34

ELECTROMAGNETIC FIELDS AND WAVES

Conceptual Questions

34.1. **(a)** Yes, up. Andre sees the same magnetic field as the laboratory field, which points up.
(b) Andre sees an electric field $\vec{E}_{\text{A}} = \vec{V}_{\text{AL}} \times \vec{B}_{\text{L}}$, which by the right-hand rule points into the page.

34.2. **(a)** A magnetic force acting down and an electric force acting up. Sharon sees the same magnetic field as Bill but also sees an electric field $\vec{E}_{\text{B}} = \vec{V}_{\text{BS}} \times \vec{B}_{\text{S}}$. (Note that the magnetic and electric forces on the charge are equal and opposite so the total force on the charge is zero.)
(b) In Bill's frame of reference the charge is at rest and therefore has no magnetic force acting on it. Since there is not an electric field present in Bill's reference frame, no forces act on the charge.

34.3. **(a)** Negative. Your thumb will point opposite to the magnetic field.
(b) Positive. Your thumb will point in the same direction as the magnetic field.

34.4. Negative. The net current through the surface is to the right, and if you curl your fingers along the arrow around the surface, your thumb points to the left.

34.5. Decreasing. Pointing the thumb of your right hand makes your fingers curl ccw, so $\dfrac{d\Phi_e}{dt}$ must be negative to get a cw $\vec{B}$.

34.6. **(a)** No. The direction of $\vec{v}_{\text{em}}$ is $\vec{E} \times \vec{B}$ which would be along the $-x$-axis.
(b) Yes. The Pointing vector requirement is satisfied.

34.7. **(a)** The right-hand rule has $\vec{E} \times \vec{B}$ point out of the page.
(b) $\vec{E} \times \vec{B}$ points up.

34.8. **(a)** Since $I \propto E_0{}^2$, the new intensity is $40\ \text{W/m}^2$.
(b) For an electromagnetic wave $E_0 \propto B_0$, so $40\ \text{W/m}^2$.
(c) For electromagnetic waves, doubling one amplitude requires that the other is doubled, so $40\ \text{W/m}^2$.
(d) Changing the frequency does not change the amplitudes and hence the intensity is unchanged at 10 W/m2.

34.9. A loop antenna works by taking advantage of the current produced by magnetic induction in Faraday's law as an electromagnetic wave passes through.

34.10. $I_d > I_c = I_e > I_b > I_a$. The intensity passing through a polarizer is $I_0 \cos^2\theta$. Polarizer *d* is aligned with the incident wave $(\theta = 0)$ while $\theta = 90°$ for polarizer *a*. Polarizers *c* and *e* are at the same angle θ from the vertical.

Exercises and Problems

Section 34.1 *E* or *B*? It Depends on Your Perspective

34.1. Model: Apply the Galilean transformation of velocity.
Solve: (a) In the laboratory frame, the speed of the proton is

$$v = \sqrt{(1.41\times10^6 \text{ m/s})^2 + (1.41\times10^6 \text{ m/s})^2} = 2.0\times10^6 \text{ m/s}$$

The angle the velocity vector makes with the positive *y*-axis is

$$\theta = \tan^{-1}\left(\frac{1.41\times10^6 \text{ m/s}}{1.41\times10^6 \text{ m/s}}\right) = 45°$$

(b) In the rocket frame, we need to first determine the vector $\vec{v}'$. Equation 34.1 yields:

$$\vec{v}_{\text{PR}} = \vec{v}_{\text{PL}} + \vec{v}_{\text{LR}} = (1.41\times10^6\hat{i} + 1.41\times10^6\hat{j}) \text{ m/s} + (-1.00\times10^6\hat{i}) \text{ m/s} = (0.41\times10^6\hat{i} + 1.41\times10^6\hat{j}) \text{ m/s}$$

The speed of the proton is

$$v_{\text{PL}} = \sqrt{(0.41\times10^6 \text{ m/s})^2 + (1.41\times10^6 \text{ m/s})^2} = 1.47\times10^6 \text{ m/s}$$

The angle the velocity vector makes with the positive *y*-axis is

$$\theta' = \tan^{-1}\left(\frac{0.41\times10^6 \text{ m/s}}{1.41\times10^6 \text{ m/s}}\right) = 16.2°$$

34.2. Model: Apply the Galilean transformation of fields.
Solve: (a) Equation 34.11 gives the Galilean field transformation equation for magnetic fields:

$$\vec{B}_{\text{B}} = \vec{B}_{\text{A}} - \frac{1}{c^2}\vec{v}_{\text{BA}} \times \vec{E}_{\text{A}}$$

$\vec{B}_{\text{A}}$ is in the positive $\hat{k}$ direction, $\vec{B}_{\text{A}} = B\hat{k}$. For $B_{\text{B}} > B_{\text{A}}$, $\vec{v}_{\text{BA}} \times \vec{E}_{\text{A}}$ *must* be in the negative $\hat{k}$ direction. Since $\vec{E}_{\text{A}} = E\hat{j}$, $\vec{v}_{\text{BA}}$ *must* be in the negative $\hat{i}$ direction, so that $\vec{v}_{\text{BA}} \times \vec{E}_{\text{A}} = -(v_{\text{BA}}\hat{i}) \times (E_{\text{A}}\hat{j}) = -v_{\text{BA}}E_{\text{A}}\hat{k}$. The rocket scientist will measure $B_{\text{B}} > B_{\text{A}}$ if the rocket moves along the –*x*-axis.
(b) For $B_{\text{B}} = B_{\text{A}}$, $\vec{v}_{\text{BA}} \times \vec{E}_{\text{A}}$ *must* be zero. The rocket scientist will measure $B_{\text{B}} = B_{\text{A}}$ if the rocket moves along either the +*y*-axis or the –*y*-axis.
(c) For $B_{\text{B}} < B_{\text{A}}$, $\vec{v}_{\text{BA}} \times \vec{E}_{\text{A}}$ *must* be in the positive $\hat{k}$ direction. The rocket scientist will measure $\vec{v}_{\text{BA}} \times \vec{E}_{\text{A}}$, if the rocket moves along the +*x*-axis.

34.3. Model: Use the Galilean transformation of fields.
Solve: Equation 34.10 gives the Galilean transformation equations for the electric and magnetic fields in different frames:

$$\vec{E}_{\text{B}} = \vec{E}_{\text{A}} + \vec{v}_{\text{BA}} \times \vec{B}_{\text{A}} \qquad \vec{B}_{\text{B}} = \vec{B}_{\text{A}} - \frac{1}{c^2}\vec{v}_{\text{BA}} \times \vec{E}_{\text{A}}$$

In a region of space where $\vec{B} = \vec{0}$, $\vec{E}_B = \vec{E}_A = -1.0\times10^6\hat{k}$ V/m. The magnetic field is

$$\vec{B}_B = \vec{0} - \frac{1}{c^2}(1.0\times10^6\hat{i}\text{ m/s})\times(-1.0\times10^6\hat{k}\text{ V/m}) = -\frac{1.0\times10^{12}}{(3.0\times10^8)^2}\hat{j}\text{ T} = -1.11\times10^{-5}\hat{j}\text{ T}$$

34.4. Model: Use the Galilean transformation of fields.

Visualize: We are given $\vec{v}_{BA} = 2.0\times10^6\hat{i}$ m/s, $B_B = 1.0\hat{j}$ T, and $\vec{E}_B = 1.0\times10^6\hat{k}$ V/m.

Solve: Equation 34.10 gives the Galilean transformation equations for the electric and magnetic fields in different frames:

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA}\times\vec{B}_A \qquad \vec{B}_B = \vec{B}_A - \frac{1}{c^2}\vec{v}_{BA}\times\vec{E}_A$$

The electric and magnetic fields viewed from earth are

$$\vec{E}_A = 1.0\times10^6\hat{k}\text{ V/m} - (2.0\times10^6\hat{i}\text{ m/s})\times(1.0\hat{j}\text{ T}) = -(1.0\times10^6\text{ V/m})\hat{k}$$

$$\vec{B}_A = 1.0\hat{j}\text{ T} + \frac{1}{c^2}(2.0\times10^6\hat{i}\text{ m/s})\times(1.0\times10^6\hat{k}\text{ V/m}) = 1.0\hat{j}\text{ T} - \frac{2.0\times10^{12}\text{ V/m}}{(3.0\times10^8\text{ m/s})^2}\hat{j} = 0.99998\hat{j}\text{ T}$$

Assess: Although $B_A < B_V$, you need five significant figures of accuracy to tell the difference between them.

34.5. Model: Use the Galilean transformation of fields.

Visualize: We are given $\vec{v}_{BA} = 1.0\times10^6\hat{i}$ m/s, $\vec{B}_A = 0.50\hat{k}$ T, and $\vec{E}_A = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)\times10^6$ V/m.

Solve: Equation 34.10 gives the Galilean transformation equation for the electric field in different frames:

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA}\times\vec{B}_A \qquad \vec{B}_B = \vec{B}_A - \frac{1}{c^2}\vec{v}_{BA}\times\vec{E}_A$$

The electric field from the moving rocket is

$$\vec{E}' = (\hat{i}+\hat{j})0.707\times10^6\text{ V/m} + (1.0\times10^6\hat{i}\text{ m/s})\times(0.50\hat{k}\text{ T}) = (0.707\times10^6\hat{i} + 0.207\times10^6\hat{j})\text{ V/m}$$

$$\theta = \tan^{-1}\left(\frac{0.207\times10^6\text{ V/m}}{0.707\times10^6\text{ V/m}}\right) = 16.3^\circ\text{ above the }x\text{-axis}$$

Section 34.2 The Field Laws Thus Far

Section 34.3 The Displacement Current

34.6. Model: The net magnetic flux over a closed surface is zero.

Solve: Because we can't enclose a "net pole" within a surface, $\Phi_m = \oint\vec{B}\cdot d\vec{A} = 0$. Since the magnetic field is uniform over each face of the box, the total magnetic flux around the box is

$$(1\text{ cm}\times2\text{ cm})(2\text{ T} - 2\text{ T} - 1\text{ T} + 3\text{ T}) + (1\text{ cm}\times1\text{ cm})(2\text{ T}) + \Phi_{\text{unknown}} = 0$$
$$\Rightarrow \Phi_{\text{unknown}} = \int\vec{B}_{\text{unknown}}\cdot d\vec{A} = -0.0006\text{ T m}^2 \Rightarrow B_{\text{unknown}}\cos\theta = -6\text{ T}$$

The angle θ must be 180°. Because θ is the angle between $\vec{B}$ and the outward normal of $d\vec{A}$, the field $\vec{B}$ is directed into the face.

34.7. Solve: The units of $\varepsilon_0(d\Phi_e/dt)$ are

$$\frac{C^2}{N\,m^2}\times\frac{(N/C)(m^2)}{s}=\frac{C}{s}=A$$

34.8. Solve: The displacement current is defined as $I_{disp}=\varepsilon_0(d\Phi_e/dt)$. The electric flux inside a capacitor with plate area A is $\Phi_e = EA$. The electric field inside a capacitor is $E=\eta/\varepsilon_0=(Q/A)/\varepsilon_0$, and thus the electric flux is

$$\Phi_e = EA = \frac{Q}{\varepsilon_0}=\frac{CV_C}{\varepsilon_0}$$

where V_C is the capacitor voltage. The capacitance C is constant, hence the displacement current is

$$I_{disp}=\varepsilon_0\frac{d\Phi_e}{dt}=\varepsilon_0\frac{d}{dt}\left(\frac{CV_C}{\varepsilon_0}\right)=C\frac{dV_C}{dt}$$

34.9. Model: Use the results of Exercise EX34.8.
Solve:

$$I_{disp}=C\frac{dV_C}{dt}\quad\Rightarrow\quad C=\frac{I_{disp}}{\frac{dV_C}{dt}}=\frac{1.0\text{ A}}{1.0\times10^6\text{ V/s}}=1.0\ \mu\text{F}$$

Assess: This is a typical capacitance.

34.10. Model: The electric field inside a parallel-plate capacitor is uniform. As the capacitor is charged, the changing electric field induces a magnetic field.
Visualize: The induced magnetic field lines are circles concentric with the capacitor. Please refer to Figure 34.18.
Solve: (a) The Ampere-Maxwell law is

$$\oint\vec{B}\cdot d\vec{s}=\varepsilon_0\mu_0\frac{d\Phi_e}{dt}=\varepsilon_0\mu_0A\frac{dE}{dt}$$

where EA is the electric flux through the circle of radius r. The magnetic field is everywhere tangent to the circle of radius r, so the integral of $\vec{B}\cdot d\vec{s}$ around the circle is simply $B(2\pi r)$. The Ampere-Maxwell law becomes

$$2\pi rB=\varepsilon_0\mu_0(\pi r^2)\frac{dE}{dt}\Rightarrow B=\varepsilon_0\mu_0\frac{r}{2}\frac{dE}{dt}$$

On the axis, $r = 0$ m, so $B = 0$ T.
(b) At $r = 3.0$ cm,

$$B=\frac{1}{(3.0\times10^8\text{ m/s})^2}\left(\frac{0.030\text{ m}}{2}\right)(1.0\times10^6\text{ V/m s})=1.67\times10^{-13}\text{ T}$$

(c) For $r > 5.0$ cm, the electric flux Φ_e is the flux through a 10-cm-diameter circle because $E = 0$ V/m outside the capacitor plates. The Ampere-Maxwell law is

$$B(2\pi r)=\varepsilon_0\mu_0\pi R^2\frac{dE}{dt}$$

$$\Rightarrow B=\varepsilon_0\mu_0\frac{R^2}{2r}\frac{dE}{dt}=\frac{1}{(3.0\times10^8\text{ m/s})^2}\frac{(0.050\text{ m})^2}{2(0.07\text{ m})}(1.0\times10^6\text{ V/m s})=1.98\times10^{-13}\text{ T}$$

34.11. Model: The displacement current is numerically equal to the current in the wires leading to and from the capacitor.
Solve: The process of charging increases the charge on the plates of a parallel-plate capacitor. The charge Q on a capacitor plate at time t is $Q = CV_C$, where V_C is the voltage across the capacitor plates. Taking the derivative,

$$I_{disp} = I = \frac{dQ}{dt} = C\frac{dV_C}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV_C}{dt} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)\pi(0.025\ \text{m})^2}{0.50\times10^{-3}\ \text{m}}(500{,}000\ \text{V/s}) = 17\ \mu\text{A}$$

Section 34.5 Electromagnetic Waves

34.12. Model: The electric and magnetic field amplitudes of an electromagnetic wave are related.
Solve: Using Equation 34.29,

$$E_0 = cB_0 = (3.0\times10^8\ \text{m/s})(2.0\times10^{-3}\ \text{T}) = 6.0\times10^5\ \text{V/m}$$

34.13. Model: The electric and magnetic field amplitudes of an electromagnetic wave are related.
Solve: Using Equation 34.29,

$$B_0 = \frac{E_0}{c} = \frac{10\ \text{V/m}}{3\times10^8\ \text{m/s}} = 3.3\times10^{-8}\ \text{Vs/m}^2 = 3.3\times10^{-8}\ \text{T}$$

34.14. Model: Electromagnetic waves are sinusoidal.
Solve: **(a)** The magnetic field is $B_z = B_0 \sin(kx - \omega t)$, where $B_0 = 3.00\ \mu\text{T}$ and $k = 1.00\times10^7\ \text{m}^{-1}$. The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.00\times10^7\ \text{m}^{-1}} = 6.28\times10^{-7}\ \text{m} = 628\ \text{nm}$$

(b) The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^8\ \text{m/s}}{6.28\times10^{-7}\ \text{m}} = 4.77\times10^{14}\ \text{Hz}$$

(c) The electric field amplitude is

$$E_0 = cB_0 = (3\times10^8\ \text{m/s})(3.00\times10^{-6}\ \text{T}) = 900\ \text{V/m}$$

34.15. Model: Electromagnetic waves are sinusoidal.
Solve: **(a)** The electric field is $E_y = E_0 \cos(kx - \omega t)$, where $E_0 = 20.0\ \text{V/m}$ and $k = 6.28\times10^8\ \text{m}^{-1}$. The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{6.28\times10^8\ \text{m}^{-1}} = 1.00\times10^{-8}\ \text{m} = 10.0\ \text{nm}$$

(b) The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^8\ \text{m/s}}{1.00\times10^{-8}\ \text{m}} = 3.00\times10^{16}\ \text{Hz}$$

(c) The magnetic field amplitude is

$$B_0 = \frac{E_0}{v_{em}} = \frac{20.0\ \text{V/m}}{3.0\times10^8\ \text{m/s}} = 6.67\times10^{-8}\ \text{T}$$

Section 34.6 Properties of Electromagnetic Waves

34.16. Model: $\vec{E}$ and $\vec{B}$ are perpendicular to each other and $\vec{E}\times\vec{B}$ is in the direction of $\vec{v}_{\text{em}}$.

Solve: At any point, the Poynting vector $\vec{S}\equiv\mu_0{}^{-1}\vec{E}\times\vec{B}$ points in the direction that an electromagnetic wave is traveling. We have $\vec{B}=B\hat{i}$ and $\vec{S}=-S\hat{j}$. So,

$$-S\hat{j}=\frac{1}{\mu_0}\vec{E}\times B\hat{i}\Rightarrow\vec{E}=-E\hat{k}$$

The electric field points in the negative z-direction.

34.17. Model: The electric and magnetic field amplitudes of an electromagnetic wave are related to each other.
Solve: (a) Using Equation 34.29,

$$B_0=\frac{1}{c}E_0=\frac{100\text{ V/m}}{3.0\times10^8\text{ m/s}}=3.33\times10^{-7}\text{ T}$$

(b) From Equation 34.36, the intensity of an electromagnetic wave is

$$I=\frac{c\varepsilon_0}{2}E_0^2=\frac{(3\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)}{2}(100\text{ V/m})^2=13.3\text{ W/m}^2$$

34.18. Model: A radio signal is an electromagnetic wave.
Solve: From Equation 34.36, the intensity of an electromagnetic wave is

$$I=\frac{c\varepsilon_0}{2}E_0^2=\frac{(3\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)}{2}(300\times10^{-6}\text{ V/m})^2=1.195\times10^{-10}\text{ W/m}^2$$

34.19. Model: The laser beam is an electromagnetic plane wave.
Solve: First compute the intensity of the beam.

$$I=\frac{P}{A}=\frac{P}{\pi r^2}=\frac{1.0\times10^{-3}\text{ W}}{\pi(0.00050\text{ m})^2}=1270\text{ W/m}^2$$

Relate this intensity to the electric field amplitude:

$$E_0=\sqrt{\frac{2I}{c\varepsilon_0}}=\sqrt{\frac{2(1270\text{ W/m}^2)}{(3.00\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)}}=978\text{ V/m}\approx980\text{ V/m}$$

Use the electric field amplitude to get the magnetic field amplitude:

$$B_0=\frac{E_0}{c}=\frac{978\text{ V/m}}{3.0\times10^8\text{ m/s}}=3.26\text{ T}\approx3.3\text{ T}$$

Assess: This is a sizable electric field, comparable to the electric field near a charged glass or plastic rod.

34.20. Model: The laser beam is an electromagnetic plane wave. Assume that the energy is uniformly distributed over the diameter of the laser beam.
Solve: (a) Using Equation 34.36, the light intensity is

$$I=\frac{P}{A}=\frac{c\varepsilon_0}{2}E_0^2\Rightarrow\frac{200\times10^6\text{ W}}{\pi(1.0\times10^{-6}\text{ m})^2}=\frac{(3\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)}{2}E_0^2\Rightarrow E_0=2.2\times10^{11}\text{ V/m}$$

(b) The electric field between the proton and the electron is

$$E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}=\frac{(9\times10^9\text{ N m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})}{(0.053\times10^{-9}\text{ m})^2}=5.1\times10^{11}\text{ V/m}$$

The ratio of the laser beam's electric field to this field is

$$\frac{2.2\times10^{11}\ \text{V/m}}{5.1\times10^{11}\ \text{V/m}}=0.43$$

The laser beam's electric field is approximately half the electric field that keeps the electron in its orbit.

34.21. Model: A radio wave is an electromagnetic wave.
Solve: (a) The energy transported per second by the radio wave is 25 kW, or 25×10^3 J/s. This energy is carried uniformly in all directions. From Equation 34.36, the light intensity is

$$I=\frac{P}{A}=\frac{P}{4\pi r^2}=\frac{25\times10^3\ \text{W}}{4\pi(30\times10^3\ \text{m})^2}=2.2\times10^{-6}\ \text{W/m}^2$$

(b) Using Equation 34.36 again,

$$I=\frac{c\epsilon_0}{2}E_0^2\Rightarrow 2.2\times10^{-6}\ \text{W/m}^2=\frac{(3\times10^8\ \text{m/s})(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)}{2}E_0^2\Rightarrow E_0=0.041\ \text{V/m}$$

34.22. Solve: Substitute $E=cB$ in the equation for intensity to get it in terms of B_0.

$$I=\frac{P}{A}=\frac{P}{\pi r^2}=\frac{E_0^2}{2c\mu_0}=\frac{(cB_0)^2}{2c\mu_0}=\frac{cB_0^2}{2\mu_0}$$

Then solve for r.

$$\frac{P}{\pi r^2}=\frac{cB_0^2}{2\mu_0}\Rightarrow r^2=\frac{2\mu_0 P}{\pi cB_0^2}\Rightarrow$$

$$r=\sqrt{\frac{2\mu_0 P}{\pi cB_0^2}}=\sqrt{\frac{2(4\pi\times10^{-7}\ \text{Tm/A})(10\ \text{W})}{\pi(3.0\times10^8\ \text{m/s})(1.0\ \mu\text{T})^2}}=16\ \text{cm}$$

34.23. Model: An object gains momentum when it absorbs electromagnetic waves.
Solve: The radiation force on an object that absorbs all the light is

$$F=\frac{P}{c}=\frac{1000\ \text{W}}{3.0\times10^8\ \text{m/s}}=3.3\times10^{-6}\ \text{N}$$

Assess: The force is independent of the size of the beam and the wavelength.

Section 34.7 Polarization

34.24. Model: A polarized radio wave is an electromagnetic wave.
Solve: (a) From Equation 34.29,

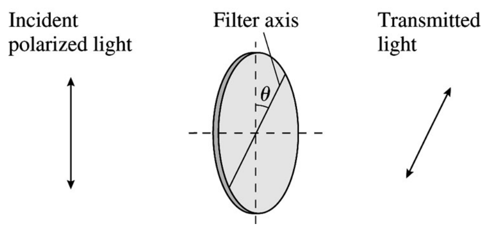
$$B_0=\frac{E_0}{c}=\frac{1000\ \text{V/m}}{3.0\times10^8\ \text{m/s}}=3.33\times10^{-6}\ \text{T}$$

(b) Likewise,

$$B=\frac{E}{c}=\frac{500\ \text{V/m}}{3.0\times10^8\ \text{m/s}}=1.67\times10^{-6}\ \text{T}$$

The direction of the wave is into the page ($-\hat{k}$ direction) and $\vec{E}$ is down ($-\hat{j}$ direction). Using the right-hand rule and $\vec{S}=\mu_0^{-1}\vec{E}\times\vec{B}$, $\vec{B}$ is to the left ($-\hat{i}$ direction).

34.25. Model: Use Malus's law for the polarized light.
Visualize:

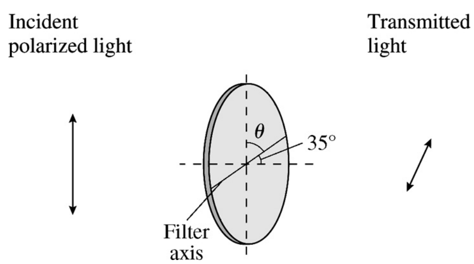


Solve: From Equation 34.41, the relationship between the incident and the transmitted polarized light is $I_{\text{transmitted}} = I_0 \cos^2\theta$ where θ is the angle between the electric field and the axis of the filter. Therefore,

$$0.25\, I_0 = I_0 \cos^2\theta \Rightarrow \cos\theta = 0.50 \Rightarrow \theta = 60°$$

Assess: Note that θ is the angle between the electric field and the axis of the filter.

34.26. Model: Use Malus's law for the polarized light.
Visualize:

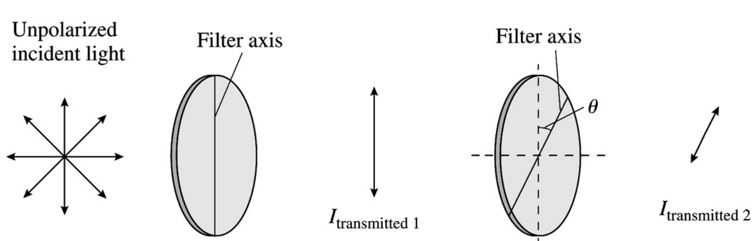


Solve: We can use Equation 34.41 for transmitted power as well as intensity if the area of the beam does not change. The transmitted polarized light is

$$P_{\text{transmitted}} = P_0 \cos^2\theta = (200\ \text{mW})\cos^2(90° - 35°) = 66\ \text{mW}$$

Assess: Note that θ is the angle between the electric field and the axis of the filter, which is $90° - 35° = 55°$.

34.27. Model: Use Malus's law for polarized light.
Visualize:



Solve: For unpolarized light, the electric field vector varies randomly through all possible values of θ. Because the *average* value of $\cos^2\theta$ is $\frac{1}{2}$, the intensity transmitted by a polarizing filter is $I_{\text{transmitted}} = \frac{1}{2}I_0$. On the other hand, for polarized light $I_{\text{transmitted}} = I_0\cos^2\theta$. Therefore,

$$I_{\text{transmitted 2}} = I_{\text{transmitted 1}}\cos^2\theta = \tfrac{1}{2}I_0\cos^2\theta = \tfrac{1}{2}(350\ \text{W/m}^2)\cos^2\theta = 131\ \text{W/m}^2 \Rightarrow$$

$$\cos^2\theta = \frac{131\ \text{W/m}^2}{\frac{1}{2}(350\ \text{W/m}^2)} \Rightarrow \theta = \cos^{-1}\sqrt{\frac{131\ \text{W/m}^2}{\frac{1}{2}(350\ \text{W/m}^2)}} = 30°$$

Assess: Note that any particular wave has a clear polarization. It is only in a "sea" of waves that the resultant wave has no polarization.

34.28. Model: Electric and magnetic fields exert forces on charged particles. Assume the fields are uniform.
Visualize: The electric field is in the direction of the positive *y*-axis and the magnetic field is in the direction of the negative *z*-axis.
Solve: Substituting into the Lorentz force law,

$$\begin{aligned}\vec{F}_{\text{net}} &= q(\vec{E} + \vec{v}\times\vec{B}) = (1.6\times10^{-19}\ \text{C})(-1.0\times10^{6}\hat{i}\ \text{V/m} + (1.0\times10^{7}\hat{i}\ \text{m/s})\times(-0.10\hat{k}\ \text{T}))\\ &= 1.6\times10^{-13}(-\hat{i} + \hat{j})\ \text{N}\end{aligned}$$

$$\Rightarrow F_{\text{net}} = \sqrt{(1.6)^2 + (1.6)^2}\times10^{-13}\ \text{N} = 2.3\times10^{-13}\ \text{N} \qquad \theta = \tan^{-1}\left(\frac{1.6\times10^{-13}\ \text{N}}{1.6\times10^{-13}\ \text{N}}\right) = 45°$$

The angle θ is measured counterclockwise from the vertical.

34.29. Model: Assume the electric and magnetic fields are uniform.
Solve: The force on the proton, which is the sum of the electric and magnetic forces, is

$$\vec{F} = \vec{F}_{\text{E}} + \vec{F}_{\text{B}} = -F\cos30°\hat{i} + F\sin30°\hat{j} = (-2.77\hat{i} + 1.60\hat{j})\times10^{-13}\ \text{N}$$

Since $\vec{v}$ points out of the page, the magnetic force is $\vec{F}_{\text{B}} = e\vec{v}\times\vec{B} = 1.60\times10^{-13}\,\hat{j}$ N. Thus

$$\vec{F}_{\text{E}} = e\vec{E} = \vec{F} - \vec{F}_{\text{B}} = -2.77\times10^{-13}\,\hat{i}\ \text{N} \Rightarrow \vec{E} = \vec{F}_{\text{E}}/e = -1.73\times10^{6}\,\hat{i}\ \text{V/m}$$

That is, the electric field is $\vec{E} = (1.73\times10^{6}$ V/m, left).

34.30. Model: Assume that the electric and magnetic fields are uniform fields.
Visualize: The magnetic force on the negative electron by the right-hand rule is directed *downward*. So that the electron is undeflected, we must apply an electric field to cause an electric force directed upward. That is, the electric field must point *downward*.
Solve: For the electron to not deflect,

$$F_{\text{B}} = F_{\text{E}} \Rightarrow e\left|\vec{v}\times\vec{B}\right| = eE \Rightarrow E = vB\sin90° = (2.0\times10^{7}\ \text{m/s})(0.010\ \text{T}) = 2.0\times10^{5}\ \text{V/m}$$

34.31. Model: Use the Galilean transformation of fields. Assume that the electric and magnetic fields are uniform inside the capacitor.
Visualize: The laboratory frame is the A frame and the proton's frame is the B frame.
Solve: (a) The electric field is directed downward, and thus the electric force on the proton is downward. The magnetic field $\vec{B}$ is oriented so that the force on the proton is directed upward. Use of the right-hand rule tells us that the magnetic field is directed into the page. The magnitude of the magnetic field is obtained from setting the magnetic force equal to the electric force, yielding the equation $evB = eE$. Solving for B,

$$B = \frac{E}{v} = \frac{1.0\times10^{5}\ \text{V/m}}{1.0\times10^{6}\ \text{m/s}} = 0.10\ \text{T}$$

Thus $\vec{B} = (0.10$ T, into page).

(b) In the B frame, the magnetic and electric fields are

$$\vec{B}_B = \vec{B}_A - \frac{1}{c^2}\vec{v}_{BA} \times \vec{E}_A = -0.10\hat{k}\ \text{T} - \frac{(1.0\times10^6\,\hat{j}\ \text{m/s})\times(1.0\times10^5\,\hat{i}\ \text{V/m})}{(3.0\times10^8\ \text{m/s})^2} \approx -0.10\hat{k}\ \text{T}$$

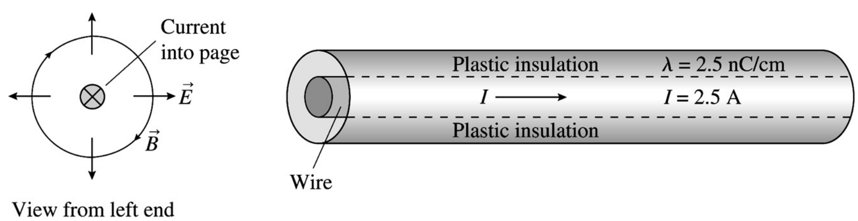
$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A = 1.0\times10^5\,\hat{i}\ \frac{\text{V}}{\text{m}} + (1.0\times10^6\,\hat{j}\ \text{m/s})\times(-0.10\hat{k}\ \text{T}) = 0\ \frac{\text{V}}{\text{m}}$$

(c) There is no electric force in the proton's frame because $E_B = 0$, and there is no magnetic force because the proton is at rest in the B frame.

34.32. Model: Electric and magnetic fields exert forces on charged particles. Assume the fields are uniform.
Solve: Substituting into the Lorentz force law,

$$\vec{F}_{net} = q(\vec{E} + \vec{v}\times\vec{B}) = (-1.6\times10^{-19}\ \text{C})\left[2.0\times10^5(\hat{i}-\hat{j})\frac{\text{V}}{\text{m}} + (5.0\times10^6\,\hat{i}\ \text{m/s})\times(-0.10\hat{k}\ \text{T})\right]$$
$$= -(3.2\times10^{-14})(\hat{i}-\hat{j})\ \text{N} - (8.0\times10^{-14}\,\hat{j})\ \text{N} = (-3.2\times10^{-14}\,\hat{i} - 4.8\times10^{-14}\,\hat{j})\ \text{N}$$

34.33. Model: Use the Galilean transformation of fields.
Visualize:



A current of 2.5 A flows to the right through the wire, and the plastic insulation has a charge of linear density $\lambda = 2.5$ n C/cm.
Solve: The magnetic field B at a distance r from the wire is

$$\vec{B} = \left(\frac{\mu_0 I}{2\pi r},\ \text{clockwise seen from left}\right)$$

On the other hand, $\vec{E}$ is radially out along $\hat{r}$, that is,

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

As the mosquito is 1.0 cm from the center of the wire at the top of the wire,

$$B = \frac{(4\pi\times10^{-7}\ \text{T m/A})(2.5\ \text{A})}{2\pi(0.010\ \text{m})} = 5.0\times10^{-5}\ \text{T}$$

$$E = \frac{(2.5\times10^{-7}\ \text{C/m})(2)(9.0\times10^9\ \text{N m}^2/\text{C}^2)}{0.010\ \text{m}} = 4.5\times10^5\ \frac{\text{V}}{\text{m}}$$

where the direction of $\vec{B}$ is out of the page and the direction of $\vec{E}$ is radially outward. In the mosquito's frame (let us call it M), we want $\vec{B}_M = \vec{0}$ T. Thus,

$$\vec{B}_M = \vec{B} - \frac{1}{c^2}\vec{v}\times\vec{E} = 0 \Rightarrow \vec{B} = \frac{1}{c^2}\vec{v}\times\vec{E}$$

Because $\vec{v}\times\vec{E}$ must be in the direction of $\vec{B}$ and $\vec{E}$ is radially outward, according to the right-hand rule $\vec{v}$ must be along the direction of the current. The magnitude of the velocity is

$$v=\frac{c^2B}{E}=\frac{(3.0\times10^8\ \text{m/s})^2(5.0\times10^{-5}\ \text{T})}{4.5\times10^5\ \text{V/m}}=1.0\times10^7\ \text{m/s}$$

The mosquito must fly at 1.0×10^7 m/s parallel to the current. This is highly unlikely to happen unless the mosquito is from Planet Krypton, like Superman.

34.34. Model: Use the Galilean transformation of fields. Assume that the wire is infinite.
Visualize: The laboratory frame is frame W (wire) and the circular loop's frame is frame L.
Solve: **(a)** From Equation 26.15, the electric field $\vec{E}$ due to a wire at rest is

$$\vec{E}=\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}=\left(\frac{\lambda}{2\pi\varepsilon_0 r},\text{ away from wire}\right)$$

The magnetic field $\vec{B}$ at a point on the loop is zero because there are no moving charges. These are the fields measured in frame L.
(b) In frame L, $\vec{E}_\text{L}=\vec{E}_\text{W}+\vec{v}_\text{LW}\times\vec{B}_W=\vec{E}_\text{W}$ because $\vec{B}_\text{W}=0$ T. The magnetic field in frame L is

$$\vec{B}_\text{L}=\vec{B}_\text{W}-\frac{1}{c^2}\vec{v}_\text{LW}\times\vec{E}_\text{W}=0\ \text{T}-\frac{1}{c^2}\vec{v}_\text{LW}\times\hat{r}\,\frac{\lambda}{2\pi\varepsilon_0 r}=\left(\frac{1}{c^2\varepsilon_0}\frac{v\lambda}{2\pi r},\text{ into the page at the top}\right)$$

(c) Consider a segment of the wire of length Δx with charge $\Delta Q=\lambda\Delta x$. The experimenter in the loop's frame sees a charge on this segment passing by him/her with a velocity v to the left. Thus,

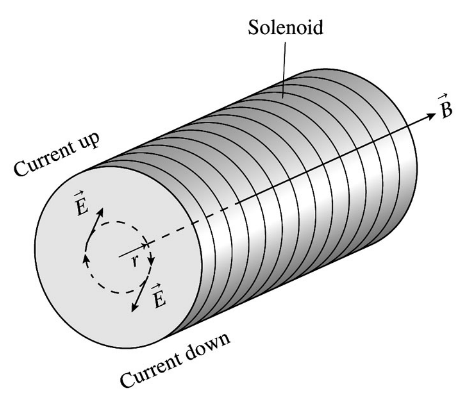
$$I=\frac{\Delta Q}{\Delta t}=\frac{\lambda\Delta x}{\Delta t}=\lambda v$$

(d) The magnetic field in the loop's frame is due to current I:

$$B_\text{L}=\frac{\mu_0 I}{2\pi r}=\frac{\mu_0\lambda v}{2\pi r}=\frac{\mu_0\lambda v}{2\pi r}\times\frac{\varepsilon_0}{\varepsilon_0}=\frac{1}{\varepsilon_0 c^2}\frac{\lambda v}{2\pi r}$$

Since the current I is moving to the left, the right-hand rule states the direction of B_L is into the page on the top. The electric field $\vec{E}_\text{L}$ which is due to the charge on the wire would be equal to $\vec{E}_\text{W}$.
(e) As we see, the results in parts (b) and (d) are the same.

34.35. Model: Use Faraday's law of electric induction and assume that the magnetic field inside the solenoid is uniform.
Visualize:



Equation 34.16 for Faraday's law is

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\left[\int \vec{B}\cdot d\vec{A}\right]$$

To solve this equation, choose a *clockwise* direction around a circle of radius r as the closed curve. The electric field vectors, as the figure shows, are everywhere tangent to the curve. The line integral of $\vec{E}$ then is

$$\oint \vec{E}\cdot d\vec{s} = E(2\pi r)$$

To do the surface integral, we need to know the sign of the flux or the integral $\int \vec{B}\cdot d\vec{A}$. Curl your right fingers around the circle in the clockwise direction. Your thumb points to the right, which is along the same direction as the magnetic field $\vec{B}$. That is, $\int \vec{B}\cdot d\vec{A} = B\pi r^2$ is positive.

Solve: **(a)** Since $B = 10.0\text{ T} + (2.0\text{ T})\sin[2\pi(10\text{ Hz})t]$, Faraday's law simplifies to

$$E(2\pi r) = -\pi r^2\frac{dB}{dt} = -\pi r^2(2.0\text{ T})[2\pi(10\text{ Hz})]\cos[2\pi(10\text{ Hz})t] \Rightarrow E = -\left(20.0\ \frac{\text{V}}{\text{m}^2}\right)\pi r\cos[2\pi(10\text{ Hz})t]$$

The field strength is maximum when the cosine function is equal to –1. Hence at $r = 1.5$ cm,

$$E_{\text{max}} = \left(20.0\ \frac{\text{V}}{\text{m}^2}\right)\pi(0.015\text{ m}) = 0.94\text{ V/m}$$

(b) E is maximum when $\cos[2\pi(10\text{ Hz})t] = -1$ which means when $\sin[2\pi(10\text{ Hz})t] = 0$. Under this condition,

$$B = (10.0\text{ T}) + (2.0\text{ T})\sin[2\pi(10\text{ Hz})t] = 10.0\text{ T}$$

That is, $B = 10.0$ T at the instant E has a maximum value of 0.94 V/m.

34.36. Model: Assume a uniform electric field inside the capacitor. The displacement current I_{disp} between the capacitor plates is numerically equal to the current I in the wires leading to and from the capacitor.

Solve: **(a)** From Equation 31.36, the voltage across the capacitor that develops during charging is $V_C = \mathcal{E}(1-e^{-t/\tau})$, where $\tau = RC$ and $\mathcal{E} = 25$ V is the battery emf. Since $Q = CV_C$, we have

$$I_{\text{disp}} = I = \frac{dQ}{dt} = C\frac{dV_C}{dt} = \frac{C\mathcal{E}}{\tau}e^{-t/\tau} \Rightarrow I_{\text{disp max}} = \frac{C\mathcal{E}}{\tau} = \frac{C\mathcal{E}}{RC} = \frac{\mathcal{E}}{R} = \frac{25\text{ V}}{150\ \Omega} = 0.17\text{ A}$$

The maximum electric flux through the capacitor plates is

$$\Phi_{\text{max}} = E_{\text{max}}A = \frac{V_{\text{max}}}{d}A = \frac{\mathcal{E}}{d}\frac{Cd}{\varepsilon_0} = \frac{\mathcal{E}C}{\varepsilon_0} = \frac{(25\text{ V})(2.5\times10^{-12}\text{ F})}{8.85\times10^{-12}\text{ C}^2/\text{N m}^2} = 7.1\text{ V m}$$

(b) As a function of time, the flux is

$$\Phi = \frac{V_C C}{\varepsilon_0} = \frac{C}{\varepsilon_0}\mathcal{E}(1-e^{-t/\tau}) = \Phi_{\text{max}}(1-e^{-t/\tau})$$

$$\Rightarrow \Phi = (7.1\text{ V m})\left[1-e^{-(0.50\times10^{-9}\text{ s})/(2.5\times10^{-12}\text{ F}\times150\ \Omega)}\right] = 5.2\text{ V m}$$

Likewise, from part (a) the displacement current is

$$I_{\text{disp}} = \frac{C\mathcal{E}}{\tau}e^{-t/\tau} = (0.167\text{ A})e^{-(0.50\times10^{-9}\text{ s})/(2.5\times10^{-12}\text{ F}\times150\ \Omega)} = 0.044\text{ A}$$

34.37. Model: Use Equation 34.20 for the definition of the displacement current.

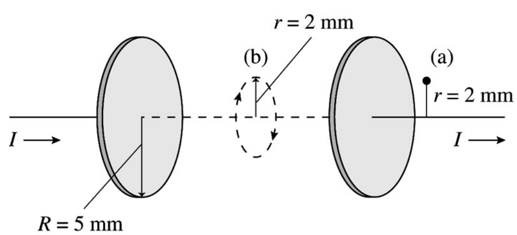
Solve: The current in a conductor arises from the electric field E in the conductor. From Equation 30.17,

$$J = \frac{I}{A} = \sigma E \Rightarrow \frac{dI}{dt} = \frac{d}{dt}(\sigma EA) = \sigma\frac{d}{dt}(EA) = \sigma\frac{d}{dt}\Phi_e = \sigma\frac{I_{\text{disp}}}{\varepsilon_0}$$

where $\Phi_e = EA$ is the electric flux through the wire and, by definition, $I_{disp} = \varepsilon_0 d\Phi_e/dt$. Thus $I_{disp} = (\varepsilon_0/\sigma)dI/dt$.
(b) Using the value for the conductivity of copper wire from Table 30.2,

$$I_{disp} = \frac{\varepsilon_0}{\sigma}\frac{dI}{dt} = \frac{8.85\times10^{-12}\ \text{C}^2/\text{N m}^2}{6.0\times10^{7}\,\Omega^{-1}\ \text{m}^{-1}}(1.0\times10^{6}\ \text{A/s}) = 1.5\times10^{-13}\ \text{A}$$

34.38. Model: Assume the electric field inside the capacitor is uniform and use the Ampere-Maxwell law.
Visualize:



Solve: (a) For a current-carrying wire, Example 32.3 yields an equation for the magnetic field strength:

$$B = \frac{\mu_0 I_{wire}}{2\pi r} = \frac{(4\pi\times10^{-7}\ \text{T m/A})(10\ \text{A})}{(2\pi)(2.0\times10^{-3}\ \text{m})} = 1.0\times10^{-3}\ \text{T}$$

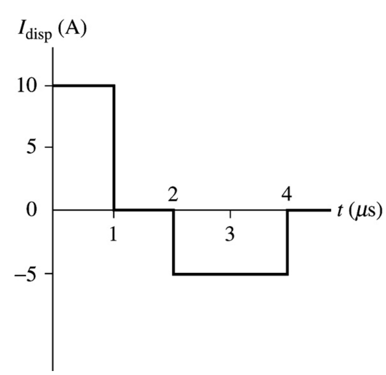
(b) Example 34.3 found that the induced magnetic field inside a charging capacitor is

$$B = \frac{\mu_0}{2\pi}\frac{r}{R^2}\frac{dQ}{dt} = \frac{\mu_0}{2\pi}\frac{r}{R^2}I_{wire}$$

where we used $I_{wire} = dQ/dt$ as the actual current in the wire leading to the capacitor. At $r = 2.0$ mm,

$$B = \frac{(4\pi\times10^{-7}\ \text{T m/A})(2.0\times10^{-3}\ \text{m})}{2\pi\,(5.0\times10^{-3}\ \text{m})^2}(10\ \text{A}) = 1.60\times10^{-4}\ \text{T}$$

34.39. Model: The displacement current through a capacitor is the same as the current in the connecting wires.
Solve:

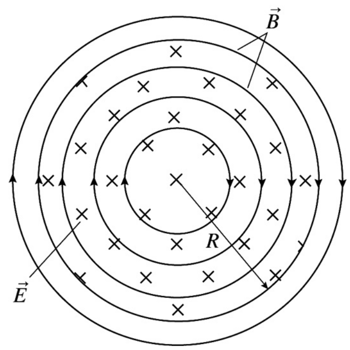


We have

$$I_{disp} = \varepsilon_0\frac{d\Phi_e}{dt} = \varepsilon_0\frac{d}{dt}\left(\frac{CV_c}{\varepsilon_0}\right) = C\frac{dV_c}{dt}$$

The displacement current is the slope of the V_c vs. t curve times C.

34.40. Model: Assume that the electric field inside the cylinder is uniform. Use the Ampere-Maxwell law to obtain the induced magnetic field.
Visualize:



Solve: (a) The electric flux through the entire cylinder is $\Phi = \int \vec{E}\cdot d\vec{A}$. For a closed curve of radius R, assuming a clockwise direction around the ring, the right-hand rule places the thumb into the page, which is the same direction as that of the electric field $\vec{E}$. Thus, we will take the flux Φ to have a positive sign:

$$\Phi = E(\pi R^2) = \left(1.0\times10^8 t^2\ \frac{\text{V}}{\text{m}}\right)[\pi(3.0\times10^{-3}\ \text{m})^2] = (2.8\times10^3 t^2)\ \text{Vm}$$

(b) From the Ampere-Maxwell law, with $I_{\text{through}} = 0$,

$$\oint \vec{B}\cdot d\vec{s} = \varepsilon_0\mu_0\frac{d\Phi_e}{dt} = \varepsilon_0\mu_0\frac{d\Phi_e}{dt}$$

Because Φ_e is positive and the field strength increases with time, $d\Phi_e/dt$ is positive. The line integral is therefore positive, which means that B is in the clockwise direction. This reasoning applies to both the $r < R$ and $r > R$ regions. Thus the field lines are clockwise, and they are shown in the figure.
(c) We can now make use of the Ampere-Maxwell law to find the magnitude of B for $r < R$. Based upon parts (a) and (b),

$$B(2\pi r) = \varepsilon_0\mu_0\frac{dE}{dt}(\pi r^2) \Rightarrow B = \varepsilon_0\mu_0\left(\frac{r}{2}\right)\frac{dE}{dt} = \frac{r}{2c^2}\left[\left(1.0\times10^8\ \frac{\text{V}}{\text{m}}\right)2t\right] = (1.11\times10^{-9}\ rt)\ \text{T}$$

where r is in m and t is in s.
At $r = 2$ mm and $t = 2.0$ s,

$$B = (1.11\times10^{-9})(2.0\times10^{-3})(2.0)\ \text{T} = 4.4\times10^{-12}\ \text{T}$$

(d) For $r > R$, the flux is confined to a circle of radius R. Thus

$$B(2\pi r) = \varepsilon_0\mu_0\left(\frac{dE}{dt}\right)\pi R^2 \Rightarrow B = \frac{R^2}{2c^2 r}\left[\left(1.0\times10^8\ \frac{\text{V}}{\text{m}}\right)2t\right] = \left(1.00\times10^{-14}\ \frac{t}{r}\right)\text{T}$$

where r is in m and t is in s.
At $r = 4.0$ mm and $t = 2.0$ s,

$$B = \frac{1.00\times10^{-14}(2.0)}{(4.0\times10^{-3})}\ \text{T} = 5.0\times10^{-12}\ \text{T}$$

34.41. Model: The displacement current through a capacitor is the same as the current in the wires.
Solve: From Chapter 32, a discharging capacitor has circuit current

$$I = I_0 e^{-t/\tau} = \frac{(\Delta V_c)_0}{R}e^{-t/\tau}$$

Identifying $\tau = 2.0\ \mu\text{s} = \text{RC}$, we get $\text{R} = 2.0\ \Omega$. Thus

$$I_0 = \frac{(\Delta V_c)_0}{R} \Rightarrow (\Delta V_c)_0 = I_0 R = (10\ \text{A})(2.0\ \Omega) = 20\ \text{V}$$

34.42. Model: $\vec{E}$ and $\vec{B}$ are perpendicular in electromagnetic waves, and their magnitudes are related.
Solve: **(a)** Since $\vec{E} \perp \vec{B}$, the dot product must be zero.

$$\vec{E} \cdot \vec{B} = 0 = (200)(7.3) + (300)(-7.3) + (-50)a$$
$$\Rightarrow a = -14.6$$

Since B_0 multiplies the complete vector $\vec{B}$ it does not effect the calculation for a. Requiring $|\vec{E}| = c|\vec{B}|$ and squaring yields

$$((200)^2 + (300)^2 + (-50)^2)(\text{V/m})^2 = B_0{}^2 (3.0\times10^8 \text{ m/s})^2((7.3)^2 + (-7.3)^2 + (-14.6)^2)(\mu\text{T})^2$$
$$\Rightarrow B_0 = 6.8\times10^{-2}$$

(b) The Poynting vector is

$$\vec{S} = \mu_0{}^{-1}\vec{E}\times\vec{B}$$
$$= \mu_0{}^{-1}B_0(10^{-6})\begin{Bmatrix} [(300)(-14.6) - (-7.3)(-50)]\hat{i} \\ +[(-50)(7.3) - (-14.6)(200)]\hat{j} \\ +[(200)(-7.3) - (7.3)(300)]\hat{k} \end{Bmatrix}$$
$$= \mu_0{}^{-1}B_0(10^{-3})[-4.75\hat{i} + 2.56\hat{j} - 3.65\hat{k}] = -260\hat{i} + 140\hat{j} - 200\hat{k} \text{ W/m}^2$$

Assess: A quick check yields $\vec{E}\cdot\vec{S} = 0$ and $\vec{B}\cdot\vec{S} = 0$.

34.43. Solve: **(a)** The electric field energy density is $u_\text{E} = \frac{1}{2}\varepsilon_0 E^2$ and the magnetic field energy density is $u_\text{B} = (1/2\,\mu_0)B^2$. In an electromagnetic wave, the fields are related by $E = cB$. Using this and the fact that $c^2 = 1/(\varepsilon_0\mu_0)$, we find

$$u_\text{E} = \frac{\varepsilon_0}{2}E^2 = \frac{\varepsilon_0}{2}(cB)^2 = \frac{c^2\varepsilon_0}{2}B^2 = \frac{\varepsilon_0}{2\varepsilon_0\mu_0}B^2 = \frac{1}{2\mu_0}B^2 = u_\text{B}$$

(b) Since the energy density is equally divided between the electric field and the magnetic field, the total energy density in an electromagnetic wave is $u_\text{tot} = u_\text{E} + u_\text{B} = 2u_\text{E}$. We also know that the wave intensity is $I = \frac{1}{2}c\varepsilon_0 E_0^2$. Thus

$$u_\text{tot} = 2u_\text{E} = \varepsilon_0 E_0^2 = \frac{2}{c}\left(\frac{1}{2}c\varepsilon_0 E_0^2\right) = \frac{2I}{c} = \frac{2(1000 \text{ W/m}^2)}{3.0\times10^8 \text{ m/s}} = 6.67\times10^{-6} \text{ J/m}^3$$

34.44. Model: Light is an electromagnetic wave.
Solve: 100 W is the energy transported per second by the electromagnetic light wave. This energy is carried in all directions. The light intensity is given by Equation 34.36:

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{c\varepsilon_0}{2}E_0^2$$
$$\Rightarrow \frac{100 \text{ W}}{4\pi(3.5\times10^{-2} \text{ m})^2} = \frac{(3\times10^8 \text{ m/s})(8.85\times10^{-12} \text{ C}^2/\text{N m}^2)}{2}E_0^2 \Rightarrow E_0 = 2212\frac{\text{V}}{\text{m}} \approx 2200 \frac{\text{V}}{\text{m}}$$
$$\Rightarrow B_0 = \frac{E_0}{c} = \left(\frac{2212 \text{ V/m}}{3.0\times10^8 \text{ m/s}}\right) \approx 7.4\times10^{-6} \text{ T}$$

34.45. Model: Sunlight is an electromagnetic wave.
Solve: **(a)** The sun's energy is transported by the electromagnetic waves in all directions. From Equation 34.36, the light intensity is

$$I = \frac{P}{A} \Rightarrow P = IA = (1360 \text{ W/m}^2)(4\pi R_\text{sun-earth}^2) = (1360 \text{ W/m}^2)4\pi(1.50\times10^{11} \text{ m})^2 = 3.85\times10^{26} \text{ W}$$

(b) The intensity of sunlight at Mars is

$$I = \frac{P}{A} = \frac{(1360\ \text{W/m}^2)(4\pi R_{\text{earth}}^2)}{4\pi R_{\text{Mars}}^2} = (1360\ \text{W/m}^2)\left(\frac{1.50\times10^{11}\ \text{m}}{2.28\times10^{11}\ \text{m}}\right)^2 = 589\ \text{W/m}^2$$

34.46. Model: The microwave beam is an electromagnetic wave. The water does not lose heat during the process.
Solve: The rate of energy transfer from the beam to the cube is

$$P = (0.80)IA = (0.80)\frac{C\varepsilon_0}{2}E_0{}^2 A$$
$$= (0.80)\frac{(3\times10^8\ \text{m/s})(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2)}{2}(11\times10^3\ \text{V/m})^2(0.10\ \text{m})^2 = 1.29\ \text{kW}$$

The amount of energy required to raise the temperature by 50 °C is

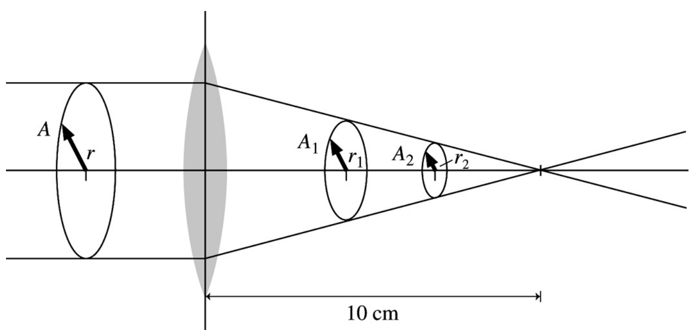
$$\Delta E = mc\Delta T = (0.10\ \text{m})^3(1000\ \text{kg/m}^3)(4186\ \text{J/kg/°C})(50\ \text{C}) = 2.09\times10^5\ \text{J}$$

The time required for the water to absorb this much energy from the microwave beam is

$$\Delta t = \frac{\Delta E}{P} = \frac{2.09\times10^5\ \text{J}}{1.29\times10^3\ \text{W}} = 162\ \text{s} \approx 160\ \text{s}$$

Assess: Raising 1 kg of water by 50°C in a microwave oven takes around 2–3 minutes, so this is reasonable.

34.47. Model: The converging lens is an ideal thin lens, and the laser beam is pointed along its principal axis.
Visualize:



Solve: **(a)** The lens will change the cross-sectional area A of the laser beam. The power of the beam does not change. Since the intensity $I \propto A^{-1}$, reducing the area by ¼ will increase I by a factor of 4. This occurs when the radius of the laser beam is halved, so that

$$A_1 = \pi r_1{}^2 = \pi\left(\frac{r}{2}\right)^2 = \frac{\pi r^2}{4} = \frac{A}{4}.$$

The distance past the lens is halfway to the focal point.
(b) Since $I \propto E_0{}^2$, quadrupling E_0 requires increasing I by a factor of 16, reducing the area by 16, requiring a reduction in radius of 4. The distance past the lens is ¾ of the way to the focal point.

34.48. Model: Radio waves are electromagnetic waves.
Solve: **(a)** The radio transmitter is radiating energy in all directions at the rate of 21 J per second. The signal intensity received at the earth is given by Equation 34.36:

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{21\ \text{W}}{4\pi(4.5\times10^{12}\ \text{m})^2} = 8.3\times10^{-26}\ \text{W/m}^2$$

(b) Using Equation 34.36 again,

$$I = \frac{c\varepsilon_0}{2}E_0^2 \Rightarrow E_0 = \sqrt{\frac{2I}{c\varepsilon_0}} = \sqrt{\frac{2(8.3\times10^{-26}\text{ W/m}^2)}{(3.0\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2\text{/N m}^2)}} = 7.9\times10^{-12}\ \frac{\text{V}}{\text{m}}$$

34.49. Model: Radio waves are electromagnetic waves. Assume that the transmitter unit radiates in all directions.
Solve: The transmitting unit radiates energy in all directions at the rate of 250 mJ per second. From Equation 34.36, the signal intensity at a distance of 42 m is

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{250\times10^{-3}\text{ W}}{4\pi(42\text{ m})^2} = 1.13\times10^{-5}\text{ W/m}^2$$

Using Equation 34.36 again,

$$I = \frac{c\varepsilon_0}{2}E_0^2 \Rightarrow E_0 = \sqrt{\frac{2I}{c\varepsilon_0}} = \sqrt{\frac{2(1.13\times10^{-5}\text{ W/m}^2)}{(3.0\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2\text{/N m}^2)}} = 0.092\ \frac{\text{V}}{\text{m}}$$

A few steps before 42 m, the field strength was 0.100 V/m and the door opened. The manufacturer's claims are correct.

34.50. Model: The laser beam is an electromagnetic wave.
Solve: The maximum intensity of the laser beam is determined by the maximum electric field strength in air. Thus the maximum power delivered by the beam is

$$\begin{aligned} P &= IA = \frac{c\varepsilon_0}{2}{E_0}^2 A \\ &= \frac{(3\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2\text{/Nm}^2)}{2}(3.0\times10^6\text{ V/m})^2\pi(0.050\text{ m})^2 \\ &= 9.4\times10^7\text{ W} \end{aligned}$$

34.51. Model: The laser beam is an electromagnetic plane wave.
Visualize: Power is energy/time: $P = 2.5\text{ mJ/10 ns} = 2.5\times10^5$ W. The beam has a circular cross section with radius $r = (0.85\text{ mm})/2 = 0.425$ mm.
Solve:

$$I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{E_0^2}{2c\mu_0} \Rightarrow$$

$$E_0 = \sqrt{\frac{2P}{c\varepsilon_0\pi r^2}} = \sqrt{\frac{2(2.5\times10^5\text{ W})}{(3.0\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2\text{/Nm}^2)\pi(0.425\text{ mm})^2}} = 1.8\times10^7\text{ V/m}$$

Assess: This is a very large field strength that accompanies powerful laser bursts.

34.52. Model: The earth is a complete absorber of sunlight. An object gains momentum when it absorbs electromagnetic waves.
Solve: **(a)** The radiation pressure on an object that absorbs all the light is

$$p_{\text{rad}} = \frac{I}{c} = \frac{1360\text{ W/m}^2}{3.0\times10^8\text{ m/s}} = 4.533\times10^{-6}\text{ Pa}$$

Seen from the sun, the earth is a circle of radius R_{earth} and area $A = \pi R_{\text{earth}}^2$. The pressure exerts a force on this area

$$F_{\text{rad}} = p_{\text{rad}}A = p_{\text{rad}}(\pi R_{\text{earth}}^2) = (4.533\times10^{-6}\text{ Pa})\pi(6.37\times10^6\text{ m})^2 = 5.78\times10^8\text{ N}$$

(b) The sun's gravitational force on the earth is

$$F_{\text{grav}} = \frac{GM_{\text{sun}}M_{\text{earth}}}{R^2_{\text{sun-earth}}} = \frac{(6.67\times10^{-11}\ \text{N m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(5.98\times10^{24}\ \text{kg})}{(1.50\times10^{11}\ \text{m})^2} = 3.53\times10^{22}\ \text{N}$$

$$\Rightarrow \frac{F_{\text{rad}}}{F_{\text{grav}}} = \frac{5.78\times10^{8}\ \text{N}}{3.53\times10^{22}\ \text{N}} = 1.64\times10^{-14}$$

That is, F_{rad} is $1.64\times10^{-12}\%$ of F_{grav}.

34.53. Visualize: Use subscript 1 for the 27 MHz waves and subscript 2 for the 2.4 GHz waves. Also recall that for EM waves $c = \lambda f$.

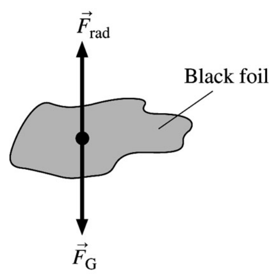
Solve: $d \propto \lambda^{1/2}$.

$$\frac{d_2}{d_1} = \left(\frac{\lambda_2}{\lambda_1}\right)^{1/2} = \left(\frac{c/f_2}{c/f_1}\right)^{1/2} = \sqrt{\frac{f_2}{f_1}}$$

$$d_2 = d_1\sqrt{\frac{f_2}{f_1}} = (14\ \text{cm})\sqrt{\frac{2.4\ \text{GHz}}{27\ \text{MHz}}} = 132\ \text{cm} \approx 1.3\ \text{m}$$

Assess: 1.3 m is farther than most of us are thick, so the 2.4 GHz waves go through people fairly well.

34.54. Model: Assume that the black foil absorbs the laser light completely. Use the particle model for the foil.
Visualize:

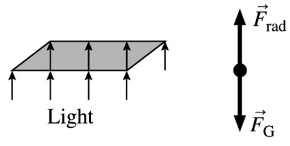


For the foil to levitate, the radiation-pressure force must be equal to the gravitational force on the foil.
Solve: (a) Using Equation 34.38,

$$F_{\text{rad}} = p_{\text{rad}}A = \frac{I}{c}A = \frac{P}{c} \Rightarrow P = cF_{\text{rad}} = cF_{\text{G}} = cmg$$

(b) $(3.0\times10^{8}\ \text{m/s})(25\times10^{-6}\ \text{kg})(9.8\ \text{m/s}^2) = 73.5\ \text{W}$

34.55. Model: Assume that the black paper absorbs the light completely. Use the particle model for the paper.
Visualize:



For the black paper to be suspended, the radiation-pressure force must be equal to the gravitational force on the paper.
Solve: From Equation 34.38, $F_{\text{rad}} = p_{\text{rad}}A = IA/c$. Hence,

$$I = \frac{c}{A}F_{\text{rad}} = \frac{c}{A}F_{\text{G}} = \frac{(3.0\times10^{8}\ \text{m/s})(1.0\times10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)}{(8.5\ \text{inch}\times11\ \text{inch})(2.54\times10^{-2}\ \text{m/inch})^2} = 4.9\times10^{7}\ \text{W/m}^2$$

34.56. Model: Assume that the block absorbs the laser light completely. Use the particle model for the block.
Solve: From Equation 34.38,

$$F_{\text{rad}} = p_{\text{rad}}A = \frac{P}{c} = \frac{25\times10^6 \text{ W}}{3.0\times10^8 \text{ m/s}} = 0.0833 \text{ N}$$

Applying Newton's second law,

$$F_{\text{rad}} = ma = (100 \text{ kg})a \Rightarrow a = \frac{F_{\text{rad}}}{100 \text{ kg}} = \frac{0.0833 \text{ N}}{100 \text{ kg}} = 8.33\times10^{-4} \text{ m/s}^2$$

From kinematics,

$$v_{\text{f}}^2 = v_{\text{i}}^2 + 2a(s_{\text{f}} - s_{\text{i}}) = 0 \text{ m}^2/\text{s}^2 + 2(8.33\times10^{-4} \text{ m/s}^2)(100 \text{ m}) \Rightarrow v_{\text{f}} = 0.408 \text{ m/s} \approx 0.41 \text{ m/s}$$

Assess: This does not seem like a promising method for launching satellites.

34.57. Model: Use the particle model for the astronaut.
Solve: According to Newton's third law, the force of the radiation on the astronaut is equal to the momentum delivered by the radiation. For this force we have

$$F = p_{\text{rad}}A = \frac{P}{c} = \frac{1000 \text{ W}}{3.0\times10^8 \text{ m/s}} = 3.333\times10^{-6} \text{ N}$$

Using Newton's second law, the acceleration of the astronaut is

$$a = \frac{3.333\times10^{-6} \text{ N}}{80 \text{ kg}} = 4.167\times10^{-8} \text{ m/s}^2$$

Using $v_{\text{f}} = v_{\text{i}} + a(t_{\text{f}} - t_{\text{i}})$ and a time equal to the lifetime of the batteries,

$$v_{\text{f}} = 0 \text{ m/s} + (4.167\times10^{-8} \text{ m/s}^2)(3600 \text{ s}) = 1.500\times10^{-4} \text{ m/s}$$

The distance traveled in the first hour is calculated as follows:

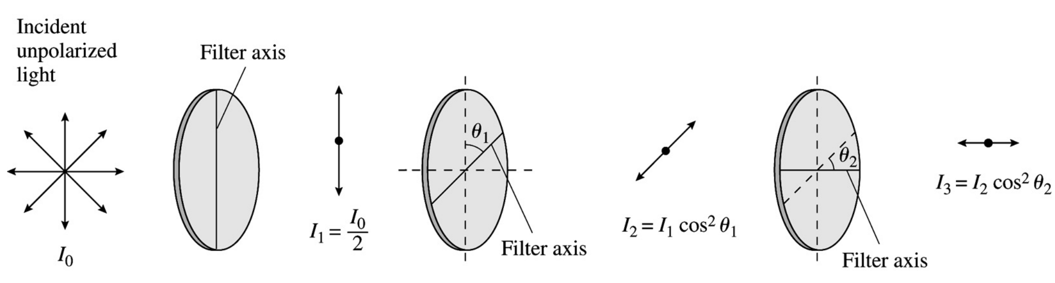
$$v_{\text{f}}^2 - v_{\text{i}}^2 = 2a(\Delta s)_{\text{first hour}}$$
$$\Rightarrow (1.500\times10^{-4} \text{ m/s})^2 - (0 \text{ m/s})^2 = 2(4.167\times10^{-8} \text{ m/s}^2)(\Delta s)_{\text{first hour}} \Rightarrow (\Delta s)_{\text{first hour}} = 0.270 \text{ m}$$

This means the astronaut must cover a distance of $5.0 \text{ m} - 0.27 \text{ m} = 4.73 \text{ m}$ in a time of 9 hours. The acceleration is zero during this time. The time it will take the astronaut to reach the space capsule is

$$\Delta t = \frac{4.73 \text{ m}}{1.500\times10^{-4} \text{ m/s}} = 31{,}533 \text{ s} = 8.76 \text{ hours} \approx 8.8 \text{ h}$$

Because this time is less than 9 hours, the astronaut is able to make it safely to the space capsule.

34.58. Model: Use Malus's law for the polarized light.
Visualize:



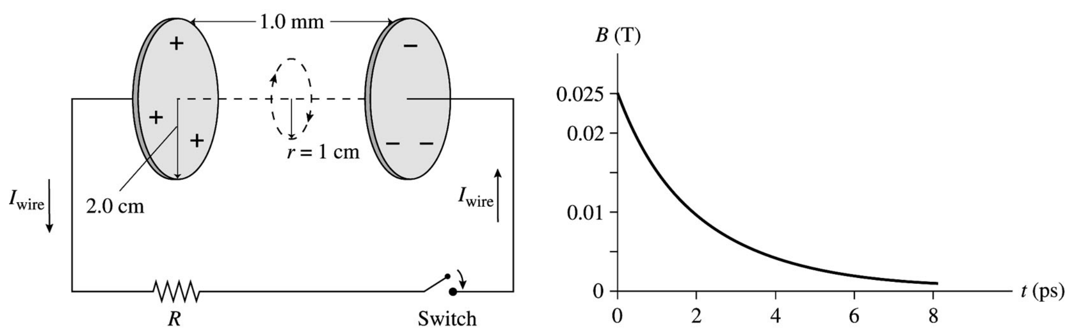
Solve: For unpolarized light, the electric field vector varies randomly through all possible values of θ. Because the average value of $\cos^2\theta$ is $\frac{1}{2}$, the intensity transmitted by a polarizing filter when the incident light is unpolarized is $I_1 = \frac{1}{2}I_0$. For polarized light, $I_{\text{transmitted}} = I_0\cos^2\theta$. Therefore,

$$I_2 = I_1\cos^2 45° \qquad I_3 = I_2\cos^2 45°$$
$$\Rightarrow I_3 = (I_1\cos^2 45°)\cos^2 45° = \tfrac{1}{2}I_0(\cos^4 45°) = \tfrac{1}{8}I_0$$

34.59. Model: Assume that the electric and magnetic fields are uniform fields.
Solve: Substituting into the Lorentz force law,

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) \Rightarrow (9.6\times10^{-14}\text{ N})(\hat{\imath} - \hat{k}) = -(1.60\times10^{-19}\text{ C})[\vec{E} + (5.0\times10^{6}\hat{\imath}\text{ m/s})\times(0.10\,\hat{\jmath}\text{ T})]$$
$$\Rightarrow \vec{E} = -(6.0\times10^{5}\text{ N/C})(\hat{\imath} - \hat{k}) - (5.0\times10^{5}\text{ N/C})\hat{k} = (-6.0\times10^{5}\hat{\imath} + 1.0\times10^{5}\hat{k})\text{ V/m}$$

34.60. Model: Assume that the electric field inside the capacitor is uniform. Use the Ampere-Maxwell law to find the magnetic field.
Visualize:



Solve: (a) This is an RC circuit with capacitance $C = \varepsilon_0 A/d = \varepsilon_0\pi R^2/d = 1.11\times10^{-11}\text{ F} = 11.1\text{ pF}$. We know from Chapter 32 that the current through the wire decays exponentially as

$$I_{\text{wire}} = \frac{\Delta V_C}{R}e^{-t/\tau} = (5000\text{ A})e^{-t/\tau}$$

where the time constant is $\tau = RC = 2.22\times10^{-12}\text{ s} = 2.22\text{ ps}$. Example 34.3 found that the induced magnetic field strength at radius r inside a charging or discharging capacitor is

$$B = \frac{\mu_0}{2\pi}\frac{r}{R^2}\frac{dQ}{dt} = \frac{\mu_0}{2\pi}\frac{r}{R^2}I_{\text{wire}}$$

where we used $I_{\text{wire}} = dQ/dt$ as the actual current in the wire leading to the capacitor. Thus

$$B = \frac{(4\pi\times10^{-7}\text{ T m/A})(1.0\times10^{-2}\text{ m})}{2\pi(2.0\times10^{-2}\text{ m})^2}(5000\text{ A})e^{-t/2.22\text{ ps}} = (0.025\text{ A})e^{-t/2.22\text{ ps}}$$

(b) The graph is shown above.

34.61. Model: The radar beam is an electromagnetic wave. There is no absorption of the beam by the atmosphere.
Solve: The intensity at the airplane is

$$I = \frac{P_{\text{source}}}{4\pi r^2} = \frac{(150\times10^3\text{ W})}{4\pi(30\times10^3\text{ m})^2} = 1.33\times10^{-5}\text{ W/m}^2$$

The airplane then acts as a source of microwaves with power

$$P = IA = (1.33\times10^{-5}\ \text{W/m}^2)(31\ \text{m}^2) = 4.11\times10^{-4}\ \text{W}$$

The intensity of the reflected radar beam at the airport is thus

$$I_{\text{airport}} = \frac{P}{4\pi r^2} = \frac{c\varepsilon_0}{2}E_0^{\,2} \Rightarrow E_0 = \sqrt{\frac{2P}{4\pi\varepsilon_0 c r^2}}$$
$$= \sqrt{\frac{2(4.11\times10^{-4}\ \text{W})}{4\pi(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2)(3\times10^8\ \text{m/s})(30\times10^3\ \text{m})^2}}$$
$$= 5.2\ \mu\text{V/m}$$

34.62. Model: Dust particles absorb sunlight completely.
Solve: Let R be the radius of a dust grain and ρ its density. The area of cross section of the dust particles is πR^2, its mass is $m = \rho\left(\frac{4\pi}{3}R^3\right)$. Letting d be the distance of the dust grain from the sun, the gravitational force on the dust grain is

$$F_G = \frac{GM_S m}{d^2}$$

On the other hand, the pressure on the grain due to the sun's electromagnetic radiation is

$$p_{\text{rad}} = \frac{I}{c} = \frac{P_{\text{sun}}}{(4\pi d^2)c} = \frac{F_{\text{rad}}}{A} = \frac{F_{\text{rad}}}{\pi R^2} \Rightarrow F_{\text{rad}} = \frac{R^2}{4d^2}\frac{P_{\text{sun}}}{c}$$

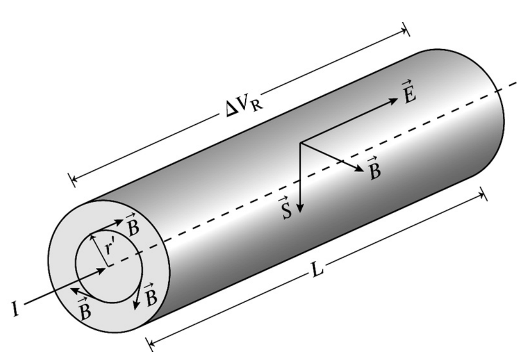
For the dust particles to remain in the solar system over long periods of time, $F_{\text{rad}} < F_G$. Hence,

$$\frac{R^2}{4d^2}\frac{P_{\text{sun}}}{c} < \frac{GM_S m}{d^2} = \frac{GM_S\rho 4\pi R^3}{3d^2} \Rightarrow R > \frac{3\,P_{\text{sun}}}{16\pi c G M_S \rho}$$

$$\Rightarrow R > \frac{3(3.9\times10^{26}\ \text{W})}{16\pi(3.0\times10^8\ \text{m/s})(6.67\times10^{-11}\ \text{N m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(2000\ \text{kg/m}^3)} = 2.9\times10^{-7}\ \text{m}$$
$$\Rightarrow \text{diameter } D = 2R < 0.58\ \mu\text{m}$$

For $D > 0.58\ \mu\text{m}$, the radiation pressure force is less than the sun's gravitational force, so the particle can orbit the sun. However, for $D < 0.58\ \mu\text{m}$, radiation pressure force is greater than the sun's gravitational force, so the particle is gradually pushed out of the solar system. Thus, the diameter of the dust particle that can remain in the solar system for a very long period of time is $0.58\ \mu\text{m}$.

34.63. Model: Use Ampere's law and assume that the current through the resistor is uniform.
Visualize:



Solve: (a) The electric field E in the resistor is

$$E = \frac{\Delta V_{\mathrm{R}}}{L} = \frac{IR}{L} = E_{\text{surface}}$$

For the magnetic field, we use the Ampere-Maxwell law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r} = B_{\text{surface}}$$

(b) The Poynting vector $\vec{S} = \mu_0^{-1}\vec{E} \times \vec{B}$ is directed into the curved surface. That is, into the wire. The magnitude is

$$S = \frac{1}{\mu_0} EB = \frac{1}{\mu_0}\frac{IR}{L}\frac{\mu_0 I}{2\pi r} = \frac{I^2 R}{2\pi r L}$$

(c) The flux over the surface of the resistor is

$$\int \vec{S} \cdot d\vec{A} = \int_{\text{faces}} \vec{S} \cdot d\vec{A} + \int_{\text{wall}} \vec{S} \cdot d\vec{A} = 0 - SA_{\text{wall}} = \frac{I^2 R}{2\pi r L}(2\pi r L) = -I^2 R$$

The Poynting vector is the power per unit area carried by electric and magnetic fields. Thus the Poynting flux (SA_{wall}) is electromagnetic power directed into the resistor. It matches the power dissipated by the resistor ($I^2 R$), which is what we expect from energy conservation.

34.64. Model: Use Malus's law for the polarized light.

Solve: For unpolarized light, the electric field vector varies randomly through all possible values of θ. Because the average value of $\cos^2\theta$ is $\frac{1}{2}$, the intensity transmitted by the first polarizing filter is $I_1 = \frac{1}{2} I_0$. For polarized light, $I_{\text{transmitted}} = I_0 \cos^2\theta$. For the second filter the transmitted intensity is $I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0 \cos^2\theta$. Similarly, $I_3 = I_2 \cos^2\theta = \frac{1}{2} I_0 (\cos^2\theta)^2$, and so on. Thus,

$$I_7 = I_1(\cos^2\theta)^{7-1} = \tfrac{1}{2} I_0 (\cos^2\theta)^6 = \tfrac{1}{2} I_0 (\cos^2 15°)^6 = 0.33 I_0$$