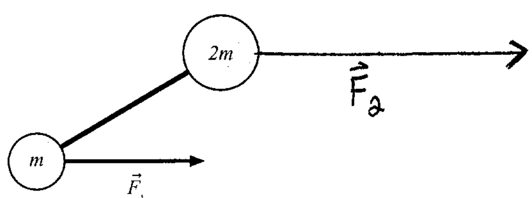
8 Equilibrium and Elasticity

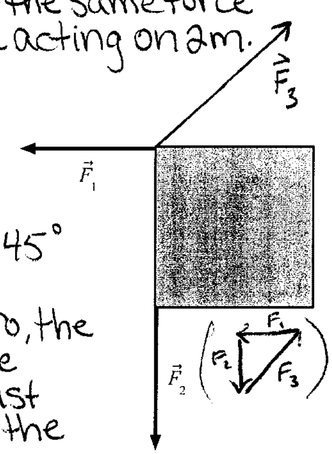
8.1 Torque and Static Equilibrium

1. The dumbbell has masses m and $2m$. Force $\vec{F}_1$ acts on mass m in the direction shown. Is there a force $\vec{F}_2$ that can act on mass $2m$ such that the dumbbell moves with pure translational motion, without any rotation? If so, draw $\vec{F}_2$, making sure that its length shows the magnitude of $\vec{F}_2$ relative to $\vec{F}_1$. If not, explain why not.



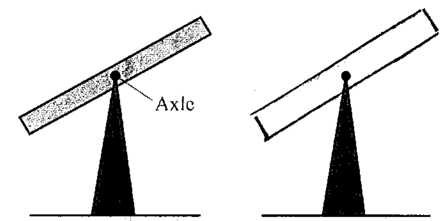
$\vec{F}_2$ must be twice the magnitude and in the same direction as $\vec{F}_1$. The center of mass of the dumbbell is 2/3 of the way from m to 2m. Thus, $\vec{F}_1$ contributes a torque about that center of mass that is twice what the same force would require acting on 2m.

2. Forces $\vec{F}_1$ and $\vec{F}_2$ have the same magnitude and are applied to the corners of a square plate. Is there a *single* force $\vec{F}_3$ that, if applied to the appropriate point on the plate, will cause the plate to be in total equilibrium? If so, draw it, making sure it has the right position, orientation, and length. If not, explain why not.



For the force to sum to zero, $\vec{F}_3$ must be applied at 45° above the rightward horizontal and have a magnitude of $\sqrt{2}\,|F_1|$. For the torques to sum to zero, the force $\vec{F}_3$ must be applied so as to cause a clockwise torque of magnitude $F_1\frac{a}{2} + F_2\frac{a}{2} = F_1 a$. Thus $\vec{F}_3$ must be applied at a distance of $a/\sqrt{2}$ from the center of the square plate.

3. A uniform rod pivots about a frictionless, horizontal axle through its center. It is placed on a stand, held motionless in the position shown, then gently released. On the right side of the figure, draw the final, equilibrium position of the rod. Explain your reasoning.

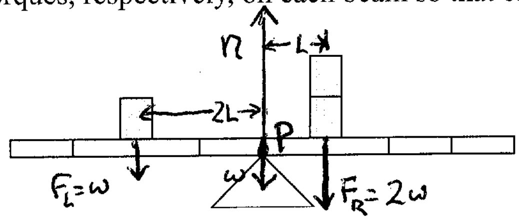


If the rod is already balanced at its center, there is no net torque to cause it to rotate. It is already in equilibrium.

Exercises 4–8: For each of the following

- Draw and label the appropriate force vectors at the locations at which each force acts on the beams and with the appropriate relative lengths so that each beam is in equilibrium. Assume each beam is uniform and has a weight w. Assume each block also has a weight w.
- Write three equilibrium equations to sum the vertical forces, the horizontal forces, and the torques, respectively, on each beam so that each beam is can be shown to be in equilibrium.

4.

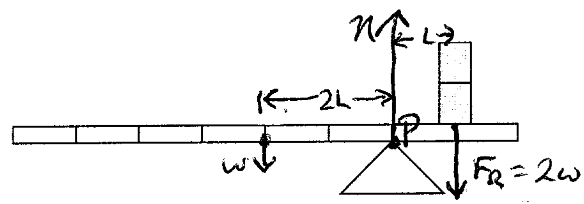


$\sum F_y = n - w - F_L - F_R = n - 4w = 0$

$\sum F_x = 0$

$\sum \tau_p = F_L(2L) - F_R(L) = 0$

5.

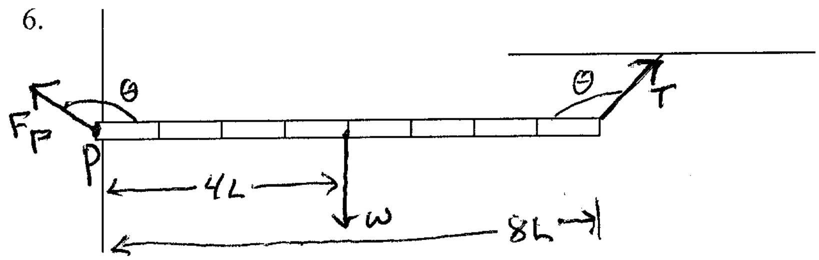


$\sum F_y = n - w - F_R = n - 3w = 0$

$\sum F_x = 0$

$\sum \tau_p = w(2L) - F_R(L) = 0$

6.

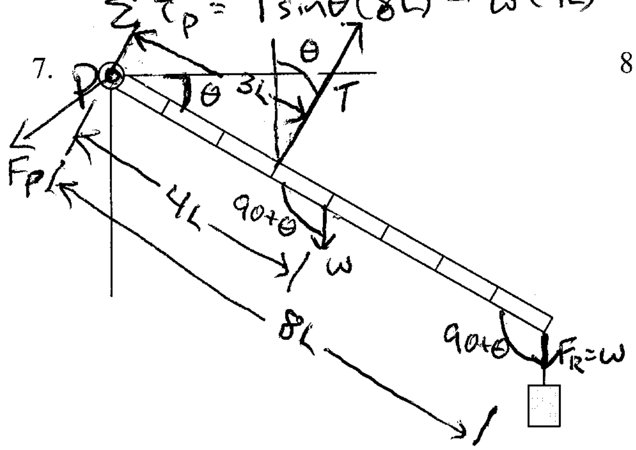


$\sum F_x = T_x + F_{px} = T\cos\theta - F_p\cos\theta = 0$

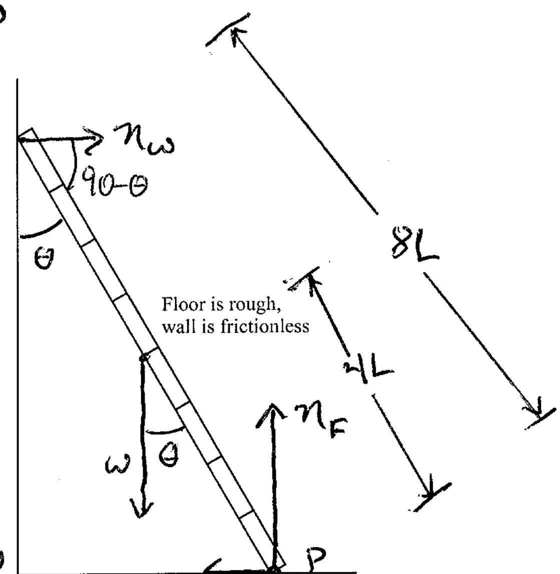
$\sum F_y = T_y + F_{py} - w = T\sin\theta + F_p\sin\theta - w = 0$

$\sum \tau_p = T\sin\theta(8L) - w(4L) = 0$

7.



8.



$\sum F_x = T_x + F_{px} = T\sin\theta - |F_{px}| = 0$

$\sum F_y = T_y + F_{py} - F_R - w = T\cos\theta - |F_{py}| - 2w = 0$

$\sum \tau_p = T(3L) - w(4L)\sin(90+\theta) - F_R(8L)\sin(90+\theta)$

$= T(3L) - w(4L)\cos\theta - w(8L)\cos\theta = 0$

$\sum F_x = n_w - f_F = 0$

$\sum F_y = n_F - w = 0$

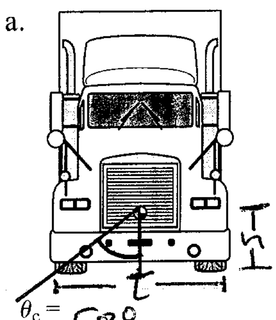
$\sum \tau_p = w(4L)\sin\theta - n_w(8L)\sin(90-\theta)$

$= w(4L)\sin\theta - n_w(8L)\cos\theta = 0$

8.2 Stability and Balance

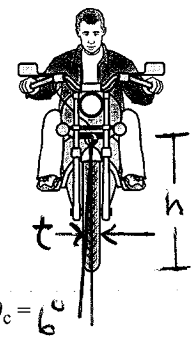
9. The center of gravity (⊙) is marked on each of the objects shown below.
- Mark and label the track width supporting the object.
- Clearly label the critical angle for each object.
- Measure the height to the center of gravity and the track width and determine the approximate value for the critical angle in each case.

a.



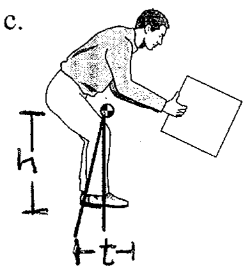
$\theta_c = 58°$

b.



$\theta_c = 6°$

c.



$\theta_c = 15°$

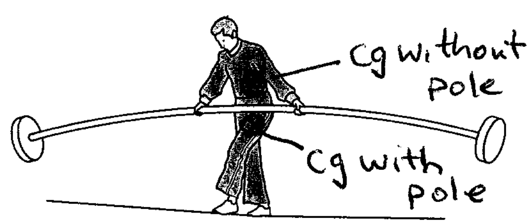
$t = 2.2cm$ $\quad \theta_c = \tan^{-1}\left(\frac{t/2}{h}\right)$ $\quad t = 0.2cm$ $\quad \theta_c = \tan^{-1}\left(\frac{t}{2h}\right)$ $\quad t = 0.7cm$ $\quad \theta_c = \tan^{-1}\left(\frac{.7}{2(1.3)}\right)$

$h = 0.7cm$ $\quad h = 1.8cm$ $\quad \approx 6°$ $\quad h = 1.3cm$ $\quad = 15°$

$\approx 58°$

10. Tight rope walkers often carry long poles that are weighted at the ends. These poles serve at least two purposes, one is to change the critical angle and the second is to increase the walker's moment of inertia.

a. Use the diagram below to describe how the critical angle is changed by the tightrope walker's use of the pole.



Because the pole is heavily weighted on its sagging ends, it lowers the center of gravity.

θ_c increases as h decreases.

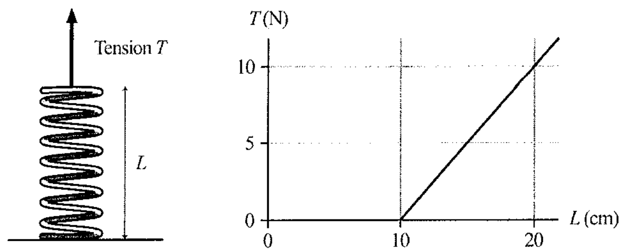
b. Why would increasing the tightrope walker's moment of inertia also help to make him less likely to fall?

$\tau = I\alpha$. A torque is caused by the weight force not passing through the track width. As I increases, the resulting angular acceleration α decreases, allowing more time for the walker to move so as to bring the center of gravity back within the track width.

(Much greater benefits are possible if the center of gravity can be lowered below the tightrope.)

8.3 Springs and Elastic Materials

11. A spring is attached to the floor and pulled straight up by a string. The spring's tension is measured. The graph shows the tension in the spring as a function of the spring's length L.



a. Does this spring obey Hooke's Law? Explain why or why not.

Yes, the plot is linear $\Delta T = k \Delta L$

b. If it does, what is the spring constant?

$k = \frac{\Delta T}{\Delta L} = \frac{10\text{ N}}{10\text{ cm}} = 1\frac{\text{N}}{\text{cm}} = 100\text{ N/m}$

12. A spring has an unstretched length of 10 cm. It exerts a restoring force F when stretched to a length of 11 cm.

a. For what length of the spring is its restoring force $3F$? 13 cm

b. At what compressed length is the restoring force $2F$? 8 cm

13. The left end of a spring is attached to a wall. When Bob pulls on the right end with a 200 N force, he stretches the spring by 20 cm. The same spring is then used for a tug-of-war between Bob and Carlos. Each pulls on his end of the spring with a 200 N force.

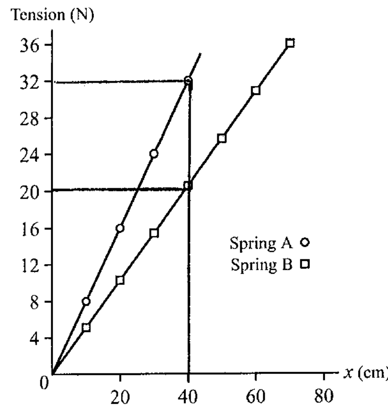
a. How far does Bob's end of the spring move? Explain.

10 cm. Though the spring stretched 20 cm originally, its center moved by 10 cm. In this case Carlos provides the opposing force previously provided by the wall, except that he moves also.

b. How far does Carlos's end of the spring move? Explain.

−10 cm. The total stretch under a 200 tension must still be 20 cm.

14. The graph below shows the stretching of two different springs, A and B, when different forces were applied.



a. Determine the spring constant for each spring.

$T_A = k_A \Delta x$ (line slope)

$k_A = \frac{T_A}{\Delta x} = \frac{32\,N}{.40\,m}$

$k_B = \frac{T_B}{\Delta x_B} = \frac{20\,N}{.40\,m}$

$k_A =$ $80\,N/m$

$k_B =$ $50\,N/m$

b. For each spring, determine the amount of tension required to stretch the spring 40.0 cm and mark that tension on the graph.

$T_A = 80\frac{N}{m}(.40m)$

$T_B = 50\frac{N}{m}(.40m)$

$T_A =$ 32N

$T_B =$ 20N

c. Can you determine the amount of tension required to stretch each spring by 400 cm? If so, what is the tension required in each case. If not, why not?

No. It would be unreasonable to use the graph for a stretch that is many times longer than any shown on the graph. The spring may deform or even break.

8.4 Stretching, Compressing and Bending Materials

15. A force stretches a wire by 1 mm.
   a. A second wire of the same material has the same cross section and twice the length. How far will it be stretched by the same force? Explain.

The second wire will be stretched by $\boxed{2mm}$ because the amount of stretching is proportional to the length of the wire.

$$\frac{F/A}{\Delta L/L} = Y \quad \Delta L = \frac{F}{A}\frac{L}{Y} \text{ so } \Delta L' = \frac{F}{A}\frac{2L}{Y} = 2\Delta L$$

   b. A third wire of the same material has the same length and twice the diameter as the first. How far will it be stretched by the same force? Explain.

Because the cross-sectional area is increased by 4 times when the diameter is doubled, the stretching will be 1/4 as much or $\Delta L' = \frac{F}{4A}\frac{L}{Y} = \frac{\Delta L}{4} = \boxed{0.25mm}$

16. A 2000 N force stretches a wire by 1 mm.
   a. A second wire of the same material is twice as long and has twice the diameter. How much force is needed to stretch it by 1 mm? Explain.

Because the cross-sectional area increases by 4 times, but the length is only doubled, a force of $2 \times 2000 N = \boxed{4000N}$ is required.

$$F = AY\frac{\Delta L}{L} \quad F' = 4AY\frac{\Delta L}{2L} = 2F$$

   b. A third wire is twice as long as the first and has the same diameter. How far is it stretched by a 4000 N force?

It is stretched by $\boxed{4mm}$.

$$\Delta L' = \frac{2F}{A}\frac{2L}{Y} = 4\Delta L$$

17. A wire is stretched right to the breaking point by a 5000 N force. A longer wire made of the same material has the same diameter. Is the force that will stretch it right to the breaking point larger than, smaller than, or equal to 5000 N? Explain.

The longer wire will also break at 5000N. The force per area is the same in both cases.