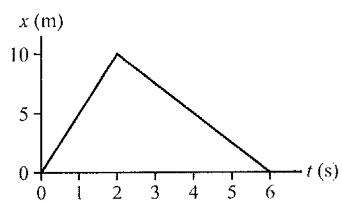
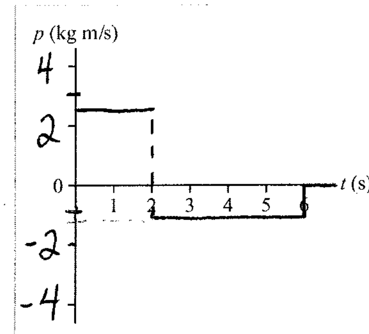
9 Momentum

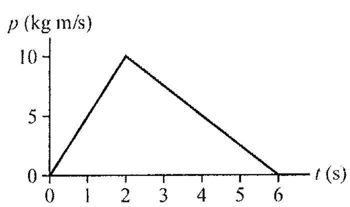
9.1 Impulse

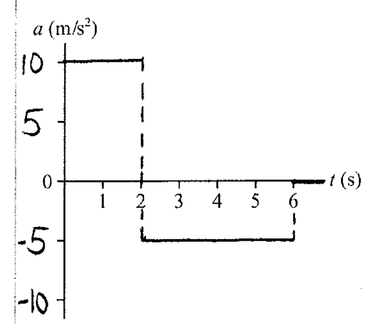
1. The position-versus-time graph is shown for a 500 g object. Draw the corresponding momentum-versus-time graph. Include an appropriate vertical scale.





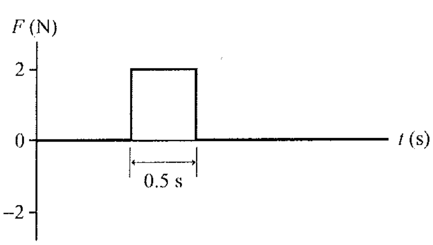
2. The momentum-versus-time graph is shown for a 500 g object. Draw the corresponding acceleration-versus-time graph. Include an appropriate vertical scale.





3. A 2 kg object is moving to the right with a speed of 1 m/s when it experiences an impulse due to the force shown in the graph. What is the object's speed and direction after the impulse?

a.

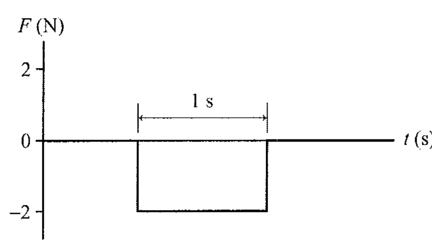


$J_x = 2N \cdot 0.5s = 1 N \cdot S = \Delta p_x$

$\Delta V_x = \Delta p_x / m = 1 N \cdot S / 2kg = 0.5\ m/s$

$V_{f_x} = V_{i_x} + \Delta V_x = 1\ m/s + 0.5\ m/s =$ **1.5 m/s right**

b.



$J_x = -2N \cdot 1s = -2N \cdot S = \Delta p_x$

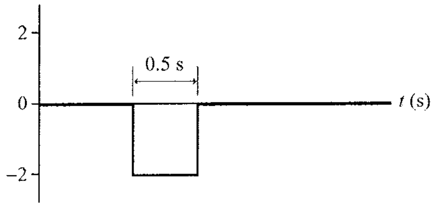
$\Delta V_x = \Delta p_x / m = -2N \cdot S / 2kg = -1\ m/s$

$V_{fx} = V_{ix} + \Delta V_x$

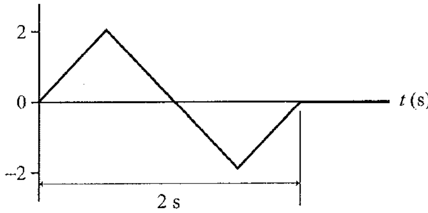
$V_{fx} = 1\ m/s - 1\ m/s =$ **0 m/s**

4. A 2 kg object is moving to the right with a speed of 1 m/s when it experiences an impulse due to the force shown in the graph. What is the object's speed and direction after the impulse?

a.



b.



$J_x = -2N \cdot 0.5 \, m/s = -1 N\cdot S = \Delta p_x$

$\Delta V_x = \Delta p_x / m = -1 N\cdot S / 2kg = -0.5 \, m/s$

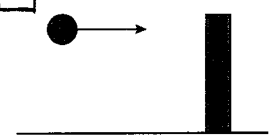
$V_{fx} = V_{ix} + \Delta V_x = 1\frac{m}{s} - 0.5 \, m/s =$ 0.5 m/s right

$J_x = 1/2 \cdot 2N \cdot 1S - 1/2 \cdot 2N \cdot 1S = 0$

$\Delta V_x = 0$

$V_{fx} =$ 1 m/s to the right

5. A carnival game requires you to knock over a wood post by throwing a ball at it. You're offered a very bouncy rubber ball and a very sticky clay ball of equal mass. Assume that you can throw them with equal speed and equal accuracy. You only get one throw.



a. Which ball will you choose? Why?

Choose the bouncy rubber ball. Because the rubber ball bounces back, it experiences a greater change in momentum than the clay ball at the same initial speed. Therefore, it imparts a greater impulse on the post and so is more likely to knock it over.

b. Let's think about the situation more carefully. Both balls have the same initial momentum p_{ix} just before hitting the post. The clay ball sticks, the rubber ball bounces off with essentially no loss of speed. What is the final momentum of each ball?

Clay ball: $p_{fx} =$ 0 Rubber ball: $p_{fx} =$ $-p_{fx}$

Hint: Momentum has a sign. Did you take the sign into account?

c. What is the *change* in the momentum of each ball?

Clay ball: $\Delta p_x =$ $-p_{fx}$ Rubber ball: $\Delta p_x =$ $-2p_{fx}$

d. Which ball experiences a larger impulse during the collision? Explain.

The bouncy rubber ball experiences a greater impulse as can be seen from its greater change in momentum.

e. From Newton's third law, the impulse that the ball exerts on the post is equal in magnitude, although opposite in direction, to the impulse that the post exerts on the ball. Which ball exerts the larger impulse on the post?

The rubber ball exerts a larger impulse because of the greater impulse on it.

f. Don't change your answer to part a, but are you still happy with that answer? If not, how would you change your answer? Why?

Still happy, we hope.

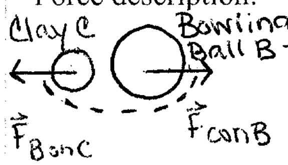
9.2 Momentum and the Impulse-Momentum Theorem

9.3 Solving Impulse and Momentum Problems

6. For each of the following situations, use both words and pictures to
   - Describe what happens in the language of force, acceleration, and action/reaction.
   - Describe what happens in the language of impulse and momentum.

a. A moving blob of clay hits a stationary bowling ball.

Force description: The force exerted on each are equal and opposite action-reaction pairs. The clay undergoes a larger acceleration, because of its smaller mass. It is difficult to quantify the acceleration and force without knowing more about the interaction.



Momentum description: The clay and bowling ball exert equal and opposite impulses on each other and so have equal and opposite changes in momentum. The clay blob loses most of its forward momentum. The bowling ball gains the momentum lost by the clay ball, though they move much more slowly due to the large mass of the bowling ball.

b. A falling rubber ball bounces off the floor.

Force description: The ball exerts a force on the floor and the floor exerts an equal, but opposite force back on the ball. The ball's acceleration is large, though the floor's is negligable because of its huge mass. It is difficult to analyze the forces and the acceleration without knowing more about the interaction.

Momentum description: Because of the impulse from the floor, the rubber ball's momentum changes by more than the magnitude of its initial momentum. The floor receives an equal impulse in the opposite direction but has no measurable velocity due to its large mass.

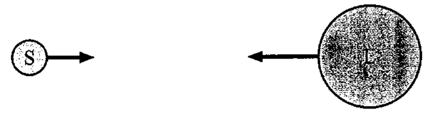
c. Two equal masses are pushed apart by a compressed spring between them.

Force description: As the spring expands, it exerts a changing force on each mass that is always equal and opposite to the force on the other mass. The masses accelerate equally due to this force, but the acceleration cannot be determined without the time over which the spring pushes.

Momentum description:

Equal and opposite impulses are given by the spring so that each mass has an equal, but opposite, momentum afterwards.

7. A small, light ball S and a large, heavy ball L move toward each other, collide, and bounce apart.



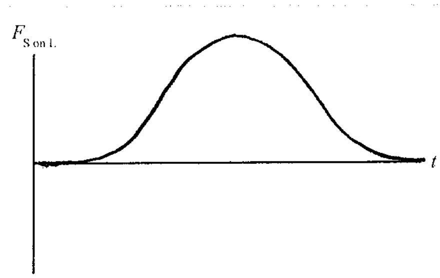
a. Compare the force that S exerts on L to the force that L exerts on S. That is, is $F_{S\ on\ L}$ larger, smaller, or equal to $F_{L\ on\ S}$? Explain.

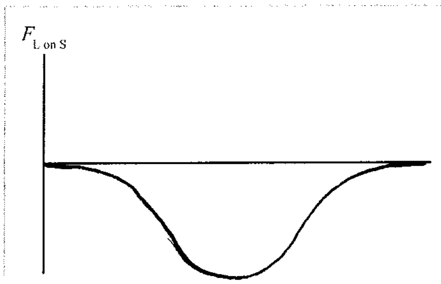
$F_{SonL} = F_{LonS}$. These two forces are an action-reaction pair, equal and opposite by Newton's third law.

b. Compare the time interval during which S experiences a force to the time interval during which L experiences a force. Are they equal, or is one longer than the other?

The time intervals are also equal.

c. Sketch a graph showing a *plausible* $F_{S\ on\ L}$ as a function of time and another graph showing a plausible $F_{L\ on\ S}$ as a function of time. Be sure think about the *sign* of each force.





d. Compare the impulse delivered to S to the impulse delivered to L.

The forces are equal and are extended over the same time interval. Therefore, the impulses are equal, but opposite in direction.

e. Compare the momentum change of S to the momentum change of L.

The momentum changes are equal in magnitude, but opposite in direction for each.

f. Compare the velocity change of S to the velocity change of L.

The velocity changes are not equal. S experiences a much greater change because its mass is much smaller.

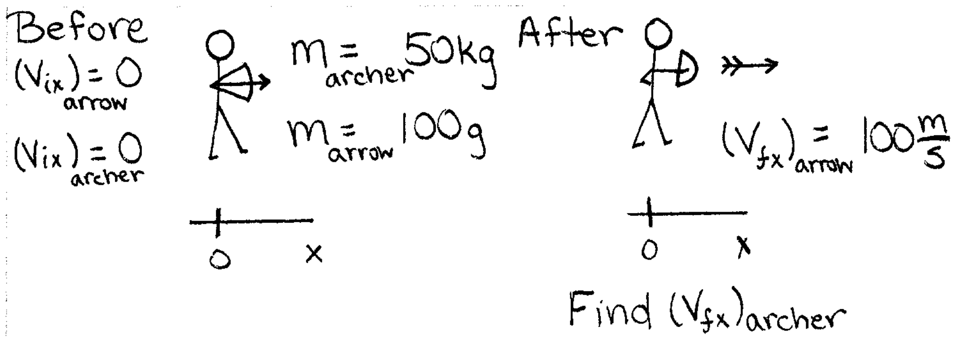
g. What is the change in the *sum* of the momenta of the two balls? Is it positive, negative, or zero?

There is no change in the sum of the momenta of the two balls, because their changes are equal and opposite.

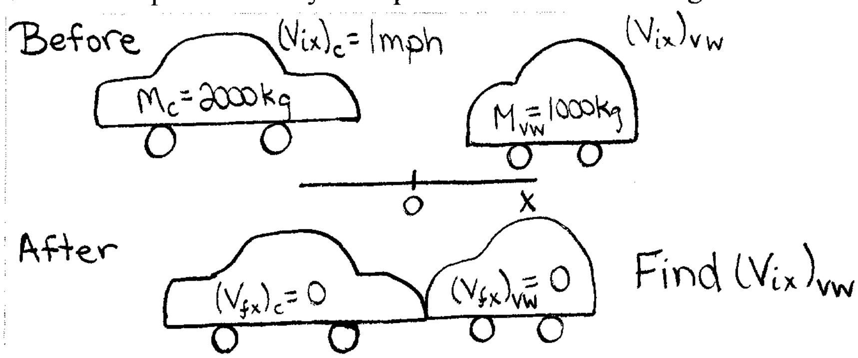
Exercises 8–11: Prepare a pictorial representation for these problems, but *do not* solve them.

- Draw pictures of "before" and "after."
- Define symbols relevant to the problem.
- List known information, and identify the desired unknown.

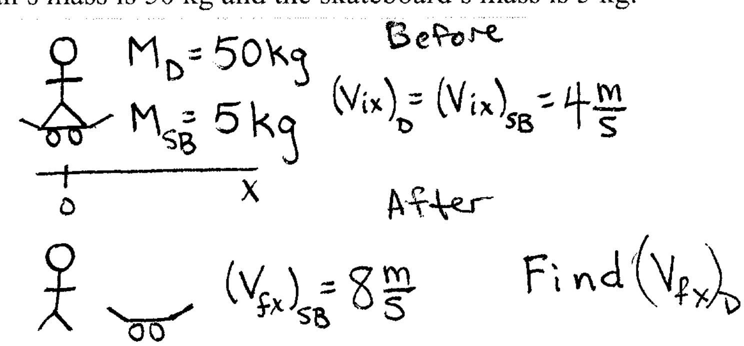
8. A 50 kg archer, standing on frictionless ice, shoots a 100 g arrow at a speed of 100 m/s. What is the recoil speed of the archer?



9. The parking brake on a 2000 kg Cadillac has failed, and it is rolling slowly, at 1 mph, toward a group of small innocent children. As you see the situation, you realize there is just time for you to drive your 1000 kg Volkswagen side-on into the Cadillac and thus save the children. With what speed should you impact the Cadillac to bring it to a halt?

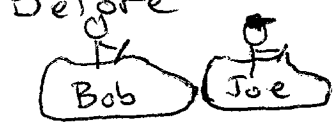


10. Dan is gliding on his skateboard at 4 m/s. He suddenly jumps backward off the skateboard, kicking the skateboard forward at 8 m/s. How fast is Dan going as his feet hit the ground? Dan's mass is 50 kg and the skateboard's mass is 5 kg.



11. While riding bumper cars at the fair, Bob's car collides directly with the back of Joe's car while both cars are moving to the right. Before the collision, Joe's car was traveling at 1.8 m/s and Bob's at 2.0 m/s. The combined mass of Joe and his car is only 80 kg, and the combined mass of Bob and his car is 100 kg. Immediately after the collision, Joe's car moves right at 2.0 m/s. How fast and in which direction does Bob's car move?

Before



$V_{i_{Bob}} = 2.0\frac{m}{s}$ $V_{i_{Joe}} = 1.8\frac{m}{s}$

0 —— x

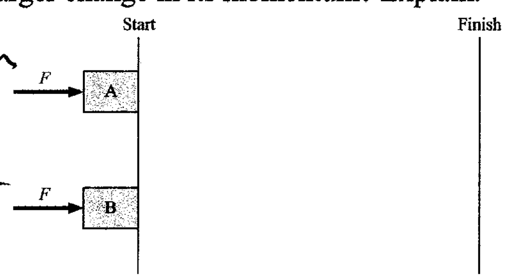
$m_{Bob} = 100kg$ $m_{Joe} = 80kg$

After

0 ⟶ x $V_{f_{Joe}} = 2.0\frac{m}{s}$

Find $V_{f_{Bob}}$

12. Identical blocks A and B are pushed to the right continuously by identical constant forces from the start to the finish shown below. At the starting line, block A is initially moving to the right, but block B is stationary. Which block undergoes a larger change in its momentum? Explain.



Block B. The momentum change is caused by the impulsive force acting over time. Because block A is initially moving to the right and the force F acts only from 'start' to 'finish', that force acts for a longer time on B.

9.4 Conservation of Momentum

13. As you release a ball, it falls—gaining speed and momentum. Is momentum conserved?
    a. Answer this question from the perspective of choosing the ball alone as the system.

The momentum of the ball is not conserved. The weight is an unbalanced force from an agent external to the system.

    b. Answer this question from the perspective of choosing ball + earth as the system.

If the earth is included, then momentum is conserved. While the ball is gaining downward momentum, the earth is gaining an equal magnitude of upward momentum, (but it can't be measured due to the earth's mass and complexity).

14. Is it possible for momentum to be conserved if the system is NOT isolated? Explain.

Yes - and no! If the system is subject to a net external force, then momentum will not be conserved. If the vector sum of the external forces is zero, then the linear momentum will be conserved.

15. Consider the interaction of two objects along a line. Show that the following three equations are equivalent if the system is isolated and the objects remain intact:

(1) $(p_f - p_i)_1 = -(p_f - p_i)_2$

(2) $(p_i)_1 + (p_i)_2 = (p_f)_1 + (p_f)_2$

(3) $F_{1 \text{ on } 2} = m_2 \dfrac{\Delta v_2}{\Delta t} = -F_{2 \text{ on } 1} = -m_1 \dfrac{\Delta v_1}{\Delta t}$

The first two are equivalent by simple algebra:

(1) becomes $(P_f)_1 - (P_i)_1 = -(P_f)_2 + (P_i)_2$ or

(2) $(P_f)_1 + (P_f)_2 = (P_i)_1 + (P_i)_2$. The third equation can be rewritten using $\Delta V = V_f - V_i$ so $\dfrac{m_2(V_f)_2 - m_2(V_i)_2}{\Delta t} = \dfrac{-m_1(V_f)_1 + m_1(V_i)_1}{\Delta t}$

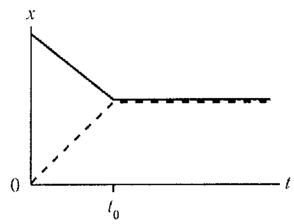
Multiplying each side by Δt and using $p = mv$ yields

$(P_f)_2 - (P_i)_2 = -(P_f)_1 + (P_i)_1$ or

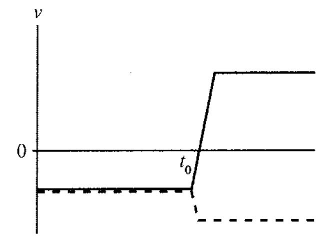
(1) $(P_f - P_i)_2 = -(P_f - P_i)_1$

9.5 Inelastic Collisions

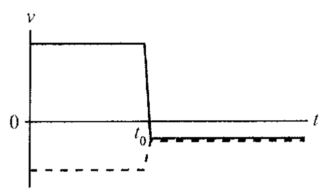
16. For each of the following graphs a–e:
    - Indicate whether the graph could represent an inelastic collision between two objects A (solid line) and B (dashed line) at time t_0 and whether momentum could be conserved in each case. (Note that graph a shows position-versus-time and the remaining graphs show velocity-versus-time.)
    - What can you infer about the relative masses of A and B for those cases in which momentum is conserved?

a.


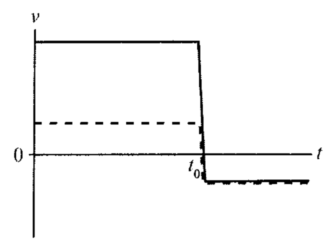
Yes, inelastic
Momentum is conserved if the masses are identical.

d.


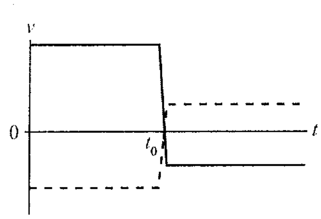
Yes, inelastic (treated as reversed in time – an explosion)
Momentum could be conserved if mass B is greater than mass A.

b.


Yes, inelastic
Momentum could be conserved if mass B is greater than mass A

e.


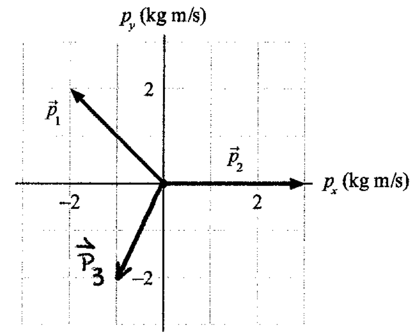
No. It appears that the objects stick together after t_0, but they have both changed from moving forward to moving backwards. The total momentum changed signs! (Impossible)

c.


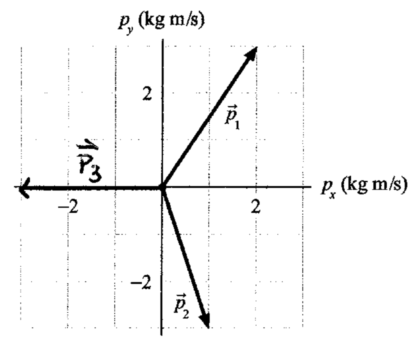
No, not inelastic.
(But momentum might still be conserved)

9.6 Momentum and Collisions in Two Dimensions

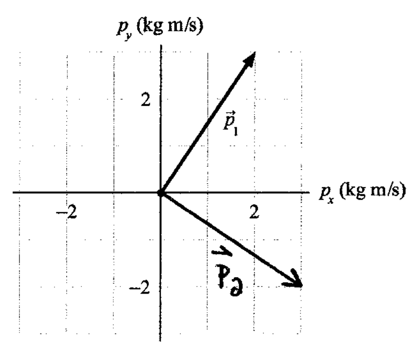
17. An object initially at rest explodes into three fragments. The momentum vectors of two of the fragments are shown. Draw the momentum vector $\vec{p}_3$ of the third fragment.



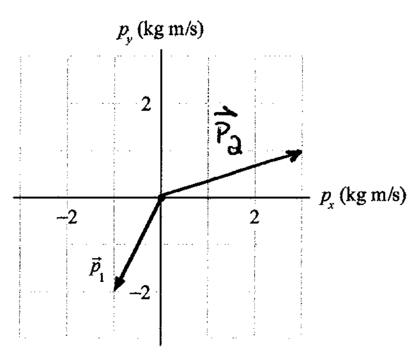
18. An object initially at rest explodes into three fragments. The momentum vectors of two of the fragments are shown. Draw the momentum vector $\vec{p}_3$ of the third fragment.



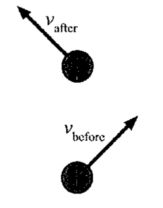
19. A 500 g ball traveling to the right at 4.0 m/s collides with and bounces off another ball. The figure shows the momentum vector $\vec{p}_1$ of the first ball after the collision. Draw the momentum vector $\vec{p}_2$ of the second ball.



20. A 500 g ball traveling to the right at 4.0 m/s collides with and bounces off another ball. The figure shows the momentum vector $\vec{p}_1$ of the first ball after the collision. Draw the momentum vector $\vec{p}_2$ of the second ball.

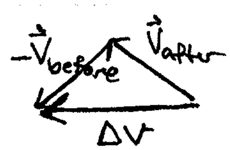


21. Consider a ball thrown at an angle against a wall with velocity vectors before and after the collision as shown at right from above.



a. Redraw the velocity vectors below to determine the *change* in velocity of the ball due to the collision and draw an arrow below to indicate that change.

$\Delta\vec{v} = \vec{v}_{after} - \vec{v}_{before}$



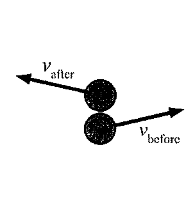
b. Draw an arrow below to indicate the direction of the impulse on the ball by the wall.

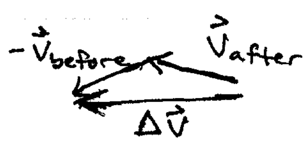
← $\vec{J}_{wall\ on\ ball}$ (no vertical component)

c. Draw an arrow below to indicate the relative magnitude and direction of the impulse by the ball on the wall.

→ $\vec{J}_{ball\ on\ wall}$ (no vertical component)

d. Repeat part a for the figure shown at right. (The velocity vectors are the same magnitude as in part a.)





e. Draw an arrow below to indicate the direction of the impulse on the ball by the wall.

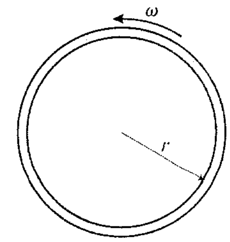
← (horizontal only)

f. Draw an arrow below to indicate the relative magnitude and direction of the impulse by the ball on the wall.

→ (horizontal only)

9.7 Angular Momentum

22. A hoop of mass m and radius r is set spinning at angular speed ω about its center axis as shown from above.



a. What would be its angular speed if its mass is suddenly doubled without changing its radius? Explain.

$\frac{\omega}{2}$. $L = I\omega$; $I = mr^2$ so $I' = 2I$

Because $L = L' = 2I\omega'$, $\omega' = \frac{\omega}{2}$.

b. What would be its angular speed if its radius is suddenly halved without changing its mass? Explain.

4ω. $L = I\omega$, $I = mr^2$ so $I' = m\left(\frac{r}{2}\right)^2 = \frac{I}{4}$

$L = L'$ so $I\omega = \frac{I}{4}\omega' \Rightarrow \omega' = 4\omega$

c. What would be its angular speed if a force F is exerted at the outside surface of the cylinder directly toward the axis of rotation for a time Δt? Explain.

ω. Because the force acts through the axis of rotation, it exerts no torque and so does not change the angular momentum or angular speed. (I is constant.)