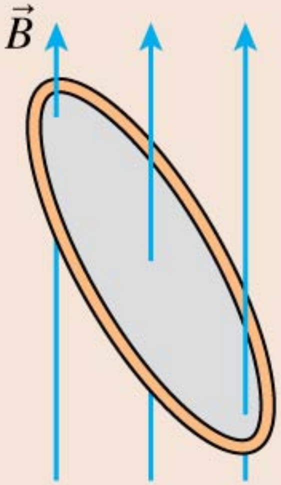
Chapter 33 Preview

Magnetic Flux

A key idea will be the amount of magnetic field passing through a loop. This is called the **magnetic flux.**

You'll find that the magnetic flux depends on the strength of the magnetic field, the area of the loop, and the angle between them.

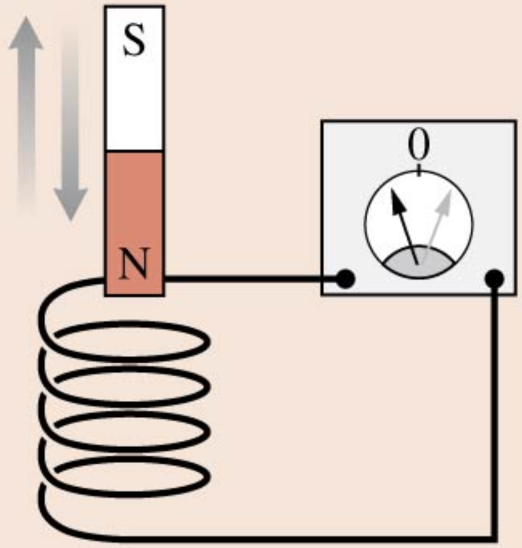


Chapter 33 Preview

Connecting *E* and *B*

We previously found that a current generates a magnetic field. In fact, the connection between electric and magnetic fields is much more profound.

You'll learn that pushing a magnet into a coil of wire, or pulling it out, causes an **induced current** in the wire. The process is called **electromagnetic induction.**



Applications

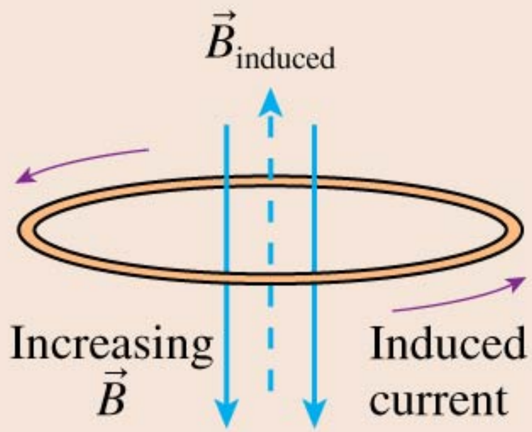
Electromagnetic induction has many applications. You'll learn about using **inductors**—coils of wire that store magnetic energy—in circuits.

You'll also learn how a generator, such as this **generator** turned by windmill blades, transforms mechanical energy into electric energy.

Lenz's Law

Lenz's law says that a current is induced in a closed loop *if and only if the magnetic flux through the loop is changing*. Simply having a magnetic flux doesn't do anything; the flux has to *change*.

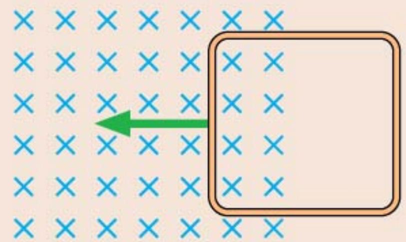
You'll learn how to use Lenz's law to determine the direction of an induced current.



Chapter 33 Preview

Faraday's Law

You'll learn to use **Faraday's law,** the most important law connecting electric and magnetic fields.

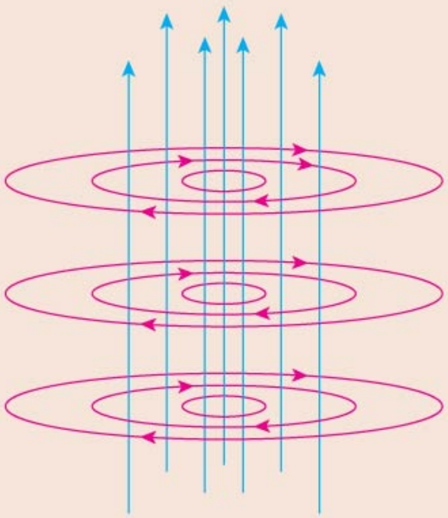


The magnetic flux through this loop is increasing as the loop moves into the field. Faraday's law allows us to compute the induced emf and the induced current. The current direction is given by Lenz's law.

Induced Fields

At its most fundamental level, Faraday's law tells us that a changing magnetic field creates an **induced electric field.** It is the induced electric field that then creates the induced current in a conducting loop.

An increasing magnetic field (blue) creates an electric field (red) that circulates in closed loops. You'll learn to calculate the strength of the induced field.



Reading Question 33.1

Currents circulate in a piece of metal that is pulled through a magnetic field. What are these currents called?

A. Induced currents.
B. Displacement currents.
C. Faraday's currents.
D. Eddy currents.
E. This topic is not covered in Chapter 33.

Reading Question 33.1

Currents circulate in a piece of metal that is pulled through a magnetic field. What are these currents called?

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C. Faraday's currents.
✔ **D. Eddy currents.**
E. This topic is not covered in Chapter 33.

Reading Question 33.2

Electromagnetic induction was discovered by

A. Faraday.
B. Henry.
C. Maxwell.
D. Both Faraday and Henry.
E. All three.

Reading Question 33.2

Electromagnetic induction was discovered by

A. Faraday.
B. Henry.
C. Maxwell.
✔ **D. Both Faraday and Henry.**
E. All three.

Reading Question 33.3

The direction that an induced current flows in a circuit is given by

A. Faraday's law.
B. Lenz's law.
C. Henry's law.
D. Hertz's law.
E. Maxwell's law.

Reading Question 33.3

The direction that an induced current flows in a circuit is given by

A. Faraday's law.

✔ **B. Lenz's law.**

C. Henry's law.

D. Hertz's law.

E. Maxwell's law.

Reading Question 33.4

After thinking about electromagnetic induction, James Clerk Maxwell was lead to propose that

A. An electric current can be induced by a changing magnetic flux.
B. A magnetic field can be produced by an electric current.
C. Light is an electromagnetic wave.
D. Moving charges accelerate in an magnetic field.
E. Nothing can travel faster than the speed of light.

Reading Question 33.4

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B. A magnetic field can be produced by an electric current.

✔ **C. Light is an electromagnetic wave.**

D. Moving charges accelerate in an magnetic field.

E. Nothing can travel faster than the speed of light.

Reading Question 33.5

A transformer

A. Boosts the maximum current provided by a battery.
B. Changes mechanical energy to electrical energy.
C. Changes the voltage of an alternating current.
D. Resists changes in current.
E. Converts alternating current to direct current.

Reading Question 33.5

A transformer

A. Boosts the maximum current provided by a battery.

B. Changes mechanical energy to electrical energy.

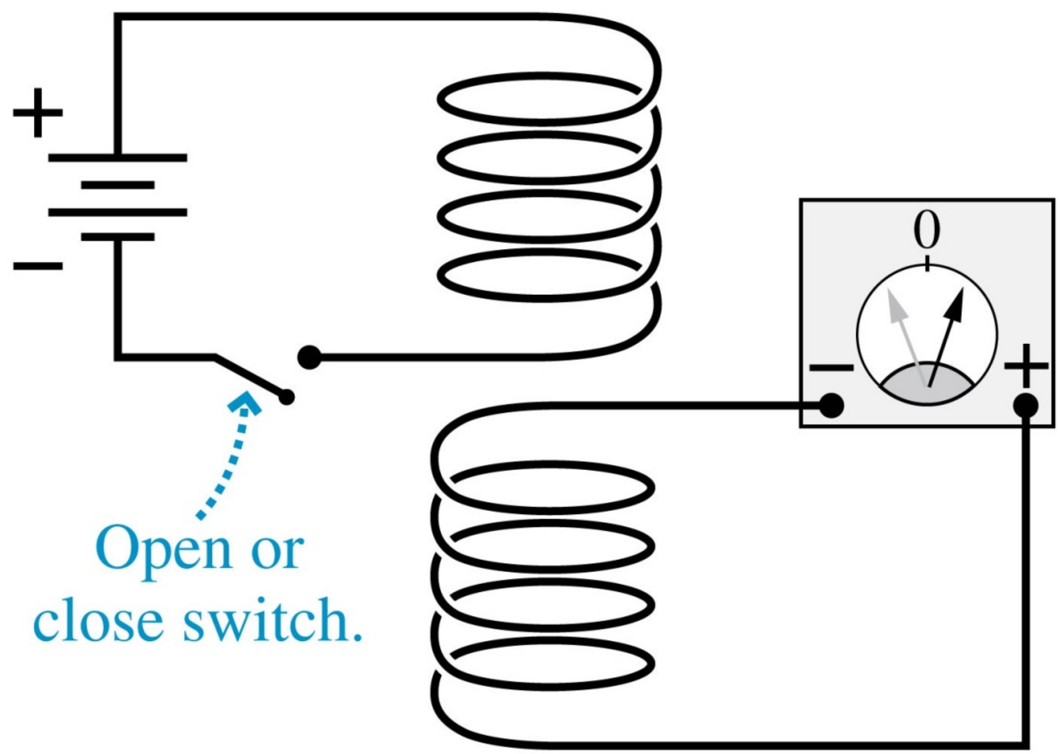
✔ **C. Changes the voltage of an alternating current.**

D. Resists changes in current.

E. Converts alternating current to direct current.

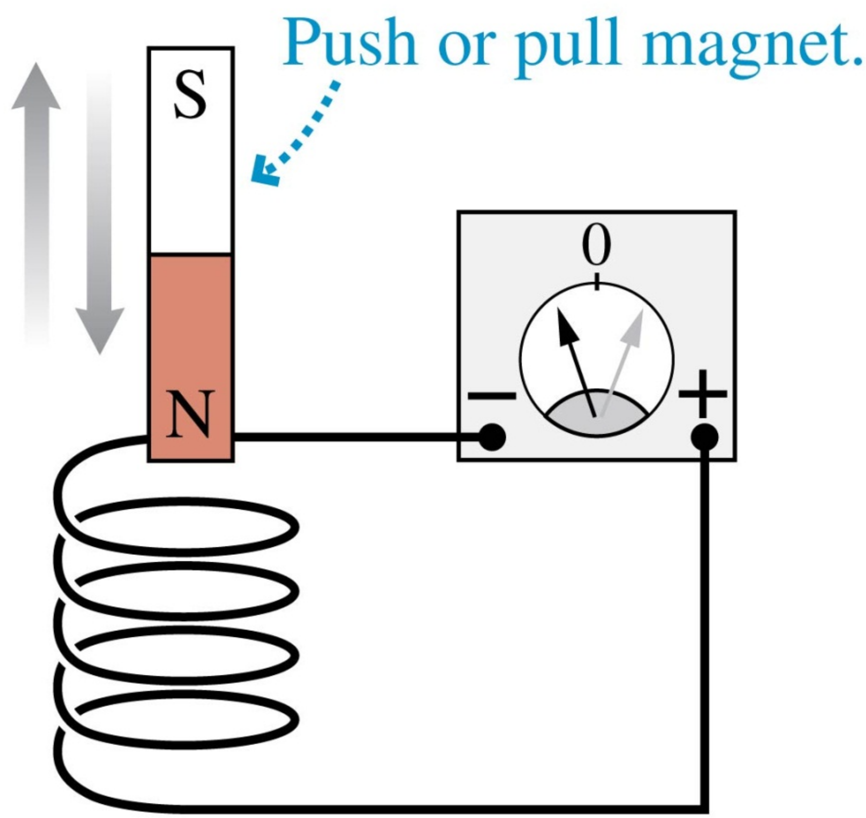
Faraday's Discovery of 1831

- When one coil is placed directly above another, there is no current in the lower circuit while the switch is in the closed position.
- A momentary current appears whenever the switch is opened or closed.



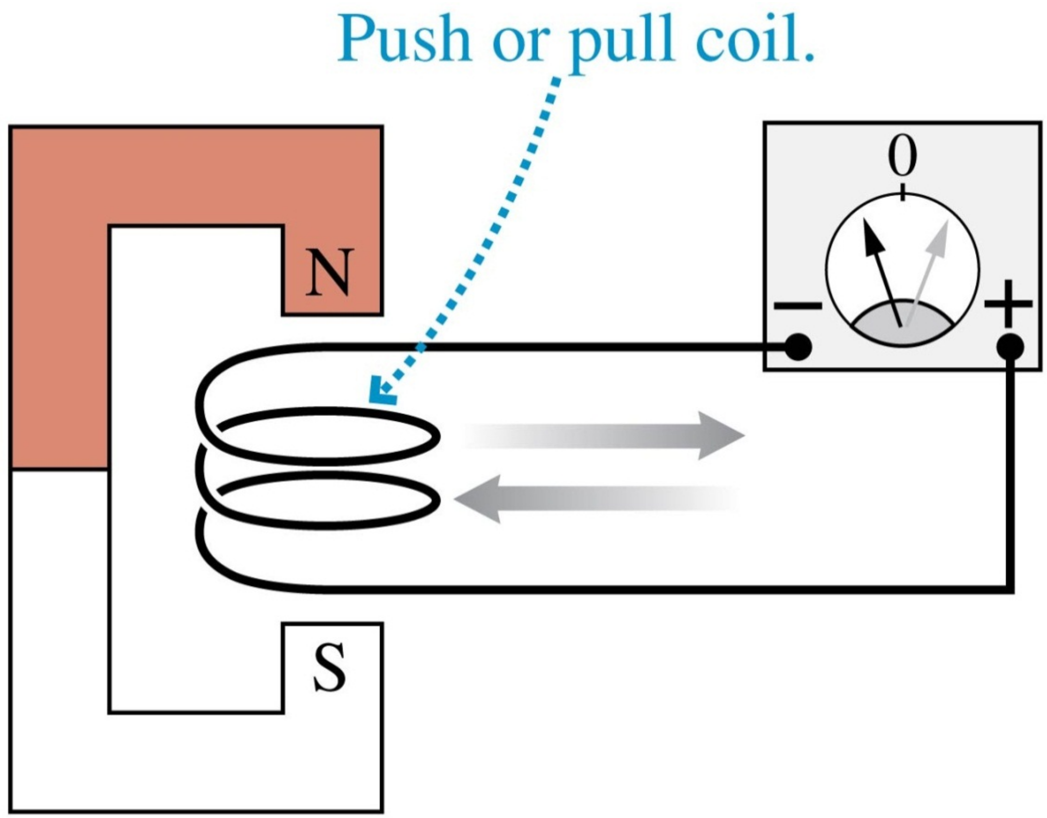
Faraday's Discovery of 1831

- When a bar magnet is pushed into a coil of wire, it causes a momentary deflection of the current-meter needle.
- Holding the magnet inside the coil has no effect.
- A quick withdrawal of the magnet deflects the needle in the other direction.

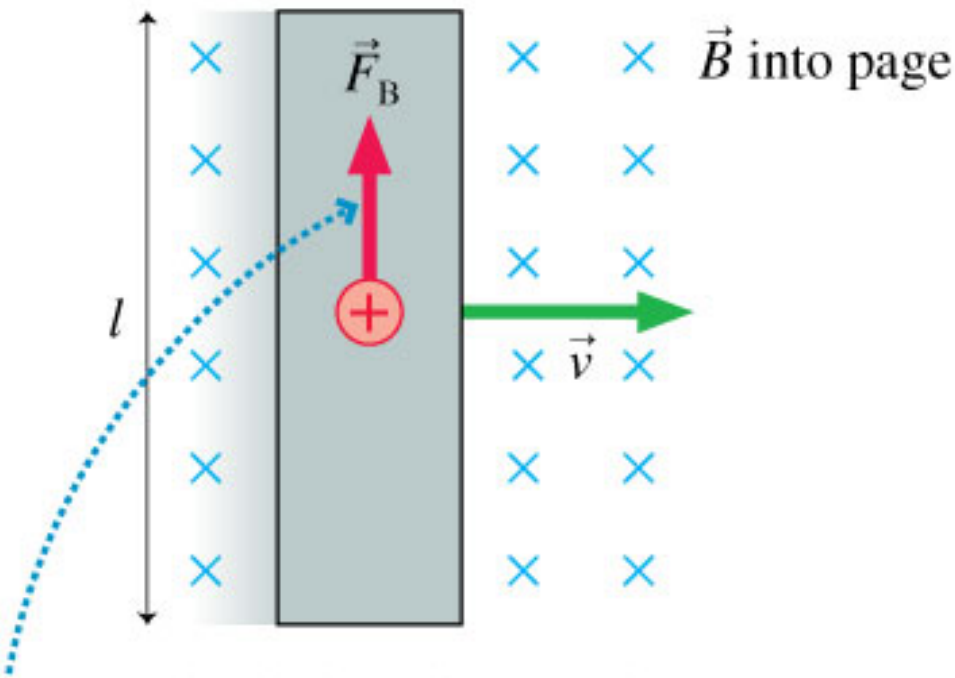


Faraday's Discovery of 1831

- A momentary current is produced by rapidly pulling a coil of wire out of a magnetic field.
- Pushing the coil into the magnet causes the needle to deflect in the opposite direction.

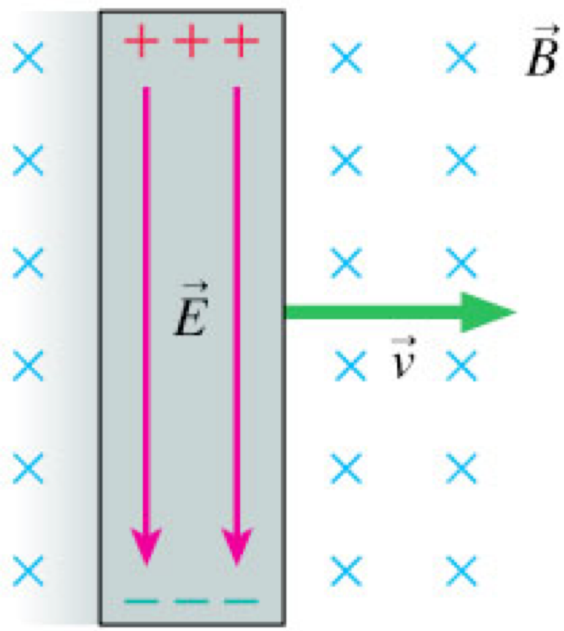


Motional emf



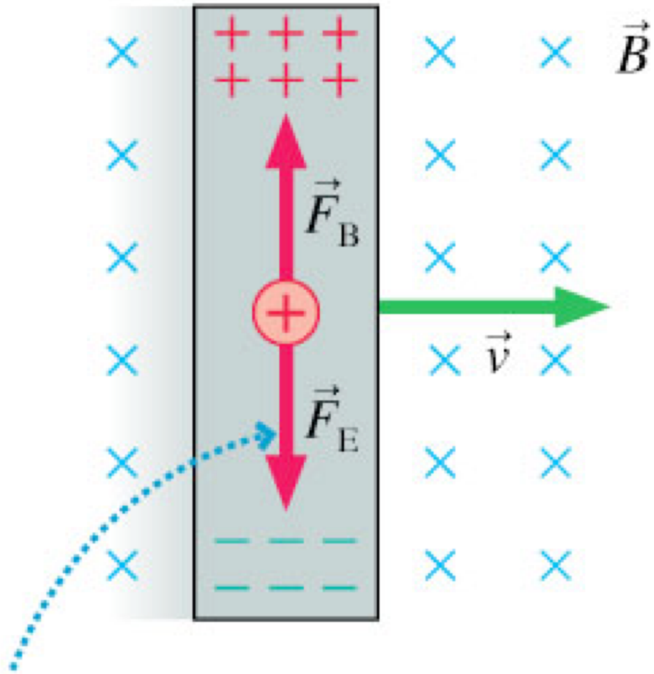
Charge carriers in the wire experience an upward force of magnitude $F_B = qvB$. Being free to move, positive charges flow upward (or, if you prefer, negative charges downward).

Motional emf



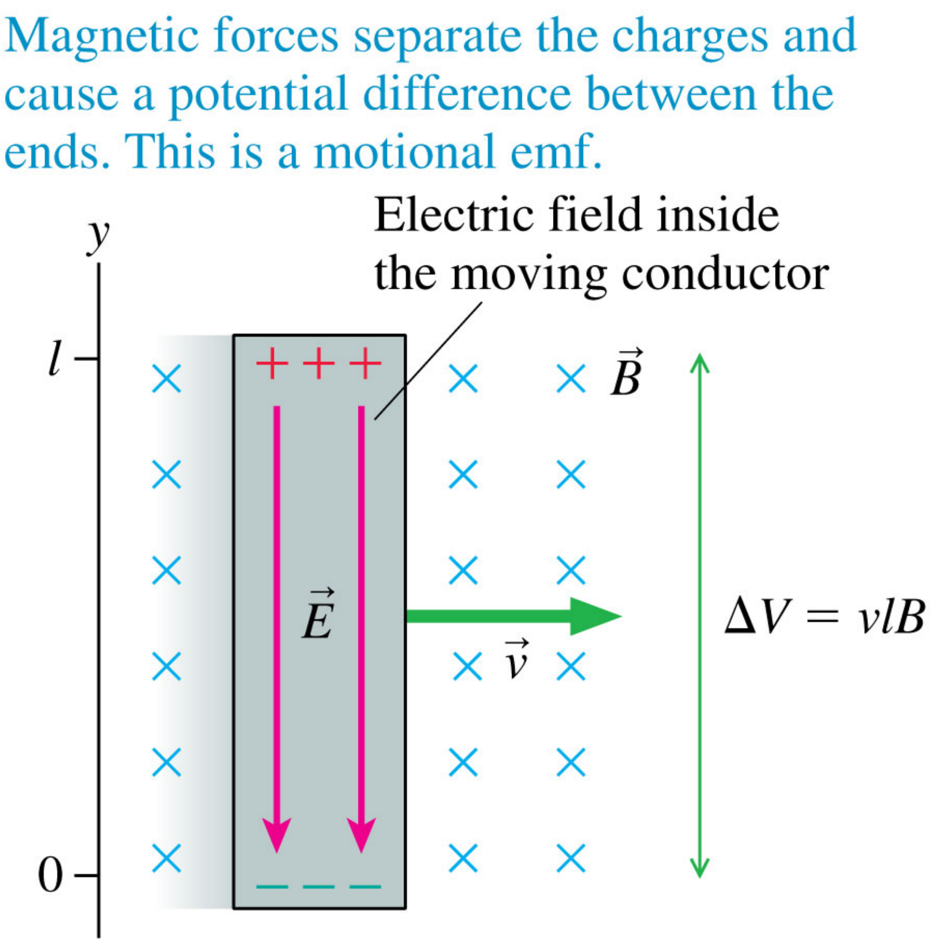
The charge separation creates an electric field in the conductor. $\vec{E}$ increases as more charge flows.

Motional emf



The charge flow continues until the downward electric force $\vec{F}_E$ is large enough to balance the upward magnetic force $\vec{F}_B$. Then the net force on a charge is zero and the current ceases.

Motional emf

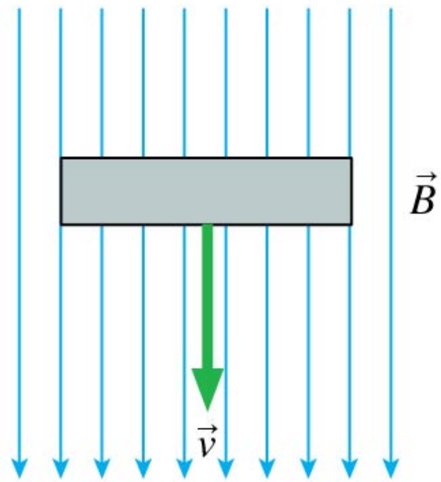


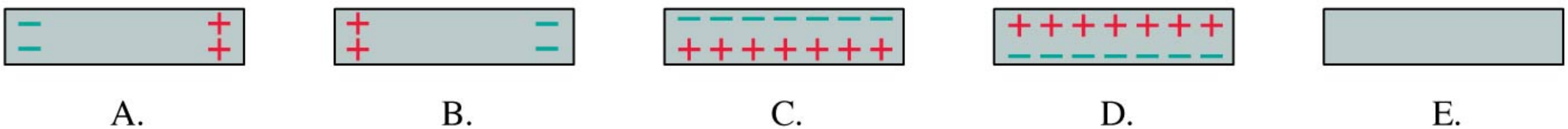
- The magnetic force on the charge carriers in a moving conductor creates an electric field of strength $E = vB$ inside the conductor.
- For a conductor of length l, the motional emf perpendicular to the magnetic field is:

$$\mathcal{E} = vlB$$

QuickCheck 33.1

A metal bar moves through a magnetic field. The induced charges on the bar are





QuickCheck 33.1

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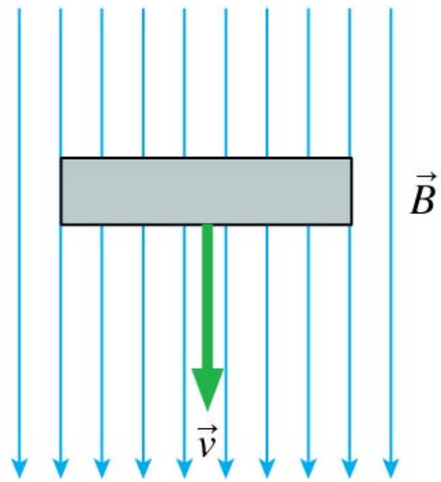
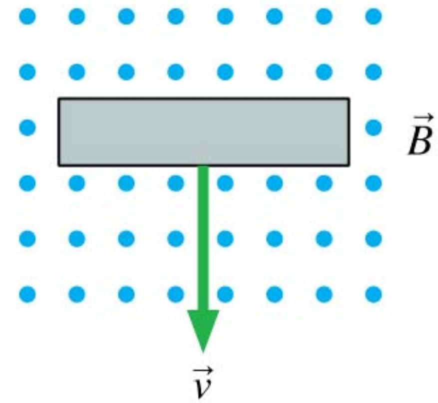
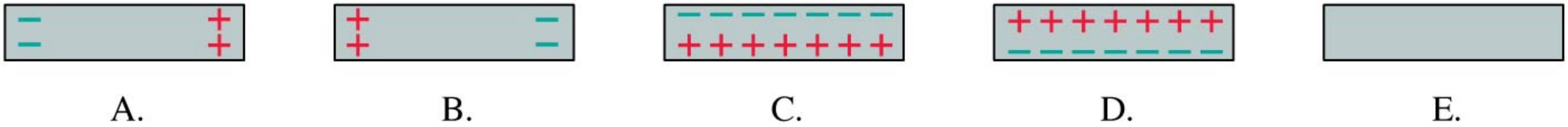


A. B. C. D. **E.** ✔

QuickCheck 33.2

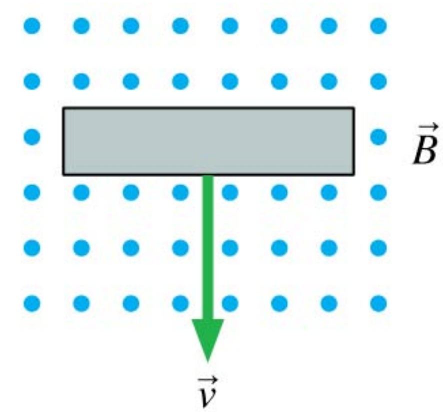
A metal bar moves through a magnetic field. The induced charges on the bar are

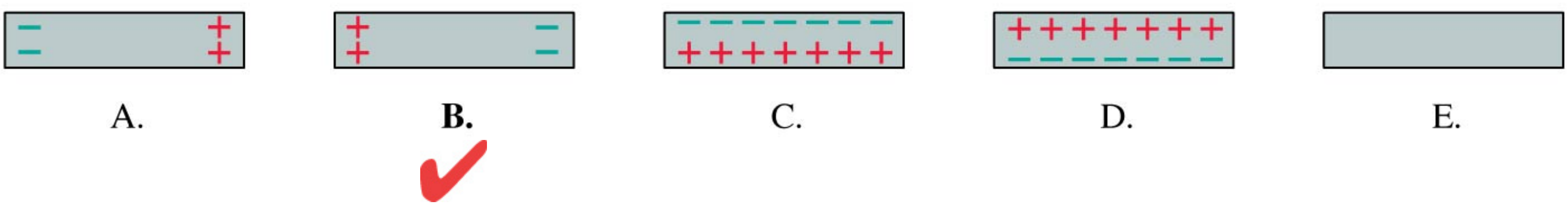




QuickCheck 33.2

A metal bar moves through a magnetic field. The induced charges on the bar are





Example 33.1 Measuring the Earth's Magnetic Field

EXAMPLE 33.1 Measuring the earth's magnetic field

It is known that the earth's magnetic field over northern Canada points straight down. The crew of a Boeing 747 aircraft flying at 260 m/s over northern Canada finds a 0.95 V potential difference between the wing tips. The wing span of a Boeing 747 is 65 m. What is the magnetic field strength there?

MODEL The wing is a conductor moving through a magnetic field, so there is a motional emf.

Example 33.1 Measuring the Earth's Magnetic Field

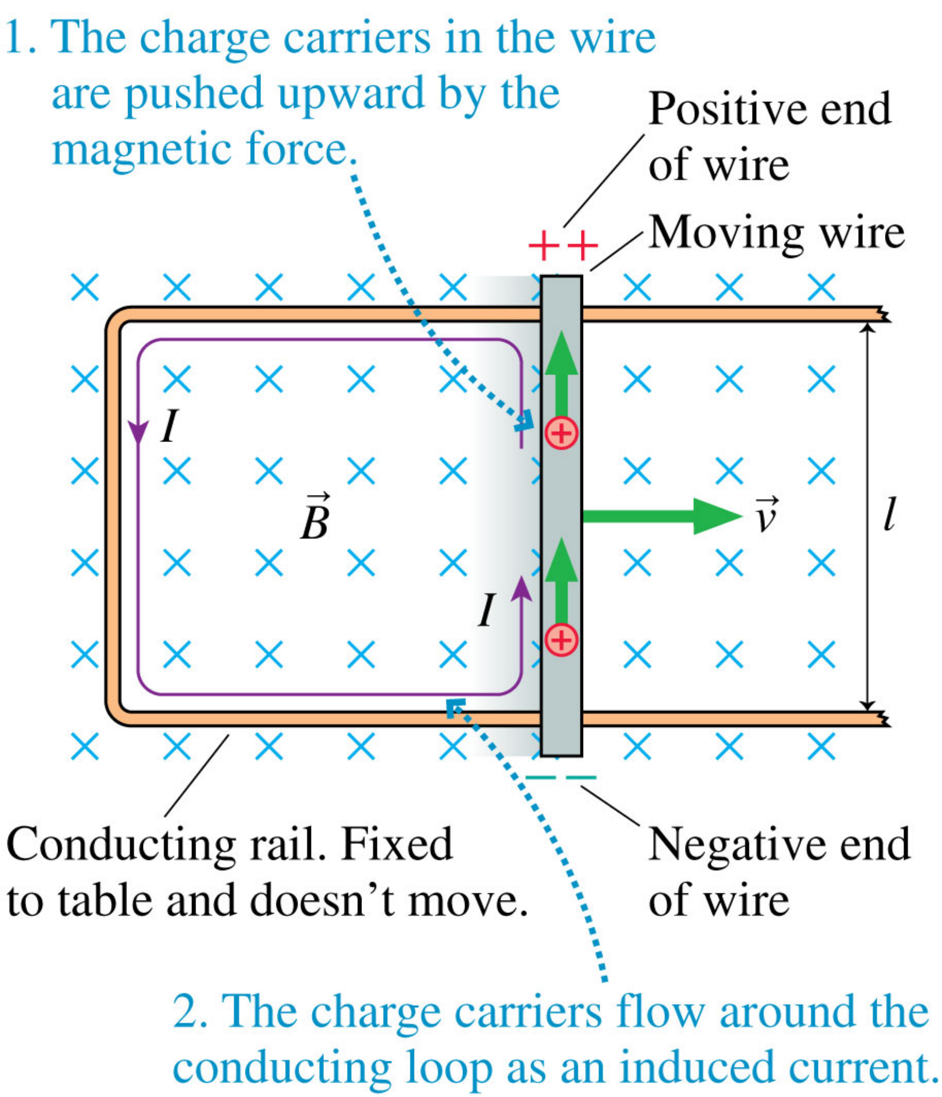
EXAMPLE 33.1 **Measuring the earth's magnetic field**

SOLVE The magnetic field is perpendicular to the velocity, so we can use Equation 33.3 to find

$$B = \frac{\mathcal{E}}{vL} = \frac{0.95\ \text{V}}{(260\ \text{m/s})(65\ \text{m})} = 5.6 \times 10^{-5}\ \text{T}$$

ASSESS Chapter 32 noted that the earth's magnetic field is roughly 5×10^{-5} T. The field is somewhat stronger than this near the magnetic poles, somewhat weaker near the equator.

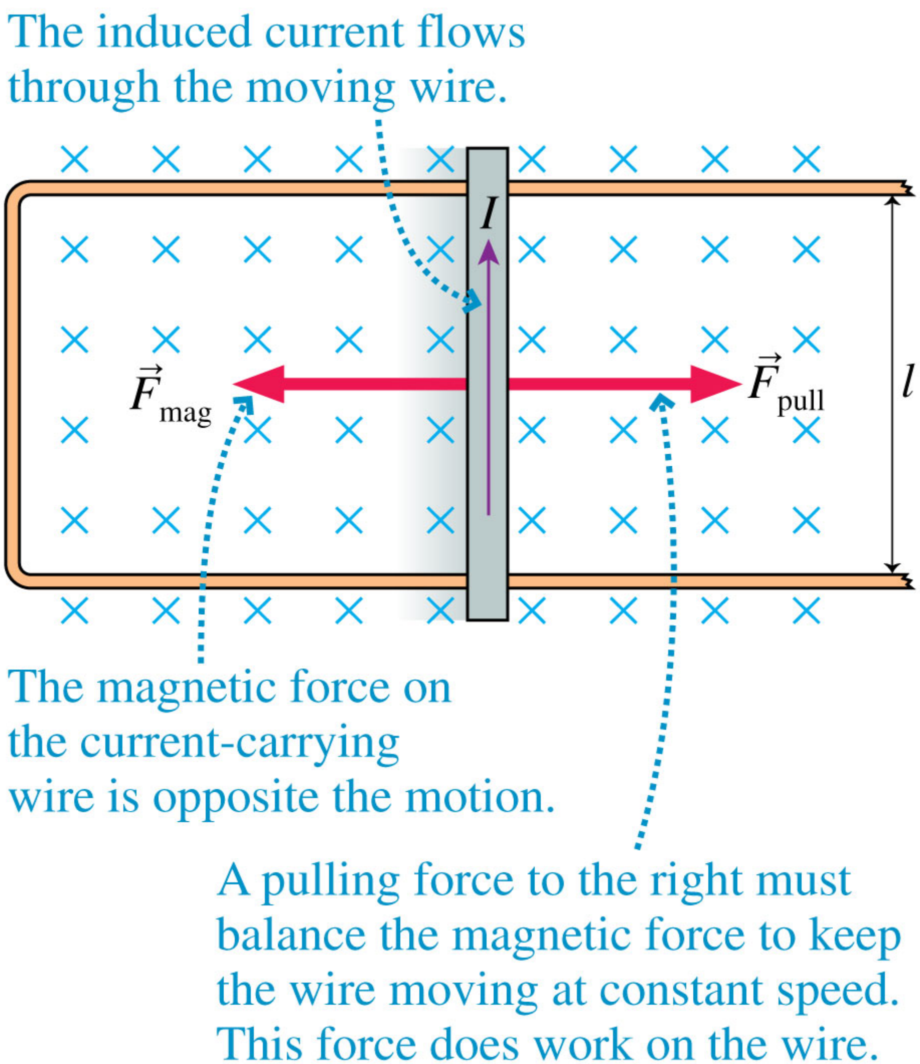
Induced Current



- If we slide a conducting wire along a U-shaped conducting rail, we can complete a circuit and drive an electric current.
- If the total resistance of the circuit is R, the *induced current* is given by Ohm's law as:

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R}$$

Induced Current

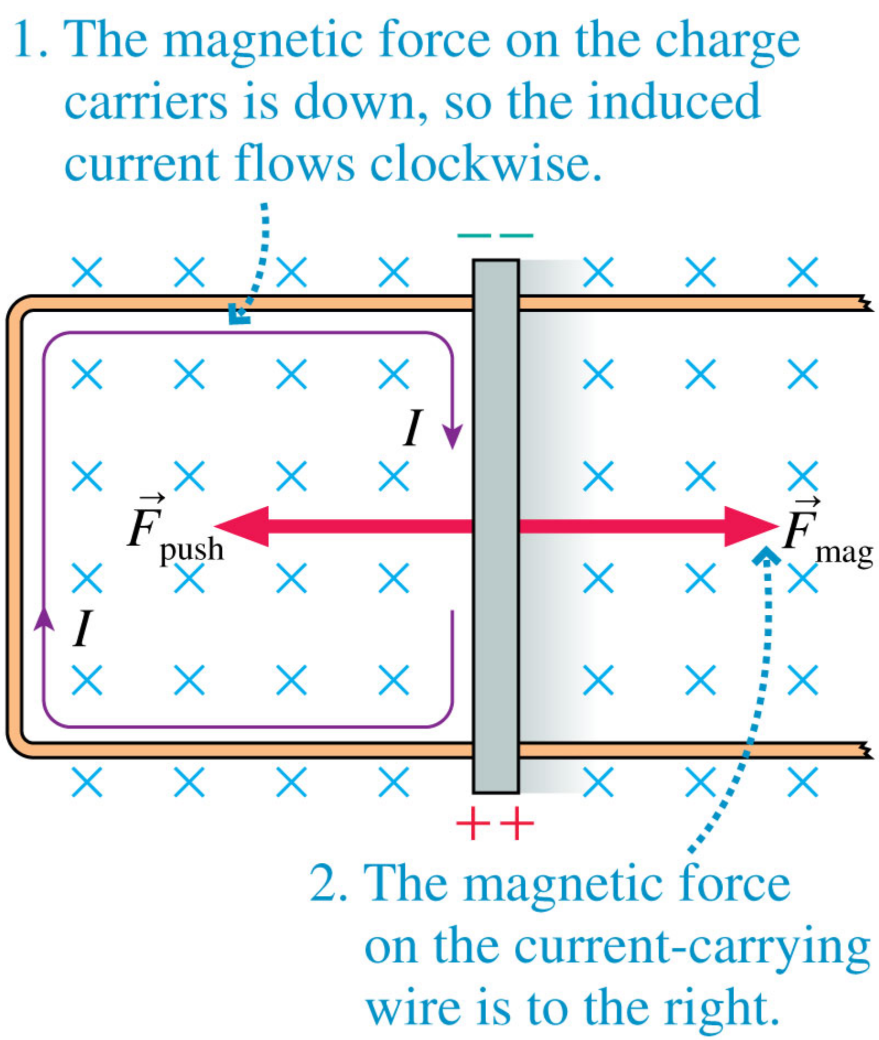


- To keep the wire moving at a constant speed v, we must apply a pulling force $F_{pull} = vl^2B^2/R$.
- This pulling force does work at a rate:

$$P_{input} = F_{pull}v = \frac{v^2l^2B^2}{R}$$

- All of this power is dissipated by the resistance of the circuit.

Induced Current

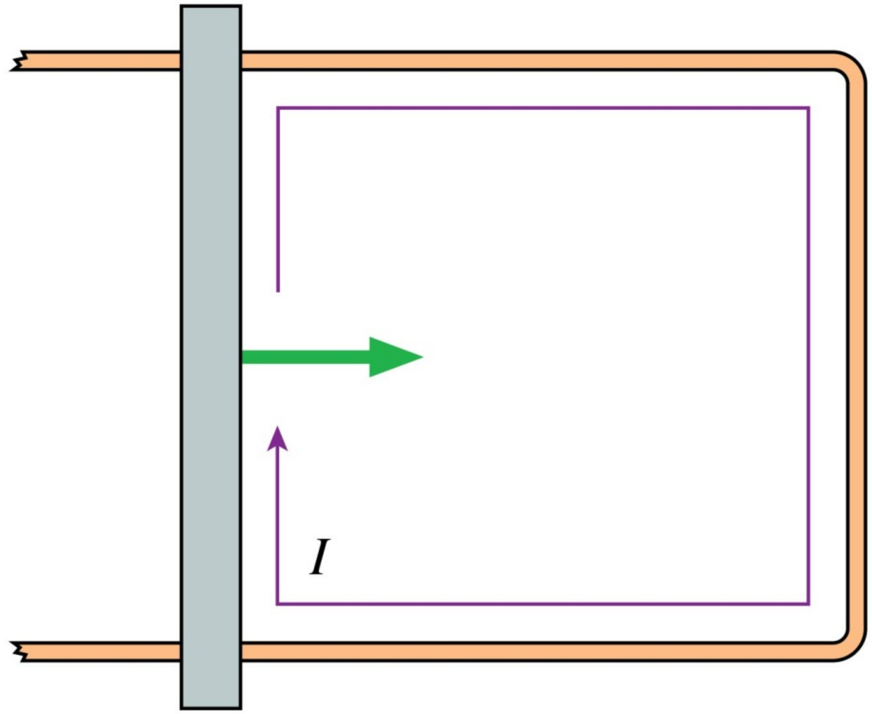


- The figure shows a conducting wire sliding to the *left*.
- In this case, a *pushing* force is needed to keep the wire moving at constant speed.
- Once again, this input power is dissipated in the electric circuit.
- A device that converts mechanical energy to electric energy is called a **generator**.

QuickCheck 33.3

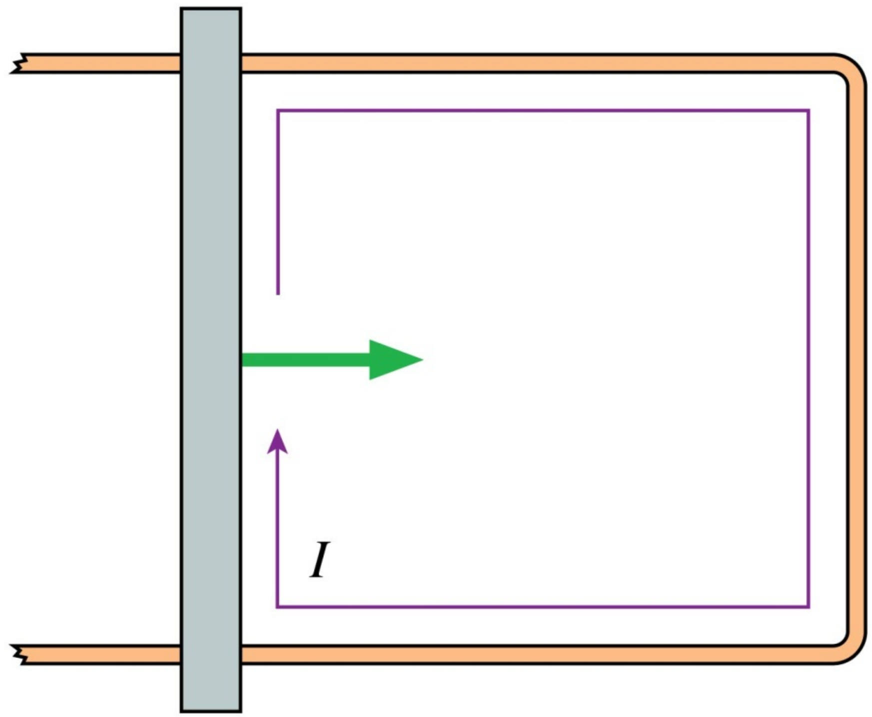
An induced current flows clockwise as the metal bar is pushed to the right. The magnetic field points

A. Up.
B. Down.
C. Into the screen.
D. Out of the screen.
E. To the right.



QuickCheck 33.3

An induced current flows clockwise as the metal bar is pushed to the right. The magnetic field points



A. Up.

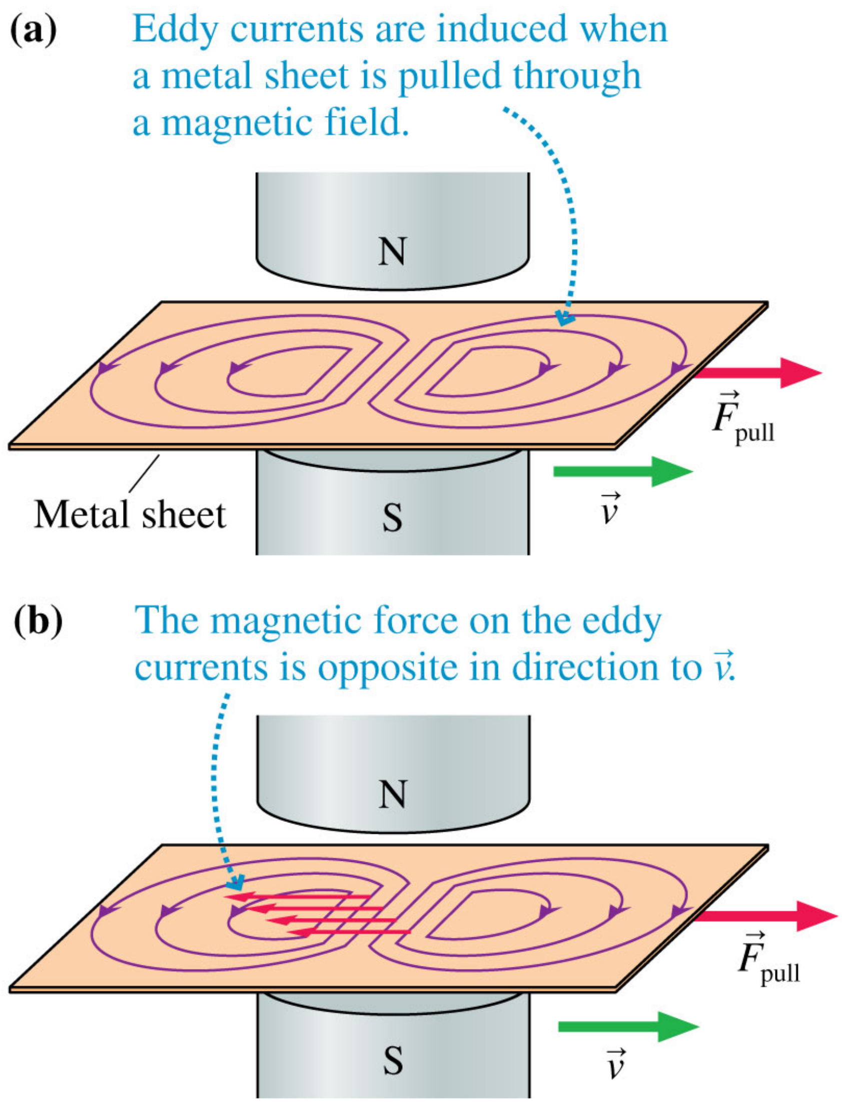
B. Down.

✔ **C. Into the screen.**

D. Out of the screen.

E. To the right.

Eddy Currents



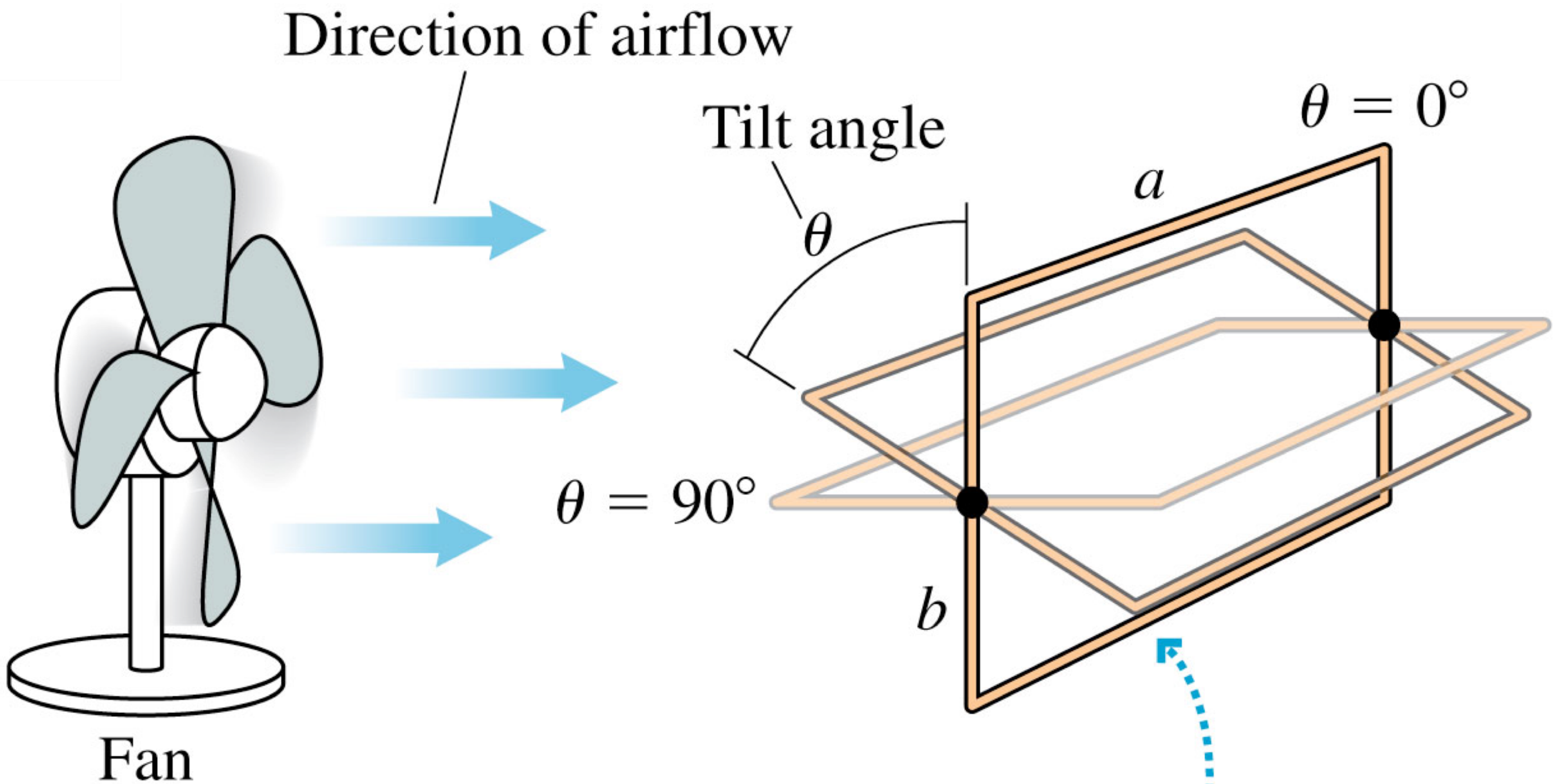
- Consider pulling a *sheet* of metal through a magnetic field.
- Two "whirlpools" of current begin to circulate in the solid metal, called **eddy currents.**
- The magnetic force on the eddy currents is a retarding force.
- This is a form of **magnetic braking.**

The Basic Definition of Flux

- Imagine holding a rectangular wire loop of area $A = ab$ in front of a fan.
- The volume of air flowing through the loop each second depends on the angle between the loop and the direction of flow.
- No air goes through the same loop if it lies parallel to the flow.
- The flow is *maximum* through a loop that is perpendicular to the airflow.
- This occurs because the effective area is greatest at this angle.
- The effective angle (as seen facing the fan) is:

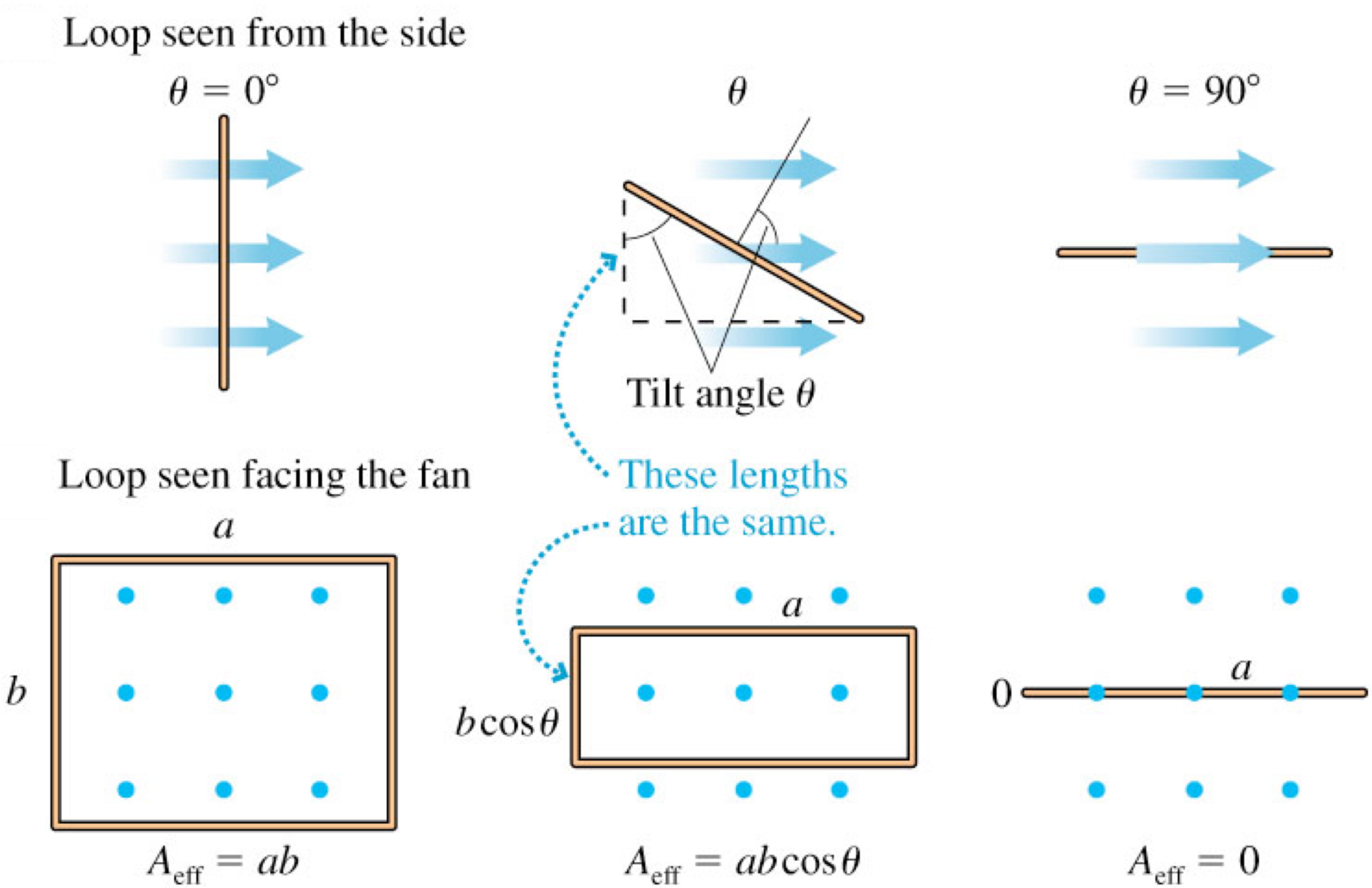
$$A_{\text{eff}} = ab\cos\theta = A\cos\theta$$

The Basic Definition of Flux

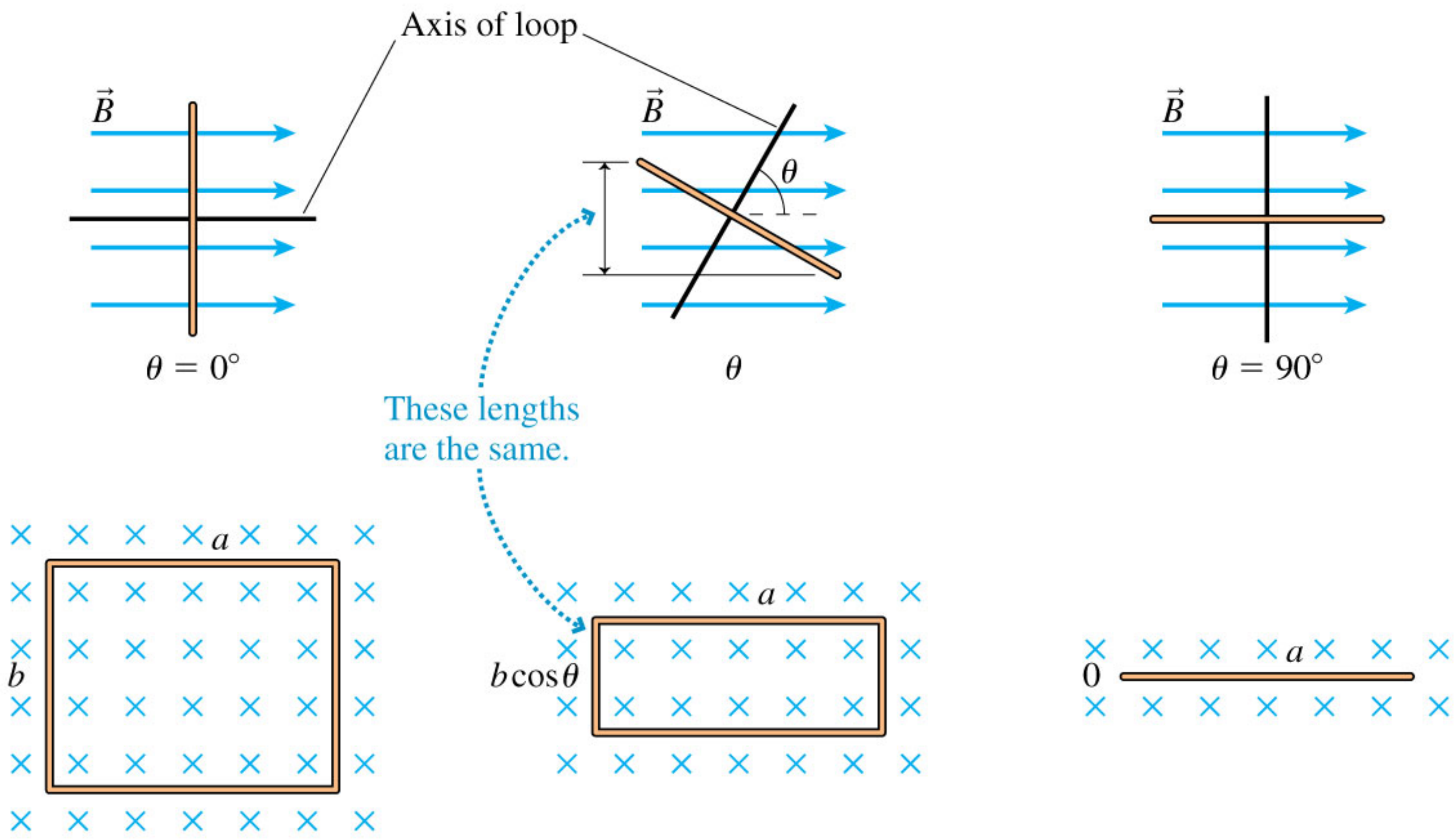


Imagine holding a rectangular loop of wire in front of a fan. Start with the loop face-on to the direction of airflow, then tilt the loop as shown until it is horizontal.

The Basic Definition of Flux



Magnetic Flux Through a Loop



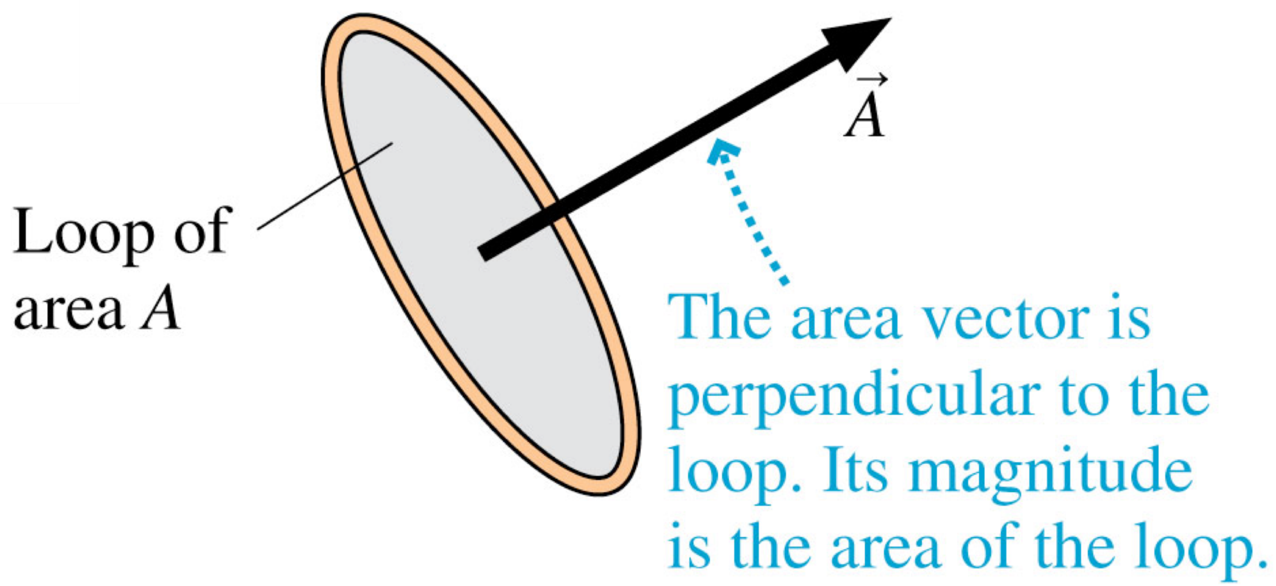
Loop perpendicular to field. Maximum number of arrows pass through.

Loop rotated through angle θ. Fewer arrows pass through.

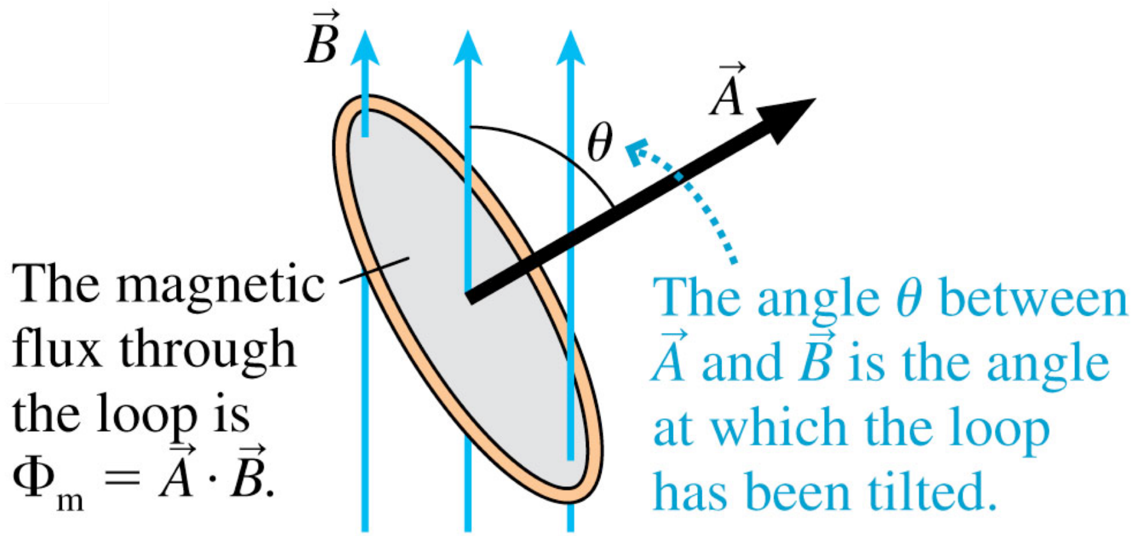
Loop rotated 90°. No arrows pass through.

The Area Vector

- Let's define an area vector $\vec{A} = A\hat{n}$ to be a vector in the direction of, perpendicular to the surface, with a magnitude A equal to the area of the surface.
- Vector $\vec{A}$ has units of m^2.



Magnetic Flux



The magnetic flux measures the amount of magnetic field passing through a loop of area A if the loop is tilted at an angle θ from the field.

$$\Phi_m = A_{eff}B = AB\cos\theta$$

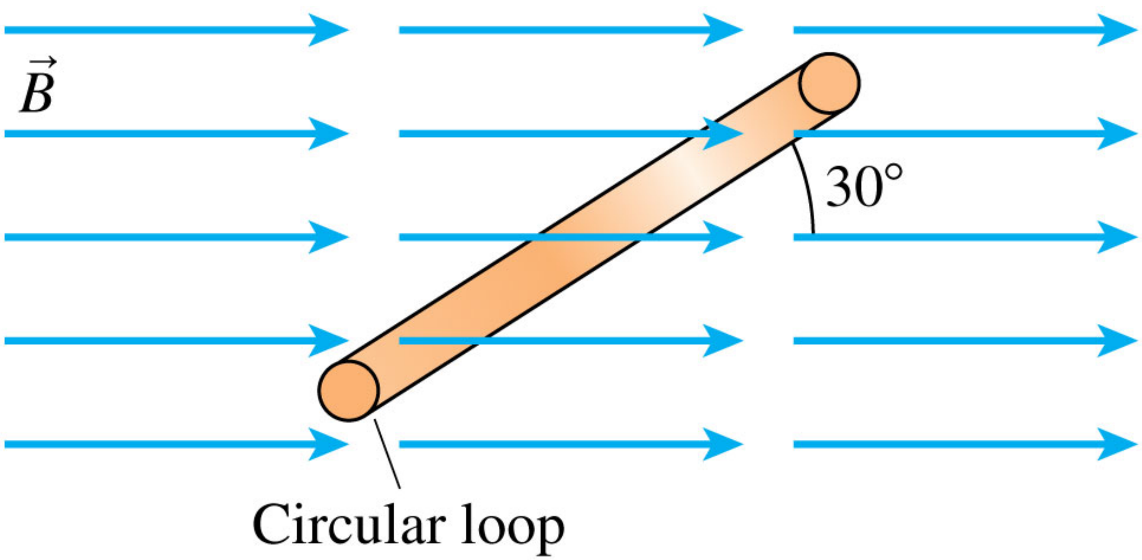
The SI unit of magnetic flux is the **weber**:

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T m}^2$$

Example 33.4 A Circular Loop in a Magnetic Field

EXAMPLE 33.4 A circular loop in a magnetic field

The figure below is an edge view of a 10-cm-diameter circular loop in a uniform 0.050 T magnetic field. What is the magnetic flux through the loop?

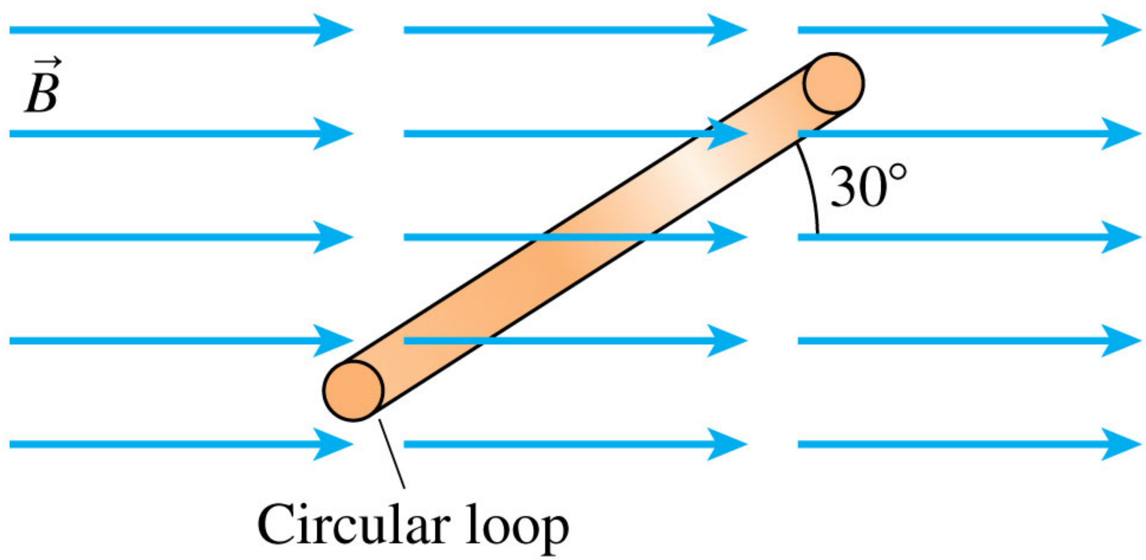


Example 33.4 A Circular Loop in a Magnetic Field

EXAMPLE 33.4 **A circular loop in a magnetic field**

SOLVE Angle θ is the angle between the loop's area vector $\vec{A}$, which is perpendicular to the plane of the loop, and the magnetic field $\vec{B}$. In this case, $\theta = 60°$, not the 30° angle shown in the figure. Vector $\vec{A}$ has magnitude $A = \pi r^2 = 7.85 \times 10^{-3}\ \text{m}^2$. Thus the magnetic flux is

$$\Phi_m = \vec{A} \cdot \vec{B} = AB\cos\theta = 2.0 \times 10^{-4}\ \text{Wb}$$



QuickCheck 33.4

Which loop has the larger magnetic flux through it?

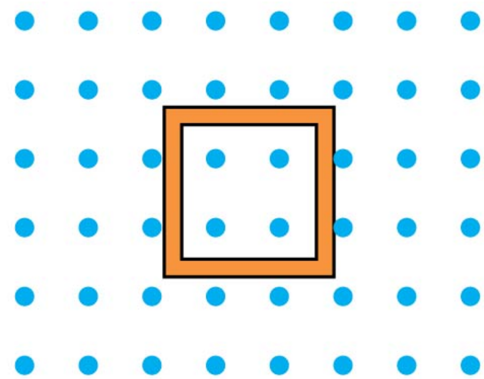
A. Loop A.

B. Loop B.

C. The fluxes are the same.

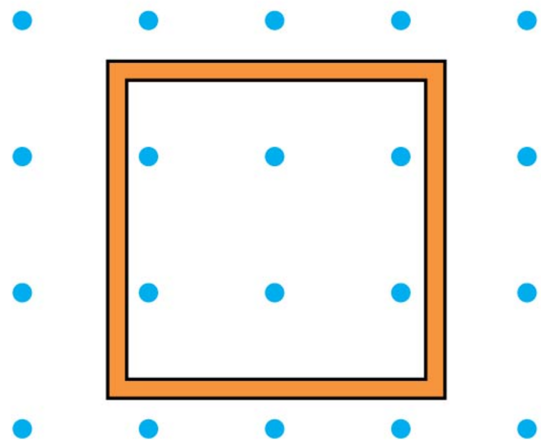
D. Not enough information to tell.

A.



This field is twice as strong.

B.



This square is twice as wide.

QuickCheck 33.4

Which loop has the larger magnetic flux through it?

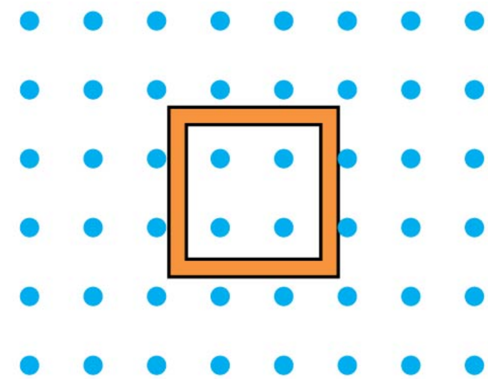
A. Loop A.

B. Loop B. ✔ $\Phi_m = L^2 B$

C. The fluxes are the same.

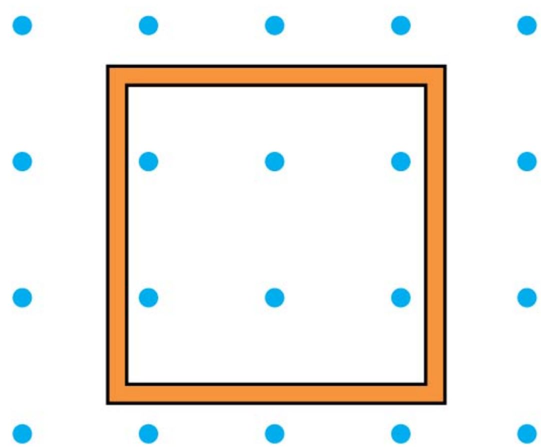
D. Not enough information to tell.

A.



This field is twice as strong.

B.



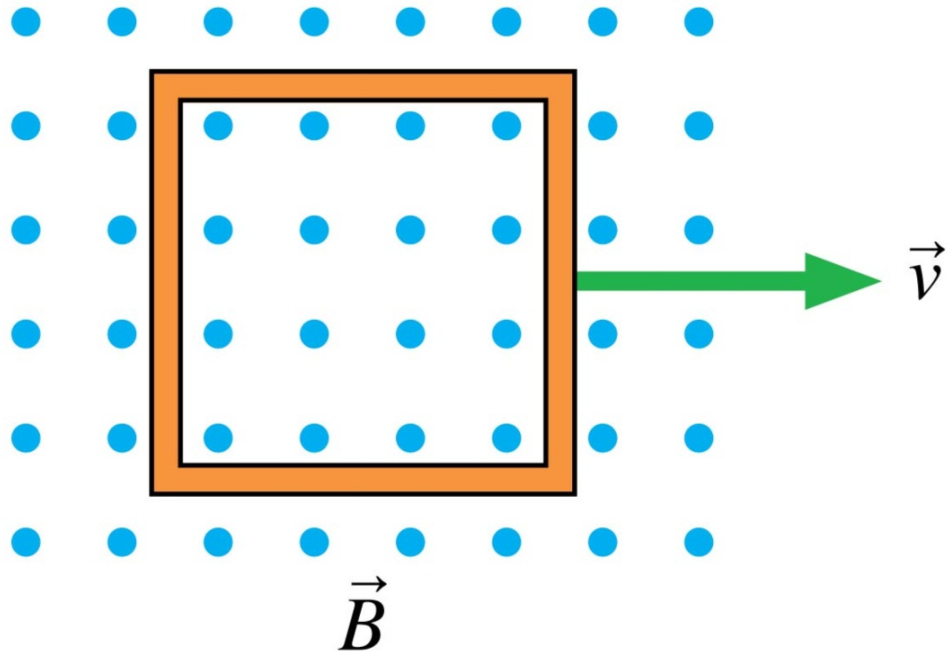
This square is twice as wide.

QuickCheck 33.5

The metal loop is being pulled through a uniform magnetic field. Is the magnetic flux through the loop changing?

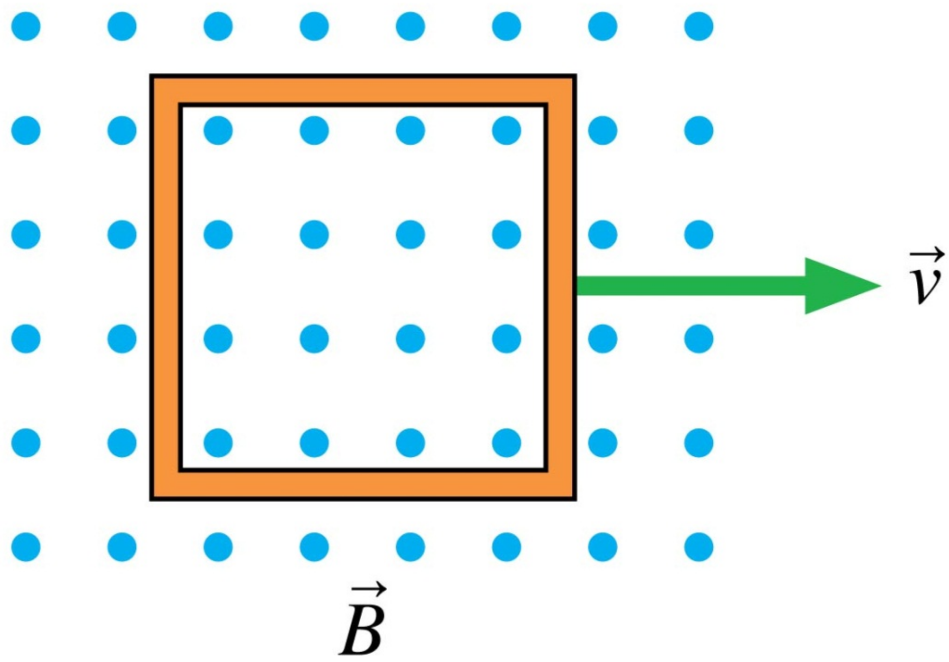
A. Yes.

B. No.



QuickCheck 33.5

The metal loop is being pulled through a uniform magnetic field. Is the magnetic flux through the loop changing?

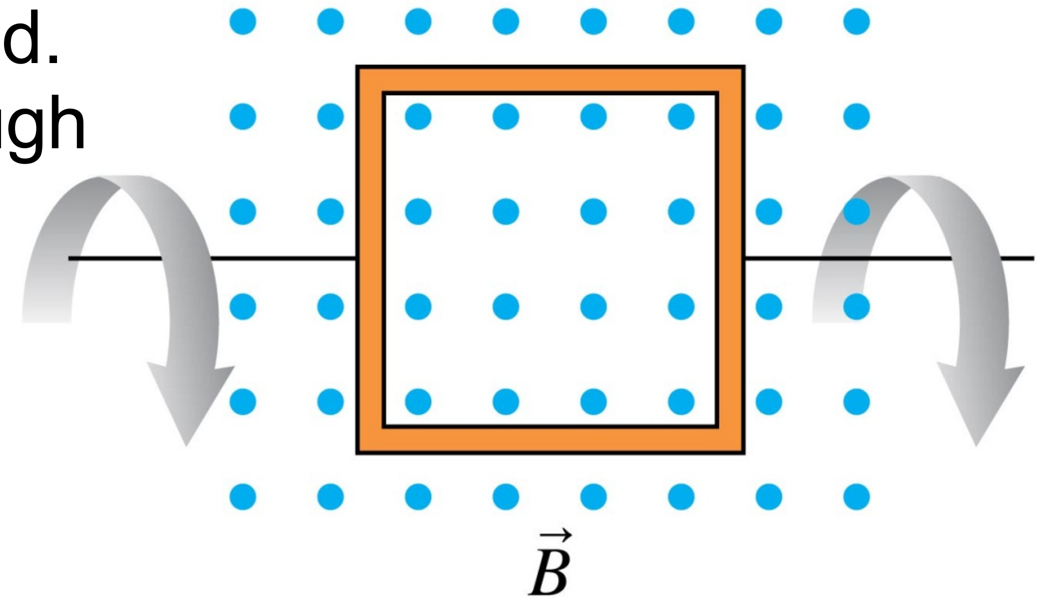


A. Yes.

✔ **B. No.**

QuickCheck 33.6

The metal loop is rotating in a uniform magnetic field. Is the magnetic flux through the loop changing?

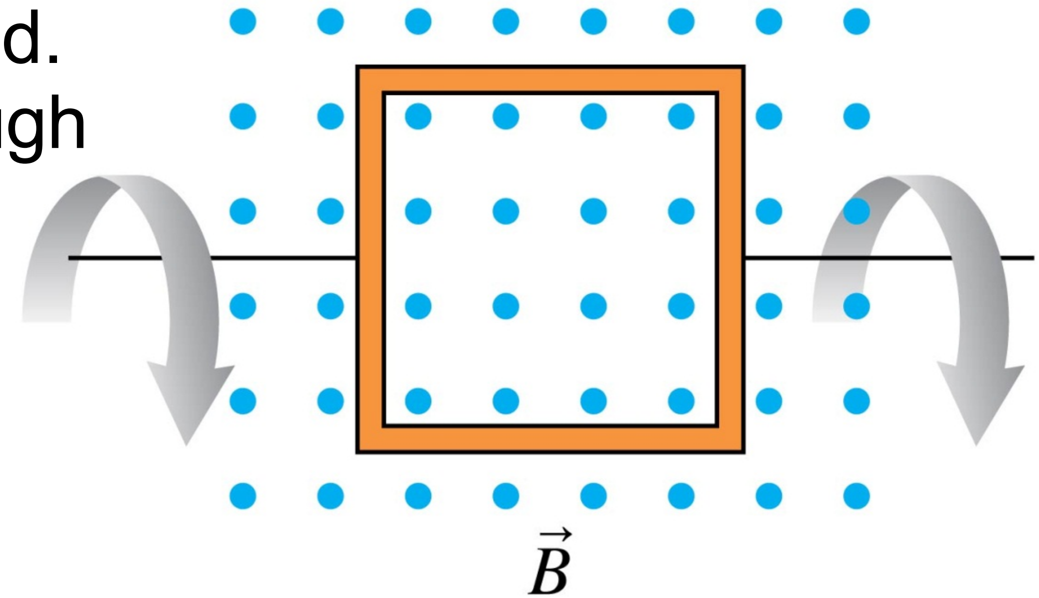


A. Yes.

B. No.

QuickCheck 33.6

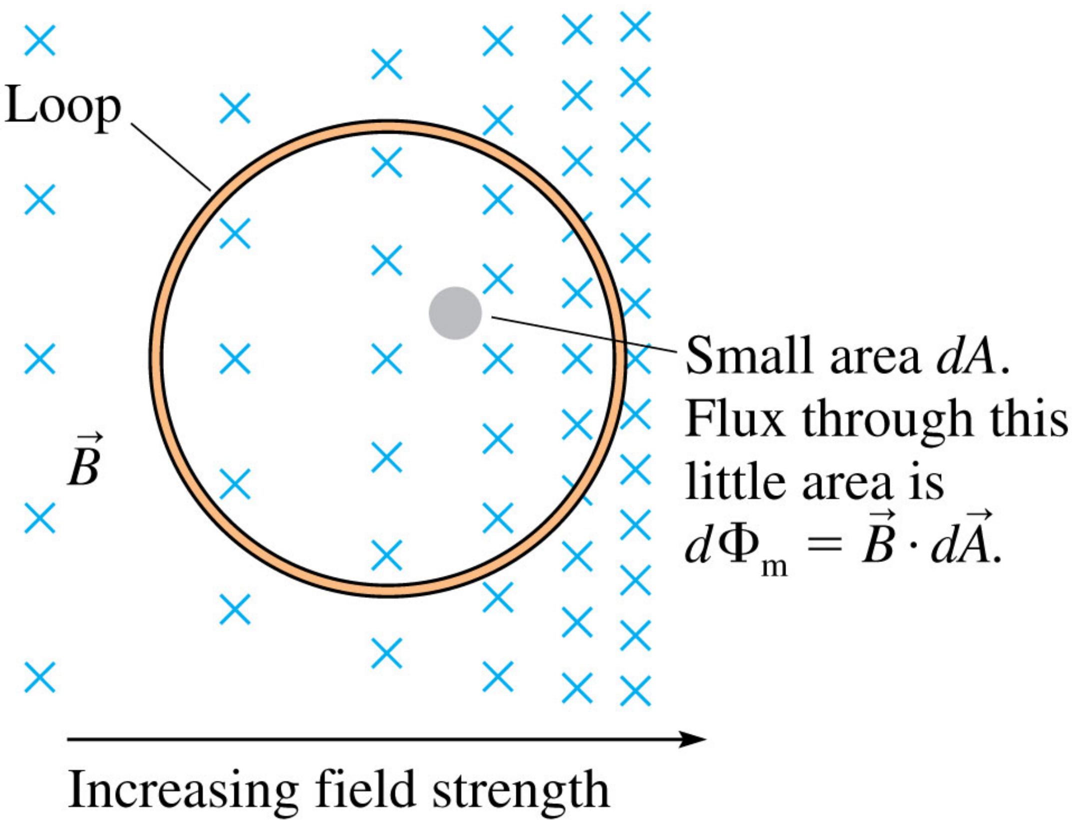
The metal loop is rotating in a uniform magnetic field. Is the magnetic flux through the loop changing?



✔ **A. Yes.**

B. No.

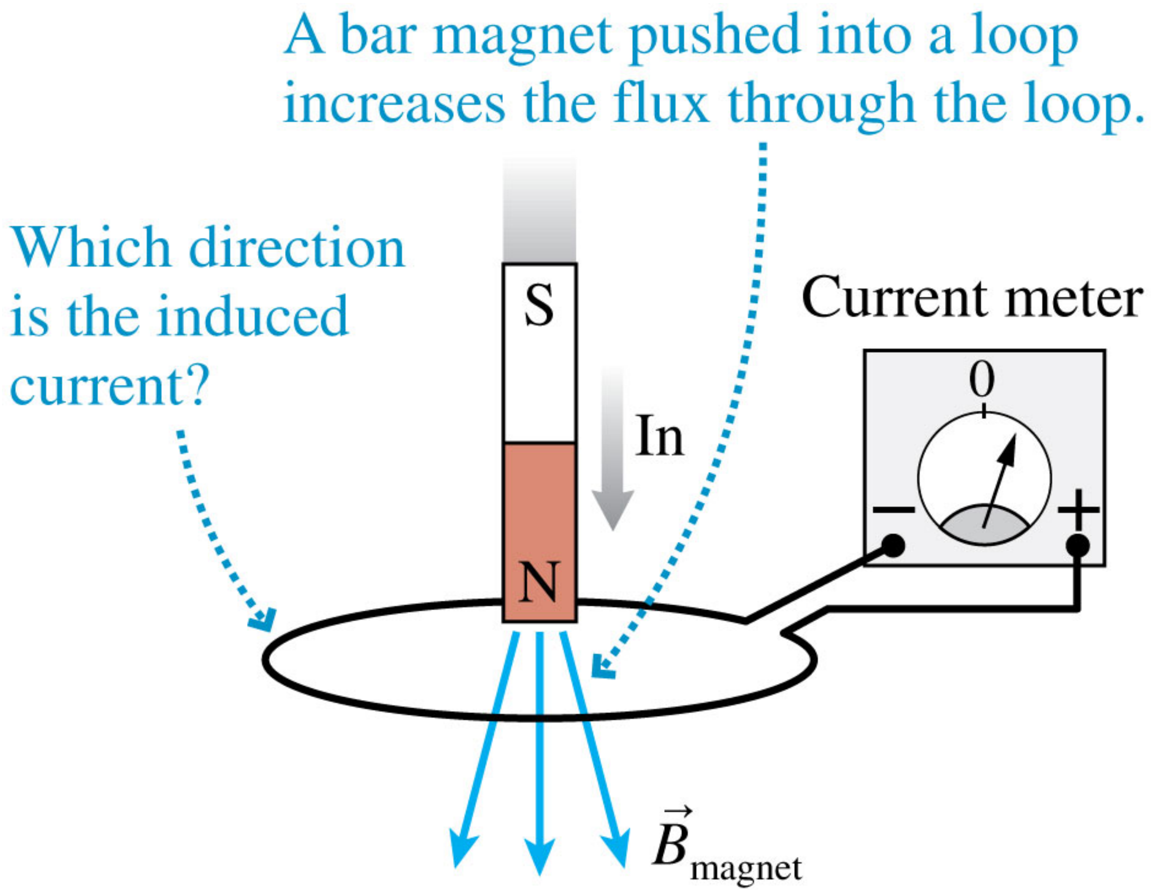
Magnetic Flux in a Nonuniform Field



- The figure shows a loop in a nonuniform magnetic field.
- The total magnetic flux through the loop is found with an *area integral.*

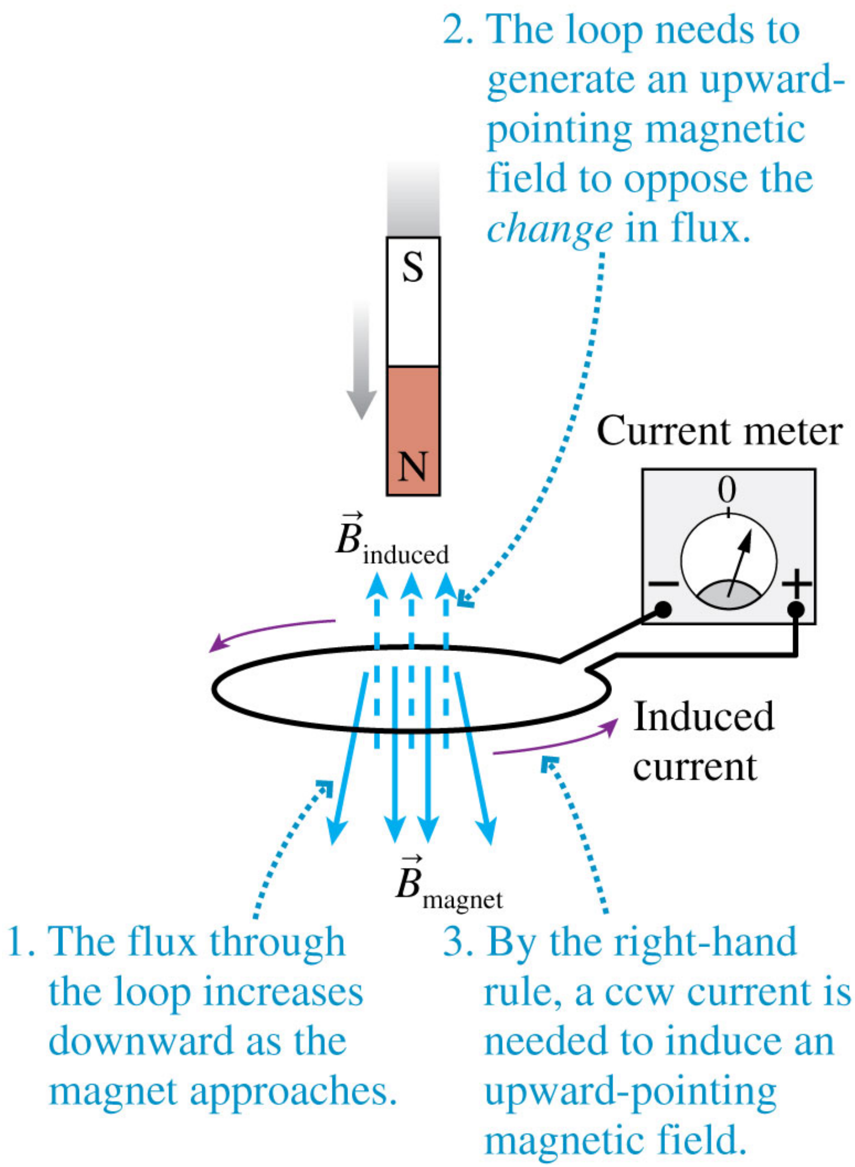
$$\Phi_m = \int_{\text{area of loop}} \vec{B} \cdot d\vec{A}$$

Lenz's Law



Lenz's law There is an induced current in a closed, conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.

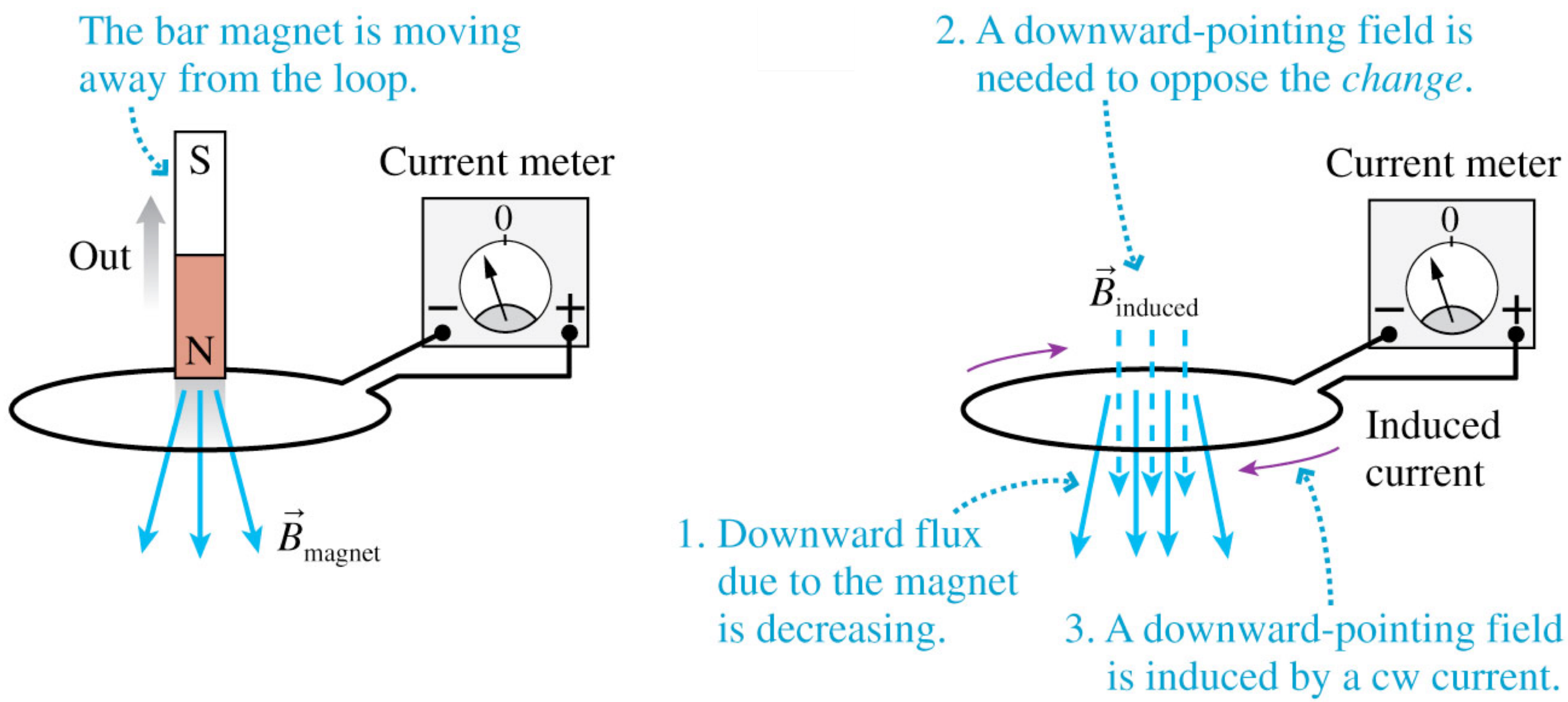
Lenz's Law



- Pushing the bar magnet into the loop causes the magnetic flux to *increase* in the downward direction.
- To oppose the *change* in flux, which is what Lenz's law requires, the loop itself needs to generate an *upward*-pointing magnetic field.
- The induced current ceases as soon as the magnet stops moving.

Lenz's Law

- Pushing the bar magnet away from the loop causes the magnetic flux to *decrease* in the downward direction.
- To *oppose this decrease*, a clockwise current is induced.



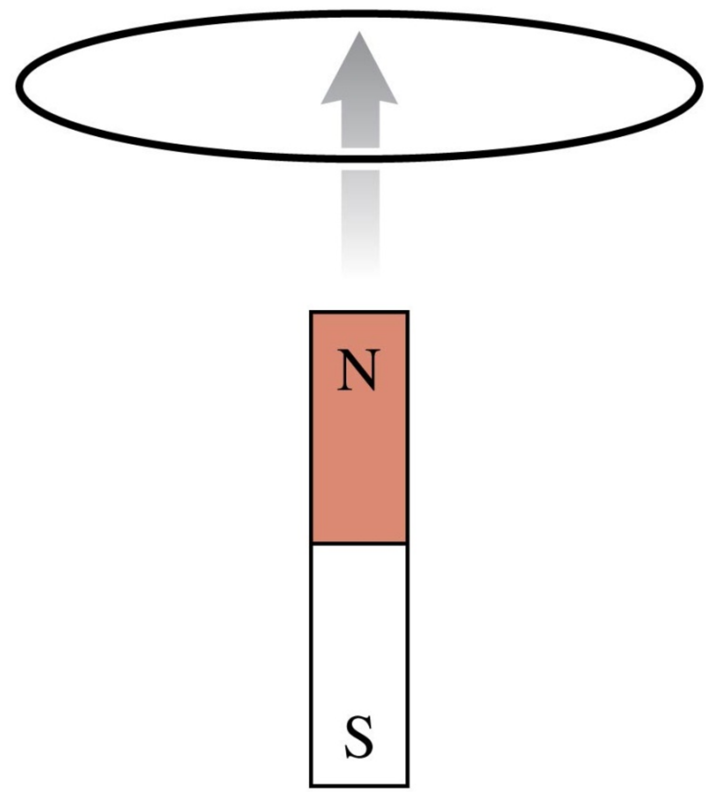
QuickCheck 33.7

The bar magnet is pushed toward the center of a wire loop. Which is true?

A. There is a clockwise induced current in the loop.

B. There is a counterclockwise induced current in the loop.

C. There is no induced current in the loop.



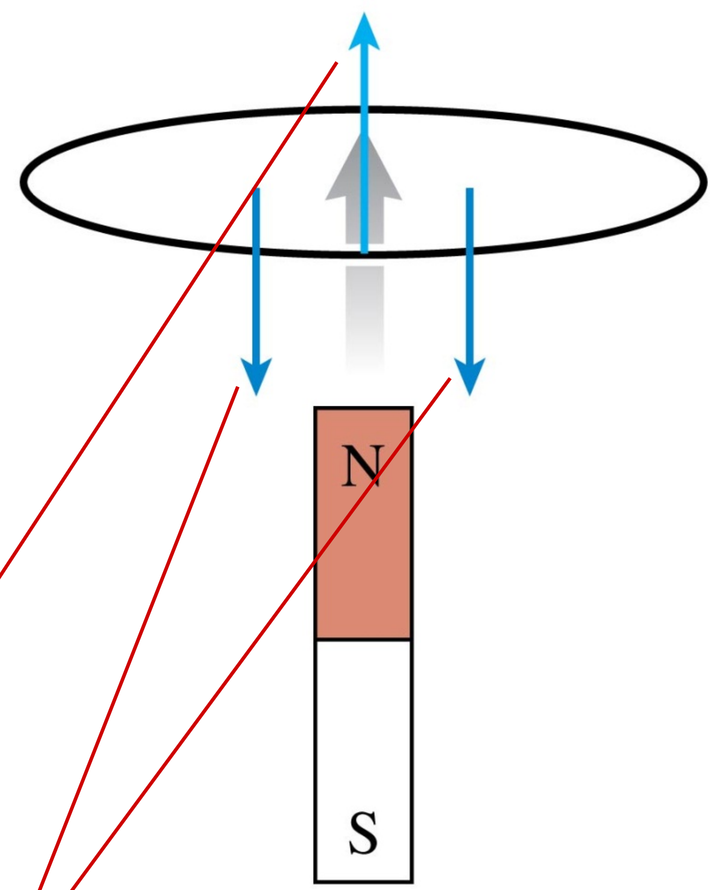
QuickCheck 33.7

The bar magnet is pushed toward the center of a wire loop. Which is true?

A. There is a clockwise induced current in the loop.

B. There is a counterclockwise induced current in the loop.

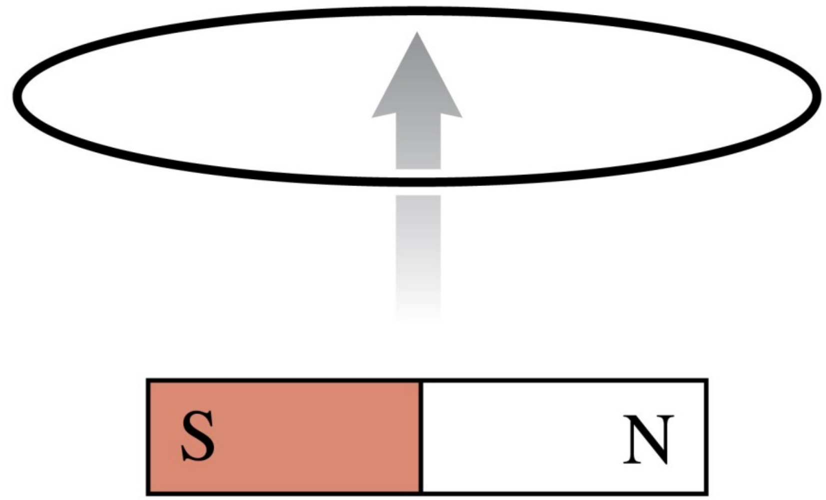
C. There is no induced current in the loop.



1. Upward flux from magnet is increasing.
2. To oppose the increase, the field of the induced current points down.
3. From the right-hand rule, a downward field needs a cw current.

QuickCheck 33.8

The bar magnet is pushed toward the center of a wire loop. Which is true?



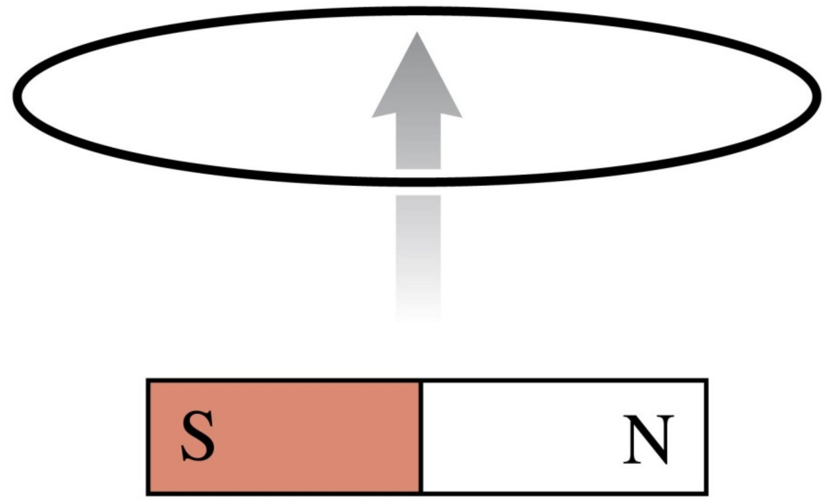
A. There is a clockwise induced current in the loop.

B. There is a counterclockwise induced current in the loop.

C. There is no induced current in the loop.

QuickCheck 33.8

The bar magnet is pushed toward the center of a wire loop. Which is true?



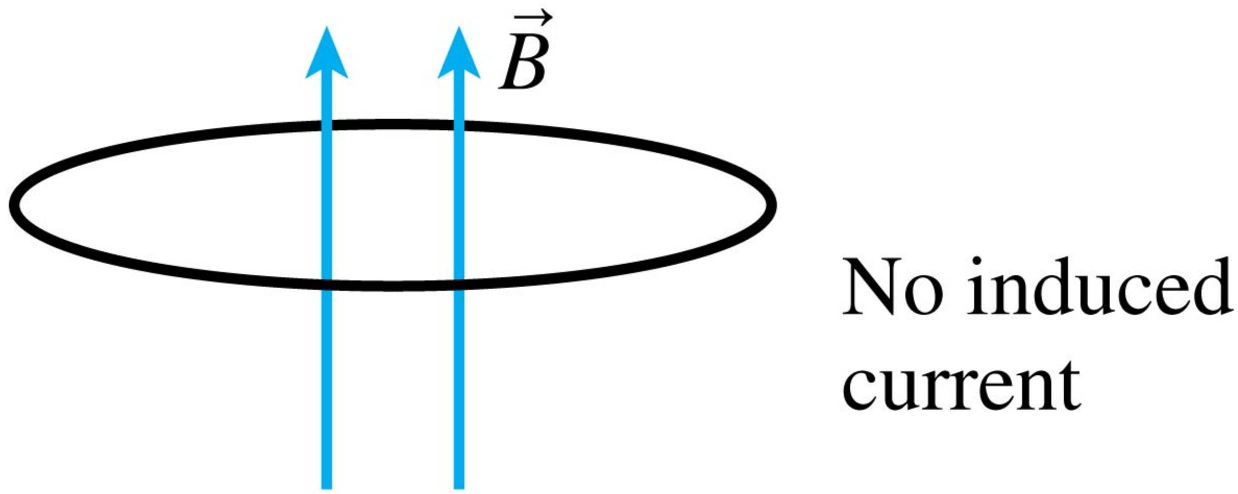
A. There is a clockwise induced current in the loop.

B. There is a counterclockwise induced current in the loop.

✔ **C. There is no induced current in the loop.**

Magnetic flux is zero, so there's no change of flux.

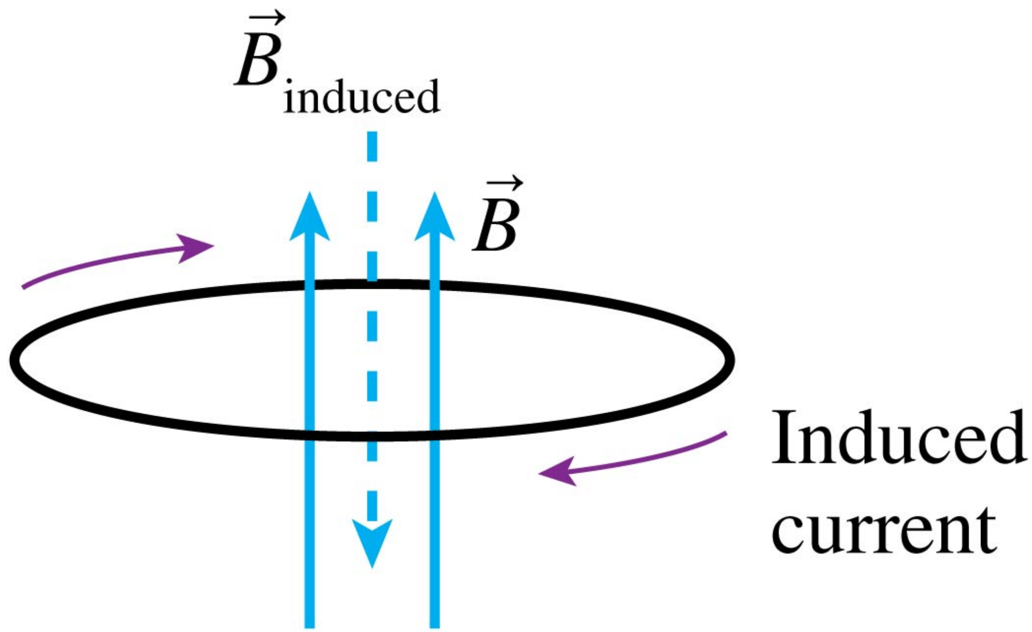
The Induced Current for Six Different Situations: Slide 1



$\vec{B}$ up and steady

- No change in flux
- No induced field
- No induced current

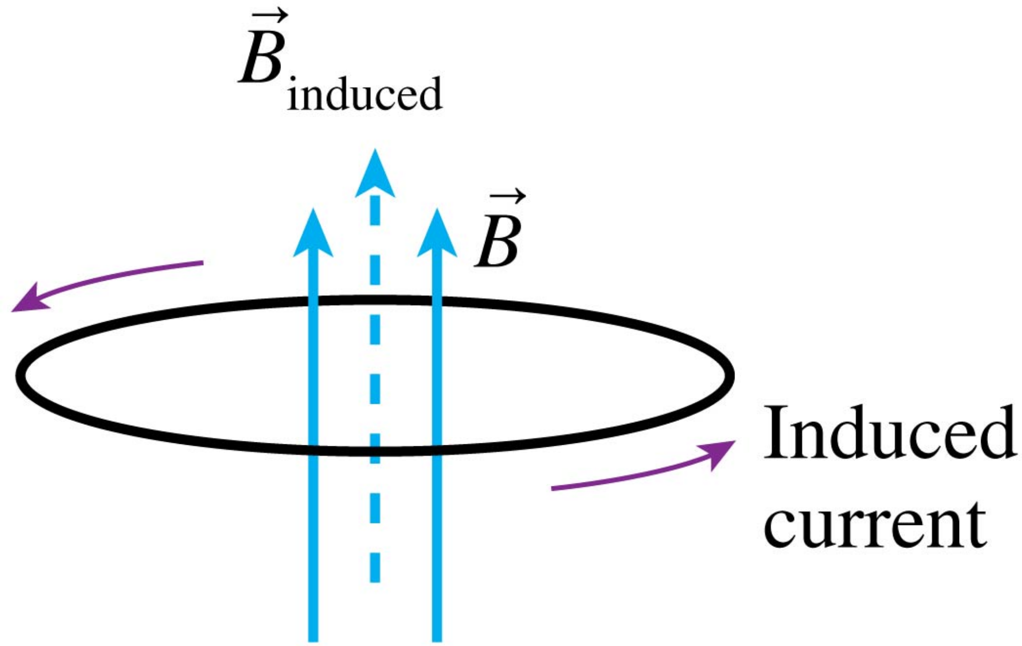
The Induced Current for Six Different Situations: Slide 2



$\vec{B}$ up and increasing

- Change in flux ↑
- Induced field ↓
- Induced current cw

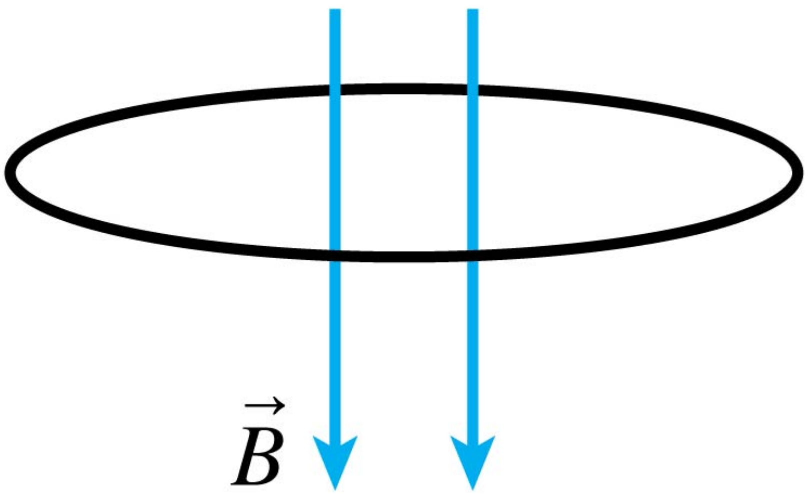
The Induced Current for Six Different Situations: Slide 3



$\vec{B}$ up and decreasing

- Change in flux ↓
- Induced field ↑
- Induced current ccw

The Induced Current for Six Different Situations: Slide 4

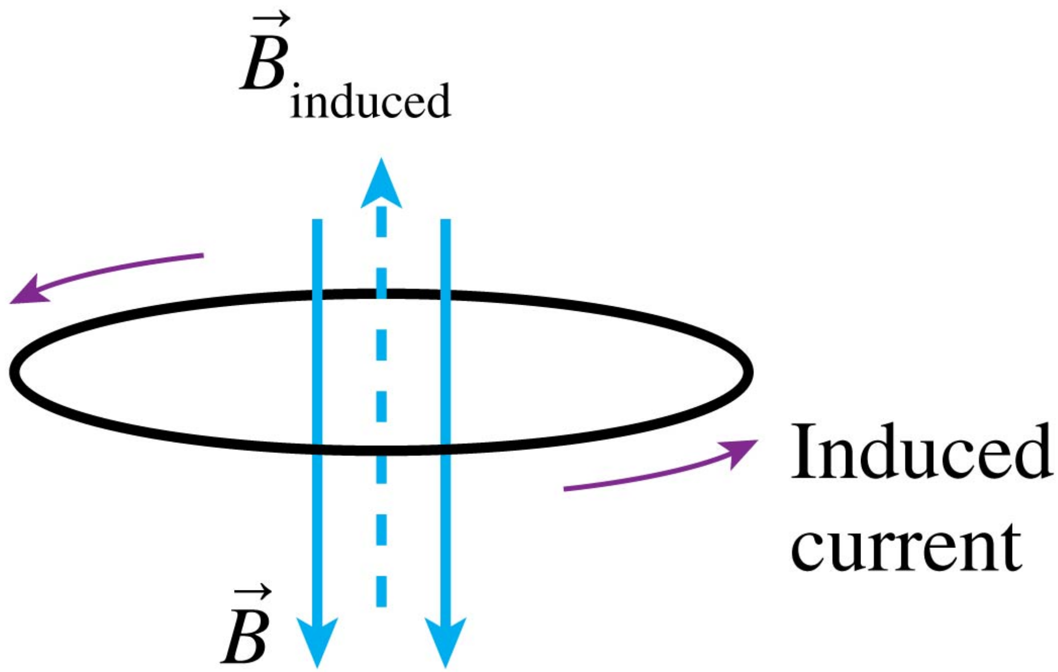


$\vec{B}$ down and steady

- No change in flux
- No induced field

No induced current

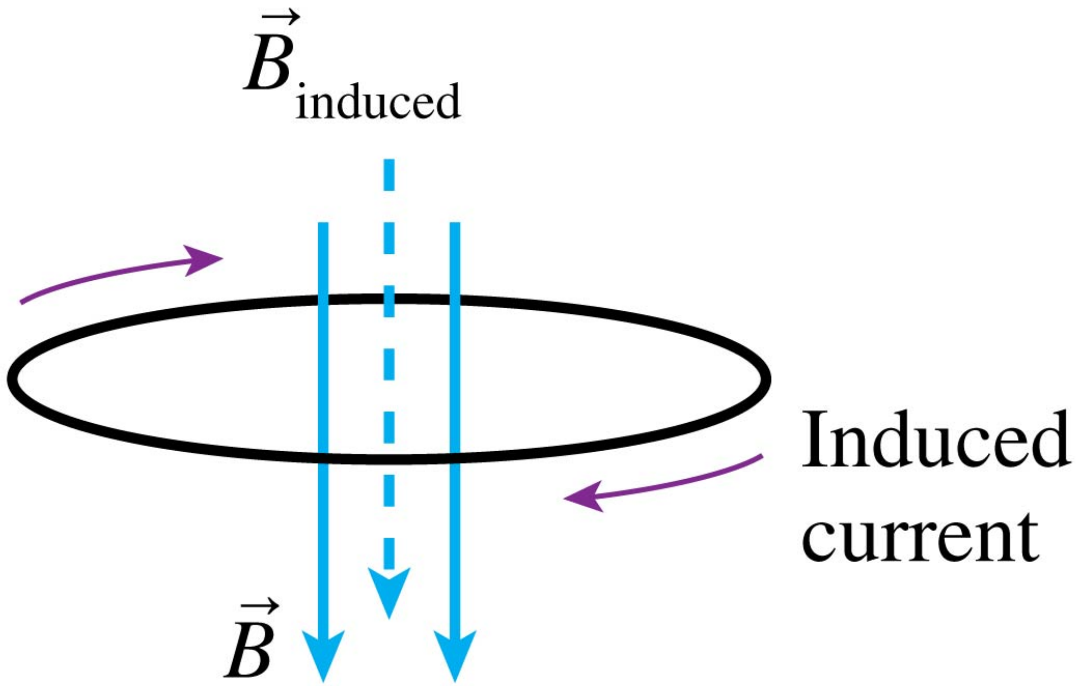
The Induced Current for Six Different Situations: Slide 5



$\vec{B}$ down and increasing

- Change in flux ↓
- Induced field ↑
- Induced current ccw

The Induced Current for Six Different Situations: Slide 6



$\vec{B}$ down and decreasing

- Change in flux ↑
- Induced field ↓
- Induced current cw

Tactics: Using Lenz's Law

TACTICS BOX 33.1 **Using Lenz's law**

❶ **Determine the direction of the applied magnetic field.** The field must pass through the loop.

❷ **Determine how the flux is changing.** Is it increasing, decreasing, or staying the same?

❸ **Determine the direction of an induced magnetic field that will oppose the *change* in the flux.**

- Increasing flux: the induced magnetic field points opposite the applied magnetic field.
- Decreasing flux: the induced magnetic field points in the same direction as the applied magnetic field.
- Steady flux: there is no induced magnetic field.

❹ **Determine the direction of the induced current.** Use the right-hand rule to determine the current direction in the loop that generates the induced magnetic field you found in step 3.

Exercises 10–14

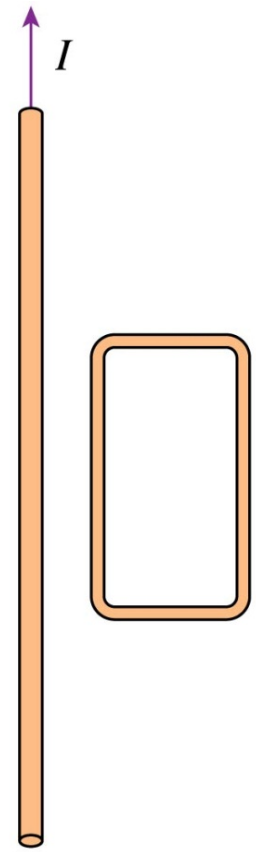
QuickCheck 33.9

The current in the straight wire is decreasing. Which is true?

A. There is a clockwise induced current in the loop.

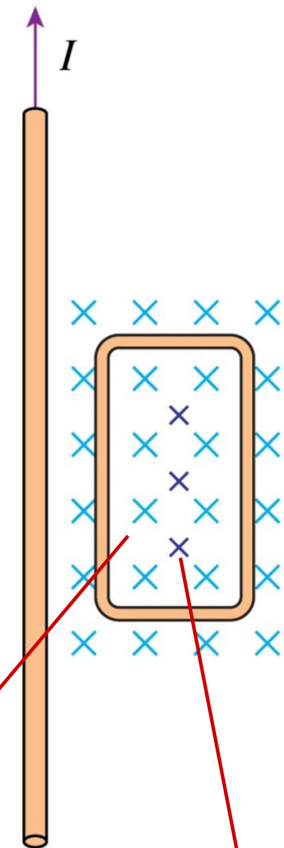
B. There is a counterclockwise induced current in the loop.

C. There is no induced current in the loop.



QuickCheck 33.9

The current in the straight wire is decreasing. Which is true?



✔ **A. There is a clockwise induced current in the loop.**

B. There is a counterclockwise induced current in the loop.

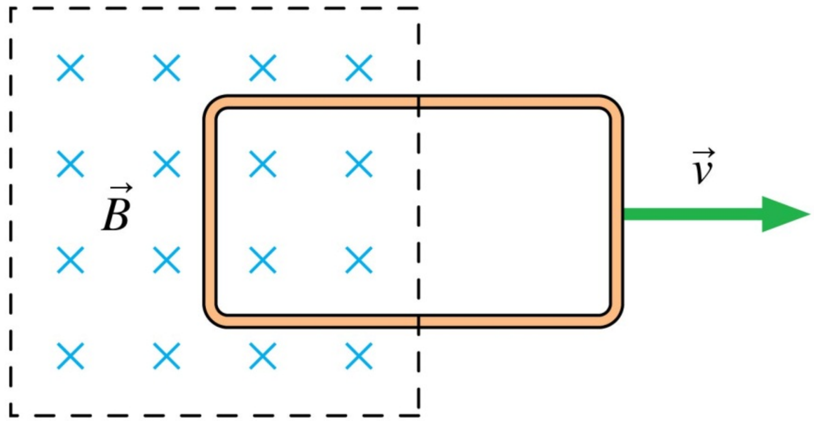
C. There is no induced current in the loop.

1. The flux from wire's field is <u>into</u> the screen and <u>decreasing</u>.
2. To oppose the <u>decrease</u>, the field of the induced current must point <u>into</u> the screen.
3. From the right-hand rule, an inward field needs a cw current.

QuickCheck 33.10

The magnetic field is confined to the region inside the dashed lines; it is zero outside. The metal loop is being pulled out of the magnetic field. Which is true?

A. There is a clockwise induced current in the loop.
B. There is a counterclockwise induced current in the loop.
C. There is no induced current in the loop.



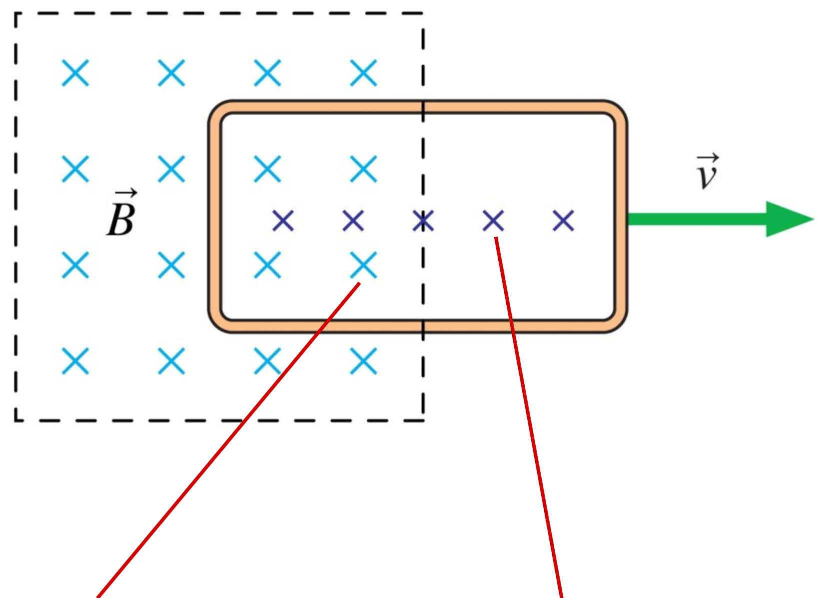
QuickCheck 33.10

The magnetic field is confined to the region inside the dashed lines; it is zero outside. The metal loop is being pulled out of the magnetic field. Which is true?

✔ **A. There is a clockwise induced current in the loop.**

B. There is a counterclockwise induced current in the loop.

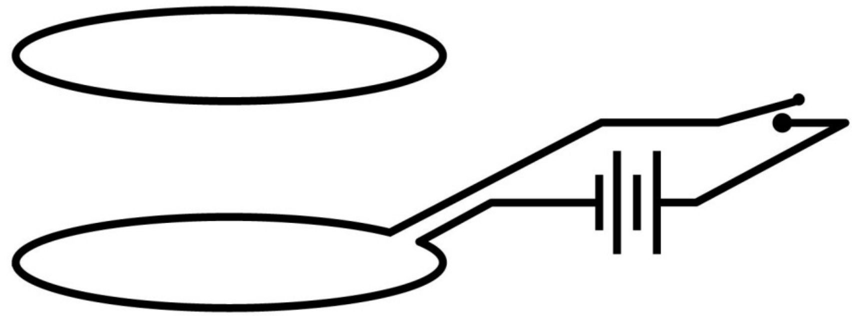
C. There is no induced current in the loop.



1. The flux through the loop is <u>into</u> the screen and <u>decreasing</u>.
2. To oppose the <u>decrease</u>, the field of the induced current must point <u>into</u> the screen.
3. From the right-hand rule, an inward field needs a cw current.

QuickCheck 33.11

Immediately after the switch is closed, the lower loop exerts _____ on the upper loop.



A. a torque

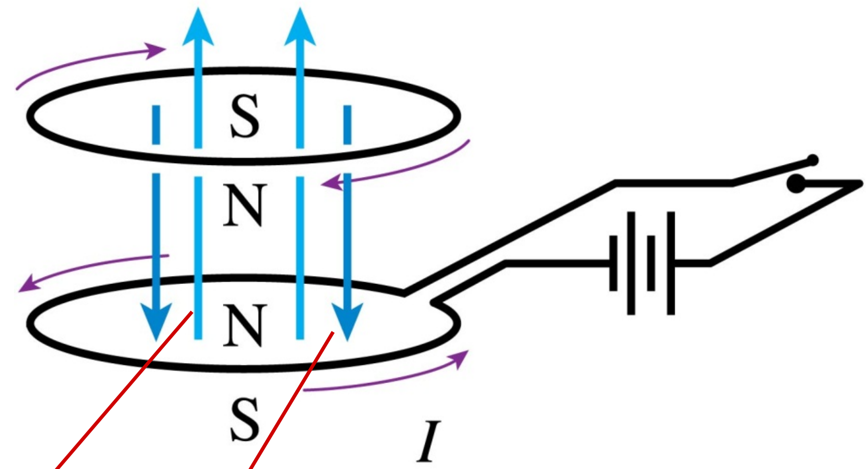
B. an upward force

C. a downward force

D. no force or torque

QuickCheck 33.11

Immediately after the switch is closed, the lower loop exerts _____ on the upper loop.



A. a torque

✔ **B. an upward force**

C. a downward force

D. no force or torque

1. The battery drives a ccw current that, briefly, increases rapidly.
2. The flux through the top loop is upward and increasing.
3. To oppose the increase, the field of the induced current must point downward.
4. From the right-hand rule, a downward field needs a cw current.
5. The ccw current in the lower loop makes the upper face a north pole. The cw induced current in the upper loop makes the lower face a north pole.
6. Facing north poles exert repulsive forces on each other.

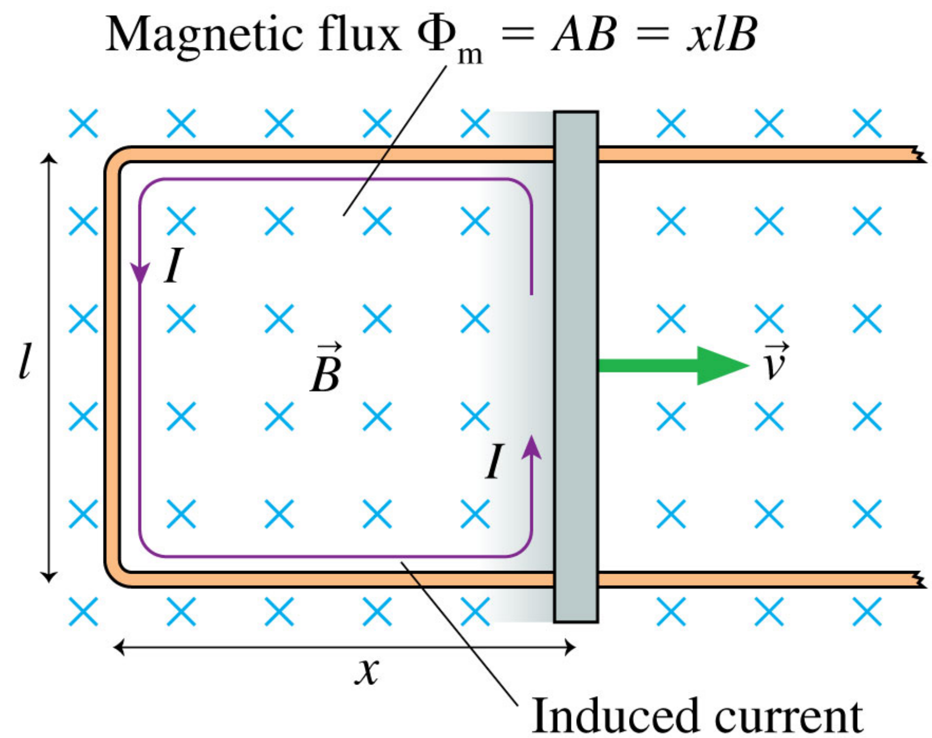
Faraday's Law

- An emf is induced in a conducting loop if the magnetic flux through the loop changes.
- The magnitude of the emf is:

$$\mathcal{E} = \left| \frac{d\Phi_{\text{m}}}{dt} \right|$$

- The direction of the emf is such as to drive an induced current in the direction given by Lenz's law.

Using Faraday's Law



- If we slide a conducting wire along a U-shaped conducting rail, we can complete a circuit and drive an electric current.
- We can find the induced emf and current by using Faraday's law and Ohm's law:

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| = \frac{d}{dt}(xlB) = \frac{dx}{dt}lB = vlB \qquad I = \frac{\mathcal{E}}{R} = \frac{vlB}{R}$$

Problem-Solving Strategy: Electromagnetic Induction

PROBLEM-SOLVING STRATEGY 33.1 **Electromagnetic induction**

MODEL Make simplifying assumptions about wires and magnetic fields.

VISUALIZE Draw a picture or a circuit diagram. Use Lenz's law to determine the direction of the induced current.

SOLVE The mathematical representation is based on Faraday's law

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right|$$

For an N-turn coil, multiply by N. The size of the induced current is $I = \mathcal{E}/R$.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 18

QuickCheck 33.12

The induced emf around this loop is

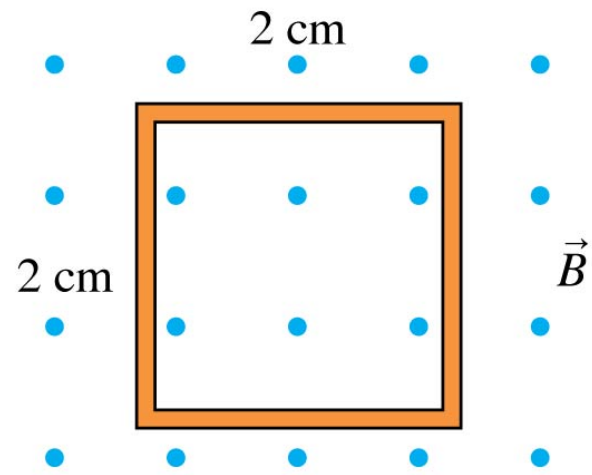
A. 200 V.

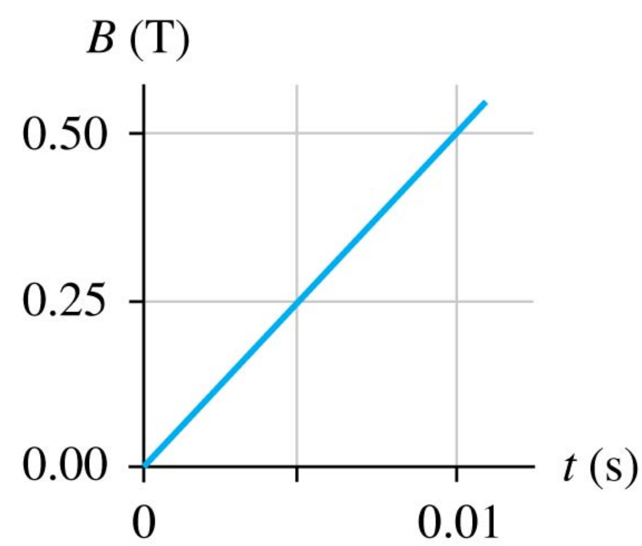
B. 50 V.

C. 2 V.

D. 0.5 V.

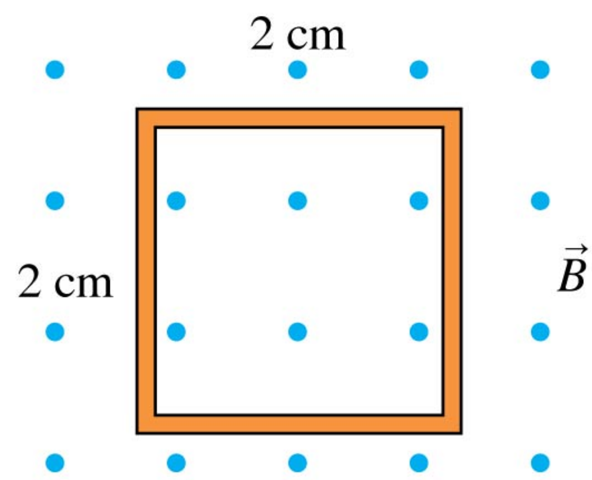
E. 0.02 V.

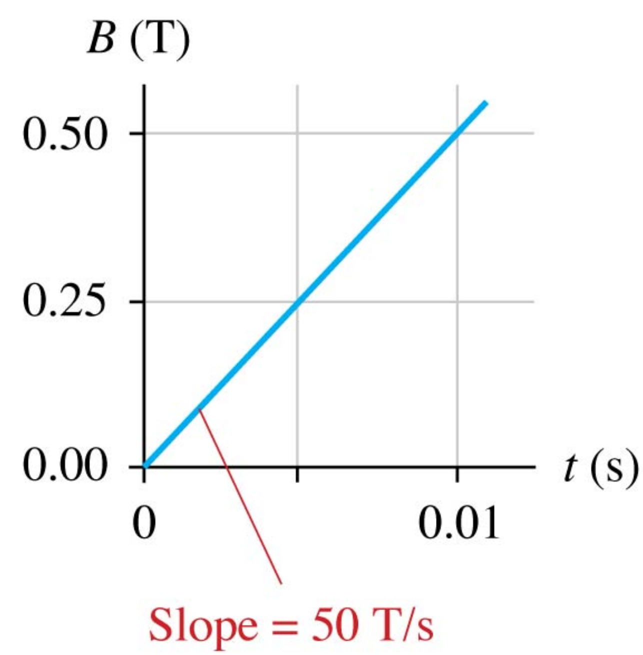




QuickCheck 33.12

The induced emf around this loop is





A. 200 V.

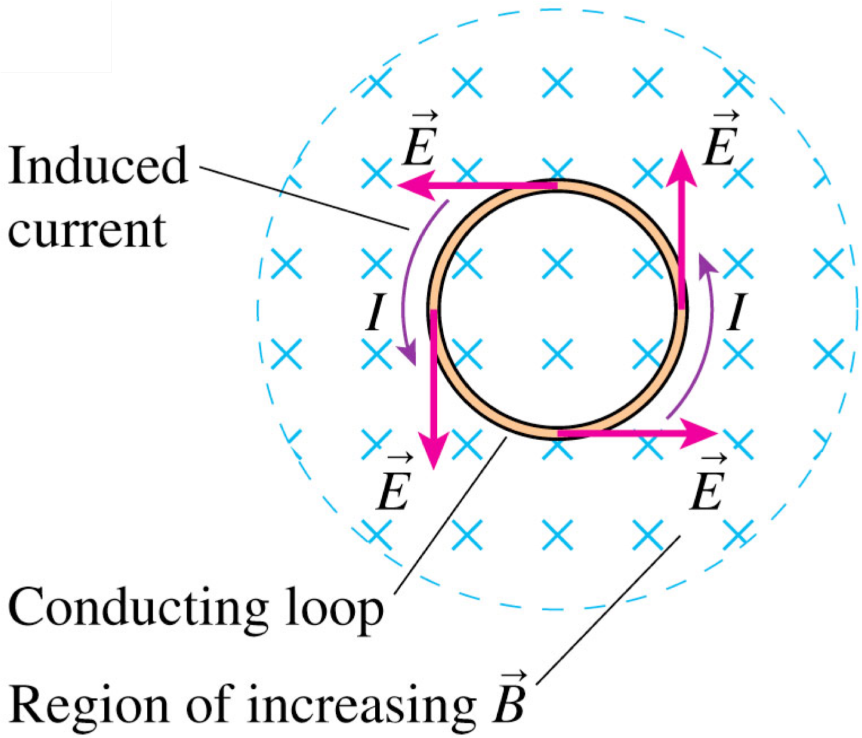
B. 50 V.

C. 2 V.

D. 0.5 V.

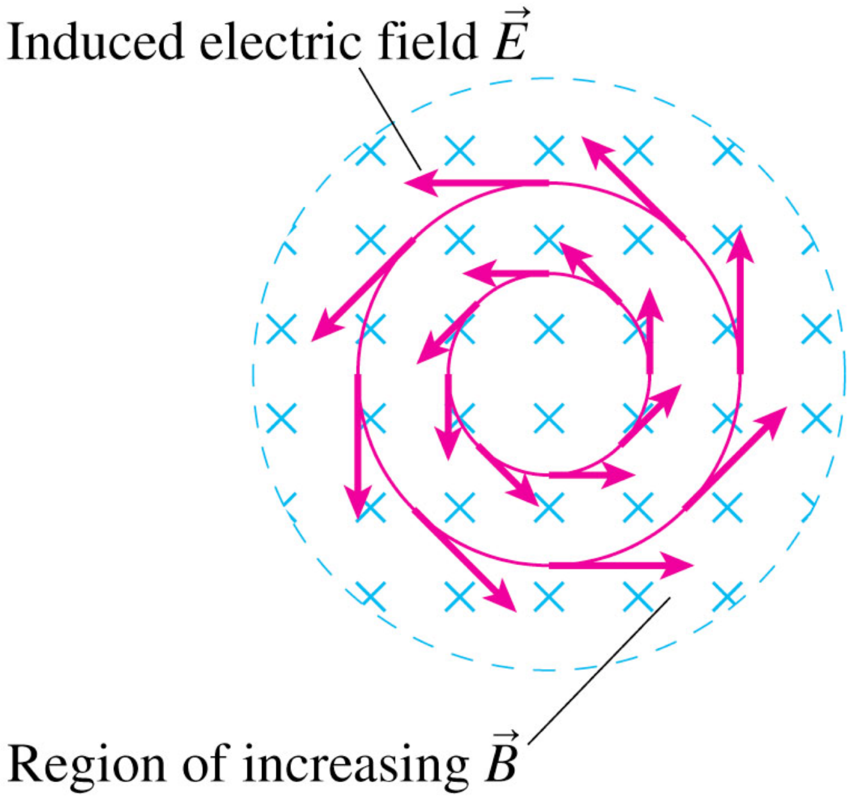
✔ **E. 0.02 V.** $\mathcal{E} = \frac{d\Phi_m}{dt} = A\frac{dB}{dt} = A \times \text{slope of graph}$

Induced Fields



- The figure shows a conducting loop in an increasing magnetic field.
- According to Lenz's law, there is an induced current in the counterclockwise direction.
- Something has to act on the charge carriers to make them move, so we infer that there must be an **induced electric field** tangent to the loop at all points.

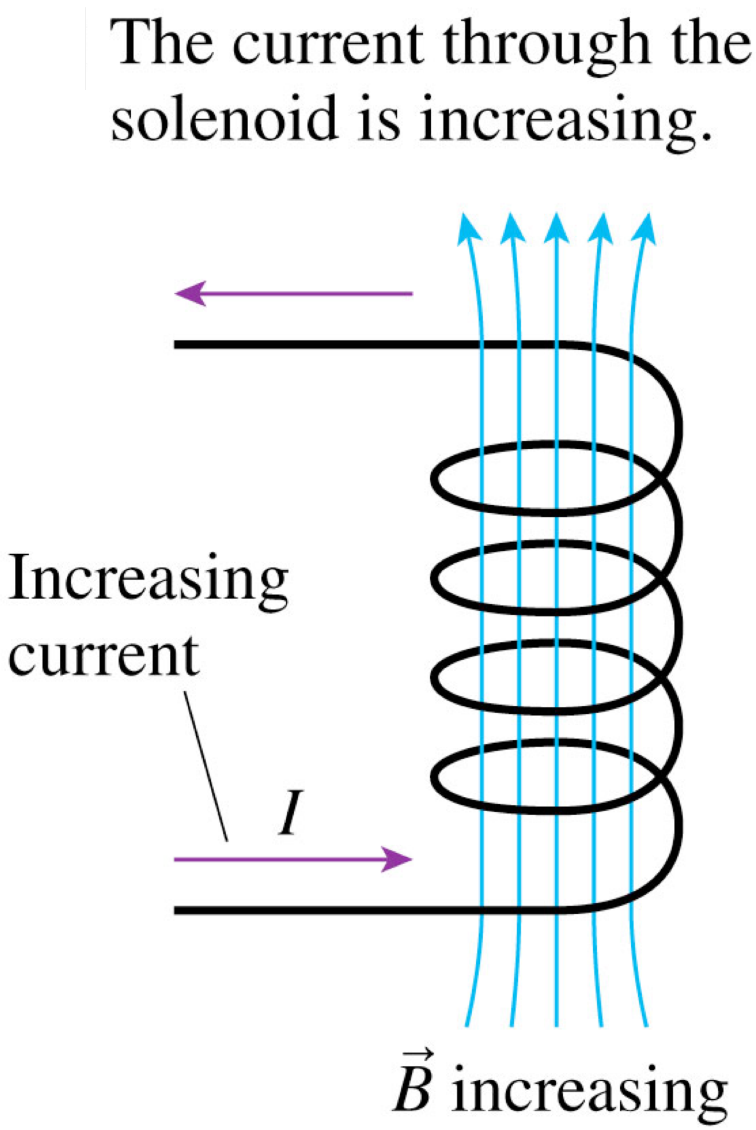
The Induced Electric Field



- When the magnetic field is increasing in a region of space, we may define a closed loop which is perpendicular to the magnetic field.
- Faraday's Law specifies the loop integral of the induced electric field around this loop:

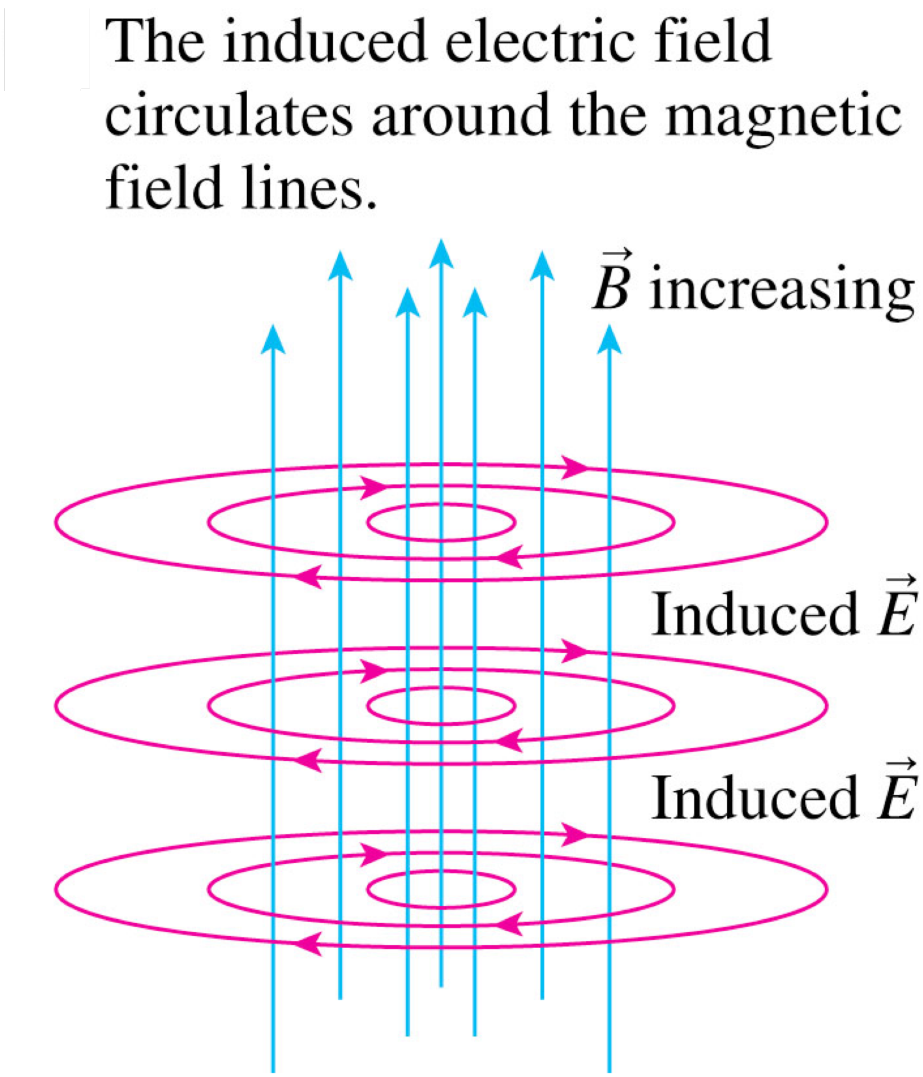
$$\oint \vec{E} \cdot d\vec{s} = A \left| \frac{dB}{dt} \right|$$

Induced Electric Field in a Solenoid Slide 1 of 3



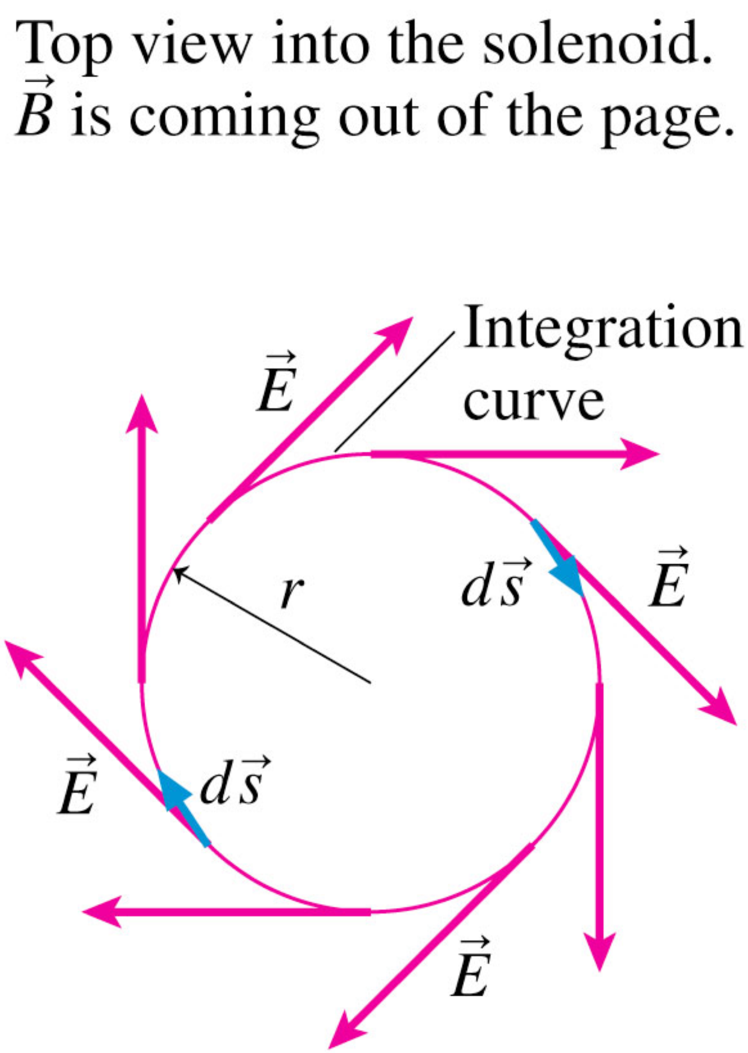
- The current through the solenoid creates an upward pointing magnetic field.
- As the current is increasing, B is increasing, so it must induce an electric field.

Induced Electric Field in a Solenoid Slide 2 of 3



- We could use Lenz's law to determine that if there were a conducting loop in the solenoid, the induced current would be clockwise.
- The induced electric field must therefore be clockwise around the magnetic field lines.

Induced Electric Field in a Solenoid Slide 3 of 3



- To use Faraday's law, integrate around a clockwise circle of radius r:

$$\oint \vec{E} \cdot d\vec{s} = 2\pi r E$$

$$= A\left|\frac{dB}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$$

- Thus the strength of the induced electric field inside the solenoid is:

$$E_{\text{inside}} = \frac{r}{2}\left|\frac{dB}{dt}\right|$$

Example 33.10 An Induced Electric Field

EXAMPLE 33.10 **An induced electric field**

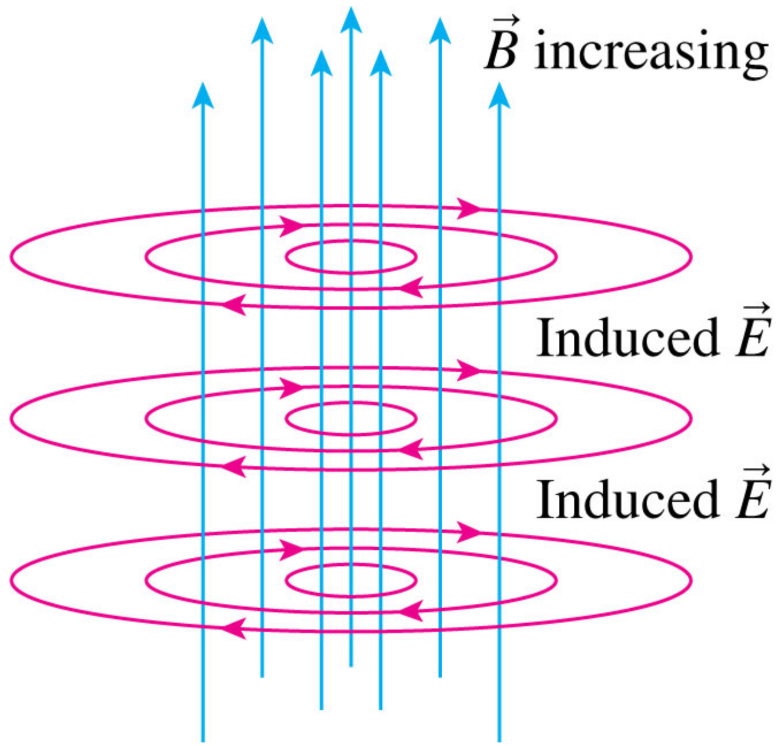
A 4.0-cm-diameter solenoid is wound with 2000 turns per meter. The current through the solenoid oscillates at 60 Hz with an amplitude of 2.0 A. What is the maximum strength of the induced electric field inside the solenoid?

MODEL Assume that the magnetic field inside the solenoid is uniform.

Example 33.10 An Induced Electric Field

EXAMPLE 33.10 An induced electric field

VISUALIZE The electric field lines are concentric circles around the magnetic field lines, as shown in the figure below. They reverse direction twice every period as the current oscillates.



Example 33.10 An Induced Electric Field

EXAMPLE 33.10 An induced electric field

SOLVE You learned in Chapter 32 that the magnetic field strength inside a solenoid with n turns per meter is $B = \mu_0 n I$. In this case, the current through the solenoid is $I = I_0 \sin \omega t$, where $I_0 = 2.0$ A is the peak current and $\omega = 2\pi(60\text{ Hz}) = 377$ rad/s. Thus the induced electric field strength at radius r is

$$E = \frac{r}{2}\left|\frac{dB}{dt}\right| = \frac{r}{2}\frac{d}{dt}(\mu_0 n I_0 \sin \omega t) = \frac{1}{2}\mu_0 n r \omega I_0 \cos \omega t$$

The field strength is maximum at maximum radius ($r = R$) *and* at the instant when $\cos \omega t = 1$. That is,

$$E_{\text{max}} = \frac{1}{2}\mu_0 n R \omega I_0 = 0.019\text{ V/m}$$

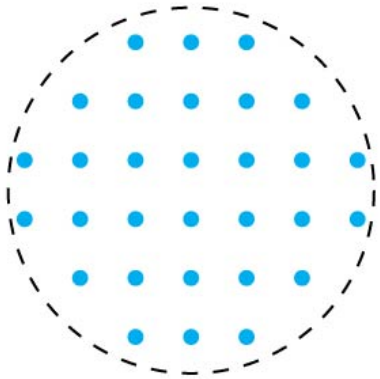
Example 33.10 An Induced Electric Field

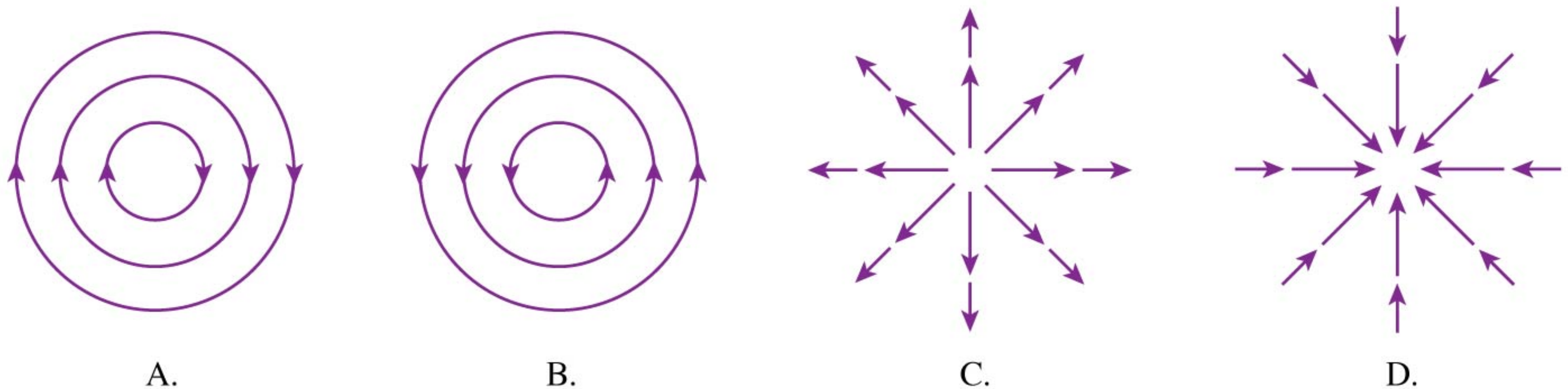
EXAMPLE 33.10 An induced electric field

ASSESS This field strength, although not large, is similar to the field strength that the emf of a battery creates in a wire. Hence this induced electric field can drive a substantial induced current through a conducting loop *if* a loop is present. But the induced electric field exists inside the solenoid whether or not there is a conducting loop.

QuickCheck 33.13

The magnetic field is decreasing.
Which is the induced electric field?

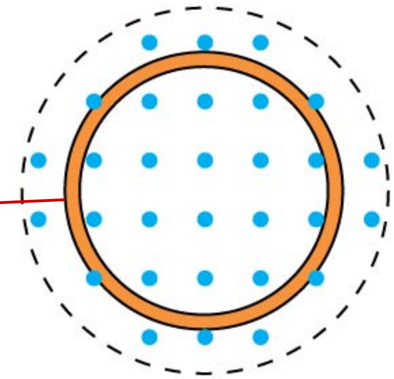




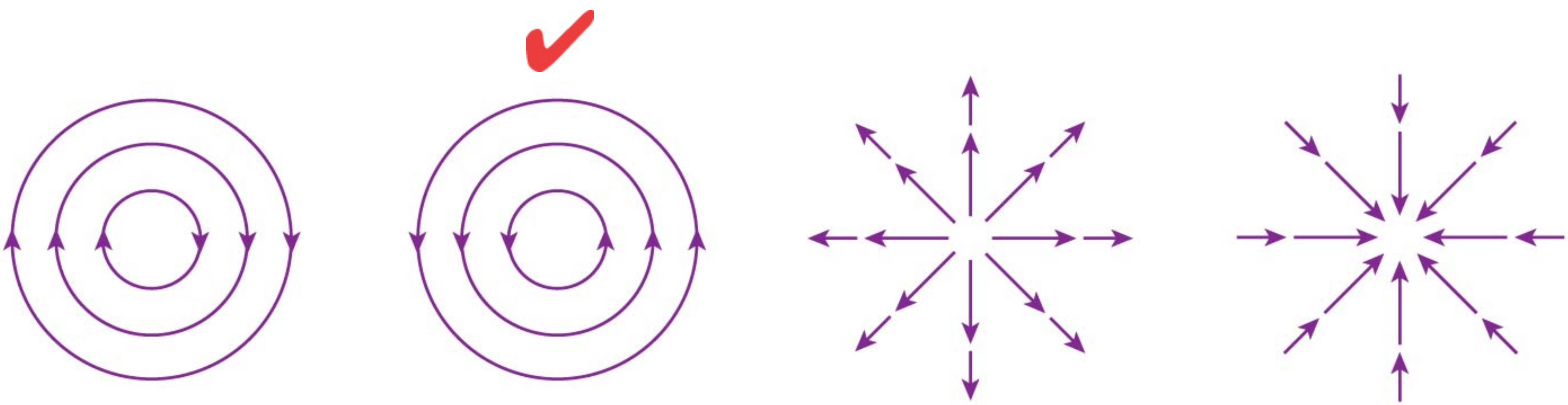
E. There’s no induced field in this case.

QuickCheck 33.13

The magnetic field is decreasing.
Which is the induced electric field?



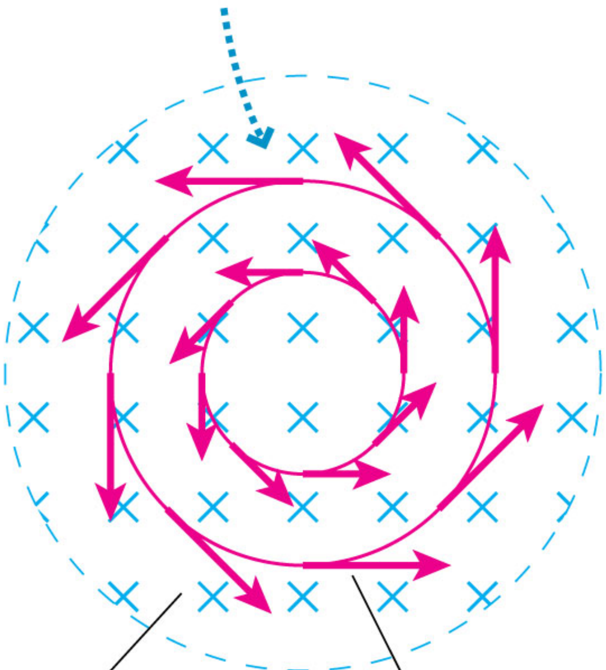
The field is the same direction as induced current would flow if there were a loop in the field.



A. B. C. D.

E. There's no induced field in this case.

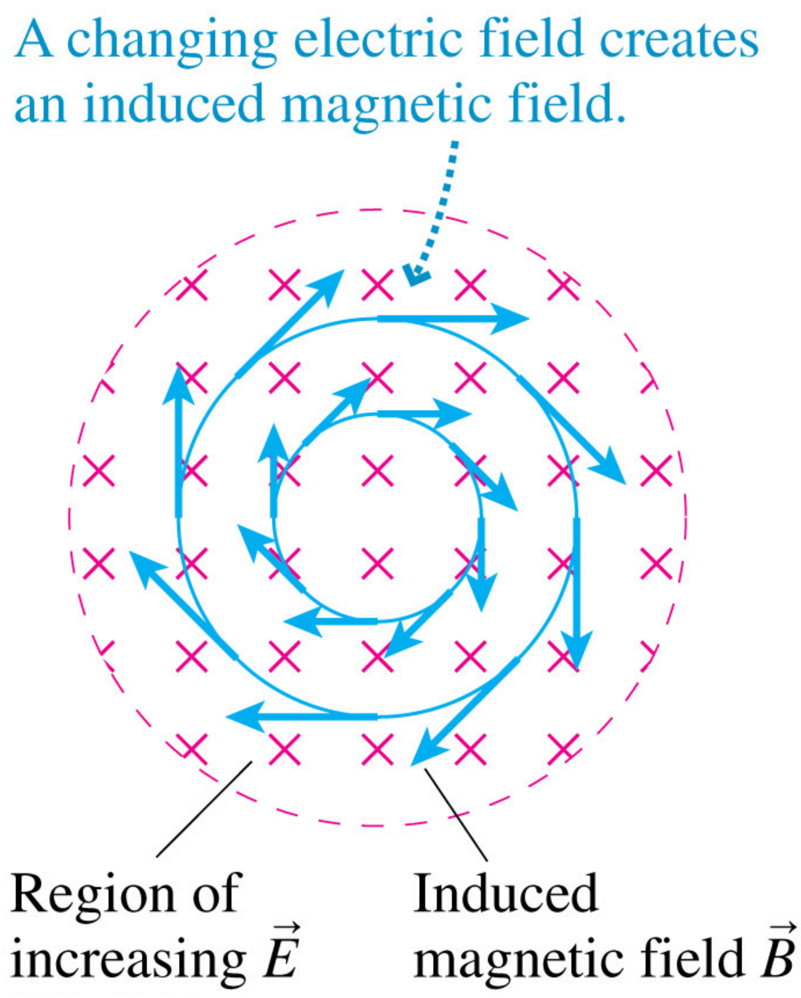
The Induced Electric Field



- Faraday's law and Lenz's law may be combined by noting that the emf must oppose the change in Φ_m.
- Mathematically, emf must have the opposite sign of dB/dt.
- Faraday's law may be written as:

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$$

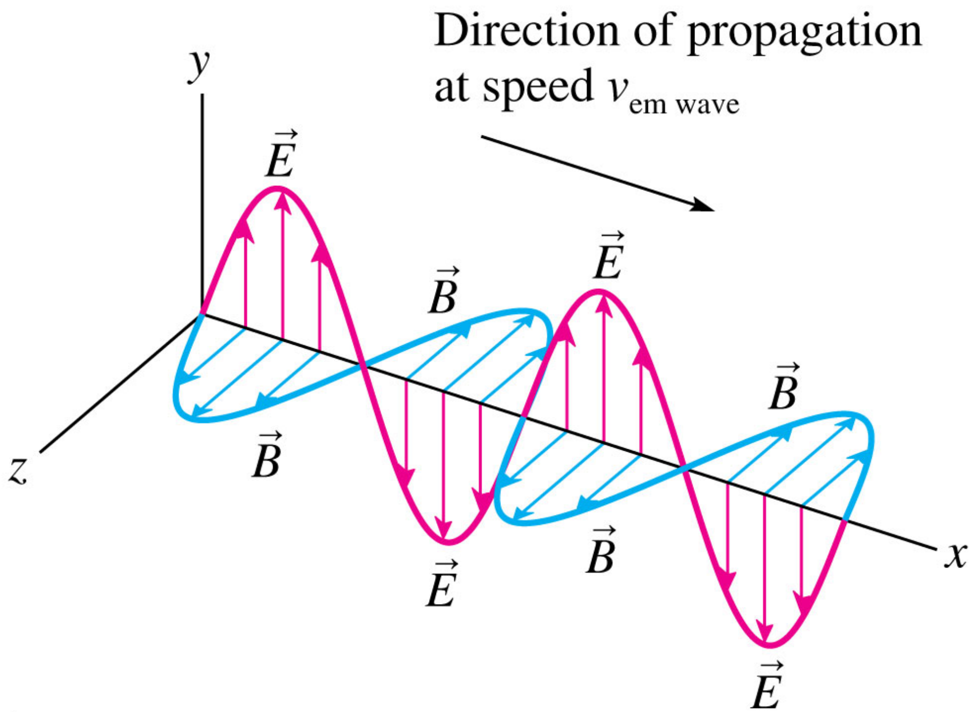
The Induced Magnetic Field



- As we know, changing the magnetic field induces a circular electric field.
- Symmetrically, changing the electric field induces a circular magnetic field.
- The **induced magnetic field** was first suggested as a possibility by James Clerk Maxwell in 1855.

Maxwell's Theory of Electromagnetic Waves

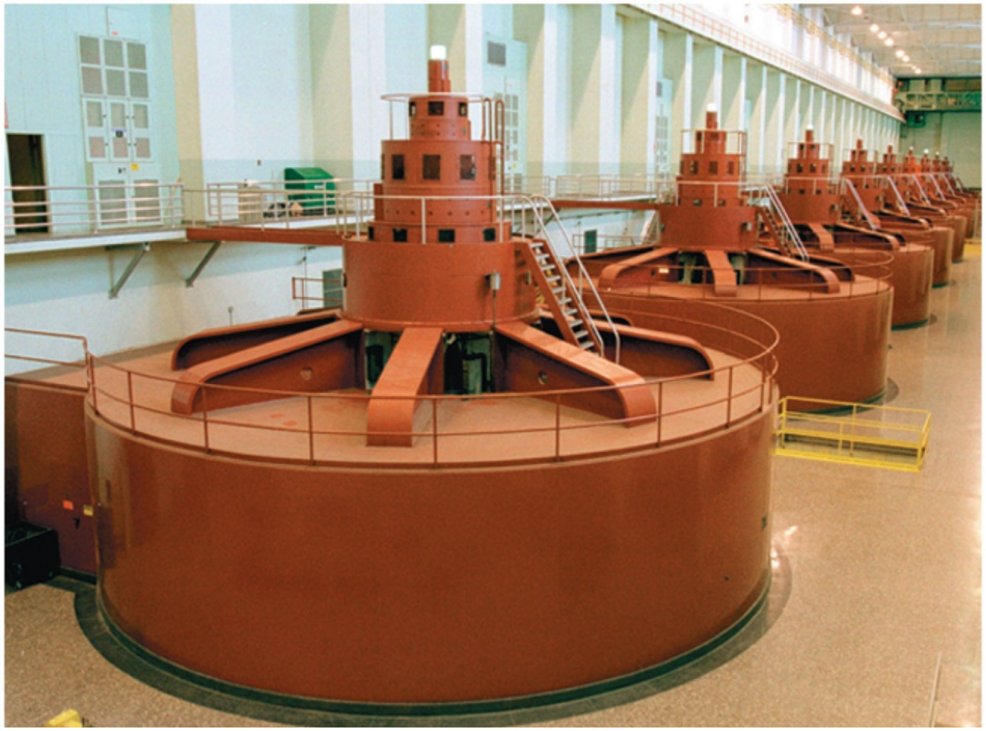
- A changing electric field creates a magnetic field, which then changes in just the right way to recreate the electric field, which then changes in just the right way to again recreate the magnetic field, and so on.
- This is an **electromagnetic wave**.



$$v_{\text{em wave}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

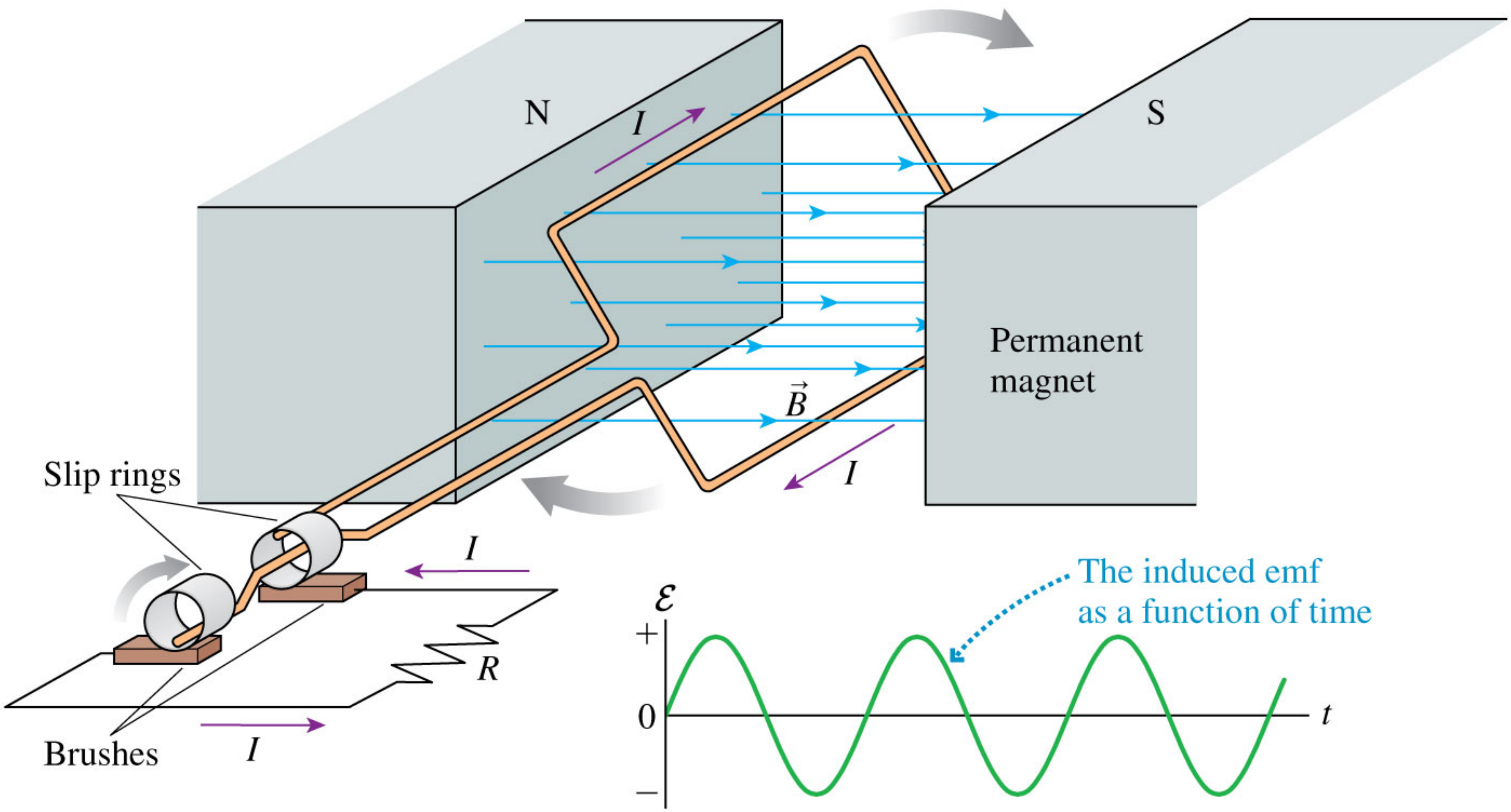
Generators

A generator is a device that transforms mechanical energy into electric energy.



A generator inside a hydroelectric dam uses electromagnetic induction to convert the mechanical energy of a spinning turbine into electric energy.

An Alternating-Current Generator



$$\mathcal{E}_{\text{coil}} = -N\frac{d\Phi_{\text{m}}}{dt} = -ABN\frac{d}{dt}(\cos\omega t) = \omega ABN\sin\omega t$$

Example 33.11 An AC Generator

EXAMPLE 33.11 **An AC generator**

A coil with area 2.0 m^2 rotates in a 0.010 T magnetic field at a frequency of 60 Hz. How many turns are needed to generate a peak voltage of 160 V?

SOLVE The coil's maximum voltage is found from Equation 33.29:

$$\mathcal{E}_{max} = \omega ABN = 2\pi fABN$$

The number of turns needed to generate $\mathcal{E}_{max} = 160$ V is

$$N = \frac{\mathcal{E}_{max}}{2\pi fAB} = \frac{160\ \text{V}}{2\pi(60\ \text{Hz})(2.0\ \text{m}^2)(0.010\ \text{T})} = 21\ \text{turns}$$

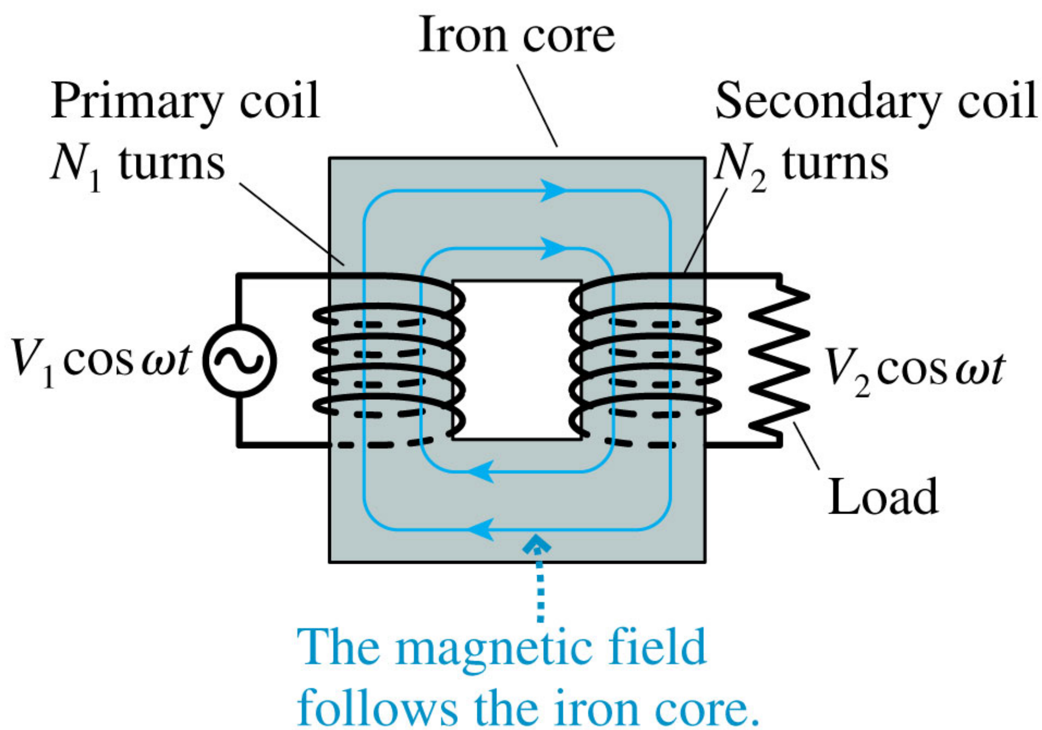
Example 33.11 An AC Generator

EXAMPLE 33.11 An AC generator

ASSESS A 0.010 T field is modest, so you can see that generating large voltages is not difficult with large (2 m^2) coils. Commercial generators use water flowing through a dam, rotating windmill blades, or turbines spun by expanding steam to rotate the generator coils. Work is required to rotate the coil, just as work was required to pull the slide wire in Section 33.2, because the magnetic field exerts retarding forces on the currents in the coil. Thus a generator is a device that turns motion (mechanical energy) into a current (electric energy). A generator is the opposite of a motor, which turns a current into motion.

Transformers

- A transformer sends an alternating emf V_1 through the primary coil.
- This causes an oscillating magnetic flux through the secondary coil and, hence, an induced emf V_2.
- The induced emf of the secondary coil is delivered to the load:

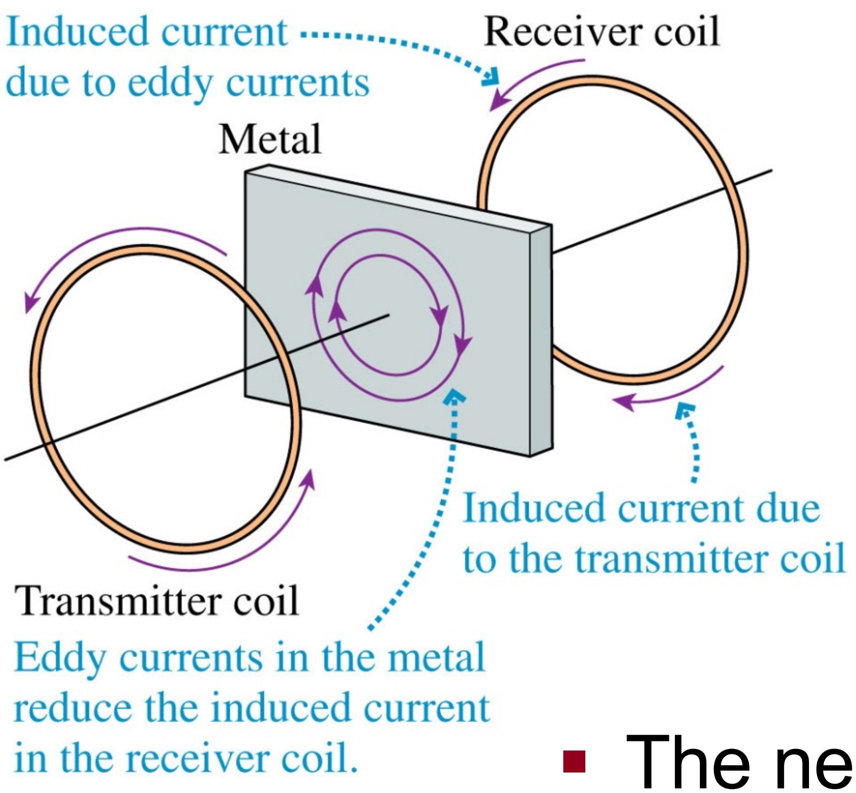


$$V_2 = \frac{N_2}{N_1} V_1$$

Transformers

- A *step-up transformer*, with $N_2 >> N_1$, can boost the voltage of a generator up to several hundred thousand volts.
- Delivering power with smaller currents at higher voltages reduces losses due to the resistance of the wires.
- High-voltage transmission lines carry electric power to urban areas, where *step-down transformers* ($N_2 << N_1$) lower the voltage to 120 V.

Metal Detectors



- A metal detector consists of two coils: a transmitter coil and a receiver coil.
- A high-frequency AC current in the transmitter coil causes a field which induces current in the receiver coil.
- The net field at the receiver decreases when a piece of metal is inserted between the coils.
- Electronic circuits detect the current decrease in the receiver coil and set off an alarm.

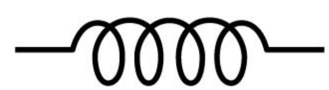
Inductors

- A coil of wire, or solenoid, can be used in a circuit to store energy in the magnetic field.
- We define the **inductance** of a solenoid having N turns, length l and cross-section area A as:

$$L_{\text{solenoid}} = \frac{\Phi_{\text{m}}}{I} = \frac{\mu_0 N^2 A}{l}$$

- The SI unit of inductance is the henry, defined as:

$$1 \text{ henry} = 1 \text{ H} = 1 \text{ Wb/A} = 1 \text{ T m2/A}$$

- A coil of wire used in a circuit for the purpose of inductance is called an **inductor.**
- The circuit symbol for an ideal inductor is  .

Example 33.12 The Length of an Inductor

EXAMPLE 33.12 The length of an inductor

An inductor is made by tightly wrapping 0.30-mm-diameter wire around a 4.0-mm-diameter cylinder. What length cylinder has an inductance of 10 μH?

Example 33.12 The Length of an Inductor

EXAMPLE 33.12 **The length of an inductor**

SOLVE The cross-section area of the solenoid is $A = \pi r^2$. If the wire diameter is d, the number of turns of wire on a cylinder of length l is $N = l/d$. Thus the inductance is

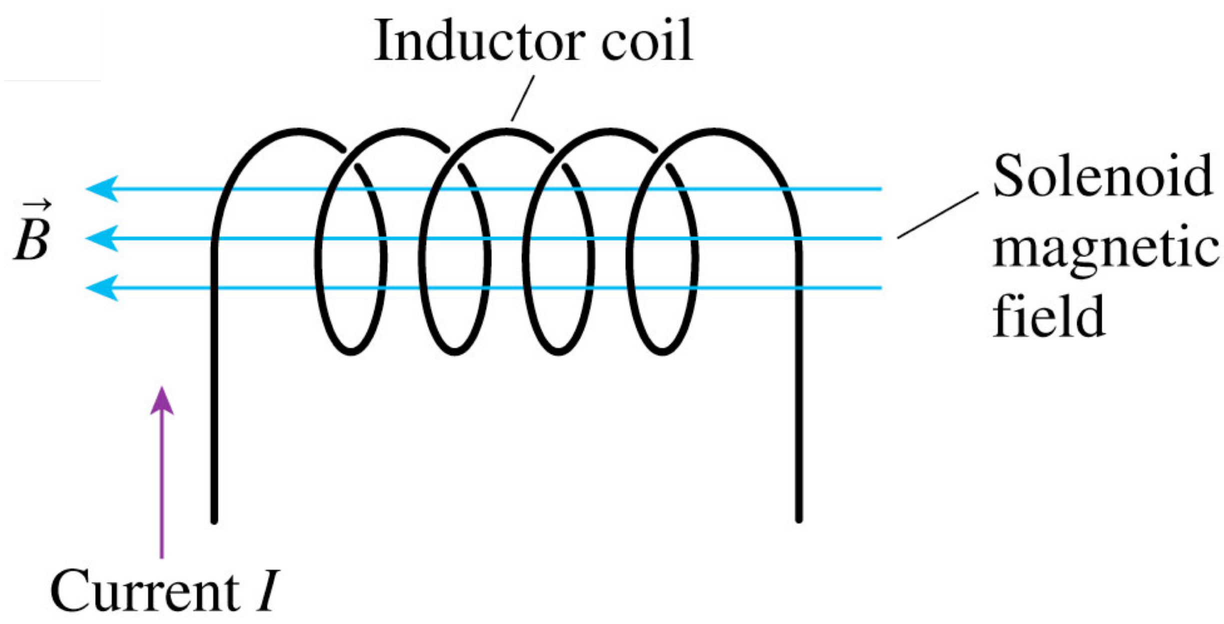
$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 (l/d)^2 \pi r^2}{l} = \frac{\mu_0 \pi r^2 l}{d^2}$$

The length needed to give inductance $L = 1.0 \times 10^{-5}$ H is

$$l = \frac{d^2 L}{\mu_0 \pi r^2} = \frac{(0.00030 \text{ m})^2 (1.0 \times 10^{-5} \text{ H})}{(4\pi \times 10^{-7} \text{ T m/A}) \pi (0.0020 \text{ m})^2}$$

$$= 0.057 \text{ m} = 5.7 \text{ cm}$$

Potential Difference Across an Inductor

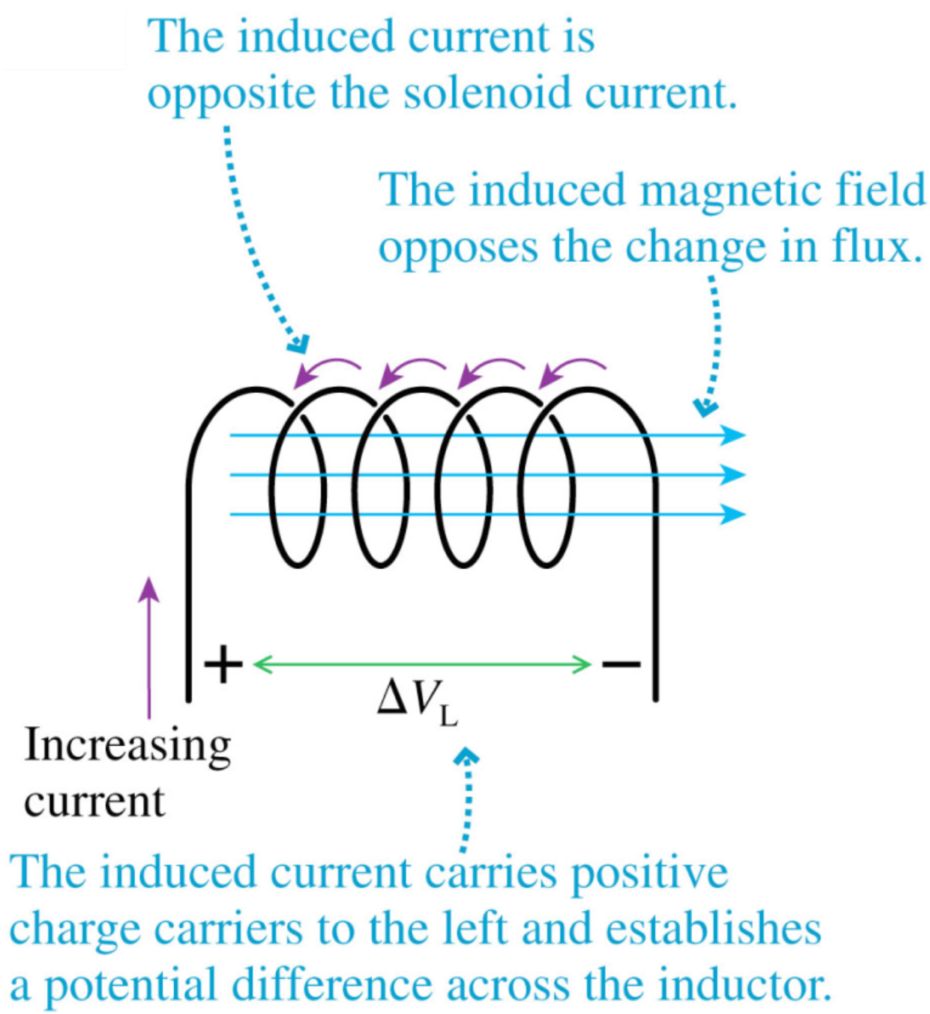


- The figure above shows a steady current into the left side of an inductor.
- The solenoid's magnetic field passes through the coils, establishing a flux.
- The next slide shows what happens if the current increases.

Potential Difference Across an Inductor

- In the figure, the current into the solenoid is increasing.
- This creates an increasing flux to the left.
- Therefore the induced magnetic field must point to the right.
- The induced emf ΔV_L must be *opposite* to the current into the solenoid:

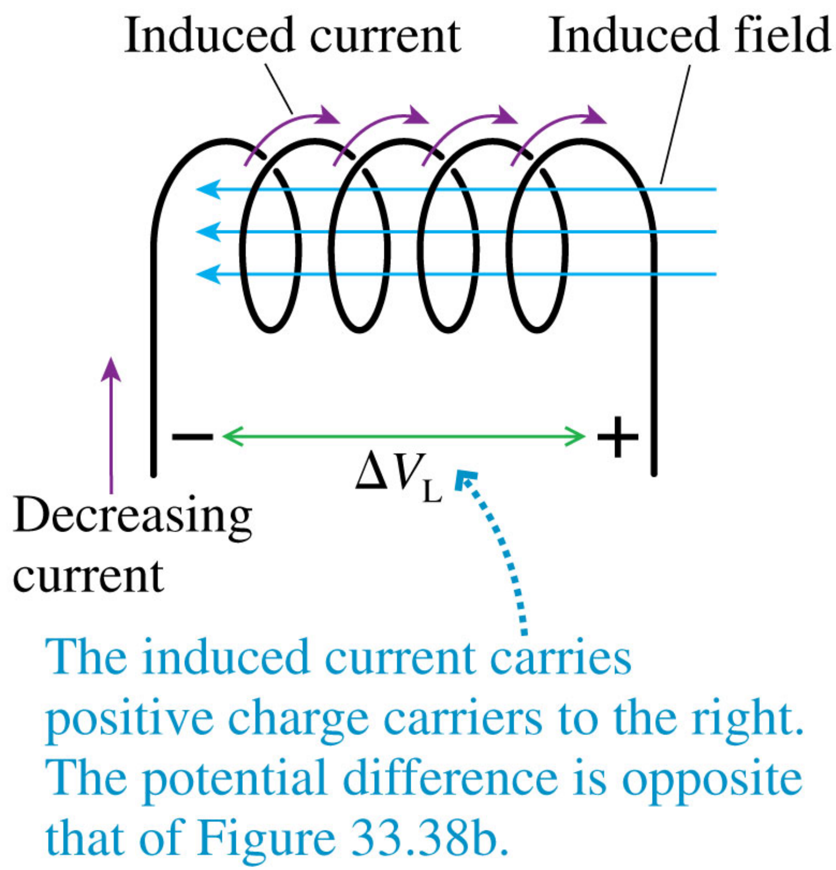
$$\Delta V_L = -L\frac{dI}{dt}$$



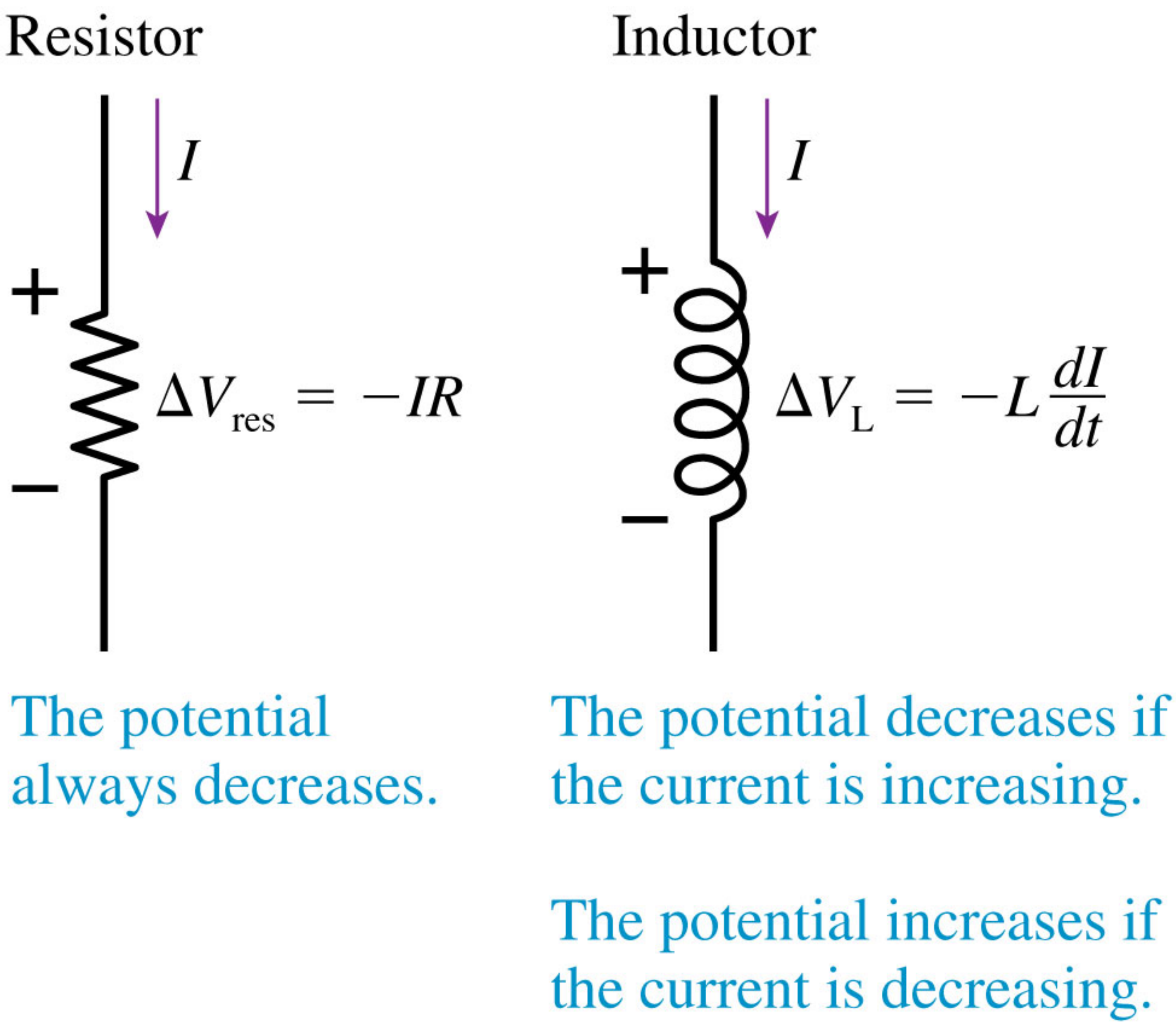
Potential Difference Across an Inductor

- In the figure, the current into the solenoid is decreasing.
- To oppose the decrease in flux, the induced emf ΔV_L is in the same direction as the input current.
- The potential difference across an inductor, *measured along the direction of the current*, is:

$$\Delta V_L = -L\frac{dI}{dt}$$



Potential Difference Across an Inductor



QuickCheck 33.14

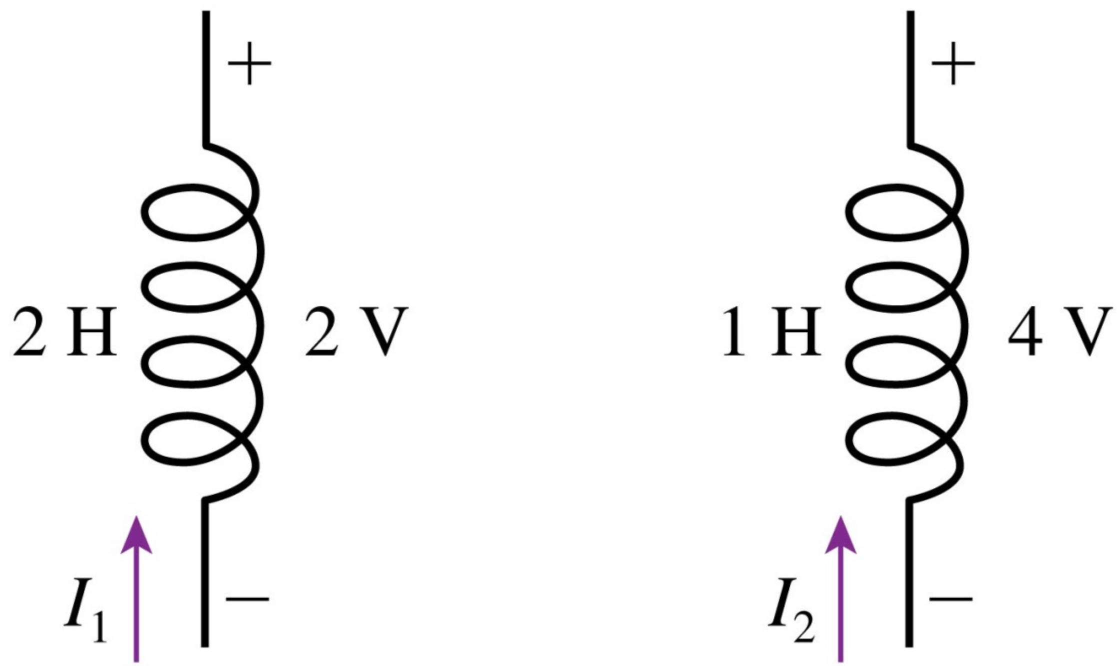
Which current is changing more rapidly?

A. Current I_1.

B. Current I_2.

C. They are changing at the same rate.

D. Not enough information to tell.



QuickCheck 33.14

Which current is changing more rapidly?

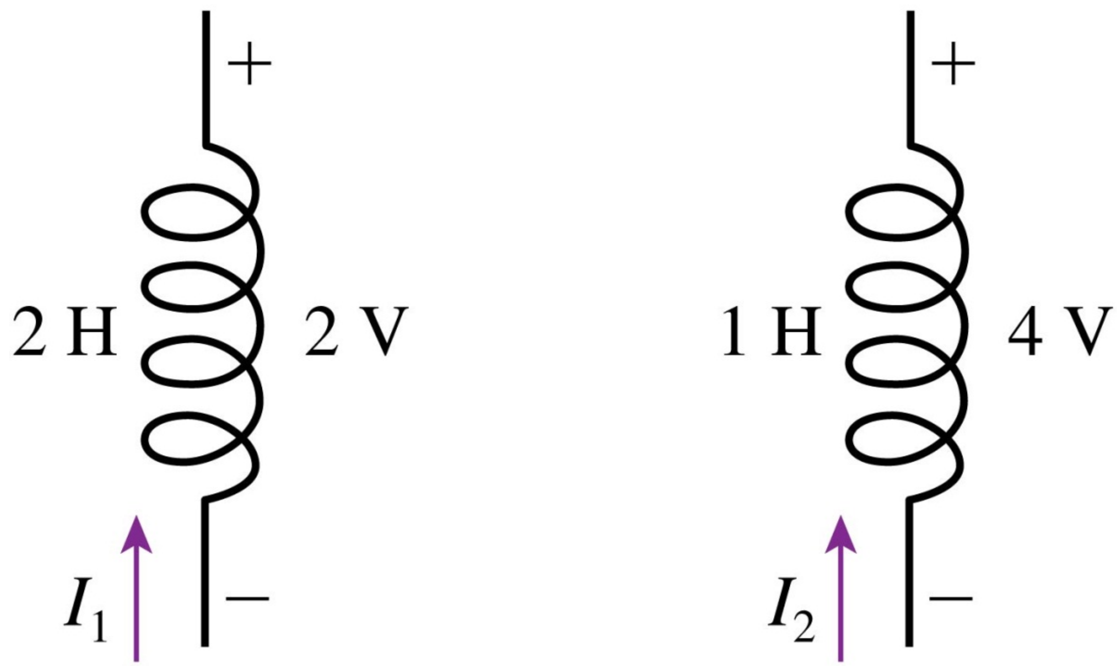
A. Current I_1.

✔ **B. Current I_2.**

C. They are changing at the same rate.

D. Not enough information to tell.

$$\Delta V_L = -L\frac{dI}{dt}$$



Example 33.13 Large Voltage Across an Inductor

EXAMPLE 33.13 Large voltage across an inductor

A 1.0 A current passes through a 10 mH inductor coil. What potential difference is induced across the coil if the current drops to zero in 5.0 μs?

MODEL Assume this is an ideal inductor, with $R = 0\ \Omega$, and that the current decrease is linear with time.

Example 33.13 Large Voltage Across an Inductor

EXAMPLE 33.13 Large voltage across an inductor

SOLVE The rate of current decrease is

$$\frac{dI}{dt} \approx \frac{\Delta I}{\Delta t} = \frac{-1.0\text{ A}}{5.0 \times 10^{-6}\text{ s}} = -2.0 \times 10^{5}\text{ A/s}$$

The induced voltage is

$$\Delta V_{\mathrm{L}} = -L\frac{dI}{dt} \approx -(0.010\text{ H})(-2.0 \times 10^{5}\text{ A/s}) = 2000\text{ V}$$

ASSESS Inductors may be physically small, but they can pack a punch if you try to change the current through them too quickly.

Energy in Inductors and Magnetic Fields

- As current passes through an inductor, the electric power is:

$$P_{\text{elec}} = I\,\Delta V_{\text{L}} = -LI\,\frac{dI}{dt}$$

- P_{elec} is negative because the current is losing energy.
- That energy is being transferred to the inductor, which is storing energy U_{L} at the rate:

$$\frac{dU_{\text{L}}}{dt} = +LI\,\frac{dI}{dt}$$

- We can find the total energy stored in an inductor by integrating:

$$U_{\text{L}} = L\int_0^I I\,dI = \frac{1}{2}LI^2$$

Energy in Inductors and Magnetic Fields

- Inside a solenoid, the magnetic field strength is $B = \mu_0 NI/l$.
- The inductor's energy can be related to B:

$$U_L = \frac{1}{2}LI^2 = \frac{\mu_0 N^2 A}{2l}I^2 = \frac{1}{2\mu_0}Al\left(\frac{\mu_0 NI}{l}\right)^2$$

$$U_L = \frac{1}{2\mu_0}AlB^2$$

- But Al is the volume inside the solenoid.
- Dividing by Al, the magnetic field energy density (energy per m^3) is:

$$u_B = \frac{1}{2\mu_0}B^2$$

Energy in Electric and Magnetic Fields

Electric fields	Magnetic fields
A capacitor stores energy	An inductor stores energy
$U_C = \frac{1}{2}C(\Delta V)^2$	$U_L = \frac{1}{2}LI^2$
Energy density in the field is	Energy density in the field is
$u_E = \frac{\epsilon_0}{2}E^2$	$u_B = \frac{1}{2\mu_0}B^2$

Example 33.14 Energy Stored in an Inductor

EXAMPLE 33.14 **Energy stored in an inductor**

The 10 μH inductor of Example 33.12 was 5.7 cm long and 4.0 mm in diameter. Suppose it carries a 100 mA current. What are the energy stored in the inductor, the magnetic energy density, and the magnetic field strength?

Example 33.14 Energy Stored in an Inductor

EXAMPLE 33.14 **Energy stored in an inductor**

SOLVE The stored energy is

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}(1.0 \times 10^{-5}\ \text{H})(0.10\ \text{A})^2 = 5.0 \times 10^{-8}\ \text{J}$$

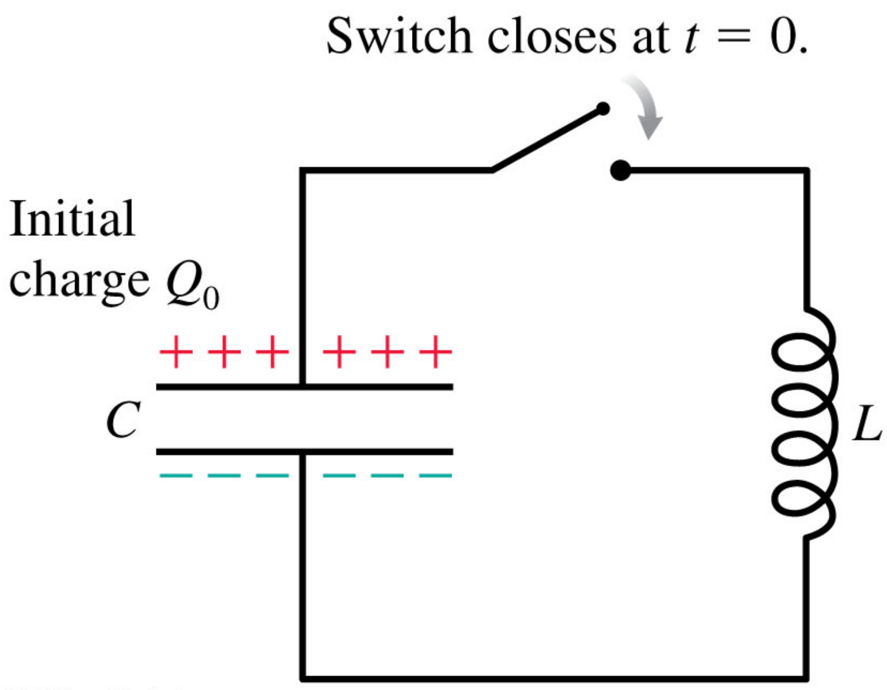
The solenoid volume is $(\pi r^2)l = 7.16 \times 10^{-7}\ \text{m}^3$. Using this gives the energy density of the magnetic field:

$$u_B = \frac{5.0 \times 10^{-8}\ \text{J}}{7.16 \times 10^{-7}\ \text{m}^3} = 0.070\ \text{J/m}^3$$

From Equation 33.42, the magnetic field with this energy density is

$$B = \sqrt{2\mu_0 u_B} = 4.2 \times 10^{-4}\ \text{T}$$

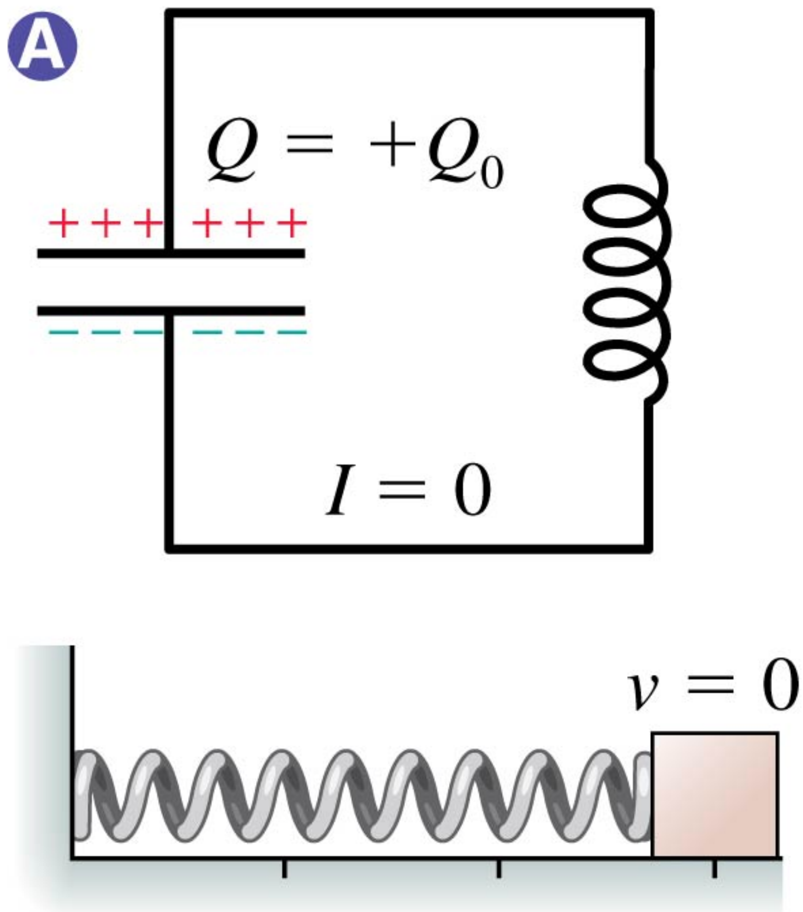
LC Circuits



- The figure shows a capacitor with initial charge Q_0, an inductor, and a switch.
- The switch has been open for a long time, so there is no current in the circuit.
- At $t = 0$, the switch is closed.
- How does the circuit respond?

The charge and current oscillate in a way that is analogous to a mass on a spring.

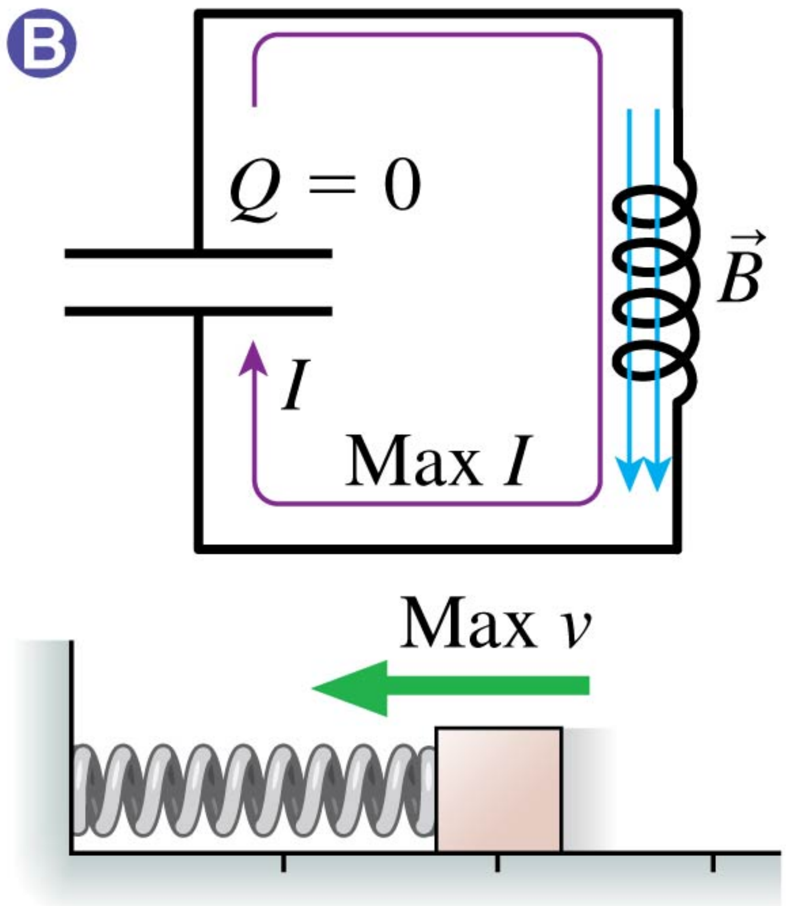
LC Circuits: Step A



Maximum capacitor charge is like a fully stretched spring.

The capacitor discharges until the current is a maximum.

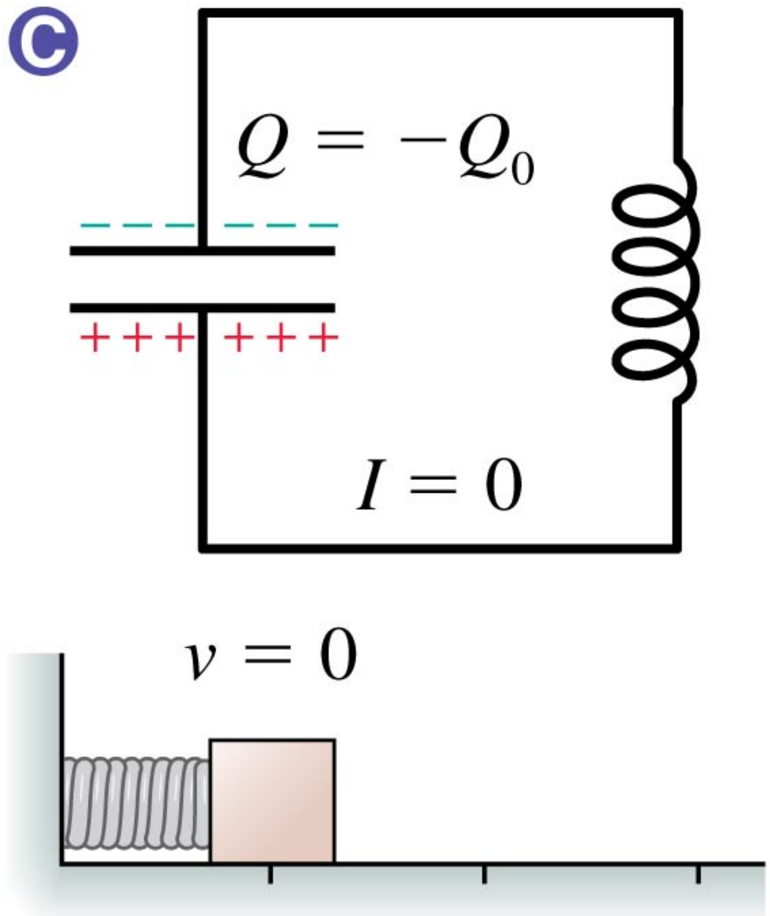
LC Circuits: Step B



The current continues until the capacitor is fully recharged with opposite polarization.

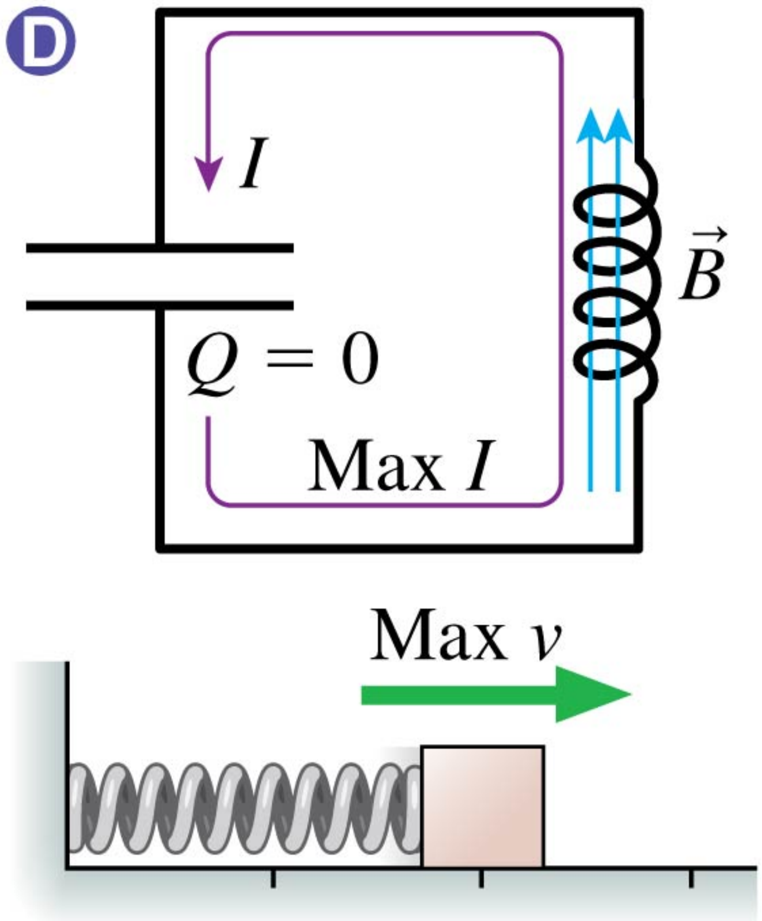
Maximum current is like the block having maximum speed.

LC Circuits: Step C



Now the discharge goes in the opposite direction.

LC Circuits: Step D

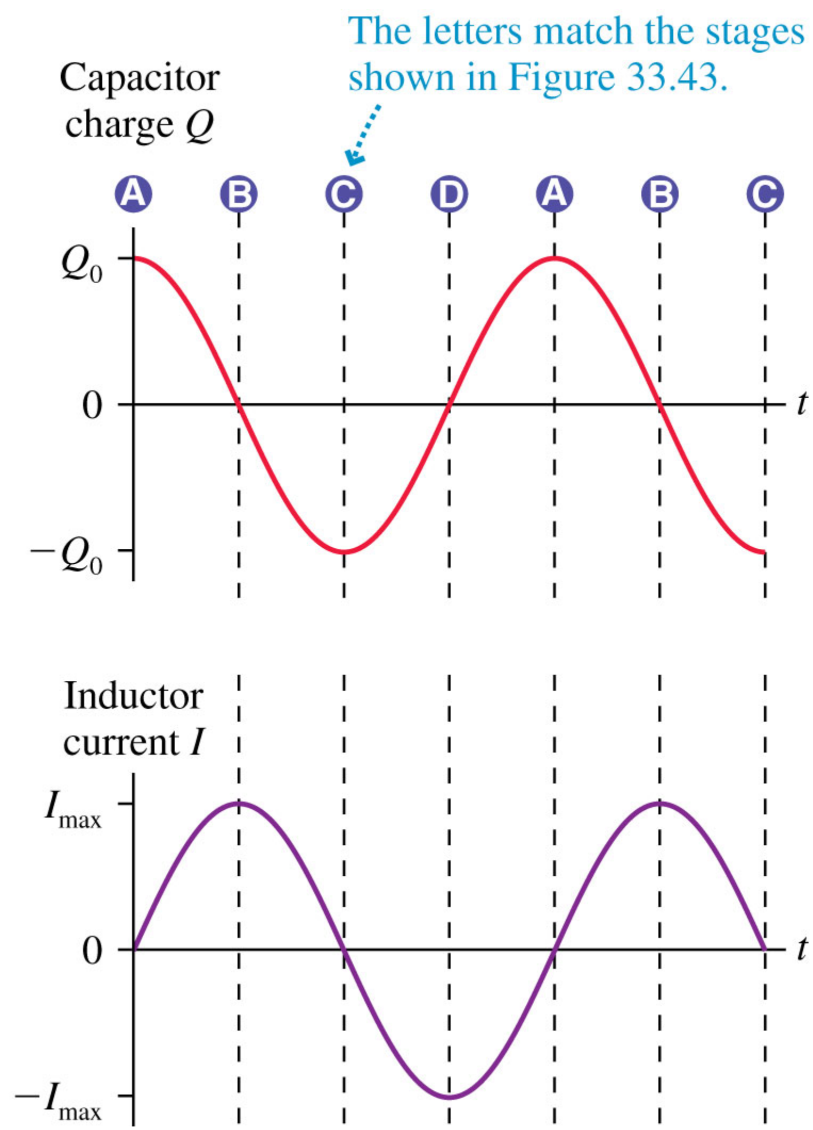


The current continues until the initial capacitor charge is restored.

LC Circuits

- An *LC* circuit is an *electric oscillator.*
- The letters on the graph correspond to the four steps in the previous slides.
- The charge on the upper plate is $Q = Q_0 \cos\omega t$ and the current through the inductor is $I = I_{max} \sin\omega t$, where:

$$\omega = \sqrt{\frac{1}{LC}}$$



QuickCheck 33.15

If the top circuit has an oscillation frequency of 1000 Hz, the frequency of the bottom circuit is

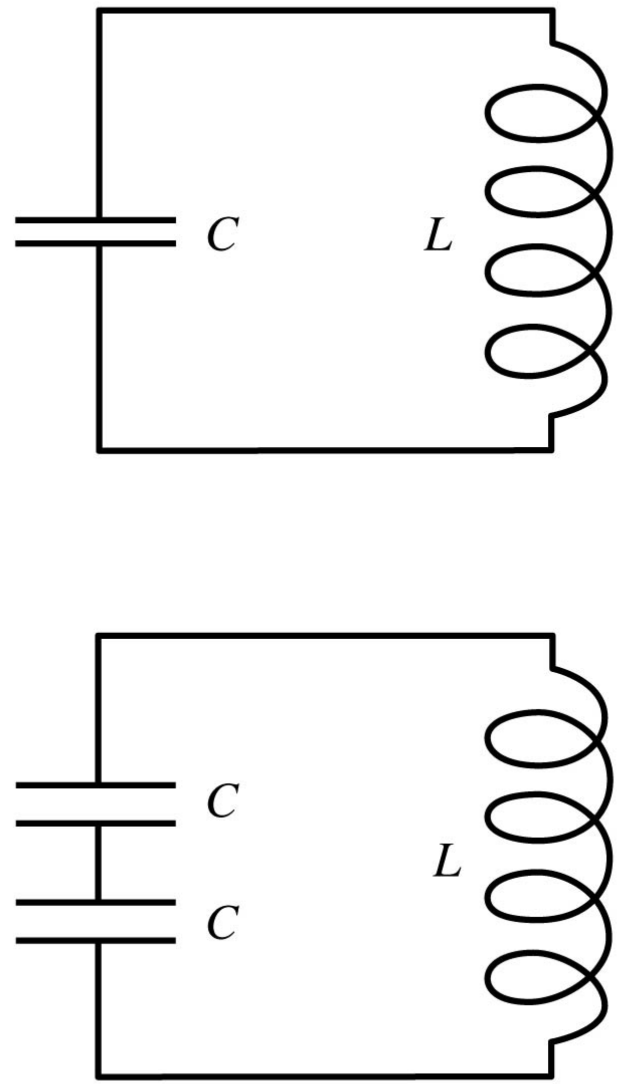
A. 500 Hz.

B. 707 Hz.

C. 1000 Hz.

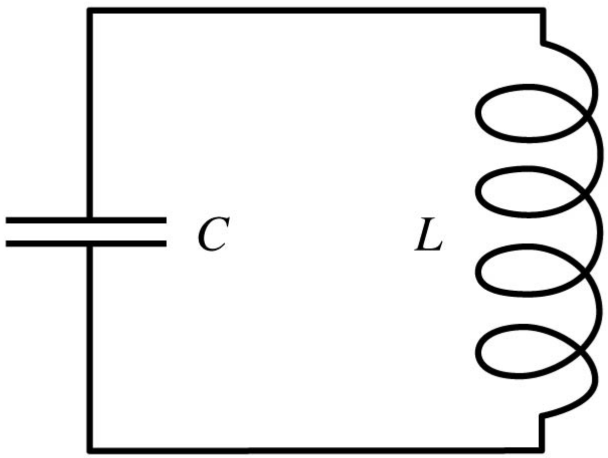
D. 1410 Hz.

E. 2000 Hz.



QuickCheck 33.15

If the top circuit has an oscillation frequency of 1000 Hz, the frequency of the bottom circuit is



A. 500 Hz.

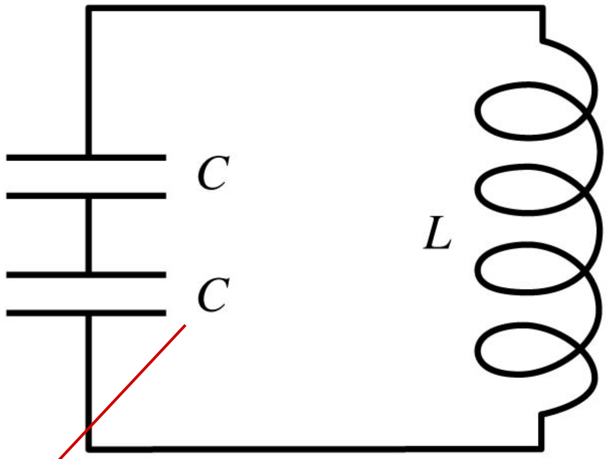
B. 707 Hz.

C. 1000 Hz.

✔ **D. 1410 Hz.**

E. 2000 Hz.

$$\omega = \sqrt{\frac{1}{LC}}$$



Series capacitors have equivalent $C/2$.

LC Circuits

A cell phone is actually a very sophisticated two-way radio that communicates with the nearest base station via high-frequency radio waves—roughly 1000 MHz. As in any radio or communications device, the transmission frequency is established by the oscillating current in an *LC* circuit.

Example 33.15 An Am Radio Oscillator

EXAMPLE 33.15 An AM radio oscillator

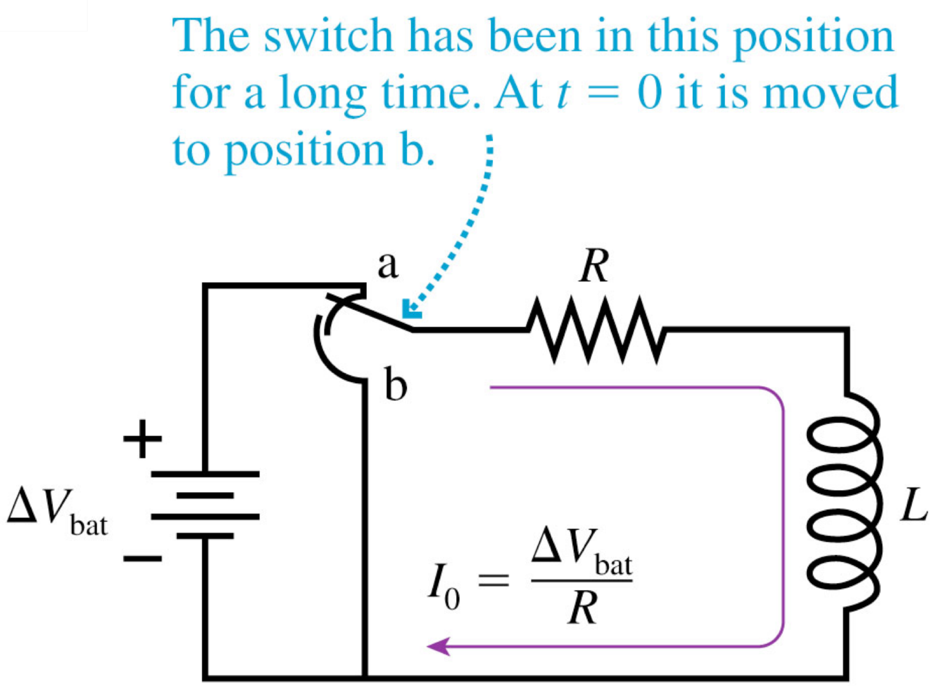
You have a 1.0 mH inductor. What capacitor should you choose to make an oscillator with a frequency of 920 kHz? (This frequency is near the center of the AM radio band.)

SOLVE The angular frequency is $\omega = 2\pi f = 5.78 \times 10^6$ rad/s. Using Equation 33.51 for ω gives the required capacitor:

$$C = \frac{1}{\omega^2 L} = \frac{1}{(5.78 \times 10^6 \text{ rad/s})^2(0.0010 \text{ H})}$$

$$= 3.0 \times 10^{-11} \text{ F} = 30 \text{ pF}$$

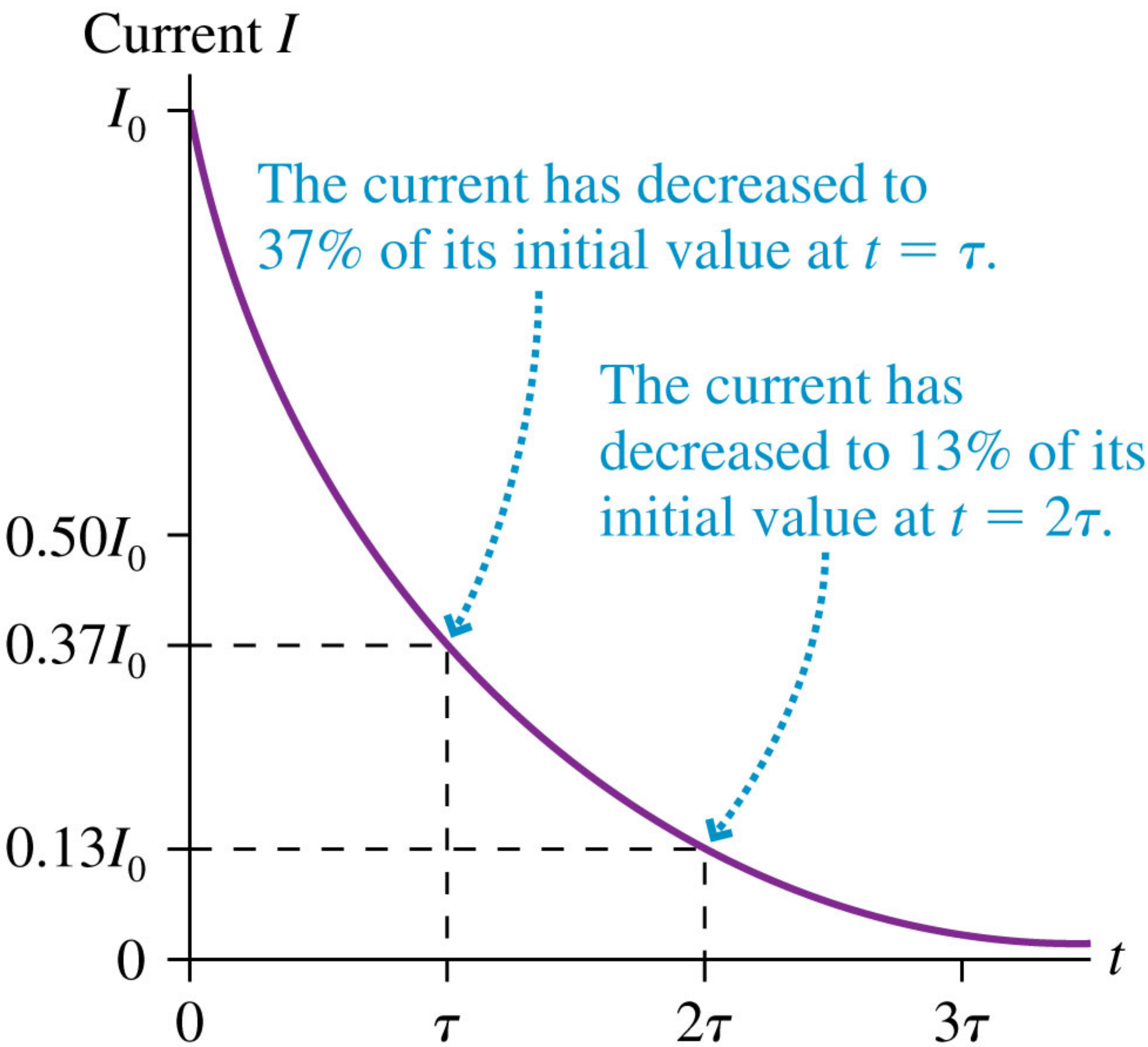
LR Circuits



- The figure shows an inductor and resistor in series.
- Initially there is a steady current I_0 being driven through the LR circuit by an external battery.
- At $t = 0$, the switch is closed.
- How does the circuit respond?

The current through the circuit decays exponentially, with a time constant $\tau = L/R$.

LR Circuits



$$I = I_0 e^{-t/(L/R)}$$

$$\tau = \frac{L}{R}$$

QuickCheck 33.16

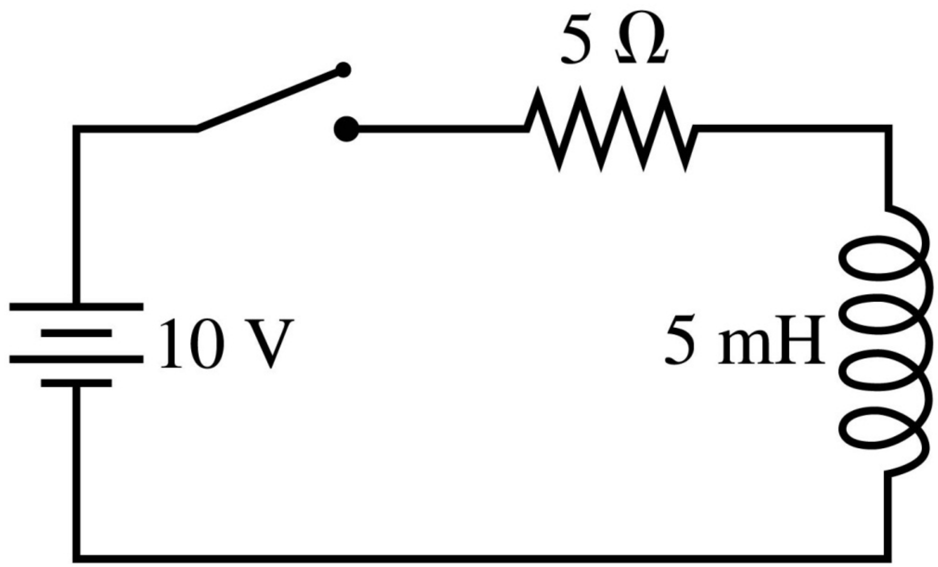
What is the battery current immediately after the switch has closed?

A. 0 A

B. 1 A

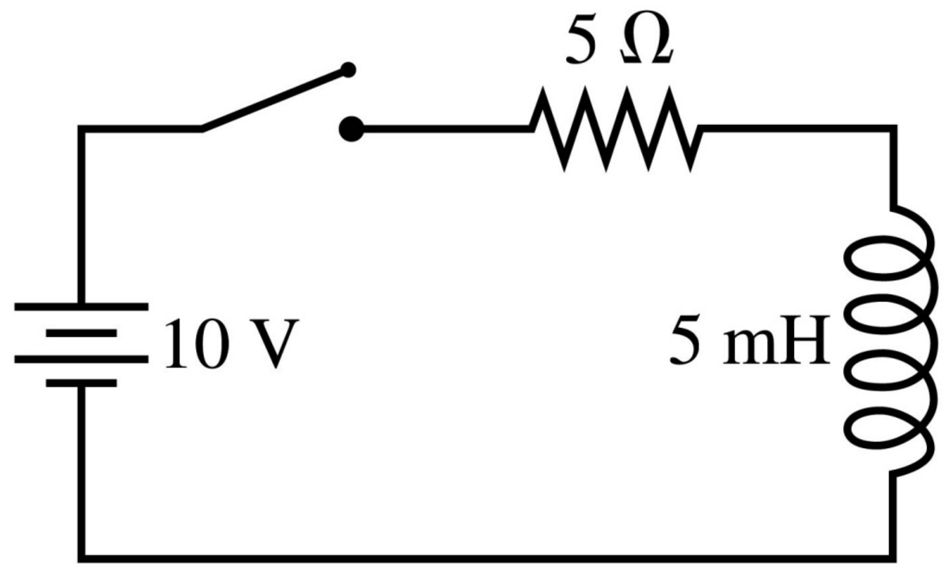
C. 2 A

D. Undefined



QuickCheck 33.16

What is the battery current immediately after the switch has closed?



✔ **A. 0 A**

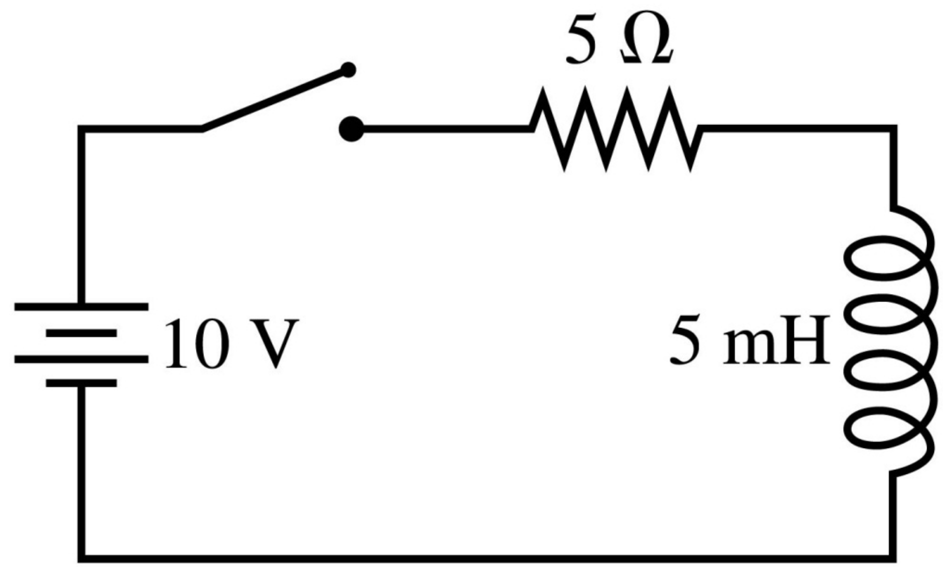
B. 1 A

C. 2 A

D. Undefined

QuickCheck 33.17

What is the battery current immediately after the switch has been closed for a very long time?



A. 0 A

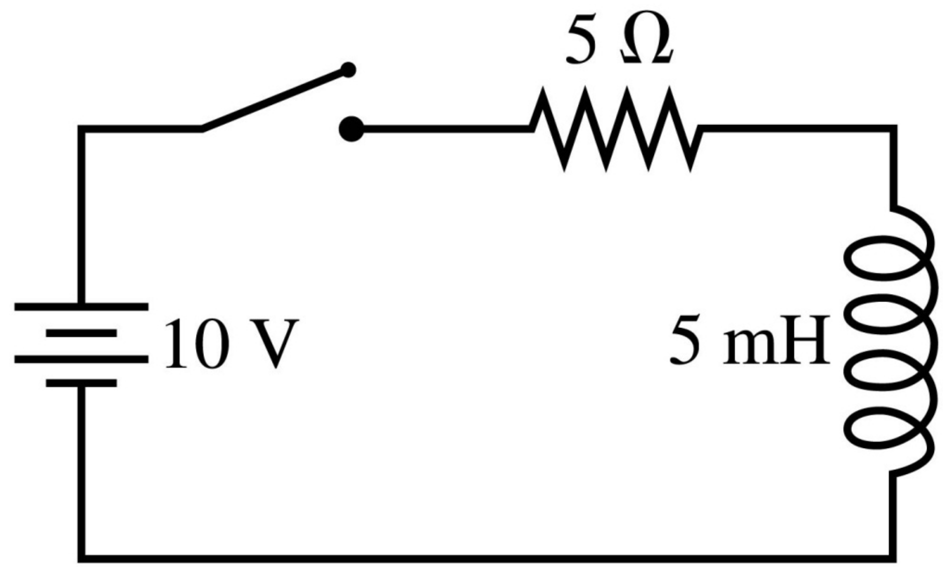
B. 1 A

C. 2 A

D. Undefined

QuickCheck 33.17

What is the battery current immediately after the switch has been closed for a very long time?



A. 0 A

B. 1 A

✔ **C. 2 A**

D. Undefined

Chapter 33 Summary Slides

General Principles

Faraday's Law

MODEL Make simplifying assumptions.

VISUALIZE Use Lenz's law to determine the direction of the **induced current.**

SOLVE The **induced emf** is

$$\mathcal{E} = \left|\frac{d\Phi_{\mathrm{m}}}{dt}\right|$$

Multiply by N for an N-turn coil.
The size of the induced current is $I = \mathcal{E}/R$.

ASSESS Is the result reasonable?

General Principles

Lenz's Law

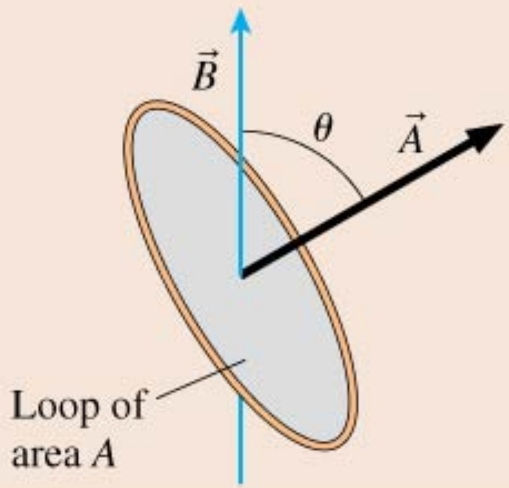
There is an induced current in a closed conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.

General Principles

Magnetic flux

Magnetic flux measures the amount of magnetic field passing through a surface.

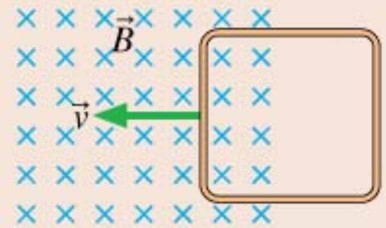
$$\Phi_m = \vec{A} \cdot \vec{B} = AB\cos\theta$$



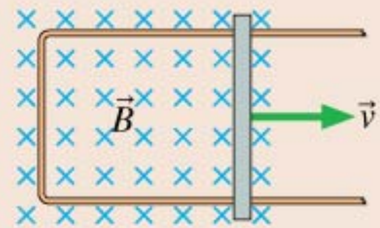
Important Concepts

Three ways to change the flux

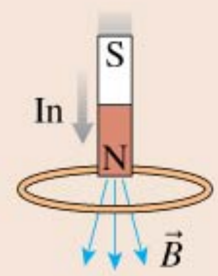
1. A loop moves into or out of a magnetic field.



2. The loop changes area or rotates.



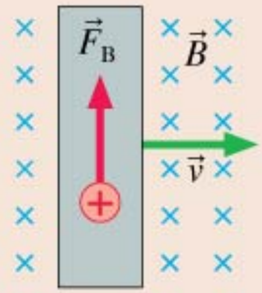
3. The magnetic field through the loop increases or decreases.



Important Concepts

Two ways to create an induced current

1. A **motional emf** is due to magnetic forces on moving charge carriers.



2. An induced electric field is due to a changing magnetic field.