31

FUNDAMENTALS OF CIRCUITS

Conceptual Questions

31.1. Calculate $I = \Delta V/R$.

$$I_a = \frac{2V}{2\Omega} = 1\text{ A} \qquad I_b = \frac{1V}{2\Omega} = 0.5\text{ A}$$

$$I_c = \frac{2V}{1\Omega} = 2\text{ A} \qquad I_d = \frac{1V}{1\Omega} = 1\text{ A}$$

$I_c > I_a = I_d > I_b$.

31.2. No. This is not a complete circuit. A connection from the outer metal case of the bulb to the negative end of the battery is required to make a complete circuit.

31.3. $\Delta V_{12} = 3$ V. There is no current through the resistor so there is no potential difference across the resistor.

31.4. R_1 dissipates more power, since $P = I^2R$ and the same current flows through each resistor.

31.5. R_2 dissipates more power, since $P = (\Delta V)^2/R$ and both resistors have the same potential drop ΔV because they are connected in parallel.

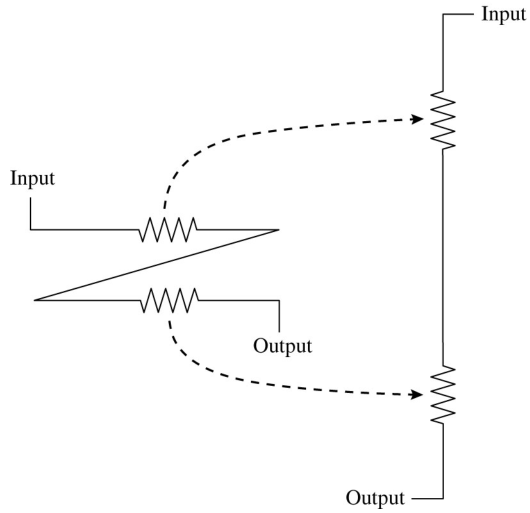
31.6. Calculate $P = (\Delta V)^2/R$.

$$P_a = \frac{(\Delta V)^2}{R} \qquad P_b = \frac{\left(\frac{1}{2}\Delta V\right)^2}{2R} = \frac{1}{8}P_a$$

$$P_c = \frac{(2\Delta V)^2}{\frac{1}{2}R} = 8P_a \qquad P_d = \frac{(2\Delta V)^2}{2R} = 2P_a$$

$P_c > P_d > P_a > P_b$.

31.7. The two resistors are in series, as can be seen by redrawing the circuit as follows:



31.8. Increase. Recall $\Delta V_{\text{bat}} = R\varepsilon/(R+r)$, where ε is the ideal battery voltage. As R increases, ΔV_{bat} also increases. This makes sense since less current flows through the circuit, thus there is a smaller potential drop in the battery due to its internal resistance. In the case of no current (battery disconnected, $R \to \infty$), $\Delta V_{\text{bat}} = \varepsilon$.

31.9. Bulb A gets brighter and bulb B goes out. When the switch is closed all the current travels along this zero resistance path rather than through bulb B. With less resistance in the circuit the current is larger, so bulb A burns brighter.

31.10. **(a)** $A > B = C$. All current flowing from the battery must go through bulb A, then the current splits at the junction with half going through B and half through C. With the same resistance for each bulb, the larger current means more brightness.
(b) Bulbs B and C go out because now there is a zero resistance wire along which all the current will flow. Now the total resistance in the circuit has decreased, so the current increases and bulb A will burn brighter.

31.11. Applying Kirchhoff's loop law around the outside edge of the circuit,

$$\sum \Delta V_1 = \varepsilon - IR - \Delta V_{12} = 0 \quad \Rightarrow \quad \Delta V_{12} = \varepsilon - IR$$

That is, the potential difference ΔV_{12} between points 1 and 2 is the potential supplied by the battery *minus* the potential lost in resistor R. When bulb B is in place, a current I flows through the resistor and bulb. In that case, ΔV_{12} is less than ε. However, if bulb B is removed, the current no longer flows and $I = 0$ A. Thus, ΔV_{12} (no bulb) $= \varepsilon$. Thus, the potential difference ΔV_{12} increases when bulb B is removed.

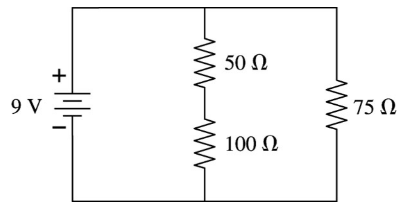
31.12. The brightness of each bulb stays the same. The ground wire is not part of a complete circuit so no current will flow down it. The current through the bulbs remains the same.

31.13. $R_2 > R_3 > R_1$. A larger resistance makes a larger time constant $t = RC$, which means it takes longer for the voltage to decrease. The size of the initial voltage makes no difference. The R_2 curve decays the most slowly, so R_2 is the largest, and the R_1 curve decays the most rapidly, so R_1 is the smallest.

Exercises and Problems

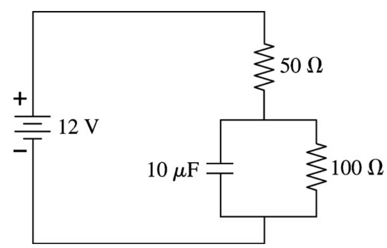
Section 31.1 Circuit Elements and Diagrams

31.1. Solve:



From the circuit in Figure EX31.1, we see that the 50 and 100 Ω resistors are connected in series across the battery. The 75 Ω resister is also connected across the battery in parallel with the first 50 and 100 Ω resistors.

31.2. Solve: In Figure EX31.2, the positive terminal of the battery is connected to the 50 Ω resistor, whose other end is connected to the 100 Ω resistor and the capacitor, which are in parallel. Thus, we have a resistor connected in series with a parallel combination of a resistor and a capacitor.



Section 31.2 Kirchhoff's Laws and the Basic Circuit

31.3. Model: Assume that the connecting wires are ideal.
Visualize: Please refer to Figure EX31.3.
Solve: The current in the 2 Ω resistor is $I_1 = 4\text{ V}/2\ \Omega = 2\text{ A}$ to the right. The current in the 5 Ω resistor is $I_2 = (15\text{ V})/(5\ \Omega) = 3\text{ A}$ downward. Let I be the current flowing out of the junction. Kirchhoff's junction law then gives

$$I = I_1 - I_2 = 2\text{ A} - 3\text{ A} = -1\text{ A}$$

Thus, 1 A of current flows into the junction (i.e., to the left).

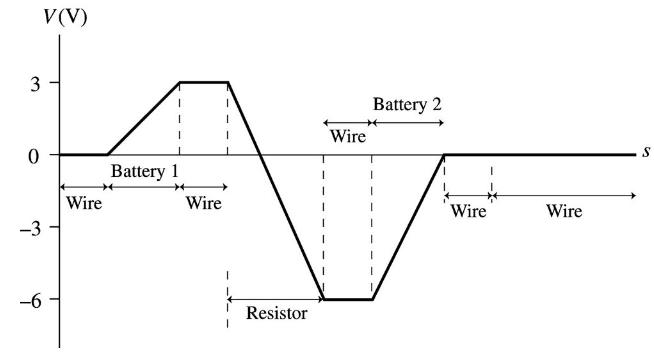
31.4. Model: Assume ideal connecting wires and an ideal battery for which $\Delta V_{\text{bat}} = \mathcal{E}$.
Visualize: Please refer to Figure EX31.4. We will choose a clockwise direction for I. Note that the choice of the current's direction is arbitrary because, with two batteries, we may not be sure of the actual current direction. The 3 V battery will be labeled 1 and the 6 V battery will be labeled 2.
Solve: **(a)** Kirchhoff's loop law, going clockwise from the negative terminal of the 3 V battery gives

$$\Delta V_{\text{closed loop}} = \sum_i (\Delta V)_i = \Delta V_{\text{bat 1}} + \Delta V_{\text{R}} + \Delta V_{\text{bat 2}} = 0$$

$$+3\text{ V} - (18\ \Omega)I + 6\text{ V} = 0 \quad \Rightarrow \quad I = \frac{9\text{ V}}{18\ \Omega} = 0.5\text{ A}$$

Thus, the current through the 18 Ω resistor is 0.5 A. Because I is positive, the current is to the right (i.e., clockwise).

(b)



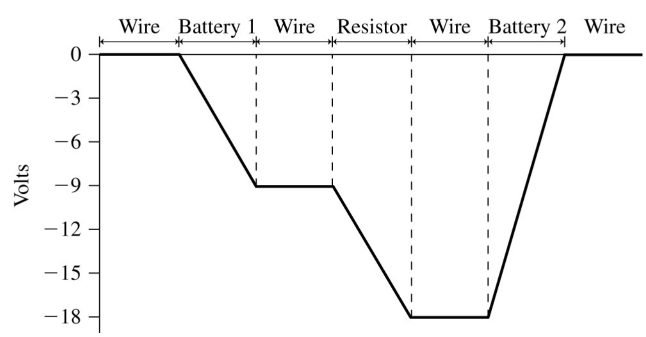
Assess: The graph shows a 3 V gain in battery 1, a −9 V loss in the resistor, and a gain of 6 V in battery 2. The final potential is the same as the initial potential, as required.

31.5. Model: The batteries and the connecting wires are ideal.
Visualize: Please refer to Figure EX31.5.
Solve: (a) Choose the current I to be in the clockwise direction. If I ends up being a positive number, then the current really does flow in this direction. If I is negative, the current really flows counterclockwise. There are no junctions, so I is the same for all elements in the circuit. With the 9 V battery labeled 1 and the 6 V battery labeled 2, Kirchhoff's loop law gives

$$\sum \Delta V_i = \Delta V_{\text{bat 1}} + \Delta V_{\text{R}} + \Delta V_{\text{bat 2}} = +\mathcal{E}_1 - IR - \mathcal{E}_2 = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R} = \frac{9\text{ V} - 18\text{ V}}{10\ \Omega} = -0.9\text{ A}$$

Note the signs: Potential is gained in battery 1, but potential is lost both in the resistor and in battery 2. Because I is negative, we can say that $I = 0.9$ A and flows from right to left through the resistor.
(b) In the graph below, we start at the lower-left corner of the circuit and travel clockwise around the circuit (i.e., against the current). We start by losing 9 V going through battery 1, then loss $\Delta V_{\text{R}} = -IR = 9$ V going through the resistor. We then gain 18 V going through battery 2. The final potential is the same as the initial potential, as required.



31.6. Visualize: Please refer to Figure EX31.6. Define the current I as a clockwise flow.
Solve: There are no junctions, so conservation of current tells us that the same current flows through each circuit element. From Kirchhoff's loop law,

$$\sum \Delta V_i = \Delta V_{\text{bat}} + \Delta V_{12} + \Delta V_{33} = 0$$

As we go around the circuit in the direction of the current, potential is gained in the battery ($\Delta V_{\text{bat}} = \mathcal{E}_{\text{bat}} = +30$ V) and potential is *lost* in the resistors ($\Delta V_{\text{res}} = -IR$). The loop law gives

$$\mathcal{E}_{\text{bat}} = -IR_1 - IR_2 = -I(R_{12} + R_{33}) \quad \Rightarrow \quad I = \frac{\mathcal{E}_{\text{bat}}}{R_{12} + R_{33}} = \frac{30\text{ V}}{45\ \Omega} = \frac{2}{3}\text{ A}$$

Now that we know the current, we can find the potential difference across each resistor:

$$\Delta V_{12} = IR_{12} = (2/3\text{ A})(12\ \Omega) = 8.0\text{ V}$$
$$\Delta V_{33} = IR_{33} = (2/3\text{ A})(33\ \Omega) = 22\text{ V}$$

Assess: The sum of the potential differences across the resistors equals the potential across the battery, as it should.

Section 31.3 Energy and Power

31.7. Model: The 1500 W rating is for operating at 120 V.
Solve: The hair dryer dissipates 1500 W at $\Delta V_R = 120$ V. Thus, the hair dryer's resistance is

$$R = \frac{(\Delta V_R)^2}{P_R} = \frac{(120\text{ V})^2}{1500\text{ W}} = 9.60\ \Omega$$

The current in the hair dryer when it is used is given by Ohm's law:

$$I = \frac{\Delta V_R}{R} = \frac{120\text{ V}}{9.60\ \Omega} = 12.5\text{ A}$$

31.8. Model: Assume ideal connecting wires and an ideal battery.
Visualize: Please refer to Figure EX31.8.
Solve: The power dissipated by each resistor can be calculated from Equation 31.14, $P_R = I^2R$, provided we can find the current through the resistors. Let us choose a clockwise direction for the current and solve for the value of I by using Kirchhoff's loop law. Going clockwise from the negative terminal of the battery gives

$$\sum_i (\Delta V)_i = \Delta V_{\text{bat}} + \Delta V_{R1} + \Delta V_{R2} = 0 \quad \Rightarrow \quad +12\text{ V} - IR_1 - IR_2 = 0$$

$$I = \frac{12\text{ V}}{R_1 + R_2} = \frac{12\text{ V}}{12\ \Omega + 18\ \Omega} = \frac{2}{5}\text{ A} = 0.40\text{ A}$$

The power dissipated by resistors R_1 and R_2 is:

$$P_{R_1} = I^2R_1 = (0.40\text{ A})^2(12\ \Omega) = 1.9\text{ W} \qquad P_{R_2} = I^2R_2 = (0.40\text{ A})^2(18\ \Omega) = 2.9\text{ W}$$

31.9. Model: Assume ideal connecting wires and that both bulbs have the same rated voltage (normally 120 V in the United States, although this value does not matter for this problem).
Visualize: Please refer to Figure EX31.9.
Solve: If the bulbs are operated at their rated voltage V_{Rating}, their resistance is

$$60\text{ W} = (\Delta V_{\text{Rating}})^2/R_{60} \quad \Rightarrow \quad R_{60} = \frac{(\Delta V_{\text{Rating}})^2}{60\text{ W}}$$

$$100\text{ W} = (\Delta V_{\text{Rating}})^2/R_{100} \quad \Rightarrow \quad R_{100} = \frac{(\Delta V_{\text{Rating}})^2}{100\text{ W}}$$

When operated with the battery of unknown voltage, the power output of the bulbs will be different. We shall call these powers P_{60} and P_{100}. We know that the same current must run through both bulbs, so we can express their power output in terms of their known resistance and the current I:

$$\left.\begin{array}{l} P_{60} = I^2R_{60} \\ P_{100} = I^2R_{100} \end{array}\right\} \quad \Rightarrow \quad \frac{P_{60}}{P_{100}} = \frac{I^2R_{60}}{I^2R_{100}} = \frac{(\Delta V_{\text{Rating}})^2/(60\text{ W})}{(\Delta V_{\text{Rating}})^2/(100\text{ W})} = \frac{100\text{ W}}{60\text{ W}} > 1$$

Therefore, the 60 W bulb emits more power (or is brighter) than the 100 W bulb, so the response is A.

31.10. Model: The 100 W rating is for operating at 120 V.
Solve: A standard bulb uses $\Delta V = 120$ V. We can use the power dissipation to find the resistance of the filament:

$$P = \frac{\Delta V^2}{R} \quad \Rightarrow \quad R = \frac{\Delta V^2}{P} = \frac{(120\text{ V})^2}{100\text{ W}} = 144\ \Omega$$

But the resistance is related to the filament's geometry:

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} \quad \Rightarrow \quad r = \sqrt{\frac{\rho L}{\pi R}} = \sqrt{\frac{(9.0\times10^{-7}\ \Omega\ \text{m})(0.070\ \text{m})}{\pi(144\ \Omega)}} = 1.18\times10^{-5}\ \text{m} = 11.8\ \mu\text{m}$$

The filament's diameter is $d = 2r = 24\ \mu$m.

31.11. Solve: (a) The average power consumed by a typical American family is

$$P_{\text{avg}} = 1000\frac{\text{kWh}}{\text{month}} = 1000\frac{\text{kWh}}{30\times24\ \text{h}} = \frac{1000}{720}\ \text{kW} = 1.389\ \text{kW}$$

Because $P = (\Delta V)I$ with ΔV being the voltage of the power line to the house,

$$I_{\text{avg}} = \frac{P_{\text{avg}}}{\Delta V} = \frac{1389\ \text{W}}{120\ \text{V}} = 11.6\ \text{A}$$

(b) Because $P = (\Delta V)^2/R$,

$$R_{\text{avg}} = \frac{(\Delta V)^2}{P_{\text{avg}}} = \frac{(120\ \text{V})^2}{1389\ \text{W}} = 10.4\ \Omega$$

31.12. Solve: The cost of running the waterbed 25% of the time for a year is

$$(0.25)(450\ \text{W})(365\ \text{days})\left(\frac{24\ \text{hr}}{\text{day}}\right)\left(\frac{\text{kW}}{1000\ \text{W}}\right)\left(\frac{\$0.12}{\text{kW hr}}\right) = \$118$$

To two significant figures, the cost is $120.

Section 31.4 Series Resistors

Section 31.5 Real Batteries

31.13. Visualize: Please refer to Figure EX31.13.
Solve: The three resistors are in series. The total resistance is $200\ \Omega$, so the unknown resistance R is

$$R_{\text{eq}} = R + 50\ \Omega + R = 2R + 50\ \Omega \quad \Rightarrow \quad R = \frac{R_{\text{eq}} - 50\ \Omega}{2} = \frac{200\ \Omega - 50\ \Omega}{2} = 75\ \Omega$$

31.14. Model: Assume ideal connecting wires and an ideal battery.
Solve: As shown in Figure EX31.14, a potential difference of 5.0 V causes a current of 100 mA through the three resistors in series. The situation is the same if we replace the three resistors with an equivalent resistor R_{eq}. That is, a potential difference of 5.0 V across R_{eq} causes a current of 100 mA through it. From Ohm's law,

$$R_{\text{eq}} = \frac{\Delta V_{\text{R}}}{I} \quad \Rightarrow \quad R + 15\ \Omega + 10\ \Omega = \frac{5.0\ \text{V}}{100\ \text{mA}} \quad \Rightarrow \quad R = 25\ \Omega$$

31.15. Visualize: Please refer to Figure EX31.15.
Solve: (a) Apply Kirchhoff's circuit law to the circuit shown in Figure EX31.15:

$$\Delta V_{\text{batt}} = 0 \quad \Rightarrow \quad \varepsilon - Ir = 0 \quad \Rightarrow \quad r = \frac{\varepsilon}{I} = \frac{1.5\ \text{V}}{2.3\ \text{A}} = 0.65\ \Omega$$

(b) The power dissipated is

$$P = I^2 r = (2.3\ \text{A})^2(0.65\ \Omega) = 3.5\ \text{W}$$

31.16. Model: Assume ideal connecting wires but not an ideal battery.
Visualize: Please refer to Figure 31.18.

Solve: From Equation 31.21, the potential difference across the battery is

$$\Delta V_{\text{bat}} = \frac{R}{R+r} \quad \Rightarrow \quad r = R\left(\frac{\mathcal{E}}{\Delta V_{\text{bat}}} - 1\right) = (20\ \Omega)\left(\frac{9.0\ \text{V}}{8.5\ \text{V}} - 1\right) = 1.2\ \Omega$$

Assess: $1\ \Omega$ is a typical internal resistance for a battery. This causes the battery's terminal voltage in the circuit to be 0.5 V less than its emf.

31.17. Model: Assume ideal connecting wires but not an ideal battery.
Visualize: The circuit for an ideal battery is the same as the circuit in Figure EX31.17, except that the $1\ \Omega$ resistor is not present.
Solve: In the case of an ideal battery, we have a battery with $\mathcal{E} = 15\ \text{V}$ connected to two series resistors of $10\ \Omega$ and $20\ \Omega$ resistance. Because the equivalent resistance is $R_{\text{eq}} = 10\ \Omega + 20\ \Omega = 30\ \Omega$ and the potential difference across R_{eq} is 15 V, the current in the circuit is

$$I = \frac{\Delta V}{R_{\text{eq}}} = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{15\ \text{V}}{30\ \Omega} = 0.50\ \text{A}$$

The potential difference across the $20\ \Omega$ resistor is

$$\Delta V_{20} = IR = (0.50\ \text{A})(20\ \Omega) = 10\ \text{V}$$

In the case of a real battery, we have a battery with $\mathcal{E} = 15\ \text{V}$ connected to three series resistors: $10\ \Omega$, $20\ \Omega$, and an internal resistance of $1\ \Omega$. Now the equivalent resistance is

$$R'_{\text{eq}} = 10\ \Omega + 20\ \Omega + 1\ \Omega = 31\ \Omega$$

The potential difference across R_{eq} is the same as before $\mathcal{E} = 15\ \text{V}$. Thus,

$$I' = \frac{\Delta V'}{R'_{\text{eq}}} = \frac{\mathcal{E}}{R'_{\text{eq}}} = \frac{15\ \text{V}}{31\ \Omega} = 0.4839\ \text{A}$$

Therefore, the potential difference across the $20\ \Omega$ resistor is

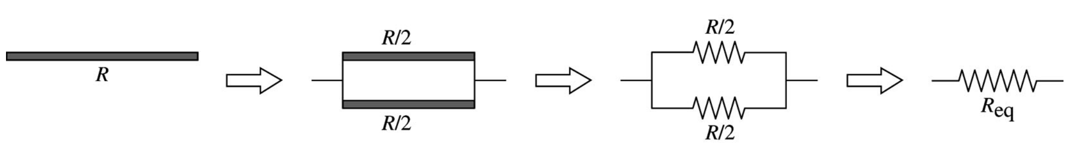
$$\Delta V'_{20} = I'R = (0.4839\ \text{A})(20\ \Omega) = 9.68\ \text{V}$$

That is, the potential difference across the $20\ \Omega$ resistor is reduced from 10 V to 9.68 V due to the internal resistance of $1\ \Omega$ of the battery. The percentage change in the potential difference is

$$\left(\frac{10.0\ \text{V} - 9.68\ \text{V}}{10.0\ \text{V}}\right) \times 100\% = 3.2\%$$

Section 31.6 Parallel Resistors

31.18. Visualize:



The figure shows a metal wire of resistance R that is cut into two pieces of equal length. This produces two wires each of resistance $R/2$.
Solve: Since these two wires are connected in parallel,

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R/2} + \frac{1}{R/2} = \frac{2}{R} + \frac{2}{R} = \frac{4}{R} \quad \Rightarrow \quad R_{\text{eq}} = \frac{R}{4}$$

31.19. Visualize: The three resistors in Figure EX31.19 are equivalent to a resistor of resistance $R_{\text{eq}} = 75\ \Omega$.

Solve: Because the three resistors are in parallel,

$$\frac{1}{R_{eq}}=\frac{1}{R}+\frac{1}{200\ \Omega}+\frac{1}{R}=\frac{2}{R}+\frac{1}{200\ \Omega}=\frac{400\ \Omega+R}{(200\ \Omega)R}\ \Rightarrow\ R_{eq}=75\ \Omega=\frac{(200\ \Omega)R}{(400\ \Omega+R)}=\frac{200\ \Omega}{1+\left(\frac{400\ \Omega}{R}\right)}$$

$$R=\frac{400\ \Omega}{\frac{200\ \Omega}{75\ \Omega}-1}=240\ \Omega$$

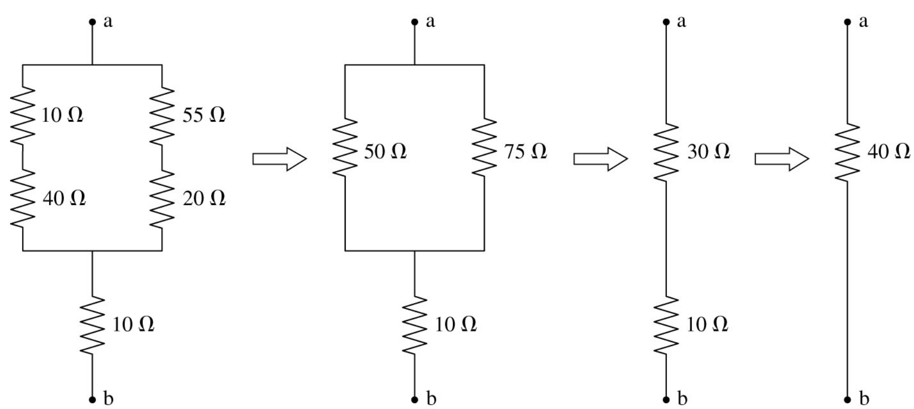
31.20. Model: Assume ideal connecting wires.
Visualize: Please refer to Figure EX31.20.
Solve: The resistance R is given by Ohm's law, $R=\Delta V_R/I_R$. To determine I_R we use Kirchhoff's junction law. The input current I splits into the three currents I_{10}, I_{15}, and I_R. That is,

$$2.0\text{ A}=I_{10}+I_{15}+I_R=\frac{8\text{ V}}{10\ \Omega}+\frac{8\text{ V}}{15\ \Omega}+I_R\ \Rightarrow\ I_R=2.0\text{ A}-\tfrac{20}{15}\text{ A}=\tfrac{2}{3}\text{ A}$$

Using this value of I_R in Ohm's law gives

$$R=\frac{8\text{ V}}{\frac{2}{3}\text{ A}}=12\ \Omega$$

31.21. Model: The connecting wires are ideal with zero resistance.
Solve:

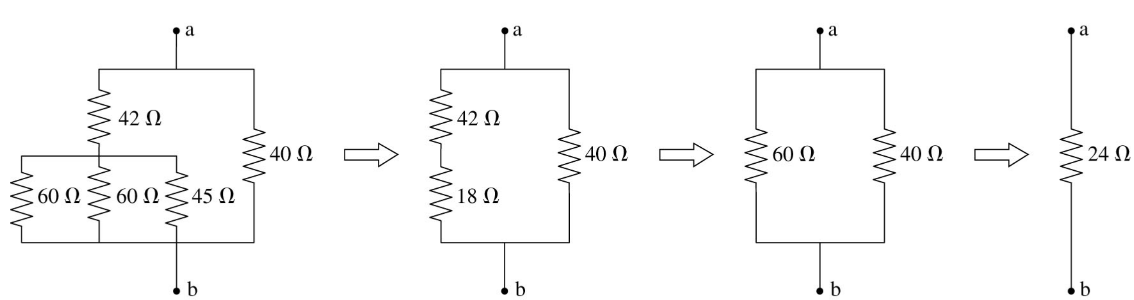


For the first step, the 10 Ω and 40 Ω resistors are in series and the equivalent resistance is 50 Ω. Likewise, the 55 Ω and 20 Ω resistors give an equivalent resistance of 75 Ω. For the second step, the 75 Ω and 50 Ω resistors are in parallel and the equivalent resistance is

$$\left[\frac{1}{50\ \Omega}+\frac{1}{75\ \Omega}\right]^{-1}=30\ \Omega$$

For the third step, the 30 Ω and 10 Ω resistors are in series and the equivalent resistance is 40 Ω.

31.22. Model: The connecting wires are ideal with zero resistance.
Solve:



For the first step, the two 60 W resistors and the $45\ \Omega$ resistor are in parallel. Their equivalent resistance is

$$\frac{1}{R_{\text{eq 1}}}=\frac{2}{60\ \Omega}+\frac{1}{45\ \Omega}=\frac{5}{90}\ \Omega \quad\Rightarrow\quad R_{\text{eq 1}}=18\ \Omega$$

For the second step, resistors $42\ \Omega$ and $R_{\text{eq 1}}=18\ \Omega$ are in series. Therefore,

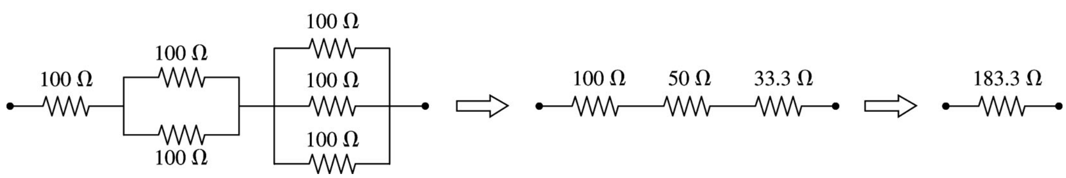
$$R_{\text{eq 2}}=R_{\text{eq 1}}+42\ \Omega=18\ \Omega+42\ \Omega=60\ \Omega$$

For the third step, the resistors $40\ \Omega$ and $R_{\text{eq 2}}=60\ \Omega$ are in parallel. So,

$$\frac{1}{R_{\text{eq 3}}}=\frac{1}{60\ \Omega}+\frac{1}{40\ \Omega} \quad\Rightarrow\quad R_{\text{eq 3}}=24\ \Omega$$

The equivalent resistance of the circuit is $24\ \Omega$.

31.23. Model: The connecting wires are ideal with zero resistance.
Solve:



For the first step, the two resistors in the middle of the circuit are in parallel, so their equivalent resistance is

$$\frac{1}{R_{\text{eq 1}}}=\frac{1}{100\ \Omega}+\frac{1}{100\ \Omega} \quad\Rightarrow\quad R_{\text{eq 1}}=50\ \Omega$$

The three $100\ \Omega$ resistors at the end are in parallel. Their equivalent resistance is

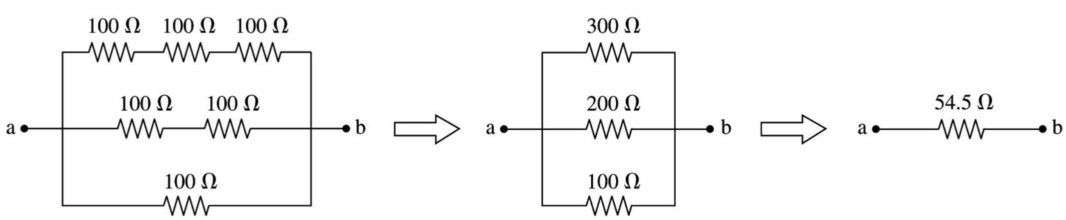
$$\frac{1}{R_{\text{eq 2}}}=\frac{1}{100\ \Omega}+\frac{1}{100\ \Omega}+\frac{1}{100\ \Omega} \quad\Rightarrow\quad R_{\text{eq 2}}=33.3\ \Omega$$

For the second step, the three resistors are in series, so their equivalent resistance is

$$100\ \Omega+50\ \Omega+33.3\ \Omega=183\ \Omega$$

The equivalent resistance of the circuit is $183\ \Omega$.

31.24. Model: The connecting wires are ideal with zero resistance.
Solve:



In the first step, the resistors $100\ \Omega$, $100\ \Omega$, and $100\ \Omega$ in the top branch are in series. Their combined resistance is $300\ \Omega$. In the middle branch, the two resistors, each $100\ \Omega$, are in series, so their equivalent resistance is $200\ \Omega$. In the second step, the three resistors are in parallel. Their equivalent resistance is

$$\frac{1}{R_{\text{eq}}}=\frac{1}{300\ \Omega}+\frac{1}{200\ \Omega}+\frac{1}{100\ \Omega} \quad\Rightarrow\quad R_{\text{eq}}=54.5\ \Omega$$

The equivalent resistance of the circuit is $54.5\ \Omega$.

Section 31.8 Getting Grounded

31.25. Model: The connecting wires and the batteries are ideal with zero resistance.
Solve: By definition, the potential at the grounded point is zero. The 9 V battery raises the potential of point a 9 V above ground, so point a is at 9 V. Now apply Kirchhoff's current law to the circuit to find the current running through it. Starting at the lower-left (grounded) corner and proceeding clockwise, this gives

$$9\text{ V} - I(2\ \Omega) - 6\text{ V} - I(1\ \Omega) = 0 \quad \Rightarrow \quad I = \frac{-3\text{ V}}{-3\ \Omega} = 1\text{ A}$$

Thus, the potential difference across the $2\ \Omega$ resistor is $\Delta V_{\text{res}} = -(1\text{ A})(2\ \Omega) = -2\text{ V}$, so the potential at the upper-right corner is $9\text{ V} - 2\text{ V} = 7\text{ V}$. The potential drop going through the right-side battery is 6 V, so point b is $7\text{ V} - 6\text{ V} = 1\text{ V}$. Thus, the potential at point a is at 9 V and the potential at point b is at 1 V.
Assess: To check, we can find the potential difference across the bottom resistor and verify that we find a potential of zero volts at the grounded point. The potential difference across the bottom resistor is $\Delta V_{\text{res}} = -(1\text{ A})(1\ \Omega) = -1\text{ V}$, which when added to the potential at point b gives zero volts, as expected.

31.26. Model: The connecting wires and the batteries are ideal with zero resistance.
Solve: By definition, the potential at the grounded point is zero. The 5 V battery raises the potential of point a 5 V above ground, so point a is at 5 V. Now apply Kirchhoff's current law to the circuit to find the current running through it. Starting at the lower-right (grounded) corner and proceeding counterclockwise, this gives

$$5\text{ V} - I(4\ \Omega) - 15\text{ V} - I(1\ \Omega) = 0 \quad \Rightarrow \quad I = \frac{10\text{ V}}{-5\ \Omega} = -2\text{ A}$$

(Note that the negative sign means the current actually flows clockwise.) Thus, the potential difference across the $2\ \Omega$ resistor is $\Delta V_{\text{res}} = -(-2\text{ A})(4\ \Omega) = 8\text{ V}$, so the potential at the upper-left corner is $5\text{ V} + 8\text{ V} = 13\text{ V}$. The potential drop going through the left-side battery is 15 V, so point b is $13\text{ V} - 15\text{ V} = -2\text{ V}$. Thus, the potential at point a is at 5 V and the potential at point b is at -2 V
Assess: To check, we can find the potential difference across the bottom resistor and verify that we find a potential of zero volts at the grounded point. The potential difference across the bottom resistor is $\Delta V_{\text{res}} = -(-2\text{ A})(1\ \Omega) = 2\text{ V}$, which when added to the potential at point b gives zero volts, as expected.

Section 31.9 *RC* Circuits

31.27. Solve: Noting that the unit of resistance is the ohm (V/A) and the unit of capacitance is the farad (C/V), the unit of *RC* is

$$RC = \frac{\text{V}}{\text{A}} \times \frac{\text{C}}{\text{V}} = \frac{\text{C}}{\text{A}} = \frac{\text{C}}{\text{C/s}} = \text{s}$$

31.28. Model: Assume ideal wires as the capacitors discharge through the two $1\text{ k}\Omega$ resistors.
Visualize: The circuit in Figure EX31.28 has an equivalent circuit with resistance R_{eq} and capacitance C_{eq}.
Solve: The equivalent capacitance is $C_{\text{eq}} = 2\ \mu\text{F} + 2\ \mu\text{F} = 4\ \mu\text{F}$, and the equivalent resistance is

$$\frac{1}{R_{\text{eq}}} = \frac{1}{1\text{ k}\Omega} + \frac{1}{1\text{ k}\Omega} \quad \Rightarrow \quad R_{\text{eq}} = 0.5\text{ k}\Omega$$

Thus, the time constant for the discharge of the capacitors is

$$\tau = R_{\text{eq}}C_{\text{eq}} = (0.5\text{ k}\Omega)(4\ \mu\text{F}) = 2\times 10^{-3}\text{s} = 2\text{ ms}$$

31.29. Model: Assume ideal wires as the capacitors discharge through the two $1\text{ k}\Omega$ resistors.
Visualize: The circuit in Figure EX31.29 has an equivalent circuit with resistance R_{eq} and capacitance C_{eq}.

Solve: The equivalent capacitance is

$$\frac{1}{C_{\text{eq}}}=\frac{1}{2\ \mu\text{F}}+\frac{1}{2\ \mu\text{F}} \quad\Rightarrow\quad C_{\text{eq}}=1\ \mu\text{F}$$

and the equivalent resistance is $R_{\text{eq}}=1\ \text{k}\Omega+1\ \text{k}\Omega=2\ \text{k}\Omega.$ Thus, the time constant for the discharge of the capacitors is

$$\tau=R_{\text{eq}}C_{\text{eq}}=(2\ \text{k}\Omega)(1\ \mu\text{F})=2\times10^{-3}\text{s}=2\ \text{ms}$$

Assess: The capacitors will be almost entirely discharged $5\tau=5\times2\ \text{ms}=10\ \text{ms}$ after the switch is closed.

31.30. Model: The capacitor discharges through a resistor. Assume that the wires are ideal.

Solve: The decay of the capacitor charge is given by the Equation 31.33: $Q=Q_0e^{-t/\tau}$. The time constant is

$$\tau=RC=(1.0\ \text{k}\Omega)(10\ \mu\text{F})=0.010\ \text{s}$$

The initial charge on the capacitor is $Q_0=20\ \mu\text{C}$ and it decays to 10 μC in time *t*. That is,

$$10\ \mu\text{C}=(20\ \mu\text{C})e^{-\tau/(0.010\ \text{s})} \quad\Rightarrow\quad \ln\left(\frac{10\ \mu\text{C}}{20\ \mu\text{C}}\right)=-\frac{\tau}{0.010\ \text{s}} \quad\Rightarrow\quad -\tau=(0.010\ \text{s})(\ln 2)=6.9\ \text{ms}$$

31.31. Model: The capacitor discharges through a resistor. Assume ideal wires.

Visualize: The switch in the circuit in Figure EX31.31 is in position a. When the switch is in position b the circuit consists of a capacitor and a resistor.

Solve: **(a)** The switch has been in position a for a long time, which means the capacitor is fully charged to a charge

$$Q_0=C\Delta V=C\varepsilon=(4\ \mu\text{F})(9\ \text{V})=36\ \mu\text{C}$$

Immediately after the switch is moved to the b position, the charge on the capacitor is $Q_0=36\ \mu\text{C}$. The current through the resistor is

$$I_0=\frac{\Delta V_{\text{R}}}{R}=\frac{9\ \text{V}}{25\ \Omega}=0.36\ \text{A}$$

Note that, as soon as the switch is closed, the potential difference across the capacitor ΔV_{C} appears across the 25 Ω resistor.

(b) The charge Q_0 decays as $Q=Q_0e^{-t/\tau}$, where

$$\tau=RC=(25\ \Omega)(4\ \mu\text{F})=100\ \mu\text{s}$$

Thus, at $t=50\ \mu\text{s}$, the charge is

$$Q=(36\ \mu\text{C})e^{-50\ \mu\text{s}/100\ \mu\text{s}}=(36\ \mu\text{C})e^{-0.5}=22\ \mu\text{C}$$

and the resistor current is

$$I=I_0e^{-t/\tau}=(0.36\ \text{A})e^{-50\ \mu\text{s}/100\ \mu\text{s}}=0.22\ \text{A}$$

(c) Likewise, at $t=200\ \mu\text{s}$, the charge is $Q=4.9\ \mu\text{C}$ and the current is $I=49\ \text{mA}$.

31.32. Model: A capacitor discharges through a resistor. Assume ideal wires.

Solve: A capacitor initially charged to Q_0 decays as $Q=Q_0e^{-t/RC}$. We wish to find *R* so that a 1.0 μF capacitor will discharge to 10% of its initial value in 2.0 ms. That is,

$$(0.10)Q_0=Q_0e^{-(2.0\ \text{ms})/R(1.0\ \mu\text{F})} \quad\Rightarrow\quad \ln(0.10)=-\frac{2.0\times10^{-3}\ \text{s}}{R(1.0\times10^{-6}\ \text{F})} \quad\Rightarrow\quad R=\frac{-2.0\times10^{-3}\ \text{s}}{(1.0\times10^{-6}\ \text{F})\ln(0.10)}=0.87\ \text{k}\Omega$$

Assess: A time constant of $\tau=RC=(870\ \Omega)(1.0\times10^{-6}\ \text{F})=0.87\ \text{ms}$ is reasonable.

31.33. Model: A capacitor discharges through a resistor. Assume ideal wires.

Solve: The discharge current or the resistor current follows Equation 31.35: $I=I_0e^{-t/RC}$. We wish to find the capacitance *C* so that the resistor current will decrease to 25% of its initial value in 2.5 ms. That is,

$$0.25I_0=I_0e^{-(2.5\ \text{ms})/(100\ \Omega)C} \quad\Rightarrow\quad \ln(0.25)=-\frac{2.5\times10^{-3}\ \text{s}}{(100\ \Omega)C} \quad\Rightarrow\quad C=18\ \mu\text{F}$$

31.34. Model: Assume ideal wires.
Solve: Because the bulbs are identical, they all have the same resistance, which we shall call R. The equivalent resistance of the left-hand branch is

$$(R_{eq})_L = R + (1/R + 1/R)^{-1} = R + R/2 = 3R/2$$

The equivalent resistance of the right-hand branch is

$$(R_{eq})_R = R + R = 2R$$

The voltage difference across the equivalent resistance of the left-hand and right-hand branches is ε, so the currents flowing through the left- and right-hand branches are

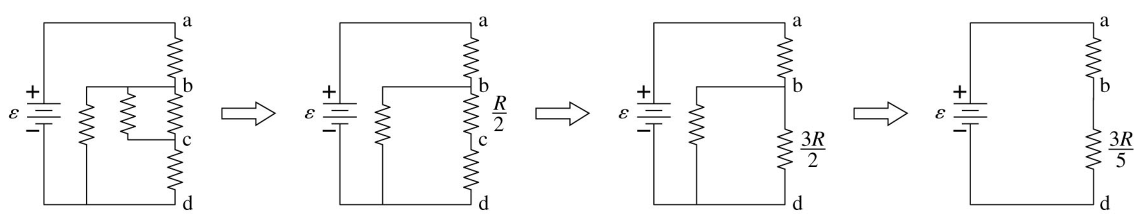
$$I_L = \frac{\varepsilon}{(R_{eq})_L} = \frac{2}{3}\frac{\varepsilon}{R}, \quad I_R = \frac{\varepsilon}{(R_{eq})_R} = \frac{1}{2}\frac{\varepsilon}{R}$$

Thus, the current flowing through bulbs S and T is I_R. The current flowing through bulb P is I_L and that flowing through bulbs Q and R is $I_L/2$. Because the brightness is proportional to the power $P = I^2R$, ordering the bulbs in terms of current will give the same result as ordering them in terms of brightness. Thus, ordering the bulbs from brightest to dimmest gives

$$P > S = T > Q = R$$

so the response is C.

31.35. Model: Assume ideal wires.
Visualize: The following equivalent circuits will be useful, where we have labeled three points in the circuits a, b, and c. The unlabeled resistors all have resistance R.



Solve: Because the bulbs are identical, they all have the same resistance, which we shall call R. The bulb brightness is proportional to the power $P = V^2/R$, so we shall calculate the voltage difference across each bulb. First, we find the potential at points a, b, and c with respect to point d (i.e., the negative terminal of the battery), which we shall assign as the zero of the potential. The potential at point a is ε, so $V_a = \varepsilon$. Considering the last equivalent circuit above, we see that the current flowing around the circuit is

$$I = \frac{V_a}{R + 3R/5} = \frac{5\varepsilon}{8R}$$

Therefore, the potential at point b is

$$V_b = V_a - IR = \varepsilon - \frac{5\varepsilon}{8} = \frac{3\varepsilon}{8}$$

Considering the second-to-last circuit, we see that the current flowing from point b to point d must be

$$I_R = \frac{V_b}{3R/2} = \frac{2}{3R}\frac{3\varepsilon}{8} = \frac{\varepsilon}{4R}$$

so, looking at the second circuit, we can find the potential at point c:

$$V_c = V_b - I_R\left(\frac{R}{2}\right) = \frac{3\varepsilon}{8} - \left(\frac{\varepsilon}{4R}\right)\left(\frac{R}{2}\right) = \frac{\varepsilon}{4}$$

In the following table, we put the bulbs and the potential difference across them:

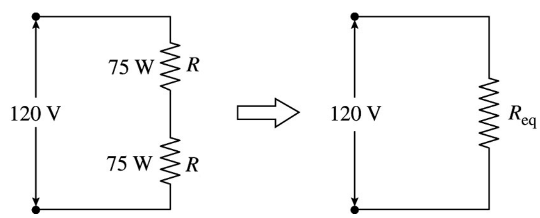
Bulb	Potential Difference	Power
P	$\Delta V = V_b - V_a = -\frac{5\varepsilon}{8}$	$\frac{25}{64}\frac{\varepsilon^2}{R}$
Q	$\Delta V = V_d - V_b = -\frac{3\varepsilon}{8}$	$\frac{9}{64}\frac{\varepsilon^2}{R}$
R	$\Delta V = V_c - V_b = -\frac{\varepsilon}{8}$	$\frac{1}{64}\frac{\varepsilon^2}{R}$
S	$\Delta V = V_c - V_b = -\frac{\varepsilon}{8}$	$\frac{1}{64}\frac{\varepsilon^2}{R}$
T	$\Delta V = V_d - V_c = -\frac{\varepsilon}{4}$	$\frac{4}{64}\frac{\varepsilon^2}{R}$

Thus, ordering the bulbs from brightest to dimmest gives

$$\text{P} > \text{Q} > \text{T} > \text{R} = \text{S}$$

so the response is D.

31.36. Model: Assume ideal connecting wires and an ideal power supply.
Visualize:



The two light bulbs are basically two resistors in series.
Solve: A 75 W (120 V) light bulb has a resistance of

$$R = \frac{\Delta V^2}{P} = \frac{(120\text{ V})^2}{75\text{ W}} = 192\ \Omega$$

The combined resistance of the two bulbs is

$$R_{eq} = R_1 + R_2 = 192\ \Omega + 192\ \Omega = 384\ \Omega$$

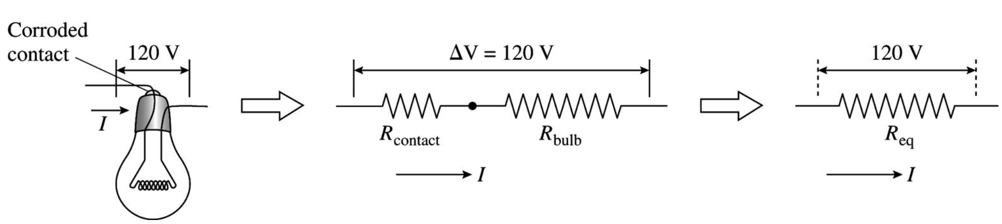
The current I flowing through R_{eq} is

$$I = \frac{\Delta V}{R_{eq}} = \frac{120\text{ V}}{384\ \Omega} = 0.3125\text{ A}$$

Because R_{eq} is a series combination of the two equivalent resistors, the current 0.3125 A flows through both resistors. Thus the power dissipated by each bulb is

$$P = I^2 R_1 = (0.3125\text{ A})^2(192\ \Omega) = 19\text{ W}$$

31.37. Model: Assume ideal connecting wires and an ideal power supply.
Visualize:



Solve: We have two resistors in series such that $R_{eq} = R_{bulb} + R_{contacts}$. R_{bulb} can be found from the fact that we have a 100 W (120 V) bulb:

$$R_{bulb} = \frac{\Delta V^2}{P} = \frac{(120\text{ V})^2}{100\text{ W}} = 144\ \Omega$$

We have a total resistance of $R_{eq} = 144\ \Omega + 5.0\ \Omega = 149\ \Omega$. The current flowing through R_{eq} is

$$I = \frac{\Delta V}{R_{eq}} = \frac{120\text{ V}}{149\ \Omega} = 0.8054\text{ A}$$

Because R_{eq} is a series combination of R_{bulb} and $R_{contacts}$, this current flows through both the bulb and the contact. Thus,

$$P_{bulb} = I^2 R_{bulb} = (0.8054\text{ A})^2(144\ \Omega) = 93\text{ W}$$

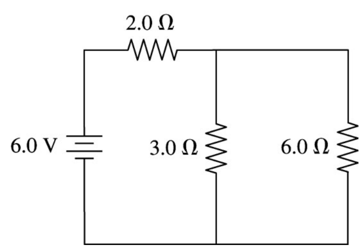
Assess: The corroded contact changes the circuit's total resistance and reduces the current below that at which the bulb was rated. So, it makes sense for it to operate at less than full power.

31.38. Model: Assume the eel is an ideal battery and that it connects to its prey by ideal connecting wires.
Solve: (a) The power delivered by the eel is $P = VI$. The energy of the pulse is $E = Pt$, or

$$E = VIt = (450\text{ V})(0.80\text{ A})(1.0\text{ ms}) = 0.36\text{ J}$$

(b) The total charge that flows is $Q = It = (0.80\text{ A})(1.0\text{ ms}) = 0.80\text{ mC}$.

31.39. Model: The wires and battery are ideal.
Visualize:

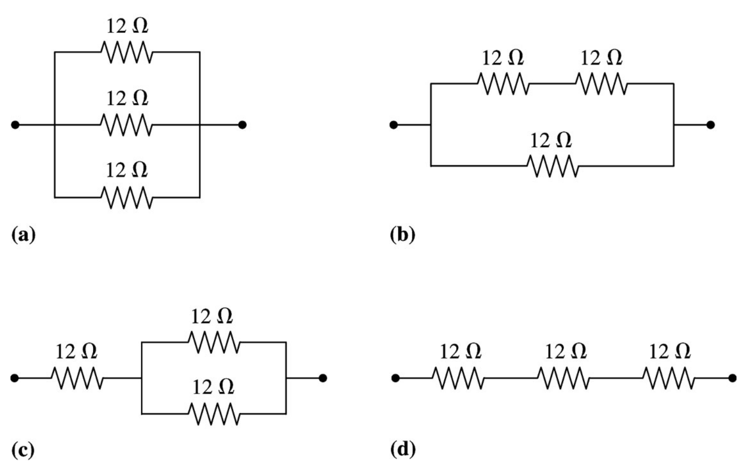


Solve: We can find the equivalent resistance necessary for the battery to deliver 9 W of power:

$$P = \frac{(\Delta V)^2}{R} \Rightarrow R = \frac{(\Delta V)^2}{P} = \frac{(6.0\text{ V})^2}{9.0\text{ W}} = 4.0\ \Omega$$

The combination of the $2.0\ \Omega$, $3.0\ \Omega$, and $6.0\ \Omega$ resistors that make $4.0\ \Omega$ is shown in the figure above. The $3.0\ \Omega$ and $6.0\ \Omega$ parallel combination has an equivalent resistance of $2.0\ \Omega$, which when added to the $2.0\ \Omega$ resistor in series totals $4.0\ \Omega$ equivalent resistance.

31.40. Visualize:



Solve: **(a)** The three resistors in parallel have an equivalent resistance of

$$\frac{1}{R_{eq}} = \frac{1}{12\ \Omega} + \frac{1}{12\ \Omega} + \frac{1}{12\ \Omega} \quad \Rightarrow \quad R_{eq} = 4.0\ \Omega$$

(b) One resistor in parallel with two series resistors has an equivalent resistance of

$$\frac{1}{R_{eq}} = \frac{1}{12\ \Omega + 12\ \Omega} + \frac{1}{12\ \Omega} = \frac{1}{24\ \Omega} + \frac{1}{12\ \Omega} = \frac{1}{8.0\ \Omega} \quad \Rightarrow \quad R_{eq} = 8.0\ \Omega$$

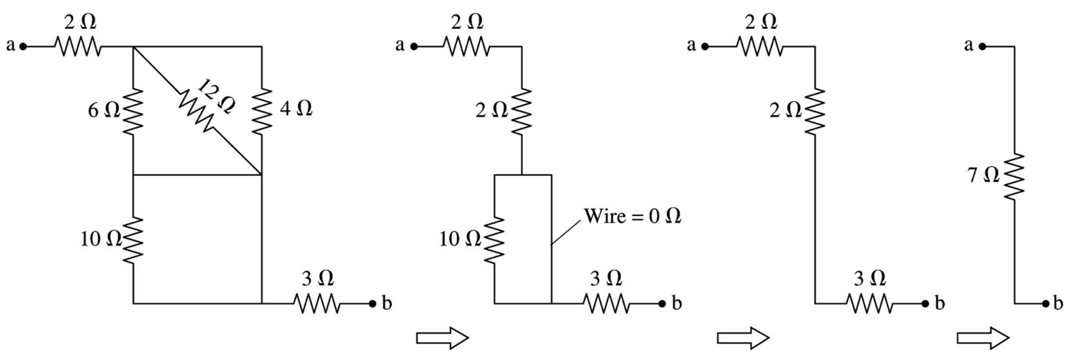
(c) One resistor in series with two parallel resistors has an equivalent resistance of

$$\frac{1}{R_{eq}} = 12\ \Omega + \left(\frac{1}{12\ \Omega} + \frac{1}{12\ \Omega}\right)^{-1} = 12\ \Omega + 6\ \Omega = 18\ \Omega$$

(d) The three resistors in series have an equivalent resistance of

$$12\ \Omega + 12\ \Omega + 12\ \Omega = 36\ \Omega$$

31.41. Model: Use the laws of series and parallel resistances.
Visualize:



Solve: Despite the diagonal orientation of the $12\ \Omega$ resistor, the $6\ \Omega$, $12\ \Omega$, and $4\ \Omega$ resistors are in parallel because they have a common connection at both the top end and at the bottom end. Their equivalent resistance is

$$R_{eq} = \left(\frac{1}{6\ \Omega} + \frac{1}{12\ \Omega} + \frac{1}{4\ \Omega}\right)^{-1} = 2\ \Omega$$

The trickiest issue is the $10\ \Omega$ resistor. It is in parallel with a *wire*, which is the same thing as a resistor with $R = 0\ \Omega$. The equivalent resistance of $10\ \Omega$ in parallel with $0\ \Omega$ is

$$R_{eq} = \left(\frac{1}{10\ \Omega} + \frac{1}{0\ \Omega}\right)^{-1} = (\infty)^{-1} = \frac{1}{\infty} = 0\ \Omega$$

In other words, the wire is a short circuit around the $10\ \Omega$, so all the current goes through the wire rather than the resistor. The $10\ \Omega$ resistor contributes nothing to the circuit. So the total circuit is equivalent to a $2\ \Omega$ resistor in series with the $2\ \Omega$ equivalent resistance in series with the final $3\ \Omega$ resistor. The equivalent resistance of these three series resistors is

$$R_{ab} = 2\ \Omega + 2\ \Omega + 3\ \Omega = 7\ \Omega$$

31.42. Model: Assume the batteries and the connecting wires are ideal.
Visualize: Please refer to Figure P31.42.
Solve: The two batteries in this circuit are oriented to "oppose" each other. The curent will flow in the direction of the battery with the greater voltage. The direction of the current is counterclockwise because the 12 V battery is greater. There are no junctions, so the same current I flows through all circuit elements. Applying Kirchhoff's loop law in the *counterclockwise* direction and starting at the lower right corner,

$$\sum \Delta V_i = 12\ \text{V} - I(12\ \Omega) - I(6\ \Omega) - 6\ \text{V} - IR = 0$$

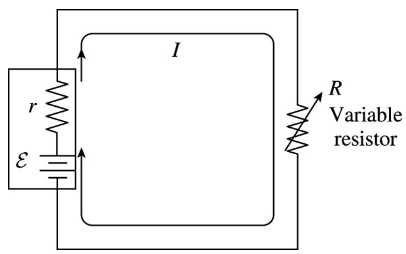
Note that the IR terms are all negative because we're applying the loop law in the direction of current flow, and the potential *decreases* as current flows through a resistor. We can solve to find the unknown resistance R:

$$6\text{ V} - I(18\ \Omega) - IR = 0 \quad \Rightarrow \quad R = \frac{6\text{ V} - (18\ \Omega)I}{I} = \frac{6\text{ V} - (18\ \Omega)(0.25\text{ A})}{0.25\text{ A}} = 6\ \Omega$$

The power is $P = I^2R = (0.25\text{ A})^2(6\ \Omega) = 0.38\text{ W}$.

Assess: The total power dissipated by the resistors should equal the sum of the net power generated in the batteries. $P_{\text{resist}} = (0.25\text{ A})^2(18\ \Omega) + 0.38\text{ W} = 1.5\text{ W}$. $P_{\text{batteries}} = (0.25\text{ A})(12\text{ V} - 6\text{ V}) = 1.5\text{ W}$. The powers are equal, so our result is reasonable.

31.43. Model: Assume that the connecting wires are ideal but the battery is not ideal.
Visualize:



Solve: The figure shows a variable resistor R connected across the terminals of a battery that has an emf ε and an internal resistance r. Using Kirchhoff's loop law and starting from the lower-left corner gives

$$+\varepsilon - Ir - IR = 0 \quad \Rightarrow \quad \varepsilon = I(r + R)$$

From the point in Figure P31.43 that corresponds to $R = 0\ \Omega$, we have

$$\varepsilon = (6\text{ A})(r + 0\ \Omega) = (6\text{ A})r$$

From the point that corresponds to $R = 10\ \Omega$, we have

$$\varepsilon = (3\text{ A})(r + 10\ \Omega)$$

Combining the two equations gives

$$(6\text{ A})r = (3\text{ A})(r + 10\ \Omega) \quad \Rightarrow \quad 2r = r + 10\ \Omega \quad \Rightarrow \quad r = 10\ \Omega$$

Also, $\varepsilon = (3\text{ A})(10\ \Omega + 10\ \Omega) = 60\text{ V}$.

Assess: With $\varepsilon = 60\text{ V}$ and $r = 10\ \Omega$, the equation $\varepsilon = I(r + R)$ is satisfied by all values of R and I on the graph in Figure P31.43.

31.44. Model: The connecting wires are ideal, but the battery is not.
Visualize: Please refer to Fig. P31.44. We will designate the current in the $5\ \Omega$ resistor I_5 and the voltage drop ΔV_5. Similar designations will be used for the other resistors.
Solve: Since the $10\ \Omega$ resistor is dissipating 40 W,

$$P_{10} = I_{10}^2 R_{10} = 40\text{ W} \quad \Rightarrow \quad I_{10} = \sqrt{\frac{P_{10}}{R_{10}}} = \sqrt{\frac{40\text{ W}}{10\ \Omega}} = 2.0\text{ A}$$

Because the $5\ \Omega$ resistor is in series with the $10\ \Omega$ resistor, the same current must run through the $5\ \Omega$ resistor. Therefore, the power dissipated by the $5\ \Omega$ resistor is

$$P_5 = I^2 R_5 = (2.0\text{ A})^2(5\ \Omega) = 20\text{ W}$$

The potential drop across the two left-hand resistors is the same as that across the right-hand resistor, so the power dissipated by the $20\ \Omega$ resistor is

$$P_{20} = \frac{V_{20}^2}{R_{20}} = \frac{(V_5 + V_{10})^2}{R_{20}} = \frac{(I_{10}R_5 + I_{10}R_{10})^2}{R_{20}} = \frac{(2.0\text{ A})^2(5\ \Omega + 10\ \Omega)^2}{20} = 45\text{ W}$$

31.45. Model: Assume that the connecting wires are ideal, but the battery is not. The battery has internal resistance. Also assume that the ammeter does not have any resistance.
Visualize: Please refer to Figure P31.45.

Solve: When the switch is open,

$$\varepsilon - Ir - I(5.0\ \Omega) = 0 \quad \Rightarrow \quad V = (1.636\ \text{A})(r + 5.0\ \Omega)$$

where we applied Kirchhoff's loop law starting from the lower-left corner. When the switch is closed, the current I comes out of the battery and splits at the junction. The current $I' = 1.565$ A flows through the $5.0\ \Omega$ resistor and the rest $(I - I')$ flows through the $10.0\ \Omega$ resistor. Because the potential differences across the two resistors are equal,

$$I'(5.0\ \Omega) = (I - I')(10.0\ \Omega) \quad \Rightarrow \quad (1.565\ \text{A})(5.0\ \Omega) = (I - 1.565\ \text{A})(10.0\ \Omega) \quad \Rightarrow \quad I = 2.348\ \text{A}$$

Applying Kirchhoff's loop law to the left loop of the closed circuit gives

$$\varepsilon - Ir - I'(5.0\ \Omega) = 0\ \text{V} \quad \Rightarrow \quad \varepsilon = (2.348\ \text{A})r + (1.565\ \text{A})(5.0\ \Omega) = (2.348\ \text{A})r + 7.825\ \text{V}$$

Combining this equation for ε with the equation obtained from the circuit when the switch was open gives

$$(2.348\ \text{A})r + 7.825\ \text{V} = (1.636\ \text{A})r + 8.18\ \text{V} \quad \Rightarrow \quad (0.712\ \text{A})r = 0.355\ \text{V} \quad \Rightarrow \quad r = 0.50\ \Omega$$

We also have $\varepsilon = (1.636\ \text{A})(0.50\ \Omega + 5.0\ \Omega) = 9.0$ V.

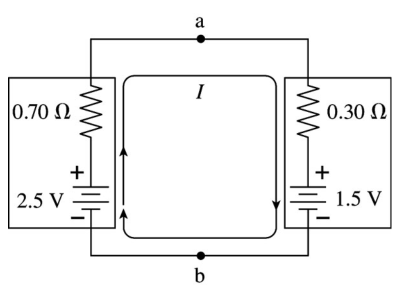
31.46. Model: Assume that the connecting wire and the battery are ideal.
Visualize: Please refer to Figure P31.46.
Solve: The middle and right branches are in parallel, so the potential difference across these two branches must be the same. The currents are known, so these potential differences are

$$\Delta V_{\text{middle}} = (3.0\ \text{A})R = \Delta V_{\text{right}} = (2.0\ \text{A})(R + 10\ \Omega)$$

This gives $R = 20\ \Omega$. The middle resistor R is connected directly across the battery, thus (for an ideal battery with no internal resistance) the potential difference ΔV_{middle} equals the emf of the battery. That is,

$$\varepsilon = \Delta V_{\text{middle}} = (3.0\ \text{A})(20\ \Omega) = 60\ \text{V}$$

31.47. Model: The connecting wires are ideal.
Visualize:



Solve: Let the current in the circuit be I. The terminal voltage of the 2.5 V battery is $V_a - V_b$. This is also the terminal voltage of the 1.5 V battery. $V_a - V_b$ can be obtained by noting that

$$V_b + 2.5\ \text{V} - I(0.70\ \Omega) = V_a \quad \Rightarrow \quad V_a - V_b = 2.5\ \text{V} - I(0.70\ \Omega)$$

To determine I, we apply Kirchhoff's loop law starting from the lower-left corner:

$$+2.5\ \text{V} - I(0.70\ \Omega) - I(0.30\ \Omega) - 1.5\ \text{V} = 0\ \text{V} \quad \Rightarrow \quad I = 1.0\ \text{A}$$

Thus, $V_a - V_b = 2.5\ \text{V} - (1.0\ \text{A})(0.70\ \Omega) = 1.8$ V. That is, the terminal voltage of the 1.5 V and the 2.5 V batteries is 1.8 V.

31.48. Model: The connecting wires are ideal.
Visualize: Please refer to Figure 31.20.
Solve: **(a)** The internal resistance r and the load resistance R are in series, so the total resistance is $R + r$ and the current flowing in the circuit due to the emf ε is $I = \varepsilon/(R + r)$. The power dissipated by the load resistance is

$$P = I^2 R = \frac{\mathcal{E}^2 R}{(R + r)^2}$$

This is not the power $\mathcal{E}I$ generated by the battery, but simply the power dissipated by the load R. The power is a function of R, so we can find the maximum power by setting $dP/dR = 0$:

$$\frac{dP}{dR} = \frac{\mathcal{E}^2}{(R+r)^2} - \frac{2\mathcal{E}^2 R}{(R+r)^3} = \frac{\mathcal{E}^2(R+r) - 2\mathcal{E}^2 R}{(R+r)^3} = \frac{\mathcal{E}^2(r-R)}{(R+r)^3} = 0$$

That is, the power dissipated by R is a maximum when $R = r$.

(b) The load's maximum power dissipation will occur when $R = r = 1.0\ \Omega$, in which case

$$P = \frac{\mathcal{E}^2 R}{(R+r)^2} = \frac{(9.0\text{ V})^2(1.0\ \Omega)}{(2.0\ \Omega)^2} = 20\text{ W}$$

(c) When R is very small ($R \to 0\ \Omega$), the current is a maximum ($I \to \varepsilon/r$) but the potential difference *across the load* is very small ($\Delta V = IR \to 0$ V). So the power dissipation of the load is also very small ($P = I\Delta V \to 0$ W). When R is very large ($R \to \infty$), the potential difference across the load is a maximum ($\Delta V \to \varepsilon$) but the current is very small ($I \to 0$ A). Once again, P is very small. If P is zero both for $R \to 0$ and for $R \to \infty$, there must be some intermediate value of R where P is a maximum.

31.49. Model: The batteries are ideal, the connecting wires are ideal, and the ammeter has a negligibly small resistance.

Visualize: Please refer to Figure P31.49.

Solve: Kirchhoff's junction law tells us that the current flowing through the $2.0\ \Omega$ resistance in the middle branch is $I_1 + I_2 = 3.0$ A. We can therefore determine I_1 by applying Kirchhoff's loop law to the left loop. Starting clockwise from the lower-left corner,

$$+9.0\text{ V} - I_1(3.0\ \Omega) - (3.0\text{ A})(2.0\ \Omega) = 0\text{ V} \quad \Rightarrow \quad I_1 = 1.0\text{ A}$$

$$I_2 = (3.0\text{ A} - I_1) = (3.0\text{ A} - 1.0\text{ A}) = 2.0\text{ A}$$

Finally, to determine the emf ε, we apply Kirchhoff's loop law to the right loop and start counterclockwise from the lower-right corner of the loop:

$$+\varepsilon - I_2(4.5\ \Omega) - (3.0\text{ A})(2.0\text{W}) = 0\text{ V} \quad \Rightarrow \quad \varepsilon - (2.0\text{ A})(4.5\ \Omega) - 6.0\text{ V} = 0\text{ V} \quad \Rightarrow \quad \varepsilon = 15\text{ V}$$

31.50. Visualize: Please refer to Figure P31.50.

Solve: Because the $4\ \Omega$ resistor is grounded at both ends, the potential difference across this resistor is zero. That is, no current flows through the $4\ \Omega$ resistor, and the negative terminals of both batteries are at zero potential. To determine the current in the $2\ \Omega$ resistor, we apply Kirchhoff's loop law. We assume that current I flows clockwise through the $2\ \Omega$ resistor. Starting from the lower-left corner, the sum of the potential differences across various elements in the circuit is

$$+9\text{ V} - I(2\ \Omega) - 3\text{ V} = 0\text{ V} \quad \Rightarrow \quad I = 3\text{ A}$$

31.51. Solve: Let the units guide you. For the incandescent bulb, the life-cycle cost p_{bulb} is

$$p_{\text{bulb}} = \$0.50 + (0.060\text{ kW})(0.10\text{ \$/kW h})(10{,}000\text{ h}) = \$60.50$$

This will give 1,000 hours, so the cost for 10,000 hours is \$65.00. For the fluorescent tube, the cost for 10,000 hours is

$$p_{\text{tube}} = \$5 + (0.015\text{ kW})(0.10\text{ \$/kW h})(10{,}000\text{ h}) = \$20$$

Assess: The lifetime cost of the fluorescent bulb is one-third that of the incandescent bulb.

31.52. Solve: (a) The cost per month of the 1000 W refrigerator is

$$(1000\text{ W})\left(\frac{1\text{ kW}}{1000\text{ W}}\right)\left(\frac{30\text{ day}}{1\text{ month}}\right)\left(\frac{24\text{ h}}{1\text{ day}}\right)(0.20)\left(\frac{\$0.10}{1\text{ kW h}}\right)(1\text{ month}) = \$14.40$$

(b) The cost per month of a refrigerator with a 800 W compressor is \$11.52. The difference in the running cost of the two refrigerators is \$2.88 per month. So, the number of months before you recover the additional cost of \$100 (of the energy-efficient refrigerator) is \$100/\$2.88 = 34.7 months.

31.53. Visualize: Please refer to Figure P31.53.

Solve: **(a)** Only bulb A is in the circuit when the switch is open. The bulb's resistance R is in series with the internal resistance r, giving a total resistance $R_{eq} = R + r$. The current is

$$I_{bat} = \frac{\varepsilon}{R+r} = \frac{1.50\text{ V}}{6.50\ \Omega} = 0.231\text{ A}$$

This is the current leaving the battery. But all of this current flows through bulb A, so $I_A = I_{bat} = 0.231$ A.

(b) With the switch closed, bulbs A and B are in parallel with an equivalent resistance $R_{eq} = \frac{1}{2}R = 3.00\ \Omega$. Their equivalent resistance is in series with the battery's internal resistance, so the current flowing *from the battery is*

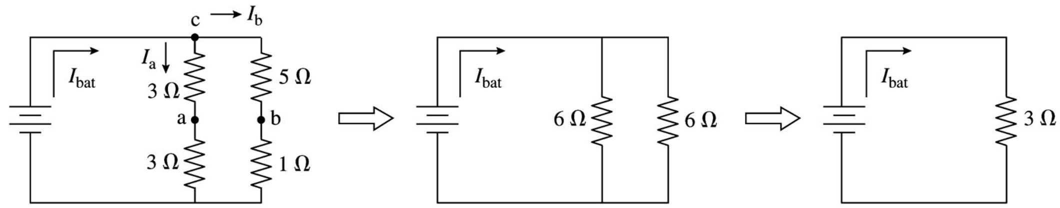
$$I_{bat} = \frac{\varepsilon}{R_{eq}+r} = \frac{1.50\text{ V}}{3.50\ \Omega} = 0.428\text{ A}$$

But only half this current goes through bulb A, with the other half through bulb B, so $I_A = \frac{1}{2}I_{bat} = 0.214$ A.

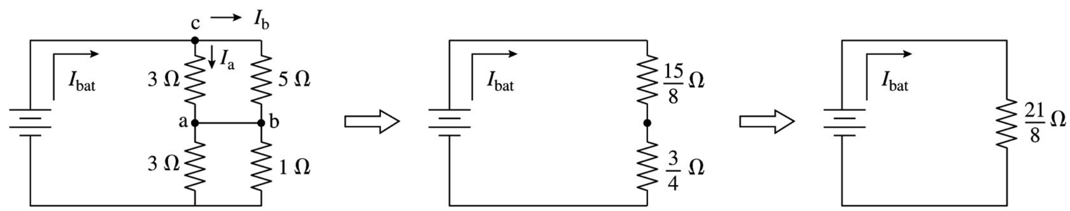
(c) The change in I_A when the switch is closed is 0.017 A. This is a decrease of 7.4%.

31.54. Model: The battery and the connecting wires are ideal.

Visualize:



(a)



(b)

The figure shows the two circuits formed from the circuit in Figure P31.54 when the switch is open and when the switch is closed.

Solve: **(a)** Using the rules of series and parallel resistors, we have simplified the circuit in two steps as shown in figure (a) above. A battery with emf $\varepsilon = 24$ V is connected to an equivalent resistor of 3 Ω. The current in this circuit is (24 V)/(3 Ω) = 8 A. Thus, the current that flows through the battery is $I_{bat} = 8$ A. To determine the potential difference ΔV_{ab}, we will find the potentials at point a and point b and then take the difference. To do this, we need the currents I_a and I_b. We note that the potential difference across the 3-Ω − 3-Ω branch is the same as the potential difference across the 5-Ω − 1-Ω branch, so

$$\varepsilon = 24\text{ V} = I_a(3\ \Omega + 3\ \Omega)I_a = 4\text{ A} = I_b$$

Now, $V_c - I_a(3\ \Omega) = V_a$, and $V_c - I_b(5\ \Omega) = V_b$. Subtracting these two equations give us ΔV_{ab}:

$$V_a - V_b = I_b(5\ \Omega) - I_a(3\ \Omega) = (4\text{ A})(5\ \Omega) - (4\text{ A})(3\ \Omega) = +8\text{ V}$$

(b) Using the rules of series and parallel resistors, we simplifi the circuit as shown in figure (b), above. A battery with emf $\varepsilon = 24$ V is connected to an equivalent resistor of $\frac{21}{8}\Omega$. The current in this circuit is $(24\text{ V})\big/\left(\frac{21}{8}\ \Omega\right) =$ 9.143 A. Thus, the current that flows through the battery is $I_{bat} = 9$ A. When the switch is closed, points a and b are connected by an ideal wire and must therefore be at the same potential. Thus $V_{ab} = 0$ V.

31.55. Model: The voltage source and the connecting wires are ideal.
Visualize: Please refer to Figure P31.55.
Solve: Let us first apply Kirchhoff's loop law starting clockwise from the lower-left corner:

$$+V_{\text{in}} - IR - I(100\ \Omega) = 0\ \text{V} \quad \Rightarrow \quad I = \frac{V_{\text{in}}}{R + 100\ \Omega}$$

The output voltage is

$$V_{\text{out}} = (100\ \Omega)I = (100\ \Omega)\left(\frac{V_{\text{in}}}{R + 100\ \Omega}\right) \quad \Rightarrow \quad \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{100\ \Omega}{R + 100\ \Omega}$$

For $V_{\text{out}} = V_{\text{in}}/10,$ the above equation can be simplified to obtain R:

$$\frac{V_{\text{in}}/10}{V_{\text{in}}} = \frac{100\ \Omega}{R + 100\ \Omega} \quad \Rightarrow \quad R + 100\ \Omega = 1000\ \Omega \quad \Rightarrow \quad R = 900\ \Omega$$

31.56. Model: Assume ideal connecting wires.
Visualize: Please refer to Figure P31.56. Because the ammeter we have shows a full-scale deflection with a current of $500\ \mu\text{A},$ we must not pass a current greater than this through the ammeter.
Solve: The maximum potential difference is 5.0 V and the maximum current is $500\ \mu\text{A}.$ Using Ohm's law,

$$\Delta V = I_{\text{A}}(R + R_{\text{ammeter}}) \quad \Rightarrow \quad 5.0\ \text{V} = (500 \times 10^{-6}\ \text{A})(R + 50.0\ \Omega) \quad \Rightarrow \quad R = 9.95\ \text{k}\Omega$$

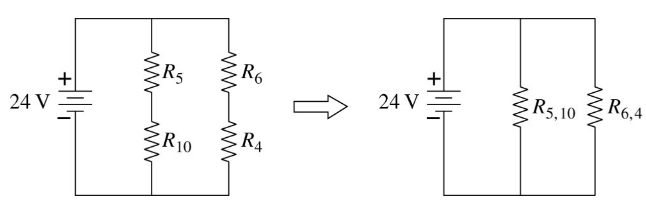
To two significant figures, the resistance is 10 kΩ.

31.57. Model: Assume ideal connecting wires.
Visualize: Please refer to Figure P31.57. Because the ammeter we have shows a full-scale deflection with a current of $500\ \mu\text{A} = 0.500\ \text{mA},$ we must not allow a current more than 0.500 mA to pass through the ammeter. Because we wish to measure a maximum current of 50 mA, we must split the current in such a way that a maximum of 0.500 mA flows through the ammeter with the remaining 49.500 mA flowing through the resistor R.
Solve: **(a)** The potential difference across the ammeter and the resistor is the same. Thus,

$$V_{\text{R}} = V_{\text{ammeter}} \quad \Rightarrow \quad (49.500 \times 10^{-3}\ \text{A})R = (0.500 \times 10^{-3}\ \text{A})(50.0\ \Omega) \quad \Rightarrow \quad R = 0.505\ \Omega$$

(b) The effective resistance is $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{0.505\ \Omega} + \dfrac{1}{50.0\ \Omega} \Rightarrow R_{\text{eq}} = 0.500\ \Omega$

31.58. Model: The battery and the connecting wires are ideal.
Visualize:



The figure shows how to simplify the circuit in Figure P31.58 using the laws of series and parallel resistances. We have labeled the resistors as $R_6 = 6\ \Omega$, $R_{10} = 10\ \Omega$, $R_4 = 4\ \Omega$, and $R_5 = 5\ \Omega$.
Solve: R_5 and R_{10} are combined to get $R_{5,10} = 15\ \Omega$, and R_6 and R_6 and R_4 are combined to obtain $R_{6,4} = 10\ \Omega$. The voltage across both branches is 24 V, so the currents through the branches are

$$I_{5,10} = \frac{\varepsilon}{R_{5,10}} = \frac{24\ \text{V}}{15\ \Omega} = 1.6\ \text{A}, \qquad I_{6,4} = \frac{\varepsilon}{R_{6,4}} = \frac{24\ \text{V}}{10\ \Omega} = 2.4\ \text{A}$$

Thus, the 1.6 A runs through both resistors R_5 and R_{10}, and 2.4 A runs through resistors R_6 and R_4. The potential drop across each resistor can be found using Ohm's law:

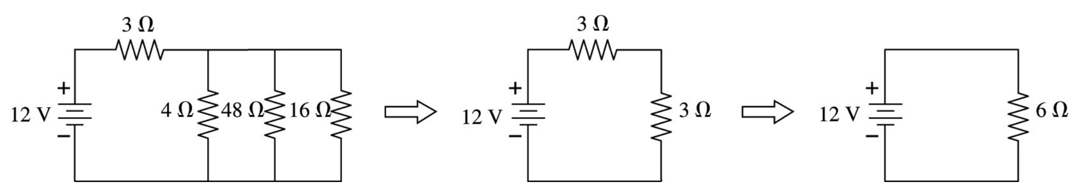
$$\Delta V_5 = I_{5,10}R_5 = (1.6\text{ A})(5\ \Omega) = 8\text{ V}$$
$$\Delta V_{10} = I_{5,10}R_{10} = (1.6\text{ A})(10\ \Omega) = 16\text{ V}$$
$$\Delta V_6 = I_{6,4}R_6 = (2.4\text{ A})(6\ \Omega) = 14.4\text{ V}$$
$$\Delta V_4 = I_{6,4}R_4 = (2.4\text{ A})(4\ \Omega) = 9.6\text{ V}$$

The table below summarizes the results:

Resistor	Potential difference (V)	Current (A)
5 Ω	8	1.6
10 Ω	16	1.6
6 Ω	14.4	2.4
4 Ω	9.6	2.4

Assess: Note that the potential differences across both branches of the circuit sum to 24 V, as required.

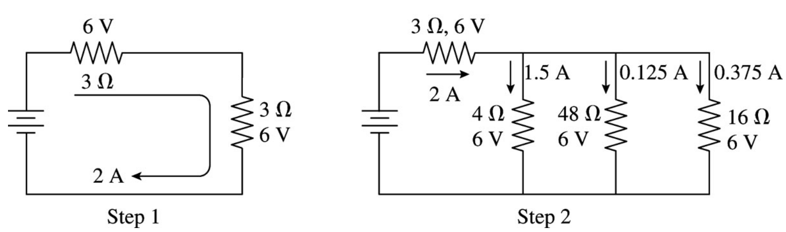
31.59. Model: The battery and the connecting wires are ideal.
Visualize:



The figure shows how to simplify the circuit in Figure P31.59 using the laws of series and parallel resistances. Having reduced the circuit to a single equivalent resistance, we will reverse the procedure and "build up" the circuit using the loop law and the junction law to find the current and potential difference across each resistor.
Solve: From the last circuit in the diagram,

$$I = \frac{\varepsilon}{6\ \Omega} = \frac{12\text{ V}}{6\ \Omega} = 2\text{ A}$$

Thus, the current through the battery is 2 A. As we rebuild the circuit, we note that series resistors *must* have the same current *I* and that parallel resistors *must* have the same potential difference ΔV.



In Step 1, the 6 Ω resistor is returned to a 3 Ω and 3 Ω resistor in series. Both resistors must have the same 2 A current as the 6 Ω resistance. We then use Ohm's law to find

$$\Delta V_3 = (2\text{ A})(3\ \Omega) = 6\text{ V}$$

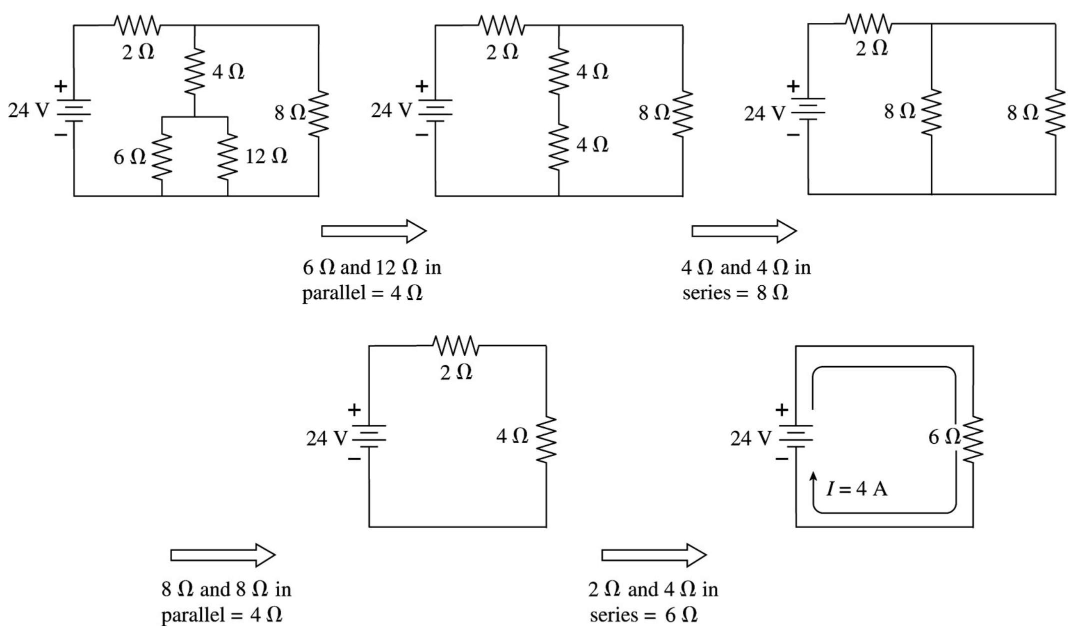
As a check, $6\text{ V} + 6\text{ V} = 12\text{ V}$, which was ΔV of the $6\ \Omega$ resistor. In Step 2, one of the two $3\ \Omega$ resistances is returned to the $4\ \Omega$, $48\ \Omega$, and $16\ \Omega$ resistors in parallel. The three resistors must have the same $\Delta V = 6\text{ V}$. From Ohm's law,

$$I_4 = \frac{6\text{ V}}{4\ \Omega} = 1.5\text{ A} \qquad I_{48} = \frac{6\text{ V}}{48\ \Omega} = \frac{1}{8}\text{ A} \qquad I_{16} = \frac{6\text{ V}}{16\ \Omega} = \frac{3}{8}\text{ A}$$

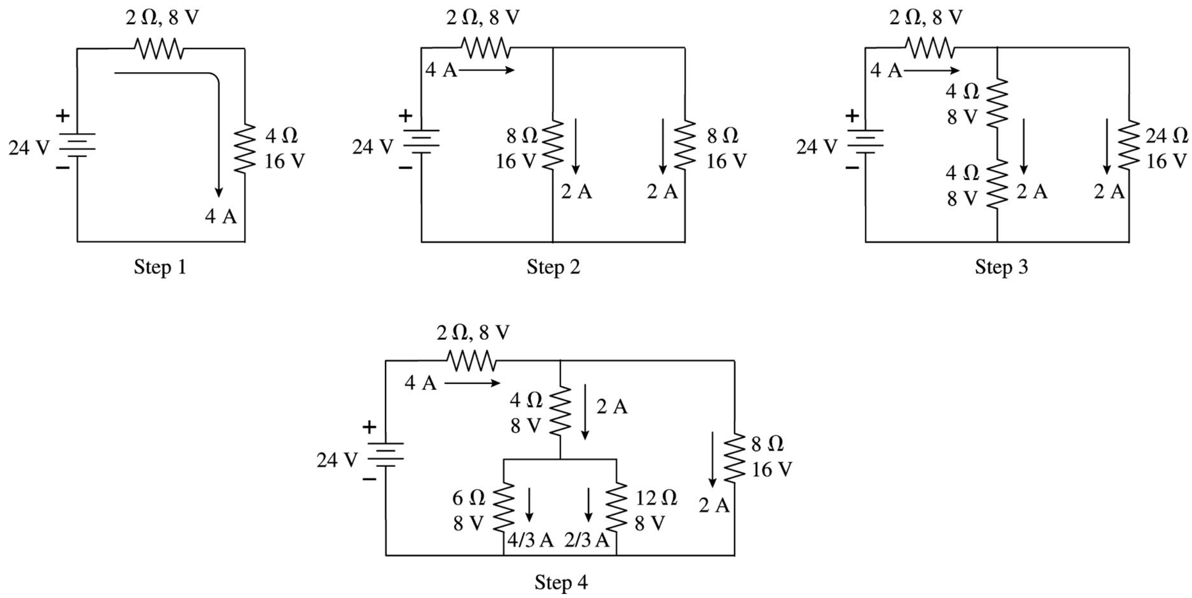
Resistor	Potential difference (V)	Current (A)
3 Ω	6	2
4 Ω	6	1.5
48 Ω	6	1/8
16 Ω	6	3/8

Assess: Note that the currents flowing through the three parallel resistors sum to the 2 A flowing through the battery, as required.

31.60. Model: The battery and the connecting wires are ideal.
Visualize:



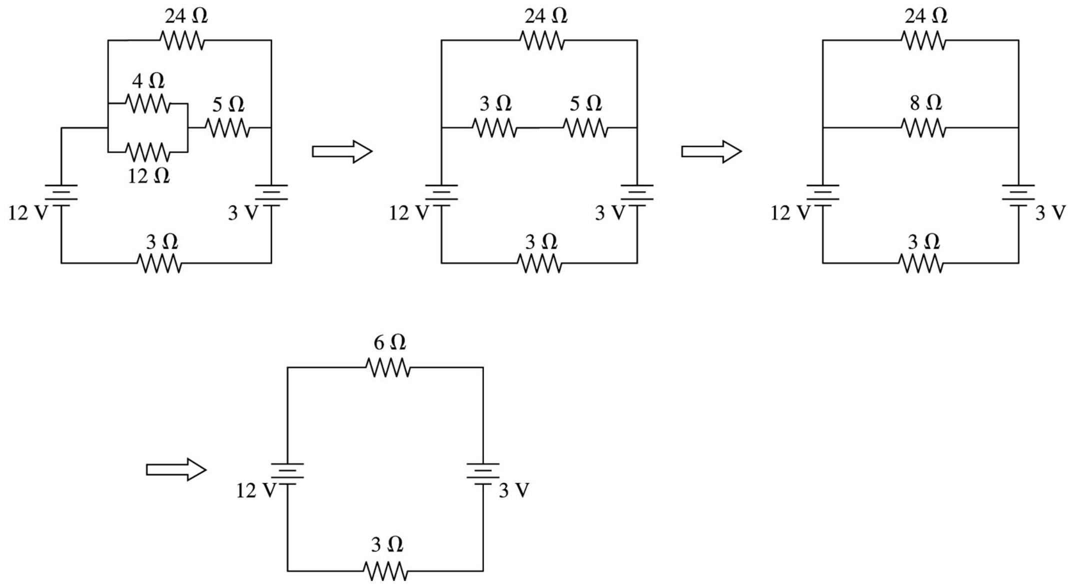
The figure shows how to simplify the circuit in Figure P31.60 using the laws of series and parallel resistances. We will reverse the procedure and "build up" the circuit using the loop law and junction law to find the current and potential difference of each resistor.
Solve: Having found $R_{eq} = 6\ \Omega$, the current from the battery is $I = (24\text{ V})/(6\ \Omega) = 4\text{ A}$. As we rebuild the circuit, note that series resistors *must* have the same current I and that parallel resistors *must* have the same potential difference ΔV.



In Step 1 of the figure above, the 6 Ω equivalent resistor is returned to 2 Ω and 4 Ω resistors in series. Both resistors must have the same 4 A as the 6 Ω equivalent resistor. We use Ohm's law to find $\Delta V_2 = 8$ V and $\Delta V_4 = 16$ V. As a check, $8\text{ V} + 16\text{ V} = 24$ V, which was ΔV of the battery. In Step 2, the 4 Ω resistor is returned to the two 8 Ω resistors in parallel. Both resistors must have the same $\Delta V = 16$ V as the 4 Ω resistor. From Ohm's law, $I_8 = (16\text{ V})/(8\ \Omega) = 2$ A. As a check, $2\text{ A} + 2\text{ A} = 4$ A, which was the current I of the 4 Ω equivalent resistor. In Step 3, the 8 Ω resistor is returned to the two 4 Ω resistors in series. Both resistors must have the same 2 A as the 8 Ω equivalent resistor. We use Ohm's law to find $\Delta V_4 = 8$ V. As a check, $8\text{ V} + 8\text{ V} = 16$ V, which was ΔV of the 8 Ω equivalent resistor. Finally, in Step 4, the lower 4 Ω resistor is returned to the 6 Ω and 12 Ω resistors in parallel. Both resistors must have the same $\Delta V = 8$ V as the 4 Ω equivalent resistor. From Ohm's law, $I_6 = (8\text{ V})/(6\ \Omega) = 4/3$ A and $I_{12} = (8\text{ V})/(12\ \Omega) = 2/3$ A. As a check, $I_6 + I_{12} = 2$ A, which was the current I of the 4 Ω equivalent resistor. The results are summarized in the table below.

Resistor	Potential difference (V)	Current (A)
2 Ω	8	4
4 Ω	8	2
6 Ω	8	4/3
8 Ω	16	2
12 Ω	8	2/3

31.61. Model: The batteries and the connecting wires are ideal.
Visualize:

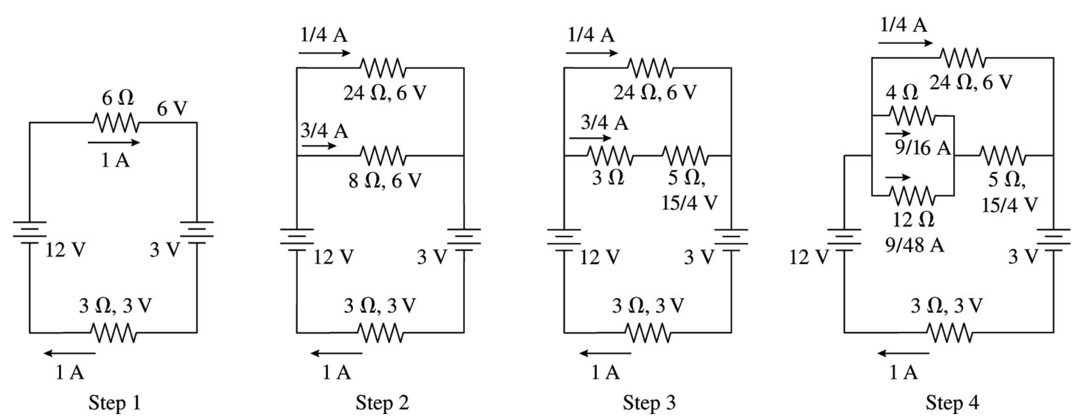


The figure shows how to simplify the circuit in Figure P31.61 using the laws of series and parallel resistances. Having reduced the circuit to a single equivalent resistance, we will reverse the procedure and "build up" the circuit using the loop law and the junction law to find the current and potential difference of each resistor.

Solve: From the last circuit in the figure and from Kirchhoff's loop law,

$$I = \frac{12\text{ V} - 3\text{ V}}{6\ \Omega + 3\ \Omega} = 1\text{ A}$$

Thus, the current through the batteries is 1 A. As we rebuild the circuit, we note that series resistors *must* have the same current I and that parallel resistors *must* have the same potential difference.



In Step 1 of the above figure, both resistors must have the same 1 A current. We use Ohm's law to find

$$\Delta V_3 = (1\text{ A})(3\ \Omega) = 3\text{ V} \qquad \Delta V_{6\text{eq}} = 6\text{ V}$$

As a check we sum the voltages around the circuit starting at the lower-left corner: $12\text{V} - 6\text{ V} - 3\text{ V} - 3\text{ V} = 0\text{ V}$, as required. In Step 2, the $6\ \Omega$ equivalent resistor is returned to the $24\ \Omega$ and $8\ \Omega$ resistors in parallel. The two resistors must have the same potential difference $\Delta V = 6\text{ V}$. From Ohm's law,

$$I_{8\text{eq}} = \frac{6\text{ V}}{8\ \Omega} = \frac{3}{4}\text{ A} \qquad I_{24} = \frac{6\text{ V}}{24\ \Omega} = \frac{1}{4}\text{ A}$$

As a check, $3/4\ \text{A} + 1/4\ \text{A} = 1\ \text{A}$ which was the current I of the $6\ \Omega$ equivalaent resistor. In Step 3, the $8\ \Omega$ equivalent resistor is returned to the $3\ \Omega$ and $5\ \Omega$ resistors in series, so the two resistors must have the same current of 3/4 A. We use Ohm's law to find

$$\Delta V_{3\text{eq}} = \left(\frac{3}{4}\ \text{A}\right)(3\ \Omega) = \frac{9}{4}\ \text{V} \qquad \Delta V_5 = \left(\frac{3}{4}\ \text{A}\right)(5\ \Omega) = \frac{15}{4}\ \text{V}$$

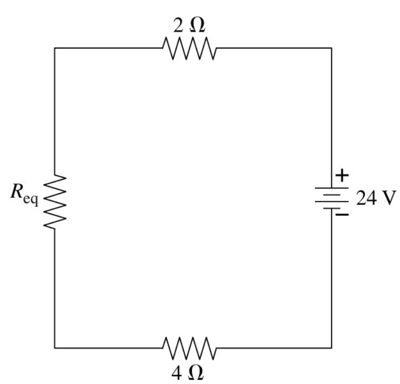
As a check, $9/4\ \text{V} + 15/4\ \text{V} = 24/4\ \text{V} = 6\ \text{V}$, which was ΔV of the $8\ \Omega$ equivalent resistor. In Step 4, the $3\ \Omega$ equivalent resistor is returned to $4\ \Omega$ and $12\ \Omega$ resistors in parallel, so the two must have the same potential difference $\Delta V = 9/4\ \text{V}$. From Ohm's law,

$$I_4 = \frac{9/4\ \text{V}}{4\ \Omega} = \frac{9}{16}\ \text{A} \qquad I_{12} = \frac{9/4\ \text{V}}{12\ \Omega} = \frac{9}{48}\ \text{A}$$

As a check, $9/16\ \text{A} + 9/48\ \text{A} = 3/4\ \text{A}$, which was the same as the current through the $3\ \Omega$ equivalent resistor. The results are summarized in the table below.

Resistor	Potential difference (V)	Current (A)
24 Ω	6	1/4
3 Ω	3	1
5 Ω	15/4	3/4
4 Ω	9/4	9/16
12 Ω	9/4	9/48

31.62. Model: The batteries and the connecting wires are ideal.
Visualize:



The figure shows how to simplify the circuit in Figure P31.62 using the laws of series and parallel resistances.
Solve: The resistance of R_{eq} is

$$R_{\text{eq}} = \left(\frac{1}{20\ \Omega} + \frac{1}{5\ \Omega}\right)^{-1} = \left(\frac{5}{20\ \Omega}\right)^{-1} = 4\ \Omega$$

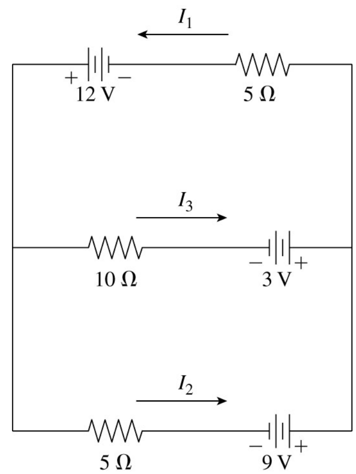
Apply Kirchhoff's loop law, starting from the lower-right corner:

$$\sum \Delta V = 100\ \text{V} - I(2\ \Omega) - I(4\ \Omega) - I(4\ \Omega) \quad \Rightarrow \quad I = \frac{100\ \text{V}}{2\ \Omega + 4\ \Omega + 4\ \Omega} = 10\ \text{A}$$

The voltage difference across the equivalent resistor is $\Delta V = IR_{\text{eq}} = (10\ \text{A})(4\ \Omega) = 40\ \text{V}$, which is also the voltage difference across both resistors that make up this equivalent resistor. Thus, the current through the $20\ \Omega$ resistor is

$$I_{20} = \frac{\Delta V}{R_{20}} = \frac{40\ \text{V}}{20\ \Omega} = 2.0\ \text{A}$$

31.63. Model: The wires and batteries are ideal.
Visualize:



Solve: Assign currents I_1, I_2, and I_3 as shown in the figure above. If I_3 turns out to be negative, we'll know it really flows right to left. Apply Kirchhoff's loop rule counterclockwise to the top loop from the top-right corner:

$$-I_1(5\ \Omega)+12\text{ V}-I_3(10\ \Omega)+3\text{ V}=0$$

Apply the loop rule counterclockwise to the bottom loop starting at the lower-left corner:

$$-I_2(5\ \Omega)+9\text{ V}-3\text{ V}+I_3(10\ \Omega)=0$$

Note that, because we went against the current direction through the $10\ \Omega$ resistor, the potential increased across this resistor. Apply the junction rule to the right middle junction:

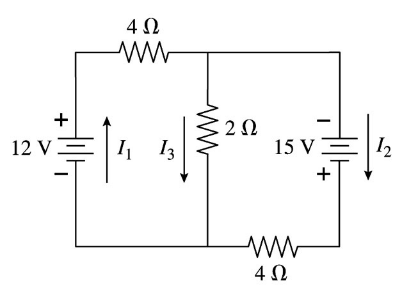
$$I_1=I_2+I_3.$$

These three equations can be solved for the current I_3:

$$(-I_1+I_2)(5\ \Omega)+9\text{ V}-2I_3(10\ \Omega)=0 \quad\Rightarrow\quad -I_3(5\ \Omega)+9\text{ V}-2I_3(10\ \Omega)=0 \quad\Rightarrow\quad I_3=\frac{9}{25}\text{ A}$$

The result is $I_3=9/25\text{ A}=0.12\text{ A}$ flowing from left to right (as shown in the figure above).

31.64. Model: The wires and batteries are ideal.
Visualize:



Solve: The circuit has been redrawn for clarity. Assign the currents I_1, I_2, and I_3 as shown in the figure. To find the power dissipated by the $2\ \Omega$ resistor, we must find the current through it. Apply Kirchhoff's loop rule clockwise to the left loop from the bottom-left corner:

$$+12\text{ V}-I_1(4\ \Omega)-I_3(2\ \Omega)=0.$$

Apply the loop rule clockwise to the right loop starting at the top-right corner:

$$+15\text{ V}-I_2(4\ \Omega)+I_3(2\ \Omega)=0$$

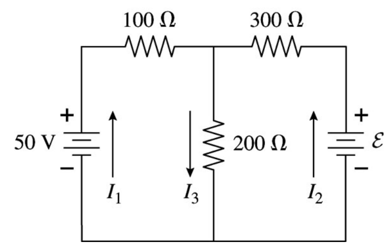
Note that, because we went against the current direction through the $2\ \Omega$ resistor, the potential increased. Apply the junction rule to the lower junction:

$$I_1 = I_2 + I_3$$

These three equations can be solved for the current I_3 by subtracting the second equation from the first, then making the substitution $I_2 - I_1 = -I_3$ which was derived from the third equation. The result is $I_3 = -3/8$ A. The power dissipated in the resistor is

$$P_{2\,\Omega} = I_3^2 R_2 = \left(-\frac{3}{8}\text{ A}\right)^2 (2\ \Omega) = \frac{9}{32}\text{ W} = 0.3\text{ W}$$

31.65. Model: The wires and batteries are ideal.
Visualize:



Solve: If no power is dissipated in the $200\ \Omega$ resistor, the current through it must be zero. To see if this is possible, set up Kirchhoff's rules for the circuit, then assume the current through the $200\ \Omega$ resistor is zero and see if there is a solution. Assume the unknown battery is oriented with its positive terminal at the top and define currents I_1, I_2, and I_3 as shown in the figure above. Apply Kirchhoff's loop rule clockwise to the left loop:

$$50\text{ V} - I_1(100) - I_3(200) = 0$$

Apply Kirchhoff's loop rule counterclockwise to the right hand loop:

$$\varepsilon - I_2(300\ \Omega) - I_3(200\ \Omega) = 0$$

The junction rule yields

$$I_1 + I_2 = I_3.$$

Now assume $I_3 = 0$ and solve for ε. In that case, the first equation gives

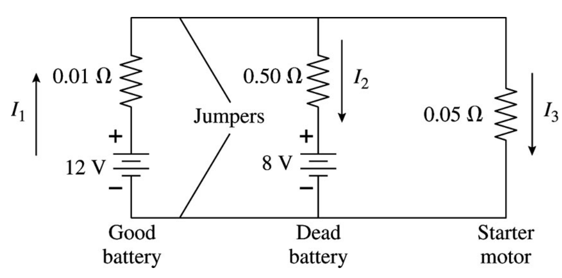
$$I_1 = \frac{50\text{ V}}{100\ \Omega} = \frac{1}{2}\text{ A}.$$

From the third equation, $I_2 = -I_1$, so the second equation gives us

$$\varepsilon = I_2(300\ \Omega) = \left(-\frac{1}{2}\text{ A}\right)(300\ \Omega) = -150\text{ V}$$

Thus $\varepsilon = 150$ V and it is oriented with the positive terminal on the bottom, opposite to our guess.

31.66. Model: The wires are ideal, but the batteries are not.
Visualize:



Solve: (a) The good battery alone can drive a current through the starter motor

$$I = \frac{12\text{ V}}{(0.01\ \Omega + 0.05\ \Omega)} = 200\text{ A}$$

(b) Alone, the dead battery drives a current

$$I = \frac{8.0\text{ V}}{(0.50\ \Omega + 0.05\ \Omega)} = 14.5\text{ A} \approx 15\text{ A}$$

(c) Let I_1, I_2, and I_3 be defined as shown in the figure above. Kirchhoff's laws applied to the good- and dead-battery loop, good-battery and starter-motor loop, and the top middle junction yield three equations in the three unknown currents:

$$12\text{ V} - I_1(0.01\ \Omega) - I_3(0.05\ \Omega) = 0$$
$$12\text{ V} - I_1(0.01\ \Omega) - I_2(0.50\ \Omega) - 8.0\text{ V} = 0$$
$$I_1 = I_2 + I_3$$

Substituting for I_1 from the third equation into the first and second equations gives

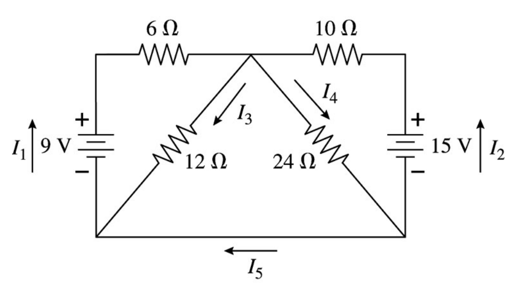
$$12\text{ V} - I_2(0.01\ \Omega) - I_3(0.06\ \Omega) = 0$$
$$4\text{ V} - I_2(0.51\ \Omega) - I_3(0.01\ \Omega) = 0$$

Solving for I_2 from the first equation,

$$I_2 = \frac{12\text{ V} - I_3(0.06\ \Omega)}{(0.01\ \Omega)}$$

Substituting into the second equation and solving for I_3 yields the current through the starter motor is 199 A. To a single significant figure, this is 200 A.
(d) Substituting the value for I_3 into the expression for I_2 yields the current through the dead battery as 3.9 A. To a single significant figure, this is 4 A.
Assess: The good battery is charging the dead battery as well as running the started motor. A total of 203 A flows through the good battery.

31.67. Model: The wires and batteries are ideal.
Visualize:



Solve: The circuit is redrawn above for clarity and the currents are shown. We must find I_5.
Repeatedly apply Kirchhoff's rules to the loops. The loop rule applied clockwise about the three triangles yields

$$\text{Left: } 9\text{ V} - I_1(6\ \Omega) - I_3(12\ \Omega) = 0 \quad\Rightarrow\quad I_1 = 1.5\text{ A} - 2I_3$$
$$\text{Center: } -I_4(24\ \Omega) + I_3(12\ \Omega) = 0 \quad\Rightarrow\quad I_4 = I_3/2$$
$$\text{Right: } 15\text{ V} - I_2(10\ \Omega) - I_4(24\ \Omega) = 0 \quad\Rightarrow\quad I_2 = 1.5\text{ A} - 2.4I_4$$

The junction rule applied at the bottom corners gives equations into which the results above may be substituted:

$$I_1 = I_3 + I_5 \quad\Rightarrow\quad 1.5\text{ A} - 2I_3 = I_3 + I_5 \quad\Rightarrow\quad I_5 = 1.5\text{ A} - 3I_3$$
$$I_4 = I_2 + I_5 \quad\Rightarrow\quad I_4 = 1.5\text{ A} - 2.4I_4 + I_5 \quad\Rightarrow\quad I_5 = 3.4I_4 - 1.5\text{ A}$$

Using $I_4 = I_3/2$ and solving for I_3 gives

$$1.5\text{ A} - 3I_3 = 3.4(I_3/2) - 1.5\text{ A} \quad\Rightarrow\quad I_3 = \frac{30}{47}\text{ A}$$

$$I_5 = 1.5\text{ A} - 3\left(\frac{30}{47}\text{ A}\right) = -\frac{201}{94}\text{ A} = -0.41\text{ A}$$

Since the result is negative, 0.40 A flows from left to right through the bottom wire.

31.68. Model: Assume ideal wires. The capacitor discharges through the resistor.

Solve: (a) The capacitor discharges through the resistor R as $Q = Q_0 e^{-t/\tau}$. For $Q = Q_0/2$,

$$\frac{Q_0}{2} = Q_0 e^{-t/10\text{ ms}} \Rightarrow \ln\left(\frac{1}{2}\right) = -\frac{t}{0.010\text{ s}} \quad\Rightarrow\quad t = -(0.010\text{ s})\ln(0.5) = 6.9\text{ ms}$$

(b) If the initial capacitor energy is U_0, we want the time when the capacitor's energy will be $U = U_0/2$. Noting that $U_0 = Q_0^2/(2C)$, this means $Q = Q_0/\sqrt{2}$. Applying the equation for the discharging capacitor gives

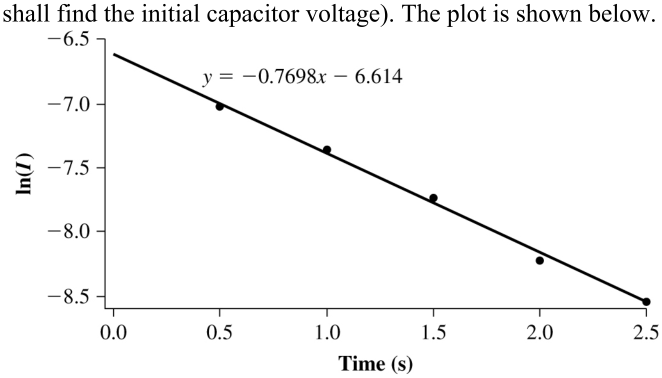
$$\frac{Q_0}{\sqrt{2}} = Q_0 e^{-t/10\text{ ms}} \quad\Rightarrow\quad \ln\left(\frac{1}{\sqrt{2}}\right) = -\frac{t}{0.010\text{ s}} \quad\Rightarrow\quad t = -(0.010\text{ s})\ln\left(\frac{1}{\sqrt{2}}\right) = 3.5\text{ ms}$$

31.69. Model: The capacitor discharges through the resistor R as $I = I_0 e^{-t/\tau}$, where $\tau = RC$. Assume negligible resistance in the connecting leads.

Solve: (a) Taking natural logs of the current equation gives

$$\ln(I) = \ln(I_0) - t/\tau$$

Therefore, if we plot ln(I) versus t, we should get a straight line with slope $s = -1/\tau$ and y-intercept equal to ln I_0. The slope s will be related to the resistance according to $s = -1/\tau$ and the initial current can be found from the y-intercept (from which we shall find the initial capacitor voltage). The plot is shown below.



The slope is $s = -0.7698$, so the resistance is

$$-0.7698\text{ s}^{-1} = -\frac{1}{\tau} = -\frac{1}{RC} \quad\Rightarrow\quad R = \frac{1}{(0.7698\text{ s}^{-1})(20\ \mu\text{F})} = 64{,}952\ \Omega = 65\text{ k}\Omega$$

(b) The y-intercept of the graph is −6.614, so the initial current is

$$\ln(I_0) = -6.614 \quad\Rightarrow\quad I_0 = e^{-6.614} = 1.341\text{ mA}$$

Because the capacitor and the resistor are in parallel, the capacitor voltage equals the resistor voltage. Using Ohm's law to find the initial resistor voltage will therefore also give us the capacitor voltage:

$$V_C = V_0 = I_0 R = (1.341\text{ mA})(64.952\text{ k}\Omega) = 87\text{ V}$$

31.70. Model: The capacitor discharges through the resistor (the patient's chest) according to $Q = Q_0 e^{-t/\tau}$ (see Equation 31.31), where $\tau = RC$. Assume negligible resistance in the connecting leads and that all the resistance is due to the patient's chest.

Solve: At time $t = 40$ ms, the capacitor has lost 95% of its charge, so it has only 5% left. Therefore $Q = 0.050Q_0$ at that time:

$$\frac{Q}{Q_0} = 0.050 = e^{-t/\tau} = e^{-t/(RC)} \quad\Rightarrow\quad R = \frac{-t}{C\ln(0.050)} = \frac{-(40\text{ ms})}{(150\ \mu\text{F})\ln(0.050)} = 89\ \Omega$$

31.71. Model: The capacitor discharges through the resistor, and the wires are ideal.

Solve: In an *RC* circuit, the charge at a given time is related to the original charge as $Q = Q_0 e^{-t/\tau}$. For a capacitor $Q = C\Delta V$, so $\Delta V = \Delta V_0 e^{-t/\tau}$. From the Figure P31.71, we note that $\Delta V_0 = 30$ V and $\Delta V = 10$ V at $t = 4$ ms. So,

$$10\text{ V} = (30\text{ V})e^{-4\text{ ms}/R(50\times10^{-6}\text{ F})} \Rightarrow \ln\left(\frac{10\text{ V}}{30\text{ V}}\right) = -\frac{4\times10^{-3}\text{ s}}{R(50\times10^{-6}\text{ F})} \Rightarrow R = -\frac{4\times10^{-3}\text{ s}}{(50\times10^{-6}\text{ F})\ln\left(\frac{1}{3}\right)} = 73\ \Omega$$

31.72. Model: The capacitor discharges through the resistors. The wires are ideal.

Solve: The charge Q on the capacitor charged to 50 V is

$$Q = C\Delta V = (0.25\ \mu\text{F})(50\text{ V}) = 12.5\ \mu\text{C}$$

When this fully charged capacitor is connected in series with a $25\ \Omega$ resistor and a $100\ \Omega$ resistor, it will dissipate all its stored energy

$$U_C = \frac{Q^2}{2C} = \frac{(12.5\ \mu\text{C})^2}{2(0.25\ \mu\text{F})} = 312.5\times10^{-6}\text{ J}$$

through the two resistors. Energy dissipated by the resistor is I^2R, which means the $100\ \Omega$ resistor will dissipate four times more energy than the $25\ \Omega$ resistor at any given time. Thus, the energy dissipated by the $25\ \Omega$ resistor is

$$312.5\times10^{-6}\left(\frac{25\ \Omega}{25\ \Omega + 100\ \Omega}\right) = 62.5\times10^{-6}\text{ J} = 63\ \mu\text{J}$$

31.73. Model: The battery and the connecting wires are ideal.

Visualize: Please refer to Figure P31.73.

Solve: (a) A very long time after the switch has closed, the potential difference ΔV_C across the capacitor is ε. This is because the capacitor charges until $\Delta V_C = \varepsilon$ while the charging current approaches zero.

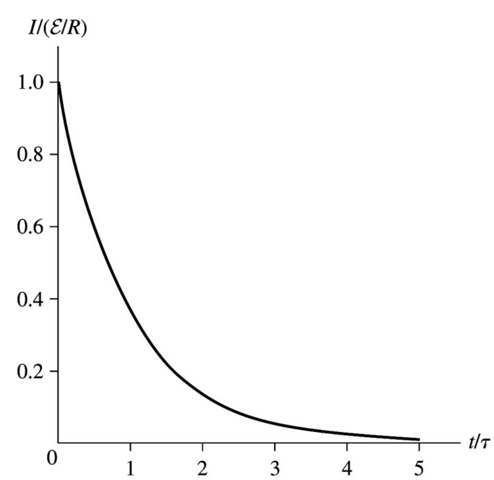
(b) The full charge of the capacitor is $Q_{max} = C(\Delta V_C)_{max} = C\varepsilon$.

(c) In this circuit, $I = +dQ/dt$ because the capacitor is charging; that is, because the charge on the capacitor is increasing.

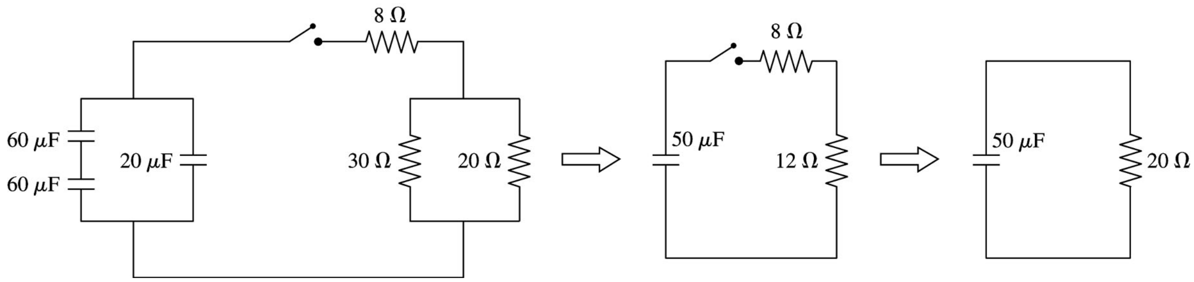
(d) From Equation 31.36, capacitor charge at time *t* is $Q = Q_{max}(1 - e^{-t/\tau})$. Therefore,

$$I = \frac{dQ}{dt} = C\varepsilon\frac{d}{dt}(1 - e^{-t/\tau}) = C\varepsilon\left(\frac{1}{\tau}\right)e^{-t/\tau} = C\varepsilon\left(\frac{1}{RC}\right)e^{-t/\tau} = \frac{\varepsilon}{R}e^{-t/\tau}$$

A graph of *I* as a function of *t* is shown below.



31.74. Model: The connecting wires are ideal. The capacitors discharge through the resistors.
Visualize:



The figure shows how to simplify the circuit in Figure P31.74 using the laws of series and parallel resistors and the laws of series and parallel capacitors.

Solve: The 30 Ω and 20 Ω resistors are in parallel and are equivalent to a 12 Ω resistor. This 12 Ω resistor is in series with the 8 Ω resistor so the equivalent resistance of the circuit $R_{eq} = 20\ \Omega$. The two 60 μF capacitors are in series producing an equivalent capacitance of 30 μF. This 30 μF capacitor is in parallel with the 20 μF capacitor so the equivalent capacitance C_{eq} of the circuit is 50 μF. The time constant of this circuit is

$$\tau = R_{eq}C_{eq} = (20\ \Omega)(50\ \mu\text{F}) = 1.0\ \text{ms}$$

The current due to the three capacitors through the 20 Ω equivalent resistor is the same as through the 8 Ω resistor. So, the voltage across the 8 Ω resistor follows the decay equation $V = V_0 e^{-t/\tau}$. For $V = V_0/2$, we get

$$\frac{V_0}{2} = V_0 e^{-t/1.0\ \text{ms}} \Rightarrow \ln\left(\frac{1}{2}\right) = -\frac{t}{1.0\ \text{ms}} \Rightarrow t = 0.69\ \text{ms}$$

31.75. Solve: The resistivity of aluminum is $2.8\times10^{-8}\ \Omega$ m and we want the wire to dissipate 7.5 W when connected to a 1.5 V battery. The resistance of the wire must be

$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(1.5\ \text{V})^2}{7.5\ \text{W}} = 0.30\ \Omega$$

Using the formula for the resistance of a wire,

$$R = \rho\frac{L}{A} \Rightarrow 0.30\ \Omega = (2.8\times10^{-8}\ \Omega\text{m})\frac{L}{\pi r^2} \Rightarrow L = (3.366\times10^7\ \text{m}^{-1})r^2$$

We need another relation connecting L and r. Making use of the mass density of aluminum gives

$$\frac{1.0\times10^{-3}\ \text{kg}}{\pi r^2 L} = 2700\ \text{kg/m}^3 \Rightarrow r^2 L = 1.179\times10^{-7}\ \text{m}^3$$

Using the value of L obtained above,

$$r^2(3.366\times10^7\ \text{m}^{-1})r^2 = 1.179\times10^{-7}\ \text{m}^3 \Rightarrow r^4 = 3.50\times10^{-15}\ \text{m}^4 \Rightarrow r = 2.43\times10^{-4}\ \text{m} = 0.243\ \text{mm}$$

Thus, the diameter of the wire is 0.49 mm and the length is

$$L = (3.366\times10^7\ \text{m}^{-1})(2.43\times10^{-4}\ \text{m})^2 = 2.0\ \text{m}$$

Assess: It is reasonable to make a 2.0 m long wire with a diameter of 0.49 mm from an aluminum block of 1.0 g.

31.76. Model: Assume the battery and the connecting wires are ideal.
Visualize: Please refer to Figure CP31.76.
Solve: (a) If the switch has been closed for a long time, the capacitor is fully charged and there is no current flowing through the right branch that contains the capacitor. Therefore, a voltage of 60 V appears across the 60 Ω resistor and a voltage of 40 V appears across the 40 Ω resistor. That is, maximum voltage across the capacitor is 40 V. Thus, the charge on the capacitor is

$$Q_0 = \varepsilon C = (40\ \text{V})(2.0\times10^{-6}\ \text{F}) = 80\ \mu\text{C}$$

(b) Once the switch is opened, the battery is disconnected from the capacitor. The capacitor C has two resistances ($10\ \Omega$ and $40\ \Omega$, which give a $50\ \Omega$ equivalent resistance) in series and discharges according to $Q = Q_0 e^{-t/RC}$. For $Q = 0.10Q_0$,

$$0.10Q_0 = Q_0 e^{-t/[(50\ \Omega)(2.0\ \mu\text{F})]} \Rightarrow \ln(0.10) = -\frac{t}{(50\ \Omega)(2.0\ \mu\text{F})}$$

$$t = -(50\ \Omega)(2.0\ \mu\text{F})\ln(0.10) = 0.23\text{ ms}$$

31.77. Solve: The capacitor's voltage is given by $V = Q/C$. The charge Q may be found by integrating the current provided by the capacitor-charging circuit:

$$Q(t = 20\text{ ms}) = \int_0^{20\text{ ms}} I(t')dt' = I_0 \int_0^{20\text{ ms}} e^{-t'/\tau}dt' = -I_0\tau e^{-t'/\tau}\Big|_0^{20\text{ ms}} = I_0\tau\left(1 - e^{-(20\text{ ms})/\tau}\right)$$

Using the given values of $I_0 = 65$ mA and $\tau = 40$ ms, we find $Q = 1.02$ mC. Thus, after 20 ms, the voltage on the capacitor is

$$V = \frac{Q}{C} = \frac{1.02 \times 10^{-3}\text{ C}}{50\ \mu\text{F}} = 20\text{ V}$$

31.78. Model: The battery and the connecting wires are ideal.
Visualize: Please refer to Figure 31.38a.
Solve: After the switch closes at $t = 0$ s, the capacitor begins to charge. At time t, let the current and the charge in the circuit be i and q, respectively. Also, assume clockwise direction for the current i. Using Kirchhoff's loop law and starting clockwise from the lower-left corner of the loop,

$$+\mathcal{E} - iR - \frac{q}{C} = 0 \Rightarrow \mathcal{E} = \frac{dq}{dt}R + \frac{q}{C} \Rightarrow RCdq = (\varepsilon C - q)dt \Rightarrow \frac{dq}{\varepsilon C - q} = \frac{dt}{RC}$$

Integrating both sides gives

$$\int_0^Q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{RC} \Rightarrow -[\ln(\varepsilon C - q)]_0^Q = \frac{t}{RC} \Rightarrow -\ln(\varepsilon C - Q) + \ln(\varepsilon C) = \frac{t}{RC}$$

$$\ln\left(\frac{\varepsilon C - Q}{\varepsilon C}\right) = -\frac{t}{RC} \Rightarrow \frac{\varepsilon C - Q}{\varepsilon C} = e^{-t/RC} \Rightarrow Q = \varepsilon C(1 - e^{-t/RC})$$

Letting $Q_{\text{max}} = \varepsilon C$ and $\tau = RC$, we get $Q = Q_{\text{max}}(1 - e^{-t/\tau})$.

31.79. Model: The battery and the connecting wires are ideal.
Visualize: Please refer to Figure 31.38a.
Solve: (a) According to Equation 31.36, the charge on the capacitor increases according to $Q = Q_{\text{max}}(1 - e^{-t/\tau})$ during charging. Therefore, the current in the circuit behaves as

$$I = \frac{dQ}{dt} = Q_{\text{max}}\left(-\frac{1}{\tau}\right)(-e^{-t/\tau}) = \frac{\varepsilon C}{RC}e^{-t/RC} = \frac{\varepsilon}{R}e^{-t/RC}$$

Using Equation 31.8, the power supplied by the battery as the capacitor is being charged is

$$P_{\text{bat}} = I\varepsilon = \left(\frac{\varepsilon}{R}e^{-t/RC}\right)\varepsilon = \frac{\varepsilon^2}{R}e^{-t/RC}$$

Because $P_{\text{bat}} = dU/dt$, we have

$$dU = P_{\text{bat}}dt \Rightarrow \int dU = \int_0^\infty P_{\text{bat}}dt = \int_0^\infty \frac{\varepsilon^2}{R}e^{-t/RC}dt = \frac{\varepsilon^2}{R}\left[-RCe^{-t/RC}\right]_0^\infty = \varepsilon^2 C$$

That is, the total energy which has been supplied by the battery when the capacitor is fully charged is $\varepsilon^2 C$.
(b) The power dissipated by the resistor as the capacitor is being charged is

$$P_{\text{resistor}} = I^2 R = \frac{\varepsilon^2}{R^2} R e^{-2t/RC}$$

Because $P_{\text{resistor}} = dU/dt$, we have

$$dU = P_{\text{resistor}} dt \quad \Rightarrow \quad \int dU = \int_0^\infty \frac{\varepsilon^2}{R} e^{-2t/RC} dt \quad \Rightarrow \quad U_{\text{resistor}} = \frac{\varepsilon^2}{R}\left[-\frac{RC}{2} e^{-2t/RC}\right]_0^\infty = \frac{\varepsilon^2 C}{2}$$

(c) The energy stored in the capacitor when it is fully charged is

$$U_{\text{C}} = \frac{1}{2}\frac{Q_{\max}^2}{C} = \frac{1}{2}\frac{C^2\mathcal{E}^2}{C} = \frac{1}{2}C\mathcal{E}^2$$

(d) For energy conservation, the energy delivered by battery is equal to the energy dissipated by the resistor R plus the energy stored in the capacitor C. This is indeed the case because $U_{\text{bat}} = \mathcal{E}^2 C$, $U_{\text{resistor}} = \frac{1}{2}\mathcal{E}^2 C$, and $U_{\text{C}} = \frac{1}{2}\mathcal{E}^2 C$.

31.80. Model: The battery and the connecting wires are ideal.
Visualize: Please refer to Figure CP31.80.
Solve: (a) During charging, when the neon gas behaves like an insulator, the charge on the capacitor increases according to Equation 31.36; that is, $Q = Q_{\max}(1 - e^{-t/\tau})$. Because $Q = C\Delta V_{\text{C}}$,

$$\Delta V_{\text{C}} = \Delta V_{\text{C}0}(1 - e^{-t/\tau}) = \varepsilon(1 - e^{-t/\tau})$$

Let us say that the period of oscillation begins when $\Delta V = V_{\text{off}}$ and it ends when $\Delta V = V_{\text{on}}$. Then,

$$V_{\text{off}} = \varepsilon(1 - e^{-t_{\text{off}}/\tau}) \quad \Rightarrow \quad \frac{\varepsilon - V_{\text{off}}}{\varepsilon} = e^{-t_{\text{off}}/\tau} \quad \Rightarrow \quad \frac{t_{\text{off}}}{\tau} = \ln\left(\frac{\varepsilon}{\varepsilon - V_{\text{off}}}\right)$$

Because the period $T = t_{\text{on}} - t_{\text{off}}$, we have

$$T = \tau\ln\left(\frac{\varepsilon}{\varepsilon - V_{\text{on}}}\right) - \tau\ln\left(\frac{\varepsilon}{\varepsilon - V_{\text{off}}}\right) = \tau\ln\left(\frac{\varepsilon - V_{\text{off}}}{\varepsilon - V_{\text{on}}}\right) = RC\ln\left(\frac{\varepsilon - V_{\text{off}}}{\varepsilon - V_{\text{on}}}\right)$$

(b) Substituting the given values into the above expression and noting that and $T = 1/f = 0.10\text{ s}$ gives

$$0.10\text{ s} = R(10\times10^{-6}\text{ F})\ln\left(\frac{90\text{ V} - 20\text{ V}}{90\text{ V} - 80\text{ V}}\right) \quad \Rightarrow \quad R = \frac{0.10\text{ s}}{(10\times10^{-6}\text{ F})\ln(7)} = 5.1\text{ k}\Omega$$

31.81. Model: The battery is ideal.
Solve: The variable-resistance wire may be thought of as an infinite series of infinitesimally small resistors of resistance

$$dR = \frac{\rho(x)dx}{A}$$

where A is the cross-sectional area of the wire. Because resistors in series add, we can integrate this expression to find the total resistance R of the wire:

$$R = \frac{1}{A}\int_0^{2.0\text{ m}} \rho(x)dx = \frac{1}{A}\int_0^{2.0\text{ m}} \left\{(2.5\times10^{-6})\left[1 + \left(\frac{x}{1.0\text{ m}}\right)^2\right]\Omega\text{ m}\right\}dx$$

$$= \frac{2.5\times10^{-6}}{\pi(0.50\times10^{-3}\text{ m})^2}\left(x + \frac{x^3}{3.0\text{ m}^2}\right)_0^{2.0\text{ m}} = \frac{2.5\times10^{-6}}{\pi(0.50\times10^{-3}\text{ m})^2}\left(2.0\text{ m} + \frac{(2.0\text{ m})^3}{3.0\text{ m}^2}\right) = 14.85\ \Omega$$

We can use Ohm's law to find the current through this wire if it were attached to a 9.0 V battery:

$$I = \frac{V}{R} = \frac{9.0\text{ V}}{14.85\ \Omega} = 0.61\text{ A}$$