## VECTORS AND COORDINATE SYSTEMS

## Conceptual Questions

3.1. The magnitude of the displacement vector is the minimum distance traveled since the displacement is the vector sum of a number of individual movements. Thus, it is not possible for the magnitude of the displacement vector to be more than the distance traveled. If the individual movements are all in the same direction, the total displacement and the distance traveled are equal. However, it is possible that the total displacement is less than the distance traveled, if the individual movements are not in the same direction.
3.2. It is possible that $C=A+B$ only if $\vec{A}$ and $\vec{B}$ both point in the same direction as in the figure below. It is not possible that $C>A+B$ because, if $\vec{A}$ and $\vec{B}$ point in different directions, putting them tip to tail gives a resultant with a shorter length (see figure below).

3.3. It is possible that $C=0$ if $\vec{A}=-\vec{B}$. It is not possible for the length of a vector to be negative, so $C \geq 0$. Even if $\vec{A}$ and $\vec{B}$ are parallel but in opposite directions, $\vec{C}$ will still have a length greater than or equal to zero.
3.4. No, it is not possible to add a scalar to a vector, since the scalar has no direction.
3.5. The zero vector $\overrightarrow{0}$ has zero length. It does not point in any direction.
3.6. A vector can have a component that is zero and still have nonzero length only if another component is nonzero. For example, consider the vector $\hat{i}=(1,0)$, which points along the $x$-axis.
3.7. If one component of a vector is nonzero then it is not possible for the vector to have zero magnitude. The magnitude of the vector depends on the sum of the squares of the components, so any component signs do not matter.
3.8. No, it is not possible for two vectors with unequal magnitudes to add to zero. To add to zero, two vectors must be antiparallel and of the same length (magnitude).
3.9. (a) False, because the size of a vector is fixed.
(b) False, because the direction of a vector in space is independent of any coordinate system.
(c) True, because the orientation of the vector relative to the axes can be different.

## Exercises and Problems

## Section 3.1 Vectors

## Section 3.2 Properties of Vectors

### 3.1. Visualize:



Solve: (a) To find $\vec{A}+\vec{B}$, we place the tail of vector $\vec{B}$ on the tip of vector $\vec{A}$ and draw an arrow from the tail of vector $\vec{A}$ to the tip of vector $\vec{B}$.
(b) Since $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$, we place the tail of the vector $-\vec{B}$ on the tip of vector $\vec{A}$ and then draw and from the tail of vector $\vec{A}$ to the tip of vector $-\vec{B}$.

### 3.2. Visualize:



Solve: (a) To find $\vec{A}+\vec{B}$, we place the tail of vector $\vec{B}$ on the tip of vector $\vec{A}$ and then draw an arrow from $\vec{A}$ 's tail to vector $\vec{B}$ 's tip.
(b) To find $\vec{A}-\vec{B}$, we note that $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$. We place the tail of vector $-\vec{B}$ on the tip of vector $\vec{A}$ and then draw an arrow from vector $\vec{A}$ 's tail to the tip of vector $-\vec{B}$.

## Section 3.3 Coordinate Systems and Vector Components

### 3.3. Visualize:



Solve: Vector $\vec{E}$ points to the left and up, so the components $E_{x}$ and $E_{y}$ are negative and positive, respectively, according to the Tactics Box 3.1.
(a) $E_{x}=-E \cos \theta$ and $E_{y}=E \sin \theta$.
(b) $E_{x}=-E \sin \varphi$ and $E_{y}=E \cos \varphi$.

Assess: Note that the role of sine and cosine are reversed because we are using a different angle. $\theta$ and $\phi$ are complementary angles.
3.4. Visualize: The figure shows the components $v_{x}$ and $v_{y}$, and the angle $\theta$.


Solve: We have, $v_{y}=-v \sin \theta$ where we have manually inserted the minus sign because $\vec{v}_{y}$ points in the negative- $y$ direction. The $x$-component is $v_{x}=v \cos \theta$. Taking the ratio $v_{x} / v_{y}$ and solving for $v_{x}$ gives $v_{x}=-v_{y}(\tan \theta)^{-1}=$ $-(-10 \mathrm{~m} / \mathrm{s})\left(\tan 40^{\circ}\right)^{-1}=12 \mathrm{~m} / \mathrm{s}$.
Assess: The $x$-component is positive since the position vector is in the fourth quadrant.
3.5. Visualize: The position vector $\vec{r}$ whose magnitude $r$ is 10 m has an $x$-component of 8 m . It makes an angle $\theta$ with the $+x$-axis in the first quadrant.


Solve: Using trigonometry, $r_{x}=r \cos \theta$, or $8 \mathrm{~m}=(10 \mathrm{~m}) \cos \theta$. This gives $\theta=36.9^{\circ}$. Thus the $y$-component of the position vector $\vec{r}$ is $r_{y}=r \sin \theta=(10 \mathrm{~m}) \sin \left(36.9^{\circ}\right)=6 \mathrm{~m}$.
Assess: The $y$-component is positive since the position vector is in the first quadrant.

### 3.6. Visualize:


(a)

(b)

(c)

We will follow rules in Tactics Box 3.1.
Solve: (a) Vector $\vec{r}$ points to the right and down, so the components $r_{x}$ and $r_{y}$ are positive and negative, respectively:

$$
r_{x}=r \cos \theta=(100 \mathrm{~m}) \cos 45^{\circ}=71 \mathrm{~m}, r_{y}=-r \sin \theta=-(100 \mathrm{~m}) \sin 45^{\circ}=-71 \mathrm{~m}
$$

(b) Vector $\vec{v}$ points to the right and up, so the components $v_{x}$ and $v_{y}$ are both positive:

$$
v_{x}=v \cos \theta=(300 \mathrm{~m} / \mathrm{s}) \cos \left(20^{\circ}\right)=280 \mathrm{~m} / \mathrm{s}, \quad v_{y}=v \sin \theta=(300 \mathrm{~m} / \mathrm{s}) \sin \left(20^{\circ}\right)=100 \mathrm{~m} / \mathrm{s}
$$

(c) Vector $\vec{a}$ has the following components:

$$
a_{x}=-a \cos \theta=-\left(5.0 \mathrm{~m} / \mathrm{s}^{2}\right) \cos 90^{\circ}=0 \mathrm{~m} / \mathrm{s}^{2}, a_{y}=-a \sin \theta=-\left(5.0 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 90^{\circ}=-5.0 \mathrm{~m} / \mathrm{s}^{2}
$$

Assess: The components have same units as the vectors. Note the minus signs we have manually inserted according to Tactics Box 3.1.

### 3.7. Visualize:


(a)

$\frac{\text { Known }}{a=20 \mathrm{~m} / \mathrm{s}^{2}, \theta=30^{\circ}}$

(b)

(c)

We will follow the rules given in Tactics Box 3.1.

Solve:
(a) $v_{x}=(10 \mathrm{~m} / \mathrm{s}) \cos \left(90.0^{\circ}\right)=0 \mathrm{~m} / \mathrm{s} \quad v_{y}=-(10 \mathrm{~cm} / \mathrm{s}) \sin \left(90.0^{\circ}\right)=-10 \mathrm{~m} / \mathrm{s}$
(b) $a_{x}=\left(20 \mathrm{~m} / \mathrm{s}^{2}\right) \cos 30^{\circ}=17 \mathrm{~m} / \mathrm{s}^{2} \quad a_{y}=-\left(20 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 30^{\circ}=-10 \mathrm{~m} / \mathrm{s}^{2}$
(c) $F_{x}=-(100 \mathrm{~N}) \sin \left(36.9^{\circ}\right)=-60 \mathrm{~N} \quad F_{y}=(100 \mathrm{~N}) \cos \left(36.9^{\circ}\right)=80 \mathrm{~N}$

Assess: The components have the same units as the vectors. Note the minus signs we have manually inserted according to Tactics Box 3.1.

### 3.8. Visualize:



The components of the vector $\vec{C}$ and $\vec{D}$, and the angles $\theta$ are shown.
Solve: For $\vec{C}$ we have $C_{x}=-(3.15 \mathrm{~m}) \cos \left(15^{\circ}\right)=-3.04 \mathrm{~m}$ and $C_{y}=(3.15 \mathrm{~m}) \sin \left(15^{\circ}\right)=0.815 \mathrm{~m}$. For $\vec{D}$ we have $D_{x}=(25.6 \mathrm{~m}) \sin \left(30^{\circ}\right)=12.8 \mathrm{~m}$ and $D_{y}=-(25.67 \mathrm{~m}) \cos \left(30^{\circ}\right)=-22.2 \mathrm{~m}$.
Assess: The components of the vectors $\vec{C}$ and $\vec{D}$ have the same units as the vectors themselves. Note the minus signs we have manually inserted, as per the rules of Tactics Box 3.1.

### 3.9. Visualize:



Solve: The magnitude of the vector is $B=\sqrt{B_{x}^{2}+B_{y}^{2}}=\sqrt{(2.0 \mathrm{~T})^{2}+(-1.0 \mathrm{~T})^{2}}=\sqrt{5.0} \mathrm{~T}=2.2 \mathrm{~T}$. The angle $\theta$ is

$$
\theta=\tan ^{-1} \frac{\left|B_{y}\right|}{B_{x}}=\tan ^{-1}\left(\frac{1.0 \mathrm{~T}}{2.0 \mathrm{~T}}\right)=27^{\circ}
$$

Assess: Since $\left|B_{y}\right|<\left|B_{x}\right|$, the angle $\theta$ made with the $+x$-axis is less than $45^{\circ} . \theta=45^{\circ}$ for $\left|B_{y}\right|=\left|B_{x}\right|$.

## Section 3.4 Vector Algebra

### 3.10. Visualize:


(a)

(b)
Known $v_{x}=-10 \mathrm{~m} / \mathrm{s}$ $v_{y}=-100 \mathrm{~m} / \mathrm{s}$
Find
(c)


(d)

Solve: (a) Using the formulas for the magnitude and direction of a vector, we have:

$$
B=\sqrt{(-4)^{2}+(4)^{2}}=5.7, \theta=\tan ^{-1}\left(\frac{4}{4}\right)=45^{\circ}
$$

(b) $r=\sqrt{(-2.0 \mathrm{~cm})^{2}+(-1.0 \mathrm{~cm})^{2}}=2.2 \mathrm{~cm}, \theta=\tan ^{-1}\left(\frac{1.0}{2.0}\right)=27^{\circ}$
(c) $v=\sqrt{(-10 \mathrm{~m} / \mathrm{s})^{2}+(-100 \mathrm{~m} / \mathrm{s})^{2}}=100 \mathrm{~m} / \mathrm{s}, \quad \theta=\tan ^{-1}\left(\frac{100}{10}\right)=84^{\circ}$
(d) $a=\sqrt{\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}+\left(20 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}}=22 \mathrm{~m} / \mathrm{s}^{2}, \quad \theta=\tan ^{-1}\left(\frac{10}{20}\right)=27^{\circ}$

### 3.11. Visualize:



Solve: (a) Using the formulas for the magnitude and direction of a vector, we have:

$$
A=\sqrt{(4)^{2}+(-6)^{2}}=7.2, \quad \theta=\tan ^{-1}\left(\frac{r_{y}}{r_{x}}\right)=\tan ^{-1}\left(\frac{6}{4}\right)=56^{\circ} \text { below the }+x \text {-axis }
$$

(b) $r=\sqrt{(50 \mathrm{~m})^{2}+(80 \mathrm{~m})^{2}}=94 \mathrm{~m} \quad \theta=\tan ^{-1}\left(\frac{80 \mathrm{~m}}{50 \mathrm{~m}}\right)=58^{\circ}$ above the $+x$-axis
(c) $v=\sqrt{(-20 \mathrm{~m} / \mathrm{s})^{2}+(40 \mathrm{~m} / \mathrm{s})^{2}}=45 \mathrm{~m} / \mathrm{s} \quad \theta=\tan ^{-1}\left(\frac{40}{20}\right)=63^{\circ}$ above the $-x$-axis
(d) $a=\sqrt{\left(2.0 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}+\left(-6.0 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}}=6.3 \mathrm{~m} / \mathrm{s}^{2} \quad \theta=\tan ^{-1}\left(\frac{2.0}{6.0}\right)=18^{\circ}$ to the right of the $-y$-axis

### 3.12. Visualize:



We have $\vec{C}=\vec{A}-\vec{B}$ or $\vec{C}=\vec{A}+(-\vec{B})$, where $-\vec{B}$ is the same as $\vec{B}$, but in the opposite direction. Look back at Tactics Box 1.2, which shows how to perform vector subtraction graphically.
Solve: To obtain vector $\vec{C}$ from $\vec{A}$ and $\vec{B}$, we place the tail of $-\vec{B}$ on the tip of $\vec{A}$, and then draw a vector arrow from the tail of $\vec{A}$ to the tip of $-\vec{B}$. Reading the coordonates off the graph, we find that $\vec{C}=-2 \hat{i}+5 \hat{j}$
3.13. Visualize: The vectors $\vec{A}, \vec{B}$, and $\vec{C}=\vec{A}+\vec{B}$ are shown.


Solve: (a) We have $\vec{A}=4 \hat{i}-2 \hat{j}$ and $\vec{B}=-3 \hat{i}+5 \hat{j}$. Thus, $\vec{C}=\vec{A}+\vec{B}=(4 \hat{i}-2 \hat{j})+(-3 \hat{i}+5 \hat{j})=1 \hat{i}+3 \hat{j}$.
(b) Vectors $\vec{A}, \vec{B}$, and $\vec{C}$ are shown in the figure above.
(c) Since $\vec{C}=1 \hat{i}+3 \hat{j}=C_{x} \hat{i}+C_{y} \hat{j}, C_{x}=1$, and $C_{y}=3$. Therefore, the magnitude and direction of $\vec{C}$ are $C=\sqrt{(1)^{2}+(3)^{2}}=\sqrt{10}=3.2$ and $\theta=\tan ^{-1}\left(C_{y} / C_{x}\right)=\tan ^{-1}(3 / 1)=72^{\circ}$, respectively.
Assess: The vector $\vec{C}$ is to the right and up, thus implying that both the $x$ and $y$ components are positive. Also $\theta>45^{\circ}$ since $\left|C_{y}\right|>\left|C_{x}\right|$.

### 3.14. Visualize:



Solve: (a) We have $\vec{A}=4 \hat{i}-2 \hat{j}, \vec{B}=-3 \hat{i}+5 \hat{j}$, and $-\vec{B}=3 \hat{i}-5 \hat{j}$. Thus, $\vec{D}=\vec{A}+(-\vec{B})=(4+3) \hat{i}+(-2-5) \hat{j}=7 \hat{i}-7 \hat{j}$.
(b) Vectors $\vec{A}, \vec{B}$, and $\vec{D}$ are shown in the figure above.
(c) Since $\vec{D}=7 \hat{i}-7 \hat{j}=D_{x} \hat{i}+D_{y} \hat{j}, D_{x}=7$ and $D_{y}=-7$. Therefore, the magnitude and direction of $\vec{D}$ are

$$
D=\sqrt{(7)^{2}+(-7)^{2}}=7 \sqrt{2}=9.9 \quad \theta=\tan ^{-1}\left(\left|D_{y}\right| / D_{x}\right)=\tan ^{-1}(7 / 7)=45^{\circ}
$$

Assess: Since $\left|D_{y}\right|=\left|D_{x}\right|$, the angle $\theta=45^{\circ}$, as expected.

### 3.15. Visualize:



Solve: (a) We have $\vec{A}=4 \hat{i}-2 \hat{j}$ and $\vec{B}=-3 \hat{i}+5 \hat{j}$. This means $4 \vec{A}=16 \hat{i}-8 \hat{j}$ and $2 \vec{B}=-6 \hat{i}+10 \hat{j}$. Thus, $\vec{E}=4 \vec{A}+2 \vec{B}=[16+(-6)] \hat{i}+[(-8)+10] \hat{j}=10 \hat{i}+2 \hat{j}$.
(b) Vectors $\vec{A}, \vec{B}$, and $\vec{E}$ are shown in the figure above.
(c) From the $\vec{E}$ vector, $E_{x}=10$ and $E_{y}=2$. Therefore, the magnitude and direction of $\vec{E}$ are

$$
E=\sqrt{(10)^{2}+(2)^{2}}=\sqrt{104}=10, \theta=\tan ^{-1}\left(E_{y} / E_{x}\right)=\tan ^{-1}(2 / 10)=11^{\circ}
$$

So $\vec{E}$ is $11^{\circ}$ above the $+x$-axis.

### 3.16. Visualize:



Solve: (a) We have $\vec{A}=4 \hat{i}-2 \hat{j}$ and $\vec{B}=-3 \hat{i}+5 \hat{j}$. This means $4 \vec{B}=-12 \hat{i}+20 \hat{j}$. Hence, $\vec{F}=\vec{A}-4 \vec{B}=[4-(-12)] \hat{i}+$ $[-2-20] \hat{j}=16 \hat{i}-22 \hat{j}=F_{x} \hat{i}+F_{y} \hat{j}$, so $F_{x}=16$ and $F_{y}=-22$.
(b) The vectors $\vec{A}, \vec{B}$, and $\vec{F}$ are shown in the above figure.
(c) The magnitude and direction of $\vec{F}$ are

$$
\begin{aligned}
& F=\sqrt{F_{x}^{2}+F_{y}^{2}}=\sqrt{(16)^{2}+(-22)^{2}}=27 \\
& \theta=\tan ^{-1}\left(\left|F_{y}\right| / F_{x}\right)=\tan ^{-1}(22 / 16)=54^{\circ}
\end{aligned}
$$

Assess: $F_{y}>F_{x}$ implies $\theta>45^{\circ}$, which is consistent with the figure.

### 3.17. Visualize:



Solve: In coordinate system I, the vector $\vec{B}$ makes an angle of $60^{\circ}$ counterclockwise from vertical, so it has an angle of $\theta=30^{\circ}$ with the negative $x$-axis. Since $\vec{B}$ points to the left and up, it has a negative $x$-component and a positive
$y$-component. Thus, $B_{x}=-(5.0 \mathrm{~m}) \cos \left(30^{\circ}\right)=-4.3 \mathrm{~m}$ and $B_{y}=+(5.0 \mathrm{~m}) \sin \left(30^{\circ}\right)=2.5 \mathrm{~m}$. Thus, $\vec{B}=-(4.3 \mathrm{~m}) \hat{i}+(2.5 \mathrm{~m}) \hat{j}$.

In coordinate system II, the vector $\vec{B}$ makes an angle of $30^{\circ}$ with the $+y$-axis and is to the left and up. This means we have to manually insert a minus sign for the $x$-component. Thus, $B_{x}=-B \sin \left(30^{\circ}\right)=-(5.0 \mathrm{~m}) \sin \left(30^{\circ}\right)=-2.5 \mathrm{~m}$, and $B_{y}=+B \cos \left(30^{\circ}\right)=(5.0 \mathrm{~m}) \cos \left(30^{\circ}\right)=4.3 \mathrm{~m}$. Thus $\vec{B}=-(2.5 \mathrm{~m}) \hat{i}+(4.3 \mathrm{~m}) \hat{j}$.
3.18. Visualize: Refer to Figure EX3.18. The velocity vector $\vec{v}$ points south and makes an angle of $30^{\circ}$ with the $-y$-axis. The vector $\vec{v}$ points to the left and down, implying that both $v_{x}$ and $v_{y}$ are negative.
Solve: We have $v_{x}=-v \sin \left(30^{\circ}\right)=-(100 \mathrm{~m} / \mathrm{s}) \sin \left(30^{\circ}\right)=-50 \mathrm{~m} / \mathrm{s}$ and $v_{y}=-v \cos \left(30^{\circ}\right)=-(100 \mathrm{~m} / \mathrm{s}) \cos \left(30^{\circ}\right)=-87 \mathrm{~m} / \mathrm{s}$.
Assess: Notice that $v_{x}$ and $v_{y}$ have the same units as $\vec{v}$.

### 3.19. Visualize: (a)



Solve: (b) The components of the vectors $\vec{A}, \vec{B}$, and $\vec{C}$ are $A_{x}=(3.0 \mathrm{~m}) \cos \left(20^{\circ}\right)=2.8 \mathrm{~m}$ and $A_{y}=-(3.0 \mathrm{~m}) \sin \left(20^{\circ}\right)=-1.0 \mathrm{~m} ; \quad B_{x}=0 \mathrm{~m}$ and $B_{y}=2.0 \mathrm{~m}$;
$C_{x}=-(5.0 \mathrm{~m}) \cos \left(70^{\circ}\right)=-1.7 \mathrm{~m}$ and $C_{y}=-(5.0 \mathrm{~m}) \sin \left(70^{\circ}\right)=-4.7 \mathrm{~m}$. This means the vectors can be written as

$$
\vec{A}=(2.8 \hat{i}+1.0 \hat{j}) \mathrm{m}, \quad \vec{B}=(2.0 \hat{j}) \mathrm{m}, \quad \vec{C}=(-1.7 \hat{i}-4.7 \hat{j}) \mathrm{m}
$$

(c) We have $\vec{D}=\vec{A}+\vec{B}+\vec{C}=(1.1 \mathrm{~m}) \hat{i}-(3.7 \mathrm{~m}) \hat{j}$. This means

$$
D=\sqrt{(1.1 \mathrm{~m})^{2}+(3.7 \mathrm{~m})^{2}}=3.9 \mathrm{~m} \quad \theta=\tan ^{-1}(3.9 / 1.09)=74^{\circ}
$$

The direction of $\vec{D}$ is south of east, $74^{\circ}$ below the $+x$-axis.
3.20. Solve: We have $\vec{E}=E_{x} \hat{i}+E_{y} \hat{j}=2 \hat{i}+3 \hat{j}$, which means $E_{x}=2$ and $E_{y}=3$. Also, $\vec{F}=F_{x} \hat{i}+F_{y} \hat{j}=2 \hat{i}-2 \hat{j}$, which means $F_{x}=2$ and $F_{y}=-2$.
(a) The magnitude of $\vec{E}$ is given by $E=\sqrt{E_{x}^{2}+E_{y}^{2}}=\sqrt{(2)^{2}+(3)^{2}}=3.6$ and the magnitude of $\vec{F}$ is given by $F=\sqrt{F_{x}^{2}+F_{y}^{2}}=\sqrt{(2)^{2}+(2)^{2}}=2.8$.
(b) Since $\vec{E}+\vec{F}=4 \hat{i}+1 \hat{j}$, the magnitude of $\vec{E}+\vec{F}$ is $\sqrt{(4)^{2}+(1)^{2}}=4.1$.
(c) Since $-\vec{E}-2 \vec{F}=-(2 \hat{i}+3 \hat{j})-2(2 \hat{i}-2 \hat{j})=-6 \hat{i}+1 \hat{j}$, the magnitude of $-\vec{E}-2 \vec{F}$ is $\sqrt{(-6)^{2}+(1)^{2}}=6.1$.
3.21. Solve: We have $\vec{r}=(5.0 \hat{i}+4.0 \hat{j}) t^{2} \mathrm{~m}$. This means that $\vec{r}$ does not change the ratio of its components as $t$ increases; that is, the direction of $\vec{r}$ is constant. The magnitude of $\vec{r}$ is given by $r=\sqrt{\left(5.0 t^{2}\right)^{2}+\left(4.0 t^{2}\right)^{2}} \mathrm{~m}=6.40 t^{2} \mathrm{~m}$.
(a) The particle's distance from the origin at $t=0 \mathrm{~s}, t=2 \mathrm{~s}$, and $t=5 \mathrm{~s}$ is $0 \mathrm{~m}, 26 \mathrm{~m}$, and 160 m .
(b) The particle's velocity is $\vec{v}(t)=\frac{d \vec{r}}{d t}=(5.0 \hat{i}+4.0 \hat{j}) \frac{d t^{2}}{d t} \mathrm{~m} / \mathrm{s}=(5.0 \hat{i}+4.0 \hat{j}) 2 t \mathrm{~m} / \mathrm{s}=(10 \hat{i}+8.0 \hat{j}) t \mathrm{~m} / \mathrm{s}$.
(c) The magnitude of the particle's velocity is given by $v=\sqrt{(10 t)^{2}+(8.0 t)^{2}}=13 \mathrm{t} \mathrm{m} / \mathrm{s}$. The particle's speed at $t=0 \mathrm{~s}$, $t=2 \mathrm{~s}$, and $t=5 \mathrm{~s}$ is $0 \mathrm{~m} / \mathrm{s}, 26 \mathrm{~m} / \mathrm{s}$, and $64 \mathrm{~m} / \mathrm{s}$.

### 3.22. Visualize:



Solve: (a) Vector $\vec{C}$ is the sum of vectors $\vec{A}$ and $\vec{B}$, which is obtained using the tip-to-tip rule of graphical addition. Its magnitude is measured to be 4.7 and its angle made with the $+x$-axis is measured to be $33^{\circ}$.
(b) The geometry of parallelograms shows that $\phi=180^{\circ}-\left(\theta_{B}-\theta_{A}\right)=180^{\circ}-\left(60^{\circ}-20^{\circ}\right)=140^{\circ}$. Combining this with the law of cosines, $C^{2}=A^{2}+B^{2}-2 A B \cos \phi$, gives

$$
C=\sqrt{(3)^{2}+(2)^{2}-2(3)(2) \cos \left(140^{\circ}\right)}=4.7
$$

Using the law of sines:

$$
\frac{\sin \alpha}{2}=\frac{\sin 140^{\circ}}{4.71} \Rightarrow \alpha=16^{\circ}
$$

Thus, $\theta_{C}=\alpha+20^{\circ}=36^{\circ}$.
(c) We have:

$$
\begin{array}{ll}
A_{x}=A \cos \theta_{A}=3 \cos 20^{\circ}=2.82 & A_{y}=A \sin \theta_{A}=3 \sin 20^{\circ}=1.03 \\
B_{x}=B \cos \theta_{B}=2 \cos 60^{\circ}=1.00 & B_{y}=B \sin \theta_{B}=2 \sin 60^{\circ}=1.73
\end{array}
$$

This means: $C_{x}=A_{x}+B_{x}=3.82$ and $C_{y}=A_{y}+B_{y}=2.76$. The magnitude and direction of $C$ are given by

$$
C=\sqrt{C_{x}^{2}+C_{y}^{2}}=\sqrt{(3.82)^{2}+(2.76)^{2}}=4.7 \quad \theta_{C}=\tan ^{-1}\left(\frac{C_{y}}{C_{x}}\right)=\tan ^{-1}\left(\frac{2.76}{3.82}\right)=36^{\circ}
$$

Assess: Using the method of vector components and their algebraic addition to find the resultant vector yields the same results as using the graphical addition of vectors.
3.23. Visualize: Refer to Figure P 3.23 in your textbook.

Solve: (a) We are given that $\vec{A}+\vec{B}+\vec{C}=1 \hat{j}$ with $\vec{A}=4 \hat{i}$, and $\vec{C}=-2 \hat{j}$. This means $\vec{A}+\vec{C}=4 \hat{i}-2 \hat{j}$. Thus, $\vec{B}=(1 \hat{j})-(\vec{A}+\vec{C})=(1 \hat{j})-(4 \hat{i}-2 \hat{j})=-4 \hat{i}+3 \hat{j}$.
(b) We have $\vec{B}=B_{x} \hat{i}+B_{y} \hat{j}$ with $B_{x}=-4$ and $B_{y}=3$. Hence, $B=\sqrt{(-4)^{2}+(3)^{2}}=5$

$$
\theta=\tan ^{-1} \frac{\left|B_{y}\right|}{\left|B_{x}\right|}=\tan ^{-1}\left(\frac{3}{4}\right)=37^{\circ}
$$

Since $\vec{B}$ has a negative $x$-component and a positive $y$-component, the vector $\vec{B}$ is in the second quadrant and the angle $\theta$ made by $\vec{B}$ is measured above the $-x$-axis.

Assess: Since $\left|B_{y}\right|<\left|B_{x}\right|, \theta<45^{\circ}$ as obtained above.

### 3.24. Visualize:



Solve: (a) $\theta_{E}=\tan ^{-1}\left(\frac{1}{1}\right)=45^{\circ}, \quad \theta_{F}=\tan ^{-1}\left(\frac{2}{1}\right)=63^{\circ}$. Thus $\phi=180^{\circ}-\theta_{E}-\theta_{F}=72^{\circ}$
(b) From the figure, $E=\sqrt{2}$ and $F=\sqrt{5}$. Using

$$
\begin{aligned}
G^{2} & =E^{2}+F^{2}-2 E F \cos \varphi=(\sqrt{2})^{2}+(\sqrt{5})^{2}-2(\sqrt{2})(\sqrt{5}) \cos \left(180^{\circ}-72^{\circ}\right) \\
G & =3
\end{aligned}
$$

Furthermore, using $\frac{\sin \alpha}{\sqrt{5}}=\frac{\sin \left(180^{\circ}-72^{\circ}\right)}{2.975} \Rightarrow \alpha=45^{\circ}$
Since $\theta_{E}=45^{\circ}$, the angle made by the vector $\vec{G}$ with the $+x$-axis is $\theta_{G}=\left(\alpha+\theta_{E}\right)=45^{\circ}+45^{\circ}=90^{\circ}$.
(c) We have

$$
\begin{aligned}
& E_{x}=+1.0, \quad \text { and } \quad E_{y}=+1.0 \\
& F_{x}=-1.0, \quad \text { and } \quad F_{y}=+2.0 \\
& G_{x}=0.0, \quad \text { and } \quad G_{y}=3.0 \\
& G=\sqrt{(0.0)^{2}+(3.0)^{2}}=3.0, \quad \text { and } \theta=\tan ^{-1} \frac{\left|G_{y}\right|}{\left|G_{x}\right|}=\tan ^{-1}\left(\frac{3.0}{0.0}\right)=90^{\circ}
\end{aligned}
$$

That is, the vector $\vec{G}$ makes an angle of $90^{\circ}$ with the $x$-axis.
Assess: The graphical solution and the vector solution give the same answer within the given significance of figures.
3.25. Visualize: Refer to Figure P3.25.

Solve: From the rules of trigonometry, we have $A_{x}=4 \cos \left(40^{\circ}\right)=3.06$ and $A_{y}=4 \sin \left(40^{\circ}\right)=2.57$. Also,
$B_{x}=-2 \cos \left(10^{\circ}\right)=-1.97$ and $B_{y}=+2 \sin \left(10^{\circ}\right)=0.35$. Since $\vec{A}+\vec{B}+\vec{C}=\overrightarrow{0}$,
$\vec{C}=-\vec{A}-\vec{B}=(-\vec{A})+(-\vec{B})=(-3.06 \hat{i}-2.57 \hat{j})+(+1.97 \hat{i}-0.35 \hat{j})=-1 \hat{i}-3 \hat{j}$.

### 3.26. Visualize:

Known

$$
A=2 \mathrm{~m}
$$

$$
B=4 \mathrm{~m}
$$

Find $\vec{D}=2 \vec{A}+\vec{B}$ D $\theta$


Solve: In the tilted coordinate system, the vectors $\vec{A}$ and $\vec{B}$ are expressed as:

$$
\vec{A}=\left[2 \sin \left(15^{\circ}\right) \mathrm{m}\right] \hat{i}+\left[2 \cos \left(15^{\circ}\right) \mathrm{m}\right] \hat{j} \text { and } \vec{B}=\left[4 \cos \left(15^{\circ}\right) \mathrm{m}\right] \hat{i}-\left[4 \sin \left(15^{\circ}\right) \mathrm{m}\right] \hat{j}
$$

Therefore, $\vec{D}=2 \vec{A}+\vec{B}=(4 \mathrm{~m})\left[\sin \left(15^{\circ}\right)+\cos \left(15^{\circ}\right)\right] \hat{i}+(4 \mathrm{~m})\left[\cos \left(15^{\circ}\right)-\sin \left(15^{\circ}\right)\right] \hat{j}=(4.9 \mathrm{~m}) \hat{i}+(2.9 \mathrm{~m}) \hat{j}$.
Assess: The magnitude of this vector is $D=\sqrt{(4.9 \mathrm{~m})^{2}+(2.9 \mathrm{~m})^{2}}=5.7 \mathrm{~m}$, and it makes an angle of $\theta=\tan ^{-1}(2.9 \mathrm{~m} / 4.9 \mathrm{~m})=31^{\circ}$ with the $+x$-axis. The resultant vector can be obtained graphically by using the rule of tail-to-tip addition.

### 3.27. Visualize:



The magnitude of the unknown vector is 1 and its direction is along $\hat{i}+\hat{j}$. Let $\vec{A}=\hat{i}+\hat{j}$ as shown in the diagram.
That is, $\vec{A}=1 \hat{i}+1 \hat{j}$ and the $x$ - and $y$-components of $\vec{A}$ are both unity. Since $\theta=\tan ^{-1}\left(A_{y} / A_{x}\right)=45^{\circ}$, the unknown vector must make an angle of $45^{\circ}$ with the $+x$-axis and have unit magnitude.
Solve: Let the unknown vector be $\vec{B}=B_{x} \hat{i}+B_{y} \hat{j}$ where

$$
B_{x}=B \cos \left(45^{\circ}\right)=\frac{1}{\sqrt{2}} B \quad \text { and } \quad B_{y}=B \sin \left(45^{\circ}\right)=\frac{1}{\sqrt{2}} B
$$

We want the magnitude of $\vec{B}$ to be 1 , so we have

$$
B=\sqrt{B_{x}^{2}+B_{y}^{2}}=1 \Rightarrow \sqrt{\left(\frac{1}{\sqrt{2}} B\right)^{2}+\left(\frac{1}{\sqrt{2}} B\right)^{2}}=1 \Rightarrow \sqrt{B^{2}}=1 \Rightarrow B=1
$$

Thus,

$$
B_{x}=B_{y}=\frac{1}{\sqrt{2}}
$$

Finally,

$$
\vec{B}=B_{x} \hat{i}+B_{y} \hat{j}=\frac{1}{\sqrt{2}} \hat{i}+\frac{1}{\sqrt{2}} \hat{j}
$$

3.28. Model: Carlos will be represented as a particle and the particle model will be used for motion under constant acceleration kinetic equations.

## Visualize:



Solve: Carlos runs at constant speed without changing direction. The total distance he travels is found from kinematics:

$$
r_{1}=r_{0}+v_{0} \Delta t=0.0 \mathrm{~m}+(5.0 \mathrm{~m} / \mathrm{s})(600 \mathrm{~s})=3000 \mathrm{~m}
$$

This displacement is north of east, or $\theta=25^{\circ}$ from the $+x$-axis. Thus the position $\vec{r}_{1}$ becomes

$$
\vec{r}_{1}=(3000 \mathrm{~m})\left[\cos \left(25^{\circ}\right) \hat{i}+\sin \left(25^{\circ}\right) \hat{j}\right]=2.7 \mathrm{~km} \hat{i}+1.3 \mathrm{~km} \hat{j}
$$

That is, Carlos ends up 1.3 km north of his starting position.
Assess: The choice of our coordinate system is such that the $x$-component of the displacement is along the east and the $y$-component is along the north. The displacement of 3.0 km is reasonable for Carlos to run in 10 minutes if he is an athlete.
3.29. Visualize: The coordinate system $(x, y, z)$ is shown here; $+x$ denotes east, $+y$ denotes north, and $+z$ denotes upward vertical. The vectors $\vec{S}_{\text {morning }}$ (shortened to $\vec{S}_{\mathrm{m}}$ ), $\vec{S}_{\text {afternoon }}$ (shortened to $\vec{S}_{\mathrm{a}}$ ), and the total displacement vector $\vec{S}_{\text {total }}=\vec{S}_{\mathrm{a}}+\vec{S}_{\mathrm{m}}$ are also shown.


Solve: $\vec{S}_{\mathrm{m}}=(2000 \hat{i}+3000 \hat{j}+200 \hat{k}) \mathrm{m}$, and $\vec{S}_{\mathrm{a}}=(-1500 \hat{i}+2000 \hat{j}-300 \hat{k}) \mathrm{m}$. The total displacement is the sum of the individual displacements.
(a) The sum of the $z$-components of the afternoon and morning displacements is $S_{\mathrm{a} z}+S_{\mathrm{m} z}=-300 \mathrm{~m}+200 \mathrm{~m}=$ -100 m ; that is, 100 m lower.
(b) $\vec{S}_{\text {total }}=\vec{S}_{\mathrm{a}}+\vec{S}_{\mathrm{m}}=(500 \hat{i}+5000 \hat{j}-100 \hat{k}) \mathrm{m}$; that is, $(500 \mathrm{~m}$ east $)+(5000 \mathrm{~m}$ north $)-(100 \mathrm{~m}$ vertical $)$. The magnitude of your total displacement is

$$
S_{\text {total }}=\sqrt{(500)^{2}+(5000)^{2}+(-100)^{2}} \mathrm{~m}=5.0 \mathrm{~km}
$$

### 3.30. Visualize:



Only the minute hand is shown in the figure.
Solve: (a) We have $\vec{S}_{8: 00}=(2.0 \mathrm{~cm}) \hat{j}$ and $\vec{S}_{8: 20}=(2.0 \mathrm{~cm}) \cos \left(30^{\circ}\right) \hat{i}-(2.0 \mathrm{~cm}) \sin \left(30^{\circ}\right) \hat{j}$. The displacement vector is

$$
\begin{aligned}
\Delta \vec{r} & =\vec{S}_{8: 20}-\vec{S}_{8: 00} \\
& =(2.0 \mathrm{~cm})\left[\cos 30^{\circ} \hat{i}-\left(\sin 30^{\circ}+1\right) \hat{j}\right] \\
& =(2.0 \mathrm{~cm})(0.87 \hat{i}-1.50 \hat{j}) \\
& =(1.7 \mathrm{~cm}) \hat{i}-(3.0 \mathrm{~cm}) \hat{j}
\end{aligned}
$$

(b) We have $\vec{S}_{8: 00}=(2.0 \mathrm{~cm}) \hat{j}$ and $\vec{S}_{9: 00}=(2.0 \mathrm{~cm}) \hat{j}$. The displacement vector is $\Delta \vec{r}=\vec{S}_{9: 00}-\vec{S}_{8: 00}=0.0 \mathrm{~cm}$.

Assess: The displacement vector in part (a) has a positive $x$-component and a negative $y$-component. The vector thus is to the right and points down, in quadrant IV. This is where the vector drawn from the tip of the 8:00 a.m. arm to the tip of the 8:20 a.m. arm will point.

### 3.31. Visualize: (a)



Note that $+x$ is along the east and $+y$ is along the north.
Solve: (b) We are given $\vec{A}=-(200 \mathrm{~m}) \hat{j}$, and can use trigonometry to obtain
$\vec{B}=-(400 \mathrm{~m}) \cos \left(45^{\circ}\right)-(400 \mathrm{~m}) \sin \left(45^{\circ}\right)=-(283 \mathrm{~m}) \hat{i}-(283 \mathrm{~m}) \hat{j}$ and
$\vec{C}=(200 \mathrm{~m}) \sin \left(30^{\circ}\right)+(200 \mathrm{~m}) \cos \left(30^{\circ}\right)=(100 \mathrm{~m}) \hat{i}+(173 \mathrm{~m}) \hat{j}$. We want $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, so
$\vec{D}=-\vec{A}-\vec{B}-\vec{C}$
$=(200 \mathrm{~m}) \hat{j}-[-(283 \mathrm{~m}) \hat{i}-(283 \mathrm{~m}) \hat{j}]-[(100 \mathrm{~m}) \hat{i}+(173 \mathrm{~m}) \hat{j}]=(183 \mathrm{~m}) \hat{i}+(310 \mathrm{~m}) \hat{j}$
The magnitude and direction of $\vec{D}$ are

$$
D=\sqrt{(183 \mathrm{~m})^{2}+(310 \mathrm{~m})^{2}}=360 \mathrm{~m} \quad \text { and } \quad \theta=\tan ^{-1}\left(\frac{D_{y}}{D_{x}}\right)=\tan ^{-1}\left(\frac{310 \mathrm{~m}}{183 \mathrm{~m}}\right)=59^{\circ}
$$

This means $\vec{D}=\left(360 \mathrm{~m} 59^{\circ}\right.$ north of east $)$.
(c) The measured length of the vector $\vec{D}$ on the graph (with a ruler) is approximately 1.75 times the measured length of vector $\vec{A}$. Since $A=200 \mathrm{~m}$, this gives $D=1.75 \times 200 \mathrm{~m}=350 \mathrm{~m}$. Similarly, the angle $\theta$ measured with the protractor is close to $60^{\circ}$. These answers are in close agreement to part (b).
3.32. Visualize: (a) The figure shows Sparky's individual displacements and his net displacement.


Solve: (b) $\vec{D}_{\text {net }}=\vec{D}_{1}+\vec{D}_{2}+\vec{D}_{3}$, where individual displacements are

$$
\begin{aligned}
& \vec{D}_{1}=(50 \mathrm{~m}) \cos \left(45^{\circ}\right) \hat{i}+(50 \mathrm{~m}) \sin \left(45^{\circ}\right) \hat{j}=(35.4 \mathrm{~m}) \hat{i}+(35.4 \mathrm{~m}) \hat{j} \\
& \vec{D}_{2}=-(70 \mathrm{~m}) \hat{i} \\
& \vec{D}_{3}=-(20 \mathrm{~m}) \hat{j}
\end{aligned}
$$

Thus, to two significant figures, Sparky's displacement is $\vec{D}_{\text {net }}=-(35 \mathrm{~m}) \hat{i}+(15 \mathrm{~m}) \hat{j}$.
(c) As a magnitude and angle,

$$
D_{\mathrm{net}}=\sqrt{\left(D_{\mathrm{net}, x}\right)^{2}+\left(D_{\mathrm{net}, y}\right)^{2}}=\sqrt{(-35 \mathrm{~m})^{2}+(15 \mathrm{~m})^{2}}=38 \mathrm{~m}, \quad \theta_{\mathrm{net}}=\tan ^{-1}\left(\frac{D_{\mathrm{net}, y}}{\left|D_{\mathrm{net}, x}\right|}\right)=24^{\circ}
$$

Sparky's net displacement is 38 m in a direction $24^{\circ}$ north of west.

### 3.33. Visualize:



Solve: We are given $\vec{A}=(5.0 \mathrm{~m}) \hat{i}$ and $\vec{C}=(-1.0 \mathrm{~m}) \hat{k}$ Using trigonometry, $\vec{B}=(3.0 \mathrm{~m}) \cos \left(45^{\circ}\right) \hat{i}-(3.0 \mathrm{~m}) \sin \left(45^{\circ}\right) \hat{j}$
The total displacement is $\vec{r}=\vec{A}+\vec{B}+\vec{C}=(7.12 \mathrm{~m}) \hat{i}-(2.12 \mathrm{~m}) \hat{j}-(1.0 \mathrm{~m}) \hat{k}$. The magnitude of $\vec{r}$ is
$r=\sqrt{(7.12 \mathrm{~m})^{2}+(2.12 \mathrm{~m})^{2}+(1.0 \mathrm{~m})^{2}}=7.5 \mathrm{~m}$.
Assess: A displacement of 7.5 m is a reasonable displacement.

### 3.34. Visualize:



Solve: We have $\vec{v}=v_{x} \hat{i}+v_{y} \hat{j}=v_{\|} \hat{i}+v_{\perp} \hat{j}=v \cos \theta \hat{i}+v \sin \theta \hat{j}$. Thus, $v_{\|}=v \cos \theta=(100 \mathrm{~m} / \mathrm{s}) \cos \left(30^{\circ}\right)=87 \mathrm{~m} / \mathrm{s}$.
Assess: For the angle of $30^{\circ}, 87 \mathrm{~m} / \mathrm{s}$ for the horizontal component seems reasonable.

### 3.35. Visualize:



Solve: (a) Since $v_{x}=v \cos \theta$, we have $2.5 \mathrm{~m} / \mathrm{s}=(3.0 \mathrm{~m} / \mathrm{s}) \cos \theta \Rightarrow \theta=\cos ^{-1}\left(\frac{2.5 \mathrm{~m} / \mathrm{s}}{3.0 \mathrm{~m} / \mathrm{s}}\right)=34^{\circ}$.
(b) The vertical component is $v_{y}=v \sin \theta=(3.0 \mathrm{~m} / \mathrm{s}) \sin \left(34^{\circ}\right)=1.7 \mathrm{~m} / \mathrm{s}$.

### 3.36. Visualize:



The coordinate system used here is tilted so that the $x$-axis is along the slope.
Solve: The component of the velocity parallel to the $x$-axis is $v_{\|}=-v \cos \left(70^{\circ}\right)=-v \sin \left(20^{\circ}\right)=-(10 \mathrm{~m} / \mathrm{s})(0.34)=$ $-3.4 \mathrm{~m} / \mathrm{s}$. This is the speed down the slope. The component of the velocity perpendicular to the slope is $v_{\perp}=-v \sin \left(70^{\circ}\right)=-v \cos \left(20^{\circ}\right)=-(10 \mathrm{~m} / \mathrm{s})(0.94)=-9.4 \mathrm{~m} / \mathrm{s}$. This is the speed toward the ground.
Assess: A final speed of approximately $10 \mathrm{~m} / \mathrm{s}$ implies a fall time of approximately 1 second under free fall. Note that $g=-9.8 \mathrm{~m} / \mathrm{s}^{2}$. This time is reasonable for a drop of approximately 5 m , or 16 feet.

### 3.37. Visualize:

(a)

(b)


Solve: (a) The river is 100 m wide. If Mary rows due north at a constant speed of $v_{\text {row }}=2.0 \mathrm{~m} / \mathrm{s}$, it will take her $(100 \mathrm{~m}) /(2.0 \mathrm{~m} / \mathrm{s})=50 \mathrm{~s}$ to row across. But while she's doing so, the current sweeps her boat sideways at a speed $v_{\text {current }}=1.0 \mathrm{~m} / \mathrm{s}$. In the 50 s it takes her to cross the river, the current sweeps here a distance $d_{\|}=\left(v_{\text {current }} \times 50 \mathrm{~s}\right)=1.0 \mathrm{~m} / \mathrm{s} \times 50 \mathrm{~s}=50 \mathrm{~m}$, so she lands 50 m east of the point that was directly across the river from her when she started.
(b) Mary's net displacement $\vec{D}_{\text {net }}$, her displacement $\vec{D}_{\text {current }}$ due to the river's current, and her displacemnt $\vec{D}_{\text {row }}$ due to her rowing are shown in the figure.
3.38. Visualize: Establish a coordinate system with origin at the tree and with the $x$-axis pointing east. Let $\vec{A}$ be a displacement vector directly from the tree to the treasure. Vector $\vec{A}$ is $\vec{A}=(100 \hat{i}+500 \hat{j})$ paces.
This describes the displacement you would undergo by walking north 500 paces, then east 100 paces. Instead, you follow the road for 300 paces and undergo displacement

$$
\vec{B}=\left[300 \sin \left(60^{\circ}\right) \hat{i}+300 \cos \left(60^{\circ}\right) \hat{j}\right] \text { paces }=(260 \hat{i}+150 \hat{j}) \text { paces }
$$



Solve: Now let $\vec{C}$ be the displacement vector from your position to the treasure. From the figure $\vec{A}=\vec{B}+\vec{C}$.
So the displacement you need to reach the treasure is $\vec{C}=\vec{A}-\vec{B}=(-160 \hat{i}+350 \hat{j})$ paces.
If $\theta$ is the angle measured between $\vec{C}$ and the $y$-axis,

$$
\theta=\tan ^{-1}\left(\frac{160}{350}\right)=25^{\circ}
$$

You should head $25^{\circ}$ west of north. You need to walk distance $C=\sqrt{C_{x}^{2}+C_{y}^{2}}=\sqrt{(-160)^{2}+(350)^{2}}$ paces $=385$ paces to get to the treasure.
3.39. Visualize: A $3 \%$ grade rises 3 m for every 100 m of horizontal distance. The angle of the ground is thus $\alpha=\tan ^{-1}(3 / 100)=1.72^{\circ}$.
Establish a tilted coordinate system with one axis parallel to the ground and the other axis perpendicular to the ground.


Solve: From the figure, the magnitude of the component vector of $\vec{v}$ perpendicular to the ground is $v_{\perp}=v \sin \alpha=15.0 \mathrm{~m} / \mathrm{s}$. But this is only the size. We also have to note that the direction of $\vec{v}_{\perp}$ is down, so the component is $\vec{v}_{\perp}=-(15 \mathrm{~m} / \mathrm{s}) \hat{j}$.
3.40. Visualize: The average velocity is the net displacement $\vec{D}_{\text {net }}$ divided by the total time, which are marked on the graph. We also mark on the graph of the bacterium's individual displacements and the time for each.


Solve: The magnitude of the net displacement is found with Pythagoreum's rule, taking the values from the graph. We have $D_{\text {net }}=\sqrt{D_{\text {net, } x}^{2}+D_{\text {net, }, y}^{2}}=\sqrt{(40 \mu \mathrm{~m})^{2}+(-20 \mu \mathrm{~m})^{2}}=45 \mu \mathrm{~m}$. The direction of this displacement is

$$
\theta=\tan ^{-1}\left(\frac{\left|D_{\text {net, }, y}\right|}{D_{\text {net }, x}}\right)=\tan ^{-1}\left(\frac{20 \mu \mathrm{~m}}{40 \mu \mathrm{~m}}\right)=27^{\circ}
$$

The total time for the displacement is the sum of the individual times, which may be found by dividing each individual distance by the bacterium's constant speed of $20 \mu \mathrm{~m} / \mathrm{s}$. This gives

$$
\begin{aligned}
& \Delta t_{A B}=D_{A B} /(20 \mu \mathrm{~m} / \mathrm{s})=\sqrt{(50 \mu \mathrm{~m})^{2}+(10 \mu \mathrm{~m})^{2}} /(20 \mu \mathrm{~m} / \mathrm{s})=(51.0 \mu \mathrm{~m}) /(20 \mu \mathrm{~m} / s)=2.55 \mathrm{~s} \\
& \Delta t_{B C}=D_{B C} /(20 \mu \mathrm{~m} / \mathrm{s})=(10 \mu \mathrm{~m}) /(20 \mu \mathrm{~m} / \mathrm{s})=0.50 \mathrm{~s} \\
& \Delta t_{C D}=D_{C D} /(20 \mu \mathrm{~m} / s)=\sqrt{(40 \mu \mathrm{~m})^{2}+(10 \mu \mathrm{~m})^{2}} /(20 \mu \mathrm{~m} / \mathrm{s})=(41.0 \mu \mathrm{~m}) /(20 \mu \mathrm{~m} / \mathrm{s})=2.06 \mathrm{~s} \\
& \Delta t_{D E}=D_{D E} /(20 \mu \mathrm{~m} / s)=\sqrt{(-50 \mu \mathrm{~m})^{2}+(-50 \mu \mathrm{~m})^{2}} /(20 \mu \mathrm{~m} / \mathrm{s})=(70.7 \mu \mathrm{~m}) /(20 \mu \mathrm{~m} / \mathrm{s})=3.54 \mathrm{~s}
\end{aligned}
$$

The total time is therefore $\Delta t_{\text {Tot }}=2.55 \mathrm{~s}+0.50 \mathrm{~s}+2.06 \mathrm{~s}+3.54 \mathrm{~s}=8.65 \mathrm{~s}$ and the magnitude of the bacterium's net velocity is

$$
v_{\mathrm{net}}=\frac{D_{\mathrm{net}}}{\Delta t_{\mathrm{Tot}}}=\frac{45 \mu \mathrm{~m}}{8.65 \mathrm{~s}}=5.2 \mu \mathrm{~m} / \mathrm{s}
$$

### 3.41. Visualize:



Solve: The resulting velocity is given by $\vec{v}=\vec{v}_{\text {fly }}+\vec{v}_{\text {wind }}$, where $\vec{v}_{\text {wind }}=(6.0 \mathrm{~m} / \mathrm{s}) \hat{i}$ and $\vec{v}_{\text {fly }}=-v \sin \theta \hat{i}-v \cos \theta \hat{j}$. Substituting the known values we get $\vec{v}=-(8.0 \mathrm{~m} / \mathrm{s}) \sin \theta \hat{i}-(8.0 \mathrm{~m} / \mathrm{s}) \cos \theta \hat{j}+(6.0 \mathrm{~m} / \mathrm{s}) \hat{i}$. We need to have $v_{x}=0$. This means $0=-(8.0 \mathrm{~m} / \mathrm{s}) \sin \theta+(6.0 \mathrm{~m} / \mathrm{s})$, so $\sin \theta=\frac{6}{8}$ or $\theta=49^{\circ}$. Thus the ducks should head $49^{\circ}$ west of south.
3.42. Model: We will treat the knot in the rope as a particle in static equilibrium.

## Visualize:



Solve: Expressing the vectors in component form, we have $\vec{F}_{1}=3.0 \hat{i}$ and $\vec{F}_{2}=-5.0 \sin \left(30^{\circ}\right) \hat{i}+5.0 \cos \left(30^{\circ}\right) \hat{j}$. Since we must have $\vec{F}_{1}+\vec{F}_{2}+\vec{F}_{3}=\overrightarrow{0}$ for the know to remain stationary, we can write $\vec{F}_{3}=-\vec{F}_{1}-\vec{F}_{2}=-0.50 \hat{i}-4.33 \hat{j}$. The magnitude of $\vec{F}_{3}$ is given by $F_{3}=\sqrt{(-0.50)^{2}+(-4.33)^{2}}=4.4$ units The angle between $\vec{F}_{3}$ and the negative $x$-axis is $\theta=\tan ^{-1}(4.33 / 0.50)=83^{\circ}$ below the negative $x$-axis.
Assess: The resultant vector has both components negative, and is therefore in quadrant III. Its magnitude and direction are reasonable. Note the minus sign that we have manually inserted with the force $\vec{F}_{2}$.

### 3.43. Visualize:



Use a tilted coordinate system such that $x$-axis is down the slope.
Solve: Expressing all three forces in terms of unit vectors, we have $\vec{F}_{1}=-(3.0 \mathrm{~N}) \hat{i}, \vec{F}_{2}=+(6.0 \mathrm{~N}) \hat{j}$, and $\vec{F}_{3}=(5.0 \mathrm{~N}) \sin \theta \hat{i}-(5.0 \mathrm{~N}) \cos \theta \hat{j}$.
(a) The component of $\vec{F}_{\text {net }}$ parallel to the floor is $\left(F_{\text {net }}\right)_{x}=-(3.0 \mathrm{~N})+0 \mathrm{~N}+(5.0 \mathrm{~N}) \sin \left(30^{\circ}\right)=-0.50 \mathrm{~N}$, or 0.50 N up the slope.
(b) The component of $\vec{F}_{\text {net }}$ perpendicular to the floor is $\left(F_{\text {net }}\right)_{y}=0 \mathrm{~N}+(6.0 \mathrm{~N})-(5.0 \mathrm{~N}) \cos \left(30^{\circ}\right)=1.67 \mathrm{~N}$, or 1.7 N to two significant figures.
(c) The magnitude of $\vec{F}_{\text {net }}$ is $F_{\text {net }}=\sqrt{\left(F_{\text {net }}\right)_{x}+\left(F_{\text {net }}\right)_{y}}=\sqrt{(-0.50 \mathrm{~N})^{2}+(1.67 \mathrm{~N})^{2}}=1.74 \mathrm{~N}$, or 1.7 N to two significant figures. The angle between $\vec{F}_{\text {net }}$ and the negative $x$-axis is

$$
\phi=\tan ^{-1} \frac{\left(F_{\text {net }}\right)_{y}}{\left|\left(\vec{F}_{\text {net }}\right)_{x}\right|}=\tan ^{-1}\left(\frac{1.67 \mathrm{~N}}{0.50 \mathrm{~N}}\right)=73^{\circ}
$$

$\vec{F}_{\text {net }}$ is $73^{\circ}$ clockwise from the $-x$-axis.

### 3.44. Visualize:



Solve: Using trigonometry to calculate $\theta$, we get $\theta=\tan ^{-1}(100 \mathrm{~cm} / 141 \mathrm{~cm})=35.3^{\circ}$. Expressing the three forces component form gives $\vec{F}_{\mathrm{B}}=-(3.0 \mathrm{~N}) \hat{i}, \vec{F}_{\mathrm{C}}=-(6.0 \mathrm{~N}) \hat{j}$, and $\vec{F}_{\mathrm{D}}=+(2.0 \mathrm{~N}) \cos \left(35.3^{\circ}\right) \hat{i}-(2.0 \mathrm{~N}) \sin \left(35.3^{\circ}\right) \hat{j}=$ $(1.63 \mathrm{~N}) \hat{i}-(1.16 \mathrm{~N}) \hat{j}$. The total force is $\vec{F}_{\mathrm{net}}=\vec{F}_{\mathrm{B}}+\vec{F}_{\mathrm{C}}+\vec{F}_{\mathrm{D}}=-1.37 \mathrm{~N} \hat{i}-7.2 \mathrm{~N} \hat{j}$. The magnitude of $\vec{F}_{\text {net }}$ is $F_{\text {net }}=\sqrt{(1.37 \mathrm{~N})^{2}+(7.2 \mathrm{~N})^{2}}=7.3 \mathrm{~N}$.

$$
\theta_{\text {net }}=\tan ^{-1} \frac{\left|\left(F_{\text {net }}\right)_{y}\right|}{\left|\left(F_{\text {net }}\right)_{x}\right|}=\tan ^{-1}\left(\frac{7.2 \mathrm{~N}}{1.37 \mathrm{~N}}\right)=79^{\circ}
$$

$\vec{F}_{\text {net }}=\left(7.3 \mathrm{~N}, 79^{\circ}\right.$ below the negative $x$-axis in quadrant III).

