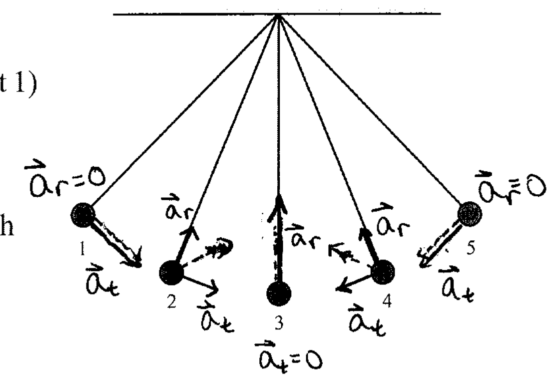
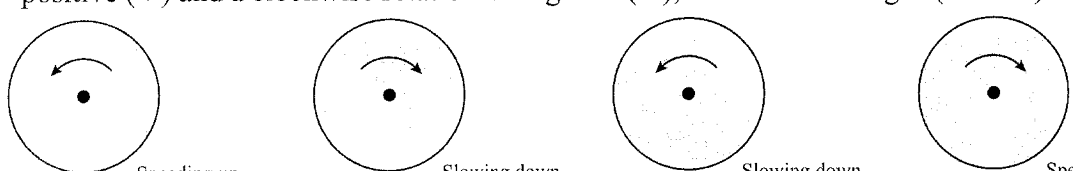
# 7 Rotational Motion

## 7.1 The Rotation of a Rigid Body

1. A pendulum swings from its end point on the left (point 1) to its end point on the right (point 5). At each of the labeled points:
   a. Use a **black** pen or pencil to draw and label the vectors $\vec{a}_c$ and $\vec{a}_t$ at each point. Make sure the length indicates the relative size of the vector. ———
   b. Use a **red** pen or pencil to draw and label the total acceleration vector $\vec{a}$. – – – →

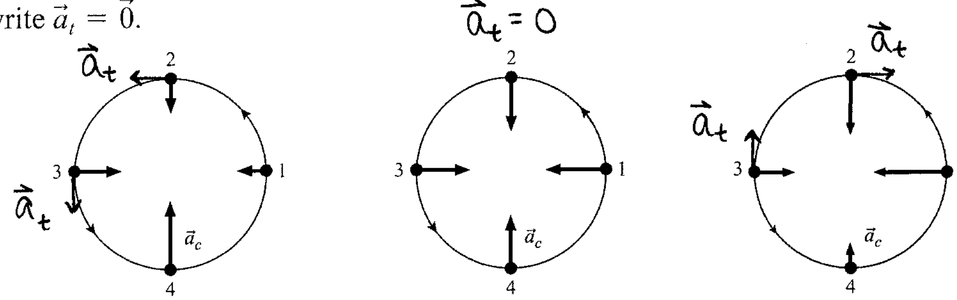



2. The following figures show a rotating wheel. If we consider a counterclockwise rotation as positive (+) and a clockwise rotation as negative (−), determine the signs (+ or −) of $\omega$ and $\alpha$.



| Speeding up | Slowing down | Slowing down | Speeding up |
|---|---|---|---|
| $\omega$ + | $\omega$ − | $\omega$ + | $\omega$ − |
| $\alpha$ + | $\alpha$ + | $\alpha$ − | $\alpha$ − |

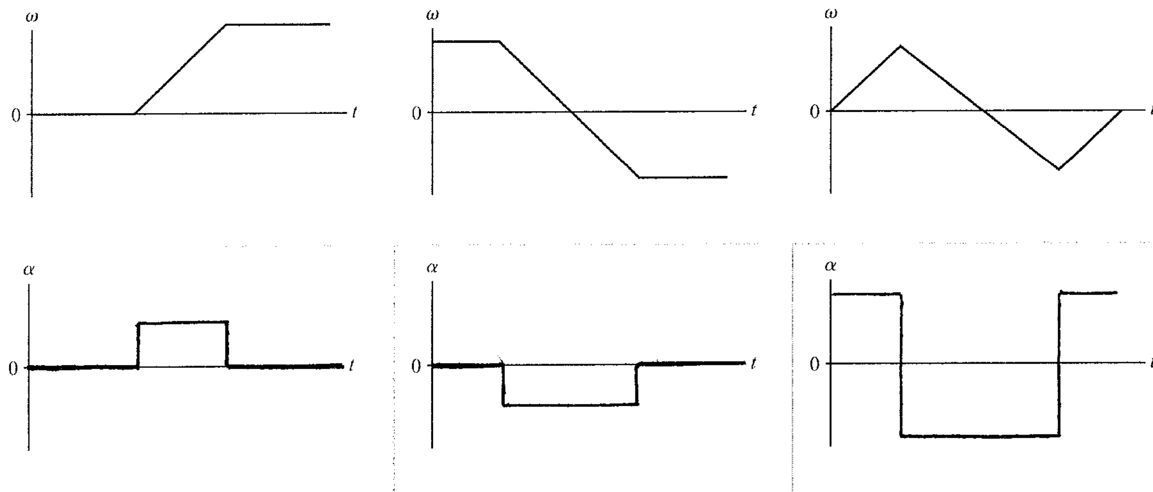
3. The figures below show the centripetal acceleration vector $\vec{a}_c$ at four successive points on the trajectory of a particle moving in a counterclockwise circle.
   a. For each, draw the tangential acceleration vector $\vec{a}_t$ at points 2 and 3 or, if appropriate, write $\vec{a}_t = \vec{0}$.



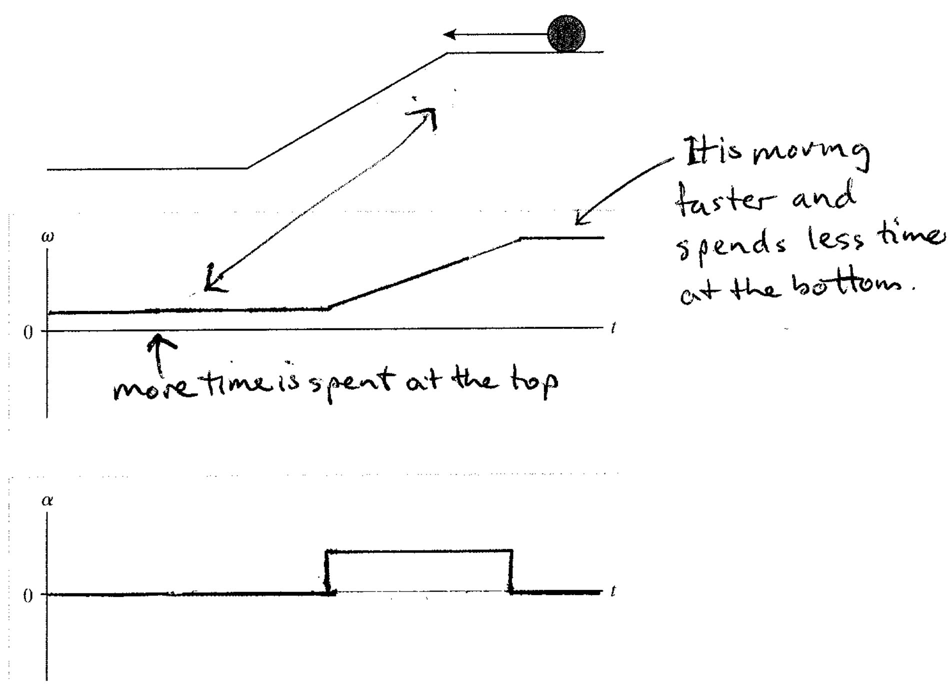

   b. If we consider a counterclockwise rotation as positive and a clockwise rotation as negative, determine if the particle's angular acceleration $\alpha$ is positive (+), negative (−), or zero (0).

   $\alpha =$ +   $\alpha =$ 0   $\alpha =$ −

4. Below are three angular velocity-versus-time graphs. For each, draw the corresponding angular acceleration-versus-time graph.

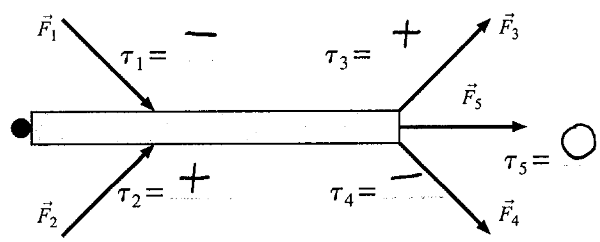



5. A wheel rolls to the left along a horizontal surface, down a ramp, then continues along the lower horizontal surface. Draw graphs for the wheel's angular velocity $\omega$ and angular acceleration $\alpha$ as functions of time.

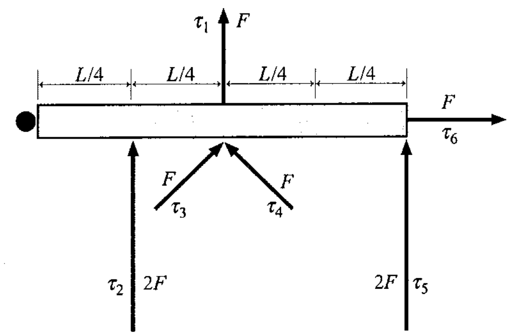


## 7.2 Torque

6. Five forces are applied to a door. For each, determine if the torque about the hinge is positive (+), negative (−), or zero (0).




7. Six forces, each of magnitude either $F$ or $2F$, are applied to a door. Rank in order, from largest to smallest, the six torques $\tau_1$ to $\tau_6$ about the hinge.




Order: $\tau_5 > \tau_1 = \tau_2 > \tau_3 = \tau_4 > \tau_5$

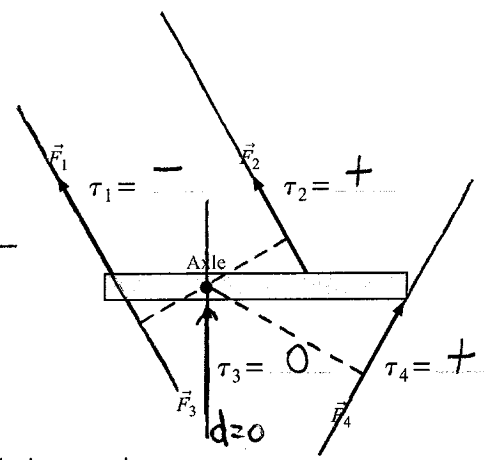
Explanation:

$\tau_5 = L(2F)$ $\quad \tau_3 = \frac{L}{2} F \sin 45°$

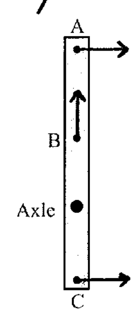
$\tau_1 = \frac{L}{2}(F)$ $\quad \tau_4 = \frac{L}{2} F \sin 45°$

$\tau_2 = \frac{L}{4}(2F)$ $\quad \tau_6 = LF \sin 0° = 0$

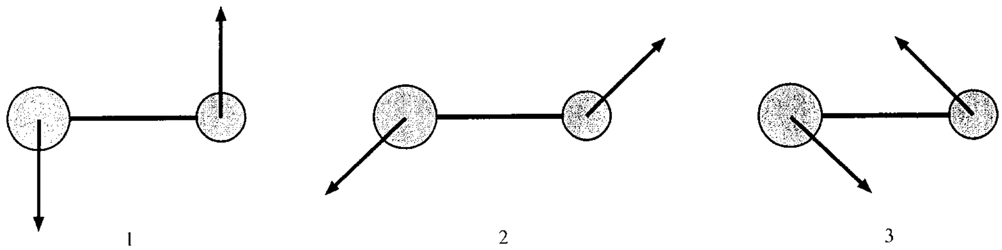
8. Four forces are applied to a rod that can pivot on an axle. For each force,
   a. Use a **black** pen or pencil to draw the line of action.
   b. Use a **red** pen or pencil to draw and label the moment arm, or state that $d = 0$.
   c. Determine if the torque about the axle is positive (+), negative (−), or zero (0).




9. a. Draw a force vector at A whose torque about the axle is negative.
   b. Draw a force vector at B whose torque about the axle is zero.
   c. Draw a force vector at C whose torque about the axle is positive.



10. The dumbbells below are all the same size, and the forces all have the same magnitude. Rank in order, from largest to smallest, the torques $\tau_1$, $\tau_2$, and $\tau_3$ about the midpoint of each connecting rod.

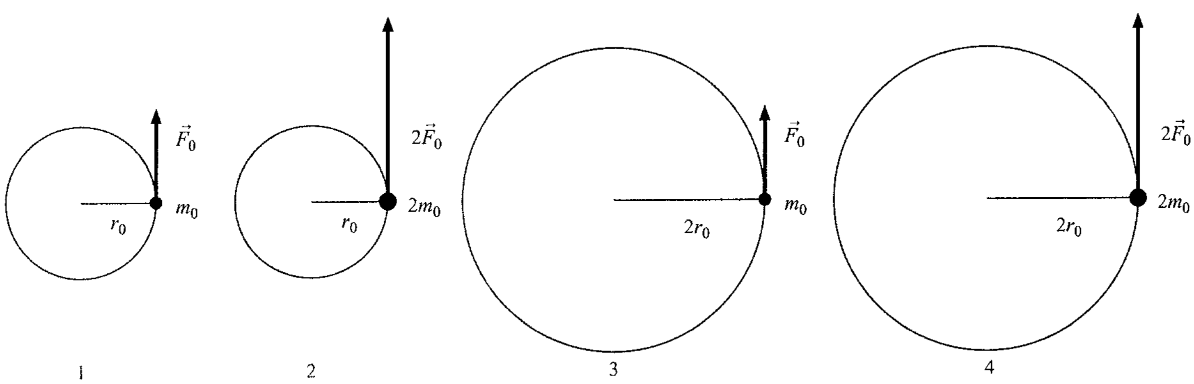



Order: $\tau_1 > \tau_2 = \tau_3$

Explanation:

An equal force will lead to a greater torque if applied at 90°. A torque applied at 90° − $\phi$ leads to the same torque if applied at 90° + $\phi$.

11. a. Rank in order, from largest to smallest, the torques $\tau_1$ to $\tau_4$.




Order: $\tau_4 > \tau_2 = \tau_3 > \tau_1$

Explanation:

$\tau = rF$, $\tau_4 = 2r_0 2F_0$, $\tau_2 = r_0 2F_0$, $\tau_3 = 2r_0 F_0$, $\tau_1 = r_0 F_0$

b. Rank in order, from largest to smallest, the angular accelerations $\alpha_1$ to $\alpha_4$.

Order: $\alpha_1 = \alpha_2 > \alpha_3 = \alpha_4$

Explanation:

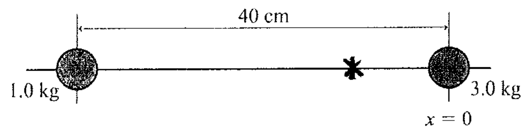
$\tau = I\alpha$ so $\alpha = \tau / I$

$I_1 = m_0 r_0^2$, $I_2 = 2m_0 r_0^2$, $I_3 = 4m_0 r_0^2$, $I_4 = 8m_0 r_0^2$

$\alpha_1 = \frac{r_0 F_0}{m_0 r_0^2} = \alpha_2 = \frac{2r_0 F_0}{2m_0 r_0^2}$ $\quad \alpha_3 = \frac{2r_0 F_0}{4m_0 r_0^2} = \alpha_4 = \frac{4r_0 F_0}{8m_0 r_0^2}$

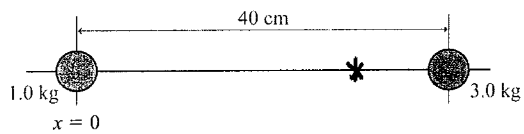
## 7.3 Gravitational Torque and the Center of Gravity

12. a. Find the coordinates for and *mark* the center of gravity for the pair of masses shown, using the center of the 3.0 kg mass as the origin.




$x_{cg}$ = 10 cm

b. Find the coordinates for and *mark* the center of gravity for the pair of masses shown, using the center of the 1.0 kg mass as the origin.

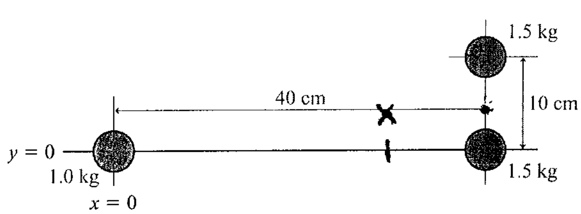



$x_{cg}$ = 30 cm

c. How do the locations for the marks in parts a and b compare? How do the coordinates compare?

The location is the same, but the coordinates are not.

d. The 3.0 kg mass from parts a and b above is separated into two 1.5 kg pieces. One of these is moved 10 cm in the $+y$-direction. Find the coordinates for and *mark* the center of gravity of the new system using the origin at the left-side mass.



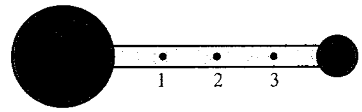

$y_{cg}$ = 3.75cm

$x_{cg}$ = 30 cm

e. What effect did separating the 3.0 kg mass along the $y$-direction have on the $x$-component of the center of gravity of the system?

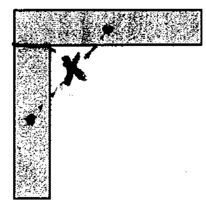
Movement of any part of the mass in the y-direction does not affect the x-component of the center of gravity.

13. Is the center of gravity of this dumbbell at point 1, 2, or 3? Explain. (Assume the end masses are of the same material.)

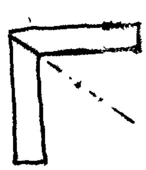



1. Assuming the larger sphere is more massive, the center of mass will be nearer to the larger sphere.

14. Mark the center of gravity of this object with an ×. Explain.



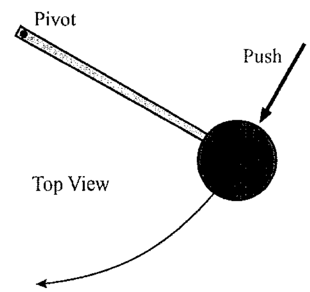

The object can be treated as a composite of two uniform rectangles. The center of gravity of each rectangle is at its geometric center and is marked here with a dot. The center of gravity of the composite must be along the line connecting these two dots, closer to the upper dot because the upper rectangle is larger. By symmetry, the center of gravity must lie along a diagonal line through the top left corners. This can also be seen by considering the object as a composite of two identical trapezoids



with a center of gravity along the line passing through their intersection.

## 7.4 Rotational Dynamics and Moment of Inertia

15. A student gives a quick push to a ball at the end of a massless, rigid rod, causing the ball to rotate clockwise in a *horizontal* circle. The rod's pivot is frictionless.




a. As the student is pushing, is the torque about the pivot positive, negative, or zero? negative

b. After the push has ended, does the ball's angular velocity

i. Steadily increase?

ii. Increase for awhile, then hold steady?

$\boxed{\text{iii. Hold steady?}}$

iv. Decrease for awhile, then hold steady?

v. Steadily decrease?

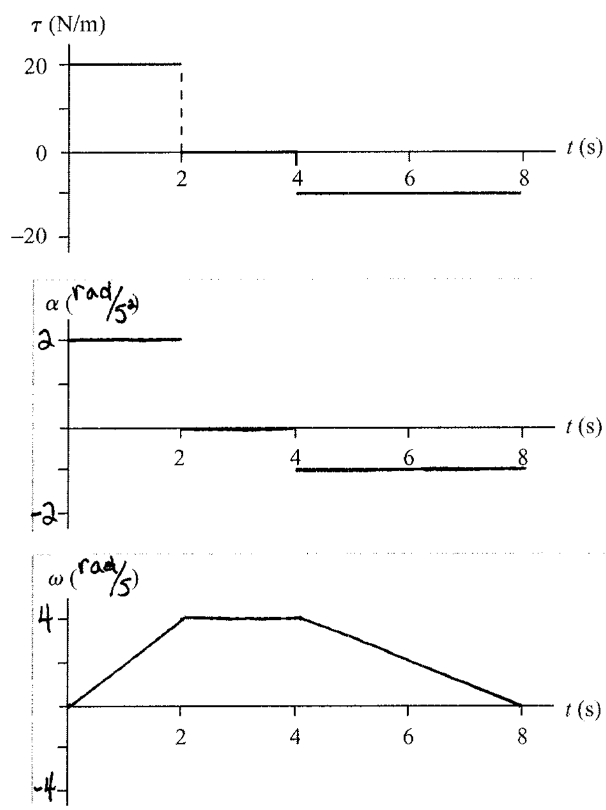
Explain the reason for your choice.

There is no more torque after the push ends. Therefore, there is no angular acceleration.

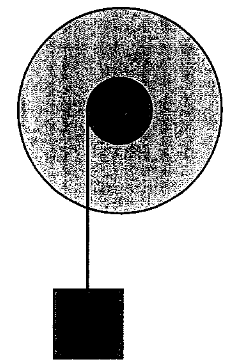
c. Right after the push has ended, is the torque positive, negative, or zero? zero

16. The top graph shows the torque on a rotating wheel as a function of time. The wheel's moment of inertia is 10 kg-m$^2$. Draw graphs of $\alpha$-versus-$t$ and $\omega$-versus-$t$, assuming $\omega_0 = 0$.

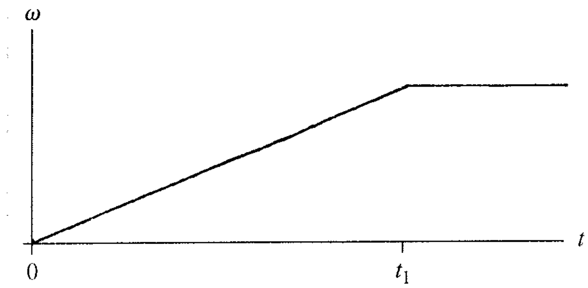
$\tau = I\alpha$ ; $\alpha = \tau / I$



17. The wheel turns on a frictionless axle. A string wrapped around the smaller diameter shaft is tied to a block. The block is released at $t = 0$ s and hits the ground at $t = t_1$.



a. Draw a graph of $\omega$-versus-$t$ for the wheel, starting at $t = 0$ s and continuing to some time $t > t_1$.

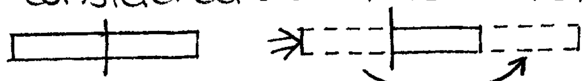



b. Is the magnitude of the block's downward acceleration greater than $g$, less than $g$, or equal to $g$? Explain.

The block's acceleration is less than g. The weight of the block must also turn the axle.

18. The moment of inertia of a uniform rod about an axis through its center is $\frac{1}{12}ML^2$. The moment of inertia about an axis at one end is $\frac{1}{3}ML^2$. Explain *why* the moment of inertia is larger about the end than about the center.

When considered about one end, then half of the mass is at a distance from the axis that is greater than any of the mass if considered about its center. The dotted mass



has effectively been moved outward as shown.

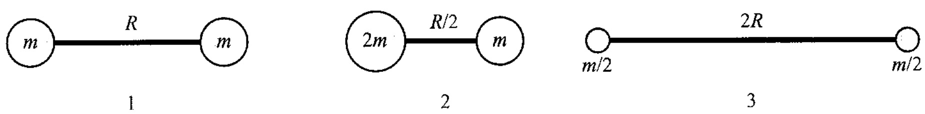
19. You have two steel spheres. Sphere 2 has twice the radius of sphere 1. By what *factor* does the moment of inertia $I_2$ of sphere 2 exceed the moment of inertia $I_1$ of sphere 1?

Because sphere 2 has twice the radius, its mass is greater by a factor of eight. ($2^3$). ($m = \frac{4}{3}\pi r^3 \rho_{steel}$). The added mass is also distributed further from the center and, so, $I \propto mr^2$ leads to $I_2 \propto (8m_1)(2r_1)^2 = \boxed{32 I_1}$

20. The professor hands you two spheres. They have the same mass, the same radius, and the same exterior surface. The professor claims that one is a solid sphere and that the other is hollow. Can you determine which is which without cutting them open? If so, how? If not, why not?

It will be easier to rotate the solid sphere because the hollow sphere's mass is generally distributed further from its center. If you roll both simultaneously down an incline, the solid sphere will win.

21. Rank in order, from largest to smallest, the moments of inertia $I_1$, $I_2$, and $I_3$ about the midpoint of each connecting rod.




Order: $I_3 > I_1 > I_2$

Explanation:

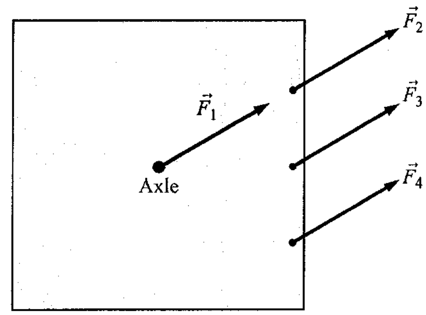
$$I_1 \sim 2m\left(\frac{R}{2}\right)^2 = \frac{mR^2}{2}$$

$$I_2 \sim 2m\left(\frac{R}{4}\right)^2 + m\left(\frac{R}{4}\right)^2 = \frac{3mR^2}{16}$$

$$I_3 \sim \frac{m}{2}R^2 + \frac{m}{2}R^2 = mR^2$$

## 7.5 Using Newton's Second Law for Rotation

22. A square plate can rotate about an axle through its center. Four forces of equal magnitude are applied to different points on the plate. The forces turn as the plate rotates, maintaining the same orientation with respect to the plate. Rank in order, from largest to smallest, the angular accelerations $\alpha_1$ to $\alpha_4$.

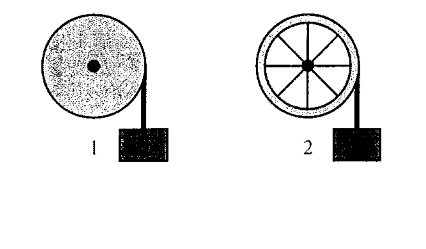



Order: $\alpha_4 > \alpha_3 > \alpha_2 = \alpha_1$

Explanation:

Both $F_2$ and $F_1$ act along a line through the axle and so cause no torque.

23. A solid cylinder and a cylindrical shell have the same mass, same radius, and turn on frictionless, horizontal axles. (The cylindrical shell has lightweight spokes connecting the shell to the axle.) A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground. Both blocks are released simultaneously. The ropes do not slip.

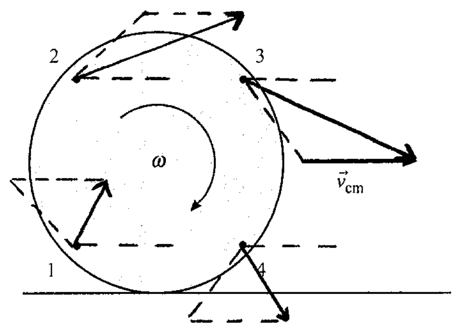



Which block hits the ground first? Or is it a tie? Explain.

The block attached to the solid cylinder hits first. The solid cylinder has a smaller moment of inertia and therefore requires less torque to cause it to rotate, allowing the block to fall faster.

## 7.6 Rolling Motion

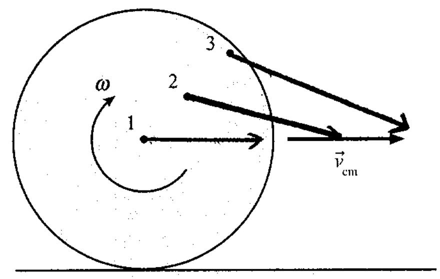
24. A wheel is rolling along a horizontal surface with the center-of-mass velocity shown. Draw the velocity vector $\vec{v}$ at points 1 to 4 on the rim of the wheel.




$|V_1| = |V_4|$

$|V_2| = |V_3|$

25. A wheel is rolling along a horizontal surface with the center-of-mass velocity shown. Draw the velocity vector $\vec{v}$ at points 1 to 3 on the wheel.




26. If a solid disk and a circular hoop of the same mass and radius are released from rest at the top of a ramp and allowed to roll to the bottom, the disk will get to the bottom first. *Without referring to equations*, explain why this is so.

The hoop has all of its mass concentrated at its rim, further from the axis of rotation, making it require a longer time to reach the same rotational speed.