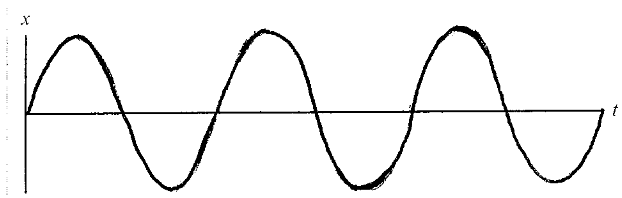
14 Oscillations

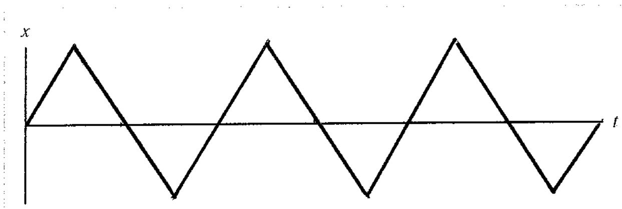
14.1 Equilibrium and Oscillation

14.2 Linear Restoring Forces and Simple Harmonic Motion

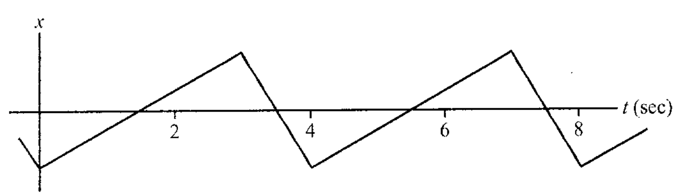
1. On the axes below, sketch three cycles of the position-versus-time graph for:
   a. A particle undergoing simple harmonic motion.

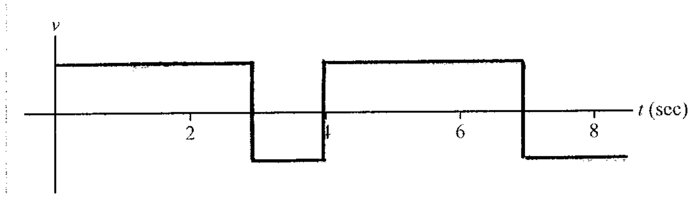


   b. A particle undergoing periodic motion that is not simple harmonic motion.



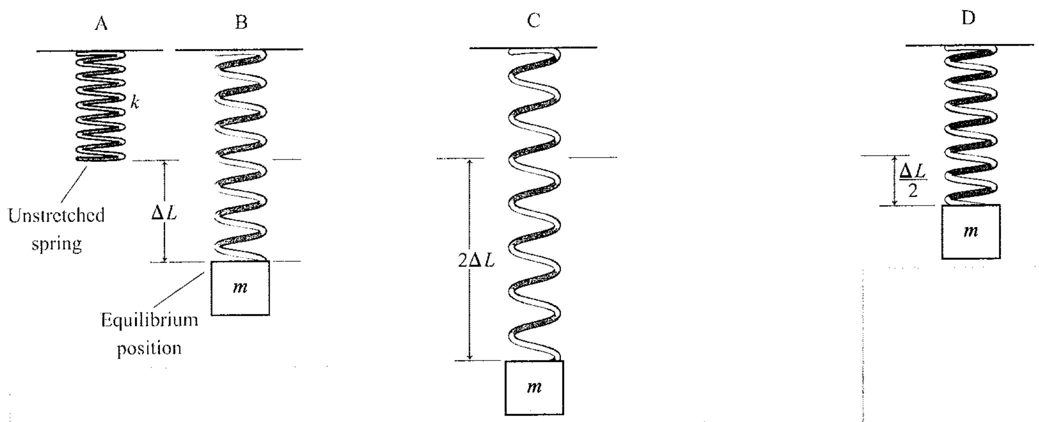
2. Consider the particle whose motion is represented by the x-versus-t graph below.

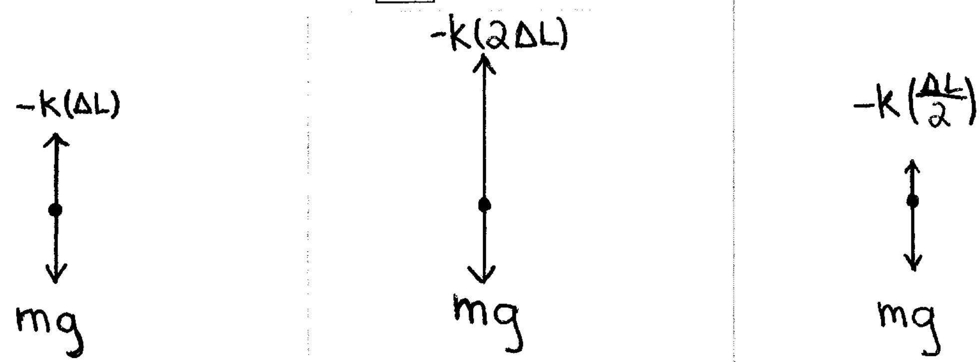




   a. Is this periodic motion? yes
   b. Is this motion SHM? no
   c. What is the period? 4 sec
   d. What is the frequency? $f = \frac{1}{T} = 0.25\,\text{Hz}$
   e. You learned in Chapter 2 to relate velocity graphs to position graphs. Use that knowledge to draw the particle's velocity-versus-time graph on the axes provided.

3. A mass hung vertically on the spring shown in A stretches the spring by ΔL as shown in B, which is in its equilibrium position. Draw a free-body diagram for the same mass-spring system shown in locations B, C (in which the spring is stretched by $2\Delta L$), and D (in which the spring is stretched by $\Delta L/2$).





14.3 Describing Simple Harmonic Motion

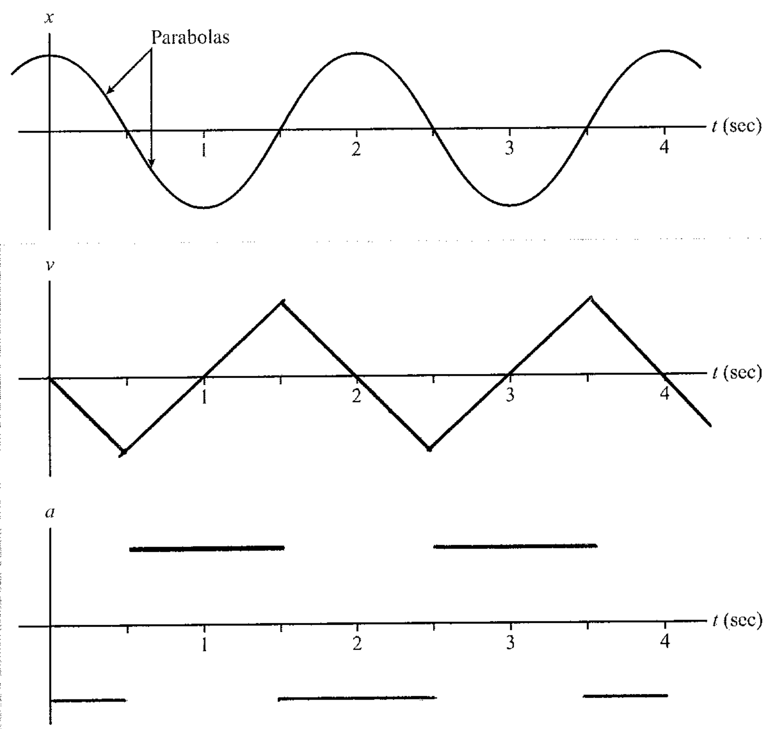
4. The graph shown is the position-versus-time graph of an oscillating particle. It is constructed of *parabolic* segments that are joined at $x = 0$.

   a. Is this simple harmonic motion? Why or why not?

   No. Sinusoidal graphs are not the same as parabolic segments. Simple harmonic motion implies sinusoidal motion.

   b. Draw the corresponding velocity-versus-time graph.
   Hint: Recall that each parabolic segment corresponds to linearly changing velocity and constant positive or negative acceleration.

   c. Draw the corresponding acceleration-versus-time graph.



   d. At what times is the position a maximum? 0, 2, 4 s
   At those times, is the velocity a maximum, a minimum, or zero? 0
   At those times, is the acceleration a maximum, a minimum, or zero? minimum

   e. At what times is the position a minimum (most negative)? 1, 3, 5 s
   At those times, is the velocity a maximum, a minimum, or zero? 0
   At those times, is the acceleration a maximum, a minimum, or zero? maximum

   f. At what times is the velocity a maximum? 1.5, 3.5 s
   At those times, where is the particle? 0

   g. Can you find a simple relationship between the *sign* of the position and the *sign* of the acceleration at the same instant of time? If so, what is it?

   Signs are opposite.

5. A particle goes around a circle 5 times at constant speed, taking a total of 2.5 seconds.

   a. Through what angle *in degrees* has the particle moved? $5 \times 360° = 1800°$

   b. Through what angle *in radians* has the particle moved? $10\pi = 31.4$ rads

   c. What is the particle's frequency f?

   $$\frac{5\text{ cycles}}{2.5\text{ s}} = 2.0\text{ Hz}$$

   d. Use your answer to part b to determine the particle's angular frequency ω.

   $$\omega = 2\pi f = 4\pi = 12.6\ \text{rad/s}$$

   e. Does ω (in rad/s) $= 2\pi f$ (in Hz)? yes

6. The quantity $\sin\theta$ is familiar from trigonometry as the ratio of a right triangle's opposite side to its hypotenuse. In that case, it is usually equally convenient to express θ in either degrees or radians. By contrast, if a sinusoidal function is used to describe simple harmonic motion, such as $x = A\sin\left(\frac{2\pi t}{T}\right)$, the angle θ is itself a function of other quantities, namely the period T and the time t, where $\theta = \frac{2\pi t}{T}$.

   a. Why is it necessary to use radians as the argument for the sine function in $x = A\sin\left(\frac{2\pi t}{T}\right)$?

   As t changes, the function repeats whenever $\Delta t/T = 1$, which corresponds to the argument of the sine function changing by 2π.

   b. How could the sine function be revised so that its argument is given in degrees?

   $$x = A\sin\left(\frac{360\,t}{T}\right)$$

   c. The function $x = A\sin\left(\frac{2\pi t}{T}\right)$ has the value 0 when $t = 0$ and has its first maximum value A when $t = T/4$. What is x at $t = T/8$?

   $$x = \frac{A}{\sqrt{2}} \simeq .707\,A$$

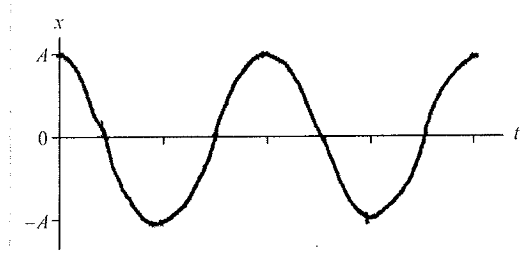
   d. At what earliest time does $x = A/2$?

   $$x = \frac{A}{2} \text{ when } \sin\frac{2\pi t}{T} = \frac{1}{2} \text{ or } \frac{t}{T} = \frac{1}{2\pi}\sin^{-1}(.5)$$

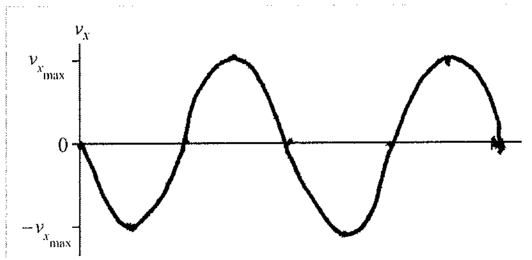
   $$t = \frac{T}{12} = .083\,T$$

7. A particle moves counterclockwise around a circle at constant speed beginning at $\phi_0 = 0$. For each of the kinematic quantities shown, sketch two cycles of the component of the particle's motion.

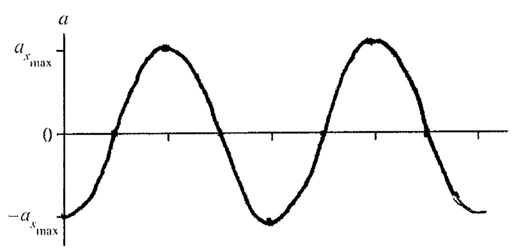
a.



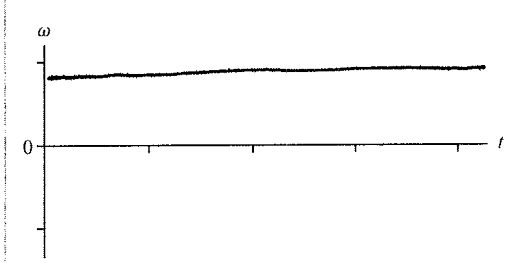
b.



c.



d.



8. A mass on a spring oscillates with period T, amplitude A, maximum speed v_{max}, and maximum acceleration a_{max}.

a. If T doubles without changing A,

i. how does v_{max} change, if at all?

V_{max} also doubles.

ii. how does a_{max} change, if at all?

a_{max} increases by a factor of 4.

b. If A doubles without changing T,

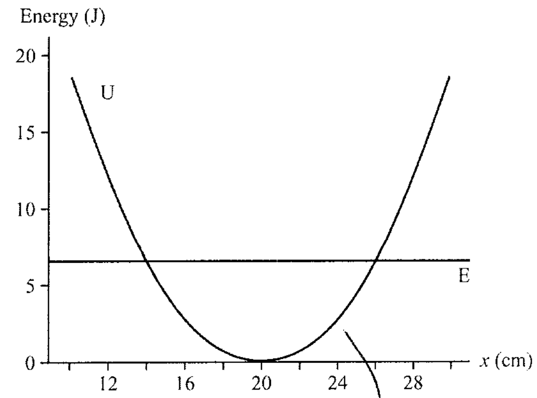
i. how does v_{max} change, if at all?

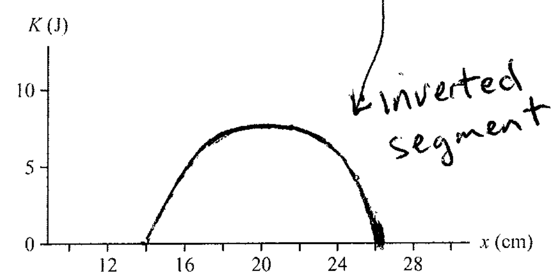
V_{max} also doubles.

ii. how does a_{max} change, if at all?

a_{max} also doubles.

14.4 Energy in Simple Harmonic Motion





inverted segment

9. The figure shows the potential-energy diagram and the total energy line of a particle oscillating on a spring.
   a. What is the spring's equilibrium length?

      20 cm

   b. Where are the turning points of the motion? Explain how you identify them.

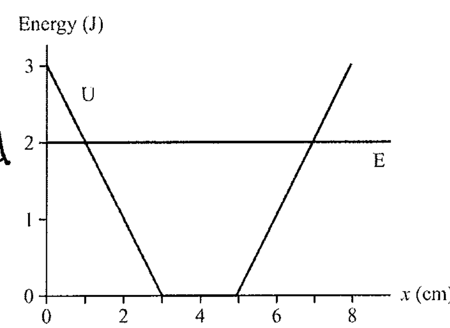
      U = E
      14cm, 26cm

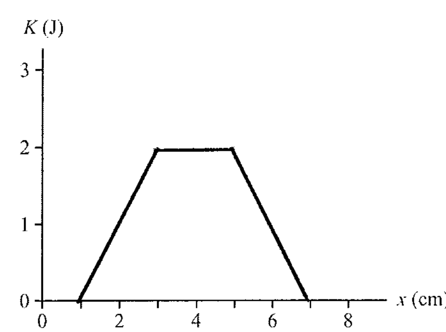
   c. What is the particle's maximum kinetic energy?

      ~7J

   d. Draw a graph of the particle's kinetic energy as a function of position.
   e. What will be the turning points if the particle's total energy is doubled?

      12cm, 28cm





10. The figure shows the potential energy diagram of a particle.
    a. Is the particle's motion periodic? How can you tell?

       Yes, There are turning points at 1cm and at 7cm and the particle oscillates between these two points.

    b. Is the particle's motion simple harmonic motion? How can you tell?

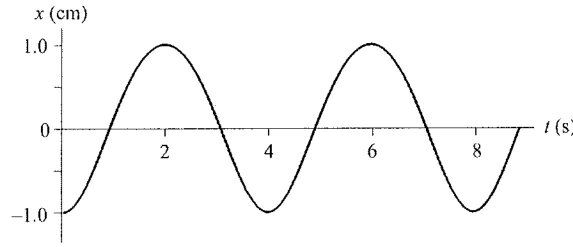
       No. The U curve is not quadratic.

    c. What is the amplitude of the motion?

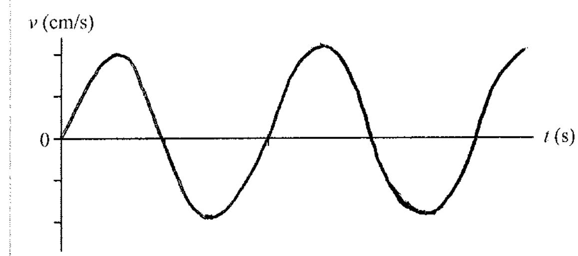
       3 cm

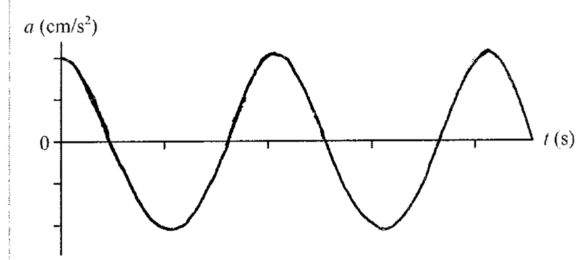
    d. Draw a graph of the particle's kinetic energy as a function of position.

11. The graph on the right is the position-versus-time graph for a simple harmonic oscillator. Be sure to include appropriate numerical values on the axes.



   a. Draw the v-versus-t and a-versus-t graphs.





   b. When x is greater than zero, is a ever greater than zero? If so, at which points in the cycle?

   No

   c. When x is less than zero, is a ever less than zero? If so, at which points in the cycle?

   No

   d. Can you make a general conclusion about the relationship between the sign of x and the sign of a?

   The signs of x and a are opposite.

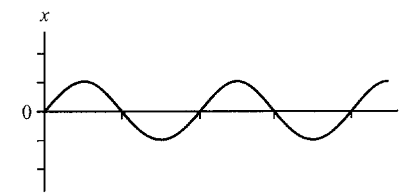
   e. When x is greater than zero, is v ever greater than zero? If so, how is the oscillator moving at those times?

   Yes. If $x>0$ and $v>0$, the object is slowing down as it approaches a turning point.

12. Equation 14.24 in the textbook states that $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$. What does this equation mean? Is there a time at which both the speed and displacement are at maximum? Explain.

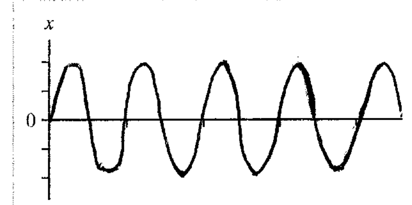
The equation means that the maximum potential energy $\frac{1}{2}kA^2$ is the same as the maximum kinetic energy $\frac{1}{2}mV_{max}^2$ but it does not imply that these maxima occur at the same time. The energy shifts back and forth so that when the potential energy is a maximum, the kinetic energy is zero and vice-versa.

13. The top graph shows the position versus time for a mass oscillating on a spring. On the axes below, sketch the position-versus-time graph for this block for the following situations:

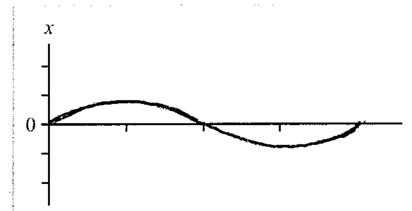
    

    Note: The changes described in each part refer back to the original oscillation, not to the oscillation of the previous part of the question. Assume that all other parameters remain constant. Use the same horizontal and vertical scales as the original oscillation graph.

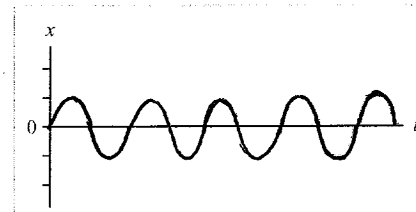
    a. The amplitude and the frequency are doubled.

    b. The amplitude is halved and the mass is quadrupled.

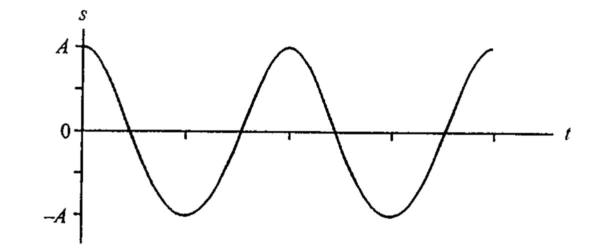
    

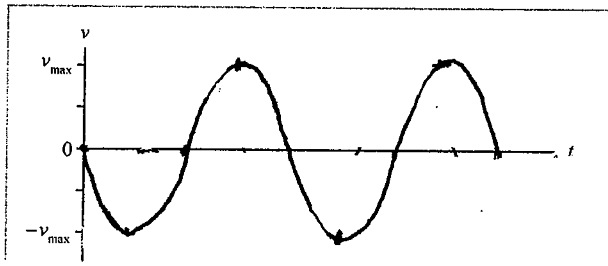
    c. The maximum speed is doubled while the amplitude remains constant.

14.5 Pendulum Motion

14. The graph shows the displacement s versus time for an oscillating pendulum.

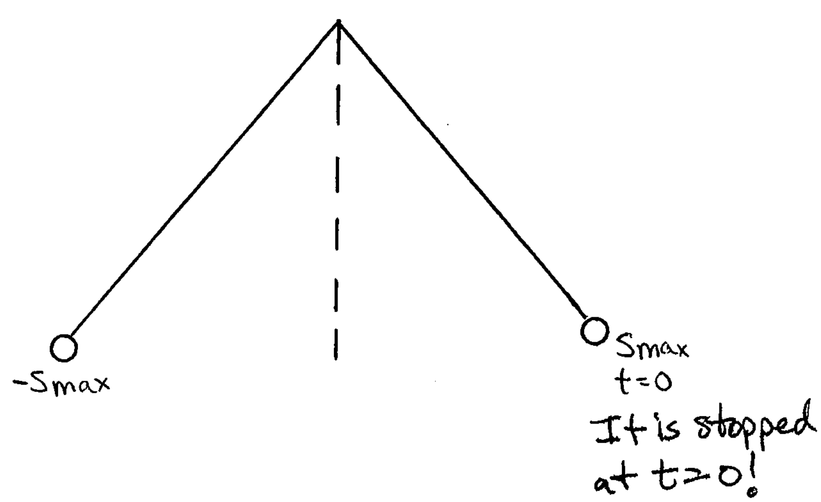




a. Draw the pendulum's velocity-versus-time graph.

b. In the space below, draw a *picture* of the pendulum that shows (and labels!)

- The extremes of its motion.
- Its position at $t = 0$ s.



14.6 Damped Oscillations

15. If the time constant τ of an oscillator is decreased,

   a. Is the drag force increased or decreased?

   Increased

   b. Do the oscillations damp out more quickly or less quickly?

   More quickly

   c. Is τ greater or less than the time it takes for the oscillation amplitude to decrease to half of its original value?

   Greater than. $\left(\frac{1}{2} = \exp(-t_{1/2}/\tau) \right.$ so $\ln 2 = t_{1/2}/\tau$ $\left. \tau \approx 1.44 t_{1/2} \right)$

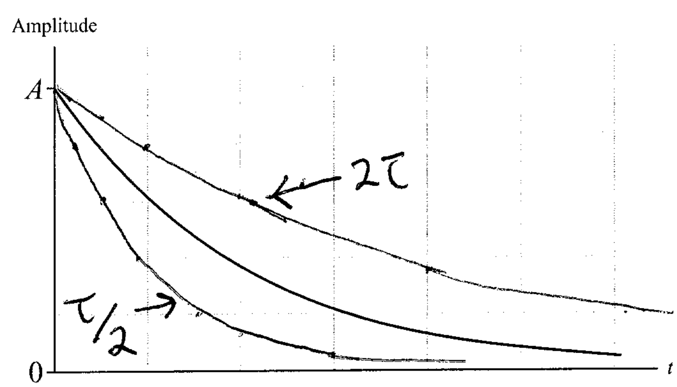
   d. After the amplitude has decayed to half of its original value, how much longer will it take for the amplitude to decay to one-fourth its original value?

   The same amount of time longer as it took to decay by $\frac{1}{2}$.

16. The figure below shows the envelope of the oscillations of a damped oscillator. On the same axes, draw the envelope of oscillations if

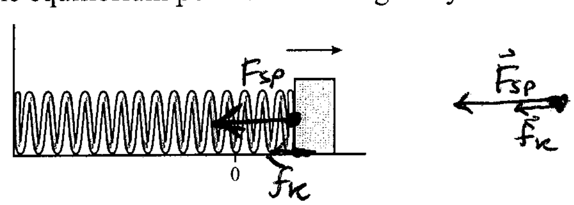
   a. The time constant is doubled.

   b. The time constant is halved.

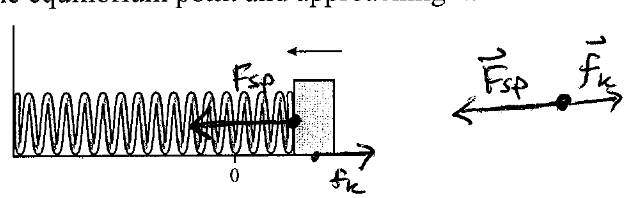


17. A block on a spring oscillates horizontally on a table *with friction*. Draw and label force vectors on the block to show all *horizontal* forces on the block.

   a. The mass is to the right of the equilibrium point and moving away from it.



   b. The mass is to the right of the equilibrium point and approaching it.



14.7 Driven Oscillations and Resonance

18. A car drives along a bumpy road on which the bumps are equally spaced. At a speed of 20 mph, the frequency of hitting bumps is equal to the natural frequency of the car bouncing on its springs.

    a. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has new shock absorbers (large damping coefficient).

    b. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has worn out shock absorbers (small damping coefficient).

    Draw both graphs on the same axes, and label them as to which is which.