32

THE MAGNETIC FIELD

Conceptual Questions

32.1. **(a)** The sphere is not effected by the magnet. Glass is not magnetic.
(b) There is no magnetic force between the glass sphere and magnet since magnetic forces act on moving charges and some metals. There is, however, a weak attraction due to a polarization of the charges if the magnet is metal.

32.2. It is attracted. Magnetic materials are attracted to both poles of a magnet. This is analogous to how neutral objects are attracted to both positively and negatively charged rods by the polarization force.

32.3. Yes, if both cylinders are magnets they will repel if one of them is turned 180°. If only one cylinder is a magnet, then they will still attract after one of them is turned around.

32.4. Because the north poles of the compass magnets point counterclockwise, the magnetic force is counterclockwise. When you point fingers of your right hand counterclockwise, the thumb points up. Thus, the current in the wire is out of the page.

32.5. The magnetic field is into the page on the left of the wire and it is out of the page on the right of the wire. Grab the wire with your right hand in such a way that your fingers point out of the page to the right of the wire. Since the thumb now points down, the current in the wire is down.

32.6. **(a)** The force on a charge moving in a magnetic field is

$$\vec{F}_{\text{on q}} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{ direction of right-hand rule})$$

A *positive* charge moving to the right with $\vec{B}$ into the page gives a force that is *up*.
(b) The direction of the force on a negative charge is opposite the direction determined by the right-hand rule. A *negative* charge moving up with $\vec{B}$ out of the page gives a force to the left.

32.7. **(a)** The force on a charge moving in a magnetic field is

$$\vec{F}_{\text{on q}} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{ direction of right-hand rule})$$

A *positive* charge moving to the right with $\vec{B}$ down gives a force *into the page*.
(b) The direction of the force on a negative charge is opposite the direction determined by the right-hand rule. Any charge moving parallel to the field has *no* force and *no* deflection.

32.8. **(a)** The force on a charge moving in a magnetic field is

$$\vec{F}_{\text{on q}} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{ direction of right-hand rule})$$

The magnetic field must be in a plane perpendicular to both the $\vec{v}$ and $\vec{F}$ vectors. Using the right-hand rule for a *positive* charge moving to the right, the $\vec{B}$ field must be *out* of the page.

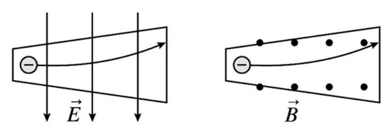
(b) The direction of the force on a negative charge is opposite the direction determined by the right-hand rule. The force $\vec{F}$ on the *negative* charge is into the page. Since the velocity is to the right, the magnetic field $\vec{B}$ must be *up*.

32.9. **(a)** The force on a charge moving in a magnetic field is

$$\vec{F}_{\text{on q}} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{ direction of right-hand rule})$$

The direction of the force on a negative charge is opposite the direction determined by the right-hand rule. Since the force $\vec{F}$ is *out of the page* and the velocity of the negative charge is to the left and up in the plane of the paper, the magnetic field $\vec{B}$ must be in the plane of the page, $45°$ clockwise from straight up.
(b) The magnetic field on the positive charge is in the plane of the page, 45° counterclockwise from straight down.

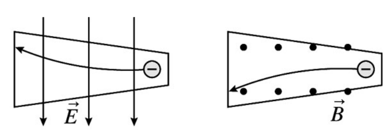
32.10. Magnetic and electric fields exert forces on a moving charge. Consider a top view of the cathode ray tube (CRT), as seen from the ceiling. If an electric field is causing the deflection, the electric field must point to the left, as seen when viewing the screen. If a magnetic field is causing the deflection, the magnetic field must point up toward the ceiling. These conclusions both depend on the fact that the electron is negative. The figure below illustrates these two possibilities.



(a) Electric field pointing to the left   Magnetic field pointing up

The first step is to turn the CRT around 180° to face the opposite wall. If the deflection is caused by an electric field, the deflection will reverse and appear as a deflection to the left. However, if the deflection is caused by a magnetic field or if the CRT is broken, the deflection will still be to the right. The figure below illustrates these two possibilities.

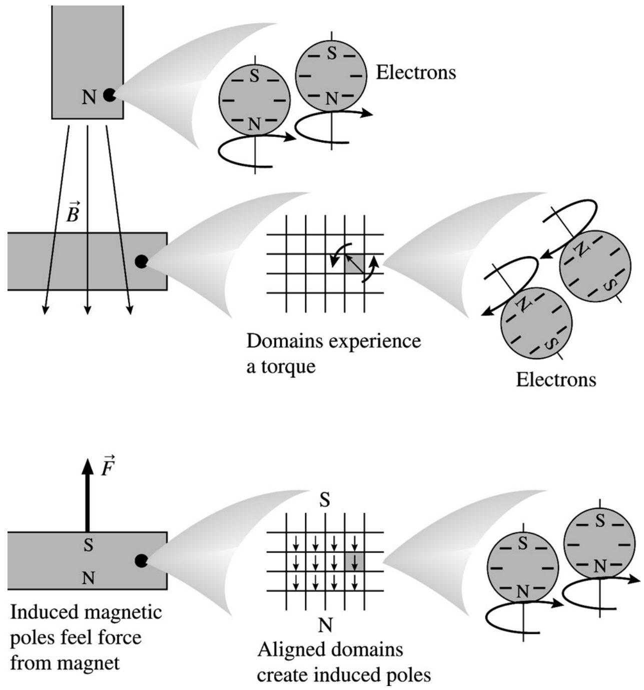
(b)



The next step is to point the CRT at the ceiling. If there is a magnetic field, the electrons are now moving parallel to the field and won't be deflected at all. So a centered spot after the second rotation means that the cause is a magnetic field. If the spot is still to the right, the CRT is broken.

32.11. The magnet is repelled. The current loop produces a magnetic dipole with the north pole on the left and the south pole on the right. The approaching south pole of the magnet is repelled by the south pole of the current loop.

32.12.



A permanent magnet creates a magnetic field. This is due to the motion of the charge of the "spinning" electrons. The unmagnetized piece of iron also has spinning electrons, but these are organized into magnetic domains that are randomly oriented and give no net magnetic moment. However, each individual domain—due to the aligned atomic spins—is a magnetic moment, so it experiences a torque in the field of the permanent magnet. This torque tends to align the domains with the external field $\vec{B}$. Now there are many aligned domains, so the piece of iron has an *induced* net magnetic moment. That is, the piece of iron has an induced north and south pole. The induced south pole of the iron is facing the permanent magnet's north pole, so there is an attractive force that lifts the iron. Although the lifting is a macroscopic phenomenon, it is due to magnetic attraction between the spinning electrons in the permanent magnet and the spinning electrons in the piece of iron.

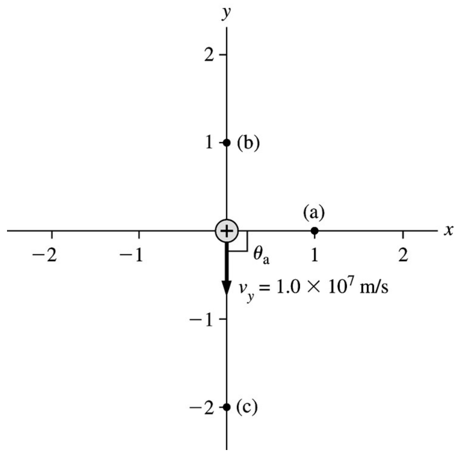
Exercises and Problems

Section 32.3 The Source of the Magnetic Field: Moving Charges

32.1. Model: A magnetic field is caused by an electric current.
Solve: The magnitude of the magnetic field at point 1 is 2.0 mT and its direction can be determined by using the right-hand rule. Grab the current-carrying wire so that your thumb points in the direction of the current. Because your fingers at point 1 point into the page, $\vec{B}_1 = (2.0 \text{ mT, into the page})$. At point 2, the magnetic field due to the bottom wire is into the page. The right-hand rule tells us that the magnetic field from the top wire is also into the page. At point 2, $\vec{B}_2 = (4.0 \text{ mT, into the page})$.

32.2. Model: A magnetic field is caused by an electric current.
Solve: The current in the wire is directed to the right. $B_2 = 20 \text{ mT} + 20 \text{ mT} = 40 \text{ mT}$ because the two overlapping wires are carrying current in the same direction and each wire produces a magnetic field having the same direction at point 2. $B_3 = 20 \text{ mT} - 20 \text{ mT} = 0 \text{ mT}$, because the two overlapping wires carry currents in opposite directions and each wire produces a field having opposite directions at point 3. The currents at 4 are also in opposite directions, but the point is to the right of one wire and to the left of the other. From the right-hand rule, the field of both currents is out of the page. Thus $B_4 = 20 \text{ mT} + 20 \text{ mT} = 40 \text{ mT}$.

32.3. Model: The magnetic field is that of a moving charged particle.
Visualize:



The first point is on the x-axis, with $\theta_a = 0°$. The second point is on the y-axis, with $\theta_b = 90°$, and the third point is on the $-y$-axis with $\theta_c = 90°$ (in the opposite senese). Use the Bio-Savart law.

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$

Solve: (a) $B_a = 0$ T because $\sin\theta_a = \sin 0° = 0$.
(b) Using the Biot-Savart law, the magnetic field strength is

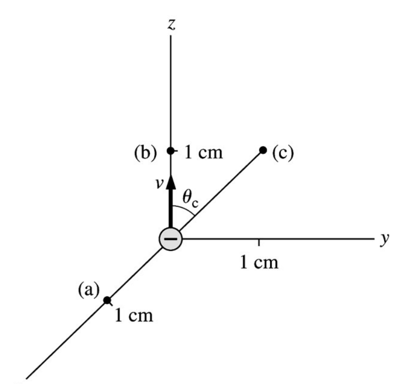
$$B_b = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2} = \frac{(10^{-7}\text{ T m/A})(1.60\times10^{-19}\text{ C})(1.0\times10^{+7}\text{ m/s})\sin 90°}{(1.0\times10^{-2}\text{ m})^2} = 1.60\times10^{-15}\text{ T}$$

To use the right-hand rule for finding the direction of $\vec{B}$, point your thumb in the direction of $\vec{v}$. The magnetic field vector $\vec{B}$ is perpendicular to the plane of $\vec{r}$ and $\vec{v}$ and points in the same direction that your fingers point. In the present case, the fingers point along the $\hat{k}$ direction. Thus, $\vec{B}_b = 1.60\times10^{-15}\hat{k}$ T.
(c) Using the Biot-Savart law, the magnetic field strength is

$$B_c = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2} = \frac{(10^{-7}\text{ T m/A})(1.60\times10^{-19}\text{ C})(1.0\times10^{+7}\text{ m/s})\sin(90°)}{(2.0\times10^{-2}\text{ m})^2} = 4.0\times10^{-16}\text{ T}$$

To use the right-hand rule for finding the direction of $\vec{B}$, point your thumb in the direction of $\vec{v}$. The magnetic field vector $\vec{B}$ is perpendicular to the plane of $\vec{r}$ and $\vec{v}$ and points in the same direction that your fingers point. In the present case, the fingers point along the $-\hat{k}$ direction. Thus, $\vec{B}_c = -4.0\times10^{-16}\hat{k}$ T.

32.4. Model: The magnetic field is that of a moving charged particle.
Visualize:



The first point is on the x-axis, with $\theta_a = 90°$. The second point is on the z-axis, with $\theta_b = 0°$, and the third point is in the yz plane with $\theta_c = 45°$.

Solve: **(a)** Using the Biot-Savart law, the magnetic field strength is

$$B_a = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2} = \frac{(10^{-7}\ \text{T m/A})(1.60\times10^{-19}\ \text{C})(2.0\times10^{+7}\ \text{m/s})\sin 90°}{(1.0\times10^{-2}\ \text{m})^2} = 3.2\times10^{-15}\ \text{T}$$

To use the right-hand rule for finding the direction of $\vec{B}_a$, point your thumb in the direction of $\vec{v}$. Your fingers point along the $-y$-axis, but since the charge is negative, $\vec{B}_a$ points along the $+y$-axis. Thus, $\vec{B}_a = 3.2\times10^{-15}\,\hat{j}$ T.

(b) $B_b = 0$ T because $\sin\theta_b = \sin 0° = 0$.

(c) For the third point,

$$B_c = \frac{(10^{-7}\ \text{T m/A})(1.60\times10^{-19}\ \text{C})(2.0\times10^{7}\ \text{m/s})\sin 45°}{(1.0\times10^{-2}\ \text{m})^2 + (1.0\times10^{-2}\ \text{m})^2} = 1.13\times10^{-15}\ \text{T}$$

The direction of $\vec{B}_c$ is perpendicular to the plane formed by $\vec{r}$ and $\vec{v}$. The right-hand rule gives the direction of $\vec{v}\times\vec{r}$ along the $-x$-axis, but because the charge is negative, $\vec{B}_c = 1.13\times10^{-15}\,\hat{i}$ T.

32.5. Model: The magnetic field is that of a moving charged particle.

Solve: Using the Biot-Savart law,

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2} = \frac{(10^{-7}\ \text{T m/A})(1.60\times10^{-19}\ \text{C})(2.0\times10^{7}\ \text{m/s})\sin 45°}{(1.0\times10^{-2}\ \text{m})^2 + (1.0\times10^{-2}\ \text{m})^2} = 1.13\times10^{-15}\ \text{T}$$

The right-hand rule applied to the *proton* points $\vec{B}$ *into* the page. Thus, $\vec{B} = -1.13\times10^{-15}\,\hat{k}$ T.

32.6. Model: The magnetic field is that of a moving charged particle.

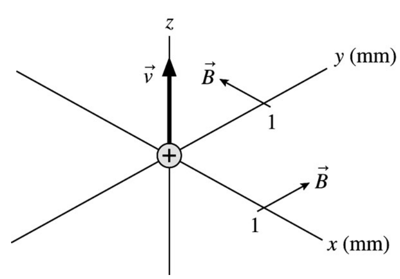
Solve: The Biot-Savart law is

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2} = \frac{(10^{-7}\ \text{T m/A})(1.60\times10^{-19}\ \text{C})(2.0\times10^{7}\ \text{m/s})\sin 135°}{(2.0\times10^{-2}\ \text{m})^2 + (2.0\times10^{-2}\ \text{m})^2} = 2.83\times10^{-16}\ \text{T}$$

The right-hand rule for the *positive charge* indicates the field points out of the page. Thus, $\vec{B} = 2.83\times10^{-16}\,\hat{k}$ T.

32.7. Model: The magnetic field is that of a moving proton.

Visualize:



The magnetic field lies in the xy-plane.

Solve: Using the right-hand rule, the charge is moving along the $+z$-direction. That is, $\vec{v} = v\hat{k}$. Using the Biot-Savart law,

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2} \Rightarrow 1.0\times10^{-13}\ \text{T} = \frac{(10^{-7}\ \text{T m/A})(1.60\times10^{-19}\ \text{C})v\sin 90°}{(1.0\times10^{-3}\ \text{m})^2} \Rightarrow v = 6.3\times10^{6}\ \text{m/s in the } +z\text{-direction}$$

Section 32.4 The Magnetic Field of a Current

32.8. Model: The magnetic field is that of an electric current in a long, straight wire.
Solve: From Example 32.3, the magnetic field strength of a long, straight wire carrying current I at a distance d from the wire is

$$B_{\text{wire}} = \frac{\mu_0}{2\pi}\frac{I}{d}$$

The current needed to produce the earth's magnetic field is calculated as follows:

$$B_{\text{earth surface}} = 5\times10^{-5}\text{ T } = \frac{(2\times10^{-7}\text{ T m/A})I}{0.010\text{ m}} \Rightarrow I = 2.5\text{ A}$$

Likewise, the currents needed for a refrigerator magnet, a laboratory magnet, and a superconducting magnet are 250 A, 5000–50,000 A, and 500,000 A.

32.9. Model: The magnetic field is that of an electric current in a long, straight wire.
Solve: From Example 32.3, the magnetic field strength of a long, straight wire carrying current I at a distance d from the wire is

$$B = \frac{\mu_0}{2\pi}\frac{I}{d}$$

The distance d at which the magnetic field is equivalent to Earth's magnetic field is calculated as follows:

$$B_{\text{earth surface}} = 5\times10^{-5}\text{ T } = (2\times10^{-7}\text{ T m/A})\frac{10\text{ A}}{d} \Rightarrow d = 4.0\text{ cm}$$

Likewise, the corresponding distances for a refrigerator magnet, a laboratory magnet, and a superconducting magnet are 0.40 mm, 20 μm to 2.0 μm, and 0.20 μm.

32.10. Model: Assume that the superconducting niobium wire is very long.
Solve: The magnetic field of a long wire carrying current I is

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi d}$$

We're interested in the magnetic field of the current right at the surface of the wire, where $d = 1.5$ mm. The maximum field is 0.10 T, so the maximum current is

$$I = \frac{(2\pi d)B_{\text{wire}}}{\mu_0} = \frac{2\pi(1.5\times10^{-3}\text{ m})(0.10\text{ T})}{4\pi\times10^{-7}\text{ T m/A}} = 750\text{ A}$$

Assess: The current density in this superconducting wire is of the order of $1\times10^{8}\text{ A/m}^2$. This is a typical value for conventional superconducting materials.

32.11. Model: The magnetic field is that of a current loop.
Solve: (a) From Equation 32.7, the magnetic field strength at the center of a loop is

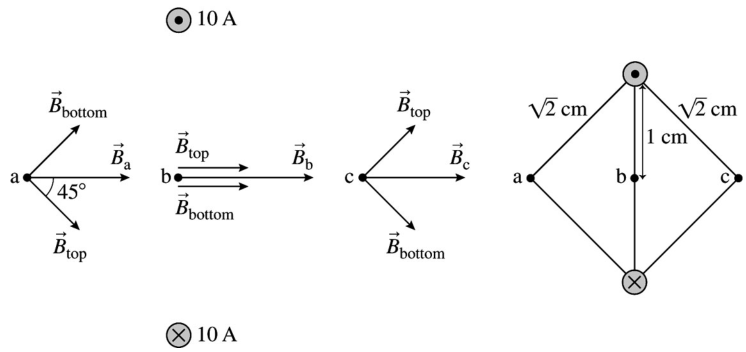
$$B_{\text{loop center}} = \frac{\mu_0 I}{2R} \Rightarrow I = \frac{2RB_{\text{loop center}}}{\mu_0} = \frac{2(0.50\times10^{-2}\text{ m})(2.5\times10^{-3}\text{ T})}{4\pi(10^{-7}\text{ T m/A})} = 20\text{ A}$$

(b) For a long, straight wire that carries a current I, the magnetic field strength is

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi d} \Rightarrow 2.5\times10^{-3}\text{ T } = \frac{4\pi(10^{-7}\text{ T m/A})(20\text{ A})}{2\pi d} \Rightarrow d = 1.6\times10^{-3}\text{ m}$$

32.12. Model: The magnetic field is the superposition of the magnetic fields of three wire segments.
Solve: The magnetic field of the horizontal wire, with current I, encircles the wire. Because the dot is on the axis of the wire, the input current creates no magnetic field at this point. The current divides at the junction, with $I/2$ traveling upward and $I/2$ traveling downward. The right-hand rule tells us that the upward current creates a field at the dot that is into the page; the downward current creates a field that is out of the page. Although we could calculate the strength of each field, the symmetry of the situation (the dot is the same distance and direction from the base of each wire) tells us that the fields of the upward and downward current must have the same strength. Since they are in opposite directions, their sum is $\vec{0}$. Altogether, then, the field at the dot is $\vec{B} = \vec{0}$ T.

32.13. Model: Assume the wires are infinitely long.
Visualize:



The field vectors are tangent to circles around the currents. The net magnetic field is the vectorial sum of the fields $\vec{B}_{\text{top}}$ and $\vec{B}_{\text{bottom}}$. Points a and c are at a distance $d=\sqrt{2}$ cm from both wires and point b is at a distance $d = 1$ cm.
Solve: The magnetic field at points a, b, and c are

$$\vec{B}_{\text{a}} = \vec{B}_{\text{top}} + \vec{B}_{\text{bottom}} = \frac{\mu_0 I}{2\pi d}(\cos 45°\hat{i} - \sin 45°\hat{j}) + \frac{\mu_0 I}{2\pi d}(\cos 45°\hat{i} + \sin 45°\hat{j})$$

$$= \frac{\mu_0 I}{2\pi d} 2\cos 45°\hat{i} = \frac{(2\times10^{-7}\text{ T m/A})(10\text{ A})}{\sqrt{2}\times10^{-2}\text{ m}} 2\left(\frac{1}{\sqrt{2}}\right)\hat{i} = 2.0\times10^{-4}\hat{i}\text{ T}$$

$$\vec{B}_{\text{b}} = \frac{\mu_0 I}{2\pi d}\hat{i} + \frac{\mu_0 I}{2\pi d}\hat{i} = 2\frac{(2\times10^{-7}\text{ T m/A})(10\text{ A})}{1\times10^{-2}\text{ m}}\hat{i} = 4.0\times10^{-4}\hat{i}\text{ T}$$

$$\vec{B}_{\text{c}} = \frac{\mu_0 I}{2\pi d}(\cos 45°\hat{i} + \sin 45°\hat{j}) + \frac{\mu_0 I}{2\pi d}(\cos 45°\hat{i} - \sin 45°\hat{j}) = 2.0\times10^{-4}\hat{i}\text{ T}$$

32.14. Model: Assume the wires are infinitely long.
Solve: The magnetic field strength at point a is

$$\vec{B}_{\text{at a}} = \vec{B}_{\text{top}} + \vec{B}_{\text{bottom}} = \left(\frac{\mu_0 I}{2\pi d}, \text{ out of page}\right)_{\text{top}} + \left(\frac{\mu_0 I}{2\pi d}, \text{ into page}\right)_{\text{bottom}}$$

$$\Rightarrow B_{\text{at a}} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{2.0\text{ cm}} - \frac{1}{(4.0+2.0)\text{ cm}}\right) = (2\times10^{-7}\text{ T m/A})(10\text{ A})\left(\frac{1}{2.0\times10^{-2}\text{ m}} - \frac{1}{6.0\times10^{-2}\text{ m}}\right)$$

$$\Rightarrow \vec{B}_{\text{at a}} = (6.7\times10^{-5}\text{ T, out of page})$$

At points b and c,

$$\vec{B}_{\text{at b}} = \left(\frac{\mu_0 I}{2\pi d}, \text{ into page}\right) + \left(\frac{\mu_0 I}{2\pi d}, \text{ into page}\right) = (2.0\times10^{-4}\text{ T, into page})$$

$$\vec{B}_{\text{at c}} = \left(\frac{\mu_0 I}{2\pi d}, \text{ into page}\right) + \left(\frac{\mu_0 I}{2\pi d}, \text{ out of page}\right) = (6.7\times10^{-5}\text{ T, out of page})$$

Section 32.5 Magnetic Dipoles

32.15. Model: Assume that the 10 cm distance is much larger than the size of the small bar magnet.
Solve: **(a)** From Equation 32.9, the on-axis field of a magnetic dipole is

$$B = \frac{\mu_0}{4\pi}\frac{2\mu}{z^3} \Rightarrow \mu = \frac{4\pi}{\mu_0}\frac{Bz^3}{2} = \frac{(5.0\times10^{-6}\text{ T})(0.10\text{ m})^3}{2(10^{-7}\text{ T m/A})} = 0.025\text{ A m}^2$$

(b) The on-axis field strength 15 cm from the magnet is

$$B = \frac{\mu_0}{4\pi}\frac{2\mu}{z^3} = (10^{-7}\ \text{T m/A})\frac{2(0.025\ \text{A m}^2)}{(0.15\ \text{m})^3} = 1.48\times10^{-6}\ \text{T} \approx 1.5\,\mu\text{T}$$

32.16. Solve: (a) The magnetic dipole moment of the superconducting ring is

$$\mu = (\pi R^2)I = \pi(1.0\times10^{-3}\ \text{m})^2(100\ \text{A}) = 3.1\times10^{-4}\ \text{A m}^2$$

(b) From Example 32.5, the on-axis magnetic field of the superconducting ring is

$$B_{\text{ring}} = \frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{3/2}} = \frac{2\pi(10^{-7}\ \text{T m/A})(100\ \text{A})(1.0\times10^{-3}\ \text{m})^2}{\left[(0.050\ \text{m})^2+(0.0010\ \text{m})^2\right]^{3/2}} = 5.0\times10^{-7}\ \text{T}$$

32.17. Model: The size of the loop is much smaller than 50 cm, so that the magnetic dipole moment of the loop is $\mu = AI$.
Solve: The magnitude of the on-axis magnetic field of the loop is

$$B = \frac{\mu_0}{4\pi}\frac{2(AI)}{z^3} = 7.5\times10^{-9}\ \text{T} \Rightarrow \text{A} = \frac{(0.50\ \text{m})^3(7.5\times10^{-9}\ \text{T})}{2(25\ \text{A})(10^{-7}\ \text{Tm/A})} = 1.88\times10^{-4}\ \text{m}^2$$

Let L be the edge-length of the square. Thus

$$L = \sqrt{A} = \sqrt{1.88\times10^{-4}\ \text{m}^2} = 1.37\times10^{-2}\ \text{m} \approx 1.4\ \text{cm}$$

Assess: The 1.4 cm edge-length is much smaller than 50 cm, as we assumed.

32.18. Model: The radius of the earth is much larger than the size of the current loop.
Solve: (a) From Equation 32.9, the magnetic field strength at the surface of the earth at the earth's north pole is

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3} = \frac{(2\times10^{-7}\ \text{T m/A})(8.0\times10^{22}\ \text{A m}^2)}{(6.38\times10^{6}\ \text{m})^3} = 6.2\times10^{-5}\ \text{T}$$

This value is close to the value of 5×10^{-5} T given in Table 32.1.
(b) The current required to produce a dipole moment like that on the earth is

$$\mu = AI = (\pi R_{\text{earth}}^2)I \Rightarrow 8.0\times10^{22}\ \text{A m}^2 = \pi(6.38\times10^{6}\ \text{m})^2 I \Rightarrow I = 6.3\times10^{8}\ \text{A}$$

Assess: This is an extremely large current to run through a wire around the equator.

Section 32.6 Ampère's Law and Solenoids

32.19. Solve: Because $\vec{B}$ is in the same direction as the integration path $\vec{s}$ from i to f, the dot product of $\vec{B}$ and $d\vec{s}$ is simply Bds. Hence the line integral

$$\int_{\text{i}}^{\text{f}}\vec{B}\cdot d\vec{s} = \int_{\text{i}}^{\text{f}}Bds = B\int_{\text{i}}^{\text{f}}ds = B\left(\sqrt{(0.50\ \text{m})^2+(0.50\ \text{m})^2}\right) = (0.10\ \text{T})\sqrt{2}(0.50\ \text{m}) = 0.071\ \text{T m}$$

32.20. Solve: The line integral of $\vec{B}$ between points i and f is

$$\int_{\text{i}}^{\text{f}}\vec{B}\cdot d\vec{s}$$

Because $\vec{B}$ is perpendicular to the integration path from i to f, the dot product is zero at all points and the line integral is zero.

32.21. Model: The magnetic field is that of the three currents enclosed by the loop.
Solve: Ampere's law gives the line integral of the magnetic field around the closed path:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}} = 3.77\times10^{-6}\ \text{T m} = \mu_0(I_1 - I_2 + I_3) = (4\pi\times10^{-7}\ \text{T m/A})(2.0\ \text{A} - 6.0\ \text{A} + I_3)$$

$$\Rightarrow (I_3 - 4.0\ \text{A}) = \frac{3.77\times10^{-6}\ \text{T m}}{4\pi\times10^{-7}\ \text{T m/A}} \Rightarrow I_3 = 7.00\ \text{A}$$

Assess: The right-hand rule was used above to assign positive signs to I_1 and I_3 and a negative sign to I_2.

32.22. Model: Only the two currents enclosed by the closed path contribute to the line integral.
Solve: Ampere's law gives the line integral of the magnetic field around the closed path:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}} = 1.38\times10^{-5}\ \text{T m} = \mu_0(I_2 + I_3) = (4\pi\times10^{-7}\ \text{T m/A})(-12\ \text{A} + I_3)$$

$$\Rightarrow (I_3 - 12\ \text{A}) = \frac{1.38\times10^{-5}\ \text{T m}}{4\pi\times10^{-7}\ \text{T m/A}} \Rightarrow I_3 = 23.0\ \text{A, into the page.}$$

Assess: The right-hand rule was used above to assign a negative sign to I_2. Since I_3 is positive, it is in the opposite direction as I_2.

32.23. Model: The magnetic field is that of a current flowing into the plane of the paper. The current-carrying wire is very long.
Solve: Divide the line integral into three parts:

$$\int_{\text{i}}^{\text{f}} \vec{B}\cdot d\vec{s} = \int_{\text{left line}} \vec{B}\cdot d\vec{s} + \int_{\text{semicircle}} \vec{B}\cdot d\vec{s} + \int_{\text{right line}} \vec{B}\cdot d\vec{s}$$

The magnetic field of the current-carrying wire is tangent to clockwise circles around the wire. $\vec{B}$ is everywhere perpendicular to the left line and to the right line, thus the first and third parts of the line integral are zero. Along the semicircle, $\vec{B}$ is tangent to the path *and* has the same magnitude $B = \mu_0 I/2\pi d$ at every point. Thus

$$\int_{\text{i}}^{\text{f}} \vec{B}\cdot d\vec{s} = 0 + BL + 0 = \frac{\mu_0 I}{2\pi d}(\pi d) = \frac{\mu_0 I}{2} = \frac{(4\pi\times10^{-7}\ \text{T m/A})(2.0\ \text{A})}{2} = 1.26\times10^{-6}\ \text{T m}$$

where $L = \pi d$ is the length of the semicircle, which is half the circumference of a circle of radius d.

32.24. Model: Assume that the solenoid is an ideal solenoid.
Solve: We can use Equation 32.16 to find the current that will generate a magnetic field of 1.5 T inside the solenoid:

$$B_{\text{solenoid}} = \frac{\mu_0 N I}{l} \Rightarrow I = \frac{B_{\text{solenoid}}\, l}{\mu_0 N}$$

Using l = 1.8 m and $N = 1.8\ \text{m}/0.0020 = 900$,

$$I = \frac{(1.5\ \text{T})(1.8\ \text{m})}{4\pi(10^{-7}\ \text{T m/A})900} = 2.4\ \text{kA}$$

Assess: Large currents can be passed through superconducting wires. The current density through the superconducting wire is $(2400\ \text{A})/\pi(0.001\ \text{m})^2 = 7.6\times10^8\ \text{A/m}^2$. This value is reasonable for a superconducting wire.

32.25. Model: Model the solenoid as ideal; the turns of wire are wrapped very closely to each other.
Visualize: The magnetic field in an ideal solenoid does not depend on the cross-sectional area of the solenoid.
Solve: The magnetic field is zero outside an ideal solenoid; inside it is $B_{\text{solenoid}} = \mu_0 n I$ where $n = N/l$ is the number of turns per unit length. The diameter of the wire is the width of one turn which is $l/N = 1/n$.

$$\frac{1}{n} = \frac{\mu_0 I}{B_{\text{solenoid}}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.5\ \text{A})}{3.0\ \text{mT}} = 1.0\ \text{mm}$$

Assess: A wire with a width of 1.0 mm seems reasonable. The units check out. We did not need to use the diameter of the solenoid nor its length.

Section 32.7 The Magnetic Force on a Moving Charge

32.26. Model: A magnetic field exerts a magnetic force on a moving charge.
Solve: (a) The force on the charge is

$$\vec{F}_{\text{on q}} = q\vec{v}\times\vec{B} = (1.60\times10^{-19}\text{ C})(1.0\times10^{7}\text{ m/s})(\cos45°\hat{i} + \sin45°\hat{k})\times(0.50\hat{i}\text{ T})$$
$$= (1.60\times10^{-19}\text{ C})(0.50\text{ T})\frac{(1.0\times10^{7}\text{ m/s})}{\sqrt{2}}(\hat{i}\times\hat{i} + \hat{k}\times\hat{i}) = +5.7\times10^{-13}\hat{j}\text{ N}$$

(b) The force on the charge is

$$\vec{F}_{\text{on q}} = q\vec{v}\times\vec{B} = (1.60\times10^{-19}\text{ C})(1.0\times10^{7}\text{ m/s})(-\hat{j})\times(0.50\hat{i}\text{ T})$$
$$= (1.60\times10^{-19}\text{ C})(0.50\text{ T})(1.0\times10^{7}\text{ m/s})(\hat{k}) = 8.0\times10^{-13}\hat{k}\text{ N}$$

32.27. Model: A magnetic field exerts a magnetic force on a moving charge.
Solve: (a) The force is

$$\vec{F}_{\text{on q}} = q\vec{v}\times\vec{B} = (-1.60\times10^{-19}\text{ C})(-1.0\times10^{7}\hat{k}\text{ m/s})\times(0.50\hat{i}\text{ T}) = 8.0\times10^{-13}\hat{j}\text{ N}$$

(b) The force is

$$\vec{F}_{\text{on q}} = (-1.60\times10^{-19}\text{ C})(1.0\times10^{7}\text{ m/s})(-\cos45°\hat{j} + \sin45°\hat{k})\times(0.50\hat{i}\text{ T}) = 5.7\times10^{-13}(-\hat{j} - \hat{k})\text{ N}$$

32.28. Solve: The cyclotron frequency of a charged particle in a magnetic field is $f_{\text{cyc}} = qB/2\pi m$. For example, the mass of N_2^+ is

$$m = m_{\text{N}} + m_{\text{N}} = 2(14.003\text{ u})(1.6605\times10^{-27}\text{ kg/u}) = 4.6503\times10^{-26}\text{ kg}$$

Note that we're given the atomic masses very accurately, and we need to retain that accuracy to tell the difference between N_2^+ and CO^+. Calculating each mass and frequency:

Ion	Mass (kg)	f (MHz)
O_2^+	5.2969×10^{-26}	1.4442
N_2^+	4.6503×10^{-26}	1.6450
CO^+	4.6485×10^{-26}	1.6457

Assess: The difference between N_2^+ and CO^+ is not large but is easily detectable.

32.29. Model: A charged particle moving perpendicular to a uniform magnetic field moves in a circle.
Solve: The frequency of revolution of a charge moving at right angles to the magnetic field is

$$f_{\text{cyc}} = \frac{qB}{2\pi m} \Rightarrow B = \frac{2\pi m f_{\text{cyc}}}{q} = \frac{2\pi(9.11\times10^{-31}\text{ kg})(45\times10^{6}\text{ Hz})}{1.60\times10^{-19}\text{ C}} = 1.6\times10^{-3}\text{ T}$$

32.30. Model: Charged particles moving perpendicular to a uniform magnetic field undergo circular motion at constant speed.
Solve: The magnetic force on a proton causes the centripetal acceleration of

$$evB = \frac{mv^2}{r} \Rightarrow B = \frac{mv}{er}$$

The maximum radius r is 0.25 m, and the desired speed v is $0.10c$. So, the required field B is

$$B = \frac{(1.67\times10^{-27}\text{ kg})(0.10)(3\times10^{8}\text{ m/s})}{(1.6\times10^{-19}\text{ C})(0.25\text{ m})} = 1.25\text{ T}$$

32.31. Model: Assume the magnetic field is uniform over the Hall probe.
Solve: The Hall voltage is given by Equation 32.24:

$$\Delta V_{\rm H} = \frac{IB}{tne} \Rightarrow \frac{\Delta V_H}{B} = \frac{I}{tne}$$

In both uses, the quantities I, t, n, and e are unchanged, so the ratio $\dfrac{\Delta V_H}{B}$ is constant. Thus

$$\frac{1.9\times10^{-6}\ \text{V}}{55\times10^{-3}\ \text{T}} = \frac{2.8\times10^{-6}\ \text{V}}{B} \Rightarrow B = 81\ \text{mT}$$

32.32. Solve:

$$\Delta V_{\rm H} = \frac{IB}{tne} \Rightarrow$$

$$B = \frac{\Delta V_{\rm H} tne}{I} = \frac{(120\ \text{mV})(1.0\ \text{mm})(2.1\times10^{21}\ \text{m}^{-3})(1.6\times10^{-19}\ \text{C})}{60\ \text{mA}} = 0.67\ \text{T}$$

Assess: The units check out. We did not need to use the width of the wire.

Section 32.8 Magnetic Forces on Current-Carrying Wires

32.33. Model: Assume that the field is uniform. The wire will float in the magnetic field if the magnetic force on the wire points upward and has a magnitude *mg*, allowing it to balance the downward gravitational force.
Solve: We can use the right-hand rule to determine which current direction experiences an upward force. The current being from right to left, the force will be *up* if the magnetic field $\vec{B}$ points out of the page. The forces will balance when

$$F = ILB = mg \Rightarrow B = \frac{mg}{IL} = \frac{(2.0\times10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)}{(1.5\ \text{A})(0.10\ \text{m})} = 0.131\ \text{T}$$

Thus $\vec{B}$ = (0.131 T, out of page).

32.34. Model: Assume that the magnetic field is uniform over the 10 cm length of the wire. Force on top and bottom pieces will cancel.
Visualize: The figure shows a 10-cm-segment of a circuit in a region where the magnetic field is directed into the page.
Solve: The current through the 10-cm-segment is

$$I = \frac{\varepsilon}{R} = \frac{15\ \text{V}}{3\ \Omega} = 5\ \text{A}$$

and is flowing *down*. The force on this wire, given by the right-hand rule, is to the right and perpendicular to the current and the magnetic field. The magnitude of the force is

$$F = ILB = (5\ \text{A})(0.10\ \text{m})(50\ \text{mT}) = 0.025\ \text{N}$$

Thus $\vec{F}$ = (0.025 N, right).

32.35. Model: Two parallel wires carrying currents in the same direction exert attractive magnetic forces on each other.
Visualize: The current in the circuit on the left is I_1 and has a clockwise direction. The current in the circuit on the right is I_2 and has a counterclockwise direction.
Solve: Since $I_1 = 9\ \text{V}/2\ \Omega = 4.5\ \text{A}$, the force between the two wires is

$$F = 5.4\times10^{-5}\ \text{N} = \frac{\mu_0 L I_1 I_2}{2\pi d} = \frac{(2\times10^{-7}\ \text{T m/A})(0.10\ \text{m})(4.5\ \text{A})I_2}{0.0050\ \text{m}}$$

$$\Rightarrow I_2 = 3.0\ \text{A} \Rightarrow R = \frac{9\ \text{V}}{3.0\ \text{A}} = 3.0\ \Omega$$

32.36. Model: Two parallel wires carrying currents in opposite directions exert repulsive magnetic forces on each other. Two parallel wires carrying currents in the same direction exert attractive magnetic forces on each other.
Solve: The magnitudes of the various forces between the parallel wires are

$$F_{2\text{ on }1} = \frac{\mu_0 L I_1 I_2}{2\pi d} = \frac{(2\times10^{-7}\ \text{T m/A})(0.50\ \text{m})(10\ \text{A})(10\ \text{A})}{0.02\ \text{m}} = 5.0\times10^{-4}\ \text{N} = F_{2\text{ on }3} = F_{3\text{ on }2} = F_{1\text{ on }2}$$

$$F_{3\text{ on }1} = \frac{\mu_0 L I_1 I_3}{2\pi d} = \frac{(2\times10^{-7}\ \text{T m/A})(0.50\ \text{m})(10\ \text{A})(10\ \text{A})}{0.04\ \text{m}} = 2.5\times10^{-4}\ \text{N} = F_{1\text{ on }3}$$

Now we can find the net force each wire exerts on the other as follows:

$$\vec{F}_{\text{on }1} = \vec{F}_{2\text{ on }1} + \vec{F}_{3\text{ on }1} = (5.0\times10^{-4}\,\hat{j})\ \text{N} + (-2.5\times10^{-4}\,\hat{j})\ \text{N} = 2.5\times10^{-4}\,\hat{j}\,\text{N} = (2.5\times10^{-4}\ \text{N, up})$$

$$\vec{F}_{\text{on }2} = \vec{F}_{1\text{ on }2} + \vec{F}_{3\text{ on }2} = (-5.0\times10^{-4}\,\hat{j})\ \text{N} + (+5.0\times10^{-4}\,\hat{j})\ \text{N} = 0\ \text{N}$$

$$\vec{F}_{\text{on }3} = \vec{F}_{1\text{ on }3} + \vec{F}_{2\text{ on }3} = (2.5\times10^{-4}\,\hat{j})\ \text{N} + (-5.0\times10^{-4}\,\hat{j})\ \text{N} = -2.5\times10^{-4}\,\hat{j}\ \text{N} = (2.5\times10^{-4}\ \text{N, down})$$

Section 32.9 Forces and Torques on Current Loops

32.37. Solve: From Equation 32.28, the torque on the loop exerted by the magnetic field is

$$\vec{\tau} = \vec{\mu}\times\vec{B} \Rightarrow \tau = \mu B\sin\theta = IAB\sin\theta = (0.500\ \text{A})(0.050\ \text{m}\times0.050\ \text{m})(1.2\ \text{T})\sin30^\circ = 7.5\times10^{-4}\ \text{N m}$$

32.38. Solve: From Equation 32.28, the torque on the bar magnet exerted by the magnetic field is

$$\vec{\tau} = \vec{\mu}\times\vec{B} \Rightarrow \mu = \frac{\tau}{B\sin\theta} = \frac{0.020\ \text{N m}}{(0.10\ \text{T})\sin45^\circ} = 0.28\ \text{A m}^2$$

32.39. Model: The torque on the current loop is due to the magnetic field produced by the current-carrying wire. Assume that the wire is very long.
Solve: (a) From Equation 32.27, the magnitude of the torque on the current loop is $\tau = \mu B\sin\theta$, where $\mu = I_{\text{loop}}$ A and B is the magnetic field produced by the current I_{wire} in the wire. The magnetic field of the wire is tangent to a circle around the wire. At the position of the loop, $\vec{B}$ points up and is $\theta = 90^\circ$ from the axis of the loop. Thus,

$$\tau = (I_{\text{loop}}A)\frac{\mu_0 I_{\text{wire}}}{2\pi d}\sin\theta = \frac{(0.20\ \text{A})\pi(0.0010\ \text{m})^2(2\times10^{-7}\ \text{T m/A})(2.0\ \text{A})\sin90^\circ}{2.0\times10^{-2}\ \text{m}} = 1.26\times10^{-11}\ \text{N m}$$

Note that the magnetic field produced by the wire on the current loop is *up* so that the angle θ between $\vec{B}$ and the normal to the loop is 90°.
(b) The loop is in equilibrium when $\theta = 0^\circ$ or 180°. That is, when the coil is rotated by $\pm 90^\circ$.

32.40. Model: Use the equation for the magnetic field for a long, straight wire.
Solve: (a) The field of a transmission line is around

$$B = \frac{\mu_0}{2\pi}\frac{I}{d} = \frac{(2\times10^{-7}\ \text{T}\cdot\text{m/A})(200\ \text{A})}{20\ \text{m}} = 2.0\times10^{-6}\ \text{T} = 2.0\ \mu\text{T}$$

(b) The earth's field is $B_{\text{earth}} = 5\times10^{-5}\ \text{T} = 50\ \mu\text{T}$, so $B_{\text{wire}}/B_{\text{earth}} = 2.0\ \mu\text{T}/(50\ \mu\text{T}) = 0.04 = 4.0\%$.
Assess: The field produced by a transmission line on the ground is much smaller than the earth's magnetic field.

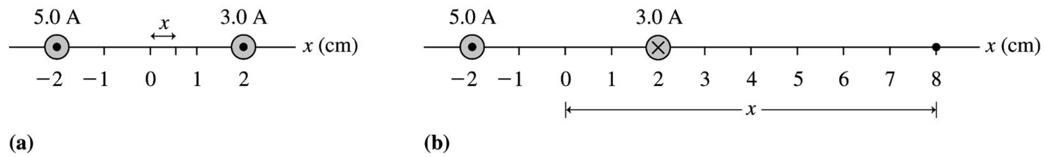
32.41. Model: Assume the axon is a long, straight wire: $B = \dfrac{\mu_0 I}{2\pi r}$.

Solve: Solve for I:

$$I = \frac{2\pi r B}{\mu_0} = \frac{(1.0\times10^{-3}\ \text{m})(8.0\times10^{-12}\ \text{T})}{(2\times10^{-7}\ \text{T}\cdot\text{m/A})} = 4.0\times10^{-8}\ \text{A} = 0.040\ \mu\text{A}$$

Assess: This is a very small current, as we would expect from an axon.

32.42. Model: The magnetic field is that of two long wires that carry current.
Visualize:



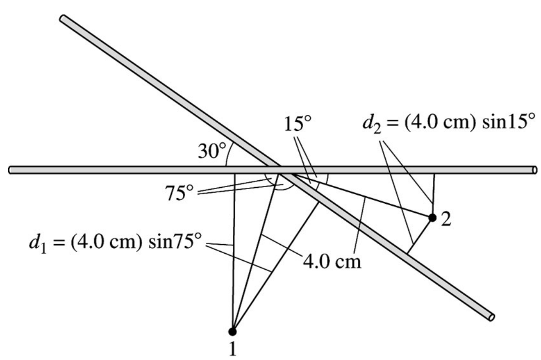
Solve: **(a)** For $x > +2$ cm and for $x < -2$ cm, the magnetic fields due to the currents in the two wires add. The point where the two magnetic fields cancel lies on the *x*-axis in between the two wires. Let that point be a distance *x* away from the origin. Because the magnetic field of a long wire is $B = \mu_0 I/2\pi r$, we have

$$\frac{\mu_0}{2\pi}\frac{(5.0\text{ A})}{(0.020\text{ m}+x)} = \frac{\mu_0}{2\pi}\frac{(3.0\text{ A})}{(0.020\text{ m}-x)} \Rightarrow 5(0.020\text{ m}-x) = 3(0.020\text{ m}+x) \Rightarrow x = 0.0050\text{ m} = 0.50\text{ cm}$$

(b) The magnetic fields due to the currents in the two wires add in the region $-2.0\text{ cm} < x < 2.0\text{ cm}$. For $x < -2.0$ cm, the magnetic fields subtract, but the field due to the 5.0 A current is always larger than the field due to the 3.0 A current. However, for $x > 2.0$ m, the two fields will cancel at a point on the *x*-axis. Let that point be a distance *x* away from the origin, so

$$\frac{\mu_0}{2\pi}\frac{5.0\text{ A}}{x+0.020\text{ m}} = \frac{\mu_0}{2\pi}\frac{3.0\text{ A}}{x-0.020\text{ m}} \Rightarrow 5(x-0.020\text{ m}) = 3(x+0.020\text{ m}) \Rightarrow x = 8.0\text{ cm}$$

32.43. Model: Assume that the wires are infinitely long and that the magnetic field is due to currents in both the wires.
Visualize: Point 1 is a distance d_1 away from the two wires and point 2 is a distant d_2 away from the two wires. A right triangle with a 75° degree angle is formed by a straight line from point 1 to the intersection and a line from point 1 that is perpendicular to the wire. Likewise, point 2 makes a 15° right triangle.



Solve: First we determine the distances d_1 and d_2 of the points from the two wires:

$$d_1 = (4.0\text{ cm})\sin 75° = 3.86\text{ cm} = 0.0386\text{ m}$$
$$d_2 = (4.0\text{ cm})\sin 15° = 1.04\text{ cm} = 0.0104\text{ m}$$

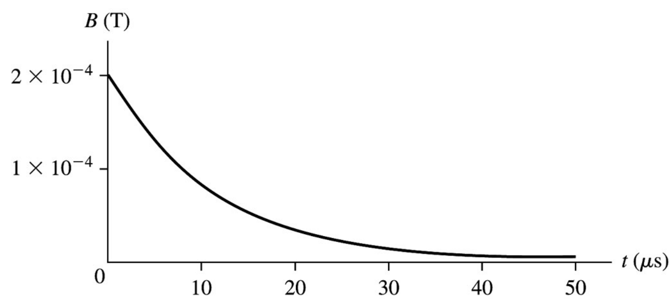
At point 1, the fields from both the wires point up and hence add. The total field is

$$B_1 = B_{\text{wire 1}} + B_{\text{wire 2}} = \frac{\mu_0}{2\pi}\frac{I_1}{d_1} + \frac{\mu_0}{2\pi}\frac{I_2}{d_1} = \frac{\mu_0}{\pi}\frac{(5.0\text{ A})}{d_1} = \frac{(4\times10^{-7}\text{ T m/A})(5.0\text{ A})}{0.0386\text{ m}} = 5.2\times10^{-5}\text{ T}$$

In vector form, $\vec{B}_1 = (5.2\times10^{-5}\text{ T, out of page})$. Using the right-hand rule at point 2, the fields are in opposite directions but equal in magnitude. So, $\vec{B}_2 = \vec{0}\text{ T}$.

32.44. Model: The resistor and capacitor form an RC circuit. Assume that the length of the wire next to the dot is much larger than 1.0 cm.

Visualize:



Solve: As the capacitor discharges through the resistor, the current through the circuit will decrease as

$$I = \frac{\varepsilon}{R}e^{-t/RC} = \frac{50\text{ V}}{5\ \Omega}e^{-t/(5\ \Omega)(2\ \mu\text{F})} = (10\text{ A})e^{-t/10\ \mu\text{s}}$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{2\pi r}(10\text{ A})e^{-t/10\ \mu\text{s}} = \frac{(2.0\times10^{-7}\text{ T m/A})(10\text{ A})}{0.010\text{ m}}e^{-t/10\ \mu\text{s}} = (2.0\times10^{-4}\text{ T})e^{-t/10\ \mu\text{s}}$$

A graph of this equation is shown above.

32.45. Solve: From Example 32.5, the on-axis magnetic field of a current loop is

$$B_{\text{loop}}(z) = \frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{\frac{3}{2}}}.$$

We want to find the value of z such that $B(z) = 2B(0)$.

$$\frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{\frac{3}{2}}} = 2\frac{\mu_0}{2}\frac{IR^2}{(R^2)^{\frac{3}{2}}}$$

$$\Rightarrow (z^2+R^2)^{\frac{3}{2}} = \frac{R^3}{z} \Rightarrow z^2+R^2 = \frac{R^2}{2^{\frac{2}{3}}} \Rightarrow z = R(2^{-\frac{2}{3}}-1)^{\frac{1}{2}} = 0.77R$$

32.46. Model: Use the Biot-Savart law for a current-carrying segment.
Solve: The Biot-Savart law (Equation 32.6) for the magnetic field of a current segment $\Delta\vec{s}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

where the unit vector $\hat{r}$ points from current segment Δs to the point, a distance r away, at which we want to evaluate the field. For the two linear segments of the wire, $\Delta\vec{s}$ is in the same direction as $\hat{r}$, so $\Delta\vec{s}\times\hat{r} = 0$. For the curved segment, $\Delta\vec{s}$ and $\hat{r}$ are always perpendicular, so $\Delta\vec{s}\times\hat{r} = \Delta s$. Thus

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r^2}$$

Now we are ready to sum the magnetic field of all the segments at point P. For all segments on the arc, the distance to point P is $r = R$. The superposition of the fields is

$$B = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int_{\text{arc}} ds = \frac{\mu_0}{4\pi}\frac{IL}{R^2} = \frac{\mu_0 I\theta}{4\pi R}$$

where $L = R\theta$ is the length of the arc.

32.47. Model: Use the Biot-Savart law for a current-carrying segment.
Visualize: The distance from P to the inner arc is r_1 and the distance from P to the outer arc is r_2.
Solve: As given in Equation 32.6, the Biot-Savart law for a current-carrying small segment $\Delta\vec{s}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

For the linear segments of the loop, $B_{\Delta s} = 0\text{ T}$ because $\Delta\vec{s}\times\hat{r} = 0$. Consider a segment $\Delta\vec{s}$ on length on the inner arc. Because $\Delta\vec{s}$ is perpendicular to the $\hat{r}$ vector, we have

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r_1^2} = \frac{\mu_0}{4\pi}\frac{Ir_1\Delta\theta}{r_1^2} = \frac{\mu_0}{4\pi}\frac{I\Delta\theta}{r_1} \Rightarrow B_{\text{arc 1}} = \int_{-\pi/2}^{\pi/2}\frac{\mu_0 Id\theta}{4\pi r_1} = \frac{\mu_0 I}{4\pi r_1}\pi = \frac{\mu_0 I}{4r_1}$$

A similar expression applies for $B_{\text{arc 2}}$. The right-hand rule indicates an out-of-page direction for $B_{\text{arc 2}}$ and an into-page direction for $B_{\text{arc 1}}$. Thus,

$$\vec{B} = \left(\frac{\mu_0 I}{4r_1}, \text{ into page}\right) + \left(\frac{\mu_0 I}{4r_2}, \text{ out of page}\right) = \left[\frac{\mu_0 I}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \text{ into page}\right]$$

The field strength is

$$B = \frac{(4\pi\times10^{-7}\text{ T m/A})(5.0\text{ A})}{4}\left(\frac{1}{0.010\text{ m}} - \frac{1}{0.020\text{ m}}\right) = 7.9\times10^{-5}\text{ T}$$

Thus $\vec{B} = (7.9\times10^{-5}\text{ T, into page})$.

32.48. Model: Assume that the wire is infinitely long.
Solve: Using Equation 32.7 and Example 32.3, the magnetic field at P is

$$B_{\text{P}} = B_{\text{loop center}} + B_{\text{wire}} = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R}$$
$$= \frac{4\pi(10^{-7}\text{ T m/A})(5.0\text{ A})}{2(0.010\text{ m})} + \frac{4\pi(10^{-7}\text{ T m/A})(5.0\text{ A})}{2\pi(0.010\text{ m})} = 4.1\times10^{-4}\text{ T}$$

The right-hand rule shows that the direction is into the page, so $B_{\text{P}} = (4.1\times10^{-4}\text{ T, into page})$.

32.49. Model: Assume that the solenoid is an ideal solenoid.
Solve: The magnetic field of a solenoid is $B_{\text{sol}} = \mu_0 NI/l$, where l is the length and N the total number of turns of wire. If the wires are wound as closely as possible, the spacing between one turn and the next is simply the diameter d_{wire} of the wire. The number of turns that will fit into length L is $N = l/d_{\text{wire}}$. For #18 wire, $N = (20\text{ cm})/(0.102\text{ cm}) = 196$ turns. The current required is

$$I_{\#18} = \frac{LB_{\text{sol}}}{\mu_0 N} = \frac{(0.20\text{ m})(5\times10^{-3}\text{ T})}{(4\pi\times10^{-7}\text{ T m/A})(196)} = 4.1\text{ A}$$

For #26 wire, $d_{\text{wire}} = 0.41\text{ mm}$, leading to $N = 488$ turns and $I_{\#26} = 1.63\text{ A}$. The current that would be needed with #26 wire exceeds the current limit of 1 A, but the current needed with #18 wire is within the current limit of 6 A. So use #18 wire with a current of 4.1 A.

32.50. Model: Assume that the solenoid is an ideal solenoid.
Solve: A solenoid field is $B_{\text{sol}} = \mu_0 NI/l$, so the necessary number of turns is

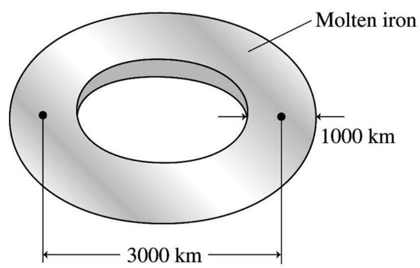
$$N = \frac{lB_{\text{sol}}}{\mu_0 I} = \frac{(0.08\text{ m})(0.10\text{ T})}{(4\pi\times10^{-7}\text{ T m/A})(2\text{ A})} = 3180$$

Problem 32.49 states that the rating is 6 A for a wire with $d_{\text{wire}} = 1.02\text{ mm}$ and the rating is 1 A for a wire having $d_{\text{wire}} = 0.41\text{ mm}$. This means the rating of a wire can be increased 1 A by increasing the diameter by approximately 0.12 mm. We can assume that a wire that can carry a 2 A current will probably have a diameter larger than 0.41 mm + 0.12 mm. Let us take the wire's diameter to be 0.6 mm. The number of turns that will fit in a length of 8 cm is

$$N_{1\text{ layer}} = \frac{8\text{ cm}}{0.06\text{ cm}} = 133$$

Thus 3180 turns would require 3180/133 = 24 layers.
Assess: With this many layers, the thickness of the layers of wires becomes larger than the radius of the solenoid. Maybe not impossible, but it is probably not feasible.

32.51. Model: A 1000-km-diameter ring makes a loop of diameter 3000 km.
Visualize:



Solve: **(a)** The current loop has a diameter of 3000 km, so its nominal area, ignoring curvature effects, is

$$A_{\text{loop}} = \pi r^2 = \pi(1500\times10^3\text{ m})^2 = 7.07\times10^{12}\text{ m}^2$$

Because the magnetic dipole moment of the earth is modeled to be due to a current flowing in such a loop, $\mu = IA_{\text{loop}}$. The current in the loop is

$$I = \frac{\mu}{A_{\text{loop}}} = \frac{8.0\times10^{22}\text{ A m}^2}{7.07\times10^{12}\text{ m}^2} = 1.13\times10^{10}\text{ A}$$

(b) The current density J in the above loop is

$$J_{\text{loop}} = \frac{I}{A} = \frac{1.13\times10^{10}\text{ A}}{\pi\left(\frac{1}{2}\times1000\times10^3\text{ m}\right)^2} = 0.014\text{ A/m}^2$$

(c) The current density in the wire is

$$J_{\text{wire}} = \frac{I}{A} = \frac{1.0\text{ A}}{\pi\left(\frac{1}{2}\times1.0\times10^{-3}\text{ m}\right)^2} = 1.3\times10^6\text{ A/m}^2$$

You can see that $J_{\text{loop}} \ll J_{\text{wire}}$. The current in the earth's core is large, but the current density is actually quite small.

32.52. Model: Model the brain current as a simple current loop of radius R.
Visualize: We know the field at an on-axis point R away from the center of the loop (the pole of the sphere).
Solve: From Example 32.5 we have the field due to a current loop.

$$B_{\text{loop}} = \frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{3/2}}$$

When $z = R$ this becomes

$$B_{\text{loop}} = \frac{\mu_0}{2}\frac{IR^2}{(2R^2)^{3/2}} = \frac{\mu_0 I}{4\sqrt{2}R}$$

Solve for the current.

$$I = \frac{4\sqrt{2}RB}{\mu_0} = \frac{4\sqrt{2}(8.0\text{ cm})(3.0\text{ pT})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 1.1\ \mu\text{A}$$

Assess: The units check out.

32.53. Model: The heart is compared to a loop of current with radius 4.0 cm and magnetic field of 90 pT at its center.
Solve: (**a**) The current needed to produce this field can be computed from the equation for a magnetic field at the center of a loop.

$$I = \frac{2BR}{\mu_0} = \frac{2(90\times10^{-12}\text{ T})(0.040\text{ m})}{(4\pi\times10^{-7}\text{T}\cdot\text{m/A})} = 5.73\times10^{-6}\text{ A} \approx 5.7\times10^{-6}\text{ A}$$

(**b**) $\mu = AI = \pi(0.040\text{ m})^2(5.73\times10^{-6}\text{ A}) = 2.88\times10^{-8}\text{ A m}^2 \approx 2.9\times10^{-8}\text{ A m}^2$

Assess: This is a small current, as we would have expected.

32.54. Model: The coils are identical, parallel, and carry equal currents in the same direction. The magnetic field is that of the currents in these coils.
Solve: (**a**) The on-axis magnetic field of an N-turn current loop at a distance z from the loop center is

$$B_{\text{loop}} = \frac{\mu_0}{2}\frac{NIR^2}{(z^2+R^2)^{3/2}}$$

If the spacing between the loops is R, then the midpoint between them is $z = R/2$. Since the currents are in the same direction, the field of each loop is in the same direction and the net field is simply twice the field due to a single loop. Thus

$$B = \frac{\mu_0 NIR^2}{(R^2/4+R^2)^{3/2}} = (1.25)^{3/2}\frac{\mu_0 NI}{R}$$

(**b**) The magnetic field is

$$B = (0.716)\frac{4\pi(10^{-7}\text{ T m/A})10(1.0\text{ A})}{0.050\text{ m}} = 1.80\times10^{-4}\text{ T}$$

32.55. Model: The magnetic field is that of a current in the wire.
Solve: As given in Equation 32.6 for a current-carrying small segment $\Delta\vec{s}$, the Biot-Savart law is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

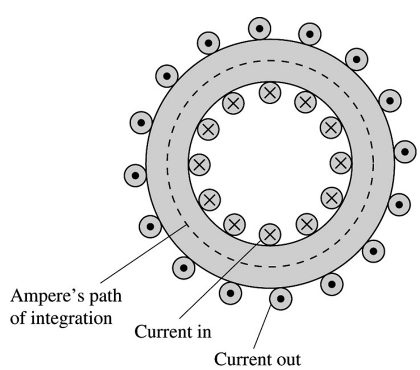
For the straight sections, $\Delta\vec{s}\times\hat{r} = 0$ because both $\Delta\vec{s}$ and $\hat{r}$ point along the same line. That is not the case with the curved section over which $\Delta\vec{s}$ and $\vec{r}$ are perpendicular. Thus,

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r^2} = \frac{\mu_0}{4\pi}\frac{IR\,d\theta}{R^2} = \frac{\mu_0 I\,d\theta}{4\pi R}$$

where we used $\Delta s = R\Delta\theta \approx R\,d\theta$ for the small arc length Δs. Integrating to obtain the total magnetic field at the center of the semicircle,

$$B = \int_{-\pi/2}^{\pi/2}\frac{\mu_0 I\,d\theta}{4\pi R} = \frac{\mu_0 I}{4\pi R}\pi = \frac{\mu_0 I}{4R}$$

32.56. Model: The toroid may be viewed as a solenoid that has been bent into a circle.
Solve: (**a**) A long solenoid has a uniform magnetic field inside and it is parallel to the axis. If we bend the solenoid to make it circular, we will have circular magnetic field lines around the inside of the toroid. However, as explained in part (c) the field is not uniform.
(**b**)



A top view of the toroid is shown. Current is into the page for the inside windings and out of the page for the outside windings. The closed Ampere's path of integration thus contains a current NI where I is the current flowing through the wire and N is the number of turns. Applied to the closed line path, the Ampere's law is

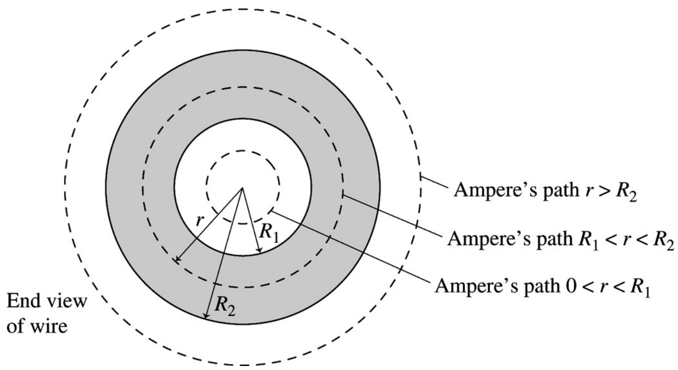
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} = \mu_0 NI$$

Because $\vec{B}$ and $d\vec{s}$ are along the same direction and B is the same along the line integral, the above simplifies to

$$B \cdot ds = B2\pi r = \mu_0 NI \Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

(c) The magnetic field for a toroid depends inversely on r which is the distance from the center of the toroid. As r increases from the inside of the toroid to the outside, B_{toroid} decreases. Thus the field is not uniform.

32.57. Model: The magnetic field is that of the current which is distributed uniformly in the hollow wire.
Visualize:



Ampere's integration paths are shown in the figure for the regions 0 m $< r < R_1$, $R_1 < r < R_2$, and $< r$.

Solve: For the region 0 m $< r < R_1$, $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$. Because the current inside the integration path is zero, $B = 0$ T. To find I_{through} in the region $R_1 < r < R_2$, we multiply the current density by the area inside the integration path that carries the current. Thus,

$$I_{\text{through}} = \frac{I}{\pi(R_2^2 - R_1^2)}\pi(r^2 - R_1^2)$$

where the current density is the first term. Because the magnetic field has the same magnitude at every point on the circular path of integration, Ampere's law simplifies to

$$\oint \vec{B} \cdot d\vec{s} = B\int ds = B(2\pi r) = \mu_0 \frac{I(r^2 - R_1^2)}{(R_2^2 - R_1^2)} \Rightarrow B = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - R_1^2}{R_2^2 - R_1^2}\right)$$

For the region $R_2 < r$, I_{through} is simply I because the loop encompasses the entire current. Thus,

$$\oint \vec{B} \cdot d\vec{s} = B\int ds = B2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

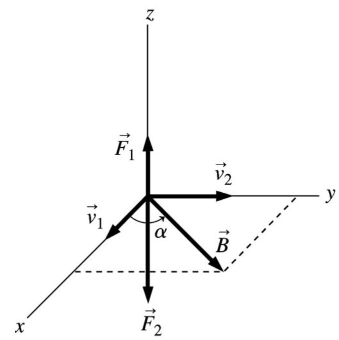
Assess: The results obtained for the regions $r > R_2$ and $R_1 < r < R_2$ yield the same result at $r = R_2$. Also note that a hollow wire and a regular wire have the same magnetic field outside the wire.

32.58. Model: The electron is a point mass undergoing cyclotron motion.

Solve: The electron's angular momentum is $L = mvr$. The cyclotron radius is $r = \dfrac{mv}{qB}$. Multiplying the latter equation by r and combining it with the expression for L, we have

$$r^2 = \frac{L}{qB} \Rightarrow \sqrt{\frac{L}{qB}} = \sqrt{\frac{8.0\times10^{-26}\ \text{kg m}^2/\text{s}}{(1.60\times10^{-19}\ \text{C})(5.0\times10^3\ \text{T})}} = 0.010\ \text{m} = 1.0\ \text{cm}$$

32.59. Model: Magnetic fields exert a force on moving charges.
Visualize:



Solve: The right-hand rule applied to the first proton requires $\vec{B}$ lie toward the +*y*-axis from $\vec{v}_1$, while when applied to the second proton requires $\vec{B}$ lie toward the +*x*-axis from $\vec{v}_2$. Thus $\vec{B}$ lies in the first quadrant of the *xy*-plane. The force on each proton is

$$F_1 = qv_1 B\sin\alpha \qquad F_2 = qv_2 B\sin(90° - \alpha) = qv_1 B\cos\alpha$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{v_1}{v_2}\tan\alpha \Rightarrow \alpha = \tan^{-1}\left(\frac{2.00\times10^6 \text{ m/s}}{1.00\times10^6 \text{ m/s}}\cdot\frac{1.20\times10^{-16} \text{ N}}{4.16\times10^{-16} \text{ N}}\right) = 30°$$

The magnetic field strength is thus

$$B = \frac{F_1}{qv_1\sin\alpha} = \frac{1.20\times10^{-16} \text{ N}}{(1.60\times10^{-19} \text{ C})(1.00\times10^6 \text{ m/s})\sin 30°} = 1.50\times10^{-3} \text{ T}$$

Thus $\vec{B} = (1.50 \text{ mT, } 30° \text{ ccw from } +x\text{-axis})$.

32.60. Solve: The electric field is

$$\vec{E} = \left(\frac{200 \text{ V}}{1 \text{ cm}}, \text{ down}\right) = (20{,}000 \text{ V/m, down})$$

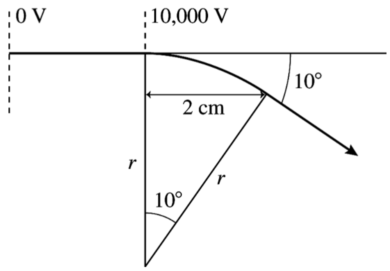
The force this field exerts on the electron is $\vec{F}_{\text{elec}} = q\vec{E} = -e\vec{E} = (3.2\times10^{-15} \text{ N, up})$. The electron will pass through without deflection *if* the magnetic field also exerts a force on the electron such that $\vec{F}_{\text{net}} = \vec{F}_{\text{elec}} + \vec{F}_{\text{mag}} = 0$ N. That is, $\vec{F}_{\text{mag}} = (3.2\times10^{-15} \text{ N, } 3.2\times10^{-15} \text{ N, down})$. In this case, the electric and magnetic forces cancel each other. For a negative charge with $\vec{v}$ to the right to have $\vec{F}_{\text{mag}}$ down requires, from the right-hand rule, that $\vec{B}$ point *into* the page. The magnitude of the magnetic force on a moving charge is $F_{\text{mag}} = qvB$, so the needed field strength is

$$B = \frac{F_{\text{mag}}}{ev} = \frac{3.2\times10^{-15} \text{ N}}{(1.60\times10^{-19} \text{ C})(1.0\times10^7 \text{ m/s})} = 2.0\times10^{-3} \text{ T} = 2.0 \text{ mT}$$

Thus, the required magnetic field is $\vec{B}$ = (2.0 mT, into page).

32.61. Model: Energy is conserved as the electron moves between the two electrodes. Assume the electron starts from rest. Once in the magnetic field, the electron moves along a circular arc.

Visualize:



The electron is deflected by 10° after moving along a circular arc of angular width 10°.
Solve: Energy is conserved as the electron moves from the 0 V electrode to the 10,000 V electrode. The potential energy is $U = qV$ with $q = -e$, so

$$K_f + U_f = K_i + U_i \Rightarrow \tfrac{1}{2}mv^2 - eV = 0 + 0$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(10{,}000\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 5.93\times10^7\text{ m/s}$$

The radius of cyclotron motion in a magnetic field is $r = mv/eB$. From the figure we see that the radius of the circular arc is $r = (2.0\text{ cm})/\sin10°$. Thus

$$B = \frac{mv}{er} = \frac{(9.11\times10^{-31}\text{ kg})(5.93\times10^7\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.020\text{ m})/\sin 10°} = 2.9\times10^{-3}\text{ T}$$

32.62. Model: Charged particles moving perpendicular to a uniform magnetic field undergo uniform circular motion at a constant speed.
Solve: **(a)** From Equation 32.20, the magnetic field is

$$B = \frac{2\pi fm}{e} = \frac{2\pi(2.4\times10^9\text{ Hz})(9.11\times10^{-31}\text{ kg})}{1.6\times10^{-19}\text{ C}} = 0.086\text{ T} = 86\text{ mT}$$

(b) The maximum kinetic energy is for an orbit with radius 1.25 cm.

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(2\pi rf)^2 = \tfrac{1}{2}(9.11\times10^{-31}\text{ kg})[2\pi(0.0125\text{ m})(2.4\times10^9\text{ Hz})]^2 = 1.62\times10^{-14}\text{ J}$$

32.63. Model: Electric and magnetic fields exert forces on a moving charge. The fields are uniform throughout the region.
Solve: We will first find the net force on the antiproton, and then find the net acceleration using Newton's second law. The magnitudes of the electric and magnetic forces are

$$F_E = eE = (1.60\times10^{-19}\text{ C})(1000\text{ V/m}) = 1.60\times10^{-16}\text{ N}$$

$$F_B = evB = (1.60\times10^{-19}\text{ C})(500\text{ m/s})(2.5\text{ T}) = 2.00\times10^{-16}\text{ N}$$

The directions of these two forces on the antiproton are opposite. $\vec{F}_E$ points *down* whereas, using the right-hand rule, $\vec{F}_B$ points up. Hence,

$$\vec{F}_{net} = (2.0\times10^{-16}\text{ N} - 1.60\times10^{-16}\text{ N, up}) \Rightarrow F_{net} = 0.40\times10^{-16}\text{ N} = ma$$

$$\Rightarrow \vec{a} = \left(\frac{0.40\times10^{-16}\text{ N}}{1.67\times10^{-27}\text{ kg}} = 2.4\times10^{10}\text{ m/s}^2\text{, up}\right)$$

32.64. Model: Charged particles moving perpendicular to a uniform magnetic field undergo uniform circular motion at constant speed.
Solve: **(a)** The magnetic force on a proton causes a centripetal acceleration:

$$evB = \frac{mv^2}{r} \Rightarrow v = \frac{eBr}{m}$$

Maximum kinetic energy is achieved when the diameter of the proton's orbit matches the diameter of the cyclotron:

$$K = \tfrac{1}{2}mv^2 = \frac{e^2B^2r^2}{2m} = \frac{(1.60\times10^{-19}\ \text{C})^2(0.75\ \text{T})^2(0.325\ \text{m})^2}{2(1.67\times10^{-27}\ \text{kg})} = 4.6\times10^{-13}\ \text{J}$$

(b) The proton accelerates through a potential difference of 500 V twice during one revolution. The energy gained per cycle is

$$2\,q\Delta V = 2e(500\ \text{V}) = 1.60\times10^{-16}\ \text{J}$$

Using the maximum kinetic energy of the proton from part (a)**,** the number of cycles before the proton attains this energy is

$$\frac{4.6\times10^{-13}\ \text{J}}{1.60\times10^{-16}\ \text{J}} = 2850$$

32.65. Model: Charged particles moving perpendicular to a uniform magnetic field undergo circular motion at constant speed.

Solve: The potential difference causes an ion of mass m to accelerate from rest to a speed v. Upon entering the magnetic field, the ion follows a circular trajectory with cyclotron radius $r = mv/eB$. To be detected, an ion's trajectory must have radius r = d/2 = 4.0000 cm. This means the ion needs the speed

$$v = \frac{eBr}{m} = \frac{eBd}{2m}$$

This speed was acquired by accelerating from potential V to potential 0. We can use the conservation of energy equation to find the voltage that will accelerate the ion:

$$K_1 + U_1 = K_2 + U_2 \Rightarrow 0\ \text{J} + e\Delta V = \tfrac{1}{2}mv^2 + 0\ \text{J} \Rightarrow \Delta V = \frac{mv^2}{2e}$$

Using the above expression for v, the voltage that causes an ion to be detected is

$$\Delta V = \frac{mv^2}{2e} = \frac{m}{2e}\left(\frac{eBd}{2m}\right)^2 = \frac{eB^2d^2}{8m}$$

For example, the mass of N_2^+ is

$$m = m_N + m_N = 2(14.003\ \text{u})(1.6605\times10^{-27}\ \text{kg/u}) = 4.6503\times10^{-26}\ \text{kg}$$

Note that we're given the atomic masses very accurately in Exercise 28. We need to retain this accuracy to tell the difference between N_2^+ and CO^+. The voltage for N_2^+ is

$$\Delta V = \frac{(1.6022\times10^{-19}\ \text{C})(0.20000\ \text{T})^2(0.080000\ \text{m})^2}{8(4.6503\times10^{-26}\ \text{kg})} = 110.07\ \text{V}$$

Ion	Mass(kg)	Accelerating voltage(V)
N_2^+	4.6503×10^{-26}	110.25
O_2^+	5.2969×10^{-26}	96.793
CO^+	4.6485×10^{-26}	110.29

Assess: The difference between N_2^+ and CO^+ is not large but is easily detectable.

32.66. Model: Assume that the magnetic field is uniform over the Hall probe.

Solve: Equation 32.24 gives the Hall voltage and Equation 32.20 gives the cyclotron frequency in terms of the magnetic field. We have

$$\Delta V_H = \frac{IB}{tne} \qquad B = \frac{2\pi m f_{cyl}}{e}$$

$$\Rightarrow tne = \frac{2\pi m f_{cyc} I}{e\Delta V_H} = \frac{2\pi(1.67\times10^{-27}\ \text{kg})(10.0\times10^6\ \text{Hz})(0.150\times10^{-3}\ \text{A})}{(1.60\times10^{-19}\ \text{C})(0.543\times10^{-3}\ \text{V})} = 0.1812\ \text{T A/V}$$

With this value of *tne*, we can once again use the Hall voltage equation to find the magnetic field:

$$B = \left(\frac{\Delta V_{\rm H}}{\rm I}\right) tne = \left(\frac{1.735\times10^{-3}\ \text{V}}{0.150\times10^{-3}\ \text{A}}\right)(0.1812\ \text{TA/V}) = 2.10\ \text{T}$$

32.67. Model: The loop will not rotate about the axle if the torque due to the magnetic force on the loop balances the torque of the weight.

Solve: The rotational equilibrium condition $\sum \vec{\tau}_{\rm net} = 0\ \text{N m}$ is about the axle and means that the torque from the weight is equal and opposite to the torque from the magnetic force. We have

$$(50\times10^{-3}\ \text{kg})g(0.025\ \text{m}) = \mu B \sin 90° = (NIA)B$$

$$\Rightarrow B = \frac{(50\times10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)(0.025\ \text{m})}{(10)(2.0\ \text{A})(0.050\ \text{m})(0.100\ \text{m})} = 0.12\ \text{T}$$

Assess: The current in the loop must be clockwise for the two torques to be equal.

32.68. Model: A magnetic field exerts a magnetic force on a length of current-carrying wire. We ignore gravitational effects, and focus on the *B* effects.

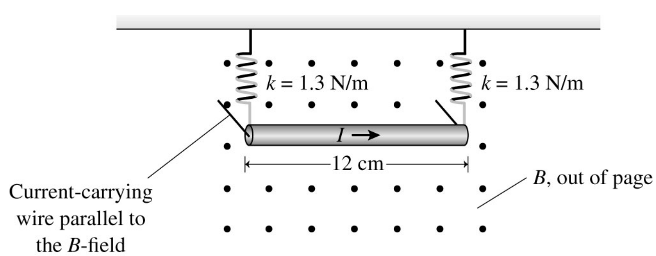
Visualize: The figure shows a wire in a magnetic field that is directed out of the page. The magnetic force on the wire is therefore to the left and will compress the springs.

Solve: In static equilibrium, the sum of the forces on the wire is zero:

$$F_{\rm B} + F_{\rm sp\,1} + F_{\rm sp\,2} = 0\ \text{N} \Rightarrow ILB + (-k\Delta x) + (-k\Delta x) \Rightarrow I = \frac{2k\Delta x}{LB} = \frac{2(10\ \text{N/m})(0.01\ \text{m})}{(0.20\ \text{m})(0.5\ \text{T})} = 2.0\ \text{A}$$

32.69. Model: Orient the page so the magnetic field comes out of the page and the current in the rod is to the right.

Visualize: When the current is on the magnetic force $F_{\rm B} = ILB$ will be directed down by the right-hand rule. This will stretch the springs, which will produce an upward force equal in magnitude to the magnetic force.



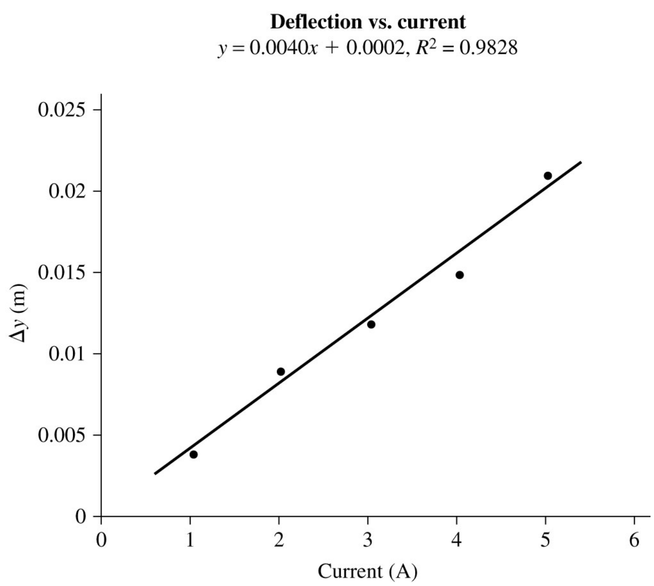
Solve: There are two springs, so the upward force due to the stretched springs will be $F_{\rm sp} = 2k(\Delta y)$. The rod is in equilibrium, so

$$2k(\Delta y) = ILB$$

Isolate Δy.

$$\Delta y = \frac{LB}{2k} I$$

This leads us to believe that a graph of Δy vs. I would produce a straight line whose slope is $\dfrac{LB}{2k}$ and whose intercept is zero.



We see from the spreadsheet that the linear fit is reasonable and that the slope is 0.0040 m/A and the intercept is very small. Solve for B:

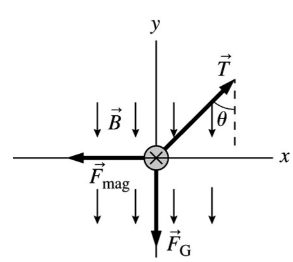
$$B = \frac{(\text{slope})(2k)}{L} = \frac{(0.0040\text{ m/A})[2(1.3\text{ N/m})]}{0.12\text{ m}} = 87\text{ mT}$$

Assess: While not immediately obvious, the units work out.

32.70. Model: The bar is a current-carrying wire in a perpendicular uniform magnetic field. The current is constant.
Solve: **(a)** The right-hand rule as described in section 32.8 requires the current to be into the page.
(b) The net force on the bar is $F = IlB,$ and is constant throughout the motion. The acceleration of the bar is thus $\frac{F}{m} = \frac{IlB}{m}$. Using constant acceleration kinematics,

$$v_f^2 = v_0^2 + 2a\Delta s = (0\text{ m/s})^2 + 2\left(\frac{IlB}{m}\right)d \Rightarrow v_f = \sqrt{\frac{2IlBd}{m}}$$

32.71. Model: A magnetic field exerts a magnetic force on a length of current-carrying wire.
Visualize:



Solve: **(a)** The above figure shows a side view of the wire, with the current moving into the page. From the right-hand rule, the magnetic field $\vec{B}$ points *down* to give a leftward force on the current. The wire is hanging in static equilibrium, so $\vec{F}_{\text{net}} = \vec{F}_{\text{mag}} + \vec{F}_G + \vec{T} = 0\text{ N}.$ Consider a segment of wire of length L. The wire's linear mass density is $\mu = 0.050$ kg/m, so the mass of this segment is $m = \mu L$ and its weight is $F_G = mg = \mu Lg.$ The magnetic force on this length of wire is $F_{\text{mag}} = ILB.$ In component form, Newton's first law is

$$(F_{\text{net}})_x = T\sin\theta - F_{\text{mag}} = T\sin\theta - ILB = 0\text{ N} \Rightarrow T\sin\theta = ILB$$
$$(F_{\text{net}})_y = T\cos\theta - F_G = T\cos\theta - \mu Lg = 0\text{ N} \Rightarrow T\cos\theta = \mu Lg$$

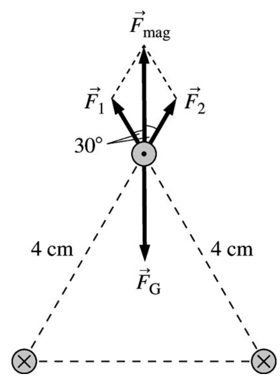
Dividing the first equation by the second,

$$\left[\frac{T\sin\theta}{T\cos\theta} = \tan\theta\right] = \left[\frac{ILB}{\mu Lg} = \frac{IB}{\mu g}\right] \Rightarrow B = \frac{\mu g\tan\theta}{I}$$

(b) $B = \dfrac{\mu g\tan\theta}{I} = \dfrac{(0.055\text{ kg/m})(9.8\text{ m/s}^2)\tan 11°}{10\text{ A}} = 0.011456\text{ T}$

The magnetic field is $\vec{B} = (11\text{ mT, down})$.

32.72. Model: The wire will float in the magnetic field if the magnetic force on the wire points upward and has a magnitude mg, allowing it to balance the downward gravitational force.
Visualize:



Solve: Each lower wire exerts a repulsive force on the upper wire because the currents are in opposite directions. The currents are of equal magnitude and the distances are equal, so $F_1 = F_2$. Consider segments of the wires of length L. Then the forces are

$$F_1 = F_2 = \frac{\mu_0 LI^2}{2\pi d}$$

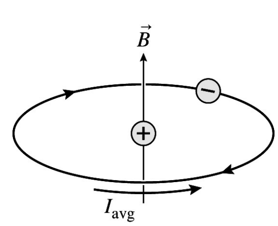
The horizontal components of these two forces cancel, so the net magnetic force is upward and of magnitude

$$F_{\text{mag}} = 2F_1\cos 30° = \frac{\mu_0 LI^2\cos 30°}{\pi d}$$

In equilibrium, this force must exactly balance the downward weight of the wire. The wire's linear mass density is $\mu = 0.050$ kg/m, so the mass of this segment is $m = \mu L$ and its weight is $w = mg = \mu Lg$. Equating these gives

$$\frac{\mu_0 LI^2\cos 30°}{\pi d} = \mu Lg \Rightarrow I = \sqrt{\frac{\mu g\pi d}{\mu_0\cos 30°}} = \sqrt{\frac{(0.050\text{ kg/m})(9.8\text{ m/s}^2)\pi(0.040\text{ m})}{(4\pi\times10^{-7}\text{ T m/A})\cos 30°}} = 238\text{ A} \approx 240\text{ A}$$

32.73. Model: A current loop produces a magnetic field.
Visualize:



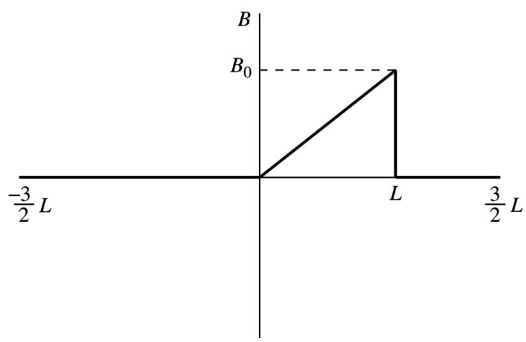
Solve: The field at the center of a current loop is $B_{\text{loop}} = \mu_0 I/2R$. The electron orbiting an atomic nucleus is, on average, a small current loop. Current is defined as $I = \Delta q/\Delta t$. During one orbital period T, the charge $\Delta q = e$ goes around the loop one time. Thus the *average* current is $I_{\text{avg}} = e/T$. For circular motion, the period is

$$T = \frac{2\pi R}{v} = \frac{2\pi(5.3\times10^{-11}\text{ m})}{2.2\times10^{6}\text{ m/s}} = 1.514\times10^{-16}\text{ s} \Rightarrow I_{\text{avg}} = \frac{1.60\times10^{-19}\text{ C}}{1.514\times10^{-16}\text{ s}} = 1.057\times10^{-3}\text{ A}$$

Thus, the magnetic field at the center of the atom is

$$B_{\text{center}} = \frac{\mu_0 I_{\text{avg}}}{2R} = \frac{(4\pi\times10^{-7}\text{ T m/A})(1.057\times10^{-3}\text{ A})}{2(5.3\times10^{-11}\text{ m})} = 12.5\text{ T} \approx 13\text{ T}$$

32.74. Model: The superposition principle holds for forces and torques.
Visualize:



Solve: **(a)** A graph of B vs. x is shown above.
(b) The magnetic force on a segment of wire Δx in length located at x_i is

$$\Delta F_i = IB(x_i)\Delta x$$

The total force on the wire is

$$F_{\text{net}} = \sum \Delta F_i = \sum IB(x_i)\Delta x \rightarrow \int IBdx = \int_0^L IB_0 \frac{x}{L} dx = \frac{1}{2} ILB_0$$

The right-hand rule gives the direction of the force along the $+y$-axis, so $\vec{F}_{\text{net}} = \frac{1}{2} ILB_0 \hat{j}$.

(c) Each segment of wire experiences a torque about the origin of

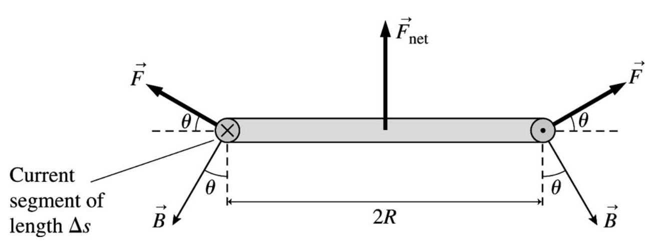
$$\tau_i = r_i \Delta F_i \sin 90° = x_i IB(x_i)\Delta x.$$

The total torque on the wire is thus

$$\tau = \sum \tau_i = \sum x_i IB(x_i)\Delta x \rightarrow \int_0^L IB_0 \frac{x^2}{L} dx = \frac{1}{3} IL^2 B_0$$

The direction of the torque is along the $+z$-axis by the right-hand rule, indicating that the magnetic field will exert a torque causing the wire to rotate counterclockwise in the xy-plane.

32.75. Model: A magnetic field exerts a force on a segment of current.
Visualize: The figure shows the forces on two small segments of current, one in which the current enters the plane of the page and one in which the current leaves the plane of the page.



Solve: **(a)** Consider a small segment of the loop of length Δs. The magnetic force on this segment is perpendicular both to the current and to the magnetic field. The figure shows two segments on opposite sides of the loop. The horizontal components of the forces cancel but the vertical components combine to give a force toward the bar magnet. The net force is the sum of the vertical components of the force on all segments around the loop. For a segment of length Δs, the magnetic force is $F = IB\Delta s$ and the vertical component is $Fy = (IB\Delta s)\sin\theta$. Thus the net force on the current loop is

$$F_{\text{net}} = \sum F_y = \sum(IB\Delta s)\sin\theta = IB\sin\theta\sum\Delta s$$

We could take $\sin\theta$ outside the summation because θ is the same for all segments. The sum of all the Δs is simply the circumference $2\pi R$ of the loop, so

$$F_{\text{net}} = 2\pi RIB\sin\theta$$

(b) The net force is

$$F_{\text{net}} = 2\pi(0.020\text{ m})(\sin 20°)(0.50\text{ A})(200\times10^{-3}\text{ T}) = 4.3\times10^{-3}\text{ N}$$

32.76. **Solve:** From Equation 32.7, the magnetic field at the center ($z = 0$ m) of an N-turn coil is

$$B_{\text{coil center}} = \frac{\mu_0}{2}\frac{NI}{R} = \frac{\mu_0}{2}\frac{1.0\text{ m}}{2\pi R}\frac{I}{R}$$

where we used the requirement that the entire length of wire be used. That is, $1.0\text{ m} = (2\pi R)N$. Solving for R,

$$R = \sqrt{\frac{\mu_0}{2}\frac{(1.0\text{ m})I}{2\pi B_{\text{coil center}}}} == \sqrt{\frac{2\pi(10^{-7}\text{ T m/A})(1.0\text{ m})(1.0\text{ A})}{2\pi(1.0\times10^{-3}\text{ T})}} = 0.010\text{ m} = 1.0\text{ cm}$$

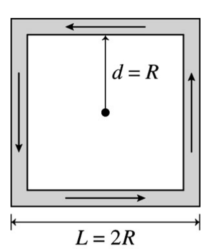
32.77. **Model:** The magnetic field is that of a finite length of current-carrying wire.
Solve: **(a)** Let the wire be on the x-axis and the point of interest be at distance d on the y-axis. The wire extends from $x = -L/2$ to $x = +L/2$. As explained in Example 32.3 and Figure 32.14, each segment dx of the wire generates a field at the observation point of strength

$$dB = \frac{\mu_0 Id\,dx}{4\pi(x^2+d^2)^{3/2}}$$

These infinitesimal fields all point the same direction, so we can add them (superposition) to find the total field. The sum becomes an integral, giving

$$B = \frac{\mu_0 Id}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{(x^2+d^2)^{3/2}} = \frac{\mu_0 Id}{4\pi}\left.\frac{x}{d^2(x^2+d^2)^{1/2}}\right|_{-L/2}^{L/2} = \frac{\mu_0 IL}{4\pi d\sqrt{(L/2)^2+d^2}}$$

(b)



A square is formed of four straight-wire segments, with each segment contributing the same field at the center. The length is $L = 2R$, and the observation point is distance $d = R$ from the center of each segment. Using the formula from part (a),

$$B_{\text{square}} = 4B_{\text{seg}} = 4\times\frac{\mu_0 I(2R)}{4\pi R\sqrt{(2R/2)^2+R^2}} = 4\times\frac{\mu_0 I}{2\sqrt{2}\pi R} = \frac{\sqrt{2}\mu_0 I}{\pi R}$$

(**c**) The ratio of the field strengths of a square loop and a circular loop, each with "radius" R, is

$$\frac{B_{\text{square}}}{B_{\text{circle}}} = \frac{\sqrt{2}\mu_0 I/\pi R}{\mu_0 I/2R} = \frac{2\sqrt{2}}{\pi} = 0.900 = 90.0\%$$

The formula for the magnetic field at the center of a circular loop is from Equation 32.7.

32.78. Model: The spinning charged disk forms a series of concentric current loops.
Solve: The magnetic field at the center of a current loop of radius r is taken from Example 32.5 with $z = 0$:

$$B_{\text{loop}} = \frac{\mu_0 I}{2r}$$

Consider a ring of charge on the disk at radius r_i with width Δr. The charge on the ring is $\Delta Q = \dfrac{2\pi r_i \Delta r}{\pi R^2}Q$. The time for one revolution of the ring is $\Delta t = \dfrac{2\pi}{\omega}$, so the ring can be considered a current loop with

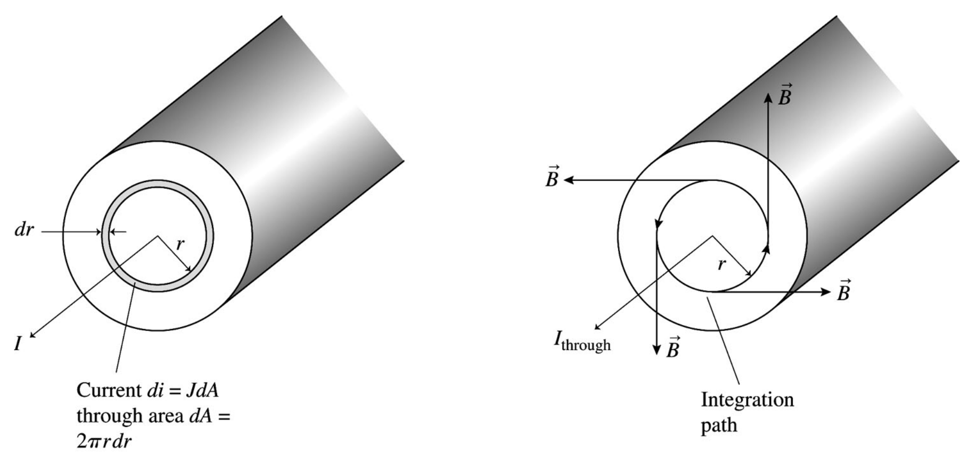
$$I_i = \frac{\Delta Q}{\Delta t} = \frac{2Qr_i\Delta r}{R^2}\left(\frac{\omega}{2\pi}\right) = \frac{\omega Q}{\pi R^2} r_i \Delta r$$

Each ring contributes a field $(B_{\text{loop}})_i$, so by superposition

$$B_{\text{center}} = \sum (B_{\text{loop}})_i = \sum \frac{\mu_0 I_i}{2r_i} = \sum \frac{\mu_0 \omega Q}{2\pi R^2}\Delta r$$

$$\to \int_0^R \frac{\mu_0 \omega Q}{2\pi R^2} dr = \frac{\mu_0 \omega Q}{2\pi R}$$

32.79. Model: The magnetic field is that of a conducting wire that has a nonuniform current density.
Visualize:



Solve: (**a**) Consider a small circular disk of width dr at a distance r from the center. The current through this disk is

$$di = JdA = J_0\left(\frac{r}{R}\right)(2\pi r)dr = \frac{2\pi J_0 r^2 dr}{R}$$

Integrating this expression, we get

$$I = \int di = \int_0^R \frac{2\pi J_0}{R} r^2\, dr = \frac{2\pi J_0}{R}\left[\frac{r^3}{3}\right]_0^R = \frac{2\pi J_0 R^2}{3} \Rightarrow J_0 = \frac{3I}{2\pi R^2}$$

(b) Applying $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$ to the circular path of integration, we note that the wire has perfect cylindrical symmetry with all the charges moving parallel to the wire. So, the magnetic field must be tangent to circles that are concentric with the wire. The enclosed current is the current within radius r. Thus,

$$\int Bds = \mu_0 \int_0^r di = \mu_0 \int_0^r \frac{2\pi J_0}{R} r^2 dr \Rightarrow B(2\pi r) = \frac{\mu_0 2\pi J_0}{R}\frac{r^3}{3} = \left(\frac{\mu_0 2\pi}{R}\right)\left(\frac{3I}{2\pi R^2}\right)\frac{r^3}{3} \Rightarrow B = \frac{\mu_0}{2\pi}\frac{Ir^2}{R^3}$$

(c) At $r = R$,

$$B = \frac{\mu_0}{2\pi}\frac{IR^2}{R^3} = \frac{\mu_0}{2\pi}\frac{I}{R}$$

This is the same result as obtained in Example 32.3 for the magnetic field of a long, straight wire.

32.80. Model: The magnetic field is that of a coaxial cable consisting of a solid inner conductor surrounded by a hollow outer conductor of essentially zero thickness.
Visualize: The solid inner conductor and the hollow outer conductor carry equal currents but in opposite directions. The coaxial cable has perfect cylindrical symmetry. So the magnetic field must be tangent to circles that are concentric with the wire.
Solve: **(a)** Ampere's law is

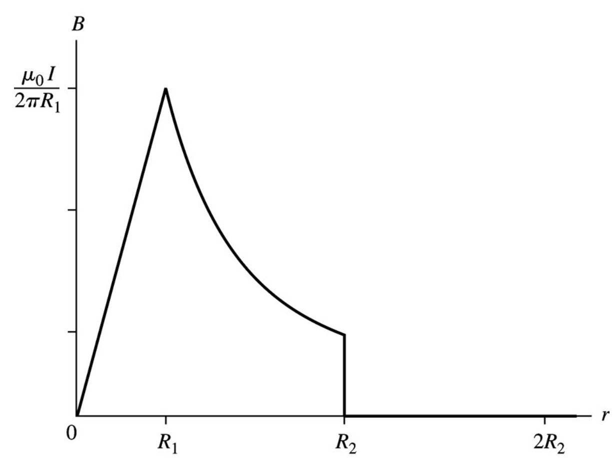
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} = B\int ds = B(2\pi r) \Rightarrow B = \frac{\mu_0 I_{\text{through}}}{2\pi r}$$

For $r < R_1$,

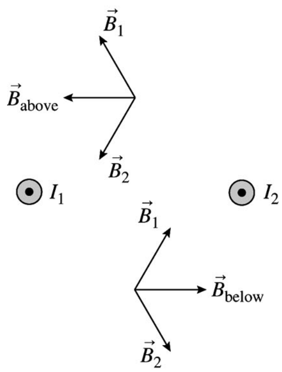
$$I_{\text{through}} = \left(\frac{I}{\pi R_1^2}\right)\pi r^2 = \frac{Ir^2}{R_1^2} \Rightarrow B = \frac{\mu_0}{2\pi}\left(\frac{Ir}{R_1^2}\right)$$

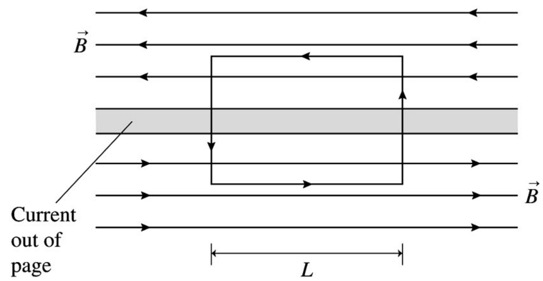
For $R_1 < r < R_2, I_{\text{through}} = I.$ Hence, $B = \mu_0 I/2\pi r.$ For $r > R_2, I_{\text{through}} = 0$ A and $B = 0$ T.

(b) In the region $r < R_1, B$ is linearly proportional to r; in the region $R_1 < r < R_2, B$ is inversely proportional to r; and in the region for $r > 2R_2, B = 0$ T. A B versus r graph in the range $r = 0$ m to $r = 2R_2$ is shown below.



32.81. Model: The current sheet extends infinitely far in both directions.
Visualize: **(a)** The shape of the magnetic field must be consistent with the symmetry of the current sheet. Consider two current-carrying wires. The magnetic field above the midpoint of the wires is horizontal and to the left. The magnetic field below the midpoint of the wires is horizontal and to the right. If we imagine the current sheet to consist of many such pairs of wires, closely spaced, then we see that the magnetic field above the sheet is everywhere horizontal and to the left while the magnetic field below the sheet is everywhere horizontal and to the right. The figure shows a closed path for using Ampere's law.





Solve: (**b**) The closed path of width L shown in the figure is parallel to the field along the top and bottom edges, perpendicular to the field along the left and right edges. Thus the line integral in Ampere's law is

$$\oint \vec{B}\cdot d\vec{s} = \int_{\text{top}} \vec{B}\cdot d\vec{s} + \int_{\text{left}} \vec{B}\cdot d\vec{s} + \int_{\text{bottom}} \vec{B}\cdot d\vec{s} + \int_{\text{right}} \vec{B}\cdot d\vec{s} = BL + 0 + BL + 0 = 2BL$$

Because the current per unit width is J_s, the amount of current in a length L of the current sheet (the current through the closed path) is $I_{\text{through}} = J_s L$. Thus Ampere's law gives

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}} \Rightarrow 2BL = \mu_0 J_s L \Rightarrow B = \tfrac{1}{2}\mu_0 J_s$$

Assess: The magnetic field strength is independent of the distance from the sheet. This is not unexpected since the electric field of an infinite plane of charge is independent of the distance from the plane.

32.82. Model: A magnetic field exerts a magnetic force on a moving charge given by $\vec{F} = q\vec{v}\times\vec{B}$. Assume the magnetic field is uniform.

Visualize: The magnetic field points in the $+z$-direction. If the charged particle is moving along $\vec{B}, F = 0$ N. If $\vec{v}$ is perpendicular to $\vec{B}$, the motion of the charged particle is a circle. However, when $\vec{v}$ makes an angle with $\vec{B}$, the motion of the charged particle is like a helix or a spiral. The perpendicular component of the velocity is responsible for the circular motion, and the parallel component is responsible for the linear motion along the magnetic field direction.

Solve: From the figure we see that $v_y = v\cos 30°$ and $v_z = v\sin 30°$. For the circular motion, the magnetic force causes a centripetal acceleration. That is,

$$ev_y B = \frac{mv_y^2}{r} \Rightarrow r = \frac{mv_y}{eB} = \frac{(9.11\times10^{-31}\text{ kg})(5.0\times10^{6}\text{ m/s})\cos 30°}{(1.60\times10^{-19}\text{ C})(0.030\text{ T})} = 0.82\text{ mm}$$

The time for one revolution is

$$T = \frac{2\pi r}{v_y} = \frac{2\pi(8.2\times10^{-4}\text{ m})}{(5.0\times10^{6}\text{ m/s})\cos 30°} = 1.19\times10^{-9}\text{ s}$$

The pitch p is the vertical distance covered in time T. We have

$$p = v_z T = (5.0\times10^{6}\text{ m/s})\sin 30°(1.19\times10^{-9}\text{ s}) = 3.0\times10^{-3}\text{ m} = 3.0\text{ mm}$$