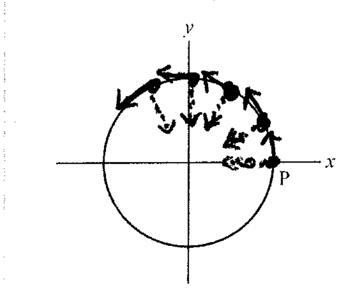
6 Circular Motion, Orbits, and Gravity

6.1 Uniform Circular Motion

6.2 Speed, Velocity and Acceleration in Uniform Circular Motion

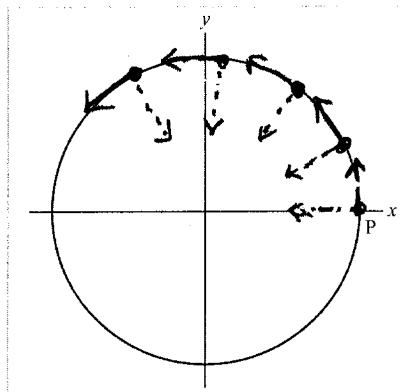
1. On each of the two circles shown below, draw a motion diagram for an object moving at a constant angular speed of $\omega = +0.5$ rad/s along the perimeter starting at point P and through an angle of *two* radians. Indicate the location of the particle every 1.0 s. Draw velocity vectors **black** and acceleration vectors **red.**

a. 1.0rad ≈ 57°



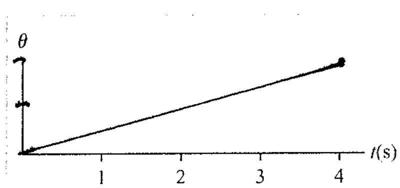
— Black ···· Red

b.

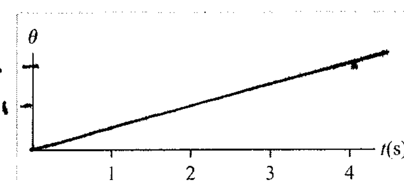


Plot the angle as a function of time for each of two motion diagrams that you have drawn above. Include an appropriate scale for the vertical axis in each case.

c.

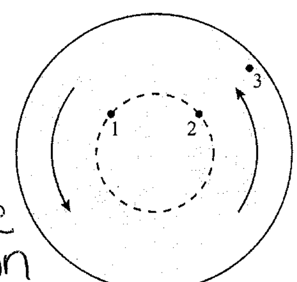


d.



(Same)

2. The figure shows three points on a steadily rotating wheel. Rank in order, from largest to smallest, the angular velocities ω_1, ω_2, and ω_3 of these points.

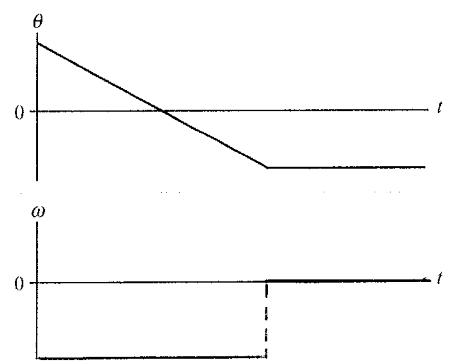


Order: $\omega_1 = \omega_2 = \omega_3$

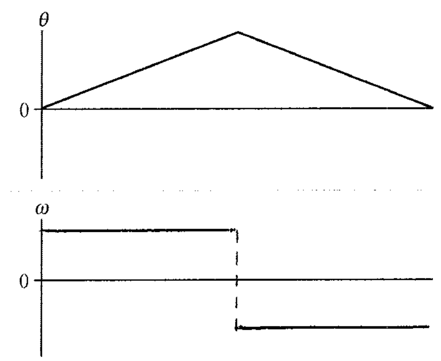
Explanation: Each point traverses the same angle in the same time. All points on the wheel rotate with the same period.

3. Below are two angular position-versus-time graphs. For each, draw the corresponding angular velocity-versus-time graph directly below it.

a.

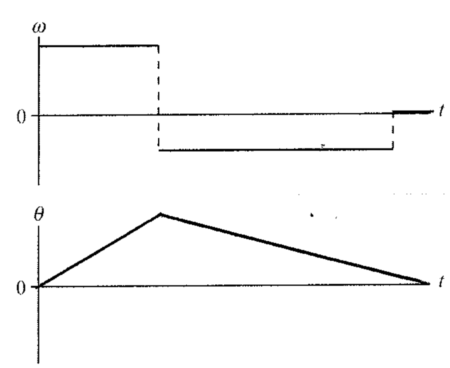


b.

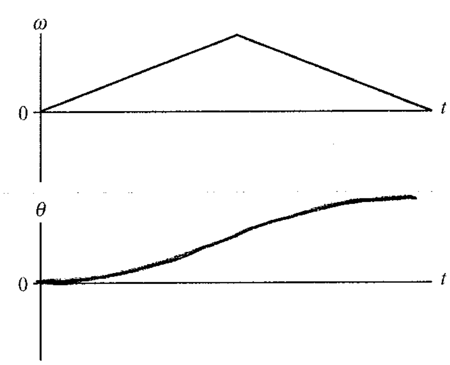


4. Below are two angular velocity-versus-time graphs. For each, draw the corresponding angular position-versus-time graph directly below it. Assume $\theta_0 = 0$ rad.

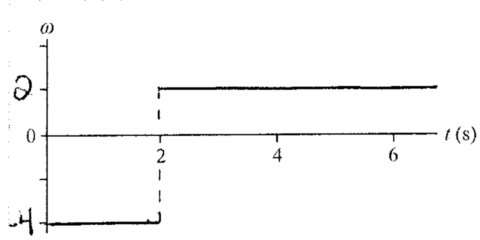
a.

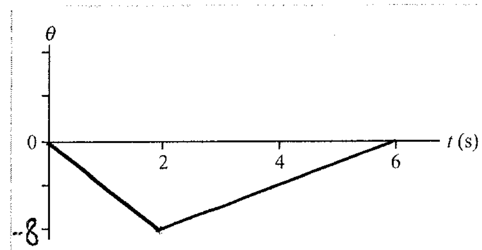


b.



5. A particle in circular motion rotates clockwise at 4 rad/s for 2 s, then counterclockwise at 2 rad/s for 4 s. The time required to change direction is negligible. Graph the angular velocity and the angular position, assuming $\theta_0 = 0$ rad.





6. A particle travels at constant speed along a circle with $a = 8\ \text{m/s}^2$. What is a if

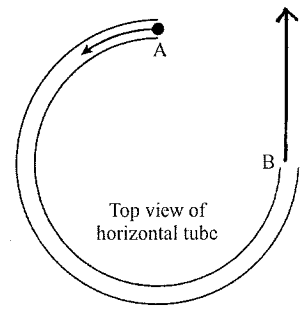
a. The radius is doubled without changing the angular velocity? $a_r = 16\ \frac{m}{s^2}$

b. The radius is doubled without changing the particle's speed? $a_r = 4\ \frac{m}{s^2}$

c. The angular velocity is doubled without changing the particle's radius? $a_r = 32\ \frac{m}{s^2}$

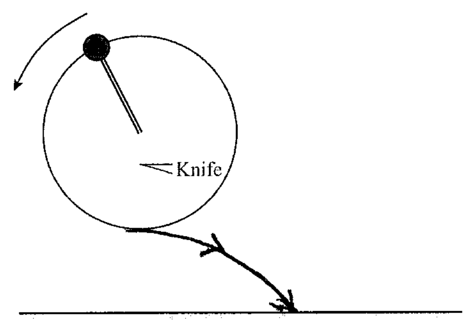
6.3 Dynamics of Uniform Circular Motion

7. The figure shows a *top view* of a plastic tube that is fixed on a horizontal table top. A marble is shot into the tube at A. Sketch the marble's trajectory after it leaves the tube at B. Explain.



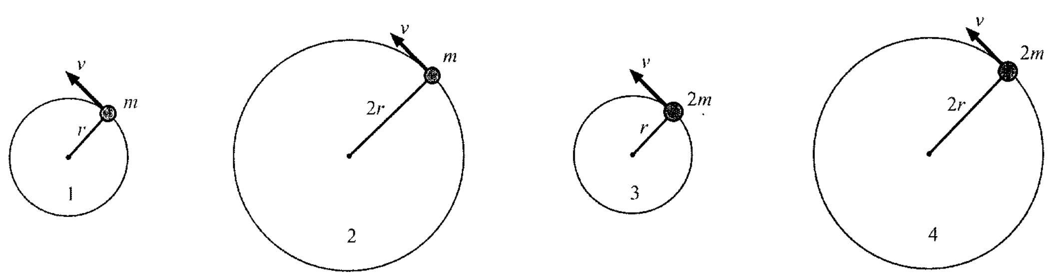
The marble continues in a straight line (towards the top of the page).

8. A ball swings in a *vertical* circle on a string. During one revolution, a very sharp knife is used to cut the string at the instant when the ball is at its lowest point. Sketch the subsequent trajectory of the ball until it hits the ground. Explain.



The trajectory is parabolic, like that of a horizontally launched projectile.

9. The figures are a bird's-eye view of particles moving in horizontal circles on a table top. All are moving at the same speed. Rank in order, from largest to smallest, the tensions T_1 to T_4.

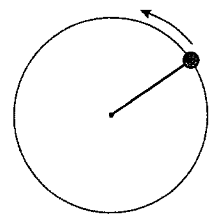


Order: $T_3 > T_1 = T_4 > T_2$

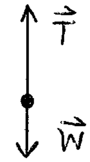
Explanation:

$T = \frac{mv^2}{r}$ Case 3 combines a larger mass and smaller r. Case 4 is the same as case 1 because both the mass and the radius are doubled.

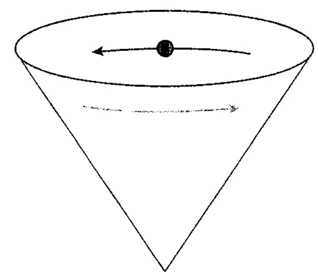
10. A ball on a string moves in a vertical circle. When the ball is at its lowest point, is the tension in the string greater than, less than, or equal to the ball's weight? Explain. (You should include a free-body diagram as part of your explanation.)

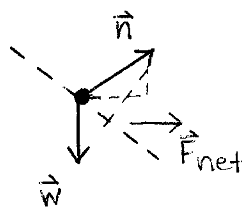


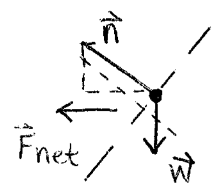
At the lowest point, the acceleration is upward. Thus, the tension must be greater than the weight for the net force to be upward.



11. A marble rolls around the inside of a cone. Draw a free-body diagram of the marble when it is on the left side of the cone and a free-body diagram of the marble when it is on the right side of the cone.



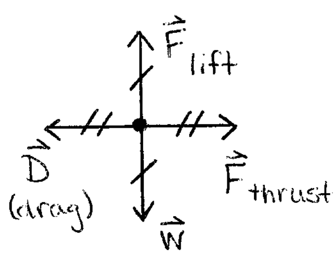




On left side

On right side

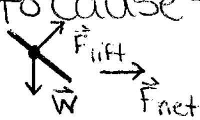
12. A jet airplane is flying on a level course at constant velocity.
   a. What is the *net* force on the plane? $\vec{F}_{net} = 0$
   b. Draw a free-body diagram and identify all of the forces acting on the plane.



$|\vec{F}_{lift}| = |\vec{w}|$

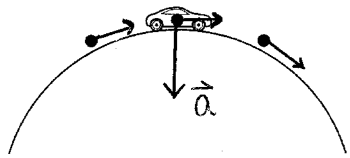
$|\vec{F}_{thrust}| = |\vec{F}_{drag}|$

   c. Airplanes bank when they turn. Explain why, in terms of forces and physical laws. Hint: What would a free-body diagram look like to an observer *behind* the plane?

When the plane banks, $\vec{F}_{lift}$ includes a horizontal component. The horizontal component of $\vec{F}_{lift}$ provides a radically inward force needed to cause the plane to turn. From behind the plane:  In this case, $|\vec{F}_{lift}| > |\vec{w}|$, if the turn is in a horizontal direction (level flight).

6.4 Apparent Forces in Circular Motion

13. The drawing shows a car moving clockwise at constant speed over the top of a circular hill.
   a. On the drawing, complete the motion diagram by showing the car's velocity vectors for each of the three motion diagram points and use these to indicate the acceleration of the car at the top of the hill.
   b. To the right of the sketch, draw a free-body diagram for the car when at the top of the hill and indicate the direction of the net force on the car.

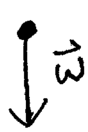


   c. Is your free-body diagram consistent with the motion diagram? Explain.

Yes. The weight is greater than the normal force so the net force is downward.

   d. For this situation, is there a maximum speed at which the car can travel over the top of the hill and not lose contact with the hill? If not, why not? If so, show how your free-body diagram would change, if at all, at that speed.

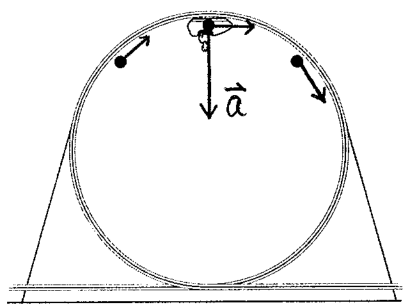
Yes. As the speed increases, the acceleration required for circular motion increases and the force causing that acceleration must also increase. The only way to increase the downward net force is to decrease the normal force. But if the normal force becomes zero, the car will lose contact at any greater speed.

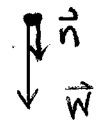


14. The drawing shows a car moving upside down while looping a circular roller coaster loop-the-loop at constant speed in the clockwise direction.

   a. On the drawing, complete the motion diagram by showing the car's velocity vectors for each of the three motion diagram points and use these to indicate the acceleration of the car at the top of the loop-the-loop.

   b. To the right of the sketch, draw a free-body diagram for the car at the top of the loop and indicate the direction of the net force on the car. (Assume the car is moving fast enough so that it would not fall, even if not attached to the roller coaster track.)



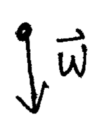


   c. Is your free-body diagram consistent with your motion diagram? Explain.

Yes. The acceleration and the net force are both downward.

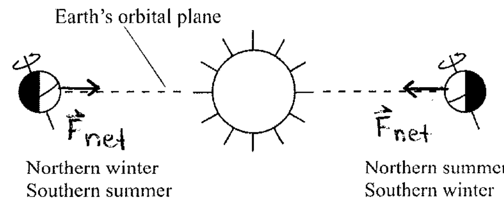
   d. For this situation, is there a minimum speed at which the car can travel over the top of the loop and not lose contact with the loop? If not, why not? If so, show how your free-body diagram would change, if at all, at that speed.

Yes. As the speed decreases, the acceleration downward must decrease as well as the net force downward. But the downward force cannot be smaller than the weight in this situation. If the weight is greater than the force needed for the acceleration at that speed, the car will fall.



6.5 Circular Orbits and Weightlessness

15. The earth has seasons because the axis of the earth's rotation is tilted 23° away from a line perpendicular to the plane of the earth's orbit. You can see this in the figure, which shows the edge of the earth's orbit around the sun. For both positions of the earth, draw a force vector to show the net force acting on the earth or, if appropriate, write $\vec{F} = \vec{0}$.



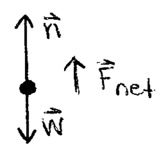
16. A small projectile is launched parallel to the ground at height $h = 1$ m with sufficient speed to orbit a completely smooth, airless planet. A bug rides in a small hole inside the projectile. Is the bug weightless? Explain.

The bug is weightless in the sense that it is in freefall with the projectile. The bug still has a weight of $\vec{W} = m_{bug}g$.

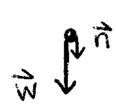
17. A stunt plane does a series of vertical loop-the-loops. At what point in the circle does the pilot feel the heaviest? Explain. Include a free-body diagram with your explanation.

The pilot feels heaviest at the bottom of the vertical loop. At that point, the normal force on the pilot is greatest, as is the apparent weight.

At bottom:



At top:



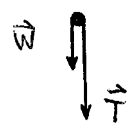
This analysis assumes the pilot is moving at comparable speed throughout.

18. A roller-coaster car goes around the inside of a loop-the-loop. Check the statement that is true when the car is at the highest point and at the lowest point in the loop.

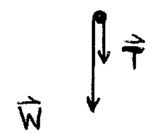
	Highest	Lowest
The apparent weight w_{app} is always less than w		
The apparent weight w_{app} is always equal to w		
The apparent weight w_{app} is always greater than w		✓
w_{app} could be less than, equal to, or greater than w	✓	

19. You can swing a ball on a string in a *vertical* circle if you swing it fast enough.

a. Draw two free-body diagrams of the ball at the top of the circle. On the left, show the ball when it is going around the circle very fast. On the right, show the ball as it goes around the circle more slowly.



Very fast



Slower

b. As you continue slowing the swing, there comes a frequency at which the string goes slack and the ball doesn't make it to the top of the circle. What condition must be satisfied for the ball to be able to complete the full circle?

$\vec{F}_{net} = m\frac{v^2}{r} = m\omega^2 r$ The minimum downward force is the weight, so $mg = mr\omega^2_{min}$ or $\omega^2_{min} = g/r$ $\omega_{min} = \sqrt{g/r}$

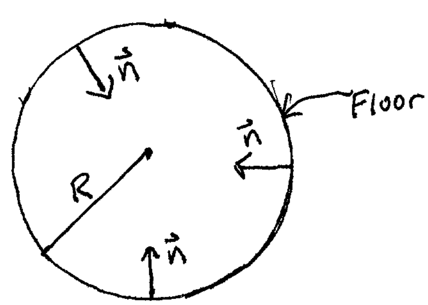
c. Suppose the ball has the smallest possible frequency that allows it to go all the way around the circle. What is the tension in the string when the ball is at the highest point? Explain.

$\vec{T} = 0$. At the smallest frequency, the only radially inward force is the force of gravity, the weight.

20. It's been proposed that future space stations create "artificial gravity" by rotating around an axis. (The space station would have to be much larger than the present space station for this to be feasible.)

a. How would this work? Explain.

The outside wall of the station would provide the floor and the normal force required to keep the occupants and contents rotating would be the apparent weight of the objects in this artificial gravity.



b. Would the artificial gravity be equally effective throughout the space station? If not, where in the space station would the residents want to live and work?

The apparent weight would be due to an inward normal force provided by the outside wall. As one moves inward, the artificial gravity would become weaker due to the smaller radius. $\vec{n} = m\omega^2 r$.

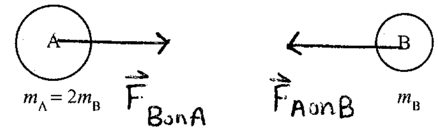
6.6 Newton's Law of Gravity

21. Is the earth's gravitational force on the sun larger than, smaller than, or equal to the sun's gravitational force on the earth? Explain.

The gravitational forces are equal and opposite. They are an action-reaction pair.

22. Star A is twice as massive as star B.

a. Draw gravitational force vectors on both stars. The length of each vector should be proportional to the size of the force.



b. Is the acceleration of star A larger than, smaller than, or equal to the acceleration of star B? Explain.

The acceleration of star A is smaller. For equal forces, the smaller mass experiences the greater acceleration because $a = F/m$.

23. The quantity y is inversely proportional to the square of x and $y = 4$ when $x = 5$.

a. Write an equation to represent this inverse-square relationship for all y and x.

$y = \frac{100}{x^2}$ $\quad (k = yx^2 = 100)$

b. Find y if $x = 2$. $y = 25$

c. Find x if $y = 100$. $x = 1$

d. By what factor must x change for the value of y to double? $1/\sqrt{2}$

e. Compare your equation in part a to the equation from your text relating the force of gravitational attraction of two objects F to the distance between them r,

$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = \frac{Gm_1m_2}{r^2}$.

Which quantity assumes the role of x? r

Which quantity assumes the role of y? F

What is the constant of proportionality relating F and r^2? Gm_1m_2

24. How far away from the earth does an orbiting spacecraft have to be in order for the astronauts inside to be "weightless"?

The astronauts can be "weightless" at any distance because an object is said to be weightless if it is in freefall (as in orbit). For the gravitational force to become zero, the spacecraft would have to be an infinite distance away.

25. The acceleration due to gravity at the surface of planet 1 is 20 m/s^2. The radius and the mass of planet 2 are twice those of planet 1. What is g on planet 2?

$g \propto \frac{m}{r^2}$ $\quad g_2 = g_1 \frac{2}{(2)^2} = \frac{g_1}{2}$

6.7 Gravity and Orbits

26. Planet X orbits the star Omega with a "year" that is 200 earth days long. Planet Y circles Omega at four times the distance of planet X. How long is a year on planet Y?

From Kepler's third law, the orbital period squared is proportional to the orbital radius cubed. $T^2 \propto r^3$. Thus, at $r_Y = 4r_X$, $T_Y^2 \propto (4r_X)^3 = 64 r_X^3 \propto (8T_X)^2$

$T_Y = 8T_X$. A year on planet Y is 1600 earth days long.

27. The mass of Jupiter is $M_{\text{Jupiter}} = 300M_{\text{earth}}$. Jupiter orbits around the sun with $T_{\text{Jupiter}} = 11.9$ years in an orbit with $r_{\text{Jupiter}} = 5.2r_{\text{earth}}$. Suppose the earth could be moved to the distance of Jupiter and placed in a circular orbit around the sun. The new period of the earth's orbit would be

a. 1 year.
b. 11.9 years.
c. Between 1 year and 11.9 years.
d. More than 11.9 years.
e. It could be anything, depending on the speed the earth is given.
f. It is impossible for a planet of earth's mass to orbit at the distance of Jupiter.

Circle the letter of the true statement. Then explain your choice.

The orbital period is independent of the mass of the orbiting body, provided that the orbiting body's mass is much less than the mass of the body being orbited.

28. The gravitational force of a star on orbiting planet 1 is F_1. Planet 2, which is twice as massive as planet 1 and orbits at twice the distance from the star, experiences gravitational force F_2.

a. What is the ratio F_2/F_1?

$$F_1 = \frac{-GMm_1}{r_1^2} \quad \text{so } F_2 = \frac{-GM(2m_1)}{(2r_1)^2} \qquad (M = \text{Mass of star})$$

$$\frac{F_2}{F_1} = \frac{\frac{-GM(2m_1)}{(2r_1)^2}}{\frac{-GMm_1}{r_1^2}} = \frac{2}{2^2} \boxed{= \frac{1}{2}}$$

b. Planet 1 orbits ~~with~~ the star with period T_1 and planet 2 with period T_1. What is the ratio T_2/T_1?

$$T_1^2 = \left(\frac{4\pi^2}{GM}\right)r_1^3 \quad \text{so } T_2^2 = \left(\frac{4\pi^2}{GM}\right)(2r_1)^3 = 8T_1^2$$

$$\frac{T_2}{T_1} = \left(\frac{T_2^2}{T_1^2}\right)^{\frac{1}{2}} = \left(\frac{8}{1}\right)^{\frac{1}{2}} = 2\sqrt{2} \boxed{= 2.83}$$

29. Satellite A orbits a planet with a speed of 10,000 m/s. Satellite B is twice as massive as satellite A and orbits at twice the distance from the center of the planet. What is the speed of satellite B?

Satellite B's mass is not relavent, but at twice the distance, its orbital period is greater by a factor of $\sqrt{(2)^3} = 2\sqrt{2}$. But the distance it must travel during each period is also greater by a factor of 2. Thus, its speed is $2/2\sqrt{2}$ less or $1/\sqrt{2}$ less.

$$V_B = .7071 V_A = 7.071 \frac{m}{s} \approx 7 \times 10^3 \frac{m}{s}$$