11

WORK

Conceptual Questions

11.1. There is not enough information to tell. The lost potential energy and the work done by the environment could increase the kinetic energy or it is possible that all the work and energy are converted to thermal energy.

11.2. There is not enough information to tell. The work done could cause some or all of the potential energy change or some of the work could be converted to thermal energy. Without more information, it is impossible to say whether a kinetic energy change is present.

11.3. The system is doing work on the environment. The total mechanical energy of the system is lower.

11.4. The ball's kinetic energy is equal to the work done on it by gravity. Since work is force × distance, the kinetic energy of the ball increases by equal amounts in equal distance intervals.

11.5. No work was done by gravity. $W_g = -m_g \Delta y$. Here, $\Delta y = 0$. Any work done during a downward part of the motion was undone during the upward parts.

11.6. The kinetic energies are equal. Equal forces are applied over equal displacements so that the same work is done on each. Thus, the change in kinetic energy is the same. Because $K_{\text{i}} = 0$, $\Delta K = K_{\text{f}}$. (The plastic will be moving 10 times faster, however.)

11.7. The work is the same in both cases, since the work done against gravity is $-m_g \Delta y$, and Δy, the change in height, is the same in both cases.

11.8. **(a)** No, the rate of change of potential energy with respect to position will be zero at that point, but the value of the potential energy is not known without specifying it at some reference point.
(b) No, the zero point for the potential energy is arbitrary. There will be a force present if the rate of change of the potential energy with position is nonzero.

11.9. The kinetic energy was dissipated as thermal energy by friction between the tires and the road and in the brakes.

11.10. Gravitational potential energy is transformed into thermal energy. There is no change in the kinetic energy.

11.11. **(a)** Push a puck with force F across a frictionless level surface. With the puck as the system, $W_{ext} = F\Delta x = \Delta K$. The gravitational potential energy does not change because $\Delta y = 0$. Since the surface is frictionless, $\Delta E_{th} = 0$.
(b) Push a box across a rough level surface at constant speed. The system is the box. Again, $\Delta y = 0$, but now $\Delta K = 0$, and friction dissipates the external work done by the push as thermal energy.

11.12. Power is energy per time. The energy required to lift a beam a height Δy is the same as the change in gravitational potential energy of the beam. Power $P = \dfrac{W}{\Delta t} = \dfrac{mg\Delta y}{\Delta t}$. So doubling Δy and halving Δt requires a different power $P' = \dfrac{mg(2\Delta y)}{(\Delta t/2)} = 4\dfrac{mg\Delta y}{\Delta t} = 4P$. The power must be increased by a factor of 4.

Exercises and Problems

Section 11.2 Work and Kinetic Energy

Section 11.3 Calculating and Using Work

11.1. Solve: (a) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (3)(2) + (4)(-6) = -18.$
(b) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (3)(6) + (-2)(4) = 10.$

11.2. Solve: (a) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (4)(-2) + (-2)(-3) = -2.$
(b) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (-4)(2) + (2)(4) = 0.$

11.3. Solve: (a) The length of $\vec{A}$ is $|\vec{A}| = A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5.$ The length of $\vec{B}$ is $B = \sqrt{(2)^2 + (-6)^2} = \sqrt{40} = 2\sqrt{10}$. Using the answer $\vec{A}\cdot\vec{B} = -18$ from Ex 11.1(a),

$$\vec{A}\cdot\vec{B} = AB\cos\alpha$$
$$-18 = (5)(2\sqrt{10})\cos\alpha$$
$$\alpha = \cos^{-1}(-18/\sqrt{40}) = 125°$$

(b) The length of $\vec{A}$ is $|\vec{A}| = A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{(3)^2 + (-2)^2} = \sqrt{13}$. The length of $\vec{B}$ is $B = \sqrt{(6)^2 + (4)^2} = \sqrt{52} = 2\sqrt{13}$. Using the answer $\vec{A}\cdot\vec{B} = 10$ from Ex 11.1(b),

$$\vec{A}\cdot\vec{B} = AB\cos\alpha$$
$$10 = (\sqrt{13})(2\sqrt{13})\cos\alpha$$
$$\alpha = \cos^{-1}(10/26) = 67°$$

11.4. Solve: (a) The length of $\vec{A}$ is $|\vec{A}| = A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{(4)^2 + (-2)^2} = \sqrt{20}.$ The length of $\vec{B}$ is $B = \sqrt{(-2)^2 + (-3)^2} = \sqrt{13}$. Using the answer $\vec{A}\cdot\vec{B} = -2$ from EX11.2(a),

$$\vec{A}\cdot\vec{B} = AB\cos\alpha$$
$$-2 = (\sqrt{20})(\sqrt{13})\cos\alpha$$
$$\alpha = \cos^{-1}(-2/\sqrt{260}) = 97°$$

(b) From EX11.2(b), $\vec{A}\cdot\vec{B} = 0$. Thus

$$\cos\alpha = 0$$
$$\alpha = 90°$$

11.5. Visualize: Please refer to Figure EX11.5.

Solve: **(a)** $\vec{A}\cdot\vec{B} = AB\cos\alpha = (5)(3)\cos 40° = 11.$

(b) $\vec{C}\cdot\vec{D} = CD\cos\alpha = (2)(3)\cos 140° = -4.6.$

(c) $\vec{E}\cdot\vec{F} = EF\cos\alpha = (3)(4)\cos 90° = 0.$

11.6. Visualize: Please refer to Figure EX11.6.

Solve: **(a)** $\vec{A}\cdot\vec{B} = AB\cos\alpha = (2)(4)\cos 110° = -2.7.$

(b) $\vec{C}\cdot\vec{D} = CD\cos\alpha = (5)(4)\cos 180° = -20.$

(c) $\vec{E}\cdot\vec{F} = EF\cos\alpha = (4)(3)\cos 30° = 10.$

11.7. Solve: **(a)** $W = \vec{F}\cdot\Delta\vec{r} = (-3.0\hat{i} + 6.0\hat{j})\cdot(2.0\hat{i})\text{ N m} = \left(-6.0\hat{i}\cdot\hat{i} + 12\overbrace{\hat{j}\cdot\hat{i}}^{=0}\right)\text{ J} = -6.0\text{ J}.$

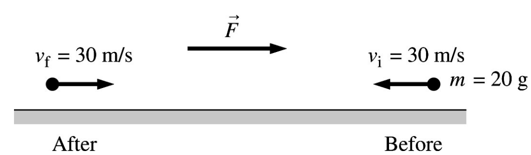
(b) $W = \vec{F}\cdot\Delta\vec{r} = (-3.0\hat{i} + 6.0\hat{j})\cdot(2.0\hat{j})\text{ N m} = \left(-6.0\overbrace{\hat{i}\cdot\hat{j}}^{=0} + 12\hat{j}\cdot\hat{j}\right)\text{ J} = 12\text{ J}.$

11.8. Solve: **(a)** $\vec{W} = \vec{F}\cdot\Delta\vec{r} = (-4.0\hat{i} - 6.0\hat{j})\cdot(-3.0\hat{i})\text{ N m} = \left(12\hat{i}\cdot\hat{i} + 18.0\overbrace{\hat{j}\cdot\hat{i}}^{=0}\right)\text{ J} = 12\text{ J}.$

(b) $\vec{W} = \vec{F}\cdot\Delta\vec{r} = (-4.0\hat{i} - 6.0\hat{j})\cdot(3.0\hat{i} - 2.0\hat{j})\text{ N m} = (-12 + 12)\text{ J} = 0\text{ J}.$

11.9. Model: Use the work-kinetic energy theorem to find the net work done on the particle.

Visualize:



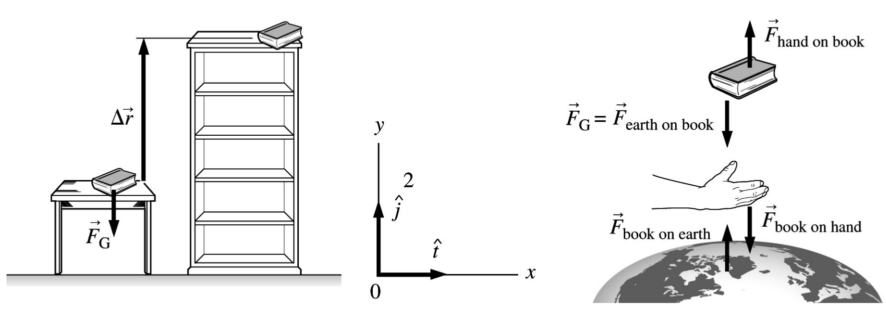
Solve: From the work-kinetic energy theorem,

$$W = \Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(v_1^2 - v_0^2) = \frac{1}{2}(0.020\text{ kg})\left[(30\text{ m/s})^2 - (-30\text{ m/s})\right]^2 = 0\text{ J}$$

Assess: Negative work is done in slowing down the particle to rest, and an equal amount of positive work is done in bringing the particle to the original speed but in the opposite direction.

11.10. Model: Work done by a force $\vec{F}$ on a particle is defined as $\vec{W} = \vec{F}\cdot\Delta\vec{r}$, where $\Delta\vec{r}$ is the particle's displacement.

Visualize:



Solve: **(a)** The work done by gravity is

$$W_g = \vec{F}_G \cdot \Delta\vec{r} = (-mg\hat{j}) \cdot (2.25 - 0.75)\hat{j} \text{ N m} = -(2.0 \text{ kg})(9.8 \text{ m/s}^2)(1.50 \text{ m}) \text{ J} = -29 \text{ J}$$

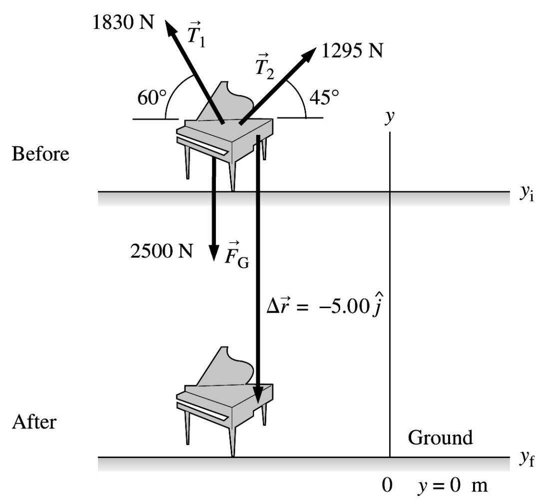
(b) The work done by hand is $W_H = \vec{F}_{\text{hand on book}} \cdot \Delta\vec{r}$. As long as the book does not accelerate,

$$\vec{F}_{\text{hand on book}} = -\vec{F}_{\text{earth on book}} = -(-mg\hat{j}) = mg\hat{j}$$

$$W_H = (mg\hat{j}) \cdot (2.25 - 0.75)\hat{j} \text{ N m} = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(1.50 \text{ m}) = 29 \text{ J}$$

11.11. Model: Model the piano as a particle and use $W = \vec{F} \cdot \Delta\vec{r}$, where W is the work done by the force $\vec{F}$ through the displacement $\Delta\vec{r}$.

Visualize:



Solve: For the force $\vec{F}_G$:

$$W = \vec{F} \cdot \Delta\vec{r} = \vec{F}_G \cdot \Delta\vec{r} = (F_g) \cdot (\Delta r)\cos(0°) = (255 \text{ kg})(9.81 \text{ m/s}^2)(5.00 \text{ m})(1.00) = 1.25 \times 10^4 \text{ J}$$

For the tension $\vec{T}_1$:

$$W = \vec{T}_1 \cdot \Delta\vec{r} = (T_1)(\Delta r)\cos(150°) = (1830 \text{ N})(5.00 \text{ m})(-0.8660) = -7.92 \times 10^3 \text{ J}$$

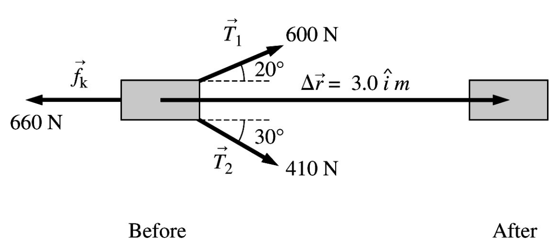
For the tension $\vec{T}_2$:

$$W = \vec{T}_2 \cdot \Delta\vec{r} = (T_2)(\Delta r)\cos(135°) = (1295 \text{ N})(5.00 \text{ m})(-0.7071) = -4.58 \times 10^3 \text{ J}$$

Assess: Note that the displacement $\Delta\vec{r}$ in all the above cases is directed downwards along $-\hat{j}$.

11.12. Model: Model the crate as a particle and use $W = \vec{F} \cdot \Delta\vec{r}$, where W is the work done by a force $\vec{F}$ on a particle and $\Delta\vec{r}$ is the particle's displacement.

Visualize:



Solve: For the force $\vec{f}_k$:

$$W = \vec{f}_k \cdot \Delta\vec{r} = f_k(\Delta r)\cos(180°) = (660\text{ N})(3.0\text{ m})(-1.0) = -2.0\text{ kJ}$$

For the tension $\vec{T}_1$:

$$W = \vec{T}_1 \cdot \Delta\vec{r} = (T_1)(\Delta r)\cos(20°) = (600\text{ N})(3.0\text{ m})(0.9397) = 1.7\text{ kJ}$$

For the tension $\vec{T}_2$:

$$W = \vec{T}_2 \cdot \Delta\vec{r} = (T_2)(\Delta r)\cos(30°) = (410\text{ N})(3.0\text{ m})(0.866) = 1.1\text{ kJ}$$

Assess: Negative work done by the force of kinetic friction $\vec{f}_k$ means that 1.95 kJ of energy has been transferred *out* of the crate.

11.13. Model: Model the 2.0 kg object as a particle, and use the work–kinetic-energy theorem.
Visualize: Please refer to Figure EX11.13. For each of the five intervals the velocity-versus-time graph gives the initial and final velocities. The mass of the object is 2.0 kg.
Solve: According to the work–kinetic-energy theorem:

$$W = \Delta K = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}m(v_f^2 - v_i^2)$$

$$\text{Interval AB: } v_i = 2\text{ m/s}, v_f = -2\text{ m/s} \quad\Rightarrow\quad W = \tfrac{1}{2}(2.0\text{ kg})\left[(-2\text{ m/s})^2 - (2\text{ m/s})^2\right] = 0\text{ J}$$

$$\text{Interval BC: } v_i = -2\text{ m/s}, v_f = -2\text{ m/s} \quad\Rightarrow\quad W = \tfrac{1}{2}(2.0\text{ kg})\left[(-2\text{ m/s})^2 - (-2\text{ m/s})\right]^2 = 0\text{ J}$$

$$\text{Interval CD: } v_i = -2\text{ m/s}, v_f = 0\text{ m/s} \quad\Rightarrow\quad W = \tfrac{1}{2}(2.0\text{ kg})\left[(0\text{ m/s})^2 - (-2\text{ m/s})^2\right] = -4\text{ J}$$

$$\text{Interval DE: } v_i = 0\text{ m/s}, v_f = 2\text{ m/s} \quad\Rightarrow\quad W = \tfrac{1}{2}(2.0\text{ kg})\left[(2\text{ m/s})^2 - (0\text{ m/s})^2\right] = +4\text{ J}$$

Assess: The work done is zero in intervals AB and BC. In the interval $\text{CD} + \text{DE}$ the total work done is zero. It is not whether v is positive or negative that counts because $K \propto v^2$. What is important is the magnitude of v and how v changes.

Section 11.4 The Work Done by a Variable Force

11.14. Model: Use the definition of work.
Visualize: Please refer to Figure EX11.14.
Solve: Work is defined as the area under the force-versus-position graph:

$$W = \int_{s_i}^{s_f} F_s\,ds = \text{area under the force curve}$$

$$\text{Interval } 0-1\text{ m: } W = (4\text{ N})(1\text{ m} - 0\text{ m}) = 4\text{ J}$$

$$\text{Interval } 1-2\text{ m: } W = (4\text{ N})(0.5\text{ m}) + (-4\text{ N})(0.5\text{ m}) = 0\text{ J}$$

$$\text{Interval } 2-3\text{ m: } W = \tfrac{1}{2}(-4.0\text{ N})(1\text{ m}) = -2\text{ J}$$

11.15. Model: Use the work–kinetic-energy theorem to find velocities.
Visualize: Please refer to Figure EX11.15.
Solve: The work–kinetic-energy theorem is

$$\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = W = \int_{x_i}^{x_f} F_x\,dx = \text{ area under the force curve from } x_i \text{ to } x_f$$

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}(0.500\text{ kg})(2.0\text{ m/s})^2 = \tfrac{1}{2}mv_f^2 - 1.0\text{ J} = \int_{0\text{ m}}^{x_f} F_x\,dx = \tfrac{5}{2}x^2\text{ N m}$$

$$v_f = \sqrt{\frac{5x^2\text{ N m}}{0.500\text{ kg}} + 4.0\text{ m}^2/\text{s}^2}$$

$$\text{At } x = 1 \text{ m:} \quad \Rightarrow \quad v_f = 3.7 \text{ m/s}$$
$$\text{At } x = 2 \text{ m:} \quad \Rightarrow \quad v_f = 6.6 \text{ m/s}$$
$$\text{At } x = 3 \text{ m:} \quad \Rightarrow \quad v_f = 9.7 \text{ m/s}$$

11.16. Model: Use the work–kinetic-energy theorem.
Visualize: Please refer to Figure EX11.16.
Solve: The work–kinetc-energy theorem is

$$\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \int_{0\text{ m}}^{x_f} F_x\, dx = 10x - \tfrac{5}{2}x^2$$

$$v_f = \sqrt{\frac{20x - 5x^2}{2.0 \text{ kg}} + 4.0 \text{ m/s}}$$

$$\text{At } x = 2 \text{ m:} \quad \Rightarrow \quad v_f = 5 \text{ m/s}$$
$$\text{At } x = 4 \text{ m:} \quad \Rightarrow \quad v_f = 4 \text{ m/s}$$

11.17. Model: Use the work–kinetic-energy theorem.
Visualize: Please refer to Figure EX11.17.
Solve: The work–kinetic-energy theorem is

$$\Delta K = W = \int_{x_i}^{x_f} F_x\, dx = \text{ area of the } F_x\text{-versus-}x \text{ graph between } x_i \text{ and } x_f$$

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(F_{max})(2 \text{ m})$$

Using $m = 0.500$ kg, $v_f = 6.0$ m/s, and $v_i = 2.0$ m/s, the above equation yields $F_{max} = 8$ N.

Assess: Problems in which the force is not a constant cannot be solved using constant-acceleration kinematic equations.

Section 11.5 Work and Potential Energy

Section 11.6 Finding Force from Potential Energy

11.18. Model: Use the definition $F_s = -dU/ds$.
Visualize: Please refer to Figure EX11.18.
Solve: F_x is the negative of the slope of the potential energy graph at position x. Between $x = 0$ cm and $x = 10$ cm the slope is

$$\text{slope} = (U_f - U_i)/(x_f - x_i) = (0 \text{ J} - 10 \text{ J})/(0.10 \text{ m} - 0.0 \text{ m}) = -100 \text{ N}$$

Thus, $F_x = 100$ N at $x = 5$ cm. The slope between $x = 10$ cm and $x = 20$ cm is zero, so $F_x = 0$ N at $x = 15$ cm. Between 20 cm and 40 cm,

$$\text{slope} = (10 \text{ J} - 0 \text{ J})/(0.40 \text{ m} - 0.20 \text{ m}) = 50 \text{ N}$$

At $x = 25$ cm and $x = 35$ cm, therefore, $F_x = -50$ N.

11.19. Model: Use the definition $F_s = -dU/ds$.
Visualize: Please refer to Figure EX11.19.
Solve: F_x is the negative of the slope of the potential energy graph at position x.

$$F_x = -\left(\frac{dU}{dx}\right)$$

Between $y = 0$ m and $x = 1$ m, the slope is

$$\text{slope} = (U_f - U_i)/(x_f - x_i) = (60 \text{ J} - 0 \text{ J})/(1 \text{ m} - 0 \text{ m}) = 60 \text{ N}$$

Thus, $F_x = -60$ N at $x = 1$ m. Between $x = 1$ m and $x = 5$ m, the slope is

$$\text{slope} = (U_f - U_i)/(x_f - x_i) = (0\text{ J} - 60\text{ J})/(5\text{ m} - 1\text{ m}) = -15\text{ N}$$

Thus, $F_x = 15$ N at $x = 4$ m.

11.20. Model: Use the negative derivative of the potential energy to determine the force acting on a particle.
Solve: The y-component of the force is

$$F_y = -\frac{dU}{dy} = -\frac{d}{dy}(4y^3\text{ J}) = -12y^2\text{ N}$$

At $y = 0$ m, $F_y = 0$ N; at $y = 1$ m, $F_y = -12$ N; and at $y = 2$ m, $F_y = -48$ N.

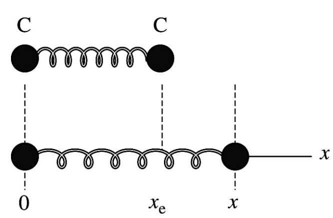
11.21. Model: Use the negative derivative of the potential energy to determine the force acting on a particle.
Solve: The x-component of the force is

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{10}{x}\text{ J}\right) = \frac{10}{x^2}\text{ N}$$

$$F(x = 2\text{ m}) = \left.\frac{10}{x^2}\right|_{x=2\text{ m}}\text{ N} = 2.5\text{ N},\quad F(x = 5\text{ m}) = \left.\frac{10}{x^2}\right|_{x=5\text{ m}}\text{ N} = 0.40\text{ N},\quad F(x = 8\text{ m}) = \left.\frac{10}{x^2}\right|_{x=8\text{ m}}\text{ N} = 0.16\text{ N}$$

Section 11.7 Thermal Energy

11.22. Model: Assume the carbon-carbon bond acts like an ideal spring that obeys Hooke's law.
Visualize:



The quantity $(x - x_e)$ is the stretching relative to the spring's equilibrium length. In the present case, bond stretching is analogous to spring stretching.
Solve: (a) The kinetic energy of the carbon atom is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.0\times10^{-26}\text{ kg})(500\text{ m/s})^2 = 2.5\times10^{-21}\text{ J}$$

(b) The energy of the spring is given by

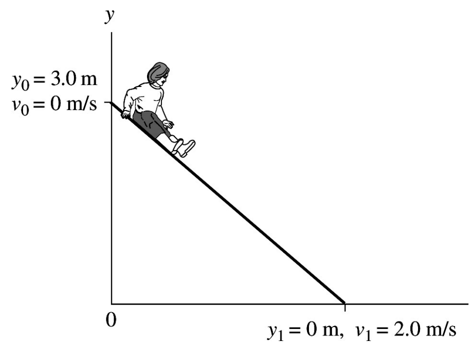
$$U_s = \frac{1}{2}k(x - x_e)^2 = K$$

$$k = \frac{2K}{(x - x_e)^2} = \frac{2(2.5\times10^{-21}\text{ J})}{(0.050\times10^{-9}\text{ m})^2} = 2.0\text{ N/m}$$

11.23. Visualize: One mole of helium atoms in the gas phase contains $N_A = 6.02\times10^{23}$ atoms.
Solve: If each atom moves with the same speed v, the microscopic total kinetic energy will be

$$K_{\text{micro}} = N_A\left(\frac{1}{2}mv^2\right) = 3700\text{ J} \quad\Rightarrow\quad v = \sqrt{\frac{2K_{\text{micro}}}{mN_A}} = \sqrt{\frac{2(3700\text{ J})}{(6.68\times10^{-27}\text{ kg})(6.02\times10^{23})}} = 1360\text{ m/s}$$

11.24. Visualize:

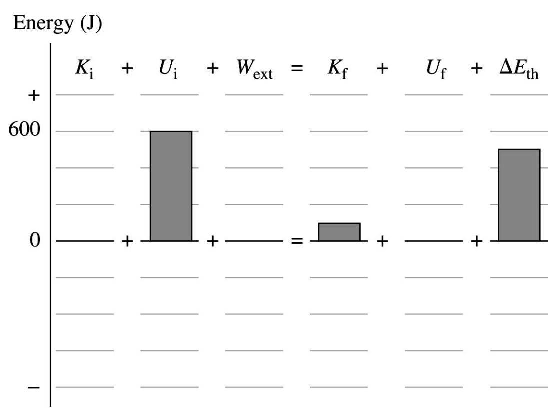


Solve: (a) $K_i = K_0 = \frac{1}{2}mv_0^2 = 0 \text{ J}$, $U_i = U_{g0} = mgy_0 = (20 \text{ kg})(9.8 \text{ m/s}^2)(3.0 \text{ m}) = 5.9\times10^2 \text{ J}$

$W_{ext} = 0 \text{ J}$, $K_f = K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(20 \text{ kg})(2.0 \text{ m/s})^2 = 40 \text{ J}$, $U_f = U_{g1} = mgy_1 = 0 \text{ J}$

At the top of the slide, the child has gravitational potential energy of 5.9×10^2 J. This energy is transformed into the thermal energy of the child's pants and the slide and the kinetic energy of the child. This energy transfer and transformation is shown on the energy bar chart.

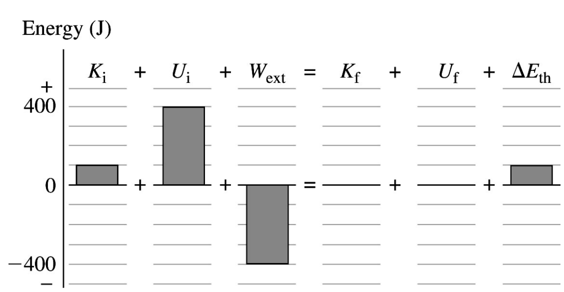
(b)



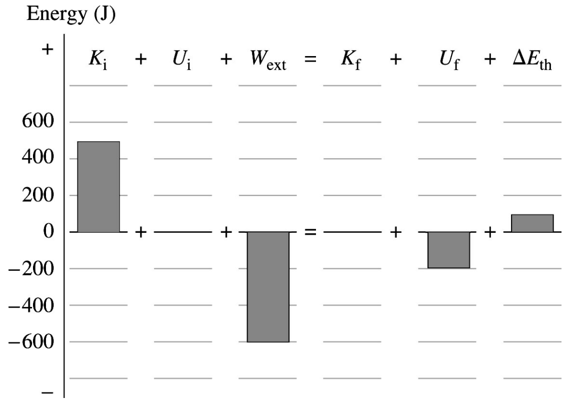
The change in the thermal energy of the slide and of the child's pants is $5.9\times10^2 \text{ J} - 40 \text{ J} = 5.5\times10^2 \text{ J}$.

Section 11.8 Conservation of Energy

11.25. Visualize: The system loses 400 J of potential energy. In the process of losing this energy, it does 400 J of work on the environment, which means $W_{ext} = -400$ J. Since the thermal energy increases 100 J, we have $\Delta E_{th} = 100$ J, which must have been 100 J of kinetic energy originally. This is shown in the energy bar chart.



11.26. Visualize:



Note that the conservation of energy equation

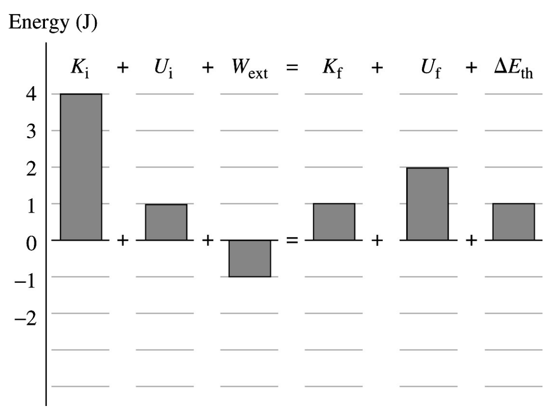
$$K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th}$$

requires that W_{ext} be equal to -400 J.

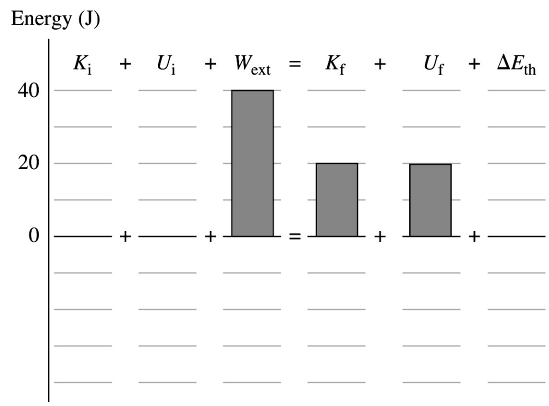
11.27. Solve: Please refer to Figure EX11.27. The energy conservation equation yields

$$K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th} \quad \Rightarrow \quad 4\text{ J} + 1\text{ J} + W_{ext} = 1\text{ J} + 2\text{ J} + 1\text{ J} \quad \Rightarrow \quad \text{W}_{ext} = -1\text{ J}$$

Thus, the work done to the environment is -1 J. In other words, 1 J of energy is transferred from the system into the environment. This is shown in the energy bar chart.



11.28. Visualize: The tension of 20.0 N in the cable is an external force that does work on the block $W_{ext} = (20.0\text{ N})(2.00\text{ m}) = 40.0\text{ J}$, increasing the gravitational potential energy of the block. We placed the origin of our coordinate system on the initial resting position of the block, so we have $U_i = 0$ J and $U_f = mgy_f = (1.02\text{ kg})(9.8\text{ m/s}^2)(2.00\text{ m}) = 20.0\text{ J}$. Also, $K_i = 0$ J, and $\Delta E_{th} = 0$ J. The energy bar chart shows the energy transfers and transformations.



Solve: The conservation of energy equation is

$$K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th} \Rightarrow 0\text{ J} + 0\text{ J} + 40.0\text{ J} = \tfrac{1}{2}mv_f^2 + 20.0\text{ J} + 0\text{ J}$$

$$v_f = \sqrt{(20.0\text{ J})(2)/(1.02\text{ kg})} = 6.26\text{ m/s}$$

Section 11.9 Power

11.29. Model: Model the elevator as a particle, and apply the conservation of energy.
Solve: The tension in the cable does work on the elevator to lift it. Because the cable is pulled by the motor, we say that the motor does the work of lifting the elevator.
(a) The energy conservation equation is $K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th}$. Using $K_i = 0$ J, $K_f = 0$ J, and $\Delta E_{th} = 0$ J gives

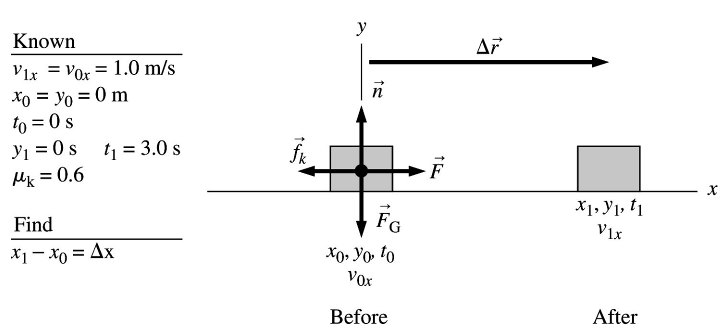
$$W_{ext} = (U_f - U_i) = mg(y_f - y_i) = (1000\text{ kg})(9.8\text{ m/s}^2)(100\text{ m}) = 9.80\times10^5\text{ J}$$

(b) The power required to give the elevator this much energy in a time of 50 s is

$$P = \frac{W_{ext}}{\Delta t} = \frac{9.80\times10^5\text{ J}}{50\text{ s}} = 1.96\times10^4\text{ W}$$

Assess: Since 1 horsepower (hp) is 746 W, the power of the motor is 26 hp. This is a reasonable amount of power to lift a mass of 1000 kg to a height of 100 m in 50 s.

11.30. Model: Model the steel block as a particle subject to the force of kinetic friction and use energy conservation.
Visualize:



Solve: (a) The work done on the block is $W_{net} = \vec{F}_{net} \cdot \Delta\vec{r}$ where $\Delta\vec{r}$ is the displacement. We will find the displacement using kinematic equations and the force using Newton's second law of motion. The displacement in the x-direction is

$$\Delta x = x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 = 0\text{ m} + (1.0\text{ m/s})(3.0\text{ s} - 0\text{ s}) + 0\text{ m} = 3.0\text{ m}$$

Thus $\Delta\vec{r} = 3.0\hat{i}$ m.

The equations for Newton's second law along the x and y components are

$$(F_{net})_y = n - F_G = 0 \text{ N} \quad \Rightarrow \quad n = F_G = mg = (10 \text{ kg})(9.8 \text{ m/s}^2) = 98.0 \text{ N}$$

$$(F_{net})_x = \vec{F} - \vec{f}_k = 0 \text{ N} \quad \Rightarrow \quad F = f_k = \mu_k n = (0.6)(98.0 \text{ N}) = 58.8 \text{ N}$$

$$W_{net} = \vec{F}_{net} \cdot \Delta\vec{r} = F\Delta x \cos(0°) = (58.8 \text{ N})(3.0 \text{ m})(1) = 176 \text{ J}$$

(b) The power required to do this much work in 3.0 s is

$$P = \frac{W}{t} = \frac{176 \text{ J}}{3.0 \text{ s}} = 59 \text{ W}$$

11.31. Solve: The power of the solar collector is the solar energy collected divided by time. The intensity of the solar energy striking the earth is the power divided by area. We have

$$P = \frac{\Delta E}{\Delta t} = \frac{150\times10^6 \text{ J}}{3600 \text{ s}} = 41,667 \text{ W and intensity} = 1000 \text{ W/m}^2$$

$$\text{Area of solar collector} = \frac{41,667 \text{ W}}{1000 \text{ W/m}^2} = 42 \text{ m}^2$$

11.32. Solve: The night light consumes more energy than the hair dryer. The calculations are

$$1.2 \text{ kW}\times10 \text{ min} = 1.2\times10^3\times10\times60 \text{ J} = 7.2\times10^5 \text{ J}$$

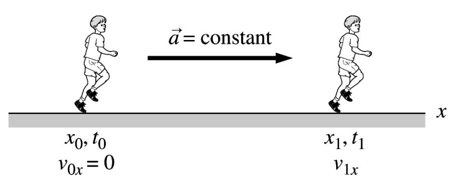
$$10 \text{ W}\times24 \text{ hours} = 10\times24\times60\times60 \text{ J} = 8.6\times10^5 \text{ J}$$

11.33. Solve: Using the conversion $746 \text{ W} = 1 \text{ hp}$, we have a power of 1492 J/s. This means $W = Pt = (1492 \text{ J/s})(1 \text{ h}) = 5.3712\times10^6 \text{ J}$ is the total work done by the electric motor in one hour. Furthermore,

$$W_{motor} = -W_g = U_{gf} - U_{gi} = mg(y_f - y_i) = mg(10 \text{ m})$$

$$m = \frac{W_{motor}}{g(10 \text{ m})} = \frac{5.3712\times10^6 \text{ J}}{(9.8 \text{ m/s}^2)(10 \text{ m})} = 5.481\times10^4 \text{ kg} = 5.481\times10^4 \text{ kg}\times\frac{1 \text{ liter}}{1 \text{ kg}} = 5.5\times10^4 \text{ liters}$$

11.34. Model: Model the sprinter as a particle, and use the constant-acceleration kinematic equations and the definition of power in terms of velocity.
Visualize:



Solve: (a) We can find the acceleration from the kinematic equations and the horizontal force from Newton's second law. We have

$$x = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 \quad \Rightarrow \quad 50 \text{ m} = 0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}a_x(7.0 \text{ s} - 0 \text{ s})^2 \quad \Rightarrow \quad a_x = 2.04 \text{ m/s}^2$$

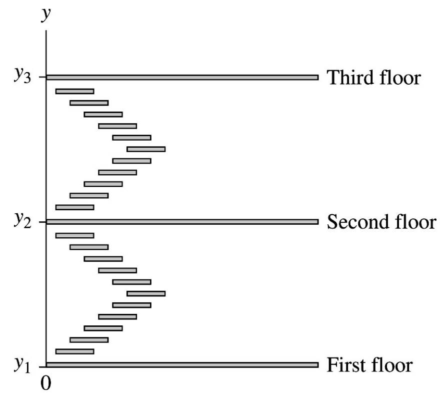
$$F_x = ma_x = (50 \text{ kg})(2.04 \text{ m/s}^2) = 10\times10^1 \text{ N}$$

(b) We obtain the sprinter's power output by using $P = \vec{F}\cdot\vec{v}$, where $\vec{v}$ is the sprinter's velocity. At $t = 2.0 \text{ s}$ the power is

$$P = (F_x)[v_{0x} + a_x(t - t_0)] = (102 \text{ N})[0 \text{ m/s} + (2.04 \text{ m/s}^2)(2.0 \text{ s} - 0 \text{ s})] = 0.42 \text{ kW}$$

The power at $t = 4.0 \text{ s}$ is 0.83 kW, and at $t = 6.0 \text{ s}$ the power is 1.3 kW.

11.35. Visualize: We place the origin of the coordinate system at the base of the stairs on the first floor.



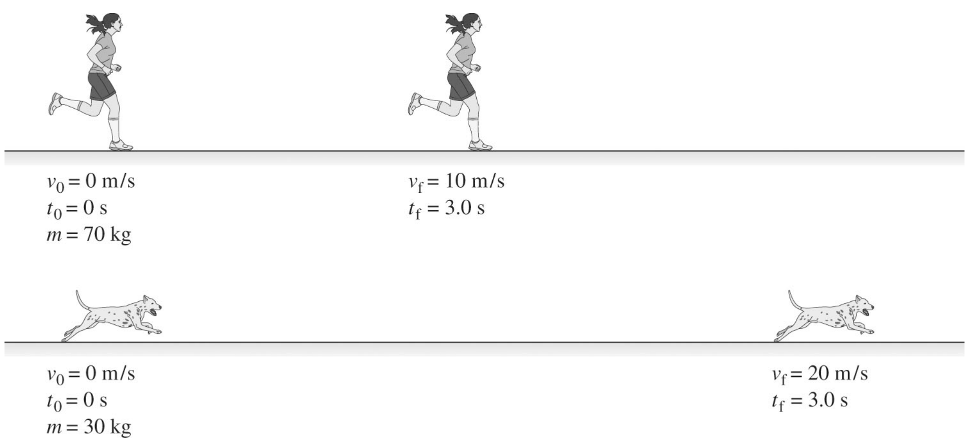
Solve: **(a)** We might estimate $y_2 - y_1 \approx 4.0 \text{ m} \approx 12 \text{ ft} \approx y_3 - y_2$, thus, $y_3 - y_1 \approx 8.0 \text{ m}$.
(b) We might estimate the time to run up these two flights of stairs to be 20 s.
(c) Estimate your mass as $m \approx 70 \text{ kg} \approx 150 \text{ lb}$. Your power output while running up the stairs is

$$\frac{\text{work done by you}}{\text{time}} = \frac{\text{change in potential energy}}{\text{time}} = \frac{mg(y_3 - y_1)}{\text{time}}$$

$$= \frac{(70 \text{ kg})(9.8 \text{ m/s}^2)(8.0 \text{ m})}{20 \text{ s}} \approx 270 \text{ W} = (270 \text{ W})\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) \approx 0.35 \text{ hp}$$

Assess: Your estimate may vary, depending on your mass and how fast you run.

11.36. Visualize: See figure below.



Solve: Average power output is the change in energy of the system divided by the time interval. For the runner, the change in energy is just the change ΔK in the kinetic energy because the potential energy remains unchanged. Thus, $\Delta E = \Delta K = K_f - K_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}(70 \text{ kg})(10 \text{ m/s})^2 = 3500 \text{ J}$. For the greyhound, the change in energy is $\Delta E = \Delta K = \frac{1}{2}(30 \text{ kg})(20 \text{ m/s})^2 = 6000 \text{ J}$. Thus, the average power output of the runner is $\bar{P} = \Delta E/\Delta t = (3500 \text{ J})/(3.0 \text{ s}) = 1.2 \text{ kW}$ and the average power output of the greyhound is $\bar{P} = (6000 \text{ J})/(3.0 \text{ s}) = 2.0 \text{ kW}$.

11.37. Model: Use the definition of work for a constant force $\vec{F}$, $W = \vec{F} \cdot \Delta\vec{s}$, where $\Delta\vec{s}$ is the displacement.
Visualize: Please refer to Figure P11.37. The force $\vec{F} = (6\hat{i} + 8\hat{j})$ N on the particle is constant.
Solve: **(a)** $W_{ABD} = W_{AB} + W_{BD} = \vec{F} \cdot (\Delta\vec{s})_{AB} + \vec{F} \cdot (\Delta\vec{s})_{BD}$

$$= (6\hat{i} + 8\hat{j}) \text{ N} \cdot (3\hat{i}) \text{ m} + (6\hat{i} + 8\hat{j}) \text{ N} \cdot (4\hat{j}) \text{ m} = 18 \text{ J} + 32 \text{ J} = 50 \text{ J}$$

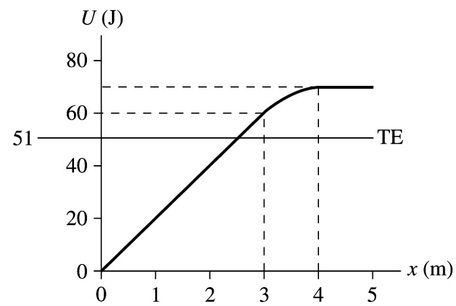
(b) $W_{ACD} = W_{AC} + W_{CD} = \vec{F} \cdot (\Delta\vec{s})_{AC} + \vec{F} \cdot (\Delta\vec{s})_{CD}$

$$= (6\hat{i} + 8\hat{j})\ \text{N} \cdot (4\hat{j})\ \text{m} + (6\hat{i} + 8\hat{j})\ \text{N} \cdot (3\hat{j})\ \text{m} = 32\ \text{J} + 18\ \text{J} = 50\ \text{J}$$

(c) $W_{AD} = \vec{F} \cdot (\Delta\vec{s})_{AD} = (6\hat{i} + 8\hat{j})\ \text{N} \cdot (3\hat{i} + 4\hat{j})\ \text{m} = 18\ \text{J} + 32\ \text{J} = 50\ \text{J}$

The force is conservative because the work done is independent of the path.

11.38. Model: The force is conservative, so it has a potential energy.
Visualize: Please refer to Figure P11.38 for the graph of the force.
Solve: (a) The definition of potential energy is $\Delta U = -W(\text{i} \to \text{f})$. In addition, work is the area under the force-versus-displacement graph. Thus $\Delta U = U_f - U_i = -$(area under the force curve). Since $U_i = 0$ at $x = 0$ m, the potential energy at position x is $U(x) = -$(area under the force curve from 0 to x). From 0 m to 3 m, the area increases linearly from 0 Nm to −60 Nm, so U increases from 0 J to 60 J. At $x = 4$ m, the area is −70 J. Thus $U = 70$ J at $x = 4$ m, and U doesn't change after that since the force is then zero. Between 3 m and 4 m, where F changes linearly, U must have a quadratic dependence on x (i.e., the potential energy curve is a parabola). This information is shown on the potential energy graph below.

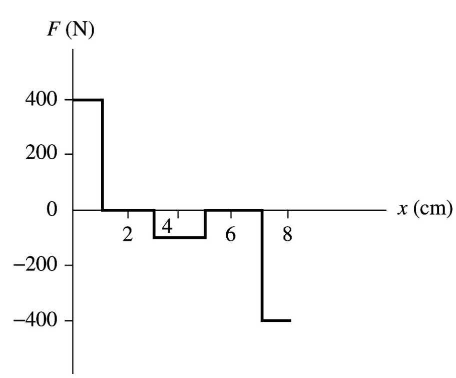


(b) Mechanical energy is $E = K + U$. From the graph, $U = 20$ J at $x = 1.0$ m.
The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.100\ \text{kg})(25\ \text{m/s})^2 = 31.25\ \text{J}$. Thus $E = 51$ J.
(c) The total energy line at 51 J is shown on the graph above.
(d) The turning point occurs where the total energy line crosses the potential energy curve. We can see from the graph that this is at approximately 2.5 m. For a more accurate value, the potential energy function is $U = 20x$ J. The TE line crosses at the point where $20x = 51.25$, which is $x = 2.6$ m.

11.39. Model: Use the relationship between force and potential energy and the work–kinetic-energy theorem.
Visualize: Please refer to Figure P11.39. We will find the slope in the following x regions: $0\ \text{cm} < x < 1$ cm, $1 < x < 3$ cm, $3 < x < 5$ cm, $5 < x < 7$ cm, and $7 < x < 8$ cm.
Solve: (a) F_x is the negative slope of the U-versus-x graph, for example, for $0\ \text{m} < x < 2$ m

$$\frac{dU}{dx} = \frac{-4\ \text{J}}{0.01\ \text{m}} = -400\ \text{N} \quad \Rightarrow \quad F_x = +400\ \text{N}$$

Calculating the values of F_x in this way, we can draw the force-versus-position graph as shown below.



(b) Since $W = \int_{x_i}^{x_f} F_x\, dx =$ area of the F_x-versus-x graph between x_i and x_f, the work done by the force as the particle moves from $x_i = 2$ cm to $x_f = 6$ cm is -2 J.

(c) The conservation of energy equation is $K_f + U_f = K_i + U_i$. We can see from the graph that $U_i = 0$ J and $U_f = 2$ J in moving from $x = 2$ cm to $x = 6$ cm. The final speed is $v_f = 10$ m/s, so

$$2\text{ J} + \tfrac{1}{2}(0.010\text{ kg})(10.0\text{ m/s})^2 = 0\text{ J} + \tfrac{1}{2}(0.010\text{ kg})v_i^2 \quad\Rightarrow\quad v_i = 22\text{ m/s}$$

11.40. Model: Use the relationship between a conservative force and potential energy.

Visualize: Please refer to Figure P11.40. We will obtain U as a function of x and F_x as a function of x by using the calculus techniques of integration and differentiation.

Solve: (a) For the interval $0\text{ m} < x < 0.5\text{ m}$, $F_x = (4x)$ N, where x is in meters. This means

$$\frac{dU}{dx} = -F_x = -4x \quad\Rightarrow\quad U = -2x^2 + C_1 = -2x^2$$

where we have used $U = 0$ J at $x = 0$ m to obtain $C_1 = 0$. For the interval $0.5\text{ m} < x < 1\text{ m}$, $F_x = (-4x + 4)$ N. Likewise,

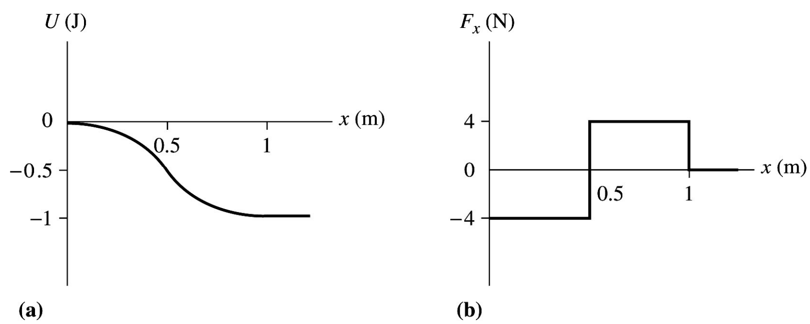
$$\frac{dU}{dx} = 4x - 4 \quad\Rightarrow\quad U = 2x^2 - 4x + C_2$$

Since U should be continuous at the junction, we have the continuity condition

$$(-2x^2)_{x=0.5\text{ m}} = (2x^2 - 4x + C_2)_{x=0.5\text{ m}} \quad\Rightarrow\quad -0.5 = 0.5 - 2 + C_2 \quad\Rightarrow\quad C_2 = 1$$

U remains constant for $x \geq 1$ m.

(b) For the interval $0\text{ m} < x < 0.5\text{ m}$, $U = +4x$, and for the interval $0.5\text{ m} < x < 1.0\text{ m}$, $U = -4x + 4$, where x is in meters. The derivatives give $F_x = -4$ N and $F_x = +4$ N, respectively. The slope is zero for $x \geq 1$ m.

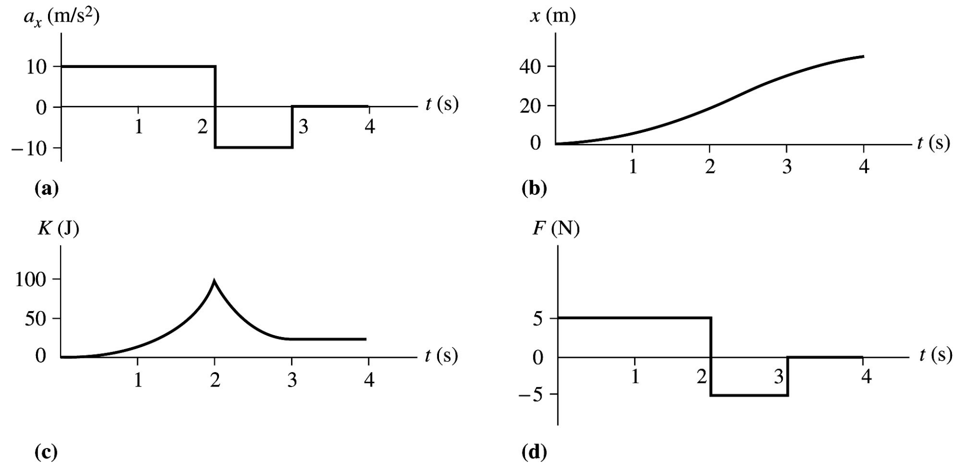


11.41. Model: Use $a_x = dv_x/dt$, $x = \int v_x\, dt$, $K = \tfrac{1}{2}mv_x^2$, and $F = ma_x$.

Visualize: Please refer to Figure P11.41. We know $a_x =$ slope of the v_x-versus-t graph and $x =$ area under the v_x-versus-x graph between 0 and x.

Solve: Using the above definitions and methodology, we can generate the following table:

t(s)	a_x (m/s2)	x(m)	K(J)	F(N)
0	10	0	0	5
0.5	10	1.25	6.25	5
1.0	10	5	25	5
1.5	10	11.25	56.25	5
2.0	+10 or −10	20	100	−5 or +5
2.5	10	28.75	56.25	5
3.0	−10 or 0	35	25	−5 or 0
3.5	0	40	25	0
4.0	0	45	25	0



(e) Let J_1 be the impulse from $t = 0$ s to $t = 2$ s and J_2 be the impulse from $t = 2$ s to $t = 4$ s. We have

$$J_1 = \int_{0\text{s}}^{2\text{s}} F_x\,dt = (5\text{ N})(2\text{ s}) = 10\text{ N}\cdot\text{s} \text{ and } J_2 = \int_{0\text{s}}^{4\text{s}} F_x\,dt = (-5\text{ N})(1\text{ s}) = -5\text{ N}\cdot\text{s}$$

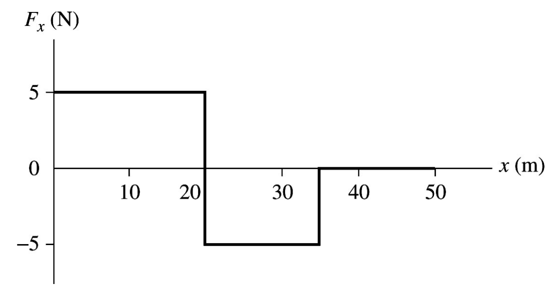
(f) $J = \Delta p = mv_\text{f} - mv_\text{i} \quad \Rightarrow \quad v_\text{f} = v_\text{i} + J/m$

At $t = 2$ s, $v_x = 0\text{ m/s} + (10\text{ N}\cdot\text{s})/(0.5\text{ kg}) = 0\text{ m/s} + 20\text{ m/s} = 20\text{ m/s}$

At $t = 4$ s, $v_x = 20\text{ m/s} + (-5\text{ N}\cdot\text{s})/(0.5\text{ kg}) = 20\text{ m/s} - 10\text{ m/s} = 10\text{ m/s}$

The v_x-versus-t graph also gives $v_x = 20$ m/s at $t = 2$ s and $v_x = 10$ m/s at $t = 4$ s.

(g)



(h) From $t = 0$ s to $t = 2$ s, $W = \int F_x\,dx = (5\text{ N})(20\text{ m}) = 100\text{ J}$

From $t = 2$ s to $t = 4$ s, $W = \int F_x\,dx = (-5\text{ N})(15\text{ m}) = -75\text{ J}$

(i) At $t = 0$ s, $v_x = 0$ m/s so the work–kinetic-energy theorem for calculating v_x at $t = 2$ s is

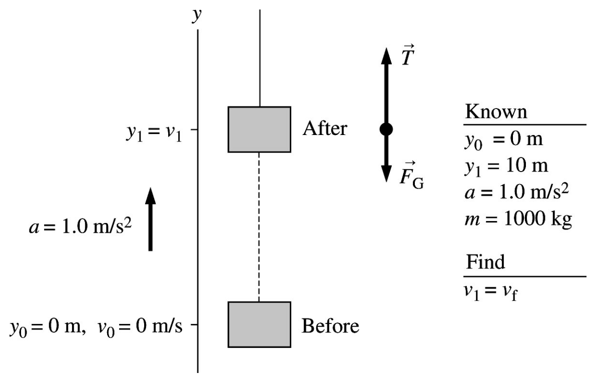
$$W = \Delta K = \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2 \quad \Rightarrow \quad 100\text{ J} = \frac{1}{2}(0.5\text{ kg})v_x^2 - \frac{1}{2}(0.5\text{ kg})(0\text{ m/s})^2 \quad \Rightarrow \quad v_x = 20\text{ m/s}$$

To calculate v_x at $t = 4$ s, we use v_x at $t = 2$ s as the initial velocity:

$$-75\text{ J} = \frac{1}{2}(0.5\text{ kg})v_x^2 - \frac{1}{2}(0.5\text{ kg})(20\text{ m/s})^2 \quad \Rightarrow \quad v_x = 10\text{ m/s}$$

Both of these values agree with the values on the velocity graph.

11.42. Model: Model the elevator as a particle.
Visualize:



Solve: **(a)** The work done by gravity on the elevator is

$$W_g = -\Delta U_g = mgy_0 - mgy_1 = -mg(y_1 - y_0) = -(1000\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = -9.8\times10^4\text{ J}$$

(b) The work done by the tension in the cable on the elevator is

$$W_T = T(\Delta y)\cos(0°) = T(y_1 - y_0) = T(10\text{ m})$$

To find T we write Newton's second law for the elevator:

$$\sum F_y = T - F_G = ma_y \quad\Rightarrow\quad T = F_G + ma_y = m(g + a_y) = (1000\text{ kg})(9.8\text{ m/s}^2 + 1.0\text{ m/s}^2)$$

$$= 1.08\times10^4\text{ N} \quad\Rightarrow\quad W_T = (1.08\times10^4\text{ N})(10\text{ m}) = 1.1\times10^5\text{ J}$$

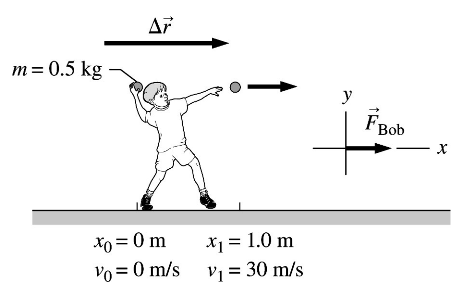
(c) The work–kinetic-energy theorem is

$$W_{net} = W_g + W_T = \Delta K = K_f - K_i = K_f - \tfrac{1}{2}mv_0^2 \Rightarrow K_f = W_g + W_T + \tfrac{1}{2}mv_0^2$$

$$K_f = (-9.8\times10^4\text{ J}) + (1.08\times10^5\text{ J}) + \tfrac{1}{2}(1000\text{ kg})(0\text{ m/s})^2 = 1.0\times10^4\text{ J}$$

(d) $K_f = \tfrac{1}{2}mv_f^2 \Rightarrow 1.0\times10^4\text{ J} = \tfrac{1}{2}(1000\text{ kg})v_f^2 \Rightarrow v_f = 4.5\text{ m/s}$

11.43. Model: Model the rock as a particle, and apply the work–kinetic-energy theorem.
Visualize:



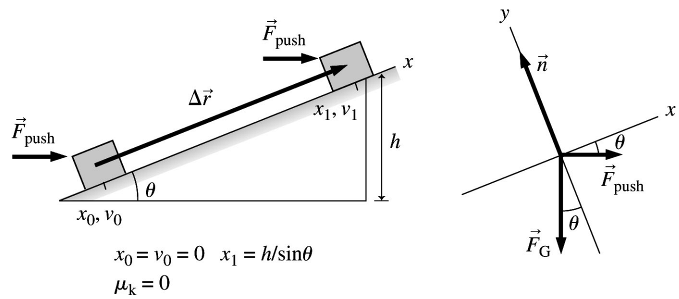
Solve: **(a)** The work done by Bob on the rock is

$$W_{Bob} = \Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(0.500\text{ kg})(30\text{ m/s})^2 = 225\text{ J} = 2.3\times10^2\text{ J}$$

(b) For a constant force, $W_{Bob} = F_{Bob}\Delta x \quad\Rightarrow\quad F_{Bob} = W_{Bob}/\Delta x = 2.3\times10^2$ N.

(c) Bob's power output is $P_{Bob} = F_{Bob}v_{rock}$ and will be a maximum when the rock has maximum speed. This is just as he releases the rock with $v_{rock} = v_1 = 30$ m/s. Thus, $P_{max} = F_{Bob}v_1 = (225\text{ J})(30\text{ m/s}) = 6750\text{ W} = 6.8\text{ kW}$.

11.44. Model: Model the crate as a particle, and use the work–kinetic-energy theorem.
Visualize:



Solve: **(a)** The work–kinetic-energy theorem is $\Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 = W_{total}$. Three forces act on the box, so $W_{total} = W_{grav} + W_n + W_{push}$. The normal force is perpendicular to the motion, so $W_n = 0$ J. The other two forces do the following amount of work:

$$W_{push} = \vec{F}\cdot\Delta\vec{r} = Fx_1\cos\theta = F\left(\frac{h}{\sin\theta}\right)\cos\theta = Fh\cot\theta$$

$$W_{grav} = \vec{F}_G\cdot\Delta\vec{r} = -mgx_1\sin\theta = -mg\left(\frac{h}{\sin\theta}\right)\sin\theta = -mgh$$

Thus, the speed at the top of the ramp is

$$v_1 = \sqrt{\frac{2W_{total}}{m}} = \sqrt{\frac{2(Fh\cot\theta - mgh)}{m}}$$

(b) Insert the given quantities into the expression for the speed to find

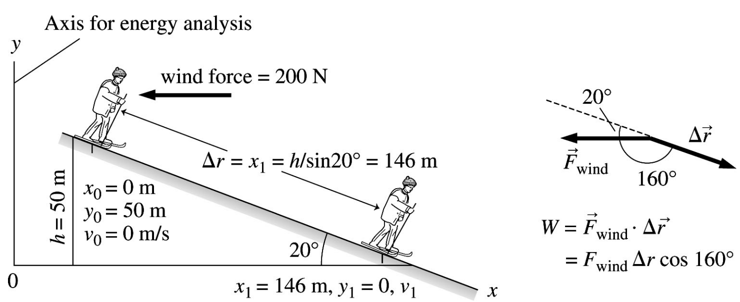
$$v_1 = \sqrt{\frac{2[(25\text{ N})(2.0\text{ m})\cot(20°) - (5.0\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m})]}{5.0\text{ kg}}} = 4.0\text{ m/s}$$

Assess: Note that $F\cos\theta > mg\sin\theta$ for the radical to remain positive. This means that the component of the pushing force up the slope must be greater that then component of gravity down the slope for the crate to move upwards, which is the assumption with which we started. Furthermore, if we take the limit $h \to 0$, we get

$$\underset{\lim h\to 0}{v_1} = \sqrt{\frac{2\left[Fh\left(\frac{1}{h}\right) - mgh\right]}{m}}\Bigg|_{h=0} = \sqrt{\frac{2F}{m}}$$

which is the expected result for pushing the crate along a horizontal frictionless surface.

11.45. Model: Model Sam strapped with skis as a particle, and apply the law of conservation of energy.
Visualize:



Solve: (a) The conservation of energy equation is

$$K_1 + U_{g1} + \Delta E_{th} = K_0 + U_{g0} + W_{ext}$$

The snow is frictionless, so $\Delta E_{th} = 0$ J. However, the wind is an external force doing work on Sam as he moves down the hill. Thus,

$$W_{ext} = W_{wind} = (K_1 + U_{g1}) - (K_0 + U_{g0})$$
$$= \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) - \left(\tfrac{1}{2}mv_0^2 + mgy_0\right) = \left(\tfrac{1}{2}mv_1^2 + 0\text{ J}\right) - (0\text{ J} + mgy_0) = \tfrac{1}{2}mv_1^2 - mgy_0$$
$$v_1 = \sqrt{2gy_0 + \frac{2W_{wind}}{m}}$$

We compute the work done by the wind as follows:

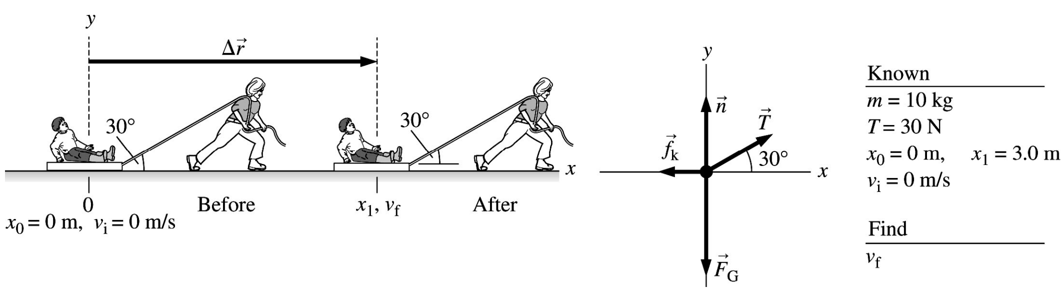
$$W_{wind} = \vec{F}_{wind} \cdot \Delta\vec{r} = F_{wind}\Delta r\cos(160°) = (200\text{ N})(146\text{ m})\cos(160°) = -27{,}400\text{ J}$$

where we have used $\Delta r = h/\sin(20°) = 146$ m. Now we can compute

$$v_1 = \sqrt{2(9.8\text{ m/s}^2)(50\text{ m}) + \frac{2(-27{,}400\text{ J})}{75\text{ kg}}} = 16\text{ m/s}$$

Assess: We used a vertical y-axis for energy analysis, rather than a tilted coordinate system, because U_g is determined by its vertical position.

11.46. Model: Model Paul and the mat as a particle, assume the mat to be massless, use the model of kinetic friction, and apply the work–kinetic-energy theorem.
Visualize:



We define the x-axis along the floor and the y-axis perpendicular to the floor.
Solve: We first need to determine f_k. Newton's second law in the y-direction gives

$$n + T\sin(30°) = F_G = mg \quad \Rightarrow \quad n = mg - T\sin(30°) = (10\text{ kg})(9.8\text{ m/s}^2) - (30\text{ N})\sin(30°) = 83.0\text{ N}.$$

Using n and the model of kinetic friction gives $f_k = \mu_k n = (0.2)(83.0\text{ N}) = 16.60$ N. The net force on Paul and the mat is therefore $F_{net} = T\cos(30°) - f_k = (30\text{ N})\cos(30°) - 16.6\text{ N} = 9.4\text{ N}$. Thus,

$$W_{net} = F_{net}\Delta r = (9.4\text{ N})(3.0\text{ m}) = 28\text{ J}$$

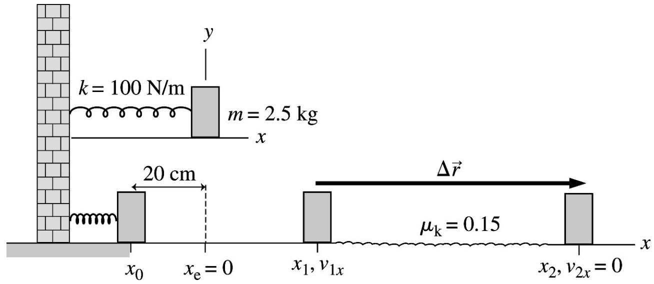
The other forces $\vec{n}$ and $\vec{F}_G$ make an angle of 90° with $\Delta\vec{r}$ and do zero work. We can now use the work–kinetic-energy theorem to find the final velocity as follows:

$$W_{net} = K_f - K_i = K_f - 0\text{ J} = \frac{1}{2}mv_f^2 \quad \Rightarrow \quad v_f = \sqrt{2W_{net}/m} = \sqrt{2(28\text{ J})/(10\text{ kg})} = 2.4\text{ m/s}$$

Assess: A speed of 2.4 m/s or 5.4 mph is reasonable for the present problem.

11.47. Model: Assume an ideal spring that obeys Hooke's law. Model the box as a particle and use the model of kinetic friction.

Visualize:



Solve: When the horizontal surface is frictionless, conservation of energy means

$$\tfrac{1}{2}k(x_0 - x_e)^2 = \tfrac{1}{2}mv_{1x}^2 = K_1 \quad \Rightarrow \quad K_1 = \tfrac{1}{2}(100\text{ N/m})(0.20\text{ m} - 0\text{ m})^2 = 2.0\text{ J}$$

That is, the box is launched with 2.0 J of kinetic energy. It will lose 2.0 J of kinetic energy on the rough surface. The net force on the box is $\vec{F}_{\text{net}} = -\vec{f}_k = -\mu_k mg\hat{i}$. The work–kinetic-energy theorem is

$$W_{\text{net}} = \vec{F}_{\text{net}} \cdot \Delta\vec{r} = K_2 - K_1 = 0\text{ J} - 2.0\text{ J} = -2.0\text{ J}$$

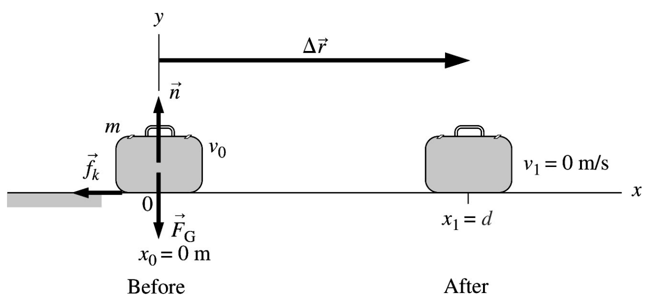
$$(-\mu_k mg)(x_2 - x_1) = -2.0\text{ J}$$

$$(x_2 - x_1) = \frac{2.0\text{ J}}{\mu_k mg} = \frac{2.0\text{ J}}{(0.15)(2.5\text{ kg})(9.8\text{ m/s}^2)} = 0.54\text{ m}$$

Assess: Because the force of friction transforms kinetic energy into thermal energy, energy is transferred out of the box into the environment. In response, the box slows down and comes to rest.

11.48. Model: Model the suitcase as a particle, use the model of kinetic friction, and use the work–kinetic-energy theorem.

Visualize:



The net force on the suitcase is $\vec{F}_{\text{net}} = \vec{f}_k$.

Solve: (a) The work–kinetic-energy theorem gives

$$W_{\text{net}} = \Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 \quad \Rightarrow \quad \vec{F}_{\text{net}} \cdot \Delta\vec{r} = \vec{f}_k \cdot \Delta\vec{r} = 0\text{ J} - \tfrac{1}{2}mv_0^2$$

$$(f_k)d\cos(180°) = -\tfrac{1}{2}mv_0^2 - \mu_k mgd = -\tfrac{1}{2}mv_0^2 \quad \Rightarrow \quad \mu_k = \frac{v_0^2}{2gd}$$

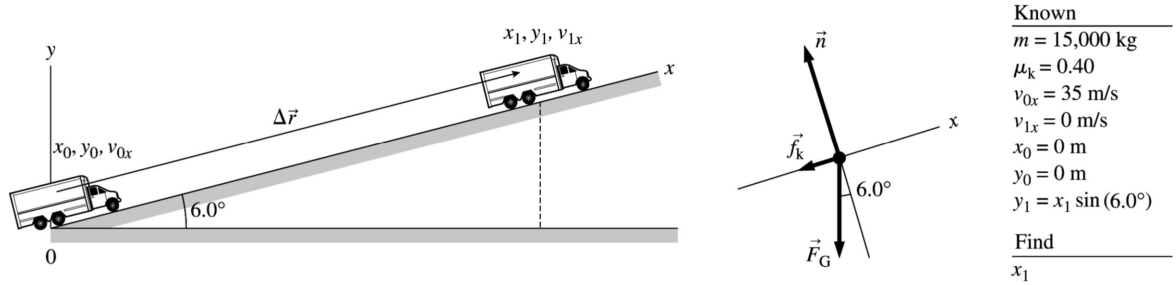
(b) Inserting the given quantities into the expression for the coefficient of kinetic friction gives

$$\mu_k = \frac{v_0^2}{2gd} = \frac{(1.2\text{ m/s})^2}{2(9.8\text{ m/s}^2)(2.0\text{ m})} = 0.037$$

Assess: Friction transforms kinetic energy of the suitcase into thermal energy. In response, the suitcase slows down and comes to rest. Notice that the coefficient of friction does not depend on the mass of the object, which is reasonable.

11.49. Model: Identify the truck and the loose gravel as the system. We need the gravel inside the system because friction increases the temperature of the truck and the gravel. We will also use the model of kinetic friction and the conservation of energy equation.

Visualize:



We place the origin of our coordinate system at the base of the ramp in such a way that the x-axis is along the ramp and the y-axis is vertical so that we can calculate potential energy. The free-body diagram of forces on the truck is shown.

Solve: The conservation of energy equation is $K_1 + U_{g1} + \Delta E_{th} = K_0 + U_{g0} + W_{ext}$. In the present case, $W_{ext} = 0$ J, $v_{1x} = 0$ m/s, $U_{g0} = 0$ J, $v_{0x} = 35$ m/s. The thermal energy created by friction is

$$\Delta E_{th} = f_k(x_1 - x_0) = (\mu_k n)(x_1 - x_0) = \mu_k mg\cos(6.0°)(x_1 - x_0)$$
$$= (0.40)(15{,}000\text{ kg})(9.8\text{ m/s}^2)\cos(6.0°)(x_1 - x_0) = (58{,}478\text{ J/m})(x_1 - x_0)$$

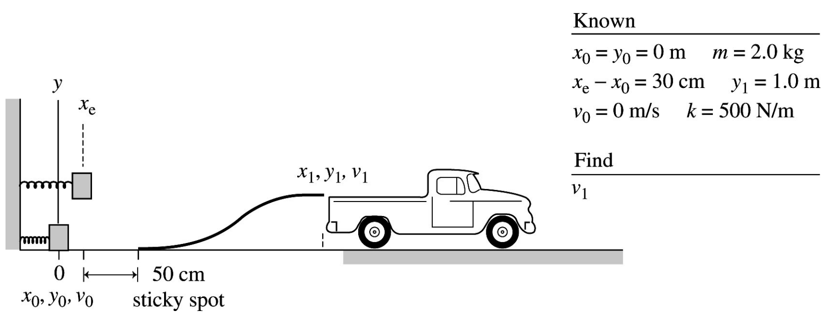
Thus, the energy conservation equation simplifies to

$$0\text{ J} + mgy_1 + (58{,}478\text{ J/m})(x_1 - x_0) = \tfrac{1}{2}mv_{0x}^2 + 0\text{ J} + 0\text{ J}$$
$$(15{,}000\text{ kg})(9.8\text{ m/s}^2)(x_1 - x_0)\sin(6.0°) + (58{,}478\text{ J/m})(x_1 - x_0) = \tfrac{1}{2}(15{,}000\text{ kg})(35\text{ m/s})^2$$
$$(x_1 - x_0) = 124\text{ m} = 0.12\text{ km}$$

Assess: A length of 124 m at a slope of 6° seems reasonable.

11.50. Model: We will use the spring, the package, and the ramp as the system. We will model the package as a particle.

Visualize:



We place the origin of our coordinate system on the end of the spring when it is compressed and is in contact with the package to be shot.

Model: **(a)** The energy conservation equation is

$$K_1 + U_{g1} + U_{s1} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$
$$\tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(x_e - x_e)^2 + \Delta E_{th} = \tfrac{1}{2}mv_0^2 + mgy_0 + \tfrac{1}{2}k(\Delta x)^2 + W_{ext}$$

Using $y_1 = 1.0$ m, $\Delta E_{th} = 0$ J (the frictionless ramp), $v_0 = 0$ m/s, $y_0 = 0$ m, $\Delta x = 30$ cm, and $W_{ext} = 0$ J, we get

$$\tfrac{1}{2}mv_1^2 + mg(1.0\text{ m}) + 0\text{ J} + 0\text{ J} = 0\text{ J} + 0\text{ J} + \tfrac{1}{2}k(0.30\text{ m})^2 + 0\text{ J}$$
$$\tfrac{1}{2}(2.0\text{ kg})v_1^2 + (2.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m}) = \tfrac{1}{2}(500\text{ N/m})(0.30\text{ m})^2$$
$$v_1 = 1.7\text{ m/s}$$

(b) How high can the package go after crossing the sticky spot? If the package can reach $y_1 \geq 1.0$ m before stopping $(v_1 = 0)$, then it makes it. But if $y_1 < 1.0$ m when $v_1 = 0$, the package does not make it. The friction of the sticky spot generates thermal energy

$$\Delta E_{\text{th}} = (\mu_k mg)\Delta x = (0.30)(2.0 \text{ kg})(9.8 \text{ m/s}^2)(0.50 \text{ m}) = 2.94 \text{ J}$$

The energy conservation equation is now

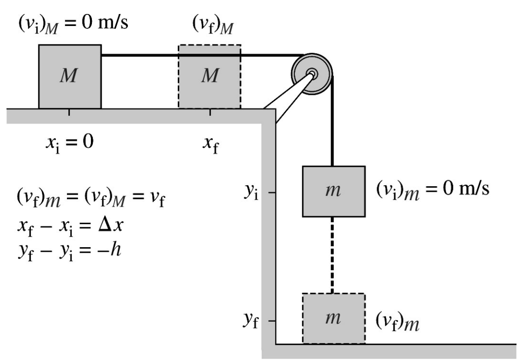
$$\tfrac{1}{2}mv_1^2 + mgy_1 + \Delta E_{\text{th}} = \tfrac{1}{2}k(\Delta x)^2$$

If we set $v_1 = 0$ m/s to find the highest point the package can reach, we get

$$y_1 = \left(\tfrac{1}{2}k\Delta x^2 - \Delta E_{\text{th}}\right)\Big/(mg) = [\tfrac{1}{2}(500 \text{ N/m})(0.30 \text{ m})^2 - 2.94 \text{ J}]/[(2.0 \text{ kg})(9.8 \text{ m/s}^2)] = 0.998 \text{ m}$$

The package does not make it. It just barely misses.

11.51. Model: Model the two blocks as particles. The two blocks make our system.
Visualize:



We place the origin of our coordinate system at the location of the 3.0 kg block.
Solve: (a) The conservation of energy equation gives $K_f + U_{gf} + \Delta E_{\text{th}} = K_i + U_{gi} + W_{\text{ext}}$. The thermal energy is $\Delta E_{\text{th}} = \mu_k Mg\Delta x$ and the external work done is $W_{\text{ext}} = 0$. The initial potential energy is $U_{gi} = my_i$, and the final potential energy is $U_{gf} = my_f$, where we have ignored the gravitational potential energy of block M because its height does not change. The initial and final kinetic energy are $K_i = 0$, and $K_f = \frac{1}{2}(M + m)v_f^2$, respectively. The energy conservation equation thus takes the form

$$\tfrac{1}{2}(m_3 + m_2)v_f^2 + m_3 y_{\text{table}} + m_2 g y_f + \mu_k m_3 g\Delta x = m_3 y_{\text{table}} + m_2 g y_i$$

Note that $\Delta x = -\Delta y = h$ because the blocks are constrained by the cable to move the same distance. Solving for v_f gives

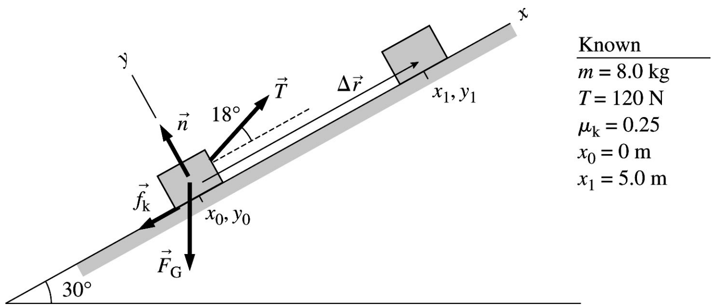
$$v_f = \sqrt{\frac{2g}{M+m}(-m\Delta y - \mu_k M\Delta x)} = \sqrt{\frac{2gh}{M+m}(m - \mu_k M)}$$

(b) If the table is frictionless, this expression takes the form

$$v_f = \sqrt{\frac{2gmh}{M+m}}$$

Assess: It is reasonable that the speed is reduced when friction is present compared with when there is no friction.

11.52. Model: Use the particle model, the definition of work $W = \vec{F} \cdot \Delta\vec{s}$, and the model of kinetic friction.
Visualize: We place the coordinate frame on the incline so that its x-axis is along the incline.



Solve: **(a)** $W_T = \vec{T} \cdot \Delta\vec{r} = T\Delta x\cos(18°) = (120\text{ N})(5.0\text{ m})\cos(18°) = 0.57\text{ kJ}$

$W_g = \vec{F}_G \cdot \Delta\vec{r} = mg\Delta x\cos(120°) = (8.0\text{ kg})(9.8\text{ m/s}^2)(5.0\text{ m})\cos(120°) = -0.20\text{ kJ}$

$W_n = \vec{n} \cdot \Delta\vec{r} = n\Delta x\cos(90°) = 0.0\text{ J}$

(b) The amount of energy transformed into thermal energy is $\Delta E_{th} = f_k\Delta x = \mu_k n\Delta x$.

To find n, we write Newton's second law as follows:

$$\sum F_y = n - F_G\cos(30°) + T\sin(18°) = 0\text{ N} \quad\Rightarrow\quad n = F_G\cos(30°) - T\sin(18°)$$

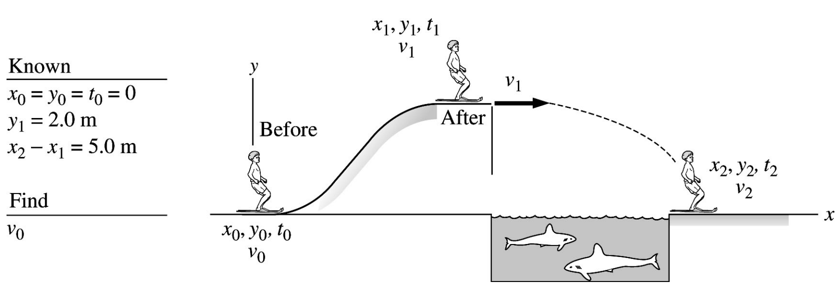
$$n = mg\cos(30°) - T\sin(18°) = (8.0\text{ kg})(9.8\text{ m/s}^2)\cos(30°) - (120\text{ N})\sin(18°) = 30.814\text{ N}$$

Thus, $\Delta E_{th} = (0.25)(30.814\text{ N})(5.0\text{ m}) = 39\text{ J}$.

Assess: Any force that acts perpendicular to the displacement does no work.

11.53. Model: Model the water skier as a particle, apply the law of conservation of mechanical energy, and use the constant-acceleration kinematic equations.

Visualize:



We placed the origin of the coordinate system at the base of the frictionless ramp.

Solve: We'll start by finding the smallest speed v_1 at the top of the ramp that allows her to clear the shark tank. From the vertical motion for jumping the shark tank,

$$y_2 = y_1 + v_{1y}\Delta t + \tfrac{1}{2}a_y\Delta t^2$$

$$0\text{ m} = 2.0\text{ m} + 0\text{ m} + \tfrac{1}{2}(9.8\text{ m/s}^2)\Delta t^2 \quad\Rightarrow\quad \Delta t = 0.639\text{ s}$$

From the horizontal motion,

$$x_2 = x_1 + v_{1x}\Delta t + \tfrac{1}{2}a_x\Delta t^2$$

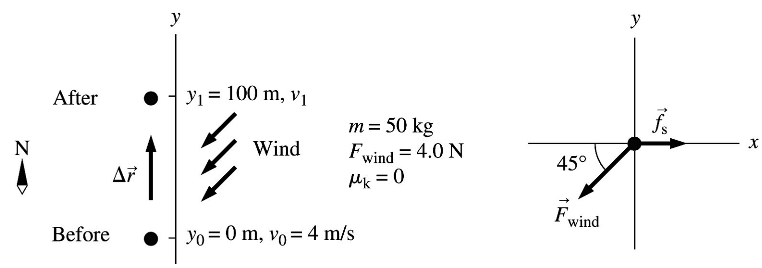
$$(x_1 + 5.0\text{ m}) = x_1 + v_1\Delta t + 0\text{ m} \quad\Rightarrow\quad v_1 = \frac{5.0\text{ m}}{0.639\text{ s}} = 7.825\text{ m/s}$$

Having found the v_1 that will take the skier to the other side of the tank, we now use the energy equation to find the minimum speed v_0. We have

$$K_1 + U_{g1} = K_0 + U_{g0} \quad\Rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_0^2 + mgy_0$$

$$v_0 = \sqrt{v_1^2 + 2g(y_1 - y_0)} = \sqrt{(7.825\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(2.0\text{ m})} = 10\text{ m/s}$$

11.54. Model: Use the particle model for the ice skater, the friction model, and the work–kinetic-energy theorem.
Visualize:



Solve: (a) The work–kinetic-energy theorem gives

$$\Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 = W_{net} = W_{wind}$$

There is no kinetic friction along her direction of motion. Static friction acts to prevent her skates from slipping sideways on the ice, but this force is perpendicular to the motion and does not contribute to a change in thermal energy. The angle between $\vec{F}_{wind}$ and $\Delta\vec{r}$ is $\theta = 135°$, so

$$W_{wind} = \vec{F}_{wind} \cdot \Delta\vec{r} = F_{wind}\Delta y\cos(135°) = (4.0\text{ N})(100\text{ m})\cos(135°) = -282.8\text{ J}$$

Thus, her final speed is

$$v_1 = \sqrt{v_0^2 + \frac{2W_{wind}}{m}} = 2.2\text{ m/s}$$

(b) If the skates don't slip, she has no acceleration in the x-direction and so $(F_{net})_x = 0$ N. That is:

$$f_s - F_{wind}\cos(45°) = 0\text{ N} \quad\Rightarrow\quad f_s = F_{wind}\cos(45°) = 2.83\text{ N}$$

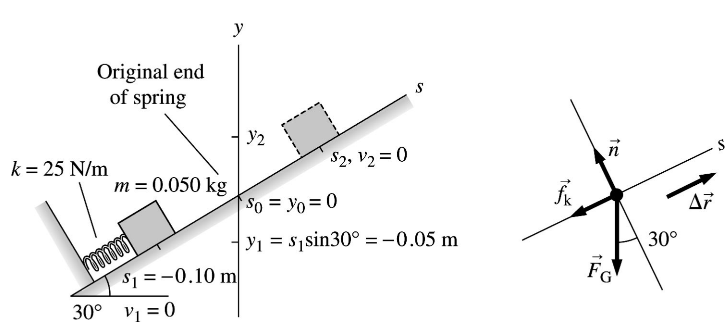
Now there is an upper limit to the static friction: $f_s \le (f_s)_{max} = \mu_s mg$. To not slip requires

$$\mu_s \ge \frac{f_s}{mg} = \frac{2.83\text{ N}}{(50\text{ kg})(9.8\text{ m/s}^2)} = 0.0058$$

Thus, the minimum value of μ_s is 0.0058.

Assess: The work done by the wind on the ice skater is negative, because the wind slows the skater down.

11.55. Model: Model the ice cube as a particle, the spring as an ideal that obeys Hooke's law, and the law of conservation of energy.
Visualize:



Solve: (a) The normal force does no work and the slope is frictionless, so mechanical energy is conserved. We've drawn two separate axes: a vertical y-axis to measure potential energy and a tilted s-axis to measure distance along the slope. Both have the same origin which is at the point where the spring is not compressed. Thus, the two axes are related by $y = s\sin\theta$. Also, this choice of origin makes the elastic potential energy simply $U_s = \frac{1}{2}k(s - s_0)^2 = \frac{1}{2}ks^2$. Because energy is conserved, we can relate the initial point—with the spring compressed—to the final point where the ice cube is at maximum height. We do *not* need to find the speed with which it leaves the spring. We have

$$K_2 + U_{g2} + U_{s2} = K_1 + U_{g1} + U_{s1}$$

$$\frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}ks_0^2 = \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}ks_1^2$$

It is important to note that at the final point, when the ice cube is at y_2, the end of the spring is only at s_0. The spring does *not* stretch to s_2, so U_{s2} is *not* $\frac{1}{2}ks_2^2$. Three of the terms are zero, leaving

$$mgy_2 = +mgy_1 + \frac{1}{2}ks_1^2 \quad \Rightarrow \quad y_2 - y_1 = \Delta y = \text{height gained} = \frac{ks_1^2}{2mg} = 0.255 \text{ m} = 25.5 \text{ cm}$$

The distance traveled is $\Delta s = \Delta y/\sin(30°) = 0.51$ m.

(b) Using the energy equation and the expression for thermal energy:

$$K_2 + U_{g2} + U_{s2} + \Delta E_{th} = K_1 + U_{g1} + U_{s1} + W_{ext},\ \Delta E_{th} = f_k\Delta s = \mu_k n\Delta s$$

From the free-body diagram,

$$(\vec{F}_{net})_y = 0 \text{ N} = n - mg\cos(30°) \quad \Rightarrow \quad n = mg\cos(30°)$$

Now, having found $\Delta E_{th} = \mu_k mg\cos(30°)\Delta s$, the energy equation can be written

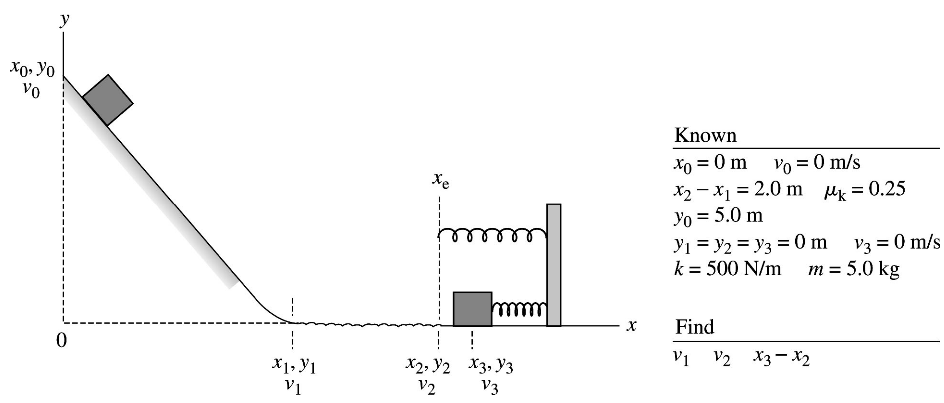
$$0 \text{ J} + mgy_2 + 0 \text{ J} + \mu_k mg\cos(30°)\Delta s = 0 \text{ J} + mgy_1 + \frac{1}{2}ks_1^2 + 0 \text{ J}$$

$$mg(y_2 - y_1) - \frac{1}{2}ks_1^2 + \mu_k mg\cos(30°)\Delta s = 0$$

Using $\Delta y = \Delta s\sin(30°)$, the above equation simplifies to

$$mg\Delta s\sin(30°) + \mu_k mg\cos(30°)\Delta s = \frac{1}{2}ks_1^2 \quad \Rightarrow \quad \Delta s = \frac{ks_1^2}{2mg[\sin(30°) + \mu_k\cos(30°)]} = 0.38 \text{ m}$$

11.56. Model: Assume an ideal spring, so Hooke's law is obeyed. Treat the box as a particle and apply the energy conservation law. Box, spring, and the ground make our system, and we also use the model of kinetic friction.
Visualize: We place the origin of the coordinate system on the ground directly below the box's starting position.



Solve: (a) The energy conservation equation gives

$$K_1 + U_{g1} + U_{s1} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\frac{1}{2}mv_1^2 + mgy_1 + 0 \text{ J} + 0 \text{ J} = \frac{1}{2}mv_0^2 + mgy_0 + 0 \text{ J} + 0 \text{ J} \quad \Rightarrow \quad \frac{1}{2}mv_1^2 + 0 \text{ J} = 0 \text{ J} + mgy_0$$

$$v_1 = \sqrt{2gy_0} = \sqrt{2(9.8 \text{ m/s}^2)(5.0 \text{ m})} = 9.9 \text{ m/s}$$

(b) The friction creates thermal energy. The energy conservation equation for this part of the problem is

$$K_2 + U_{g2} + U_{s2} + \Delta E_{th} = K_1 + U_{g1} + U_{s1} + W_{ext},\ \tfrac{1}{2}mv_2^2 + 0\text{ J} + 0\text{ J} + \mu_k mg(x_2 - x_1) = \tfrac{1}{2}mv_1^2 + 0\text{ J} + 0\text{ J} + 0\text{ J}$$

$$\tfrac{1}{2}mv_2^2 + \mu_k n(x_2 - x_1) = \tfrac{1}{2}mv_1^2 \quad \Rightarrow \quad \tfrac{1}{2}mv_2^2 + \mu_k mg(x_2 - x_1) = \tfrac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2\mu_k g(x_2 - x_1)} = \sqrt{(9.9\text{ m/s})^2 - 2(0.25)(9.8\text{ m/s}^2)(2.0\text{ m})} = 9.4\text{ m/s}$$

(c) To find how much the spring is compressed, we apply the energy conservation once again:

$$K_3 + U_{g3} + U_{s3} + \Delta E_{th} = K_2 + U_{g2} + U_{s2} + W_{ext},\ 0\text{ J} + 0\text{ J} + \frac{1}{2}k(x_3 - x_2)^2 + 0\text{ J} = \frac{1}{2}mv_2^2 + 0\text{ J} + 0\text{ J} + 0\text{ J}$$

Using $v_2 = 9.4$ m/s, $k = 500$ N/m, and $m = 5.0$ kg, the above equation yields $(x_3 - x_2) = \Delta x = 0.94$ m.

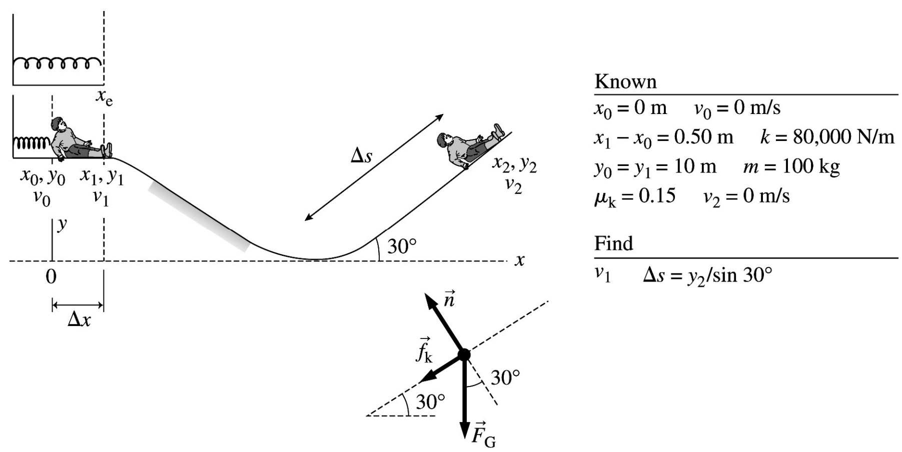
(d) The initial energy $= mgy_0 = (5.0\text{ kg})(9.8\text{ m/s}^2)(5.0\text{ m}) = 254$ J. The energy transformed to thermal energy during each passage is

$$\mu_k mg(x_2 - x_1) = (0.25)(5.0\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m}) = 24.5\text{ J}$$

The number of passages is equal to 245 J/24.5 J or 10.

11.57. Model: Assume an ideal spring, so Hooke's law is obeyed. Treat the physics student as a particle and apply the law of conservation of energy. Our system is comprised of the spring, the student, and the ground. We also use the model of kinetic friction.

Visualize: We place the origin of the coordinate system on the ground directly below the end of the compressed spring that is in contact with the student.



Solve: **(a)** The energy conservation equation gives

$$K_1 + U_{g1} + U_{s1} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(x_1 - x_e)^2 + 0\text{ J} = \tfrac{1}{2}mv_0^2 + mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2 + 0\text{ J}$$

Since $y_1 = y_0 = 10$ m, $x_1 = x_e$, $v_0 = 0$ m/s, $k = 80{,}000$ N/m, $m = 100$ kg, and $(x_1 - x_0) = 0.5$ m,

$$\tfrac{1}{2}mv_1^2 = \tfrac{1}{2}k(x_1 - x_0)^2 \quad \Rightarrow \quad v_1 = \sqrt{\frac{k}{m}}(x_1 - x_0) = \sqrt{\frac{80{,}000\text{ N}/m}{100\text{ kg}}}(0.50\text{ m}) = 14\text{ m/s}$$

(b) Friction creates thermal energy. Applying the conservation of energy equation once again:

$$K_2 + U_{g2} + U_{s2} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\tfrac{1}{2}mv_2^2 + mgy_2 + 0\text{ J} + f_k\Delta s = 0\text{ J} + mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2 + 0\text{ J}$$

With $v_2 = 0$ m/s and $y_2 = \Delta s \sin(30°)$, the above equation is simplified to

$$mg\Delta s \sin(30°) + \mu_k n\Delta s = mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2$$

From the free-body diagram for the physics student, we see that $n = F_G \cos(30°) = mg\cos(30°)$. Thus, the conservation of energy equation gives

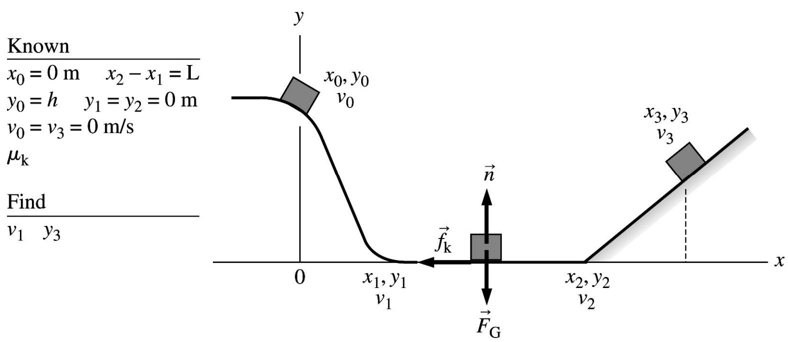
$$\Delta s[mg\sin(30°) + \mu_k mg\cos(30°)] = mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2$$

Using $m = 100$ kg, $k = 80{,}000$ N/m, $(x_1 - x_0) = 0.50$ m, $y_0 = 10$ m, and $\mu_k = 0.15$, we get

$$\Delta s = \frac{mgy_0 + \frac{1}{2}k(x_1 - x_0)^2}{mg[\sin(30°) + \mu_k\cos(30°)]} = 32 \text{ m}$$

Assess: $y_2 = \Delta s\sin(30°) = 16$ m, which is greater than $y_0 = 10$ m. The higher value is due to the transformation of the spring energy into gravitational potential energy.

11.58. Model: Treat the block as a particle, use the model of kinetic friction, and apply the energy conservation law. The block and the incline comprise our system.
Visualize: We place the origin of the coordinate system directly below the block's starting position at the same level as the horizontal surface. On the horizontal surface the model of kinetic friction applies.



Solve: (a) For the first incline, the conservation of energy equation gives

$$K_1 + U_{g1} + \Delta E_{th} = K_0 + U_{g0} + W_{ext}, \quad \tfrac{1}{2}mv_1^2 + 0\text{ J} + 0\text{ J} = 0\text{ J} + mgy_0 + 0\text{ J} \quad \Rightarrow \quad v_1 = \sqrt{2gy_0} = \sqrt{2gh}$$

(b) The friction creates thermal energy. Applying once again the conservation of energy equation, we have

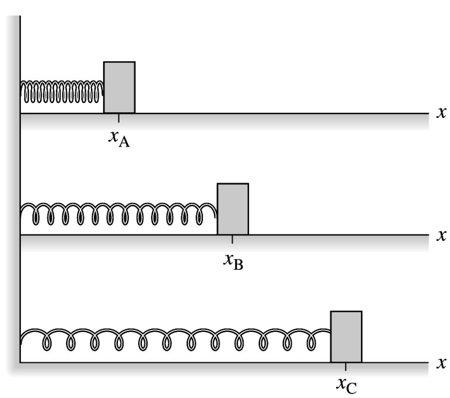
$$K_3 + U_{g3} + \Delta E_{th} = K_1 + U_{g1} + W_{ext}, \quad \tfrac{1}{2}mv_3^2 + mgy_3 + \mu_k mg(x_2 - x_1) = \tfrac{1}{2}mv_1^2 + mgy_1 + W_{ext}$$

Using $v_3 = 0$ m/s, $y_1 = 0$ m, $W_{ext} = 0$ J, $v_1 = \sqrt{2gh}$, and $(x_2 - x_1) = L$, we get

$$mgy_3 + \mu_k mgL = \tfrac{1}{2}m(2gh) \quad \Rightarrow \quad y_3 = h - \mu_k L$$

Assess: For $\mu_k = 0$, $y_3 = h$ which is predicted by the law of the conservation of energy.

11.59. Model: Assume an ideal spring, so Hooke's law is obeyed.
Visualize:



Solve: For a conservative force the work done on a particle as it moves from an initial to a final position is independent of the path. We will show that $W_{A\to C\to B} = W_{A\to B}$ for the spring force. Work done by a spring force $F = -kx$ is given by

$$W = \int F dx = -\int_{x_i}^{x_f} kx\, dx$$

This means

$$W_{A\to B} = -\int_{x_A}^{x_B} kx\, dx = -\frac{k}{2}(x_B^2 - x_A^2),\; W_{A\to C} = -\int_{x_A}^{x_C} kx\, dx = -\frac{k}{2}(x_C^2 - x_A^2), \text{ and } W_{C\to B} = -\int_{x_C}^{x_B} kx\, dx = -\frac{k}{2}(x_B^2 - x_C^2)$$

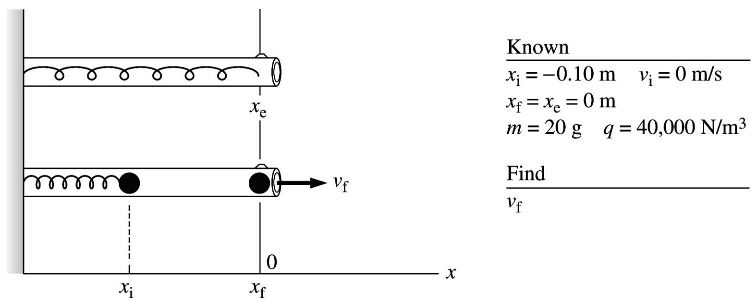
Adding the last two:

$$W_{A\to C\to B} = W_{A\to C} + W_{C\to B} = -\frac{k}{2}(x_C^2 - x_A^2 + x_B^2 - x_C^2) = W_{A\to B}$$

Assess: Because the paths are arbitrary, we have shown that the work done on a particle is independent of the path, so the spring force is conservative.

11.60. Model: A "sprong" obeys the force law $F_x = -q(x - x_e)^3$, where q is the sprong constant and x_e is the equilibrium position.

Visualize: We place the origin of the coordinate system on the free end of the sprong, that is, $x_e = x_f = 0$ m.



Solve: **(a)** The units of q are N/m^3.

(b) Since $F_x = -dU/dx$, we have $U(x) = -\int F_x dx = -\int_0^x (-qx^3)dx = qx^4/4$.

(c) Applying the energy conservation equation to the ball and sprong system gives

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}mv_f^2 + 0 \text{ J} = 0 \text{ J} + \tfrac{1}{4}qx_i^4$$

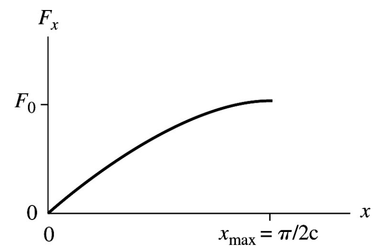
$$v_f = \sqrt{\frac{qx^4}{2m}} = \sqrt{\frac{(40{,}000 \text{ N/m}^3)(-0.10 \text{ m})^4}{2(0.020 \text{ kg})}} = 10 \text{ m/s}$$

11.61. Solve: **(a)** Because sin (cx) is dimensionless, F_0 must have units of force in newtons.

(b) The product cx is an angle because we are taking the sine of it. An angle is dimensionless. If x has units of m and the product cx is dimensionless, then c has to have units of m^{-1}.

(c) The force is a maximum when $\sin(cx) = 1$. This occurs when $cx = \pi/2$, or for $x_{max} = \pi/(2c)$.

(d) The graph is the first quarter of a sine curve.



(e) We can find the velocity v_f at $x_f = x_{max}$ from the work–kinetic-energy theorem:

$$\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2 = W \quad \Rightarrow \quad v_f = \sqrt{v_0^2 + \frac{2W}{m}}$$

This is a variable force. As the particle moves from $x_i = 0$ m to $x_f = x_{max} = \pi/(2c)$ the work done on it is

$$W = \int_{x_i}^{x_f} F(x)dx = F_0 \int_{x_i}^{x_f} \sin(cx)dx = -\frac{F_0}{c}\cos(cx)\Big|_0^{\pi/(2c)} = -\frac{F_0}{c}\left[\cos(\pi/2) - \cos(0)\right] = \frac{F_0}{c}$$

Thus, the particle's speed at $x_f = x_{max} = \pi/(2c)$ is $v_f = \sqrt{v_0^2 + 2F_0/(mc)}$.

11.62. Solve: The average power output during the push-off period is equal to the work done by the cat divided by the time the cat applied the force. Since the force on the floor by the cat is equal in magnitude to the force on the cat by the floor, work done by the cat can be found using the work–kinetic-energy theorem during the push-off period; $W_{net} = W_{floor} = \Delta K$. We do not need to explicitly calculate W_{cat}, since we know that the cat's kinetic energy is transformed into its potential energy during the leap. That is,

$$\Delta U_g = mg(y_2 - y_1) = (5.0 \text{ kg})(9.8 \text{ m/s}^2)(0.95 \text{ m}) = 46.55 \text{ J}$$

Thus, the average power output during the push-off period is

$$P = \frac{W_{net}}{t} = \frac{46.55 \text{ J}}{0.20 \text{ s}} = 0.23 \text{ kW}$$

11.63. Model: The heart provides the pressure to move blood through the body and therefore does work on the blood. We assume all the work goes into pushing the blood through the body.

Solve: (a) Using the hint, $W = PAd = PV = (1.3\times10^4 \text{ N/m}^2)(6.0\times10^{-3} \text{ m}^3) = 78 \text{ J}$ (in this equation, P represents pressure, not power).

(b) Using *P* to represent power now, we can calculate the average power output of the heart as follows:

$$\overline{P} = \frac{W}{\Delta t} = \frac{78 \text{ J}}{60 \text{ s}} = 1.3 \text{ W}$$

Assess: This power is much less than that of an ordinary lightbulb (≈ 75 W).

11.64. Model: We will ignore rolling friction because it is much less than the drag force (and becaue we are not given the mass of the bicyclist + bicycle). Therefore, model the system as a particle with the given cross-sectional area and that is moving through the air at the given speed.

Solve: (a) From Eq. 6.16, we know that the drag force *D* has the magnitude

$$D = \tfrac{1}{2}C\rho Av^2$$

where $C = 0.90$, $A = 0.45 \text{ m}^2$, and $\rho = 1.2 \text{ kg/m}^3$ (the density of air). To overcome this force, the cyclist must generate the force $\vec{F} = -\vec{D}$, or a power $P = \vec{F}\cdot\vec{v} = Dv$, where the last equality follows because the drag force acts in parallel to the velocity. Thus, the power isi

$$P = \tfrac{1}{2}C\rho Av^3 = \tfrac{1}{2}(0.90)(1.2 \text{ kg/m}^3)(0.45 \text{ m}^2)(7.3 \text{ m/s})^3 = 95 \text{ W}$$

(b) The metabolic power output P_M is $P_M = P/0.25 = 3.8\times10^2$ W.

(c) The number of food calories c burned riding for one hour is

$$c = (378\text{ W})\left(\frac{1\text{ cal}}{4190\text{ J}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)(1\text{ h}) = 3.2\times10^2\text{ cal}$$

Assess: The 320 cal in part (c) is about what you would get from drinking two 8-oz glasses of whole milk.

11.65. Solve: (a) The change in the potential energy of 1.0 kg of water in falling 25 m is

$$\Delta U_g = -mgh = -(1.0\text{ kg})(9.8\text{ m/s}^2)(25\text{ m}) = -245\text{ J} \approx -0.25\text{ kJ}$$

(b) The power required of the dam is

$$P = \frac{W}{t} = \frac{W}{1\text{ s}} = 50\times10^6\text{ Watts} \Rightarrow W = 50\times10^6\text{ J}$$

That is, 50×10^6 J of energy is required per second for the dam. Out of the 245 J of lost potential energy, $(245\text{ J})(0.80) = 196\text{ J}$ is converted to electrical energy. Thus, the amount of water needed per second is $(50\times10^6\text{ J})(1.0\text{ kg}/196\text{ J}) = 255{,}000\text{ kg} \approx 2.6\times10^5\text{ kg}$.

11.66. Solve: The force required to tow a water skier at a speed v is $F_{tow} = Av$. Since power $P = Fv$, the power required to tow the water skier is $P_{tow} = F_{tow}v = Av^2$. We can find the constant A by noting that a speed of $v = 2.5$ mph requires a power of 2 hp. Thus,

$$2\text{ hp} = A(2.5\text{ mph})^2 \Rightarrow A = 0.32\frac{\text{hp}}{\text{mph}^2}$$

Now, the power required to tow a water skier at 7.5 mph is

$$P_{tow} = Av^2 = 0.32\frac{\text{hp}}{\text{mph}^2}(7.5\text{ mph})^2 = 18\text{ hp}$$

Assess: Since $P \propto v^2$, a three-fold increase in velocity leads to a nine-fold increase in power.

11.67. Model: Use the model of static friction, kinematic equations, and the definition of power.
Solve: (a) The rated power of the Porsche is $217\text{ hp} = 161{,}882\text{ W}$ and the gravitational force on the car is $(1480\text{ kg})(9.8\text{ m/s}^2) = 14{,}504\text{ N}$. The amount of that force on the drive wheels is $(14{,}504)(2/3) = 9670\text{ N}$. Because the static friction of the tires on road pushes the car forward,

$$F_{max} = f_{s,max} = \mu_s n = \mu_s mg = (1.00)(9670\text{ N}) = ma_{max}$$

$$a_{max} = \frac{9670\text{ N}}{1480\text{ kg}} = 6.53\text{ m/s}^2$$

(b) Only 70% of the power generated by the motor is applied at the wheels.

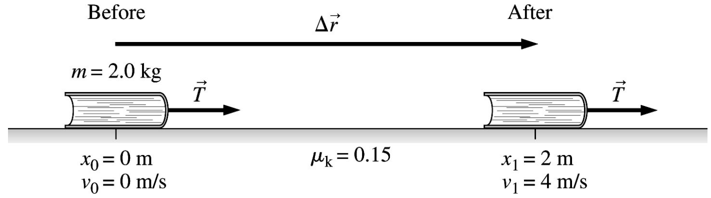
$$P = Fv_{max} \Rightarrow v_{max} = \frac{P}{F} = \frac{(0.70)(161{,}882\text{ W})}{9670\text{ N}} = 11.7\text{ m/s}$$

(c) Using the kinematic equation, $v_{max} = v_0 + a_{max}(t_{min} - t_0)$ with $v_0 = 0$ m/s and $t_0 = 0$ s, we obtain

$$t_{min} = \frac{v_{max}}{a_{max}} = \frac{11.7\text{ m/s}}{6.53\text{ m/s}^2} = 1.79\text{ s}$$

Assess: An acceleration time of 1.79 s for the Porsche to reach a speed of ≈ 26 mph from rest is reasonable.

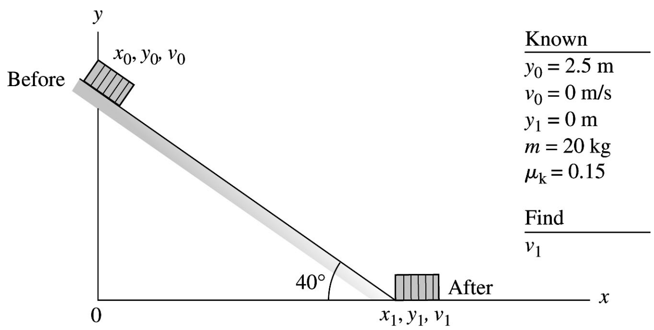
11.68. Solve: (a) A student uses a string to pull her 2.0 kg physics book, starting from rest, across a 2.0-m-long lab bench. The coefficient of kinetic friction between the book and the lab bench is 0.15. If the book's final speed is 4.0 m/s, what is the tension in the string?
(b)



(c) The tension does external work W_{ext}. This work increases the book's kinetic energy and also causes an increase ΔE_{th} in the thermal energy of the book and the lab bench. Solving the equation gives $T = 11$ N.

11.69. **(a)** A 20 kg chicken crate slides down a 2.5-m-high, 40° ramp from the back of a truck to the ground. The coefficient of kinetic friction between the crate and the ramp bench is 0.15. How fast are the chickens going at the bottom of the ramp?

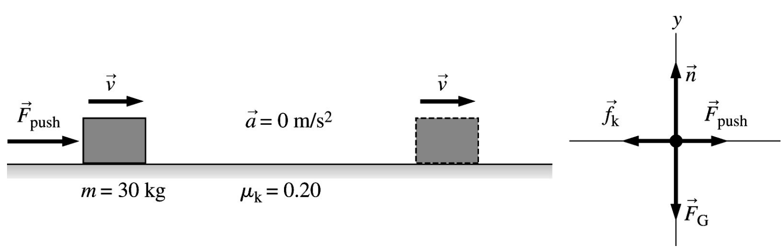
(b)



(c) $v_1 = 6.3$ m/s.

11.70. **(a)** If you expend 75 W of power to push a 30 kg sled on a surface where the coefficient of kinetic friction between the sled and the surface is $\mu_k = 0.20$, what speed will you be able to maintain?

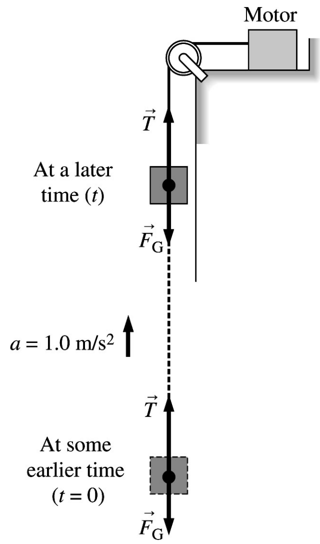
(b)



(c) $F_{\text{push}} = (0.20)(30 \text{ kg})(9.8 \text{ m/s}^2) = 58.8 \text{ N} \quad \Rightarrow \quad 75 \text{ W} = (58.8 \text{ N})v \quad \Rightarrow \quad v = \dfrac{75 \text{ W}}{58.8 \text{ N}} = 1.3 \text{ m/s}$

11.71. **(a)** A 1500 kg object is being accelerated upward at 1.0 m/s^2 by a rope. How much power must the motor supply at the instant when the velocity is 2.0 m/s?

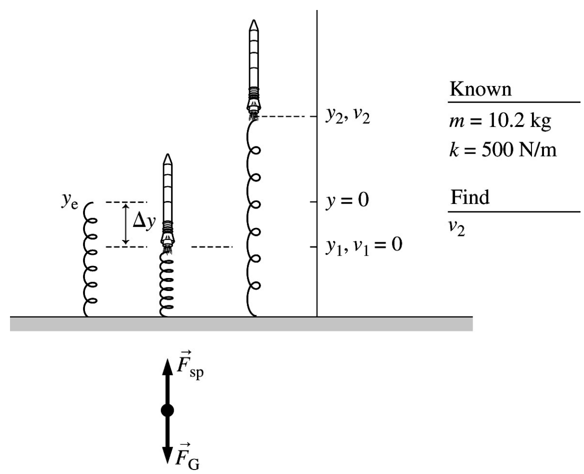
(b)



(c) $T = (1500\text{ kg})(9.8\text{ m/s}^2) + 1500\text{ kg}(1.0\text{ m/s}^2) = 16{,}200\text{ N} = 16.2\text{ kN}$

$P = T(2\text{ m/s}) = (16{,}200\text{ N})(2.0\text{ m/s}) = 32{,}400\text{ W} = 32\text{ kW}$

11.72. Model: Assume the spring is ideal so that Hooke's law is obeyed, and model the weather rocket as a particle.
Visualize:



The origin of the coordinate system is placed on the free end of the spring. Note that the bottom of the spring is anchored to the ground.

Solve: (a) The rocket is initially at rest. The free-body diagram on the rocket helps us write Newton's second law as

$$\left(\sum F_y\right) = 0\text{ N} \quad \Rightarrow \quad F_{sp} = F_G = mg \quad \Rightarrow \quad k\Delta y = mg$$

$$\Delta y = \frac{mg}{k} = \frac{(10.2\text{ kg})(9.81\text{ m/s}^2)}{(500\text{ N/m})} = 20.0\text{ cm}$$

(b) The thrust does work. Using the energy conservation equation when $y_2 - y_e = 40\text{ cm} = 0.40\text{ m}$:

$$K_2 + U_{g2} + U_{sp2} = K_1 + U_{g1} + U_{sp1} + W_{ext}$$

$$W_{ext} = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}k(y_2 - y_e)^2 = \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(y_1 - y_e)^2 + (200\text{ N})(0.60\text{ m})$$

$$(5.10\text{ kg})v_2^2 + 40.0\text{ J} + 40.0\text{ J} = 0 - 20.0\text{ J} + 10.0\text{ J} + 120\text{ J} \quad \Rightarrow \quad v_2 = 2.43\text{ m/s}$$

If the rocket were not attached to the spring, the energy conservation equation would not involve the spring energy term U_{sp2}. That is,

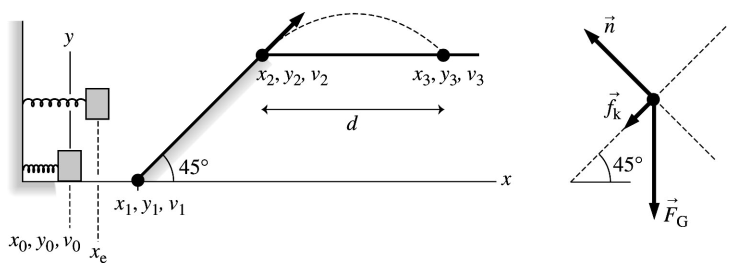
$$K_2 + U_{g2} = K_1 + U_{g1} + U_{sp1} + W_{ext}$$

$$\tfrac{1}{2}(10.2\text{ kg})v_2^2 + (10.2\text{ kg})(9.81\text{ m/s}^2)(0.40\text{ m}) = 0\text{ J} - (10.2\text{ kg})(9.81\text{ m/s}^2)(0.20\text{ m})$$
$$+\tfrac{1}{2}(500\text{ N/m})(0.20\text{ m})^2 + (200\text{ N})(0.60\text{ m})$$

$$(5.10\text{ kg})v_2^2 = 70.0\text{ J} \quad\Rightarrow\quad v_2 = 3.70\text{ m/s}$$

Assess: (a) The rocket has greater speed at y_2 when it is not attached to the spring because, as the spring extends, it contributes a downward force to the rocket.

11.73. Model: Assume the spring to be ideal that obeys Hooke's law, and model the block as a particle.
Visualize: We place the origin of the coordinate system on the free end of the compressed spring which is in contact with the block. Because the horizontal surface at the bottom of the ramp is frictionless, the spring energy appears as kinetic energy of the block until the block begins to climb up the incline.



Solve: Although we could find the speed v_1 of the block as it leaves the spring, we don't need to. We can use energy conservation to relate the initial potential energy of the spring to the energy of the block as it begins projectile motion at point 2. However, friction requires us to calculate the increase in thermal energy. The energy equation is

$$K_2 + U_{g2} + \Delta E_{th} = K_0 + U_{g0} + W_{ext} \quad\Rightarrow\quad \tfrac{1}{2}mv_2^2 + mgy_2 + f_k\Delta s = \tfrac{1}{2}k(x_0 - x_e)^2$$

The distance along the slope is $\Delta s = y_2/\sin(45°)$. The friction force is $f_k = \mu_k n$, and we can see from the free-body diagram that $n = mg\cos(45°)$. Thus

$$v_2 = \sqrt{\frac{k}{m}(x_0 - x_e)^2 - 2gy_2 - 2\mu_k g y_2 \cot(45°)}$$

$$= \left[\frac{1000\text{ N/m}}{0.20\text{ kg}}(0.15\text{ m})^2 - 2(9.8\text{ m/s}^2)(2.0\text{ m}) - 2(0.20)(9.8\text{ m/s}^2)(2.0\text{ m})\cot(45°)\right]^{1/2} = 8.091\text{ m/s}$$

Having found the velocity v_2, we can now find $(x_3 - x_2) = d$ using the kinematic equations of projectile motion:

$$y_3 = y_2 + v_{2y}(t_3 - t_2) + \frac{1}{2}a_{2y}(t_3 - t_2)^2$$

$$2.0\text{ m} = 2.0\text{ m} + v_2\sin(45°)(t_3 - t_2) + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t_3 - t_2)^2$$

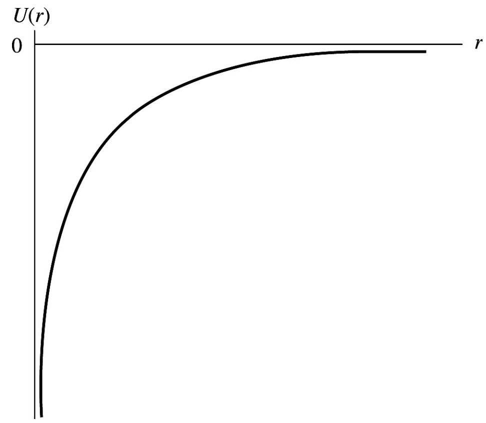
$$t_3 - t_2 = 0\text{ s and } 1.168\text{ s}$$

Finally,

$$x_3 = x_2 + v_{2x}(t_3 - t_2) + \frac{1}{2}a_{2x}(t_3 - t_2)^2$$

$$d = (x_3 - x_2) = v_2\cos(45°)(1.168\text{ s}) + 0\text{ m} = 6.7\text{ m}$$

11.74. Solve: (a)



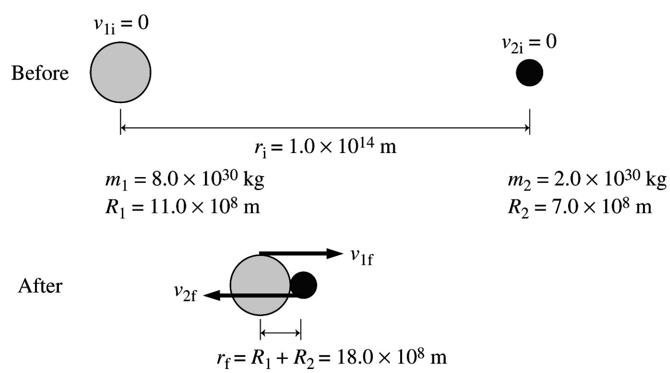
The graph is a hyperbola.

(b) The separation for zero potential energy is $r = \infty$, since

$$U = -\frac{Gm_1m_2}{r} \rightarrow 0 \text{ J as } r \rightarrow \infty$$

This makes sense because two masses don't interact at all if they are infinitely far apart.

(c) Due to the absence of nonconservative forces in our system of two particles, the mechanical energy is conserved.



The equations of energy and momentum conservation are

$$K_f + U_{gf} = K_i + U_{gi} \quad \tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 + \left(-\frac{Gm_1m_2}{r_f}\right) = \tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 + \left(-\frac{Gm_1m_2}{r_i}\right)$$

$$\tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 = Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$p_f = p_i \quad \Rightarrow \quad m_1v_{1f} + m_2v_{2f} = 0 \text{ kg m/s} \quad \Rightarrow \quad v_{2f} = -\frac{m_1}{m_2}v_{1f}$$

Substituting this expression for v_{2f} into the energy equation, we get

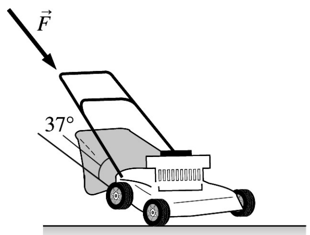
$$\tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2\left(\frac{m_1}{m_2}v_{1f}\right)^2 = Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right) \Rightarrow v_{1f}^2 = \frac{2Gm_2}{(1 + m_1/m_2)}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

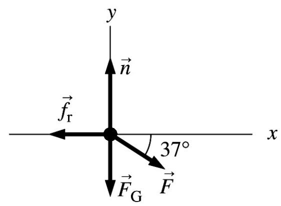
With $G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m(kg)}^{-2}$, $r_f = R_1 + R_2 = 18 \times 10^8$ m, $r_i = 1.0 \times 10^{14}$ m, $m_1 = 8.0 \times 10^{30}$ kg, and $m_2 = 2.0 \times 10^{30}$ kg, the above equation can be simplified to yield

$$v_{1f} = 1.72 \times 10^5 \text{ m/s, and } v_{2f} = -\frac{m_1}{m_2}v_{1f} = \left(\frac{8.0 \times 10^{30}\text{ kg}}{2.0 \times 10^{30}\text{ kg}}\right)(1.72 \times 10^5 \text{ m/s}) = 6.89 \times 10^5 \text{ m/s}$$

The speed of the heavier star is 1.7×10^5 m/s. That of the lighter star is 6.9×10^5 m/s.

11.75. Model: Model the lawnmower as a particle and use the model of kinetic friction.
Visualize:





We placed the origin of our coordinate system on the lawnmower and drew the free-body diagram of forces.
Solve: The normal force $\vec{n}$, which is related to the frictional force, is not equal to $\vec{F}_G$. This is due to the presence of $\vec{F}$. The rolling friction is $f_r = \mu_r n$, or $n = f_r/\mu_r$. The lawnmower moves at constant velocity, so $\vec{F}_{net} = \vec{0}$. The two components of Newton's second law are

$$(\sum F_y) = n - F_G - F\sin(37°) = ma_y = 0\text{ N} \Rightarrow f_r/\mu_r - mg - F\sin(37°) = 0\text{ N} \quad \Rightarrow \quad f_r = \mu_r mg + \mu_r F\sin 37°$$

$$(\sum F_x) = F\cos(37°) - f_r = 0\text{ N} \quad \Rightarrow \quad F\cos(37°) - \mu_r mg - \mu_r F\sin(37°) = 0\text{ N}$$

$$F = \frac{\mu_r mg}{\cos(37°) - \mu_r\sin(37°)} = \frac{(0.15)(12\text{ kg})(9.8\text{ m/s}^2)}{0.7986 - (0.15)(0.6018)} = 24.9\text{ N}$$

Thus, the power supplied by the gardener in pushing the lawnmower at a constant speed of 1.2 m/s is $P = \vec{F}\cdot\vec{v} = Fv\cos\theta = (24.9\text{ N})(1.2\text{ m/s})\cos(37°) = 24\text{ W}$.