10 Energy and Work

10.1 A "Natural Money" Called Energy

1. One month, John has income of \$3000, expenses of \$2500, and he sells \$300 of stocks.

   a. Can you determine John's liquid assets L at the end of the month? If so, what is L? If not, why not?

   No, we cannot determine L because we don't know with what liquid assets John started the month.

   b. Can you determine the amount by which John's liquid assets *changed* during the month? If so, what is ΔL?

   Yes. I = income E = expenses S = stock sales

   $\Delta L = I - E + S$

   $= \$3000 - \$2500 + \$300 = \boxed{+\$800}$

2. John begins the month with \$2000 of liquid assets and \$5000 of savings. His financial activity for the month is as follows:

   Labels: L = liquid assets
   S = savings

Day of Month		Activity
1	+L	Receives a \$3000 paycheck; deposits it in checking
3	−L	Spends \$500
8	−L, +S	Buys a \$1000 savings bond
10	−L	Pays bills totaling \$1000
15	+L	Receives a \$100 birthday present from Grandma
23	+L, −S	Sells \$1500 of stock
28	−L	Buys a \$1200 bicycle

   a. What are John's liquid assets and saved assets at the end of the month?

   $L_{initial} = \$2000$

ΔL	Day
+3000	1
−500	3
−1000	8
−1000	10
+100	15
+1500	23
−1200	28

   $S_{initial} = \$5000$

ΔS	Day
+1000	8
−1500	23

   $L_{final} = L_{initial} + \Delta L_{net} = \boxed{\$2900}$

   $S_{final} = S_{initial} + \Delta S_{net} = \boxed{\$4500}$

   b. Show that John's monetary relationship $\Delta W = I - E$ is satisfied.

   $W_{initial} = \$5000 + \$2000$
   $= \$7000$

   $W_{final} = \$4500 + \$2900$
   $= \$7400$

I	E	Day
\$3000		1
	500	3
1000	1000	8
	1000	10
100		15
1500	1500	23
	1200	28

   $\Delta W = I - E$
   $= \$5600 - \$5200$
   $\Delta W = +\$400$
   $W_f - W_i = \$7400 - \$7000$
   $= +\$400$

10.2 The Basic Energy Model

3. What are the two primary processes by which energy can be transferred from the environment to a system?

1) mechanical (forces doing work)
2) nonmechanical (heat)

4. Identify the energy transformations in each of the following processes (e.g., $K \rightarrow U_g \rightarrow E_{th}$)

a. A ball is dropped from atop a tall building.

$U_g \rightarrow K$

b. A helicopter rises from the ground at constant speed.

$E_{chem} \rightarrow U_g$

c. An arrow is shot from a bow and stops in the center of its target.

$K (+U_g) \rightarrow E_{th}$

d. A pole vaulter runs, plants his pole, and vaults up over the bar.

$K \rightarrow U_{sp} \rightarrow U_g$

5. The kinetic energy of a system decreases and its potential energy is unchanged. What is doing work on what? That is, does the environment do work on the system, or does the system do work on the environment? Explain.

The system is doing work on the environment.
The total mechanical energy of the system is lower.

10.3 The Law of Conservation of Energy

6. Identify an appropriate system for applying conservation of energy to each of the following:
   a. A spring is used to launch a ball into the air.

   System: spring, ball, earth

   b. A spring is used to push a car on an air track.

   System: spring, car

   c. A spring is used to slide a block across a table where it stops.

   System: spring, block, table

   d. A car moving on an air track collides with a spring and rebounds at essentially the same speed with which it hit the spring.

   System: spring, car

7. What is meant by an *isolated system*?

   An isolated system is a system in which no external forces do work.

8. a. A process occurs in which a system's potential energy decreases while the environment does work on the system. Does the system's kinetic energy increase, decrease, or stay the same? Or is there not enough information to tell? Explain.

   There is not enough information to tell. The lost potential energy and the work done by the environment could increase the kinetic energy or it is possible that all the work and energy are converted to thermal energy.

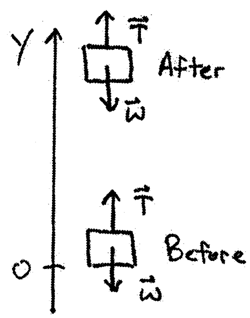
   b. A process occurs in which a system's potential energy increases while the environment does work on the system. Does the system's kinetic energy increase, decrease, or stay the same? Or is there not enough information to tell? Explain.

   There is not enough information to tell. The work done could cause some or all of the potential energy change or some of the work could be converted to thermal energy. Without more information, it is impossible to say whether a kinetic energy change is present.

10.4 Work

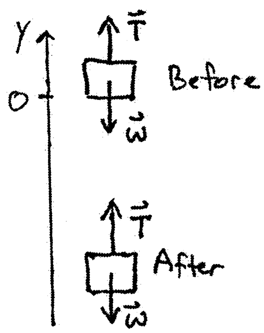
9. For each situation described below:
   - Draw a before-and-after diagram, similar to Figures 10.8 and 10.11 in the textbook.
   - Identify *all* forces acting on the particle.
   - Determine if the work done by each of these forces is positive (+), negative (–), or zero (0). Make a little table beside the figure showing *every* force and the sign of its work.

a. An elevator moves upward.



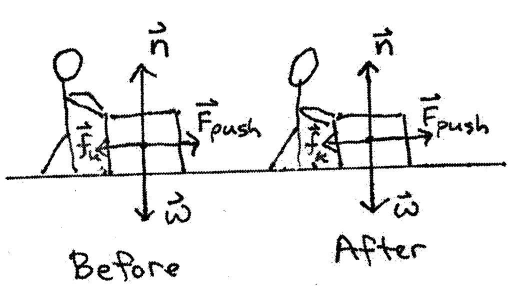
Force	Work	Sign
Tension	W_T	+
Weight	W_g	−

b. An elevator moves downward.



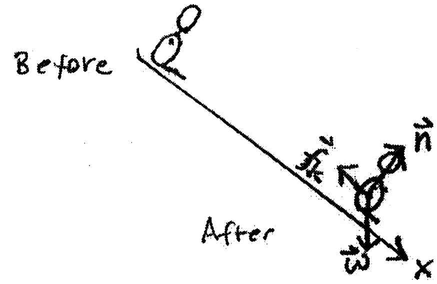
Force	Work	Sign
Tension	W_T	−
Weight	W_g	+

c. You push a box across a rough floor.



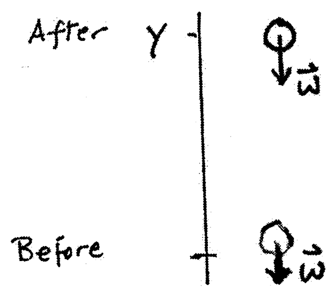
Force	Work	Sign
Push	W_P	+
Friction	W_f	−
Weight	W_g	0
Normal	W_n	0

d. You slide down a steep hill.



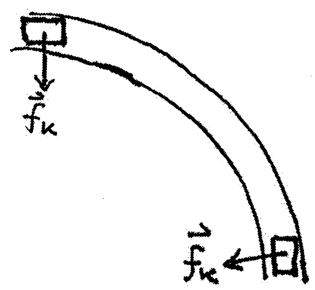
Force	Work	sign
Weight	W_g	+
friction	W_f	−
normal	W_n	0

e. A ball is thrown straight up. Consider the ball from one microsecond after it leaves your hand until the highest point of its trajectory.



Force	Work	sign
Weight	W_g	−

f. A car turns a corner at constant speed.

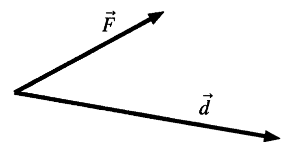


Force	Work	Sign
friction	W_f	0
Weight	W_g	0
normal	W_n	0

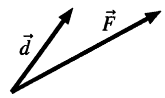
10. A 0.2 kg plastic cart and a 20 kg lead cart both roll without friction on a horizontal surface. Equal forces are used to push both carts forward a distance of 1 m, starting from rest. After traveling 1 m, is the kinetic energy of the plastic cart greater than, less than, or equal to the kinetic energy of the lead cart? Explain.

The kinetic energies are equal. Equal forces are applied over equal displacements so that the same work is done on each. Thus, the change in kinetic energy is the same because $K_i = 0$, $\Delta K = K_f$ (However, the plastic cart will be moving 10 × faster.)

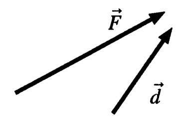
11. An object experiences a force while undergoing the displacement shown. Is the work done positive (+), negative (−), or zero (0)?

a. 

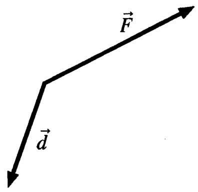
Sign = +

b. 

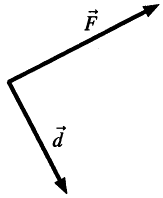
Sign = +

c. 

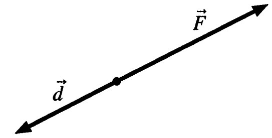
Sign = +

d. 

Sign = −

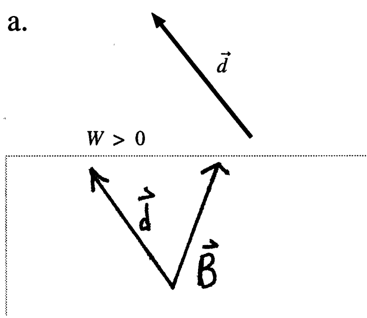
e. 

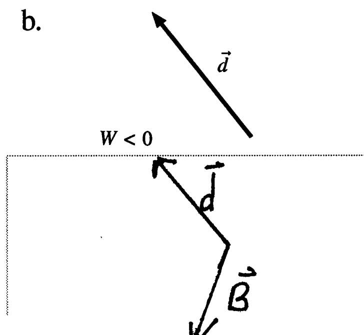
Sign = 0

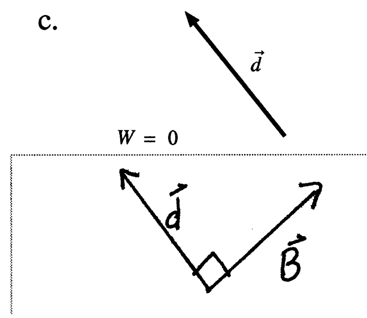
f. 

Sign = −

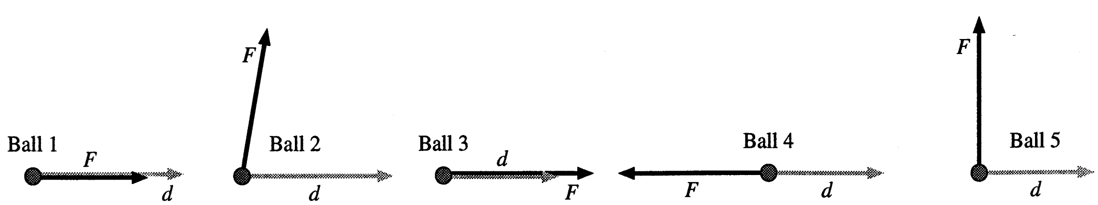
12. Each of the diagrams below shows a displacement vector for an object. Draw and label a force vector that will do work on the object with the sign indicated.

a. 

b. 

c. 

13. Five balls with equal initial kinetic energies experience different forces acting over different displacements, as shown by the arrows. Rank in order, from largest to smallest, the kinetic energies of the balls after being acted on by these forces.



Order: 1 = 3 > 2 > 5 > 4

Explanation: 1 = 3 because the product of $\vec{F} \cdot \vec{d}$ is the same and these are greatest because the force acts along the displacement. Next 2 has only a small component of the force along the displacement of the kinetic energy increases a little. Ball 5 has no change in KE because F is perpendicular to d. Ball 4's KE is less because the work done is negative.

10.5 Kinetic Energy

14. Can kinetic energy ever be negative? No

Give a plausible *reason* for your answer without making use of any formulas.

Kinetic energy is energy of motion. motion may stop, but it can't be negative. Speed has no direction and cannot be negative.

15. a. If a particle's speed increases by a factor of three, by what factor does its kinetic energy change?

$K_i = \frac{1}{2} m v^2 \quad K_f = \frac{1}{2} m (3v)^2 = 9 K_i$

K increases by a factor of $\boxed{9.}$

b. Particle A has half the mass and eight times the kinetic energy of particle B. What is the speed ratio v_A/v_B?

$K_A = 8 K_B \quad m_A = \frac{m_B}{2},$

$\frac{1}{2} m_A v_A^2 = 8 \frac{1}{2} m_B v_B^2$

$\frac{1}{2} \frac{m_B}{2} v_A^2 = 8 \frac{1}{2} m_B v_B^2 \quad \left(\frac{v_A}{v_B}\right)^2 = 16 \quad \boxed{\frac{v_A}{v_B} = 4}$

c. If a rotating skater triples her rate of rotation by decreasing her moment of inertia by 1/3, by what factor does her rotational kinetic energy change?

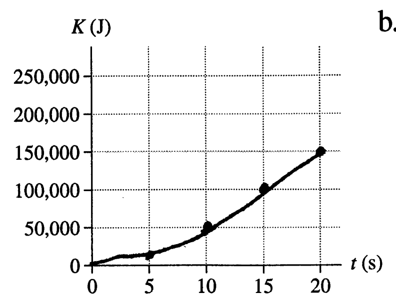
$K_{rot_i} = \frac{1}{2} I_i \omega_i^2 \quad K_{rot_f} = \frac{1}{2} \frac{I_i}{3} (3\omega_i)^2 = 3\left[\frac{1}{2} I_i \omega_i^2\right]$

Her rotational kinetic energy increases by a factor of $\boxed{3}$

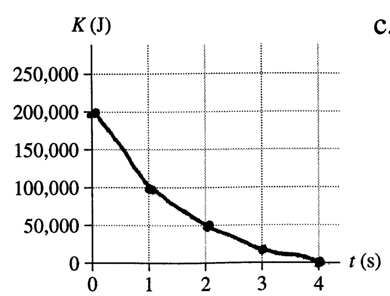
16. On the axes below, draw graphs of the kinetic energy of
    a. A 1000 kg car that uniformly accelerates from 0 to 20 m/s in 20 s.
    b. A 1000 kg car moving at 20 m/s that brakes to a halt with uniform deceleration in 4 s.
    c. A 1000 kg car that drives once around a 40-m-diameter circle at a speed of 20 m/s.

    Calculate K at several times, plot the points, and draw a smooth curve between them.

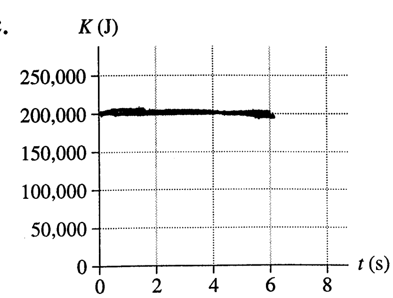
a.



b.



c.



$a = \frac{20\frac{m}{s} - 0}{20s} = 1\frac{m}{s^2}$

$v = at \quad K_5 = \frac{1}{2} 1000kg (5\frac{m}{s})^2$

$K_{10} = \frac{1}{2} 1000kg (10\frac{m}{s})^2$

$K_{15} = \frac{1}{2} 1000kg (15\frac{m}{s})^2$

$K_{20} = \frac{1}{2} 1000kg (20\frac{m}{s})^2$

$a = -5\frac{m}{s^2} \quad V = V_0 - at$

$K_0 = \frac{1}{2} 1000kg (20\frac{m}{s})^2$

$K_1 = \frac{1}{2} 1000kg (15\frac{m}{s})^2$

$K_2 = \frac{1}{2} 1000kg (10\frac{m}{s})^2$

$K_3 = \frac{1}{2} 1000kg (5\frac{m}{s})^2$

$K_4 = \frac{1}{2} 1000kg (0\frac{m}{s})^2$

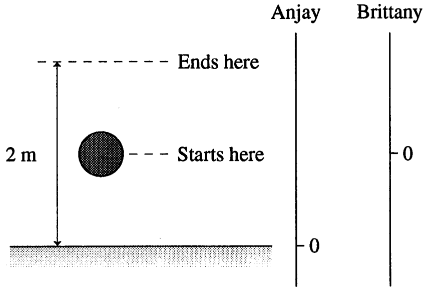
$s = vt$

$t = \frac{s}{v} = \frac{\pi (40m)}{20 m/s}$

$t = 6.3 s$

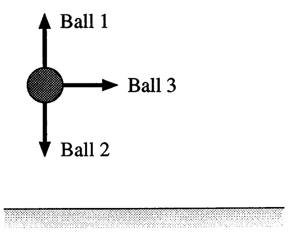
10.6 Potential Energy

17. Below we see a 1 kg object that is initially 1 m above the ground and rises to a height of 2 m. Anjay and Brittany each measure its position but use a different coordinate system to do so. Fill in the table to show the initial and final gravitational potential energies and ΔU as measured by Anjay and Brittany.



	U_i	U_f	ΔU
Anjay	9.8J	19.6J	9.8J
Brittany	0	9.8J	9.8J

18. Three balls of equal mass are fired simultaneously with *equal* speeds from the same height above the ground. Ball 1 is fired straight up, ball 2 is fired straight down, and ball 3 is fired horizontally. Rank in order, from largest to smallest, their speeds v_1, v_2, and v_3 as they hit the ground.

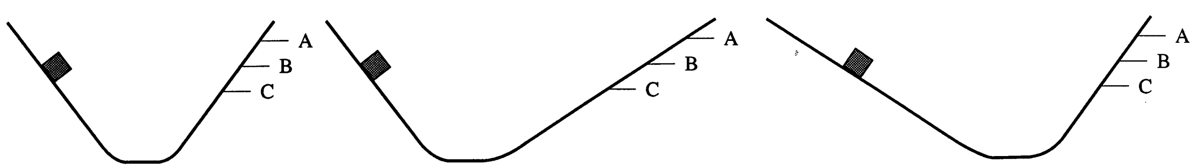


Order: $V_1 = V_2 = V_3$

Explanation:

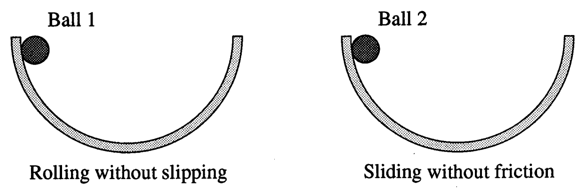
They each start with the same kinetic energy and they each have the same change in potential energy, so they end with the same kinetic energy, and thus, the same speed.

19. Below are shown three frictionless tracks. A block is released from rest at the position shown on the left. To which point does the block make it on the right before reversing direction and sliding back? Point B is the same height as the starting position.



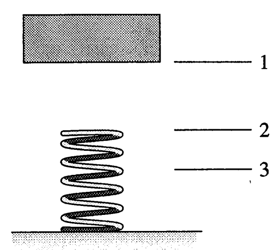
Makes it to B   Makes it to B   Makes it to B

20. Two balls are released from just below the rim of two identical bowls. Ball 1 rolls down without slipping while ball 2 slides down without friction. Which ball will reach the higher point on the other side before reversing direction? Explain.



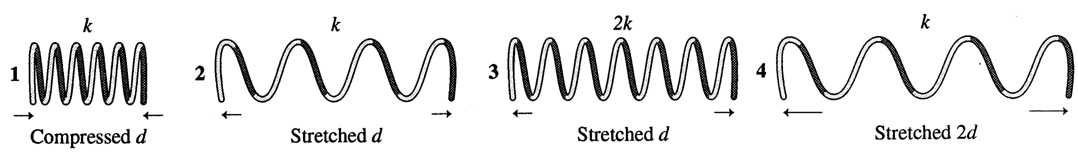
The balls reach the same height. The rolling ball converts gravitational potential energy to both translational and rotational kinetic energy but then converts these back to the same gravitational energy as it stops at the same height on the other side of the bowl.

21. A heavy object is released from rest at position 1 above a spring. It falls and contacts the spring at position 2. The spring achieves maxiumum compression at position 3. Fill in the table below to indicate whether each of the quantities are +, –, or 0 during the intervals 1→2, 2→3, and 1→3.



	1→2	2→3	1→3
ΔK	+	–	0
ΔU_g	–	–	–
ΔU_s	0	+	+

22. Rank in order, from most to least, the amount of elastic potential energy $(U_s)_1$ to $(U_s)_4$ stored in each of these springs.



Order: $(U_s)_4 > (U_s)_3 > (U_s)_2 = (U_s)_1$

Explanation:

$U_s = \frac{1}{2} k (\Delta s)^2$ Increasing the stretch by a factor of 2 increases the stored energy by a factor of 4.

10.7 Thermal Energy

23. A car traveling at 60 mph slams on its brakes and skids to a halt. What happened to the kinetic energy the car had just before stopping?

It was dissipated as thermal energy.

24. What energy transformations occur as a skier glides down a gentle slope at constant speed?

Potential energy due to gravity is transformed into thermal energy, most of which is dissipated.

25. Give a *specific* example of a situation in which:

a. $W \rightarrow K$ with $\Delta U = 0$ and $\Delta E_{th} = 0$.

Push a puck across a frictionless surface.
(system: puck)

$$W = \vec{F} \cdot \Delta\vec{r} = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

b. $W \rightarrow U$ with $\Delta K = 0$ and $\Delta E_{th} = 0$.

Lift a book at constant speed.
(system: book/earth)

$$W = \vec{F} \cdot \Delta\vec{y} = mg\Delta y = \Delta U_g$$

c. $K \rightarrow U$ with $W = 0$ and $\Delta E_{th} = 0$.

A ball thrown upward, just after release until reaching its peak.
(system: ball/earth)

$$K = \frac{1}{2} m v^2 = mg\Delta y = \Delta U_g$$

d. $W \rightarrow E_{th}$ with $\Delta K = 0$ and $\Delta U = 0$.

A box pushed at constant speed along a horizontal surface. (system: box/surface)

$$W = \Delta E_{th}$$

e. $U \rightarrow E_{th}$ with $\Delta K = 0$ and $W = 0$.

A box slides at constant speed down a ramp with friction.

(system: box/surface/earth)

$$\Delta U = mg\Delta y = \Delta E_{th}$$

10.8 Further Examples of Conservation of Energy

26. If a solid disk and a circular hoop of the same mass and radius are released from rest at the top of a ramp and allowed to roll to the bottom, the disk will get to the bottom first. *Without referring to equations*, explain why this is so.

They both convert the same initial gravitational potential energy to kinetic energy, but the hoop has more rotational kinetic energy giving it less translational kinetic energy due to its larger moment of inertia. Thus, the hoop moves more slowly.

10.9 Energy in Collisions

27. Ball 1 with an initial speed of 14 m/s has a perfectly elastic collision with ball 2 that is initially at rest. Afterward, the speed of ball 2 is 21 m/s.

a. What will be the speed of ball 2 if the initial speed of ball 1 is doubled?

$(V_{fx})_2 = \frac{2m_1}{m_1+m_2}(V_{ix})_1$ Therefore, doubling $(V_{ix})_1$ will double $(V_{fx})_2$.

$(V_{fx})_2 = 2 \times 21\ \text{m/s} = \boxed{42\ \text{m/s}}$

b. What will be the speed of ball 2 if the mass of ball 1 is doubled?

From part a. $21\ \text{m/s} = \frac{2m_1}{m_1+m_2}(14\ \text{m/s})$ multiply by $\frac{1/m_1}{1/m_1}$

$21\ \text{m/s} = \frac{2}{1+m_2/m_1}(14\ \text{m/s})$ multiply by $\frac{m_2}{m_1}$

$\frac{m_2}{m_1} = \frac{1}{3}$

Doubling m_1 yields $\frac{m_2}{m_1} = \frac{1}{6}$, so $(V_{fx})_2 = \frac{2}{1+1/6}(14\ \frac{m}{s}) = \boxed{24\ \frac{m}{s}}$

28. You can dive into a swimming pool of water from a high diving board without being hurt, but to dive into an empty pool from a much lower distance might be fatal? Why the difference?

The work done to stop you is essentially the same, but the work must be done over a much shorter distance without the water present. So a much greater force is required.

29. Consider a perfectly elastic collision in which a moving ball 1 strikes an initially stationary ball 2. Is ball 1 more likely to recoil backwards if it is moving very fast (large forward momentum) or moving slowly (small forward momentum) assuming that ball 2 is identical in each case? Or does the speed of ball 1 matter? Explain.

The speed of the ball is not relevant because ball 2 has no momentum initially. Only the relative masses of the 2 balls will matter.

30. Consider a perfectly elastic collision in which a moving ball 1 strikes a initially stationary ball 2.

a. Under what circumstances, if any, will ball 1 come to a stop?

Ball 1 will stop in a head on collision if the balls have the same mass.

b. Under what circumstances, if any, will ball 1 recoil backwards?

Ball 1 will recoil backwards if ball 2 is more massive.

c. Under what circumstances, if any, will ball 1 continue moving forward?

$\text{Mass}_{\text{ball 1}} > \text{Mass}_{\text{ball 2}}$

d. Is it possible for ball 1 to move forward or backwards at a greater speed than its speed just before the collision?

No. The collision supplies no additional energy. Only in a perfectly elastic collision with an infinitely massive object will ball 1 move back at even an equal speed.

10.10 Power

31. a. If you push an object 10 m with a 10 N force in the direction of motion, how much work do you do on it?

$W = \vec{F} \cdot \Delta\vec{r} = 10\,N - 10\,m = \boxed{100\,J}$

b. How much power must you provide to push the object in 1 s? In 10 s? In 0.1 s?

$P = \frac{W_{ext}}{\Delta t}$ In 1 s, $P = \boxed{100\,W}$

In 10 s, $P = \frac{100\,J}{10\,s} = \boxed{10\,W}$

In 0.1 s, $P = \frac{100\,J}{0.1\,s} = \boxed{1000\,W}$