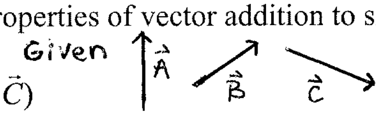
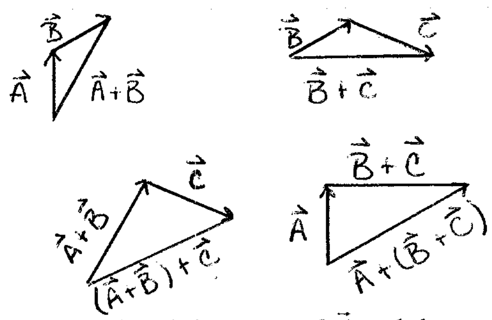
3 Vectors and Motion in Two Dimensions

3.1 Using Vectors

1. Use a figure and the properties of vector addition to show that vector addition is associative. That is, show that

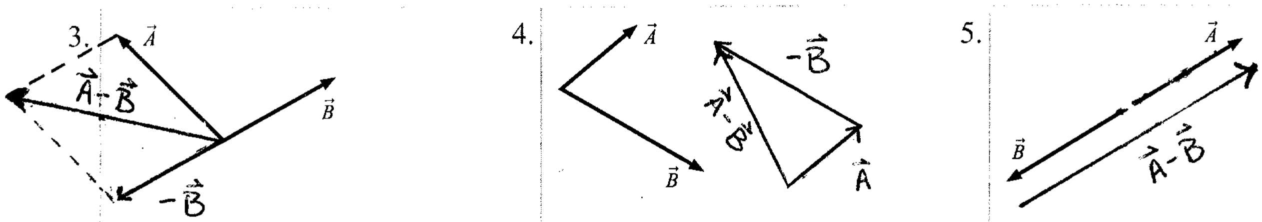
$(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$



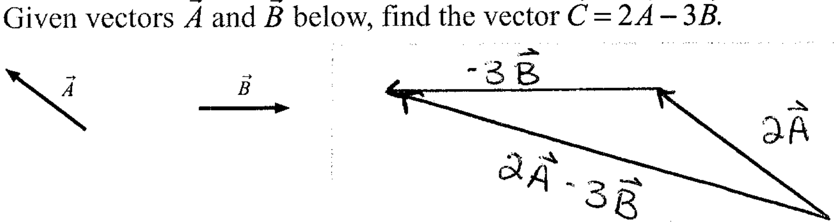


2. Draw and label the vector $2\vec{A}$ and the vector $\frac{1}{2}\vec{A}$.

Exercises 3–5: Draw and label the vector difference $\vec{A}-\vec{B}$.

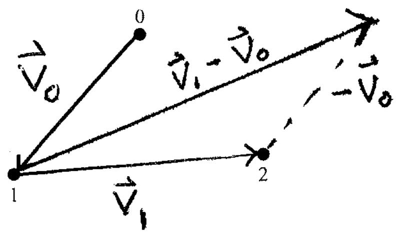


6. Given vectors $\vec{A}$ and $\vec{B}$ below, find the vector $\vec{C}=2\vec{A}-3\vec{B}$.

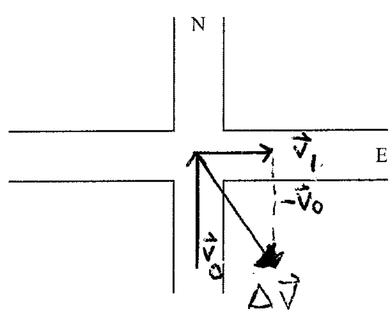


3.2 Using Vectors on Motion Diagrams

7. The figure below shows the positions of a moving object in three successive frames of film. Draw and label the velocity vector $\vec{v}_0$ for the motion from 0 to 1 and the vector $\vec{v}_1$ for the motion from 1 to 2. Then draw the vector $\vec{v}_1 - \vec{v}_0$ with its tail on point 1.



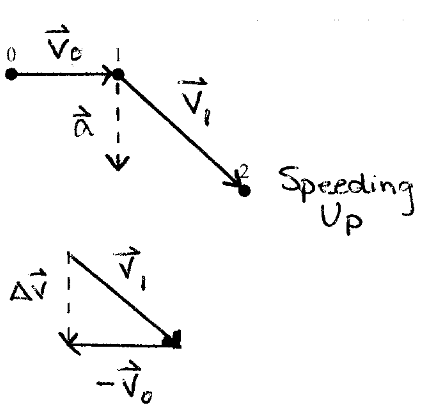
8. A car enters an icy intersection traveling at 16 m/s due north. After a violent collision with a truck, the car slides away moving 12 m/s due east. Draw arrows on the picture below to show (i) the car's velocity $\vec{v}_0$ when entering the intersection, (ii) its velocity $\vec{v}_1$ when leaving, and (iii) the car's change in velocity $\Delta\vec{v} = \vec{v}_1 - \vec{v}_0$ due to the collision.



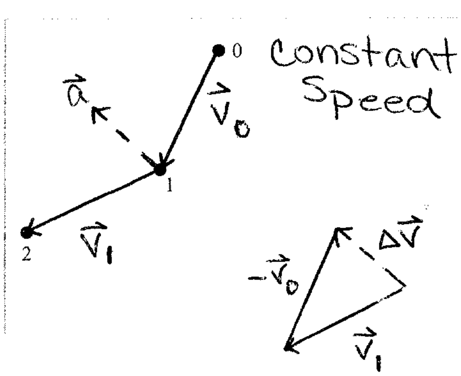
Exercises 9–10: The figures below show an object's position in three successive frames of film. The object is moving in the direction 0 → 1 → 2. For each diagram:

- Draw and label the initial and final velocity vectors $\vec{v}_0$ and $\vec{v}_1$. Use **black**.
- Use the steps of Tactics Box 3.2 to find the change in velocity $\Delta\vec{v}$.
- Draw and label $\vec{a}$ at the proper location on the motion diagram. Use **red**.
- Determine whether the object is speeding up, slowing down, or moving at a constant speed. Write your answer beside the diagram.

9.



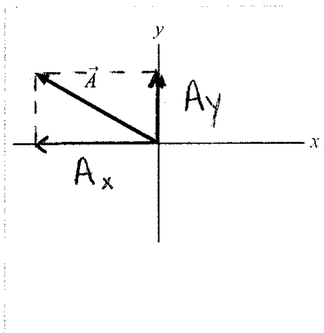
10.

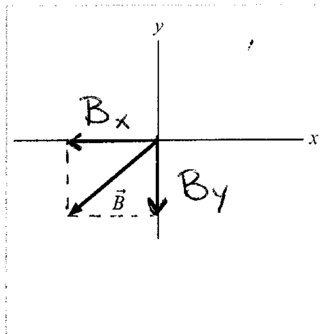


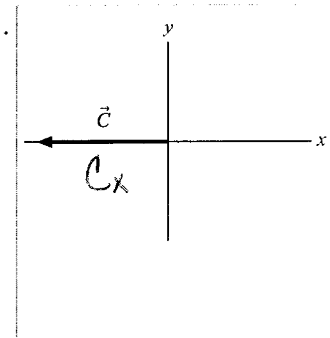
3.3 Coordinate Systems and Vector Components

3.4 Motion on a Ramp

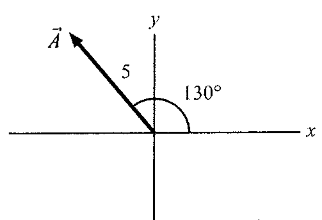
Exercises 11–13: Draw and label the *x*- and *y*-component vectors of the vector shown.

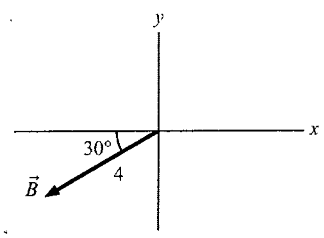
11.


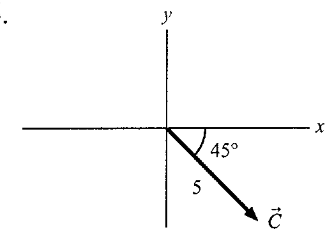
12.


13.


Exercises 14–16: Determine the numerical values of the *x*- and *y*-components of each vector.

14.


15.


16.


To 3 significant figures:

$A_x = 5\cos(130^\circ) = -3.21$ $B_x = -4\cos(30^\circ) = -3.46$ $C_x = 5\cos(45^\circ) = 3.54$

$A_y = 5\sin(130^\circ) = 3.83$ $B_y = -4\sin(30^\circ) = -2.00$ $C_y = 5\sin(45^\circ) = -3.54$

17. What is the vector sum $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ of the three vectors defined in Exercises 14–16?

$D_x =$ -3.13 $D_y =$ -1.71

18. Can a vector have a component equal to zero and still have nonzero magnitude? Explain.

Yes, so long as one of its components is nonzero.

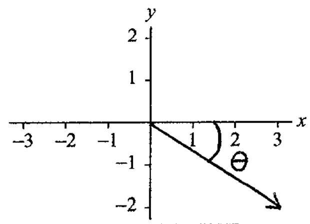
19. Can a vector have zero magnitude if one of its components is nonzero? Explain.

No, if one component is nonzero, then no other component can detract from it because the components are in perpendicular directions.

Exercises 20–22: For each vector:

- Draw the vector on the axes provided.
- Draw and label an angle θ to describe the direction of the vector.
- Find the magnitude and the angle of the vector.

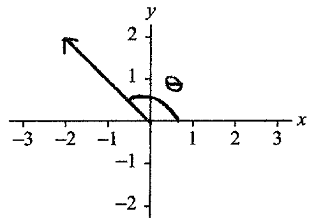
20. $A_x = 3, A_y = -2$



$A = \sqrt{3^2 + (-2)^2} = 3.61$

$\theta = -33.6°$

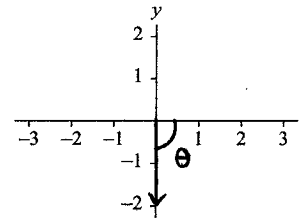
21. $B_x = -2, B_y = 2$



$B = \sqrt{(-2)^2 + 2^2} = 2.83$

$\theta = 135°$

22. $C_x = 0, C_y = -2$

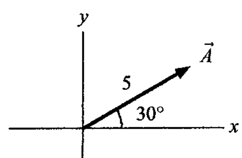


$C = 2$

$\theta = -90°$

Exercises 23–25: Define vector $\vec{A}$ = (5, 30° above the horizontal). Determine the components A_x and A_y in the three coordinate systems shown below. Show your work below the figure.

23.



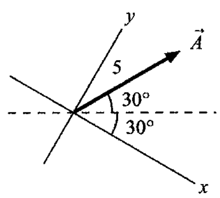
$A_x = 5\cos 30°$

$A_y = 5\sin 30°$

$A_x = 4.33$

$A_y = 2.50$

24.



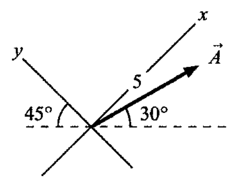
$A_x = 5\cos(30° + 30°)$

$A_y = 5\sin(30° + 30°)$

$A_x = 2.50$

$A_y = 4.33$

25.



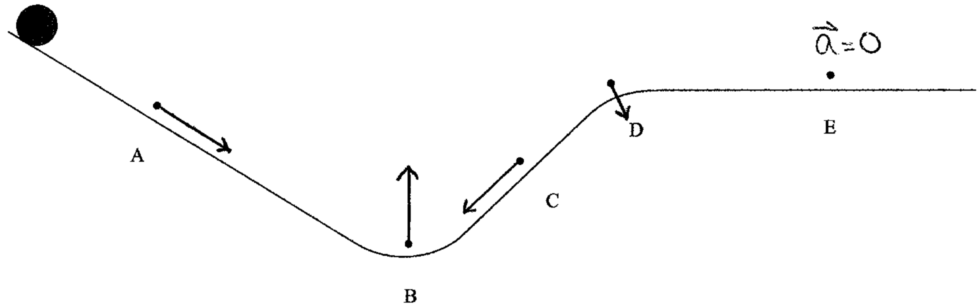
$A_x = 5\cos(45° - 30°)$

$A_y = -5\sin(45° - 30°)$

$A_x = 4.80$

$A_y = -1.29$

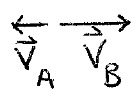
26. The figure shows a ramp and a ball that rolls along the ramp. Draw vector arrows on the figure to show the ball's acceleration at each of the lettered points A to E (or write $\vec{a} = \vec{0}$, if appropriate).



3.5 Relative Motion

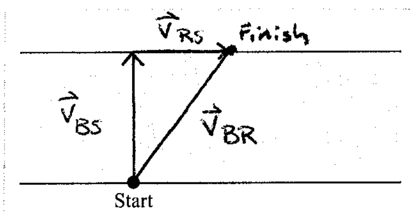
27. On a ferry moving steadily forward in still water at 5 m/s, a passenger walks towards the back of the boat at 2 m/s. (i) Write a symbolic equation to find the velocity of the passenger with respect to the water using $(v_x)_{AB}$ notation. (ii) Substitute the appropriate values into your equation to determine the value of that velocity.

$\vec{V}_P = \vec{V}_A + \vec{V}_B = 2 + (-5) = -3\frac{m}{s}$

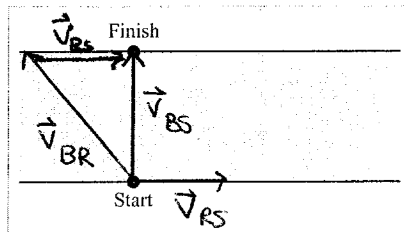


28. A boat crossing a river can move at 5 m/s with respect to the water. The river is flowing to the right at 3 m/s. In (a), the boat points straight across the river and is carried downstream by the water. In (b), the boat is angled upstream by the amount needed for it to travel straight across the river. For each situation, draw the velocity vectors $\vec{v}_{RS}$ of the river with respect to the shore, $\vec{v}_{BS}$ of the boat with respect to the river, and $\vec{v}_{BS}$ of the boat with respect to the shore,

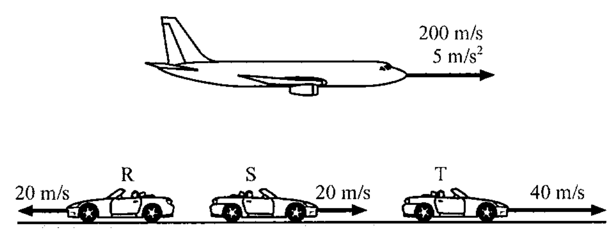
a.



b.



29. Ryan, Samantha, and Tomas are driving their convertibles. At the same instant, they each see a jet plane with an instantaneous velocity of 200 m/s and an acceleration of 5 m/s^2.



a. Rank in order, from largest to smallest, the jet's *speed* v_R, v_S, and v_T according to Ryan, Samantha, and Tomas. Explain.

$V_R > V_S > V_T$

The jet's speed, according to Ryan, is 200 m/s − (−20 m/s) = 220 m/s.
The jet's speed, according to Samantha, is 200 m/s − 20 m/s = 180 m/s.
The jet's speed, according to Tomas, is 200 m/s − 40 m/s = 160 m/s.

b. Rank in order, from largest to smallest, the jet's *acceleration* a_R, a_S, and a_T according to Ryan, Samantha, and Tomas. Explain.

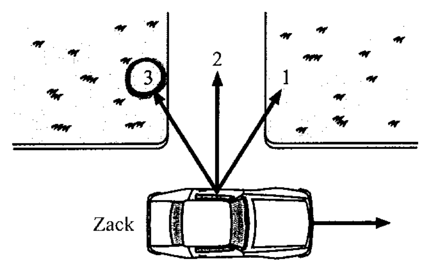
$a_R = a_S = a_T$

Because Ryan, Samantha, and Tomas are moving at constant speed, they perceive the same rate of change in speed for the jet. The acceleration of the jet is the same in all intertial reference frames.

30. An electromagnet on the ceiling of an airplane holds a steel ball. When a button is pushed, the magnet releases the ball. The experiment is first done while the plane is parked on the ground, and the point where the ball hits the floor is marked with an X. Then the experiment is repeated while the plane is flying level at a steady 500 mph. Does the ball land slightly in front of the X (toward the nose of the plane), on the X, or slightly behind the X (toward the tail of the plane)? Explain.

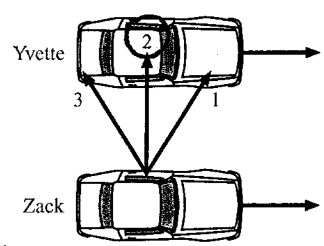
The ball still lands on the X. In the second experiment, the ball's initial velocity relative to the plane is still zero. Although someone at rest on the ground would perceive both the ball and the plane to be moving past at 500 mph, to someone on the plane the ball still falls straight down.

31. Zack is driving past his house. He wants to toss his physics book out the window and have it land in his driveway. If he lets go of the book exactly as he passes the end of the driveway, should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain. throw 3



When Zack throws the book, it also has the forward motion of the car. If he wants the book to follow path 2 into the driveway, he needs to include a backward component to the velocity from his throw that is equal and opposite to the forward velocity of the car.

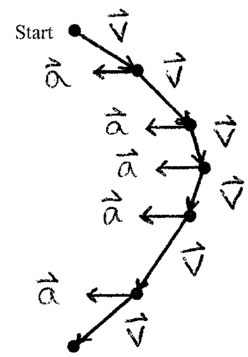
32. Yvette and Zack are driving down the freeway side by side with their windows rolled down. Zack wants to toss his physics book out the window and have it land in Yvette's front seat. Should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain. throw 2



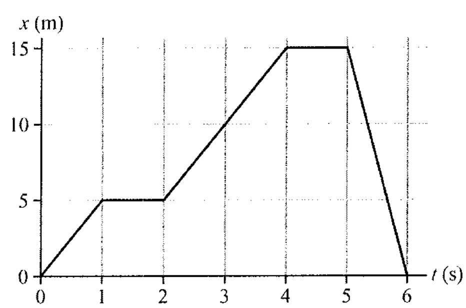
Yvette's car is not moving relative to Zack's car. From the perspective of an observer at rest with to the freeway, the book already has a forward velocity equal to Yvette's and, therefore, only needs to be thrown straight outward.

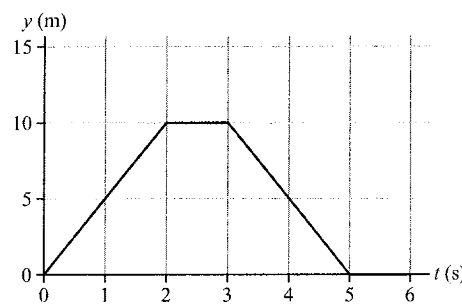
3.6 Motion in Two Dimensions: Projectile Motion

33. Complete the motion diagram for this trajectory, showing velocity and acceleration vectors.

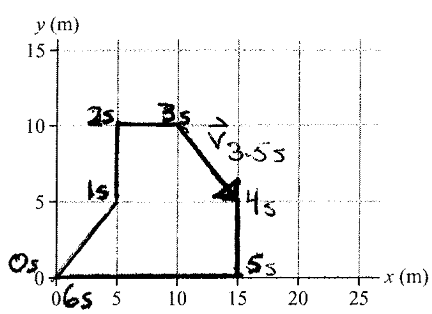


34. A particle moving along a trajectory in the xy-plane has the x-versus-t graph and the y-versus-t graph shown below.



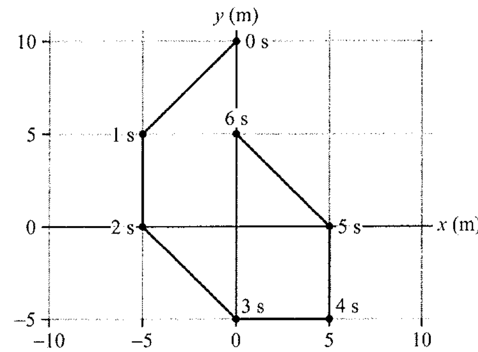


a. Use the grid below to draw a y-versus-x graph of the trajectory.

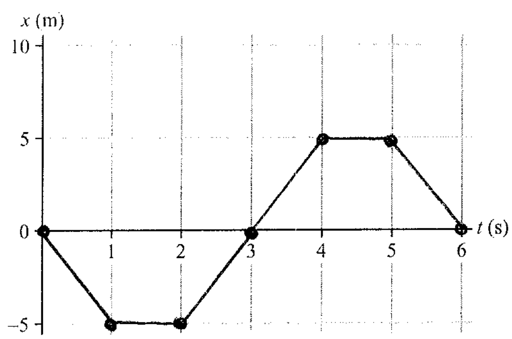


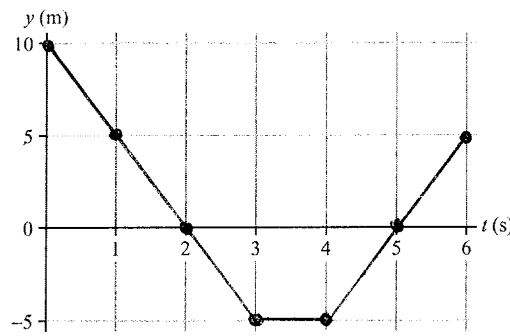
b. Draw the particle's velocity vector at $t = 3.5$ s on your graph.

35. The trajectory of a particle is shown below. The particle's position is indicated with dots at 1-second intervals. The particle moves between each pair of dots at constant speed.



a. Draw x-versus-t and y-versus-t graphs for the particle.





b. Is the particle's speed between $t = 5$ s and $t = 6$ s greater than, less than, or equal to its speed between $t = 1$ s and $t = 2$ s? Explain.

$|V_{1-2}| = \sqrt{0^2+(5\frac{m}{s})^2} = 5\frac{m}{s}$ The line between t=5 and t=6 is longer.

$|V_{5-6}| = \sqrt{(5\frac{m}{s})^2+(5\frac{m}{s})^2} = \sqrt{50\frac{m^2}{s^2}} = 7.1\frac{m}{s}$

36. A projectile is launched over horizontal ground at an angle between 0° and 90°.

a. Is there any point on the trajectory where $\vec{v}$ and $\vec{a}$ are parallel to each other? If so, where?

No, unless the launch angle is 90°, $\vec{a}$ is always vertically downward. $\vec{v}$ will be parallel to $\vec{a}$ only if $\vec{v}$ has no horizontal component, which is only possible if the launch angle is 90°.

b. Is there any point where $\vec{v}$ and $\vec{a}$ are perpendicular to each other? If so, where?

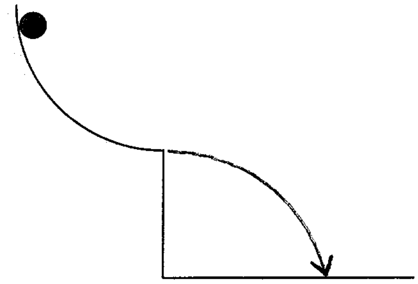
At the top of the trajectory, $\vec{v}$ is horizontal, $\vec{a}$ is vertical.

c. Which of the following remain constant throughout the entire trajectory: x, y, v, v_x, v_y, a_x, a_y?

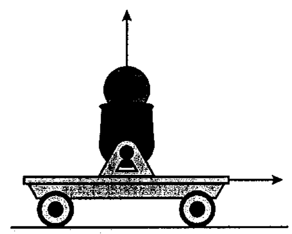
v_x, a_x, a_y ($a_x = 0$, $a_y = -g$)

3.7 Projectile Motion: Solving Problems

37. The figure shows a ball that rolls down a quarter-circle ramp, then off a cliff. Sketch the ball's trajectory from the instant it is released until it hits the ground.



38. a. A cart that is rolling at constant velocity fires a ball straight up. When the ball comes back down, will it land in front of the launching tube, behind the launching tube, or directly in the tube? Explain.

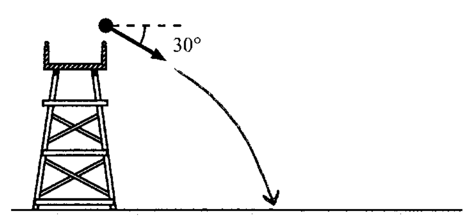


The ball will land directly in the cart. The ball is already moving horizontally with the cart when launched. Though the force on the ball when launched is vertical, the ball retains the same constant horizontal velocity as the cart and lands in it.

b. Will your answer change if the cart is accelerating in the forward direction? If so, how?

If the cart is accelerating, then the ball will land behind the cart. The ball's horizontal velocity when launched is the same as the carts at that moment, but the cart is speeding up and gets ahead of the ball.

39. A rock is thrown from a bridge at an angle 30° below horizontal.



a. Sketch the rock's trajectory on the figure.

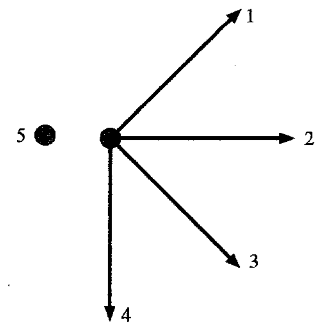
b. Immediately after the rock is released, is the magnitude of its acceleration greater than, less than, or equal to g? Explain.

$= g$ The acceleration of the rock is independent of the initial velocity of the rock.

c. At the instant of impact, is the rock's speed greater than, less than, or equal to the speed with which it was thrown? Explain.

The rock's speed is greater at the instant of impact. The horizontal speed does not change but the vertical speed increases due to gravity.

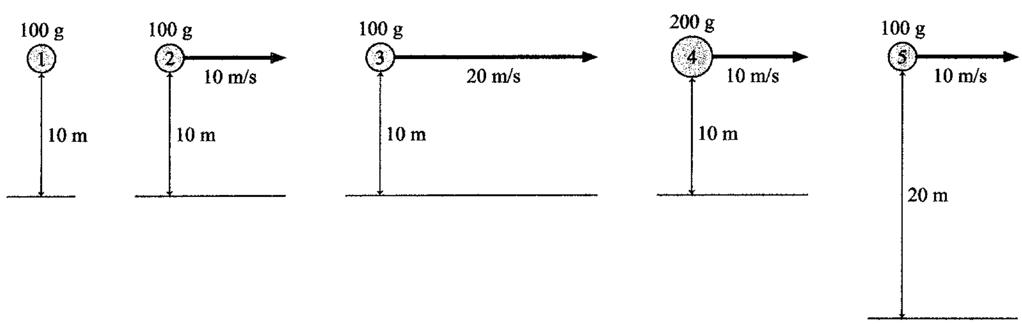
40. Four balls are simultaneously launched with the same speed from the same height h above the ground. At the same instant, ball 5 is released from rest at the same height. Rank in order, from shortest to longest, the amount of time it takes each of these balls to hit the ground. (Some may be simultaneous.)



Order: 4, 3, 2 = 5, 1

Explanation: Because 1-4 each have the same initial speed, we need only to consider the vertical component of their initial velocity. Ball 4 has the greatest initial downward velocity component, ball 1 the least. Both balls 2 and 5 have no initial velocity in the y-direction and hit simultaneously.

41. Rank in order, from shortest to longest, the amount of time it takes each of these projectiles to hit the ground. (Some may be simultaneous.)



Order: 1 = 2 = 3 = 4, 5

Explanation:

None of the projectiles have any initial velocity in the y-direction. The time to fall depends only on the height above the ground. Balls 1-4 begin at the same height.

3.8 Motion in Two Dimensions: Circular Motion

42. a. The crankshaft in your car rotates at 3000 rpm. What is the frequency in revolutions per second?

$$\frac{3000\text{ rev}}{1\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = 50\frac{\text{rev}}{\text{s}}$$

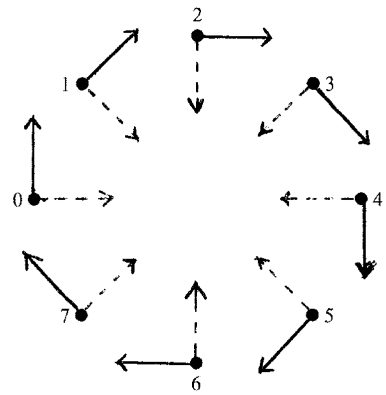
b. A record turntable rotates at 33.3 rpm. What is the period in seconds?

$$\frac{33.3\text{ rev}}{1\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = 0.555\frac{\text{rev}}{\text{s}}$$

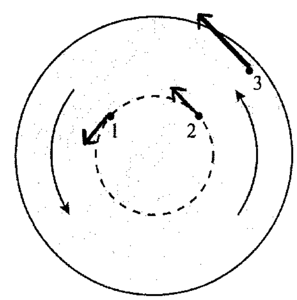
$$T = \frac{1}{0.555}\text{ s} = 1.8\text{ s}$$

43. The dots of a motion diagram are shown below for an object in uniform circular motion. Carefully complete the diagram.
    - Draw and label the velocity vectors $\vec{v}$. Use a **black** pen or pencil.
    - Draw and label the acceleration vectors $\vec{a}$. Use a **red** pen or pencil.

—— Black
- - - - Red



44. The figure shows three points on a steadily rotating wheel.
    a. Draw the velocity vectors at each of the three points.
    b. Rank in order, from largest to smallest, the speeds v_1, v_2, and v_3 of these points.



Order: $\omega_1 = \omega_2 = \omega_3$

Explanation:

Each point traverses the same angle in the same time. All points on the wheel rotate with the same period.

45. An object is traveling in a circle of radius r at constant speed v.
    a. By what factor does the object's acceleration change if its speed is doubled and the radius is unchanged? 4 $a_c = \frac{v^2}{r}$

    b. By what factor does the acceleration change if the radius of the circle is doubled and its speed is unchanged? 1/2

    c. By what factor does the acceleration change if the period of the motion is doubled without changing the size of the circle? 1/4