# AP ${ }^{\circledR}$ Physics C: Mechanics 2013 Free-Response Questions 


#### Abstract

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TABLE OF INFORMATION DEVELOPED FOR 2012

## CONSTANTS AND CONVERSION FACTORS

| CONSTANTS AND CONVERSION FACTORS |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Proton mass, $m_{p}=1.67 \times 10^{-27} \mathrm{~kg}$ <br> Neutron mass, $m_{n}=1.67 \times 10^{-27} \mathrm{~kg}$ <br> Electron mass, $m_{e}=9.11 \times 10^{-31} \mathrm{~kg}$ <br> Avogadro's number, $N_{0}=6.02 \times 10^{23} \mathrm{~mol}^{-1}$ <br> Universal gas constant, $\quad R=8.31 \mathrm{~J} /(\mathrm{mol} \cdot \mathrm{K})$ <br> Boltzmann's constant, $\quad k_{B}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ |  |  | Electron charge magnitude,$\quad e=1.60 \times 10^{-19} \mathrm{C}$ <br> 1 electron volt, $1 \mathrm{eV}=1.60 \times 10^{-19} \mathrm{~J}$ <br> Speed of light, $\quad c=3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$ <br> $\begin{array}{r}\text { Universal gravitational } \\ \text { constant, }\end{array} \quad G=6.67 \times 10^{-11} \mathrm{~m}^{3} / \mathrm{kg} \cdot \mathrm{s}^{2}$ <br> Acceleration due to gravity at Earth's surface, $\quad g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ |  |  |  |  |
| 1 unified atomic mass unit, 1 u $=1.66 \times 10^{-27} \mathrm{~kg}=931 \mathrm{MeV} / \mathrm{c}^{2}$ <br> Planck's constant, $h$ $=6.63 \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}=4.14 \times 10^{-15} \mathrm{eV} \cdot \mathrm{s}$ <br>  $h c$ $=1.99 \times 10^{-25} \mathrm{~J} \cdot \mathrm{~m}=1.24 \times 10^{3} \mathrm{eV} \cdot \mathrm{nm}$ <br> Vacuum permittivity, $\epsilon_{0}$ $=8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{m}^{2}$ <br> Coulomb's law constant, $k=1 / 4 \pi \epsilon_{0}$ $=9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$  <br> Vacuum permeability, $\mu_{0}$ $=4 \pi \times 10^{-7}(\mathrm{~T} \cdot \mathrm{~m}) / \mathrm{A}$ <br> Magnetic constant, $k^{\prime}=\mu_{0} / 4 \pi$ $=1 \times 10^{-7}(\mathrm{~T} \cdot \mathrm{~m}) / \mathrm{A}$ <br> 1 atmosphere pressure, 1 atm $=1.0 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}=1.0 \times 10^{5} \mathrm{~Pa}$ |  |  |  |  |  |  |  |
| UNIT <br> SYMBOLS | meter, m <br> kilogram, kg <br> second, s <br> ampere, A <br> kelvin, K | mole, hertz, newton, pascal, joule, | $\begin{gathered} \hline \mathrm{mol} \\ \mathrm{~Hz} \\ \mathrm{~N} \\ \mathrm{~Pa} \\ \mathrm{~J} \\ \hline \end{gathered}$ | watt, coulomb, volt, ohm, henry, | W <br> C <br> V <br>  <br> H | farad, tesla, degree Celsius, electron-volt | F T ${ }^{\circ} \mathrm{C}$ eV |


| PREFIXES |  |  |
| :---: | :---: | :---: |
| Factor | Prefix | Symbol |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |


| VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\theta$ | $0^{\circ}$ | $30^{\circ}$ | $37^{\circ}$ | $45^{\circ}$ | $53^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ |
| $\sin \theta$ | 0 | 1/2 | 3/5 | $\sqrt{2} / 2$ | 4/5 | $\sqrt{3} / 2$ | 1 |
| $\cos \theta$ | 1 | $\sqrt{3} / 2$ | 4/5 | $\sqrt{2} / 2$ | 3/5 | 1/2 | 0 |
| $\tan \theta$ | 0 | $\sqrt{3} / 3$ | 3/4 | 1 | 4/3 | $\sqrt{3}$ | $\infty$ |

The following conventions are used in this exam.
I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.
II. The direction of any electric current is the direction of flow of positive charge (conventional current).
III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

## MECHANICS

| $v=v_{0}+a t$ | $\begin{aligned} & a=\text { acceleration } \\ & F=\text { force } \end{aligned}$ |
| :---: | :---: |
| $x=x_{0}+v_{0} t+\frac{1}{2} a t^{2}$ | $\begin{aligned} & f=\text { frequency } \\ & h=\text { height } \end{aligned}$ |
| $v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right)$ | $\begin{aligned} & I=\text { rotational inertia } \\ & J=\text { impulse } \end{aligned}$ |
| $\sum \mathbf{F}=\mathbf{F}_{\text {net }}=m \mathbf{a}$ | $\begin{aligned} & K=\text { kinetic energy } \\ & k=\text { spring constant } \end{aligned}$ |
| $\mathbf{F}=\frac{d \mathbf{p}}{d t}$ | $\begin{aligned} \ell & =\text { length } \\ L & =\text { angular momentum } \\ m & =\text { mass } \end{aligned}$ |
| $\mathbf{J}=\int \mathbf{F} d t=\Delta \mathbf{p}$ | $\begin{aligned} & N=\text { normal force } \\ & P=\text { power } \end{aligned}$ |
| $\mathbf{p}=m \mathbf{v}$ | $\begin{aligned} p & =\text { momentum } \\ r & =\text { radius or distance } \end{aligned}$ |
| $F_{\text {fric }} \leq \mu N$ | $\begin{aligned} \mathbf{r} & =\text { position vector } \\ T & =\text { period } \end{aligned}$ |
| $W=\int \mathbf{F} \cdot d \mathbf{r}$ | $\begin{aligned} & t=\text { time } \\ & U=\text { potential energy } \end{aligned}$ |
| $K=\frac{1}{2} m v^{2}$ | $\begin{aligned} v & =\text { velocity or speed } \\ W & =\text { work done on a system } \\ x & =\text { position } \end{aligned}$ |
| $P=\frac{d W}{d t}$ | $\begin{aligned} \mu & =\text { coefficient of friction } \\ \theta & =\text { angle } \end{aligned}$ |
| $P=\mathbf{F} \cdot \mathbf{v}$ | $\begin{aligned} & \tau=\text { torque } \\ & \omega=\text { angular speed } \end{aligned}$ |
| $\Delta U_{g}=m g h$ | $\begin{aligned} & \alpha=\text { angular acceleration } \\ & \phi=\text { phase angle } \end{aligned}$ |
| $a_{c}=\frac{v^{2}}{r}=\omega^{2} r$ | $\mathbf{F}_{s}=-k \mathbf{x}$ |
| $\tau=\mathbf{r} \times \mathbf{F}$ $\sum \tau=\tau_{\text {net }}=I \boldsymbol{\alpha}$ | $U_{S}=\frac{1}{2} k x^{2}$ |
| $\begin{aligned} & I=\int r^{2} d m=\sum m r^{2} \\ & \mathbf{r}_{c m}=\sum m \mathbf{r} / \sum m \end{aligned}$ | $\begin{aligned} & x=x_{\max } \cos (\omega t+\phi) \\ & T=\frac{2 \pi}{\omega}=\frac{1}{f} \end{aligned}$ |
| $v=r \omega$ | $T_{s}=2 \pi \sqrt{\frac{m}{k}}$ |

$\mathbf{L}=\mathbf{r} \times \mathbf{p}=I \boldsymbol{\omega}$
$K=\frac{1}{2} I \omega^{2}$
$\omega=\omega_{0}+\alpha t$
$\theta=\theta_{0}+\omega_{0} t+\frac{1}{2} \alpha t^{2}$

## ELECTRICITY AND MAGNETISM

$$
\begin{aligned}
& F=\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}} \\
& \mathbf{E}=\frac{\mathbf{F}}{q} \\
& \oint \mathbf{E} \cdot d \mathbf{A}=\frac{Q}{\epsilon_{0}} \\
& E=-\frac{d V}{d r} \\
& V=\frac{1}{4 \pi \epsilon_{0}} \sum_{i} \frac{q_{i}}{r_{i}} \\
& U_{E}=q V=\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r} \\
& C=\frac{Q}{V}
\end{aligned}
$$

$$
C=\frac{\kappa \epsilon_{0} A}{d}
$$

$$
C_{p}=\sum_{i} C_{i}
$$

$$
\frac{1}{C_{s}}=\sum_{i} \frac{1}{C_{i}}
$$

$$
I=\frac{d Q}{d t}
$$

$$
U_{c}=\frac{1}{2} Q V=\frac{1}{2} C V^{2}
$$

$$
R=\frac{\rho \ell}{A}
$$

$$
\mathbf{E}=\rho \mathbf{J}
$$

$$
I=N e v_{d} A
$$

$$
V=I R
$$

$$
R_{s}=\sum_{i} R_{i}
$$

$$
\frac{1}{R_{p}}=\sum_{i} \frac{1}{R_{i}}
$$

$$
P=I V
$$

$A=$ area
$B=$ magnetic field
$C=$ capacitance
$d=$ distance
$E=$ electric field
$\boldsymbol{\mathcal { E }}=\mathrm{emf}$
$F=$ force
$I=$ current
$J=$ current density
$L=$ inductance
$\ell=$ length
$n=$ number of loops of wire per unit length
$N=$ number of charge carriers per unit volume
$P=$ power
$Q=$ charge
$q=$ point charge
$R=$ resistance
$r=$ distance
$t=$ time
$U=$ potential or stored energy
$V=$ electric potential
$v=$ velocity or speed
$\rho=$ resistivity
$\phi_{m}=$ magnetic flux
$\kappa=$ dielectric constant
$\oint \mathbf{B} \cdot d \boldsymbol{\ell}=\mu_{0} I$
$d \mathbf{B}=\frac{\mu_{0}}{4 \pi} \frac{I d \ell \times \mathbf{r}}{r^{3}}$
$\mathbf{F}=\int I d \ell \times \mathbf{B}$
$B_{s}=\mu_{0} n I$
$\phi_{m}=\int \mathbf{B} \cdot d \mathbf{A}$
$\boldsymbol{\varepsilon}=\oint \mathbf{E} \cdot d \boldsymbol{\ell}=-\frac{d \phi_{m}}{d t}$
$\varepsilon=-L \frac{d I}{d t}$
$U_{L}=\frac{1}{2} L I^{2}$

$$
\mathbf{F}_{M}=q \mathbf{v} \times \mathbf{B}
$$



# 2013 AP ${ }^{\circledR}$ PHYSICS C: MECHANICS FREE-RESPONSE QUESTIONS 

PHYSICS C: MECHANICS

## SECTION II

Time- 45 minutes
3 Questions
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.


Figure 1


Figure 2
Mech 1.
A student places a 0.40 kg glider on an air track of negligible friction and holds it so that it touches an uncompressed ideal spring, as shown in Figure 1 above. The student then pushes the glider back to compress the spring by 0.25 m , as shown in Figure 2. At time $t=0$, the student releases the glider, and a motion sensor begins recording the velocity of the reflector at the front of the glider as a function of time. The data points are shown in the table below. At time $t=0.79 \mathrm{~s}$, the glider loses contact with the spring.

| Time $(\mathrm{s})$ | 0 | 0.25 | 0.50 | 0.75 | 1.00 | 1.50 | 2.00 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Velocity $(\mathrm{m} / \mathrm{s})$ | 0 | 0.25 | 0.43 | 0.48 | 0.50 | 0.49 | 0.51 |

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(a) On the axes below, plot the data points for velocity $v$ as a function of time $t$ for the glider, and draw a smooth curve that best fits the data. Be sure to label an appropriate scale on the vertical axis.

(b) The student wishes to use the data to plot position $x$ as a function of time $t$ for the glider.
i. Describe a method the student could use to do this.
ii. On the axes below, sketch the position $x$ as a function of time $t$ for the glider. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

(c) Calculate the time at which the glider makes contact with the bumper at the far right.
(d) Calculate the force constant of the spring.
(e) The experiment is run again, but this time the glider is attached to the spring rather than simply being pushed against it.
i. Determine the amplitude of the resulting periodic motion.
ii. Calculate the period of oscillation of the resulting periodic motion.

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## Mech 2.

A box of mass $m$ initially at rest is acted upon by a constant applied force of magnitude $F_{A}$, as shown in the figure above. The friction between the box and the horizontal surface can be assumed to be negligible, but the box is subject to a drag force of magnitude $k v$ where $v$ is the speed of the box and $k$ is a positive constant. Express all your answers in terms of the given quantities and fundamental constants, as appropriate.
(a) The dot below represents the box. Draw and label the forces (not components) that act on the box.
(b) Write, but do not solve, a differential equation that could be used to determine the speed $v$ of the box as a function of time $t$. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).
(c) Determine the magnitude of the terminal velocity of the box.
(d) Use the differential equation from part (b) to derive the equation for the speed $v$ of the box as a function of time $t$. Assume that $v=0$ at time $t=0$.
(e) On the axes below, sketch a graph of the speed $v$ of the box as a function of time $t$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.


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Note: Figure not drawn to scale.
Mech 3.
A disk of mass $M=2.0 \mathrm{~kg}$ and radius $R=0.10 \mathrm{~m}$ is supported by a rope of negligible mass, as shown above. The rope is attached to the ceiling at one end and passes under the disk. The other end of the rope is pulled upward with a force $F_{A}$. The rotational inertia of the disk around its center is $M R^{2} / 2$.
(a) Calculate the magnitude of the force $F_{A}$ necessary to hold the disk at rest.

At time $t=0$, the force $F_{A}$ is increased to 12 N , causing the disk to accelerate upward. The rope does not slip on the disk as the disk rotates.
(b) Calculate the linear acceleration of the disk.
(c) Calculate the angular speed of the disk at $t=3.0 \mathrm{~s}$.
(d) Calculate the increase in total mechanical energy of the disk from $t=0$ to $t=3.0 \mathrm{~s}$.
(e) The disk is replaced by a hoop of the same mass and radius. Indicate whether the linear acceleration of the hoop is greater than, less than, or the same as the linear acceleration of the disk.
_ Greater than _ Less than _ The same as
Justify your answer.

## STOP

## END OF EXAM

