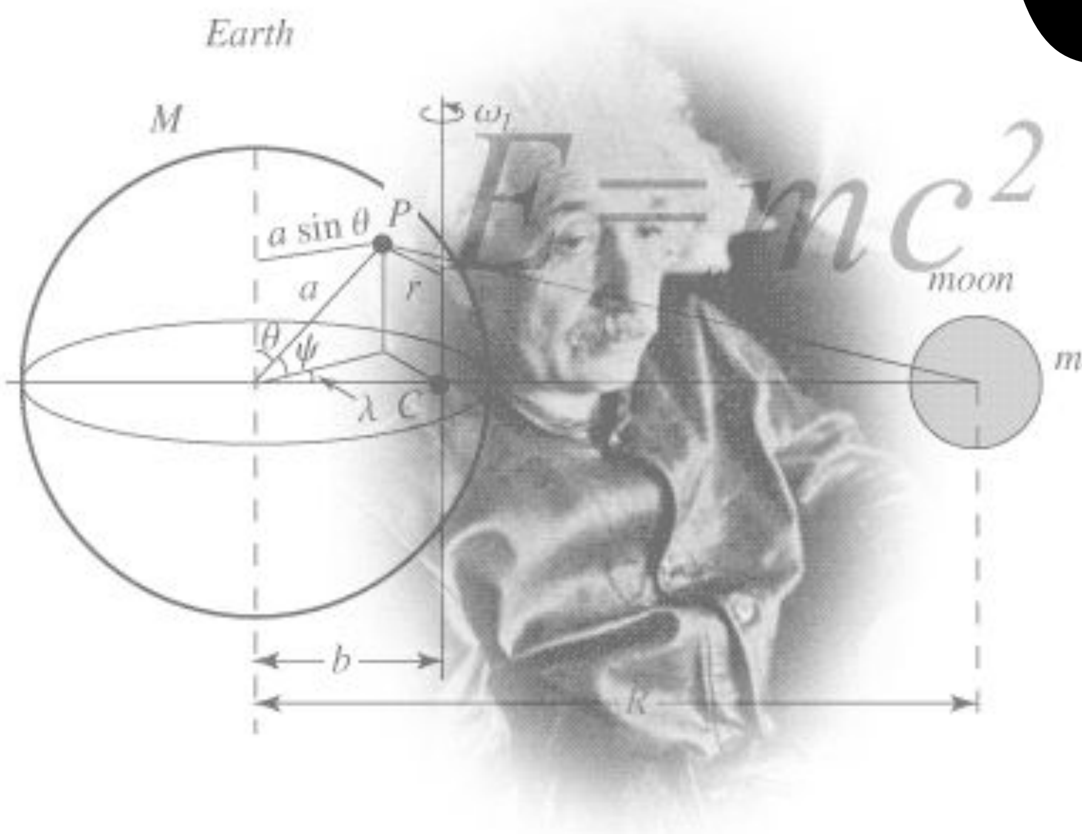
# AP Physics C – Practice Workbook – Book 2

## Electricity and Magnetism

C






2012-2013

# Table of Contents

This book is a compilation of all the problems published by College Board in AP Physics C organized by topic.

The problems vary in level of difficulty and type and this book represents an invaluable resource for practice and review and should be used… often. Whether you are struggling or confident in a topic, you should be doing these problems as a reinforcement of ideas and concepts on a scale that could never be covered in the class time allotted.

The answers as presented are not the only method to solving many of these problems and physics teachers may present slightly different methods and/or different symbols and variables in each topic, but the underlying physics concepts are the same and we ask you read the solutions with an open mind and use these differences to expand your problem solving skills.

Finally, we *are* fallible and if you find any typographical errors, formatting errors or anything that strikes you as unclear or unreadable, please let us know so we can make the necessary announcements and corrections.

CollegeBoard AP

# Table of Information and Equation Tables for AP Physics Exams

The accompanying Table of Information and Equation Tables will be provided to students when they take the AP Physics Exams. Therefore, students may NOT bring their own copies of these tables to the exam room, although they may use them throughout the year in their classes in order to become familiar with their content. **Check the Physics course home pages on AP Central for the latest versions of these tables (apcentral.collegeboard.com).**

## Table of Information

For both the Physics B and Physics C Exams, the Table of Information is printed near the front cover of the multiple-choice section and on the green insert provided with the free-response section. The tables are identical for both exams except for one convention as noted.

## Equation Tables

For both the Physics B and Physics C Exams, the equation tables for each exam are printed only on the green insert provided with the free-response section. The equation tables may be used by students when taking the free-response sections of both exams but NOT when taking the multiple-choice sections.

The equations in the tables express the relationships that are encountered most frequently in AP Physics courses and exams. However, the tables do not include all equations that might possibly be used. For example, they do not include many equations that can be derived by combining other equations in the tables. Nor do they include equations that are simply special cases of any that are in the tables. Students are responsible for understanding the physical principles that underlie each equation and for knowing the conditions for which each equation is applicable.

The equation tables are grouped in sections according to the major content category in which they appear. Within each section, the symbols used for the variables in that section are defined. However, in some cases the same symbol is used to represent different quantities in different tables. It should be noted that there is no uniform convention among textbooks for the symbols used in writing equations. The equation tables follow many common conventions, but in some cases consistency was sacrificed for the sake of clarity.

Some explanations about notation used in the equation tables:

1. The symbols used for physical constants are the same as those in the Table of Information and are defined in the Table of Information rather than in the right-hand columns of the tables.
2. Symbols in bold face represent vector quantities.
3. Subscripts on symbols in the equations are used to represent special cases of the variables defined in the right-hand columns.
4. The symbol $\Delta$ before a variable in an equation specifically indicates a change in the variable (i.e., final value minus initial value).
5. Several different symbols (e.g., $d, r, s, h, \ell$) are used for linear dimensions such as length. The particular symbol used in an equation is one that is commonly used for that equation in textbooks.

**TABLE OF INFORMATION DEVELOPED FOR 2012 (see note on cover page)**

CONSTANTS AND CONVERSION FACTORS

| | | | |
|---|---|---|---|
| Proton mass, | $m_p = 1.67 \times 10^{-27}$ kg | Electron charge magnitude, | $e = 1.60 \times 10^{-19}$ C |
| Neutron mass, | $m_n = 1.67 \times 10^{-27}$ kg | 1 electron volt, | $1\text{ eV} = 1.60 \times 10^{-19}$ J |
| Electron mass, | $m_e = 9.11 \times 10^{-31}$ kg | Speed of light, | $c = 3.00 \times 10^{8}$ m/s |
| Avogadro's number, | $N_0 = 6.02 \times 10^{23}$ mol$^{-1}$ | Universal gravitational constant, | $G = 6.67 \times 10^{-11}$ m$^3$/kg•s$^2$ |
| Universal gas constant, | $R = 8.31$ J/(mol•K) | Acceleration due to gravity at Earth's surface, | $g = 9.8$ m/s$^2$ |
| Boltzmann's constant, | $k_B = 1.38 \times 10^{-23}$ J/K | | |

| | |
|---|---|
| 1 unified atomic mass unit, | $1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\text{ MeV}/c^2$ |
| Planck's constant, | $h = 6.63 \times 10^{-34}\text{ J}\cdot\text{s} = 4.14 \times 10^{-15}\text{ eV}\cdot\text{s}$ |
| | $hc = 1.99 \times 10^{-25}\text{ J}\cdot\text{m} = 1.24 \times 10^{3}\text{ eV}\cdot\text{nm}$ |
| Vacuum permittivity, | $\epsilon_0 = 8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$ |
| Coulomb's law constant, | $k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2$ |
| Vacuum permeability, | $\mu_0 = 4\pi \times 10^{-7}\text{ (T}\cdot\text{m)/A}$ |
| Magnetic constant, | $k' = \mu_0/4\pi = 1 \times 10^{-7}\text{ (T}\cdot\text{m)/A}$ |
| 1 atmosphere pressure, | $1\text{ atm} = 1.0 \times 10^{5}\text{ N/m}^2 = 1.0 \times 10^{5}\text{ Pa}$ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| UNIT SYMBOLS | meter, | m | mole, | mol | watt, | W | farad, | F |
| | kilogram, | kg | hertz, | Hz | coulomb, | C | tesla, | T |
| | second, | s | newton, | N | volt, | V | degree Celsius, | °C |
| | ampere, | A | pascal, | Pa | ohm, | Ω | electron-volt, | eV |
| | kelvin, | K | joule, | J | henry, | H | | |

PREFIXES

| Factor | Prefix | Symbol |
|---|---|---|
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | μ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

| $\theta$ | $0°$ | $30°$ | $37°$ | $45°$ | $53°$ | $60°$ | $90°$ |
|---|---|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $1/2$ | $3/5$ | $\sqrt{2}/2$ | $4/5$ | $\sqrt{3}/2$ | 1 |
| $\cos\theta$ | 1 | $\sqrt{3}/2$ | $4/5$ | $\sqrt{2}/2$ | $3/5$ | $1/2$ | 0 |
| $\tan\theta$ | 0 | $\sqrt{3}/3$ | $3/4$ | 1 | $4/3$ | $\sqrt{3}$ | $\infty$ |

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

*IV. For mechanics and thermodynamics equations, *W* represents the work done on a system.

*Not on the Table of Information for Physics C, since Thermodynamics is not a Physics C topic.

# ADVANCED PLACEMENT PHYSICS B EQUATIONS DEVELOPED FOR 2012

## NEWTONIAN MECHANICS

$$\upsilon = \upsilon_0 + at$$

$$x = x_0 + \upsilon_0 t + \frac{1}{2}at^2$$

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0)$$

$$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$F_{fric} \le \mu N$$

$$a_c = \frac{\upsilon^2}{r}$$

$$\tau = rF \sin\theta$$

$$\mathbf{p} = m\mathbf{v}$$

$$\mathbf{J} = \mathbf{F}\Delta t = \Delta\mathbf{p}$$

$$K = \frac{1}{2}m\upsilon^2$$

$$\Delta U_g = mgh$$

$$W = F\Delta r \cos\theta$$

$$P_{avg} = \frac{W}{\Delta t}$$

$$P = F\upsilon\cos\theta$$

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2}kx^2$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T = \frac{1}{f}$$

$$F_G = -\frac{Gm_1m_2}{r^2}$$

$$U_G = -\frac{Gm_1m_2}{r}$$

$a$ = acceleration
$F$ = force
$f$ = frequency
$h$ = height
$J$ = impulse
$K$ = kinetic energy
$k$ = spring constant
$\ell$ = length
$m$ = mass
$N$ = normal force
$P$ = power
$p$ = momentum
$r$ = radius or distance
$T$ = period
$t$ = time
$U$ = potential energy
$\upsilon$ = velocity or speed
$W$ = work done on a system
$x$ = position
$\mu$ = coefficient of friction
$\theta$ = angle
$\tau$ = torque

## ELECTRICITY AND MAGNETISM

$$F = \frac{kq_1q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$U_E = qV = \frac{kq_1q_2}{r}$$

$$E_{avg} = -\frac{V}{d}$$

$$V = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\right)$$

$$C = \frac{Q}{V}$$

$$C = \frac{\epsilon_0 A}{d}$$

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

$$R = \frac{\rho\ell}{A}$$

$$V = IR$$

$$P = IV$$

$$C_p = C_1 + C_2 + C_3 + \ldots$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

$$R_s = R_1 + R_2 + R_3 + \ldots$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

$$F_B = q\upsilon B\sin\theta$$

$$F_B = BI\ell\sin\theta$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$\phi_m = BA\cos\theta$$

$$\mathcal{E}_{avg} = -\frac{\Delta\phi_m}{\Delta t}$$

$$\mathcal{E} = B\ell\upsilon$$

$A$ = area
$B$ = magnetic field
$C$ = capacitance
$d$ = distance
$E$ = electric field
$\mathcal{E}$ = emf
$F$ = force
$I$ = current
$\ell$ = length
$P$ = power
$Q$ = charge
$q$ = point charge
$R$ = resistance
$r$ = distance
$t$ = time
$U$ = potential (stored) energy
$V$ = electric potential or potential difference
$\upsilon$ = velocity or speed
$\rho$ = resistivity
$\theta$ = angle
$\phi_m$ = magnetic flux

## ADVANCED PLACEMENT PHYSICS B EQUATIONS DEVELOPED FOR 2012

### FLUID MECHANICS AND THERMAL PHYSICS

$\rho = m/V$

$P = P_0 + \rho g h$

$F_{buoy} = \rho V g$

$A_1 \upsilon_1 = A_2 \upsilon_2$

$P + \rho g y + \frac{1}{2}\rho \upsilon^2 = \text{const.}$

$\Delta \ell = \alpha \ell_0 \Delta T$

$H = \dfrac{kA\,\Delta T}{L}$

$P = \dfrac{F}{A}$

$PV = nRT = Nk_B T$

$K_{avg} = \frac{3}{2} k_B T$

$\upsilon_{rms} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3k_B T}{\mu}}$

$W = -P\Delta V$

$\Delta U = Q + W$

$e = \left|\dfrac{W}{Q_H}\right|$

$e_c = \dfrac{T_H - T_C}{T_H}$

- $A$ = area
- $e$ = efficiency
- $F$ = force
- $h$ = depth
- $H$ = rate of heat transfer
- $k$ = thermal conductivity
- $K_{avg}$ = average molecular kinetic energy
- $\ell$ = length
- $L$ = thickness
- $m$ = mass
- $M$ = molar mass
- $n$ = number of moles
- $N$ = number of molecules
- $P$ = pressure
- $Q$ = heat transferred to a system
- $T$ = temperature
- $U$ = internal energy
- $V$ = volume
- $\upsilon$ = velocity or speed
- $\upsilon_{rms}$ = root-mean-square velocity
- $W$ = work done on a system
- $y$ = height
- $\alpha$ = coefficient of linear expansion
- $\mu$ = mass of molecule
- $\rho$ = density

### ATOMIC AND NUCLEAR PHYSICS

$E = hf = pc$

$K_{\max} = hf - \phi$

$\lambda = \dfrac{h}{p}$

$\Delta E = (\Delta m)c^2$

- $E$ = energy
- $f$ = frequency
- $K$ = kinetic energy
- $m$ = mass
- $p$ = momentum
- $\lambda$ = wavelength
- $\phi$ = work function

### WAVES AND OPTICS

$\upsilon = f\lambda$

$n = \dfrac{c}{\upsilon}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sin\theta_c = \dfrac{n_2}{n_1}$

$\dfrac{1}{s_i} + \dfrac{1}{s_0} = \dfrac{1}{f}$

$M = \dfrac{h_i}{h_0} = -\dfrac{s_i}{s_0}$

$f = \dfrac{R}{2}$

$d\sin\theta = m\lambda$

$x_m \approx \dfrac{m\lambda L}{d}$

- $d$ = separation
- $f$ = frequency or focal length
- $h$ = height
- $L$ = distance
- $M$ = magnification
- $m$ = an integer
- $n$ = index of refraction
- $R$ = radius of curvature
- $s$ = distance
- $\upsilon$ = speed
- $x$ = position
- $\lambda$ = wavelength
- $\theta$ = angle

### GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

Rectangular Solid
$V = \ell w h$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r \ell + 2\pi r^2$

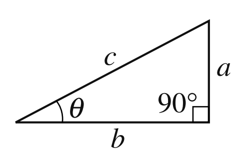
Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

Right Triangle
$a^2 + b^2 = c^2$

$\sin\theta = \dfrac{a}{c}$

$\cos\theta = \dfrac{b}{c}$

$\tan\theta = \dfrac{a}{b}$




- $A$ = area
- $C$ = circumference
- $V$ = volume
- $S$ = surface area
- $b$ = base
- $h$ = height
- $\ell$ = length
- $w$ = width
- $r$ = radius

# ADVANCED PLACEMENT PHYSICS C EQUATIONS DEVELOPED FOR 2012

## MECHANICS

$v = v_0 + at$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(x - x_0)$

$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_{fric} \le \mu N$

$W = \int \mathbf{F} \cdot d\mathbf{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dW}{dt}$

$P = \mathbf{F} \cdot \mathbf{v}$

$\Delta U_g = mgh$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2\,dm = \sum mr^2$

$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$

$v = r\omega$

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$a$ = acceleration
$F$ = force
$f$ = frequency
$h$ = height
$I$ = rotational inertia
$J$ = impulse
$K$ = kinetic energy
$k$ = spring constant
$\ell$ = length
$L$ = angular momentum
$m$ = mass
$N$ = normal force
$P$ = power
$p$ = momentum
$r$ = radius or distance
$\mathbf{r}$ = position vector
$T$ = period
$t$ = time
$U$ = potential energy
$v$ = velocity or speed
$W$ = work done on a system
$x$ = position
$\mu$ = coefficient of friction
$\theta$ = angle
$\tau$ = torque
$\omega$ = angular speed
$\alpha$ = angular acceleration
$\phi$ = phase angle

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2}kx^2$

$x = x_{\max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

$U_G = -\frac{Gm_1m_2}{r}$

## ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa\epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$

$R = \frac{\rho\ell}{A}$

$\mathbf{E} = \rho\mathbf{J}$

$I = Nev_d A$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

$A$ = area
$B$ = magnetic field
$C$ = capacitance
$d$ = distance
$E$ = electric field
$\mathcal{E}$ = emf
$F$ = force
$I$ = current
$J$ = current density
$L$ = inductance
$\ell$ = length
$n$ = number of loops of wire per unit length
$N$ = number of charge carriers per unit volume
$P$ = power
$Q$ = charge
$q$ = point charge
$R$ = resistance
$r$ = distance
$t$ = time
$U$ = potential or stored energy
$V$ = electric potential
$v$ = velocity or speed
$\rho$ = resistivity
$\phi_m$ = magnetic flux
$\kappa$ = dielectric constant

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$

$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$

$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 nI$

$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$

$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

## ADVANCED PLACEMENT PHYSICS C EQUATIONS DEVELOPED FOR 2012

### GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

$A$ = area
$C$ = circumference
$V$ = volume
$S$ = surface area
$b$ = base
$h$ = height
$\ell$ = length
$w$ = width
$r$ = radius

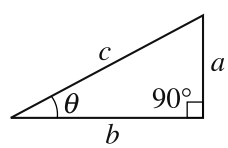
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$




### CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

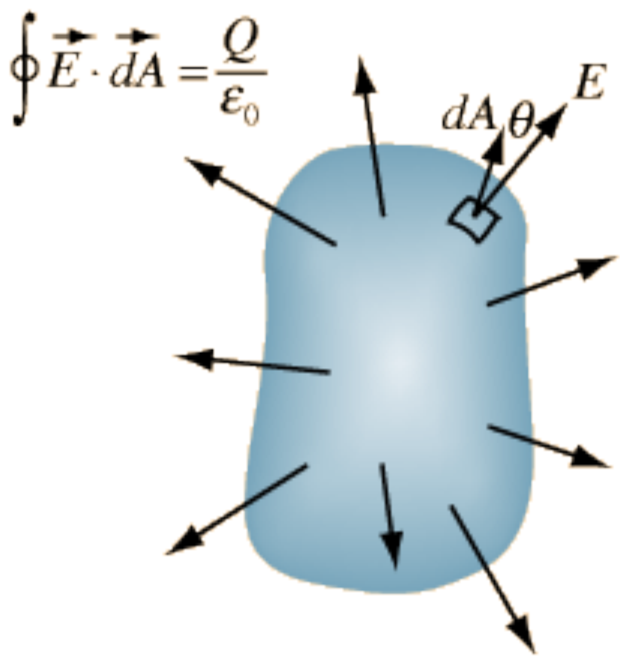
$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

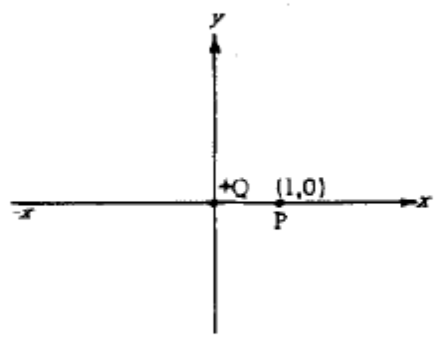
# Chapter 8

## Electrostatics

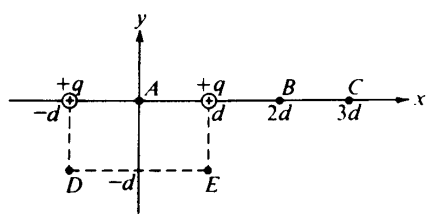


# SECTION A – Coulomb's Law and Coulomb's Law Methods

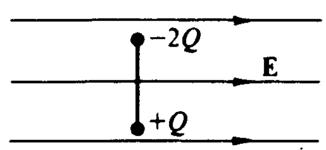
1. A conducting sphere with a radius of 0.10 meter has $1.0 \times 10^{-9}$ coulomb of charge deposited on it. The electric field just outside the surface of the sphere is
(A) zero (B) 450 V/m (C) 900 V/m (D) 4,500 V/m (E) 90,000 V/m



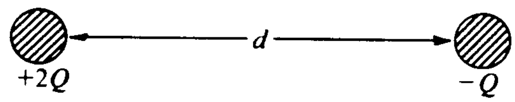

2. A positive charge +Q located at the origin produces an electric field $E_o$ at point P (x = +1, y = 0). A negative charge -2Q is placed at such a point as to produce a net field of zero at point P. The second charge will be placed on the
(A) x-axis where x > 1 (B) x-axis where 0 < x < 1 (C) x-axis where x < 0 (D) y-axis where y > 0
(E) y-axis where y < 0




3. Two positive charges of magnitude q are each a distance d from the origin A of a coordinate system, as shown above. At which of the following points is the electric field <u>least</u> in magnitude?
(A) A (B) B (C) C (D) D (E) E




4. A rigid insulated rod, with two unequal charges attached to its ends, is placed in a uniform electric field E as shown above. The rod experiences a
(A) net force to the left and a clockwise rotation
(B) net force to the left and a counterclockwise rotation
(C) net force to the right and a clockwise rotation
(D) net force to the right and a counterclockwise rotation
(E) rotation, but no net force

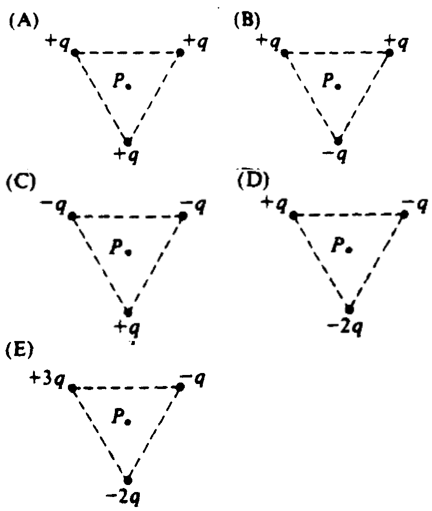



5. Two identical conducting spheres are charged to +2Q and -Q. respectively, and are separated by a distance d (much greater than the radii of the spheres) as shown above. The magnitude of the force of attraction on the left sphere is $F_1$ After the two spheres are made to touch and then are reseparated by distance d the magnitude of the force on the left sphere is $F_2$. Which of the following relationships is correct?
(A) $2F_1 = F_2$ (B) $F_1 = F_2$ (C) $F_1 = 2F_2$ (D) $F_1 = 4F_2$ (E) $F_1 = 8F_2$

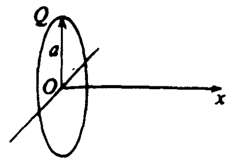
6. Two small spheres have equal charges q and are separated by a distance d. The force exerted on each sphere by the other has magnitude F. If the charge on each sphere is doubled and d is halved, the force on each sphere has magnitude
(A) F (B) 2F (C) 4F (D) 8F (E) 16F

7. A charged particle traveling with a velocity **v** in an electric field **E** experiences a force **F** that must be
(A) parallel to **v** (B) perpendicular to **v** (C) parallel to **v** x **E** (D) parallel to **E** (E) perpendicular to **E**

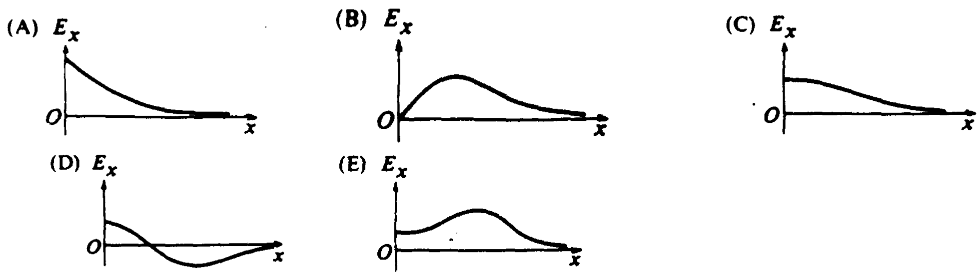
<u>Questions 8-9</u> relate to the following configurations of electric charges located at the vertices of an equilateral triangle. Point P is equidistant from the charges.




8. In which configuration is the electric field at P equal to zero?
(A) (B) (C) (D) (E)
9. In which configuration is the electric field at P pointed at the midpoint between two of the charges?
(A) (B) (C) (D) (E)

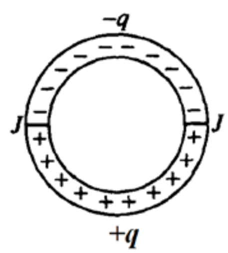



10. Positive charge Q is uniformly distributed over a thin ring of radius a that lies in a plane perpendicular to the x-axis. with its center at the origin 0, as shown above. Which of the following graphs best represents the electric field along the positive x-axis?



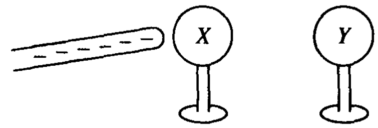

11. From the electric field vector at a point, one can determine which of the following?
    I. The direction of the electrostatic force on a test charge of known sign at that point
    II. The magnitude of the electrostatic force exerted per unit charge on a test charge at that point
    III. The electrostatic charge at that point

    A) I only B) III only C) I and II only D) II and III only E) I, II, and III



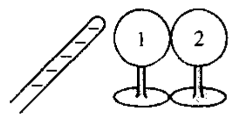

12. A circular ring made of an insulating material is cut in half. One half is given a charge $-q$ uniformly distributed along its arc. The other half is given a charge $+q$ also uniformly distributed along its arc. The two halves are then rejoined with insulation at the junctions J, as shown above. If there is no change in the charge distributions, what is the direction of the net electrostatic force on an electron located at the center of the circle?
    A) Toward the top of the page B) Toward the bottom of the page C) To the right
    D) To the left E) Into the page.

13. A conducting sphere of radius R carries a charge Q. Another conducting sphere has a radius R/2, but carries the same charge. The spheres are far apart. The ratio of the electric field near the surface of the smaller sphere to the field near the surface of the larger sphere is most nearly
    A) 1/4 B) 1/2 C) 1 D) 2 E) 4



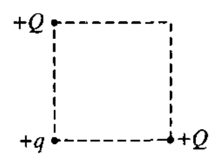

14. Two metal spheres that are initially uncharged are mounted on insulating stands, as shown above. A negatively charged rubber rod is brought close to, but does not make contact with, sphere X. Sphere Y is then brought close to X on the side opposite to the rubber rod. Y is allowed to touch X and then is removed some distance away. The rubber rod is then moved far away from X and Y. What are the final charges on the spheres?

| | Sphere X | Sphere Y |
|---|---|---|
| A) | Zero | Zero |
| B) | Negative | Negative |
| C) | Negative | Positive |
| D) | Positive | Negative |
| E) | Positive | Positive |




15. Two initially uncharged conductors, 1 and 2, are mounted on insulating stands and are in contact, as shown above. A negatively charged rod is brought near but does not touch them. With the rod held in place, conductor 2 is moved to the right by pushing its stand, so that the conductors are separated. Which of the following is now true of conductor 2?
(A) It is uncharged. (B) It is positively charged. (C) It is negatively charged.
(D) It is charged, but its sign cannot be predicted.
(E) It is at the same potential that it was before the charged rod was brought near.

<u>Questions 16-17</u>




As shown above, two particles, each of charge +Q, are fixed at opposite corners of a square that lies in the plane of the page. A positive test charge +q is placed at a third corner.

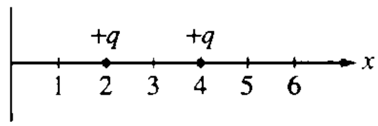
16. What is the direction of the force on the test charge due to the two other charges?

(A) ↗ (B) → (C) ↘ (D) ↓ (E) ↙

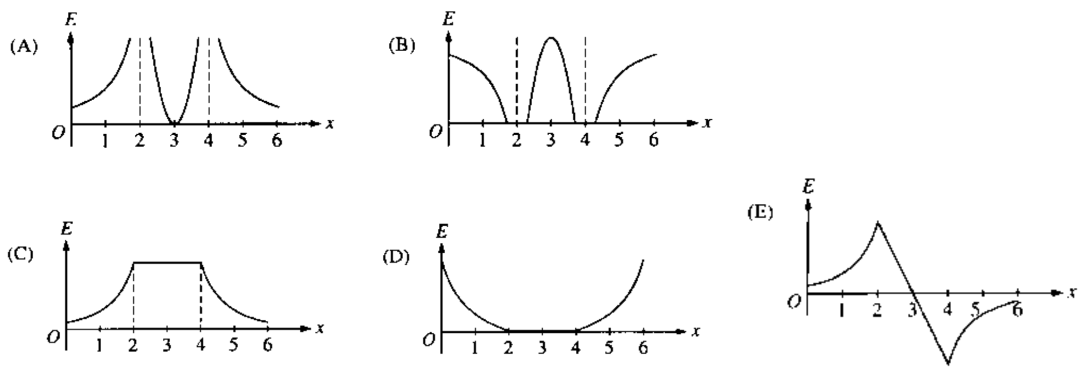
17. If F is the magnitude of the force on the test charge due to only one of the other charges, what is the magnitude of the net force acting on the test charge due to both of these charges?

(A) Zero (B) $\frac{F}{\sqrt{2}}$ (C) F (D) $\sqrt{2}F$ (E) 2

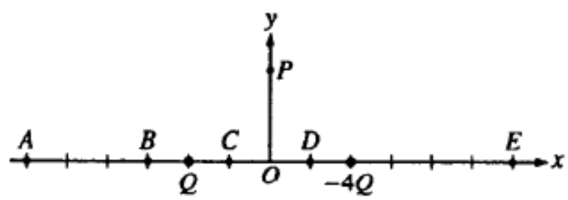
18. If the only force acting on an electron is due to a uniform electric field, the electron moves with constant
(A) acceleration in a direction opposite to that of the field
(B) acceleration in the direction of the field
(C) acceleration in a direction perpendicular to that of the field
(D) speed in a direction opposite to that of the field
(E) speed in the direction of the field




19. Two charged particles, each with a charge of +q, are located along the x-axis at x = 2 and x = 4, as shown above. Which of the following shows the graph of the magnitude of the electric field along the x-axis from the origin to x = 6?



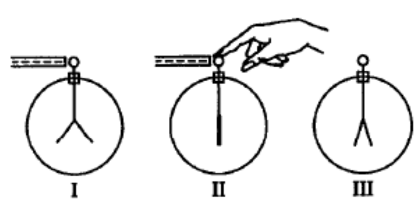

Questions 20-21




Particles of charge Q and -4Q are located on the x-axis as shown in the figure above. Assume the particles are isolated from all other charges.

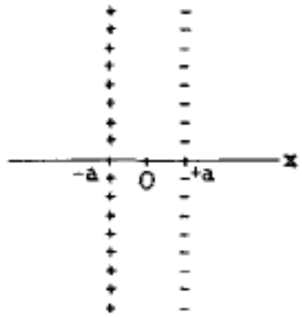
20. Which of the following describes the direction of the electric field at point P ?
(A) +x (B) +y (C)-y
(D) Components in both the -x- and +y-directions
(E) Components in both the +x- and -y-directions

21. At which of the labeled points on the x-axis is the electric field zero?
(A) A (B) B (C) C (D) D (E) E

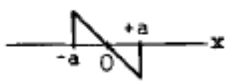


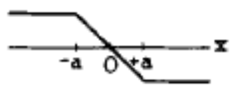

22. When a negatively charged rod is brought near, but does not touch, the initially uncharged electroscope shown above, the leaves spring apart (I). When the electroscope is then touched with a finger, the leaves collapse (II). When next the finger and finally the rod are removed, the leaves spring apart a second time (III). The charge on the leaves is
(A) positive in both I and III (B) negative in both I and III (C) positive in I, negative in III
(D) negative in I, positive in III (E) impossible to determine in either I or III

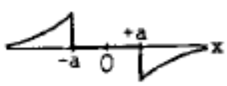
## SECTION B – Gauss’s Law

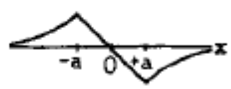


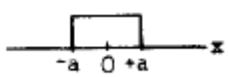

23. Two infinite parallel sheets of charge perpendicular to the x-axis have equal and opposite charge densities as shown above. The sheet that intersects x = -a has uniform positive surface charge density; the sheet that intersects x = +a has uniform negative surface charge density. Which graph best represents the plot of electric field as a function of x ?

(A) 


(B) 

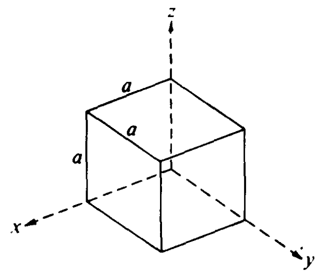

(C) 


(D) 

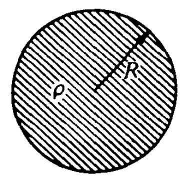

(E) 


24. A point charge is placed at the center of an uncharged, spherical, conducting shell of radius R. The electric fields inside and outside the sphere are measured. The point charge is then moved off center a distance R/2 and the fields are measured again. What is the effect on the electric fields?
(A) Changed neither inside nor outside (B) Changed inside but not changed outside
(C) Not changed inside but changed outside (D) Changed inside and outside
(E) It cannot be determined without further information.

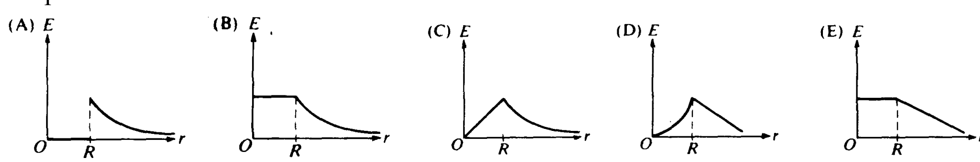
25. The electric field E just outside the surface of a charged conductor is
(A) directed perpendicular to the surface (B) directed parallel to the surface
(C) independent of the surface charge density (D) zero (E) infinite

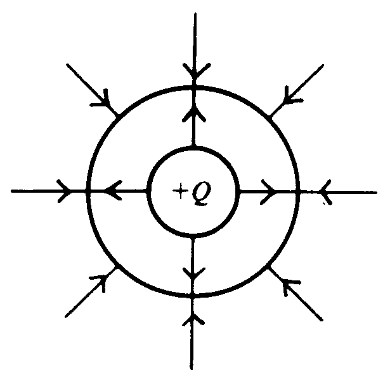



26. A closed surface, in the shape of a cube of side a, is oriented as shown above in a region where there is a constant electric field of magnitude E parallel to the x-axis. The total electric flux through the cubical surface is
(A) $-Ea^2$ (B) zero (C) $Ea^2$ (D) $2Ea^2$ (E) $6Ea^2$




27. The figure above shows a spherical distribution of charge of radius R and constant charge density ρ. Which of the following graphs best represents the electric field strength E as a function of the distance r from the center of the sphere?

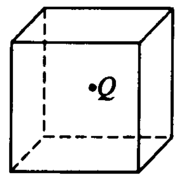



28. The electric field of two long coaxial cylinders is represented by lines of force as shown above. The charge on the inner cylinder is +Q. The charge on the outer cylinder is
(A) +3Q (B) +Q (C) 0 (D) –Q (E) –3Q

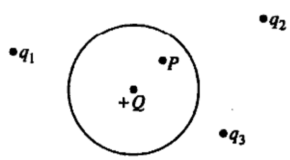
29. The net electric flux through a closed surface is
A) infinite only if there are no charges enclosed by the surface
B) infinite only if the net charge enclosed by the surface is zero
C) zero if only negative charges are enclosed by the surface
D) zero if only positive charges are enclosed by the surface
E) zero if the net charge enclosed by the surface is zero

30. A solid nonconducting sphere of radius R has a charge Q uniformly distributed throughout its volume. A Gaussian surface of radius r with r < R is used to calculate the magnitude of the electric field E at a distance r from the center of the sphere. Which of the following equations results from a correct application of Gauss's law for this situation?
A) $E(4\pi R^2) = Q/\varepsilon_o$ B) $E(4\pi r^2) = Q/\varepsilon_o$ C) $E(4\pi r^2) = (Q3r^3)/(\varepsilon_o 4\pi R)$
D) $E(4\pi r^2) = (Qr^3)/(\varepsilon_o R^3)$ E) $E(4\pi r^2) = 0$




31. The point charge Q shown above is at the center of a metal box that is isolated, ungrounded, and uncharged. Which of the following is true?
A) The net charge on the outside surface of the box is Q.
B) The potential inside the box is zero.
C) The electric field inside the box is constant.
D) The electric field outside the box is zero everywhere.
E) The electric field outside the box is the same as if only the point charge (and not the box) were there.

32. Gauss's law provides a convenient way to calculate the electric field outside and near each of the following isolated charged conductors EXCEPT a
(A) large plate (B) sphere (C) cube (D) long, solid rod (E) long, hollow cylinder



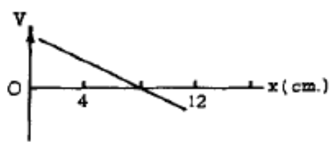

33. A point charge +Q is inside an uncharged conducting spherical shell that in turn is near several isolated point charges, as shown above. The electric field at point P inside the shell depends on the magnitude of
(A) Q only
(B) the charge distribution on the sphere only
(C) Q and the charge distribution on the sphere
(D) all of the point charges
(E) all of the point charges and the charge distribution on the sphere

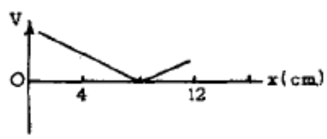
34. A uniform spherical charge distribution has radius *R*.. Which of the following is true of the electric field strength due to this charge distribution at a distance r from the center of the charge?
(A) It is greatest when $r = 0$.
(B) It is greatest when $r = R/2$.
(C) It is directly proportional to $r$ when $r > R$.
(D) It is directly proportional to $r$ when $r < R$.
(E) It is directly proportional to $r^2$.

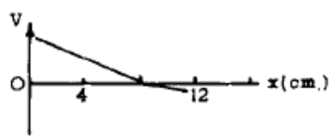
## SECTION C – Electric Potential and Energy

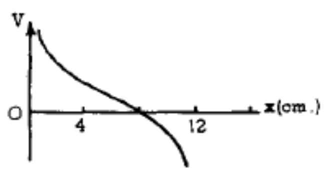
35. A distribution of charge is confined to a finite region of space. The difference in electric potential between any two points $P_1$ and $P_2$ due to this charge distribution depends only upon the
(A) charges located at the points $P_1$ and $P_2$
(B) magnitude of a test charge moved from $P_1$ to $P_2$
(C) value of the electric field at $P_1$ and $P_2$
(D) path taken by a test charge moved from $P_1$ to $P_2$
(E) value of the integral $-\int_{P_1}^{P_2} \boldsymbol{E} \cdot \boldsymbol{dr}$

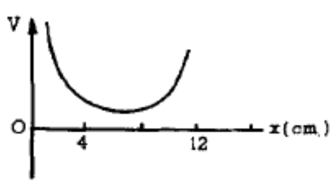
36. Two small spheres having charges of +2Q and –Q are located 12 centimeters apart. The potential of points lying on a line joining the charges is best represented as a function of the distance x from the positive charge by which of the following?

(A)


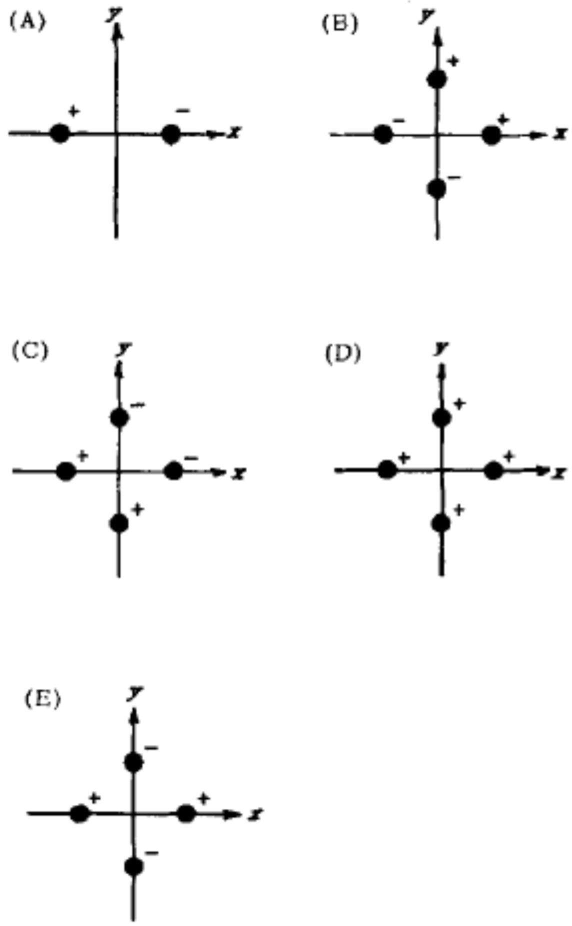

(B)



(C)



(D)


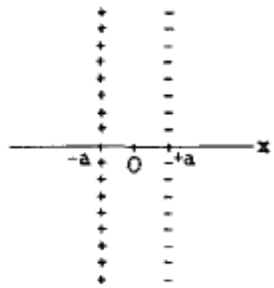

(E)


Questions 37-38 refer to five different charge configurations on the xy-plane using two or four point charges of equal magnitude having signs as indicated below. All charges are the same distance from the origin. The electric potential infinitely far from the origin is defined to be zero.

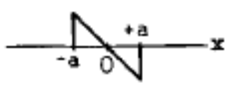


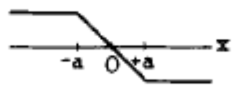

37. In which configuration are both the electric field and the electric potential at the origin equal to zero?
    (A) A (B) B (C) C (D) D (E) E

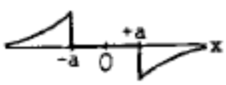
38. In which configuration is the value of the electric field at tile origin equal to zero, but the electric potential at the origin not equal to zero?
    (A) A (B) B (C) C (D) D (E) E

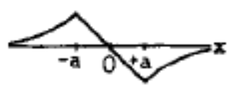


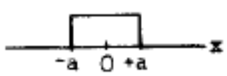

39. Two infinite parallel sheets of charge perpendicular to the x-axis have equal and opposite charge densities as shown above. The sheet that intersects x = -a has uniform positive surface charge density; the sheet that intersects x = +a has uniform negative surface charge density. Which graph best represents the plot of electric potential as a function of x ?

(A)


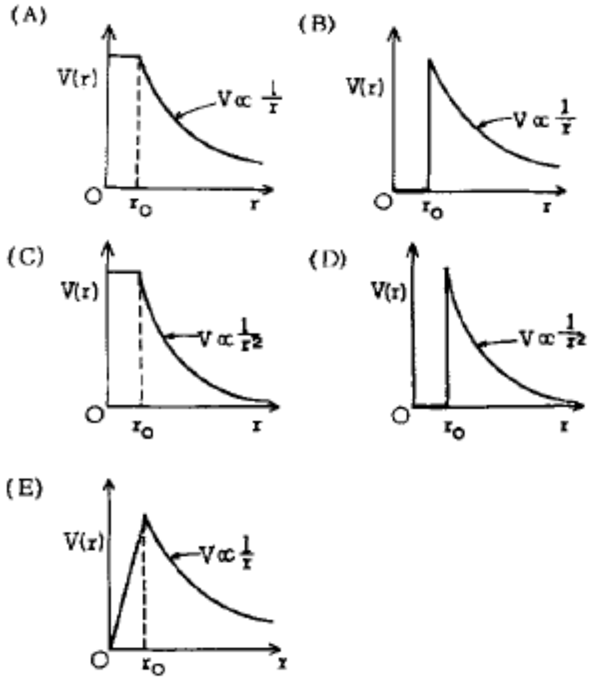

(B)


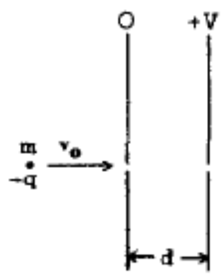

(C)



(D)


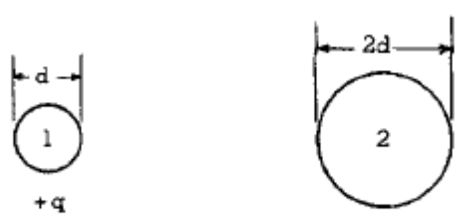

(E)


40. An insulated spherical conductor of radius $r_o$ carries a charge q. The electric potential due to this system varies as a function of the distance r from the center of the sphere in which of the following ways? (The potential is taken to be zero at $r = \infty$)

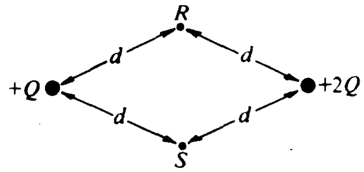


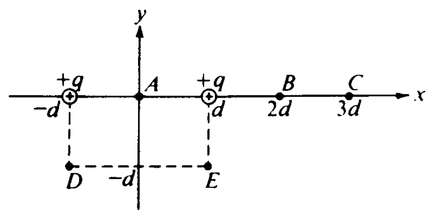

41. As shown in the diagram above, a charged particle having mass m and charge -q is projected into the region between two parallel plates with a speed $v_o$ to the right. The potential difference between the plates is V and they are separated by a distance d. What is the net change in kinetic energy of the particle during the time it takes the particle to traverse the distance d ?
(A) $+\frac{1}{2}mv_0^2$ (B) –qV/d (C) $\frac{+2qV}{mv_0^2}$ (D) +qV (E) None of the above



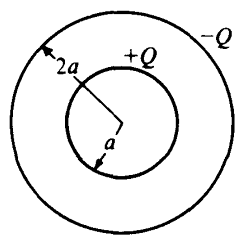

42. Two conducting spheres, one having twice the diameter of the other, are shown above. The smaller sphere initially has a charge +q. When the spheres are connected by a thin wire, which of the following is true?
(A) 1 and 2 are both at the same potential. (B) 2 has twice the potential of 1.
(C) 2 has half the potential of 1. (D) 1 and 2 have equal charges. (E) All of the charge is dissipated.



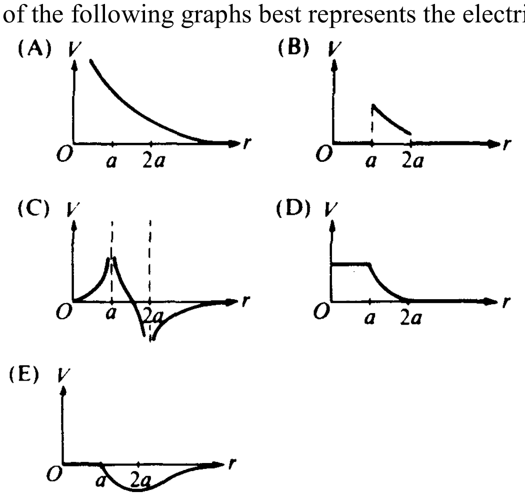

43. Points R and S are each the same distance d from two unequal charges, +Q and +2Q, as shown above. The work required to move a charge -Q from point R to point S is
(A) dependent on the path taken from R to S (B) directly proportional to the distance between R and S
(C) positive (D) zero (E) negative




44. Two positive charges of magnitude q are each a distance d from the origin A of a coordinate system, as shown above. At which of the following points is the electric potential <u>greatest</u> in magnitude?
(A) A (B) B (C) C (D) D (E) E




45. Concentric conducting spheres of radii a and 2a bear equal but opposite charges +Q and -Q. respectively. Which of the following graphs best represents the electric potential V as a function of r ?




46. Which of the following statements about conductors under electrostatic conditions is true?
(A) Positive work is required to move a positive charge over the surface of a conductor.
(B) Charge that is placed on the surface of a conductor always spreads evenly over the surface.
(C) The electric potential inside a conductor is always zero.
(D) The electric field at the surface of a conductor is tangent to the surface.
(E) The surface of a conductor is always an equipotential surface.

47. A positive charge of $3.0 \times 10^{-8}$ coulomb is placed in an upward directed uniform electric field of $4.0 \times 10^{4}$ N/C. When the charge is moved 0.5 meter upward, the work done by the electric force on the charge is
(A) $6 \times 10^{-4}$ J (B) $12 \times 10^{-4}$ J (C) $2 \times 10^{4}$ J (D) $8 \times 10^{4}$ J (E) $12 \times 10^{4}$ J


48. Two conducting spheres, X and Y. have the same positive charge +Q, but different radii ($r_x > r_y$) as shown above. The spheres are separated so that the distance between them is large compared with either radius. If a wire is connected between them, in which direction will current be directed in the wire?
(A) From X to Y
(B) From Y to X
(C) There will be no current in the wire.
(D) It cannot be determined without knowing the magnitude of Q.
(E) It cannot be determined without knowing whether the spheres are solid or hollow.

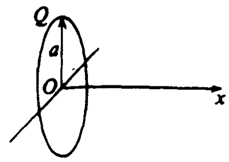
Questions 49-50 refer to a sphere of radius R that has positive charge Q uniformly distributed on its surface

49. Which of the following represents the magnitude of the electric field E and the potential V as functions of r, the distance from the center of the sphere, when r < R ?

| | E | V |
|---|---|---|
| (A) | 0 | kQ/R |
| (B) | 0 | kQ/r |
| (C) | 0 | 0 |
| (D) | $kQ/r^2$ | 0 |
| (E) | $kQ/R^2$ | 0 |

50. Which of the following represents the magnitude, of the electric field E and the potential V as functions of r, the distance from the center of sphere, when r > R ?

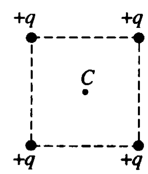
| | E | V |
|---|---|---|
| (A) | $kQ/R^2$ | kQ/R |
| (B) | kQ/R | kQ/R |
| (C) | kQ/R | kQ/r |
| (D) | $kQ/r^2$ | kQ/r |
| (E) | $kQ/r^2$ | $kQ/r^2$ |




51. Positive charge Q is uniformly distributed over a thin ring of radius a that lies in a plane perpendicular to the x-axis. with its center at the origin 0, as shown above. The potential V at points on the x-axis is represented by which of the following functions?

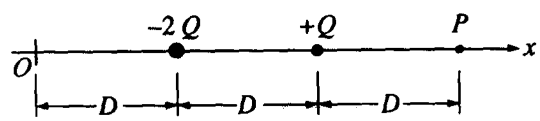
(A) $V(x) = \dfrac{kQ}{x^2 + a^2}$ (B) $V(x) = \dfrac{kQ}{\sqrt{a^2 + x^2}}$

(C) $V(x) = \dfrac{kQ}{x^2}$ (D) $V(x) = \dfrac{kQ}{x}$ (E) $V(x) = \dfrac{kQ}{a + x}$




52. Four positive charges of magnitude q are arranged at the corners of a square, as shown above. At the center C of the square, the potential due to one charge alone is $V_o$ and the electric field due to one charge alone has magnitude $E_o$. Which of the following correctly gives the electric potential and the magnitude of the electric field at the center of the square due to all four charges?

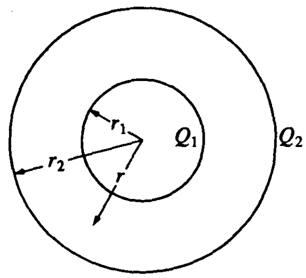
| | Electric Potential | Electric Field |
|---|---|---|
| A) | Zero | Zero |
| B) | Zero | $2E_o$ |
| C) | $2 V_o$ | $4E_o$ |
| D) | $4 V_o$ | Zero |
| E) | $4 V_o$ | $2E_o$ |




53. Two charges, -2Q and +Q, are located on the x-axis, as shown above. Point P, at a distance of 3D from the origin O, is one of two points on the positive x-axis at which the electric potential is zero. How far from the origin O is the other point?
A) (2/3) D B) D C) 3/2 D D) 5/3 D E) 2D

54. What is the radial component of the electric field associated with the potential $V = ar^{-2}$ where a is a constant?
A) $-2ar^{-3}$ B) $-2ar^{-1}$ C) $ar^{-1}$ D) $2ar^{-1}$ E) $2ar^{-3}$

**Questions 55-56**

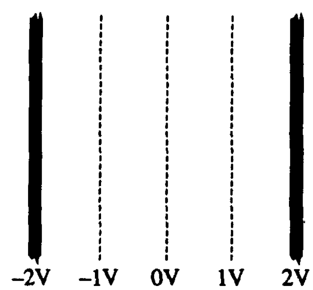



Two concentric, spherical conducting shells have radii $r_1$ and $r_2$ and charges $Q_1$ and $Q_2$, as shown above. Let r be the distance from the center of the spheres and consider the region $r_1 < r < r_2$.

55. In this region the electric field is proportional to
A) $Q_1/r^2$ B) $(Q_1 + Q_2)/r^2$ C) $(Q_1 + Q_2)/r$ D) $Q_1/r_1 + Q_2/r$ E) $Q_1/r + Q_2/r_2$

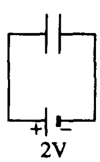
56. In this region the electric potential relative to infinity is proportional to
A) $Q_1/r^2$ B) $(Q_1 + Q_2)/r^2$ C) $(Q_1 + Q_2)/r$ D) $Q_1/r_1 + Q_2/r$ E) $Q_1/r + Q_2/r_2$

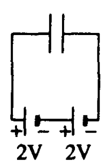
Questions 57-58

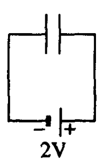


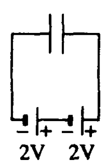

A battery or batteries connected to two parallel plates produce the equipotential lines between the plates shown above.

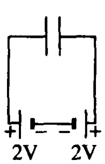
57. Which of the following configurations is most likely to produce these equipotential lines?

(A)



(B)



(C)



(D)



(E)


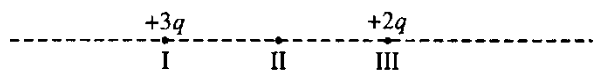
58. The force on an electron located on the 0-volt potential line is
A) 0 N
B) 1 N, directed to the right
C) 1 N, directed to the left
D) directed to the right, but its magnitude cannot be determined without knowing the distance between the lines
E) directed to the left, but its magnitude cannot be determined without knowing the distance between the lines

59. The potential of an isolated conducting sphere of radius R is given as a function of the charge q on the sphere by the equation V = kq/R. If the sphere is initially uncharged, the work W required to gradually increase the total charge on the sphere from zero to Q is given by which of the following expressions?

A) W = kQ/R B) W = $kQ^2/R$ C) W = $\int_0^Q (kq/R)dq$ D) W = $\int_0^Q (kq^2/R)dq$

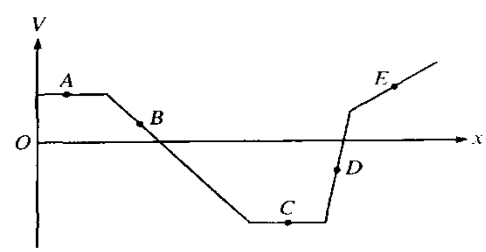
E) W = $\int_0^Q (kq/R^2)dq$

Questions 60-61 refer to two charges located on the line shown in the figure below, in which the charge at point I is +3q and the charge at point III is +2q. Point II is halfway between points I and III.




60. Other than at infinity, the electric field strength is zero at a point on the line in which of the following ranges?
(A) To the left of I (B) Between I and II (C) Between II and III (D) To the right of III
(E) None; the field is zero only at infinity.

61. The electric potential is negative at some points on the line in which of the following ranges?
(A) To the left of I (B) Between I and II (C) Between II and III (D) To the right of III
(E) None; this potential is never negative.




62. The graph above shows the electric potential V in a region of space as a function of position along the x-axis. At which point would a charged particle experience the force of greatest magnitude?
(A) A (B) B (C) C (D) D (E) E:

63. The work that must be done by an external agent to move a point charge of 2 mC from the origin to a point 3 m away is 5 J. What is the potential difference between the two points?
(A) $4 \times 10^{-4}$ V (B) $10^{-2}$ V (C) $2.5 \times 10^{3}$ V (D) $2 \times 10^{6}$ V (E) $6 \times 10^{6}$ V

64. Suppose that an electron (charge -e) could orbit a proton (charge +e) in a circular orbit of constant radius R. Assuming that the proton is stationary and only electrostatic forces act on the particles, which of the following represents the kinetic energy of the two-particle system?

(A) $\frac{1}{4\pi\varepsilon_0}\frac{e}{R}$ (B) $\frac{1}{8\pi\varepsilon_0}\frac{e^2}{R}$ (C) $-\frac{1}{8\pi\varepsilon_0}\frac{e^2}{R}$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}$ (E) $-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}$

Questions 65-66

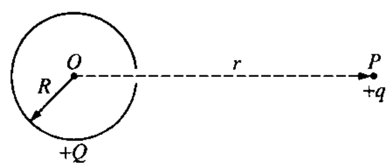
In a region of space, a spherically symmetric electric potential is given as a function of $r$, the distance from the origin, by the equation $V(r) = kr^2$, where $k$ is a positive constant.

65. What is the magnitude of the electric field at a point a distance $r_0$ from the origin?
(A) Zero (B) $kr_0$ (C) $2kr_0$ (D) $kr_0^2$ (E) $2kr_0^3/3$

66. What is the direction of the electric field at a point a distance $r_0$ from the origin and the direction of the force on an electron placed at this point?

| | Electric Field | Force on Electron |
|---|---|---|
| (A) | Toward origin | Toward origin |
| (B) | Toward origin | Away from origin |
| (C) | Away from origin | Toward origin |
| (D) | Away from origin | Away from origin |
| (E) | Undefined, since the field is zero | Undefined, since the force is zero |

67. A positive electric charge is moved at a constant speed between two locations in an electric field, with no work done by or against the field at any time during the motion. This situation can occur only if the
(A) charge is moved in the direction of the field
(B) charge is moved opposite to the direction of the field
(C) charge is moved perpendicular to an equipotential line
(D) charge is moved along an equipotential line
(E) electric field is uniform




68. The nonconducting hollow sphere of radius R shown above carries a large charge +Q, which is uniformly distributed on its surface. There is a small hole in the sphere. A small charge +q is initially located at point P. a distance r from the center of the sphere. If $k = 1/4\pi\varepsilon_o$, what is the work that must be done by an external agent in moving the charge +q from P through the hole to the center O of the sphere?
(A) Zero (B) kqQ/r (C) kqQ/R (D) kq(Q-q)/r (E) kqQ(1/R - 1/r)

69. In a certain region, the electric field along the x-axis is given by

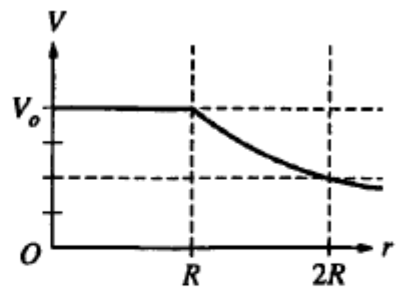
$$E = ax + b, \text{ where } a = 40 \text{ V/m}^2 \text{ and } b = 4 \text{ V/m}.$$

The potential difference between the origin and x = 0.5 m is
(A) -36 V (B) -7 V (C) -3 V (D) 10 V (E) 16 V

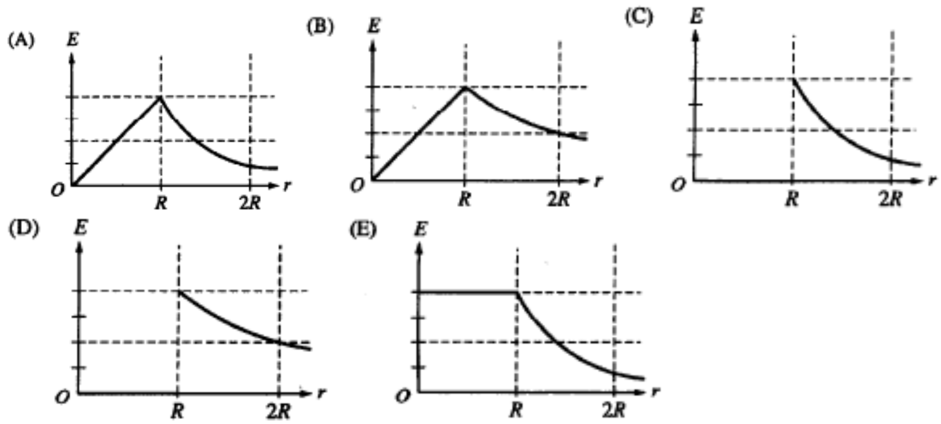
70. A 20 μF parallel-plate capacitor is fully charged to 30 V. The energy stored in the capacitor is most nearly
(A) $9 \times 10^3$ J (B) $9 \times 10^{-3}$ J (C) $6 \times 10^{-4}$ J (D) $2 \times 10^{-4}$ J (E) $2 \times 10^{-7}$ J

71. A potential difference $V$ is maintained between two large, parallel conducting plates. An electron starts from rest on the surface of one plate and accelerates toward the other. Its speed as it reaches the second plate is proportional to

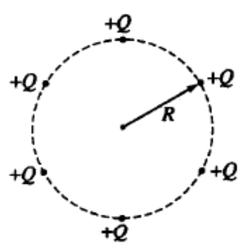
(A) $1/V$ (B) $1/\sqrt{V}$ (C) $\sqrt{V}$ (D) $V$ (E) $V^2$




72. A solid metallic sphere of radius $R$ has charge $Q$ uniformly distributed on its outer surface. A graph of electric potential $V$ as a function of position $r$ is shown above. Which of the following graphs best represents the magnitude of the electric field $E$ as a function of position $r$ for this sphere?




Questions 73-74




As shown in the figure above, six particles, each with charge $+Q$, are held fixed and ate equally spaced around the circumference of a circle of radius $R$.

73. What is the magnitude of the resultant electric field at the center of the circle?

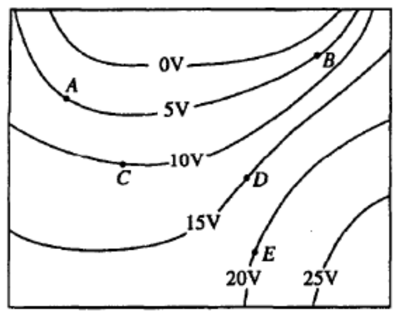
(A) 0 (B) $\frac{\sqrt{6}}{4\pi\varepsilon_0}\frac{Q}{R^2}$ (C) $\frac{2\sqrt{3}}{4\pi\varepsilon_0}\frac{Q}{R^2}$ (D) $\frac{3\sqrt{2}}{4\pi\varepsilon_0}\frac{Q}{R^2}$ (E) $\frac{3}{2\pi\varepsilon_0}\frac{Q}{R^2}$

74. With the six particles held fixed, how much work would be required to bring a seventh particle of charge + Q from very far away and place it at the center of the circle?

(A) 0 (B) $\frac{\sqrt{6}}{4\pi\varepsilon_0}\frac{Q}{R}$ (C) $\frac{3}{2\pi\varepsilon_0}\frac{Q^2}{R^2}$

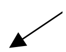
(D) $\frac{3}{2\pi\varepsilon_0}\frac{Q^2}{R}$ (E) $\frac{9}{\pi\varepsilon_0}\frac{Q^2}{R}$

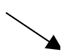
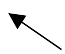
Questions 75-77




The diagram above shows equipotential lines produced by an unknown charge distribution. A, B, C, D, and E are points in the plane.

75. Which vector below best describes the direction of the electric field at point A ?

(A)  (B) 

(C)  (D) 

(E) None of these; the field is zero.

76. At which point does the electric field have the greatest magnitude?
(A) A (B) B (C) C (D) D (E) E

77. How much net work must be done by an external force to move a -1 μC point charge from rest at point C to rest at point E ?
(A) -20 μJ (B) -10 μJ (C) 10 μJ (D) 20 μJ (E) 30 μJ

78. A physics problem starts: "A solid sphere has charge distributed uniformly throughout. . . " It may be correctly concluded that the
(A) electric field is zero everywhere inside the sphere
(B) electric field inside the sphere is the same as the electric field outside
(C) electric potential on the surface of the sphere is not constant
(D) electric potential in the center of the sphere is zero
(E) sphere is not made of metal

# SECTION D – Capacitance

79. The two plates of a parallel-plate capacitor are a distance d apart and are mounted on insulating supports. A battery is connected across the capacitor to charge it and is then disconnected. The distance between the insulated plates is then increased to 2d. If fringing of the field is still negligible, which of the following quantities is doubled?
(A) The capacitance of the capacitor
(B) The total charge on the capacitor
(C) The surface density of the charge on the plates of the capacitor
(D) The energy stored in the capacitor
(E) The intensity of the electric field between the plates of the capacitor

80. A parallel-plate capacitor has a capacitance $C_o$. A second parallel-plate capacitor has plates with twice the area and twice the separation. The capacitance of the second capacitor is most nearly
(A) ¼$C_o$ (B) ½$C_o$ (C) $C_o$ (D) $2C_o$ (E) $4C_o$

Questions 81-82
Three 6-microfarad capacitors are connected in series with a 6-volt battery.

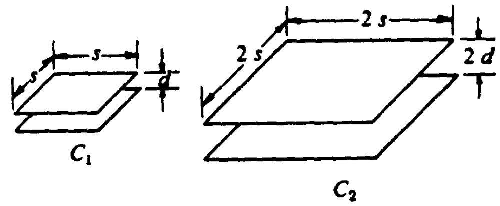
81. The equivalent capacitance of the set of capacitors is
(A) 0.5 μF (B) 2 μF (C) 3 μF (D) 9 μF (E) 18 μF

82. The energy stored in each capacitor is
(A) 4 μJ (B) 6 μJ (C) 12 μJ (D) 18 μJ (E) 36 μJ

83. An isolated capacitor with air between its plates has a potential difference $V_o$ and a charge $Q_o$. After the space between the plates is filled with oil, the difference in potential is V and the charge is Q. Which of the following pairs of relationships is correct?
(A) $Q=Q_o$ and $V>V_o$ (B) $Q=Q_o$ and $V<V_o$ (C) $Q>Q_o$ and $V=V_o$ (D) $Q< Q_o$ and $V<V_o$
(E) $Q>Q_o$ and $V>V_o$

84. When two identical parallel-plate capacitors are connected in series, which of the following is true of the equivalent capacitance?
(A) It depends on the charge on each capacitor.
(B) It depends on the potential difference across both capacitors.
(C) It is larger than the capacitance of each capacitor.
(D) It is smaller than the capacitance of each capacitor.
(E) It is the same as the capacitance of each capacitor.

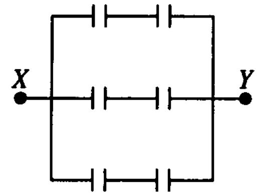
85. Which of the following can be used along with fundamental constants, but no other quantities, to calculate the magnitude of the electric field between the plates of a parallel-plate capacitor whose plate dimensions and spacing are not known?
(A) The flux between the plates
(B) The total charge on either plate
(C) The potential difference between the plates
(D) The surface charge density on either plate
(E) The total energy stored in the capacitor




86. Two square parallel-plate capacitors of capacitances $C_1$ and $C_2$ have the dimensions shown in the diagrams above. The ratio of $C_1$ to $C_2$ is
(A) 1 to 4 (B) 1 to 2 (C) 1 to 1 (D) 2 to 1 (E) 4 to 1

87. A sheet of mica is inserted between the plates of an isolated charged parallel-plate capacitor. Which of the following statements is true?
(A) The capacitance decreases.
(B) The potential difference across the capacitor decreases.
(C) The energy of the capacitor does not change.
(D) The charge on the capacitor plates decreases
(E) The electric field between the capacitor plates increases.

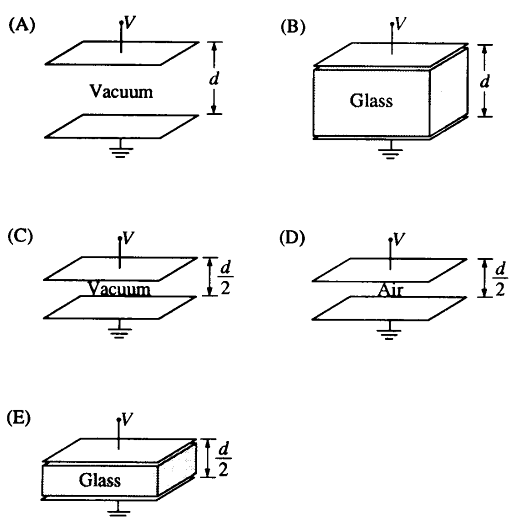
<u>Questions 88-89</u> refer to the system of six 2-microfarad capacitors shown below.




88. The equivalent capacitance of the system of capacitors is
A) 2/3μF B) 4/3 μF C) 3 μF D) 6 μF E) 12 μF

89. What potential difference must be applied between points X and Y so that the charge on each plate of each capacitor will have magnitude 6 microcoulombs?
A) 1.5 V B) 3V C) 6 V D) 9 V E) 18 V

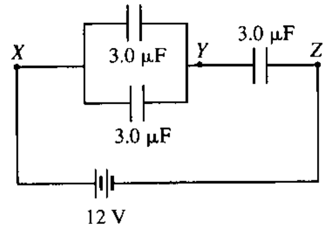
90. Which of the following capacitors, each of which has plates of area A, would store the most charge on the top plate for a given potential difference V ?



91. A parallel-plate capacitor has charge $+Q$ on one plate and charge $-Q$ on the other. The plates, each of area $A$, are a distance $d$ apart and are separated by a vacuum. A single proton of charge $+e$, released from rest at the surface of the positively charged plate, will arrive at the other plate with kinetic energy proportional to

(A) $\frac{edQ}{A}$ (B) $\frac{Q^2}{eAd}$ (C) $\frac{AeQ}{d}$ (D) $\frac{Q}{ed}$ (E) $\frac{eQ^2}{Ad}$

Questions 92-93




Three identical capacitors, each of capacitance 3.0 μF, are connected in a circuit with a 12 V battery as shown above.

92 The equivalent capacitance between points X and Z is
(A) 1.0 μF (B) 2.0 μF (C) 4.5 μF (D) 6.0 μF (E) 9.0 μF

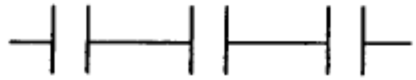
93. The potential difference between points Y and Z is
(A) zero (B) 3 V (C) 4 V (D) 8 V (E) 9 V

Questions 94-95

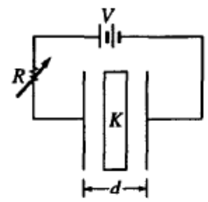
A capacitor is constructed of two identical conducting plates parallel to each other and separated by a distance d. The capacitor is charged to a potential difference of $V_o$ by a battery, which is then disconnected.

94. If any edge effects are negligible, what is the magnitude of the electric field between the plates?
(A) $V_o d$ (B) $V_o/d$ (C) $d/V_o$ (D) $V_o/d^2$ (E) $V_o^2/d$

95. A sheet of insulating plastic material is inserted between the plates without otherwise disturbing the system. What effect does this have on the capacitance?
(A) It causes the capacitance to increase.
(B) It causes the capacitance to decrease.
(C) None; the capacitance does not change.
(D) Nothing can be said about the effect without knowing the dielectric constant of the plastic.
(E) Nothing can be said about the effect without knowing the thickness of the sheet.

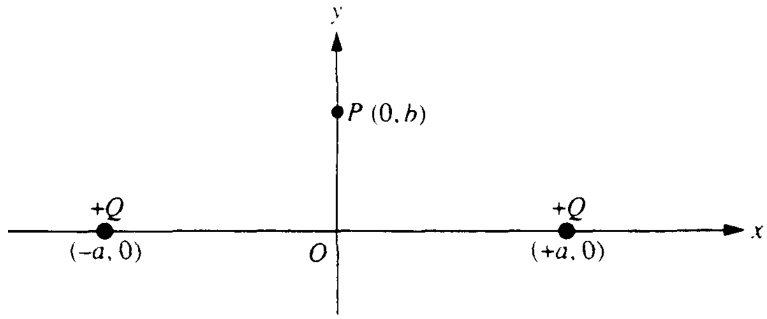


96. Three 1/2 μF capacitors are connected in series as shown in the diagram above. The capacitance of the combination is (A) 0.1 μF (B) 1 μF (C) 2/3 μF (D) ½ μF (E) 1/6 μF




97. The plates of a parallel-plate capacitor of cross sectional area *A* are separated by a distance *d*, as shown above. Between the plates is a dielectric material of constant *K*. The plates are connected in series with a variable resistance *R* and a power supply of potential difference *V*. The capacitance *C* of this capacitor will increase if which of the following is decreased?
(A) *A* (B) *R* (C) *K* (D) *d* (E) *V*

# SECTION A – Coulomb's Law and Coulomb's Law Methods

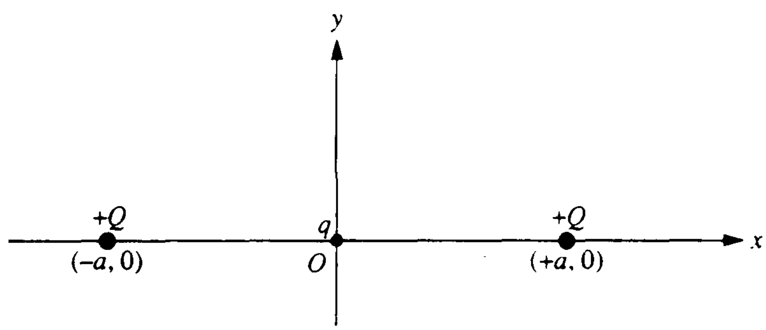



1991E1. Two equal positive charges Q are fixed on the x-axis. one at +a and the other at -a, as shown above. Point P is a point on the y-axis with coordinates (0, b ). Determine each of the following in terms of the given quantities and fundamental constants.

a. The electric field E at the origin O

b. The electric potential V at the origin O.

c. The magnitude of the electric field E at point P.

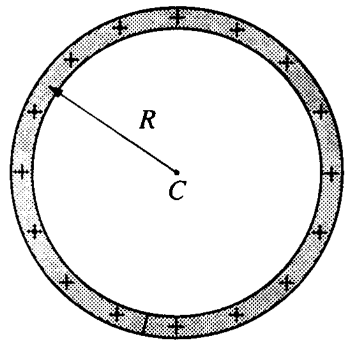



A small particle of charge $q$ ($q << Q$ ) and mass m is placed at the origin, displaced slightly, and then released. Assume that the only subsequent forces acting are the electric forces from the two fixed charges Q. at x = +a and x = -a. and that the particle moves only in the xy -plane. In each of the following cases, describe briefly the motion of the charged particle after it is released. Write an expression for its speed when far away if the resulting force pushes it away from the origin.

d. q is positive and is displaced in the +x direction.

e. q is positive and is displaced in the +y direction.

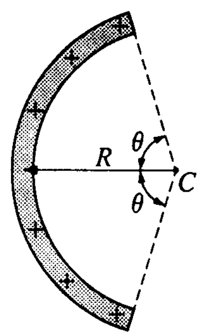
f. q is negative and is displaced in the +y direction.




1994E1. A thin nonconducting rod that carries a uniform charge per unit length of λ is bent into a circle of radius R. as shown above. Express your answers in terms of λ, R. and fundamental constants.

a. Determine the electric potential V at the center C of the circle.

b. Determine the magnitude E of the electric field at the center C of the circle.

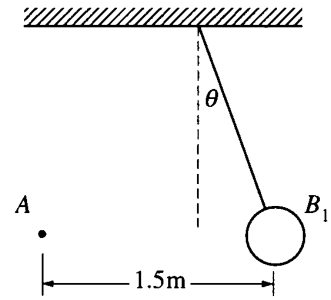



Another thin nonconducting rod that carries the same uniform charge per unit length λ is bent into an arc of a circle of radius R. which subtends an angle of $2\theta$, as shown above. Express your answers in terms of λ and the quantities given above.

c. Determine the total charge on the rod.

d. Determine the electric potential V at the center of curvature C of the arc.

e. Determine the magnitude E of the electric field at the center of curvature C of the arc. Indicate the direction of the electric field on the diagram above.



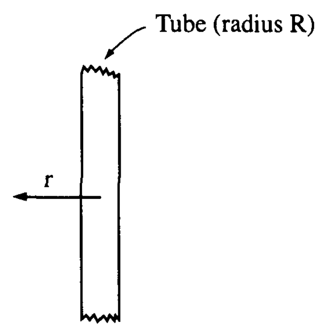

Note: Figure not drawn to scale.

1998E1. The small sphere A in the diagram above has a charge of 120 μ C. The large sphere $B_1$ is a thin shell of nonconducting material with a net charge that is uniformly distributed over its surface. Sphere $B_1$ has a mass of 0.025 kg, a radius of 0.05 m, and is suspended from an uncharged, nonconducting thread. Sphere $B_1$ is in equilibrium when the thread makes an angle θ = 20° with the vertical. The centers of the spheres are at the same vertical height and are a horizontal distance of 1.5 m apart, as shown.

a. Calculate the charge on sphere $B_1$.

b. Suppose that sphere $B_1$ is replaced by a second suspended sphere $B_2$ that has the same mass, radius, and charge, but that is conducting. Equilibrium is again established when sphere A is 1.5 m from sphere $B_2$ and their centers are at the same vertical height. State whether the equilibrium angle $\theta_2$ will be less than, equal to, or greater than 20°. Justify your answer.

The sphere $B_2$ is now replaced by a very long, horizontal, nonconducting tube, as shown in the top view below. The tube is hollow with thin walls of radius R = 0.20 m and a uniform positive charge per unit length of λ = +0.10 μC/m.

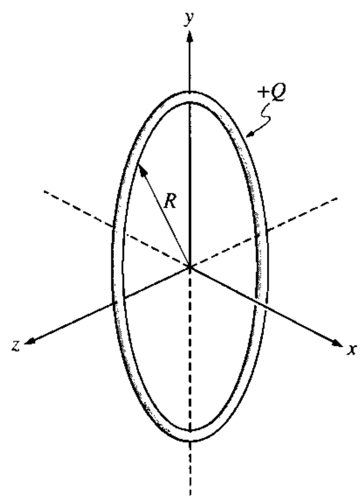



Top View

c. Use Gauss's law to show that the electric field at a perpendicular distance r from the tube is given by the expression $E = (1.8 \times 10^3)/r$ N/C, where r > R and r is in meters.

d. The small sphere A with charge 120 μC is now brought into the vicinity of the tube and is held at a distance of r = 1.5 m from the center of the tube. Calculate the repulsive force that the tube exerts on the sphere.

e. Calculate the work done against the electrostatic repulsion to move sphere A toward the tube from a distance r = 1.5 m to a distance r = 0.3 m from the tube.




1999E3. The nonconducting ring of radius R shown above lies in the yz-plane and carries a uniformly distributed positive charge Q.

a. Determine the electric potential at points along the x-axis as a function of x.

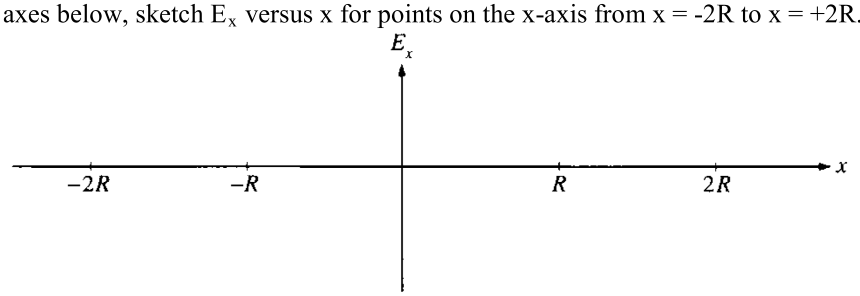
b. i. Show that the x-component of the electric field along the x-axis is given by

$$E_x = \frac{Qx}{4\pi\varepsilon_0\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

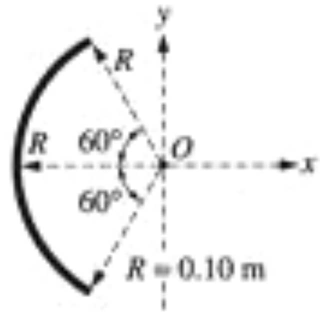
ii. What are the y- and z- components of the electric field along the x-axis?

c. Determine the following.
i. The value of x for which $E_x$ is a maximum
ii. The maximum electric field $E_{x\,max}$

d. On the axes below, sketch $E_x$ versus x for points on the x-axis from x = -2R to x = +2R.




e. An electron is placed at x = R/2 and released from rest. Qualitatively describe its subsequent motion.




2002E1. A rod of uniform linear charge density $\lambda = +1.5 \times 10^{-5}$ C/m is bent into an arc of radius R = 0.10 m. The arc is placed with its center at the origin of the axes shown above.

a. Determine the total charge on the rod.
b. Determine the magnitude and direction of the electric field at the center O of the arc.
c. Determine the electric potential at point O.

A proton is now placed at point O and held in place. Ignore the effects of gravity in the rest of this problem.

d. Determine the magnitude and direction of the force that must be applied in order to keep the proton at rest.
e. The proton is now released. Describe in words its motion for a long time after its release.

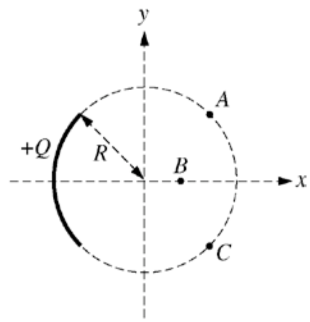



Figure I

2010E1. A charge $+Q$ is uniformly distributed over a quarter circle of radius $R$, as shown above. Points $A$, $B$, and $C$ are located as shown, with $A$ and $C$ located symmetrically relative to the $x$-axis. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Rank the magnitude of the electric potential at points $A$, $B$, and $C$ from greatest to least, with number 1 being greatest. If two points have the same potential, give them the same ranking.

____*VA* ____*VB* ____*VC*

Justify your rankings.

Point $P$ is at the origin, as shown below, and is the center of curvature of the charge distribution.

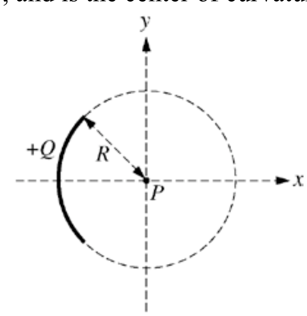
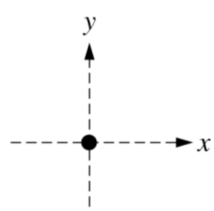



Figure II

b. Determine an expression for the electric potential at point $P$ due to the charge $Q$.

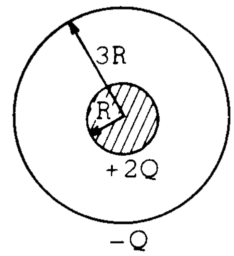
c. A positive point charge $q$ with mass $m$ is placed at point $P$ and released from rest. Derive an expression for the speed of the point charge when it is very far from the origin.

d. On the dot representing point $P$ below, indicate the direction of the electric field at point $P$ due to the charge $Q$.



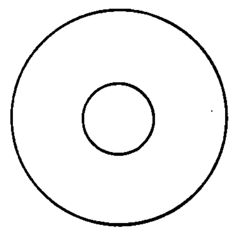

e. Derive an expression for the magnitude of the electric field at point $P$.

## SECTION B – Gauss's Law




1976E1. A solid metal sphere of radius R has charge +2Q. A hollow spherical shell of radius 3R placed concentric with the first sphere has net charge -Q.

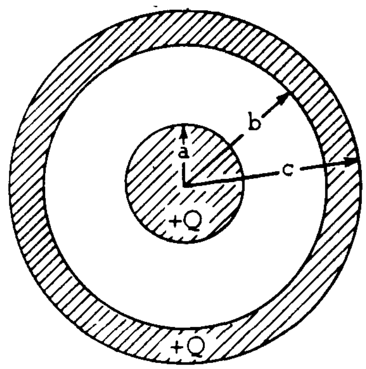
a. On the diagram below, make a sketch of the electric field lines inside and outside the spheres.



b. Use Gauss's law to find an expression for the magnitude of the electric field between the spheres at a distance r from the center of the inner sphere ($R < r < 3R$).

c. Calculate the potential difference between the two spheres.

d. What would be the final distribution of the charge if the spheres were joined by a conducting wire?



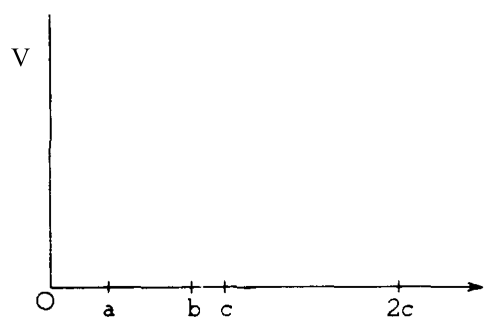

1979E1. A solid conducting sphere of radius a is surrounded by a hollow conducting shell of inner radius b and outer radius c as shown above. The sphere and the shell each have a charge +Q. Express your answers to parts (a), (b) and (e) in terms of Q, a, b, c, and the Coulomb's law constant.

a. Using Gauss's law, derive an expression for the electric field magnitude at $a < r < b$, where r is the distance from the center of the solid sphere.

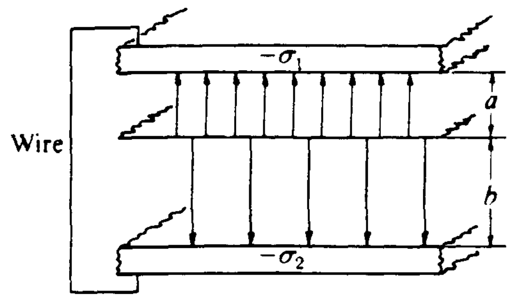
b. Write expressions for the electric field magnitude at $r > c$, $b < r < c$, and $r < a$. Full credit will be given for statements of the correct expressions. It is not necessary to show your work on this part.

c. On the axes below, sketch a graph of the electric field magnitude E vs. distance r from the center of the solid sphere.


d. On the axes below, sketch a graph of potential V vs. distance r from the center of the solid sphere. (The potential V is zero at $r = \infty$.)



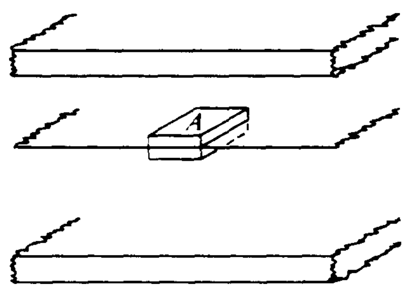

e. Determine the Potential at $r = b$.



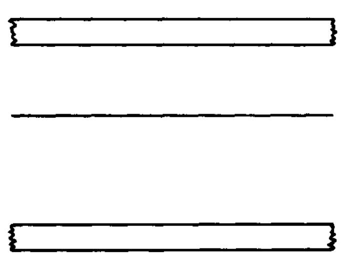

1984E2. Two large, parallel conducting plates are joined by a wire, as shown above, so that they are at the same potential. Between the plates, at a distance a from the upper plate and a distance b from the lower plate, is a thin, uniformly charged sheet whose charge per unit area is σ The electric fields between the plates above and below the sheet have magnitudes $E_1$ and $E_2$, respectively.

a. Determine the ratio $E_1/E_2$ so that the potential difference between the outer plates is zero.

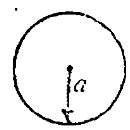
b. The Gaussian surface in the diagram immediately below has faces of area A parallel to the charged sheet. By applying Gauss's law to this surface, develop a relationship among $E_1$, $E_2$, a, and any appropriate fundamental constants.




c. By applying Gauss's law to an appropriately chosen Gaussian surface, show that the sum of the induced charge densities, $\sigma_1$ and $\sigma_2$, on the inner surfaces of the conducting plates equals $-\sigma$. Indicate clearly on the diagram below, the Gaussian surface you used.

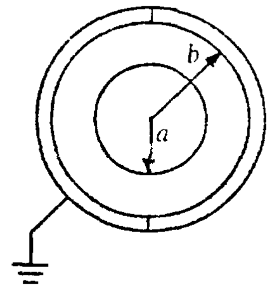


d. Develop an expression for the potential difference V between the charged sheet and the conducting plates in terms of σ, a, b, and any appropriate fundamental constants.




1988E1. The isolated conducting solid sphere of radius a shown above is charged to a potential V.

a. Determine the charge on the sphere.

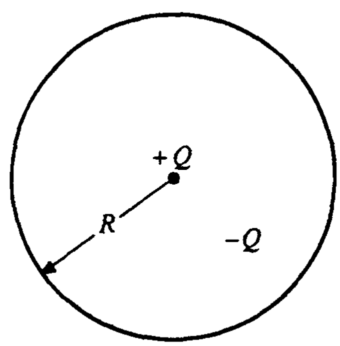



Two conducting hemispherical shells of inner radius b are then brought up and, without contacting the solid sphere are connected to form a spherical shell surrounding and concentric with the solid sphere as shown below The outer shell is then grounded.

b. By means of Gauss's law, determine the electric field in the space between the solid sphere and the shell at a distance r from the center.

c. Determine the potential of the solid sphere relative to ground.

d. Determine the capacitance of the system in terms of the given quantities and fundamental constants.



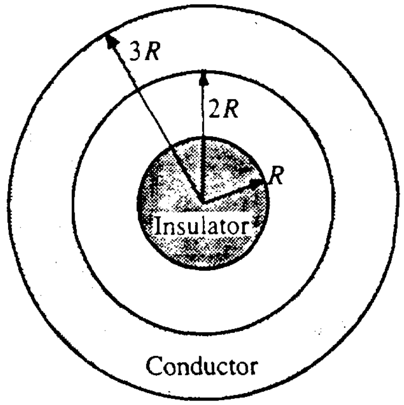

1989E1. A negative charge - Q is uniformly distributed throughout the spherical volume of radius R shown above. A positive point charge + Q is at the center of the sphere. Determine each of the following in terms of the quantities given and fundamental constants.

a. The electric field E outside the sphere at a distance r > R from the center

b. The electric potential V outside the sphere at a distance r > R from the center

c. The electric field inside the sphere at a distance r < R from the center

d. The electric potential inside the sphere at a distance r < R from the center




1990E1. A sphere of radius R is surrounded by a concentric spherical shell of inner radius *2R* and outer radius *3R,* as shown above. The inner sphere is an insulator containing a net charge + Q distributed uniformly throughout its volume. The spherical shell is a conductor containing a net charge + q different from + Q.

Use Gauss's law to determine the electric field for the following values of r, the distance from the center of the insulator.

a. $0 < r < R$
b. $R < r < 2R$
c. $2R < r < 3R$

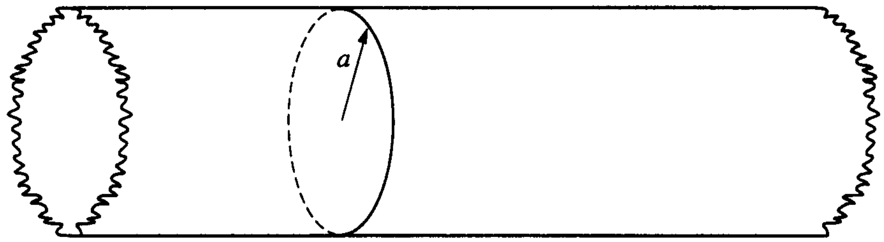
Determine the surface charge density (charge per unit area) on

d. the inside surface of the conducting shell;
e. the outside surface of the conducting shell.

---

1992E1. A positive charge distribution exists within a nonconducting spherical region of radius a. The volume charge density $\rho$ is <u>not</u> uniform but varies with the distance r from the center of the spherical charge distribution, according to the relationship $\rho = \beta r$ for $0 \leq r \leq a$, where $\beta$ is a positive constant, and $\rho=0$, for $r > a$.

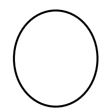
a. Show that the total charge Q in the spherical region of radius a is $\beta\pi a^4$
b. In terms of $\beta$, r, a, and fundamental constants, determine the magnitude of the electric field at a point a distance r from the center of the spherical charge distribution for each of the following cases.
   i. $r > a$ ii. $r = a$ iii. $0 < r < a$
c. In terms of $\beta$, a, and fundamental constants, determine the electric potential at a point a distance r from the center of the spherical charge distribution for each of the following cases.
   i. $r = a$ ii. $r = 0$



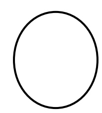

1995E1. A very long nonconducting rod of radius a has positive charge distributed throughout its volume. The charge distribution is cylindrically symmetric, and the total charge per unit length of the rod is $\lambda$ .

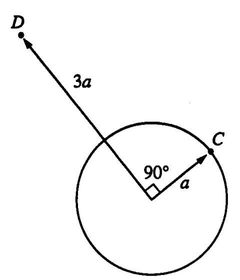
a. Use Gauss's law to derive an expression for the magnitude of the electric field E outside the rod.

b. The diagrams below represent cross sections of the rod. On these diagrams, sketch the following.
i. Several equipotential lines in the region r > a



ii. Several electric field lines in the region r > a

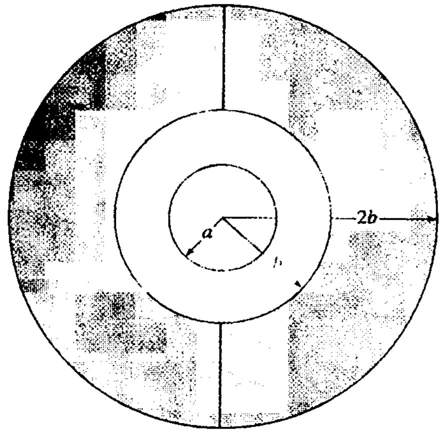





c. In the diagram above, point C is a distance a from the center of the rod (i.e., on the rod's surface), and point D is a distance 3a from the center on a radius that is 90° from point C. Determine the following.
i. The potential difference $V_C - V_D$ between points C and D
ii. The work required by an external agent to move a charge + Q from rest at point D to rest at point C

Inside the rod (r < a), the charge density $\rho$ is a function of radial distance r from the axis of the rod and is given by $\rho=\rho_o(r/a)^{½}$, where $\rho_o$ is a constant.

d. Determine the magnitude of the electric field E as a function of r for r < a. Express your answer in terms of $\rho_o$, a, and fundamental constants.




1996E1. A solid metal sphere of radius a is charged to a potential $V_o > 0$ and then isolated from the charging source. It is then surrounded by joining two uncharged metal hemispherical shells of inner radius b and outer radius 2b, as shown above, without touching the inner sphere or any source of charge.

a. In terms of the given quantities and fundamental constants, determine the initial charge $Q_o$ on the solid sphere before it was surrounded by the outer shell.

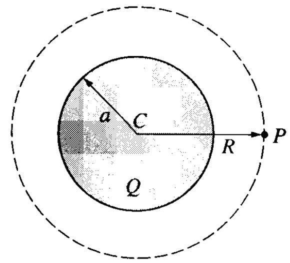
b. Indicate the induced charge on the following after the outer shell is in place.
   i. The inner surface of the shell
   ii. The outer surface of the shell

c. Indicate the magnitude of the electric field as a function of r and the direction (if any) of the field for the regions indicated below. Write your answers on the appropriate lines.

| | | |
|---|---|---|
| i. $r < a$ | Magnitude ____________________ | Direction ____________________ |
| ii. $a < r < b$ | Magnitude ____________________ | Direction ____________________ |
| iii. $b< r< 2b$ | Magnitude ____________________ | Direction ____________________ |
| iv. $2b < r$ | Magnitude ____________________ | Direction ____________________ |

d. Does the inner sphere exert a force on the uncharged hemispheres while the shell is being assembled? Why or why not?

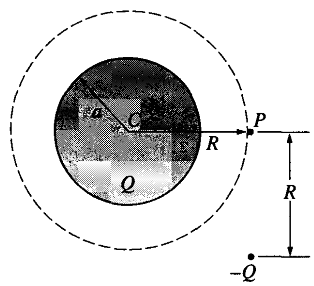
e. Although the charge on the inner solid sphere has not changed, its potential has. In terms of $V_o$, a, and b, determine the new potential on the inner sphere. Be sure to show your work.




1997E2. A nonconducting sphere with center C and radius a has a spherically symmetric electric charge density. The total charge of the object is $Q > 0$.

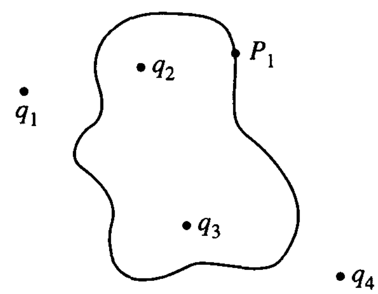
a. Determine the magnitude and direction of the electric field at point P, which is a distance $R > a$ to the right of the sphere's center.

b. Determine the flux of the electric field through the spherical surface centered at C and passing through P.




A point particle of charge -Q is now placed a distance R below point P. as shown above.

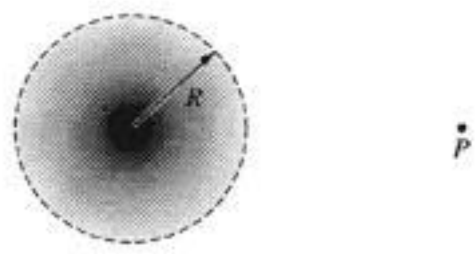
c. Determine the magnitude and direction of the electric field at point P.




d. Now consider four point charges, $q_1$, $q_2$, $q_3$, and $q_4$, that lie in the plane of the page as shown in the diagram above. Imagine a three-dimensional closed surface whose cross section in the plane of the page is indicated.
   i. Which of these charges contribute to the net electric flux through the surface?
   ii. Which of these charges contribute to the electric field at point $P_1$ ?
   iii. Are your answers to i and ii the same or are they different? Explain why this is so.

e. If the net charge enclosed by a surface is zero, does this mean that the field is zero at all points on the surface? Justify your answer.

f. If the field is zero at all points on a surface, does this mean there is no net charge enclosed by the surface? Justify your answer.




2003E1. A spherical cloud of charge of radius R contains a total charge $+Q$ with a nonuniform volume charge density that varies according to the equation

$$\rho(r) = \rho_o(1 - r/R) \text{ for } r \leq R \text{ and}$$
$$\rho = 0 \text{ for } r > R,$$

where $r$ is the distance from the center of the cloud. Express all algebraic answers in terms of $Q$, $R$, and fundamental constants.

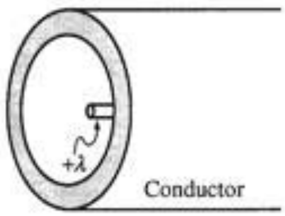
a. Determine the following as a function of $r$ for $r > R$ .
 i. The magnitude $E$ of the electric field
ii. The electric potential $V$

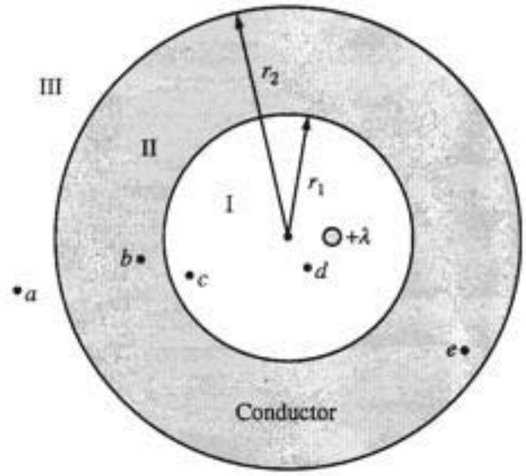
b. A proton is placed at point $P$ shown above and released. Describe its motion for a long time after its release.

c. An electron of charge magnitude $e$ is now placed at point $P$, which is a distance $r$ from the center of the sphere, and released. Determine the kinetic energy of the electron as a function of $r$ as it strikes the cloud.

d. Derive an expression for $\rho_o$ .

e. Determine the magnitude $E$ of the electric field as a function of $r$ for $r \leq R$ .



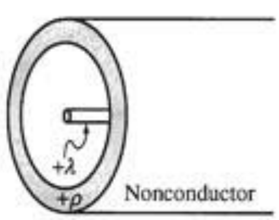

Cross Section

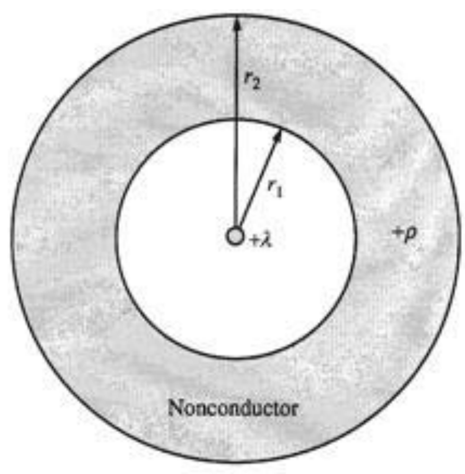
2004E1. The figure above left shows a hollow, infinite, cylindrical, uncharged conducting shell of inner radius $r_1$ and outer radius $r_2$. An infinite line charge of linear charge density $+\lambda$ is parallel to its axis but off center. An enlarged cross section of the cylindrical shell is shown above right.

a. On the cross section above right,
   i. sketch the electric field lines, if any, in each of regions I, II, and III and
   ii. use + and - signs to indicate any charge induced on the conductor.

b. In the spaces below, rank the electric potentials at points *a, b, c, d*, and *e* from highest to lowest (1 = highest potential). If two points are at the same potential, give them the same number.

____$V_a$
____$V_b$
____$V_c$
____$V_d$
____$V_e$

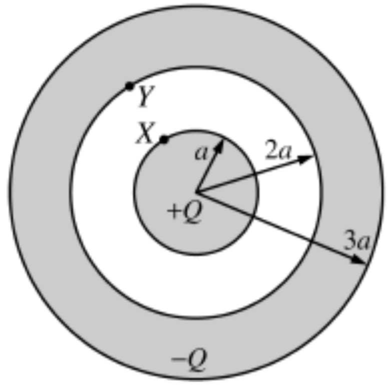



Cross Section

c. The shell is replaced by another cylindrical shell that has the same dimensions but is nonconducting and carries a uniform volume charge density $+\rho$. The infinite line charge, still of charge density $+\lambda$, is located at the center of the shell as shown above. Using Gauss's law, calculate the magnitude of the electric field as a function of the distance $r$ from the center of the shell for each of the following regions. Express your answers in terms of the given quantities and fundamental constants.

i. $r < r_1$
ii. $r_1 \le r \le r_2$
iii. $r > r_2$




2007E2. In the figure above, a nonconducting solid sphere of radius $a$ with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.

(a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius $r$ in the following regions.

i. Within the solid sphere ($r < a$)

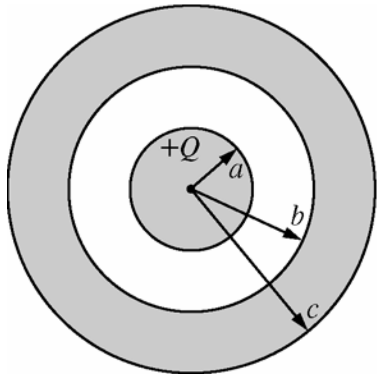
ii. Between the solid sphere and the spherical shell ($a < r < 2a$)

iii. Within the spherical shell ($2a < r < 3a$)

iv. Outside the spherical shell ($r > 3a$)

(b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.

(c) Derive an expression for the electric potential difference $V_x - V_y$ between points $X$ and $Y$ shown in the figure.




2008E1. A metal sphere of radius $a$ contains a charge $+Q$ and is surrounded by an uncharged, concentric, metallic shell of inner radius $b$ and outer radius $c$, as shown above. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the induced charge on each of the following and explain your reasoning in each case.

i. The inner surface of the metallic shell

ii. The outer surface of the metallic shell

(b) Determine expressions for the magnitude of the electric field $E$ as a function of $r$, the distance from the center of the inner sphere, in each of the following regions.

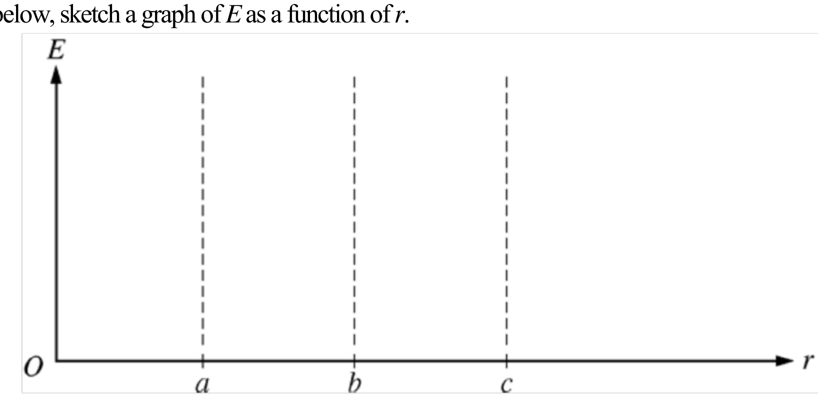
i. $r < a$

ii. $a < r < b$

iii. $b < r < c$

iv. $c$

(c) On the axes below, sketch a graph of $E$ as a function of $r$.




(d) An electron of mass $m_e$ carrying a charge $-e$ is released from rest at a very large distance from the spheres. Derive an expression for the speed of the particle at a distance $10c$ from the center of the spheres.

2011E1.

A nonconducting, thin, spherical shell has a uniform surface charge density $\sigma$ on its outside surface and no charge anywhere else inside.

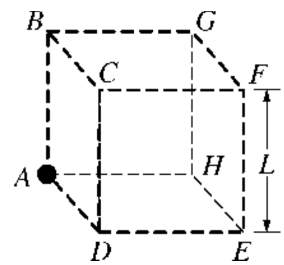
(a) Use Gauss's law to prove that the electric field inside the shell is zero everywhere. Describe the Gaussian surface that you use.

(b) The charges are now redistributed so that the surface charge density is no longer uniform. Is the electric field still zero everywhere inside the shell?

____ Yes ____ No ____ It cannot be determined from the information given.

Justify your answer.

Now consider a small conducting sphere with charge $+Q$ whose center is at corner $A$ of a cubical surface, as shown below.




(c) For which faces of the surface, if any, is the electric flux through that face equal to zero?

____ *ABCD* ____ *CDEF* ____ *EFGH* ____ *ABGH* ____ *BCFG* ____ *ADEH*

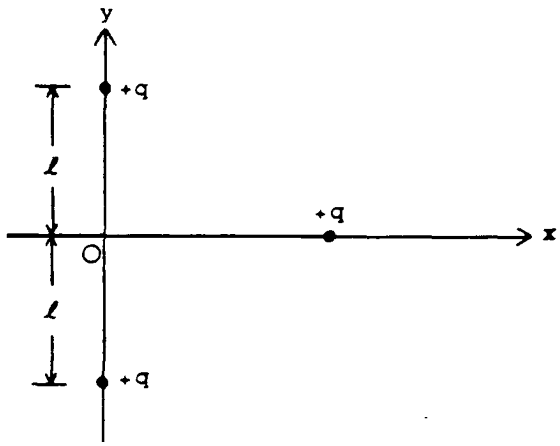
Explain your reasoning.

(d) At which corner(s) of the surface does the electric field have the least magnitude?

(e) Determine the electric field strength at the position(s) you have indicated in part (d) in terms of $Q$, $L$, and fundamental constants, as appropriate.

(f) Given that one-eighth of the sphere at point $A$ is inside the surface, calculate the electric flux through face $CDEF$.

# SECTION C – Electric Potential and Energy




1975E1. Two stationary point charges +q are located on the y-axis as shown above. A third charge +q is brought in from infinity along the x-axis.

a. Express the potential energy of the movable charge as a function of its position on the x-axis.

b. Determine the magnitude and direction of the force acting on the movable charge when it is located at the position $x = l$

c. Determine the work done by the electric field as the charge moves from infinity to the origin.

---

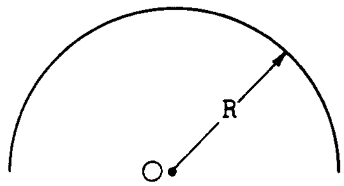
1977E1. A charge +Q is uniformly distributed around a wire ring of radius R. Assume that the electric potential is zero at x = infinity, with the origin 0 of the x-axis at the center of the ring.

a. What is the electric potential at a point P on the x-axis?

b. Where along the x-axis is the electric potential the greatest? Justify your answer.

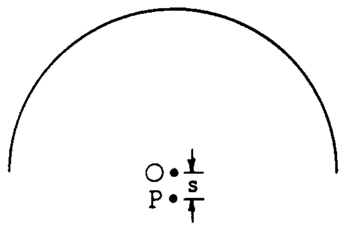
c. What is the magnitude and direction of the electric field **E** at point P?

d. On the axes below, make a sketch of **E** as a function of the distance along the x-axis showing significant features.



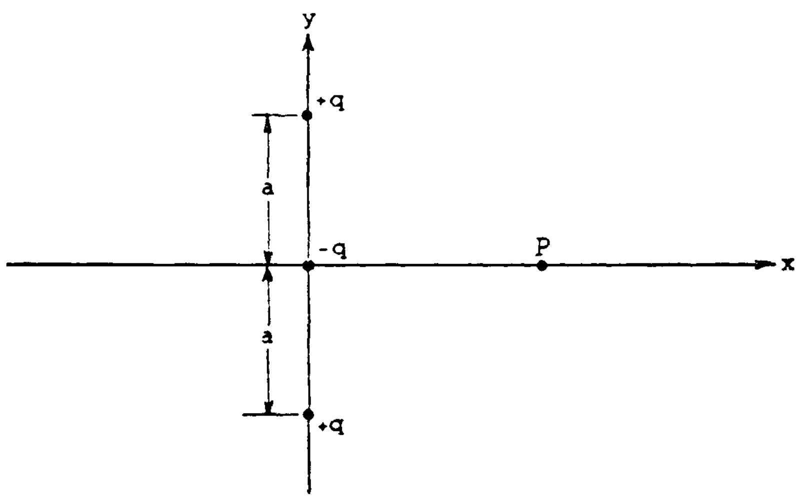

1980E1. A thin plastic rod has uniform linear positive-charge density $\lambda$. The rod is bent into a semicircle of radius R as shown above.

a. Determine the electric potential $V_o$ at point 0, the center of the semicircle.
b. Indicate on the diagram above the direction of the electric field at point 0. Explain your reasoning.
c. Calculate the magnitude $E_o$ of the electric field at point 0.



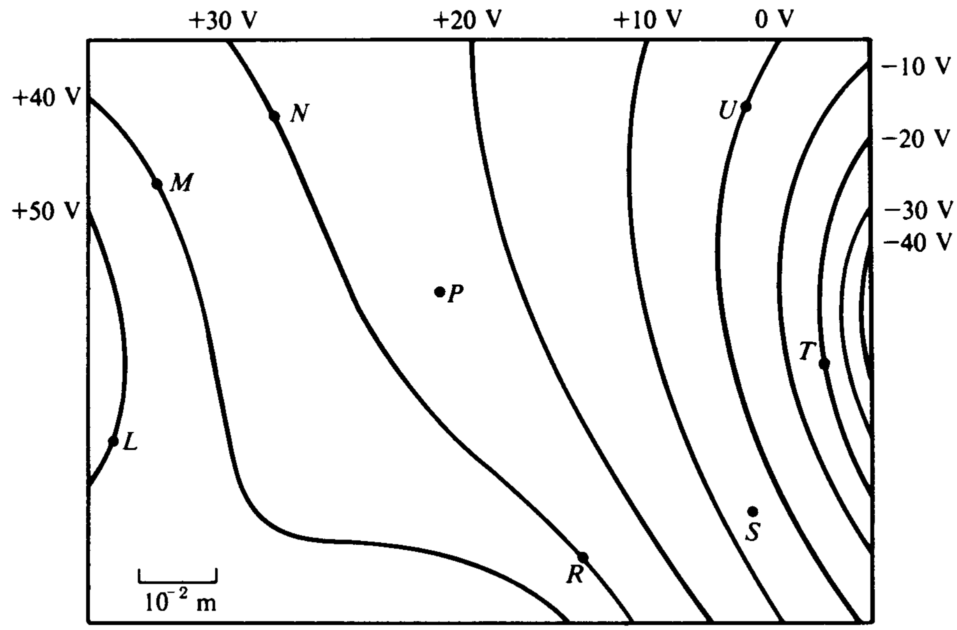

d. Write an approximate expression, in terms of q, $V_o$, and $E_o$, for the work required to bring a positive point charge q from infinity to point P, located a small distance s from point O as shown in the diagram above.

---




1982E1. Three point charges are arranged on the y-axis as shown above. The charges are +q at (0, a), -q at (0, 0), and +q at (0, -a). Any other charge or material is infinitely far away.

a. Determine the point(s) on the x-axis where the electric potential due to this system of charges is zero.
b. Determine the x and y components of the electric field at a point P on the x-axis at a distance x from the origin.
c. Using Gauss's law, determine the net electric flux through a spherical surface of radius r = 2a centered at the origin.




1986E1. Three point charges produce the electric equipotential lines shown on the diagram above.

a. Draw arrows at points L, N. and U on the diagram to indicate the direction of the electric field at these points.
b. At which of the lettered points is the electric field E greatest in magnitude? Explain your reasoning.
c. Compute an approximate value for the magnitude of the electric field E at point P.
d. Compute an approximate value for the potential difference, $V_M - V_S$, between points M and S.
e. Determine the work done by the field if a charge of $+5 \times 10^{-12}$ coulomb is moved from point M to point R.
f. If the charge of $+5 \times 10^{-12}$ coulomb were moved from point M first to point S, and then to point R, would the answer to (e) be different, and if so, how?

---

1987E1. A total charge Q is distributed uniformly throughout a spherical volume of radius R. Let r denote the distance of a point from the center of the sphere of charge. Use Gauss's law to derive an expression for the magnitude of the electric field at a point

a. outside the sphere, $r > R$;
b. inside the sphere, $r < R$.

The electrostatic potential is assumed to be zero at an infinite distance from the sphere.

c. What is the potential at the surface of the sphere?
d. Determine the potential at the center of the sphere.

2000E2. Three particles, A, B, and C, have equal positive charges Q and are held in place at the vertices of an equilateral triangle with sides of length f, as shown in the figures below. The dotted lines represent the bisectors for each side. The base of the triangle lies on the x-axis, and the altitude of the triangle lies on the y-axis.

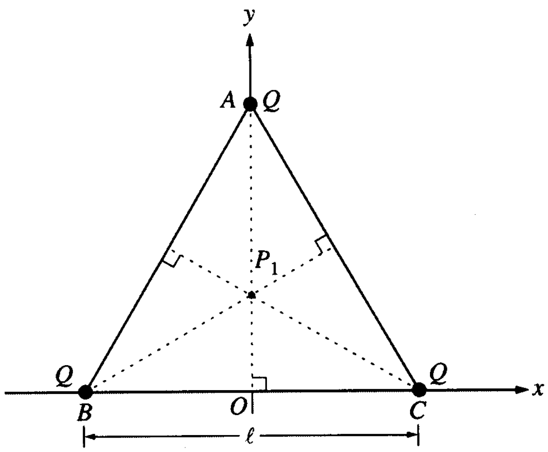


Figure 1

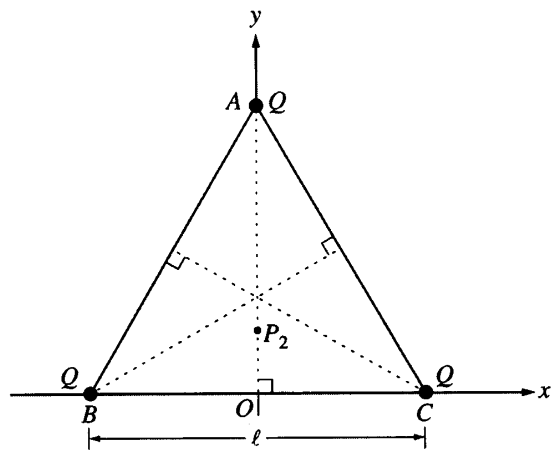


Figure 2

a.

i. Point $P_1$, the intersection of the three bisectors, locates the geometric center of the triangle and is one point where the electric field is zero. On Figure 1 above, draw the electric field vectors $E_A$, $E_B$, and $E_C$ at P, due to each of the three charges. Be sure your arrows are drawn to reflect the relative magnitude of the fields.

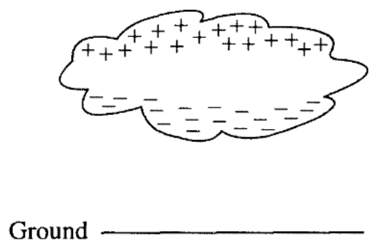
ii. Another point where the electric field is zero is point $P_2$ at (0, $y_2$). On Figure 2 above, draw electric field vectors $E_A$, $E_B$, and $E_C$ at $P_2$ due to each of the three point charges. Indicate below whether the magnitude of each of these vectors is greater than, less than, or the same as for point $P_1$.

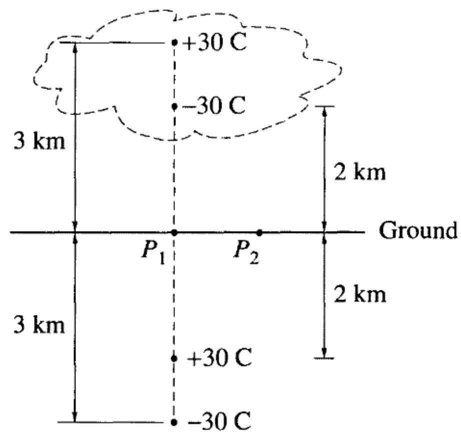
| | **Greater than at $P_1$** | **Less than at $P_1$** | **The same as at $P_1$** |
|---|---|---|---|
| $E_A$ | | | |
| $E_B$ | | | |
| $E_C$ | | | |

b. Explain why the x-component of the total electric field is zero at any point on the y-axis.

c. Write a general expression for the electric potential V at any point on the y-axis inside the triangle in terms of Q, $l$, and y.

d. Describe how the answer to part (c) could be used to determine the y-coordinates of points $P_1$ and $P_2$ at which the electric field is zero. (You do not need to actually determine these coordinates.)




Note: Figures not drawn to scale.

2001E1. A thundercloud has the charge distribution illustrated above left. Treat this distribution as two point charges, a negative charge of -30 C at a height of 2 km above ground and a positive charge of +30 C at a height of 3 km. The presence of these charges induces charges on the ground. Assuming the ground is a conductor, it can be shown that the induced charges can be treated as a charge of +30 C at a depth of 2 km below ground and a charge of -30 C at a depth of 3 km, as shown above right. Consider point $P_1$, which is just above the ground directly below the thundercloud, and point $P_2$, which is 1 km horizontally away from $P_1$.

a. Determine the direction and magnitude of the electric field at point $P_1$.

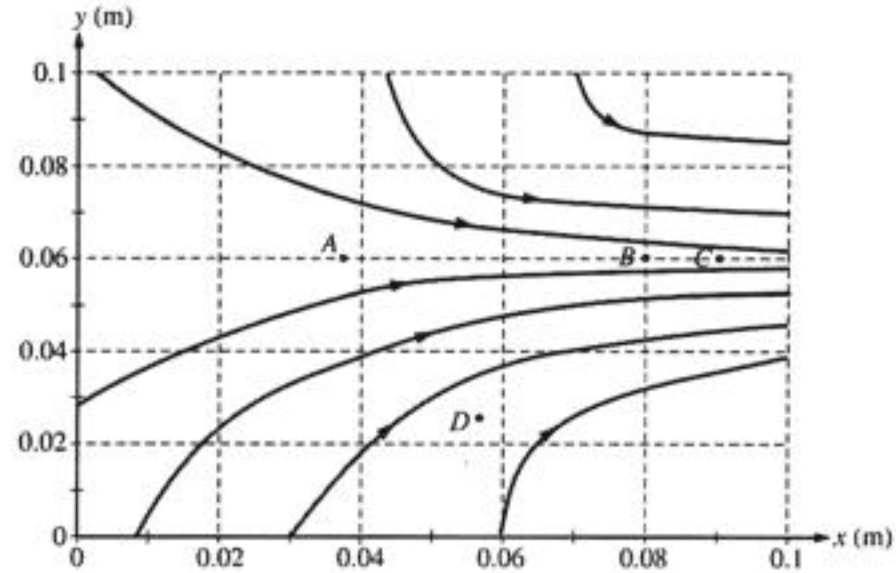
b. i. On the diagram on the previous page, clearly indicate the direction of the electric field at point $P_2$
ii. How does the magnitude of the field at this point compare with the magnitude at point $P_1$? Justify your answer:

______Greater ______Equal ______ Less

c. Letting the zero of potential be at infinity, determine the potential at these points.
i. Point $P_1$
ii. Point $P_2$

d. Determine the electric potential at an altitude of 1 km directly above point $P_1$.

e. Determine the total electric potential energy of this arrangement of charges.




2005E1. Consider the electric field diagram above.

a. Points $A$, $B$, and C are all located at y = 0.06 m .

i. At which of these three points is the magnitude of the electric field the greatest? Justify your answer.

ii. At which of these three points is the electric potential the greatest? Justify your answer.

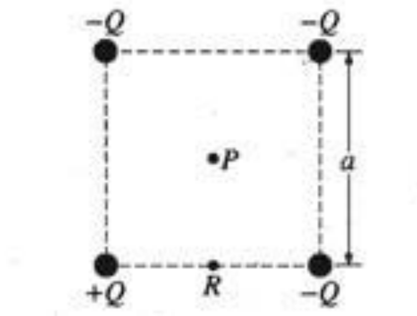
b. An electron is released from rest at point $B$.

i. Qualitatively describe the electron's motion in terms of direction, speed, and acceleration.

ii. Calculate the electron's speed after it has moved through a potential difference of 10 V.

c. Points $B$ and $C$ are separated by a potential difference of 20 V. Estimate the magnitude of the electric field midway between them and state any assumptions that you make.

d. On the diagram, draw an equipotential line that passes through point $D$ and intersects at least three electric field lines.




2006E1. The square of side a above contains a positive point charge $+Q$ fixed at the lower left corner and negative point charges $-Q$ fixed at the other three corners of the square. Point $P$ is located at the center of the square.

a. On the diagram, indicate with an arrow the direction of the net electric field at point $P$.

b. Derive expressions for each of the following in terms of the given quantities and fundamental constants.

   i. The magnitude of the electric field at point $P$

   ii. The electric potential at point $P$

c. A positive charge is placed at point $P$. It is then moved from point $P$ to point $R$, which is at the midpoint of the bottom side of the square. As the charge is moved, is the work done on it by the electric field positive, negative, or zero?

______ Positive ______ Negative ______ Zero

Explain your reasoning.

d.

   i: Describe one way to replace a single charge in this configuration that would make the electric field at the center of the square equal to zero. Justify your answer.

   ii. Describe one way to replace a single charge in this configuration such that the electric potential at the center of the square is zero but the electric field is not zero. Justify your answer.

2009E1.

A spherically symmetric charge distribution has net positive charge $Q_0$ distributed within a radius of $R$. Its electric potential $V$ as a function of the distance $r$ from the center of the sphere is given by the following.

$$V(r) = \frac{Q}{4\pi\varepsilon_0 R}\left[-2+3\left(\frac{r}{R}\right)^2\right] \quad \text{for } r < R$$

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r} \quad \text{for } r > R$$

Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) For the following regions, indicate the direction of the electric field $E(r)$ and derive an expression for its magnitude.

i. $r<R$

____ Radially inward ____ Radially outward

ii. $r>R$

____ Radially inward ____ Radially outward

(b) For the following regions, derive an expression for the enclosed charge that generates the electric field in that region, expressed as a function of $r$.

i. $r<R$

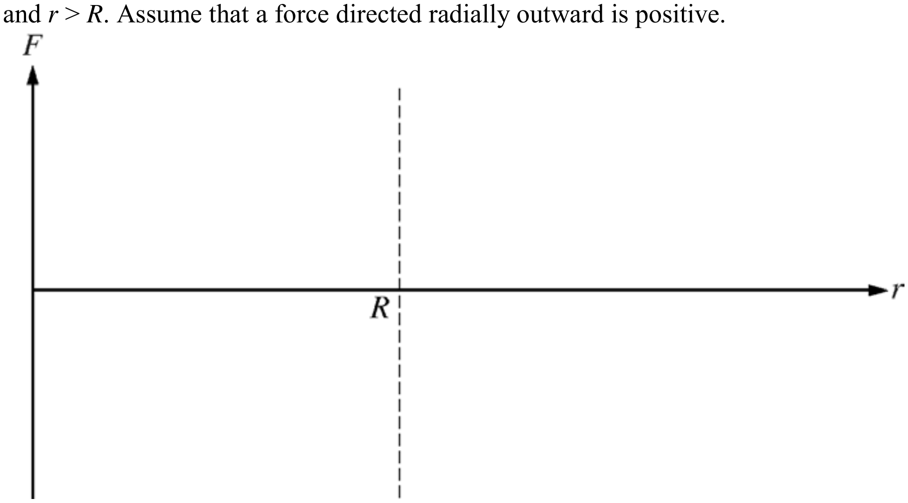
ii. $r>R$

(c) Is there any charge on the surface of the sphere ($r = R$)?

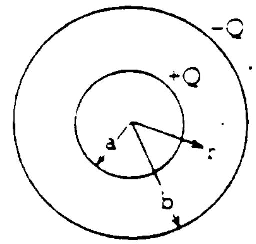
____ Yes ____ No

If there is, determine the charge. In either case, explain your reasoning.

(d) On the axes below, sketch a graph of the force that would act on a positive test charge in the regions $r < R$ and $r > R$. Assume that a force directed radially outward is positive.



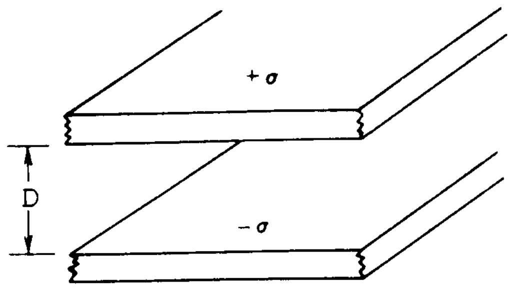
## SECTION D – Capacitance




1978E3. A capacitor is composed of two concentric spherical shells of radii a and b, respectively, that have equal and opposite charges as shown above. Just outside the surface of the inner shell, the electric field is directed radially outward and has magnitude $E_o$.

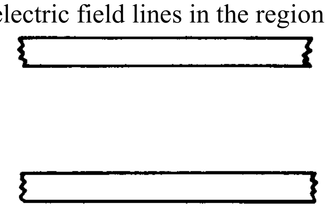
a. With the use of Gauss's law, express the charge +Q on the inner shell as a function of $E_o$ and a.
b. Write an expression for the electric field strength E between the shells as a function of $E_o$, a, and r.
c. What is the potential difference V between the shells as a function of $E_o$, a, and b?
d. Express the energy U stored in this capacitor as a function of $E_o$, a, and b.
e. Determine the value of a that should be chosen in order to maximize U, if $E_o$ and b are fixed.

---

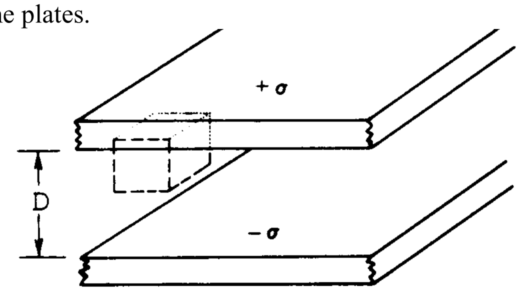



1980E2. A parallel-plate capacitor consists of two conducting plates separated by a distance D as shown above. The plates may be considered very large so that the effects of the edges may be ignored. The two plates have an equal but opposite surface charge per unit area, σ. The charge on either plate resides entirely on the inner surface facing the opposite plate.

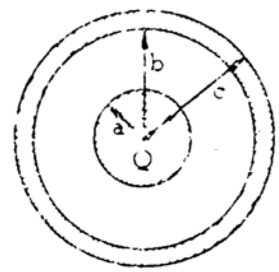
a. On the diagram below draw the electric field lines in the region between the plates.



b. By applying Gauss's law to the rectangular box whose upper surface lies entirely within the top conducting plate, as shown in the following diagram, determine the magnitude of the electric field E in the region between the plates.




c. A dielectric is inserted and fills the region between the plates. Is the electric field greater than, less than, or equal to the electric field when there is no dielectric? Describe the mechanism responsible for this effect. Recognize that the plates are not connected to a battery.

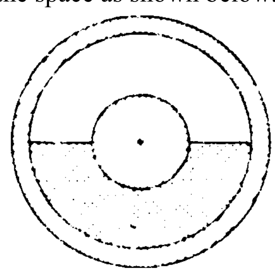



1981E1. A conducting sphere of radius a and charge Q is surrounded by a concentric conducting shell of inner radius b and outer radius c as shown above. The outer shell is first grounded; then the grounding wire is removed.

a. Using Gauss's law, determine the electric field in the region a < r < b, where r is the distance from the center of the inner sphere.

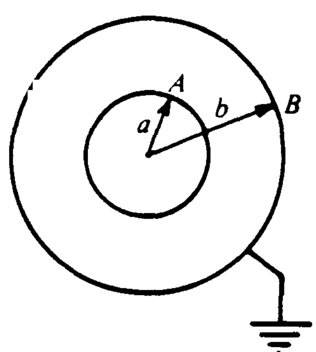
b. Develop an expression for the capacitance $C_o$ of the system of the two spheres.

A liquid dielectric with a dielectric constant of 4 is then inserted in the space between the conducting spheres and the shell, filling half of the space as shown below.



c. Determine the capacitance C of the system in terms of $C_{o.}$

---

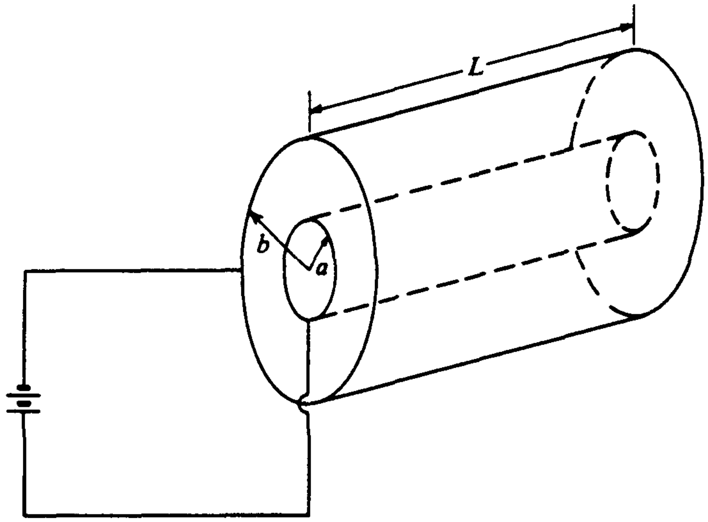



1983E1. Two concentric, conducting spherical shells, A and B, have radii a and b, respectively, (a < b). Shell B is grounded, whereas shell A is maintained at a positive potential $V_o$

a. Using Gauss's law, develop an expression for the magnitude E of the electric field at a distance r from the center of the shells in the region between the shells. Express your answer in terms of the charge Q on the inner shell.

b. By evaluating an appropriate integral, develop an expression for the potential $V_o$ in terms of Q, a, and b.

c. Develop an expression for the capacitance of the system in terms of a and b



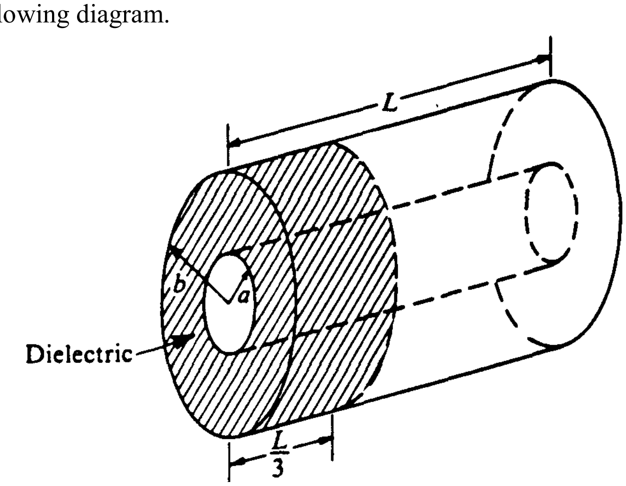

1985E1. A capacitor consisting of conducting coaxial cylinders of radii a and *b*, respectively, and length L is connected to a source of emf, as shown above. When the capacitor is charged, the inner cylinder has a charge + Q on it. Neglect end effects and assume that the region between the cylinders is filled with air. Express your answers in terms of the given quantities.

a. Use Gauss's law to determine an expression for the electric field at a distance r from the axis of the cylinder where $a < r < b$.

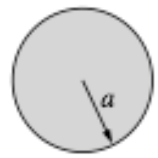
b. Determine the potential difference between the cylinders.

c. Determine the capacitance $C_o$ of the capacitor.

One third of the length of the capacitor is then filled with a dielectric of dielectric constant k = 2, as shown in the following diagram.

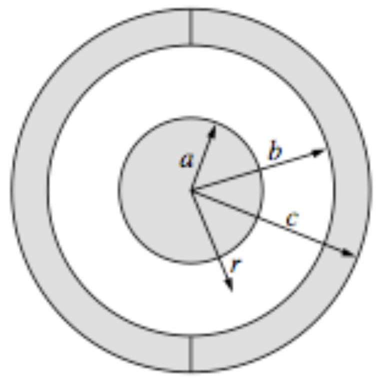



d. Determine the new capacitance C in terms of $C_o$.




1999E1. An isolated conducting sphere of radius a = 0.20 m is at a potential of -2,000 V.

a. Determine the charge $Q_0$ on the sphere.




The charged sphere is then concentrically surrounded by two uncharged conducting hemispheres of inner radius b = 0.40 m and outer radius c = 0.50 m, which are joined together as shown above, forming a spherical capacitor. A wire is connected from the outer sphere to ground, and then removed.

b. Determine the magnitude of the electric field in the following regions as a function of the distance r from the center of the inner sphere.
   i. $r < a$
   ii. $a < r < b$
   iii. $b < r < c$
   iv. $r > c$

c. Determine the magnitude of the potential difference between the sphere and the conducting shell.

d. Determine the capacitance of the spherical capacitor.

ANSWERS - AP Physics C Multiple Choice Practice – Electrostatics

## SECTION A – Coulomb's Law and Coulomb's Law Methods

| | Solution | Answer |
|---|---|---|
| 1. | Outside a point charge $E = kQ/r^2$ | C |
| 2. | The electric field due to +Q point to the right along the x axis, for an opposing field (pointing to the left along the x axis) we need a negative charge to the left of point P. Since the magnitude of the negative charge is larger, it needs to be farther away for the field from the negative charge to have the same magnitude as the field from the positive charge. | C |
| 3. | E fields cancel from the two charges at the midpoint | A |
| 4. | The force on the bottom is to the right, the force on the top is to the left and larger. There is a non-zero net force and a net torque. | B |
| 5. | The force is proportional to the product of the charges. Before touching, this product is $2Q^2$. After touching and separating, the new charges are $(+2Q - Q)/2 = Q/2$. The new force is proportional to the new product $(Q/2)^2 = Q^2/4$, one-eighth of the original product | E |
| 6. | F is proportional to $q \times q/d^2$ | E |
| 7. | $\mathbf{F} = \mathbf{E}q$, no relevance to velocity | D |
| 8. | Electric field lines point away from + charges and toward – charges. Symmetry is required for the fields to cancel at the center | A |
| 9. | Electric field lines point away from + charges and toward – charges. Construct each field vector | C |
| 10. | At the center of the ring, the field is zero due to symmetry/cancellation. Only choice B has this feature. | B |
| 11. | $\mathbf{E} = \mathbf{F}/q$, without knowing the force, you cannot know the charge | C |
| 12. | From symmetry, the electron will be attracted to the center of the positive piece and repelled from the center of the negative piece | B |
| 13. | $E = kQ/r^2$ | E |
| 14. | Negative charges in sphere X are driven into sphere Y | D |
| 15. | Negative charges in sphere 1 are driven into sphere 2 | C |
| 16. | The test charge is repelled to the left and down | E |
| 17. | Each force is equal and they are at right angles (Pythagorean theorem) | D |
| 18. | If there is a single force on an object, it must be accelerating. Negative charges experience forces opposite the direction of electric field lines. | A |
| 19. | E = 0 at the midpoint and is large near each of the charges, growing to infinity as the charge is approached | A |
| 20. | E points directly away from the positive charge and toward the (larger) negative charge | E |
| 21. | To be zero, the point must be closer to the smaller magnitude of charge (left of the origin) and where the vectors point in opposite directions (outside the charges – to the left of Q). Since the negative charge is 4 times greater, the point must be twice as far from the -4Q than from the Q. | A |
| 22. | Charge separation in I, Charging by induction in III (opposite charge of the rod) | D |

## SECTION B – Gauss's Law

| | | |
|---|---|---|
| 23. | The field from an infinite sheet of charge is uniform and, in this case, equal in magnitude, pointing away from the sheet on the left and toward the sheet on the right. The E field cancels outside the sheets. | E |
| 24. | The arrangement of the field lines inside will change as the charge is moved as it is due to that charge. Outside the sphere, the induced surface charge is evenly distributed around the outer surface. | B |
| 25. | By definition. | A |
| 26. | No net charge is enclosed | B |
| 27. | Linear (proportional to r) inside, proportional to $1/r^2$ outside | C |
| 28. | If four lines is proportional to +Q, then 8 inward pointing lines is proportional to -2Q. This would be the *net* charge enclosed | E |
| 29. | By Gauss's Law, the flux is proportional to the net charge enclosed | E |
| 30. | Left side: $EA = E(4\pi r^2)$ (area of the Gaussian surface) Right side: Q enclosed = fraction of Q inside = Q×(volume of Gaussian surface/volume of sphere) | D |
| 31. | The charge +Q induces a charge –Q on the inner surface of the box, inducing a charge +Q on the outer surface | A |
| 32. | Gauss's Law needs some symmetry and regularity to the electric field for a convenient Gaussian surface to be drawn and used. | C |
| 33. | E fields from external charges will not permeate into a conductor | A |
| 34. | Linear (proportional to r) inside, proportional to $1/r^2$ outside | D |

## SECTION C – Electric Potential and Energy

| | | |
|---|---|---|
| 35. | By definition | E |
| 36. | $V = \Sigma kQ/r$, positive and approaching infinity as it nears the positive charge and negative and approaching negative infinity near the negative charge. Since the positive charge is larger, the zero point is closer to the smaller charge. | D |
| 37. | For potential to be zero, we need two positive and two negative charges. For the electric field to be zero, we need symmetry about the origin to cancel the fields. | E |
| 38. | With all positive charges, the potential can never be zero at the origin, while the symmetry allows the electric fields to cancel | D |
| 39. | The field from an infinite sheet of charge is uniform and, in this case, equal in magnitude, pointing away from the sheet on the left and toward the sheet on the right. The E field cancels outside the sheets. With E = 0, the potential is uniform outside the sheets, positive on the left and negative on the right, with a linear transition between as E is uniform. | B |
| 40. | E = 0 inside a conductor, which means V is constant. Outside the sphere, V varies as 1/r | A |
| 41. | $W_{field} = -q\Delta V$ | D |

42. When charged conductors are connected, charge flows when there is a difference in potential, until there is no longer a difference in potential. A

43. $V = \Sigma kQ/r$, with the symmetry $V_R = V_S$. $W = q\Delta V = 0$ D

44. $V = \Sigma kQ/r$, point A represents the largest sum of Q/r for the two charges A

45. Outside the spheres E = 0 so V is constant (and zero relative to infinity). Once inside the negative shell, the potential is that for a positively charged conducting sphere (constant inside, proportional to 1/r outside) D

46. E = 0 in and on conductors which means V is constant throughout E

47. $W = q\Delta V = qEd$ A

48. V = kQ/r so the smaller sphere is at a higher potential. Current flows from higher to lower potential. B

49. For a spherical shell of charge, E = 0 inside, which means V is constant, equal to its value on the surface A

50. Standard spherical charge distribution formulae D

51. $V = \Sigma kQ/r$, every point on the ring is equidistant from any given point on the x axis so V = kQ/r, where r is the distance from a point on the ring to a point on the x axis (Pythagorean theorem) B

52. $V = \Sigma kQ/r$ and all are positive so they all add. The electric field vectors cancel. D

53. There is a locus of points around +Q that satisfy the condition $k(-2Q)/r_1 + k(+Q)/r_2 = 0$. On the x axis, one is at point P, the other is between the charges (and closer to the smaller charge) D

54. $E = –dV/dr$ E

55. From the spherical symmetry, the electric field between the shells is only dependent on the inside shell. A

56. Relative to infinity, on the outer surface of the larger shell, the potential is $k(Q_1 + Q_2)/r_2$. Once inside there is no more change to the potential due to $Q_2$, but still varies as 1/r due to $Q_1$ until the final position is reached. E

57. The potential difference between the plates is 4V, this can be produced with two 2V batteries in series (note the positive plate is on the left) D

58. Electrons are forced from low potential toward high potential. The electric field strength is necessary to know the magnitude. D

59. The incremental amount of work required to bring a small amount of charge dq is dW = V(dq) where V is the potential relative to infinity at that time, which is kq/R (q being the amount of charge currently on the sphere) C

60. E is zero closer to the smaller charge and where the vectors will point in opposite directions C

61. $V = \Sigma kQ/r$, with no negative charges in the vicinity, V can never be zero E

62. E is proportional to the gradient of V, in this case, the slope. F is largest where E is largest which is where the greatest slope occurs. D

63. $\Delta V = W/q$ (distance is not needed) C

64. $F = ke^2/R^2 = mv^2/R$ and $K = ½ mv^2$ (K > 0) B

65. $E = –dV/dr$ C

66. Since $E = -dV/dr = -2kr$ the field points toward the origin and electrons experience forces opposite in direction to electric fields — B

67. With no work done by the field, the charge must be moving along an equipotential, which is perpendicular to E fields. $W = -q\Delta V$ with $\Delta V = 0$ — D

68. No work is required to move the charge inside the sphere so the only work done is to move the charge to the surface. $W = q\Delta V = q(V_R - V_r)$ where $V_R = kQ/R$ and $V_r = kQ/r$ — E

69. $V = -\int_0^{0.5} \boldsymbol{E} \cdot \boldsymbol{dx}$ — B

70. (misplaced question) $U_C = ½ CV^2$ — B

71. $q\Delta V = ½ mv^2$ — C

72. The magnitude of E is the slope of the graph, which is zero for $r < R$ and since V is proportional to $1/r$ for $r > R$, then E is proportional to $1/r^2$ for $r > R$ — C

73. Due to symmetry, all fields cancel — A

74. The potential at the center is $V = \Sigma kQ/R = 6kQ/R$ and $W = Q\Delta V$ — D

75. E points from high to low potential and E lines are perpendicular to equipotential lines — A

76. E has the greatest magnitude where V has the largest gradient (the lines are closest) — B

77. $W = q\Delta V$ — B

78. For charge to be distributed throughout an object, it must not be a conductor, otherwise the charge would move to the surface of the object — E

## SECTION D – Capacitance

79. Once disconnected and isolated, the charge on the capacitor remains constant. Doubling the plate separation halves the capacitance ($C \propto 1/d$). $V = Q/C$ and $U_C = Q^2/2C$ or $1/2\ QV$ — D

80. $C \propto A/d$ — C

81. In series $C_{total} = (\Sigma(1/C))^{-1}$. For N identical capacitors in series $C_{total} = C/N$ — B

82. Each capacitor has 2 volts across it. $U_C = 1/2\ CV^2$ — C

83. An isolated capacitor has constant charge. Adding a dielectric increases the capacitance and $V = Q/C$ — B

84. Like resistors in parallel, the mathematics dictate that the total is less than the smallest capacitance — D

85. From Gauss's Law: $E \propto \sigma$ for a sheet of charge — D

86. $C \propto A/d$ — B

87. An isolated capacitor has constant charge. Adding a dielectric increases the capacitance and $V = Q/C$ — B

88. Each of the three branches has an equivalent capacitance of $C_{total} = C/N = 2\ \mu F/2 = 1\ \mu F$. In parallel, the total capacitance is the sum of the individual capacitances (branches in this case) — C

89. For a 2 μF capacitor to have a charge of 6 μC it needs a voltage of $V = Q/C = 3$ V. Since each branch has two capacitors in series the branch should have a total voltage of 6 V — C

90. $C \propto \kappa A/d$ where $\kappa_{glass} > 1$ E

91. $K = W = q\Delta V$ where $V = Q/C$ and $C \propto A/d$ A

92. The two capacitors in parallel have an equivalent capacitance of 6 μF. In series with the 3 μF, the total capacitance is $C_{total} = (C_1 \times C_2)/(C_1 + C_2)$ B

93. Between the set of two parallel capacitors with an equivalent capacitance of 6 μF and the 3 μF capacitor, the 12 V splits in the ratio of 1:2 (8 V and 4 V) with the larger voltage across the smaller capacitance D

94. $E = V/d$ B

95. $C \propto \kappa A/d$ where $\kappa > 1$ A

96. In series $C_{total} = (\Sigma(1/C))^{-1}$. For N identical capacitors in series $C_{total} = C/N$ E

97. $C \propto \kappa A/d$ D

## SECTION A – Coulomb's Law and Coulomb's Law Methods

1991E1

a. From each charge $E = kQ/r^2$, but at the origin, the vectors from the two charges point in opposite directions and cancel so $E = 0$
b. $V = kQ/a + kQ/a = 2kQ/a$
c. Due to each charge $E = kQ/r^2 = kQ/(a^2 + b^2)$, but the x components cancel so we only need to add the y components $E_y = [kQ/(a^2 + b^2)] \sin\theta = [kQ/(a^2 + b^2)]\, b/(a^2 + b^2)^{1/2}$ which gives $E = 2kQb/(a^2 + b^2)^{3/2}$
d. The particle will repel from the closer charge and move through the origin until it repelled and reversed by the charge on the other side of the origin. The particle will oscillate about the origin.
e. The particle will move away from the origin
   $U = 2kQq/a = ½\, mv^2$ giving $v = 2(kQq/ma)^{1/2}$
f. The particle will oscillate about the origin

---

1994E1

a. $V = \Sigma kQ/r$ and since all the charge is equidistant from the center we get $V = kQ/R$ where $Q = \lambda L = \lambda 2\pi R$ so $V = 2\pi k\lambda$
b. The electric field from each infinitesimal piece of the ring is cancelled from a piece on the opposite side of the ring. $E = 0$
c. The total charge on the rod is the charge per unit length times the length of the rod: $L = 2R\theta$ so $Q = 2R\theta\lambda$
d. As in part a, the charge is all equidistance so we substitute the new amount of charge into the expression $V = 2k\theta\lambda$
e. $E = kQ/R^2$, all of the charge is equidistant, but we must take direction into account. The y components will cancel so we only need to consider the x (horizontal) component from each infinitesimal element $dq = \lambda R\, d\theta$

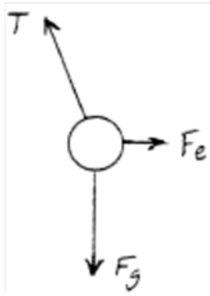
$$E = \int_{-\theta}^{\theta} \frac{kdq}{R^2} cos\theta d\theta = \int_{-\theta}^{\theta} \frac{kd\lambda R}{R^2} cos\theta d\theta = \int_{-\theta}^{\theta} \frac{k\lambda}{R} cos\theta d\theta = \frac{k\lambda}{R} sin\theta|_{-\theta}^{\theta} = \frac{2k\lambda}{R} sin\theta$$

pointing to the right

---

1998E1

a.




From the FBD: T cos θ = mg and T sin θ = $F_e = kQ_AQ_B/r^2$ solving the two equations gives Q = $1.9 \times 10^{-7}$ C

b. The angle is smaller since the charge will move on a conductor so the spacing between the charges is farther apart, reducing the electric force.

c.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

Left side: E(2πrL)

Right side: $\lambda L/\varepsilon_0$

This gives $E = \lambda/2\pi r\varepsilon_0$

d. F = qE = 0.14 N

e.

$$W = q\Delta V = -q\int Edr = -q\int_{1.5}^{0.3}\frac{1.8\times10^3}{r}dr = -.216[\ln r]_{1.5}^{0.3} = 0.35\,J$$

---

1999E3

a. The charge on any section of the ring is equidistant from a point on the x axis so $V = kQ/r = kQ/(x^2 + R^2)^{1/2}$

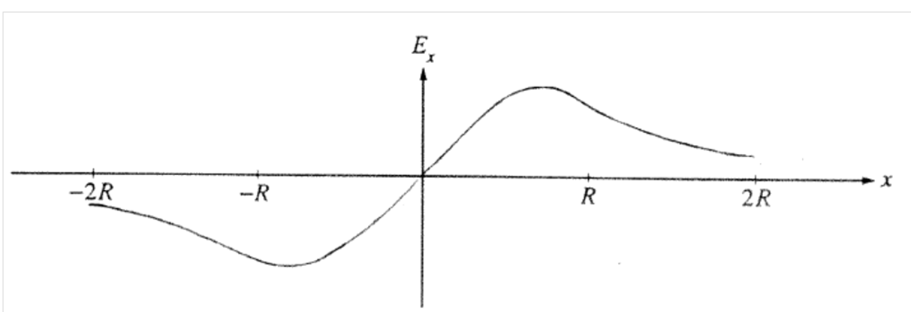
b. i. $E = -dV/dr = kQx/(x^2 + R^2)$

ii. zero (the components cancel)

c. i. Set $dE_x/dx = 0$ gives $x = R/\sqrt{2}$

ii. Plugging in the value found above gives $E = kQ/(3\sqrt{3})R^2$

d.




e. The electron oscillates about the origin (when displaced in the +x direction there is a force in the –x direction and vice versa

---

2002E1

a. The total charge is $q = \lambda L = \lambda(2\pi R/3) = 3.1 \times 10^{-6}$ C

b. $E = kQ/R^2$, all of the charge is equidistant, but we must take direction into account. The y components will cancel so we only need to consider the x (horizontal) component from each infinitesimal element $dq = \lambda R\, d\theta$

$$E = \int_{120}^{240} \frac{kdq}{R^2} cos\theta d\theta = \int_{120}^{240} \frac{kd\lambda R}{R^2} cos\theta d\theta = \int_{120}^{240} \frac{k\lambda}{R} cos\theta d\theta = \frac{k\lambda}{R} sin\theta|_{120}^{240} = 2.3 \times 10^6 N/C$$

To the right

c. $V = kQ/R = 2.8 \times 10^5$ V (all charge is equidistant from point O)

d. $F = qE = 3.7 \times 10^{-13}$ N

e. The proton moves off to the right, but as the force decreases the proton's acceleration decreases, all the while speeding up to the right asymptotic to some value

---

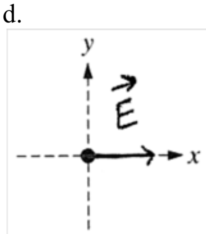
2010E1

a. Compared to points *A* and *C*, point *B* is closer to most, and possibly all, points along the charge distribution. Since potential varies inversely with distance, point *B* has the highest potential. Points *A* and *C* have the same potential by symmetry.

b. All points on the arc are a distance *R* from point *P*. Since potential is a scalar quantity, the potential will be the same as that of a point charge with charge *Q* located a distance *R* away
$V = kQ/R$

c. Energy is conserved: $U_i = K_f$
$qV = q(kQ/R) = ½ mv^2$ gives $v = (2kqQ/mR)^{1/2}$

d.



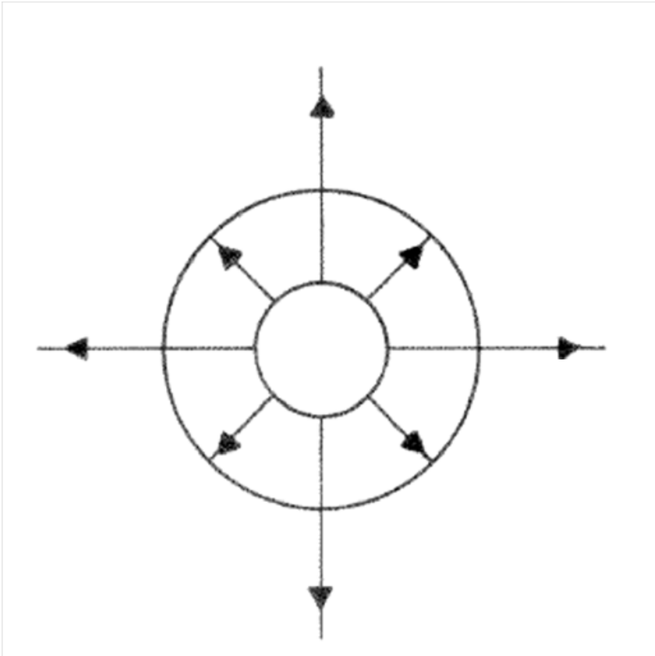

e. $E = kQ/R^2$, all of the charge is equidistant, but we must take direction into account. The y components will cancel so we only need to consider the x (horizontal) component from each infinitesimal element dq
where $dq = (2Q/\pi)d\theta$

$$|E| = \int_{-\pi/4}^{\pi/4} \frac{2kQ}{\pi R^2} cos\theta d\theta = \frac{2kQ}{\pi R^2} sin\theta|_{-\pi/4}^{\pi/4} = \frac{2\sqrt{2}kQ}{\pi R^2}$$

# SECTION B – Gauss's Law

1976E1

a.



b.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

E $(4\pi r^2) = +2Q/\varepsilon_0$

$E = Q/2\pi\varepsilon_0 r^2$

c.

$$\Delta V = -\int E \cdot dr = -\int_R^{3R} \frac{2kQ}{r^2} dr = \left.\frac{2kQ}{r}\right|_R^{3R} = \frac{4kQ}{3R}$$

We can actually ignore the sign since the question asks for the general potential difference between the spheres

d. The total charge would distribute on the outer surface, leaving no charge on the inner sphere and +Q on the outer.

---

1979E1

a.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

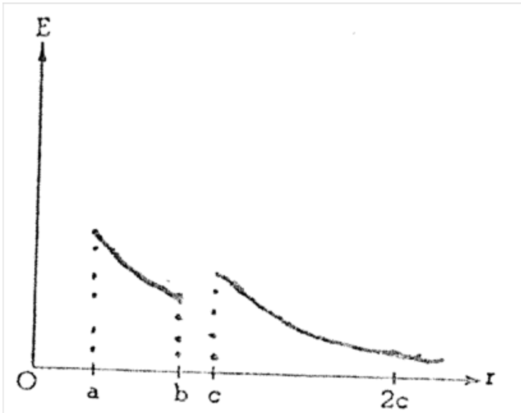
$E = kQ/r^2$

b. $r > c$: $Q_{enc} = 2Q$ so $E = 2kQ/r^2$

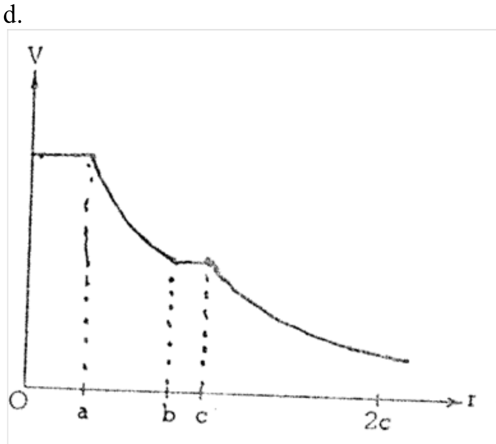
$b < r < c$: inside a conductor $E = 0$

$r < a$: inside a conductor $E = 0$

c.




d.




e.

$$\Delta V = -\int E \cdot dr = -\int_{\infty}^{b} E \cdot dr = -\int_{\infty}^{c} \frac{2kQ}{r^2} dr - \int_{c}^{b} 0\, dr = \frac{2kQ}{c}$$

---

1984E2

a. $V_1 = E_1 a$ and $V_2 = E_2 b$ so $E_1/E_2 = b/a$

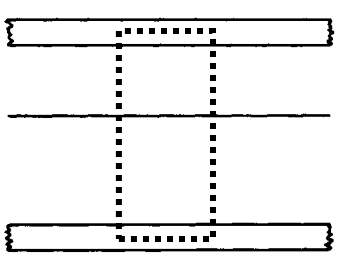
b.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

Left side: $E_1A + E_2A$ Right side: $Q_{enc} = \sigma A$

So $E_1 + E_2 = \sigma/\varepsilon_0$

c.



E = 0 in a conductor so $Q_{enc} = 0$ so $\sigma_1 A + \sigma_2 A + \sigma A = 0$, or $\sigma_1 + \sigma_2 = -\sigma$

d. $E_1 + E_2 = \sigma/\varepsilon_0$ where $E_1 = V/a$ and $E_2 = V/b$
$V/a + V/b = \sigma/\varepsilon_0$ gives $V = \sigma ab/\varepsilon_0(a + b)$

---

1988E1

a. $V = kQ/a$ so $Q = aV/k$

b.
$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$
$E(4\pi r^2) = Q/\varepsilon_0$
$E = aV/r^2$

c.
$$\Delta V = -\int E \cdot dr = -\int_b^a E \cdot dr = -\int_b^a \frac{kQ}{r^2} dr \ = kQ\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{V(b-a)}{b}$$

d. $Q = C\Delta V$
$aV/k = CV(b - a)/b$
$C = ab/k(b - a)$

---

1989E1

a. $E = 0$ since the net charge is zero

b. $V = 0$ because the net charge is zero

c.
$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$
The portion of the negative charge within the Gaussian surface is $-Qr^3/R^3$ (volume ratio)
The net charge enclosed is therefore $Q_{enc} = Q - Qr^3/R^3$
Applying this to Gauss's Law gives $E = kQ(1/r^2 - r/R^3)$

d.
$$\Delta V = -\int E \cdot dr = -\int_R^r E \cdot dr = -\int_R^r \frac{kQ}{r^2} - \frac{kQr}{R^3} dr \ = -kQ\left(-\frac{1}{r} - \frac{r^2}{2R^3}\right) = kQ\left(\frac{1}{r} + \frac{r^2}{2R^3} - \frac{3}{2R}\right)$$

---

1990E1

a.
$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$
The portion of the charge within the Gaussian surface is $Qr^3/R^3$ (volume ratio)
$E(4\pi r^2) = Qr^3/R^3$ giving $E = kQr/R^3$

b. $Q_{enc} = Q$ so $E = kQ/r^2$

c. inside a conductor $E = 0$

d. Since $E = 0$ for r inside the conductor, the total charge enclosed by such a sphere must be zero. The inner surface therefore has induced charge $-Q$ over its surface of area $4\pi(2R)^2$ which gives $\sigma = -Q/16\pi R^2$

e. Since the shell has a net charge $+q$, the outer surface charge combined with the inner surface charge shoud add to this value: $q_{outer} + q_{inner} = +q = q_{outer} - Q$ so $q_{outer} = q + Q$ distributed over its surface of area $4\pi(3R)^2$ which gives $\sigma_{outer} = (q + Q)/36\pi R^2$

---

1992E1

a. From a given volume charge density: $Q = \int \rho \, dV$
$$Q = \int_0^a \rho 4\pi r^2 dr = \int_0^a 4\pi\beta r^3 dr = 4\pi\beta \left.\frac{r^4}{4}\right|_0^a = \beta\pi a^4$$

b. i. for r > a the sphere can be treated as a point charge where $E = kQ/r^2 = \beta a^4/4\varepsilon_0 r^2$

ii. R = a is a limiting case of (i) above so just substitute r = a, which gives $E = \beta a^2/4\varepsilon_0$

iii. Using Gauss's Law: left side = $E4\pi r^2$; right side:

$$Q_{enc} = \int_0^r \rho 4\pi r^2 dr = \int_0^r 4\pi\beta r^3 dr = 4\pi\beta \left.\frac{r^4}{4}\right|_0^r = \beta\pi r^4$$

giving $E = \beta r^2/4\varepsilon_0$

c. i. This is a limiting case of r > a, where the sphere can be treated as a point charge and $V = kQ/r = \beta a^3/4\varepsilon_0$

ii.

$$\Delta V = -\int E \cdot dr = -\int_\infty^0 E \cdot dr = -\int_\infty^a \frac{\beta a^4}{4\varepsilon_0 r^2} dr - \int_a^0 \frac{\beta r^2}{4\varepsilon_0}\, dr = V(r = a) - \frac{\beta}{4\varepsilon_0}\left.\frac{r^3}{3}\right|_a^0 = \frac{\beta a^3}{3\varepsilon_0}$$

---

<u>1995E1</u>

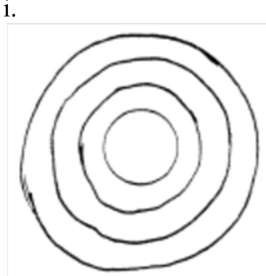
a.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

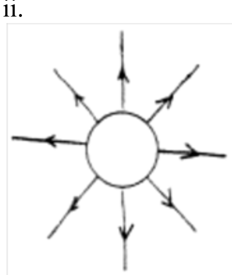
$E(2\pi rL) = Q_{enc}/\varepsilon_0 = \lambda L/\varepsilon_0$

$E = \lambda/2\pi\varepsilon_0 r$

b. i.



ii.



c. i.

$$\Delta V = -\int E \cdot dr$$

$$V_C - V_D = -\int_D^C E \cdot dr = \int_C^D E \cdot dr = \int_C^D \frac{\lambda}{2\pi\varepsilon_0}\frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0}\ln r|_a^{3a} = \frac{\lambda}{2\pi\varepsilon_0}\ln 3$$

ii. $W = Q\Delta V = (Q\lambda \ln 3)/2\pi\varepsilon_0$

d.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$\varepsilon_0 E(2\pi rl) = \int_0^r \rho_0 \left(\frac{r}{a}\right)^{\frac{1}{2}} (2\pi rl)dr = \frac{2\pi l\rho_0}{\sqrt{a}} \int_0^r r^{\frac{3}{2}} dr = \frac{4\pi l\rho_0 r^{\frac{5}{2}}}{5\sqrt{a}}$$

$$E = \frac{2\rho_0 r^{\frac{3}{2}}}{5\varepsilon_0\sqrt{a}}$$

1996E1

a. The sphere is metal, all charge resides on the outer surface and can be treated as a point charge where $V = kQ/r$
$V_0 = kQ_0/a$ so $Q_0 = 4\pi\varepsilon_0 V_0 a$ or $V_0 a/k$

b. i. E inside a conductor is zero, therefore the inner surface of the shell must carry a charge equal and opposite to that of the sphere to cancel the field from the sphere: $Q_{inner} = -Q_0$
ii. The net charge on the shell is zero so the charge on the outer surface must cancel that on the inner surface so $Q_{outer} = +Q_0$

c. i. E = 0 inside a conductor (zero field has no direction)
ii. Only the charge inside the region contributes to the field. It can be treated as a point charge: $E = kQ_0/r^2$ directed radially outward
iii. E = 0 inside a conductor (zero field has no direction)
iv. Equivalent to the situation in (ii) $E = kQ_0/r^2$ directed radially outward

d. Yes, the charges induced on the inner and outer surfaces appear as the shell is being assembled and the fact that the negative charge is closer means there is a net attractive force.

e.

$$\Delta V = -\int E \cdot dr = -\int_\infty^{2b} \frac{kQ_0}{r^2} dr - \int_b^a \frac{kQ_0}{r^2} dr = kQ_0\left(\frac{1}{r}\Big|_\infty^{2b} + \frac{1}{r}\Big|_b^a\right) = kQ_0\left(\frac{2b-a}{2ab}\right) = V_0\left(\frac{2b-a}{2b}\right)$$

---

1997E2

a. The sphere can be treated as a point charge: $E = kQ/R^2$ to the right

b. The flux is represented by either side of Gauss's Law: $Q/\varepsilon_0$ or $4\pi R^2 E$

c. The new field is the vector sum of the fields from the sphere and the point charge
From the sphere: $kQ/R^2$ to the right   from the point charge: $kQ/R^2$ downward
from the Pythagorean theorem: $E_{net} = \sqrt{2}kQ/R^2$ 45° from the horizontal, down and to the right

d. i. Only those charges inside contribute to the net electric flux: $q_2$ and $q_3$
ii. All four charges contribute to the value of the electric field at point $P_1$
iii. Different, the electric field is the sum of the individual fields while the flux is the sum of components over the whole surface

e. No. Charges may exist outside the surface that would contribute to a field on the surface or, for example, the surface may enclose a dipole, which would cause a net field at some points

f. Yes. A zero net field means the flux is also zero. Since charges inside the surface contribute to the net flux, there can be no net charge inside the surface.

---

2003E1

a. The sphere can be treated as a point charge
i. $E = kQ/r^2$
ii. $V = kQ/r$

b. The proton will move away from the sphere, its velocity will increase to reach some final value asymptotically while the acceleration decreases

c. $K = U_r - U_R = (-keQ/r) - (-keQ/R) = keQ(1/R - 1/r)$

d. $\rho_0$ can be determined by integrating the volume distribution and setting it equal to the total charge Q

$$Q = \int_0^R \rho(r)\, dV = \int_0^R \rho_0\left(1 - \frac{r}{R}\right) 4\pi r^2 dr = 4\pi\rho_0 \frac{R^3}{12} \; so \; \rho_0 = \frac{3Q}{\pi R^3}$$

e.

$$Q_{enc} = \int_0^r \rho(r)\, dV = \int_0^r \frac{3Q}{\pi R^3}\left(1 - \frac{r}{R}\right) 4\pi r^2 dr = \frac{Q}{R^3}\left(4r^3 - \frac{3r^4}{R}\right)$$

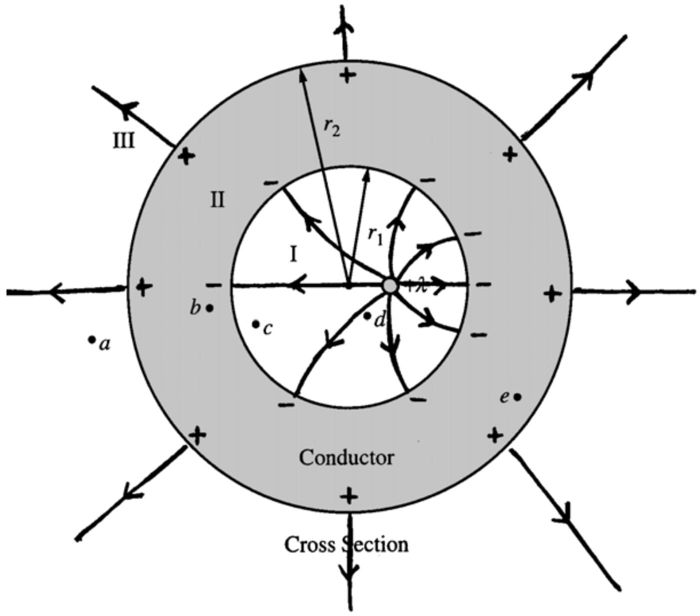
Substituting into Gauss's Law gives

$$E = \frac{kQr}{R^3}\left(4 - \frac{3r}{R}\right)$$

---

2004E1

a.




b. 4-3-2-1-3 respectively (b and e within the conductor are at the same potential)

c. i.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$E(2\pi rL) = Q_{enc}/\varepsilon_0 = \lambda L/\varepsilon_0$

$E = \lambda/2\pi\varepsilon_0 r$

ii. $Q_{enc} = Q_{line} + Q_{shell,inside}$ where the portion of the shell's charge within the Gaussian surface is $\rho(\pi r^2 l - \pi r_1^{\,2} l)$ so combining the electric fields from the line and shell gives $E = \lambda/2\pi\varepsilon_0 r + (\rho/2\varepsilon_0 r)(r^2 - r_1^{\,2})$

iii. Outside the shell, we can just replace r with $r_2$, enclosing all the charge of the shell:

$E = \lambda/2\pi\varepsilon_0 r + (\rho/2\varepsilon_0 r)(r_2^{\,2} - r_1^{\,2})$

---

2007E2

a. For all parts, the left side of Gauss's Law will be $E4\pi r^2$, the right side will be related to the net charge enclosed

i. The portion of the charge within the Gaussian surface is $Qr^3/a^3$ (volume ratio)

$E(4\pi r^2) = Qr^3/a^3$ giving $E = kQr/a^3$

ii. Between the sphere and the shell we can treat the sphere as a point charge where $E = kQ/r^2$

iii. $Q_{enc} = Q - \rho_0 V_0$ where $\rho_0$ is the charge density of the outer sphere and $V_0$ is the volume of the outer sphere enclosed by the Gaussian surface

$$\rho_0 = \frac{Q}{\frac{4}{3}\pi(3a)^3 - \frac{4}{3}\pi(2a)^3} = \frac{Q}{\frac{4}{3}\pi a^3(19)}$$

$$V_0 = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi(2a)^3 = \frac{4}{3}\pi(r^3 - 8a^3)$$

$$Q_{enc} = Q - \frac{Q}{\frac{4}{3}\pi a^3(19)}\frac{4}{3}\pi(r^3 - 8a^3) = \frac{Q}{19}\left(27 - \frac{r^3}{a^3}\right)$$

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0} = E(4\pi r^2)$$

$$E = \frac{Q}{76\pi\varepsilon_0 r^2}\left(27 - \frac{r^3}{a^3}\right)$$

iv. No charge is enclosed so $E = 0$

b. Since $V = \int E dr$ and $E = 0$ from infinity to 3a, it follows that $V = 0$

c.

$$\Delta V = -\int E \cdot dr = -\int_{2a}^{a} E \cdot dr = -\int_{2a}^{a} \frac{kQ}{r^2} dr = -kQ - \frac{1}{r}\bigg|_{2a}^{a} = kQ\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{Q}{8\pi\varepsilon_0 a}$$

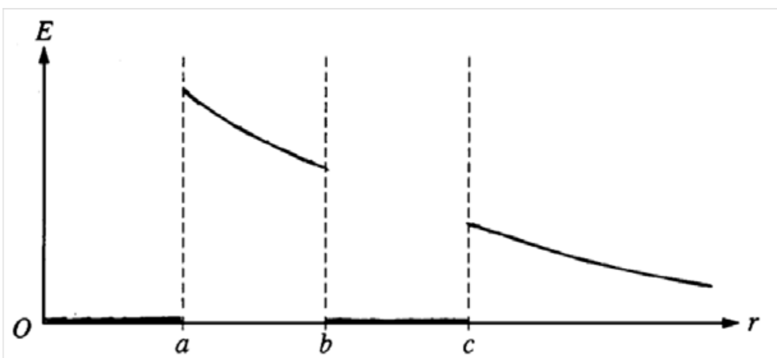
---

2008E1

a. i. Applying Gauss's Law to a Gaussian surface within the shell gives $Q_{enc} = 0$ since the field in the conductor is zero. Therefore the charge on the inner surface is –Q.
ii. The net charge on the shell is zero, therefore the charge on the outer surface must be equal and opposite to the charge on the inner surface, $Q_{outer} = +Q$

b. i. Since the sphere is a conductor, all the charge lies on the outer surface, applying Gauss's Law to any Gaussian surface inside the sphere gives $Q_{enc} = 0$ therefore E = 0
ii. We can treat the sphere as a point charge where $E = kQ/r^2$
iii. Inside a conductor E = 0
iv. The net charge enclosed is +Q (treat both objects as point charges): $E = kQ/r^2$

c.




d. K + U = 0

$$\frac{1}{2}m_e v^2 + \frac{kQ(-e)}{10r} = 0 \text{ gives } v = \sqrt{\frac{kQe}{5m_e r}}$$

---

2011E1

a.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

The Gaussian surface is a sphere, concentric with the charged shell, and with a radius less than the radius of the shell. The enclosed charge *Q* is zero for all radii of the Gaussian surface; therefore, the electric field *E* is also zero everywhere inside the sphere.

b. No. With an asymmetric distribution, the fields from individual charges no longer have the net effect of completely cancelling out.

c. ABCD, ABGH, ADEH
The electric field from the sphere is radial, so it is parallel to the three faces selected

d. Corner A is inside the conducting sphere so the electric field there is zero.

e. E = 0

f. total flux = $Q_{enc}/\varepsilon_0$, the cube encloses 1/8 of the charge so $Q_{enc} = Q/8$
The flux is the same through each of the three nonzero flux sides and is therefore each equal to 1/3 of the total flux through the cube so the flux through CDEF = $Q/24\varepsilon_0$

# SECTION C – Electric Potential and Energy

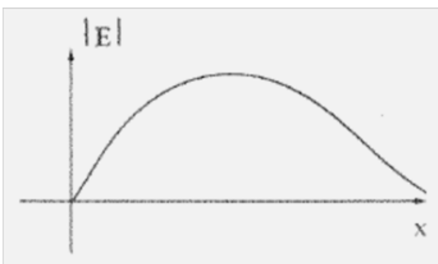
## 1975E1

a. $U = \Sigma qV = \Sigma kqq/r = 2kq^2/r = 2kq^2/(x^2 + \ell^2)$
b. Each $F = kq^2/r^2 = kq^2/(x^2 + \ell^2)$
The y components of the forces cancel, leaving just the x components (and $x = \ell$). This gives $F = kq^2/\sqrt{2}\ell^2$
c. $W_{field} = -q\Delta V = -(U_0 - U_\infty) = -2kq^2/\ell - 0 = -2kq^2/\ell$

---

## 1977E1

a. All parts of the ring are equidistant from point P and $V = kQ/r = kQ/(R^2 + x^2)^{1/2}$
b. V is a maximum where $(R^2 + x^2)$ is at a minimum, which is at $x = 0$
c. All parts of the ring are equidistant from point P and $E = kQ/r^2$, but only the x components are relevant since the components perpendicular to the x axis cancel. The net field is therefore $E = E_x = (kQ/r^2)\cos\theta$
and $\cos\theta = x/(x^2 + R^2)^{1/2}$, which gives $E = kQx/(R^2 + X^2)^{3/2}$
d.




---

## 1980E1

a. Each part of the ring is equidistant from the center and $V_o = kQ/R$ where $Q = \lambda L = \lambda\pi R$ giving $V_o = k\lambda\pi$
b. Because of the symmetry of the charge distribution, all horizontal components cancel, thus the field is directed downward (toward the bottom of the page)
c. Since we only need to consider vertical components we can use $dE_y = dE\cos\theta$ where $\theta$ is the angle from our differential charge element and the y axis. The differential charge element $dq = \lambda dl = \lambda R d\theta$

$$dE_y = \frac{\lambda cos\theta d\theta}{4\pi\varepsilon_0 R}$$

$$E = \frac{\lambda}{4\pi\varepsilon_0 R} 2\int_0^{\frac{\pi}{2}} cos\theta\, d\theta = \frac{2k\lambda}{R}$$

d. The work required to being a charge form infinity to point P is $W = qV_P$
The work required to bring a charge from P to O is $W = q(V_o - V_P)$
If the field is assumed to be approximately constant between O and P the work is also given by $W_{OP} = qE_o s$
Therefore $W_P = qV_P = q(V_o - E_o s)$

---

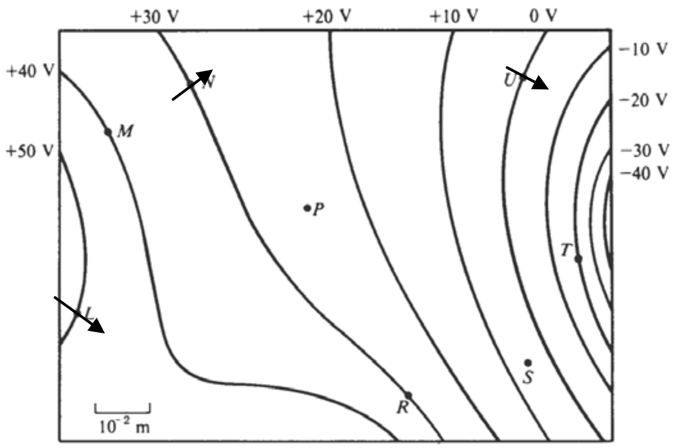
## 1982E1

a. $V = \Sigma kq/r = V_1 + V_2 + V_3 = kq/(a^2 + x^2)^{1/2} + kq/(a^2 + x^2)^{1/2} - kq/x = 0$ which gives $x = \pm a/\sqrt{3}$
b. By symmetry $E_y = 0$, $E_x = E\cos\theta = E(x/(a^2 + x^2)^{1/2}$
$E_x = \Sigma E\cos\theta = E_{1x} + E_{2x} + E_{3x} = 2kqx/(a^2 + x^2)^{1/2} - kq/x^2$
c. By Gauss's Law the net flux $= Q_{enc}/\varepsilon_0 = q/\varepsilon_0$ (or $4\pi kq$)

---

1986E1

a.




b. E is greatest at point T because the equipotentials are closest together (largest gradient)
c. $E = \Delta V/\Delta x = 10\ \text{V}/0.02\ \text{m} = 500\ \text{V/m}$
d. $V_M - V_S = 40\ \text{V} - 5\ \text{V} = 35\ \text{V}$
e. $W_{\text{field}} = -q\Delta V = -(5\ \text{pC})(30\ \text{V} - 40\ \text{V}) = 5 \times 10^{-11}\ \text{J}$
f. No, work does not depend on path

---

1987E1

a. Outside the sphere we treat the sphere as a point charge: $E = kQ/r^2$
b. The portion of the charge within the Gaussian surface is $Qr^3/R^3$ (volume ratio)
$E(4\pi r^2) = Qr^3/R^3$ giving $E = kQr/R^3$
c. The surface of the sphere is a limiting case for the potential outside the sphere, which can be treated as a point charge, where $V = kQ/r$ so on the surface $V = kQ/R$
d. $V_{\text{center}} = V_{\text{surface}} + \int E dr = kQ/R + kQ/R^3[R^2/2] = 3kQ/2R$

---

2000E2

a. i.

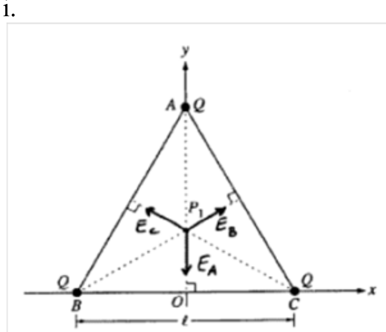



Figure 1

ii.

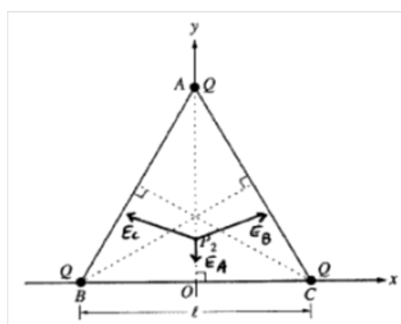



Figure 2

| | Greater than at $P_1$ | Less than at $P_1$ | The same as at $P_1$ |
|---|---|---|---|
| $E_A$ | | ✓ | |
| $E_B$ | ✓ | | |
| $E_C$ | ✓ | | |

b. The x components of the field vectors due to particles C and B cancel each other due to the symmetry created by having a vertex of the triangle on the y axis.

c. $V = \Sigma kQ/r = k(Q_A/r_A + Q_B/r_B + Q_C/r_C) = k(Q_A/r_A + 2Q/r_B)$, with the proper substitutions this gives

$$V = k\left(\frac{Q}{\frac{\sqrt{3}l}{2} - y} + \frac{2Q}{\sqrt{\frac{l^2}{4} + y^2}}\right)$$

d. Since $E_y = -dV(y)/dy$, to find the y coordinate of the point on the y-axis at which the electric field is zero, take the derivative of the expression in part (c) with respect to y, set the expression equal to zero and solve for y

---

2001E1

a. Since the charges are all aligned directly above and below point $P_1$ we can write $E = \Sigma kq/r^2$, letting up be positive and down be negative. We get $E = -k(30)/(3000)^2 + k(30)/(2000)^2 + k(30)/(2000)^2 - k(30)/(3000)^2$
$E = 75{,}000$ N/C upward

b. i. The electric field points directly upward
ii. Less, since $P_2$ is farther from all the charges than $P_1$

c. i. $V = \Sigma kQ/r = k(30/3000 - 30/2000 + 30/2000 - 30/3000) = 0$
ii. V = 0 here as well

d. $V = \Sigma kQ/r = k(30/2000 - 30/1000 + 30/3000 - 30/4000) = -1.12 \times 10^8$ V

e. $U = \Sigma kq_iq_j/r_{ij} = k[(30)(-30)/1000 + (30)(30)/5000 + (30)(-30)/6000 + (30)(-30)/4000 + (-30)(-30)/5000 + (30)(-30)/1000] = -1.6 \times 10^{10}$ J

---

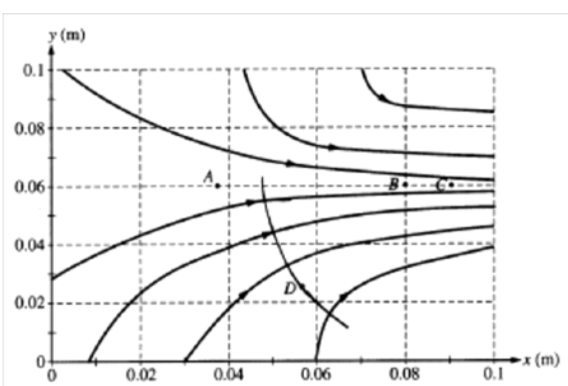
2005E1

a. i. The field is greatest at point C where the field lines are closest together.
ii. The potential is greatest at point A, electric fields point in the direction of decreasing potential.

b. i. The electric moves to the left with increasing speed and decreasing acceleration
ii. $\frac{1}{2} mv^2 = q\Delta V$ gives $v = (2q\Delta V/m)^{1/2} = 1.9 \times 10^6$ m/s

c. If we assume the field is uniform: $E = -\Delta V/r = 20$ V/0.01 m = 2000 V/m

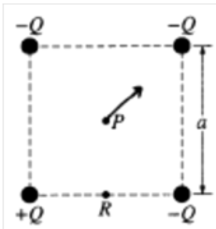
d.




---

2006E1

a.




b. i. The field due to the upper left and lower right charges are equal in magnitude and opposite in direction so they cancel out. The fields due to the other two charges are equal in magnitude and in the same direction so they add $E = 2kQ/r^2$ where $r^2 = a^2/2$ which gives $E = 4kQ/a^2$

ii. $V = \Sigma kQ/r = k(-Q-Q-Q+Q)/r = -2kQ/r$ where $r = a/\sqrt{2}$ giving $V = -2\sqrt{2}kQ/a$

c. Negative. The field is directed generally from R to P (away from the positive charge and toward the negatives) and the charge moves in the opposite direction, thus the field does negative work on the charge.

d. i. Replace the top right negative charge with a positive charge *or* replace the bottom left positive charge with a negative charge,. The vectors will all then cancel from oppositely located same charge pairs.

ii. Replace the top left negative charge with a positive charge *or* replace the bottom right negative charge with a positive charge. The scalar potentials all cancel (2 positive and 2 negative) but the fields do not.

---

2009E1

a. i.

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\frac{kQ}{R}\left[-2 + 3\left(\frac{r}{R}\right)^2\right] = -\frac{kQ}{R}\left[(3)(2)\left(\frac{r}{R}\right)\left(\frac{1}{R}\right)\right] = -\frac{6kQ_0 r}{R^3}$$

$$|E| = \frac{6kQ_0 r}{R^3}$$

Directed inward ($E < 0$)

ii. $E = -dV/dr = kQ_0/r^2$ directed outward

b. i.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E 4\pi r^2 = Q_{enc}/\varepsilon_0$$

$$E_{inside} = \frac{Q_{enclosed,r<R}}{4\pi\varepsilon_0 r^2} = -\frac{6kQ_0 r}{R^3}$$

$$Q_{enclosed,r<R} = -\frac{6Q_0 r^3}{R^3}$$

ii.

$$E 4\pi r^2 = Q_{enc}/\varepsilon_0$$

$$E_{outside} = \frac{Q_0}{4\pi\varepsilon_0 r^2} = \frac{Q_{enclosed,r>R}}{4\pi\varepsilon_0 r^2}$$

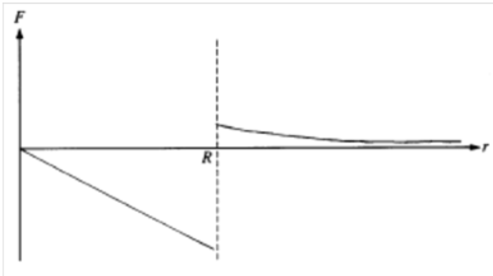
$$Q_{enclosed,r>R} = +Q_0$$

c. Yes. The total enclosed charge equals the charge at the surface plus all the charge inside the sphere

$Q_{enclosed,r>R} = Q_{surface} + Q_{enclosed,r<R \text{ at } r = R}$

$Q_{surface} = Q_0 - (-6Q_0R^3/R^3) = 7Q_0$

d.




## SECTION D – Capacitance

1978E3

a.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$\varepsilon_0 E_0 4\pi a^2 = Q$

b. $E = kQ/r^2 = E_0a^2/r^2$

c. $V = kQ/r$ $\quad kQ = E_{0a}{}^2$ $\quad V(a) = kQ/a$ $\quad V(b) = kQ/b$

$\Delta V = E_0a^2(1/a - 1/b)$

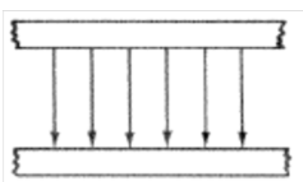
d. $U = ½ Q\Delta V = 2\pi\varepsilon_0 E_0{}^2(a^3 - a^4/b)$

e. setting $dU/da = 0$ gives $a = ¾b$

---

1980E2

a.



b.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$Q_{enc} = \sigma A$ and E through the top of the box is zero, only the bottom of the box has a non-zero flux

$EA = \sigma A/\varepsilon_0$ gives $E = \sigma/\varepsilon_0$

c. The electric field is less. The bound charge distribution in the dielectric has a net negative charge on the top surface and positive on the bottom due to induction. Thus the combined charge is less and so is the electric field.

---

1981E1

a.

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$E(4\pi r^2) = Q/\varepsilon_0$

$E= Q/4\pi\varepsilon_0 r^2$

b. $C = Q/\Delta V$ where $\Delta V = kQ/a - kQ/b$ giving $C_0 = 4\pi\varepsilon_0(ab/b - a)$

c. Consider the system as two capacitors in parallel $C_{top} = C_0/2$ (half the area) and $C_{bottom} = \kappa C_0/2 = C_0/2 \times 4$
In parallel $C_{total} = C_{top} + C_{bottom} = 5C_0/2$

---

1983E1

a.
$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$
$E(4\pi r^2) = Q/\varepsilon_0$
$E = Q/4\pi\varepsilon_0 r^2$

b. $\Delta V = kQ/a - kQ/b = kQ(b - a)/ab$

c. $C = Q/\Delta V$ giving $C = 4\pi\varepsilon_0(ab/b - a)$

---

1985E1

a.
$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$
$E(2\pi rL) = Q_{enc}/\varepsilon_0 = Q/\varepsilon_0$
$E = Q/2\pi\varepsilon_0 Lr$

b. i.
$$\Delta V = -\int E \cdot dr$$
$$V_C - V_D = -\int_D^C E \cdot dr = \int_C^D E \cdot dr = \int_C^D \frac{Q}{2\pi\varepsilon_0 L}\frac{dr}{r} = \frac{Q}{2\pi\varepsilon_0 L}\ln r\big|_a^b = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{b}{a}$$

c. $C_0 = Q/V = 2\pi\varepsilon_0 L/\ln(b/a)$

d. Consider the system as two capacitors in parallel, $C = C_1 + C_2$
$C_{left} = \kappa C_0/3$ and $C_{right} = 2/3\ C_0$ giving $C = 4/3\ C_0$

---

1999E1

a. $V = kQ/r$, or $Q = rV/k = -4.4 \times 10^{-8}$ C

b. i. E = 0 inside a conductor
ii. Treat the sphere as a point charge $E = kQ/r^2 = 396/r^2$ N/C
iii. E = 0inside a conductor
iv. E = 0 since grounding the outer sphere gives it a charge of $-Q_0$ induced to its inner surface

c. $\Delta V = V(a) - V(b) = kQ/a - kQ/b = 1000$ V

d. $C = Q_0/V = 4.4 \times 10^{-11}$ F