## Conceptual Questions

27.1. No. The cube lacks sufficient symmetry to deduce the shape of the field. In particular, the cube lacks both translational and rotational symmetry. It is symmetric under $90^{\circ}$ rotations but, unlike a sphere, not symmetric for arbitrary rotations.
27.2. (a) No net charge is enclosed. There is no flux through the bottom surface, and the flux into the left-hand side is cancelled by the flux out of the right-hand side.
(b) Net positive charge is enclosed, since the flux is outward on all surfaces.
(c) Net negative charge is enclosed since the flux is inward on all surfaces.
27.3. From Equation 27.3, we know that $\Phi_{\text {square }}=\vec{E}_{\text {square }} \cdot \vec{A}_{\text {square }}=E_{\text {square }} A_{\text {square }}$ and $\Phi_{\text {circle }}=\vec{E}_{\text {circle }} \cdot \vec{A}_{\text {circle }}=$ $E_{\text {circle }} A_{\text {circle }}$, where the last equality holds because the electric field is parallel to the surface normal. We also know $E_{\text {square }}=E_{\text {circle }}$ and $A_{\text {square }}>A_{\text {circle }}$, so

$$
\frac{\Phi_{\text {square }}}{\Phi_{\text {circle }}}=\frac{\mathrm{E}_{\text {square }} \mathrm{A}_{\text {square }}}{\mathrm{E}_{\text {circle }} \mathrm{A}_{\text {circle }}}=\frac{\mathrm{A}_{\text {square }}}{\mathrm{A}_{\text {circle }}}>1 \Rightarrow \Phi_{\text {square }}>\Phi_{\text {circle }}
$$

27.4. $\Phi_{1}=\Phi_{2}$. In the absence of a net enclosed charge, any flux into surface 1 must come out of surface 2 because the two surfaces form a single closed surface.
27.5. (a) $+q / \epsilon_{0}$ (b) $-q / \epsilon_{0}$ (c) 0
27.6. $\Phi_{\mathrm{A}}=+4 q / \epsilon_{0}, \Phi_{\mathrm{B}}=-4 q / \epsilon_{0}, \Phi_{\mathrm{C}}=0, \Phi_{\mathrm{D}}=+3 q / \epsilon_{0}, \Phi_{\mathrm{E}}=0$
27.7. The charge lies on the surface of the hollow balloon.

Point 1: Stays the same. Gauss's Law with $Q_{\text {in }}=0$ implies $E=0$ in both situations.
Point 2: Decreases. $E>0$ initially when the point is outside the balloon but $E=0$ at the end when the point is inside the balloon.
Point 3: Stays the same. The electric field of the balloon looks like that of a point charge at the geometric center of the balloon both before and after the balloon is blown up.
27.8. Student 1 is correct. The two spheres are Gaussian surfaces enclosing the same amount of charge, so the flux through the two surfaces is equal, no matter their size. In this case, one can see that the area increases as $r^{2}$ and the electric field strength decreases as $1 / r^{2}$ so the flux is the same through spheres A and B.
27.9. Student 2 is correct. The sphere and ellipsoid are Gaussian surfaces enclosing the same amount of charge, so the flux through the two surfaces is equal, no matter their shape.
27.10. The hole that the conducting wire passes through is so small that it can be ignored.
(a) Positive charge has been transferred to the small sphere, so it has a positive charge.
(b) Negative. A Gaussian surface through the conductor of the larger sphere must contain a net charge of zero, so the inner surface of the larger conductor must be negatively charged to balance the positive charge on the small sphere.
(c) The negative charge on the inner surface of the larger sphere came from its outer surface (all excess charge lies on the surface of a conductor) so the outside of the larger sphere has a positive charge. This can also be seen by considering a Gaussian surface outside the larger sphere. It contains a net positive charge (i.e., the charge on the smaller sphere, since the larger sphere is overall neutral). Since the charge on the inner surface is equal and opposite to that on the smaller sphere, the outside of the larger sphere must be positively charged.

## Exercises and Problems

## Section 27.1 Symmetry

### 27.1. Visualize:



As discussed in Section 27.1, the symmetry of the electric field must match the symmetry of the charge distribution. In particular, the electric field of a cylindrically symmetric charge distribution cannot have a component parallel to the cylinder axis. Also, the electric field of a cylindrically symmetric charge distribution cannot have a component tangent to the circular cross section. The only shape for the electric field that matches the symmetry of the charge distribution with respect to (i) translation parallel to the cylinder axis, (ii) rotation by an angle about the cylinder axis, and (iii) reflections in any plane containing or perpendicular to the cylinder axis is the one shown in the figure.

### 27.2. Visualize:



The object has spherical symmetry, so the electric field is radial. The net charge is positive, so the electric field is nonzero and points radially outward outside the outer sphere.

### 27.3. Visualize:

$$
\begin{aligned}
& \frac{\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow}{+++++++++++++++++} \\
& \vec{E}=0 \mathrm{~N} / \mathrm{C} \\
& \frac{+++++++++++++++++}{\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow}
\end{aligned}
$$

Figure 27.6 shows the electric field for an infinite plane of charge. For two parallel planes, this is the only shape of the electric field vectors that matches the symmetry of the charge distribution. Because the charge density is equal on each plane, there can be no electric field between the planes.

## Section 27.2 The Concept of Flux

27.4. Model: The electric flux "flows" out of a closed surface around a region of space containing a net positive charge and into a closed surface surrounding a net negative charge.
Visualize: Please refer to Figure EX27.4. Let $A$ be the area in $\mathrm{m}^{2}$ of each of the six faces of the cube and let the direction of $\vec{A}$ be outward from the surfaces.
Solve: The electric flux is defined as $\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=E A \cos \theta$, where $\theta$ is the angle between the electric field and a line perpendicular to the plane of the surface. The electric flux out of the closed cube surface is

$$
\Phi_{\text {out }}=(15 \mathrm{~N} / \mathrm{C}+20 \mathrm{~N} / \mathrm{C}+10 \mathrm{~N} / \mathrm{C}) A \cos \left(0^{\circ}\right)=(45 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

Similarly, the electric flux into the closed cube surface is

$$
\Phi_{\text {in }}=(20 \mathrm{~N} / \mathrm{C}+15 \mathrm{~N} / \mathrm{C}+15 \mathrm{~N} / \mathrm{C}) A \cos \left(180^{\circ}\right)=-(50 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

The net electric flux is $(45 A) \mathrm{Nm}^{2} / \mathrm{C}-(50 A) \mathrm{Nm}^{2} / \mathrm{C}=-(5 A) \mathrm{Nm}^{2} / C$. Since the net electric flux is negative (i.e., inward), the closed box contains a negative charge.
27.5. Model: The electric flux "flows" out of a closed surface around a region of space containing a net positive charge and into a closed surface surrounding a net negative charge.
Visualize: Please refer to Figure EX27.5. Let $A$ be the area of each of the six faces of the cube and let the direction of $\vec{A}$ be outward from the surfaces.
Solve: The electric flux is defined as $\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=E A \cos \theta$, where $\theta$ is the angle between the electric field and a line perpendicular to the plane of the surface (i.e., the "normal" to the surface). The electric flux out of the closed cube surface is

$$
\Phi_{\text {out }}=(10 \mathrm{~N} / \mathrm{C}+10 \mathrm{~N} / \mathrm{C}+20 \mathrm{~N} / \mathrm{C}) A \cos \left(0^{\circ}\right)=(40 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

Similarly, the electric flux into the closed cube surface is

$$
\Phi_{\mathrm{in}}=(15 \mathrm{~N} / \mathrm{C}+20 \mathrm{~N} / \mathrm{C}+5 \mathrm{~N} / \mathrm{C}) A \cos \left(180^{\circ}\right)=-(40 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

Thus, $\Phi_{\text {out }}+\Phi_{\text {in }}=0 \mathrm{Nm}^{2} / \mathrm{C}$. Since there is no net electric flux passing through the surface of the closed box, it contains no charge.
27.6. Model: The electric flux "flows" out of a closed surface around a region of space containing a net positive charge and into a closed surface surrounding a net negative charge.
Visualize: Please refer to Figure EX27.6. Let $A$ be the area of each of the six faces of the cube and let the direction of $\vec{A}$ be outward from the surfaces.
Solve: The electric flux is defined as $\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=E A \cos \theta$, where $\theta$ is the angle between the electric field and a line perpendicular to the plane of the surface. The electric flux out of the closed cube surface is

$$
\Phi_{\text {out }}=(20 \mathrm{~N} / \mathrm{C}+10 \mathrm{~N} / \mathrm{C}+10 \mathrm{~N} / \mathrm{C}) A \cos \left(0^{\circ}\right)=(40 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

Similarly, the electric flux into the closed cube surface is

$$
\Phi_{\mathrm{in}}=(20 \mathrm{~N} / \mathrm{C}+15 \mathrm{~N} / \mathrm{C}) A \cos \left(180^{\circ}\right)=-(35 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

Because the cube contains negative charge, $\Phi_{\text {out }}+\Phi_{\text {in }}$ must be negative. This means $\Phi_{\text {out }}+\Phi_{\text {in }}+\Phi_{\text {unknown }}<$ $0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}$. Therefore,

$$
\begin{gathered}
(40 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}+(-35 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}+\Phi_{\text {unknown }}<0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C} \\
\Phi_{\text {unknown }}<(-5 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
\end{gathered}
$$

That is, the unknown electric field vector points into the front face of the cube and its field strength must exceed 5 N/C.
27.7. Model: The electric flux "flows" out of a closed surface around a region of space containing a net positive charge and into a closed surface surrounding a net negative charge.
Visualize: Please refer to Figure EX27.7. Let $A$ be the area of each of the six faces of the cube and let the direction of $\vec{A}$ be outward from the surfaces.
Solve: The electric flux is defined as $\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=E A \cos \theta$, where $\theta$ is the angle between the electric field vector and an outward unit vector that is perpendicular to the plane of the surface. The electric flux out of the closed cube surface is

$$
\Phi_{\text {out }}=(20 \mathrm{~N} / \mathrm{C}+10 \mathrm{~N} / \mathrm{C}+10 \mathrm{~N} / \mathrm{C}) A \cos \left(0^{\circ}\right)=(40 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

Similarly, the electric flux into the closed cube surface is

$$
\Phi_{\text {in }}=(20 \mathrm{~N} / \mathrm{C}+15 \mathrm{~N} / \mathrm{C}) A \cos \left(180^{\circ}\right)=-(35 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

Because the cube contains negative charge, $\Phi_{\text {out }}+\Phi_{\text {in }}$ must be negative. This means $\Phi_{\text {out }}+\Phi_{\text {in }}+\Phi_{\text {unknown }}<0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}$. Therefore,

$$
\begin{gathered}
(40 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}+(-35 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}+\Phi_{\text {unknown }}<0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C} \\
\Phi_{\text {unknown }}<(-5 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
\end{gathered}
$$

Therefore, to produce negative flux, the unknown electric field vector must point into the front face of the cube, and its field strength must be greater than $5 \mathrm{~N} / \mathrm{C}$.
27.8. Model: The electric flux "flows" out of a closed surface around a region of space containing a net positive charge and into a closed surface surrounding a net negative charge.
Visualize: Please refer to Figure EX27.8. Let $A$ be the area of each of the six faces of the cube and let the direction of $\vec{A}$ be outward from the surfaces.
Solve: The electric flux is defined as $\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=E A \cos \theta$, where $\theta$ is the angle between the electric field vector and an outward unit vector that is perpendicular to the plane of the surface. The electric flux out of the closed cube surface is

$$
\Phi_{\text {out }}=(15 \mathrm{~N} / \mathrm{C}+15 \mathrm{~N} / \mathrm{C}+10 \mathrm{~N} / \mathrm{C}) A \cos \left(0^{\circ}\right)=(40 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

Similarly, the electric flux into the closed cube surface is

$$
\Phi_{\mathrm{in}}=(20 \mathrm{~N} / \mathrm{C}+15 \mathrm{~N} / \mathrm{C}) A \cos \left(180^{\circ}\right)=-(35 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

Because the cube contains no charge, the total flux is zero. This means $\Phi_{\text {out }}+\Phi_{\text {in }}+\Phi_{\text {unknown }}=0 \mathrm{Nm}^{2} / \mathrm{C}$, so

$$
(40 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}-(35 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}+\Phi_{\text {unknown }}=0 \Rightarrow \Phi_{\text {unknown }}=-(5 A) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}
$$

The unknown electric field vector on the front face points in, and the electric field strength is $5 \mathrm{~N} / \mathrm{C}$.

## Section 27.3 Calculating Electric Flux

27.9. Model: The electric field is uniform over the entire surface.

Visualize: Please refer to Figure EX27.9. The electric field vectors make an angle of $30^{\circ}$ with the planar surface. Because the normal $\hat{n}$ to the planar surface is at an angle of $90^{\circ}$ with the surface, the angle between $\hat{n}$ and $\vec{E}$ is $\theta=60^{\circ}$.
Solve: The electric flux is

$$
\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=E A \cos \theta=(200 \mathrm{~N} / \mathrm{C})\left(1.0 \times 10^{-2} \mathrm{~m}^{2}\right) \cos \left(60^{\circ}\right)=1.0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
$$

27.10. Model: The electric field is uniform over the entire surface.

Visualize: Please refer to Figure EX27.10. The electric field vectors make an angle of $30^{\circ}$ below the surface. Because the normal $\hat{n}$ to the planar surface is at an angle of $90^{\circ}$ relative to the surface, the angle between $\hat{n}$ and $\vec{E}$ is $\theta=120^{\circ}$.
Solve: The electric flux is

$$
\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=E A \cos \theta=(180 \mathrm{~N} / \mathrm{C})\left(15 \times 10^{-2} \mathrm{~m}\right)^{2} \cos \left(120^{\circ}\right)=-2.3 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
$$

27.11. Model: The electric field is uniform over the entire surface.

Visualize: Please refer to Figure EX27.11. The electric field vectors make an angle of $60^{\circ}$ above the surface. Because the normal $\hat{n}$ to the planar surface is at an angle of $90^{\circ}$ relative to the surface, the angle between $\hat{n}$ and $\vec{E}$ is $\theta=30^{\circ}$.
Solve: The electric flux is

$$
\begin{aligned}
\Phi_{\mathrm{e}} & =\vec{E} \cdot \vec{A}=E A \cos \theta \\
E & =\frac{\Phi_{\mathrm{e}}}{A \cos \theta}=\frac{25 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}}{\left(10 \times 10^{-2} \mathrm{~m}\right)\left(20 \times 10^{-2} \mathrm{~m}\right) \cos \left(30^{\circ}\right)}=1.4 \times 10^{3} \mathrm{~N} / \mathrm{C}
\end{aligned}
$$

27.12. Model: The electric field is uniform over the rectangle in the $x y$ plane.

Solve: (a) The area vector is perpendicular to the $x y$ plane and points in the $\hat{k}$ direction. Thus

$$
\vec{A}=(2.0 \mathrm{~cm} \times 3.0 \mathrm{~cm}) \hat{k}=\left(6.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{k}
$$

The electric flux through the rectangle is

$$
\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=(100 \hat{i}+50 \hat{k}) \cdot\left(6.0 \times 10^{-4} \hat{k}\right) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}=3.0 \times 10^{-2} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
$$

(b) The electric flux is

$$
\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=(100 \hat{i}+50 \hat{j}) \cdot\left(6.0 \times 10^{-4} \hat{k}\right) \mathrm{N} \mathrm{~m}^{2} / \mathrm{C}=0.0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
$$

Assess: In (b), $\vec{E}$ is in the plane of the rectangle, which is why the flux is zero.
27.13. Model: The electric field over the rectangle in the $x z$ plane is uniform.

Solve: (a) The area vector is perpendicular to the $x z$ plane and points in the $\hat{j}$ direction. Thus

$$
\vec{A}=(2.0 \mathrm{~cm} \times 3.0 \mathrm{~cm}) \hat{j}=\left(6.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{j}
$$

The electric flux through the rectangle is

$$
\begin{aligned}
\Phi_{\mathrm{e}} & =\vec{E} \cdot \vec{A}=(100 \hat{i}+50 \hat{k}) \mathrm{N} / \mathrm{C} \cdot\left(6.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{j} \\
& =\left(600 \times 10^{-4} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)(\hat{i} \cdot \hat{j})+\left(300 \times 10^{-4} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)(\hat{k} \cdot \hat{j})=0.0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
\end{aligned}
$$

(b) The flux is

$$
\begin{aligned}
\Phi_{\mathrm{e}} & =\vec{E} \cdot \vec{A}=(100 \hat{i}+50 \hat{j}) \mathrm{N} / \mathrm{C} \cdot\left(6.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{j} \\
& =\left(600 \times 10^{-4} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)(\hat{i} \cdot \hat{j})+\left(300 \times 10^{-4} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right) \hat{j} \cdot \hat{j} \\
& =0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}+\left(3.0 \times 10^{-2} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)=3.0 \times 10^{-2} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
\end{aligned}
$$

27.14. Model: The electric field over the circle in the $x z$ plane is uniform.

Solve: The area vector of the circle is

$$
\vec{A}=\pi r^{2} \hat{k}=\pi(0.015 \mathrm{~m})^{2} \hat{j}=\left(7.07 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{j}
$$

Thus, the flux through the area of the circle is

$$
\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=\vec{E}=(1500 \hat{i}+1500 \hat{j}-1500 \hat{k}) \mathrm{N} / \mathrm{C} \cdot\left(7.07 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{j}
$$

Using $\hat{i} \cdot \hat{j}=\hat{j} \cdot \hat{k}=0$ and $\hat{j} \cdot \hat{j}=1$, we find

$$
\Phi_{\mathrm{e}}=(1500 \mathrm{~N} / \mathrm{C})\left(7.07 \times 10^{-4} \mathrm{~m}^{2}\right)=1.1 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
$$

27.15. Model: The electric field is uniform, and we take the area vectors to point outward from the box.

Visualize: In the figure below, the box is positioned with its edges aligned with the $x y z$ axes, and the electric field is evaluated at the input face and the exit face.


Solve: The area vectors of the six box faces are $\vec{A}_{1}=\left(1.0 \times 10^{-2} \mathrm{~m}\right)^{2} \hat{i}=\left(1.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{i}, \quad \vec{A}_{2}=-\left(1.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{i}$, $\vec{A}_{3}=\left(1.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{j}, \quad \vec{A}_{4}=-\left(1.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{j}, \quad \vec{A}_{5}=\left(1.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{k}$, and $\vec{A}_{6}=-\left(1.0 \times 10^{-4} \mathrm{~m}^{2}\right) \hat{k}$. The net flux through the box is

$$
\begin{aligned}
\Phi & =\vec{E} \cdot \vec{A}=\sum_{i=1}^{6} \vec{E} \cdot \vec{A}_{i}=\vec{E}(x=0.01 \mathrm{~m}) \cdot \vec{A}_{1}+\vec{E}(x=0.0 \mathrm{~m}) \cdot \vec{A}_{2} \\
& =(153.5 \mathrm{~N} / \mathrm{C})\left(1.0 \times 10^{-4} \mathrm{~m}^{2}\right)-(150 \mathrm{~N} / \mathrm{C})\left(1.0 \times 10^{-4} \mathrm{~m}^{2}\right) \\
& =3.5 \times 10^{-4} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
\end{aligned}
$$

27.16. Model: The electric field through the two cylinders is uniform.

Visualize: Please refer to Figure EX27.16. Let $A=\pi R^{2}$ be the area of the end of the cylinder and let the area vector point outward from the cylinder ends. $E$ is the electric field strength.
Solve: (a) There's no flux through the side walls of the cylinder because $\vec{E}$ is parallel to the wall. On the right end, where $\vec{E}$ points outward, $\Phi_{\text {right }}=E A \cos \left(0^{\circ}\right)=\pi R^{2} E$. The field points inward on the left, so $\Phi_{\text {left }}=E A \cos \left(180^{\circ}\right)=-\pi R^{2} E$. Altogether, the net flux is $\Phi_{\mathrm{e}}=0 \mathrm{Nm}^{2} / \mathrm{C}^{2}$.
(b) The only difference from part (a) is that $\vec{E}$ points outward on the left end, making $\Phi_{\text {left }}=E A \cos \left(0^{\circ}\right)=\pi R^{2} E$. Thus the net flux through the cylinder is $\Phi_{\mathrm{e}}=2 \pi R^{2} E \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}^{2}$.

## Section 27.4 Gauss's Law

## Section 27.5 Using Gauss's Law

### 27.17. Visualize:


(c)

For any closed surface that encloses a total charge $Q_{\mathrm{in}}$, the net electric flux through the closed surface is $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \epsilon_{0}$.

### 27.18. Visualize:



For any closed surface that encloses a total charge $Q_{\mathrm{in}}$, the net electric flux through the closed surface is $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \epsilon_{0}$.
27.19. Visualize: Please refer to Figure EX27.19.

Solve: For any closed surface that encloses a total charge $Q_{i n}$, the net electric flux through the surface is $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \varepsilon_{0}$. We can write three equations from the three closed surfaces in the figure:

$$
\begin{gathered}
\Phi_{\mathrm{A}}=-\frac{q}{\epsilon_{0}}=\frac{q_{1}+q_{3}}{\epsilon_{0}} \Rightarrow q_{1}+q_{3}=-q \quad \Phi_{\mathrm{B}}=\frac{3 q}{\epsilon_{0}}=\frac{q_{1}+q_{2}}{\epsilon_{0}} \Rightarrow q_{1}+q_{2}=3 q \\
\Phi_{\mathrm{C}}=\frac{-2 q}{\epsilon_{0}}=\frac{q_{2}+q_{3}}{\epsilon_{0}} \Rightarrow q_{2}+q_{3}=-2 q
\end{gathered}
$$

Subtracting third equation from the first gives

$$
q_{1}-q_{2}=+q
$$

Adding second equation to this equation,

$$
2 q_{1}=+4 q q_{1}=2 q
$$

That is, $q_{1}=+2 q, q_{2}=+q$, and $q_{3}=-3 q$.
27.20. Visualize: Please refer to Figure EX27.20. For any closed surface that encloses a total charge $Q_{\text {in }}$, the net electric flux through the closed surface is $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \epsilon_{0}$. For the closed surface of the torus, $Q_{\text {in }}$ includes only the -1 nC charge. Thus, the net flux through the torus is due only to this charge:

$$
\Phi_{\mathrm{e}}=\frac{-1 \times 10^{-9} \mathrm{C}}{8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}}=-0.11 \mathrm{kN} \mathrm{~m}^{2} / \mathrm{C}
$$

This is inward flux.
27.21. Visualize: Please refer to Figure EX27.21. For any closed surface that encloses a total charge $Q_{\mathrm{in}}$, the net electric flux through the closed surface is $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \epsilon_{0}$. The cylinder encloses the +1 nC charge only as both the +100 nC and the -100 nC charges are outside the cylinder. Thus,

$$
\Phi_{\mathrm{e}}=\frac{1 \times 10^{-9} \mathrm{C}}{8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}}=0.11 \mathrm{kN} \mathrm{~m}^{2} / \mathrm{C}
$$

This is outward flux.
27.22. Solve: For any closed surface enclosing a total charge $Q_{\mathrm{in}}$, the net electric flux through the surface is

$$
\Phi_{\mathrm{e}}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \Rightarrow Q_{\mathrm{in}}=\epsilon_{0} \Phi_{\mathrm{e}}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left(-1000 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)=-8.85 \mathrm{nC}
$$

27.23. Solve: For any closed surface that encloses a total charge $Q_{\mathrm{in}}$, the net electric flux through the surface is

$$
\Phi_{\mathrm{e}}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}=\frac{\left(55.3 \times 10^{6}\right)\left(-1.60 \times 10^{-19} \mathrm{C}\right)}{8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}}=-1.00 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
$$

## Section 27.6 Conductors in Electrostatic Equilibrium

27.24. Model: A copper penny is a conductor. Assume the penny to be a flat disc of radius much, much greater than the distance from the surface at which we are measuring the electric field.
Solve: The excess charge on a conductor resides on the surface. The electric field at the surface of a charged conductor is

$$
\begin{aligned}
& \vec{E}_{\text {surface }}=\left(\frac{\eta}{\epsilon_{0}}, \text { perpendicular to surface }\right) \\
& \eta=\epsilon_{0} E_{\text {surface }}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)(2000 \mathrm{~N} / \mathrm{C})=17.7 \times 10^{-9} \mathrm{C} / \mathrm{m}^{2}
\end{aligned}
$$

Assess: Because the actual surface of a penny is not flat, the surface charge density will not be uniform. The result above gives the average surface charge density, far from the edges of the coin.
27.25. Model: The excess charge on a conductor resides on the outer surface.

Solve: The electric field at the surface of a charged conductor is

$$
\begin{aligned}
& \vec{E}_{\text {surface }}=\left(\frac{\eta}{\epsilon_{0}}, \text { perpendicular to surface }\right) \\
& \eta=\epsilon_{0} E_{\text {surface }}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left(3.0 \times 10^{6} \mathrm{~N} / \mathrm{C}\right)=2.7 \times 10^{-5} \mathrm{C} / \mathrm{m}^{2}
\end{aligned}
$$

Assess: It is the air molecules just above the surface that "break down" when the $E$-field becomes strong enough to accelerate stray charges to approximately 15 eV between collisions, thus causing collisional ionization. It does not make any difference whether $E$ points toward or away from the surface.
27.26. Model: The excess charge on a conductor resides on the outer surface.

Visualize: Please refer to Figure EX27.26.
Solve: Point 1 is at the surface of a charged conductor, hence

$$
\vec{E}_{\text {surface }}=\left(\frac{\eta}{\epsilon_{0}}, \text { perpendicular to surface }\right) \Rightarrow E_{\text {surface }}=\frac{\left(5.0 \times 10^{10}\right)\left(1.60 \times 10^{-19} \mathrm{C} / \mathrm{m}^{2}\right)}{8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}}=0.90 \mathrm{kN} / \mathrm{C}
$$

At point 2 the electric field strength is zero because this point lies inside the conductor. The electric field strength at point 3 is zero because there is no excess charge on the interior surface of the box. This can be quickly seen by considering a Gaussian surface just inside the interior surface of the box as shown in Figure 27.31.
27.27. Model: The copper plate is a conductor. The excess charge (consisting of electrons, which are negative) resides on the surface of the plate. Ignore the charge that resides on the edge of the plate because the plate's thickness is much, much less than the radius.
Solve: (a) One-half of the charge is located on the top surface and one-half on the bottom surface of the copper plate, so the surface charge density is

$$
\eta=\frac{q}{A}=\frac{(3.5 / 2 \mathrm{nC})}{\pi(0.10 / 2 \mathrm{~m})^{2}}=2.23 \times 10^{-7} \mathrm{C} / \mathrm{m}^{2}
$$

Thus, the strength of the electric field at the surface of the plate is

$$
E=\frac{\eta}{\epsilon_{0}}=\frac{2.23 \times 10^{-7} \mathrm{C} / \mathrm{m}^{2}}{8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \mathrm{~m}^{2}}=2.5 \times 10^{4} \mathrm{~N} / \mathrm{C}
$$

Because the charge on the plate is negative, the direction of the electric field is toward the plate, which is downward.
(b) The center of mass of the plate is in the interior of the plate, so $E=0 \mathrm{~N} / \mathrm{C}$ because the electric field within a conductor is zero.
(c) The electric field strength is the same as it is above the plate, $E=2.5 \times 10^{4} \mathrm{~N} / \mathrm{C}$, and the field is still directed toward the plate, which is upward in this case.
27.28. Visualize: Please refer to Figure EX27.28.

Solve: For any closed surface that encloses a total charge $Q_{\text {in }}$, the net electric flux through the closed surface is $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \varepsilon_{0}$. In the present case, the conductor is neutral and there is a point charge $Q$ inside the cavity. Thus $Q_{\text {in }}=Q$ and the flux is

$$
\Phi_{\mathrm{e}}=\frac{Q}{\epsilon_{0}}
$$

27.29. Model: The electric field over the four faces of the cube is uniform.

Visualize: Please refer to Figure P27.29.
Solve: (a) The electric flux through a surface area $\vec{A}$ is $\Phi=\vec{E} \cdot \vec{A}=E A \cos \theta$, where $\theta$ is the angle between the electric field and the vector $\vec{A}$ that points outward from the surface. Thus

$$
\begin{aligned}
& \Phi_{1}=E A \cos \theta_{1}=(500 \mathrm{~N} / \mathrm{C})\left(9.0 \times 10^{-4} \mathrm{~m}^{2}\right) \cos \left(150^{\circ}\right)=-0.39 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C} \\
& \Phi_{2}=E A \cos \theta_{2}=(500 \mathrm{~N} / \mathrm{C})\left(9.0 \times 10^{-4} \mathrm{~m}^{2}\right) \cos \left(60^{\circ}\right)=0.23 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C} \\
& \Phi_{3}=E A \cos \theta_{3}=(500 \mathrm{~N} / \mathrm{C})\left(9.0 \times 10^{-4} \mathrm{~m}^{2}\right) \cos \left(30^{\circ}\right)=0.39 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C} \\
& \Phi_{4}=E A \cos \theta_{4}=(500 \mathrm{~N} / \mathrm{C})\left(9.0 \times 10^{-4} \mathrm{~m}^{2}\right) \cos \left(120^{\circ}\right)=-0.23 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
\end{aligned}
$$

(b) The net flux through these four sides is $\Phi_{\mathrm{T}}=\Phi_{1}+\Phi_{2}+\Phi_{3}+\Phi_{4}=0.0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}$.

Assess: The net flux through the four faces of the cube is zero because there is no enclosed charge.
27.30. Model: The electric field over the five surfaces is uniform.

Visualize: Please refer to Figure P27.30.
Solve: The electric flux through a surface area $\vec{A}$ is $\Phi_{\mathrm{e}}=\vec{E} \cdot \vec{A}=E A \cos \theta$ where $\theta$ is the angle between the electric field and a line perpendicular to the plane of the surface. The electric field is perpendicular to side 1 and is parallel to sides 2,3 , and 5. Also the angle between $\vec{E}$ and $\vec{A}_{4}$ is $60^{\circ}$. The electric fluxes through these five surfaces are

$$
\begin{aligned}
& \Phi_{1}=E_{1} A_{1} \cos \theta_{1}=(400 \mathrm{~N} / \mathrm{C})(2.0 \mathrm{~m})(4.0 \mathrm{~m}) \cos \left(180^{\circ}\right)=-3.2 \mathrm{kN} \mathrm{~m}^{2} / \mathrm{C} \\
& \Phi_{2}=E_{2} A_{2} \cos \left(90^{\circ}\right)=\Phi_{3}=\Phi_{5}=0.0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C} \\
& \Phi_{4}=E_{4} A_{4} \cos \theta_{4}=(400 \mathrm{~N} / \mathrm{C})\left(\frac{2.0 \mathrm{~m}}{\sin \left(30^{\circ}\right)}\right)(4.0 \mathrm{~m}) \cos \left(60^{\circ}\right)=+3.2 \mathrm{kN} \mathrm{~m}^{2} / \mathrm{C}
\end{aligned}
$$

Assess: Because the flux into these five faces is equal to the flux out of the five faces, the net flux is zero, as we found.
27.31. Model: Because the tetrahedron contains no charge, the net flux through the tetrahedron is zero. Visualize:


Solve: (a) The area of the base of the tetrahedron is $\frac{\sqrt{3}}{4} a^{2}$, where $a=20 \mathrm{~cm}$ is the length of one of the sides. Because the base of the tetrahedron is parallel to the ground and the vertical uniform electric field passes upward through the base, the angle between $\vec{E}$ and $\vec{A}$ is $180^{\circ}$. Thus,

$$
\Phi_{\text {base }}=\vec{E} \cdot \vec{A}_{\text {base }}=E A_{\text {base }} \cos 180^{\circ}=-E \frac{\sqrt{3}}{4} a^{2}=-(200 \mathrm{~N} / \mathrm{C}) \frac{\sqrt{3}}{4}(0.20 \mathrm{~m})^{2}=-3.464 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
$$

The electric flux through the base is $-3.5 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}$.
(b) Since $\vec{E}$ is perpendicular to the base, the other three sides of the tetrahedron share the flux equally. Because the tetrahedron contains no charge,

$$
\begin{gathered}
\Phi_{\text {net }}=\Phi_{\text {base }}+3 \Phi_{\text {side }}=0.0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C} \Rightarrow-3.46 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}+3 \Phi_{\text {side }}=0.0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C} \\
\Phi_{\text {side }}=1.2 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
\end{gathered}
$$

Assess: The results are given to two significant figures and so may appear not to sum to zero net flux. To show that the net flux is zero, these results should be treated as intermediate results, so more significant figures would be retained:

$$
\Phi_{\text {net }}=\Phi_{\text {base }}+\Phi_{1}+\Phi_{2}+\Phi_{3}=-3.4641 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}+3\left(1.1547 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)=0.0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}
$$

27.32. Solve: (a) When centered at the origin the sphere encloses both $q_{1}$ and $q_{2}$. For any closed surface that encloses a total charge $Q_{\text {in }}$, the net electric flux through the closed surface is

$$
\Phi_{\mathrm{e}}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}=\frac{(-4 Q+2 Q)}{\epsilon_{0}}=\frac{-2 Q}{\epsilon_{0}}
$$

(b) When centered at $x=2 a$, the sphere encloses only $q_{2}$ located at $x=+a$. The net electric flux through the closed surface is thus

$$
\Phi_{\mathrm{e}}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}=\frac{2 Q}{\epsilon_{0}}
$$

27.33. Solve: For any closed surface that encloses a total charge $Q_{\text {in }}$, the net electric flux through the closed surface is $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \epsilon_{0}$. The flux through the top surface of the cube is one-sixth of the total:

$$
\Phi_{\mathrm{e} \text { surface }}=\frac{Q_{\mathrm{in}}}{6 \epsilon_{0}}=\frac{10 \times 10^{-9} \mathrm{C}}{6\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)}=0.19 \mathrm{kN} \mathrm{~m}^{2} / \mathrm{C}
$$

27.34. Solve: For any closed surface that encloses a total charge $Q_{i n}$, the net electric flux through the closed surface is $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \epsilon_{0}$. The charge in the box is therefore

$$
\Phi_{\mathrm{e} \text { surface }}=2\left(300 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)+4\left(100 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \Rightarrow Q_{\mathrm{in}}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left(1000 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)=8.85 \mathrm{nC}
$$

Because we did not use dimensions of the box, which are given to two significant figures, we report the answer to three significant figures.
27.35. Solve: (a) The electric field is

$$
\vec{E}=\left(\frac{200}{0.10}\right) \hat{r} \mathrm{~N} / \mathrm{C}=2.0 \hat{r} \mathrm{kN} / \mathrm{C}
$$

So the electric field strength is $2.0 \mathrm{kN} / \mathrm{C}$.
(b) The area of the spherical surface is $A_{\text {sphere }}=4 \pi(0.10 \mathrm{~m})^{2}=0.1257 \mathrm{~m}^{2}$. Hence, the flux is

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=E A_{\text {sphere }}=(2000 \mathrm{~N} / \mathrm{C})\left(0.1257 \mathrm{~m}^{2}\right)=0.25 \mathrm{kN} \mathrm{~m}^{2} / \mathrm{C}
$$

(c) Because $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \epsilon_{0}$,

$$
Q_{\mathrm{in}}=\epsilon_{0} \Phi_{\mathrm{e}}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left(250 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)=2.2 \times 10^{-9} \mathrm{C}=2.2 \mathrm{nC}
$$

27.36. Solve: (a) The electric field at $r=20 \mathrm{~cm}=0.20 \mathrm{~m}$ is

$$
\vec{E}=\left(5000 r^{2}\right) \hat{r} \mathrm{~N} / \mathrm{C}=5000(0.20)^{2} \hat{r} \mathrm{~N} / \mathrm{C}=200 \hat{r} \mathrm{~N} / \mathrm{C}
$$

So the electric field strength is $0.20 \mathrm{kN} / \mathrm{C}$.
(b) The area of the surface is $A_{\text {sphere }}=4 \pi(0.20 \mathrm{~m})^{2}=0.5027 \mathrm{~m}^{2}$. Thus, the electric flux is

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=E A_{\text {sphere }}=(200 \mathrm{~N} / \mathrm{C})\left(0.5027 \mathrm{~m}^{2}\right)=0.10 \mathrm{kN} \mathrm{~m}^{2} / \mathrm{C}
$$

(c) Because $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \epsilon_{0}$,

$$
Q_{\mathrm{in}}=\epsilon_{0} \Phi_{\mathrm{e}}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left(100.5 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}\right)=89 \mathrm{nC}
$$

27.37. Model: The excess charge on a conductor resides on the outer surface.

## Visualize:



Solve: (a) Consider a Gaussian surface surrounding the cavity just inside the conductor. The electric field inside a conductor in electrostatic equilibrium is zero, so $\vec{E}$ is zero at all points on the Gaussian surface. Thus $\Phi_{\mathrm{e}}=0$. Gauss's law tells us that $\Phi_{\mathrm{e}}=Q_{\mathrm{in}} / \epsilon_{0}$, so the net charge enclosed by this Gaussian surface is $Q_{\text {in }}=Q_{\text {point }}+Q_{\text {wall }}=0$. We know that $Q_{\text {point }}=+100 \mathrm{nC}$, so $Q_{\text {wall }}=-100 \mathrm{nC}$. The positive charge in the cavity attracts an equal negative charge to the inside surface.
(b) The conductor started out neutral. If there is -100 nC on the wall of the cavity, then the exterior surface of the conductor was initially +100 nC . Transferring -50 nC to the conductor reduces the exterior surface charge by 50 nC , leaving it at +50 nC .
Assess: The electric field inside the conductor stays zero.
27.38. Model: The excess charge on a conductor resides on the outer surface. The field inside, outside, and within the hollow metal sphere has spherical symmetry.

## Visualize:



The figure shows spherical Gaussian surfaces with radii $r \leq a, a<r<b$, and $r \geq b$. These surfaces match the symmetry of the charge distribution.
Solve: (a) For $r \leq a$, Gauss's law is

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}=\frac{+Q}{\epsilon_{0}}
$$

Notice that the electric field is everywhere perpendicular to the spherical surface. Because of the spherical symmetry of the charge, the electric field magnitude $E$ is the same at all points on the Gaussian surface. Thus,

$$
\Phi_{\mathrm{e}}=E A_{\text {sphere }}=E\left(4 \pi r^{2}\right)=\frac{Q}{\epsilon_{0}} \Rightarrow E=\frac{Q}{4 \pi \epsilon_{0} r^{2}} \Rightarrow \vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}} \hat{r}
$$

where we made use of the fact that $E$ is directed radially outward. The field depends only on the enclosed charge, not on the charge on the outer sphere.
For $a<r<b$, Gauss's law is

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=E A_{\text {sphere }}=E\left(4 \pi r^{2}\right)=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}
$$

Here $Q_{\mathrm{in}}=0 \mathrm{C}$. It is not $+Q$, because the charge in the cavity polarizes the metal sphere in such a way that $E=0$ in the metal. Thus a charge $-Q$ moves to the inner surface. Because the hollow sphere has a net charge of $+2 Q$, the exterior surface now has a charge of $+3 Q$. Thus, the electric field $E=0.0 \mathrm{~N} / \mathrm{C}$.
For $r \geq b$,

$$
Q_{\text {in }}=Q_{\text {exterior }}+Q_{\text {interior }}+Q_{\text {cavity }}=+3 Q+(-Q)+(+Q)=+3 Q
$$

Gauss's law applied to the Gaussian surface at $r \geq b$ yields:

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=E A_{\text {sphere }}=E\left(4 \pi r^{2}\right)=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}=\frac{+3 Q}{\epsilon_{0}} \Rightarrow E=\frac{1}{4 \pi \epsilon_{0}}\left(\frac{3 Q}{r^{2}}\right) \Rightarrow \vec{E}=\frac{1}{4 \pi \epsilon_{0}}\left(\frac{3 Q}{r^{2}}\right) \hat{r}
$$

(b) As determined in part (a), the inside surface of the hollow sphere has a charge of $-Q$, and the exterior surface of the hollow sphere has a charge of $+3 Q$.
Assess: The hollow sphere still has the same charge $+2 Q$ as given in the problem, although the sphere is polarized.
27.39. Model: The ball is uniformly charged. The charge distribution is spherically symmetric.

## Visualize:



The figure shows spherical Gaussian surfaces with radii $r=5 \mathrm{~cm}, 10 \mathrm{~cm}$, and 20 cm . These surfaces match the symmetry of the charge distribution. So, $\vec{E}$ is perpendicular to the Gaussian surface and the value of the field strength is the same at all points on the surface.
Solve: (a) Because the ball is uniformly charged, its charge density is

$$
\rho=\frac{Q}{V}=\frac{Q}{\frac{4}{3} \pi r^{3}}=\frac{3\left(80 \times 10^{-9} \mathrm{C}\right)}{4 \pi(0.20 \mathrm{~m})^{3}}=2.387 \times 10^{-6} \mathrm{C} / \mathrm{m}^{3} \approx 2.4 \times 10^{-6} \mathrm{C} / \mathrm{m}^{3}
$$

(b) Once again, because the ball is uniformly charged,

$$
Q=\rho V=\rho\left(\frac{4 \pi}{3} r^{3}\right)=\left(2.387 \times 10^{-6} \mathrm{C} / \mathrm{m}^{3}\right)\left(\frac{4 \pi}{3} r^{3}\right)=\left(1.00 \times 10^{-5} \mathrm{C} / \mathrm{m}^{3}\right) r^{3}
$$

When $r=5 \mathrm{~cm}, Q_{5}=\left(1.00 \times 10^{-5} \mathrm{C} / \mathrm{m}^{3}\right)(0.05 \mathrm{~m})^{3}=1 \mathrm{nC}$. When $r=10 \mathrm{~cm}$ and $20 \mathrm{~cm}, Q_{10}=10 \mathrm{nC}$ and $Q_{20}=80 \mathrm{nC}$.
(c) Gauss's law is $\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=E A=Q_{\mathrm{in}} / \epsilon_{0}$. For the 5 -cm-radius Gaussian spherical surface,

$$
\frac{Q_{5}}{\epsilon_{0}}=E_{5} A_{5} \Rightarrow E_{5}=\frac{Q_{5}}{\epsilon_{0} A_{5}}=\frac{1.25 \times 10^{-9} \mathrm{C}}{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left[4 \pi(0.05 \mathrm{~m})^{2}\right]}=4.5 \times 10^{3} \mathrm{~N} / \mathrm{C} \approx 5 \mathrm{kN} / \mathrm{C}
$$

Similarly, we can apply Gauss's law to the 10 cm -radius and 20 cm -radius spherical surfaces and obtain $E_{10}=9.0 \times 10^{3} \mathrm{~N} / \mathrm{C}$ and $E_{20}=1.8 \times 10^{4} \mathrm{~N} / \mathrm{C}$.
Assess: We note that $E_{20}=2 E_{10}=4 E_{5}$. This result is consistent with the result of Example 27.4 according to which

$$
E_{\text {inside }}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{R_{3}} r
$$

27.40. Model: The excess charge on a conductor resides on the outer surface. The charge distributions on the two spheres are spherically symmetric.
Visualize: Please refer to Figure P27.40. The Gaussian surfaces with radii $r=8 \mathrm{~cm}, 10 \mathrm{~cm}$, and 17 cm match the symmetry of the charge distribution. Therefore, $\vec{E}$ is perpendicular to these Gaussian surfaces and the field strength has the same value at all points on the Gaussian surface.
Solve: (a) Gauss's law is $\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=Q_{\mathrm{in}} / \epsilon_{0}$. Applying it to a Gaussian surface of radius 8 cm gives

$$
Q_{\text {in }}=-\epsilon_{0} E A_{\text {sphere }}=-\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)(15,000 \mathrm{~N} / \mathrm{C})\left[4 \pi(0.08 \mathrm{~m})^{2}\right]=-1 \times 10^{-8} \mathrm{C}
$$

Because the excess charge on a conductor resides on its outer surface and because we have a solid metal sphere inside our Gaussian surface, $Q_{\text {in }}$ is the charge that is located on the exterior surface of the inner sphere.
(b) In electrostatics, the electric field within a conductor is zero. Applying Gauss's law to a spherical Gaussian surface concentric to the spheres and with a radius just slightly larger than 10 cm , gives

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \Rightarrow Q_{\mathrm{in}}=0 \mathrm{C}
$$

That is, there is no net charge inside the Gaussian sphere. Because the inner sphere has a charge of $-1 \times 10^{-8} \mathrm{C}$, the inside surface of the hollow sphere must have a charge of $+1 \times 10^{-8} \mathrm{C}$.
(c) Applying Gauss's law to a Gaussian surface at $r=17 \mathrm{~cm}$ gives

$$
Q_{\text {in }}=\epsilon_{0} \oint \vec{E} \cdot d \vec{A}=\epsilon_{0} E A_{\text {sphere }}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)(15,000 \mathrm{~N} / \mathrm{C}) 4 \pi(0.17 \mathrm{~m})^{2}=4.8 \times 10^{-8} \mathrm{C}
$$

This value includes the charge on the inner sphere, the charge on the inside surface of the hollow sphere, and the charge on the exterior surface of the hollow sphere due to polarization. Thus,

$$
\begin{aligned}
Q_{\text {exterior hollow }}+ & \left(1 \times 10^{-8} \mathrm{C}\right)+\left(-1 \times 10^{-8} \mathrm{C}\right)=4.8 \times 10^{-8} \mathrm{C} \\
Q_{\text {exterior hollow }}= & 4.8 \times 10^{-8} \mathrm{C}
\end{aligned}
$$

27.41. Model: The charge distribution at the surface of the earth is assumed to be uniform and to have spherical symmetry.

## Visualize:



Due to the symmetry of the charge distribution, $\vec{E}$ is perpendicular to the Gaussian surface and the field strength has the same value at all points on the surface.
Solve: Gauss's law is $\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=Q_{\mathrm{in}} / \epsilon_{0}$. The electric field points inward (negative flux), hence

$$
Q_{\text {in }}=-\epsilon_{0} E A_{\text {sphere }}=-\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)(100 \mathrm{~N} / \mathrm{C}) 4 \pi\left(6.37 \times 10^{6} \mathrm{~m}\right)^{2}=-4.51 \times 10^{5} \mathrm{C}
$$

27.42. Model: The electric field inside the box is $\overrightarrow{0}$.

Visualize: Please refer to Figure 27.32.
Solve: The charge on the capacitor plates produces a uniform electric field pointing to the right. When the metal box is added, the field inside the box is $\vec{E}_{\text {net }}=\overrightarrow{0}$ because there is no charge enclosed within the box. The principle of superposition tells us that $\vec{E}_{\text {net }}=\vec{E}_{\text {capacitor }}+\vec{E}_{\text {box }}$, where $\vec{E}_{\text {box }}$ is the electric field due to the charge on the exterior surface of the box. The net surface charge is zero, because the box as a whole is neutral, but the box is polarized by the capacitor's electric field to where the left end of the box is negative and the right end is positive. Since $\vec{E}_{\text {net }}=\overrightarrow{0}$, we're led to conclude that the field due to the charge on the surface of the box is $\vec{E}_{\text {box }}=-\vec{E}_{\text {capacitor }}$, which gives a uniform field pointing to the left. Note that this is the electric field inside the box due to the surface charge on the box. The field outside the box is much more complex. If we freeze the surface charges and remove the box from the capacitor, we're left with $\vec{E}_{\text {net }}=\vec{E}_{\mathrm{box}}=$ a uniform field pointing to the left. This is shown in the picture below.

27.43. Model: The hollow metal sphere is charged such that the charge distribution is spherically symmetric. Visualize:


The figure shows spherical Gaussian surfaces at $r=4 \mathrm{~cm}$, at $r=8 \mathrm{~cm}$, and at $r=12 \mathrm{~cm}$. These surfaces match the symmetry of the spherical charge distribution, so $\vec{E}$ is perpendicular to the Gaussian surface and the field strength has the same value at all points on a given Gaussian surface.
Solve: The charge on the inside surface is

$$
Q_{\text {inside }}=\left(-100 \mathrm{nC} / \mathrm{m}^{2}\right) 4 \pi(0.06 \mathrm{~m})^{2}=-4.524 \mathrm{nC}
$$

This charge is caused by polarization. That is, the inside surface can be charged only if there is a charge of +4.524 nC at the center that polarizes the metal sphere. Applying Gauss's law to the 4.0 -cm-radius Gaussian surface, which encloses the +4.524 nC charge gives

$$
\begin{gathered}
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=E A_{\text {sphere }}=\frac{Q_{\text {in }}}{\varepsilon_{0}} \\
E=\frac{Q_{\text {in }}}{A \epsilon_{0}}=\frac{4.524 \times 10^{-9} \mathrm{C}}{4 \pi \epsilon_{0}(0.040 \mathrm{~m})^{2}}=\frac{\left(4.524 \times 10^{-9} \mathrm{C}\right)\left(9.0 \times 10^{9} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}{(0.040 \mathrm{~m})^{2}}=2.54 \times 10^{4} \mathrm{~N} / \mathrm{C}
\end{gathered}
$$

Thus, at $r=4.0 \mathrm{~cm}, \vec{E}=\left(2.5 \times 10^{4} \mathrm{~N} / \mathrm{C}\right.$, outward $)$.
There is no electric field inside a conductor in electrostatic equilibrium. So at $r=8 \mathrm{~cm}, E=0 \mathrm{~N} / \mathrm{C}$. Applying Gauss's law to a $12-\mathrm{cm}$-radius sphere,

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=E A_{\text {sphere }}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}
$$

The charge on the outside surface is

$$
\begin{gathered}
Q_{\text {outside }}=\left(+100 \mathrm{nC} / \mathrm{m}^{2}\right) 4 \pi(0.10 \mathrm{~m})^{2}=1.257 \times 10^{-8} \mathrm{C} \\
Q_{\text {in }}=Q_{\text {outside }}+Q_{\text {inside }}+Q_{\text {center }}=1.257 \times 10^{-8} \mathrm{C}-0.452 \times 10^{-8} \mathrm{C}+0.452 \times 10^{-8} \mathrm{C}=1.257 \times 10^{-8} \mathrm{C} \\
E=\frac{Q_{\text {in }}}{\epsilon_{0} A}=\frac{1.257 \times 10^{-8} \mathrm{C}}{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right) 4 \pi(0.12 \mathrm{~m})^{2}}=7.86 \times 10^{3} \mathrm{~N} / \mathrm{C}
\end{gathered}
$$

At $r=12 \mathrm{~cm}, \vec{E}=\left(7.9 \times 10^{3} \mathrm{~N} / \mathrm{C}\right.$, outward $)$.
27.44. Model: The charged hollow spherical shell is assumed to have a spherically symmetric charge distribution. Visualize:


The figure shows two spherical Gaussian surfaces at $r<R$ and $r>R$. These surfaces match the symmetry of the spherical charge distribution, so $\vec{E}$ is everywhere perpendicular to the Gaussian surface and the field strength has the same value at all points on a given surface.
Solve: (a) Gauss's law applied to the Gaussian surface inside the sphere $(r<R)$ gives

$$
\begin{gathered}
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=E A_{\text {sphere }}=\frac{Q_{\text {in }}}{\epsilon_{0}} \Rightarrow E\left(4 \pi r^{2}\right)=\frac{+q}{\epsilon_{0}} \Rightarrow E=\frac{1}{4 \pi \epsilon_{0}} \frac{+q}{r^{2}} \\
\vec{E}=\left(\frac{1}{4 \pi \epsilon_{0}} \frac{+q}{r^{2}}, \text { outward }\right)=\frac{1}{4 \pi \epsilon_{0}} \frac{q}{r^{2}} \hat{r}
\end{gathered}
$$

(b) Gauss's law applied to the Gaussian surface outside the sphere $(r>R)$ is

$$
\begin{gathered}
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=-E A_{\text {sphere }}=\frac{Q_{\text {in }}}{\epsilon_{0}}=\frac{-2 q+q}{\epsilon_{0}}=\frac{-q}{\epsilon_{0}} \Rightarrow E=\frac{1}{4 \pi \epsilon_{0}} \frac{+q}{r^{2}} \\
\vec{E}=\left(\frac{1}{4 \pi \epsilon_{0}} \frac{q}{r^{2}}, \text { inward }\right)=-\frac{1}{4 \pi \epsilon_{0}} \frac{q}{r^{2}} \hat{r}
\end{gathered}
$$

Assess: A uniform spherical shell of charge has the same electric field at $r>R$ as a point charge placed at the center of the shell. Additionally, the electric field for $r<R$ is zero, meaning that the shell of charge exerts no electric force on a charged particle inside the shell.
27.45. Model: The hollow plastic ball has a charge uniformly distributed on its outer surface. This distribution leads to a spherically symmetric electric field. Assume $Q>0$.

## Visualize:



The figure shows Gaussian surfaces at $r<R$ and $r>R$.
Solve: (a) Gauss's law for the Gaussian surface for $r<R$ where $Q_{\mathrm{in}}=0$ is

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\mathrm{in}}}{\varepsilon_{0}}=0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C} \Rightarrow E=0 \mathrm{~N} / \mathrm{C}
$$

(b) Gauss's law for the Gaussian surface for $r>R$ is

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=E A_{\text {sphere }}=\frac{Q_{\text {in }}}{\epsilon_{0}}=\frac{Q}{\epsilon_{0}} \Rightarrow E A_{\text {sphere }}=\frac{Q}{\epsilon_{0}} \Rightarrow E=\frac{Q}{\epsilon_{0} A_{\text {sphere }}}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}}
$$

Because $Q>0$, the electric field points radially outward, so $\vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}} \hat{r}$
Assess: A uniform spherical shell of charge has the same electric field at $r>R$ as a point charge placed at the center of the sphere. Additionally, the shell of charge exerts no electric force on a charged particle inside the shell.
27.46. Model: The charge distributions of the ball and the metal shell are assumed to have spherical symmetry. Visualize:


Spherical Gaussian surfaces
The spherical symmetry of the charge distribution tells us that the electric field points radially inward or outward. We will therefore choose Gaussian surfaces to match the spherical symmetry of the charge distribution and the field. The figure shows four Gaussian surfaces in the four regions: $r \leq a, a<r<b, b \leq r \leq c$, and $r>c$.

Solve: Gauss's law is $\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=Q_{\mathrm{in}} / \epsilon_{0}$. Applying it to the region $r \leq a$, where the charge is negative so $\vec{E}$ points inward, we get

$$
-E A_{\text {sphere }}=-E\left(4 \pi r^{2}\right)=\frac{+\rho\left(\frac{4 \pi}{3} r^{3}\right)}{\epsilon_{0}} \Rightarrow E=\frac{-\rho r}{3 \epsilon_{0}}
$$

Here $\rho=-Q /\left(\frac{4 \pi}{3} a^{3}\right)$ is the charge density $\left(\mathrm{C} / \mathrm{m}^{3}\right)$. Thus

$$
\vec{E}=\left(\frac{1}{4 \pi \varepsilon_{0}} \frac{Q r}{a^{3}}, \text { inward }\right)=-\frac{1}{4 \pi \varepsilon_{0}} \frac{Q r}{a^{3}} \hat{r}
$$

Applying Gauss's law to the region $a<r<b$,

$$
-E A_{\text {sphere }}=\frac{-Q}{\epsilon_{0}} \Rightarrow E=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}} \Rightarrow \vec{E}=-\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}} \hat{r}
$$

$\vec{E}=\overrightarrow{0}$ in the region $b \leq r \leq \mathrm{c}$ because this is a conductor in electrostatic equilibrium.
To apply Gauss's law to the region $r>c$, we use $Q_{\mathrm{in}}=-Q+2 Q=+Q$. Thus,

$$
E A_{\text {sphere }}=\frac{Q}{\epsilon_{0}} \Rightarrow E=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}} \Rightarrow \vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}} \hat{r}
$$

27.47. Model: The three planes of charge are infinite planes.

Visualize:


With planar symmetry the electric field can point straight toward or away from the symmetry plane. The three planes are labeled as P (top), $\mathrm{P}^{\prime}$, and $\mathrm{P}^{\prime \prime}$ (bottom).
Solve: From Example 27.6, the electric field of an infinite charged plane of charge density $\eta$ is

$$
E_{\text {plane }}=\frac{\eta}{2 \epsilon_{0}} \Rightarrow E_{\mathrm{P}}=E_{\mathrm{P}^{\prime \prime}}=\frac{\eta}{4 \epsilon_{0}}=\frac{E_{\mathrm{P}^{\prime}}}{2}
$$

In region 1 the three electric fields are

$$
\vec{E}_{\mathrm{P}}=-\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime}}=\frac{\eta}{2 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime \prime}}=-\frac{\eta}{4 \epsilon_{0}} \hat{j}
$$

Adding the three contributions, we get $\vec{E}_{\text {net }}=\overrightarrow{0} \mathrm{~N} / \mathrm{C}$.
In region 2 the three electric fields are

$$
\vec{E}_{\mathrm{P}}=\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime}}=\frac{\eta}{2 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime \prime}}=\frac{-\eta}{4 \epsilon_{0}} \hat{j}
$$

Thus, $\vec{E}_{\text {net }}=\left[\eta /\left(2 \epsilon_{0}\right)\right] \hat{j}$.
In region 3,

$$
\vec{E}_{\mathrm{P}}=\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime}}=-\frac{\eta}{2 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime \prime}}=-\frac{\eta}{4 \epsilon_{0}} \hat{j}
$$

Thus, $\vec{E}_{\text {net }}=-\left[\eta /\left(2 \epsilon_{0}\right)\right] \hat{j}$.
In region 4,

$$
\vec{E}_{\mathrm{P}}=\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime}}=-\frac{\eta}{2 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime \prime}}=\frac{\eta}{4 \epsilon_{0}} \hat{j}
$$

Thus $\vec{E}_{\text {net }}=\overrightarrow{0} \mathrm{~N} / \mathrm{C}$.
27.48. Model: The charge has planar symmetry, so the electric field must point toward or away from the slab. Furthermore, the field strength must be the same at equal distances on either side of the center of the slab.

## Visualize:



Choose Gaussian surfaces to be cylinders of length $2 z$ centered on the $z=0$ plane. The ends of the cylinders have area $A$.
Solve: (a) For the Gaussian cylinder inside the slab, with $z<z_{0}$, Gauss's law is

$$
\oint \vec{E} \cdot d \vec{A}=\int_{\text {top }} \vec{E} \cdot d \vec{A}+\int_{\text {bottom }} \vec{E} \cdot d \vec{A}+\int_{\text {sides }} \vec{E} \cdot d \vec{A}=\frac{Q_{\text {in }}}{\epsilon_{0}}
$$

The field is parallel to the sides, so the third integral is zero. The field emerges from both ends, so the first two integrals are the same. The charge enclosed is the volume of the cylinder multiplied by the charge density, or $Q_{\text {in }}=\rho V=\rho(2 z A)$. Thus

$$
\frac{Q_{\text {in }}}{\epsilon_{0}}=\frac{\rho(2 z A)}{\epsilon_{0}}=\int_{\text {top }} \vec{E} \cdot d \vec{A}+\int_{\text {bottom }} \vec{E} \cdot d \vec{A}+0=2 E A \Rightarrow E=\frac{\rho z}{\epsilon_{0}}
$$

The field increases linearly with distance from the center. The sign of $\rho$ determines the direction of the electric field,
so $\vec{E}\left(|z| \leq z_{0}\right)=\frac{\rho z}{\epsilon_{0}} \hat{k}$.
(b) The analysis is the same for the cylinder that extends outside the slab, with $z>z_{0}$, except that the enclosed charge $Q=\rho\left(2 z_{0} A\right)$ is that within a cylinder of length $2 z_{0}$ rather than $2 z$. Thus

$$
\frac{Q_{\text {in }}}{\epsilon_{0}}=\frac{\rho\left(2 z_{0} A\right)}{\epsilon_{0}}=\int_{\text {top }} \vec{E} \cdot d \vec{A}+\int_{\text {bottom }} \vec{E} \cdot d \vec{A}+0=2 E A \Rightarrow E=\frac{\rho z_{0}}{\epsilon_{0}}
$$

The field strength outside the slab is constant, and it matches the result of part (a) at the boundary. The sign of $r$ determines the direction of the electric field, so $\vec{E}\left(z \geq z_{0}\right)=\frac{\rho z_{0}}{\epsilon_{0}} \hat{k}$
(c)

27.49. Model: The infinitely wide plane of charge with surface charge density $\eta$ polarizes the infinitely wide conductor.

## Visualize:



Because $\vec{E}=\overrightarrow{0}$ in the metal there will be an induced charge polarization. The face of the conductor adjacent to the plane of charge is negatively charged. This makes the other face of the conductor positively charged. We thus have three infinite planes of charge. These are P (top conducting face), $\mathrm{P}^{\prime}$ (bottom conducting face), and $\mathrm{P}^{\prime \prime}$ (plane of charge).
Solve: Let $\eta_{1}, \eta_{2}$, and $\eta_{3}$ be the surface charge densities of the three surfaces with $\eta_{2}<0$. The electric field due to a plane of charge with surface charge density $\eta$ is $E=\eta /\left(2 \epsilon_{0}\right)$. Because the electric field inside a conductor is zero (region 2),

$$
\vec{E}_{\mathrm{P}}+\vec{E}_{\mathrm{P}^{\prime}}+\vec{E}_{\mathrm{P}^{\prime \prime}}=\overrightarrow{0} \mathrm{~N} / \mathrm{C} \Rightarrow-\frac{\eta_{1}}{2 \epsilon_{0}} \hat{j}+\frac{\eta_{2}}{2 \epsilon_{0}} \hat{j}+\frac{\eta_{3}}{2 \epsilon_{0}} \hat{j}=\overrightarrow{0} \mathrm{~N} / \mathrm{C} \Rightarrow-\eta_{1}+\eta_{2}+\eta=0 \mathrm{C} / \mathrm{m}^{2}
$$

We have made the substitution $\eta_{3}=\eta$. Also note that the field inside the conductor is downward from planes P and $\mathrm{P}^{\prime}$ and upward from $\mathrm{P}^{\prime \prime}$. Because $\eta_{1}+\eta_{2}=0 \mathrm{C} / \mathrm{m}^{2}$, because the conductor is neutral, $\eta_{2}=-\eta_{1}$. The above equation becomes

$$
-\eta_{1}-\eta_{1}+\eta=0 \mathrm{C} / \mathrm{m}^{2} \Rightarrow \eta_{1}=\frac{1}{2} \eta \Rightarrow \eta_{2}=-\frac{1}{2} \eta
$$

We are now in a position to find electric field in regions 1-4.
For region 1,

$$
\vec{E}_{\mathrm{P}}=\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime}}=-\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime \prime}}=\frac{\eta}{2 \epsilon_{0}} \hat{j}
$$

The electric field is $\vec{E}_{\text {net }}=\vec{E}_{\mathrm{P}}+\vec{E}_{\mathrm{P}^{\prime}}+\vec{E}_{\mathrm{P}^{\prime \prime}}=\left[\eta /\left(2 \epsilon_{0}\right)\right] \hat{j}$.
In region $2, \vec{E}_{\text {net }}=\overrightarrow{0} \mathrm{~N} / \mathrm{C}$. In region 3,

$$
\vec{E}_{\mathrm{P}}=-\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime}}=\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime \prime}}=\frac{\eta}{2 \epsilon_{0}} \hat{j}
$$

The electric field is $\vec{E}_{\text {net }}=\left[\eta /\left(2 \epsilon_{0}\right)\right] \hat{j}$.
In region 4,

$$
\vec{E}_{\mathrm{P}}=-\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime}}=\frac{\eta}{4 \epsilon_{0}} \hat{j} \quad \vec{E}_{\mathrm{P}^{\prime \prime}}=-\frac{\eta}{2 \epsilon_{0}} \hat{j}
$$

The electric field is $\vec{E}_{\text {net }}=-\left[\eta /\left(2 \epsilon_{0}\right)\right] \hat{j}$.
27.50. Model: Assume the metal slabs are large enough to model as infinite planes. Then the electric field has planar symmetry, pointing either toward or away from the planes.
Visualize:


One Gaussian surface is a cylinder with end-areas $a$ extending past the two metal slabs. A second Gaussian surface ends in region 3, the space between the slabs.
Solve: (a) Because these are metals, we immediately know that $\vec{E}_{2}=\vec{E}_{4}=\overrightarrow{0}$. To find the fields outside the slabs, consider the Gaussian surface on the left. The electric field everywhere points up or down, so there is no flux through the sides of this cylinder. Because both slabs are positive, the electric fields in regions 1 and 5 point outward. Gauss's law gives

$$
\oint \vec{E} \cdot d \vec{A}=\int_{\text {top }} \vec{E} \cdot d \vec{A}+\int_{\text {bottom }} \vec{E} \cdot d \vec{A}+\int_{\text {sides }} \vec{E} \cdot d \vec{A}=E_{1} a+E_{5} a+0=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}
$$

The Gaussian surface encloses both the upper and lower surfaces of the top slab. The total charge per unit area on both surfaces of this slab is $Q_{1} / A=Q / A$, so the charge enclosed within the cylinder is $Q a / A$. (For this calculation we don't need to know how the charge is distributed between the two surfaces.) Similarly, the enclosed charge on the lower slab is $Q_{2} a / A=2 Q a / A$. Thus

$$
E_{1} a+E_{5} a=\frac{Q a}{\epsilon_{0} A}+\frac{2 Q a}{\epsilon_{0} A} \Rightarrow E_{1}+E_{5}=\frac{3 Q}{\epsilon_{0} A}
$$

Fields $\vec{E}_{1}$ and $\vec{E}_{5}$ are both a superposition of the fields of four sheets of surface charge. Because the field of a plane of charge is independent of distance from the plane, the superposition at points above the top plane must be the same magnitude, but opposite direction, as the superposition at points below the bottom plane. Consequently, $E_{1}=E_{5}$ (same field strengths). This is a rather subtle point in the reasoning and one worth thinking about. Thus $E_{1}=E_{5}=3 Q /\left(2 \epsilon_{0} A\right)$.
Now consider the Gaussian surface on the right. The lower slab is more positive than the upper slab, so the electric field in region 3 must point upward, into the lower face of this cylinder. Thus

$$
\oint \vec{E} \cdot d \vec{A}=\int_{\text {top }} \vec{E} \cdot d \vec{A}+\int_{\text {bottom }} \vec{E} \cdot d \vec{A}+\int_{\text {sides }} \vec{E} \cdot d \vec{A}=E_{1} a-E_{3} a+0=\frac{Q_{\text {in }}}{\epsilon_{0}}
$$

where the minus sign with $E_{3}$ is because of the direction. We know $E_{1}$, and we've already determined that $Q_{\text {in }}=Q a / A$. Thus

$$
E_{3}=E_{1}-\frac{Q}{\epsilon_{0} A}=\frac{3 Q}{2 \epsilon_{0} A}-\frac{Q}{\epsilon_{0} A}=\frac{Q}{2 \epsilon_{0} A}
$$

Summarizing, $E_{1}=3 Q /\left(2 \epsilon_{0} A\right), \quad E_{2}=0, \quad E_{3}=Q /\left(2 \epsilon_{0} A\right), \quad E_{4}=0$, and $E_{5}=3 Q /\left(2 \epsilon_{0} A\right)$.
(b) The electric field at the surface of a conductor is $E=\eta / \epsilon_{0}$. We can use the known fields and $\eta=\epsilon_{0} E$ to find the four surface charge densities. At surface a, $\vec{E}_{1}$ points away from the surface. Thus

$$
\eta_{\mathrm{a}}=+\epsilon_{0} E_{1}=+\epsilon_{0} \frac{3 Q}{2 \varepsilon_{0} A}=+\frac{3}{2} \frac{Q}{A}
$$

At surface b, $\vec{E}_{3}$ points toward the surface. Thus

$$
\eta_{\mathrm{b}}=-\epsilon_{0} E_{3}=-\epsilon_{0} \frac{Q}{2 \epsilon_{0} A}=-\frac{1}{2} \frac{Q}{A}
$$

Surface c is opposite to surface b , because the field points away from the surface, so $\eta_{\mathrm{c}}=+Q /(2 A)$. Finally, at surface d, the field points away from the surface and has the same strength as $\vec{E}_{1}$, hence $\eta_{\mathrm{d}}=+3 Q /(2 A)$.
Assess: Notice that $\eta_{\mathrm{a}}+\eta_{\mathrm{b}}=Q / A$ and $\eta_{\mathrm{c}}+\eta_{\mathrm{d}}=2 Q / A$. This is the expected "net" surface charge density for slabs with total charge $Q$ and $2 Q$.
27.51. Model: A long, charged wire can be modeled as an infinitely long line of charge.

## Visualize:




End view

The figure shows an infinitely long line of charge that is surrounded by a hollow metal cylinder of radius $R$. The symmetry of the situation indicates that the only possible shape of the electric field is to point straight in or out from the wire. The shape of the field suggests that we choose our Gaussian surface to be a cylinder of radius $r$ and length $L$, centered on the wire.
Solve: (a) For the region $r<R$, Gauss's law is

$$
\begin{aligned}
\Phi_{\mathrm{e}} & \oint \vec{E} \cdot d \vec{A}=\frac{Q_{\text {in }}}{\epsilon_{0}} \Rightarrow \int_{\text {top }} \vec{E} \cdot d \vec{A}+\int_{\text {bottom }} \vec{E} \cdot d \vec{A}+\int_{\text {side }} \vec{E} \cdot d \vec{A}=\frac{\lambda L}{\epsilon_{0}} \\
& 0 \mathrm{Nm}^{2} / \mathrm{C}+0 \mathrm{Nm}^{2} / \mathrm{C}+\vec{E} \cdot \vec{A}_{\text {side }}=\frac{\lambda L}{\epsilon_{0}} \Rightarrow E(2 \pi r) L=\frac{\lambda L}{\epsilon_{0}} \\
& E=\frac{\lambda}{2 \pi \epsilon_{0}} \frac{1}{r} \Rightarrow \vec{E}=\left(\frac{\lambda}{2 \pi \epsilon_{0}} \frac{1}{r}, \text { outward }\right)=\frac{\lambda}{2 \pi \epsilon_{0}} \frac{\hat{r}}{r}
\end{aligned}
$$

(b) Applying Gauss's law to the Gaussian surface at $r>R$,

$$
\oint \vec{E} \cdot d \vec{A}=\int_{\text {top }} \vec{E} \cdot d \vec{A}+\int_{\text {bottom }} \vec{E} \cdot d \vec{A}+\int_{\text {side }} \vec{E} \cdot d \vec{A}=\frac{Q_{\text {in }}}{\epsilon_{0}}
$$

$$
\begin{gathered}
0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}+0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}+\vec{E} \cdot \vec{A}_{\text {wall }}=\frac{Q_{\text {in }}}{\epsilon_{0}} \\
E(2 \pi r L)=\frac{Q_{\text {line }}+Q_{\text {cylinder }}}{\epsilon_{0}}=\frac{\lambda L+2 \lambda L}{\epsilon_{0}} \Rightarrow E=\frac{3 \lambda}{2 \pi \epsilon_{0}} \frac{1}{r} \Rightarrow \vec{E}=\frac{3 \lambda}{2 \pi \epsilon_{0}} \frac{\hat{r}}{r}
\end{gathered}
$$

27.52. Model: A long, charged cylinder is assumed to be infinite and to have linear symmetry. Visualize:


The cylindrical symmetry of the situation indicates that the only possible shape of the electric field is to point straight in or out from the cylinder. The shape of the field suggests that we choose our Gaussian surface to be a cylinder of radius $r$ and length $L$, which is concentric with the charged cylinder.
Solve: (a) For $r \geq R$, Gauss's law gives

$$
\begin{gathered}
\oint \vec{E} \cdot d \vec{A}=\int_{\text {left }} \vec{E} \cdot d \vec{A}+\int_{\text {right }} \vec{E} \cdot d \vec{A}+\int_{\text {wall }} \vec{E} \cdot d \vec{A}=\frac{Q_{\text {in }}}{\epsilon_{0}} \\
0 \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}+0 \mathrm{Nm}^{2} / \mathrm{C}+E A=\frac{Q_{\text {in }}}{\epsilon_{0}} \Rightarrow E=\frac{\lambda L}{(2 \pi r L) \epsilon_{0}}=\frac{\lambda}{2 \pi \epsilon_{0} r} \Rightarrow \vec{E}=\left(\frac{\lambda}{2 \pi \epsilon_{0}} \frac{1}{r}\right) \hat{r}
\end{gathered}
$$

(b) For $r \leq R$, Gauss's law is

$$
E A=\frac{Q_{\text {in }}}{\epsilon_{0}} \Rightarrow E=\frac{Q_{\text {in }}}{A \epsilon_{0}}=\frac{\rho\left(\pi r^{2} L\right)}{(2 \pi r L) \epsilon_{0}}
$$

An expression for the volume charge density $\rho$ in terms of the linear charge density can be calculated by considering the charge on a cylinder of length $d$ and radius $R$ :

$$
\begin{gathered}
\lambda d=\rho\left(\pi R^{2} d\right) \Rightarrow \rho=\frac{\lambda}{\pi R^{2}} \\
E=\left(\frac{\lambda}{\pi R^{2}}\right) \frac{\pi r^{2} L}{(2 \pi r L) \epsilon_{0}}=\frac{1}{2 \pi \epsilon_{0}} \frac{\lambda r}{R^{2}} \Rightarrow \vec{E}=\frac{1}{2 \pi \epsilon_{0}} \frac{\lambda r}{R^{2}} \hat{r}
\end{gathered}
$$

(c) At $r=R$,

$$
E_{r=R}=\frac{1}{2 \pi \epsilon_{0}} \frac{\lambda R}{R^{2}} \hat{r}=\frac{1}{2 \pi \epsilon_{0}} \frac{\lambda}{R} \hat{r}
$$

27.53. Model: The charge distribution in the shell has spherical symmetry. Visualize:


The spherical surfaces of radii $r \geq R_{\text {out }}, r \leq R_{\text {in }}$, and $R_{\text {in }} \leq r \leq R_{\text {out }}$, concentric with the spherical shell, are Gaussian surfaces.
Solve: (a) Gauss's law for the Gaussian surface $r \geq R_{\text {out }}$ is

$$
\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \Rightarrow E\left(4 \pi r^{2}\right)=\frac{Q}{\epsilon_{0}} \Rightarrow E=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}} \Rightarrow \vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}} \hat{r}
$$

The vector form comes from the fact that the field is directed radially outward.
(b) For $r \leq R_{\text {in }}$, Gauss's law is

$$
\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}=\frac{0 \mathrm{C}}{\epsilon_{0}} \Rightarrow \vec{E}=\overrightarrow{0}
$$

(c) For $R_{\text {in }} \leq r \leq R_{\text {out }}$, Gauss's law is

$$
\begin{gathered}
\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \Rightarrow E\left(4 \pi r^{2}\right)=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \\
Q_{\mathrm{in}}=\frac{4 \pi}{3}\left(r^{3}-R_{\mathrm{in}}^{3}\right) \rho=\frac{4 \pi}{3} \frac{\left(r^{3}-R_{\mathrm{in}}^{3}\right) Q}{\frac{4 \pi}{3}\left(R_{\mathrm{out}}^{3}-R_{\mathrm{in}}^{3}\right)}=Q\left(\frac{r^{3}-R_{\mathrm{in}}^{3}}{R_{\mathrm{out}}^{3}-R_{\mathrm{in}}^{3}}\right) \\
E=\frac{1}{4 \pi r^{2}} \frac{Q}{\epsilon_{0}} \frac{r^{3}-R_{\mathrm{in}}^{3}}{R_{\mathrm{out}}^{3}-R_{\mathrm{in}}^{3}} \Rightarrow \vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}}\left(\frac{r^{3}-R_{\mathrm{in}}^{3}}{R_{\mathrm{out}}^{3}-R_{\mathrm{in}}^{3}}\right) \hat{r}
\end{gathered}
$$

(d) The result obtained in part (c) for the electric field simplifies to $\vec{E}=\overrightarrow{0}$, when $r=R_{\text {in }}$ which is the result obtained in part (b). Furthermore, at $r=R_{\text {out }}$, the electric field obtained in part (c) becomes

$$
\vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{R_{\text {out }}^{2}} \hat{r}
$$

which is the same as the electric field obtained in part (a).
27.54. Model: Assume that the negative charge uniformly distributed in the atom has spherical symmetry. Visualize:


The nucleus is a positive point charge $+Z e$ at the center of a sphere of radius $R$. The spherical symmetry of the charge distribution tells us that the electric field must be radial. We choose a spherical Gaussian surface to match the spherical symmetry of the charge distribution and the field. The Gaussian surface is at $r<R$, which means that we will calculate the amount of charge contained in this surface.
Solve: (a) Gauss's law is $\oint \vec{E} \cdot d \vec{A}=Q_{\mathrm{in}} / \epsilon_{0}$. The amount of charge inside is

$$
\begin{gathered}
Q_{\mathrm{in}}=\rho\left(\frac{4 \pi}{3} r^{3}\right)+Z e=\frac{(-Z e)}{\left(\frac{4 \pi}{3} R^{3}\right)}\left(\frac{4 \pi}{3} r^{3}\right)+Z e=-(Z e) \frac{r^{3}}{R^{3}}+Z e=Z e\left[1-\frac{r^{3}}{R^{3}}\right] \\
E_{\text {in }}\left(4 \pi r^{2}\right)=\frac{Z e}{\epsilon_{0}}\left[1-\frac{r^{3}}{R^{3}}\right] \Rightarrow E_{\text {in }}=\frac{Z e}{4 \pi \epsilon_{0}}\left[\frac{1}{r^{2}}-\frac{r}{R^{3}}\right]
\end{gathered}
$$

(b) At the surface of the atom, $r=R$. Thus,

$$
E_{\text {in }}=\frac{Z e}{4 \pi \epsilon_{0}}\left[\frac{1}{R^{2}}-\frac{R}{R^{3}}\right]=0 \mathrm{~N} / \mathrm{C}
$$

This is an expected result, which can be quickly obtained from Gauss's law. Applying Gauss's law to a Gaussian surface just outside $r=R$. Because the atom is electrically neutral, $Q_{i n}=0$. Thus

$$
\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}=0 \Rightarrow E=0 \mathrm{~N} / \mathrm{C}
$$

(c) For a uranium atom, the electric field strength at $r=\frac{1}{2} R=0.050 \mathrm{~nm}$ is

$$
E_{\text {in }}=92\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(9.0 \times 10^{9} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left[\frac{1}{(0.050 \mathrm{~nm})^{2}}-\frac{(0.050 \mathrm{~nm})}{(0.10 \mathrm{~nm})^{3}}\right]=4.6 \times 10^{13} \mathrm{~N} / \mathrm{C}
$$

27.55. Model: The long thin wire is assumed to be an infinite line of charge.

Visualize: Please refer to Figure CP27.55. The cube of edge length $L$ is centered on the line charge with a linear charge density $\lambda$. Although the line charge has cylindrical symmetry, we will take the cube as our Gaussian surface.
Solve: (a) The electric flux through an area $d \vec{A}$ in the $y z$ plane is $d \Phi=\vec{E} \cdot d \vec{A}$. The electric field $\vec{E}$ due to an infinite line of charge at a distance $s$ from the line charge is

$$
\vec{E}=\frac{\lambda}{2 \pi \epsilon_{0}} \frac{1}{r} \hat{r}=\frac{\lambda}{2 \pi_{0}} \frac{1}{\sqrt{y^{2}+(L / 2)^{2}}} \hat{r}
$$

Also $d \vec{A}=L d y \hat{i}$. Thus,

$$
\begin{aligned}
d \Phi & =\frac{\lambda}{2 \pi \epsilon_{0}} \frac{1}{\sqrt{y^{2}+(L / 2)^{2}}} L d y(\hat{r} \cdot \hat{i})=\frac{\lambda L d y}{2 \pi \epsilon_{0}} \frac{1}{\sqrt{y^{2}+(L / 2)^{2}}} \cos \theta \\
& =\frac{\lambda L d y}{2 \pi \epsilon_{0} \sqrt{y^{2}+(L / 2)^{2}}} \frac{L / 2}{\sqrt{y^{2}+(L / 2)^{2}}}=\frac{\lambda L^{2} d y}{4 \pi \epsilon_{0}\left[y^{2}+(L / 2)^{2}\right]}
\end{aligned}
$$

(b) The expression for the flux $d \Phi$ can now be integrated to obtain the total flux through this face as follows:

$$
\begin{aligned}
\Phi & =\int d \Phi=\int_{-L / 2}^{L / 2} \frac{\lambda L^{2} d y}{4 \pi \epsilon_{0}\left[y^{2}+(L / 2)^{2}\right]}=\frac{\lambda L^{2}}{4 \pi \epsilon_{0}}\left[\frac{2}{L} \tan ^{-1}\left(\frac{2 y}{L}\right)\right]_{-L / 2}^{L / 2} \\
& =\frac{\lambda L}{2 \pi \epsilon_{0}}\left[\tan ^{-1}(1)-\tan ^{-1}(-1)\right]=\frac{\lambda L}{2 \pi \epsilon_{0}}\left[\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right]=\frac{\lambda L}{4 \epsilon_{0}}
\end{aligned}
$$

(c) Because there are four faces through which the flux flows, the net flux through the cube is

$$
\Phi_{\mathrm{e}}=4 \Phi=\frac{\lambda L}{\epsilon_{0}}=\frac{\left(Q_{\mathrm{in}} / L\right) L}{\epsilon_{0}}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}
$$

27.56. Model: The field has cylindrical symmetry.

## Visualize:




Solve: (a) The volume charge density $\rho(r)=r \rho_{0} / R$ is linearly proportional to $r$. The graph is shown above.
(b) Consider the cylindrical shell of length $L$, radius $r$, and thickness $d r$ shown in the diagram. The charge within a small volume $d V$ is

$$
d q=\rho d V=\frac{\rho_{0} r}{R}(2 \pi r) d r L=\frac{2 \pi \rho_{0} L}{R} r^{2} d r
$$

Integrating this expression to obtain the total charge in the cylinder:

$$
Q=\int d q=\frac{2 \pi \rho_{0} L}{R} \int_{0}^{R} r^{2} d r=\frac{2 \pi \rho_{0} L}{R}\left[\frac{r^{3}}{3}\right]_{0}^{R}=\frac{2 \pi \rho_{0} L R^{3}}{3 R}=\lambda L \Rightarrow \rho_{0}=\frac{3 \lambda}{2 \pi R^{2}}
$$

(c) Consider the cylindrical Gaussian surface of length $l$ at $r<R$ shown in the figure. Gauss's law is $\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=Q_{\mathrm{in}} / \epsilon_{0}$. The charge on the inside of the Gaussian surface is

$$
\begin{gathered}
Q_{\mathrm{in}}=\int d q=\int \rho d V=\int_{0}^{r} \frac{2 \pi \rho_{0} l}{R} r^{2} d r=\frac{2 \pi \rho_{0} l r^{3}}{3 R}=\frac{2 \pi}{3 R}\left(\frac{3 \lambda}{2 \pi R^{2}}\right) l r^{3}=\lambda l\left(\frac{r^{3}}{R^{3}}\right) \\
\oint \vec{E} \cdot d \vec{A}=E(2 \pi r l)=\frac{Q_{\mathrm{in}}}{\epsilon_{0}}=\frac{\lambda l}{\epsilon_{0}}\left(\frac{r^{3}}{R^{3}}\right) \Rightarrow E=\frac{\lambda}{2 \pi \epsilon_{0}}\left(\frac{r^{2}}{R^{3}}\right)
\end{gathered}
$$

The last step is justified because the electric field has radial dependence. The direction of the electric field is determined by the sign of 1 , so $\vec{E}=\frac{\lambda}{2 \pi \epsilon_{0}}\left(\frac{r^{2}}{R^{3}}\right) \hat{r}$.
(d) The expression for the electric field strength simplifies at $r=R$ to

$$
E=\frac{\lambda}{2 \pi \epsilon_{0}} \frac{R^{2}}{R^{3}}=\frac{\lambda}{2 \pi \epsilon_{0} R}
$$

This is the same result as obtained in Example 27.5 for a long, charged wire.

### 27.57. Visualize:



Solve: (a) Consider the spherical shell of radius $r$ and thickness $d r$ shown in the figure. The charge $d q$ within a small volume $d V$ is

$$
d q=\rho d V=\frac{C}{r^{2}}\left(4 \pi r^{2}\right) d r=4 \pi C d r
$$

Integrating this expression to obtain the total charge in the sphere:

$$
Q=\int d q=\int_{0}^{R} 4 \pi C d r=4 \pi C R \quad \Rightarrow \quad C=\frac{Q}{4 \pi R}
$$

(b) Consider the spherical Gaussian surface at $r<R$ shown in the figure. Gauss's law applied to this surface is

$$
\Phi_{\mathrm{e}}=\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \Rightarrow E\left(4 \pi r^{2}\right)=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \Rightarrow E=\frac{Q_{\mathrm{in}}}{4 \pi \epsilon_{0} r^{2}}
$$

Using the results from part (a),

$$
\begin{aligned}
Q_{\mathrm{in}}=\int d q & =\int \rho d V=\int_{0}^{r} \frac{C}{r^{2}} 4 \pi r^{2} d r=\int_{0}^{r}\left(\frac{Q}{4 \pi R}\right) 4 \pi d r=\frac{Q}{R} r \\
E & =\left(\frac{Q}{R} r\right) \frac{1}{4 \pi \epsilon_{0} r^{2}} \Rightarrow \vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{R r} \hat{r}
\end{aligned}
$$

(c) At $r=R$, the equation for the electric field obtained in part (b) simplifies to

$$
\vec{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{R^{2}} \hat{r}
$$

This is the same result as obtained in Example 27.3. The result was expected because a spherical charge behaves, for $r \geq R$, as if the entire charge were at the center.
27.58. Model: The charge density within the sphere is nonuniform. We will assume that the distribution has spherical symmetry.

## Visualize:



Solve: (a) Consider the thin shell of width $d r$ at a distance $r$ from the center shown in the figure. The volume of this thin shell is $d V=\left(4 \pi r^{2}\right) d r$. The charge contained in this volume is

$$
\begin{aligned}
Q & =\int d q=\int_{0}^{R} 4 \pi r^{2} \rho d r=\int_{0}^{R} 4 \pi r^{2} \rho_{0}\left(1-\frac{r}{R}\right) d r=4 \pi \rho_{0} \int_{0}^{R}\left(r^{2}-\frac{r^{3}}{R}\right) d r \\
& =4 \pi \rho_{0}\left[\frac{r^{3}}{3}-\frac{r^{4}}{4 R}\right]_{0}^{R}=4 \pi \rho_{0} \frac{R^{3}}{12} \Rightarrow \rho_{0}=\frac{3 Q}{\pi R^{3}}
\end{aligned}
$$

(b) Gauss's law for the Gaussian surface shown in the figure at $r<R$ is

$$
\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\text {in }}}{\epsilon_{0}} \Rightarrow E\left(4 \pi r^{2}\right)=\frac{Q_{\text {in }}}{\epsilon_{0}}
$$

The charge inside the Gaussian surface is

$$
Q_{\text {in }}=\int d q=\int_{0}^{r} 4 \pi r^{2} \rho d r=\int_{0}^{r} 4 \pi r^{2} \rho_{0}\left(1-\frac{r}{R}\right) d r=4 \pi \rho_{0} \int_{0}^{r}\left(r^{2}-\frac{r^{3}}{R}\right) d r=4 \pi \rho_{0}\left[\frac{r^{3}}{3}-\frac{r^{4}}{4 R}\right]
$$

Gauss's law becomes

$$
E\left(4 \pi r^{2}\right)=\frac{4 \pi \rho_{0}}{\epsilon_{0}}\left[\frac{r^{3}}{3}-\frac{r^{4}}{4 R}\right] \Rightarrow E=\frac{\rho_{0}}{\epsilon_{0}}\left[\frac{r}{3}-\frac{r^{2}}{4 R}\right] \Rightarrow=\left(\frac{3 Q}{\pi R^{3}}\right) \frac{1}{\epsilon_{0}} \frac{r}{3}\left[1-\frac{3 r}{4 R}\right]=\frac{Q r}{4 \pi \epsilon_{0} R^{3}}\left(4-\frac{3 r}{R}\right)
$$

The direction of the field is radially outward.
(c) At $r=R$, the above expression for the electric field reduces to

$$
E=\frac{Q R}{4 \pi \epsilon_{0} R^{3}}\left(4-\frac{3 R}{R}\right)=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{R^{2}}
$$

This is an expected result, because all charge $Q$ is inside $R$ and the field looks like that of a point charge.

### 27.59. Visualize:



Solve: (a) It is clear that $E(r) \propto r^{4}$ up to $r=R$. That is, the maximum value of $E(r)$ occurs at $r=R$. At this point,

$$
E=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{R^{2}}
$$

because all the charge is inside $R$ and the charge is spherically distributed about a point at the center. Thus,

$$
E=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{R^{2}}=E_{\max }\left(\frac{R^{4}}{R^{4}}\right) \Rightarrow E_{\max }=\frac{Q}{4 \pi \epsilon_{0} R^{2}}
$$

(b) Applying Gauss's law to the surface at $r<R$ shown in the figure,

$$
\oint \vec{E} \cdot d \vec{A}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \Rightarrow\left(E_{\max } \frac{r^{4}}{R^{4}}\right) 4 \pi r^{2}=\frac{Q_{\mathrm{in}}}{\epsilon_{0}} \Rightarrow Q_{\mathrm{in}}=\left(\frac{Q}{4 \pi \epsilon_{0} R^{2}}\right) \epsilon_{0} \frac{r^{4}}{R^{4}} 4 \pi r^{2}=Q\left(\frac{r^{6}}{R^{6}}\right)
$$

To find $Q_{\text {in }}$, we consider the thin spherical shell of width $d r$ and charge $d q$ at a distance $r$ from the center shown in the figure. Thus,

$$
d q=\rho d V=\rho\left(4 \pi r^{2}\right) d r \Rightarrow \int d q=Q_{\mathrm{in}}=\int \rho(r)\left(4 \pi r^{2}\right) d r
$$

Using this form of $Q_{\text {in }}$ in the equation obtained from Gauss's law,

$$
Q_{\mathrm{in}}=Q \frac{r^{6}}{R^{6}}=\int \rho(r) 4 \pi r^{2} d r \Rightarrow \int \rho(r) r^{2} d r=\frac{Q}{4 \pi} \frac{r^{6}}{R^{6}}
$$

Taking the position derivative on both sides

$$
\frac{d}{d r} \int \rho(r) r^{2} d r=\frac{Q}{4 \pi R^{6}} \frac{d}{d r}\left(r^{6}\right) \Rightarrow \rho(r) r^{2}=\frac{Q}{4 \pi R^{6}}\left(6 r^{5}\right) \Rightarrow \rho(r)=\frac{3 Q r^{3}}{2 \pi R^{6}}
$$

(c) Consider a spherical shell of width $d r$ and charge $d q$ at a distance $r$ from the center. Then

$$
d q=\rho(r) d V=\frac{3 Q r^{3}}{2 \pi R^{6}} 4 \pi r^{2} d r \Rightarrow d q=6 Q \frac{r^{5}}{R^{6}} d r \Rightarrow Q=\int d q=\frac{6 Q}{R^{6}} \int_{0}^{R} r^{5} d r=\frac{6 Q}{R^{6}} \frac{R^{6}}{6}=Q
$$

