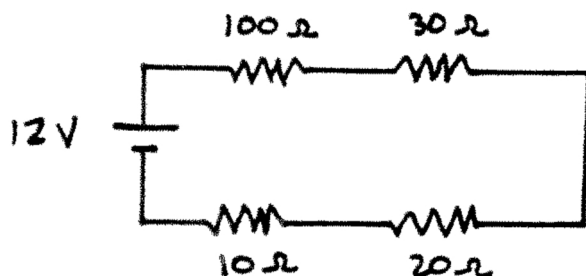


The bulb, needing only 12 V must have a resistor in series with it and the toaster, requiring 120 V must be connected directly to the power supply.

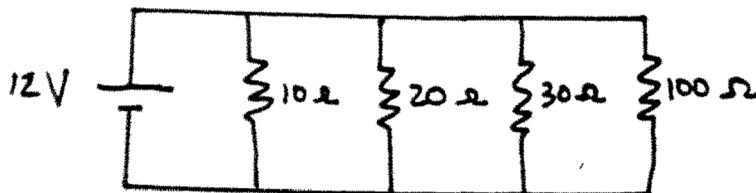
- The current through the bulb is  $I = P/V = 0.5 \text{ A}$ , which is also the current in the resistor, which must have 108 V across it to provide the light bulb only 12 V.  $R = V/I = (108 \text{ V})/(0.5 \text{ A}) = 216 \Omega$
- If the resistance of the resistor is increased, the current through the branch will decrease, decreasing the brightness of the bulb.
  - Since the toaster operates in its own parallel branch, nothing will change for the toaster.

1996B4

- For the smallest current, place the resistors in series



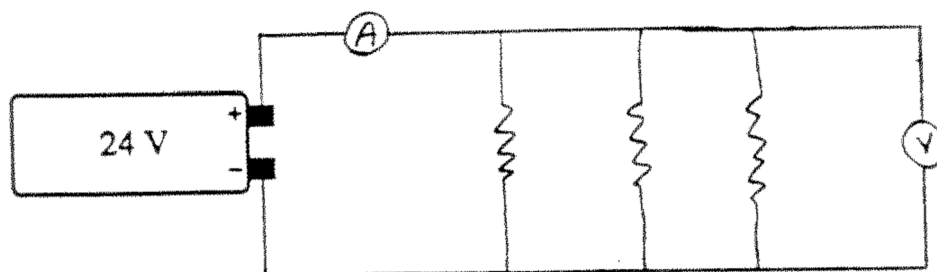
- For the largest current, place the resistors in parallel



- The  $20 \Omega$  and  $30 \Omega$  resistors combine in series as a  $50 \Omega$  resistor, which is in parallel with the  $100 \Omega$  resistor making their effective resistance  $33.3 \Omega$ . Adding the  $10 \Omega$  resistor in the main branch in series gives a total circuit resistance of  $43 \Omega$ . The current in the  $10 \Omega$  resistor is the total current delivered by the battery  $\mathcal{E}/R = 0.28 \text{ A}$
  - $P = \mathcal{E}^2/R = 3.35 \text{ W}$
- $E = Pt$ , or  $t = E/P = (10 \times 10^3 \text{ J})/(3.35 \text{ W}) = 3 \times 10^3 \text{ seconds}$

1997B4

- a.
  - i. In series  $R_T = 90 \, \Omega$  and  $P = V^2/R = 6.4 \, \text{W}$
  - ii. In parallel  $R_T = 10 \, \Omega$  and  $P = 57.6 \, \text{W}$
- b. The fastest heating occurs with a parallel connection

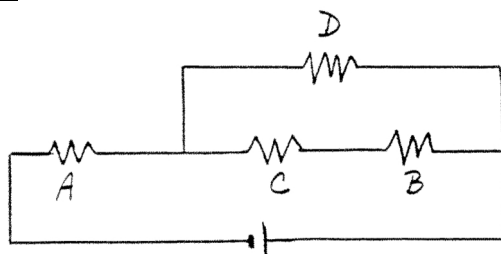


2002B3B

- a. The resistance of the  $6 \, \Omega$  and  $3 \, \Omega$  resistors in parallel is  $(6 \times 3)/(6 + 3) = 2 \, \Omega$ . Adding the  $3 \, \Omega$  resistor in the main branch gives a total circuit resistance of  $5 \, \Omega$ . The current in bulb A in the main branch is the total current delivered by the battery  $I = \mathcal{E}/R = (9 \, \text{V})/(5 \, \Omega) = 1.8 \, \text{A}$
- b. Bulb A is the brightest. In the main branch, it receives the most current. You can also calculate the power of each resistor where  $P_A = 9.7 \, \text{W}$ ,  $P_B = 2.2 \, \text{W}$  and  $P_C = 4.3 \, \text{W}$
- c.
  - i. Removing Bulb C from the circuit changes the circuit to a series circuit, increasing the total resistance and decreasing the total current. With the total current decreased, bulb A is dimmer.
  - ii. Since bulb A receives less current, the potential drop is less than the original value and being in a loop with bulb B causes the voltage of bulb B to increase, making bulb B brighter. The current through bulb B is greater since it is no longer sharing current with bulb C.
  - iii. The current through bulb C is zero, bulb C goes out.

1998B4

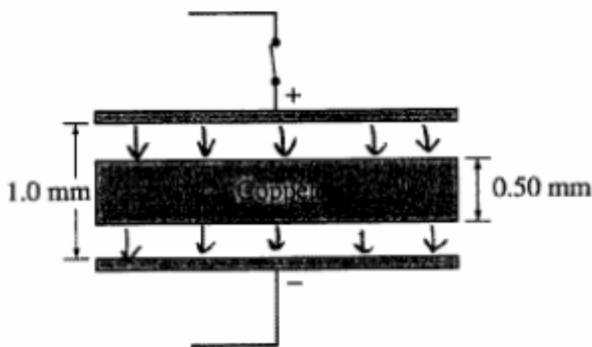
a.



- b.  $A > D > B = C$   
Bulb A has the largest current through it, making it brightest. The voltage across bulb D is the same as that across bulbs B and C combined, so it is next brightest, leaving B and C as least bright. Bulbs B and C are in series, and thus have the same current through them, so they must be equally bright.
- c.
  - i. The brightness of bulb A decreases. The total resistance of the circuit increases so the current in bulb A decreases.
  - ii. The brightness of bulb B increases. The current (and the voltage) across B increases. Even though the total current decreases, it is no longer splitting to go through the branch with bulb D. Another way to look at it is since A has less current, the potential difference across A is decreased, this allows a larger share of the battery voltage to be across B and C.

2000B3

- The equivalent resistance of the two resistors in parallel is  $R/2$ , which is  $1/2$  the resistance of the resistor in the main branch, so the parallel combination will receive half the potential difference of the main branch resistor. The 30 V of the battery will then divide into 20 V for the main branch resistor (and across the voltmeter) and 10 V each for the resistors in parallel.
- After the switch has been closed for a long time, the voltage across the capacitor will be 30 V.  
 $Q = CV = 3 \times 10^8 \text{ C}$
- The 30 V battery is still connected across the capacitor so the potential difference remains 30 V.
  - $E = 0$  inside a conductor in electrostatic equilibrium
  -



- $E = V/d$  and you can use the entire gap or just one of the two gaps;  $E = 30 \text{ V}/(0.5 \text{ mm})$  or  $15 \text{ V}/(0.25 \text{ mm})$   
 $E = 60 \text{ V/mm}$  or  $60,000 \text{ V/m}$

2002B3

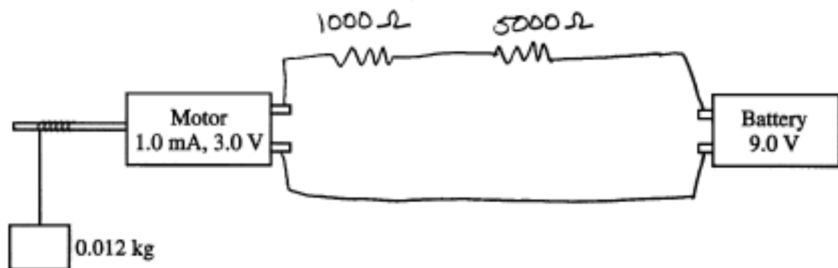
- $P = V^2/R$  gives  $R = 480 \Omega$  and  $V = IR$  gives  $I = 0.25 \text{ A}$
  - $P = V^2/R$  gives  $R = 360 \Omega$  and  $V = IR$  gives  $I = 0.33 \text{ A}$
- i./ii. The resistances are unchanged =  $480 \Omega$  and  $360 \Omega$ . The total resistance in series is  $480 \Omega + 360 \Omega = 840 \Omega$  making the total current  $I = V/R = 0.14 \text{ A}$  which is the same value for both resistors in series
- The bulbs are brightest in parallel, where they provide their labeled values of 40 W and 30 W. In series, it is the larger resistor (the 30 W bulb) that glows brighter with a larger potential difference across it in series. This gives the order from top to bottom as **2 1 3 4**
- In parallel, they each operate at their rated voltage so they each provide their rated power and  $P_T = 30 \text{ W} + 40 \text{ W} = 70 \text{ W}$
  - In series  $P_T = V_T^2/R_T = 17 \text{ W}$

2003B2

- For two capacitors in series the equivalent capacitance is  $(6 \times 12)/(6 + 12) = 4 \mu\text{F}$
- The capacitors are fully charged so current flows through the resistors but not the capacitors.  $R_T = 30 \Omega$  and  $I = V/R = 0.2 \text{ A}$
- The potential difference between A and B is the voltage across the  $20 \Omega$  resistor.  $V = IR = 4 \text{ V}$
- The capacitors in series store the same charge as a single  $4 \mu\text{F}$  capacitor.  $Q = CV = (4 \mu\text{F})(4 \text{ V}) = 16 \mu\text{C}$
- Remains the same. No current is flowing from A to P to B therefore breaking the circuit at point P does not affect the current in the outer loop, and therefore will not affect the potential difference between A and B.

2003B2B

- $P = IV = 3 \text{ mW} = 3 \times 10^{-3} \text{ W}$
- $E = Pt = 0.180 \text{ J}$
- $e = \frac{\text{"what you get"}}{\text{"what you are paying for"}} = \frac{(\text{power lifting the mass})}{(\text{power provided by the motor})}$   
 $P_{\text{lifting}} = Fv = mgv = mgd/t = 1.96 \text{ mW}$  so the efficiency is  $1.96/3 = 0.653$  or  $65.3 \%$
- To reduce the battery voltage of  $9 \text{ V}$  to the motor's required voltage of  $3 \text{ V}$ , we need  $6 \text{ V}$  across the resistors. The required resistance is then  $V/I = (6 \text{ V})/(1 \text{ mA}) = 6000 \Omega$ . This is done with a  $1000 \Omega$  and a  $5000 \Omega$  resistor in series.



2007B3

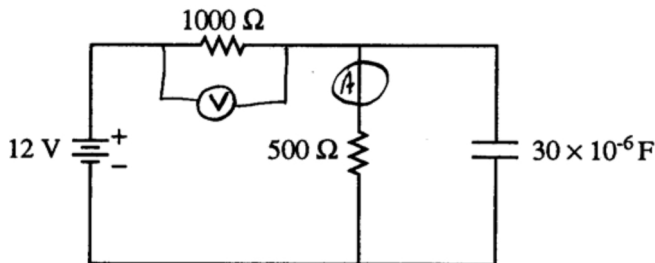
- $\frac{1}{2} I_A$      $\frac{3}{2} I_B$      $\frac{2}{3} I_C$
  - The total current flows through  $R_A$  and gets divided between the other two resistors with the smaller resistor  $R_C$  getting a larger current
- $\frac{1}{2} V_A$      $\frac{2}{3} V_B$      $\frac{2}{3} V_C$
  - No resistor is greater than  $R_A$  and  $R_A$  has the full current through it.  $R_B$  and  $R_C$  are in parallel and therefore have the same potential difference.
- For the two resistors in parallel, the equivalent resistance is  $(2R \times R)/(2R + R) = 2/3 R = 133 \Omega$ . Adding  $R_A$  in series with the pair gives  $R_T = 400 \Omega + 133 \Omega = 533 \Omega$
- $I_T = I_A = \mathcal{E}/R_T = 0.0225 \text{ A}$ . The potential drop across A is  $V = IR = 9 \text{ V}$  which leaves  $3 \text{ V}$  for the two branches in parallel.  $I_C = V_C/R_C = 0.015 \text{ A}$
- In the new circuit,  $I_B = 0$  at equilibrium and the circuit behaves as a simple series circuit with a total resistance of  $600 \Omega$  and a total current of  $\mathcal{E}/R = 0.02 \text{ A}$ . The voltage across the capacitor is the same as the voltage across resistor C and  $V_C = IR_C = 4 \text{ V}$  and  $Q = CV = 8 \times 10^{-6} \text{ C}$

1975E2

- $Q = C\mathcal{E} = 12 \mu\text{F} \times 100 \text{ V} = 1200 \mu\text{C}$
- Connecting the two capacitors puts them in parallel with the same voltage so  $V_1 = V_2$  and  $V = Q/C$  which gives  $Q_1/C_1 = Q_2/C_2$  or  $Q_1/12 = Q_2/24$  and  $Q_2 = 2Q_1$ . We also know the total charge is conserved so  $Q_1 + Q_2 = 1200 \mu\text{C}$  so we have  $Q_1 + 2Q_1 = 1200 \mu\text{C}$  so  $Q_1 = 400 \mu\text{C}$
- $V = Q/C = 33.3 \text{ V}$
- When the battery is reconnected, both capacitors charge to a potential difference of  $100 \text{ V}$  each. The total charge is then  $Q = Q_1 + Q_2 = (C_1 + C_2)V = 3600 \mu\text{C}$  making the *additional* charge from the battery  $2400 \mu\text{C}$ .

2007B3B

- a. In their steady states, no current flows through the capacitor so the total resistance is  $1500\ \Omega$  and the total current is  $\mathcal{E}/R_T = 8.0 \times 10^{-3}\text{ A}$
- b.



- c. The voltage across the capacitor is the same as the voltage across the  $500\ \Omega$  resistor  $= IR = 4\text{ V}$  so we have  $Q = CV = 1.2 \times 10^{-4}\text{ C}$
- d.  $P = I^2R = 6.4 \times 10^{-2}\text{ W}$
- e. Larger. Replacing the  $50\ \Omega$  resistor with a larger resistor lowers the steady state current, causing the voltage across the  $1000\ \Omega$  resistor to decrease and the voltage across the replacement resistor to increase.

1988E2

- a. In their steady states, no current flows through the capacitor so the effective resistance of the branch on the right is  $8\ \Omega + 4\ \Omega = 12\ \Omega$ . This is in parallel with the  $4\ \Omega$  resistor making their effective resistance  $(12 \times 4)/(12 + 4) = 3\ \Omega$ . Adding the  $9\ \Omega$  resistor in the main branch gives a total circuit resistance of  $12\ \Omega$  and a total current of  $\mathcal{E}/R = 10\text{ A}$ . This is the current in the  $9\ \Omega$  resistor as it is in the main branch.
- b. With  $10\text{ A}$  across the  $9\ \Omega$  resistor, the potential drop across it is  $90\text{ V}$ , leaving  $30\text{ V}$  across the two parallel branches on the right. With  $30\text{ V}$  across the  $12\ \Omega$  effective resistance in the right branch, we have a current through that branch (including the  $8\ \Omega$  resistor) of  $V/R = 2.5\text{ A}$
- c.  $V_C = V_4 = IR = (2.5\text{ A})(4\ \Omega) = 10\text{ V}$
- d.  $U_C = \frac{1}{2} CV^2 = 1500\ \mu\text{J}$

1985E2

- a. Immediately after the switch is closed, the capacitor begins charging with current flowing to the capacitor as if it was just a wire. This short circuits  $R_2$  making the total effective resistance of the circuit  $5 \times 10^6\ \Omega$  and the total current  $\mathcal{E}/R_{\text{eff}} = 0.006\text{ A}$
- b. When the capacitor is fully charged, no current flows through that branch and the circuit behaves as a simple series circuit with a total resistance of  $15 \times 10^6\ \Omega$  and a total current of  $\mathcal{E}/R = 0.002\text{ A}$
- c. The voltage across the capacitor is equal to the voltage across the  $10\text{ M}\Omega$  resistor as they are in parallel.  $V_C = V_{10\text{M}} = IR = 2000\text{ V}$  and  $Q = CV = 0.01\text{ C}$
- d.  $U_C = \frac{1}{2} CV^2 = 10\text{ J}$

1986E2

- a. The resistance of the two parallel branches are equal at  $40\ \Omega$  each making the equivalent resistance of the two branches  $20\ \Omega$ . Adding the  $5\ \Omega$  resistance in the main branch gives a total circuit resistance of  $25\ \Omega$  and a total current of  $\mathcal{E}/R = 1\text{ A}$  which will split evenly between the two equal branches giving  $I_R = 0.5\text{ A}$
- b. After the capacitor is charged, no current flows from A to B, making the circuit operate as it did initially when the capacitor was not present. Therefore the current through R is the same as calculated above at  $0.5\text{ A}$
- c. Consider the voltage at the junction above resistor R. The potential drop from this point to point A is  $V = IR = (0.5\text{ A})(10\ \Omega) = 5\text{ V}$  and to point B is  $(0.5\text{ A})(30\ \Omega) = 15\text{ V}$  making the potential difference across the plates of the capacitor  $15\text{ V} - 5\text{ V} = 10\text{ V}$ .  $Q = CV = (10\ \mu\text{F})(10\text{ V}) = 100\ \mu\text{C}$



1989E3

- a. When charged, the potential difference across the capacitor is 20 V.  $U_C = \frac{1}{2} CV^2 = 1200 \mu\text{J}$
- b. Given that the charge is initially unchanged, the work done is the change in the energy stored in the capacitor. Increasing the distance between plates to 4 times the initial value causes the capacitance to decrease to  $\frac{1}{4}$  its initial value ( $C \propto 1/d$ ). Since  $Q_i = Q_f$  we have  $C_i V_i = C_f V_f$  so  $V_f = 4V_i$   
 $W = \Delta U_C = \frac{1}{2} C_f V_f^2 - \frac{1}{2} C_i V_i^2 = \frac{1}{2} (\frac{1}{4} C(4V)^2) - \frac{1}{2} CV^2 = 3600 \mu\text{J}$
- c. After the spacing is increased, the capacitor acts as a battery with a voltage of  $4V = 80 \text{ V}$  with its emf opposite that of the 20 V battery making the effective voltage supplied to the circuit  $80 \text{ V} - 20 \text{ V} = 60 \text{ V}$ .  
 $I = \mathcal{E}_{\text{eff}}/R = 2 \times 10^{-4} \text{ A}$
- d. The charge on the capacitor initially was  $Q = CV = 120 \mu\text{C}$  and after the plates have been separated and a new equilibrium is reached  $Q = (\frac{1}{4}C)V = 30 \mu\text{C}$  so the charge that flowed back through the battery is  $120 \mu\text{C} - 30 \mu\text{C} = 90 \mu\text{C}$
- e. For the battery  $U = Q_{\text{added}}V = 1800 \mu\text{J}$

1992E2

- a. i.  $Q = CV = 4 \times 10^{-3} \text{ C}$   
 ii.  $U_C = \frac{1}{2} CV^2 = 4 \text{ J}$
- b. When the switch is closed, there is no charge on the  $6 \mu\text{F}$  capacitor so the potential difference across the resistor equals that across the  $2 \mu\text{F}$  capacitor, or 2000 V and  $I = V/R = 2 \times 10^{-3} \text{ A}$
- c. In equilibrium, charge is no longer moving so there is no potential difference across the resistor therefore the capacitors have the same potential difference.  $V_2 = V_6$  gives  $Q_2/C_2 = Q_6/C_6$  giving  $Q_6 = 3Q_2$  and since total charge is conserved we have  $Q_2 + Q_6 = Q_2 + 3Q_2 = 4Q_2 = 4 \times 10^{-3} \text{ C}$  so  $Q_2 = 1 \times 10^{-3} \text{ C}$  and  $Q_6 = 3 \times 10^{-3} \text{ C}$
- d.  $U_C = U_2 + U_6 = Q_2^2/2C_2 + Q_6^2/2C_6 = 1 \text{ J}$ . This is less than in part a. ii. Part of the energy was converted to heat in the resistor.

1995E2

- a.  $C = \kappa \epsilon_0 A/d$  so  $\kappa = Cd/\epsilon_0 A = 5.65$
- b. i. When the switch is closed, the voltage across the capacitor is zero thus all the voltage appears across the resistor and  $I = \mathcal{E}/R = 1.5 \times 10^{-5} \text{ A}$
- c. When fully charged, the current has stopped flowing and all the voltage now appears across the capacitor and  $Q = CV = 1.5 \times 10^{-6} \text{ C}$  and since the bottom plate is connected to the negative terminal of the battery the charge on that plate is also negative.
- d.  $U_C = \frac{1}{2} CV^2 = 2.25 \times 10^{-5} \text{ J}$
- e. Since the capacitor is isolated, the charge on it remains the same. Removing the plastic reduces the capacitance to  $C' = \epsilon_0 A/d = C_{\text{original}}/\kappa$  and  $V = Q/C' = 170 \text{ V}$
- f.  $U' = Q^2/2C' = Q^2/2(C/\kappa) = \kappa(Q^2/2C) = \kappa U > U_{\text{original}}$ . The increase came from the work that had to be done to remove the plastic from the capacitor.

1996E2

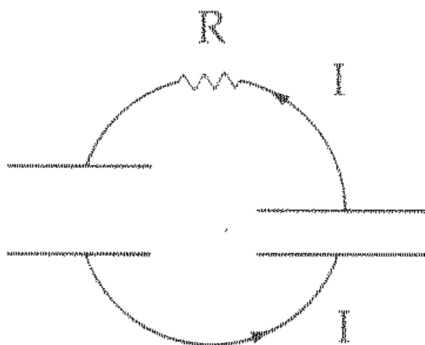
- a. The initial charge on  $C_1$  is  $Q = CV_0 = 200 \mu\text{C}$ . In equilibrium, charge is no longer moving so there is no potential difference across the resistor therefore the capacitors have the same potential difference.  $V_1 = V_2$  gives  $Q_1/C_1 = Q_2/C_2$  giving  $Q_2 = 3Q_1$  and since total charge is conserved we have  $Q_1 + Q_2 = Q_1 + 3Q_1 = 4Q_1 = 200 \mu\text{C}$  so  $Q_1 = 50 \mu\text{C}$  and  $Q_2 = 150 \mu\text{C}$
- b.  $\Delta U = U_f - U_i = (Q_1^2/2C_1 + Q_2^2/2C_2) - \frac{1}{2} C_1 V_0^2 = -3750 \mu\text{J}$

2008E2

- With a  $50\ \Omega$  resistor, the right branch has a total resistance of  $150\ \Omega$ , making the parallel combination with the  $300\ \Omega$  resistor equal to  $(150 \times 300)/(150 + 300) = 100\ \Omega$ . Adding  $R_1$  from the main branch in series with the branches gives a total circuit resistance of  $300\ \Omega$  and a total current of  $\mathcal{E}/R = 5\ \text{A}$ . The potential difference across  $R_1$  is then  $V = IR = 1000\text{V}$ , leaving  $500\ \text{V}$  across the two parallel branches and across  $R_2$ .
- When the switch is closed with a capacitor between points A and B, the voltage across the capacitor is zero and the current flows through the branch as if the capacitor was a wire. This gives the effective resistance of the parallel resistors as  $(100 \times 300)/(100 + 300) = 75\ \Omega$  and the total resistance =  $275\ \Omega$ , the total current =  $\mathcal{E}/R = 5.45\ \text{A}$ , the voltage across  $R_1 = IR = 1090\ \text{V}$  and  $V_2 = 1500\ \text{V} - 1090\ \text{V} = 410\ \text{V}$

1978B3

- $V = Ed = Es$
- Since the field points from the power plate to the upper plate, the lower plate is positive and the upper plate is negative.
- 

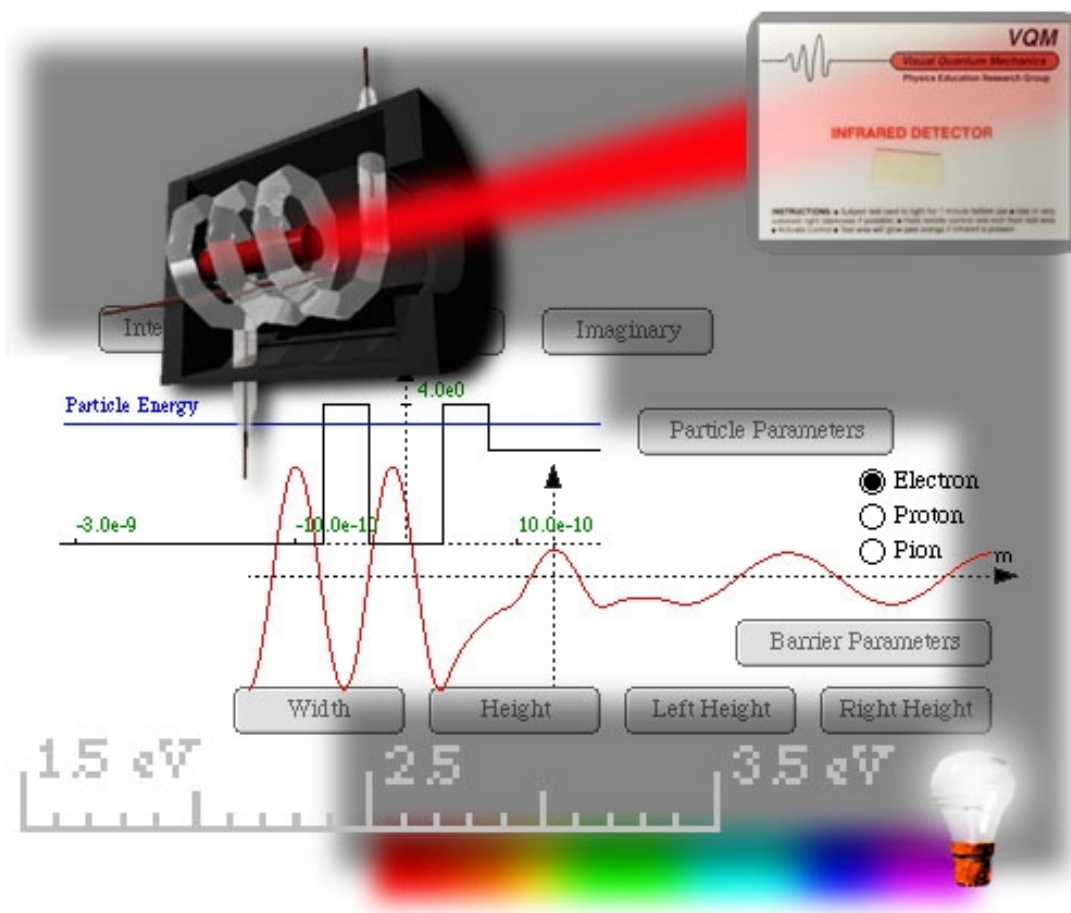


When the potential difference is the same on the two capacitors, charge will stop flowing as charge will flow only when there is a difference in potential.

- The capacitor on the left has the smaller capacitance and since the two capacitors are in parallel, they have the same voltage.  $Q = CV$  so the larger capacitor (on the right) contains more charge.
- The energy lost has been converted to heat through the resistor.

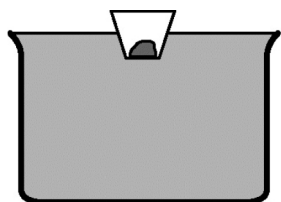
## APPENDIX 2

### Experimental Questions



## AP LAB-BASED QUESTIONS

### FLUIDS



#### BUOYANT FORCE/DENSITY (2010B2)

A large pan is filled to the top with oil of density  $\rho_o$ . A plastic cup of mass  $m_C$ , containing a sample of known mass  $m_S$ , is placed in the oil so that the cup and sample float, as shown above. The oil that overflows from the pan is collected, and its volume is measured. The procedure is repeated with a variety of samples of different mass, and the pan is refilled each time.

- Draw and label the forces (not components) that act on the cup-sample system when it is floating on the surface of the oil.
- Derive an expression for the overflow volume  $V_O$  (the volume of oil that overflows due to the floating system) in terms of  $\rho_o$ ,  $m_S$ ,  $m_C$ , and fundamental constants. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a). Assume that the following data are obtained for the overflow volume  $V_O$  for several sample masses  $m_S$

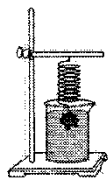
Sample mass $m_S$ (kg)	0.020	0.030	0.040	0.050	0.060	0.070
Overflow volume $V_O$ ( $\text{m}^3$ )	$29 \times 10^{-6}$	$38 \times 10^{-6}$	$54 \times 10^{-6}$	$62 \times 10^{-6}$	$76 \times 10^{-6}$	$84 \times 10^{-6}$

- Graph the data plotting the overflow volume as a function of sample mass. Place numbers and units on both axes. Draw a straight line that best represents the data.
- Use the slope of the best-fit line to calculate the density of the oil.
- What is the physical significance of the intercept of your line with the vertical axis?

**BUOYANT FORCE/SPRING/DENSITY (2002B6)** In the laboratory, you are given a cylindrical beaker containing a fluid and you are asked to determine the density  $\rho$  of the fluid. You are to use a spring of negligible mass and unknown spring constant  $k$  attached to a stand. An irregularly shaped object of known mass  $m$  and density ( $D \gg \rho$ ) hangs from the spring. You may also choose from among the following items to complete the task.

\_\_\_\_\_ A metric ruler                      \_\_\_\_\_ A stopwatch                      \_\_\_\_\_ String

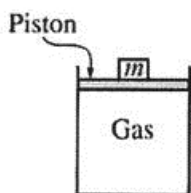
- Explain how you could experimentally determine the spring constant  $k$ .



- The spring-object system is now arranged so that the object (but none of the spring) is immersed in the unknown fluid, as shown. Describe any changes that are observed in the spring-object system and explain why they occur.

- Explain how you could experimentally determine the density of the fluid.
- Show explicitly, using equations, how you will use your measurements to calculate the fluid density  $\rho$ . Start by identifying any symbols you use in your equations.

## THERMODYNAMICS



Note: Figure not drawn to scale.

**THERMODYNAMICS (2005B6b)** You are given a cylinder of cross-sectional area  $A$  containing  $n$  moles of an ideal gas. A piston fitting closely in the cylinder is lightweight and frictionless, and objects of different mass  $m$  can be placed on top of it, as shown in the figure. In order to determine  $n$ , you perform an experiment that consists of adding 1 kg masses one at a time on top of the piston, compressing the gas, and allowing the gas to return to room temperature  $T$  before measuring the new volume  $V$ .

The data collected are given in the table below.

$m$ (kg)	$V$ ( $\text{m}^3$ )	$1/V$ ( $\text{m}^{-3}$ )	$P$ (Pa)
0	$6.0 \times 10^{-5}$	$1.7 \times 10^4$	
1	$4.5 \times 10^{-5}$	$2.2 \times 10^4$	
2	$3.6 \times 10^{-5}$	$2.8 \times 10^4$	
3	$3.0 \times 10^{-5}$	$3.3 \times 10^4$	
4	$2.6 \times 10^{-5}$	$3.8 \times 10^4$	

- a. Write a relationship between total pressure  $P$  and volume  $V$  in terms of the given quantities and fundamental constants that will allow you to determine  $n$ .

You also determine that  $A = 3.0 \times 10^{-4} \text{ m}^2$  and  $T = 300 \text{ K}$ .

- b. Calculate the value of  $P$  for each value of  $m$  and record your values in the data table above.  
 c. Plot the data on the graph below, labeling the axes with appropriate numbers to indicate the scale.  
 d. Using your graph in part (c), calculate the experimental value of  $n$ .

## ELECTROSTATICS

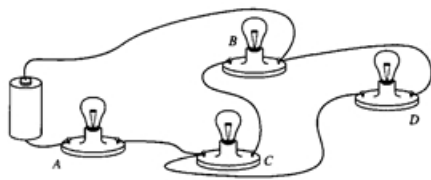
**ELECTRIC FIELD (2011 B2b).** You are to determine the magnitude and direction of the electric field at a point between two large parallel conducting plates. The two plates have equal but opposite charges, but it is not known which is positive and which is negative. The plates are mounted vertically on insulating stands.

- a. A small ball of known mass  $m$ , with a small charge  $+q$  of known magnitude, is provided. The ball is attached to an insulating string. The additional laboratory equipment available includes only those items listed below, plus stands and clamps as needed. Choose the equipment you would use to make measurements needed to determine the magnitude and direction of the electric field between the two plates.

<input type="checkbox"/> Wooden meterstick	<input type="checkbox"/> Protractor	<input type="checkbox"/> Screen
<input type="checkbox"/> Spring scale	<input type="checkbox"/> Stopwatch	<input type="checkbox"/> Bright light
<input type="checkbox"/> Metal rod	<input type="checkbox"/> Camera (still or video)	<input type="checkbox"/> Binoculars

- b. Sketch a diagram of the experimental setup and label the pieces of equipment used.  
 c. Outline the experimental procedure you would use, including a list of quantities you would measure. For each quantity, identify the equipment you would use to make the measurement.

## ELECTRICITY



**DC CIRCUITS (1998B4)** In the circuit shown, A, B, C, and D are identical light bulbs. Assume that the battery maintains a constant potential difference between its terminals (i.e., the internal resistance of the battery is assumed to be negligible) and the resistance of each light bulb remains constant.

- a. Draw a diagram of the circuit in the box below. **Use and label** the resistors symbols as A, B, C, and D to refer to the corresponding light bulbs.
- b. List the bulbs in order of brightness, from brightest to least bright. If any two or more bulbs have the same brightness, state which ones. **Justify your answer.**
- c. Bulb D is then removed from its socket.
  - i. Describe the change in the brightness, if any, of bulb A when bulb D is removed from its socket.

**Justify your answer.**

- ii. Describe the change in the brightness, if any, of bulb B when bulb D is removed from its socket.

**Justify your answer.**

**CIRCUITS (2003Bb2)** A student is asked to design a circuit to supply an electric motor with 1.0 mA of current at 3.0 V potential difference.

- a. Determine the power to be supplied to the motor.
- b. Determine the electrical energy to be supplied to the motor in 60 s.
- c. Operating as designed above, the motor can lift a 0.012 kg mass a distance of 1.0 m in 60 s at constant velocity. Determine the efficiency of the motor.

To operate the motor, the student has available only a 9.0 V battery to use as the power source and the following five resistors.



- d. In the space below, complete a schematic diagram of a circuit that shows how one or more of these resistors can be connected to the battery and motor so that 1.0 mA of current and 3.0 V of potential difference are supplied to the motor. Be sure to label each resistor in the circuit with the correct value of its resistance.

