

19-20 AP Physics C Practice Problems Ch 4-6

[View Basic/Answers](#)**Practice Problems Ch 4-6 Begin Date:** 9/29/2019 12:01:00 AM -- **Due Date:** 10/8/2019 11:59:00 PM **End Date:** 10/8/2019 11:59:00 PM**Problem 1 - c4.1.1 :**

Two athletes run in different directions at 10 mph. One heads east and one heads south.

Part (a) Are their speeds the same?

Speed depends only on the rate of motion. Since both of them are moving at 10 mph, they are both moving at the same speed.

 Yes**Part (b) Are their velocities the same?**

Unlike speed, velocity is a vector quantity. This means that velocity depends on both your speed and your direction. As a consequence, if the two athletes are not direction, they do not have the same velocity.

 No**Problem 2 - c4.1.2 :**

Answer the following question.

Part (a) Can a car that has only been traveling at different speeds in a straight line east have accelerated north?

If the car accelerates north while it is traveling to the east, then it would turn slightly northward and no longer be moving directly east. Since the car travels in a straight line, it never curves off of traveling directly east. This means that it is impossible for the car to have accelerated north.

 No**Problem 3 - c4.1.3 :**

Answer the following question.

Part (a) Can a car that has been traveling at different speeds in a straight line east have accelerated west?

Let's consider this situation. If a car that was moving to the east accelerated to the west, it would start to slow down. However, if it didn't accelerate to the west fast enough, then this acceleration would just result in the car slowing down. Since the car is stated to have traveled at different speeds, there is actually no problem with it. As such, the answer is

 Yes**Problem 4 - c4.1.4 :**

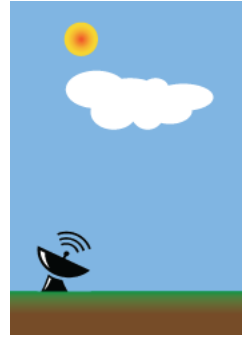
A ball is tied to a string and swung so that the ball follows a circular path that lies in a horizontal plane.

Part (a) Which of the following statements is true in an inertial reference frame, i.e. a reference frame that is at rest with respect to the Earth?

Since the ball is moving in a circle, the direction it travels is constantly changing. While we don't know anything about the speed for certain, we do know that velocity means that direction is a part of it. As such, since the ball is changing direction, the velocity is changing. The acceleration is similar, in that if the ball only accelerated, it wouldn't be able to keep spinning and instead would eventually go off in one general direction. Therefore, the direction of the acceleration must be constantly changing. Since velocity, acceleration is a vector, so changing direction is the same thing as changing the acceleration. We thus see that both the velocity and acceleration are constantly changing. The answer is

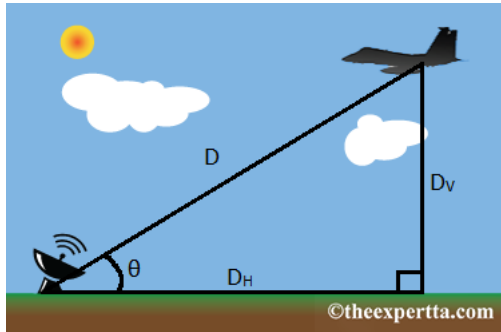
 The ball has changing velocity and changing acceleration.**Problem 5 - 4.1.1 :**

A plane flies towards a ground-based radar dish. Radar locates the plane at a distance $D = 11$ km from the dish, at an angle $\theta = 31^\circ$ above horizontal.



Part (a) What is the plane's horizontal distance, D_H in meters, from the radar dish?

We can begin this problem by drawing a triangle that shows the location of the plane relative to the radar.



Now, recall that the cosine function of an angle in a right triangle gives the adjacent side divided by the hypotenuse. We can use this information to create a relation to solve for the horizontal distance.

$$\cos(\theta) = \frac{D_H}{D}$$

$$D \cdot \cos(\theta) = D_H$$

$$D_H = D \cdot \cos(\theta)$$

$$D_H = 11 \text{ km} \cdot \cos(31^\circ)$$

$$D_H = 9.43 \text{ km}$$

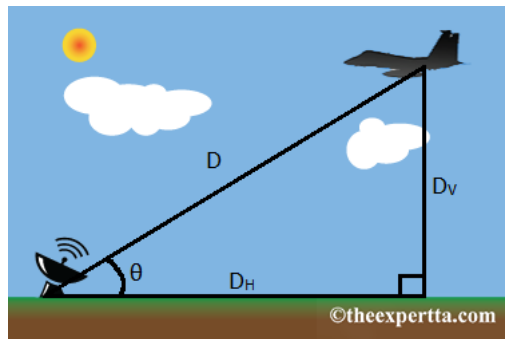
Now we have an answer in kilometers, but we were asked for an answer in meters. We must now convert our result to get the correct answer.

$$D_H = (9.43 \cdot 1000) \text{ m}$$

$$D_H = 9430.394 \text{ m}$$

Part (b) What is the plane's vertical distance, D_V in meters, above the radar dish?

As in part (a), let's begin by looking at an image that shows the location of the plane relative to the radar.



Now, recall that the sine function of an angle in a right triangle gives the opposite side divided by the hypotenuse. We can use this relationship to solve for the vertical distance to the plane.

$$\sin(\theta) = \frac{D_V}{D}$$

$$D \cdot \sin(\theta) = D_V$$

$$D_V = 11 \text{ km} \cdot \sin(31^\circ)$$

$$D_V = 5.663 \text{ km}$$

As in part (a), we are asked to get an answer in meters rather than kilometers. This means that we must convert our result from kilometers to meters to get the correct answer.

$$D_V = (5.663 \cdot 1000) \text{ m}$$

$$D_V = 5662.832 \text{ m}$$

Part (c) Write an expression for the distance vector, D , in rectangular form in terms of D and θ , in a coordinate system with the dish at the origin and the horizontal and vertical directions. (to the right and up)

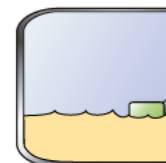
We already found expressions for the horizontal and vertical distance to the plane in parts (a) and (b). If we add the horizontal and vertical vectors together, then we get the distance vector. Recall that the unit vector "i" refers to movement to the right and "j" refers to upward movement. This means that the horizontal component will be multiplied by "i" and the vertical component will be multiplied by "j".

$$D = D_H + D_V$$

$$D = D \cos(\theta) \mathbf{i} + D \sin(\theta) \mathbf{j}$$

Problem 6 - 4.1.2 :

The fuel tank on a car is $d = 0.31$ m tall. The fuel level in the tank is detected by a $L = 0.605$ m arm that is free to rotate about a pivot at an upper fuel tank corner. Its sensor end floats at the surface of the fuel as indicated in the diagram



Part (a) Derive an expression for the sensor height, h , above the horizontal tank bottom as a function of L , d and θ (the angle between the arm and the v

In this problem, we want a function that we can use to say how high above the bottom of the tank the sensor is floating. We can use our knowledge of trigonometry to say how far down the tank in height the sensor is as follows.

$$h_d = L \cos \theta$$

What we want, however, is how high from the bottom the sensor is, not how far from the top it is. To account for this, we can subtract the distance from the top to the total height of the tank. This will give us the height from the bottom of the tank to the sensor.

$$h = d - L \cos \theta$$

Part (b) Use logic to deduce the angle, θ_{full} in degrees, associated with a full fuel tank, without performing any calculations.

We know that the arm is connected to the top corner of the fuel tank. If the tank was full, then the sensor would be at the same height as the arm. This means that the line perpendicular to the wall of the tank out to the sensor. Since the arm would be perpendicular to the wall, the angle between the wall and the sensor w

$$\theta_{full} = 90^\circ$$

Part (c) Calculate the angle, θ_{half} in degrees, associated with a half-full fuel tank.

Using our results from part (a), we can set up an equation for the angle and solve it for this scenario. The important thing to realize is that we know that if the tank is half full, the height of the sensor must be $\frac{d}{2}$.

$$h = d - L \cos \theta_{half}$$

$$\frac{d}{2} = d - L \cos \theta_{half}$$

$$\frac{d}{2} - d = -L \cos \theta_{half}$$

$$-\frac{d}{2} = -L \cos \theta_{half}$$

$$\frac{d}{2L} = \cos \theta_{half}$$

$$\arccos\left(\frac{d}{2L}\right) = \theta_{half}$$

$$\theta_{half} = \arccos\left(\frac{0.31 \text{ m}}{2 \cdot 0.605 \text{ m}}\right)$$

$$\theta_{half} = 75.194^\circ$$

Part (d) What angle, θ_{empty} in degrees, is associated with an empty fuel tank?

At first, one might be tempted to think that since the tank is empty, the arm would swing all the way back to the wall making an angle of zero degrees. However, if we compare the length of the arm to the height of the tank, we see that the arm is longer than the height of the tank such that it's impossible for it to be parallel with the wall. Instead, we use the results from part (a) to set up an equation for the angle in this case. The important thing to realize is that since the tank is empty, the height of the sensor above the bottom of the tank is zero.

$$h = d - L \cos \theta_{empty}$$

$$0 = d - L \cos \theta_{empty}$$

$$-d = -L \cos \theta_{empty}$$

$$\frac{d}{L} = \cos \theta_{empty}$$

$$\arccos\left(\frac{d}{L}\right) = \theta_{empty}$$

$$\theta_{empty} = \arccos\left(\frac{0.31 \text{ m}}{0.605 \text{ m}}\right)$$

$$\theta_{empty} = 59.176^\circ$$

Problem 7 - 4.1.3 :

A car is moving on a straight road in a fixed direction at a constant speed of $v = 31$ km/h with respect to the road. You wish to state the kinematic vectors of the motion of the car by using a Cartesian coordinate system whose positive x -axis is pointed in the direction of the motion of the car and the origin is fixed at some point on the road.

Part (a) What is the expression for the velocity of the car, using the speed v and the unit vectors i, j , and k ?

With i denoting the unit vector in the positive x -direction and v denoting the speed in the positive x -direction, the expression for the velocity vector is

$$v = vi$$

Part (b) What is the x -component of the position vector, in units of kilometers, at time $t = 0.02$ hr?

The car has constant speed v in the positive x -direction. We use the appropriate kinematic equation to find the x -component of its position vector at time t after leaving the origin.

$$x = vt = (31 \text{ km/h})(0.02 \text{ h}).$$

$$x = 0.62 \text{ km}$$

Problem 8 - 4.1.4 :

In a particular Cartesian coordinate system, the y and z -components of the velocity of a point-like object are zero and the x -component varies as given by the following function:

$$v_x(t) = 2 + 5t,$$

where t is in seconds and v_x is in meters per second.

Part (a) Find the instantaneous velocity, in meters per second, at $t = 0.55$ s.

We substitute the given time value into the given function for the x -component of the velocity.

$$v_x(t_1) = 2 + 5t_1 = 2 + 5(0.55 \text{ s}).$$

$$v_x(t_1) = 4.75 \text{ m/s}$$

Part (b) Find the instantaneous velocity, in meters per second, at $t = 1.5$ s.

We substitute the given time value into the given function for the x-component of the velocity.

$$v_x(t_2) = 2 + 5t_2 = 2 + 5(1.5 \text{ s}).$$

$$v_x(t_2) = 9.5 \text{ m/s}$$

Part (c) Find the instantaneous velocity, in meters per second, at $t = 2.5$ s.

We substitute the given time value into the given function for the x-component of the velocity.

$$v_x(t_3) = 2 + 5t_3 = 2 + 5(2.5 \text{ s}).$$

$$v_x(t_3) = 14.5 \text{ m/s}$$

Part (d) Find the instantaneous acceleration, in meters per square second, at $t = 0.55$ s.

The instantaneous acceleration, which in the present case is in the x-direction, is the time derivative of the x-component of the velocity function.

$$a_x(t) = 5.$$

The acceleration is thus constant

$$a_x(t) = 5 \text{ m/s}^2,$$

so its value at the given time is

$$a_x(t_1) = 5 \text{ m/s}^2$$

Part (e) Find the instantaneous acceleration, in meter per square second, at $t = 1.5$ s.

Since the acceleration is constant

$$a_x(t) = 5 \text{ m/s}^2,$$

its value at the given time is

$$a_x(t_2) = 5 \text{ m/s}^2$$

Part (f) Find the instantaneous acceleration, in meters per square second, at $t = 2.5$ s.

Since the acceleration is constant

$$a_x(t) = 5 \text{ m/s}^2,$$

its value at the given time is

$$a_x(t_3) = 5 \text{ m/s}^2$$

Part (g) If the particle was at $x = 0$ at $t = 0$, find the position, in meters, of the particle at $t = 0.55$ s.

The x-component of the position vector is the time integral of the x-component of the velocity vector, taking the initial condition into account. Integration gives us

$$x(t) = 2t + \frac{5t^2}{2} + \text{const.}$$

The given initial condition means that the integration constant is zero and we have

$$x(t) = 2t + \frac{5t^2}{2}.$$

We substitute the given time.

$$x(t_1) = 2(0.55 \text{ s}) + \frac{5(0.55 \text{ s})^2}{2}.$$

$$x(t_1) = 1.856 \text{ m}$$

Part (h) If the particle was at $x = 0$ at $t = 0$, find the position, in meters, of the particle at $t = 1.5$ s.

We use the function for the x-component of the position vector that we found in part (g),

$$x(t) = 2t + \frac{5t^2}{2},$$

and substitute the given time.

$$x(t_2) = 2(1.5 \text{ s}) + \frac{5(1.5 \text{ s})^2}{2}.$$

$$x(t_2) = 8.625 \text{ m}$$

Part (i) If the particle was at $x = 0$ at $t = 0$, find the position in meters of the particle at $t = 2.5$ s.

We use the function for the x-component of the position vector that we found in part (g),

$$x(t) = 2t + \frac{5t^2}{2},$$

and substitute the given time.

$$x(t_3) = 2(2.5 \text{ s}) + \frac{5(2.5 \text{ s})^2}{2}.$$

$$x(t_3) = 20.625 \text{ m}$$

Problem 9 - 4.1.5 :

In a particular Cartesian coordinate system, the y and z-components of the velocity of a point-like object are zero and the x-component varies as given by the following function:

$$v_x(t) = 2 + 5t - 3t^2,$$

where t is in seconds and v_x is in meters per second.

Part (a) Find the instantaneous velocity, in meters per second, at $t = 0.55$ s.

We substitute the given time value into the given function for the x-component of the velocity.

$$v_x(t_1) = 2 + 5t_1 - 3t_1^2 = 2 + 5(0.55 \text{ s}) - 3(0.55 \text{ s})^2.$$

$$v_x(t_1) = 3.843 \text{ m/s}$$

Part (b) Find the instantaneous velocity, in meters per second, at $t = 1.5$ s.

We substitute the given time value into the given function for the x-component of the velocity.

$$v_x(t_2) = 2 + 5t_2 - 3t_2^2 = 2 + 5(1.5 \text{ s}) - 3(1.5 \text{ s})^2.$$

$$v_x(t_2) = 2.75 \text{ m/s}$$

Part (c) Find the instantaneous velocity, in meters per second, at $t = 2.5$ s.

We substitute the given time value into the given function for the x-component of the velocity.

$$v_x(t_3) = 2 + 5t_3 - 3t_3^2 = 2 + 5(2.5 \text{ s}) - 3(2.5 \text{ s})^2.$$

$$v_x(t_3) = -4.25 \text{ m/s}$$

Part (d) Find the instantaneous acceleration, in meters per square second, at $t = 0.55$ s.

The instantaneous acceleration, which in the present case is in the x-direction, is the time derivative of the x-component of the velocity function,

$$a_x(t) = 5 - 6t.$$

We substitute the given time value.

$$a_x(t_1) = 5 - 6(0.55 \text{ s})$$

$$a_x(t_1) = 1.7 \text{ m/s}^2$$

Part (e) Find the instantaneous acceleration, in meters per square second, at $t = 1.5$ s.

We use the expression for the instantaneous acceleration that we derived in part (d),

$$a_x(t) = 5 - 6t.$$

We substitute the given time value.

$$a_x(t_2) = 5 - 6(1.5 \text{ s}).$$

$$a_x(t_2) = -4 \text{ m/s}^2$$

Part (f) Find the instantaneous acceleration, in meters per square second, at $t = 2.5$ s.

We use the expression for the instantaneous acceleration that we derived in part (d),

$$a_x(t) = 5 - 6t.$$

We substitute the given time value.

$$a_x(t_3) = 5 - 6(2.5 \text{ s}).$$

$$a_x(t_3) = -10 \text{ m/s}^2$$

Part (g) If the particle was at $x = 0$ at $t = 0$, find the position, in meters, of the particle at $t = 0.55$ s.

The x-component of the position vector is the time integral of the x-component of the velocity vector, taking the initial condition into account. Integration gives us

$$x(t) = 2t + \frac{5t^2}{2} - t^3 + \text{const.}$$

The given initial condition means that the integration constant is zero and we have

$$x(t) = 2t + \frac{5t^2}{2} - t^3.$$

We substitute the given time.

$$x(t_1) = 2(0.55 \text{ s}) + \frac{5(0.55 \text{ s})^2}{2} - (0.55 \text{ s})^3.$$

$$x(t_1) = 1.69 \text{ m}$$

Part (h) If the particle was at $x = 0$ at $t = 0$, find the position, in meters, of the particle at $t = 1.5$ s.

We use the expression for the x-component of the position vector that we found in part (g),

$$x(t) = 2t + \frac{5t^2}{2} - t^3,$$

and substitute the given time.

$$x(t_2) = 2(1.5 \text{ s}) + \frac{5(1.5 \text{ s})^2}{2} - (1.5 \text{ s})^3.$$

$$x(t_2) = 5.25 \text{ m}$$

Part (i) If the particle was at $x = 0$ at $t = 0$, find the position, in meters, of the particle at $t = 2.5$ s.

We use the expression for the x-component of the position vector that we found in part (g),

$$x(t) = 2t + \frac{5t^2}{2} - t^3,$$

and substitute the given time.

$$x(t_3) = 2(2.5 \text{ s}) + \frac{5(2.5 \text{ s})^2}{2} - (2.5 \text{ s})^3.$$

$$x(t_3) = 5 \text{ m}$$

Problem 10 - 4.1.6 :

In a particular Cartesian coordinate system, the y and z-components of the acceleration are zero and the x-component varies as given by the following function: $a_x(t) = -2 + 3t$, where t is in seconds and a_x is in meters per square second. The particle's velocity at $t = 0$ was pointed towards the positive x-axis and has a magnitude of 11 m/s.

Part (a) Find the instantaneous acceleration, in meters per square second, at $t = 1$ sec.

We substitute the given time value into the given function for the x-component of the acceleration.

$$a_x(t) = (-2 \text{ m/s}^2) + (31 \text{ m/s}^3)(1 \text{ s}).$$

$$a_x(t) = 29 \text{ m/s}^2$$

Part (b) Find the instantaneous acceleration, in meters per square second, at $t = 2$ sec.

We substitute the given time value into the given function for the x-component of the acceleration.

$$a_x(t) = (-2 \text{ m/s}^2) + (31 \text{ m/s}^3)(2 \text{ s}).$$

$$a_x(t) = 60 \text{ m/s}^2$$

Part (c) Find the instantaneous acceleration, in meters per square second, at $t = 3$ sec.

We substitute the given time value into the given function for the x-component of the acceleration.

$$a_x(t) = (-2 \text{ m/s}^2) + (31 \text{ m/s}^3)(3 \text{ s}).$$

$$a_x(t) = 91 \text{ m/s}^2$$

Part (d) Find the instantaneous velocity, in meters per second, at $t = 1$ sec.

To find the x-component of the instantaneous velocity as a function of time, we integrate the given function for the x-component of the acceleration, taking into a condition for the velocity.

$$v_x(t) = -2t + \frac{31t^2}{2} + \text{const.}$$

The initial condition tells us that $\text{const.} = 11 \text{ m/s}$, so the x-component of the velocity is

$$v_x(t) = \frac{31t^2}{2} - 2t + 11.$$

Now we substitute the given time value into the function for the x-component of the velocity.

$$v_x(t) = \frac{(31 \text{ m/s}^3)(1 \text{ s})^2}{2} - (2 \text{ m/s}^2)(1 \text{ s}) + (11 \text{ m/s}).$$

$$v_x(t) = 24.5 \text{ m/s}$$

Part (e) Find the instantaneous velocity, in meters per second, at $t = 2$ sec.

We substitute the given time value into the function for the x-component of the velocity that we found in part (d).

$$v_x(t) = \frac{(31 \text{ m/s}^3)(2 \text{ s})^2}{2} - (2 \text{ m/s}^2)(2 \text{ s}) + (11 \text{ m/s}).$$

$$v_x(t) = 69 \text{ m/s}$$

Part (f) Find the instantaneous velocity, in meters per second, at $t = 3$ sec.

We substitute the given time value into the function for the x-component of the velocity that we found in part (d).

$$v_x(t) = \frac{(31 \text{ m/s}^3)(3 \text{ s})^2}{2} - (2 \text{ m/s}^2)(3 \text{ s}) + (11 \text{ m/s}).$$

$$v_x(t) = 144.5 \text{ m/s}$$

Part (g) If the particle was at $x = 0$ at $t = 0$, find the position in meters of the particle at $t = 1$ s.

To find the x-component of the position vector as a function of time, we integrate the function for the x-component of the velocity that we found in part (d), taking condition for x.

$$x(t) = \frac{31t^3}{6} - \frac{2t^2}{2} + 11t + \text{const.}$$

The initial condition for x tells us that $\text{const.} = 0$, so

$$x(t) = \frac{31t^3}{6} - \frac{2t^2}{2} + 11t.$$

We now substitute the given time value into this function.

$$x(t) = \frac{(31 \text{ m/s}^3)(1 \text{ s})^3}{6} - \frac{(2 \text{ m/s}^2)(1 \text{ s})^2}{2} + (11 \text{ m/s})(1 \text{ s}).$$

$$x(t) = 15.167 \text{ m}$$

Part (h) If the particle was at $x = 0$ at $t = 0$, find the position in meters of the particle at $t = 2$ s.

We substitute the given time value into the function for x that we found in part (g).

$$x(t) = \frac{(31 \text{ m/s}^3)(2 \text{ s})^3}{6} - \frac{(2 \text{ m/s}^2)(2 \text{ s})^2}{2} + (11 \text{ m/s})(2 \text{ s}).$$

$$x(t) = 59.333 \text{ m}$$

Part (i) If the particle was at $x = 0$ at $t = 0$, find the position in meters of the particle at $t = 3$ s.

We substitute the given time value into the function for x that we found in part (g).

$$x(t) = \frac{(31 \text{ m/s}^3)(3 \text{ s})^3}{6} - \frac{(2 \text{ m/s}^2)(3 \text{ s})^2}{2} + (11 \text{ m/s})(3 \text{ s}).$$

$$x(t) = 163.5 \text{ m}$$

Problem 11 - 4.1.7 :

An object undergoing two-dimensional motion in the xy plane is shown in the figure as a motion diagram. The position of the object is shown after two equal time intervals of Δt each. The position at point A is $(0,0)$, the position at point B is (x_B, y_B) , and the position at point C is (x_C, y_C) .

A

Part (a) What is the average velocity of the object between position A and position B? Enter your answer as a vector in terms of the variables given above.

The velocity components in each direction are treated separately. Each is determined by taking the difference, final value minus the initial value, of either the x coordinate or the y coordinate for the j direction, respectively, and dividing by the elapsed time.

$$v_{AB} = \frac{(x_B - x_A)}{(\Delta t)} \mathbf{i} + \frac{(y_B - y_A)}{(\Delta t)} \mathbf{j}$$

Since point A has coordinates (0,0), we write the average velocity as

$$v_{AB} = \frac{x_B}{(\Delta t)} \mathbf{i} + \frac{y_B}{(\Delta t)} \mathbf{j}$$

Part (b) What is the average velocity of the object between position B and position C? Enter your answer as a vector in terms of the variables given above.

The velocity components in each direction are treated separately. Each is determined by taking the difference, final value minus the initial value, of either the x coordinate or the y coordinate for the j direction, respectively, and dividing by the elapsed time.

$$v_{BC} = \frac{(x_C - x_B)}{(\Delta t)} \mathbf{i} + \frac{(y_C - y_B)}{(\Delta t)} \mathbf{j}$$

Part (c) Now assume that the velocity of the particle in each interval is constant, with values equal to the average velocity you found in parts (a) and (b). What is the acceleration of the object over the entire interval shown in the figure? Enter your answer as a vector in terms of the variables given above and the unit vector \mathbf{j} .

Acceleration is the change in velocity with time. Using the velocities determined for the two time intervals in parts (a) and (b), we find

$$\begin{aligned} a_{ave} &= \frac{(v_{BC} - v_{AB})}{(2\Delta t)} \\ &= \frac{\left(\left(\frac{(x_C - x_B)}{(\Delta t)} \mathbf{i} + \frac{(y_C - y_B)}{(\Delta t)} \mathbf{j} \right) - \left(\frac{x_B}{(\Delta t)} \mathbf{i} + \frac{y_B}{(\Delta t)} \mathbf{j} \right) \right)}{(2\Delta t)} \end{aligned}$$

$$a_{ave} = \frac{((x_C - 2x_B) \mathbf{i} + (y_C - 2y_B) \mathbf{j})}{(2\Delta t)^2}$$

Part (d) If $x_B = 0.35$ m, $y_B = 0.31$ m, $x_C = 0.21$ m, $y_C = 1.7$ m, and the time interval $\Delta t = 0.6$ s, what is the x-component of this acceleration, in meters per second squared?

Use the x-component from part (c) and substitute the given values.

$$\begin{aligned} a_{ave,x} &= \text{Unmatched Grouping: ,) expected match} \\ &= \frac{((0.21 \text{ m}) - 2(0.35 \text{ m}))}{(2(0.6 \text{ s})^2)} \end{aligned}$$

$$a_{\text{ave},x} = -0.680555555555556 \text{ m/s}^2$$

Part (e) Using these same values, what is the y-component of the acceleration, in meters per second squared?

Use the y-component from part (c) and substitute the given values.

$$\begin{aligned} a_{\text{ave},y} &= \frac{(v_c - 2v_B)}{(2\Delta t)^2} \\ &= \frac{((1.7 \text{ m}) - 2(0.31 \text{ m}))}{(2(0.6 \text{ s})^2)} \end{aligned}$$

$$a_{\text{ave},y} = 1.5 \text{ m/s}^2$$

Part (f) What quadrant is this acceleration in?

The quadrants are numbered 1, 2, 3, 4, where quadrant 1 is the upper right where both x and y values are positive, then proceeding counterclockwise. In the case of for the acceleration in this problem, the x-component is negative and the y-component is positive. Therefore, the acceleration is in...

The second quadrant

Problem 12 - 4.1.8 :

In a particular Cartesian coordinate system, the y and z-components of the acceleration are zero and the x-component varies as given by the following function: $a_x(t) = 3t - 3t^2 + 20e^{-t/F}$, where t is in seconds, a_x is in meters per square second, and the constant F is in seconds. At $t = 0$, the particle was at position $x = 2$ m with a velocity pointing towards the positive x-axis and having magnitude 20 m/s. In the following problem you can take the constant $F = 1.0$ s.

Part (a) Find the instantaneous acceleration, in meters per second squared, at $t = 1$ s.

We substitute the given time value into the given function for the x-component of the acceleration.

$$a_x(t) = (3 \text{ m/s}^3)(1 \text{ s}) - (3 \text{ m/s}^4)(1 \text{ s})^2 + (20 \text{ m/s}^2)e^{-\frac{1 \text{ s}}{1.0 \text{ s}}}$$

$$a_x(t) = 7.358 \text{ m/s}^2$$

Part (b) Find the instantaneous acceleration, in meters per second squared, at $t = 2$ s.

We substitute the given time value into the given function for the x-component of the acceleration.

$$a_x(t) = (3 \text{ m/s}^3)(2 \text{ s}) - (3 \text{ m/s}^4)(2 \text{ s})^2 + (20 \text{ m/s}^2)e^{-\frac{2 \text{ s}}{1.0 \text{ s}}}$$

$$a_x(t) = -3.293 \text{ m/s}^2$$

Part (c) Find the instantaneous acceleration, in meters per second squared, at $t = 3$ s.

We substitute the given time value into the given function for the x-component of the acceleration.

$$a_x(t) = (3 \text{ m/s}^3)(3 \text{ s}) - (3 \text{ m/s}^4)(3 \text{ s})^2 + (20 \text{ m/s}^2)e^{-\frac{3 \text{ s}}{1.0 \text{ s}}}$$

$$a_x(t) = -17.004 \text{ m/s}^2$$

Part (d) Find the instantaneous velocity, in meters per second, at $t = 1$ s.

To find the instantaneous velocity as a function of time, we integrate the given function for the x-component of the acceleration, taking account of the initial condition. Integration gives us

$$v_x(t) = \frac{3 \text{ m/s}^3}{2} t^2 - \frac{3 \text{ m/s}^4}{3} t^3 - (20 \text{ m/s}^2) F e^{-\frac{t}{F}} + \text{const.}$$

The initial condition on the velocity is

$$v_x(0) = 20 \text{ m/s.}$$

Thus

$$\text{const.} = (20 \text{ m/s}) + (20 \text{ m/s}^2) F,$$

and the x-component of the velocity, which is the instantaneous velocity, as a function of time is

$$v_x(t) = \frac{3 \text{ m/s}^3}{2} t^2 - \frac{3 \text{ m/s}^4}{3} t^3 - (20 \text{ m/s}^2) F e^{-\frac{t}{F}} + (20 \text{ m/s}) + (20 \text{ m/s}^2) F.$$

Now substitute the given time value into the x-component of the velocity.

$$v_x(t) = \frac{3 \text{ m/s}^3}{2} (1 \text{ s})^2 - \frac{3 \text{ m/s}^4}{3} (1 \text{ s})^3 - (20 \text{ m/s}^2) (1.0 \text{ s}) e^{-\frac{1.0 \text{ s}}{1.0 \text{ s}}} + (20 \text{ m/s}) + (20 \text{ m/s}^2) (1.0 \text{ s}).$$

$$v_x(t) = 33.142 \text{ m/s}$$

Part (e) Find the instantaneous velocity, in meters per second, at $t = 2$ s.

Substitute the given time value into the function for the x-component of the velocity that we found in part (d).

$$v_x(t) = \frac{3 \text{ m/s}^3}{2} (2 \text{ s})^2 - \frac{3 \text{ m/s}^4}{3} (2 \text{ s})^3 - (20 \text{ m/s}^2) (1.0 \text{ s}) e^{-\frac{2 \text{ s}}{1.0 \text{ s}}} + (20 \text{ m/s}) + (20 \text{ m/s}^2) (1.0 \text{ s}).$$

$$v_x(t) = 35.293 \text{ m/s}$$

Part (f) Find the instantaneous velocity, in meters per second, at $t = 3$ s.

Substitute the given time value into the function for the x-component of the velocity that we found in part (d).

$$v_x(t) = \frac{3 \text{ m/s}^3}{2} (3 \text{ s})^2 - \frac{3 \text{ m/s}^4}{3} (3 \text{ s})^3 - (20 \text{ m/s}^2) (1.0 \text{ s}) e^{-\frac{3 \text{ s}}{1.0 \text{ s}}} + (20 \text{ m/s}) + (20 \text{ m/s}^2) (1.0 \text{ s}).$$

$$v_x(t) = 25.504 \text{ m/s}$$

Part (g) Find the position, in meters, of the particle at $t = 1$ s.

To find the position as a function of time, we integrate the function for the x-component of the velocity that we found in part (d), taking account of the initial condition.

$$x(t) = \frac{3 \text{ m/s}^3}{6} t^3 - \frac{3 \text{ m/s}^4}{12} t^4 + (20 \text{ m/s}^2) F^2 e^{-\frac{t}{F}} + ((20 \text{ m/s}) + (20 \text{ m/s}^2) F) t + \text{const.}$$

The initial condition on x is

$$x(0) = 2 \text{ m.}$$

Thus

$$\text{const.} = (2 \text{ m}) - (20 \text{ m/s}^2) F^2,$$

and the position as a function of time is

$$x(t) = \frac{3 \text{ m/s}^3}{6} t^3 - \frac{3 \text{ m/s}^4}{12} t^4 + (20 \text{ m/s}^2) F^2 e^{(-\frac{t}{F})} + \\ + ((20 \text{ m/s}) + (20 \text{ m/s}^2) F) t + (2 \text{ m}) - (20 \text{ m/s}^2) F^2.$$

Now substitute the given time into the function for x .

$$x(t) = \frac{3 \text{ m/s}^3}{6} (1 \text{ s})^3 - \frac{3 \text{ m/s}^4}{12} (1 \text{ s})^4 + (20 \text{ m/s}^2) (1.0 \text{ s})^2 e^{(-\frac{1 \text{ s}}{1.0 \text{ s}})} + \\ + ((20 \text{ m/s}) + (20 \text{ m/s}^2) (1.0 \text{ s})) (1 \text{ s}) + (2 \text{ m}) - (20 \text{ m/s}^2) (1.0 \text{ s})^2.$$

$$x(t) = 29.608 \text{ m}$$

Part (h) Find the position, in meters, of the particle at $t = 2 \text{ s}$.

Substitute the given time into the function for x that we found in part (g).

$$x(t) = \frac{3 \text{ m/s}^3}{6} (2 \text{ s})^3 - \frac{3 \text{ m/s}^4}{12} (2 \text{ s})^4 + (20 \text{ m/s}^2) (1.0 \text{ s})^2 e^{(-\frac{2 \text{ s}}{1.0 \text{ s}})} + \\ + ((20 \text{ m/s}) + (20 \text{ m/s}^2) (1.0 \text{ s})) (2 \text{ s}) + (2 \text{ m}) - (20 \text{ m/s}^2) (1.0 \text{ s})^2.$$

$$x(t) = 64.707 \text{ m}$$

Part (i) Find the position, in meters, of the particle at $t = 3 \text{ s}$.

Substitute the given time into the function for x that we found in part (g).

$$x(t) = \frac{3 \text{ m/s}^3}{6} (3 \text{ s})^3 - \frac{3 \text{ m/s}^4}{12} (3 \text{ s})^4 + (20 \text{ m/s}^2) (1.0 \text{ s})^2 e^{(-\frac{3 \text{ s}}{1.0 \text{ s}})} + \\ + ((20 \text{ m/s}) + (20 \text{ m/s}^2) (1.0 \text{ s})) (3 \text{ s}) + (2 \text{ m}) - (20 \text{ m/s}^2) (1.0 \text{ s})^2.$$

$$x(t) = 96.246 \text{ m}$$

Problem 13 - 4.1.9 :

Full solution not currently available at this time.

The position of an object is given to be $\mathbf{r} = 5.0 t^2 \mathbf{i} + 4.0 t^2 \mathbf{j}$ m, where t is in seconds.

Part (a) What is the magnitude of the particle's distance from the origin at $t = 0 \text{ s}$?

$$d(t=0\text{s}) = 0$$

Tolerance: ± 0

Part (b) What is the magnitude of the particle's distance from the origin at $t = 2 \text{ s}$?

$$d(t=2\text{s}) = 25.6$$

Tolerance: ± 0.768

Part (c) Provide an expression for the velocity as a vector (use i, j notation like r)

$$v = 10 t i + 8 t j$$

Part (d) What is the speed of the object when $t = 0$ s?

$$v(t=0s) = 0$$

Tolerance: ± 0

Part (e) What is the speed of the object when $t = 2$ s?

$$v(t=2s) = 25.6$$

Tolerance: ± 0.768

Problem 14 - 4.1.10 :

Full solution not currently available at this time.

The position of a particle changes from $r_1 = (1.01 \text{ cm})i + (3.01 \text{ cm})j$ to $r_2 = (-5.99 \text{ cm})i + (3.01 \text{ cm})j$.

Part (a) What is the magnitude of the particle's displacement, in cm?

$$d = r_{x1} - r_{x2}$$

$$d = 1.01 - (-5.99)$$

$$d = 7$$

Tolerance: ± 0.21

Problem 15 - 4.1.11 :

Full solution not currently available at this time.

A golfer hits two consecutive shots in which the ball's displacements were $\Delta r_1 = (250 \text{ m})i$ and $\Delta r_2 = (150 \text{ m})i + (150 \text{ m})j$.

Part (a) What is the magnitude of the total displacement, in meters?

$$d = \sqrt{(r_{x1} + r_{x2})^2 + r_{y2}^2}$$

$$d = \sqrt{(250 + 150)^2 + 150^2}$$

$$d = 427.2$$

Tolerance: ± 12.816

Part (b) What angle, in degrees, does the total displacement make with the y axis?

$$\theta = 90 - \text{atan}(r_{y2} / (r_{x1} + r_{x2})) / 0.0174533$$

$$\theta = 90 - \text{atan}(150 / (250 + 150)) / 0.0174533$$

$$\theta = 69.444$$

Tolerance: ± 2.08332

Problem 16 - 4.1.12 :

Full solution not currently available at this time.

The position of a particle is given by the following expression, where t is time measured in seconds: $r(t) = [(3.01 \text{ m/s}^2)t^2]i + (-4.99 \text{ m})j + [(2.01 \text{ m/s}^3)t^3]k$.

Part (a) What is the magnitude of the velocity of the particle, in m/s, at $t = 0.00$ s?

$$v = 0$$

Tolerance: ± 0

Part (b) What is the magnitude of the velocity of the particle, in m/s, at $t = 1.01$ s?

$$v = \sqrt{(2 * r_x * t)^2 + (3 * r_z * t^2)^2}$$

$$v = \sqrt{(2 * 3.01 * 1.01)^2 + (3 * 2.01 * 1.01^2)^2}$$

$$v = 8.649$$

Tolerance: ± 0.25947

Part (c) What angle, in degrees, does the velocity of the particle make with the $+z$ axis at $t = 1.01$ s?

$$\theta = \text{atan}((2 * r_x * t) / (3 * r_z * t^2)) / 0.0174533$$

$$\theta = \text{atan}((2 * 3.01 * 1.01) / (3 * 2.01 * 1.01^2)) / 0.0174533$$

$$\theta = 44.667$$

Tolerance: ± 1.34001

Part (d) What is the magnitude of the average velocity, in m/s, between $t = 0.00$ s and $t = 1.01$ s?

$$v = \sqrt{(rx*t)^2 + (rz*t^2)^2}$$

$$v = \sqrt{(3.01*1.01)^2 + (2.01*1.01^2)^2}$$

$$v = 3.667$$

Tolerance: ± 0.11001

Part (e) What angle, in degrees, does the average velocity between $t = 0.00$ s and $t = 1.01$ s make with the z axis?

$$\theta = \text{atan}((rx*t)/(rz*t^2))/0.0174533$$

$$\theta = \text{atan}((3.01*1.01)/(2.01*1.01^2))/0.0174533$$

$$\theta = 56.002$$

Tolerance: ± 1.68006

Problem 17 - 4.1.13 :

Full solution not currently available at this time.

In a time of 2.5 h, a bird flies a distance of 70.1 km in a direction 15 degrees east of north. Take north to be the positive y direction and east to be the positive x direction. Express your answers in km/h.

Part (a) What is the x component of the bird's average velocity?

$$x = d*\sin(\theta*0.0174533)/t$$

$$x = 70.1*\sin(15*0.0174533)/2.5$$

$$x = 7.257$$

Tolerance: ± 0.21771

Part (b) What is the y component of the bird's average velocity?

$$y = d*\cos(\theta*0.0174533)/t$$

$$y = 70.1*\cos(15*0.0174533)/2.5$$

$$y = 27.085$$

Tolerance: ± 0.81255

Problem 18 - 4.1.14 :

Full solution not currently available at this time.

A football player runs for a distance $d_1 = 7.01$ m in 1.01 s, at an angle of $\theta = 40.1$ degrees to the 50-yard line, then turns left and runs a distance $d_2 = 10.01$ m in 1.2 s, in a direction perpendicular to the 50-yard line. The diagram shows these two displacements relative to an x-y coordinate system, where the x axis is parallel to the 50-yard line, and the y axis is perpendicular to the 50-yard line.



Part (a) What is the magnitude of the total displacement, in meters?

$$d = \sqrt{(d_1*\cos(\theta*0.0174533))^2 + (d_1*\sin(\theta*0.0174533) + d_2)^2}$$

$$d = \sqrt{(7.01*\cos(40.1*0.0174533))^2 + (7.01*\sin(40.1*0.0174533) + 10.01)^2}$$

$$d = 15.483$$

Tolerance: ± 0.46449

Part (b) What angle, in degrees, does the displacement make with the y axis? (Note that the angle θ was given as measured from the x axis rather than the y axis.)

$$\theta_d = \text{atan}((d_1*\cos(\theta*0.0174533))/(d_1*\sin(\theta*0.0174533) + d_2))/0.0174533$$

$$\theta_d = \text{atan}((7.01*\cos(40.1*0.0174533))/(7.01*\sin(40.1*0.0174533) + 10.01))/0.0174533$$

$$\theta_d = 20.262$$

Tolerance: ± 0.60786

Part (c) What is the magnitude of the average velocity, in m/s?

$$v = \sqrt{(d1 \cdot \cos(\theta) \cdot 0.0174533)^2 + (d1 \cdot \sin(\theta) \cdot 0.0174533 + d2)^2} / (t1 + t2)$$

$$v = \sqrt{(7.01 \cdot \cos(40.1 \cdot 0.0174533))^2 + (7.01 \cdot \sin(40.1 \cdot 0.0174533) + 10.01)^2} / (1.01 + 1.2)$$

$$v = 7.006$$

Tolerance: ± 0.21018

Part (d) What angle, in degrees does the average velocity make with the y axis? (Note that the angle θ was given as measured from the x axis rather than

$$\theta_v = \text{atan}((d1 \cdot \cos(\theta) \cdot 0.0174533) / (d1 \cdot \sin(\theta) \cdot 0.0174533 + d2)) / 0.0174533$$

$$\theta_v = \text{atan}((7.01 \cdot \cos(40.1 \cdot 0.0174533)) / (7.01 \cdot \sin(40.1 \cdot 0.0174533) + 10.01)) / 0.0174533$$

$$\theta_v = 20.262$$

Tolerance: ± 0.60786

Problem 19 - 4.1.15 :

Full solution not currently available at this time.

The position of a particle is given by the following expression, where t is time measured in seconds: $\mathbf{r}(t) = [(2.01 \text{ m/s}^2)t^2]\mathbf{i} + (3.01 \text{ m})\mathbf{j} + [(3.01 \text{ m/s})t]\mathbf{k}$.

Part (a) What is the magnitude of the velocity, in m/s, of the particle at $t = 1.01$ s?

$$v = \sqrt{(2 \cdot r_x \cdot t)^2 + (r_z)^2}$$

$$v = \sqrt{(2 \cdot 2.01 \cdot 1.01)^2 + (3.01)^2}$$

$$v = 5.054$$

Tolerance: ± 0.15162

Part (b) What angle, in degrees, does the velocity of the particle make with the x axis at $t = 1.01$ s?

$$\theta = \text{atan}(r_z / (2 \cdot r_x \cdot t)) / 0.0174533$$

$$\theta = \text{atan}(3.01 / (2 \cdot 2.01 \cdot 1.01)) / 0.0174533$$

$$\theta = 36.551$$

Tolerance: ± 1.09653

Part (c) What is the magnitude of the particle's acceleration, in m/s^2 ?

$$a = 2 \cdot r_x$$

$$a = 2 \cdot 2.01$$

$$a = 4.02$$

Tolerance: ± 0.1206

Problem 20 - 4.1.16 :

Full solution not currently available at this time.

A particle has a constant acceleration of $\mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j}$ and at $t = 0$ it is at rest at the origin.

Part (a) What is the particle's position as a function of time?

$$\mathbf{r}(t) = 0.5 a_x t^2 \mathbf{i} + 0.5 a_y t^2 \mathbf{j}$$

Part (b) What is the particle's velocity as a function of time?

$$\mathbf{v}(t) = a_x t \mathbf{i} + a_y t \mathbf{j}$$

Part (c) What is the particle's path, expressed as x as a function of y ?

$$y = a_y / a_x x$$

Problem 21 - 4.1.17 :

Full solution not currently available at this time.

The position of a particle is given by $\mathbf{r} = (at^2)\mathbf{i} + (bt^3)\mathbf{j} + (ct^{-2})\mathbf{k}$, where a , b , and c are constants.

Part (a) What is the velocity as a function of time?

$$\mathbf{v}(t) = 2 a t \mathbf{i} + 3 b t^2 \mathbf{j} - 2 c t^{-3} \mathbf{k}$$

Part (b) What is the acceleration as a function of time?

$$\mathbf{a}(t) = 2 a \mathbf{i} + 6 b t \mathbf{j} + 6 c t^{-4} \mathbf{k}$$

Part (c) Suppose $a = 4.01 \text{ m/s}^2$, $b = -2.5 \text{ m/s}^3$, and $c = -75 \text{ ms}^2$. What is the particle's speed, in m/s, at $t = 1.5 \text{ s}$?

$$v = \sqrt{(2*a*t)^2 + (3*b*t^2)^2 + (2*c/(t^3))^2}$$

$$v = \sqrt{(2*4.01*1.5)^2 + (3*2.5*1.5^2)^2 + (2*75/(1.5^3))^2}$$

$$v = 49.039$$

Tolerance: ± 1.47117

Part (d) Referring to the values given in part (c), what is the magnitude of the particle's acceleration, in m/s^2 , at $t = 1.5 \text{ s}$?

$$a = \sqrt{(2*a)^2 + (6*b*t)^2 + (6*c/(t^4))^2}$$

$$a = \sqrt{(2*4.01)^2 + (6*2.5*1.5)^2 + (6*75/(1.5^4))^2}$$

$$a = 92.042$$

Tolerance: ± 2.76126

Problem 22 - 4.1.18 :

Full solution not currently available at this time.

The position of a particle is given by $\mathbf{r}(t) = A\cos(\omega t)\mathbf{i} + A\sin(\omega t)\mathbf{j} + ct\mathbf{k}$, where A is a constant with units of meters, ω is a constant with units of rad/s, and c is a constant with units of m/s.

Part (a) What is the velocity vector?

$$\mathbf{v}(t) = -\omega A \sin(\omega t) \mathbf{i} + \omega A \cos(\omega t) \mathbf{j} + c \mathbf{k}$$

Part (b) What is the acceleration vector?

$$\mathbf{a}(t) = -\omega^2 A \cos(\omega t) \mathbf{i} - \omega^2 A \sin(\omega t) \mathbf{j}$$

Problem 23 - 4.1.21 :

Full solution not currently available at this time.

At $t = 0$, a truck is traveling east at a constant speed of $s = 70.1 \text{ km/h}$. At an intersection $d = 32 \text{ km}$ ahead, a car is traveling north at constant speed of $v = 40.1 \text{ km/h}$

Part (a) Write an expression for the distance r between the truck and the car as a function of time. Use the variables from the problem statement for you

$$r = \sqrt{(d - st)^2 + (vt)^2}$$

Part (b) Write an expression for the time at which the distance between the car and the truck as its minimum value. Use the variables from the problem : equation.

$$t = d / (s^2 + v^2)$$

Part (c) What is the time, in hours, at which the distance between the car and the truck as its minimum value?

$$t = d*s / (v^2 + s^2)$$

$$t = 32*70.1 / (40.1^2 + 70.1^2)$$

$$t = 0.3439$$

Tolerance: ± 0.010317

Part (d) What is the minimum distance, in kilometers, between the car and the truck?

$$r = \sqrt{(d - s*(d*s/(v^2 + s^2)))^2 + v^2*(d*s/(v^2 + s^2))^2}$$

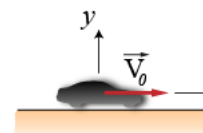
$$r = \sqrt{(32 - 70.1*(32*70.1/(40.1^2 + 70.1^2)))^2 + 40.1^2*(32*70.1/(40.1^2 + 70.1^2))^2}$$

$$r = 15.889$$

Tolerance: ± 0.47667

Problem 24 - 4.2.2 :

A car moves along a horizontal road with constant velocity $\mathbf{v}_0 = v_0\mathbf{i}$ until it encounters a smooth inclined hill, which it climbs with constant velocity $\mathbf{v}_1 = v_{1x}\mathbf{i} + v_{1y}\mathbf{j}$ as indicated in the figure. The car uniformly and instantaneously changes its velocity at the road-hill intersection. Let the origin of the Cartesian coordinate system be at the car's initial position.



Part (a) If the car moves for equal times along the road and hill, create an expression for its average velocity vector v_{ave} in terms of v_{0x} , v_{1x} , and v_{1y} and the unit vectors i and j .

The average velocity is given by the change in distance divided by the change in time. The change in distance will be given by how far the car travels, which can following equation:

$$\Delta x = t_1 v_{0x} i + t_2 (v_{1x} i + v_{1y} j)$$

Since the car travels for the same amount of time during both phases of its motion, both the time terms have the same value.

$$\Delta x = t v_{0x} i + t (v_{1x} i + v_{1y} j)$$

$$\Delta x = t (v_{0x} i + v_{1x} i + v_{1y} j)$$

The average velocity can now be found by dividing this result by the change in time, which will equal to the total time the car has traveled.

$$v_{ave} = \frac{\Delta x}{\Delta t}$$

$$v_{ave} = \frac{t (v_{0x} i + v_{1x} i + v_{1y} j)}{2t}$$

$$v_{ave} = \frac{v_{0x} i + v_{1x} i + v_{1y} j}{2}$$

$$v_{ave} = 0.5 (v_{0x} + v_{1x}) i + 0.5 v_{1y} j$$

Part (b) Create an expression for the direction of the car's average acceleration in terms of v_{0x} , v_{1x} , and v_{1y} during the transition between the horizontal. Express the answer in terms of $\tan(\theta)$, where θ is the angle of the average acceleration vector relative to the horizontal.

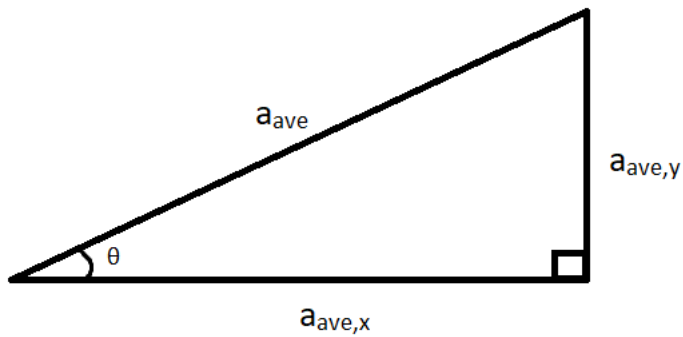
The average acceleration is given by the change in velocity divided by the change in time. The change in velocity will be given by the final velocity minus the initial velocity, meanwhile, will be given by the total amount of time the car has traveled. We can set up the following equation:

$$a_{ave} = \frac{\Delta v}{\Delta t}$$

$$a_{ave} = \frac{(v_{1x} i + v_{1y} j) - v_{0x} i}{\Delta t}$$

$$a_{ave} = \frac{v_{1x} - v_{0x}}{\Delta t} i + \frac{v_{1y}}{\Delta t} j$$

Now, let's draw a triangle using the horizontal and vertical components of the average acceleration:



Recall that the tangent function gives the opposite side divided by the adjacent side in a right triangle. We can therefore state the following relation based on our i

$$\tan(\theta) = \frac{a_{ave,y}}{a_{ave,x}}$$

We can now substitute in our results for the average acceleration, noting that the term with a coefficient of i represents the x-component of the average acceleration and the coefficient of j represents the y-component of the average acceleration.

$$\tan(\theta) = \frac{\left(\frac{v_{1y}}{\Delta t}\right)}{\left(\frac{v_{1x} - v_{0x}}{\Delta t}\right)}$$

$$\tan(\theta) = \frac{v_{1y}}{v_{1x} - v_{0x}}$$

Problem 25 - 4.2.3 :

A car moves along a horizontal road with constant velocity $\mathbf{v}_0 = v_{0x}\mathbf{i}$ until it encounters a smooth inclined hill, which it climbs with constant velocity $\mathbf{v}_1 = v_{1x}\mathbf{i} + v_{1y}\mathbf{j}$ as indicated in the figure. The period of time during which the car changes its velocity is Δt .

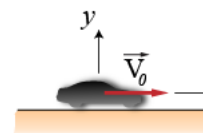
Randomized Variables

$$v_{0x} = 31 \text{ m/s}$$

$$v_{1x} = 25 \text{ m/s}$$

$$v_{1y} = 1.1 \text{ m/s}$$

$$\Delta t = 1.1 \text{ s}$$



Part (a) Give a vector expression for the average acceleration of the car during the given time period in terms of the variables in the problem and unit vectors.

This problem involves the relationship between **velocity and acceleration in two dimensions**. The solution requires the application of: (1) **vector algebra** to determine the vector, and (2) **Pythagorean's theorem** to find the magnitude of the acceleration vector.

In this part, we are asked to find an expression for the average acceleration as a vector, given three quantities: the initial velocity, the final velocity, and the time t .

The average acceleration, written as a vector, is

$$\vec{a}_{\text{ave}} = \frac{\Delta \vec{v}}{\Delta t}$$

Here, we have

$$\begin{aligned} \vec{a}_{\text{ave}} &= \frac{\vec{v}_1 - \vec{v}_0}{\Delta t} \\ &= \frac{1}{\Delta t} [(v_{1x} \hat{i} + v_{1y} \hat{j}) - (v_{0x} \hat{i})] \end{aligned}$$

It is *very important* to remember that **we can only combine vector components along the same direction**; that is, we can only combine those vector terms with

$$\vec{a}_{\text{ave}} = \frac{(v_{1x} - v_{0x}) \hat{i}}{\Delta t} + \frac{v_{1y} \hat{j}}{\Delta t}$$

$$\boxed{\vec{a}_{\text{ave}} = \frac{(v_{1x} - v_{0x}) \hat{i}}{\Delta t} + \frac{v_{1y} \hat{j}}{\Delta t}}$$

Part (b) What is the magnitude of the car's acceleration during the time period in question, in m/s^2 ?

Here, we apply **Pythagorean's theorem** to evaluate the magnitude of our result from *Part (a)*.

The average acceleration, written as a vector, is

$$\vec{a}_{\text{ave}} = \frac{(v_{1x} - v_{0x}) \hat{i}}{\Delta t} + \frac{v_{1y} \hat{j}}{\Delta t}$$

The magnitude of this acceleration is found quite simply using Pythagorean's theorem, where the x -component is one side of a right triangle, the y -component is the other side, and the magnitude is the hypotenuse!

$$\begin{aligned} |\vec{a}_{\text{ave}}| &= \sqrt{\left(\frac{(v_{1x} - v_{0x})}{\Delta t}\right)^2 + \left(\frac{v_{1y}}{\Delta t}\right)^2} \\ &= \frac{1}{\Delta t} \sqrt{(v_{1x} - v_{0x})^2 + (v_{1y})^2} \\ &= \frac{1}{1.1 \text{ s}} \sqrt{(25 \text{ m/s} - 31 \text{ m/s})^2 + (1.1 \text{ m/s})^2} \end{aligned}$$

$$= 5.545 \text{ m/s}^2$$

Voila!

$$|\vec{a}_{\text{ave}}| = 5.545 \text{ m/s}^2$$

Problem 26 - 4.2.4 :

An airplane starts at rest and accelerates at 5.6 m/s^2 at an angle of 25° south of west.

Part (a) After 7 s, how far in the westerly direction has the airplane traveled?

We are told that the airplane begins at rest and travel at an angle south of west. We must determine the horizontal component (due west) of its displacement.

Let "x" represent motion towards the west. The horizontal component can be found using the following kinematic equation:

$$x(t) = \frac{1}{2}a_x t^2 + v_{0x}t$$

where $v_{0x} = 0$ is the initial velocity component.

$$\begin{aligned} x &= \frac{1}{2}a \cos(\theta) t^2 \\ &= \frac{1}{2} \left(5.6 \frac{\text{m}}{\text{s}^2} \right) \cos(25^\circ) (7 \text{ s})^2 \end{aligned}$$

$$x = 124.345 \text{ m}$$

Part (b) After 7 s, how far in the southerly direction has the airplane traveled?

Let "y" represent motion towards the south. The appropriate kinematic equation for this situation is

$$y(t) = \frac{1}{2}a_y t^2 + v_{0y}t$$

where $v_{0y} = 0$ is the initial velocity component. So along the y-axis we have }

$$\begin{aligned} y &= \frac{1}{2}a \sin(\theta) t^2 \\ &= \frac{1}{2} \left(5.6 \frac{\text{m}}{\text{s}^2} \right) \sin(25^\circ) (7)^2 \end{aligned}$$

$$y = 57.983 \text{ m}$$

Problem 27 - 4.2.6 :

Full solution not currently available at this time.

A boat leaves the dock at $t = 0.00 \text{ s}$ and, starting from rest, maintains a constant acceleration of $(0.201 \text{ m/s}^2)\mathbf{i}$ relative to the water. Due to currents, however, the water itself is moving with a velocity of $(0.201 \text{ m/s})\mathbf{i} + (1.01 \text{ m/s})\mathbf{j}$.

Part (a) How fast is the boat moving, in m/s, at $t = 4.01 \text{ s}$?

$$\begin{aligned} v &= \sqrt{(a \cdot t + v_x)^2 + v_y^2} \\ v &= \sqrt{(0.201 \cdot 4.01 + 0.201)^2 + 1.01^2} \\ v &= 1.426 \end{aligned}$$

Tolerance: ± 0.04278

Part (b) How far, in meters, is the boat from the dock at $t = 4.01$ s?

$$s = \sqrt{((0.5*a*t^2+v_x*t)^2+(v_y*t)^2)}$$

$$s = \sqrt{((0.5*0.201*4.01^2+0.201*4.01)^2+(1.01*4.01)^2)}$$

$$s = 4.719$$

Tolerance: ± 0.14157

Problem 28 - 4.2.7 :

Full solution not currently available at this time.

The acceleration of a particle is constant. At $t = 0$, the particle is at the origin and the velocity of the particle is $\mathbf{v}_0 = v_1\mathbf{i} + v_2\mathbf{j}$. At time $t = T$, the velocity of the particle is $\mathbf{v} = v_3\mathbf{j}$. Here v_1 , v_2 , and v_3 are constants with dimensions of length divided by time. All answers should be written in terms of v_1 , v_2 , v_3 , T , and the unit vectors \mathbf{i} and \mathbf{j} .

Part (a) What is the particle's acceleration vector?

$$\mathbf{a} = -v_1/T \mathbf{i} + (v_3 - v_2)/T \mathbf{j}$$

Part (b) What is the particle's position vector at $t = 2T$?

$$\mathbf{r} = 2 v_3 T \mathbf{j}$$

Part (c) What is the particle's velocity vector at $t = 2T$?

$$\mathbf{v} = -v_1 \mathbf{i} + (2 v_3 - v_2) \mathbf{j}$$

Problem 29 - 4.2.8 :

Full solution not currently available at this time.

A fighter jet takes off from the horizontal deck of an aircraft carrier. Starting from rest, it achieves a speed of 55 m/s as it rolls 80.1 m on the deck to the end of the runway, at which point the jet leaves the deck and its acceleration changes to 4.01 m/s² at 30.1 degrees above horizontal.

Part (a) What is the magnitude of the acceleration, in m/s², of the jet while it is rolling on the deck?

$$a = (v^2)/(2*d)$$

$$a = (55^2)/(2*80.1)$$

$$a = 18.883$$

Tolerance: ± 0.56649

Part (b) What is the speed of the jet, in m/s, 3.8 s after it leaves the deck of the aircraft carrier?

$$v = \sqrt{(v+a*\cos(\theta)*t)^2+(a*\sin(\theta)*t)^2}$$

$$v = \sqrt{(55+4.01*\cos(30.1*0.0174533)*3.8)^2+(4.01*\sin(30.1*0.0174533)*3.8)^2}$$

$$v = 68.61$$

Tolerance: ± 2.0583

Part (c) What is the altitude (measured in meters, relative to the deck) of the jet 3.8 s after it leaves the deck of the aircraft carrier?

$$h = 0.5*a*\sin(\theta)*t^2$$

$$h = 0.5*4.01*\sin(30.1*0.0174533)*3.8^2$$

$$h = 14.52$$

Tolerance: ± 0.4356

Part (d) How far, horizontally in meters, is the jet from the end of the runway 3.8 s after it leaves the deck of the aircraft carrier?

$$d = v*t+0.5*a*\cos(\theta)*t^2$$

$$d = 55*3.8+0.5*4.01*\cos(30.1*\pi/180)*3.8^2$$

$$d = 234.048$$

Tolerance: ± 7.02144

Problem 30 - 4.2.9 :

Full solution not currently available at this time.

A spaceship is traveling at a velocity of $\mathbf{v}_0 = (20.1 \text{ m/s})\mathbf{i}$ when its rockets fire, giving it an acceleration of $\mathbf{a} = (2.01 \text{ m/s}^2)\mathbf{i} + (4.01 \text{ m/s}^2)\mathbf{k}$.

Part (a) How fast, in m/s, is the rocket moving 3.01 s after the rockets fire?

$$v = \sqrt{(v_0 + ax)^2 + (ay)^2}$$

$$v = \sqrt{(20.1 + 2.01 \cdot 3.01)^2 + (4.01 \cdot 3.01)^2}$$

$$v = 28.801$$

Tolerance: ± 0.86403

Problem 31 - 4.2.10 :**Full solution not currently available at this time.**

An airplane flies horizontally at a speed of **301** km/h and drops a crate that falls to the horizontal ground below. Neglect air resistance.

Part (a) If the altitude of the plane was **501** m, then how far, horizontally in meters, did the crate move as it fell to the ground?

$$x = \sqrt{2 \cdot h / 9.8} \cdot (v_0 / 3.6)$$

$$x = \sqrt{2 \cdot 501 / 9.8} \cdot (301 / 3.6)$$

$$x = 845.444$$

Tolerance: ± 25.36332

Part (b) What was the speed of the crate, in m/s, just before it hit the ground?

$$v = \sqrt{2 \cdot 9.8 \cdot h + (v_0 / 3.6)^2}$$

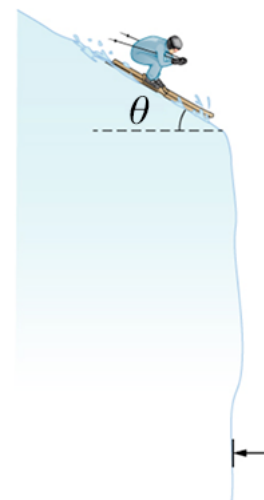
$$v = \sqrt{2 \cdot 9.8 \cdot 501 + (301 / 3.6)^2}$$

$$v = 129.655$$

Tolerance: ± 3.88965

Problem 32 - 4.2.11 :**Full solution not currently available at this time.**

A secret agent skis off a slope inclined at $\theta = 25$ degrees below horizontal at a speed of $v_0 = 12$ m/s. He must clear a gorge, and the slope on the other side of the gorge is $h = 10.1$ m below the edge of the upper slope.



Part (a) What is the maximum width, w , of the gorge (in meters) so that the agent clears it?

$$w = v_0 \cos(\theta) \left(\frac{v_0 \sin(\theta)}{g} \sqrt{2 + 19.6 \cdot h} - \frac{v_0 \sin(\theta)}{g} \right)$$

$$w = 12 \cos(25) \left(\frac{12 \sin(25)}{9.8} \sqrt{2 + 19.6 \cdot 10.1} - \frac{12 \sin(25)}{9.8} \right)$$

$$w = 10.969$$

Tolerance: ± 0.32907

Problem 33 - c4.3.1 :

You throw a ball horizontally from the top of a building, with a speed of 3 m/s. In this problem, you can neglect the force of air resistance.

Part (a) After two seconds, what is the ball's horizontal velocity?

In order to change an object's velocity in a given direction, forces must act on the object, and those forces must add as vectors to give a non-zero result in that same direction. Since there are no forces in the horizontal direction, the velocity in the horizontal direction must remain the same as it was initially. Although there is a force of gravity acting on the object, it does not act in the horizontal direction, and so it only produces a changing velocity in the vertical direction. This changing vertical velocity but constant horizontal velocity is what gives projectiles their characteristic parabolic shape.

Problem 34 - c4.3.2 :

Consider the following situation concerning projectile motion.

Part (a) In a situation where there is no air resistance, can a projectile be thrown in such a way such that its velocity and acceleration vectors are perpendicular at some point along its path?

Let's examine the physics of this problem for a moment. The acceleration of the thrown ball will be straight down due to gravity. Therefore, in order for the velocity vectors to be perpendicular, the ball must be going straight forward. If you throw a ball forward and at an upward angle, then at the exact moment the ball reaches its peak, it will have no velocity in the y-direction, but will still have velocity in the x-direction, meaning that such a throw creates a moment where the velocity and acceleration vectors are perpendicular. You could also create this situation by throwing the ball straight forward such that velocity and acceleration are perpendicular the moment the ball is thrown. We have seen that it is clearly possible to create this situation, so the answer is

Yes.

Problem 35 - c4.3.3 :

You throw a rock off of a cliff in the horizontal direction with a velocity, v . One second later you throw an identical rock with the same velocity, v , in the horizontal direction.

Part (a) What is true about the distance between the two rocks beginning after you throw the second one?

Let's think about the physics of this situation. The rock that was thrown first will have gained a downward velocity due to gravitational acceleration before the second rock is thrown, both rocks will accrue downward velocity at the same rate because they are both equally affected by gravity. However, the first rock will have a larger downward velocity than the second from the effects of gravity on it during its head start. Since the first rock always has a higher downward velocity, the second rock will gradually fall behind, widening the distance between them. The answer is therefore

The distance between the rocks will increase as time passes.

Problem 36 - c4.3.4 :

A ball is thrown off of a cliff with an initial velocity that is horizontal. There is no air resistance and it follows the path shown (typical projectile motion).



Part (a) What direction is the acceleration of the ball?

Even though the ball continues moving to the right, this does not necessarily mean that it experiences any acceleration to the right. Rather, it could simply be that it has its initial horizontal velocity. Since there is no air resistance in this case, the only thing that is accelerating the ball is gravity. Therefore, the ball is indeed not experiencing any horizontal acceleration and is merely being accelerated downward.



Problem 37 - c4.3.5 :**Full solution not currently available at this time.**

A projectile is launched from ground level at an angle of 30 degrees above the horizontal. Neglect air resistance and consider the motion from just after the moment it is launched to just before the moment it lands on the ground.

Part (a) When is the projectile's velocity equal to zero?

The projectile's velocity is never zero.

Part (b) When does the projectile have the smallest speed?

At the highest point.

Part (c) When does the projectile's speed equal its launch speed?

Just before landing on the ground.

Part (d) After launch, when does the projectile's velocity equal its launch velocity?

The projectile's velocity is never equal to its launch velocity after launch.

Problem 38 - c4.3.6 :**Full solution not currently available at this time.**

A projectile is launched from ground level at an angle of 30 degrees above the horizontal. Neglect air resistance and consider the motion from just after the moment it is launched to just before the moment it lands on the ground.

Part (a) When is the projectile's acceleration equal to zero?

The projectile's acceleration is never zero.

Part (b) At what point do the velocity and the acceleration have the same direction?

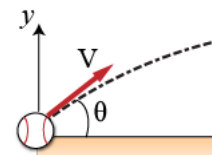
The projectile's velocity and acceleration never have the same direction.

Part (c) At what point are the velocity and the acceleration perpendicular?

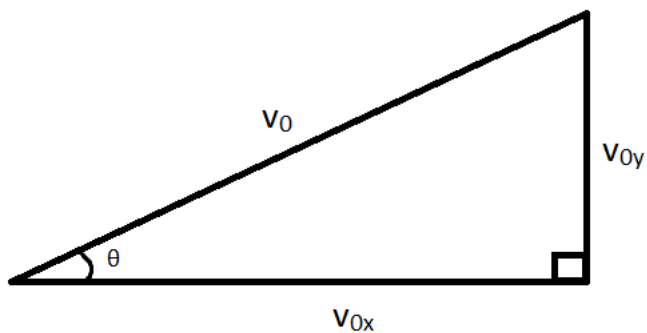
At the highest point.

Problem 39 - 4.3.1 :

During a baseball game, a baseball is struck at ground level by a batter. The ball leaves the baseball bat with an initial speed $v_0 = 25$ m/s at an angle $\theta = 15^\circ$ above horizontal. Let the origin of the Cartesian coordinate system be the ball's position the instant it leaves the bat. Air resistance may be ignored throughout this problem.

**Part (a)** Express the magnitude of the ball's initial horizontal velocity v_{0x} in terms of v_0 and θ .

Let's start by drawing a triangle from the horizontal and vertical components of the initial velocity:



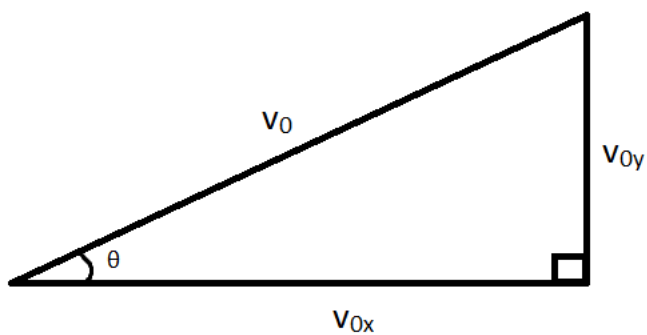
Recall that the sine function of an angle in a right triangle gives the opposite side divided by the hypotenuse. We can use this to write an equation to relate the angle and the initial vertical velocity.

$$\frac{v_{0y}}{v_0} = \sin(\theta)$$

$$v_{0y} = v_0 \sin(\theta)$$

Part (b) Express the magnitude of the ball's initial vertical velocity v_{0y} in terms of v_0 and θ .

Let's start by drawing a triangle from the horizontal and vertical components of the initial velocity:



Recall that the cosine function of an angle in a right triangle gives the adjacent side divided by the hypotenuse. We can use this to write an equation to relate the angle and the initial horizontal velocity.

$$\frac{v_{0x}}{v_0} = \cos(\theta)$$

$$v_{0x} = v_0 \cos(\theta)$$

Part (c) Find the ball's maximum vertical height h_{max} in meters above the ground.

To find the maximum vertical height of the ball, we only need to look at the vertical motion of the ball. We will need to use a kinematic equation for this, so let's choose our coordinate system to have the origin at the ground and up as the positive direction. Now, let's go over our known values. We found an expression for the initial vertical velocity v_{0y} in part (a), we know that the ball will experience acceleration downward due to gravity, and we know that the ball will have a final vertical acceleration of zero at the maximum height. We can therefore use the following kinematic equation to find a value for how high the ball traveled:

$$v^2 = v_0^2 + 2ad$$

Now, let's plug values for our system into this equation and solve for the maximum height.

$$(0 \text{ m/s})^2 = (25 \text{ m/s} \cdot \sin(15^\circ))^2 + 2 \cdot -9.8 \text{ m/s}^2 \cdot h_{max}$$

$$0 = (25 \text{ m/s} \cdot \sin(15^\circ))^2 - 19.6 \text{ m/s}^2 \cdot h_{max}$$

$$19.6 \text{ m/s}^2 \cdot h_{max} = (25 \text{ m/s} \cdot \sin(15^\circ))^2$$

$$h_{max} = \frac{(25 \text{ m/s} \cdot \sin(15^\circ))^2}{19.6 \text{ m/s}^2}$$

$$h_{max} = 2.132 \text{ m}$$

Part (d) Enter an expression in terms of v_0 , θ , and g for the time t_{max} it takes the ball to travel to its maximum vertical height.

As in part (c), we will only need to concern ourselves with the vertical motion of the ball to solve this problem. We can use the same coordinate system we used in part (c) with the same known variables with the addition of the ball's maximum height. Based on our known values, we can use the following kinematic equation:

$$v = v_0 + at$$

Now, let's plug in our variables, recalling that the velocity of the ball in the vertical direction will be zero when it reaches its maximum height.

$$0 \text{ m/s} = v_0 \sin(\theta) - gt$$

$$-v_0 \sin(\theta) = -gt$$

$$t = \frac{v_0 \sin(\theta)}{g}$$

Part (e) Calculate the horizontal distance x_{max} in meters that the ball has traveled when it returns to ground level.

To solve this problem, we will obviously need to use another kinematic equation. We can let our coordinate system be the same as it was in parts (c) and (d). We know the initial velocity of the ball from part (a). We also know that the ball experiences no horizontal acceleration during its flight. Finally, in part (d) we found an expression for the time it takes for the ball to reach its maximum height. Due to the symmetry of this problem, the ball will take just as long to land as it did to go up, meaning that the total time of flight is equal to twice the expression we found in part (d). With this information, we can set up the following kinematic equation for the ball's horizontal motion:

$$d = v_0 t + \frac{1}{2} a t^2$$

Let's begin plugging in variables (remembering that the time will be twice the expression that we found in part (d)) and solve for the total horizontal distance.

$$x_{max} = v_0 \cos(\theta) \cdot \left(2 \cdot \frac{v_0 \sin(\theta)}{g}\right) + \frac{1}{2} \cdot 0 \text{ m/s}^2 \cdot \left(2 \cdot \frac{v_0 \sin(\theta)}{g}\right)^2$$

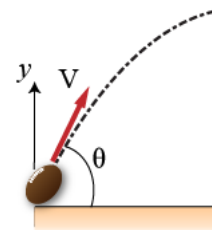
$$x_{max} = 2 \cdot \frac{v_0^2 \cos(\theta) \sin(\theta)}{g}$$

$$x_{max} = 2 \cdot \frac{(25 \text{ m/s})^2 \cdot \cos(15^\circ) \cdot \sin(15^\circ)}{9.81 \text{ m/s}^2}$$

$$x_{max} = 31.841 \text{ m}$$

Problem 40 - 4.3.2 :

A quarterback throws a football with an initial velocity v at an angle θ above horizontal. Assume the ball leaves the quarterback's hand at ground level and moves without air resistance. All portions of this problem will produce algebraic expressions in terms of v , θ , and g . Let the origin of the Cartesian coordinate system be the ball's initial position.



Part (a) Write an expression for the magnitude of the football's initial vertical velocity v_{0y} .

The velocity vector has both horizontal and vertical components. Knowing the angle and the magnitude of the velocity vector, the vertical component is found us

$$v_{0y} = v \sin(\theta)$$

Part (b) Find an expression for the magnitude of the football's initial horizontal velocity v_{0x} .

The velocity vector has both horizontal and vertical components. Knowing the angle and the magnitude of the velocity vector, the horizontal component is found

$$v_{0x} = v \cos(\theta)$$

Part (c) Write an expression for the total time, t_{total} , the football is in the air.

Using the equations of kinematics, we can find the time of flight because the football is under the constant acceleration due to gravity.

$$y = v_{0y} t + \frac{1}{2} g t^2$$

The ball starts and ends its flight at $y = 0$. Therefore, we can solve this for t .

$$v_{0y} t = \frac{1}{2} g t^2$$

$$v_{0y} = \frac{1}{2}gt$$

$$t = 2\frac{v_{0y}}{g}$$

$$t = 2\frac{(v\sin(\theta))}{g}$$

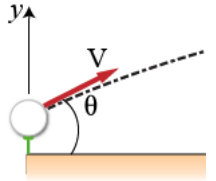
Problem 41 - 4.3.3 :

A famous golfer strikes a golf ball on the ground, giving it an initial velocity $\mathbf{v} = v_{0x}\mathbf{i} + v_{0y}\mathbf{j}$. Assume the ball moves without air resistance and its motion is described using a Cartesian coordinate system with its origin located at the ball's initial position.

Randomized Variables

$$v_{0x} = 15 \text{ m/s}$$

$$v_{0y} = 10.1 \text{ m/s}$$

**Part (a) Determine the maximum height above the ground, h_{max} in meters, attained by the golf ball.**

When the ball reaches its maximum height, it is momentarily at rest. We can use this information and the kinetic equations to find the maximum height, $h = y_{max}$

$$v_y^2 = v_{0y}^2 + 2ay$$

$$0 = v_{0y}^2 + 2gh$$

$$h = \frac{v_{0y}^2}{2g}$$

$$= \frac{(10.1 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)}$$

$$h = 5.199 \text{ m}$$

Part (b) Express the total horizontal distance, x_{max} , the ball travels until it returns to ground level in terms of v_{0x} , v_{0y} , and g .

As the ball flies upward and then back toward the Earth, it moves horizontally with a constant speed, since there is assumed to be no acceleration of the ball in the horizontal direction. The horizontal distance the ball travels is

$$x_{max} = v_{0x}t$$

The time of flight can be found as twice the time it takes the ball to reach the maximum height.

$$v_y = v_{0y} + at$$

$$t = \frac{(v_y - v_{0y})}{g} = -\frac{v_{0y}}{g}$$

$$x_{max} = v_{0x}t = \frac{v_{0x}(-2v_{0y})}{g}$$

Note that when we substitute the value for the acceleration due to gravity that we will include the minus sign representing the downward direction. The result is to write the magnitudes for each of the variables.

$$x_{max} = \frac{2v_{0x}v_{0y}}{g}$$

Part (c) Evaluate the total horizontal distance, x_{max} in meters, the ball travels until it returns to ground level.

$$\begin{aligned} x_{max} &= \frac{v_{0x}(-2v_{0y})}{g} \\ &= \frac{2(15 \text{ m/s})(-10.1 \text{ m/s})}{(-9.81 \text{ m/s}^2)} \end{aligned}$$

$$x_{max} = 30.887 \text{ m}$$

Problem 42 - 4.3.4 :

A soccer ball is kicked from the ground at an angle of $\theta = 58$ degrees with respect to the horizontal. The hang time of the ball is $t_m = 2.4$ s.

Randomized Variables

$\theta = 58$ degrees

$t_m = 2.4$ s

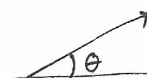
Part (a) Numerically, what is the total horizontal distance, d_m in meters, traveled by the ball in the time, t_m ?

Numeric : A numeric value is expected and not an expression.

$d_m =$

The distance is given by $x = \frac{1}{2}at^2 + v_0t + x_0 \rightarrow d_m = v_{0x}t_m$. Don't know v_{0x} , but $v_{0x} = v_0 \cos \theta$ for initial velocity v_0 , and if we use the equation $v = v_0 + at$ in the y -direction at the top of the path we have

$$0 = v_0 \sin \theta - g \frac{t_m}{2} \rightarrow v_0 = \frac{g t_m}{2 \sin \theta}$$



$$\text{Then } v_{0x} = \left(\frac{g t_m}{2 \sin \theta} \right) \cos \theta = \frac{g t_m}{2} \cot \theta$$

$$\text{So } d_m = \left(\frac{g t_m}{2} \cot \theta \right) t_m = \frac{1}{2} g t_m^2 \cot \theta, \quad \boxed{d_m \approx 17.6 \text{ m}}$$

Problem 43 - 4.3.5 :

While competing in the long jump, a person leaps over a smooth horizontal sand surface. She lands on the surface with speed $v_f = 5.25$ m/s at an angle $\theta = 30.5^\circ$ below horizontal. Assume that the person moves without air resistance. Use a Cartesian coordinate system with the origin at her final position. The positive x -axis is directed from her initial to her final position, and the positive y -axis is directed vertically upwards.

Part (a) Enter an expression for the jumper's initial velocity vector in terms of v_f , θ , g , and the unit vectors i and j .

Let's think about the jumper's trajectory for a moment. She experiences no air resistance or other forces that could accelerate her horizontally, so her horizontal v is constant throughout the jump. She also falls just as far as she goes up. As a consequence of this, her final vertical velocity should be the same magnitude and opposite vertical velocity. As a result, we can determine her initial velocity from her final velocity. Due to the symmetric nature of her trajectory, we can therefore use a bit with the angle and velocity with which she lands to determine her initial velocity.

$$v_0 = v_f \cos(\theta) i + v_f \sin(\theta) j$$

Part (b) Calculate the maximum height, in meters, that the jumper reaches above ground level.

Here, we want to model the vertical path from the beginning of the jump to its peak. We found what the initial vertical velocity is in part (a), and we know that the peak of the jump will be zero. We also know that the acceleration will simply be the acceleration due to gravity. Given these known variables, we want to pick a kinematic equation that can use these to solve for the distance traveled. We will want to use the following equation:

$$v_f^2 = v_i^2 + 2ad$$

Now, we can plug the values from our scenario into this equation and solve it for the final height.

$$0 = (v_f \sin(\theta))^2 + 2(-g)h_{max}$$

$$0 = (v_f \sin(\theta))^2 - 2gh_{max}$$

$$2gh_{max} = (v_f \sin(\theta))^2$$

$$h_{max} = \frac{(v_f \sin(\theta))^2}{2g}$$

$$h_{max} = \frac{(5.25 \text{ m/s} \cdot \sin(30.5^\circ))^2}{2 \cdot 9.81 \text{ m/s}^2}$$

$$h_{max} = 0.3615 \text{ m}$$

Part (c) What x position, in meters, did the jumper begin her long jump?

Let's start by examining our scenario. First, we want to find the distance from where the jumper landed to where she started. We took the direction of the jump to be the positive x-direction, so the distance we are finding is in the negative x-direction. As a result, we should expect a negative answer. Second, there is no acceleration in the x-direction, so as a consequence, there is no way to find the distance traveled unless we know the amount of time the jump took. Our first goal, therefore, is to find out how long it took to find this, we will want to figure out how long it took for her to complete the vertical part of the jump, as that will tell us how long she was in the air. For the portion of the jump from the ground to the maximum height, we know the initial velocity, acceleration due to gravity, and the fact that the final velocity at the top of the jump will be zero. We can use the following kinematic equation that tells us how long it took for her to reach her maximum height. Let's use the following equation:

$$v_f = v_i + at_u$$

Now let's plug in the variables for this scenario.

$$0 = v_f \sin(\theta) - gt_u$$

$$gt_u = v_f \sin(\theta)$$

$$t_u = \frac{v_f \sin(\theta)}{g}$$

Due to the symmetry of the jump, the total time of the jump will be twice the time taken to reach the maximum height. We can now express the total time in terms of t_u .

$$t = 2t_u$$

$$t = 2 \frac{v_f \sin(\theta)}{g}$$

Now that we know how long the jumper was in the air, we can use the following kinematic equation to solve for how far she traveled:

$$(x_f - x_0) = v_i t + \frac{1}{2} a t^2$$

Now let's plug in our values and solve.

$$(0 - x_0) = v_f \cos(\theta) \cdot \left(2 \frac{v_f \sin(\theta)}{g} \right) + 0$$

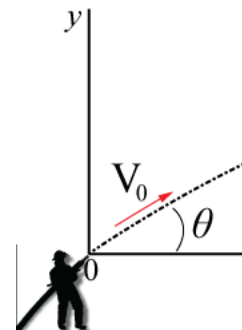
$$-x_0 = 5.25 \text{ m/s} \cdot \cos(30.5^\circ) \cdot 2 \frac{5.25 \text{ m/s} \cdot \sin(\theta)}{9.81 \text{ m/s}^2}$$

$$x_0 = - \left(5.25 \text{ m/s} \cdot \cos(30.5^\circ) \cdot 2 \frac{5.25 \text{ m/s} \cdot \sin(\theta)}{9.81 \text{ m/s}^2} \right)$$

$$x_0 = -2.457 \text{ m}$$

Problem 44 - 4.3.10 :

Water leaves a fireman's hose (held near the ground) with an initial velocity $v_0 = 10.5$ m/s at an angle $\theta = 25^\circ$ above horizontal. Assume the water acts as a projectile that moves without air resistance. Use a Cartesian coordinate system with the origin at the hose nozzle position.



Part (a) Using v_0 , θ , and g , write an expression for the time, t_{\max} , the water travels to reach its maximum vertical height.

At the maximum height, the water will momentarily have no vertical component of its velocity. We can apply the equations of kinematics to find the time the water reaches its maximum height.

$$v_y = v_{0y} + at = v_{0y} - gt_{\max} = 0$$

Solving for the elapsed time gives,

$$t_{\max} = \frac{v_{0y}}{g}$$

$$t_{\max} = \frac{v_0 \sin(\theta)}{g}$$

Part (b) At what horizontal distance d from the building base, where should the fireman place the hose for the water to reach its maximum height as it travels this distance, d , in terms of v_0 , θ , and g .

In part a, we found the time it takes for the water to reach the maximum height. Now, we want to know how far it moves horizontally during that time. Since the horizontal direction is constant, we can use the definition of average speed to get the horizontal distance.

$$d = v_x t_{\max}$$

The horizontal component of the initial velocity remains constant and is written,

$$v_x = v_{0x} = v_0 \cos(\theta)$$

Substitution this into the first equation and using the answer from part a gives,

$$d = v_0 \cos(\theta) t_{\max} = (v_0 \cos(\theta)) \left(v_0 \frac{\sin(\theta)}{g} \right)$$

$$d = v_0^2 \frac{\cos(\theta) \sin(\theta)}{g}$$

Problem 45 - 4.3.9 :

A soccer ball is kicked from ground level across a level soccer field with initial velocity vector $v_0 = 6$ m/s at $\theta = 15^\circ$ above horizontal. The soccer ball feels wind resistance which causes it to slow horizontally with constant acceleration magnitude $a_x = 0.52$ m/s², while leaving its vertical motion unchanged.

Assume any other air resistance is negligible. Choose the positive direction of x from initial point towards final point of flight. Use a Cartesian coordinate system with the origin at the ball's initial position.

Part (a) The ball travels through the air until it returns to the soccer field. Calculate the ball's time of flight, t_f in seconds.

Here, we want to find how long the ball was in the air. As a result, we can ignore the velocity and acceleration in the x -direction and focus on the y -direction. The up, and no other forces besides gravity are acting on it. As such, it will take the ball equally long to go from the ground to its highest point as it does for the ball to the ground. This means that we simply need to find the time it takes the ball to reach its highest point to find the entire time it was in the air. To begin, let's decompose into a horizontal and vertical components using trigonometry.

$$v_0 = v_0 \cos(\theta) \mathbf{i} + v_0 \sin(\theta) \mathbf{j}$$

Now that we have found the vertical component of the initial velocity, we can begin setting up a kinematic equation for the y -direction. We just found the initial velocity, we know that the velocity in the y -direction at the ball's highest point will be zero, and we know that the acceleration in the y -direction is just gravity. Given these, we can pick an equation of motion to solve for time. Let's use the following equation:

$$v_f = v_i + at$$

Now let's plug in values for our scenario.

$$0 = v_0 \sin(\theta) - gt$$

$$gt = v_0 \sin(\theta)$$

$$t = \frac{v_0 \sin(\theta)}{g}$$

This gives us an answer for how long it takes for the ball to reach its maximum height. As mentioned earlier, the time that it takes for the ball to land will be just twice as long as it takes for the ball to reach its highest point, so the time the entire flight takes will just be twice as long as it takes for the ball to reach its maximum height. Thus, we can now solve for the air.

$$t_f = 2t$$

$$t_f = 2 \frac{v_0 \sin(\theta)}{g}$$

$$t_f = 2 \frac{6 \text{ m/s} \cdot \sin(15^\circ)}{9.81 \text{ m/s}^2}$$

$$t_f = 0.3168 \text{ s}$$

Part (b) Calculate the horizontal distance, x_{max} in meters, the ball travels before it returns to the soccer field.

Here, we want to find how far the ball travels in the x -direction. We found the x -component of the initial velocity along with the time in part (a), and we know what case is. With this information, we can choose which equation of motion to use to find the total distance.

$$d = v_i t + \frac{1}{2} a t^2$$

Now, let's plug in our values. The only thing to watch out for is the fact that the acceleration in this case is opposing the direction of motion. Therefore, the acceleration value.

$$x_{max} = v_0 \cos(\theta) t_f + \frac{1}{2} (-a_x) t_f^2$$

$$x_{max} = v_0 \cos(\theta) t_f - \frac{1}{2} a_x t_f^2$$

$$x_{max} = 6 \text{ m/s} \cdot \cos(15^\circ) \cdot 0.3168 \text{ s} - \frac{1}{2} \cdot 0.52 \text{ m/s}^2 \cdot (0.3168 \text{ s})^2$$

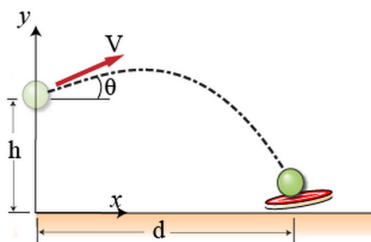
$x_{max} = 1.808 \text{ m}$

Problem 46 - 4.3.6 (alt) :

A student throws a water balloon with speed v_0 from a height $h = 1.96 \text{ m}$ at an angle $\theta = 38^\circ$ above horizontal toward a target on the ground. The target is located a horizontal distance $d = 9.5 \text{ m}$ from the student's feet. Assume that the balloon moves without air resistance. Use a Cartesian coordinate system with the origin at the balloon's initial position.

Randomized Variables

$\theta = 38$ degrees
 $h = 1.96 \text{ m}$
 $d = 9.5 \text{ m}$



Part (a) What is the position vector, $\mathbf{R}_{\text{target}}$, that originates from the balloon's original position and terminates at the target? Put this in terms of h and d , and represent it as a vector using \mathbf{i} and \mathbf{j} .

Expression :

$\mathbf{R}_{\text{target}} =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\theta)$, $\sin(\theta)$, α , β , θ , \mathbf{i} , \mathbf{j} , d , g , h , m , \mathbf{P} , r , t , v_0

Part (b) In terms of the variables in the problem, determine how long, t , after the launch it takes the balloon to reach the target. Your answer should *not* include h .

Expression :

$t =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\theta)$, $\sin(\theta)$, α , β , θ , d , g , h , \mathbf{i} , \mathbf{k} , m , \mathbf{P} , r , t , v_0

Part (c) Create an expression for the balloon's vertical position as a function of time, $y(t)$, in terms of t , v_0 , g , and θ .

Expression :

$y(t) =$ _____

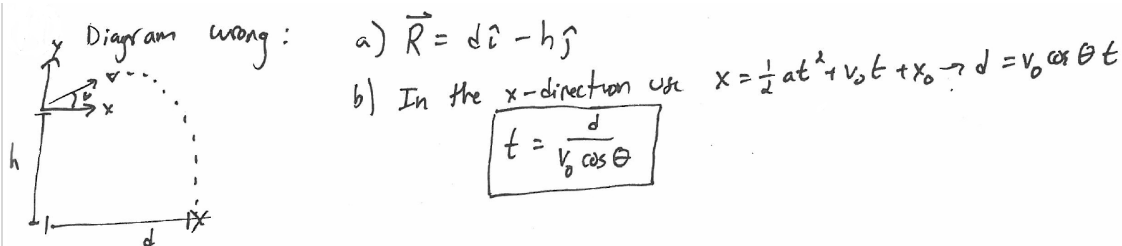
Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\theta)$, $\sin(\theta)$, α , β , θ , \mathbf{i} , \mathbf{j} , d , g , h , m , \mathbf{P} , r , t , v_0

Part (d) Determine the magnitude of the balloon's initial velocity, v_0 , in meters per second, by eliminating t from the previous two expressions.

Numeric : A numeric value is expected and not an expression.

$v_0 =$ _____



c) In the y-direction $y = \frac{1}{2}at^2 + v_0t + y_0 \rightarrow y(t) = -\frac{1}{2}gt^2 + v_0 \sin \theta t$

d) Plug the result from part (b) into the result from part (c). $y(t) = -h$ now since the time from part (b) was calculated at the point of impact.

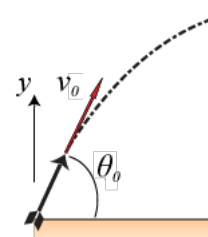
$$-h = -\frac{1}{2}g \left(\frac{d^2}{v_0^2 \cos^2 \theta} \right) + v_0 \sin \theta \left(\frac{d}{v_0 \cos \theta} \right)$$

$$-h - d \tan \theta = -\frac{1}{2}g \frac{d^2}{v_0^2 \cos^2 \theta} \rightarrow \frac{2 \cos^2 \theta}{g d^2} (h + d \tan \theta) = \frac{1}{v_0^2}$$

$$\rightarrow v_0 = \sqrt{\frac{g d}{2 \cos^2 \theta} \left(\frac{h}{d} + \tan \theta \right)^{-1}}, \quad v_0 \approx 8.71 \text{ m/s}$$

Problem 47 - 4.3.11 :

An arrow is fired with an initial velocity v_0 at an angle θ_0 above horizontal. Assume the arrow moves without air resistance. Use a Cartesian coordinate system with the origin at the arrow's initial position to analyze the arrow's motion.



Part (a) Some time later the arrow makes an angle of $\theta = 15$ degrees with respect to the horizontal. Write an expression for the time, t , that passes between fired and this later point.

Since there is no acceleration in the horizontal direction, we can write the expression for the horizontal component of the initial velocity as

$$v_x = v_{0x} = v \cos(\theta) = v_0 \cos(\theta_0)$$

For the vertical direction, the acceleration due to gravity acts downward on the arrow throughout its flight. Therefore, using the equations of kinematics, we write

$$v_y = v \sin(\theta) = v_0 \sin(\theta_0) - gt$$

We can solve the first equation above for an expression for v , then substitute that into the second equation and solve for the time t .

$$v = v_0 \frac{\cos(\theta_0)}{\cos(\theta)}$$

$$v \sin(\theta) = v_0 \sin(\theta_0) - gt$$

$$t = \frac{(v_0 \sin(\theta_0) - v \sin(\theta))}{g}$$

$$= \frac{\left(v_0 \sin(\theta_0) - \left(v_0 \frac{\cos(\theta_0)}{\cos(\theta)} \right) \sin(\theta) \right)}{g}$$

$$t = \frac{v_0}{g}(\sin(\theta_0) - \cos(\theta_0) \tan(\theta))$$

Problem 48 - 4.3.12 :

The cork from a champagne bottle slips through the hands of a waiter opening it, moving with an initial velocity $v_0 = 11$ m/s at an angle $\theta = 75^\circ$ above horizontal. A diner is sitting a horizontal distance d away when this happens. Assume the cork leaves the waiter's hands at the same vertical level as the diner and that the cork falls back to this vertical level when it reaches the diner. Use a Cartesian coordinate system with the origin at the cork's initial position.

Part (a) Calculate the time, t_d in seconds, for the cork to reach the diner.

We know from the problem statement that the cork reaches the diner at the moment when it falls back to its initial height. As such, we can find how long it takes to reach the diner based on how long it takes for it to reach its maximum height and return to its initial height. Furthermore, since the cork falls the same distance that it goes up on it, the amount of time it will take the cork to fall back down to its original height is the same amount of time that it takes the cork to reach its maximum height. The amount of time that the cork takes to reach its maximum height and double that value to find the amount of time that it took for the cork to reach the diner. The first thing to do is to decompose our initial velocity into horizontal and vertical components.

$$v_0 = v_0 \cos(\theta) \mathbf{i} + v_0 \sin(\theta) \mathbf{j}$$

Now we have the initial vertical velocity. We also know that, if we are trying to find the time for the cork to reach its maximum height, that the final velocity will be zero. We know the downward acceleration due to gravity. Taking our known variables into account, we need to pick a kinematic equation that will allow us to find the time.

$$v_f = v_i + at$$

Now that we have chosen an equation, let's plug in the values for this equation and calculate the time it takes for the cork to reach its highest point.

$$0 = v_0 \sin(\theta) - gt$$

$$gt = v_0 \sin(\theta)$$

$$t = \frac{v_0 \sin(\theta)}{g}$$

$$t = \frac{11 \text{ m/s} \cdot \sin(75^\circ)}{9.81 \text{ m/s}^2}$$

Now that we have a value for the time it takes for the cork to reach its highest point, we can double it to get the time of the entire trip.

$$t_d = 2t$$

$$t_d = 2 \frac{11 \text{ m/s} \cdot \sin(75^\circ)}{9.81 \text{ m/s}^2}$$

$$t_d = 2.166 \text{ s}$$

Part (b) Reacting quickly to avoid being struck, the diner moves 2.00 m horizontally directly toward the waiter opening the champagne bottle. Determine the distance, in meters, between the person and the diner at the time the cork reaches where the diner had previously been sitting.

According to the description, the diner is just two meters closer to the waiter than the original distance. This means that the first thing we want to do is solve for t from part (a), we found both the amount of time that the cork's journey takes and a decomposition of the initial velocity. We also know that there is nothing acting on the cork's horizontal velocity, so the acceleration is zero. Given all this information, we want to choose a kinematic equation that allows us to solve for the distance.

$$d = v_i t + \frac{1}{2} a t^2$$

Now let's plug our known values for this problem into the equation, using the horizontal component of velocity that we found in part (a) as our velocity.

$$d_i = v_0 \cos(\theta) t_d + 0$$

$$d_i =$$

$$d_i = 6.181 \text{ m}$$

This solves for the initial distance. Now all we need to do is subtract the two meters that the diner moved towards the waiter to get a final answer.

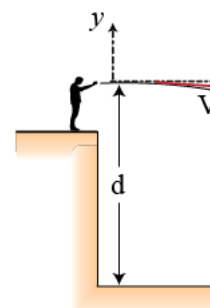
$$d = d_i - 2.00$$

$$d = 6.181 \text{ m} - 2.00 \text{ m}$$

$$d = 4.181 \text{ m}$$

Problem 49 - 4.3.13 :

A student standing on a cliff that is a vertical height $d = 8.0 \text{ m}$ above the level ground throws a stone with velocity $v_0 = 15 \text{ m/s}$ at an angle $\theta = 15^\circ$ below horizontal. The stone moves without air resistance; use a Cartesian coordinate system with the origin at the stone's initial position.



Part (a) With what speed, v_f in meters per second, does the stone strike the ground?

The horizontal component of the stone's velocity throughout its flight does not change, since there is no acceleration. The expression describing this is

$$v_x = v_0 \cos(\theta)$$

The vertical component of the stone's velocity does vary with time because of the downward acceleration due to gravity that acts on it throughout its flight. Apply kinematic equation gives the relationship between the vertical component of the velocity and the height from which the stone originates.

$$v_y^2 = v_0^2 \sin^2(\theta) + 2gd$$

Note that the acceleration vector and the vertical displacement vector both point downward, so the minus signs associated with g and d cancel each other.

The magnitude of the stone's final velocity is found using the Pythagorean theorem.

$$v_f = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 \cos^2(\theta) + v_0^2 \sin^2(\theta) + 2gd}$$

Note that $\cos^2(\theta) + \sin^2(\theta) = 1$, which simplifies the above equation.

$$\begin{aligned} v_f &= \sqrt{v_0^2 + 2gd} \\ &= \sqrt{(15 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(8.0 \text{ m})} \end{aligned}$$

$$v_f = 19.544 \text{ m/s}$$

Part (b) If the stone had been thrown from the cliff top with the same initial speed and the same angle, but *above* the horizontal, would its impact velocity

From part a, we see that angle that the stone is thrown doesn't enter into the final equation for the final velocity.

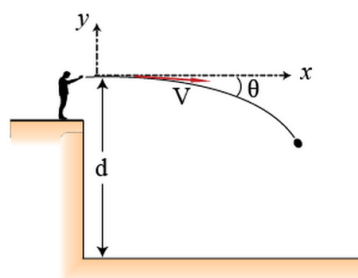
$$v_f = \sqrt{v_0^2 + 2gd}$$

Therefore, the answer will be the same no matter how the stone is shown with that initial velocity. Later, you'll learn about the conservation of mechanical energy further clarified.

No

Problem 50 - 4.3.13 (alt) :

A student throws a stone from a cliff top position $d = 8 \text{ m}$ above a horizontal plane with initial velocity $v_0 = 16 \text{ m/s}$ at an angle $\theta = 16^\circ$ below horizontal. The stone moves without air resistance; use a Cartesian coordinate system with the origin at the stone's initial position.



Randomized Variables

$$\theta = -16 \text{ degrees}$$

$$v_0 = 16 \text{ m/s}$$

Part (a) With what speed, v_f in m/s, does the stone strike the ground?

Numeric : A numeric value is expected and not an expression.

$$v_f = \underline{\hspace{2cm}}$$

Part (b) If the stone had been thrown from the cliff top in the same manner but upward instead of downward, would its impact velocity be different?

MultipleChoice :

- 1) Yes
- 2) No

a) The stone's x-velocity does not change, $v_x = v_0 \cos \theta$. The y-velocity can be found with $v_f^2 = v_0^2 + 2a \Delta x$:

$$v_y^2 = v_0^2 \sin^2 \theta + 2g d \quad (a = -g \text{ and } \Delta x = -d \text{ so negatives cancel!})$$

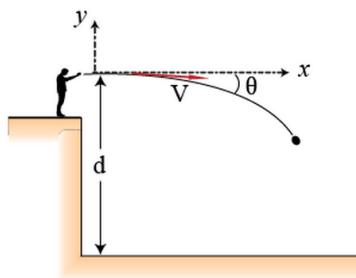
$$\text{Final velocity } v_f = \sqrt{v_x^2 + v_y^2} = \left[v_0^2 \cos^2 \theta + v_0^2 \sin^2 \theta + 2g d \right]^{1/2}$$

$$= \left[v_0^2 + 2g d \right]^{1/2}, \quad \boxed{v_f \approx 20.3 \text{ m/s}}$$

b) No. If thrown upward, on its way down when it reached a height of 8 meters we would be back to the original problem (also conservation of energy!)

Problem 51 - 4.3.13 (alt2) :

A student throws a stone from a cliff top position $d = 8$ m above a horizontal plane with initial velocity $v_0 = 16$ m/s at an angle $\theta = 16^\circ$ below horizontal. The stone moves without air resistance; use a Cartesian coordinate system with the origin at the stone's initial position.



Randomized Variables

$\theta = -16$ degrees
 $v_0 = 16$ m/s

Part (a) With what speed, v_f in m/s, does the stone strike the ground?

Numeric : A numeric value is expected and not an expression.

$v_f =$ _____

Part (b) If the stone had been thrown from the cliff top in the same manner but upward instead of downward, would its impact velocity be different?

MultipleChoice :

- 1) Yes
- 2) No

a) The stone's x-velocity does not change, $v_x = v_0 \cos \theta$. The y-velocity can be found with $v_f^2 = v_0^2 + 2a \Delta x$:

$$v_y^2 = v_0^2 \sin^2 \theta + 2g d \quad (a = -g \text{ and } \Delta x = -d \text{ so negatives cancel!})$$

$$\text{Final velocity } v_f = \sqrt{v_x^2 + v_y^2} = \left[v_0^2 \cos^2 \theta + v_0^2 \sin^2 \theta + 2g d \right]^{1/2}$$

$$= \left[v_0^2 + 2g d \right]^{1/2}, \quad \boxed{v_f \approx 20.3 \text{ m/s}}$$

b) No. If thrown upward, on its way down when it reached a height of 8 meters we would be back to the original problem (also conservation of energy!)

Problem 52 - 4.3.14 :

Apollo 14 astronaut Alan B. Shepard Jr. used an improvised six-iron to strike two golf balls while on the Fra Mauro region of the moon's surface, making what some consider the longest golf drive in history. Assume one of the golf balls was struck with initial velocity $v_0 = 25$ m/s at an angle $\theta = 15^\circ$ above the horizontal. The gravitational acceleration on the moon's surface is approximately 1/6 that on the earth's surface. Use a Cartesian coordinate system with the origin at the ball's initial position.

Randomized Variables

$v_0 = 25$ m/s
 $\theta = 15$ degrees

Part (a) What horizontal distance, R in meters, did this golf ball travel before returning to the lunar surface?

From the equations of kinematics, we can find the horizontal position of the golf ball as a function of time.

$$x = v_{0x} t + \frac{1}{2} a t^2$$

Since there is no acceleration in the horizontal direction, this simplifies to $x = v_{0x} t$. In this equation, we know the magnitude and direction of the initial velocity, time of flight. That can be determined by the ball's vertical motion. At the maximum height, the ball's vertical velocity component is momentarily equal to zero and acceleration is that due to the Moon's gravity, which is downward with a magnitude of $\frac{g}{6}$.

$$v_y = 0 = v_{0y} - \left(\frac{g}{6}\right) \left(\frac{t}{2}\right) = v_{0y} - \frac{gt}{12}$$

The elapsed time when the ball reaches the maximum height is one half the time of flight.

$$t = 12 \frac{v_{0y}}{g} = 12v_0 \frac{\sin(\theta)}{g}$$

Inserting this time into the equation for the horizontal distance R at the time of flight gives,

$$\begin{aligned} R &= v_{0x}t \\ &= v_0 \cos(\theta) t \\ &= (v_0 \cos(\theta)) \left(12v_0 \frac{\sin(\theta)}{g} \right) \\ &= 12v_0^2 \frac{\cos(\theta)\sin(\theta)}{g} \\ &= 12(25 \text{ m/s})^2 \frac{\cos(15^\circ)\sin(15^\circ)}{(9.81 \text{ m/s}^2)} \end{aligned}$$

$$R = 191.044 \text{ m}$$

Part (b) What is the numeric value of the ratio, f , of the horizontal distance traveled on the moon to the horizontal distance the ball would travel on Earth conditions?

In part a, we saw that the distance traveled is

$$R = 2 \left(v_0^2 \frac{\cos(\theta)\sin(\theta)}{a} \right)$$

where a is the downward acceleration due to gravity. On the Earth, the acceleration due to gravity is represented by g . The acceleration due to gravity on the Moon is $\frac{g}{6}$. Therefore, the ratio of the horizontal range on the Moon to that on Earth is

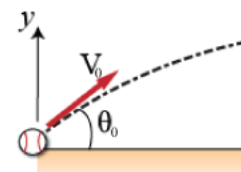
$$\begin{aligned} f &= \frac{R_{\text{Moon}}}{R_{\text{Earth}}} \\ &= \frac{\left(2v_0^2 \frac{\cos(\theta)\sin(\theta)}{\left(\frac{g}{6}\right)} \right)}{\left(2v_0^2 \frac{\cos(\theta)\sin(\theta)}{g} \right)} \end{aligned}$$

Simplifying this gives,

$$f = 6$$

Problem 53 - 4.3.15 :

A baseball slugger hits a pitch and watches the ball fly into the bleachers for a home run, landing $h = 5.5$ m higher than it was struck. When visiting with the fan that caught the ball, he learned the ball was moving with final velocity $v_f = 30.15$ m/s at an angle $\theta_f = 25^\circ$ below horizontal when caught. Assume the ball encountered no air resistance, and use a Cartesian coordinate system with the origin located at the ball's initial position.



Part (a) Create an expression for the ball's initial horizontal velocity, v_{0x} , in terms of the variables given in the problem statement.

The initial velocity vector of the baseball has components in both the horizontal and vertical directions. The horizontal component is given by

$$v_{0x} = v_0 \cos(\theta_0)$$

Since there is no acceleration of the ball in the horizontal direction after it leaves the bat and is flying through the air (because we are not including the effects due to air resistance), the horizontal component of the ball's velocity at the end of its flight must be the same as that for the initial velocity.

$$v_{0x} = v_f \cos(\theta_f)$$

Part (b) Calculate the ball's initial vertical velocity, v_{0y} , in m/s.

The vertical component is given by

$$v_{0y} = v_0 \sin(\theta_0)$$

The final y-component is given by

$$v_{fy} = v_f \sin(\theta_f)$$

Using kinematics, we can relate the initial and final vertical velocity components.

$$v_{fy}^2 = v_{0y}^2 - 2gh$$

$$\begin{aligned} v_{0y} &= \sqrt{v_{fy}^2 + 2gh} \\ &= \sqrt{v_f^2 \sin^2(\theta) + 2gh} \\ &= \sqrt{(30.15 \text{ m/s}^2)^2 \sin^2(25^\circ) + 2(9.81 \text{ m/s}^2)(5.5 \text{ m})} \end{aligned}$$

$$v_{0y} = 12.091 \text{ m/s}$$

Part (c) Calculate the magnitude of the ball's initial velocity, v_0 , in m/s.

The vertical component of the initial velocity is given in terms of the final vertical component as

$$v_{0y} = \sqrt{v_{fy}^2 + 2gh}$$

The horizontal component of the initial velocity is given by

$$v_{0x} = v_{fx} = v_f \cos(\theta)$$

since there is no acceleration in the horizontal direction.

Using Pythagorean theorem, we can find the initial velocity from its components.

$$\begin{aligned} v_0 &= \sqrt{v_{0x}^2 + v_{0y}^2} \\ &= \sqrt{v_f^2 \cos^2(\theta) + \left(\sqrt{v_{fy}^2 + 2gh}\right)^2} \\ &= \sqrt{v_f^2 \cos^2(\theta) + v_{fy}^2 + 2gh} \\ &= \sqrt{v_f^2 \cos^2(\theta) + v_f^2 \sin^2(\theta) + 2gh} \\ &= \sqrt{v_f^2 + 2gh} \\ &= \sqrt{(30.15 \text{ m/s}^2)^2 + 2(9.81 \text{ m/s}^2)(5.5 \text{ m})} \end{aligned}$$

Note that: $\sin^2(\theta) + \cos^2(\theta) = 1$.

$$v_0 = 31.889 \text{ m/s}$$

Part (d) Find the angle θ_0 in degrees above the horizontal at which the ball left the bat.

The vertical component of the initial velocity is given in terms of the final vertical component as

$$v_{0y} = \sqrt{v_{fy}^2 + 2gh}$$

The horizontal component of the initial velocity is given by

$$v_{0x} = v_{fx} = v_f \cos(\theta)$$

since there is no acceleration in the horizontal direction.

The initial angle can be found by taking the inverse tangent, or arctangent, of the ratio of the vertical component to the horizontal component.

$$\begin{aligned} \theta_0 &= \arctan\left(\frac{v_{0y}}{v_{0x}}\right) \\ &= \arctan\left(\frac{\sqrt{v_f^2 \sin^2(\theta) + 2gh}}{v_f \cos(\theta)}\right) \\ &= \arctan\left(\frac{\sqrt{(30.15 \text{ m/s})^2 \sin^2(25^\circ) + 2(9.81 \text{ m/s}^2)(5.5 \text{ m})}}{(30.15 \text{ m/s}) \cos(25^\circ)}\right) \end{aligned}$$

$$\theta_0 = 31.033^\circ$$

Problem 54 - 4.3.18 (alt) :

A basketball player shoots a free-throw with initial velocity $v_0 = 9.5 \text{ m/s}$ at an angle $\theta = 34^\circ$ above horizontal.

Use a Cartesian coordinate system with the origin located at the position the ball was released, with the ball's horizontal velocity component in the positive direction. Assume the basketball encounters no air resistance.

Randomized Variables

$$\begin{aligned} v_0 &= 9.5 \text{ m/s} \\ \theta &= 34 \text{ degrees} \end{aligned}$$

Part (a) Create an expression for the basketball's initial velocity vector, \mathbf{v}_0 , in rectangular form in terms of \mathbf{i} , \mathbf{j} , v_0 , θ , and g .

Expression :

$$\mathbf{v}_0 = \underline{\hspace{10em}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , θ , g , \mathbf{i} , \mathbf{j} , \mathbf{m} , \mathbf{t} , \mathbf{v} , v_0

Part (b) Determine the maximum vertical height h_{\max} , in meters, the ball attains above the release point.

Numeric : A numeric value is expected and not an expression.

$$h_{\max} = \underline{\hspace{10em}}$$

Part (c) What is the acceleration vector, \mathbf{a} , in meters per square second?

Expression :

$$\mathbf{a} = \underline{\hspace{10em}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , θ , g , \mathbf{i} , \mathbf{j} , \mathbf{m} , \mathbf{t} , \mathbf{v} , v_0

Part (d) Express the time, t , the basketball takes to reach its maximum vertical height in terms of v_0 , θ , and g .

Expression :

Part (d) Express the time, t , the basketball takes to reach its maximum vertical height in terms of v_0 , θ , and g .

Expression :

$$t = \underline{\hspace{10em}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , θ , g , \mathbf{i} , \mathbf{j} , \mathbf{m} , \mathbf{t} , \mathbf{v} , v_0

Part (e) Determine the time, t in seconds, the basketball takes to reach its maximum vertical height.

Numeric : A numeric value is expected and not an expression.

a) $\vec{V}_0 = (v_0 \cos \theta, v_0 \sin \theta)$

b) At maximum height $v_y = 0$ so $v^2 = v_0^2 + 2a\Delta y \rightarrow 0 = v_0^2 \sin^2 \theta - 2g h_{\max}$, $h_{\max} = \frac{v_0^2 \sin^2 \theta}{2g}$
 $h_{\max} \approx 1.43 \text{ m}$

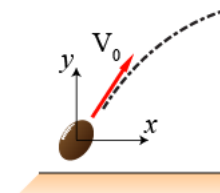
c) The only acceleration is gravity, so $\vec{a} = -9.81 \hat{j} \text{ (m/s}^2\text{)}$

d) At maximum height, $v_y = 0$ so $v = v_0 + at \rightarrow 0 = v_0 \sin \theta - g t \rightarrow t = \frac{v_0 \sin \theta}{g}$

e) $t \approx 0.54 \text{ s}$

Problem 55 - 4.3.19 :

A punted football is observed to have velocity components $v_{\text{horizontal}} = 15 \text{ m/s}$ to the right and $v_{\text{vertical}} = 1.25 \text{ m/s}$ directed downward at a height $h = 1.1 \text{ m}$ above the ground. Use a Cartesian coordinate system with the origin located on the ground at the position the football was punted and assume it encountered no air resistance.



Part (a) Determine the football's initial horizontal velocity magnitude v_{0x} in terms of $v_{\text{horizontal}}$, v_{vertical} , g , and h .

There is no acceleration in the x-direction, so the velocity in the x-direction is constant.

$$v_{0x} = v_{\text{horizontal}}$$

Part (b) Calculate the time, t in seconds, the football moved between being punted and observed.

The velocity is related to the acceleration and time by the expression

$$v_f = (v_0 + at) \text{ m/s}$$

where v_0 is the initial velocity in m/s, a is the acceleration in m/s^2 , and t is time in s. We know the final velocity but not the time or the initial velocity. To find the relate it to the acceleration, final velocity, and distance by the expression

$$v_f^2 - v_0^2 = 2a\Delta x$$

where Δx is the change in distance from the initial and final positions. In this case, the distance is h and the acceleration is from gravity. Solving for the initial ve

$$v_0 = (v_f^2 - 2a\Delta x)^{0.5} = (v_{\text{vertical}}^2 - 2 \cdot (-g) \cdot h)^{0.5} = (v_{\text{vertical}}^2 + 2 \cdot g \cdot h)^{0.5}$$

Substituting into the first equation and being mindful of the directions,

$$v_f = -v_{\text{vertical}} = v_0 - gt = (v_{\text{vertical}}^2 + 2 \cdot g \cdot h)^{0.5} - gt$$

Solving for the time,

$$t = \frac{\left((v_{\text{vertical}}^2 + 2 \cdot g \cdot h)^{0.5} + v_{\text{vertical}} \right)}{g}$$

Plugging in numbers and converting units as needed,

$$t = \frac{\left((1.25 \text{ m/s})^2 + 2 \cdot 9.81 \text{ m/s}^2 \cdot 1.1 \text{ m} \right)^{0.5} + 1.25 \text{ m/s}}{9.81 \text{ m/s}^2}$$

$$t = 0.6178 \text{ s}$$

Problem 56 - 4.3.20-alt :**Full solution not currently available at this time.**

An object rolls off a tabletop with a horizontal velocity $v_{0x} = 4 \text{ m/s}$. The table is at a height $y_0 = 0.25 \text{ m}$, above the floor. Use a coordinate system with its origin on the floor directly beneath the point where the object rolls off the table, its horizontal x -axis lying directly beneath the object's trajectory, and its vertical y -axis pointing up.

Part (a) How long, in seconds, is the object falling before it hits the floor?

$$t = (2 \cdot y / 9.81)^{0.5}$$

$$t = (2 \cdot 0.25 / 9.81)^{0.5}$$

$$t = 0.2258$$

$$\text{Tolerance: } \pm 0.006774$$

Part (b) What's the horizontal distance, in meters, from the edge of the tabletop to where the object lands?

$$x = v_0 \cdot (2 \cdot y / 9.81)^{0.5}$$

$$x = 4 \cdot (2 \cdot 0.25 / 9.81)^{0.5}$$

$$x = 0.903$$

$$\text{Tolerance: } \pm 0.02709$$

Part (c) What is the vertical component of velocity, in meters per second, when the object hits the ground?

$$v_y = -9.81 \cdot (2 \cdot y / 9.81)^{0.5}$$

$$v_y = -9.81 \cdot (2 \cdot 0.25 / 9.81)^{0.5}$$

$$v_y = -2.215$$

$$\text{Tolerance: } \pm 0.06645$$

Part (d) What is the magnitude of the velocity (it's speed) when it hits the floor?

$$|v| = (v_0^2 + (-9.81 \cdot (2 \cdot y / 9.81)^{0.5})^2)^{0.5}$$

$$|v| = (4^2 + (-9.81 \cdot (2 \cdot 0.25 / 9.81)^{0.5})^2)^{0.5}$$

$$|v| = 4.572$$

$$\text{Tolerance: } \pm 0.13716$$

Problem 57 - 4.3.20 :

An object rolls off a tabletop with a horizontal velocity $v_{0x} = 1.1 \text{ m/s}$. The table is at a height $y_0 = 0.15 \text{ m}$, above the floor. Use a coordinate system with its origin on the floor directly beneath the point where the object rolls off the table, its horizontal x -axis lying directly beneath the object's trajectory, and its vertical y -axis pointing up.

Part (a) How long, in seconds, is the object falling before it hits the floor?

This problem involves the application of the **equations of motion in two dimensions** to a case of **projectile motion**, where the only force acting is the force due important new concept to remember is that the motion along the x and y directions are *independent* of one another.

In this part, we are asked to find the time it takes for the object to reach the floor, so we can restrict our analysis to the motion along the y direction.

The solution involves three steps:

- determining which equation of motion to use
- making substitutions, using the **appropriate signs** for our known variables, into the selected equation of motion
- solving for time

Choosing an Equation of Motion

When choosing which equation of motion is the most appropriate (or easiest) to use for a problem, it's important to identify which variable we want and which variables we want the time it takes the object to reach the floor, and we know 3 variables that apply to the motion along y : the initial height above the floor (*i.e.*, the total initial y displacement), the initial y velocity, and the acceleration along y .

The equation that gives us time with the known variables is

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

Making Substitutions

The problem tells us that the origin is on the floor and the $+y$ direction is upward, meaning that the floor is located at

$$y = 0$$

and the total y displacement is therefore $-y_0$.

In addition, because the initial velocity is entirely along the x direction, the initial y component of the velocity is zero,

$$v_{0y} = 0$$

Finally, we know that the only force experienced by the object as it falls is the force of gravity, meaning that the object is in free fall, and

$$a_y = -g$$

where the negative sign indicates that the acceleration due to gravity points *downward*. With these substitutions, our equation for motion becomes

$$-y_0 = -\frac{1}{2}gt^2$$

or, more simply,

$$y_0 = \frac{1}{2}gt^2$$

Solving for t

We can easily rearrange the above equation to solve for the time it takes the object to reach the floor,

$$t = \sqrt{\frac{2y_0}{g}}$$

Note that the time taken is *independent* of the initial horizontal speed of the object. That is, for the same initial height, it takes the same amount of time for an object as it does for an object that is traveling quickly along the tabletop!

Using our known values, we have

$$t = \sqrt{\frac{2(0.15 \text{ m})}{9.81 \text{ m/s}^2}} = 0.1749 \text{ s}$$

$t = 0.1749 \text{ s}$

Part (b) How far, in meters, does the object land from the edge of the tabletop?

In this part, we are asked to find how far the object travels horizontally (once it has left the tabletop) before striking the ground.

From *Part (a)*, we know the time it takes for the object to hit the ground, regardless of its initial horizontal velocity. As it turns out, *time* is the variable that connects the two motions. For this part, we will use the t from *Part (a)* and restrict our analysis to the motion along the x direction.

The solution involves three steps:

- determining which equation of motion to use
- making substitutions, using the **appropriate signs** for our known variables, into the selected equation of motion
- solving for displacement along the x direction

Choosing an Equation of Motion

We want to know the object's displacement along x , and we know 3 variables: the time it takes to hit the floor, the initial x velocity, the the acceleration along x .

The equation that gives us displacement with the known variables is

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

Making Substitutions

The problem tells us that the origin is just beneath the point at which the object leaves the table, and $+x$ direction is along the direction of travel, meaning that ini object is

$$x_0 = 0$$

In addition, we are given the initial x velocity,

$$v_{0y} = 1.1 \text{ m/s}$$

Because the only force experienced by the object as it falls is the force of gravity, the object does not accelerate along x ,

$$a_x = 0$$

Thus, we have

$$x = v_{0x}t$$

Using our result from *Part (a)* for t

$$x = v_{0x} \left(\sqrt{\frac{2y_0}{g}} \right)$$

Evaluating x

The total x displacement is thus

$$x = (1.1 \text{ m/s}) \left(\sqrt{\frac{2(0.15 \text{ m})}{9.81 \text{ m/s}^2}} \right) = 0.1924 \text{ m}$$

$$x = 0.1924 \text{ m}$$

Part (c) What is the vertical component of velocity, in meters per second, when the object hits the ground? Recall that the positive y -direction is upwards

In this part, we are asked to find the vertical component of the velocity just before striking the ground.

When the object rolls off the tabletop, it moves with both horizontal and vertical components to its velocity. Because the only force acting on the object is the for vertical component of the velocity changes. Because the acceleration due to gravity acts along $-y$, the vertical component of the velocity, initially zero, increases

$$v_y = -gt$$

We know the time of the fall from the expression found in part a.

$$\begin{aligned} v_y &= -g\sqrt{\frac{2y}{g}} \\ &= -(9.81 \text{ m/s}^2) \sqrt{\frac{2(0.15 \text{ m})}{9.81 \text{ m/s}^2}} \end{aligned}$$

$$v_y = -1.716 \text{ m/s}$$

Part (d) What is the magnitude of the velocity (it's speed) when it hits the floor?

In this part, we are asked to find the *magnitude* of the object's velocity just before it strikes the ground.

Because the object moves "over and down" as it falls, the final velocity has both x and y components! To find the magnitude of the velocity, v , we simply apply **I** using the final x velocity as one side of a right triangle, and the final y velocity as the other side. Then, v is the hypotenuse. *Voila!*

(A very important side note: This is generally true for the magnitude of any vector quantity with x and y components.)

Our final velocity is thus given by

$$v = \sqrt{v_x^2 + v_y^2}$$

Because the only force acting on the object is the force of gravity, there is no acceleration along the x direction, meaning

$$v_x = v_{0x}$$

And, as we found in *Part (c)*,

$$v_y = -\sqrt{2gy_0}$$

Thus, the magnitude of the final velocity is

$$v = \sqrt{v_{0x}^2 + (-\sqrt{2gy_0})^2} = \sqrt{v_{0x}^2 + 2gy_0} = \sqrt{(1.1 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(0.15 \text{ m})} = 2.038 \text{ m/s}$$

$v = 2.038 \text{ m/s}$

Part (e) Find the angle of impact, in degrees below the horizontal.

In this part, we are asked to find the angle at which the object strikes the ground. We're not looking for just any angle! The problem asks us to find the angle of impact. That is, the problem really asking us the following: What is the angle between the final velocity vector and the $+x$ direction, as measured below the $+x$ axis?

If we sketch a right triangle, with the length of the top (horizontal) side of the triangle given by

v_x and the length of the right (vertical) side of the triangle given by

v_y , the hypotenuse is

v . It doesn't take long to realize that the angle we are looking for, with a triangle drawn in this manner, is the angle between the top side of the triangle and the hypotenuse. We want the angle that satisfies the following relationships

$$v_y = v \sin(\theta)$$

$$v_x = v \cos(\theta)$$

A simple way of solving for

θ without needing to calculate

v is dividing the two equations above,

$$\frac{v_y}{v_x} = \frac{v \sin(\theta)}{v \cos(\theta)} = \tan(\theta)$$

With this,

$$\theta = \arctan\left(\frac{v_y}{v_x}\right)$$

where the

x
- and
 y

- components of the final velocity are the initial velocity and the expression found in part c, respectively. It's important to note that, because this angle is measured $+x$, it should be negative. In addition, the question asks us to report the angle in degrees.

$$\theta = \tan^{-1} \left(\frac{\left(\frac{g \sqrt{\frac{2y}{g}}}{v_{0x}} \right)}{\left(\frac{1.1 \text{ m}}{s} \right)} \right) = \tan^{-1} \left(\frac{\left(\frac{\left(9.81 \frac{\text{m}}{\text{s}^2} \right) \sqrt{\frac{2(0.15 \text{ m})}{\left(9.81 \frac{\text{m}}{\text{s}^2} \right)}}}{\left(\frac{1.1 \text{ m}}{s} \right)} \right)}{\left(\frac{1.1 \text{ m}}{s} \right)} \right)$$

$$\theta = 57.332^\circ$$

Part (f) Enter an expression for the height of the object y , in terms of x , v_{0x} , y_0 , and g . This expression should *not* depend on the time.

In this part, we are asked to find $y(x)$, a function giving the vertical position of the falling object as a function of its horizontal position.

The origin of our coordinate system is indicated in the original problem statement: It's on the floor, directly beneath the point at which the ball leaves the table top $y_0 = 0.15\text{m}$ and $x_0 = 0$.

Considering our equations of motion for the y , let's start with the following

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

If we make the following substitutions

$$v_{0y} = 0$$

$$a_y = -g$$

we have

$$y - y_0 = -\frac{1}{2}gt^2$$

Solving for y ,

$$y = y_0 - \frac{1}{2}gt^2$$

The result is a function of t rather than x , precisely what is not wanted. The natural question, then, is:

Can we write t in terms of x ?

Let's look at our equations of motion for the x . This one looks perfect!

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

Our substitutions, in this case, are

$$x_0 = 0$$

$$a_x = 0$$

We now have a very simple equation

$$x = v_{0x}t$$

and we can easily solve for t in terms of x

$$t = \frac{x}{v_{0x}}$$

Putting our result for t into our equation for y ,

$$y = y_0 - \frac{1}{2}gt^2 = y_0 - \frac{1}{2}g\left(\frac{x}{v_{0x}}\right)^2 = y_0 - \frac{gx^2}{2v_{0x}^2}$$

Wow! We have now written an equation for the trajectory of the object as it falls! Note that it is a portion of an upside down parabola ($y \propto x^2$), as one would expect.

$$y = y_0 - \frac{gx^2}{2v_{0x}^2}$$

Problem 58 - 4.3.21 (alt) :

At the local swimming pool, the diving board is elevated $h = 7.5$ m above the pool surface and overhangs the pool edge by $L = 2$ m. A diver runs horizontally along the diving board with a speed of $v_0 = 3.6$ m/s and then falls into the pool.

Neglect air resistance. Use a Cartesian coordinate system with its origin at the position of the diver just before falling. Let the direction that the diver falls be the negative y direction.

Randomized Variables

$$h = 7.5 \text{ m}$$

$$v_0 = 3.6 \text{ m/s}$$

Part (a) Express the time t_w it takes the diver to move off the end of the diving board to the pool surface in terms of v_0 , h , L , and g .

Expression :

$$t_w = \underline{\hspace{2cm}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \theta, a, b, c, d, g, h, i, j, k, L, m, n, P, r, t, T, v_0$

Part (b) Calculate the time, t_w in seconds, it takes the diver to move off the end of the diving board to the pool surface.

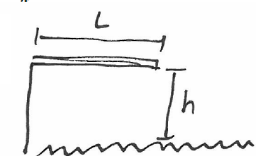
Numeric : A numeric value is expected and not an expression.

$$t_w = \underline{\hspace{2cm}}$$

Part (c) Determine the horizontal distance, d_w in meters, from the edge of the pool to where the diver enters the water.

Numeric : A numeric value is expected and not an expression.

$$d_w = \underline{\hspace{2cm}}$$



a) Kinematic equation $y = \frac{1}{2}at^2 + v_0t + y_0 \rightarrow -h = -\frac{1}{2}gt_w^2$

$$t_w = \sqrt{\frac{2h}{g}}$$

b) $t_w \approx 1.24$

c) Kinematic equation $x = \frac{1}{2}at^2 + v_0t + x_0 \rightarrow x = v_0t_w$ (from the end of the board)

Total distance is $L + x = L + v_0t_w$, $\approx 6.46\text{m}$

Problem 59 - 4.3.21 :

At the local swimming pool, the diving board is elevated $h = 7.5$ m above the pool surface and overhangs the pool edge by $L = 2$ m. A diver runs horizontally along the diving board with a speed of $v_0 = 3.6$ m/s and then falls into the pool. Neglect air resistance. Use a Cartesian coordinate system with its origin at the position of the diver just before falling. Let the direction that the diver falls be the negative y direction.

Randomized Variables

$$h = 7.5 \text{ m}$$

$$v_0 = 3.6 \text{ m/s}$$

Part (a) Express the time t_w it takes the diver to move off the end of the diving board to the pool surface in terms of v_0 , h , L , and g .

Expression :

$$t_w = \underline{\hspace{2cm}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \theta, a, b, c, d, g, h, i, j, k, L, m, n, P, r, t, T, v_0$

Part (b) Calculate the time, t_w in seconds, it takes the diver to move off the end of the diving board to the pool surface.

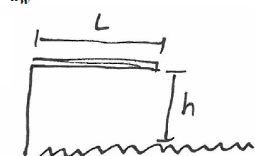
Numeric : A numeric value is expected and not an expression.

$$t_w = \underline{\hspace{2cm}}$$

Part (c) Determine the horizontal distance, d_w in meters, from the edge of the pool to where the diver enters the water.

Numeric : A numeric value is expected and not an expression.

$$d_w = \underline{\hspace{2cm}}$$



$$b) \boxed{t_w \approx 1.24}$$

$$a) \text{ Kinematic equation } y = \frac{1}{2}at^2 + v_0t + y_0 \rightarrow -h = -\frac{1}{2}gt_w^2$$

$$\boxed{t_w = \sqrt{\frac{2h}{g}}}$$

$$c) \text{ Kinematic equation } x = \frac{1}{2}at^2 + v_0t + x_0 \rightarrow x = v_0t_w \text{ (from the end of the board)}$$

$$\text{Total distance is } L + x = L + v_0t_w, \boxed{\approx 6.46\text{m}}$$

Problem 60 - 4.3.26 :

A quarterback can run with a speed of $v_q = 15$ mph. He throws a ball with a speed of $v_b = 35$ mph at some unknown angle θ between 30 and 60 degrees, which is measured from horizontal. Neglect air resistance.

Part (a) Is it possible for him to catch his own pass?

For the quarterback to catch the pass, he must make the ball's horizontal velocity less than or equal to his own. So,

$$v_q \geq v_b \cos(\theta) \text{ m/s}$$

where $v_{q,b}$ are the velocities of the quarterback and ball in m/s and θ is the angle the ball is thrown in rad. Solving for the angle and noting that arccosine decreases

$$\theta \geq \arccos\left(\frac{v_q}{v_b}\right) = \arccos\left(\frac{15 \text{ miles/h}}{35 \text{ miles/h}}\right) = 64.6^\circ$$

He has to throw at an angle equal to or over 64.6 deg.

Part (b) A receiver runs past the quarterback (just beside him) traveling at speed v_r . If the quarterback releases his pass at that instant what would be the angle θ he would have to throw the ball for the receiver to catch it?

For the receiver to catch the pass, the quarterback must make the ball's horizontal velocity equal to the receiver's. So,

$$v_r = v_b \cos(\theta) \text{ m/s}$$

where $v_{r,b}$ are the velocities of the receiver and ball in m/s and θ is the angle the ball is thrown in rad. Solving for the angle,

$$\theta = \arccos\left(\frac{v_r}{v_b}\right) = \arccos\left(\frac{15 \text{ miles/h}}{35 \text{ miles/h}}\right)$$

$$\theta = 64.6^\circ$$

Problem 61 - 4.3.27 :

Ball A is rolled across the top of a table and as it reaches the edge it has a horizontal velocity, v . Just as it rolls off the edge another ball, Ball B, is dropped from rest at the same height.

Part (a) Which ball will hit the ground first?

The initial horizontal motion of ball A does not affect its motion in the vertical direction as it rolls off the table. Thus, since both balls begin with an initial velocity component, they will both accelerate from rest vertically and reach the ground below at the same time.

Both will hit the ground at the same time.

Part (b) At the time of impact for each, which ball will have the greatest speed?

The magnitude of the velocity for each ball as it reaches the ground will be equal to

$$v = \sqrt{v_x^2 + v_y^2}$$

The vertical component will be the same value for each ball, but only ball A has a horizontal component of the final velocity, therefore its value will be greater than

Ball A

Problem 62 - 4.3.28 :

A projectile is launched at ground level with an initial speed of **35** m/s, at an angle of **25°** above the horizontal. It strikes a target above the ground **2.1** seconds later.

Part (a) What is the horizontal distance, in metres, from where the projectile was launched to where it lands?

The horizontal distance the projectile travels is the product of the elapsed time and the horizontal component of the initial velocity, which doesn't change throughout is assumed to be no acceleration in the horizontal direction.

$$\begin{aligned} x &= v_0 \cos(\theta) t \\ &= (35 \text{ m/s}) \cos(25^\circ) (2.1 \text{ s}) \end{aligned}$$

$$x = 66.614 \text{ m}$$

Part (b) What is the vertical distance, in meters, from where the projectile was launched to where it lands?

The vertical distance the projectile travels can be found by applying the equations of kinematics, since the constant downward acceleration due to gravity always

$$\begin{aligned} y &= v_{0y} t + \frac{1}{2} a t^2 \\ &= v_0 \sin(\theta) t - \frac{1}{2} g t^2 \\ &= (35 \text{ m/s}) \sin(25^\circ) (2.1 \text{ s}) - \frac{1}{2} (9.81 \text{ m/s}^2) (2.1 \text{ s})^2 \end{aligned}$$

$$y = 9.431 \text{ m}$$

Problem 63 - 4.3.28 (alt) :

A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

Solution

$$\text{Range of projectile on level ground: } R = \frac{v_0^2}{g} \sin 2\theta = \frac{(50.0 \text{ m/s})^2}{9.80 \text{ m/s}^2} \sin 60.0^\circ = 221 \text{ m}$$

The time in air is given as 3.00 s, so projectile landed above level ground. Find the position relative to the launching point:

$$x = v_{0x}t = (50.0 \text{ m/s})(\cos 30.0^\circ)(3.00 \text{ s}) = 1.30 \times 10^2 \text{ m}$$

$$y = v_{0y}t + \frac{1}{2}at^2 = (50.0 \text{ m/s})(\sin 30.0^\circ)(3.00 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(3.00 \text{ s})^2 = 30.9 \text{ m}$$

Therefore, the projectile landed $1.30 \times 10^2 \text{ m}$ horizontally and 30.9 m vertically from the launching point.

Problem 64 - 4.3.29 :

A ball is kicked at ground level with an initial velocity of 15 m/s in the horizontal direction and 10.5 m/s in the vertical direction.

Part (a) At what speed does the ball hit the ground in m/s?

Due to the symmetry of the problem, the magnitude of the final vertical component of the initial velocity will be the same as that of the initial vertical component. does not change during the motion, assuming no acceleration in the horizontal direction. Therefore, the magnitude of the final velocity is given by the Pythagorean

$$\begin{aligned} v_f &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{(15 \text{ m/s})^2 + (10.5 \text{ m/s})^2} \end{aligned}$$

$$v_f = 18.31 \text{ m/s}$$

Part (b) For how long does the ball remain in the air in seconds?

Due to the symmetry of the problem, the total time of flight can be found by first finding the time to reach the highest point, then multiplying that result by 2. Equ half the total elapsed time, t . We can apply the equations of kinematics again to find this time because at the highest point, the speed of the ball is momentarily zero

$$v_y = v_{0y} + a_y \left(\frac{t}{2} \right) = v_{0y} - g \left(\frac{t}{2} \right)$$

Solving for the time gives,

$$\begin{aligned} t &= \frac{2v_{0y}}{g} \\ &= \frac{2(10.5 \text{ m/s})}{9.81 \text{ m/s}^2} \end{aligned}$$

$$t = 2.141 \text{ s}$$

Part (c) What maximum height is attained by the ball in meters?

The vertical distance from the ground to the highest point can be found by applying the equations of kinematics using the fact that at the highest point, the speed is zero in the vertical direction.

$$v_y^2 = v_{0y}^2 + 2a_y y = v_{0y}^2 - 2gy$$

Solving for the vertical distance when the final velocity is zero gives,

$$\begin{aligned} y &= \frac{v_{0y}^2}{2g} \\ &= \frac{(10.5 \text{ m/s})^2}{(2(9.81 \text{ m/s}^2))} \end{aligned}$$

$$y = 5.619 \text{ m}$$

Problem 65 - 4.3.30 :

A ball is thrown horizontally from the top of a **55 m** building and lands **105 m** from the base of the building. Ignore air resistance, and use a coordinate system whose origin is at the top of the building, with positive y upwards and positive x in the direction of the throw.

Part (a) How long is the ball in the air in seconds?

Let the initial position of the ball be at $y = 0 \text{ m}$, then the ground is at $y = h$. Apply the equations of kinematics to find the time.

$$h = v_{0y} - \frac{1}{2}gt^2$$

Solving for the time gives using an initial vertical component of the velocity equal to zero,

$$\begin{aligned} t &= \sqrt{\frac{2h}{g}} \\ &= \sqrt{\frac{2(55 \text{ m})}{9.81 \text{ m/s}^2}} \end{aligned}$$

$$t = 3.35 \text{ s}$$

Part (b) What must have been the initial horizontal component of the velocity, in meters per second?

Assuming there is no acceleration in the horizontal direction, the distance the ball travels is given by

$$d = v_{0x}t$$

Solving for the initial horizontal component of the velocity gives,

$$\begin{aligned} v_{0x} &= \frac{d}{t} \\ &= \frac{d}{\sqrt{\frac{2h}{g}}} \\ &= \frac{(105 \text{ m})}{\sqrt{\frac{2(55 \text{ m})}{(9.81 \text{ m/s}^2)}}} \end{aligned}$$

$$v_{0x} = 31.341 \text{ m/s}$$

Part (c) What is the vertical component of the velocity just before the ball hits the ground, in meters per second?

The vertical component of the final velocity is given by

$$\begin{aligned} v_y &= v_{0y} - gt \\ &= 0 - g\sqrt{\frac{2h}{g}} \\ &= -(9.81 \text{ m/s}^2) \sqrt{\frac{2(55 \text{ m})}{(9.81 \text{ m/s}^2)}} \end{aligned}$$

$$v_y = -32.866 \text{ m/s}$$

Part (d) What is the magnitude of the velocity of the ball just before it hits the ground, in meters per second?

The magnitude of the final velocity is found using the Pythagorean theorem.

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{\left(\frac{d}{\sqrt{\frac{2h}{g}}}\right)^2 + \left(-g\sqrt{\frac{2h}{g}}\right)^2} \\ &= \sqrt{\left(\frac{(105 \text{ m})}{\sqrt{\frac{2(55 \text{ m})}{(9.81 \text{ m/s}^2)}}}\right)^2 + \left(- (9.81 \text{ m/s}^2) \sqrt{\frac{2(55 \text{ m})}{(9.81 \text{ m/s}^2)}}\right)^2} \end{aligned}$$

$$v = 45.413 \text{ m/s}$$

Problem 66 - 4.3.31 :

A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a 32° ramp, measured from the horizontal, at a speed of 40.0 m/s (144 km/h). The top of the ramp is at the same height as the roofs of the buses and each bus is 20.0 m long.

Part (a) How many buses can he clear?

We can find the number of buses by determining the horizontal distance the motorcycle and rider travel during the jump. Once that distance is determined, divide find the number of buses. The range formula can be used or derived from the equations of kinematics.

$$\begin{aligned} R &= v_0^2 \frac{\sin(2\theta)}{g} \\ &= (40.0 \text{ m/s})^2 \frac{\sin(64^\circ)}{(9.81 \text{ m/s}^2)} \\ &= 146.6 \text{ m} \end{aligned}$$

Dividing the range by 20.0 m, the length of a bus, gives 7.33 buses. Only whole buses count here, so

$$n = 7 \text{ buses}$$

Part (b) By how many meters does he clear the last bus?

In part (a), we found the range to be 146.6 m. The 7 buses account for 140.0 m of the horizontal displacement of the motorcycle, so there is an additional 6.6 m b

$$d = 6.6 \text{ m}$$

Problem 67 - 4.3.32 :

An archer shoots an arrow at a target that is a horizontal distance $d = 55$ m away; the bull's-eye of the target is at same height as the release height of the arrow.

Randomized Variables

$d = 55$ m

Part (a) At what angle, in degrees above the horizontal, must the arrow be released to hit the bull's-eye if the arrow's initial speed is 33 m/s?

The horizontal distance the arrow travels during its flight is given by the range formula.

$$d = v_0^2 \frac{\sin(2\theta)}{g}$$

Solving this for the angle gives

$$\begin{aligned}\theta &= \frac{1}{2} \arcsin\left(\frac{gd}{v_0^2}\right) \\ &= \frac{1}{2} \arcsin\left(\frac{(9.81 \text{ m/s}^2)(55 \text{ m})}{(33 \text{ m/s})^2}\right)\end{aligned}$$

$$\theta = 14.85^\circ$$

Problem 68 - 4.3.33 :

A rugby player passes the ball 6.2 m across the field, where the ball is caught at the same height as it left his hand.

Part (a) At what angle, in degrees above the horizontal, was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible

The horizontal distance the ball travels during its flight is given by the range formula.

$$d = v_0^2 \frac{\sin(2\theta_0)}{g}$$

Solving this for the angle gives

$$\begin{aligned}\theta_0 &= \frac{1}{2} \arcsin\left(\frac{gd}{v_0^2}\right) \\ &= \frac{1}{2} \arcsin\left(\frac{(9.81 \text{ m/s}^2)(6.2 \text{ m})}{(12.0 \text{ m/s})^2}\right)\end{aligned}$$

$$\theta_0 = 12.492^\circ$$

Part (b) What other angle, in degrees above the horizontal, gives the same range?

In part (a), the angle given from the range formula is the smaller of two possible angles for the same range. This occurs because two different angles can have the sine function. We know that the two angles we are looking for must be less than 90 degrees. It turns out that their sum is equal to 90 degrees.

$$\begin{aligned}\theta_0' &= 90 - \frac{1}{2} \arcsin\left(\frac{gd}{v_0^2}\right) \\ &= 90 - \frac{1}{2} \arcsin\left(\frac{(9.81 \text{ m/s}^2)(6.2 \text{ m})}{(12.0 \text{ m/s})^2}\right)\end{aligned}$$

$$\theta_0' = 77.508^\circ$$

Part (c) How long, in seconds, did the pass of part (a) take?

From the equations of kinematics, the horizontal distance the ball travels is equal to

$$d = v_{0x}t = v_0 \cos(\theta_0) t$$

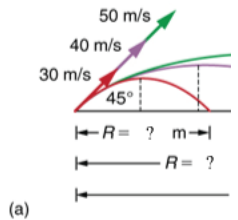
Using the expression for the angle found in part (a), we can solve for the elapsed time.

$$\begin{aligned} t &= \frac{d}{v_0 \cos(\theta)} \\ &= \frac{d}{v_0 \cos\left(\frac{1}{2} \arcsin\left(\frac{gd}{v_0^2}\right)\right)} \\ &= \frac{(6.2 \text{ m})}{(12.0 \text{ m/s}) \cos\left(\frac{1}{2} \arcsin\left(\frac{(9.80 \text{ m/s}^2)(6.2 \text{ m})}{(12.0 \text{ m/s})^2}\right)\right)} \end{aligned}$$

$$t = 0.5292 \text{ s}$$

Problem 69 - 4.3.34 :

Find the ranges for the projectiles shown in the figure at an elevation angle of 45° and the given initial speeds.



Part (a) Find the range, in meters, for the projectile with a speed of 30 m/s.

The horizontal distance the projectile travels during its flight is given by the range formula.

$$\begin{aligned} R &= v_0^2 \frac{\sin(2\theta)}{g} \\ &= (30 \text{ m/s})^2 \frac{\sin(2 \cdot 45^\circ)}{(9.8 \text{ m/s}^2)} \end{aligned}$$

$$R = 91.8 \text{ m}$$

Part (b) Find the range, in meters, for the projectile with a speed of 40 m/s.

The horizontal distance the projectile travels during its flight is given by the range formula.

$$\begin{aligned} R &= v_0^2 \frac{\sin(2\theta)}{g} \\ &= (40 \text{ m/s})^2 \frac{\sin(2 \cdot 45^\circ)}{(9.8 \text{ m/s}^2)} \end{aligned}$$

$$R = 163 \text{ m}$$

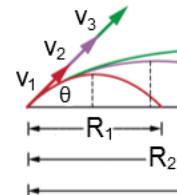
Part (c) Find the range, in meters, for the projectile with a speed of 50 m/s.

$$\begin{aligned} R &= v_0^2 \frac{\sin(2\theta)}{g} \\ &= (50 \text{ m/s})^2 \frac{\sin(2 \cdot 45^\circ)}{(9.8 \text{ m/s}^2)} \end{aligned}$$

$$R = 255 \text{ m}$$

Problem 70 - 4.3.34-alt :

Find the ranges for the projectiles in the figure for the elevation angle of $\theta = 30.1^\circ$ and the initial speeds given in the data below.

**Part (a) Find the range, in meters, for the projectile with a speed of 5.5 m/s.**

The horizontal distance the projectile travels during its flight is given by the range formula.

$$\begin{aligned} R_1 &= v_0^2 \frac{\sin(2\theta)}{g} \\ &= (5.5 \text{ m/s})^2 \frac{\sin(2 \cdot 30.1^\circ)}{(9.8 \text{ m/s}^2)} \end{aligned}$$

$$R_1 = 2.679 \text{ m}$$

Part (b) Find the range, in meters, for the projectile with a speed of 15 m/s.

The horizontal distance the projectile travels during its flight is given by the range formula. Since $v_2 > v_1$, we expect that the range will be larger than that from 1

$$\begin{aligned} R_2 &= v_0^2 \frac{\sin(2\theta)}{g} \\ &= (15 \text{ m/s})^2 \frac{\sin(2 \cdot 30.1^\circ)}{(9.8 \text{ m/s}^2)} \end{aligned}$$

$$R_2 = 19.923 \text{ m}$$

Part (c) Find the range, in meters, for the projectile with a speed of 25 m/s.

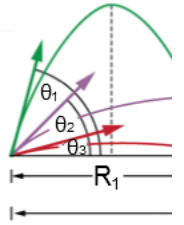
The horizontal distance the projectile travels during its flight is given by the range formula. Since $v_3 > v_2 > v_1$, we expect that the range will be larger than that

$$\begin{aligned} R_3 &= v_0^2 \frac{\sin(2\theta)}{g} \\ &= (25 \text{ m/s})^2 \frac{\sin(2 \cdot 30.1^\circ)}{(9.8 \text{ m/s}^2)} \end{aligned}$$

$$R_3 = 55.342 \text{ m}$$

Problem 71 - 4.3.35-alt :

Consider a projectile being launched with an initial speed of 40.5 m/s at a variety of initial angles. Refer to the figure.



Part (a) What is the range, in meters, of the projectile if it is launched at an angle of $\theta_1 = 70.1^\circ$?

The horizontal distance the projectile travels during its flight is given by the range formula.

$$R_1 = v_0^2 \frac{\sin(2\theta)}{g}$$

$$= (40.5 \text{ m/s})^2 \frac{\sin(2 \cdot 70.1^\circ)}{(9.8 \text{ m/s}^2)}$$

$$R_1 = 107.137 \text{ m}$$

Part (b) What is the range, in meters, of the projectile if it is launched at an angle of $\theta_2 = 35^\circ$?

The horizontal distance the projectile travels during its flight is given by the range formula.

$$R_2 = v_0^2 \frac{\sin(2\theta)}{g}$$

$$= (40.5 \text{ m/s})^2 \frac{\sin(2 \cdot 35^\circ)}{(9.8 \text{ m/s}^2)}$$

$$R_2 = 157.279 \text{ m}$$

Part (c) What is the range, in meters, of the projectile if it is launched at an angle of $\theta_3 = 90 - 70.1^\circ$, the complement of θ_1 ?

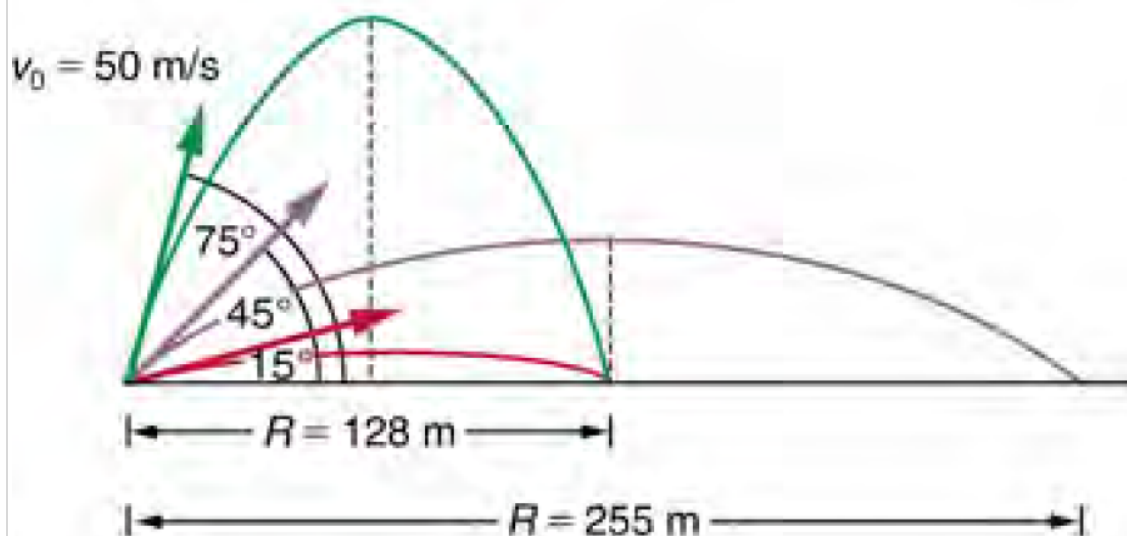
Due to the properties of the sine function, the value of $\sin(2\theta) = \sin(2(90 - \theta))$, so we expect to get the same answer as that found in part (a).

$$R_3 = v_0^2 \frac{\sin(2(90 - \theta))}{g}$$

$$= (40.5 \text{ m/s})^2 \frac{\sin(2(90 - 70.1^\circ))}{(9.8 \text{ m/s}^2)}$$

$$R_3 = 107.137 \text{ m}$$

Problem 72 - 4.3.35 :



Verify the ranges shown for the projectiles in the figure for an initial velocity of 50 m/s at the given initial angles.

Solution

$$R = \frac{(50.0 \text{ m/s})^2 \sin 2\theta_0}{g} \text{ for } v_0 = 50 \text{ m/s}: R = 128 \text{ m for } \theta = 15^\circ \text{ and } 75^\circ$$

$$R = 255 \text{ m for } \theta = 45^\circ.$$

Problem 73 - 4.3.36 :

The cannon on a battleship can fire a shell a maximum distance of 32.0 km .

Part (a) Calculate the initial speed of the shell in meters per second. Neglect air resistance to make the problem easier.

To achieve the maximum distance, the cannon must be fired at an angle of 45 degrees above the horizontal axis. If we assume that the initial and final heights of the shell are the same, we can use the range formula.

$$R = \frac{v_0^2}{g} \sin(2\theta)$$

Twice the angle equals 90 degrees, so the sine term is equal to 1 .

Solving for the initial speed, we find

$$v_0 = \sqrt{gR}$$

$$= \sqrt{(9.80 \text{ m/s}^2)(32000 \text{ m})}$$

$$v_0 = 560 \text{ m/s}$$

Part (b) What maximum height does it reach in meters?

To find the maximum height h , we can use the equations of kinematics, knowing that at the maximum height, $v_y = 0 \text{ m/s}$. The initial velocity was found in part (a).

$$v_y^2 = v_{0y}^2 + 2gh$$

$$h = \frac{v_{0y}^2}{(2g)}$$

$$= \frac{(v_0 \sin(45^\circ))^2}{(2g)}$$

$$= \frac{(560 \text{ m/s} \sin(45^\circ))^2}{(2(9.80 \text{ m/s}^2))}$$

$$h = 8000 \text{ m}$$

Part (c) The ocean is not flat, because the Earth is curved. Take the radius of the Earth to be 6.38×10^3 km. How many meters lower will its surface be 3 measured along a line that is tangent to the surface of the Earth at the location of the ship?

The first part of the problem assumes the shell starts and ends at the same height. Because the shell is traveling a great distance, the curvature of the Earth is an in Consider the following image illustrating the situation. The shell is launched at a point on the right side of the circle, which represents Earth. The hypotenuse of distance from the center of the Earth to where the shell would end, if it only traveled in a straight line.

The angle alpha can be found,

$$\alpha = \arctan\left(\frac{32.0 \text{ km}}{6380 \text{ km}}\right) = 0.2874 \text{ degrees}$$

$$x = \frac{R}{\cos(\alpha)}$$

$$= \frac{6380 \text{ km}}{\cos(0.2874 \text{ degrees})}$$

$$= 6380.080 \text{ km}$$

The distance above the surface of the ocean is then,

$$d = x - R = 6380.080 \text{ km} - 6380.000 \text{ km}$$

$$d = 80 \text{ m}$$

Problem 74 - 4.3.36-alt :

The cannon on a battleship can fire a shell a maximum distance of 25 km.

Part (a) Calculate the initial speed of the shell, in meters per second. Neglect air resistance to make the problem easier.

To achieve the maximum distance, the cannon must be fired at an angle of 45 degrees above the horizontal axis. If we assume that the initial and final heights of t can use the range formula.

$$R = \frac{v_0^2}{g} \sin(2\theta)$$

Twice the angle equals 90 degrees, so the sine term is equal to 1.

Solving for the initial speed, we find

$$v_0 = \sqrt{gR}$$

$$= \sqrt{(9.80 \text{ m/s}^2) (25 \text{ km}) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right)}$$

$$v_0 = 494.975 \text{ m/s}$$

Part (b) What maximum height, in meters, does it reach?

To find the maximum height h , we can use the equations of kinematics, knowing that at the maximum height, $v_y = 0$ m/s. The initial velocity was found in part a

$$v_y^2 = v_{0y}^2 + 2gh$$

$$h = \frac{v_{0y}^2}{(2g)}$$

$$= \frac{(v_0 \sin(45^\circ))^2}{2g}$$

$$= \frac{gR}{2g} \sin^2(45^\circ)$$

$$= \frac{R}{2} \sin^2(45^\circ)$$

$$= \frac{(25 \text{ km}) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right)}{2} \sin^2(45^\circ)$$

$$h = 6249.992 \text{ m}$$

Part (c) The ocean is not flat, because the Earth is curved. Take the radius of the Earth to be 6.38×10^3 km. How much lower, in meters, will its surface be measured along a line that is tangent to the surface of the Earth at the location of the ship?

The first part of the problem assumes the shell starts and ends at the same height. Because the shell is traveling a great distance, the curvature of the Earth is an issue. Consider the following image illustrating the situation. The shell is launched at a point on the right side of the circle, which represents Earth. The hypotenuse of distance from the center of the Earth to where the shell would end, if it only traveled in a straight line.

The angle alpha can be found,

$$\alpha = \arctan\left(\frac{R}{r}\right)$$

where r is the radius of the Earth.

$$x = \frac{r}{\cos(\arctan(\alpha))}$$

The distance above the surface of the ocean is then,

$$d = x - r$$

$$= \frac{6380.000 \text{ km}}{\cos(\arctan(\frac{25 \text{ km}}{6380 \text{ km}}))} - 6380.000 \text{ km}$$

$$d = 48.981 \text{ m}$$

Problem 75 - 4.3.37 :

An arrow is shot from a height of 1.5 m toward a cliff of height H . It is shot with a velocity of 30 m/s at an angle of 60° above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?

Solution

$$(a) \quad y = v_{0y}t + \frac{1}{2}at^2 = (30 \text{ m/s})(\sin 60^\circ)(4.0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(4 \text{ s})^2 = 25.5 \text{ m}$$

$$H = 1.5 \text{ m} + y = \underline{27.0 \text{ m}}$$

$$(b) \quad v^2 = v_0^2 + 2ay$$

$$0 = (30.0 \text{ m/s})^2 (\sin 60^\circ)^2 + 2(-9.8 \text{ m/s}^2)y$$

$$y = 34.44 \text{ m}$$

$$H = 34.44 \text{ m} + 1.5 \text{ m} = \underline{36.0 \text{ m}}$$

$$(c) \quad v_{fy} = v_{0y} + at = (30 \text{ m/s})(\sin 60^\circ) + (-9.8 \text{ m/s}^2)(4.0 \text{ s}) = 13.2 \text{ m/s}$$

$$v^2 = v_x^2 + v_y^2$$

$$v = \sqrt{(30 \cos 60^\circ)^2 + (13.2)^2} (\text{m/s}) = \underline{20 \text{ m/s}}$$

Problem 76 - 4.3.38 :

In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.55 m and the acceleration achieved during the time the jumper is extending their legs is 1.05 times the acceleration due to gravity, g .

Part (a) How far can they jump in meters? Assume the person leaves at an angle of 45° and on level ground.

We begin by using the information given to determine the initial velocity of the person jumping. The person starts from rest and moves through the distance d with

$$v_0^2 = 2ad$$

The person is launched at a 45 degree angle and reaches a maximum height at time t , which is half of the elapsed time for the jump.

$$v_{y,f} = v_{y,0} - gt = 0$$

$$t = \frac{v_{y,0}}{g} = \frac{v_0 \sin(45^\circ)}{g}$$

The distance of the jump is

$$x = v_{0,x}(2t)$$

$$= 2v_0 \cos(45^\circ) \left(\frac{v_0}{g} \sin(45^\circ) \right)$$

$$= 2 \frac{v_0^2}{g} \left(\frac{\sqrt{2}}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$$

$$= \frac{2ad}{g}$$

$$= \frac{2(1.05g)(0.55 \text{ m})}{g}$$

$$x = 1.155 \text{ m}$$

Problem 77 - 4.3.39 :

The world long jump record is 8.95 m (Mike Powell, USA, 1991).

Part (a) Treated as a projectile, what is the maximum range, in meters, obtainable by a person if he or she has a take-off speed of 8.2 m/s? Assume the initial velocity makes an angle of 45° with the horizontal.

The maximum distance, or range, of the jump can be derived using the kinematic formulas. Assuming no acceleration in the horizontal direction, the range is

$$R = v_0 \cos(45^\circ) t$$

The elapsed time is not known, however it is twice the time it takes the jumper to reach the maximum height, where the vertical component of the velocity is mor

$$v_y = 0 = v_{0y} - gT$$

$$T = \frac{v_{0y}}{g} = \frac{(v_0 \sin(45^\circ))}{g}$$

The distance of the jump is then

$$\begin{aligned} R &= v_0 \cos(45^\circ) (2T) \\ &= \frac{2(v_0 \cos(45^\circ))(v_0 \sin(45^\circ))}{g} \\ &= \frac{2\left(v_0 \frac{\sqrt{2}}{2}\right)\left(v_0 \frac{\sqrt{2}}{2}\right)}{g} \\ &= \frac{v_0^2}{g} \\ &= \frac{(8.2 \text{ m/s})^2}{9.8 \text{ m/s}^2} \end{aligned}$$

$$R = 6.861 \text{ m}$$

Problem 78 - 4.3.40 :

Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The service line is 11.9 m from the net, which is 0.91 m high.

Part (a) Find the angle θ , in degrees, at which the ball just crosses the net.

The ball, though it's moving very fast, does not follow a straight line. Therefore, one must apply the equations of kinematics in both the horizontal and vertical coordinate system with the origin directly below the center of the ball at 0.91 m above the ground, so the initial height is at $y_0 = 2.50 - 0.91 \text{ m} = 1.59 \text{ m}$. The height of the ball with respect to this origin is

$$y = y_0 - v_0 \sin(\theta) t - \frac{1}{2} g t^2$$

In the horizontal direction,

$$x = v_0 \cos(\theta) t$$

When the ball has reached the top of the net, it has traveled 11.9 m in the x-direction and is at $y = 0$ m. The initial velocity of the ball is 170 km/h, which is equal information, we can complete the solution.

$$t = \frac{L}{(v_0 \cos(\theta))}$$

$$y = 0 = y_0 - v_0 \sin(\theta) t - \frac{1}{2} g \left(\frac{x}{(v_0 \cos(\theta)) t^2} \right)$$

$$y_0 = v_0 \sin(\theta) \left(\frac{L}{v_0 \cos(\theta)} \right) + \frac{1}{2} g \left(\frac{L}{v_0 \cos(\theta)} \right)^2$$

$$= L \tan(\theta) + \frac{1}{2} \left(\frac{gL^2}{v_0^2 \cos^2(\theta)} \right)$$

A useful trigonometric identity is

$$1 + \tan^2(\theta) = \frac{1}{\cos^2(\theta)}$$

Applying this, we can rewrite the equation for y_0 as

$$\left(\frac{gL^2}{2v_0^2} - y_0 \right) + L \tan(\theta) + \left(\frac{gL^2}{2v_0^2} \right) \tan^2(\theta) = 0$$

Now, apply the quadratic formula to solve for $\tan(\theta)$.

$$\tan(\theta) = \frac{\left(-L + \sqrt{L^2 - 4 \left(\frac{gL^2}{2v_0^2} - y_0 \right) \left(\frac{gL^2}{2v_0^2} \right)} \right)}{2 \left(\frac{gL^2}{2v_0^2} - y_0 \right)}$$

$$\theta = \arctan \left(\frac{\left(-L + \sqrt{L^2 - 4 \left(\frac{gL^2}{2v_0^2} - y_0 \right) \left(\frac{gL^2}{2v_0^2} \right)} \right)}{2 \left(\frac{gL^2}{2v_0^2} - y_0 \right)} \right)$$

$$= \arctan \left(\frac{\left(-(11.9 \text{ m}) + \sqrt{(11.9 \text{ m})^2 - 4 \left(\frac{(9.80 \text{ m/s}^2)(11.9 \text{ m})^2}{2(47.2 \text{ m/s})^2} - (2.50 - 0.91 \text{ m}) \right) \left(\frac{(9.80 \text{ m/s}^2)(11.9 \text{ m})^2}{2(47.2 \text{ m/s})^2} \right)} \right)}{2 \left(\frac{(9.80 \text{ m/s}^2)(11.9 \text{ m})^2}{2(47.2 \text{ m/s})^2} - (2.50 - 0.91 \text{ m}) \right)} \right)$$

$$\theta = 6.1^\circ$$

Part (b) At what distance, in meters, from the service line does the ball land?

$$R = v_0 \cos(\theta) t$$

$$y = 0 = y_0 - v_0 \sin(\theta) t - \frac{1}{2} g t^2$$

$$t = \frac{\left(v_0 \sin(\theta) \pm \sqrt{(v_0 \sin(\theta))^2 - 4(y_0) \left(-\frac{g}{2} \right)} \right)}{2(y_0)}$$

$$= \frac{\left((47.2 \text{ m/s}) \sin(6.1^\circ) \pm \sqrt{((47.2 \text{ m/s}) \sin(6.1^\circ))^2 - 4(2.5 \text{ m}) \left(-\frac{(9.80 \text{ m/s}^2)}{2} \right)} \right)}{2(2.5 \text{ m})}$$

$$= 0.367 \text{ s}$$

$$R = v_0 \cos(\theta) t$$

$$= (47.2 \text{ m/s}) \cos(6.1^\circ) (0.367 \text{ s})$$

$$R = 17 \text{ m}$$

Problem 79 - 4.3.41 :

A football quarterback is moving straight backward at a speed of **3.1** m/s when he throws a pass to a player **15** m straight downfield.

Part (a) If the ball is thrown at an angle of 25° relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground?

To determine the initial velocity of the ball, we can apply the range formula, since the distance thrown is given and the launch angle θ .

$$\text{range} = d = v_0^2 \frac{\sin(2\theta)}{g}$$

Solving for the initial velocity gives

$$\begin{aligned} v_0 &= \sqrt{\frac{dg}{\sin(2\theta)}} \\ &= \sqrt{\frac{(15 \text{ m})(9.80 \text{ m/s}^2)}{\sin(50^\circ)}} \end{aligned}$$

$$v_0 = 13.853 \text{ m/s}$$

Part (b) How long does it take to get to the receiver in s?

In the horizontal direction, the ball moves through a distance

$$d = v_{0x}t = v_0 \cos(\theta) t$$

Using the result from part a and solving for the elapsed time at which the ball is caught gives

$$\begin{aligned} t &= \frac{d}{v_0 \cos(\theta)} \\ &= \frac{d}{\left(\sqrt{\frac{dg}{\sin(2\theta)}} \cos(\theta)\right)} \\ &= \frac{(15 \text{ m})}{\left(\sqrt{\frac{(15 \text{ m})(9.80 \text{ m/s}^2)}{\sin(50^\circ)}} \cos(25^\circ)\right)} \end{aligned}$$

$$t = 1.195 \text{ s}$$

Part (c) What is its maximum height above its point of release in m?

At the maximum height, the vertical component of the ball's velocity is momentarily equal to zero m/s.

$$v_y^2 = v_{0y}^2 + 2gh = 0$$

Solving for the height, we find

$$\begin{aligned} h &= \frac{(v_0 \sin(\theta))^2}{2g} \\ &= \frac{\left(\sqrt{\frac{dg}{\sin(2\theta)}} \sin(\theta)\right)^2}{2g} \\ &= \frac{\left(\sqrt{\frac{(15 \text{ m})(9.80 \text{ m/s}^2)}{\sin(50^\circ)}} \sin(25^\circ)\right)^2}{(2(9.80 \text{ m/s}^2))} \end{aligned}$$

$$h = 1.749 \text{ m}$$

Problem 80 - 4.3.43 :

An eagle is flying horizontally at a speed of 2.5 m/s when the fish in her talons wiggles loose and falls into the lake 4.2 m below.

Part (a) Calculate the magnitude of the velocity of the fish relative to the water when it hits the water in m/s.

To determine the magnitude of the final velocity of the fish, we must use the pythagorean theorem, knowing the final vertical and horizontal velocity components. moving horizontally at the same speed as the eagle. Also, there is assumed to be no acceleration in the horizontal direction.

$$v_x = v_{0x} = 2.5 \text{ m/s}$$

There is initially no y-component for the velocity, but the fish accelerates downward due to the gravitational acceleration.

$$v_y^2 = v_{0y}^2 + 2gh$$

$$v_y = \sqrt{2gh}$$

where h is the vertical distance that the fish falls to the surface of the lake.

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{v_x^2 + 2gh} \\ &= \sqrt{(2.5 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(4.2 \text{ m})} \end{aligned}$$

$$v = 9.411 \text{ m/s}$$

Part (b) Calculate the angle, in degrees, by which the fish's velocity is directed below the horizontal when the fish hits the water.

Using the x- and y-components from part a, we can find the angle.

$$\begin{aligned} \theta &= \arctan\left(\frac{v_y}{v_x}\right) \\ &= \arctan\left(\frac{\sqrt{2gh}}{v_x}\right) \\ &= \arctan\left(\frac{\sqrt{2(9.80 \text{ m/s}^2)(4.2 \text{ m})}}{2.5 \text{ m/s}}\right) \end{aligned}$$

$$\theta = 74.595 \text{ degrees}$$

Problem 81 - 4.3.44 :

An owl is carrying a mouse to the chicks in its nest. The owl is at that time is 4 m west and 10.5 m above the center of the 25 cm diameter nest, and flying east at 3.1 m/s at an angle 25° below the horizontal when it accidentally drops the mouse.

Part (a) Calculate the horizontal position of the mouse when it has fallen 10.5 m, assuming the nest is at the origin of a coordinate system with east being

The initial position and initial velocity of the mouse is the same as that of the owl. To solve this problem, we use a coordinate system centered on the nest.

The horizontal position as a function of time is

$$x = x_0 + v_0 \cos(\theta) t$$

The only thing we do not know in this equation is the elapsed time, which can be determined from the motion in the vertical direction. The mouse is initially at a and its final position is at $y = 0\text{m}$.

$$y = 0 = h - v_0 \sin(\theta) t - \frac{1}{2} g t^2$$

Rearranging this and applying the quadratic formula gives,

$$t = -v_0 \sin(\theta) \pm \frac{\sqrt{v_0^2 \sin^2(\theta) - 4\left(\frac{g}{2}\right)(-h)}}{2\left(\frac{g}{2}\right)}$$

Only positive times make sense here.

$$t = -v_0 \sin(\theta) + \frac{\sqrt{v_0^2 \sin^2(\theta) + 2gh}}{g}$$

$$\begin{aligned} x &= x_0 + v_0 \cos(\theta) \left(-v_0 \sin(\theta) + \frac{\sqrt{v_0^2 \sin^2(\theta) + 2gh}}{g} \right) \\ &= x_0 - \frac{v_0^2}{g} \cos(\theta) \sin(\theta) + v_0 \frac{\cos(\theta) \sqrt{v_0^2 \sin^2(\theta) + 2gh}}{g} \\ &= -4 - \frac{(3.1 \text{ m/s})^2}{(9.80 \text{ m/s}^2)} \cos(25^\circ) \sin(25^\circ) + \frac{(3.1 \text{ m/s}) \cos(25^\circ) \sqrt{(3.1 \text{ m/s})^2 \sin^2(25^\circ) + 2(9.80 \text{ m/s}^2)(10.5 \text{ m})}}{(9.80 \text{ m/s}^2)} \end{aligned}$$

$$x = -0.245714271878358 \text{ m}$$

Part (b) How far from the east edge of the nest did the mouse fall, in centimeters?

The answer in part a is the distance toward the west that the mouse is relative to the center of the nest. Here we are asked for the distance from the east edge, so the nest to the absolute value of the answer from part a, in centimeters.

$$d = |x| + r$$

$$\begin{aligned} &= \left| x_0 - \frac{v_0^2}{g} \cos(\theta) \sin(\theta) + v_0 \frac{\cos(\theta) \sqrt{v_0^2 \sin^2(\theta) + 2gh}}{g} \right| + r \\ &= \left| -4 - \frac{(3.1 \text{ m/s})^2}{(9.80 \text{ m/s}^2)} \cos(25^\circ) \sin(25^\circ) + \frac{(3.1 \text{ m/s}) \cos(25^\circ) \sqrt{(3.1 \text{ m/s})^2 \sin^2(25^\circ) + 2(9.80 \text{ m/s}^2)(10.5 \text{ m})}}{(9.80 \text{ m/s}^2)} \right| \left(\frac{100 \text{ cm}}{1 \text{ m}} \right) + \frac{25 \text{ cm}}{2} \end{aligned}$$

$$d = 37.071 \text{ cm}$$

Problem 82 - 4.3.46 :

Suppose a goalkeeper can give the ball a speed of **25** m/s.

Part (a) What is the maximum horizontal distance the ball could go in meters?

The maximum range for a projectile occurs when the launch angle is 45 degrees. The range of the ball is given by

$$\begin{aligned} R &= v_0^2 \frac{\sin(2\theta)}{g} \\ &= (25 \text{ m/s})^2 \frac{\sin(90^\circ)}{(9.80 \text{ m/s}^2)} \end{aligned}$$

$$R = 63.776 \text{ m}$$

Problem 83 - 4.3.47 :

The free throw line in basketball is 4.570 m (15 ft) from the basket, which is 3.050 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 7.157 m/s, releasing it at a height of 2.440 m above the floor.

Part (a) At what angle above the horizontal must the ball be thrown to exactly hit the basket? Give your answer in degrees.

The ball is moving in both the horizontal and vertical directions during the elapsed time. The appropriate kinematic formulas for the position as a function of time

$$y = y_0 + v_0 \sin(\theta) t - \frac{1}{2} g t^2$$

$$x = v_0 \cos(\theta) t$$

The elapsed time for the ball to travel the horizontal distance L to the basket is

$$t = \frac{L}{v_0 \cos(\theta)}$$

During this time the ball moves upward a distance

$$h = y - y_0 = v_0 \sin(\theta) t - \frac{1}{2} g t^2$$

$$h = y - y_0$$

$$= v_0 \sin(\theta) t - \frac{1}{2} g t^2$$

$$= v_0 \sin(\theta) \left(\frac{L}{v_0 \cos(\theta)} \right) - \frac{1}{2} g \left(\frac{L}{v_0 \cos(\theta)} \right)^2$$

$$= L \tan(\theta) - \frac{1}{2} \left(\frac{g L^2}{v_0^2 \cos^2(\theta)} \right)$$

A useful trigonometric identity is

$$1 + \tan^2(\theta) = \frac{1}{\cos^2(\theta)}$$

Applying this, we can rewrite the equation for h as

$$h + \frac{1}{2} \left(\frac{g L^2}{v_0^2} \right) - L \tan(\theta) + \frac{1}{2} \left(\frac{g L^2}{v_0^2} \right) \tan^2(\theta) = 0$$

Now, apply the quadratic formula to solve for $\tan(\theta)$.

$$\tan(\theta) = \frac{\left(-(-L) \pm \sqrt{-L^2 - 4 \left(\frac{g L^2}{2 v_0^2} \right) \left(h + \frac{g L^2}{2 v_0^2} \right)} \right)}{2 \left(\frac{g L^2}{2 v_0^2} \right)}$$

$$\theta = \tan^{-1} \left(\frac{\left(L \pm \sqrt{-L^2 - 4 \left(\frac{g L^2}{2 v_0^2} \right) \left(h + \frac{g L^2}{2 v_0^2} \right)} \right)}{2 \left(\frac{g L^2}{2 v_0^2} \right)} \right)$$

$$= \tan^{-1} \left(\frac{\left(4.57 \text{ m} \pm \sqrt{-(4.57 \text{ m})^2 - 4 \left(\frac{(9.80 \text{ m/s}^2)(4.57 \text{ m})^2}{2(7.157 \frac{\text{m}}{\text{s}})^2} \right) \left((0.610 \text{ m}) + \frac{(9.80 \text{ m/s}^2)(4.57 \text{ m})^2}{2 v_0^2} \right)} \right)}{2 \left(\frac{(9.80 \text{ m/s}^2)(4.57 \text{ m})^2}{2(7.157 \frac{\text{m}}{\text{s}})^2} \right)} \right)$$

$$\theta = 48.74^\circ$$

Problem 84 - 4.3.49 :

A basketball player is running at **4.15** m/s toward the basket when he jumps into the air to dunk the ball. He maintains the same horizontal velocity while in the air.

Part (a) What vertical velocity does he need to rise **0.750** m above the floor in m/s?

From the information given, the equations of kinematics can be used to determine the vertical component of the initial velocity of the player.

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

$$\begin{aligned} v_{0y} &= \sqrt{v_y^2 + 2g(y - y_0)} \\ &= \sqrt{(0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(0.75 \text{ m})} \end{aligned}$$

$$v_{0y} = 3.834 \text{ m/s}$$

Part (b) At what horizontal distance from the basket, in meters, must he start his jump to reach his maximum height at the same time as he reaches the 1

Time is the parameter that links the motion in the horizontal and vertical directions. In the horizontal direction,

$$x = v_{0x}t$$

The time is unknown, but it can be found from the motion in the vertical direction.

$$v_y = v_{0y} - gt$$

$$t = \frac{(v_{0y} - v_y)}{g}$$

$$\begin{aligned} x &= v_{0x}t \\ &= v_{0x} \frac{(v_{0y} - v_y)}{g} \\ &= \frac{(4.15 \text{ m/s})(3.83 \text{ m/s})}{(9.80 \text{ m/s}^2)} \end{aligned}$$

$$x = 1.622 \text{ m}$$

Problem 85 - 4.3.50 :

A football player punts the ball from the ground at a 45.0° angle above the horizontal. Without an effect from the wind, the ball would travel **61** m before falling back to the ground.

Part (a) What is the initial speed of the ball in m/s?

From the range of the ball's motion we can find the ball's initial velocity. the range is given by

$$R = v_0^2 \frac{\sin(2\theta)}{g}$$

$$\begin{aligned} v_0 &= \sqrt{\frac{gR}{\sin(2\theta)}} \\ &= \sqrt{\frac{(9.80 \text{ m/s}^2)(61 \text{ m})}{\sin(2(45^\circ))}} \end{aligned}$$

$$v_0 = 24.45 \text{ m/s}$$

Part (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by **1.1** m/s. What total distance, horizontally before returning to the ground?

The horizontal component of the velocity is reduced by the wind gust for the second half of the ball's flight. The time of flight is not affected by the wind gust. During flight, the ball travels, $d_1 = \frac{61}{2}$ m. The time for half the flight is

$$t_1 = \frac{d_1}{v_{0x}} = \frac{d_1}{v_0 \cos(\theta)}$$

The distance the ball travels during the second half of its flight is

$$d_2 = v_x t = v_x \frac{d_1}{v_0 \cos(\theta)}$$

The total distance the ball travels is then

$$\begin{aligned} x &= d_1 + d_2 \\ &= d_1 + v_x \frac{d_1}{v_0 \cos(45^\circ)} \\ &= d_1 + \frac{(v_{0x} - v_{\text{gun}}) d_1}{v_0 \cos(45^\circ)} \\ &= d_1 + \frac{(v_0 \cos(45^\circ) - v_{\text{gun}}) d_1}{v_0 \cos(45^\circ)} \\ &= \frac{61}{2} \text{ m} + \frac{\left(\sqrt{\frac{(9.80 \text{ m/s}^2)(61 \text{ m})}{\sin(2(45^\circ))}} \cos(45^\circ) - 1.1 \text{ m/s} \right) \left(\frac{61}{2} \text{ m} \right)}{\left(\left(\sqrt{\frac{(9.80 \text{ m/s}^2)(61 \text{ m})}{\sin(2(45^\circ))}} \right) (\cos(45^\circ)) \right)} \end{aligned}$$

$$x = 59.059 \text{ m}$$

Problem 86 - 4.3.52 :

Full solution not currently available at this time.

A bullet is shot horizontally over level ground. The initial height is **1.3** m, and its initial speed is **150** m/s.

Part (a) How much time, in seconds, elapses before the bullet hits the ground?

$$t = \sqrt{2 \cdot y / 9.8}$$

$$t = \sqrt{2 \cdot 1.3 / 9.8}$$

$$t = 0.5151$$

$$\text{Tolerance: } \pm 0.015453$$

Part (b) How far does the bullet travel horizontally, in meters, before hitting the ground?

$$x = \sqrt{2 \cdot y / 9.8} \cdot v_0$$

$$x = \sqrt{2 \cdot 1.3 / 9.8} \cdot 150$$

$$x = 77.262$$

$$\text{Tolerance: } \pm 2.31786$$

Problem 87 - 4.3.53 :

Full solution not currently available at this time.

A marble rolls off a horizontal tabletop that is **0.91** m high and hits the floor at a point that is a horizontal distance of **2.5** m from the edge of the table.

Part (a) How much time, in seconds, was the marble in the air?

$$t = \sqrt{2 \cdot h / 9.8}$$

$$t = \sqrt{2 \cdot 0.91 / 9.8}$$

$$t = 0.4309$$

$$\text{Tolerance: } \pm 0.012927$$

Part (b) What is the speed of the marble, in m/s, as it rolled off the table?

$$v = r/(\sqrt{2h/9.8})$$

$$v = 2.5/(\sqrt{2*0.91/9.8})$$

$$v = 5.801$$

$$\text{Tolerance: } \pm 0.17403$$

Part (c) What was the marble's speed, in m/s, just before hitting the floor?

$$v = \sqrt{(r/(\sqrt{2h/9.8}))^2 + (9.8*\sqrt{2h/9.8})^2}$$

$$v = \sqrt{(2.5/(\sqrt{2*0.91/9.8}))^2 + (9.8*\sqrt{2*0.91/9.8})^2}$$

$$v = 7.176$$

$$\text{Tolerance: } \pm 0.21528$$

Problem 88 - 4.3.55 :

Full solution not currently available at this time.

At a particular instant, a hot air balloon is **40.1** m above a horizontal field and descending at a rate of **2.1** m/s. At this instant, a girl throws a ball horizontally (relative to herself) with an speed of **12** m/s.

Part (a) When the balloon lands on the ground, how far is she from where the ball landed? Express your answer in meters and neglect air resistance.

$$x = (v_0x/9.8)*(\sqrt{v_0y^2+19.6h}-v_0y)$$

$$x = (12/9.8)*(\sqrt{2.1^2+19.6*40.1}-2.1)$$

$$x = 31.853$$

$$\text{Tolerance: } \pm 0.95559$$

Problem 89 - 4.3.56 :

Full solution not currently available at this time.

A man rides a motorcycle on a horizontal road at a speed s_x . At a particular instant, he throws a ball upward (relative to himself) with a speed of s_y . Let h be the height of the ball at the moment it was released. Consider an x - y coordinate system, where positive x is horizontal and forward, and positive y is vertically upward and $y = 0$ is at ground level. Assume the ball was released at $x = 0$ and $y = h$.

Part (a) Write an equation for y as a function of g (9.80 m/s^2), x , s_x , s_y , and h .

$$y = h + x s_y/s_x - (g/2) (x/s_x)^2$$

Problem 90 - 4.3.58 :

Full solution not currently available at this time.

The maximum horizontal distance that Jean can throw a baseball is **25** m.

Part (a) Assuming she can throw with the same initial speed at all launch angles, to what maximum height (measured in meters above the release point) throws it straight upward?

$$h = x/2$$

$$h = 25/2$$

$$h = 12.5$$

$$\text{Tolerance: } \pm 0.375$$

Problem 91 - 4.3.59 :

Full solution not currently available at this time.

A rock is thrown off a vertical cliff at an angle of **40.1**° above horizontal. The cliff is **60.1** m high and the ground extends horizontally from the base. The initial speed of the rock is **18** m/s. Neglect air resistance.

Part (a) To what maximum height, in meters, does the rock rise above the edge of the cliff?

$$h = (v_0*\sin(\theta*0.0174533))^2/9.8-4.9*(v_0*\sin(\theta*0.0174533)/9.8)^2$$

$$h = (18*\sin(40.1*0.0174533))^2/9.8-4.9*(18*\sin(40.1*0.0174533)/9.8)^2$$

$$h = 6.858$$

$$\text{Tolerance: } \pm 0.20574$$

Part (b) How far, in meters, has the rock moved horizontally at the moment it reaches its maximum height?

$$x = (v_0*\cos(\theta*0.0174533))*(v_0*\sin(\theta*0.0174533)/9.8)$$

$$x = (18*\cos(40.1*0.0174533))*(18*\sin(40.1*0.0174533)/9.8)$$

$$x = 16.289$$

$$\text{Tolerance: } \pm 0.48867$$

Part (c) How long, in seconds, after being thrown does the rock hit the ground?

$$t = (v_0 \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 19.6h}) / 9.8$$

$$t = (18 \sin(40.1) + \sqrt{(18 \sin(40.1))^2 + 19.6(60.1)}) / 9.8$$

$$t = 4.88$$

Tolerance: ± 0.1464

Part (d) How far from the base of the cliff, in meters, is the landing point of the rock?

$$x = v_0 \cos(\theta) (v_0 \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 19.6h}) / 9.8$$

$$x = 18 \cos(40.1) (18 \sin(40.1) + \sqrt{(18 \sin(40.1))^2 + 19.6(60.1)}) / 9.8$$

$$x = 67.187$$

Tolerance: ± 2.01561

Part (e) How high above the ground, in meters, is the rock 1.2 s before it hits the ground?

$$h = h_0 + v_0 \sin(\theta) (v_0 \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 19.6h}) / 9.8 - t - 4.9 (v_0 \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 19.6h}) / 9.8 - 1.2 - 4.9 (18 \sin(40.1) + \sqrt{(18 \sin(40.1))^2 + 19.6(60.1)}) / 9.8 - 1.2)^2$$

$$h = 60.1 + 18 \sin(40.1) (18 \sin(40.1) + \sqrt{(18 \sin(40.1))^2 + 19.6(60.1)}) / 9.8 - 1.2 - 4.9 (18 \sin(40.1) + \sqrt{(18 \sin(40.1))^2 + 19.6(60.1)}) / 9.8 - 1.2)^2$$

$$h = 36.416$$

Tolerance: ± 1.09248

Part (f) How far horizontally from the cliff, in meters, is the rock 1.2 s before it hits the ground?

$$x = v_0 \cos(\theta) (v_0 \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 19.6h}) / 9.8 - t$$

$$x = 18 \cos(40.1) (18 \sin(40.1) + \sqrt{(18 \sin(40.1))^2 + 19.6(60.1)}) / 9.8 - 1.2$$

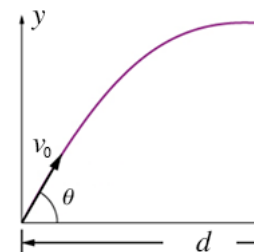
$$x = 50.664$$

Tolerance: ± 1.51992

Problem 92 - 4.3.61 :

Full solution not currently available at this time.

A projectile is shot toward a hill that is $d = 250$ m away. The launch angle is $\theta = 45$ degrees above horizontal with an initial speed of $v_0 = 70.1$ m/s. The hill can be approximated as a plane sloped at $\phi = 18$ degrees. Neglect air resistance.



Part (a) Write an equation for y as a function of x , d , and ϕ for the line that defines the slope of the hill.

$$y = x \tan(\phi) - d \tan(\phi)$$

Part (b) Write an equation for y as a function of x , g (9.80 m/s^2), v_0 , and θ of the trajectory of the projectile.

$$y = x \sin(\theta) / \cos(\theta) - 0.5 g x^2 / (v_0 \cos(\theta))^2$$

Part (c) What is the x coordinate, in meters, of the landing spot of the projectile?

$$x = 0.5 * ((\tan(\theta) - \tan(\phi)) + \sqrt{(\tan(\theta) - \tan(\phi))^2 + 4 * (4.9 / (\cos(\theta) * \cos(\phi)) * v_0^2)}) * (d * \tan(\phi))) / (4.9 / (\cos(\theta) * \cos(\phi)) * v_0^2)$$

$$x = 0.5 * ((\tan(45) - \tan(18)) + \sqrt{(\tan(45) - \tan(18))^2 + 4 * (4.9 / (\cos(45) * \cos(18)) * 70.1^2)}) * (250 * \tan(18)) / (4.9 / (\cos(45) * \cos(18)) * 70.1^2)$$

$$x = 432.649$$

Tolerance: ± 12.97947

Problem 93 - 4.3.62 :

Full solution not currently available at this time.

An astronaut on a distant planet kicks a soccer ball at an angle of 30.1 degrees above horizontal, with an initial speed of 10.1 m/s. The ball is kicked over flat ground, and the acceleration due to gravity on the planet is 2.01 m/s^2 . The planet has no atmosphere.

Part (a) How far horizontally does the ball travel, in meters, while it is in flight?

$$x = 2 * v_0^2 * \sin(\theta) * \cos(\theta) / a$$

$$x = 2 * 10.1^2 * \sin(30.1) * \cos(30.1) / 2.01$$

$$x = 44.04$$

$$\text{Tolerance: } \pm 1.3212$$

Problem 94 - 4.3.64 :**Full solution not currently available at this time.**

A robot cheetah can jump over obstacles. Suppose the launch speed is $v_0 = 3.01$ m/s, and the launch angle is $\theta = 20.1$ degrees above horizontal.

Part (a) What is the maximum height h in terms of v_0 , θ , and g (9.80 m/s²)?

$$h = v_0^2 \sin^2(\theta) / (2g)$$

Part (b) What is the maximum height in meters?

$$h = (\sin(20.1) * 3.01)^2 / (2 * 9.8)$$

$$h = 0.05459$$

$$\text{Tolerance: } \pm 0.0016377$$

Part (c) Given the same launch speed, what launch angle, in degrees, would yield a maximum height that is **30.1%** greater than that calculated in part (b)?

$$\theta = \arcsin\left(\frac{v_0 \sqrt{2g(h + h_0)}}{v_0}\right)$$

$$\theta = 23.078$$

$$\text{Tolerance: } \pm 0.69234$$

Problem 95 - 4.3.65 :**Full solution not currently available at this time.**

Mt. Asama, Japan, is an active volcano complex. In 2009, an eruption threw solid volcanic rocks that landed far from the crater. Suppose that one such rock was launched at an angle of $\theta = 30.1$ degrees above horizontal, and landed a horizontal distance $d = 510$ m from the crater, and a vertical distance $h = 410$ m below the crater.

Part (a) Write an expression for v_0 , the initial speed of the rock in terms of θ , d , and h .

$$v_0 = \sqrt{\frac{d^2 g}{2 \sin(\theta) \cos(\theta) + 2 h \cos^2(\theta)}}$$

Part (b) What is the initial speed of the rock in m/s?

$$v_0 = \sqrt{\frac{d^2 g}{2 \sin(\theta) \cos(\theta) + 2 h \cos^2(\theta)}}$$

$$v_0 = 49.123$$

$$\text{Tolerance: } \pm 1.47369$$

Part (c) How long, in seconds, was the rock in the air?

$$t = \frac{d}{v_0 \cos(\theta)}$$

$$t = 12$$

$$\text{Tolerance: } \pm 0.36$$

Problem 96 - 4.3.67 :**Full solution not currently available at this time.**

The Lunar Roving Vehicle used in NASA's late Apollo missions reached an unofficial lunar land speed of 5.00 m/s. Suppose, while traveling at that speed, it hit a bump that launched it at an angle of 15 degrees above the horizontal lunar surface.

Part (a) How long, in seconds, would the vehicle be off the ground? The acceleration of gravity on the Moon has a value of 1.62 m/s².

$$t = \frac{2 v_0 \sin(\theta)}{g}$$

$$t = 1.598$$

$$\text{Tolerance: } \pm 0.04794$$

Problem 97 - 4.3.68 :**Full solution not currently available at this time.**

A soccer goal is 2.44 m high. A player kicks the ball from a horizontal distance of 8.1 m from the goal, and the ball hits the crossbar at the top of the goal. The launch angle was 20.1 degrees above horizontal.

Part (a) What was the initial speed of the ball, in m/s?

$$v = \sqrt{d^2 * 9.8 / (2 * d * \sin(\theta * 0.0174533) * \cos(\theta * 0.0174533) - 2 * 2.44 * (\cos(\theta * 0.0174533))^2)}$$

$$v = \sqrt{(8.1^2 * 9.8 / (2 * 8.1 * \sin(20.1 * 0.0174533) * \cos(20.1 * 0.0174533) - 2 * 2.44 * (\cos(20.1 * 0.0174533))^2))}$$

$$v = 26.371$$

Tolerance: ± 0.79113

Problem 98 - 4.3.70 :

Full solution not currently available at this time.

A daredevil motorcycle jump spans a horizontal distance of 40.1 m and the landing height is the same as the takeoff height.

Part (a) If the speed at takeoff was 25 m/s, then what was the launch angle, in degrees? Ignore air resistance.

$$\theta = 57.296 * 0.5 * \text{asin}(9.8 * d / v_0^2)$$

$$\theta = 57.296 * 0.5 * \text{asin}(9.8 * 40.1 / 25^2)$$

$$\theta = 19.48$$

Tolerance: ± 0.5844

Problem 99 - 4.3.71 :

Full solution not currently available at this time.

You throw a baseball with an initial speed of $v_{01} = 12$ m/s at a particular launch angle here on Earth where the acceleration of gravity is $g_1 = 9.80$ m/s².

Assume the launch height is the same as the landing height.

Part (a) All else being equal, write an expression for the launch speed, v_{02} , that would result in the same range on a planet whose acceleration of gravity is m/s^2 .

$$v_{02} = v_{01} (g_2 / g_1)^{0.5}$$

Part (b) What is value of the answer to part (a) in m/s?

$$v_{02} = v_{01} * \sqrt{g_2 / 9.8}$$

$$v_{02} = 12 * \sqrt{2.01 / 9.8}$$

$$v_{02} = 5.435$$

Tolerance: ± 0.16305

Problem 100 - 4.3.73 :

Full solution not currently available at this time.

A long jumper can jump a distance of 5.01 m when he takes off at an angle of 45.0 degrees.

Part (a) Assuming he can jump with the same initial speed at all angles, how much distance (in meters) does he lose by taking off at 20.1 degrees?

$$\Delta x = x * (1 - \sin(2 * 0.0174533 * \theta))$$

$$\Delta x = 5.01 * (1 - \sin(2 * 0.0174533 * 20.1))$$

$$\Delta x = 1.776$$

Tolerance: ± 0.05328

Problem 101 - 4.3.74 :

Full solution not currently available at this time.

On planet Acron, the maximum horizontal distance a projectile launched at 5.01 m/s travels over flat ground is 20.1 m.

Part (a) What is the acceleration of gravity, in m/s², on planet Acron?

$$a = v^2 / x$$

$$a = 5.01^2 / 20.1$$

$$a = 1.249$$

Tolerance: ± 0.03747

Problem 102 - 4.3.76 :**Full solution not currently available at this time.**

The tee of the world's longest par 3 sits atop South Africa's Hanglip Mountain at 400.0 m above the green and can only be reached by helicopter. The horizontal distance to the green is 359.0 m. Neglect air resistance.

Part (a) If a golfer launches a shot that is 15 degrees with respect to the horizontal, what initial speed, in m/s, must she give the ball to land it on the green?

$$v = (359/\cos(\theta*0.0174533))*\sqrt{4.9/(400+359*\tan(\theta*0.0174533))}$$

$$v = (359/\cos(15*0.0174533))*\sqrt{4.9/(400+359*\tan(15*0.0174533))}$$

$$v = 36.934$$

Tolerance: ± 1.10802

Part (b) What is the time, in seconds, that it takes the ball to reach the green?

$$t = 359/((359/\cos(\theta*0.0174533))*\sqrt{4.9/(400+359*\tan(\theta*0.0174533))}*\cos(\theta*0.0174533))$$

$$t = 359/((359/\cos(15*0.0174533))*\sqrt{4.9/(400+359*\tan(15*0.0174533))}*\cos(15*0.0174533))$$

$$t = 10.063$$

Tolerance: ± 0.30189

Problem 103 - 4.3.77 :**Full solution not currently available at this time.**

When a field goal kicker kicks a football as hard as he can at 45.0 degrees to the horizontal, the ball just clears the 3.00-m-high crossbar of the goalposts 35 m away.

Part (a) What is the initial speed, in m/s, imparted on the football by the kick?

$$v = x*\sqrt{9.8/(x-3)}$$

$$v = 35*\sqrt{9.8/(35-3)}$$

$$v = 19.369$$

Tolerance: ± 0.58107

Part (b) In addition to clearing the crossbar, the football must be high enough in the air early during its flight to clear the reach of the onrushing defensive lineman is 1.01 m away. How high is the ball, in meters, when it reaches the horizontal position of the lineman?

$$h = 0.7071*(x^2/(x*\sqrt{4.9/(x-3)}))*x*\sqrt{9.8/(x-3)}-4.9*(x^2/(x*\sqrt{4.9/(x-3)}))^2$$

$$h = 0.7071*(1.01/(35*\sqrt{4.9/(35-3)}))*35*\sqrt{9.8/(35-3)}-4.9*(1.01/(35*\sqrt{4.9/(35-3)}))^2$$

$$h = 0.9833$$

Tolerance: ± 0.029499

Problem 104 - c4.4.1 :

Consider a ball moving in uniform circular motion.

Part (a) The acceleration of this ball is directed towards the center of the circle. Why is this?

Uniform circular motion means that the speed of the object and the radius of curvature remain constant as the motion takes place. At any given moment, the velocity is tangent to the circle, and hence changes from moment to moment. Although the speed doesn't change, the direction of the velocity does, and hence the velocity does. The direction of the velocity is pointed towards the center of the circle. Since acceleration is rate of change of velocity, acceleration is also directed towards the center of the circle.

Problem 105 - c4.4.2 :

You are driving in a car with your friend, who is speeding around the corner of a highway. You are worried for your safety, so you ask your friend how fast they are going.

Part (a) Your friend responds, "I am going around this corner at a constant velocity of 65 mph." Is their statement physically correct?

Your friend is using physics jargon incorrectly. Although the speed might be a constant 65 mph, the direction the car is moving is changing as it turns. Since velocity is a vector, it changes not only by the speed, but also by the direction of motion, a change in only one (in this case direction) is sufficient to change the velocity. Hence, velocity is changing although speed is constant.

Problem 106 - 4.4.4 :

A car is traveling at constant speed of $v = 10.1$ m/s. Its tires have a diameter of $d = 0.51$ m.

Randomized Variables

$$v = 10.1 \text{ m/s}$$

$$d = 0.51 \text{ m}$$

Part (a) Consider a point on the outer edge of the tire. What is the centripetal acceleration, a_c , at this point in m/s^2 ?

The centripetal acceleration for an object in spinning at a constant speed can be found with the following equation:

$$a_c = \frac{v^2}{r}$$

In this case, we are given the diameter of the tire rather than its radius. Since the radius is equal to half of the diameter, we can substitute in $0.5d$ for r in our equation for centripetal acceleration.

$$a_c = \frac{v^2}{0.5d}$$

$$a_c = \frac{(10.1 \text{ m/s})^2}{0.5 \cdot 0.51 \text{ m}}$$

$$a_c = 400.039 \text{ m/s}^2$$

Part (b) If the car was traveling twice as fast, what would be the numerical value of the ratio of the new centripetal acceleration, a_n , to the old centripetal acceleration, a_c ?

Recall that the centripetal acceleration is given by the following equation:

$$a_c = \frac{v^2}{r}$$

We can use this information to find a value for the ratio of the new centripetal acceleration to the original centripetal acceleration.

$$a_n / a_c = \frac{\left(\frac{(2v)^2}{r}\right)}{\left(\frac{v^2}{r}\right)}$$

$$a_n / a_c = \frac{\left(\frac{4v^2}{r}\right)}{\left(\frac{v^2}{r}\right)}$$

$$a_n / a_c = 4$$

Problem 107 - 4.4.5 :

A runner runs around a circular track. He completes one lap at a time of $t = 201 \text{ s}$ at a constant speed of $v = 3.1 \text{ m/s}$.

Randomized Variables

$$t = 201 \text{ s}$$

$$v = 3.1 \text{ m/s}$$

Part (a) What is the radius, r in meters, of the track?

To begin, let's find the total distance that the runner travels. Since there is no acceleration, we can use the relation between distance, speed, and time for movement:

$$d = vt$$

This distance will be equal to the circumference of the track. We can now use the relation between the circumference and the radius of a circle in order to solve for

$$d = 2\pi r$$

$$vt = 2\pi r$$

$$\frac{vt}{2\pi} = r$$

$$r = \frac{3.1 \text{ m/s} \cdot 201 \text{ s}}{2\pi}$$

$$r = 99.22 \text{ m}$$

Part (b) What was the runner's centripetal acceleration, a_c in m/s^2 , during the run?

The centripetal acceleration of someone moving in uniform circular motion is given by:

$$a_c = \frac{v^2}{r}$$

We found the radius in part (a) and we know the tangential velocity. Using this information, we can solve for the centripetal acceleration.

$$a_c = \frac{v^2}{\left(\frac{vt}{2\pi}\right)}$$

$$a_c = \frac{2\pi v}{t}$$

$$a_c = \frac{2\pi \cdot 3.1 \text{ m/s}}{201 \text{ s}}$$

$$a_c = 0.09686 \text{ m/s}^2$$

Problem 108 - 4.4.12 :

(a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s , assuming it is ideally banked? (b) Calculate the centripetal acceleration. (c) Does this acceleration seem large to you?

Solution (a) For an ideally banked curve:

$$\theta = \tan^{-1} \frac{v^2}{rg}, \text{ so that } r = \frac{v^2}{g \tan \theta} = \frac{(30.0 \text{ m/s})^2}{(9.80 \text{ m/s}^2) \tan 75.0^\circ} = \underline{24.6 \text{ m}}$$

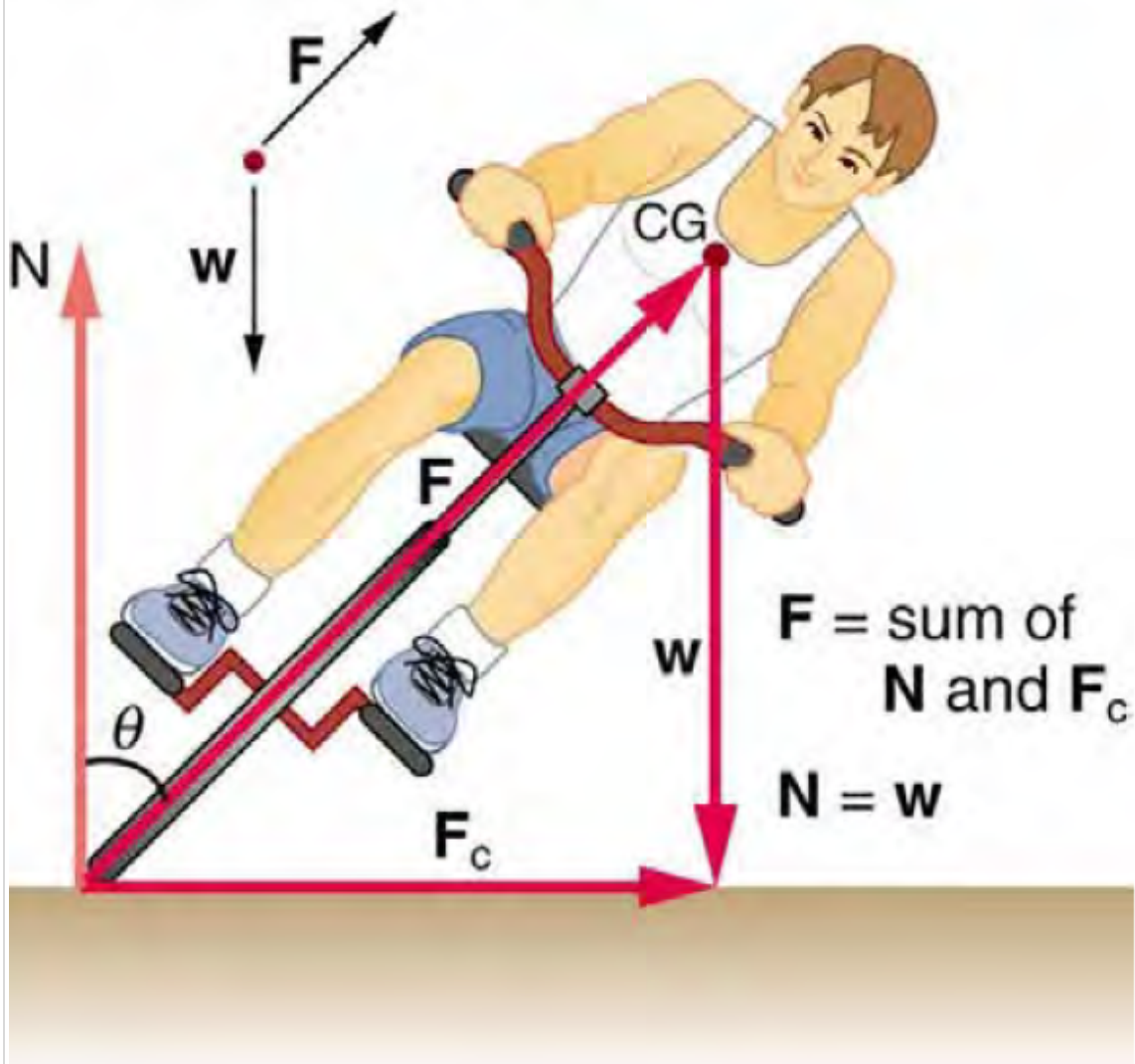
$$(b) a_c = \frac{v^2}{r} = \frac{(30.0 \text{ m/s})^2}{24.6 \text{ m}} = \underline{36.6 \text{ m/s}^2}$$

$$(c) \frac{a_c}{g} = \frac{36.6 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 3.73 \Rightarrow \underline{a_c = 3.73 g}$$

This does not seem too large, but it is clear that bobsledders feel a lot of force on them going through sharply banked turns!

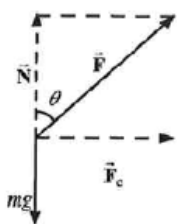
Problem 109 - 4.4.13 :

Free-body diagram



Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen in the figure. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force), and the vertical normal force (which must equal the system's weight). (a) Show that θ (as defined in the figure) is related to the speed v and radius of curvature r of the turn in the same way as for an ideally banked roadway—that is, $\theta = \tan^{-1} v^2 / rg$ (b) Calculate θ for a 12.0 m/s turn of radius 30.0 m (as in a race).

Solution



$$(a) F \sin \theta = F_c ; F \cos \theta = N = mg$$

$$\frac{F \sin \theta}{F \cos \theta} = \frac{F_c}{mg} = \frac{mv^2/r}{mg}$$

$$\tan \theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$

$$(b) \theta = \tan^{-1} \left(\frac{(12.0 \text{ m/s})^2}{30.0 \text{ m} \times 9.80 \text{ m/s}^2} \right) = \underline{26.1^\circ}$$

Problem 110 - 4.4.16 :

A fairground ride spins its occupants inside a flying-saucer-shaped vehicle.

Part (a) If the horizontal circular path the riders follow has a radius of 5.5 m, at how many revolutions per minute will the riders be subjected to a centripetal acceleration that due to gravity?

To begin this problem, let's use the relationship between angular velocity and centripetal acceleration to find an expression for the angular velocity.

$$a_c = f^2 r$$

$$\frac{a_c}{r} = f^2$$

$$\sqrt{\frac{a_c}{r}} = f$$

$$f = \sqrt{\frac{1.25 \text{ m/s}^2}{5.5 \text{ m}}}$$

$$f = 1.492 \text{ rad/s}$$

This gives us an answer in radians per second. In order to get an answer in revolutions per minute, we need to convert the units.

$$f = 1.492 \text{ rad/s} \cdot \frac{1 \text{ rotation}}{2\pi \text{ rad}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = \left(1.492 \cdot \frac{60}{2\pi}\right) \text{ rpm}$$

$$f = 14.251 \text{ rpm}$$

Problem 111 - 4.4.17 :

A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of **25 m**.

Randomized Variables

$$r = 25 \text{ m}$$

$$t = 20.1 \text{ s}$$

Part (a) If he completes the 200 m dash in **20.1 s** and runs at constant speed throughout the race, what is his centripetal acceleration as he runs the curve in m/s^2 ?

To begin, we will need to find the speed of the runner. As the runner moves at a constant speed through the entire race, we can use the relationship between distance and constant speed motion to find the runner's speed.

$$d = vt$$

$$\frac{d}{t} = v$$

The runner's speed will be his tangential velocity while he is in the curve. We can thus use the relationship between centripetal acceleration, tangential velocity, and a runner's centripetal acceleration on the curve.

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{\left(\frac{d}{t}\right)^2}{r}$$

$$a_c = \frac{\left(\frac{200 \text{ m}}{20.1 \text{ s}}\right)^2}{25 \text{ m}}$$

$$a_c = 3.96 \text{ m/s}^2$$

Problem 112 - 4.4.19 :

A workshop grindstone has a radius of **6.5 cm** and rotates at **6500 rev/min**.

Randomized Variables

$$r = 6.5 \text{ cm}$$

$$\text{rpm} = 6500 \text{ rpm}$$

Part (a) Calculate the centripetal acceleration at its edge in multiples of g .

We can find the centripetal acceleration using the relation between centripetal acceleration, angular velocity, and radius for uniform circular motion. To do this, we need to convert the angular velocity from revolutions per minute to radians per second.

$$\omega = 6500 \text{ rev/min} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = \left(6500 \cdot \frac{2\pi}{60}\right) \text{ rad/s}$$

We can now use the relationship between centripetal acceleration, angular velocity, and radius to find the centripetal acceleration. As we do so, we must also convert centimeters to meters.

$$a_c = \omega^2 r$$

$$a_c = \left(\left(6500 \cdot \frac{2\pi}{60}\right) \text{ rad/s} \right) \cdot 6.5 \cdot 10^{-2} \text{ m}$$

$$a_c = 30115.95 \text{ m/s}^2$$

This problem wants our answer in terms of multiples of g , however. We must therefore rewrite our solution in terms of g to get the final answer.

$$a_c = \frac{30115.95 \text{ m/s}^2}{9.8 \text{ m/s}^2} g$$

$$a_c = 3073.056g$$

Part (b) What is the linear speed of a point on its edge in m/s?

The linear speed can be found by multiplying the angular speed (in radians per second) by the radius. We found an expression for the linear speed in rad/s in part together with the given radius to find the linear speed. Since the given radius is in centimeters, it must be converted to meters in order to get an answer of meters

$$v = r\omega$$

$$v = 6.5 \cdot 10^{-2} \text{ m} \cdot \left(6500 \cdot \frac{2\pi}{60}\right) \text{ rad/s}$$

$$v = 44.244 \text{ m/s}$$

Problem 113 - 4.4.20 :

Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip.

Randomized Variables

$$r = 3.2 \text{ m}$$

$$rpm = 250 \text{ rpm}$$

Part (a) Calculate the centripetal acceleration at the tip of a 3.2 m long helicopter blade that rotates at 250 rev/min in m/s^2 .

To solve this problem, we can use the relationship between centripetal acceleration, angular velocity, and radius to find the centripetal acceleration:

$$a_c = \omega^2 r$$

In order to use this formula, however, we need to find a value for the angular velocity in radians per second. We must therefore convert our given angular velocity minute to radians per second.

$$\omega = 250 \text{ rev/min} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = \left(250 \cdot \frac{2\pi}{60}\right) \text{ rad/s}$$

Now we can plug this into our equation for the centripetal acceleration.

$$a_c = \left(\left(250 \cdot \frac{2\pi}{60}\right) \text{ rad/s}\right)^2 \cdot 3.2 \text{ m}$$

$$a_c = 2193.242 \text{ m/s}^2$$

Part (b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s). Give your answer in percent of the speed of sound.

To begin, we need to calculate the linear speed of the tip of the propeller. We can use the fact that the angular velocity (in rad/s) multiplied by the radius gives the problem. As we found an expression for the angular velocity in rad/s in part (a), we can use it again here.

$$V_{tip} = \omega r$$

$$V_{tip} = \left(250 \cdot \frac{2\pi}{60}\right) \text{ rad/s} \cdot 3.2 \text{ m}$$

Now that we have an expression for the linear speed of the tip, we can divide this by the speed of sound and multiply the result by one hundred in order to find what percent of the speed of sound this is.

$$V_{tip}/V_{sound} (\%) = \left(\frac{\left(250 \cdot \frac{2\pi}{60}\right) \text{ rad/s} \cdot 3.2 \text{ m}}{340 \text{ m/s}} \cdot 100\right) \%$$

$$V_{tip}/V_{sound} (\%) = 24.64\%$$

Problem 114 - 4.4.22 :

A rotating space station is said to create “artificial gravity”—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise and maintain muscle and bone strength more naturally than in non-rotating space environments.

Randomized Variables

$$d = 180 \text{ m}$$

Part (a) If the space station is 180 m in diameter, what angular velocity would produce an “artificial gravity” of 9.80 m/s² at the rim? Give your answer in rad/s.

Let's first consider what the problem is asking. Since the artificial gravity is generated by the centripetal acceleration, we want to find what angular velocity produces an acceleration of 9.80 m/s². We can use the relationship between the centripetal acceleration, angular velocity, and radius to solve this problem. Since we aren't directly given the radius, we will make use of the fact that the radius is equal to half the diameter in our equation.

$$a_c = \omega^2 r$$

$$a_c = \omega^2 \cdot 0.5d$$

$$\frac{a_c}{0.5d} = \omega^2$$

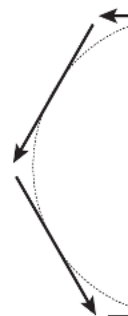
$$\sqrt{\frac{a_c}{0.5d}} = \omega$$

$$\omega = \sqrt{\frac{9.80 \text{ m/s}^2}{0.5 \cdot 180 \text{ m}}}$$

$$\omega = 0.33 \text{ rad/s}$$

Problem 115 - 4.4.23 :**Full solution not currently available at this time.**

In this problem we are going to try and understand where the acceleration in uniform circular motion comes from by considering some linear cases.



Part (a) First consider a particle moving with a velocity v_1 at an angle of θ_1 with respect to the x-axis. During a time period Δt , the particle suddenly turns to an angle of θ_2 with respect to the x-axis. Give an expression for the average acceleration vector of this motion in terms of unit vectors \mathbf{i} and \mathbf{j} , assuming the particle moves in the counterclockwise direction.

$$\mathbf{a}_{\text{ave}} = (v_2 \cos(\theta_2) - v_1 \cos(\theta_1)) \mathbf{i} / \Delta t + (v_2 \sin(\theta_2) - v_1 \sin(\theta_1)) \mathbf{j} / \Delta t$$

Part (b) Now consider the figure shown, where we approximate the circular motion of a particle of velocity 1.1 m/s by a hexagon (6-sided regular polygon). What is the magnitude of the acceleration, in m/s^2 ? You may assume that the velocity satisfies $v = 2\pi r / T$ for the period T .

$$|a_{\text{ave},6}| = 6 * v^2 / (3.14159 * r / 100) * ((1 - \cos(3.14159/3)) / 2)^{0.5}$$

$$|a_{\text{ave},6}| = 6 * 1.1^2 / (3.14159 * 11 / 100) * ((1 - \cos(3.14159/3)) / 2)^{0.5}$$

$$|a_{\text{ave},6}| = 10.504$$

$$\text{Tolerance: } \pm 0.31512$$

Part (c) What is the magnitude of the acceleration (in m/s^2) for an 11 -sided polygon?

$$|a_{\text{ave},11}| = (11 / (3.14159)) * (v^2 / (r / 100)) * ((1 - \cos(2 * 3.14159 / 11)) / 2)^{0.5}$$

$$|a_{\text{ave},11}| = (11 / (3.14159)) * (1.1^2 / (11 / 100)) * ((1 - \cos(2 * 3.14159 / 11)) / 2)^{0.5}$$

$$|a_{\text{ave},11}| = 10.851$$

$$\text{Tolerance: } \pm 0.32553$$

Part (d) What is the magnitude of the acceleration (in m/s^2) for uniform circular motion with a velocity of 1.1 m/s in a circle of radius 11 cm?

$$|a_c| = v^2 / (r / 100)$$

$$|a_c| = 1.1^2 / (11 / 100)$$

$$|a_c| = 11$$

$$\text{Tolerance: } \pm 0.33$$

Problem 116 - 4.4.25 :**Full solution not currently available at this time.**

A particle travels in a circular path of radius 10.1 m at a constant speed of 20.1 m/s.

Part (a) What is the magnitude of the acceleration of the particle in m/s^2 ?

$$a_c = v^2/r$$

$$a_c = 20.1^2/10.1$$

$$a_c = 40.001$$

$$\text{Tolerance: } \pm 1.20003$$

Problem 117 - 4.4.26 :

Full solution not currently available at this time.

A well-thrown football is spinning at **4.01** rev/s. The widest point of the football is at the center of the laces, where the radius is 8.50 cm.

Part (a) What is the magnitude of the centripetal acceleration of the laces in m/s^2 ?

$$a_c = 3.356 * w^2$$

$$a_c = 3.356 * 4.01^2$$

$$a_c = 53.965$$

$$\text{Tolerance: } \pm 1.61895$$

Problem 118 - 4.4.28 :

Full solution not currently available at this time.

A jet travels around the Earth, along the equator, just above the surface.

Part (a) What should be the jet's speed, in m/s, so that its centripetal acceleration has a magnitude equal to **40.1%** of the acceleration of gravity (**9.80** m/s²)?

$$v = 790.16 * \sqrt{p}$$

$$v = 790.16 * \sqrt{40.1}$$

$$v = 5003.653$$

$$\text{Tolerance: } \pm 150.10959$$

Problem 119 - 4.4.29 :

Full solution not currently available at this time.

The blades of a fan are rotating at **201** rev/min.

Part (a) What is the magnitude of the centripetal acceleration, in m/s^2 , of a point on one of the blades that is **10.1** cm from the axis of rotation?

$$a_c = 0.00010966 * r * w^2$$

$$a_c = 0.00010966 * 10.1 * 201^2$$

$$a_c = 44.747$$

$$\text{Tolerance: } \pm 1.34241$$

Problem 120 - 4.4.31 :

Full solution not currently available at this time.

A Formula One race car is traveling at **50.1** m/s along a straight track enters a turn on the race track with radius of curvature of **180** m.

Part (a) What magnitude centripetal acceleration, in m/s^2 , must the car have to stay on the track?

$$a_c = v^2/r$$

$$a_c = 50.1^2/180$$

$$a_c = 13.945$$

$$\text{Tolerance: } \pm 0.41835$$

Problem 121 - 4.4.32 :

Full solution not currently available at this time.

A particle is moving in a circular path in the x - y plane. The center of the circle is at the origin and the rotation is counterclockwise at a rate (angular speed) of $\omega = 2.01$ rad/s. At time $t = 0$, the particle is at $y = 0$ and $x = 2.01$ m.

Part (a) What is the x coordinate of the particle, in meters, at $t = 10.1$ s?

$$x = r * \cos(w * t)$$

$$x = 2.01 * \cos(2.01 * 10.1)$$

$$x = 0.2393$$

Tolerance: ± 0.007179

Part (b) What is the y coordinate of the particle, in meters, at $t = 10.1$ s?

$$y = r \sin(\omega t)$$

$$y = 2.01 \sin(2.01 \cdot 10.1)$$

$$y = 1.996$$

Tolerance: ± 0.05988

Part (c) What is the x component of the particle's velocity, in m/s, at $t = 10.1$ s?

$$v_x = -\omega r \sin(\omega t)$$

$$v_x = -2.01 \cdot 2.01 \sin(2.01 \cdot 10.1)$$

$$v_x = -4.011$$

Tolerance: ± 0.12033

Part (d) What is the y component of the particle's velocity, in m/s, at $t = 10.1$ s?

$$v_y = \omega r \cos(\omega t)$$

$$v_y = 2.01 \cdot 2.01 \cos(2.01 \cdot 10.1)$$

$$v_y = 0.4811$$

Tolerance: ± 0.014433

Part (e) What is the x component of the particle's acceleration, in m/s^2 , at $t = 10.1$ s?

$$a_x = -\omega^2 r \cos(\omega t)$$

$$a_x = -2.01^2 \cdot 2.01 \cos(2.01 \cdot 10.1)$$

$$a_x = -0.966912561343697$$

Tolerance: ± 0.0290073768403109

Part (f) What is the y component of the particle's acceleration, in m/s^2 , at $t = 10.1$ s?

$$a_y = -\omega^2 r \sin(\omega t)$$

$$a_y = -2.01^2 \cdot 2.01 \sin(2.01 \cdot 10.1)$$

$$a_y = -8.063$$

Tolerance: ± 0.24189

Problem 122 - 4.4.34 :

Full solution not currently available at this time.

A long rod rotates at 1.5 rev/s (revolutions per second) about an axis at one end.

Part (a) What is the magnitude of the centripetal acceleration, in m/s^2 , of a point 1.01 m from the axis?

$$a_c = 39.48 r \omega^2$$

$$a_c = 39.48 \cdot 1.01 \cdot 1.5^2$$

$$a_c = 89.718$$

Tolerance: ± 2.69154

Part (b) What is the magnitude of the centripetal acceleration, in m/s^2 , of a point 3.01 m from the axis?

$$a_c = 39.48 r \omega^2$$

$$a_c = 39.48 \cdot 3.01 \cdot 1.5^2$$

$$a_c = 267.378$$

Tolerance: ± 8.02134

Part (c) What is the direction of the centripetal acceleration of any point on the rod?

Toward the axis

Problem 123 - 4.4.35 :

Full solution not currently available at this time.

A red car has a speed of 15 m/s and is driving on a circular track with a radius of 90.1 m. A blue car has a speed of 8.1 m/s and is driving on a circular track with a radius of 51 m.

Part (a) What is the ratio of the centripetal acceleration of the red car to that of the blue car?

$$a_{\text{red}}/a_{\text{blue}} = rb*vr^2/(rr*vb^2)$$

$$a_{\text{red}}/a_{\text{blue}} = 51*15^2/(90.1*8.1^2)$$

$$a_{\text{red}}/a_{\text{blue}} = 1.941$$

$$\text{Tolerance: } \pm 0.05823$$

Problem 124 - 4.5.1 :

Consider a horizontal spinning disc of radius $R = 3$ m with a ball of mass $m = 2.1$ kg attached with a clamp to the outer edge of the disc. The disc starts at rest and experiences a constant angular acceleration $\alpha = 3.2$ rad/s². The size of the ball is negligible.

Part (a) Consider the disc at time $t = 10.1$ s. Calculate the speed v , in meters per second, of the attached ball?

We begin by finding the angular velocity of the disk at this moment in time. Since the angular acceleration is a constant $3.2 \frac{\text{rad}}{\text{s}^2}$,

$$\omega = 3.2t$$

At this moment, that angular velocity is equal to

$$\omega = 3.2 \cdot 10.1 = 32.32 \frac{\text{rad}}{\text{s}}$$

The tangential velocity of a point on the disk, including the point where the ball is attached, is given by

$$v = r\omega$$

where r is measured from the center of the disk to the point of interest. For the ball, we have

$$v = 3 \cdot 32.32$$

$$v = 96.96 \frac{\text{m}}{\text{s}}$$

Part (b) At this same time $t = 10.1$ s, calculate the instantaneous tension F_T , in newtons, that is required by the clamp to keep the ball attached to the disc

In this part, we are asked to find a force, given a description of the motion that this force creates. In other words, we will use Newton's 2nd Law, $F = ma$, to find F disk's center, given a in that same direction.

Since the distance of the ball from the center of the disk is not changing, the acceleration of the ball in the direction towards the center of the disk is described ent

$$a_r = \frac{v^2}{r}$$

where v is the velocity of the ball found in part a and r is the distance from the ball to the center of the disk. Explicitly,

$$a_r = \frac{96.96^2}{3} = 3133.747 \frac{\text{m}}{\text{s}^2}$$

By Newton's 2nd Law, we have

$$F_T = ma_r = 2.1 \cdot 3133.747$$

$$F_T = 6580.869N$$

Part (c) What is the net force F acting on the ball at $t = 10.1$ s, in newtons?

As we had to observe in part a, the ball is not only accelerating in the radial direction (which we calculated in part b). In fact, the rotation rate is increasing as we see in the figure, this is caused by gravity acting on the ball as well as by a tangential force that the clamp exerts. Here, we are asked to calculate that total tangential force along with the force we found in part b. Like in part b, we use the motion to infer the forces present. The acceleration in the tangential direction is given by

$$a = r\alpha = 3 \cdot 3.2 = 9.6 \frac{m}{s^2}$$

Again, by $F = ma$, there must be a tangential component of the net force acting on the ball of

$$F_{tan} = 2.1 \cdot 9.6 = 20.16N$$

Since this force is perpendicular to the force found in part b, we find the magnitude of the net force by taking the square root of the sum of the squares of these two forces

$$F = \sqrt{(F_{tan}^2 + F_T^2)} = \sqrt{(20.16^2 + 6580.869^2)}$$

$$F = 6580.9N$$

Problem 125 - 4.5.2 :

A NASA space probe is traveling from the Earth (average orbital radius, $r_E = 149.6 \times 10^6$ km; period of revolution, $T_E = 1.00$ yrs) to Mars (average orbital radius, $r_M = 227.9 \times 10^6$ km; period of revolution, $T_M = 1.881$ yrs). Treat the orbit of Mars and Earth as circles, with the Sun at the center. The trip takes $t_{EM} = 9.5$ months. The average length of a month is 30.436875 days.

Part (a) Calculate the orbital velocity, v_E , of Earth in m/s.

The velocity is related to distance and time by the expression

$$v = \frac{d}{t} \text{ m/s}$$

where d is the distance in m and t is the time in s. The earth will follow a circular path in its orbit around the sun. Therefore, the earth travels a distance of

$$d = 2\pi r_E$$

in a time of

$$t = T_E$$

Thus, the orbital velocity is

$$v_E = \frac{d}{t} = 2\pi \frac{r_E}{T_E}$$

Plugging in numbers and converting units as needed,

$$v_E = 2 \cdot \pi \cdot \frac{(149.6 \cdot 10^9 \text{ m})}{(1 \text{ yr} \cdot 12 \text{ month/yr} \cdot 30.436875 \text{ day/month} \cdot 24 \text{ hr/day} \cdot 60 \text{ min/hr} \cdot 60 \text{ s/min})}$$

$$v_E = 29783.388 \text{ m/s}$$

Part (b) Calculate the orbital velocity, v_M , of Mars in m/s.

The velocity is related to distance and time by the expression

$$v = \frac{d}{t} \text{ m/s}$$

where d is the distance in m and t is the time in s. Mars will follow a circular path in its orbit around the sun. Therefore, Mars travels a distance of

$$d = 2\pi r_M$$

in a time of

$$t = T_M$$

Thus, the orbital velocity is

$$v_M = \frac{d}{t} = 2\pi \frac{r_M}{T_M}$$

Plugging in numbers and converting units as needed,

$$v_M = 2 \cdot \pi \cdot \frac{(227.9 \cdot 10^9 \text{ m})}{(1.881 \text{ yr} \cdot 12 \text{ month/yr} \cdot 30.436875 \text{ day/month} \cdot 24 \text{ hr/day} \cdot 60 \text{ min/hr} \cdot 60 \text{ s/min})}$$

$$v_M = 24121.152 \text{ m/s}$$

Part (c) Assume that the probe starts at rest on the surface of the Earth and comes to rest on the surface of Mars. Calculate the average tangential acceleration over its journey.

The change in tangential acceleration is related to the change in tangential velocity and change in time by the expression

$$a_{tan} = \frac{(\Delta v)}{(\Delta t)} = \frac{(v_f - v_i)}{(t_f - t_i)} \text{ m/s}^2$$

where $v_{f,i}$ are the final and initial tangential velocities in m/s and $t_{f,i}$ are the final and initial times in s. When on earth, the probe is moving at the tangential velocity of Mars, it is moving at the tangential velocity of Mars. The time it takes is given in the description. Therefore, the change in tangential acceleration is

$$a_{tan} = \frac{(v_M - v_E)}{T_{EM}}$$

Plugging in numbers and converting units as needed,

$$a_{tan} = a_{pt} = \frac{\left(2 \cdot \pi \cdot \frac{(227.9 \cdot 10^9 \text{ m})}{(1.881 \text{ yr} \cdot 1.881 \text{ yr} \cdot 3.15576 \cdot 10^7 \text{ s/yr})} - 2 \cdot \pi \cdot \frac{(149.6 \cdot 10^9 \text{ m})}{(1 \text{ yr} \cdot 1.881 \text{ yr} \cdot 3.15576 \cdot 10^7 \text{ s/yr})}\right)}{(9.5 \text{ month} \cdot 30.436875 \text{ day/month} \cdot 24 \text{ hr/day} \cdot 60 \text{ min/hr} \cdot 60 \text{ s/min})}$$

$$a_{pt} = -0.000226625438212077 \text{ m/s}^2$$

Part (d) Calculate the change in the radial acceleration, Δa_r , in m/s^2 , of the probe relative to the Sun, as it travels from Earth to Mars.

The change in radial acceleration is related to the velocity and distance by the expression

$$a_{rad} = \Delta \left(\frac{v^2}{d} \right) = \frac{v_f^2}{d_f} - \frac{v_i^2}{d_i} \text{ m/s}^2$$

where $v_{f,i}$ are the final and initial tangential velocities in m/s and $d_{f,i}$ are the final and initial radial distances in m. Therefore, the change in radial acceleration is

$$a_{rad} = \frac{v_M^2}{r_M} - \frac{v_E^2}{r_E} = \frac{(2\pi \frac{r_M}{T_M})^2}{r_M} - \frac{(2\pi \frac{r_E}{T_E})^2}{r_E} = 4\pi^2 \left(\frac{r_M}{T_M^2} - \frac{r_E}{T_E^2} \right)$$

Plugging in numbers and converting units as needed,

$$a_{rad} = a_{pr} = 4 \cdot \pi^2 \cdot \left(\frac{(227.9 \cdot 10^9 \text{ m})}{(1.881 \text{ yr} \cdot 3.15576 \cdot 10^7 \text{ s/yr})^2} - \frac{(149.6 \cdot 10^9 \text{ m})}{(1 \text{ yr} \cdot 1.881 \text{ yr} \cdot 3.15576 \cdot 10^7 \text{ s/yr})^2} \right)$$

$$a_{pr} = -0.00337647449979061 \text{ m/s}^2$$

Problem 126 - 4.5.4 :

A rotating platform of radius $R = 1.5$ cm starts from rest and accelerates with uniform angular acceleration $\alpha = 12$ rad/s².

Randomized Variables

$$R = 1.5 \text{ cm}$$

$$\alpha = 12 \text{ rad/s}^2$$

Part (a) Write an expression that gives the centripetal acceleration on the edge of the platform as a function of time, $a_c(t)$.

The centripetal acceleration can be found using the relation between the centripetal acceleration, angular velocity, and radius.

$$a_c = \omega^2 R$$

In order to proceed, we need to find an expression for the angular velocity in terms of the angular acceleration and the amount of time that has passed. To find this we use an angular kinematic equation. We know that the platform starts rotating from rest, the value of the angular acceleration of the platform, and that we want to solve for if we are given a value for time. Given this information, we can use the following rotational kinematic equation:

$$\omega = \alpha t$$

Now, let's plug this in to our equation for centripetal acceleration to get an equation in terms of the permitted variables.

$$a_c(t) = (\alpha t)^2 R$$

$$a_c(t) = \alpha^2 t^2 R$$

Part (b) Write an expression for the time at which the magnitudes of the centripetal and tangential accelerations at the edge will be equal.

The tangential acceleration is equal to the angular acceleration multiplied by the radius.

$$a_t = \alpha R$$

We found an expression for the centripetal acceleration in part (a). We can set this result equal to the tangential acceleration and then solve for t .

$$a_c(t) = a_t$$

$$\alpha^2 t^2 R = \alpha R$$

$$t^2 = \frac{1}{\alpha}$$

$$t = \sqrt{\frac{1}{\alpha}}$$

$$t = \frac{1}{\alpha^{0.5}}$$

Part (c) Calculate the time, t in seconds, for when the magnitudes of the centripetal and tangential accelerations are equal.

Since we already found an equation for the time at which the centripetal and tangential accelerations are equal in part (b), all we need to do is substitute values in

$$t = \frac{1}{\alpha^{0.5}}$$

$$t = \frac{1}{(12 \text{ rad/s}^2)^{0.5}}$$

$$t = 0.2887 \text{ s}$$

Problem 127 - 4.5.5 :

Full solution not currently available at this time.

Consider a horizontal spinning disc of radius $R = 3$ m with a ball of mass $m = 2.1$ kg attached with a clamp to the outer edge of the disc. The disc starts at rest, and the ball at the edge of the disk experiences a tangential acceleration of $a_t = 21$ m/s². The size of the ball is negligible.

Part (a) Consider the disc at time $t = 10.1$ s. Calculate the speed v , in meters per second, of the attached ball?

$$\begin{aligned} v &= a \cdot t \\ v &= 21 \cdot 10.1 \\ v &= 212.1 \\ \text{Tolerance: } &\pm 6.363 \end{aligned}$$

Part (b) At this same time $t = 10.1$ s, calculate the instantaneous tension F_t , in newtons, that is required by the clamp to keep the ball attached to the disc

$$\begin{aligned} F_t &= m \cdot (a \cdot t)^2 / R \\ F_t &= 2.1 \cdot (21 \cdot 10.1)^2 / 3 \\ F_t &= 31490.487 \\ \text{Tolerance: } &\pm 944.71461 \end{aligned}$$

Part (c) What is the net force F acting on the ball at $t = 10.1$ s, in newtons?

$$\begin{aligned} F &= m \cdot ((a \cdot t)^2 / R)^2 + (a)^2)^{0.5} \\ F &= 2.1 \cdot (((21 \cdot 10.1)^2 / 3)^2 + (21)^2)^{0.5} \\ F &= 31490.518 \\ \text{Tolerance: } &\pm 944.71554 \end{aligned}$$

Problem 128 - 4.5.7 :

Full solution not currently available at this time.

A particle travels in a circular orbit of radius $r = 50.1$ m. Its speed is changing at a rate of $a_t = 11$ m/s² at an instant when its speed is $v = 30.1$ m/s.

Part (a) Write an expression for the magnitude a of the total acceleration of the particle in terms of the variables from the problem statement.

$$a = (a_t^2 + v^4 / r^2)^{0.5}$$



Part (b) What is the magnitude of the total acceleration of the particle, in m/s^2 .

$$a = \sqrt{(v^2/r)^2 + at^2}$$

$$a = \sqrt{(30.1^2/50.1)^2 + 11^2}$$

$$a = 21.167$$

$$\text{Tolerance: } \pm 0.63501$$

Problem 129 - 4.5.8 :

Full solution not currently available at this time.

The driver of a car moving at 90.0 km/h presses down on the brake as the car enters a circular curve of radius 101 m .

Part (a) If the speed of the car is decreasing at a rate of 5.01 km/h each second, what is the magnitude of the total acceleration of the car, in units of m/s , 50.1 km/h ?

$$a = \sqrt{(at/3.6)^2 + ((v/3.6)^2/(r))^2}$$

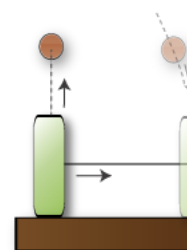
$$a = \sqrt{(5.01/3.6)^2 + ((50.1/3.6)^2/(101))^2}$$

$$a = 2.369$$

$$\text{Tolerance: } \pm 0.07107$$

Problem 130 - c4.6.1 :

A ball is loaded into a cylindrical device that can launch it vertically upward. This device is placed on a table and then a mass is attached to it via pulley. The mass is released and the apparatus begins to accelerate. At some point the ball is launched vertically, with velocity v .



Part (a) Which of the following statements is true regarding where the ball will land?

The horizontal velocity of the ball will always be

$$v_{Bx} = \text{constant}$$

but the velocity of the cylinder will be

$$v_c = v_{Bx} + at$$

and the cylinder is always accelerating. Therefore, the ball falls behind the cylinder.

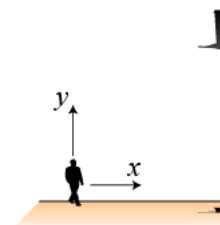
Problem 131 - 4.6.1 :

An airplane's altimeter measures its altitude to increase at a speed of $v_{\text{vertical}} = 10.5 \text{ m/s}$. An observer on the ground sees the plane's shadow moving along the ground at $v_{\text{horizontal}} = 81 \text{ m/s}$ while the sun and plane are directly overhead. Use a standard Cartesian coordinate origin located at the observer's position on the ground, with the plane's horizontal velocity in the x direction.

Randomized Variables

$$v_{\text{vertical}} = 10.5 \text{ m/s}$$

$$v_{\text{horizontal}} = 81 \text{ m/s}$$



Part (a) Express the plane's velocity vector, \mathbf{v} , in component form in terms of \mathbf{i} , \mathbf{j} , v_{vertical} and $v_{\text{horizontal}}$.

The horizontal velocity is in the horizontal direction (\mathbf{i}), and the vertical velocity is in the vertical direction (\mathbf{j}).

$$\mathbf{v} = v_{\text{vertical}}\mathbf{j} + v_{\text{horizontal}}\mathbf{i}$$

Part (b) Calculate the plane's airspeed, v in m/s.

The speed is given by Pythagorean's Theorem.

$$v = (v_{\text{horizontal}}^2 + v_{\text{vertical}}^2)^{0.5}$$

Plugging in numbers and converting units as needed,

$$v = ((81 \text{ m/s})^2 + (10.5 \text{ m/s})^2)^{0.5}$$

$$v = 81.678 \text{ m/s}$$

Part (c) At what angle, θ in degrees, above horizontal is the plane climbing?

The angle by which it is above the horizontal is related to the velocities by the trigonometric relation

$$\tan\theta = \frac{v_{\text{vertical}}}{v_{\text{horizontal}}}$$

$$\theta = \arctan\left(\frac{v_{\text{vertical}}}{v_{\text{horizontal}}}\right)$$

Plugging in numbers and converting units as needed,

$$\theta = \arctan\left(\frac{(10.5 \text{ m/s})}{(81 \text{ m/s})}\right) \cdot \frac{180}{3.14159}^\circ$$

$$\theta = 7.386 \text{ deg}$$

Problem 132 - 4.6.12 :

A ship sets sail from Rotterdam, The Netherlands, intending to head due north at 5.5 m/s relative to the water. However, the local ocean current is 1.50 m/s in a direction 40.0° north of east and changes the ship's intended motion.

Part (a) In what direction would the ship have to travel in order to have a resultant velocity straight north relative to the earth, assuming the speed relative to the water is 5.5 m/s ? Specify the angle west of north, relative to the earth (i.e. A stationary observer on the shore).

This problem involves the concept of relative velocity in two dimensions.

The problem details the case of a ship traveling through water, and we are concerned with the velocity of the ship relative to the Earth, \vec{v}_{SE} , which is the sum of the ship relative to the water \vec{v}_{SW} and the velocity of the water relative to the Earth, \vec{v}_{WE} .

That is,

$$\vec{v}_{SE} = \vec{v}_{SW} + \vec{v}_{WE}$$

If we take $+y$ as north and $+x$ as east, the velocity of the water relative to the Earth is

$$\vec{v}_{WE} = v_{WE} \cos(\theta_{WE}) \mathbf{i} + v_{WE} \sin(\theta_{WE}) \mathbf{j}$$

where we are given the magnitude of this velocity and θ_{WE} in the problem statement.

In this part, we are asked to find the angle, counterclockwise relative to $+y$ (*i. e.*, west of north), required for the ship's velocity relative to the water to produce a along $+y$ only.

Let's call the angle we are asked to find θ . Then, with simple trigonometry, we can express the ship's velocity relative to the water as

$$\vec{v}_{SW} = -v_{SW} \sin(\theta) \mathbf{i} + v_{SW} \cos(\theta) \mathbf{j}$$

Then,

$$\vec{v}_{SE} = \vec{v}_{SW} + \vec{v}_{WE} = (-v_{SW} \sin(\theta) \mathbf{i} + v_{SW} \cos(\theta) \mathbf{j}) + (v_{WE} \cos(\theta_{WE}) \mathbf{i} + v_{WE} \sin(\theta_{WE}) \mathbf{j}) = (-v_{SW} \sin(\theta) + v_{WE} \cos(\theta_{WE})) \mathbf{i} + (v_{SW} \cos(\theta) + v_{WE} \sin(\theta_{WE})) \mathbf{j}$$

With this, it is easy to see that to produce a \vec{v}_{SE} that has no component along \vec{x} , the following must be true

$$-v_{SW} \sin(\theta) + v_{WE} \cos(\theta_{WE}) = 0$$

Rearranging to solve for θ' , we have

$$\theta = \sin^{-1} \left(\left(\frac{v_{WE}}{v_{SW}} \right) \cos(\theta_{WE}) \right)$$

With our known values,

$$\theta = \sin^{-1} \left(\left(\frac{1.50 \frac{\text{m}}{\text{s}}}{5.5 \frac{\text{m}}{\text{s}}} \right) \cos(40^\circ) \right)$$

$\theta = 12.059^\circ$ west of north

Part (b) What would the ship's speed be relative to the Earth, in meters per second?

In this part, we're asked to find the magnitude of the velocity of the ship relative to the Earth.

Because this part is a continuation of *Part (a)*, the velocity has no x component,

$$\vec{v}_{SE} = \vec{v}_{SW} + \vec{v}_{WE}$$

$$\vec{v}_{SE} = (v_{SW} \cos(\theta) + v_{WE} \sin(\theta_{WE})) \mathbf{j}$$

Hence, the magnitude of \vec{v}_{SE} is simply

$$v_{SE} = v_{SW} \cos(\theta) + v_{WE} \sin(\theta_{WE})$$

Substituting our result for θ from part a,

$$v_{SE} = v_{SW} \cos\left(\sin^{-1}\left(\left(\frac{v_{WE}}{v_{SW}}\right) \cos(\theta_{WE})\right)\right) + v_{WE} \sin(\theta_{WE}) = \left(5.5 \frac{\text{m}}{\text{s}}\right) \cos\left(\sin^{-1}\left[\left(\frac{1.50 \frac{\text{m}}{\text{s}}}{5.5 \frac{\text{m}}{\text{s}}}\right) \cos\left(40^\circ\left(\frac{\pi}{180^\circ}\right)\right)\right]\right) + \left(1.50 \frac{\text{m}}{\text{s}}\right) \sin\left(40^\circ\right)$$

$$v_{SE} = 6.343 \frac{\text{m}}{\text{s}}$$

Problem 133 - 4.6.13 :

A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship: (a) relative to the ship and (b) relative to a stationary observer on shore. (c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

Solution (a) $v_y^2 = -2(gy - y_0) \Rightarrow v_y = \pm(-2gy)^{1/2}$
 $= -\left[2(9.80 \text{ m/s}^2)(-15.0 \text{ m})\right]^{1/2} = -17.15 \text{ m/s}$
 $v_y = \underline{17.1 \text{ m straight down}}$

(b) $v = (v_x^2 + v_y^2)^{1/2} = \left[(1.75 \text{ m/s})^2 + (-17.15 \text{ m/s})^2\right]^{1/2} = \underline{17.2 \text{ m/s}}$

$$\theta = \tan^{-1} \frac{|v_y|}{|v_x|} = \underline{84.2^\circ \text{ below horizontal and to the south.}}$$

(c) The sandal hits the ship going straight down (according to the ship), but the ship is moving south, so the observer on the shore sees the sandal moving mainly down,

but also a bit to the south.

Problem 134 - c5.1.1(v2) :

Given Newton's First Law of Motion, what do we reasonably expect an object to do given the following scenarios?

Part (a) An object sits at rest with no unbalanced forces acting upon it. What do we expect this object to do?

According to Newton's First Law of Motion, the object will remain at rest.

Part (b) An object is traveling with a constant velocity with no unbalanced forces acting upon it. What do we expect this object to do?

According to Newton's First Law of Motion, the object will continue traveling in a straight line (i.e., in the same direction) at the same speed.

Part (c) An object sits at rest with an unbalanced force acting on it. What do we expect this object to do?

According to Newton's First Law of Motion, the object will not remain at rest. According to Newton's Second Law of Motion, as long as the same unbalanced force on the object will move with increasing velocity.

Problem 135 - c5.1.1 :

Given Newton's First Law of Motion, what do we reasonably expect an object to do given the following scenarios?

Part (a) An object sits at rest with no unbalanced forces acting upon it. What do we expect this object to do?

According to Newton's First Law of Motion, the object will remain at rest.

Part (b) An object is traveling with a constant velocity with no unbalanced forces acting upon it. What do we expect this object to do?

According to Newton's First Law of Motion, the object will continue traveling in a straight line (i.e., in the same direction) at the same speed.

Part (c) An object sits at rest with an unbalanced force acting upon it. What should we not expect this object to do? Asking another way, which of the following is the correct outcome?

According to Newton's First Law of Motion, the object will not remain at rest. Since we don't expect the object to remain at rest, the correct choice to answer the question is "The object will not remain at rest."

Problem 136 - c5.1.2 :

Full solution not currently available at this time.

For this problem, assume that Earth is an inertial reference frame. (Strictly speaking, Earth cannot be an inertial reference frame because of its rotation and movement in its orbit about the Sun, but for most purposes it can be thought of as an inertial reference frame.)

Part (a) Which of the following objects are also inertial reference frames? Check all that apply.

A car moving in a straight line at constant speed , An elevator moving upward at constant speed

Problem 137 - c5.1.3 :

Full solution not currently available at this time.

A woman has just left home for her morning commute and has forgotten that her coffee mug is on the roof of her car. At one point, as she is driving at a constant speed of 35 km/h, she sees a squirrel running in front of the car. She slams on the brakes and the coffee mug slides across the car's roof and falls onto the windshield and hood. An observer standing on the roadside witnesses this event. For this problem, consider the motion of the coffee mug from the moment the brakes are touched to just before it falls off the roof. (Assume that it slides across the roof with little to no friction.)

Part (a) In whose reference frame does the coffee mug accelerate forward?

The driver's

Part (b) In whose reference frame does the coffee mug move at constant velocity?

The roadside observer's

Problem 138 - c5.1.4 :

Full solution not currently available at this time.

A rock is thrown straight upward.

Part (a) At which of the following points is the net force on the rock equal to zero?

At no point in the rock's trajectory is the net force equal to zero.

Problem 139 - 5.1.1 :

Full solution not currently available at this time.

Two forces of $\vec{F}_1 = 6 \frac{(\hat{i} - \hat{j})}{\sqrt{2}}$ N and $\vec{F}_2 = 101 \frac{(\hat{i} - \hat{j})}{\sqrt{2}}$ N act on an object.

Part (a) What is the resultant force in terms of the force vectors

\vec{F}_1 and
 \vec{F}_2 ?

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$$

Part (b) What is the magnitude of an equal and opposite force, F_3 , which balances the first two forces?

$$F_3 = F_2 + F_1$$

$$F_3 = 101 + 6$$

$$F_3 = 107$$

$$\text{Tolerance: } \pm 3.21$$

Part (c) Given your observations from parts a and b, if working with vectors in 3 dimensions

$(\hat{i}, \hat{j}, \hat{k})$; what will the normalizing term n be for the two vectors

$$\vec{F}_1 = c_1 \frac{(\hat{i} - \hat{j} + \hat{k})}{\sqrt{n}} \text{ N and}$$

$$\vec{F}_2 = c_2 \frac{(\hat{i} - \hat{j} + \hat{k})}{\sqrt{n}} \text{ N.}$$

$$n = 3$$

$$\text{Tolerance: } \pm 0.09$$

Part (d) Given the normalizing term n found in the previous part in 3 dimensions

$(\hat{i}, \hat{j}, \hat{k})$; what will the magnitude of the resultant vector

$|\vec{F}_{\text{res}}|$ be in terms of the constants c_1 and c_2 .

The vectors in question are

$$\vec{F}_1 = c_1 \frac{(\hat{i} - \hat{j} + \hat{k})}{\sqrt{n}} \text{ N and}$$

$$\vec{F}_2 = c_2 \frac{(\hat{i} - \hat{j} + \hat{k})}{\sqrt{n}}.$$

$$|\vec{F}_{\text{res}}| = c_1 + c_2$$

Problem 140 - 5.1.2 :

Full solution not currently available at this time.

While sliding a couch across a floor, Hannah and Andrea exerts forces \vec{F}_H and \vec{F}_A on the couch. Hannah's force is due north with a magnitude of $F_H = 11$ N and Andrea's force is $\theta = 22^\circ$ east of north with a magnitude of $F_A = 101$ N.

Part (a) Find the net force in the y-direction in Newtons.

$$\Sigma F_y = F_H + F_A \cdot \cos(\theta * \pi/180)$$

$$\Sigma F_y = 11 + 101 \cdot \cos(22 * \pi/180)$$

$$\Sigma F_y = 104.646$$

Tolerance: ± 3.13938

Part (b) Find the net force in the x-direction in Newtons.

$$\Sigma F_x = F_a \cdot \sin(\theta \cdot \pi/180)$$

$$\Sigma F_x = 101 \cdot \sin(22 \cdot \pi/180)$$

$$\Sigma F_x = 37.835$$

Tolerance: ± 1.13505

Part (c) Calculate the angle in degrees north of east of the net force exerted on the couch by Hannah and Andrea,

\vec{F}_{HA} .

$$\theta = \text{atan}((F_h + F_a \cdot \cos(\theta \cdot \pi/180))/(F_a \cdot \sin(\theta \cdot \pi/180))) \cdot 180/\pi$$

$$\theta = \text{atan}((11 + 101 \cdot \cos(22 \cdot \pi/180))/(101 \cdot \sin(22 \cdot \pi/180))) \cdot 180/\pi$$

$$\theta = 70.122$$

Tolerance: ± 2.10366

Part (d) Hannah and Andrea's housemates, David and Stephanie disagree with the move and want to prevent its relocation. Their combined force

\vec{F}_{DS} must be equal and opposite to that of

\vec{F}_{HA} . What is the magnitude in Newtons of the force

\vec{F}_{DS} which will prevent the relocation?

$$F_{DS} = ((F_a \cdot \sin(\theta \cdot \pi/180))^2 + (F_h + F_a \cdot \cos(\theta \cdot \pi/180))^2)^{0.5}$$

$$F_{DS} = ((101 \cdot \sin(22 \cdot \pi/180))^2 + (11 + 101 \cdot \cos(22 \cdot \pi/180))^2)^{0.5}$$

$$F_{DS} = 111.275$$

Tolerance: ± 3.33825

Problem 141 - 5.2.2 :

Full solution not currently available at this time.

A fireman has mass m ; he hears the fire alarm and slides down the pole with acceleration a (which is less than g in magnitude).

Part (a) Using the variables in the problem statement, write an equation giving the vertical force F resisting the force of gravity on the firefighter.

$$F = m g - m a$$

Part (b) If his mass $m = 50.1$ -kg and he accelerates at $a = 2.1$

$\frac{m}{s^2}$, what is the magnitude of the force of resistance in Newtons?

$$F = m \cdot 9.81 - m \cdot a$$

$$F = 50.1 \cdot 9.81 - 50.1 \cdot 2.1$$

$$F = 386.271$$

Tolerance: ± 11.58813

Problem 142 - 5.2.3 :

Full solution not currently available at this time.

A baseball catcher is performing a stunt for a television commercial. He will catch a baseball with mass $m = 100.1$ g, dropped from a height $\Delta x = 10.1$ m above his glove. The glove brings the ball to a complete stop in time $t_{\text{glove}} = 0.02$ seconds after the ball hits the glove.

Part (a) Use your understanding of kinematic equations to write a formula for the time that the ball is falling before it hits the glove, t_{flight} . Use variables statement alongside g for acceleration due to gravity to write your equation.

$$t_{\text{flight}} = (2 \Delta x/g)^{0.5}$$

Part (b) What is the amount of time t_{flight} in seconds the ball takes to hit the glove after it is dropped?

$$t_{\text{flight}} = (x \cdot 2/9.81)^{0.5}$$

$$t_{\text{flight}} = (10.1 \cdot 2/9.81)^{0.5}$$

$$t_{\text{flight}} = 1.435$$

Tolerance: ± 0.04305

Part (c) Write an equation for the speed of the ball just before it hits the glove, v_f . Use g for the acceleration due to gravity and t_{flight} for the time it takes after it is dropped.

$$v_f = g t_{\text{flight}}$$

Part (d) Find the speed of the ball v_g at the moment it first touches the glove.

$$v_g = (2 * 9.81 * x)^{0.5}$$

$$v_g = (2 * 9.81 * 10.1)^{0.5}$$

$$v_g = 14.077$$

Tolerance: ± 0.42231

Part (e) Considering the amount of time t_{glove} it took for the glove to stop the ball, find the magnitude of the net force on the ball in Newtons while it is in

$$F_{net} = (2 * 9.81 * x)^{0.5} / t * m / 1000$$

$$F_{net} = (2 * 9.81 * 10.1)^{0.5} / 0.02 * 100.1 / 1000$$

$$F_{net} = 70.455$$

Tolerance: ± 2.11365

Problem 143 - 5.2.4 :

Full solution not currently available at this time.

When the Moon is directly overhead at sunset, the force by the Earth on the Moon, F_{EM} , is essentially 90° to the force by the sun on the moon, F_{SM} , as depicted in the image. Let $F_{EM} = 1.01 \times 10^{20}$ N, $F_{SM} = 4.01 \times 10^{20}$ N, all other forces on the Moon be negligible, and the mass of the Moon be $m = 7.35 \times 10^{22}$ kg.



Part (a) What is the magnitude of the net force exerted by the Earth and Sun on the Moon in Newtons?

$$F_{net} = (F_{em}^2 + F_{sm}^2)^{0.5} * 10^{20}$$

$$F_{net} = (1.01^2 + 4.01^2)^{0.5} * 10^{20}$$

$$F_{net} = 4.13523880809803E+20$$

Tolerance: $\pm 1.24057164242941E+19$

Part (b) What is the magnitude of the Moon's acceleration in

$$\frac{m}{s^2} ?$$

$$a = (F_{em}^2 + F_{sm}^2)^{0.5} * 10^{20} / (7.35 * 10^{22})$$

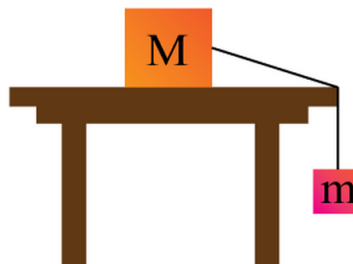
$$a = (1.01^2 + 4.01^2)^{0.5} * 10^{20} / (7.35 * 10^{22})$$

$$a = 0.005626$$

Tolerance: ± 0.00016878

Problem 144 - c5.3.1 :

Two blocks of unequal mass are tied together with a massless string that does not stretch. The blocks are placed on a frictionless table top and then one of the blocks is pushed over the edge.

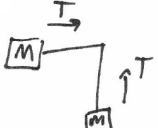


Part (a) The blocks accelerate at the same rate since they are connected. What is the acceleration? (Choose the correct answer.)

MultipleChoice :

- 1) A value between zero and g .
- 2) Cannot be Determined.
- 3) A value greater than g .
- 4) g
- 5) $g/2$
- 6) Zero

To do this carefully:



$$mg - T = ma$$

$$T = Ma$$

$$\rightarrow mg - Ma = ma$$

$$mg = (M+m)a$$

$$a = \frac{m}{M+m}g$$

For $m \ll M$: $\frac{m}{m+M} = \frac{m}{M} \left(\frac{1}{1 + \frac{m}{M}} \right) \approx \frac{m}{M} \left(1 - \frac{m}{M} \right)$

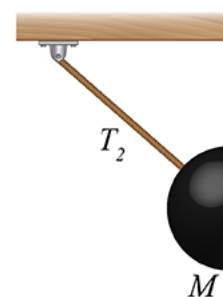
so $a \approx \frac{m}{M}g$, $a < g$ (if $m=0$, $a=0$)

$m \gg M$: $\frac{m}{m+M} = \frac{1}{1 + \frac{M}{m}} \approx \left(1 - \frac{M}{m} \right)$, $a \approx g$

so $0 < a < g$

Problem 145 - c5.3.2 :

Consider a bowling ball of mass M attached to two ropes. The bowling ball is in equilibrium, with one rope tied to the ceiling and the second rope pulling horizontally. Refer to the figure on the right.

**Part (a) How is the tension T_2 related to the weight of the bowling ball?**

The ball is being pulled to the right. Whether it is at rest in equilibrium or moving to the right and up as a result of the pulling, the magnitude of the vertical component of T_2 must be equal to the weight of the bowling ball, due to Newton's second law. In addition, the magnitude of the horizontal component of T_2 is greater than zero, because T_2 is from being freely pulled to the right. Therefore, the magnitude of T_2 must be greater than the ball's weight. The correct choice is " T_2 is greater than the object's weight."

Problem 146 - c5.3.3 :

Consider the block shown in the figure, which has a mass m and is sitting at rest on a ramp making an angle of θ with respect to the floor.

**Part (a) How big is the normal force acting on the block due to the ramp?**

Since the block is in equilibrium in the direction perpendicular to the ramp, the normal force of the ramp on the block must equal the component of the block's weight perpendicular to the ramp, according to Newton's second law. The magnitude of that component is $mg \cos(\theta)$. Thus, assuming θ is less than 90° , the ramp's normal force on the block is less than the block's weight.

Problem 147 - c5.3.4 :

A ball of mass m is tossed straight up and reaches a height of 15 ft.

Part (a) At the top, 15 ft up, what is the magnitude of the net force on the ball?

The only force acting on the ball is the force of gravity, so

$$F_{net} = mg$$

Problem 148 - c5.3.5 :

Consider a jet plane, which is flying in a general direction under its own power.

Part (a) If we break the acceleration of this jet into two components, in the vertical and horizontal direction, what statement correctly characterizes the s

Newton's second law tells us that acceleration is caused by forces acting on the accelerating object, added together as vectors. Because the law is a vector relation acting in the horizontal direction to horizontal acceleration. Similarly, vertical forces can only contribute to causing vertical acceleration. The force of gravity on the earth (vertically). However, a plane's propulsion system (it's thrust) acts in whatever direction the plane is facing. If the plane is angled upward, the thrust will upward, as well as horizontally.

Problem 149 - c5.3.6 :

Full solution not currently available at this time.

Consider applying Newton's second law to a man riding an elevator (perhaps to calculate his acceleration).

Part (a) Which of the following forces must be considered? Choose all that apply.

The force that the elevator floor exerts on the man's feet , The force of gravity on the man

Problem 150 - c5.3.7 :

Full solution not currently available at this time.

A particle is moving to the right.

Part (a) Is it possible that the net force on the particle is directed to the left?

Yes

Part (b) Is it possible that the net force on the particle is directed downward (perpendicular to the particle's velocity)?

Yes

Part (c) In general, the direction of the net force on a particle is always the same as the direction of its velocity.

FALSE

Part (d) In general, the direction of the net force on a particle is always the same as the direction of its acceleration.

TRUE

Part (e) In general, acceleration and velocity are necessarily in the same direction.

FALSE

Problem 151 - 5.3.1 (alt) :

A boxer's fist and glove have a mass of $m = 0.82$ kg. The boxer's fist can obtain a speed of $v = 6.5$ m/s in a time of $t = 0.16$ s.

Randomized Variables

$m = 0.82$ kg
 $v = 6.5$ m/s
 $t = 0.16$ s

Part (a) Write a symbolic expression for the magnitude of the average acceleration, a_{ave} , of the boxer's fist, in terms of the variables provided.

Expression :

$a_{\text{ave}} =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \theta, a, d, g, h, i, j, k, l, m, P, t, v$

Part (b) Find the magnitude of the average acceleration, a_{ave} , in meters per square second.

Numeric : A numeric value is expected and not an expression.

$a_{\text{ave}} =$ _____

Part (c) Write an expression for the magnitude of the average net force, F_b , that the boxer must apply to his fist to achieve the given velocity.

(Write the expression in terms of m, v and t .)

Expression :

$F_b =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \theta, a, d, g, h, i, j, k, l, m, P, t, v$

Part (d) What is the numerical value of F_b , in newtons?

Numeric : A numeric value is expected and not an expression.

$F_b =$ _____

a) average acceleration $\vec{a}_m = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} \rightarrow |\vec{a}_{av}| = \frac{v}{t}$

b) $a \approx 40.6 \text{ m/s}^2$

c) Net force $\vec{F}_b = m\vec{a} \rightarrow F_b = m \frac{v}{t}$

d) $F_b \approx 33.3 \text{ N}$

Problem 152 - 5.3.1 :

A boxer's fist and glove have a mass of $m = 0.72$ kg. The boxer's fist can obtain a speed of $v = 5.25$ m/s in a time of $t = 0.11$ s.

Randomized Variables

$m = 0.72$ kg
 $v = 5.25$ m/s
 $t = 0.11$ s

Part (a) Write a symbolic expression for the magnitude of the average acceleration, a_{ave} , of the boxer's fist, in terms of the variables provided.

The average acceleration is given by the change in velocity divided by the change in time.

$$a_{\text{ave}} = \frac{v}{t}$$

Part (b) Find the magnitude of the average acceleration, a_{ave} , in meters per square second.

Here, we need to plug in variables to the equation we found in part (a).

$$a_{\text{ave}} = \frac{v}{t}$$

$$a_{ave} = \frac{5.25 \text{ m/s}}{0.11 \text{ s}}$$

$$a_{ave} = 47.727 \text{ m/s}^2$$

Part (c) Write an expression for the magnitude of the average net force, F_b , that the boxer must apply to his fist to achieve the given velocity. (Write the expression in terms of m and t .)

Force is given by Newton's Second law as:

$$F = ma$$

We found an expression for the average acceleration in part (a). Plugging that into the above equation gives us an expression for the average force.

$$F_b = m \left(\frac{v}{t} \right)$$

Part (d) What is the numerical value of F_b , in newtons?

We found an expression for the average force the boxer must exert in part (c). To complete this problem, we just need to substitute values into that equation and solve.

$$F_b = m \left(\frac{v}{t} \right)$$

$$F_b = 0.72 \text{ kg} \cdot \frac{5.25 \text{ m/s}}{0.11 \text{ s}}$$

$$F_b = 34.364 \text{ N}$$

Problem 153 - 5.3.2 :

A high-performance dragster with a mass of $m = 1001$ kg can accelerate at a rate of $a = 20.5$ m/s².

Randomized Variables

$m = 1001$ kg
 $a = 20.5$ m/s²

Part (a) Write an expression for the magnitude of the net force, F_{NET} , that propels the dragster forward in terms of the variables provided.

Newton's Second Law tells us the relation between force, mass, and acceleration.

$$F_{NET} = ma$$

Part (b) If the track has length L and the dragster starts from rest, select the correct symbolic equation for how fast v_f the dragster is traveling when it finishes the track that it accelerates at the same rate along the full length of the track.)

We will need to use a kinematic equation to solve for the final velocity. In this case, we know that the dragster starts from rest, that it travels a distance L , and that it accelerates at a . With this information, we can set up the following kinematic equation:

$$V_f^2 = V_i^2 + 2aL$$

$$V_f^2 = 0^2 + 2aL$$

$$V_f^2 = 2aL$$

$$V_f = \sqrt{2aL}$$

Part (c) If the track is $L=400$ m long, what is the numerical value of the dragster's final speed, v_f in m/s?

Here, we need to plug values into the equation we found in part (b) and solve for the final velocity.

$$V_f = \sqrt{2aL}$$

$$V_f = \sqrt{2 \cdot 20.5 \text{ m/s}^2 \cdot 400 \text{ m}}$$

$$V_f = 128.062 \text{ m/s}$$

Problem 154 - 5.3.3 :

A bullet with a mass of $m = 10.5$ g is shot out of a rifle that has length $L = 0.72$ m. The bullet spends $t = 0.11$ s in the barrel.

Part (a) Write an expression, in terms of the given quantities, for the magnitude of the bullet's acceleration, a , as it travels through the rifle's barrel. You acceleration is constant throughout the motion.

To find the bullet's acceleration, we will need to use a kinematic equation. We know that the bullet has no initial velocity, we know the length of the rifle's barrel, the bullet takes to travel through the barrel. With this information, we can set up the following kinematic equation.

$$d = vt + \frac{1}{2}at^2$$

Now, let's rewrite this in terms of the variables for this problem and adjust it to isolate the acceleration on the left side.

$$L = 0 \cdot t + \frac{1}{2}at^2$$

$$L = \frac{1}{2}at^2$$

$$\frac{2L}{t^2} = a$$

$$a = \frac{2L}{t^2}$$

Part (b) Calculate the numerical value for the magnitude of the bullet's acceleration, a in m/s^2 .

We already found an equation for the velocity in part (a). To find the solution, we simply need to plug values into our previous equation and solve it.

$$a = \frac{2L}{t^2}$$

$$a = \frac{2 \cdot 0.72 \text{ m}}{(0.11 \text{ s})^2}$$

$$a = 119.008 \text{ m/s}^2$$

Part (c) What is the numerical value of the net force F_{NET} in newtons acting on the bullet?

Newton's Second Law tells us that the force acting on an object is given by the equation:

$$F = ma$$

Since we found the acceleration previously, we can plug values into this equation to find the net force. In so doing, we need to be careful to convert the mass from order to get a correct answer.

$$F_{\text{NET}} = m \cdot \frac{2L}{t^2}$$

$$F_{\text{NET}} = 10.5 \cdot 10^{-3} \text{ kg} \cdot \frac{2 \cdot 0.72 \text{ m}}{(0.11 \text{ s})^2}$$

$$F_{\text{NET}} = 1.25 \text{ N}$$

Problem 155 - 5.3.3 (alt) :

A bullet with a mass of $m = 10.5 \text{ g}$ is shot out of a rifle that has length $L = 0.72 \text{ m}$. The bullet spends $t = 0.11 \text{ s}$ in the barrel.

Part (a) Calculate the magnitude of the bullet's acceleration, in meters per second squared, as it travels through the rifle's barrel. You may assume the acceleration is constant throughout the motion.

We have the distance the bullet traveled, the time it traveled, its initial velocity (from rest), and its initial displacement (0 m). Using the kinematic equation for displacement:

$$y = \left(\frac{1}{2}at^2 + v_0t + y_0 \right) \text{ m}$$

where a is the acceleration in m/s^2 , t is the time in s, v_0 is the initial velocity in m/s , and y_0 is the initial displacement. Therefore,

$$y = \frac{1}{2}at^2 + 0 \cdot t + 0 = \frac{1}{2}at^2$$

Solving for the acceleration and converting units as needed,

$$a = \frac{2y}{t^2} = \frac{2L}{t^2} = \frac{(2 \cdot 0.72 \text{ m})}{(0.11 \text{ s})^2}$$

$$a = 119.008 \text{ m/s}^2$$

Part (b) What is the numerical value of the net force F_{NET} in newtons acting on the bullet?

The net force is related to the acceleration by the expression

$$F_{\text{NET}} = ma \text{ N}$$

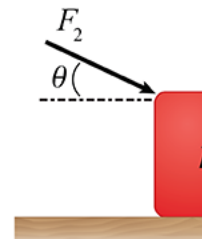
where m is the mass in kg and a is the acceleration in m/s^2 . Substituting in for both, plugging in numbers, and converting units if needed,

$$F_{NET} = ma = m \cdot 2 \cdot \frac{L}{t^2} = \frac{10.5}{1000} \text{ kg} \cdot \frac{(2 \cdot 0.72 \text{ m})}{(0.11 \text{ s})^2}$$

$$F_{NET} = 1.25 \text{ N}$$

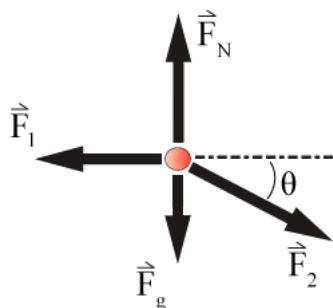
Problem 156 - 5.3.5 :

A block with a mass of $m = 11$ kg rests on a frictionless surface and is subject to two forces acting on it. The first force is directed in the negative x -direction with a magnitude of $F_1 = 8.5$ N. The second has a magnitude of $F_2 = 18$ N and acts on the body at an angle $\theta = 11^\circ$ measured from horizontal, as shown.



Part (a) Please select the correct free body diagram from the choices below.

The correct free-body diagram will need to include both of the forces that are described in the problem statement. Further, the correct diagram must account for the force of gravity on the block. This eliminates all but two of the possible choices. Looking closely at the two choices, it becomes apparent that one of them has the correct answer is therefore the one with all four of the forces placed as well as having θ labeled correctly.



Part (b) Write an expression for the component of net force, $F_{net,x}$, in the x -direction, in terms of the variables given in the problem statement.

The net force will be given by the x -component of F_2 minus F_1 .

$$F_{net,x} = F_2 \cos(\theta) - F_1$$

Part (c) Write an expression for the magnitude of the normal force, F_N , acting on the block, in terms of F_2 and the other variables of the problem. Assume the block is rigid.

The magnitude of the normal force must be equal to the total downward force on the block. As such, the normal force will be equal to the y -component of F_2 plus the weight of the block.

$$F_N = F_2 \sin(\theta) + F_g$$

$$F_N = F_2 \sin(\theta) + mg$$

Part (d) Find the block's acceleration in the x-direction, a_x , in meters per second squared.

Newton's Second Law gives us the following relation between force, mass, and acceleration:

$$F = ma$$

$$\frac{F}{m} = a$$

As we found the force in the x-direction in part (b), we can plug in values and solve for the acceleration.

$$a = \frac{F_{net,x}}{m}$$

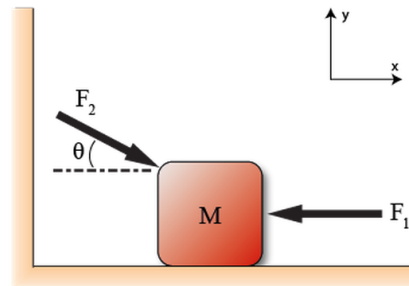
$$a = \frac{F_2 \cos(\theta) - F_1}{m}$$

$$a = \frac{18 \text{ N} \cdot \cos(11^\circ) - 8.5 \text{ N}}{11 \text{ kg}}$$

$$a = 0.8336 \text{ m/s}^2$$

Problem 157 - 5.3.5(iFBD) :

A block with a mass of $m = 30$ kg rests on a frictionless surface and is subject to two forces acting on it. The first force is directed in the negative x -direction with a magnitude of $F_1 = 10$ N. The second has a magnitude of $F_2 = 21$ N and acts on the body at an angle $\theta = 12^\circ$ up from the horizontal as shown.

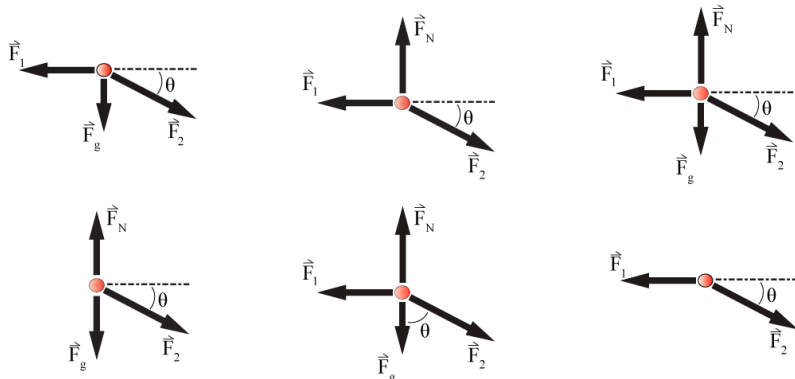


Randomized Variables

- $m = 30$ kg
- $F_1 = 10$ N
- $F_2 = 21$ N
- $\theta = 12^\circ$

Part (a) Please select the correct free body diagram from the choices below.

SchematicChoice :



Part (b) Write an expression for the component of net force, $F_{\text{net},x}$, in the x -direction, in terms of the variables given in the problem statement.

Expression :

$F_{\text{net},x} =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , θ , d , F_1 , F_2 , g , m , t

Part (c) Write an expression for the magnitude of the normal force, F_N , acting on the block, in terms of F_2 and the other variables of the problem. Assume that the surface it rests on is rigid.

Expression :

$F_N =$ _____

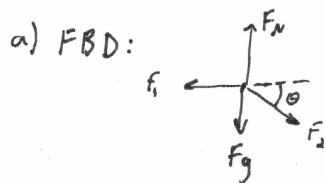
Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , θ , d , F_1 , F_2 , g , m , t

Part (d) Find the block's acceleration in the x -direction, a_x , in meters per square second.

Numeric : A numeric value is expected and not an expression.

$a_x =$



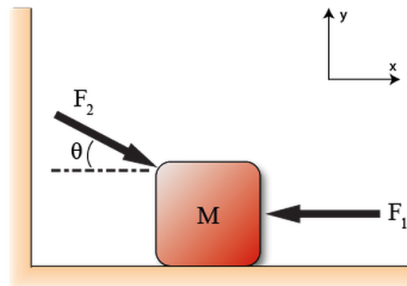
b) $\sum F_x = F_{2x} + F_1 = F_2 \cos \theta - F_1$, $F_{\text{net},x} = F_2 \cos \theta - F_1$

c) $\sum \vec{F}_y = \vec{F}_g + \vec{F}_N + \vec{F}_{2y} \rightarrow F_{\text{net},y} = -mg + F_N - F_2 \sin \theta = 0$
 $F_N = F_2 \sin \theta + mg$

d) $F_{\text{net},x} = ma_x \rightarrow F_2 \cos \theta - F_1 = ma_x$, $a_x = \frac{F_2 \cos \theta - F_1}{m}$ $a_x \approx 0.35 \text{ m/s}^2$

Problem 158 - 5.3.5 (alt) :

A block with a mass of $m = 30$ kg rests on a frictionless surface and is subject to two forces acting on it. The first force is directed in the negative x -direction with a magnitude of $F_1 = 10$ N. The second has a magnitude of $F_2 = 21$ N and acts on the body at an angle $\theta = 12^\circ$ up from the horizontal as shown.

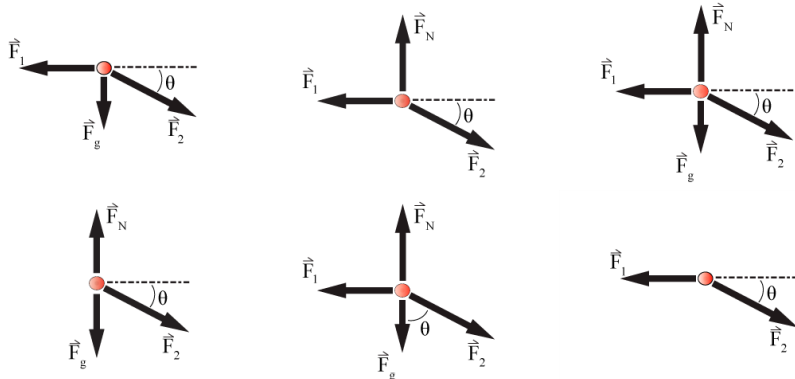


Randomized Variables

- $m = 30$ kg
- $F_1 = 10$ N
- $F_2 = 21$ N
- $\theta = 12^\circ$

Part (a) Please select the correct free body diagram from the choices below.

SchematicChoice :



Part (b) Write an expression for the component of net force, $F_{\text{net},x}$, in the x -direction, in terms of the variables given in the problem statement.

Expression :

$F_{\text{net},x} =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , θ , d , F_1 , F_2 , g , m , t

Part (c) Write an expression for the magnitude of the normal force, F_N , acting on the block, in terms of F_2 and the other variables of the problem. Assume that the surface it rests on is rigid.

Expression :

$F_N =$ _____

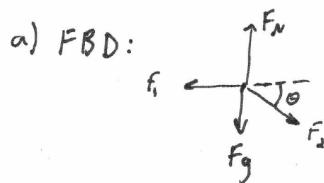
Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , θ , d , F_1 , F_2 , g , m , t

Part (d) Find the block's acceleration in the x -direction, a_x , in meters per square second.

Numeric : A numeric value is expected and not an expression.

$a_x =$



b) $\sum F_x = F_{2x} + F_1 = F_2 \cos \theta - F_1$, $F_{\text{net},x} = F_2 \cos \theta - F_1$

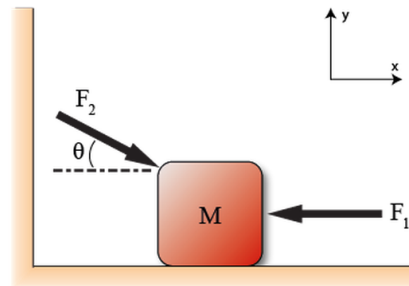
c) $\sum \vec{F}_y = \vec{F}_g + \vec{F}_N + \vec{F}_{2y} \rightarrow F_{\text{net},y} = -mg + F_N - F_2 \sin \theta = 0$

$F_N = F_2 \sin \theta + mg$

d) $F_{\text{net},x} = ma_x \rightarrow F_2 \cos \theta - F_1 = ma_x$, $a_x = \frac{F_2 \cos \theta - F_1}{m}$ $a_x \approx 0.35 \text{ m/s}^2$

Problem 159 - 5.3.5 (alt2) :

A block with a mass of $m = 30$ kg rests on a frictionless surface and is subject to two forces acting on it. The first force is directed in the negative x -direction with a magnitude of $F_1 = 10$ N. The second has a magnitude of $F_2 = 21$ N and acts on the body at an angle $\theta = 12^\circ$ up from the horizontal as shown.

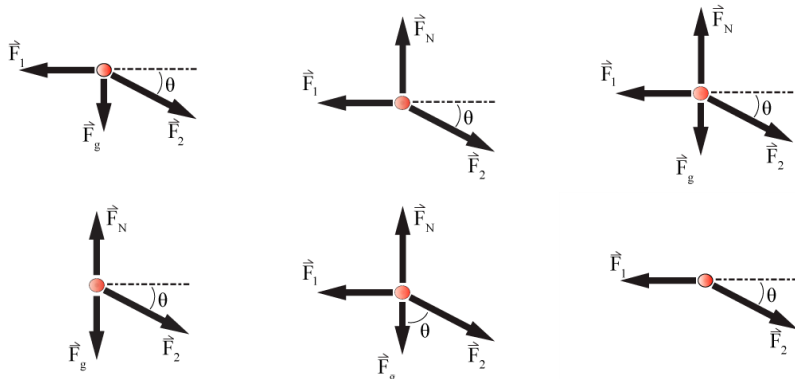


Randomized Variables

- $m = 30$ kg
- $F_1 = 10$ N
- $F_2 = 21$ N
- $\theta = 12^\circ$

Part (a) Please select the correct free body diagram from the choices below.

SchematicChoice :



Part (b) Write an expression for the component of net force, $F_{\text{net},x}$, in the x -direction, in terms of the variables given in the problem statement.

Expression :

$F_{\text{net},x} =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , θ , d , F_1 , F_2 , g , m , t

Part (c) Write an expression for the magnitude of the normal force, F_N , acting on the block, in terms of F_2 and the other variables of the problem. Assume that the surface it rests on is rigid.

Expression :

$F_N =$ _____

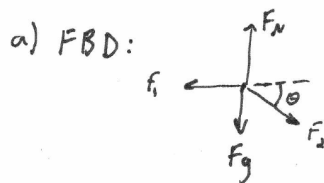
Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , θ , d , F_1 , F_2 , g , m , t

Part (d) Find the block's acceleration in the x -direction, a_x , in meters per square second.

Numeric : A numeric value is expected and not an expression.

$a_x =$



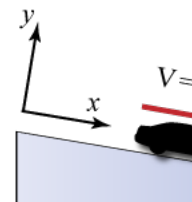
b) $\sum F_x = F_{2,x} + F_1 = F_2 \cos \theta - F_1$, $F_{\text{net},x} = F_2 \cos \theta - F_1$

c) $\sum \vec{F}_y = \vec{F}_g + \vec{F}_N + \vec{F}_{2,y} \rightarrow F_{\text{net},y} = -mg + F_N - F_2 \sin \theta = 0$
 $F_N = F_2 \sin \theta + mg$

d) $F_{\text{net},x} = ma_x \rightarrow F_2 \cos \theta - F_1 = ma_x$, $a_x = \frac{F_2 \cos \theta - F_1}{m}$ $a_x \approx 0.35 \text{ m/s}^2$

Problem 160 - 5.3.6 :

A toy car rolls down a ramp at a constant velocity. The car's mass is $m = 1.1$ kg and the ramp makes an angle of $\theta = 11$ degrees with respect to the horizontal. Assume the rolling resistance is negligible.



Part (a) What is the magnitude of the car's acceleration, a in m/s^2 ?

The problem statement notes that the toy car moves down the ramp at a constant velocity. As a consequence, there must be no acceleration.

$$a = 0$$

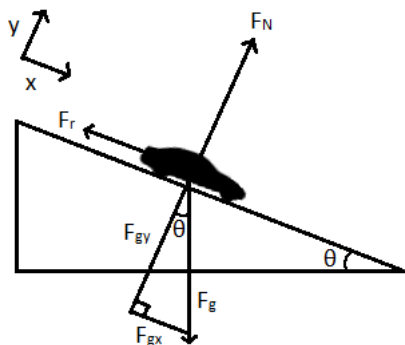
Part (b) What is the numeric value for the sum of the forces in the x -direction, ΣF_x , in Newtons?

In part (a), we noted that the car has no net acceleration since it is moving at a constant velocity. This means that the forces on the car must be balanced in order to satisfy Newton's First Law. As such, there must not be any net force on the car in the x -direction.

$$\Sigma F_x = 0$$

Part (c) Assuming the car experiences only air resistance in opposition to its motion, with magnitude F_r . Write an expression for the sum of the forces in the x -direction, ΣF_x , in Newtons.

To begin, let's draw a free body diagram of the forces on the car.



We see that the only forces acting along the chosen x -axis are the x -component of the gravitational force acting in the positive x -direction and the air resistance acting in the negative x -direction. Using trigonometry, we can construct the following equation for the sum of the forces in the x -direction:

$$\Sigma F_x = F_{gx} - F_r$$

$$\Sigma F_x = F_g \sin(\theta) - F_r$$

$$\Sigma F_x = mg \sin(\theta) - F_r$$

Part (d) What is the magnitude of the force caused by air resistance, F_r , in Newtons? (Maintain the assumption that the car's velocity is constant.)

To solve for the magnitude of the force of air resistance, we can combine the equation we found for net force in part (b) with the equation for net force we found in part (a).

$$\Sigma F_x = 0 = mg \sin(\theta) - F_r$$

$$0 = mg \sin(\theta) - F_r$$

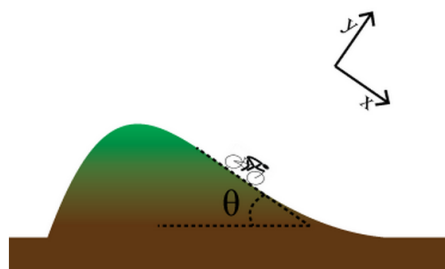
$$F_r = mg \sin(\theta)$$

$$F_r = 1.1 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \sin(11^\circ)$$

$$F_r = 2.059 \text{ N}$$

Problem 161 - 5.3.7 (alt) :

A bicycle and rider with mass m rolls down a hill at constant velocity under the influence of gravity with negligible rolling resistance. The hill makes an angle of $\theta = 19$ degrees with respect to the horizontal. The force of air resistance, termed "drag", has a magnitude of $F_d = 93$ N. Assume that the x-direction is parallel to the slope of the hill and the y-direction is perpendicular to the hill as shown.



Randomized Variables

$\theta = 19^\circ$
 $F_d = 93 \text{ N}$

Part (a) What is the sum of the forces in the y direction, ΣF_y , in Newtons?

Numeric : A numeric value is expected and not an expression.
 $\Sigma F_y =$ _____

Part (b) Input an expression for the sum of the forces in the x-direction, ΣF_x , in terms of the quantities given and variables available in the palette.

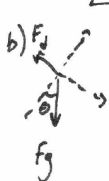
Expression :
 $\Sigma F_x =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.
 $\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , θ , d , F_d , g , h , m , t

Part (c) Using the information above, what is the total mass of the bike and rider m in kg?
Numeric : A numeric value is expected and not an expression.
 $m =$ _____

a) There is no acceleration in the y-direction (since the cyclist is staying on the hill),

so $\Sigma F_y = 0$

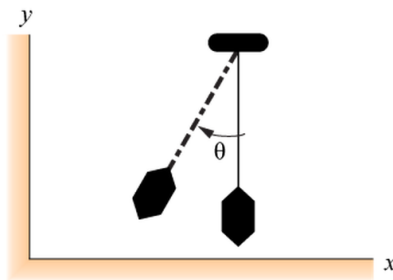


$$\Sigma F_x = mg \sin \theta - F_d$$

c) At constant velocity, $\Sigma F_x = 0$ so $mg \sin \theta = F_d \rightarrow m = \frac{F_d}{g \sin \theta}$, $m \approx 29.1 \text{ N}$

Problem 162 - 5.3.8 :

Attached to the rear-view mirror of a car is a small crystal on a string. When the car is stopped at a light, the crystal hangs vertically. When the light turns green, the driver accelerates and notices the crystal makes an angle of $\theta = 14$ degrees with respect to the vertical.

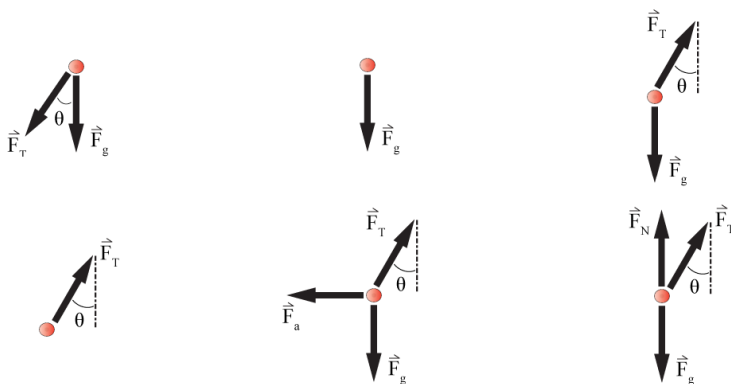


Randomized Variables

$\theta = 14$ degrees

Part (a) Please select the correct free body diagram, using an inertial coordinate system fixed to the road, given F_g is the force due to gravity, F_T is the tension in the string, F_a is the centrifugal acceleration and F_N is the normal force.

SchematicChoice :



Part (b) Input an expression for the magnitude of the car's acceleration in terms of the variables provided - the acceleration due to gravity and θ .

Expression :

$a =$ _____

Select from the variables below to write your expression. Note that all variables may not be required. $\cos(\alpha)$, $\cos(\phi)$, $\cotan(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\tan(\theta)$, a , β , θ , $\cos(\theta)$, $\cot(\theta)$, d , g , h , j , k , m , P , $\sin(\theta)$, t

Part (c) What is the car's acceleration in m/s^2 ?

Numeric : A numeric value is expected and not an expression.

$a =$ _____

Part (d) When the car is no longer accelerating, what is the angle, θ in degrees?

Numeric : A numeric value is expected and not an expression.

$\theta =$

a) b) In the radial direction, $\sum F_r = F_T \sin \theta = ma_c$ (x-direction)
 In the vertical direction, $\sum F_z = F_T \cos \theta - mg = 0$ (y-direction)

$$\left(\frac{mg}{\cos \theta}\right) \sin \theta = ma_c \rightarrow a_c = g \tan \theta \rightarrow F_T = \frac{mg}{\cos \theta}$$

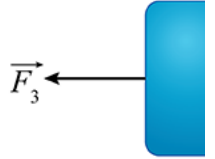
b) $a \approx 2.45 \text{ m/s}^2$

c) without acceleration, the car must be moving at constant velocity, with no net force on the crystal. $\sum F = 0$ so the FBD must be $\theta = 0$

Problem 163 - 5.3.9 :

Full solution not currently available at this time.

An object of mass m has these three forces acting on it (there is no normal force, "no surface"). $F_1 = 2 \text{ N}$, $F_2 = 7 \text{ N}$, and $F_3 = 2 \text{ N}$. When answering the questions below, assume the x -direction is to the right, and the y -direction is straight upwards.



Part (a) What is the net force in component form, in terms of F_1 , F_2 , F_3 , and the unit vectors i and j

$$\mathbf{F} = (F_2 - F_3) \mathbf{i} + (-F_1) \mathbf{j}$$

Part (b) What is the magnitude of the net force, in newtons?

$$|\mathbf{F}| = ((f_2 - f_3)^2 + f_1^2)^{0.5}$$

$$|\mathbf{F}| = ((7 - 2)^2 + 2^2)^{0.5}$$

$$|\mathbf{F}| = 5.385$$

$$\text{Tolerance: } \pm 0.16155$$

Part (c) What is the angle θ , in degrees, of the net force, measured from the $+x$ -axis? Enter an angle between -180° and 180° .

$$\theta = \text{atan}(-f_1 / (f_2 - f_3)) * (180 / 3.14159)$$

$$\theta = \text{atan}(-2 / (7 - 2)) * (180 / 3.14159)$$

$$\theta = -21.801$$

$$\text{Tolerance: } \pm 0.65403$$

Part (d) What is the magnitude, $|\mathbf{a}|$ of the acceleration, in meters per square second, if the block has a mass of 5.1 kg?

$$|\mathbf{a}| = ((f_2 - f_3)^2 + (f_1)^2)^{0.5} / (m)$$

$$|\mathbf{a}| = ((7 - 2)^2 + (2)^2)^{0.5} / (5.1)$$

$$|\mathbf{a}| = 1.056$$

$$\text{Tolerance: } \pm 0.03168$$

Problem 164 - 5.3.10 :

A chandelier is suspended by two identical, vertical chains side by side. The chandelier's mass is $m = 5.1$ kg.

Part (a) If the tension in one chain is represented by T , write an expression for the magnitude of the tension in terms of m and g , the gravitational acceleration.

Let's begin by drawing a free-body diagram of the chandelier.



Due to the symmetry of the system, the chains will both exert the same force of tension on the chandelier. Since the chandelier is stationary, there will be no net force. We can therefore set up the following equation:

$$F_{net} = T + T - F_g$$

$$0 = T + T - F_g$$

$$F_g = 2T$$

$$mg = 2T$$

$$\frac{mg}{2} = T$$

$$T = \frac{mg}{2}$$

Part (b) What is the tension in one chain, T , in newtons?

We found an equation for the force of tension on one chain in part (a). Here, we simply need to solve that equation.

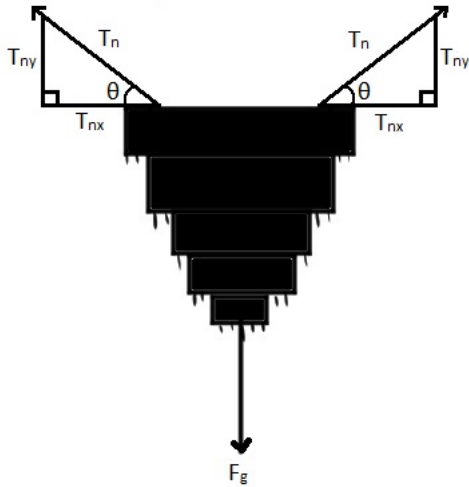
$$T = \frac{mg}{2}$$

$$T = \frac{5.1 \text{ kg} \cdot 9.8 \text{ m/s}^2}{2}$$

$$T = 24.99 \text{ N}$$

Part (c) If the tops of the chains are separated so that the chains are no longer vertical, does the tension increase or decrease?

Let's begin by drawing a new free-body diagram for this situation.



As we can see, the vertical components of the two forces of tension must be the same as the total tension in part (a), as the vertical components of the tension must force to counteract the force of gravity. However, they now also have horizontal components to the force that did not exist previously. This means that the force of

Increase

Problem 165 - 5.3.10 (alt) :

A chandelier is suspended by two vertical chains side by side. The chandelier's mass is $m = 5.6$ kg.

Randomized Variables

$m = 5.6$ kg

Part (a) If the tension in one chain is represented by T , write an expression for this tension in terms of m and g , the gravitational acceleration on the Earth's surface. Pick a coordinate system so that this tension is positive.

Expression :

$T =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \theta, a, b, d, g, h, i, j, k, m, P, S, t$

Part (b) What is the tension in one chain, T , in newtons?

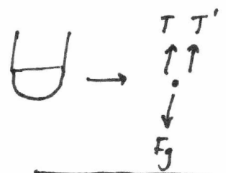
Numeric : A numeric value is expected and not an expression.

$T =$ _____

Part (c) If the tops of the chains are separated so that the chains are no longer vertical, does the tension increase or decrease?

MultipleChoice :

- 1) Decrease
- 2) Increase



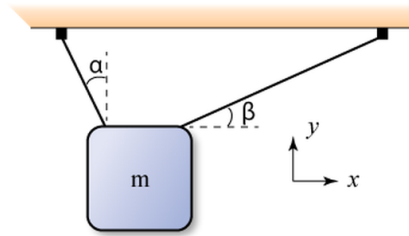
a) $\Sigma F = \underbrace{T + T'}_{\text{same}} - F_g = 2T - mg = 0 \rightarrow T = \frac{1}{2} Mg$

b) $T \approx 27.5 N$

c) If the tops are separated, only the y-component will balance out the mass so we would have $2T' \cos \theta = mg$ $T' = \frac{mg}{2 \cos \theta}$, and as $\theta > 0$, $\cos \theta < 1$ and $T' > \frac{mg}{2} = T \rightarrow \text{increase}$

Problem 166 - 5.3.12 (iFBD) :

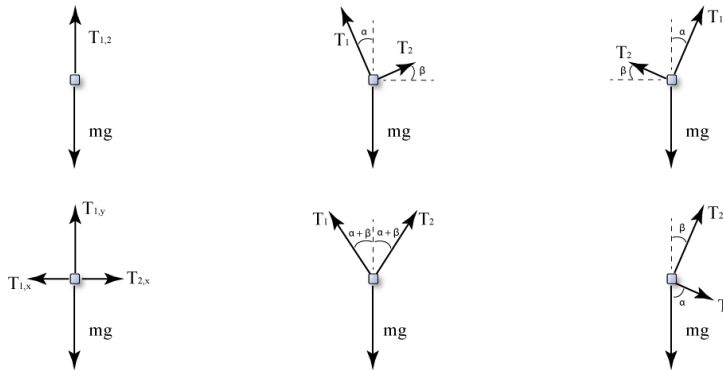
A block having a mass of $m = 18.5 \text{ kg}$ is suspended via two cables as shown in the figure. The angles shown in the figure are as follows: $\alpha = 19^\circ$ and $\beta = 34^\circ$.



Randomized Variables

$m = 18.5 \text{ kg}$
 $\alpha = 19^\circ$
 $\beta = 34^\circ$

Part (a) From the images below, choose the correct free body diagram.
SchematicChoice :



Part (b) Write an expression for the sum of forces in the x direction in terms of T_1 , T_2 , m , g , α , and β . Use the specified coordinate system.

Expression :
 $\Sigma F_x =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.
 $\cos(\beta)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\beta)$, $\sin(\phi)$, $\sin(\theta)$, α , θ , $\cos(\alpha)$, g , m , $\sin(\alpha)$, t , T_1 , T_2

Part (c) Write an expression for the sum of forces in the y direction in terms of T_1 , T_2 , m , g , α , and β . Use the specified coordinate system.

Expression :
 $\Sigma F_y =$ _____

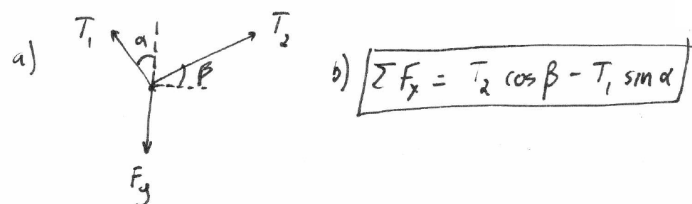
Select from the variables below to write your expression. Note that all variables may not be required.
 $\cos(\beta)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\beta)$, $\sin(\phi)$, $\sin(\theta)$, α , θ , $\cos(\alpha)$, g , m , $\sin(\alpha)$, t , T_1 , T_2

Part (d) Solve for the numeric value of T_1 , in newtons.

Numeric : A numeric value is expected and not an expression.
 $T_1 =$ _____

Part (e) Solve for the numeric value of T_2 , in newtons.

Numeric : A numeric value is expected and not an expression.
 $T_2 =$ _____



b)
$$\sum F_x = T_2 \cos \beta - T_1 \sin \alpha$$

c)
$$\sum F_y = T_1 \cos \alpha + T_2 \sin \beta - Mg$$

d) Both of these are zero, so start by solving for T_2 and eliminating:

$$T_2 = T_1 \frac{\sin \alpha}{\cos \beta}, \quad \sum F_y = T_1 \cos \alpha + T_1 \frac{\sin \alpha}{\cos \beta} \sin \beta - mg = 0$$

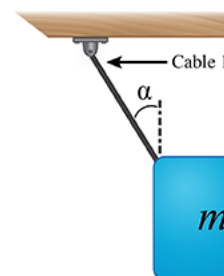
$$T_1 (\cos \alpha + \sin \alpha \tan \beta) = mg, \quad T_1 = \frac{mg}{\cos \alpha + \sin \alpha \tan \beta} \quad T_1 \approx 155.8 \text{ N}$$

e) Use result from (b): $T_2 = T_1 \frac{\sin \alpha}{\cos \beta}, \quad T_2 \approx 61.2 \text{ N}$

Problem 167 - 5.3.12 (alt2) :

Full solution not currently available at this time.

A block having a mass of $m = 10.5$ kg is suspended via two cables as shown in the figure. The angles shown in the figure are as follows: $\alpha = 11^\circ$ and $\beta = 25^\circ$.



Part (a) From the images below, choose the correct free body diagram.



Part (b) Solve for the numeric value of T_1 , in newtons.

$$T_1 = m * 9.81 / (\cos(\alpha * 3.14159 / 180) + \sin(\alpha * 3.14159 / 180) * \tan(\beta * 3.14159 / 180))$$

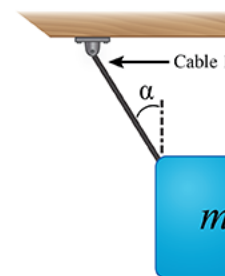
$$T_1 = 10.5 * 9.81 / (\cos(11 * 3.14159 / 180) + \sin(11 * 3.14159 / 180) * \tan(25 * 3.14159 / 180))$$

$$T_1 = 96.212$$

Tolerance: ± 2.88636

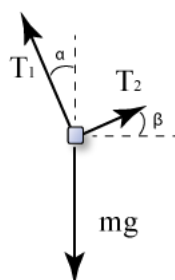
Problem 168 - 5.3.12 :

A block having a mass of $m = 10.5$ kg is suspended via two cables as shown in the figure. The angles shown in the figure are as follows: $\alpha = 11^\circ$ and $\beta = 25^\circ$. We will label the tension in Cable 1 as T_1 and the tension in Cable 2 as T_2 .



Part (a) From the images below, choose the correct free body diagram.

The block is subject to three forces: the tension from cable 1, the tension from cable 2, and the force of gravity. The correct free-body diagram needs to have all three forces placed with the correct angles.



Part (b) Write an expression for the sum of forces in the x direction in terms of T_1 , T_2 , m , g , α , and β . Use the specified coordinate system.

Looking at the free-body diagram, we see the only forces acting in the x-direction are the x-components of the two tensions, with T_1 acting in the negative x-direction. As such, we can find the following equation for the net force in the x-direction with trigonometry:

$$\Sigma F_x = T_2 \cos(\beta) - T_1 \sin(\alpha)$$

Part (c) Write an expression for the sum of forces in the y direction in terms of T_1 , T_2 , m , g , α , and β . Use the specified coordinate system.

Looking at the free-body diagram chosen in part (a), we see that the y-components of the two forces of tension both act in the positive y-direction and the force of gravity acts in the negative y-direction. Using trigonometry, we can get the following equation for the net force in the y-direction:

$$\Sigma F_y = T_2 \sin(\beta) + T_1 \cos(\alpha) - mg$$

Part (d) Solve for the numeric value of T_1 , in newtons.

To find a value for T_1 , we will need to use a system of equations using the equations we found in parts (b) and (c), noting that there is no net force on the block in either direction.

$$\Sigma F_x = T_2 \cos(\beta) - T_1 \sin(\alpha)$$

$$0 = T_2 \cos(\beta) - T_1 \sin(\alpha)$$

$$T_1 \sin(\alpha) = T_2 \cos(\beta)$$

$$\frac{T_1 \sin(\alpha)}{\cos(\beta)} = T_2$$

Now that we have an equation for T_2 in terms of T_1 , we can plug this into our equation for the net force in the y-direction to solve for T_1

$$\Sigma F_y = T_2 \sin(\beta) + T_1 \cos(\alpha) - mg$$

$$0 = T_2 \sin(\beta) + T_1 \cos(\alpha) - mg$$

$$0 = \left(\frac{T_1 \sin(\alpha)}{\cos(\beta)} \right) \sin(\beta) + T_1 \cos(\alpha) - mg$$

$$mg = T_1 \sin(\alpha) \tan(\beta) + T_1 \cos(\alpha)$$

$$mg = T_1 [\sin(\alpha) \tan(\beta) + \cos(\alpha)]$$

$$\frac{mg}{\sin(\alpha) \tan(\beta) + \cos(\alpha)} = T_1$$

$$T_1 = \frac{10.5 \text{ kg} \cdot 9.81 \text{ m/s}^2}{\sin(11^\circ) \cdot \tan(25^\circ) + \cos(11^\circ)}$$

$$T_1 = 96.212 \text{ N}$$

Part (e) Solve for the numeric value of T_2 , in newtons.

We found an equation for T_1 in part (d). We can use this together with the equation from part (b) and the fact that there is no net force in the x-direction to solve for T_2

$$\Sigma F_x = T_2 \cos(\beta) - T_1 \sin(\alpha)$$

$$0 = T_2 \cos(\beta) - T_1 \sin(\alpha)$$

$$T_1 \sin(\alpha) = T_2 \cos(\beta)$$

$$T_1 \cdot \frac{\sin(\alpha)}{\cos(\beta)} = T_2$$

Now we can plug in the value for T_1 that we found in part (d) and solve for T_2

$$\left(\frac{mg}{\sin(\alpha) \tan(\beta) + \cos(\alpha)} \right) \cdot \frac{\sin(\alpha)}{\cos(\beta)} = T_2$$

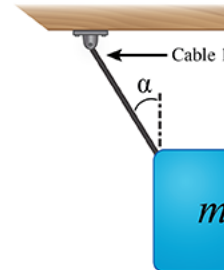
$$\frac{mg}{\sin(\beta) + \cot(\alpha) \cos(\beta)} = T_2$$

$$T_2 = \frac{10.5 \text{ kg} \cdot 9.81 \text{ m/s}^2}{\sin(25^\circ) + \cot(11^\circ) \cos(25^\circ)}$$

$$T_2 = 20.256 \text{ N}$$

Problem 169 - 5.3.12 (alt) :

A block having a mass of $m = 10.5$ kg is suspended via two cables as shown in the figure. The angles shown in the figure are as follows: $\alpha = 11^\circ$ and $\beta = 25^\circ$.

**Part (a) From the images below, choose the correct free body diagram.**

For the free body diagram, the tensions in the cables will point along the cables at the same angles depicted and the force of gravity will point straight down. Only one is correct.

Part (b) Solve for the numeric value of T_1 , in newtons.

From the free body diagram, the components of all the forces along the y and x axes can be determined. Since the block is stationary, the sum of all these forces is zero.

$$\sum F_x = T_2 \cos(\beta) - T_1 \sin(\alpha) \quad \sum F_y = T_1 \cos(\alpha) + T_2 \sin(\beta) - mg$$

$$= 0$$

$$= 0$$

so

$$T_2 \cos(\beta) = T_1 \sin(\alpha) \quad T_1 \cos(\alpha) + T_2 \sin(\beta) = mg$$

where the tensions are measured in N, the angles in rad, m is the mass in kg, and g is the acceleration of gravity in m/s^2 . First solve for the second tension in the first equation and substitute that into the second equation.

$$T_2 = T_1 \frac{\sin(\alpha)}{\cos(\beta)}$$

$$T_1 \cos(\alpha) + T_1 \frac{\sin(\alpha)}{\cos(\beta)} \sin(\beta) = mg$$

Solving for T_1

$$T_1 (\cos(\alpha) + \sin(\alpha) \tan(\beta)) = mg$$

$$T_1 = \frac{mg}{\cos(\alpha) + \sin(\alpha) \tan(\beta)}$$

Plugging in numbers and converting units as needed,

$$T_1 = \frac{(10.5 \text{ kg} \cdot 9.81 \text{ m/s}^2)}{\left(\cos\left(11^\circ \cdot \frac{\pi \text{ rad}}{180^\circ}\right) + \sin\left(11^\circ \cdot \frac{\pi \text{ rad}}{180^\circ}\right) \tan\left(25^\circ \cdot \frac{\pi \text{ rad}}{180^\circ}\right)\right)}$$

$$T_1 = 96.212 \text{ N}$$

Part (c) Solve for the numeric value of T_2 , in newtons.

From the free body diagram, the components of all the forces along the y and x axes can be determined. Since the block is stationary, the sum of all these forces r

$$\begin{aligned}\sum F_x &= T_2 \cos(\beta) - T_1 \sin(\alpha) & \sum F_y &= T_1 \cos(\alpha) + T_2 \sin(\beta) - mg \\ &= 0 & &= 0\end{aligned}$$

so

$$T_2 \cos(\beta) = T_1 \sin(\alpha) \quad T_1 \cos(\alpha) + T_2 \sin(\beta) = mg$$

where the tensions are measured in N, the angles in rad, m is the mass in kg, and g is the acceleration of gravity in m/s^2 . First solve for the second tension in the 1 substitute what was found in the previous part for the other tension.

$$T_2 = T_1 \frac{\sin(\alpha)}{\cos(\beta)} = \frac{(mg)}{(\cos(\alpha) + \sin(\alpha) \tan(\beta)) \cos(\beta)} = \frac{(mg)}{(\cot(\alpha) \cos(\beta) + \sin(\beta))}$$

Plugging in numbers and converting units as needed,

$$T_2 = \frac{(10.5 \text{ kg} \cdot 9.81 \text{ m/s}^2)}{(\cot(11^\circ \cdot \frac{\pi \text{ rad}}{180^\circ}) \cos(25^\circ \cdot \frac{\pi \text{ rad}}{180^\circ}) + \sin(25^\circ \cdot \frac{\pi \text{ rad}}{180^\circ}))}$$

$$T_2 = 20.256 \text{ N}$$

Problem 170 - 5.3.16 :

Since astronauts in orbit are in free fall and thus apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net force of **45** N is exerted and the astronaut's acceleration is measured to be **0.875** m/s^2 .<

Part (a) Calculate her mass, in kilograms.

In general, the net external force on one object is expressed by

$$F_{net} = \sum_{i=1}^n m a_i \text{ N}$$

where m is the mass of the object in kg, a_i is the i-th acceleration experienced by the object in m/s^2 and n is the total number of accelerations. In this case,

$$n = 1$$

$$F_{net} = ma$$

$$m = \frac{F_{net}}{a}$$

Plugging in numbers and converting units as needed,

$$m = \frac{(45 \text{ N})}{0.875 \text{ m/s}^2}$$

$$m = 51.429 \text{ kg}$$

Problem 171 - 5.3.17 :

In the figure, the net external force on the 24 kg mower is known to be 51 N.

Randomized Variables

$$f = 21 \text{ N}$$

$$v = 1.1 \text{ m/s}$$



Part (a) If the force of friction opposing the motion is 21 N, what force F (in newtons) is the person exerting on the mower?

In general, the net external force on one object is expressed by

$$F_{net} = \sum_{i=1}^n ma_i \text{ N}$$

where m is the mass of the object in kg, a_i is the i -th acceleration experienced by the object in m/s^2 and n is the total number of accelerations. In this case,

$$n = 2$$

$$F_{net} = F_m - F_f$$

where F_m is the mower's force in N and F_f is the friction force in N. The minus sign accounts for the direction. Solving for the mower's force

$$F_m = F_{net} + F_f$$

Plugging in numbers and converting units as needed,

$$F_m = 51 \text{ N} + 21 \text{ N}$$

$$F_m = 72 \text{ N}$$

Part (b) Suppose the mower is moving at 1.1 m/s when the force F is removed. How far will the mower go before stopping in m?

Under a constant acceleration, a velocity is related to the acceleration, initial velocity, and distance by the expression

$$v^2 = (v_0^2 + 2ad) \text{ m/s}$$

where v_0 is the initial velocity in m/s, a is the constant acceleration in m/s^2 , and d is the distance in m. The final velocity is zero so

$$d = -\frac{v_0^2}{2a}$$

The acceleration is from the friction and related to the force and mass by the relation

$$F = ma$$

$$a = \frac{F}{m}$$

where F is the force in N and m is the mass in kg. Substituting in,

$$d = -\frac{v_0^2}{2\left(\frac{F}{m}\right)} = -\frac{mv_0^2}{2F}$$

Plugging in numbers and converting units as needed,

$$d = x = \frac{(-24 \text{ kg} \cdot (1.1 \text{ m/s})^2)}{(2 \cdot 21 \text{ N})}$$

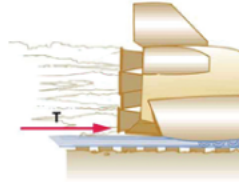
$$x = 0.6914 \text{ m}$$

Problem 172 - 5.3.19 :

The mass of the system of the rocket sled shown in the figure is 2100 kg, and the force of friction opposing the motion is known to be 625 N. The thrust for the rocket sled is $2.59 \times 10^4 \text{ N}$.

Randomized Variables

$$f = 625 \text{ N}$$



Part (a) What is its acceleration in m/s^2 ?

In general, the net external force on one object is expressed by

$$F_{net} = \sum_{i=1}^n ma_i \text{ N}$$

where m is the mass of the object in kg, a_i is the i -th acceleration experienced by the object in m/s^2 and n is the total number of accelerations. In this case,

$$n = 2$$

$$F_{net} = F_r - F_f$$

where F_r is the rocket's force in N and F_f is the friction force in N. The minus sign accounts for the direction. Therefore,

$$F_{net} = ma_{net} = F_r - F_f$$

$$a_{net} = \frac{(F_r - F_f)}{m}$$

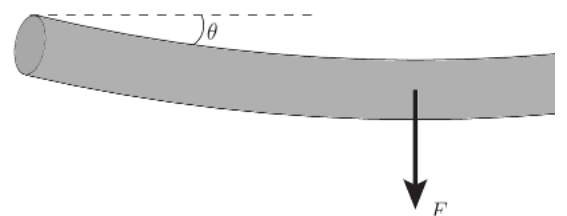
Plugging in numbers and converting units as needed,

$$a_{net} = \frac{(2.59 \cdot 10^4 \text{ N} - 625 \text{ N})}{2100 \text{ kg}}$$

$$a_{net} = 12.036 \text{ m/s}^2$$

Problem 173 - 5.3.20 :

A force exerted on a flexible object at its center, perpendicular to its length, gives rise to a tension on the object.



Part (a) What equation represents the tension on the object?

Write an equation that represents the tension on the object.

Begin by writing expression for the forces acting on the center of the object where the force is applied.

$$\Sigma \mathbf{F} = m\mathbf{a}$$

The object is in static equilibrium.

$$\Sigma \mathbf{F} = 0$$

Break the vector equation into two component equations.

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

Each half of the object pulls with tension T, but only a small component of that force pulls in the direction parallel to the force F.

The y component of each force T is $T \cdot \sin(\theta)$.

$$\Sigma F_y = 0 = T \cdot \sin(\theta) + T \cdot \sin(\theta) - F$$

$$0 = 2 \cdot T \cdot \sin(\theta) - F$$

$$2 \cdot T \cdot \sin(\theta) = F$$

$$T = \frac{F}{2\sin(\theta)}$$

Part (b) If you hang a car of mass 1500 kg from a steel beam, you observe that the beam bends 0.02°. What is the tension in this beam, in newtons?

A car of mass 1500 kg is hung from a beam that deflects 0.02°. What is the tension in this beam in Newtons?

Begin with the expression derived in part a.

$$T = \frac{F}{2\sin(\theta)}$$

In this case, the applied force F is the force of gravity exerted on the car.

$$T = \frac{mg}{2\sin(\theta)}$$

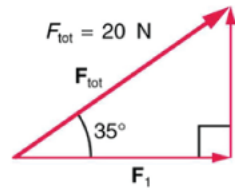
$$T = \frac{\left((1500\text{kg}) \cdot \left(9.81 \frac{\text{m}}{\text{s}^2}\right)\right)}{\left(2 \cdot \sin(0.02^\circ)\right)}$$

$$T = \frac{\left((1500\text{kg}) \cdot \left(9.81 \frac{\text{m}}{\text{s}^2}\right)\right)}{\left(2 \cdot 0.0003491\right)}$$

$$T = 21077685.316\text{N}$$

Problem 174 - 5.3.21 :

Consider the forces shown in the figure.



Part (a) Find the magnitude of the force F_1 shown in the figure in Newtons.

The forces 1 and 2 make up a right triangle with the total force as the hypotenuse. The first force is related to the hypotenuse by the expression

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{F_1}{F_{tot}}$$

where the forces are depicted in the diagram in N and θ is the angle between F_1 and F_{tot} in rad. Solving for the first force,

$$F_1 = F_{tot} \cos(\theta)$$

Plugging in numbers and converting units as needed,

$$F_1 = 20 \text{ N} \cdot \cos(35^\circ)$$

$$F_1 = 16.383 \text{ N}$$

Part (b) Find the magnitude of the force F_2 shown in the figure in Newtons.

The forces 1 and 2 make up a right triangle with the total force as the hypotenuse. The second force is related to the hypotenuse by the expression

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{F_2}{F_{tot}}$$

where the forces are depicted in the diagram in N and θ is the angle between F_1 and F_{tot} in rad. Solving for the second force,

$$F_2 = F_{tot} \sin(\theta)$$

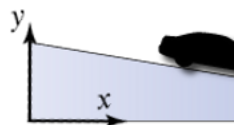
Plugging in numbers and converting units as needed,

$$F_2 = 20 \text{ N} \cdot \sin(35^\circ)$$

$$F_2 = 11.471 \text{ N}$$

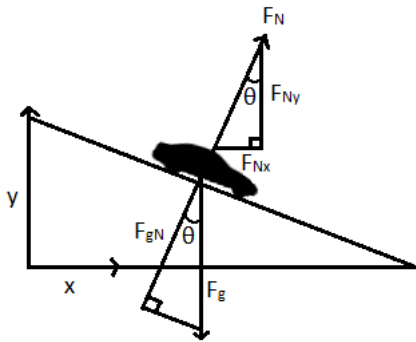
Problem 175 - 5.3.26 :

A toy car is rolling down the ramp as shown in the figure. The car's mass is $m = 1.1$ kg and the ramp makes an angle of $\theta = 11$ degrees with respect to the horizontal. Assume the car is rolling without friction.



Part (a) Using the coordinate system specified, give an expression for the acceleration of the car in terms of θ , g , and the unit vectors \mathbf{i} and \mathbf{j} .

Let's begin by drawing a free-body diagram for the forces.



As we can see, the gravitational force acts along the y-axis in the negative direction while the normal force acts in both the positive y and positive x-directions. To find a formula for the normal force in terms of the force of gravity. Since the force we have labeled F_{gN} acts into the ramp, the normal force must be equal to it opposite in direction. Using trigonometry, we can get the following value for F_{gN} and, by extension, the normal force.

$$F_{gN} = F_N = F_g \cos(\theta)$$

$$F_N = mg \cos(\theta)$$

Now that we have a value for the normal force, we can once again use trigonometry to find its components in the x and y-directions and the sum the total forces in

$$F = F_N \sin(\theta) \mathbf{i} + [F_N \cos(\theta) - F_g] \mathbf{j}$$

$$F = mg \cos(\theta) \sin(\theta) \mathbf{i} + (mg \cos(\theta) \cos(\theta) - F_g) \mathbf{j}$$

$$F = mg \cos(\theta) \sin(\theta) \mathbf{i} + (mg \cos^2(\theta) - mg) \mathbf{j}$$

$$F = mg \cos(\theta) \sin(\theta) \mathbf{i} + (mg (\cos^2(\theta) - 1)) \mathbf{j}$$

$$F = mg \cos(\theta) \sin(\theta) \mathbf{i} - (mg (1 - \cos^2(\theta))) \mathbf{j}$$

$$F = mg \cos(\theta) \sin(\theta) \mathbf{i} - mg \sin^2(\theta) \mathbf{j}$$

Now that we have an equation for the force, we can apply Newton's Second Law and note that:

$$F = ma$$

$$\frac{F}{m} = a$$

Now, let's apply this and divide our value for force through by the mass of the toy car.

$$\frac{F}{m} = \frac{mg \cos(\theta) \sin(\theta) \mathbf{i} - mg \sin^2(\theta) \mathbf{j}}{m}$$

$$a = g \cos(\theta) \sin(\theta) \mathbf{i} - g \sin^2(\theta) \mathbf{j}$$

Part (b) What is the magnitude of this acceleration in m/s^2 ?

We found an equation for the force in terms of its components in part (a). Here, all we need to do is take the magnitude.

$$|a| = \sqrt{(g \cos(\theta) \sin(\theta))^2 + (-g \sin^2(\theta))^2}$$

$$|a| = \sqrt{g^2 \cos^2(\theta) \sin^2(\theta) + g^2 \sin^4(\theta)}$$

$$|a| = g \sin(\theta) \sqrt{\cos^2(\theta) + \sin^2(\theta)}$$

$$|a| = g \sin(\theta) \sqrt{1}$$

$$|a| = g \sin(\theta)$$

$$|a| = 9.81 \text{ m/s}^2 \cdot \sin(11^\circ)$$

$$|a| = 1.872 \text{ m/s}^2$$

Problem 176 - 5.3.25 :

Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in the figure. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.)

**Part (a) Find the magnitude of the total force on the Achilles tendon in Newtons.**

From the diagram, the force applied on the tendon is the sum of the vertical components of the forces applied by the muscles. The forces are related to the hypotenuse

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

so

$$\cos(\theta) = \frac{T_1}{F_1}$$

$$\cos(\theta) = \frac{T_2}{F_2}$$

where the $F_{1,2}$ are the forces the muscles apply and $T_{1,2}$ are corresponding components of the force applied to the tendon in N and θ is the angle between the force going straight up in rad. The total force applied is therefore

$$F = T_1 + T_2 = F_1 \cos(\theta) + F_2 \cos(\theta)$$

Plugging in numbers and converting units as needed,

$$F = 200 \text{ N} \cdot \cos(20^\circ) + 200 \text{ N} \cdot \cos(20^\circ)$$

$$F = 375.822 \text{ N}$$

Problem 177 - 5.3.28 :

A particle of mass 0.55 kg is subject to a force that is always pointed towards the East but whose magnitude changes linearly with time t . The magnitude of the force is given as $F = 2t$, and has units of newtons. Let the x -axis point towards the East.

Part (a) Determine the change in the velocity Δv , in meters per second, of the particle between $t = 0$ and $t = 1.1 \text{ sec}$.

A particle of mass 0.55 kg is subject to a force that is always pointed towards the East. The force changes linearly with time according to the expression $F(t) = 2t$.

What is the change in velocity Δv of the particle between $t = 0$ and $t = 1.1 \text{ sec}$?

The force is changing as a function of time, thus the acceleration is also changing.

We can not use the conventional kinematic equations and must resort to calculus.

Begin with Newton's Second Law

$$F = ma$$

Acceleration is the rate of change of velocity, so $a = \frac{dv}{dt}$

$$F = m \frac{dv}{dt}$$

Bring the differential dt to the other side of the equation and integrate.

$$F dt = m dv$$

$$\int_{t_i}^{t_f} F dt = \int_{v_i}^{v_f} m dv$$

Before integration, substitute the expression $F(t) = 2t$

F is a function of time so it must be integrated.

$$\int_{t_i}^{t_f} 2t dt = \int_{v_i}^{v_f} m dv$$

$$\left(\frac{2}{2}\right) t^2 \Big|_{t_i}^{t_f} = m v \Big|_{v_i}^{v_f}$$

In this case, $t_i = 0$.

$$\left(\frac{2}{2}\right) t^2 \Big|_0^{t_f} = m v \Big|_{v_i}^{v_f}$$

$$\left(\frac{2}{2}\right) (t_f)^2 = m (v_f - v_i)$$

$$m (v_f - v_i) = \left(\frac{2}{2}\right) (t_f)^2$$

$$v_f - v_i = \left(\frac{2}{2m}\right) (t_f)^2$$

We are looking for change in velocity anyway. $\Delta v = v_f - v_i$

$$\Delta v = \left(\frac{2}{2m}\right) (t_f)^2$$

From the original expression for the force, you can demonstrate that the constant 2 actually has units of $kg \cdot \frac{m}{s^3}$

$$\Delta v = \left(\frac{2 \left(kg \cdot \frac{m}{s^3} \right)}{2 (0.55 kg)} \right) (1.1 s)^2$$

The units of kg will cancel. The units of s^2 will also cancel.

$$\Delta v = \left(\frac{2 \left(\frac{m}{s} \right)}{2 (0.55)} \right) (1.1)^2$$

$$\Delta v = (1.818) \cdot (1.21) \left(\frac{m}{s} \right)$$

$$\Delta v = 2.2 \left(\frac{m}{s} \right)$$

Part (b) Determine the change in x-coordinate in meters of the particle Δx between $t = 0$ and $t = 1.1$ if the initial velocity is 10.1 m/s, and pointed in the

A particle of mass $0.55 kg$ is subject to a force that is always pointed towards the East. The force changes linearly with time according to the expression $F(t) = 2$

What is the change in the x coordinate (Δx) of the particle between $t = 0$ and $t = 1.1$ sec? The initial velocity is $v_I = 10.1 \frac{m}{s}$

Velocity in the x direction is the rate of change of the x coordinate.

$$v = \frac{dx}{dt}$$

Bring the differential dt to the other side of the equation and integrate.

$$v dt = dx$$

$$\int_{t_I}^{t_F} (v) dt = \int_{x_I}^{x_F} dx$$

You will need the equation for velocity as a function of time. The following equation was found in part a.

$$v_F - v_I = \left(\frac{2}{2m} \right) (t_F)^2$$

Take v_I to the other side of the equation to get an expression for velocity as a function of time.

$$v_F = \left(\frac{2}{2m} \right) (t_F)^2 + v_I$$

$$v(t) = \left(\frac{2}{2m} \right) (t)^2 + v_I$$

Substitute this expression for velocity into the integral.

$$\int_{t_I}^{t_F} \left(\left(\frac{2}{2m} \right) (t)^2 + v_I \right) dt = \int_{x_I}^{x_F} dx$$

$$\left(\left(\frac{2}{6m} \right) (t)^3 + v_I \cdot t \right) \Big|_{t_I}^{t_F} = x \Big|_{x_I}^{x_F}$$

Note that $t_I = 0$

$$\left(\left(\frac{2}{6m} \right) (t_F)^3 + v_I \cdot t_F \right) - \left(\left(\frac{2}{6m} \right) (0)^3 + v_I \cdot 0 \right) = x_F - x_I$$

$$\left(\left(\frac{2}{6m} \right) (t_F)^3 + v_I \cdot t_F \right) = x_F - x_I$$

$$x_F - x_I = \left(\left(\frac{2}{6m} \right) (t_F)^3 + v_I \cdot t_F \right)$$

$$\Delta x = \left(\left(\frac{2}{6m} \right) (t_F)^3 + v_I \cdot t_F \right)$$

At this point, you can plug in numbers. Remember that the constant 2 has units.

$$\Delta x = \left(\frac{\left(2 \left(\frac{\text{kg}\cdot\text{m}}{\text{s}^3} \right) \right)}{\left(6 \cdot (0.55\text{kg}) \right)} \right) (1.1\text{s})^3 + \left(\frac{10.1\text{m}}{\text{s}} \right) \cdot (1.1\text{s})$$

In the first term, kg and s^3 will cancel.

$$\Delta x = \left(\frac{2(\text{m})}{6 \cdot (0.55)} \right) (1.1)^3 + (10.1\text{m}) \cdot (1.1)$$

Both terms now have units of meters. This is good because Δx should have units of meters.

$$\Delta x = 0.8067\text{m} + 11.11\text{m}$$

$$\Delta x = 11.917\text{m}$$

Problem 178 - 5.3.29 :

A particle of mass 0.51 kg is subject to a force that is always pointed towards the North but whose magnitude changes quadratically with time. Let the y -axis point towards the North. The magnitude of the force is given as $F = 2t^2$, and has units of newtons

Part (a) Determine the change in the velocity Δv , in meters per second, of the particle between $t = 0$ and $t = 1.1\text{ s}$.

A particle of mass 0.51 kg is subject to a force that is always pointed towards the North.

The force changes quadratically with time according to the expression $F(t) = 2t^2$

What is the change in velocity Δv of the particle between $t = 0$ and $t = 1.1\text{ sec}$?

The force is changing as a function of time, thus the acceleration is also changing.

We can not use the conventional kinematic equations and must resort to calculus.

Begin with Newton's Second Law

$$F = ma$$

Acceleration is the rate of change of velocity, so $a = \frac{dv}{dt}$

$$F = m \frac{dv}{dt}$$

Bring the differential dt to the other side of the equation and integrate.

$$F dt = m dv$$

$$\int_{t_i}^{t_f} F dt = \int_{v_i}^{v_f} m dv$$

Before integration, substitute the expression $F(t) = 2t^2$

F is a function of time so it must be integrated.

$$\int_{t_i}^{t_f} 2t^2 dt = \int_{v_i}^{v_f} m dv$$

$$\left(\frac{2}{3}\right) t^3 \Big|_{t_i}^{t_f} = mv \Big|_{v_i}^{v_f}$$

In this case, $t_i = 0$.

$$\left(\frac{2}{3}\right) t^3 \Big|_0^{t_f} = mv \Big|_{v_i}^{v_f}$$

$$\left(\frac{2}{3}\right) (t_f)^3 = m(v_f - v_i)$$

$$m(v_f - v_i) = \left(\frac{2}{3}\right) (t_f)^3$$

$$(v_f - v_i) = \left(\frac{2}{3m}\right) (t_f)^3$$

We are looking for change in velocity anyway. $\Delta v = v_f - v_i$

$$\Delta v = \left(\frac{2}{3m}\right) (t_f)^3$$

From the original expression for the force, you can demonstrate that the constant 2 actually has units of $kg \cdot \frac{m}{s^4}$

$$\Delta v = \left(\frac{2 \left(kg \cdot \frac{m}{s^4}\right)}{3 (0.51kg)}\right) (1.1s)^3$$

The units of kg cancel. Units of s^3 will also cancel.

$$\Delta v = \left(\frac{2 \left(\frac{m}{s}\right)}{3 (0.51)}\right) (1.1)^3$$

$$\Delta v = (1.307) \cdot (1.331) \left(\frac{m}{s}\right)$$

$$\Delta v = 1.74 \left(\frac{m}{s}\right)$$

Part (b) Determine the change in y-coordinate, in meters, of the particle Δy between $t = 0$ and $t = 1.1$ if the initial velocity = 10.1 m/s and directed North the force.

A particle of mass $0.51kg$ is subject to a force that is always pointed towards the North.

The force changes quadratically with time according to the expression $F(t) = 2t^2$

What is the change in the y coordinate (Δy) of the particle between $t = 0$ and $t = 1.1$ sec?

The initial velocity in the y direction is $v_i = 10.1 \frac{m}{s}$

Velocity in the y direction is the rate of change of the y coordinate.

$$v = \frac{dy}{dt}$$

Bring the differential dt to the other side of the equation and integrate.

$$v dt = dy$$

$$\int_{t_i}^{t_f} (v) dt = \int_{y_i}^{y_f} dy$$

You will need the equation for velocity as a function of time. The following equation was found in part a.

$$(v_F - v_I) = \left(\frac{2}{3m}\right)(t_F)^3$$

Take v_I to the other side of the equation to get an expression for velocity as a function of time.

$$v_F = \left(\frac{2}{3m}\right)(t_F)^3 + v_I$$

$$v(t) = \left(\frac{2}{3m}\right)(t)^3 + v_I$$

Substitute this expression for velocity into the integral.

$$\int_{t_i}^{t_f} \left(\left(\frac{2}{3m}\right)(t)^3 + v_I\right) dt = \int_{y_i}^{y_f} dy$$

$$\left(\left(\frac{2}{12m}\right)(t)^4 + v_I \cdot t\right) \Big|_{t_i}^{t_f} = y \Big|_{y_i}^{y_f}$$

Note that $t_I = 0$

$$\left(\left(\frac{2}{12m}\right)(t_F)^4 + v_I \cdot t_F\right) - \left(\left(\frac{2}{12m}\right)(0)^4 + v_I \cdot 0\right) = y_F - y_I$$

$$\left(\left(\frac{2}{12m}\right)(t_F)^4 + v_I \cdot t_F\right) = y_F - y_I$$

$$y_F - y_I = \left(\left(\frac{2}{12m}\right)(t_F)^4 + v_I \cdot t_F\right)$$

$$\Delta y = \left(\left(\frac{2}{12m}\right)(t_F)^4 + v_I \cdot t_F\right)$$

At this point, you can plug in numbers. Remember that the constant 2 has units.

$$\Delta y = \left(\left(\frac{\left(2 \left(\frac{\text{kg} \cdot \frac{\text{m}}{\text{s}^4}\right)\right)}{12(0.51\text{kg})}\right)(1.1\text{s})^4 + \left(10.1 \left(\frac{\text{m}}{\text{s}}\right)\right) \cdot (1.1\text{s})\right)$$

$$\Delta y = \left(\left(\frac{2(m)}{12(0.51)}\right)(1.1)^4 + (10.1(m)) \cdot (1.1)\right)$$

Both terms now have units of meters. This is good because Δy should have units of meters.

$$\Delta y = 0.4785m + 11.11m$$

$$\Delta y = 11.588m$$

Problem 179 - 5.3.31 :

A particle of mass 0.51 kg begins at rest and is then subject to a force that changes with time as given by the following function: $F = mg[1 - e^{-2.1t}]$, where g is the acceleration due to gravity.

Part (a) Determine the change in the velocity Δv of the particle between $t = 0$ and $t = 1.1$ sec.

This problem involves the application of **Newton's second law** and the **integration of a time-dependent acceleration** and a **time-dependent velocity** to analyze subject to a time-dependent force.

In this part, we are asked to find the change in the velocity of the particle within a specified time interval.

The solution involves two steps:

- using Newton's second law to determine the acceleration of the particle
- integrating the acceleration to find Δv

Applying Newton's Second Law

Newton's second law tells us that if we have an unbalanced force on an object, the object will undergo an acceleration

$$a = \frac{F}{m}$$

In this case, the force is time-dependent,

$$F = F(t) = mg [1 - e^{-ct}]$$

where $c = 2.1$.

As a result, our acceleration is also time-dependent,

$$a = a(t) = g [1 - e^{-ct}]$$

Integrating $a(t)$ to find Δv

To find the change in velocity in a given time interval, we must first integrate our function for acceleration,

$$\Delta v = \int_{t_0}^{t_1} a(t) dt = \int_{t_0}^{t_1} g [1 - e^{-ct}] dt$$

We can factor out g and write this as the sum of two integrals to see it more clearly,

$$\Delta v = g \left[\int_{t_0}^{t_1} dt - \int_{t_0}^{t_1} e^{-ct} dt \right]$$

Integrating,

$$\Delta v = g \left[(t_1 - t_0) + \frac{1}{c} (e^{-ct_1} - e^{-ct_0}) \right]$$

and, because $t_0 = 0$, we can simplify the result

$$\Delta v = g \left[t_1 + \frac{1}{c} (e^{-ct_1} - 1) \right]$$

Now, we can easily calculate Δv !

$$\Delta v = (9.81) \left[1.1 + \frac{1}{2.1} (e^{(-2.1)(1.1)} - 1) \right] \text{ m/s} = 6.583 \text{ m/s}$$

$\Delta v = 6.583 \text{ m/s}$

Part (b) Determine the change in x-coordinate of the particle Δx between $t = 0$ and $t = 1.1$.

Now, we are asked to find the displacement of the particle during the same time interval, assuming it moves entirely along the x direction.

To determine the displacement, we must integrate velocity as a function of time,

$$\Delta x = \int_{t_0}^{t_1} v(t) dt$$

Let's return to our integration of the acceleration to find $v(t)$ for a general t .

$$v(t) = \int_0^t a(t) dt = \int_0^t g [1 - e^{-ct}] dt = g \left(t + \frac{1}{c} (e^{-ct} - 1) \right)$$

where, again, $c = 2.1$.

Hence, we have

$$\Delta x = \int_{t_0}^{t_1} g \left(t + \frac{1}{c} (e^{-ct} - 1) \right) dt$$

Simplifying the integration and noting that $t_0 = 0$

$$\Delta x = g \left[\int_0^{t_1} t dt + \frac{1}{c} \int_0^{t_1} e^{-ct} dt - \frac{1}{c} \int_0^{t_1} dt \right]$$

Integrating,

$$\Delta x = g \left[\frac{1}{2} t_1^2 - \frac{1}{c^2} (e^{-ct_1} - 1) - \frac{t_1}{c} \right]$$

Voilà!

$$\Delta x = (9.81) \left[\frac{1}{2} (1.1)^2 - \frac{1}{(2.1)^2} (e^{-(2.1)(1.1)} - 1) - \frac{1.1}{2.1} \right] \text{ m} = 2.8 \text{ m}$$

$\Delta x = 2.8 \text{ m}$

As a side note: Notice that we didn't need the mass to solve this problem! This is good practice, as in most real scenarios we must determine which of the known need.

Problem 180 - 5.3.32 :

A particle of mass m is subject to a force that is proportional to the speed and pointed opposite to the direction of velocity: $F = -bv$, where b is a constant which takes only positive values. The direction of the velocity is along the positive x -axis.

Part (a) Write an expression for the velocity v at time t if the initial velocity has the magnitude v_0 at $t = 0$ and points in the positive x -direction.

We start with Newton's second law,

$$F = ma = m \frac{dv}{dt}.$$

We substitute the given expression for the force,

$$F = -bv,$$

and obtain the differential equation for the velocity function $v = v(t)$,

$$m \frac{dv}{dt} = -bv.$$

Separate the variables v and t to obtain

$$\frac{dv}{v} = -\frac{b}{m} dt.$$

Integrate both sides

$$\ln(v) = -\frac{b}{m}t + C.$$

Solve for $v = v(t)$.

$$v(t) = e^{(-\frac{bt}{m}+C)} = e^{(C)} e^{(-\frac{bt}{m})}.$$

Now we apply the given initial condition on the velocity, $v(0) = v_0$, and obtain

$$v_0 = e^{(C)}.$$

The velocity function is thus

$$v(t) = v_0 e^{(-\frac{bt}{m})}$$

Part (b) Write an expression for the position x at time t if the initial position is x_0 at $t = 0$.

We take the velocity function we found in part (a) and express it in the form

$$\frac{dx}{dt} = v_0 e^{(-\frac{bt}{m})}.$$

Separate the variables x and t to obtain

$$dx = v_0 e^{(-\frac{bt}{m})} dt.$$

Integrate both sides.

$$x(t) = -\frac{mv_0}{b} e^{(-\frac{bt}{m})} + C.$$

Apply the given initial condition on the position, $x(0) = x_0$, and obtain

$$x_0 = -\frac{mv_0}{b} + C,$$

$$C = \frac{mv_0}{b} + x_0.$$

The position as a function of time is thus

$$x(t) = \frac{mv_0}{b} \left(1 - e^{(-\frac{bt}{m})}\right) + x_0$$

Problem 181 - 5.3.34 :

A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder.

Part (a) What is the magnitude of the average force, in newtons, exerted on a 0.0275-kg bullet to accelerate it to a speed of 575 m/s in a time of 1.75 ms?

The average force is related to the change in momentum and the change in time by the expression

$$F_{ave} = \frac{(\Delta p)}{(\Delta t)} = \frac{(m\Delta v)}{(\Delta t)} = \frac{(m(v_f - v_i))}{(t_f - t_i)} \text{ N}$$

where m is the mass in kg, $v_{f,i}$ are the final and initial velocities in m/s, and $t_{f,i}$ are the final and initial times in s. The bullet starts from rest and is accelerated to t_f time interval. Therefore,

$$v_i = 0$$

$$v_f = v$$

$$(t_f - t_i) = t$$

Substituting in

$$F_{ave} = \frac{mv}{t}$$

Plugging in numbers and converting units as needed,

$$F_{ave} = 0.0275 \text{ kg} \cdot \frac{575 \text{ m/s}}{\left(\frac{1.75}{1000} \text{ s}\right)}$$

$$F_{ave} = 9035.714 \text{ N}$$

Problem 182 - 5.3.35 :

Grains from a grain hopper fall at a rate of **8.1** kg/s vertically onto a freight car that is moving horizontally at a constant speed **1.1** m/s on a straight track.

Part (a) What force, in newtons, is needed to keep the freight car moving at a constant velocity?

This problem involves the application of **Newton's second law** expressed as **the time rate of change of momentum** in one dimension.

Here, we're asked to determine the additional horizontal force needed to keep a freight car from slowing while its mass is increasing.

We can solve this problem most easily by expressing Newton's second law more generally, in terms of momentum,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Because we are considering only the motion of the car, which is along the horizontal, we can restrict our analysis to one dimension,

$$F = \frac{dp}{dt}$$

where $p = mv$.

Let's evaluate F by invoking the product rule,

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = v \frac{dm}{dt} + m \frac{dv}{dt}$$

We know that our solution requires the change in velocity with time (*i.e.*, the acceleration of the car) to be zero, so we have,

$$F = v \frac{dm}{dt}$$

Aha! We are given both the (constant) car velocity and the rate of change of the car's mass in the problem. With this information, we can calculate the additional

$$F = v \frac{dm}{dt} = (1.1 \text{ m/s})(8.1 \text{ kg/s}) = 8.91 \text{ N}$$

$$F = 8.91 \text{ N}$$

Problem 183 - 5.3.36(sym) :

Full solution not currently available at this time.

An astronaut of mass m is riding in a rocket sled that is sliding along an inclined plane. The sled has a horizontal component of acceleration of a_h and a

downward component of a_v .

Part (a) Write an equation for the magnitude of the force on the rider by the sled. Use m for the mass of the astronaut, a_h for the horizontal acceleration, and g for acceleration due to gravity. (Hint: Remember that gravitational acceleration must be considered when writing your equation.)

$$F = m (a_h^2 + (g - a_v)^2)^{0.5}$$

Problem 184 - 5.3.37 :

Full solution not currently available at this time.

Two ropes are attached to a tree, and forces of $\vec{F}_1 = 0.6 \hat{i} + 3.1 \hat{j}$ N and $\vec{F}_2 = 2.5 \hat{i} + 5.1 \hat{j}$ N are applied. The forces are coplaner (in the same plane).

Part (a) What is the resultant (net force) in Newtons of the two force vectors in the x-direction?

$$\Sigma \vec{F}_x = f1_i + f2_i$$

$$\Sigma \vec{F}_x = 0.6 + 2.5$$

$$\Sigma \vec{F}_x = 3.1$$

Tolerance: ± 0.093

Part (b) What is the resultant (net force) in Newtons of the two force vectors in the y-direction?

$$\Sigma \vec{F}_y = f1_j + f2_j$$

$$\Sigma \vec{F}_y = 3.1 + 5.1$$

$$\Sigma \vec{F}_y = 8.2$$

Tolerance: ± 0.246

Part (c) Find the magnitude of the forces of the ropes acting on the tree in Newtons.

$$\Sigma F_{\text{net}} = ((f1_i + f2_i)^2 + (f1_j + f2_j)^2)^{0.5}$$

$$\Sigma F_{\text{net}} = ((0.6 + 2.5)^2 + (3.1 + 5.1)^2)^{0.5}$$

$$\Sigma F_{\text{net}} = 8.766$$

Tolerance: ± 0.26298

Part (d) Find the angle in degrees as measured clockwise from the positive x-axis at which the forces of the ropes act on the tree.

$$\theta = \text{atan}((f1_j + f2_j)/(f1_i + f2_i)) * 180/\pi$$

$$\theta = \text{atan}((3.1 + 5.1)/(0.6 + 2.5)) * 180/\pi$$

$$\theta = 69.291$$

Tolerance: ± 2.07873

Problem 185 - 5.3.39 :

Full solution not currently available at this time.

Two forces with magnitudes $F_1 = 25$ N and $F_2 = 45$ N act on an object. Their directions differ by $\theta = 45$ degrees. The resulting acceleration has a magnitude of $a = 10.1$ m/s².

Part (a) Write an expression for the object's mass m in terms of the symbols given in the problem statement.

$$m = (1/a) ((F_1 + F_2 \cos(\theta))^2 + (F_2 \sin(\theta))^2)^{0.5}$$

Part (b) What is the object's mass, in kg?

$$m = \text{sqrt}((f1+f2*\cos(th*0.0174533))^2+(f2*\sin(th*0.0174533))^2)/a$$

$$m = \text{sqrt}((25+45*\cos(45*0.0174533))^2+(45*\sin(45*0.0174533))^2)/10.1$$

$$m = 6.448$$

Tolerance: ± 0.19344

Problem 186 - 5.3.40 :

Full solution not currently available at this time.

A swimmer has just jumped off a diving board. The swimmer has a mass of $m = 45$ kg and jumps off a board that is $h = 5.01$ m above the water. Exactly $T = 2.5$ seconds after entering the water, her downward motion is stopped.

Part (a) Write an expression for the magnitude of the average upward force F_w exerted on her by the water in terms of the variables given in the problem (m/s²).

$$F_w = m g + (m/T) (2 g h)^{0.5}$$

Part (b) What is the magnitude of the average upward force F_w (in N) exerted on her by the water?

$$F_w = m(9.8 + \sqrt{19.6h})/T$$

$$F_w = 45(9.8 + \sqrt{19.6 \cdot 5.01})/2.5$$

$$F_w = 619.369$$

$$\text{Tolerance: } \pm 18.58107$$

Problem 187 - 5.3.42 :

Full solution not currently available at this time.

A sailboat has a mass of **1500** kg and experiences two horizontal forces. One is due to the action of the water on the hull of the boat and is directed east and has a magnitude of **1200** N. The other is due to the action of the wind on the sails and is directed northeast with a magnitude of **2200** N.

Part (a) What is the magnitude of the acceleration of the sailboat in m/s^2 ?

$$a = (1/m) \sqrt{(f_1 + f_2 \cdot 0.707107)^2 + (f_2 \cdot 0.707106)^2}$$

$$a = (1/1500) \sqrt{(1200 + 2200 \cdot 0.707107)^2 + (2200 \cdot 0.707106)^2}$$

$$a = 2.11$$

$$\text{Tolerance: } \pm 0.0633$$

Part (b) What is the direction of the acceleration of the sailboat, expressed as an angle in degrees north of east?

$$\theta = 57.296 \cdot \text{atan}((f_2 \cdot 0.707106)/(f_1 + f_2 \cdot 0.707107))$$

$$\theta = 57.296 \cdot \text{atan}((2200 \cdot 0.707106)/(1200 + 2200 \cdot 0.707107))$$

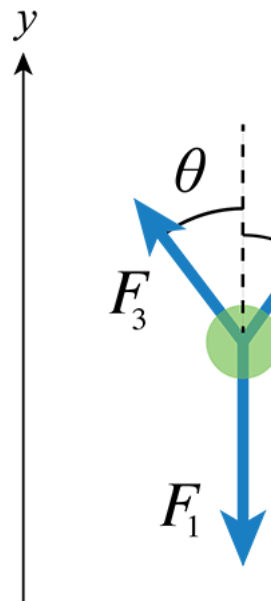
$$\theta = 29.446$$

$$\text{Tolerance: } \pm 0.88338$$

Problem 188 - 5.3.43 :

Full solution not currently available at this time.

The diagram shows all of the forces acting on a body of mass $m = 2.01$ kg. The three forces have magnitudes $F_1 = 10.1$ N, $F_2 = 20.1$ N, and $F_3 = 12$ N, with directions as indicated in the diagram, where $\theta = 25$ degrees and the dashed line is parallel to the y axis.



Part (a) Write an expression for the x component of the acceleration in terms of the symbols in the problem statement.

$$a_x = (\sin(\theta)/m) (F_2 - F_3)$$

Part (b) Write an expression for the y component of the acceleration in terms of the symbols in the problem statement.

$$a_y = (F_3 \cos(\theta) + F_2 \cos(\theta) - F_1)/m$$

Part (c) What is the magnitude of the acceleration, in m/s^2 ?

$$a = (1/m) \sqrt{((f_2 - f_3) \sin(\theta)) ^2 + ((f_2 + f_3) \cos(\theta) - f_1)^2}$$

$$a = (1/2.01) \sqrt{((20.1 - 12) \sin(25 \cdot 0.0174533)) ^2 + ((20.1 + 12) \cos(25 \cdot 0.0174533) - 10.1)^2}$$

$$a = 9.601$$

$$\text{Tolerance: } \pm 0.28803$$

Part (d) What angle, in degrees, does the acceleration make with the x axis?

$$\theta = 57.296 * \text{atan}(((f2+f3) * \cos(\text{th} * 0.0174533) - f1) / ((f2-f3) * \sin(\text{th} * 0.0174533)))$$

$$\theta = 57.296 * \text{atan}(((20.1+12) * \cos(25 * 0.0174533) - 10.1) / ((20.1-12) * \sin(25 * 0.0174533)))$$

$$\theta = 79.783$$

$$\text{Tolerance: } \pm 2.39349$$

Problem 189 - c5.4.1 :

Full solution not currently available at this time.

Consider the quantities weight and mass.

Part (a) Which of the following statements is true?

An object has the same mass on the Moon and the Earth.

Problem 190 - c5.4.2 :

Full solution not currently available at this time.

Select whether the following statements are true or false.

Part (a) Mass and weight are the same quantities with different units.

FALSE

Part (b) Weight is proportional to mass.

TRUE

Part (c) Both weight and mass can be considered vector quantities.

FALSE

Part (d) An object (not necessarily on Earth) can have zero weight, but non-zero mass.

TRUE

Problem 191 - c5.4.3 :

Full solution not currently available at this time.

A 51-kg astronaut takes off from the Earth, eventually reaching the limits of our solar system, far from the reaches of gravity.

Part (a) What is her mass, in kg, far so from Earth?

$$m = m$$

$$m = 51$$

$$\text{Tolerance: } \pm 1.53$$

Part (b) What is her weight, in N, so far from Earth?

$$w = 0$$

$$\text{Tolerance: } \pm 0$$

Problem 192 - c5.4.4 :

Full solution not currently available at this time.

Take upward to be the positive y direction and horizontally to the right to be the positive x direction.

Part (a) Write an expression, as a vector in component form, for the weight of an object on Earth in terms of its mass m and the gravitational constant g

$$w = - m g j$$

Problem 193 - 5.4.1 :

A cardboard box rests on the floor of an elevator. The box has a mass $m = 1.25$ kg and the elevator has an upward acceleration of a .

Part (a) Select the correct Free Body Diagram for the system.

Let's consider the system. Since the elevator is accelerating upward, this means that the box on the floor must be accelerating upward as well. If the box is accelerating upward, it cannot be balanced. In this case, there will be two forces acting on the box; a force of gravity acting downwards and a normal force acting upwards. Since the elevator is accelerating upwards, the magnitude of the normal force must be greater than the magnitude of the force of gravity. This means that the correct free-body diagram has a normal force vector pointing up and a gravitational force vector pointing down as well as having the normal force vector drawn longer than the gravitational force vector.

**Part (b) Write an expression for the sum of the forces acting on the box in the y-direction, ΣF_y , given that up is the positive y-direction. Your answer should be in terms of F_N and F_g .**

Looking at our free-body diagram, we see that the normal force is acting in the positive y-direction and the force of gravity is acting in the negative y-direction. The net force in the y-direction is therefore given by the following equation:

$$\Sigma F_y = F_N - F_g$$

$$\Sigma F_y = F_N - mg$$

Part (c) Write an expression for the normal force, F_N , that the block experiences in terms of the elevator's acceleration, the block's mass, and the acceleration a .

Let's begin with the equation we found in part (b) and rewrite ΣF_y using Newton's Second Law in order to find an equation for the normal force.

$$\Sigma F_y = F_N - mg$$

$$ma = F_N - mg$$

$$ma + mg = F_N$$

$$F_N = m(a + g)$$

Part (d) If the elevator's acceleration has a magnitude of g in the downward direction, what would the normal force, F_{N1} be in Newtons?

To find the answer, we can use the same equation that we found in part (c). Since the elevator is accelerating down with a magnitude of g , this means that $a = -g$.

$$F_{N1} = m(a + g)$$

$$F_{N1} = m(-g + g)$$

$$F_{N1} = m(0)$$

$$F_{N1} = 0$$

Part (e) If the elevator's acceleration had a magnitude of g in the upward direction, what would the normal force F_{N2} be in Newtons?

As in part (d), we can use the equation we found in part (c) to solve this. In this case, the elevator will be moving up with an acceleration of g , so $a = g$ for our e

$$F_{N2} = m(a + g)$$

$$F_{N2} = m(g + g)$$

$$F_{N2} = 2mg$$

$$F_{N2} = 2 \cdot 1.25 \text{ kg} \cdot 9.81 \text{ m/s}^2$$

$$F_{N2} = 24.525 \text{ N}$$

Problem 194 - 5.4.3 :

A block with mass $m = 2$ kg is sitting on a horizontal surface and not moving. The free-fall acceleration is $g = 9.81$ m/s². Please answer the following questions.

Part (a) Write an expression for the magnitude of the force of gravity F_g on the block.

The force of gravity is given by the mass multiplied by the acceleration due to gravity.

$$F_g = mg$$

Part (b) Calculate the magnitude of the force of gravity F_g on the block in Newtons.

Here, we need to solve the equation that we found in part (a).

$$F_g = mg$$

$$F_g = 2 \text{ kg} \cdot 9.81 \text{ m/s}^2$$

$$F_g = 19.62 \text{ N}$$

Part (c) In what direction is the force of gravity in this problem?

Since the force of gravity will pull the block towards the center of the Earth, the direction of gravity is

Downwards.

Part (d) What is the magnitude of the normal force F_N in Newtons?

Since the block is at rest, there cannot be any net force on it. As the normal force is the only force pushing the block upwards, it must be equal in magnitude to the two cancel out. Therefore, we can say that the magnitude of the normal force has the same value as the magnitude of the gravitational force we found in part (

$$F_N = F_g = mg$$

$$F_N = 2 \text{ kg} \cdot 9.81 \text{ m/s}^2$$

$$F_N = 19.62 \text{ N}$$

Part (e) In what direction does the normal force act?

Since the normal force in this case is the contact force exerted by the floor to prevent the box from falling through it, the normal force must logically be acting

Upwards.

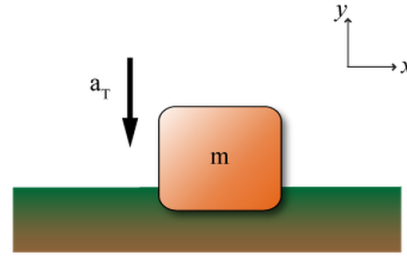
Problem 195 - 5.4.4 :

A block with mass $m = 1$ kg is sinking with constant acceleration $a_T = 0.5$ m/s² into the ground. The acceleration due to gravity is $g = 9.81$ m/s². Please answer the following questions.

Randomized Variables

$$m = 1 \text{ kg}$$

$$a_T = 0.5 \text{ m/s}^2$$



Part (a) Write an expression for the magnitude of the force of gravity on the block, F_g , in terms of the given quantities and variables available in the palette.

Expression :

$$F_g = \underline{\hspace{10em}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \pi, \rho, \theta, a_T, d, F_N, g, h, j, k, m, P, t$

Part (b) Calculate the magnitude of the force of gravity on the block, F_g in Newtons.

Numeric : A numeric value is expected and not an expression.

$$F_g = \underline{\hspace{10em}}$$

Part (c) In what direction does the force of gravity act?

MultipleChoice :

- 1) Downwards.
- 2) Force doesn't have direction.
- 3) All of these choices.
- 4) None of these choices.
- 5) Sideways.
- 6) Upwards.

Part (d) Write an expression for the magnitude of the total force of the system in the y -direction, F_T , in terms of the forces of the system.

Expression :

$$F_T = \underline{\hspace{10em}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \pi, \rho, \theta, a_T, d, F_g, F_N, g, h, j, m, P, t$

Part (e) Write an expression for the magnitude of the normal force F_N , in terms of m , a_T , and g .

Expression :

$$F_N = \underline{\hspace{10em}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \pi, \rho, \theta, a_T, d, F_N, g, h, j, k, m, P, t$

Part (f) What is the magnitude of the normal force in N?

Numeric : A numeric value is expected and not an expression.

$$F_N = \underline{\hspace{10em}}$$

Part (g) In what direction is the normal force?

MultipleChoice :

- 1) Upwards.
- 2) Force does not have direction.
- 3) All of these choices.
- 4) None of these choices.
- 5) Sideways.
- 6) Downwards.

a) $F_g = mg$ b) $F_g \approx 9.81 N$ c) Force of gravity is downwards.

d) The total (net) force on the system is $\Sigma F = F_T = F_N - mg = ma_T$
 System expects $F_T = F_N - mg$ ← actually system wants F_g here.

e) $F_N - F_g = -ma_T$ $F_N = -ma_T + F_g$ $F_N = m(g - a_T)$

f) $F_N \approx 9.31 N$ g) The normal force is directed upwards.

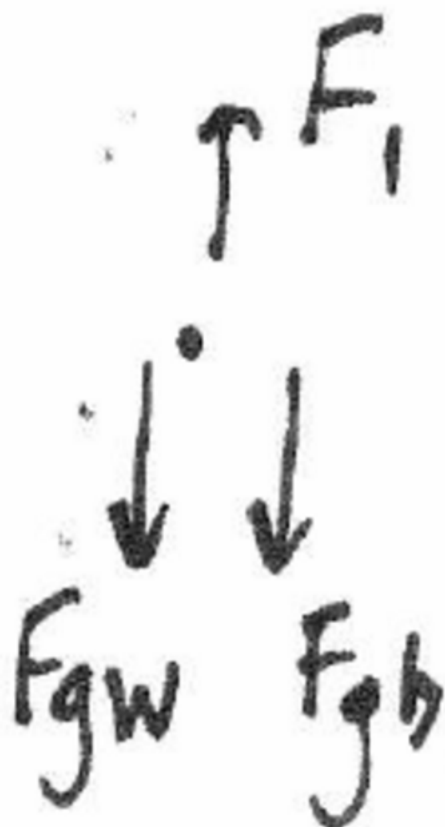
Problem 196 - 5.4.5 :

A farmer is using a rope and pulley to lift a bucket of water from the bottom of a well. The farmer uses a force $F_1 = 45 \text{ N}$ to pull the bucket of water upwards at a constant speed. The bucket, when empty, has a mass of $m_b = 0.9 \text{ kg}$.



Part (a) Identify the correct Free Body Diagram for this situation. F_{gw} is the weight of the water and F_{gb} is the weight of the empty bucket. (Note that they are not necessarily drawn to scale.)

Let's consider the directions of all the forces acting upon the bucket. The weight of the water and the weight of the bucket itself will both act downwards towards the farmer exerts, meanwhile, will act to pull the bucket upwards. The correct free-body diagram will therefore have F_1 pointed upwards and the forces F_{gw} and



Part (b) Calculate the mass of the water in the bucket, m_w in kg.

Since the bucket moves upwards at a constant speed, there is no acceleration. This means that the forces must be balanced. This allows us to write the following equation for the forces in the y-direction and solve for the mass of the water:

$$0 = F_1 - F_{gw} - F_{gb}$$

$$F_{gw} = F_1 - F_{gb}$$

$$m_w g = F_1 - m_b g$$

$$m_w = \frac{F_1 - m_b g}{g}$$

$$m_w = \frac{45 \text{ N} - 0.9 \text{ kg} \cdot 9.81 \text{ m/s}^2}{9.81 \text{ m/s}^2}$$

$$m_w = 3.687 \text{ kg}$$

Part (c) Calculate the volume of the water in the bucket, V_w in cm^3 . Assume the density of the water, ρ_w , is 1.00 g/cm^3 .

We can use the relation between mass, volume, and density to solve for the volume of the water in the bucket. As we do so, we will want to convert our answer for the water from kilograms to grams in order to make sure our units are correct.

$$\rho = \frac{m}{V}$$

$$V\rho = m$$

$$V = \frac{m}{\rho}$$

$$V = \frac{\left(\frac{F_1 - m_b g}{g}\right)}{\rho}$$

$$V = \frac{F_1 - m_b g}{g\rho}$$

$$V = \frac{45 \text{ N} - (0.9 \cdot 10^3) \text{ g} \cdot 9.81 \text{ m/s}^2}{9.81 \text{ m/s}^2 \cdot 1.00 \text{ g/cm}^3}$$

$$V = 3687.156 \text{ cm}^3$$

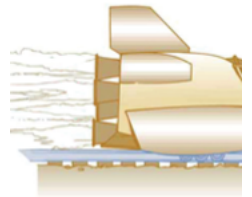
Problem 197 - 5.4.6 :

The rocket sled shown in the figure accelerates in the positive direction at a rate of 45 m/s^2 . Its passenger has a mass of 71 kg .

Randomized Variables

$$a = 45 \text{ m/s}^2$$

$$m = 71 \text{ kg}$$



Part (a) Calculate the horizontal component of the force the seat exerts against his body in N.

In general, the net external force on one object is expressed by

$$F_{net} = \sum_{i=1}^n m a_i \text{ N}$$

where m is the mass of the object in kg, a_i is the i -th acceleration experienced by the object in m/s^2 and n is the total number of accelerations. In this case,

$$n = 1$$

$$F_{net} = m a$$

Plugging in numbers and converting units as needed,

$$F_{net} = F_h = 71 \text{ kg} \cdot 45 \text{ m/s}^2$$

$$F_h = 3195 \text{ N}$$

Part (b) What is the ratio of horizontal force to the force of gravity? Give your answer in terms of times greater than weight.

In general, the weight on one object is expressed by

$$W = mg \text{ N}$$

where m is the mass of the object in kg and g is the acceleration from gravity. From part a,

$$F_h = ma$$

Taking the ratio

$$\frac{F_h}{W} = \frac{ma}{mg} = \frac{a}{g}$$

Plugging in numbers and converting units as needed,

$$\frac{F_h}{w} = \frac{45 \text{ m/s}^2}{9.81 \text{ m/s}^2}$$

$$\frac{F_h}{w} = 4.592$$

Part (c) Calculate angle of net force the seat exerts against his body. Give your answer in degrees from horizontal.

There are two forces acting on the passenger: the force from the rocket and the force from gravity. They each make up the sides of a right triangle with gravity along the vertical and the rocket along the horizontal. These forces are related to the angle the resultant force is with respect to the horizontal by the expression

$$\tan(\theta) = \frac{F_g}{F_h} = \frac{mg}{ma} = \frac{g}{a}$$

where g and a have been defined previously. Therefore,

$$\theta = \arctan\left(\frac{g}{a}\right)$$

Plugging in numbers and converting units as needed,

$$\theta = \arctan\left(\frac{9.81 \text{ m/s}^2}{45 \text{ m/s}^2}\right)$$

$$\theta = 12.298 \text{ deg}$$

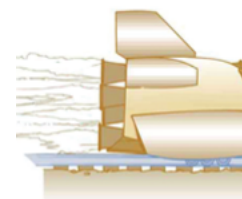
Problem 198 - 5.4.7 :

The rocket sled shown in the figure decelerates at a rate of 150 m/s^2 . Its passenger has a mass of 71 kg . Use a coordinate system in which the sled is moving in the positive direction, and assume the contact forces with the seat are the only forces acting on the passenger.

Randomized Variables

$$a = 150 \text{ m/s}^2$$

$$m = 71 \text{ kg}$$



Part (a) Calculate the horizontal component of the force that the seat exerts against his body, in newtons.

In general, the net external force on one object is expressed by

$$F_{net} = \sum_{i=1}^n ma_i \text{ N}$$

where m is the mass of the object in kg, a_i is the i -th acceleration experienced by the object in m/s^2 and n is the total number of accelerations. In this case,

$$n = 1$$

$$F_{net} = ma$$

Plugging in numbers and converting units as needed (note the acceleration points in the negative direction),

$$F_{net} = F_h = 71 \text{ kg} \cdot -150 \text{ m/s}^2$$

$$F_h = -10650 \text{ N}$$

Part (b) What is the ratio of the magnitude of the horizontal force to the magnitude of the force of gravity?

In general, the weight on one object is expressed by

$$W = mg \text{ N}$$

where m is the mass of the object in kg and g is the acceleration from gravity. From part a, the magnitude of the horizontal force is

$$|F_h| = ma$$

Taking the ratio

$$\frac{|F_h|}{W} = \frac{ma}{mg} = \frac{a}{g}$$

Plugging in numbers and converting units as needed,

$$\frac{F_h}{w} = \frac{150 \text{ m/s}^2}{9.81 \text{ m/s}^2}$$

$$\frac{F_h}{w} = 15.306$$

Part (c) Find the angle of the total force that the seat exerts against his body. Give your answer in degrees from the negative horizontal direction, with po

There are two forces acting on the passenger: the force from the rocket and the force from gravity. They each make up the sides of a right triangle with gravity along the vertical and rocket along the horizontal. These forces are related to the angle the resultant force is with respect to the horizontal by the expression

$$\tan(\theta) = \frac{F_g}{F_h} = \frac{mg}{ma} = \frac{g}{a}$$

where g and a have been defined previously. Therefore,

$$\theta = \arctan\left(\frac{g}{a}\right)$$

Plugging in numbers and converting units as needed,

$$\theta = \arctan\left(\frac{9.81 \text{ m/s}^2}{150 \text{ m/s}^2}\right)$$

$$\theta = 3.742 \text{ deg}$$

Problem 199 - 5.4.8 :

The weight of an astronaut plus his space suit on the Moon is only **225 N**.

Randomized Variables

$$W = 225 \text{ N}$$

Part (a) How much do they weigh on Earth, in newtons, assuming the acceleration due to gravity on the moon is 1.67 m/s^2 ?

In general, the weight on one object is expressed by

$$W = mg \text{ N}$$

where m is the mass of the object in kg and g is the acceleration from gravity. Therefore, on the moon

$$W_m = mg_m$$

where g_m is the gravity on the moon. The mass is

$$m = \frac{W_m}{g_m}$$

of which he would weigh on earth

$$W_e = mg_e = \frac{W_m}{g_m} g_e$$

where g_e is the acceleration of gravity on earth. Plugging in numbers and converting units as needed,

$$W_e = W_{Earth} = \frac{225 \text{ N}}{1.67 \text{ m/s}^2} \cdot 9.81 \text{ m/s}^2$$

$$W_{Earth} = 1320.359 \text{ N}$$

Part (b) What is the mass of the astronaut and his space suit on the Moon, in kilograms?

In general, the weight on one object is expressed by

$$W = mg \text{ N}$$

where m is the mass of the object in kg and g is the acceleration from gravity. Therefore, on the moon

$$W_m = mg_m$$

where g_m is the gravity on the moon. The mass is

$$m = \frac{W_m}{g_m}$$

Plugging in numbers and converting units as needed,

$$m = m_{Moon} = \frac{225 \text{ N}}{1.67 \text{ m/s}^2}$$

$$m_{Moon} = 134.731 \text{ kg}$$

Part (c) What is the mass of the astronaut and his spacesuit on the Earth, in kilograms?

In general, the weight on one object is expressed by

$$W = mg \text{ N}$$

where m is the mass of the object in kg and g is the acceleration from gravity. Therefore, on the earth

$$W_e = mg_e$$

where g_e is the gravity on the earth. The mass is

$$m = \frac{W_e}{g_e}$$

Plugging in numbers and converting units as needed,

$$m = m_{Earth} = \frac{225 \text{ N}}{1.67 \text{ m/s}^2} \cdot \frac{9.81 \text{ m/s}^2}{9.81 \text{ m/s}^2}$$

$$m_{Earth} = 134.731 \text{ kg}$$

Problem 200 - 5.4.9 :

Suppose the mass of a fully loaded module in which astronauts take off from the Moon is **7500** kg. The thrust of its engines is **27500** N.

Randomized Variables

$$m = 7500 \text{ kg}$$

$$f = 27500 \text{ N}$$

Part (a) Calculate the magnitude of its acceleration in a vertical takeoff from the Moon in meters per square second, assuming the acceleration due to gravity is 1.67 m/s^2 .

In general, the net external force on one object is expressed by

$$F_{net} = \sum_{i=1}^n ma_i \text{ N}$$

where m is the mass of the object in kg, a_i is the i -th acceleration experienced by the object in m/s^2 and n is the total number of accelerations. In this case,

$$n = 2$$

$$F_{net} = ma_{net} = F_r - F_m = F_r - mg_m$$

where F_r is the force of the rocket in N, m is the mass in kg, and g_m is the acceleration of gravity on the moon in m/s^2 . Therefore,

$$a_{net} = \frac{F_r}{m} - g_m$$

Plugging in numbers and converting units as needed,

$$a_{net} = a = \frac{27500 \text{ N}}{7500 \text{ kg}} - 1.67 \text{ m/s}^2$$

$$a = 1.997 \text{ m/s}^2$$

Problem 201 - 5.4.9 (alt) :

Full solution not currently available at this time.

Suppose the mass of a fully loaded module in which astronauts take off from the Moon is $m = 1.01 \times 10^4$ kg. The thrust of the engines is $F_T = 3.01 \times 10^4$ N. The acceleration due to gravity on the Moon is $g_{Moon} = 1.62 \frac{m}{s^2}$, and the acceleration due to gravity on Earth is $g_{Earth} = 9.81 \frac{m}{s^2}$.

Part (a) Considering the weight w of the spacecraft system, as well as the variables mentioned in the problem statement, what is the force balance the shi launch? Treat up as the positive direction for your equation.

$$F_{\text{net}} = F_T - w$$

Part (b) Calculate the module's magnitude of acceleration in $\frac{m}{s^2}$ during a vertical takeoff from the Moon.

$$\begin{aligned} a &= FT/m - 1.62 \\ a &= 3.01/1.01 - 1.62 \\ a &= 1.36 \\ \text{Tolerance: } &\pm 0.0408 \end{aligned}$$

Part (c) Calculate the module's magnitude of acceleration in $\frac{m}{s^2}$ during a vertical takeoff if the spacecraft were on Earth.

$$\begin{aligned} a &= 9.81 - FT/m \\ a &= 9.81 - 3.01/1.01 \\ a &= 6.83 \\ \text{Tolerance: } &\pm 0.2049 \end{aligned}$$

Part (d) True or False. The rocket has enough thrust, F_T , to take off from the Earth.

FALSE

Problem 202 - 5.4.10 (alt) :

What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at 7.50 m/s^2 ? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

Solution
$$F_{\text{net}} = +F_t - mg = ma \Rightarrow F_t = m(a + g) = (45.0 \text{ kg})(7.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = \underline{779 \text{ N}}$$

Problem 203 - 5.4.10 :

A gymnast with a mass of 41-kg jumps on a trampoline.

Part (a) What force does a trampoline have to apply to accelerate her straight up at 7.1 m/s^2 in Newtons?

In general, the net external force on one object is expressed by

$$F_{\text{net}} = \sum_{i=1}^n ma_i \text{ N}$$

where m is the mass of the object in kg, a_i is the i -th acceleration experienced by the object in m/s^2 and n is the total number of accelerations. In this case,

$$n = 2$$

$$F_{\text{net}} = ma_{\text{net}} = F_t - F_e = F_t - mg_e$$

where F_t is the force of the trampoline in N, m is the mass in kg, and g_e is the acceleration of gravity on the earth in m/s^2 . Therefore,

$$F_t = ma_{\text{net}} + mg_e = m(a_{\text{net}} + g_e)$$

Plugging in numbers and converting units as needed,

$$F_t = 41 \text{ kg} \cdot (7.1 \text{ m/s}^2 + 9.81 \text{ m/s}^2)$$

$$F_t = 692.9 \text{ N}$$

Problem 204 - 5.4.11 :

Unreasonable Results (a) What is the initial acceleration of a rocket that has a mass of 1.50×10^6 kg at takeoff, the engines of which produce a thrust of 2.00×10^6 N? Do not neglect gravity. (b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.) (c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)

Solution (a) $\text{net } F = ma = T - mg \Rightarrow$

$$a = \frac{T - mg}{m} = \frac{2.00 \times 10^6 \text{ N} - (1.50 \times 10^6 \text{ kg})(9.80 \text{ m/s}^2)}{1.50 \times 10^6 \text{ kg}} = \underline{-8.47 \text{ m/s}^2}$$

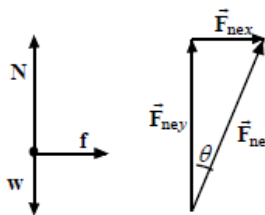
(b) There is not enough thrust to take off ($a < 0$).

(c) The thrust is not large enough for the given rocket mass to leave the ground.

Problem 205 - 5.4.12 :

A flea jumps by exerting a force of 1.20×10^{-5} N straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of 0.500×10^{-6} N on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is 6.00×10^{-7} kg. Do not neglect the gravitational force.

Solution



$$\text{net } F_y = N - w = N - mg = 1.20 \times 10^{-5} \text{ N} - (6.00 \times 10^{-7} \text{ kg})(9.80 \text{ m/s}^2) = 6.12 \times 10^{-6} \text{ N}$$

$$\text{net } F_x = f = 0.500 \times 10^{-6} \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{\text{net } F_x}{\text{net } F_y} \right) = 4.67^\circ$$

$$\text{net } F = \sqrt{(\text{net } F_x)^2 + (\text{net } F_y)^2} \\ = \left[(0.500 \times 10^{-6} \text{ N})^2 + (6.12 \times 10^{-6} \text{ N})^2 \right]^{1/2} = 6.14 \times 10^{-6} \text{ N, so that}$$

Problem 206 - 5.4.13 :

Consider the following table, which lists the four fundamental forces in our universe. It gives the quantum particles which carry the force, the relative strength (relative to the strongest, the strong nuclear force), the range, and the

direction the force acts in. Whenever we talk about the relative strengths of these forces, we are actually talking about approximate strengths only, and usually for the current epoch of the universe. In the early universe, we expect that (except for the gravitational force), the forces all had the same strength.

Force	Carrier Particle	Relative Strength
Gravitational	Graviton (hypothetical)	10^{-38}
Electromagnetic	Photon	10^{-2}
Weak Nuclear	W and Z Bosons	10^{-13}
Strong Nuclear	Gluon	1

Part (a) What is the strength of the weak nuclear force relative to the strong nuclear force?

From the table, the Relative Strength column has the weak nuclear force relative to the strong nuclear force. Therefore,

$$\frac{F_w}{F_s} = 10^{-13}$$

$$\frac{F_w}{F_s} = 1E - 13$$

Part (b) What is the strength of the weak nuclear force relative to the electromagnetic force?

From the table, the Relative Strength column has the weak nuclear force relative to the strong nuclear force and the electromagnetic force relative to the strong nuclear force.

$$\frac{F_w}{F_s} = 10^{-13}$$

and

$$\frac{F_{em}}{F_s} = 10^{-2}$$

Taking the ratio

$$\frac{\left(\frac{F_w}{F_s}\right)}{\left(\frac{F_{em}}{F_s}\right)} = \frac{F_w}{F_{em}}$$

Plugging in numbers and converting units as needed,

$$\frac{F_w}{F_{em}} = \frac{10^{-13}}{10^{-2}}$$

$$\frac{F_w}{F_{em}} = 1E - 11$$

Problem 207 - 5.4.14 :

Consider the following table, which lists the four fundamental forces in our universe. Whenever we talk about the relative strengths of these forces, we are actually talking about approximate strengths only, and usually for the current epoch of the universe. In the early universe, we expect that (except for the gravitational force), the forces all had the same strength. This problem will explore these relative strengths.

Force	Carrier Particle	Relative Strength
Gravitational	Graviton (hypothetical)	10^{-38}
Electromagnetic	Photon	10^{-2}
Weak Nuclear	W and Z Bosons	10^{-13}
Strong Nuclear	Gluon	1

Part (a) What is the ratio of the strength of the gravitational force to that of the strong nuclear force?

From the table, the Relative Strength column has the gravitational force relative to the strong nuclear force. Therefore,

$$\frac{F_g}{F_s} = 10^{-38}$$

$$\frac{F_g}{F_s} = 1E - 38$$

Part (b) What is the ratio of the strength of the gravitational force to that of the weak nuclear force?

From the table, the Relative Strength column has the weak nuclear force relative to the strong nuclear force and the gravitational force relative to the strong nucle

$$\frac{F_w}{F_s} = 10^{-13}$$

and

$$\frac{F_g}{F_s} = 10^{-38}$$

Taking the ratio

$$\frac{\left(\frac{F_g}{F_s}\right)}{\left(\frac{F_w}{F_s}\right)} = \frac{F_g}{F_w}$$

Plugging in numbers and converting units as needed,

$$\frac{F_g}{F_w} = \frac{10^{-38}}{10^{-13}}$$

$$\frac{F_g}{F_w} = 1E - 25$$

Part (c) What is the ratio of the strength of the gravitational force to that of the electromagnetic force?

From the table, the Relative Strength column has the electromagnetic force relative to the strong nuclear force and the gravitational force relative to the strong nu

$$\frac{F_{em}}{F_s} = 10^{-2}$$

and

$$\frac{F_g}{F_s} = 10^{-38}$$

Taking the ratio

$$\frac{\left(\frac{F_g}{F_s}\right)}{\left(\frac{F_{em}}{F_s}\right)} = \frac{F_g}{F_{em}}$$

Plugging in numbers and converting units as needed,

$$\frac{F_g}{F_{em}} = \frac{10^{-38}}{10^{-2}}$$

$$\frac{F_g}{F_{em}} = 1E-36$$

Part (d) What do your answers imply about the influence of the gravitational force on atomic nuclei?

The gravitational force is very weak in comparison to the other forces so doesn't affect atomic nuclei much.

Problem 208 - 5.4.15 :

Full solution not currently available at this time.

In building a house, carpenters use nails from a large box. The box is suspended from a spring twice during the day to measure the usage of nails. At the beginning of the day, the spring stretches **40.1** cm. At the end of the day, the spring stretches **20.1** cm.

Part (a) What fraction of the nails have been used?

Fraction of nails used = $1 - d_2/d_1$
 Fraction of nails used = $1 - 20.1/40.1$
 Fraction of nails used = **0.4988**
 Tolerance: ± 0.014964

Problem 209 - 5.4.16 :

Full solution not currently available at this time.

An airborne body of mass $m = 1.1$ -kg is pushed straight upward by a vertical force $F_v = 2.1$ N.

Part (a) What is the force balance in the vertical direction? Let up be the positive direction and use the force mentioned in the problem statement and th write your answer.

$$F_{\text{net}} = F_v - w$$

Part (b) What is the magnitude of the net force exerted on the object in Newtons?

$F_{\text{net}} = ((F_v - m * 9.81)^2)^{0.5}$
 $F_{\text{net}} = ((2.1 - 1.1 * 9.81)^2)^{0.5}$
 $F_{\text{net}} = 8.691$
 Tolerance: ± 0.26073

Part (c) What is the acceleration of the body in

$\frac{m}{s^2}$?

$a = (F_v - m * 9.81)/m$
 $a = (2.1 - 1.1 * 9.81)/1.1$
 $a = -7.901$
 Tolerance: ± 0.23703

Problem 210 - 5.4.17 :**Full solution not currently available at this time.**

A car weighing $w = 10001$ N starts from rest and accelerates with acceleration a to $v_f = 70.1 \frac{km}{h}$ in time $t = 5.1$ seconds. The force resisting its motion is $f = 1001$ N.

Part (a) What is the definition of weight in scalar form? Use g for the acceleration due to gravity.

$$w = m g$$

Part (b) What is the mass of the car in kg?

$$m = w/9.81$$

$$m = 10001/9.81$$

$$m = 1019.47$$

$$\text{Tolerance: } \pm 30.5841$$

Part (c) Finish the following equation for final velocity v_f using variables from the problem statement and remembering that the car begins at rest.

$$v_f = a t$$

Part (d) What is the acceleration of the vehicle in

$$\frac{m}{s^2} ?$$

$$a = v_f * 1000/3600/t$$

$$a = 70.1 * 1000/3600/5.1$$

$$a = 3.818$$

$$\text{Tolerance: } \pm 0.11454$$

Part (e) Let the car engine provide a force F in the positive direction. Write an equation for the net horizontal force based on F and the force variable f

$$F_{\text{net}} = F - f$$

Part (f) What is the force F applied by the engine in Newtons?

$$F = (w/9.81) * (v_f * 1000/3600/t) + f$$

$$F = (10001/9.81) * (70.1 * 1000/3600/5.1) + 1001$$

$$F = 4893.421$$

$$\text{Tolerance: } \pm 146.80263$$

Problem 211 - 5.4.18 :**Full solution not currently available at this time.**

Jennifer, a $m = 40.1$ kg gymnast, jumps on a trampoline which provides force F_T and accelerates her straight up with acceleration $a = 7.01 \frac{m}{s^2}$.

Part (a) Write an expression for the net force Jennifer experiences using variables from the problem statement and w for her weight.

$$F_{\text{net}} = F_T - w$$

Part (b) What is the force, F_T , applied to Jennifer by the trampoline? Give your answer in Newtons.

$$F_T = m * (a + 9.81)$$

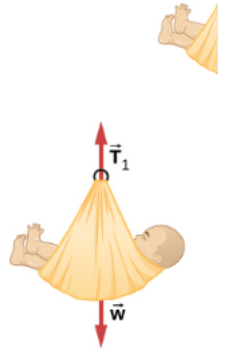
$$F_T = 40.1 * (7.01 + 9.81)$$

$$F_T = 674.482$$

$$\text{Tolerance: } \pm 20.23446$$

Problem 212 - 5.4.19 :**Full solution not currently available at this time.**

Consider the baby being weighed in the top figure. The scale weighing the infant and the basket reads $\vec{w} = 50.1$ N.



Part (a) What is the mass m of the infant and basket in kg based on the scale reading?

$$m = w/9.81$$

$$m = 50.1/9.81$$

$$m = 5.107$$

$$\text{Tolerance: } \pm 0.15321$$

Part (b) What is the tension T_1 in Newtons in the cord attaching the baby to the scale?

$$T_1 = w$$

$$T_1 = 50.1$$

$$T_1 = 50.1$$

$$\text{Tolerance: } \pm 1.503$$

Part (c) Write an equation for tension T_2 in the cord attaching the scale to the ceiling. Consider the scale mass to be m_{scale} .

$$T_2 = T_1 + m_{\text{scale}} g$$

Part (d) What is the tension T_2 in Newtons of the cord attaching the scale to the ceiling if the scale has a mass of $m_{\text{scale}} = 0.2$ kg?

$$T_2 = w + m_{\text{scale}} * 9.81$$

$$T_2 = 50.1 + 0.2 * 9.81$$

$$T_2 = 52.062$$

$$\text{Tolerance: } \pm 1.56186$$

Problem 213 - 5.4.20 :

Full solution not currently available at this time.

Suppose Kevin, an $m = 55$ kg gymnast climbs a rope.

Part (a) Write an equation for the tension in the rope T during Kevin's constant velocity climb.

$$T = m g$$

Part (b) What is the value of tension in the rope T in Newtons while Kevin climbs at constant speed?

$$T = m * 9.81$$

$$T = 55 * 9.81$$

$$T = 539.55$$

$$\text{Tolerance: } \pm 16.1865$$

Part (c) If Kevin has an acceleration a while climbing, write an equation for F_{net} in terms of tension T and weight w . For your equation, let up be the positive direction.

$$F_{\text{net}} = T - w$$

Part (d) What is the value of tension T in the rope in Newtons if Kevin accelerates with $a = 1.01$

$\frac{m}{s}$?

$$T = m * (9.81 + a)$$

$$T = 55 * (9.81 + 1.01)$$

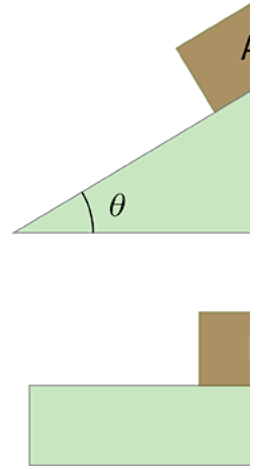
$$T = 595.1$$

$$\text{Tolerance: } \pm 17.853$$

Problem 214 - 5.4.21 :

Full solution not currently available at this time.

Crates A and B have equal mass. Crate A is at rest on an incline that makes an angle of $\theta = 20.1$ degrees to horizontal, while crate B is at rest on a horizontal surface.



Part (a) Write an expression for the ratio of the normal forces, A to B, in terms of θ .

$$N_A/N_B = \cos(\theta)$$

Part (b) What is the ratio of the normal forces, A to B?

$$N_A/N_B = \cos(20.1)$$

$$N_A/N_B = 0.9391$$

$$N_A/N_B = 0.9391$$

$$\text{Tolerance: } \pm 0.028173$$

Part (c) In which case is the normal force greater?

B

Problem 215 - 5.4.23 :

Full solution not currently available at this time.

Two identical springs, A and B, each with spring constant $k = 20.1$ N/m, support an object with a weight $W = 10.1$ N. Each spring makes an angle of $\theta = 18$ degrees to vertical, as shown in the diagram.



Part (a) Write an expression for the tension in spring A (which is equal to the tension in spring B) in terms of W and θ .

$$T = W / (2 \cos(\theta))$$

Part (b) By how much is spring A stretched, in meters?

$$x = w / (2 * k * \cos(\theta * 0.0174533))$$

$$x = 10.1 / (2 * 20.1 * \cos(18 * 0.0174533))$$

$$x = 0.2642$$

$$\text{Tolerance: } \pm 0.007926$$

Problem 216 - c5.5.1 :

A car and a dump truck are involved in an accident and crash into each other.

Part (a) Assuming the only force acting is the force of collision, which one experiences the most force during the crash - the car or the dump truck?

Assuming the only force acting is the car-truck interaction, they experience the same force.

Problem 217 - c5.5.2 :

A car and a dump truck are involved in an accident and crash into each other.

Part (a) Which one will have the greatest magnitude of acceleration during the crash? (Assume the dump truck has a mass that is 4 times larger than the car. The problem are those between the two vehicles.)

Newton's Third Law tells us that the force exerted on both the car and the truck will be equal. Newton's Second Law, meanwhile, gives us the following equation

$$F = ma$$

This means that the mass of the vehicle times the acceleration it experiences gives the force. Since both vehicles experience the same force, and the truck has four times the mass of the car, the acceleration the truck experiences must be one-fourth as high as that of the car. Therefore, the car experiences a greater magnitude of acceleration during

Problem 218 - c5.5.3 :

Newton's third law can be summarized as "every action has an equal and opposite reaction". In this problem, consider the action of the entire Earth on a person standing at rest on the ground and the opposite reaction, where 'action' and 'reaction' are understood to mean 'force' and 'reaction force'

Part (a) What is the "equal and opposite" reaction force to the gravitational force of the Earth acting on the person?

To find the "equal and opposite" reaction force to any force, simply switch the roles of the object exerting the force and the object that the force is acting on. In the "equal and opposite" reaction force to the force of the Earth acting on the person is the force of the person acting on the Earth. So the correct choice is "The gravitational force of the person pulling on the Earth."

Yes, the person pulls the Earth with the same amount of force as the Earth pulls the person. However, according to Newton's second law, the Earth is affected by a much smaller acceleration than the person is, since its mass is so much greater than the person's.

Problem 219 - c5.5.4 :**Full solution not currently available at this time.**

A man has a weight of 735 N, and the ground pushes up on him with 735 N.

Part (a) What can be said about these two forces?

The forces have equal magnitude.

Problem 220 - c5.5.5 :**Full solution not currently available at this time.**

An astronaut on the Moon jumps upward.

Part (a) Which of the following is true during this action?

Both the Moon and the astronaut accelerate away from each other, but with accelerations of vastly different magnitudes.

Problem 221 - c5.5.6 :**Full solution not currently available at this time.**

A force is specified in each part of this problem. Identify the force that is equal in magnitude, but opposite in direction to the specified force due to Newton's Third Law.

Part (a) The attractive force exerted on the Moon by the Earth (due to gravity)

The attractive force exerted on the Earth by the Moon

Part (b) The force a boy's hand exerts on a baseball while he is throwing it to a teammate

The force the baseball exerts on the boy's hand

Part (c) The force that a spinning boat propeller exerts on the water

The force that the water exerts on the boat's spinning propeller

Part (d) The upward force exerted by the ground on a police officer standing on a street corner

The force with which the police officer pushes down on the ground

Problem 222 - c5.5.7 :**Full solution not currently available at this time.**

You are holding a cup of coffee. Consider the following forces: (1) The weight of the cup of coffee, (2) the force that your hand exerts on the coffee cup, and (3) the force the coffee cup exerts on your hand.

Part (a) Which two of these forces have equal magnitude, but opposite direction because of Newton's Third Law?

2 and 3

Part (b) Which two of these forces have equal magnitude, but opposite direction because of Newton's Second (or First) Law?

1 and 2

Problem 223 - c5.5.8 :

Full solution not currently available at this time.

Consider the following forces that arise when a rifle is fired: (1) The exploding charge (gunpowder) pushes forward on the bullet, (2) the exploding charge pushes backward on the rifle, (3) the bullet pushes backward on the exploding charge, and (4) the rifle pushes forward on the exploding charge.

Part (a) Which force is equal in magnitude, but opposite in direction to (2) because of Newton's Third Law?

4

Part (b) Which force is equal in magnitude, but opposite in direction to (3) because of Newton's Third Law?

1

Problem 224 - 5.5.1 :

A teenager of mass $m_1 = 51$ kg pushes backward against the ground with his foot as he rides his skateboard. This exerts a horizontal force of magnitude $F_{foot} = 10.5$ N. The skateboard has $m_2 = 2.1$ kg.

Randomized Variables

$$F_{foot} = 10.5 \text{ N}$$

$$m_1 = 51 \text{ kg}$$

$$m_2 = 2.1 \text{ kg}$$

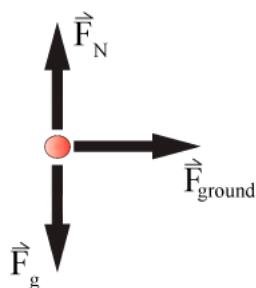
Part (a) Write an expression for the magnitude of the horizontal component of force that the ground exerts on the teenager's foot, F_{ground} .

Newton's Third Law tells us that the force exerted by the ground will be equal and opposite the force exerted on the ground. This means that the magnitudes of the force exerted by the ground and the force exerted by the ground are equal.

$$F_{ground} = F_{foot}$$

Part (b) Choose the correct free-body diagram for the system made up of teenager and his skateboard. F_N is the normal force and F_g is the weight of the skateboard.

For this diagram, we want to include all the forces acting on the system consisting of the teenager and the skateboard. This means that the correct free-body diagram includes the normal force, the force of gravity, and the force that the ground exerts back on the skateboard. While we did talk about the force the teenager's foot exerts on the ground, that force is acting on the ground rather than our system so we should not include it in our free-body diagram. The correct diagram is the only one that contains the



Part (c) Write an expression in terms of given quantities for the magnitude of the skateboard's acceleration, a , while the teenager is pushing backwards.

Newton's Second Law gives us the following equation:

$$F = ma$$

Since we are looking only at forces in the horizontal direction, the only force we need to concern ourselves with is the force the ground exerts on the skateboard. Meanwhile, the force of gravity, the force of gravity on the skateboarder and his board. As we found in part (a) that the force exerted by the ground will have the same magnitude as the force exerted by the teenager's foot, we can now plug in variables and solve for the acceleration.

$$F_{\text{foot}} = (m_1 + m_2) a$$

$$a = \frac{F_{\text{foot}}}{m_1 + m_2}$$

Part (d) What is the numerical value for the magnitude of the acceleration, a , in m/s^2 ?

Here, we simply need to plug numbers into the equation we found in part (c) and solve.

$$a = \frac{F_{\text{foot}}}{m_1 + m_2}$$

$$a = \frac{10.5 \text{ N}}{51 \text{ kg} + 2.1 \text{ kg}}$$

$$a = 0.1977 \text{ m/s}^2$$

Problem 225 - 5.5.2 :

A construction worker hits a chunk of concrete with a sledgehammer. The sledgehammer delivers a force of 750 lbs and breaks the concrete.

Part (a) With what force does the concrete act back on the sledgehammer during the impact?

Newton's Third Law tells us that when a force is exerted, a force with opposite direction and equal magnitude is exerted back. This means that the force the sledgehammer exerts on the concrete is the same as the force the concrete exerts on the sledgehammer.

$$750 \text{ lbs.}$$

Problem 226 - 5.5.3 :

An aircraft carrier uses a device called a catapult to help accelerate jets to the speed needed for take off. The flight decks on these carriers have length $d = 71 \text{ m}$. A jet with a mass of $m = 10001 \text{ kg}$ can be accelerated from rest to a speed of $v = 41 \text{ m/s}$ by the end of the flight deck.

Randomized Variables

$$d = 71 \text{ m}$$

$$m = 10001 \text{ kg}$$

$$v = 41 \text{ m/s}$$

Part (a) Select the correct symbolic expression for the magnitude of the average net force, F_L , acting on the jet during its launch.

Let's start with the equation given by Newton's Second Law.

$$F_L = ma$$

To employ this equation, we need an expression for the acceleration of the jet. To find it, we will have to use a kinematic equation. We know the distance the catapult travels, the final velocity of the jets, and the fact that the jets begin from rest. With this information, we can set up the following kinematic equation:

$$v^2 = v_0^2 + 2ad$$

Since we start from rest, we can set the v_0 term equal to zero to remove it. From there, we can rewrite this equation into an expression for the acceleration.

$$v^2 = 2ad$$

$$\frac{v^2}{2d} = a$$

Now we can take this expression for the acceleration and plug it into Newton's Second Law in order to find a solution.

$$F_L = m \left(\frac{v^2}{2d} \right)$$

$$F_L = \frac{mv^2}{2d}$$

Part (b) Calculate the numerical value of the magnitude of the force F_L in Newtons?

Here, we simply need to plug in values and solve the equation we found in part (a).

$$F_L = \frac{mv^2}{2d}$$

$$F_L = \frac{10001 \text{ kg} \cdot (41 \text{ m/s})^2}{2 \cdot 71 \text{ m}}$$

$$F_L = 118392.12 \text{ N}$$

Part (c) What is the numerical value of the ratio R of the launch force F_L to the jet's weight?

The ratio of the force to the weight will be given by the force divided by the weight. The weight is given by the mass multiplied by the force of gravity:

$$w = mg$$

Now, we use our results from part (a) to write an expression for this ratio.

$$R = \frac{F_L}{w}$$

$$R = \frac{\left(\frac{mv^2}{2d} \right)}{mg}$$

$$R = \frac{v^2}{2dg}$$

$$R = \frac{(41 \text{ m/s})^2}{2 \cdot 71 \text{ m} \cdot 9.81 \text{ m/s}^2}$$

$$R = 1.207$$

Problem 227 - 5.5.4 :

A **1020** kg artillery shell is fired from a battleship. While it is in the barrel of the gun, it experiences an acceleration of **2.1** $\times 10^4$ m/s².

Randomized Variables

$$m = 1020 \text{ kg}$$

$$a = 2.1 \times 10^4 \text{ m/s}^2$$

Part (a) What net force is exerted on the artillery shell before it leaves the barrel of the gun (in Newtons)?

We can use Newton's Second Law to find the relation between the net force on the shell and the mass and acceleration of the shell.

$$F_{net} = ma$$

$$F_{net} = 1020 \text{ kg} \cdot 2.1 \cdot 10^4 \text{ m/s}^2$$

$$F_{net} = 21420000 \text{ N}$$

Part (b) What is the magnitude of the force exerted on the ship by the artillery shell in Newtons?

Newton's Third Law tells us that the force exerted by the shell on the ship will have the same magnitude and opposite direction of the force exerted by the ship on the force on the ship is therefore the same as the net force on the shell that we found in part (a).

$$|F_{ship}| = 21420000 \text{ N}$$

Problem 228 - 5.5.5 :

A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of **720** N on him. The mass of the losing player plus equipment is **82** kg, and he is accelerating at **1.2** m/s² backward.

Randomized Variables

$$f = 720 \text{ N}$$

$$m_1 = 82 \text{ kg}$$

$$m_2 = 102 \text{ kg}$$

$$a = 1.2 \text{ m/s}^2$$

Part (a) What is the magnitude of the force of friction, in newtons, between the losing player's feet and the grass as he slides backwards?

Newton's Second Law tells us the following:

$$F_{net} = ma$$

In this case, the net force will be given by the force exerted on the rugby player minus the force of friction. We can now set up an equation to solve for the force of friction.

$$F - F_1 = ma$$

$$-F_1 = ma - F$$

$$F_1 = F - ma$$

$$F_1 = 720 \text{ N} - 82 \text{ kg} \cdot 1.2 \text{ m/s}^2$$

$$F_1 = 621.6 \text{ N}$$

Part (b) What is the magnitude of the force, in newtons, that the winning player exerts on the ground to move forward if his mass plus equipment is **102 kg?**

player has the same acceleration as the first.

Let's begin this problem by thinking about Newton's Third Law. If the winning player is exerting 720 newtons of force on the losing player, then the losing player exerts 720 newtons of force back on the winning player. With this in mind, let's now use Newton's Second Law.

$$F_{net} = ma$$

In this case, the net force will be the force the winning player is exerting on the ground minus the force exerted on him by the losing player.

$$F_2 - F = ma$$

$$F_2 = ma + F$$

$$F_2 =$$

$$F_2 = 842.4 \text{ N}$$

Problem 229 - 5.5.6 :

Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of **64** kg and exerts an average force of **1345** N horizontally, on the ground. Each of the second team's members has an average mass of **71** kg and exerts an average force of **1360** N horizontally, on the ground.

Part (a) What is the magnitude of acceleration of the two teams in meters per square second?

To solve this problem, we will need to use Newton's Second Law.

$$F_{net} = ma$$

In this case, the net force will be the sum of all the forces exerted by members of the second team minus the sum of all the forces exerted by members of the first team. The net force will be given by the average mass of team one multiplied by the number of members plus the average mass of the second team multiplied by the number of members. The following equation:

$$(9 \cdot F_2 - 9 \cdot F_1) = (9 \cdot m_1 + 9 \cdot m_2) a$$

$$9(F_2 - F_1) = 9(m_1 + m_2) a$$

$$\frac{F_2 - F_1}{m_1 + m_2} = a$$

$$a = \frac{1360 \text{ N} - 1345 \text{ N}}{64 \text{ kg} + 71 \text{ kg}}$$

$$a = 0.1111 \text{ m/s}^2$$

Part (b) What is the tension in the rope between the teams, in newtons?

To solve for the tension in the rope, let's consider the forces acting on the first team. Each of the nine members are pulling backwards with a force of 1345 newtons, and are pulled forward by the force of tension in the rope. The net force on the first team is therefore given by the force of tension minus the force exerted by the members. We can use this information to apply Newton's Second Law to the first team and find the tension in the rope.

$$F_{net} = ma$$

$$T - 9F_1 = 9m_1 a$$

$$T = 9F_1 + 9m_1 + a$$

We can now plug in our results for acceleration from part (a).

$$T = 9F_1 + 9m_1 \cdot \frac{F_2 - F_1}{m_1 + m_2}$$

$$T = 9 \cdot 1345 \text{ N} + 9 \cdot 64 \text{ kg} \cdot \frac{1360 \text{ N} - 1345 \text{ N}}{64 \text{ kg} + 71 \text{ kg}}$$

$$T = 12169 \text{ N}$$

Problem 230 - 5.5.7 :

Consider a **65** kg high-jumper.

Part (a) Calculate the magnitude of the force, in newtons, the jumper must exert on the ground to produce an upward acceleration **4.00** times the acceler

Let's begin with a statement of Newton's Second Law.

$$F_{net} = ma$$

Now, the net force acting on the jumper will be the force she exerts on the ground to move upward minus the force gravity exerts downward on her. The force that can also be found using Newton's Second Law; the force of gravity must be equal to the mass of the jumper times the acceleration due to gravity. With this information, we can use the equation to solve for the force of the jump.

$$F - mg = m(4.00 \cdot g)$$

$$F = 4.00 \cdot mg + mg$$

$$F = 5.00 \cdot mg$$

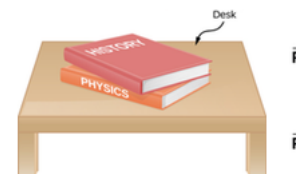
$$F = 5.00 \cdot 65 \text{ kg} \cdot 9.8 \text{ m/s}^2$$

$$F = 3185 \text{ N}$$

Problem 231 - 5.5.9 :

Full solution not currently available at this time.

A history book is lying on top of a physics book on a desk, as displayed on the left; a free-body diagram is also shown on the right. The history and physics textbooks weigh $F_h = 10.1 \text{ N}$ and $F_p = 10.1 \text{ N}$ respectively.



Part (a) Look at the free body diagram for the history book. Consider the normal force of the physics book force vector, \vec{F}_{ph} acting upwards to the history book to be positive. Using knowledge of Newton's third law, what is \vec{F}_{ph} equal to in terms of variables depicted in the diagram?

$$F_{ph} = -F_{eh}$$

Part (b) If

\vec{F}_{ph} is the normal force holding up the history textbook, what is its magnitude in Newtons?

$$F_{ph} = F_h$$

$$F_{ph} = 10.1$$

Tolerance: ± 0.303

Part (c) What is the magnitude of the force exerted on the physics book by Earth's gravity

\vec{F}_{ep} in Newtons?

$$F_{ep} = F_p$$

$$F_{ep} = 10.1$$

Tolerance: ± 0.303

Part (d) What is the magnitude of the force that the history book acts down upon the physics book with,

\vec{F}_{hp} ? Give your answer in Newtons.

$$F_{hp} = F_h$$

$$F_{hp} = 10.1$$

Tolerance: ± 0.303

Part (e) What is the normal force with which the desk acts up on the physics book,

\vec{F}_{dp} , in Newtons?

$$F_{dp} = F_h + F_p$$

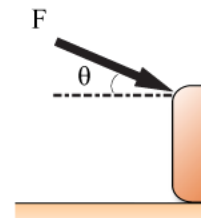
$$F_{dp} = 10.1 + 10.1$$

$$F_{dp} = 20.2$$

Tolerance: ± 0.606

Problem 232 - c5.6.1 :

A box rests on a horizontal surface. You apply a force on the box of $F = 100$ N at an angle, θ , below the horizontal and it slides at a constant velocity.



Part (a) The friction force that acts on the box is:

In order for the box to move a constant velocity, there can be no acceleration. Since the net force is equal to acceleration times mass, this means that there is no net force on the box, we can see that the force of friction must be equal in magnitude to the horizontal component of the force exerted on the box. Using trigonometry, we see that the formula for the force pushing the box in the horizontal direction is equal to:

$$\cos(\theta) = \frac{F_x}{F}$$

$$F \cos(\theta) = F_x$$

Unless $\theta = 0$, then the horizontal component of the force is going to be less than the 100 newton force with which the box is pushed diagonally down. Since the magnitude of the horizontal force on the box, the force of friction must also be less than 100 newtons. The answer is therefore:

less than 100 Newtons.

Problem 233 - c5.6.2 :

A ball is launched directly upward and ultimately reaches a height of 40 ft on a day when the wind is gusting in different directions. From the time the ball

is launched until it reaches a height of 20 ft off the ground the wind is blowing at constant 20 mph to the right. From that time to the time the ball has reached the top and traveled back down to a height of 20 ft the wind is blowing at constant 20 mph to the left. As it travels from 20 ft high back to the ground the wind again blows at a constant 20 mph to the right.

Part (a) Where will the ball land?

Let's find the time it takes to reach half its max height. From the kinematic equation

$$y = \left(\frac{1}{2}at^2 + v_0t + y_0 \right) \text{ m}$$

a is the acceleration in m/s^2 , t is the time in s , v_0 is the initial velocity in m/s , and y_0 is the initial displacement in m . The initial velocity can be determined from

$$v_f^2 - v_0^2 = 2ad(\text{m/s})^2$$

where v_f is the final velocity, a is the same as before, and d is the displacement in m . We know that at the max height (h), the ball stops. The only force acting on negative direction (downwards) so

$$0^2 - v_0^2 = 2 \cdot (-g) \cdot h$$

Therefore,

$$v_0 = (2gh)^{0.5}$$

Also, the initial height is 0 m off the ground. Substituting in for the distance of half the height,

$$\frac{h}{2} = -\frac{g}{2}t^2 + (2gh)^{0.5}t + 0$$

$$-\frac{g}{2}t^2 + (2gh)^{0.5}t - \frac{h}{2} = 0$$

This is a quadratic equation with solutions

$$t = \frac{(-2gh)^{0.5} \pm (2gh - 4 \cdot (-\frac{g}{2}) \cdot (-\frac{h}{2}))^{0.5}}{2 \cdot (-\frac{g}{2})} = \frac{(-2gh)^{0.5} \pm (gh)^{0.5}}{-g} = \left(\frac{h}{g}\right)^{0.5} (2^{0.5} \pm 1)$$

$$t_1 = \left(\frac{h}{g}\right)^{0.5} (2^{0.5} - 1)$$

$$t_2 = \left(\frac{h}{g}\right)^{0.5} (2^{0.5} + 1)$$

where t_1 is the time going up to the halfway point and t_2 is the time to go up and come back down to the halfway point. The total time it is being blown to the right is

$$t_{rtot} = 2t_1$$

and the total time it is being blown to the left is

$$t_{ltot} = t_2 - t_1$$

Taking the difference

$$\begin{aligned} t_{ltot} - t_{rtot} &= t_2 - t_1 - 2t_1 \\ &= t_2 - 3t_1 \\ &= \left(\frac{h}{g}\right)^{0.5} (2^{0.5} + 1) - 3 \cdot \left(\frac{h}{g}\right)^{0.5} (2^{0.5} - 1) \\ &= \left(\frac{h}{g}\right)^{0.5} (2^{0.5} + 1 - 3 \cdot 2^{0.5} + 3) \end{aligned}$$

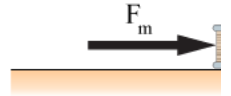
$$= \left(\frac{h}{g}\right)^{0.5} (-2 \cdot 2^{0.5} + 4)$$

$$= 1.171573 \cdot \left(\frac{h}{g}\right)^{0.5}$$

This quantity is greater than zero so it spent more time being blown left than right. It will land to the left.

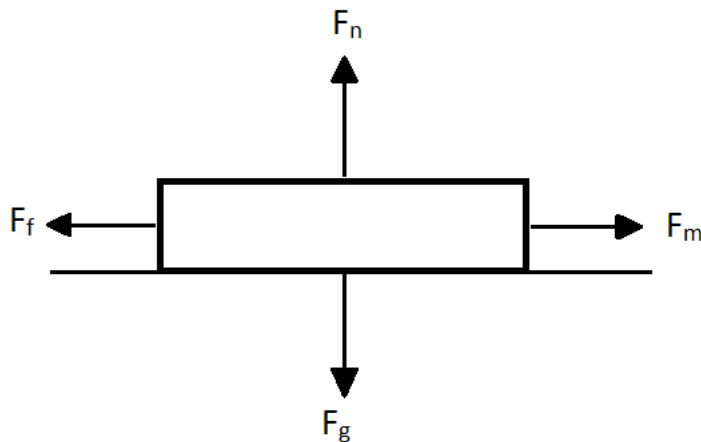
Problem 234 - 5.6.2 :

A book with mass $m = 1.25$ kg rests on the surface of a table. The coefficient of static friction between the book and the table is $\mu_s = 0.61$ and the coefficient of kinetic friction is $\mu_k = 0.21$.



Part (a) Write an expression for F_m the minimum force required to produce movement of the book on the surface of the table.

Let's begin by examining a free-body diagram of the of the system.



The minimum force required to move the book will be just enough to overcome the force of static friction. The force of static friction will be given by the normal coefficient of static friction. Since the vertical forces must be balanced, the normal force is equal to the weight of the book. Using all this information, we can write

$$F_m = \mu_s F_n$$

$$F_m = \mu_s mg$$

Part (b) Solve numerically for the magnitude of the force F_m in Newtons.

Here, we simply need to plug in values and solve the equation we created in part (a).

$$F_m = \mu_s mg$$

$$F_m = 0.61 \cdot 1.25 \text{ kg} \cdot 9.8 \text{ m/s}^2$$

$$F_m = 7.473 \text{ N}$$

Part (c) Write an expression for a , the book's acceleration, after it begins moving. (Assume the minimum force, F_m , continues to be applied.)

The acceleration of the book can be found using Newton's Second Law:

$$F_{net} = ma$$

In this case, the net force will be given by the force we found in part (a) minus the force of kinetic friction. The normal force will be equal in magnitude to the weight as in part (a), so the force of kinetic friction will be equal to the coefficient of kinetic friction multiplied by the weight of the book. Using this information, we can find the acceleration.

$$F_m - \mu_k mg = ma$$

$$\mu_s mg - \mu_k mg = ma$$

$$(\mu_s - \mu_k) mg = ma$$

$$(\mu_s - \mu_k) g = a$$

$$a = (\mu_s - \mu_k) g$$

Part (d) Solve numerically for the acceleration, a in m/s^2 .

Here, we simply need to plug in variables to the equation we found in part (c) and solve it.

$$a = (\mu_s - \mu_k) g$$

$$a = (0.61 - 0.21) \cdot 9.8 \text{ m/s}^2$$

$$a = 3.92 \text{ m/s}^2$$

Problem 235 - 5.6.3 :

A crate with a mass of $m = 110$ kg rests on the horizontal deck of a ship. The coefficient of static friction between the crate and the deck is $\mu_s = 0.71$. The coefficient of kinetic friction is $\mu_k = 0.41$.

Randomized Variables

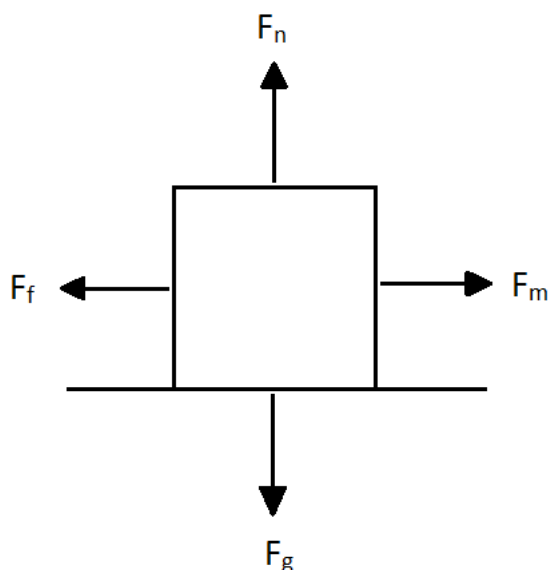
$$m = 110 \text{ kg}$$

$$\mu_s = 0.71$$

$$\mu_k = 0.41$$

Part (a) Write an expression for the minimum force, F_m , that must be applied to get the block moving from rest.

Let's begin by examining a free-body diagram of the of the system.



The minimum force required to move the block will be just enough to overcome the force of static friction. The force of static friction will be given by the normal coefficient of static friction. Since the vertical forces must be balanced, the normal force is equal to the weight of the block. Using all this information, we can write

$$F_m = \mu_s F_n$$

$$F_m = \mu_s mg$$

Part (b) What is the magnitude of the force F_m in newtons?

Here, we simply need to plug in values and solve the equation we created in part (a).

$$F_m = \mu_s mg$$

$$F_m = 0.71 \cdot 110 \text{ kg} \cdot 9.81 \text{ m/s}^2$$

$$F_m = 766.161 \text{ N}$$

Part (c) Write an expression for the force F_v that must be applied to keep the block moving at a constant velocity.

For the block to keep moving at a constant velocity, there must be no net force so there is no acceleration. This means that the force of kinetic friction must be equal to the force pushing on the block. The force of kinetic friction is given by the normal force multiplied by the coefficient of kinetic friction. As in part (a), the normal force is equal to the weight of the block. We can set up the following equation:

$$F_v = \mu_k F_n$$

$$F_v = \mu_k mg$$

Part (d) What is the magnitude of the force F_v in newtons?

To solve this problem we can plug values into the equation we found in part (c).

$$F_v = \mu_k mg$$

$$F_v = 0.41 \cdot 110 \text{ kg} \cdot 9.81 \text{ m/s}^2$$

$$F_v = 442.431$$

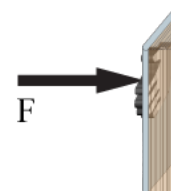
Problem 236 - 5.6.4 :

A woman holds a book by placing it between her hands such that she presses at right angles to the front and back covers. The book has a mass of $m = 0.6$ kg and the coefficient of static friction between her hand and the book is $\mu_s = 0.51$.

Randomized Variables

$$m = 0.6 \text{ kg}$$

$$\mu_s = 0.51$$

**Part (a) What is the weight of the book, F_{gb} in Newtons?**

The weight of the book will be equal to the mass of the book multiplied by the acceleration due to gravity.

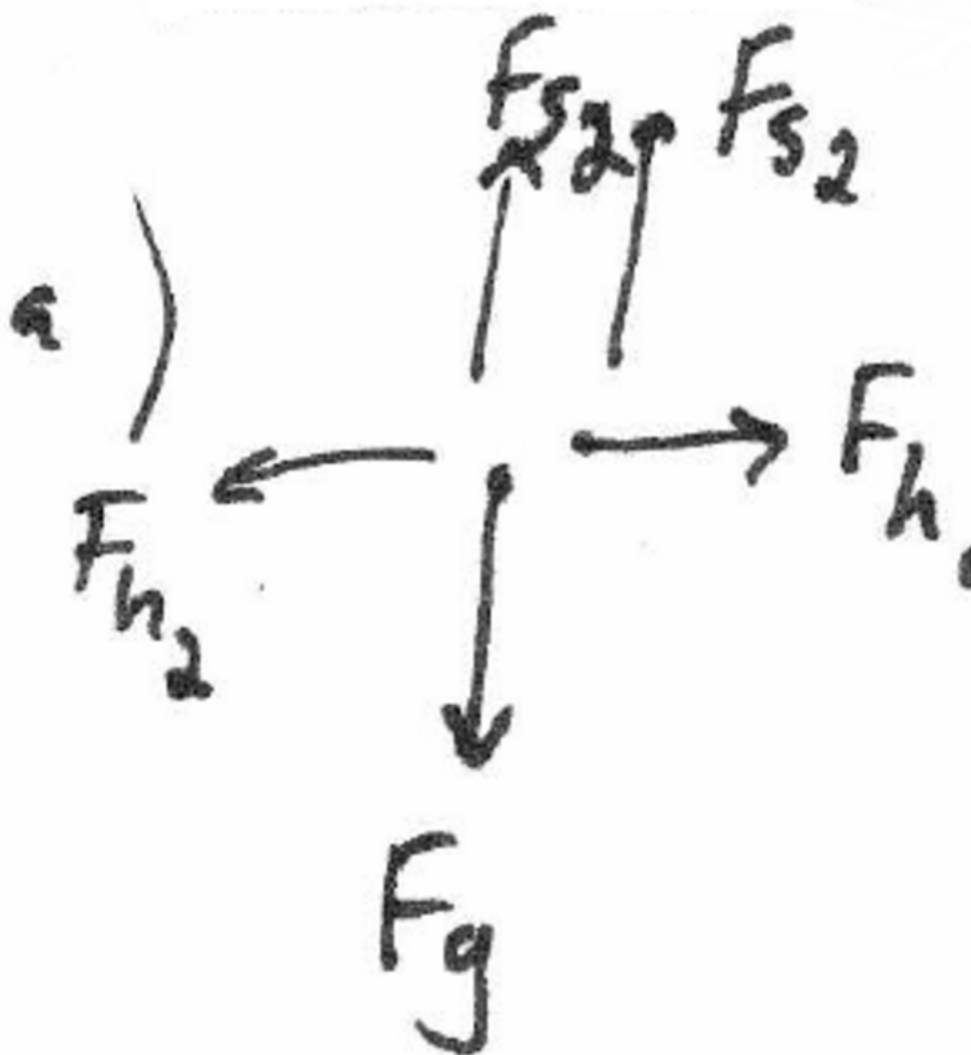
$$F_{gb} = mg$$

$$F_{gb} = 0.6 \text{ kg} \cdot 9.8 \text{ m/s}^2$$

$$F_{gb} = 5.88 \text{ N}$$

Part (b) What is the minimum force she must apply with each of her hands, F_{min} in Newtons, to keep the book from falling?

Let's start with a free-body diagram of the book.



In order for the book to remain in position, the maximum force of static friction must have at least the same magnitude as the weight of the book. The total force given by the coefficient of static friction multiplied by the total normal force. Assuming that the woman is exerting the minimum force possible on the book, the force is equal to the minimum possible force. We can use this information to set up the following equation:

$$F_{min}\mu_s + F_{min}\mu_s = F_{gb}$$

$$2F_{min}\mu_s = mg$$

$$F_{min} = \frac{mg}{2\mu_s}$$

$$F_{min} = \frac{0.6 \text{ kg} \cdot 9.8 \text{ m/s}^2}{2 \cdot 0.51}$$

$$F_{min} = 5.765 \text{ N}$$

Problem 237 - 5.6.1 :

A horizontal force, $F_1 = 55 \text{ N}$, and a force, $F_2 = 10.1 \text{ N}$ acting at an angle of θ to the horizontal, are applied to a block of mass $m =$

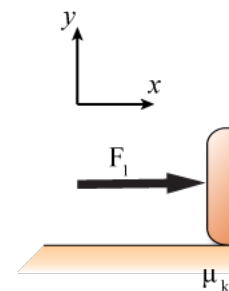
2.1 kg. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.2$. The block is moving to the right.

Randomized Variables

$$F_1 = 55 \text{ N}$$

$$F_2 = 10.1 \text{ N}$$

$$m = 2.1 \text{ kg}$$



Part (a) Solve numerically for the magnitude of the normal force, F_N in Newtons, that acts on the block if $\theta = 30^\circ$.

Looking at the diagram we are provided with, we see that the force on the block downward will be equal to the force exerted on it by gravity plus the the downward force F_2 . Since the block is not moving up or down, the normal force must be equal in magnitude and opposite in direction to the downward forces. We can thus set up the

$$F_N = mg + F_2 \sin(\theta)$$

$$F_N = 2.1 \text{ kg} \cdot 9.81 \text{ m/s}^2 + 10.1 \text{ N} \cdot \sin(30^\circ)$$

$$F_N = 25.651 \text{ N}$$

Part (b) Solve numerically for the magnitude of acceleration of the block, a in m/s^2 , if $\theta = 30^\circ$.

Looking at the forces on the block, we see that F_1 pushes the block right and the horizontal component of F_2 pushes the block left. Since the block is moving to the right, we must account for the frictional force that will resist the motion and exert force to the left on the block. We know F_1 , can find the horizontal component of F_2 , and can find the friction force by multiplying the normal force we found in part (a) by μ_k . We can use this information together with Newton's Second Law to make an equation for the

$$F_{net} = ma$$

$$F_1 - F_2 \cos(\theta) - \mu_k F_N = ma$$

$$F_1 - F_2 \cos(\theta) - \mu_k [mg + F_2 \sin(\theta)] = ma$$

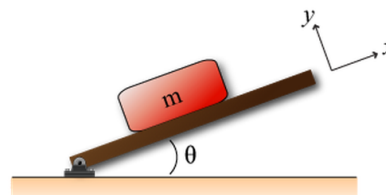
$$\frac{F_1 - F_2 \cos(\theta) - \mu_k [mg + F_2 \sin(\theta)]}{m} = a$$

$$a = \frac{55 \text{ N} - 10.1 \text{ N} \cdot \cos(30^\circ) - 0.2 \cdot [2.1 \text{ kg} \cdot 9.81 \text{ m/s}^2 + 10.1 \text{ N} \cdot \sin(30^\circ)]}{2.1 \text{ kg}}$$

$$a = 19.582 \text{ m/s}^2$$

Problem 238 - 5.6.5 (iFbd) :

A block with a mass of $m = 1 \text{ kg}$ rests on a wooden plank. The coefficient of static friction between the block and the plank is $\mu_s = 0.6$. One end of the board is attached to a hinge so that the other end can be lifted forming an angle, θ , with respect to the ground. Assume the x-axis is along the plank as shown in the figure.

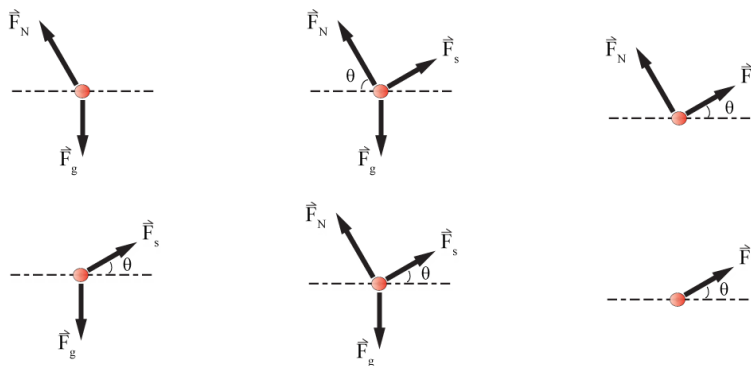


Randomized Variables

$\mu_s = 0.6$
 $m = 1 \text{ kg}$

Part (a) Please select the correct free body diagram given F_g is the force due to gravity, F_s is the static friction force and F_N is the normal force. Assume the block is at rest.

SchematicChoice :



Part (b) Assuming the x-direction is along the plank as shown, find an expression for the magnitude of the force of gravity in the y-direction, F_{gy} , perpendicular to the plank in terms of given quantities and variables available in the palette.

Expression :

$F_{gy} =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

acotan(μ_s), atan(μ_s), cos(α), cos(ϕ), cos(θ), sin(α), sin(ϕ), sin(θ), tan(θ), α , β , μ_k , μ_s , θ , b , d , g , h , m , t

Part (c) Write an expression for the magnitude of the maximum friction force along the surface, F_s , in terms of given quantities and variables available in the palette.

Expression :

$F_s =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

acotan(μ_s), atan(μ_s), cos(α), cos(ϕ), cos(θ), sin(α), sin(ϕ), sin(θ), α , μ_k , μ_s , b , g , m , t

Part (d) Assuming the static friction is maximized, write an expression, using only the given parameters and variables available in the palette, for the sum of the forces along the plank, ΣF_x .

Expression :

$\Sigma F_x =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

acotan(μ_s), atan(μ_s), cos(α), cos(ϕ), cos(θ), sin(α), sin(ϕ), sin(θ), α , μ_k , μ_s , b , g , m , t

Part (e) Write an expression for the maximum angle, θ_m , that the board can make with respect to the horizontal before the block starts moving. (Write in terms of the given parameters and variables available in the palette.)

Expression :

$\theta_m =$ _____

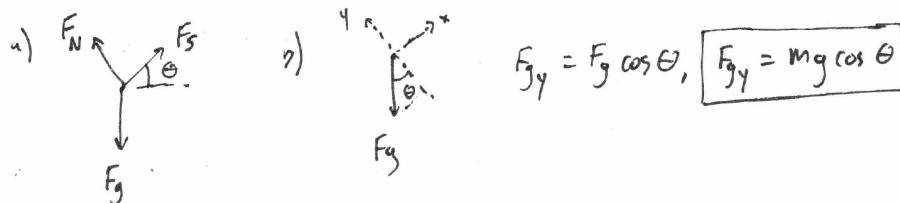
Select from the variables below to write your expression. Note that all variables may not be required.

acotan(μ_s), atan(μ_s), cos(α), cos(ϕ), cos(θ), sin(α), sin(ϕ), sin(θ), tan(θ), α , μ_k , μ_s , θ , b , t

Part (f) Solve numerically for the maximum angle, θ_m , in degrees.

Numeric : A numeric value is expected and not an expression.

$\theta_m =$ _____



b) Static friction is $F_s \leq \mu_s F_N$, so the maximum value will be $F_s = \mu_s F_N$.
 The normal force is given by Newton's 2nd law,

$$\sum F_y = F_N - F_{gy} = 0 \rightarrow F_N = mg \cos \theta, \text{ so } \boxed{F_s = \mu_s mg \cos \theta}$$

d) $\sum F_x = F_s - F_g \sin \theta = 0$ (if not moving)

Assuming the static friction is maximized, $\boxed{\sum F_x = \mu_s mg \cos \theta - mg \sin \theta}$

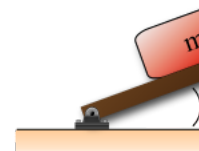
e) At maximum value the board is not moving, so $\sum F_x = 0$ and we can solve

for θ_m : $\mu_s mg \cos \theta_m = mg \sin \theta_m \rightarrow \mu_s = \tan \theta_m \rightarrow \boxed{\theta_m = \arctan(\mu_s)}$

f) $\boxed{\theta_m \approx 31.0^\circ}$

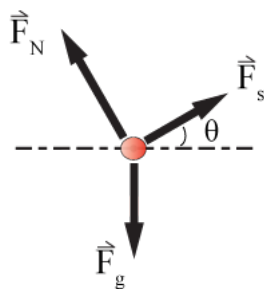
Problem 239 - 5.6.5 :

A block with a mass of $m = 1$ kg rests on a wooden plank. The coefficient of static friction between the block and the plank is $\mu_s = 0.42$. One end of the board is attached to a hinge so that the other end can be lifted forming an angle, θ , with respect to the ground. Assume the x-axis is along the plank as shown in the figure.



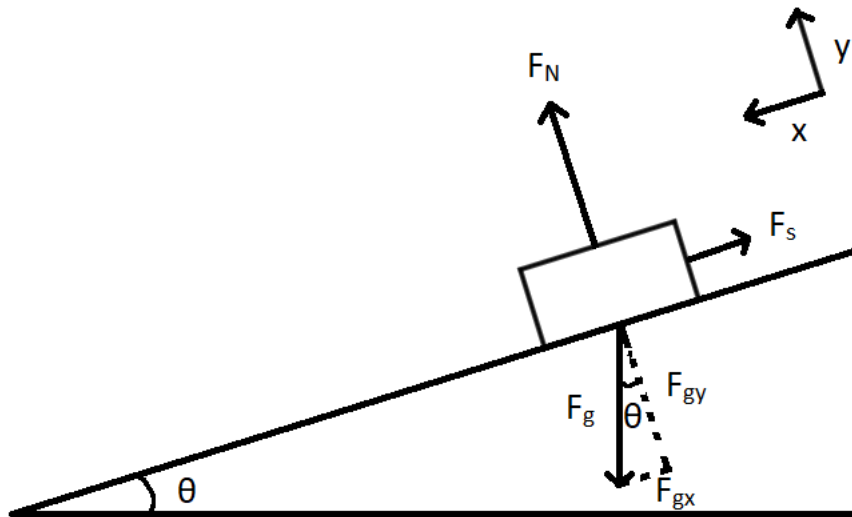
Part (a) Please select the correct free body diagram given F_g is the force due to gravity, F_s is the static friction force and F_N is the normal force. Assume

The block should experience a frictional force in the x-direction to resist it sliding down the plank. It should also experience a normal force tangential to the plank. Finally, the block will also experience a gravitational force pulling it straight down. The correct free-body diagram must contain all these forces as well as placing it in the example picture. This means that the correct answer is:



Part (b) Assuming the x-direction is along the plank as shown, find an expression for the component of the force of gravity in the y-direction, F_{gy} , perper terms of given quantities and variables available in the palette.

Let's begin by examining a more detailed free-body diagram:



Here, we can see that the y-component of the force will be given by:

$$\cos(\theta) = \frac{F_{gy}}{F_g}$$

$$F_g \cos(\theta) = F_{gy}$$

$$F_{gy} = mg \cos(\theta)$$

Part (c) Write an expression for the magnitude of the maximum friction force along the surface, F_s , in terms of given quantities and variables available in the problem.

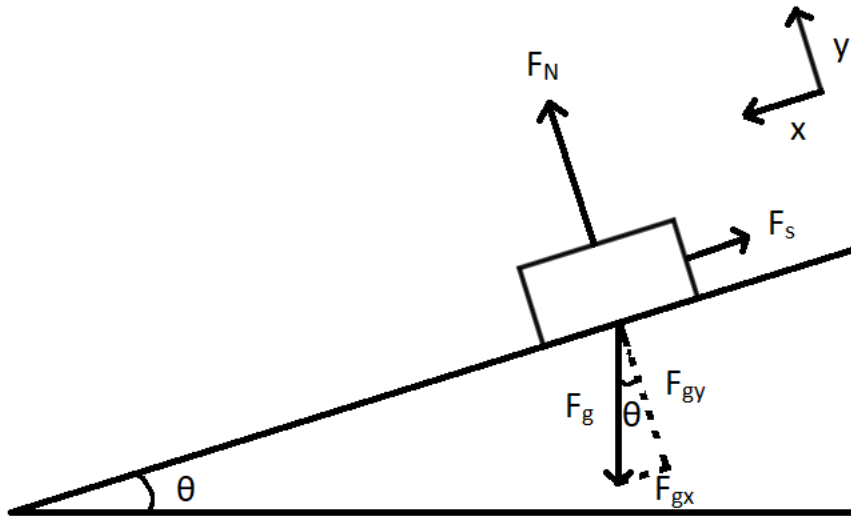
To find the maximum frictional force, we must first find the normal force. We found the component of gravity pushing the block against the plank in part (b). Since the block is not accelerating in the y-direction, this means that the normal force must be equal in magnitude and opposite in direction to the force we found in part (b). Now that we can find the maximum value of static friction by multiplying the normal force by the coefficient of static friction.

$$F_s = F_N \mu_s$$

$$F_s = mg \cos(\theta) \mu_s$$

Part (d) Assuming the static friction is maximized, write an expression, using only the given parameters and variables available in the palette, for the sum of forces in the x-direction, ΣF_x .

Let's take another look at the detailed free-body diagram we examined in part (b):



We can see that the force of static friction is acting in the positive x-direction and the x-component of the gravitational force is acting in the negative x-direction. The following equation for the sum of forces in the x-direction:

$$\Sigma F_x = F_s - F_{gx}$$

$$\Sigma F_x = mg \cos(\theta) \mu_s - mg \sin(\theta)$$

Part (e) Write an expression for the maximum angle, θ_m , that the board can make with respect to the horizontal before the block starts moving. (Write in terms of parameters and variables available in the palette.)

The block will begin moving if the x-component of the gravitational force exceeds the force of static friction. The angle just before they start moving can therefore be found by setting the sum of forces in the x-direction equal to zero and solving for the angle. We can begin with the equation we found in part (d) using θ_m in place of θ .

$$\Sigma F_x = mg \cos(\theta_m) \mu_s - mg \sin(\theta_m)$$

$$0 = mg\cos(\theta_m)\mu_s - mg\sin(\theta_m)$$

$$mg\sin(\theta_m) = mg\cos(\theta_m)\mu_s$$

$$\frac{\sin(\theta_m)}{\cos(\theta_m)} = \mu_s$$

$$\tan(\theta_m) = \mu_s$$

$$\theta_m = \arctan(\mu_s)$$

Part (f) Solve numerically for the maximum angle, θ_m , in degrees.

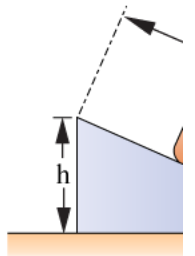
Here, we simply need to solve the equation we found in part (e).

$$\theta_m = \arctan(\mu_s)$$

$$\theta_m = 22.794^\circ$$

Problem 240 - 5.6.6 :

A block of mass $m = 10.5$ kg rests on an inclined plane with a coefficient of static friction of $\mu_s = 0.06$ between the block and the plane. The inclined plane is $L = 6.1$ m long and it has a height of $h = 3.05$ m at its tallest point.



Part (a) What angle, θ in degrees, does the plane make with respect to the horizontal?

Looking at the diagram, we see that for the triangle formed by the inclined plane, h gives the side opposite θ and L gives the hypotenuse. We can use this information to determine the value of the angle θ .

$$\sin(\theta) = \frac{h}{L}$$

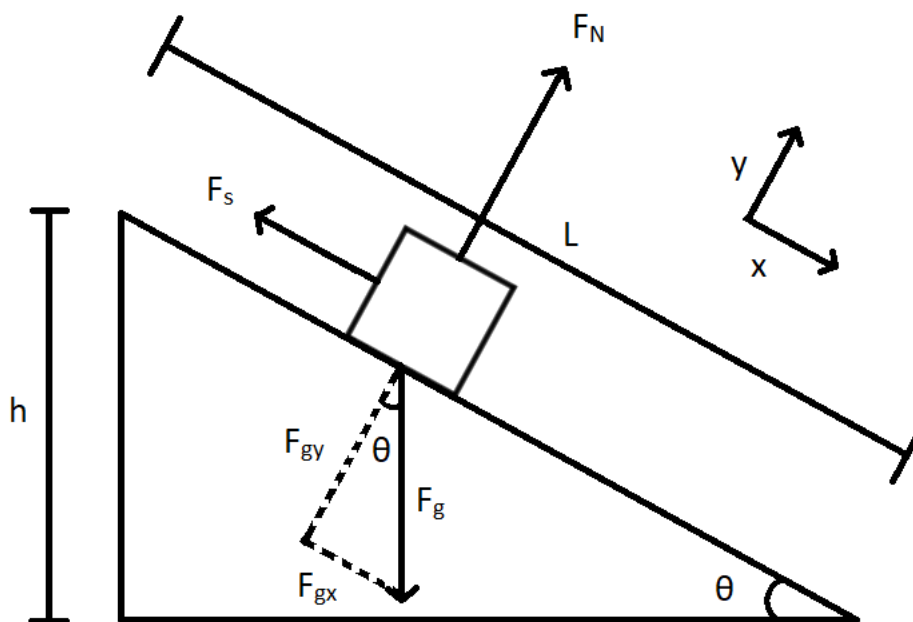
$$\theta = \arcsin\left(\frac{h}{L}\right)$$

$$\theta = \arcsin\left(\frac{3.05 \text{ m}}{6.1 \text{ m}}\right)$$

$$\theta = 30^\circ$$

Part (b) What is the magnitude of the normal force, F_N in newtons, that acts on the block?

To begin, let's draw a free-body diagram of the system formed by the block and inclined plane.



Looking at this image, we see the only forces acting on the block perpendicular to the inclined plane are F_{gy} and F_N . Since the block will not experience any acceleration perpendicular to the incline, these forces must be balanced. We can therefore set up the following equation to find the normal force:

$$F_N = F_{gy}$$

$$F_N = F_g \cos(\theta)$$

$$F_N = mg \cos(\theta)$$

We can now use our results for θ from part (a) to continue.

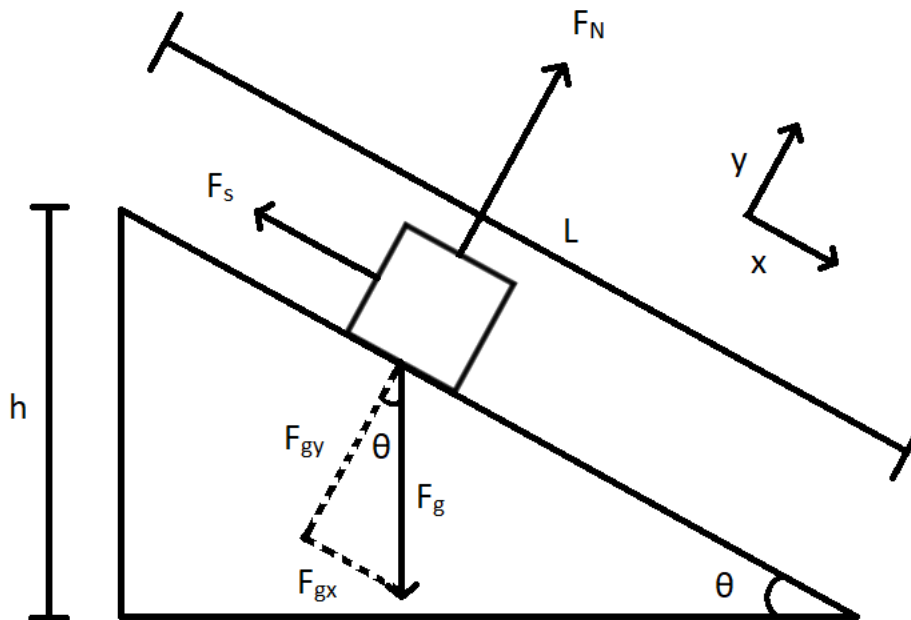
$$F_N = mg \cos\left(\arcsin\left(\frac{h}{L}\right)\right)$$

$$F_N = 10.5 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \cos\left(\arcsin\left(\frac{3.05 \text{ m}}{6.1 \text{ m}}\right)\right)$$

$$F_N = 89.205 \text{ N}$$

Part (c) What is the component of the force of gravity along the plane, F_{gx} in newtons?

Let's begin by looking at a free-body diagram:



We see that the force of gravity in the horizontal direction can be given by the following formula:

$$F_{gx} = F_g \sin(\theta)$$

$$F_{gx} = mg \sin(\theta)$$

Now we can plug in our results from part (a) in for θ .

$$F_{gx} = mg \sin\left(\arcsin\left(\frac{h}{L}\right)\right)$$

$$F_{gx} = mg \frac{h}{L}$$

$$F_{gx} = 10.5 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \frac{3.05 \text{ m}}{L}$$

$$F_{gx} = 51.503 \text{ N}$$

Part (d) Write an expression, in terms of θ , the mass m , the coefficient of static friction μ_s , and the gravitational constant g , for the magnitude of the force just before the block begins to slide.

Just before the block begins to slide, the static friction will be acting with the maximum possible force. The maximum force of static friction is given by the coefficient multiplied by the normal force. In part (b), we found an equation for the normal force which we can use to find the magnitude of the force of static friction. However, h or L in the list of variables we can use, we will need to use θ for our angle rather than the value of θ that we found in part (a).

$$F_s = F_N \mu_s$$

$$F_s = mg \cos(\theta) \mu_s$$

Part (e) Will the block slide?

To determine whether or not the block will slide, we must compare the force of gravity in the x-direction to the maximum force of static friction. If the x-component force is greater than the maximum force of static friction, then the block will accelerate down the ramp. Otherwise, it will stay still. To find this, we can solve the (d) and compare the result to our answer from part (c).

$$F_s = mg \cos(\theta) \mu_s$$

$$F_s = mg \cos\left(\arcsin\left(\frac{h}{L}\right)\right) \mu_s$$

$$F_s = 10.5 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \cos\left(\arcsin\left(\frac{3.05 \text{ m}}{6.1 \text{ m}}\right)\right) \cdot 0.06$$

$$F_s = 5.352 \text{ N}$$

Now, let's look at the value we found for F_{gx} in part (c):

$$F_{gx} = 51.503 \text{ N}$$

As we can see, the value of the force of gravity in the x-direction is greater than the maximum force of static friction, meaning that the block will indeed begin to therefore:

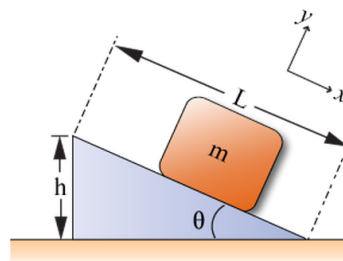
Yes

Problem 241 - 5.6.6 (alt) :

A block of mass $m = 12$ kg rests on an inclined plane with a coefficient of static friction of $\mu_s = 0.08$ between the block and the plane. The inclined plane is $L = 4.4$ m long and it has a height of $h = 3.2$ m at its tallest point.

Randomized Variables

$m = 12$ kg
 $\mu_s = 0.08$
 $L = 4.4$ m
 $h = 3.2$ m



Part (a) What angle, θ in degrees, does the plane make with respect to the horizontal?

Numeric : A numeric value is expected and not an expression.
 $\theta =$ _____

Part (b) What is the magnitude of the normal force, F_N in newtons, that acts on the block?

Numeric : A numeric value is expected and not an expression.
 $F_N =$ _____

Part (c) What is the force of gravity along the plane, F_{gx} in newtons?

Numeric : A numeric value is expected and not an expression.
 $F_{gx} =$ _____

Part (d) Write an expression in terms of θ for the magnitude of the force due to static friction, F_s , just before the block begins to slide.

Expression :
 $F_s =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.
 $\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , μ_s , θ , d , g , h , m , t

Part (e) Will the block slide?

MultipleChoice :

- 1) There is not enough information.
- 2) No
- 3) Yes

a) $\sin \theta = \frac{h}{L}$, $\theta = \arcsin\left(\frac{h}{L}\right)$, $\theta \approx 46.7^\circ$

b) $\sum F_y = F_N - F_{gy} = F_N - mg \cos \theta = 0$, $F_N \approx 80.7$ N

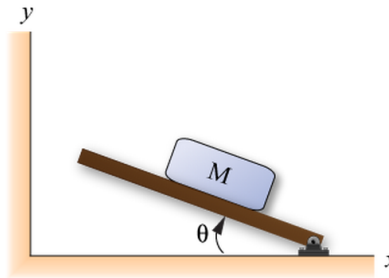
c) $F_{gx} = mg \sin \theta$, $F_{gx} \approx 85.7$ N

d) $\sum F_x = F_{gx} - F_s = 0$, $F_s = F_{gx} \leq \mu_s F_N$, the largest value it can take is the equality so $F_s = \mu_s mg \cos \theta$

e) $F_s \approx 6.46$ N, and $F_s < F_{gx}$ so yes.

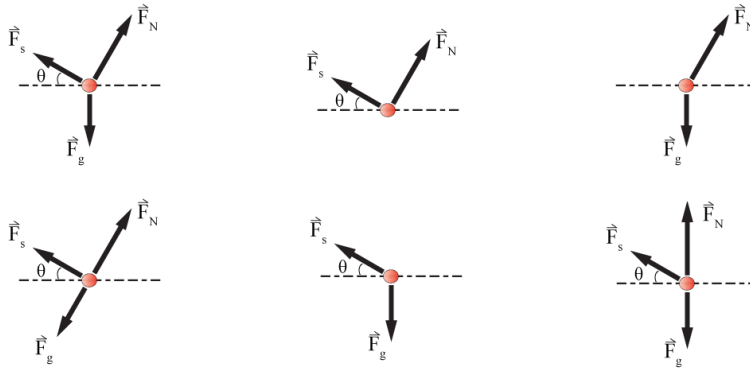
Problem 242 - 5.6.7 (iFBD) :

A block is resting on a wooden plank. On one end of the plank is a hinge so the other end may be lifted to create an angle, θ , with respect to the horizontal. The plank has a coefficient of static friction of μ_s .



Part (a) Please select the correct Free Body Diagram, where F_g is the force due to gravity, F_N is the normal force, and F_s is the static friction force.

SchematicChoice :



Part (b) The angle θ is slowly increased. Write an expression for the angle at which the block begins to move in terms of μ_s .

Expression :

$\theta =$ _____

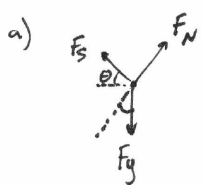
Select from the variables below to write your expression. Note that all variables may not be required.

$\text{acos}(\mu_s), \text{acotan}(\mu_s), \text{asin}(\mu_s), \text{atan}(\mu_s), \text{cos}(\mu_s), \text{cos}(\theta), \text{sin}(\mu_s), \text{sin}(\theta), \alpha, \beta, \theta, d, g, m, t$

Part (c) If a student measures that the block begins to move at an angle of $\theta = 42^\circ$, what is the numerical value of the coefficient of static friction, μ_s ?

Numeric : A numeric value is expected and not an expression.

$\mu_s =$ _____



b) Take a coordinate system along the ramp. Then

$$\sum F_x = F_s - F_{g_x} = 0 \rightarrow F_s = F_{g_x} \leq \mu_s F_N$$

$$\sum F_y = F_N - F_{g_y} = F_N - mg \cos \theta = 0, F_N = mg \cos \theta$$

$$F_{g_x} \leq \mu_s F_N \rightarrow mg \sin \theta \leq \mu_s mg \cos \theta \rightarrow \tan \theta \leq \mu_s. \text{ The equality occurs}$$

at maximum, so $\theta = \text{arctan}(\mu_s)$

c) $\mu_s = \tan \theta$ so $\mu_s \approx 0.90$

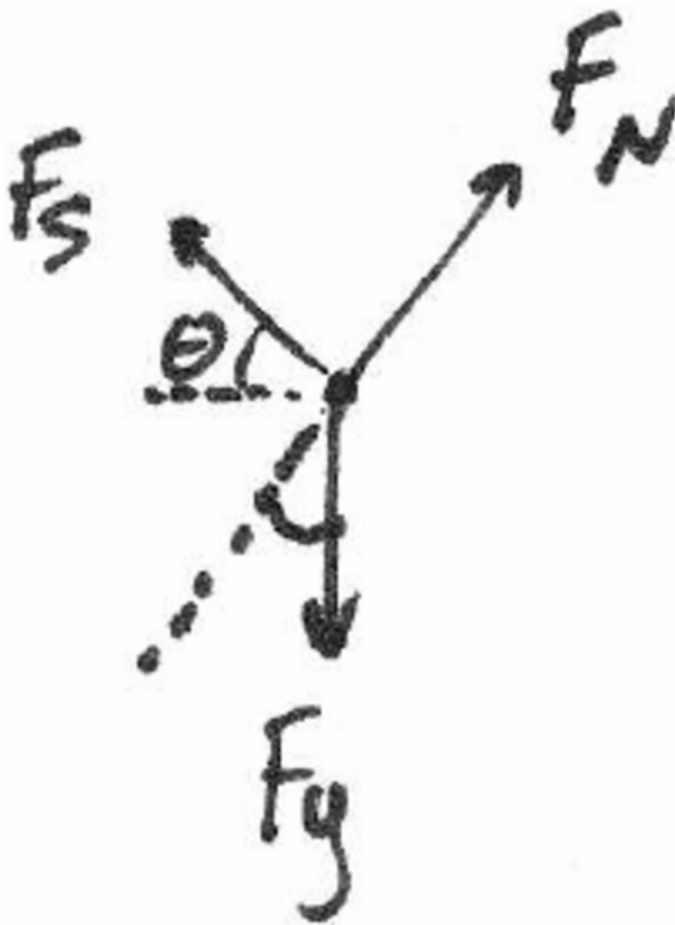
Problem 243 - 5.6.7 :

A block is resting on a wooden plank. There is a hinge on one end of the plank which allows the other end to be lifted to create an angle, θ , with respect to the horizontal as shown in the figure. The coefficient of static friction between the block and the plank is μ_s .



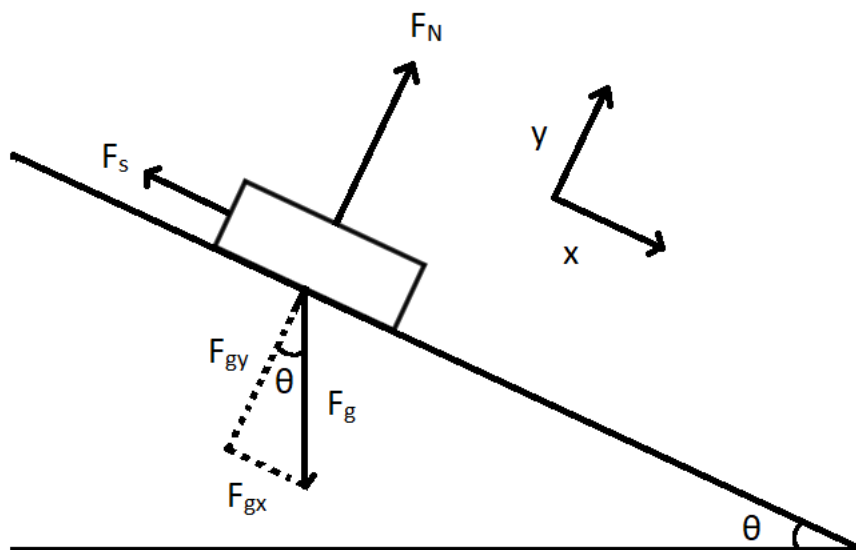
Part (a) Please select the correct Free Body Diagram, where F_g is the force due to gravity, F_N is the normal force, and F_s is the static friction force.

Examining the forces on the block, it must experience a force of gravity straight down, a normal force perpendicular to the plank, and a force of friction parallel to the block sliding down. The correct free-body diagram will feature all of these along with the angle θ correctly placed between the plank and the force of friction.



Part (b) The angle θ is slowly increased. Write an expression for the angle at which the block begins to move in terms of μ_s .

To solve this problem, let's begin by drawing a more detailed free-body diagram.



Note that in this free-body diagram, we have set our coordinate axis such that the x-direction is parallel to the plank and the y-direction is perpendicular to it. Since we move along the y-axis, the forces in the y-direction must be balanced. This allows us to set up the following equation:

$$F_N = F_{gy}$$

$$F_N = mg \cos(\theta)$$

Now that we have a value for the normal force, we can write an equation to solve for the maximum force of static friction.

$$F_s = F_N \mu_s$$

$$F_s = mg \cos(\theta) \mu_s$$

Now, the block will begin to move when the angle increases past the point where the force of static friction is equal to the x-component of the gravitational force. We set the x-component of the gravitational force equal to the maximum force of static friction and solving for θ .

$$F_{gx} = F_s$$

$$mg \sin(\theta) = mg \cos(\theta) \mu_s$$

$$\frac{\sin(\theta)}{\cos(\theta)} = \mu_s$$

$$\tan(\theta) = \mu_s$$

$$\theta = \arctan(\mu_s)$$

Part (c) If a student measures that the block begins to move at an angle of $\theta = 20.5^\circ$, what is the numerical value of the coefficient of static friction, μ_s ?

We can begin with our result from part (b) and rearrange it so that we can solve for the coefficient of static friction.

$$\theta = \arctan(\mu_s)$$

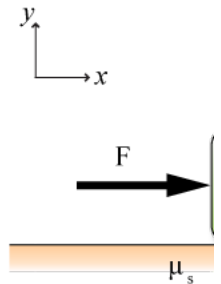
$$\tan(\theta) = \mu_s$$

$$\mu_s = \tan(20.5^\circ)$$

$$\mu_s = 0.3739$$

Problem 244 - 5.6.12 :

A block that has a mass of $m = 3.5$ kg rests on a horizontal plane. The coefficient of static friction, μ_s , is 0.15 . A horizontal force, F , is applied to the block, and it is just large enough to get the block to begin moving.



Part (a) Choose the correct Free Body Diagram from the choices below given that F_f is the static friction force, F_N is the normal force and F_g is the weight.

The forces acting on the block are the normal force (pointing up), the force of gravity (pointing down), the force of friction (pointing to the left), and the horizontal force (pointing to the right). Only one diagram correctly depicts this.

Part (b) Write an expression for the sum of the forces in the x -direction using the variables from the above Free Body Diagram.

The forces acting in the horizontal direction are the ones in the x -direction. Therefore,

$$\sum F_x = (F - F_f) \text{ N}$$

where F is the horizontal force (positive direction) and F_f is the frictional force (negative direction).

$$\sum F_x = (F - F_f) \text{ N}$$

Part (c) Given the coordinate system specified in the problem statement, write an expression for the sum of the forces in the y-direction.

The forces acting in the vertical direction are the ones in the y-direction. Therefore,

$$\sum F_y = (F_N - F_g) \text{ N}$$

where F_N is the normal force (positive direction) and F_g is the gravitational force (negative direction).

$$\sum F_y = (F_N - F_g) \text{ N}$$

Part (d) Write an expression to show the relationship between the maximum friction force, F_f , and the normal force, F_N .

Static friction force satisfies the relation

$$F_f \leq \mu_s F_N \text{ N}$$

where F_N is the normal force in N and μ_s is the static coefficient of friction. So at maximum

$$F_f = \mu_s F_N \text{ N}$$

$$F_f = \mu_s F_N \text{ N}$$

Part (e) Calculate the magnitude of F , in Newtons, if F_f is at its maximum.

From part b,

$$F - F_f = 0 \text{ N}$$

so

$$F = F_f$$

Substituting in for the frictional force from part d,

$$F = \mu_s F_N \text{ N}$$

and also from c,

$$F_N - F_g = 0$$

so

$$F_N = F_g = mg$$

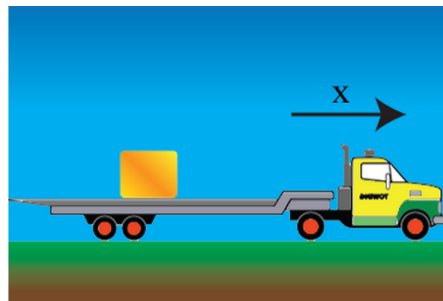
where m is the mass in kg and g is the acceleration of gravity in m/s^2 . Substituting in and plugging in numbers (converting units as needed),

$$F = \mu_s mg = 0.15 \cdot 3.5 \text{ kg} \cdot 9.81 \text{ m/s}^2$$

$$F = 5.15 \text{ N}$$

Problem 245 - 5.6.13 :

A crate is placed on flatbed truck but is not tied down. The truck accelerates in the positive x-direction (to the right, as shown) too fast and the crate falls off the back of the truck. There is friction between the truck bed and the crate.



Part (a) When the crate falls off, in what direction is it moving relative to the ground?

MultipleChoice :

- 1) To the right (positive x-direction).
- 2) To the left (negative x-direction).
- 3) It is not moving with respect to the ground.

Part (b) A friction force acts on the crate. Which of the following statements is true?

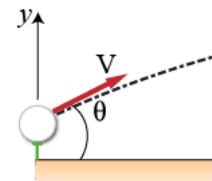
MultipleChoice :

- 1) The friction force is in the same direction as the motion of the crate relative to the ground.
- 2) The friction force is in the opposite direction of the motion of the crate relative to the ground. (Friction always opposes motion).
- 3) The friction force is perpendicular to the motion of the crate.

- a) There is only 1 force on the crate in the x-direction, that of friction. This is in the positive x-direction, so the crate moves in the positive x-direction.
- b) As stated above, the friction force acts in the same direction as the crate relative to the ground.

Problem 246 - 5.6.14 :

A golf ball is hit from ground level on a horizontal fairway with initial velocity vector $\mathbf{v}_0 = v_{0x}\mathbf{i} + v_{0y}\mathbf{j}$, where $v_{0x} = 11$ m/s and $v_{0y} = 5.25$ m/s. Throughout its trajectory, the golf ball encounters a strong wind which causes the ball to experience an acceleration in the horizontal direction with a magnitude of 1.05 m/s² as shown in the figure. Assume that any other air resistance is negligible. Use a Cartesian coordinate system with the origin at the ball's initial position as shown in the figure.



Part (a) Calculate the maximum height, h_{max} in meters, that the ball achieves.

At maximum height, the velocity in the y-direction is zero. The only force acting in the y-direction is gravity. Therefore, from the kinematic equation

$$v_f^2 - v_0^2 = 2a\Delta x \text{ (m/s)}^2$$

where $v_{f,i}$ are the final and initial velocities in m/s, a is the acceleration in m/s², and Δx is the change in distance in m,

$$0^2 - v_{0y}^2 = 2 \cdot (-g) \cdot h_{max}$$

$$h_{max} = \frac{v_{0y}^2}{(2g)}$$

Plugging in numbers and converting units as needed,

$$h_{max} = \frac{(5.25 \text{ m/s})^2}{(2 \cdot 9.81 \text{ m/s}^2)}$$

$$h_{max} = 1.405 \text{ m}$$

Part (b) How long, t_{total} in seconds, is the ball in the air before it returns to the ground?

When the ball returns to the ground, it has the same velocity it had when it started (in the y-direction), but in the opposed direction. The only force acting in the from the kinematic expression

$$v_f = (v_0 + at) \text{ m/s}$$

where v_0 is the initial velocity in m/s, a is the acceleration in m/s^2 , and t is the time in s,

$$-v_{0y} = v_{0y} - gt_{total}$$

and the total time is

$$t_{total} = \frac{2v_{0y}}{g}$$

Plugging in numbers and converting units as needed,

$$t_{total} = \frac{2(5.25 \text{ m/s})}{(9.81 \text{ m/s}^2)}$$

$$t_{total} = 1.07 \text{ m}$$

Part (c) Describe the ball's horizontal position as a function of time, $x(t)$, (in terms of t , v_{0x} , v_{0y} , a_{wind} , and g) before it returns to the fairway.

The only force acting in the horizontal or x-direction is the wind. From the kinematic equation

$$x(t) = \left(\frac{1}{2}at^2 + v_0t + x_0 \right) \text{ m}$$

where a is the acceleration in m/s^2 , t is the time in s, v_0 is the initial velocity in m/s, and x_0 is the initial displacement,

$$x(t) = \frac{1}{2} \cdot (-a_{wind})t^2 + v_{0x}t + 0 = v_{0x}t - \frac{1}{2} \cdot a_{wind}t^2$$

$$x(t) = (v_{0x}t - 0.5a_{wind}t^2) \text{ m}$$

Part (d) Calculate the horizontal distance, x_{max} in meters, the ball travels before it returns to ground level.

Using the total time found in b and the distance in c,

$$x(t_{total}) = v_{0x}t_{total} - \frac{1}{2} \cdot a_{wind}t_{total}^2 = v_{0x} \cdot \left(\frac{2v_{0y}}{g} \right) - \frac{1}{2} \cdot a_{wind} \cdot \left(\frac{2v_{0y}}{g} \right)^2 = \frac{2v_{0y}}{g} \cdot \left(v_{0x} - a_{wind} \cdot \frac{v_{0y}}{g} \right)$$

Plugging in numbers and converting units as needed,

$$x(t_{total}) = x_{max} = 2 \cdot \frac{5.25 \text{ m/s}}{9.81 \text{ m/s}^2} \cdot \left(11 \text{ m/s} - 1.05 \text{ m/s}^2 \cdot \frac{5.25 \text{ m/s}}{9.81 \text{ m/s}^2} \right)$$

$$x_{max} = 11.172 \text{ m}$$

Problem 247 - 5.6.16 :

Unreasonable Results (a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s. (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

Solution

$$(a) \mu_s = \frac{v^2}{rg} = \frac{(30.0 \text{ m/s})^2}{50.0 \text{ m} \times 9.80 \text{ m/s}^2} = \underline{1.84}$$

(b) A coefficient of friction this much greater than 1 is unreasonable. For example, the value for rubber on dry concrete is 1.0.

(c) It is unreasonable to go around an unbanked, tight curve so fast.

Problem 248 - 5.6.19 :

A $4.75 \cdot 10^5$ -kg rocket is accelerating straight up. Its engines produce $1.05 \cdot 10^7$ N of thrust, and air resistance is $4.25 \cdot 10^6$ N .

Randomized Variables

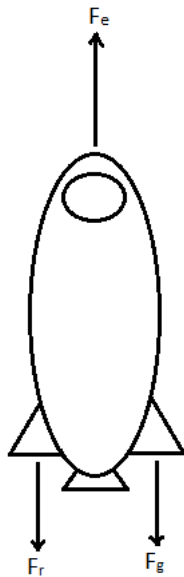
$$m = 4.75 \cdot 10^5 \text{-kg}$$

$$f = 1.05 \cdot 10^7 \text{ N}$$

$$f_r = 4.25 \cdot 10^6 \text{ N .}$$

Part (a) What is the rocket's acceleration, using a coordinate system where up is positive?

Let's begin by drawing a free-body diagram.



We see that the force of the engines is directed up, while the forces of gravity and air resistance are directed downward. We can now write an expression for the net

$$F_{net} = F - F_r - F_g$$

$$F_{net} = F - F_r - mg$$

Now that we have an expression for the net force, let's use Newton's Second Law to get an expression for the acceleration in terms of force and mass.

$$F = ma$$

$$\frac{F}{m} = a$$

Now, let's apply this to our net force to find the acceleration of the rocket.

$$a = \frac{F_{net}}{m} = \frac{F - F_r - mg}{m}$$

$$a = \frac{1.05 \cdot 10^7 \text{ N} -}{4.75 \cdot 10^5 \text{ kg}}$$

$$a = 3.358 \text{ m/s}^2$$

Problem 249 - 5.6.20 :

The wheels of a midsize car exert a force of **2050** N backward on the road to accelerate the car in the forward direction. The force of friction including air resistance is **210** N and the acceleration of the car is **1.5** m/s².

Randomized Variables

$$f = 2050 \text{ N}$$

$$f_r = 210 \text{ N}$$

$$a = 1.5 \text{ m/s}^2$$

Part (a) What is the mass of the car plus its occupants in kg?

Let's begin by summing the total forces on the car in order to find the net force.

$$F_{net} = F - F_r$$

Now that we have an expression for the net force we can use Newton's Second Law to solve for the total mass of the car.

$$F_{net} = ma$$

$$\frac{F_{net}}{a} = m$$

$$m = \frac{F - F_r}{a}$$

$$m = \frac{2050 \text{ N} - 210 \text{ N}}{1.5 \text{ m/s}^2}$$

$$m = 1226.667 \text{ kg}$$

Problem 250 - 5.6.21 :

A freight train consists of two $8.05 \cdot 10^4$ kg engines and 45 cars with average masses of $5.5 \cdot 10^4$ kg each.

Randomized Variables

$$m = 8.05 \cdot 10^4 \text{ kg}$$

$$m_2 = 5.5 \cdot 10^4 \text{ kg}$$

$$f = 7.25 \cdot 10^5 \text{ N}$$

Part (a) What is the magnitude of the force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \cdot 10^{-2} \text{ m/s}^2$ if the force exerted by the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently, efficient transportation systems.

Let's begin by finding the net force required to accelerate the train, letting m refer to the weight of an engine and m_2 refer to the weight of a car.

$$F_{net} = m_{total}a$$

$$F_{net} = (2m + 45m_2) a$$

Now that we have an expression for the net force on the train, we can write an equation for the sum of all forces. As we do so, recall that there are two engines that exert force on the train.

$$F_{net} = 2F - F_f$$

$$(2m + 45m_2) a = 2F - F_f$$

$$(2m + 45m_2) a + F_f = 2F$$

$$\frac{(2m + 45m_2) a + F_f}{2} = F$$

$$F = \frac{(2 \cdot 8.05 \cdot 10^4 \text{ kg} + 45 \cdot 5.5 \cdot 10^4 \text{ kg}) \cdot 5.00 \cdot 10^{-2} \text{ m/s}^2 + 7.25 \text{ N}}{2}$$

$$F = 428400 \text{ N}$$

Part (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass distributed among all of the cars and engines?

The coupling between the 37th and 38th cars needs to exert enough net force to accelerate the 38th through 45th cars at the same speed as the rest of the train. They can write the following equation for the net force required:

$$F'_{net} = 8m_2a$$

To write an equation for the sum of forces acting on these last eight cars, we need to know the force of friction. As the frictional force is distributed evenly between the cars, the friction force on these cars will be 8/47ths of the total frictional force the train experiences. We can now write the following equation:

$$F'_{net} = F' - \frac{8}{47}F_f$$

$$8m_2a = F' - \frac{8}{47}F_f$$

$$8m_2a + \frac{8}{47}F_f = F'$$

$$F' = 8 \cdot 5.5 \cdot 10^4 \text{ kg} \cdot 5.00 \cdot 10^{-2} \text{ m/s}^2 + \frac{8}{47} \cdot 7.25 \cdot 10^5 \text{ N}$$

$$F' = 145404.255 \text{ N}$$

Problem 251 - 5.6.22 :

Commercial airplanes are sometimes pulled out of the passenger loading area by a tractor.

Part (a) An 1800 kg tractor exerts a force of $1.75 \times 10^4 \text{ N}$ backward on the pavement, and the system experiences forces resisting motion that totals **2400 0.11 m/s²**, what is the mass of the airplane in kilograms?

To begin, let's write an equation for the net force acting on the tractor.

$$F_{net} = F - F_r$$

Now, let's write Newton's Second Law for this system and use the formula we just found to substitute for the net force.

$$F_{net} = m_{total}a$$

$$F - F_r = (m_a + m_t) a$$

$$\frac{F - F_r}{a} = m_a + m_t$$

$$\frac{F - F_r}{a} - m_t = m_a$$

$$m_a = \frac{1.75 \cdot 10^4 \text{ N} - 2400 \text{ N}}{0.11 \text{ m/s}^2} - 1800 \text{ kg}$$

$$m_a = 135472.727 \text{ kg}$$

Part (b) Calculate the force, in newtons, exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane.

To begin, let's write Newton's Second Law for the airplane to get a formula for the net force.

$$F = m_a a$$

We can now write a sum of the forces on the airplane. To find the net force the tractor exerts on it, we will need to substitute our answer from part (a) in for the m

$$F = netF - F_f$$

$$m_a a = netF - F_f$$

$$m_a a + F_f = netF$$

$$netF = \left(\frac{1.75 \cdot 10^4 \text{ N} - 2400 \text{ N}}{0.11 \text{ m/s}^2} - 1800 \text{ kg} \right) \cdot 0.11 \text{ m/s}^2 + 2200 \text{ N}$$

$$netF = 17102 \text{ N}$$

Problem 252 - 5.6.23 :

A **1050** kg car pulls a boat on a trailer. The mass of the boat plus trailer is 700 kg.

Randomized Variables

$$m = 1050 \text{ kg}$$

$$f = 1750 \text{ N}$$

$$a = 0.51 \text{ m/s}^2$$

Part (a) What is the magnitude of the force which resists the motion of the car, boat, and trailer, if the car exerts a **1750** N force on the road and produce m/s^2 in Newtons?

To find the resisting force, let's begin by using Newton's Second Law to express the net force in terms of the mass and acceleration.

$$F_{net} = ma$$

$$F_{net} = (m_1 + m_2) a$$

Now, let's write an equation for the sum of the forces on the car, boat, and trailer.

$$F_{net} = F - f$$

$$(m_1 + m_2) a = F - f$$

$$(m_1 + m_2) a - F = -f$$

$$f = 1750 \text{ N} - (1050 \text{ kg} + 700 \text{ kg}) \cdot 0.51 \text{ m/s}^2$$

$$f = 857.5 \text{ N}$$

Part (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer in Newtons?

The force exerted by the hitch must be enough to accelerate the boat and the trailer. We can use Newton's Second Law to write the net force exerted by the hitch i boat and trailer multiplied by the acceleration.

$$F_{net} = m_2 a$$

Now, let's set up an equation for the sum of the forces on the boat and trailer, noting that the force exerted by the hitch will act in the positive x-direction and 80% found in part (a) will act in the negative x-direction.

$$F_{net} = F' - 0.8f$$

$$m_2 a = F' - 0.8f$$

$$m_2 a + 0.8f = F'$$

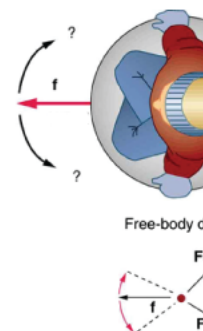
$$m_2 a + 0.8 [F - (m_1 + m_2) a] = F'$$

$$F' = 700 \text{ kg} \cdot 0.51 \text{ m/s}^2 + 0.8 \cdot [1750 \text{ N} - (1050 \text{ kg} + 700 \text{ kg}) \cdot 0.51 \text{ m/s}^2]$$

$$F' = 1043 \text{ N}$$

Problem 253 - 5.6.24 :

Two children pull a third child backward on a snow saucer sled exerting forces $F_1 = 7.5$ and $F_2 = 4.5$ as shown in the figure. Note that the direction of the friction force $f = 5.1$ N is unspecified; it will be opposite in direction to the sum of the other two forces.



Part (a) Find the magnitude of the acceleration of the 41 kg sled and child system, in meters per second squared.

To begin, we need to find the magnitude of the force resulting from the two children pulling on the sled. Let's begin by writing an expression the sum of those two

$$F_x = F_1 \cos(45^\circ) + F_2 \cos(30^\circ)$$

Let's do this for the y-direction now.

$$F_y = F_1 \sin(45^\circ) - F_2 \sin(30^\circ)$$

Now that we have these two equations, we can find the magnitude of the force pulling the sled.

$$|F| = \sqrt{(F_1 \cos(45^\circ) + F_2 \cos(30^\circ))^2 + (F_1 \sin(45^\circ) - F_2 \sin(30^\circ))^2}$$

Next, we will need to incorporate friction in order to find the net force. Since friction acts in the opposite direction of motion, the force of friction will be directly forces. Because of this, we can directly subtract it from the magnitude of the pulling force.

$$|F_{net}| = |F| - f$$

$$|F_{net}| = \sqrt{(F_1 \cos(45^\circ) + F_2 \cos(30^\circ))^2 + (F_1 \sin(45^\circ) - F_2 \sin(30^\circ))^2} - f$$

To go from a force to an acceleration, we will need to divide through by the mass. From there, we can substitute values into the equation and solve.

$$a = \frac{\sqrt{(F_1 \cos(45^\circ) + F_2 \cos(30^\circ))^2 + (F_1 \sin(45^\circ) - F_2 \sin(30^\circ))^2} - f}{m}$$

$$a = \frac{\sqrt{(7.5 \text{ N} \cdot \cos(45^\circ) + 4.5 \text{ N} \cdot \cos(30^\circ))^2 + (7.5 \text{ N} \cdot \sin(45^\circ) - 4.5 \text{ N} \cdot \sin(30^\circ))^2} - 5.1 \text{ N}}{41 \text{ kg}}$$

$$a = 0.112 \text{ m/s}^2$$

Part (b) Assuming the sled starts at rest, find the direction of the sled and child system in degrees north of east.

Since the force of friction is acting directly opposite the direction of motion, it does not change the angle at which the sled moves. As such, we need only to find the two forces pulling the sled. To do this, we can begin by examining the components of the pulling force in the x and y-directions.

$$F_x = F_1 \cos(45^\circ) + F_2 \cos(30^\circ)$$

$$F_y = F_1 \sin(45^\circ) - F_2 \sin(30^\circ)$$

We can use trigonometry to set up a relation between the tangent function of the angle of the force and the x and y components. From there, we can simplify and

$$\tan(\theta) = \frac{F_y}{F_x}$$

$$\theta = \arctan\left(\frac{F_y}{F_x}\right)$$

$$\theta = \arctan\left(\frac{F_1 \sin(45^\circ) - F_2 \sin(30^\circ)}{F_1 \cos(45^\circ) + F_2 \cos(30^\circ)}\right)$$

$$\theta = \arctan\left(\frac{7.5 \text{ N} \cdot \sin(45^\circ) - 4.5 \text{ N} \cdot \sin(30^\circ)}{7.5 \text{ N} \cdot \cos(45^\circ) + 4.5 \text{ N} \cdot \cos(30^\circ)}\right)$$

$$\theta = 18.359^\circ$$

Problem 254 - 5.6.25 :

A nurse pushes a cart by exerting a force on the handle at a downward angle 31° below the horizontal. The loaded cart has a mass of 28 kg, and the force of friction is 55 N.

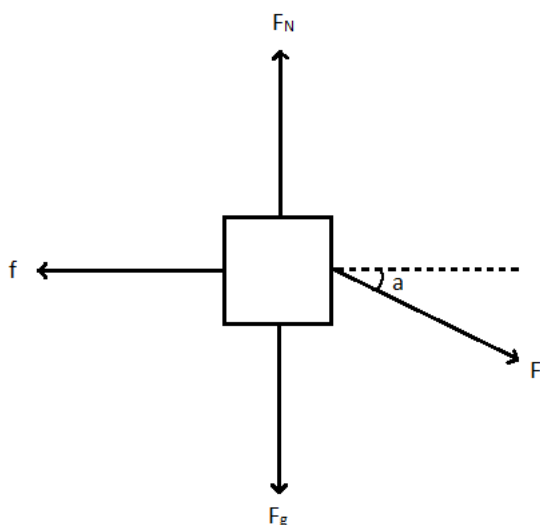
Randomized Variables

$$f = 55 \text{ N}$$

$$a = 31^\circ$$

Part (a) What is the magnitude of the force the nurse must exert to move at a constant velocity in Newtons?

To begin, let's draw a free-body diagram.



Now, we know that the forces in the y-direction will naturally cancel out, as the cart will not leave the floor when it is being pushed. This means that we need only balancing forces in the x-direction. We can write an expression for the net force in the x-direction, noting that the net force must be zero for the cart to move at a

$$F_{net} = F \cos(\theta) - f$$

$$0 = F \cos(a) - f$$

$$f = F \cos(a)$$

$$\frac{f}{\cos(a)} = F$$

$$F = \frac{55 \text{ N}}{\cos(31^\circ)}$$

$$F = 64.165 \text{ N}$$

Problem 255 - 5.6.26 :

A 1100 kg car pulls a boat on a trailer. The mass of the boat plus trailer is 700 kg.

Part (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900 N force on the road and produces an acceleration of 1.2 m

In this problem, we are told that a truck exerts a force of 1,900 N on the road. It does this by having the engine turn the wheels which the road "catch" via a friction force. The road exerts a force in the opposite direction on the tires. This force is in the direction of motion, consistent with Newton's Second Law. It might be strange to think of friction as a force that prevents or stops motion, but, in fact, the friction here is necessary for the truck to move forward. We can rationalize this if we imagine a truck with very low friction. With the low friction force, the engine would make the wheels spin, but they would no longer get "caught" by the road, and the truck would not accelerate forward. We can also see that this force of 1900 N of friction is the ONLY force propelling the truck forward. That is, the trailer, which is just being pulled by the truck and does its own thing, will not lead to any additional propulsion force on

the truck with trailer system. The only force responsible for the acceleration is that 1900 N friction force. As the problem statement points out, there, generally, with motion, from air resistance and from slight deformation of the tires (more extreme if the tires aren't well-inflated!). We can solve for this resistive force by using the following.

$$F_{net} = ma$$

where m is the mass of the system accelerating, taken here to be the combined mass of the truck, trailer, and boat, 1,800 kg. Substituting into the above yields

$$1900N - F = 1,800kg \cdot 1.2 \frac{m}{s^2}$$

We solve for F to be

$$F = -260 \text{ N}$$

$$F = -260 \text{ N}$$

Part (b) What is unreasonable about this situation?

Hopefully, your solution to part a raised a red flag and you thought something like "wait a minute! What does it mean for F to be negative? Since I subtracted it assuming it to be a resistive force, doesn't a negative value mean it is actually pushing the truck along? Is this truck driving through a hurricane that is also helping something?!" In this part b, we are asked to consider what is unreasonable about the problem set-up that could have led to the unreasonable result of a negative re unreasonable to tow something massive. In fact, the truck could pull something even more massive than it, just as you can push (or pull) your car when it breaks bring it off the road. The force the tires exert on the road to propel the truck forward depend upon the strength of the truck's engine, assuming the coefficient of friction prevent sliding. However, with such a massive system and relatively small friction force, we should expect a smaller acceleration. In the limit of there being no re would have a maximum acceleration of only

$$1900 = 1,800a$$

$$a = 1.05 \frac{m}{s^2}$$

It is unreasonable to have this acceleration

Problem 256 - 5.6.27 :

A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only **0.15 N**.

Randomized Variables

$$f = 0.15 \text{ N}$$

Part (a) Knowing the coefficient of kinetic friction between the two materials is about 0.04, he quickly calculates the normal force. What is it in newtons?

To solve this problem, we can use the fact that the force of friction is equal to the normal force multiplied by the coefficient of kinetic friction.

$$f = \mu_k N$$

$$\frac{f}{\mu_k} = N$$

$$N = \frac{0.15 \text{ N}}{0.04}$$

$$N = 3.75 \text{ N}$$

Problem 257 - 5.6.28 :

When rebuilding her car's engine, a physics major must exert **275 N** of force to insert a dry steel piston into a steel cylinder.

Randomized Variables

$$f = 275 \text{ N}$$

Part (a) What is the normal force between the piston and cylinder in newtons? You may assume the coefficient of kinetic friction here is 0.3.

To find the normal force, we can use the fact that the force of friction is equal to the coefficient of kinetic friction multiplied by the normal force.

$$F_f = \mu_k f$$

$$\frac{F_f}{\mu_k} = f$$

$$f = \frac{275 \text{ N}}{0.3}$$

$$f = 916.667 \text{ N}$$

Part (b) What force would she have to exert if the steel parts were oiled (decreasing the coefficient of friction by a factor of 10) in Newtons?

Here, we will need to use the relation between the normal force, coefficient of friction, and the force of friction once again. In this case, we will need to use our normal force to solve for the minimum required force to overcome friction.

$$f = \mu_k N$$

$$f = \frac{0.3}{10} \cdot \frac{275 \text{ N}}{0.3}$$

$$f = 27.5 \text{ N}$$

Problem 258 - 5.6.29 :

During strenuous exercise it is possible to exert forces to the joints that are ten times greater than the weight being supported when not in motion. The coefficients of friction for joints are generally very small - for this problem, assume the coefficient of static friction in a knee joint is 0.016, while for kinetic friction it is 0.015.

Randomized Variables

$$m = 61 \text{ kg}$$

Part (a) What is the frictional force in the knee joint of a person who is standing still and supporting 61 kg of her mass on a knee?

First, note that the normal force in this case will be equal to the person's weight. Further, since the person is standing still, we will want to use the coefficient of static friction. The maximum force of friction is equal to the coefficient of friction multiplied by the normal force, so we can set up the following equation:

$$f_{max} = \mu_s N$$

$$f_{max} = \mu_s mg$$

$$f_{max} = 0.016 \cdot 61 \text{ kg} \cdot 9.8 \text{ m/s}^2$$

$$f_{max} = 9.565 \text{ N}$$

Part (b) What is the maximum force of friction if this person begins strenuous exercise?

The problem statement notes that it is possible to exert ten times as much force as a person weighs during strenuous exercise. The normal force in this case will be ten times the person's weight. Since strenuous exercise will involve movement, we will need to use the coefficient of kinetic friction to find the friction in this case. Note that the force of friction is equal to the coefficient of friction multiplied by the normal force to set up the following equation:

$$f = \mu_k N$$

$$f = \mu_k \cdot 10mg$$

$$f = 0.015 \cdot 10 \cdot 61 \text{ kg} \cdot 9.81 \text{ m/s}^2$$

$$f = 89.762 \text{ N}$$

Problem 259 - 5.6.34 :

Full solution not currently available at this time.

Shown to the right is a block of mass $m = 5.1$ kg sitting on a ramp that makes an angle $\theta = 21^\circ$ with the horizontal. This block is being pushed by a horizontal force $F = 201$ N. The coefficient of kinetic friction between the two surfaces is $\mu = 0.41$.



Part (a) Write an equation for the acceleration of the block up the ramp using variables from the problem statement together with g for the acceleration

$$a = (F \cos(\theta) - m g \sin(\theta) - m g \mu \cos(\theta) - F \mu \sin(\theta)) / m$$

Part (b) Find the acceleration of the block up the ramp in

$$\frac{m}{s^2}.$$

$$a = (F \cos(\theta) - m g \sin(\theta) - m g \mu \cos(\theta) - F \mu \sin(\theta)) / m$$

$$a = (201 \cos(21^\circ) - 5.1 \cdot 9.8 \sin(21^\circ) - 5.1 \cdot 9.8 \cdot 0.41 \cos(21^\circ) - 201 \cdot 0.41 \sin(21^\circ)) / 5.1$$

$$a = 23.74$$

$$\text{Tolerance: } \pm 0.7122$$

Problem 260 - 5.6.34(sym) :

Full solution not currently available at this time.

Shown to the right is a block of mass m sitting on a ramp that makes an angle θ with the horizontal. This block is being pushed by a horizontal force F . The coefficient of kinetic friction between the two surfaces is μ .



Part (a) Write an equation for the acceleration of the block up the ramp using variables from the problem statement together with g for the acceleration

$$a = (F \cos(\theta) - m g \sin(\theta) - m g \mu \cos(\theta) - F \mu \sin(\theta)) / m$$

Problem 261 - 5.6.33 :

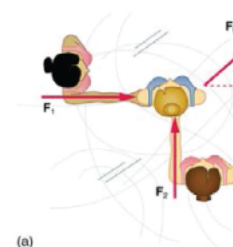
Consider the 61 kg ice skater being pushed by two others shown in the figure. The coefficient of static friction is $\mu_s = 0.4$ and kinetic is $\mu_k = 0.02$.

Randomized Variables

$$m = 61 \text{ kg}$$

$$F_1 = 230 \text{ N}$$

$$F_2 = 160 \text{ N}$$



Part (a) Find the magnitude of F_{tot} , the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are **230 N** and **160 N**, respectively.

Looking at the diagram, we see that F_1 is exerted in the x-direction and F_2 is exerted in the y-direction. To find the magnitude of the sum of these two forces, we use the Pythagorean theorem to solve for the resultant vector.

$$F_{tot} = \sqrt{F_1^2 + F_2^2}$$

$$F_{tot} = \sqrt{(230 \text{ N})^2 + (160 \text{ N})^2}$$

$$F_{tot} = 280.179 \text{ N}$$

Part (b) Find the direction of F_{tot} (in degrees relative to the horizontal), the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are respectively.

Looking at the diagram for this problem, we see that F_1 is applied in the x-direction and F_2 is applied in the y-direction. Using basic trigonometry, we can create the tangent function of the angle above the horizontal and these two forces.

$$\tan(\theta) = \frac{F_2}{F_1}$$

$$\theta = \arctan\left(\frac{F_2}{F_1}\right)$$

$$\theta = \arctan\left(\frac{160 \text{ N}}{230 \text{ N}}\right)$$

$$\theta = 34.825^\circ$$

Part (c) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of F_{tot} in m/s^2 ?

The force of friction that the skater experiences when she starts moving from rest will be equal to the coefficient of static friction multiplied by the normal force on her. Since she is standing on level ground, the normal force will be equal to her weight. Using this information together with the results from part (a) for the magnitude of the force pushing her, we can set up the following equation:

$$F_{net} = F_{tot} - \mu_s F_N$$

$$F_{net} = \sqrt{F_1^2 + F_2^2} - \mu_s mg$$

Now that we have an expression for the net force that the skater experiences, we can find her acceleration by dividing the net force by her mass.

$$a = \frac{\sqrt{F_1^2 + F_2^2} - \mu_s mg}{m}$$

$$a = \frac{\sqrt{(230 \text{ N})^2 + (160 \text{ N})^2} - 0.4 \cdot 61 \text{ kg} \cdot 9.8 \text{ m/s}^2}{61 \text{ kg}}$$

$$a = 0.6731 \text{ m/s}^2$$

Part (d) What is her acceleration assuming she is already moving in the direction of F_{tot} in m/s^2 ?

This problem is almost identical to the one we worked in part (c). The only difference is that the skater does not start from rest, meaning that we will use the coef find the force of friction. As in part (c), the normal force is still equal to the weight of the skater and the force pushing her is still equal to the value we found in p equation for the net force on the skater.

$$F_{\text{net}} = F_{\text{tot}} - \mu_k F_N$$

$$F_{\text{net}} = \sqrt{F_1^2 + F_2^2} - \mu_k mg$$

Now that we have an expression for the net force that the skater experiences, we can find her acceleration by dividing the net force by her mass.

$$a = \frac{\sqrt{F_1^2 + F_2^2} - \mu_k mg}{m}$$

$$a = \frac{\sqrt{(230 \text{ N})^2 + (160 \text{ N})^2} - 0.02 \cdot 61 \text{ kg} \cdot 9.8 \text{ m/s}^2}{61 \text{ kg}}$$

$$a = 4.397 \text{ m/s}^2$$

Problem 262 - 5.6.39 :

Consider a car heading down a 5.5° slope (one that makes an angle of 5.5° with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved —that is, the tires are not allowed to slip during the acceleration. Use a coordinate system in which down the slope is positive acceleration.

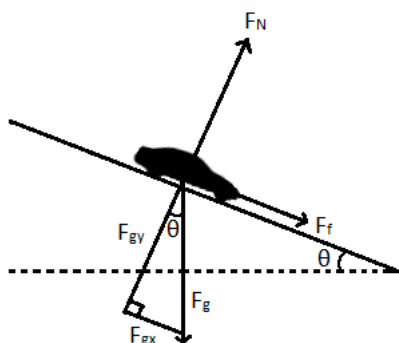
Randomized Variables

$$\theta = 5.5^\circ$$

System	s
Rubber on dry concrete	
Rubber on wet concrete	
Steel on steel(dry)	
Steel on steel (oiled)	
Shoes on wood	
Shoes on ice	
Steel on ice	

Part (a) Calculate the maximum acceleration for the car on dry concrete, in meters per square second.

Let's start this problem by drawing a free-body diagram of the car.



We see that there are two forces acting on the car in the y -direction we have assigned; a normal force moving the car in the positive direction and a y -component moving it in the negative direction. Since the car will not be moving off the incline, there will be no net force in the y -direction. We can use this information to write the sum of forces in the y -direction.

$$F_{y,net} = F_N - F_{gy}$$

$$0 = F_N - F_{gy}$$

$$F_{gy} = F_N$$

$$F_g \cos(\theta) = F_N$$

Now, let's look at the forces in the x -direction. In the x -direction, the wheels spinning against the ground cause static friction to push the car in the positive x -direction, attaining the maximum acceleration when the maximum force of static friction is exerted on it. At the same time, a component of the gravitational force also acts in the positive x -direction. Using the expression we just found for normal force, we can write the following expression for the forces in the x -direction.

$$F_{x,net} = F_f + F_{gx}$$

$$F_{x,net} = \mu_s F_g \cos(\theta) + F_g \sin(\theta)$$

$$F_{x,net} = \mu_s mg \cos(\theta) + mg \sin(\theta)$$

To convert our expression for the net force in the x -direction to acceleration in the x -direction, we need to recall Newton's Second Law.

$$F = ma$$

$$\frac{F}{m} = a$$

This equation shows that we can find the acceleration of the car by dividing the net force it experiences by its mass.

$$a_{max} = \frac{\mu_s mg \cos(\theta) + mg \sin(\theta)}{m}$$

$$a_{max} = \mu_s g \cos(\theta) + g \sin(\theta)$$

$$a_{max} = g(\mu_s \cos(\theta) + \sin(\theta))$$

$$a_{max} = 9.8 \text{ m/s}^2 \cdot (1.0 \cdot \cos(5.5^\circ) + \sin(5.5^\circ))$$

$$a_{max} = 10.694 \text{ m/s}^2$$

Part (b) Calculate the maximum acceleration on wet concrete, in meters per square second.

As compared to part (a), all that has changed is the coefficient of static friction. This means that the problem is otherwise completely identical. We can therefore find in part (a) and replace the coefficient of static friction with the one for wet concrete in order to find the solution.

$$a_{max} = g(\mu_s \cos(\theta) + \sin(\theta))$$

$$a_{max} = 9.8 \text{ m/s}^2 \cdot (0.7 \cdot \cos(5.5^\circ) + \sin(5.5^\circ))$$

$$a_{max} = 7.768 \text{ m/s}^2$$

Part (c) Calculate the maximum acceleration for the car on ice, in meters per square second, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

As compared to part (a), all that has changed is the coefficient of static friction. This means that the problem is otherwise completely identical. We can therefore find in part (a) and replace the coefficient of static friction with the one for ice to find the solution.

$$a_{max} = g(\mu_s \cos(\theta) + \sin(\theta))$$

$$a_{max} = 9.8 \text{ m/s}^2 \cdot (0.100 \cdot \cos(5.5^\circ) + \sin(5.5^\circ))$$

$$a_{max} = 1.915 \text{ m/s}^2$$

Problem 263 - 5.6.44 :

A contestant in a winter sporting event pushes a **41** kg block of ice in the positive direction across a frozen lake as shown in the figure. Assume the coefficients of static and kinetic friction are $\mu_s=0.1$ and $\mu_k=0.03$.

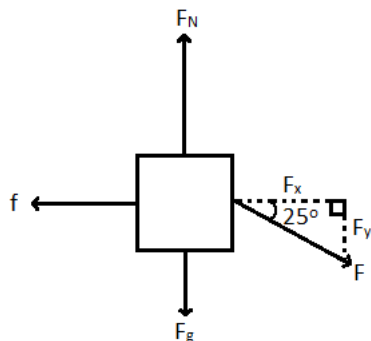
Randomized Variables

$$m = \mathbf{41} \text{ kg}$$



Part (a) Calculate the minimum force **F** he must exert to get the block sliding across the ice in newtons.

First, we'll draw a free-body diagram of this system.



Let's proceed by summing the forces in the y-direction. The normal force acts in the positive y-direction and the force of gravity and the y-component of the force act in the negative y-direction. Since the net force on the block in the y-direction must be zero (as it will not accelerate up or down), we can set up the following equation:

$$F_{net,y} = F_N - F_g - F_{min,y}$$

$$0 = F_N - mg - F_{min}\sin(25^\circ)$$

$$mg + F_{min}\sin(25^\circ) = F_N$$

Now, let's repeat this process for the x-direction. The only forces acting in the x-direction are the x-component of the force the contestant exerts in the positive direction and friction acting in the negative direction. Since we want to find the point at which the block just begins to move, we will want static friction to be exerting its maximum value. Since the net force in the x-direction must be equal to zero, this gives us the following equation:

$$F_{net,x} = F_{min,x} - f$$

$$0 = F_{min}\cos(25^\circ) - \mu_s F_N$$

We can solve this problem by substituting in the expression that we found earlier for the normal force and solving for F_{min} .

$$0 = F_{min}\cos(25^\circ) - \mu_s(mg + F_{min}\sin(25^\circ))$$

$$0 = F_{min}\cos(25^\circ) - \mu_s mg - \mu_s F_{min}\sin(25^\circ)$$

$$\mu_s mg = F_{min}\cos(25^\circ) - \mu_s F_{min}\sin(25^\circ)$$

$$\mu_s mg = F_{min}(\cos(25^\circ) - \mu_s \sin(25^\circ))$$

$$\frac{\mu_s mg}{\cos(25^\circ) - \mu_s \sin(25^\circ)} = F_{min}$$

$$F_{min} = \frac{0.1 \cdot 41 \text{ kg} \cdot 9.8 \text{ m/s}^2}{\cos(25^\circ) - 0.1 \cdot \sin(25^\circ)}$$

$$F_{min} = 46.502 \text{ N}$$

Part (b) What is its acceleration once it starts to move, if that force is maintained in m/s^2 ?

To begin this problem, we will need to sum the forces in the y-direction just like we did in part (a).

$$F_{net,y} = F_N - F_g - F_{min,y}$$

$$0 = F_N - mg - F_{min} \sin(25^\circ)$$

$$mg + F_{min} \sin(25^\circ) = F_N$$

Now let's write an equation for the forces in the x-direction. While this is mostly identical to what we did in part (a), we will use the coefficient of kinetic friction will not be zero in this case.

$$F_{net,x} = F_{min,x} - f$$

$$F_{net,x} = F_{min} \cos(25^\circ) - \mu_k F_N$$

To proceed, let's substitute in the expression we found for the normal force.

$$F_{net,x} = F_{min} \cos(25^\circ) - \mu_k (mg + F_{min} \sin(25^\circ))$$

We can now find an acceleration by applying Newton's Second Law.

$$F_{net,x} = ma$$

$$\frac{F_{net,x}}{m} = a$$

$$a = \frac{F_{min} \cos(25^\circ) - \mu_k (mg + F_{min} \sin(25^\circ))}{m}$$

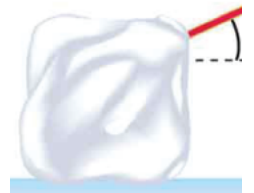
Finally, to complete the problem, we can substitute in our answer from part (a) for the minimum force and solve for the acceleration.

$$a = \frac{46.502 \text{ N} \cdot \cos(25^\circ) - 0.03 (41 \text{ kg} \cdot 9.8 \text{ m/s}^2 + 46.502 \text{ N} \cdot \sin(25^\circ))}{41 \text{ kg}}$$

$$a = 0.7196 \text{ m/s}^2$$

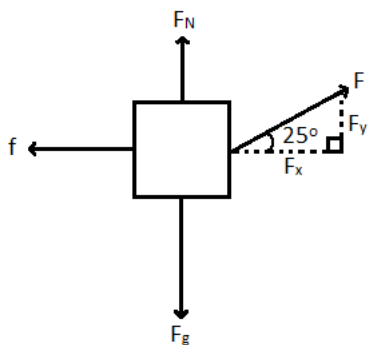
Problem 264 - 5.6.45 :

A contestant in a winter sporting event pulls a **41** kg block of ice in the positive horizontal direction with a rope over his shoulders across a frozen lake as shown in the figure. Assume the coefficients of static and kinetic friction are $\mu_s=0.1$ and $\mu_k=0.03$.



Part (a) Calculate the minimum force F he must exert to get the block sliding in newtons.

First, we'll draw a free-body diagram of this system.



Let's proceed by summing the forces in the y-direction. The normal force and the y-component of the force the contestant exerts act in the positive direction while in the negative direction. Since the net force on the block in the y-direction must be zero (as it will not accelerate up or down), we can set up the following equation:

$$F_{net,y} = F_N + F_{min,y} - F_g$$

$$0 = F_N + F_{min}\sin(25^\circ) - mg$$

$$mg - F_{min}\sin(25^\circ) = F_N$$

Now, let's repeat this process for the x-direction. The only forces acting in the x-direction are the x-component of the force the contestant exerts in the positive direction and friction acting in the negative direction. Since we want to find the point at which the block just begins to move, we will want static friction to be exerting its maximum value. This gives us the following equation:

$$F_{net,x} = F_{min,x} - f$$

$$0 = F_{min}\cos(25^\circ) - \mu_s F_N$$

We can solve this problem by substituting in the expression that we found earlier for the normal force and solving for F .

$$0 = F_{min}\cos(25^\circ) - \mu_s (mg - F_{min}\sin(25^\circ))$$

$$0 = F_{min}\cos(25^\circ) - \mu_s mg + \mu_s F_{min}\sin(25^\circ)$$

$$\mu_s mg = F_{min}\cos(25^\circ) + \mu_s F_{min}\sin(25^\circ)$$

$$\mu_s mg = F_{min}(\cos(25^\circ) + \mu_s \sin(25^\circ))$$

$$\frac{\mu_s mg}{\cos(25^\circ) + \mu_s \sin(25^\circ)} = F_{min}$$

$$F_{min} = \frac{0.1 \cdot 41 \text{ kg} \cdot 9.8 \text{ m/s}^2}{\cos(25^\circ) + 0.1 \cdot \sin(25^\circ)}$$

$$F_{min} = 42.359 \text{ N}$$

Part (b) What is its acceleration in m/s^2 once it starts to move, if that force is maintained?

To begin this problem, we will need to sum the forces in the y-direction just like we did in part (a).

$$F_{net,y} = F_N + F_{min,y} - F_g$$

$$0 = F_N + F_{min} \sin(25^\circ) - mg$$

$$mg - F_{min} \sin(25^\circ) = F_N$$

Now let's write an equation for the forces in the x-direction. While this is mostly identical to what we did in part (a), we will use the coefficient of kinetic friction will not be zero in this case.

$$F_{net,x} = F_{min,x} - f$$

$$F_{net,x} = F_{min} \cos(25^\circ) - \mu_k F_N$$

To proceed, let's substitute in the expression we found for the normal force.

$$F_{net,x} = F_{min} \cos(25^\circ) - \mu_k (mg - F_{min} \sin(25^\circ))$$

We can now find an acceleration by applying Newton's Second Law.

$$F_{net,x} = ma$$

$$\frac{F_{net,x}}{m} = a$$

$$a = \frac{F_{min} \cos(25^\circ) - \mu_k (mg - F_{min} \sin(25^\circ))}{m}$$

Finally, to complete the problem, we can substitute in our answer from part (a) for the minimum force and solve for the acceleration.

$$a = \frac{42.359 \text{ N} \cdot \cos(25^\circ) - 0.03 (41 \text{ kg} \cdot 9.8 \text{ m/s}^2 - 42.359 \text{ N} \cdot \sin(25^\circ))}{41 \text{ kg}}$$

$$a = 0.6554 \text{ m/s}^2$$

Problem 265 - 5.6.46 :

The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid.

Randomized Variables

$$m = 75 \text{ kg}$$

$$A = 0.11 \text{ m}^2$$

Part (a) Find the terminal velocity (in meters per second) of an 75-kg skydiver falling in a pike (headfirst) position with a surface area of 0.11 m² and a C_d may assume the density of air is 1.21 kg/m³.

Terminal velocity occurs when the force of drag reaches a magnitude equal to the force of gravity. We can use the following equation to solve for the terminal vel

$$v_t = \sqrt{\frac{2mg}{C_d \rho A}}$$

$$v_t = \sqrt{\frac{2 \cdot 75 \text{ kg} \cdot 9.8 \text{ m/s}^2}{0.7 \cdot 1.21 \text{ kg/m}^3 \cdot 0.11 \text{ m}^2}}$$

$$v_t = 125.609 \text{ m/s}$$

Problem 266 - 5.6.47 :

A **52** kg and a **82** kg skydiver jump from an airplane at an altitude of 6000 m, both falling in the diving/headfirst position. Assume their surface area is **0.105** m² and the drag coefficient is 0.70.

Randomized Variables

$$m_1 = 52 \text{ kg}$$

$$m_2 = 82 \text{ kg}$$

$$A = 0.105 \text{ m}^2$$

Part (a) How long will it take for the first skydiver to reach the ground in seconds (assuming the time to reach terminal velocity is small and the density of air is 1.21 kg/m³)?

Since we are assuming that the skydiver reaches terminal velocity quickly, we can assume that they travel the entire distance at terminal velocity. To begin, let's find the terminal velocity of this skydiver.

$$v_{t1} = \sqrt{\frac{2m_1g}{C\rho A}}$$

Now we can use the relation between velocity, displacement, and time for an object moving with no acceleration to solve for the time.

$$v_{t1} = \frac{d}{t_1}$$

$$t_1 v_{t1} = d$$

$$t_1 = \frac{d}{v_{t1}}$$

$$t_1 = \frac{d}{\sqrt{\frac{2m_1g}{C\rho A}}}$$

$$t_1 = \frac{6000 \text{ m}}{\sqrt{\frac{2 \cdot 52 \text{ kg} \cdot 9.8 \text{ m/s}^2}{0.70 \cdot 1.21 \text{ kg/m}^3 \cdot 0.105 \text{ m}^2}}}$$

$$t_1 = 56.048 \text{ s}$$

Part (b) How long will it take for the second skydiver to reach the ground in seconds (assuming the time to reach terminal velocity is small)?

We can use the same procedure here that we used in part (a). First, let's write an expression for the terminal velocity of the second skydiver.

$$v_{t2} = \sqrt{\frac{2m_2g}{C\rho A}}$$

Now let's use the relation between velocity, displacement, and time for an object moving with no acceleration to solve for the time.

$$v_{t2} = \frac{d}{t_2}$$

$$t_2 v_{t2} = d$$

$$t_2 = \frac{d}{v_{t2}}$$

$$t_2 = \frac{d}{\sqrt{\frac{2m_2g}{C\rho A}}}$$

$$t_2 = \frac{6000 \text{ m}}{\sqrt{\frac{2 \cdot 82 \text{ kg} \cdot 9.8 \text{ m/s}^2}{0.70 \cdot 1.21 \text{ kg/m}^3 \cdot 0.105 \text{ m}^2}}}$$

$$t_2 = 44.633 \text{ s}$$

Problem 267 - 5.6.50 :

Suppose a car accelerates from **61** to **102** km/h?

Randomized Variables

$s_1 = 61$ km/h

$s_2 = 102$ km/h

Part (a) By what percent does the drag force on a car increase?

The percent by which the drag force increases can be found by first dividing the change in drag force by the initial drag force. We will then need to multiply by 100 fraction into a percent. This gives us the following equation:

$$\%increase = \frac{F_{D2} - F_{D1}}{F_{D1}} \cdot 100$$

Now, let's plug the formula for drag force into the above equation.

$$\%increase = \frac{0.5C\rho A s_2^2 - 0.5C\rho A s_1^2}{0.5C\rho A s_1^2} \cdot 100$$

$$\%increase = \frac{s_2^2 - s_1^2}{s_1^2} \cdot 100$$

$$\%increase = \frac{102^2 - 61^2}{61^2} \cdot 100$$

$$\%increase = 179.602\%$$

Problem 268 - 5.6.51 :

Calculate the velocity a spherical rain drop would achieve falling (taking downward as positive) from **4.2** km in the following situations.

Randomized Variables

$$h = 4.2 \text{ km}$$

$$l = 3.2 \text{ mm}$$

$$d = 1.15 \text{ kg/m}^3$$

Part (a) Calculate the velocity in the absence of air drag in m/s.

In the absence of a drag force, we can solve this problem with a basic kinematic equation. Defining down as the positive direction, we see that the initial velocity falling is zero, the distance the drop falls is equal to the height it starts from, and the acceleration is simply the acceleration due to gravity. Remembering to convert kilometers to meters, we can set up and solve the following kinematic equation for the final velocity:

$$v^2 = v_0^2 + 2ad$$

$$v^2 = 0 + 2gh$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \cdot 9.8 \text{ m/s}^2 \cdot 4.2 \cdot 10^3 \text{ m}}$$

$$v = 286.915 \text{ m/s}$$

Part (b) Calculate the velocity with air drag in m/s. Take the size across of the drop to be 3.2 mm, the density of air to be 1.15 kg/m³, the density of water surface area to be πr^2 , and the drag coefficient to be 1.0.

When something falls with drag forces present, it will eventually reach a terminal velocity. Therefore, to solve this problem, we need to solve for the terminal velocity by writing out the equation for terminal velocity.

$$v_t = \sqrt{\frac{2mg}{C\rho_a A}}$$

As we are told to assume that the surface area of the raindrop is equal to $2\pi r$, all that we need to know to complete this problem is the raindrop's mass. Since we know the radius of the sphere from the radius, and we know the density, we can use the density equation to find an expression for the raindrop's mass.

$$\rho_w = \frac{m}{V}$$

$$\rho_w V = m$$

$$\rho_w \cdot \frac{4}{3}\pi r^3$$

Now, let's plug this expression for mass into our previous equation along with the expression for the area of the drop's falling surface and solve for the terminal velocity.

$$v_t = \sqrt{\frac{2\rho_w \cdot \frac{4}{3}\pi r^3 \cdot g}{C\rho_a \cdot \pi r^2}}$$

$$v_t = \sqrt{\frac{8\rho_w r g}{3\rho_a C}}$$

To finish this problem, we will need to substitute in values and solve. As we do so, we must be careful to convert the diameter of the raindrop both to a radius and meters.

$$v_t = \sqrt{\frac{8 \cdot 1000 \text{ kg/m}^3 \cdot \left(\frac{3.2 \cdot 10^{-3}}{2}\right) \text{ m} \cdot 9.8 \text{ m/s}^2}{3 \cdot 1.15 \text{ kg/m}^3 \cdot 1.0}}$$

$$v_t = 6.03 \text{ m/s}$$

Problem 269 - 5.6.53 :

Stokes' law describes sedimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density $7.5 \times 10^3 \text{ kg/m}^3$, radius 0.65 mm) is dropped in a container of motor oil. It takes 10.5 s to fall a distance of 0.45 m .

Randomized Variables

$$d = 7.5 \times 10^3 \text{ kg/m}^3$$

$$r = 0.65 \text{ mm}$$

$$t = 10.5 \text{ s}$$

$$h = 0.45 \text{ m}$$

Part (a) Calculate the viscosity of the oil assuming the ball achieves terminal velocity immediately.

The viscosity can be found by writing an equation for the net force and solving for the viscosity at which the net force would be zero, as this is the condition for Stokes' law for the drag force and letting down be the positive direction, we can write the following equation for the sum of forces:

$$F_{net} = F_g - F_D$$

$$0 = mg - 6\pi a\eta v$$

$$6\pi a\eta v = mg$$

$$\eta = \frac{mg}{6\pi a v}$$

To proceed, we need to find the velocity and the mass of the ball bearing. Since it reaches terminal velocity quickly, we can treat the ball bearing as falling the entire distance at that velocity. As such, we can use the relationship between velocity, distance, and time traveled for an object that does not experience acceleration to get an expression for velocity:

$$v = \frac{h}{t}$$

To find the mass, we can use the relation between density, mass, and volume, given that we know the density and can find the volume of a sphere from its radius.

$$\rho = \frac{m}{V}$$

$$\rho V = m$$

$$\rho \cdot \frac{4}{3}\pi a^3 = m$$

Now, we can plug these expressions for velocity and mass into our earlier equation and then solve for the viscosity. While we do so, we must be careful to convert millimeters to meters.

$$\eta = \frac{\rho \cdot \frac{4}{3}\pi a^3 g}{6\pi a \left(\frac{h}{l}\right)}$$

$$\eta = \frac{2\rho a^2 g l}{9h}$$

$$\eta = \frac{2 \cdot 7.5 \cdot 10^3 \text{ kg/m}^3 \cdot (0.65 \cdot 10^{-3} \text{ m})^2 \cdot 9.8 \text{ m/s}^2 \cdot 10.5 \text{ s}}{9 \cdot 0.45 \text{ m}}$$

$$\eta = 0.161 \text{ kg/ (m} \cdot \text{s)}$$

Problem 270 - 5.6.56 :

A penny is placed a distance r from the center of a record spinning at $\omega = 45$ rpm. The coefficient of static friction between the penny and the record is $\mu_s = 0.17$.

Randomized Variables

$$\mu_s = 0.17$$

Part (a) Select an expression for the maximum distance, r , the penny can be placed from the center and not move.

SchematicChoice :

$$r = \frac{\mu_s}{\omega^2}$$

$$r = \frac{\mu_s g}{\omega^2}$$

$$r = \frac{\mu_s}{\omega}$$

$$r = \frac{\mu_s g}{\omega}$$

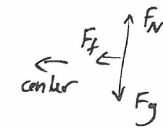
$$r = \frac{g}{\omega^2}$$

$$r = \frac{\mu_k g}{\omega^2}$$

Part (b) What is the distance, r in meters?

Numeric : A numeric value is expected and not an expression.

$r =$

a) FBD  $\sum F_r = F_f \leq \mu_s F_N = \mu_s m g = m \frac{v^2}{r} \rightarrow r = \frac{v^2}{\mu_s g}$
for maximum

with $\omega = \frac{v}{r}$, $r = \frac{\omega^2 r^2}{\mu_s g} \rightarrow \frac{1}{r} = \frac{\omega^2}{\mu_s g} \rightarrow r = \frac{\mu_s g}{\omega^2}$

b) convert $45 \frac{\text{rev}}{\text{min}} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = \frac{3}{2} \pi \text{ rad/s}$, $r = 0.075 \text{ m}$

Problem 271 - 5.6.52 :

Find the terminal velocity of a spherical bacterium (diameter $2.00 \mu\text{m}$) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be $1.10 \times 10^3 \text{ kg/m}^3$.

Solution Using Stokes' law, we can find the terminal velocity by equating the drag force and the weight of the bacterium. Solving for velocity, we obtain:

$$v = \frac{mg}{6\pi r\eta} = \frac{\rho_{\text{bac}}(4/3)\pi r^3 g}{6\pi r\eta} = \frac{2\rho_{\text{bac}}r^2 g}{9\eta}$$

$$= \frac{2(1100 \text{ kg/m}^3)(1 \times 10^{-6} \text{ m})^2(9.8 \text{ m/s}^2)}{9(1.005 \times 10^{-3} \text{ kg/m} \cdot \text{s})} = \underline{2.38 \times 10^{-6} \text{ m/s}}$$

Problem 272 - 5.6.57 :

Full solution not currently available at this time.

A crate of mass $m = 91 \text{ kg}$ rests on a rough surface inclined at an angle of $\theta = 31^\circ$ with the horizontal. A massless rope to which a force can be applied parallel to the surface is attached to the crate and leads to the top of the incline. In its present state, the crate is just ready to slip and start to move down the plane. The coefficient of kinetic friction is given by $\mu_k = f\mu_s$ where μ_s refers to the coefficient of static friction and $f = 0.75$. For all parts of this problem, consider up the surface to be the positive direction of motion.

Part (a) Write an equation for the coefficient of static friction in terms of variables from the problem statement.

$$\mu_s = \tan(\theta)$$

Part (b) Calculate the coefficient of static friction.

$$\mu_s = \tan(\theta \cdot \pi / 180)$$

$$\mu_s = \tan(31 \cdot \pi / 180)$$

$$\mu_s = 0.6009$$

$$\text{Tolerance: } \pm 0.018027$$

Part (c) Write an equation for the maximum force the rope can exert parallel to the surface of the plane before the crate begins to slide upwards. Give your variables from the problem statement together with g for the acceleration due to gravity.

$$F = 2 m g \sin(\theta)$$

Part (d) Calculate the maximum force that can be applied upward along the plane by the rope and not move the crate. Give your answer in newtons.

$$F = 2 \cdot 9.81 \cdot m \cdot \sin(\theta \cdot \pi / 180)$$

$$F = 2 \cdot 9.81 \cdot 91 \cdot \sin(31 \cdot \pi / 180)$$

$$F = 919.559$$

$$\text{Tolerance: } \pm 27.58677$$

Part (e) With a slightly greater applied force than what you found in the previous part, the crate will slide up the plane. Write an equation for the acceleration begins to slide upwards. Give your equation in terms of variables from the problem statement together with g for acceleration due to gravity and μ_s for the friction.

$$a = g \sin(\theta) - g f \mu_s \cos(\theta)$$

Part (f) Find the acceleration of the crate (in $\frac{m}{s^2}$) when it begins to slide upwards.

$$a = 9.81 \cdot \sin(\theta \cdot \pi / 180) \cdot (1 - f)$$

$$a = 9.81 \cdot \sin(31 \cdot \pi / 180) \cdot (1 - 0.75)$$

$$a = 1.263$$

$$\text{Tolerance: } \pm 0.03789$$

Part (g) After the crate begins to move, we reduce the force on the rope. Write an equation for the force exerted upwards by the rope at which the moving crate moves at a constant speed. Give your equation in terms of variables from the problem statement together with g for acceleration due to gravity and μ_s for friction.

$$F = m g \sin(\theta) + m g f \mu_s \cos(\theta)$$

Part (h) Calculate the force that must be exerted upwards by the rope for the moving crate to stop accelerating and move at a constant speed. Give your

$$F = 9.81 * m * \sin(\theta * \pi / 180) * (1 + f)$$

$$F = 9.81 * 91 * \sin(31 * \pi / 180) * (1 + 0.75)$$

$$F = 804.614$$

$$\text{Tolerance: } \pm 24.13842$$

Part (i) Consider a case where no force is being exerted by the rope and the crate is at rest. Write an equation for the acceleration of the crate down the plane to get it started. Give your answer in terms of variables from the problem statement together with g for acceleration due to gravity and μ_s for the coefficient of static friction.

$$a = g f \mu_s \cos(\theta) - g \sin(\theta)$$

Part (j) Calculate the acceleration of the crate if it is given a slight nudge down the ramp from rest. Give your answer in

$$\frac{m}{s^2}.$$

$$a = 9.81 * \sin(\theta * \pi / 180) * (f - 1)$$

$$a = 9.81 * \sin(31 * \pi / 180) * (0.75 - 1)$$

$$a = -1.263$$

$$\text{Tolerance: } \pm 0.03789$$

Part (k) After the crate has begun sliding downwards, the rope is pulled in order to keep the crate moving at a constant velocity. Write a formula for the force exerted by the rope in terms of variables from the problem statement together with g for acceleration due to gravity and μ_s for the coefficient of static friction.

$$F = m g \sin(\theta) - m g f \mu_s \cos(\theta)$$

Part (l) Find the force that must be exerted by the rope in order to stop the crate from accelerating as it slides down the inclined surface. Give your answer in terms of variables from the problem statement together with g for acceleration due to gravity and μ_s for the coefficient of static friction.

$$F = 9.81 * m * \sin(\theta * \pi / 180) * (1 - f)$$

$$F = 9.81 * 91 * \sin(31 * \pi / 180) * (1 - 0.75)$$

$$F = 114.945$$

$$\text{Tolerance: } \pm 3.44835$$

Problem 273 - 5.6.57(sym) :

Full solution not currently available at this time.

A crate of mass m rests on a rough surface inclined at an angle of θ with the horizontal. A massless rope to which a force can be applied parallel to the surface is attached to the crate and leads to the top of the incline. In its present state, the crate is just ready to slip and start to move down the plane. The coefficient of kinetic friction is given by $\mu_k = f \mu_s$ where μ_s refers to the coefficient of static friction and f is a constant. For all parts of this problem, consider up the surface to be the positive direction of motion.

Part (a) Write an equation for the coefficient of static friction in terms of variables from the problem statement.

$$\mu_s = \tan(\theta)$$

Part (b) Write an equation for the maximum force the rope can exert parallel to the surface of the plane before the crate begins to slide upwards. Give your answer in terms of variables from the problem statement together with g for the acceleration due to gravity.

$$F = 2 m g \sin(\theta)$$

Part (c) With a slightly greater applied force than what you found in the previous part, the crate will slide up the plane. Write an equation for the acceleration of the crate as it begins to slide upwards. Give your equation in terms of variables from the problem statement together with g for acceleration due to gravity and μ_s for the coefficient of static friction.

$$a = g \sin(\theta) - g f \mu_s \cos(\theta)$$

Part (d) After the crate begins to move, we reduce the force on the rope. Write an equation for the force exerted upwards by the rope at which the crate moves at a constant speed. Give your equation in terms of variables from the problem statement together with g for acceleration due to gravity and μ_s for the coefficient of static friction.

$$F = m g \sin(\theta) + m g f \mu_s \cos(\theta)$$

Part (e) Consider a case where no force is being exerted by the rope and the crate is at rest. Write an equation for the acceleration of the crate down the plane to get it started. Give your answer in terms of variables from the problem statement together with g for acceleration due to gravity and μ_s for the coefficient of static friction.

$$a = g f \mu_s \cos(\theta) - g \sin(\theta)$$

Part (f) After the crate has begun sliding downwards, the rope is pulled in order to keep the crate moving at a constant velocity. Write a formula for the force exerted by the rope in terms of variables from the problem statement together with g for acceleration due to gravity and μ_s for the coefficient of static friction.

$$F = m g \sin(\theta) - m g f \mu_s \cos(\theta)$$

Problem 274 - 5.6.58 :**Full solution not currently available at this time.**

A car is moving at high speed along a highway when the driver makes an emergency braking. The wheels become locked (stop rolling), and the resulting skid marks are $d = 30.1$ meters long. The coefficient of kinetic friction between tires and road is $\mu = 0.55$ and the acceleration was constant during braking.

Part (a) Write an equation for the speed of the car when its wheels became locked using variables from the problem statement. Use g for acceleration due

$$v = (2 g \mu d)^{0.5}$$

Part (b) Calculate the speed of the car at the moment that the wheels became locked. Give your answer in $\frac{m}{s}$.

$$v = (2 * 9.81 * \mu * d)^{0.5}$$

$$v = (2 * 9.81 * 0.55 * 30.1)^{0.5}$$

$$v = 18.022$$

$$\text{Tolerance: } \pm 0.54066$$

Problem 275 - 5.6.58(sym) :**Full solution not currently available at this time.**

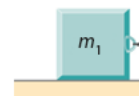
A car is moving at high speed along a highway when the driver makes an emergency braking. The wheels become locked (stop rolling), and the resulting skid marks have length d . The coefficient of kinetic friction between tires and road is μ and the acceleration was constant during braking.

Part (a) Write an equation for the speed of the car when its wheels became locked using variables from the problem statement. Use g for acceleration due

$$v = (2 g \mu d)^{0.5}$$

Problem 276 - 5.6.59 :**Full solution not currently available at this time.**

Two blocks connected by a string are pulled across a horizontal surface by a force applied to one of the blocks, as shown to the right. The mass of the left block $m_1 = 0.6$ kg and the mass of the right block $m_2 = 3.1$ kg. The angle between the applied force and the horizontal is $\theta = 51^\circ$. The coefficient of kinetic friction between the blocks and the surface is $\mu = 0.15$. Each block has an acceleration of $a = 1.1$ m/s² to the right.

**Part (a)** Write an equation for the magnitude of the applied force F acting on the second block. Use variables from the problem statement as well as use acceleration.

$$F = (m_1 a + m_2 a + \mu m_1 g + \mu m_2 g) / (\cos(\theta) + \mu \sin(\theta))$$

Part (b) Calculate the magnitude of the applied force F . Give your answer in newtons.

$$F = (m_1 * a + m_2 * a + \mu * m_1 * 9.81 + \mu * m_2 * 9.81) / (\cos(\theta * \pi / 180) + \mu * \sin(\theta * \pi / 180))$$

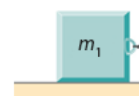
$$F = (0.6 * 1.1 + 3.1 * 1.1 + 0.15 * 0.6 * 9.81 + 0.15 * 3.1 * 9.81) / (\cos(51 * \pi / 180) + 0.15 * \sin(51 * \pi / 180))$$

$$F = 12.756$$

$$\text{Tolerance: } \pm 0.38268$$

Problem 277 - 5.6.59(sym) :**Full solution not currently available at this time.**

Two blocks connected by a string are pulled across a horizontal surface by a force applied to one of the blocks, as shown to the right. As shown, the mass of the left block is m_1 and the mass of the right block is m_2 . The angle between the string and the horizontal is θ . The coefficient of kinetic friction between the blocks and the surface is μ . Each block has an acceleration a to the right.

**Part (a)** Write an equation for the magnitude of the force exerted by the string. Use variables from the problem statement as well as using g for the gravi

$$F = (m_1 a + m_2 a + \mu m_1 g + \mu m_2 g) / (\cos(\theta) + \mu \sin(\theta))$$

Problem 278 - 5.6.60 :**Full solution not currently available at this time.**

Two blocks are stacked as shown to the right and rest on a frictionless surface. There is friction between the two blocks (coefficient of friction μ). An external force is applied to the top block at an angle θ to the horizontal.



Part (a) What is the maximum force F that can be applied for the two blocks to move together? Give your answer in terms of the variables from the problem to g for gravitational acceleration.

$$F = \mu m_1 g (m_1 + m_2) / (m_2 \cos(\theta) - \mu \sin(\theta) (m_1 + m_2))$$

Problem 279 - 5.6.61 :**Full solution not currently available at this time.**

A car of mass $m = 910$ kg is traveling along a level road at $v = 91$ km/h when its brakes are applied. The coefficient of friction of the tires is $\mu = 0.41$.

Part (a) Neglecting air resistance, write an equation for the stopping distance. Write this equation in terms of the variables in the problem statement to g acceleration due to gravity. (Hint: since the distance traveled is of interest rather than the time, x is the desired independent variable and not t . Use the Chain Rule to change the variable:

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$$

$$d = v^2 / (2 \mu g)$$

Part (b) Using the equation you found in part (a), calculate the stopping distance in meters.

$$d = (v^2) / (2 \mu g)$$

$$d = (91^2) / (2 * 0.41 * 9.81)$$

$$d = 79.432$$

$$\text{Tolerance: } \pm 2.38296$$

Problem 280 - 5.6.61(sym) :**Full solution not currently available at this time.**

A car of mass m is traveling along a level road at a speed v when its brakes are applied.

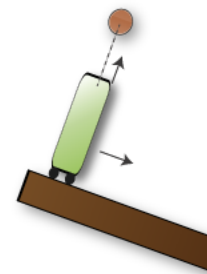
Part (a) Neglecting air resistance, write an equation for the stopping distance. Write this equation in terms of the variables in the problem statement to g acceleration due to gravity and μ for the coefficient of kinetic friction for the tires on the road. (Hint: since the distance traveled is of interest rather than t independent variable and not t . Use the Chain Rule to change the variable:

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$$

$$d = v^2 / (2 \mu g)$$

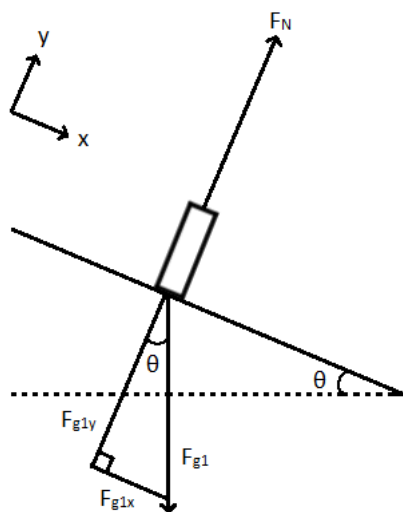
Problem 281 - c5.7.1 :

A ball is launched from a cylindrical device that has been set on a frictionless incline and turned loose.



Part (a) What can be determined about where the ball will land?

First, let's define a coordinate axis where down the ramp is the positive x-direction. Let's now consider the initial velocity of the ball. We don't know the speed in the y-direction of our coordinate axis, but we do know that, since the ball is moving with the launcher until it is launched, it will have the same speed in the x-direction at the moment it is launched. Since both the ball and launcher have the same initial speed in the x-direction, the comparative acceleration of the launcher and the ball will determine where the ball lands relative to the launcher. If the ball has more acceleration in the x-direction than the launcher, it will land in front of it. If it has equal acceleration in the x-direction as the launcher, it will land in the launcher. Finally, if the ball has less acceleration in the x-direction than the launcher, it will land behind it. To figure out the accelerations, let's begin by drawing a free-body diagram for the launcher.



As the only force acting in the x-direction on the launcher is the component of gravity acting in the x-direction, we can write the following equation for the force

$$F_{net1} = F_{g1x}$$

$$F_{net1} = m_1 g \sin(\theta)$$

We can now apply Newton's Second Law to find the acceleration of the launcher.

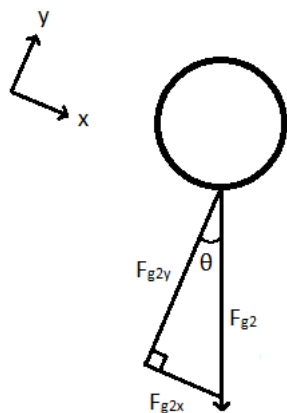
$$F_{net1} = m_1 a_1$$

$$\frac{F_{net1}}{m_1} = a_{1x}$$

$$\frac{m_1 g \sin(\theta)}{m_1} = a_{1x}$$

$$a_{1x} = g \sin(\theta)$$

Now that we have an expression for the acceleration of the launcher in the x-direction, let's repeat this process for the ball starting with a free-body diagram.



Again, the only force acting on the ball in the x-direction is a component of the gravitational force. We can therefore write the following equation:

$$F_{net2} = F_{g2x}$$

$$F_{net2} = m_2 g \sin(\theta)$$

As before, let's apply Newton's Second Law to find an expression for the acceleration.

$$F_{net2} = m_2 a_2$$

$$\frac{F_{net2}}{m_2} = a_{2x}$$

$$\frac{m_2 g \sin(\theta)}{m_2} = a_{2x}$$

$$a_{2x} = g \sin(\theta)$$

Now, let's compare the acceleration in the x-direction of the launcher and the ball.

$$a_{1x} = g \sin(\theta) = a_{2x}$$

$$a_{1x} = a_{2x}$$

Since the initial velocities and accelerations in the x-direction are both equal, the ball will always be at the same x-position as the launcher, meaning that no matter how far the ball is launched, it will inevitably land back in the launcher. The correct answer is therefore:

The ball will land back in the cylinder.

Problem 282 - c5.7.2 :

Consider a box sitting in the back of a pickup. The pickup accelerates to the right, and because the bed of the pickup is sticky, the box does not slide around the truck when this happens.

Part (a) What direction is the force acting on the box due to the truck (choose all that apply)?

Since the box is stuck to the truck bed, it is accelerating to the right together with the truck. So according to Newton's second law, the truck exerts on the box a horizontal force to the right. In addition, the truck bed prevents the box from falling to the ground. Thus the truck also exerts an upward vertical force on the box. The correct choice is "To the right and up".

Problem 283 - c5.7.2 (alt) :

Consider a box sitting in the back of a pickup. The pickup accelerates to the right, and because the bed of the pickup is sticky, the box does not slide around the truck when this happens.

Part (a) What direction is the force acting on the box due to the truck (choose all that apply)?

Since the box is stuck to the truck bed, it is accelerating to the right together with the truck. So according to Newton's second law, the truck exerts on the box a horizontal force to the right. In addition, the truck bed prevents the box from falling to the ground. Thus the truck also exerts an upward vertical force on the box. The correct choice is "To the right and up".

Part (b) Identify the forces acting on the box (select all that apply).

Let's imagine a free-body diagram for the box and consider all the forces acting on the box.

The only long-range force acting on the box is the gravitational force of Earth pulling the box downward.

Then there are the contact forces on the box:

There is the normal force of the truck's bed, which balances Earth's pull and keeps the box from falling to the ground.

There is also the horizontal force of static friction between the truck's bed and the box. That is the force accelerating the box to the right. The friction is not kinetic sliding around the truck.

The correct choice is "Normal Force, Static Friction, Gravitational Force".

Problem 284 - c5.7.3 :

Full solution not currently available at this time.

Consider a problem involving forces.

Part (a) When analyzing a particular object, what forces should be included in the free-body diagram?

The forces acting on the object

Part (b) Which of Newton's laws should be applied to a free-body diagram in order to calculate the object's acceleration?

Newton's second law

Problem 285 - c5.7.4 :

Full solution not currently available at this time.

A clam is dropped from the mouth of a seagull and falls to the ground.

Part (a) How many forces should be included in the free-body diagram of the falling clam if we neglect air resistance?

1

Part (b) How many forces should be included in the free-body diagram of the falling clam if we do not neglect air resistance?

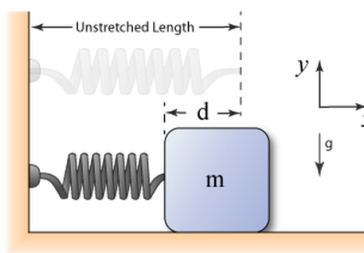
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Problem 286 - 5.7.1 (iFBD) :

A spring with a spring constant of $k = 180 \text{ N/m}$ is initially compressed by a block a distance $d = 0.32 \text{ m}$. The block is oriented horizontally and has a mass of $m = 6 \text{ kg}$.

Randomized Variables

$k = 180 \text{ N/m}$
 $d = 0.32 \text{ m}$
 $m = 6 \text{ kg}$



Part (a) Assuming the block is moving to the right, input an expression for the sum of the forces in the x-direction in the configuration shown above, using the variables provided.

Expression :
 $\Sigma F_x =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \mu_k, \theta, a, d, g, h, i, j, k, m, P, S, t$

Part (b) Using μ_s to represent the coefficient of static friction, how large would μ_s need to be to keep the block from moving?

Numeric : A numeric value is expected and not an expression.
 $\mu_s =$ _____

Part (c) Assuming the block has just begun to move and the coefficient of kinetic friction is $\mu_k = 0.2$, what is the block's acceleration in m/s^2 ?

Numeric : A numeric value is expected and not an expression.
 $a =$ _____

a) Assuming the block is moving (to the right), at the moment shown in the figure the forces are $\Sigma F_x = kd - \mu_k mg$

b) $\Sigma F_x = kd - F_s = 0 \rightarrow kd = F_s \leq \mu_s mg, \mu_s \geq \frac{kd}{mg}$

At maximum, $\mu_s \approx 0.979$

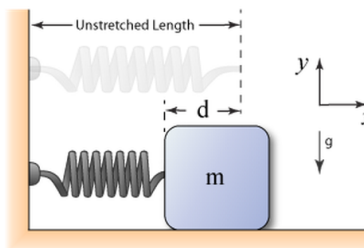
c) From part a, $kd - \mu_k mg = ma \rightarrow a = \frac{kd}{m} - \mu_k g, a \approx 7.64 \text{ m/s}^2$

Problem 287 - 5.7.1 (alt) :

A spring with a spring constant of $k = 180 \text{ N/m}$ is initially compressed by a block a distance $d = 0.32 \text{ m}$. The block is oriented horizontally and has a mass of $m = 6 \text{ kg}$.

Randomized Variables

$k = 180 \text{ N/m}$
 $d = 0.32 \text{ m}$
 $m = 6 \text{ kg}$



Part (a) Assuming the block is moving to the right, input an expression for the sum of the forces in the x-direction in the configuration shown above, using the variables provided.

Expression :
 $\Sigma F_x =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \mu_k, \theta, a, d, g, h, i, j, k, m, P, S, t$

Part (b) Using μ_s to represent the coefficient of static friction, how large would μ_s need to be to keep the block from moving?

Numeric : A numeric value is expected and not an expression.
 $\mu_s =$ _____

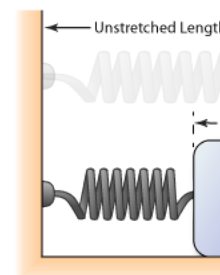
Part (c) Assuming the block has just begun to move and the coefficient of kinetic friction is $\mu_k = 0.2$, what is the block's acceleration in m/s^2 ?

Numeric : A numeric value is expected and not an expression.
 $a =$ _____

- a) Assuming the block is moving to the right, at the moment shown in the figure the forces are $\sum F_x = kd - \mu_k mg$
- b) $\sum F_x = kd - F_s = 0 \rightarrow kd = F_s \leq \mu_s mg$, $\mu_s \geq \frac{kd}{mg}$
 At maximum, $\mu_s \approx 0.979$
- c) From part a, $kd - \mu_k mg = ma \rightarrow a = \frac{kd}{m} - \mu_k g$, $a \approx 7.64 \text{ m/s}^2$

Problem 288 - 5.7.1 :

A spring with a spring constant of $k = 150 \text{ N/m}$ is initially compressed by a block a distance $d = 0.21 \text{ m}$ from its unstretched length. The block is on a horizontal surface with coefficients of static and kinetic friction μ_s and μ_k and has a mass of $m = 2 \text{ kg}$. Refer to the figure.



Part (a) The block is released from the initial position and begins to move to the right. Enter an expression for the sum of the forces in the x-direction in above, in terms of defined quantities and g .

The forces in the x-direction include the restoring force of the spring (pointing right) and the kinetic friction force (pointing left). Therefore,

$$\sum F_x = (F_s - F_k) \text{ N}$$

where F_s is the force from the spring in N and F_k is the kinetic friction force. Both of these force can be expressed by the equations

$$F_s = kd$$

$$F_k = \mu_k F_N$$

where k is the spring constant in N/m, d is the distance in m, μ_k is the coefficient of kinetic friction, and F_N is the normal force. In this case, the normal force is

$$F_N = mg$$

since it is on a horizontal surface (m is mass in kg and g is the acceleration of gravity in m/s^2). Substituting everything in,

$$\sum F_x = (k \text{ N/m}) \cdot (d \text{ m}) - \mu_k \cdot (m \text{ kg}) \cdot (g \text{ m/s}^2)$$

$$\sum F_x = (kd - \mu_k mg) \text{ N}$$

Part (b) Calculate the smallest value for the coefficient of static friction μ_s that would keep the block from moving.

To keep it from moving, the forces in the x-direction would be the restoring force of the spring (pointing right) and the static friction force (pointing left). Therefore

$$\sum F_x = F_s - F_f = 0 \text{ N}$$

where F_s is the force from the spring in N and F_f is the static friction force. Both of these force can be expressed by the equations

$$F_s = kd$$

$$F_f \leq \mu_s F_N$$

where k is the spring constant in N/m, d is the distance in m, μ_s is the coefficient of static friction, and F_N is the normal force. In this case, the normal force is

$$F_N = mg$$

since it is on a horizontal surface (m is mass in kg and g is the acceleration of gravity in m/s^2). For the static friction to just keep the block from moving, the prev an equality. Substituting everything in,

$$\sum F_x = k \cdot d - \mu_s \cdot m \cdot g = 0$$

$$\mu_s = \frac{kd}{mg}$$

Plugging in numbers and converting units as needed,

$$\mu_s = \frac{(150 \text{ N/m} \cdot 0.21 \text{ m})}{(2 \text{ kg} \cdot 9.81 \text{ m/s}^2)}$$

$$\mu_s = 1.606$$

Part (c) Assuming the block has just begun to move and the coefficient of kinetic friction is $\mu_k = 0.2$, what is the block's acceleration in meters per second

From part a,

$$\sum F_x = k \cdot d - \mu_k \cdot m \cdot g = ma$$

$$a = k \cdot \frac{d}{m} - \mu_k \cdot g$$

Plugging in numbers and converting units as needed,

$$a = \frac{(150 \text{ N/m} \cdot 0.21 \text{ m})}{(2 \text{ kg})} - 0.2 \cdot 9.81 \text{ m/s}^2$$

$$a = 13.788 \text{ m/s}^2$$

Problem 289 - 5.7.2 :

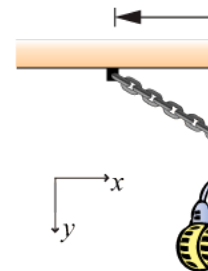
A chandelier hangs $h = 0.52$ m down from two chains of equal length. The chains are separated from one another by a length $L = 0.25$ m at the ceiling. The chandelier has a mass of $m = 11$ kg.

Randomized Variables

$$h = 0.52 \text{ m}$$

$$L = 0.25 \text{ m}$$

$$m = 11 \text{ kg}$$

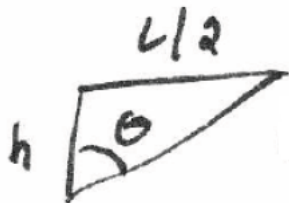


Part (a) Choose the correct Free Body Diagram given the gravitational force, F_g , the force exerted by the chains, F_T , and the normal force, F_N .

The gravitational force points straight down and each chain has a force pointing along the chain from the chandelier to the ceiling at the angle depicted. Only one this.

Part (b) What is the angle, θ in degrees, between one of the chains and the vertical where it contacts the chandelier?

Using the triangle shown below,



we have

$$\tan(\theta) = \frac{\left(\frac{L}{2}\right)}{h}$$

Therefore

$$\theta = \arctan\left(\frac{L}{2h}\right)$$

where L is the separation length in m and h is the hanging distance in m. Plugging in numbers and converting numbers as needed,

$$\theta = \arctan\left(\frac{0.25 \text{ m}}{(2 \cdot 0.52 \text{ m})}\right)$$

$$\theta = 13.523 \text{ deg}$$

Part (c) Write an expression for $F_{T,y}$, the magnitude of the y-component of the tension in one chain, in terms of the given information and variables available.

In the y-direction, there is the force of gravity and the y-components of both chains, which are equal. Therefore,

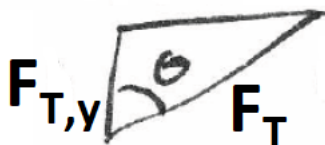
$$\sum F_y = F_{chain1} + F_{chain2} - F_g = 2F_{T,y} - F_g = 2F_{T,y} - mg = 0$$

$$F_{T,y} = \frac{1}{2} \cdot (m \text{ kg}) \cdot (g \text{ m/s}^2)$$

$$F_{T,y} = \frac{1}{2}mg \text{ N}$$

Part (d) Using your previous results, find the tension, F_T in Newtons, in one chain.

Using the triangle in the image below,



the y-component of the tension is related to the total force by the expression

$$F_{T,y} = F_T \cos(\theta)$$

So

$$F_T = \frac{F_{T,y}}{\cos(\theta)} = \frac{mg}{2 \cos\left(\arctan\left(\frac{L}{2h}\right)\right)}$$

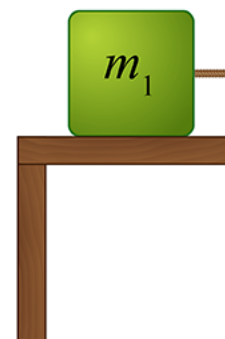
Plugging in numbers and converting units as needed,

$$F_T = \frac{(11 \text{ kg} \cdot 9.81 \text{ m/s}^2)}{\left(2 \cdot \cos\left(\arctan\left(\frac{0.25 \text{ m}}{(2 \cdot 0.52 \text{ m})}\right)\right)\right)}$$

$$F_T = 55.492 \text{ N}$$

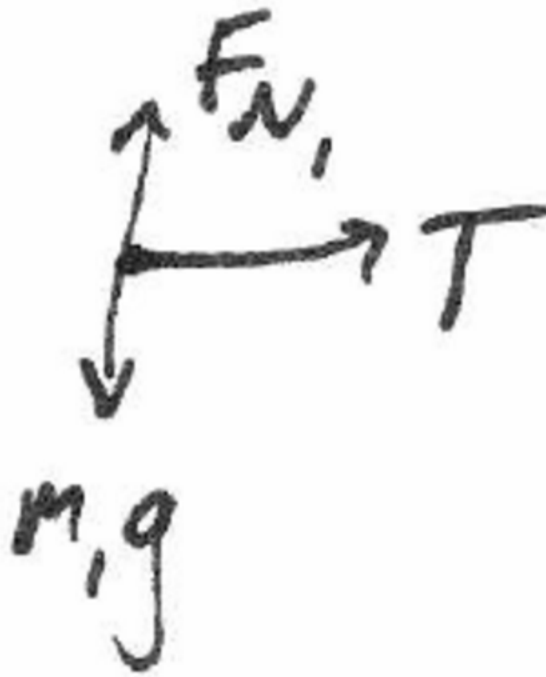
Problem 290 - 5.7.3 :

Two blocks are connected by a massless rope. The rope passes over an ideal (frictionless and massless) pulley such that one block with mass $m_1 = 11 \text{ kg}$ is on a horizontal table and the other block with mass $m_2 = 5.5 \text{ kg}$ hangs vertically. Both blocks experience gravity and the tension force, T . Use the coordinate system specified in the diagram.



Part (a) Assuming friction forces are negligible, write an expression, using only the variables provided, for the acceleration that the block of mass m_1 experiences. Your answer should involve the tension, T .

Let's begin by drawing a free-body diagram of the first block.



As we can see, the only force acting on the block in the x-direction is the force of tension. We can now use Newton's Second Law to find the acceleration in the x

$$F = ma$$

$$T = m_1 a_1$$

$$\frac{T}{m_1} = a_1$$

$$a_1 = \frac{T}{m_1}$$

Part (b) Under the same assumptions, write an expression for the acceleration, a_2 , the block of mass m_2 experiences in the y-direction. Your answer should include tension, T and m_2 .

Let's begin by drawing a free-body diagram of the second block.



This block experiences a force of tension acting in the positive y-direction and a force of gravity acting in the negative y-direction. We can therefore write the following net force on this block in the y-direction:

$$F_{net,y} = T - F_{g2}$$

$$F_{net,y} = T - m_2g$$

Now that we have an expression for the net force in the y-direction, we can use Newton's Second Law to find an expression for its acceleration in the y-direction.

$$F = ma$$

$$F_{net,y} = m_2a_2$$

$$T - m_2g = m_2a_2$$

$$\frac{T - m_2g}{m_2} = a_2$$

$$a_2 = \frac{T - m_2g}{m_2}$$

Part (c) Carefully consider how the accelerations a_1 and a_2 are related. Solve for the magnitude of the acceleration, a_1 , of the block of mass m_1 , in meters per second squared.

To begin, let's consider the physics of this problem. Since the second block is connected to the first by a taut rope, it isn't possible for the second block to fall faster than the first. This means that the acceleration of the first block in the positive x-direction must be equal to the acceleration of the second block in the negative y-direction. Let's restate the expressions we found in parts (a) and (b) using $-a_1$ in place of a_2 in our answer from part (b).

$$a_1 = \frac{T}{m_1}$$

$$-a_1 = \frac{T - m_2g}{m_2}$$

To solve for the acceleration, let's begin by rewriting the equation for the first block such that we get an expression for tension.

$$a_1 = \frac{T}{m_1}$$

$$a_1 m_1 = T$$

Now, we can use the expression we just found to eliminate the tension term from the second block's equation so that we can solve for a_1 in terms of known variables.

$$-a_1 = \frac{T - m_2g}{m_2}$$

$$-a_1 = \frac{a_1 m_1 - m_2g}{m_2}$$

$$-a_1 m_2 = a_1 m_1 - m_2g$$

$$-a_1 m_2 - a_1 m_1 = -m_2g$$

$$a_1 m_2 + a_1 m_1 = m_2g$$

$$a_1 (m_2 + m_1) = m_2g$$

$$a_1 = \frac{m_2g}{m_2 + m_1}$$

$$a_1 = \frac{5.5 \text{ kg} \cdot 9.8 \text{ m/s}^2}{5.5 \text{ kg} + 11 \text{ kg}}$$

$$a_1 = 3.267 \text{ m/s}^2$$

Part (d) Find the magnitude of the tension in the rope, T , in newtons.

To find a value for the tension of the rope, we can start with our equation from part (a) and then substitute in our answer from part (c) for the acceleration.

$$a_1 = \frac{T}{m_1}$$

$$a_1 m_1 = T$$

$$\left(\frac{m_2g}{m_2 + m_1} \right) m_1 = T$$

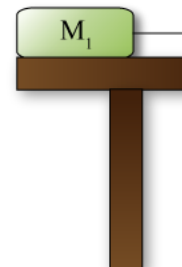
$$\frac{m_1 m_2 g}{m_2 + m_1} = T$$

$$T = \frac{11 \text{ kg} \cdot 5.5 \text{ kg} \cdot 9.8 \text{ m/s}^2}{5.5 \text{ kg} + 11 \text{ kg}}$$

$$T = 35.933 \text{ N}$$

Problem 291 - 5.7.4 :

A block with mass $m_1 = 7.1$ kg rests on the surface of a horizontal table which has a coefficient of kinetic friction of $\mu_k = 0.51$. A second block with a mass $m_2 = 8.1$ kg is connected to the first by an ideal pulley system such that the second block is hanging vertically. The second block is released and motion occurs.



Part (a) Using the variable T to represent tension, write an expression for the sum of the forces in the y-direction, ΣF_y , for block 2.

To begin, let's draw a free-body diagram for block 2.



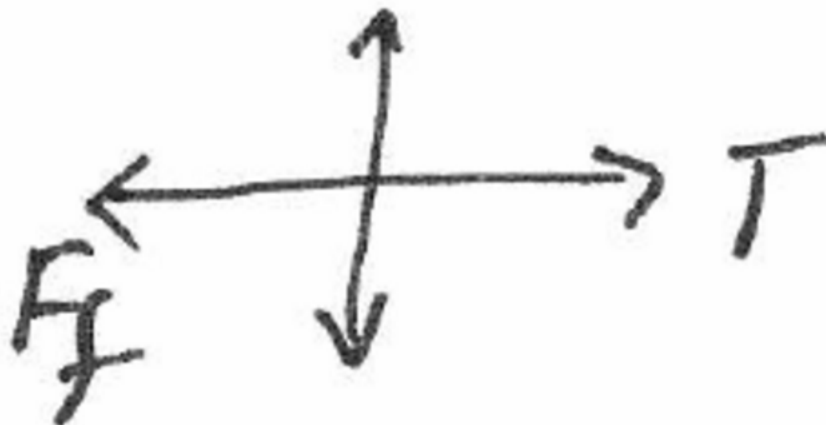
We see that there are two forces acting on this block in the y-direction; a force of tension acting in the positive y-direction and a force of gravity acting in the negative y-direction. We can write an expression for the net force in the y-direction.

$$\Sigma F_y = T - F_{g2}$$

$$\Sigma F_y = T - m_2 g$$

Part (b) Using the variable T to represent tension, write an expression for the sum of the forces in the x direction, ΣF_x for block 1.

Let's begin by drawing a free-body diagram of block 1.



In the x-direction, we see that friction is exerting a negative force and tension is exerting a positive force. Let's write an expression for the sum of forces in the x-

$$\Sigma F_x = T - F_f$$

To get this equation in terms of our known variables, we will need to find an expression for the force of friction. This means that we will need to find the normal force. The net force must be zero as the block will not experience any vertical motion. This information allows us to write an equation for the forces in the y-direction that the normal force.

$$\Sigma F_y = F_N - F_g$$

$$0 = F_N - m_1 g$$

$$m_1 g = F_N$$

Now that we have an expression for the normal force, we can rewrite the force of friction in our equation for the force in the x-direction using this expression together with kinetic friction.

$$\Sigma F_x = T - \mu_k F_N$$

$$\Sigma F_x = T - \mu_k m_1 g$$

Part (c) Write an expression for the magnitude of the acceleration of block 2, a_2 , in terms of the acceleration of block 1, a_1 . (Assume the cable connecting

If the cable is ideal, then it will not stretch any further as the two blocks move. This means that block two cannot fall faster than block one moves to the right. As which block 2 accelerates downwards must be equal to the rate at which block 1 accelerates to the right. Taking our signs into account, this means that the acceleration of block 2 is equal to the negative value of block one's acceleration. Since the only difference is the sign, the magnitudes of these two accelerations are equal allowing us to state

$$a_2 = a_1$$

Part (d) Write an expression using the variables provided for the magnitude of the tension force, T .

To begin, let's use Newton's Second Law to find expressions for the acceleration of each block using our results from part (a) and (b). For block one, we get:

$$T - \mu_k m_1 g = m_1 a_1$$

$$\frac{T - \mu_k m_1 g}{m_1} = a_1$$

Now, let's repeat this process for block two.

$$T - m_2 g = m_2 a_2$$

$$\frac{T - m_2 g}{m_2} = a_2$$

Looking at our results from part (c), we can note that the magnitudes of the two accelerations is equal. Taking into account the signs present, however, we can state two accelerations as:

$$a_2 = -a_1$$

Next, let's restate the acceleration of block two in terms of a_1 .

$$\frac{T - m_2 g}{m_2} = -a_1$$

$$\frac{m_2 g - T}{m_2} = a_1$$

We now have two expressions for a_1 . We can set these equal to one another to find an expression for the force of tension.

$$\frac{T - \mu_k m_1 g}{m_1} = \frac{m_2 g - T}{m_2}$$

$$\frac{T}{m_1} - \frac{\mu_k m_1 g}{m_1} = \frac{m_2 g}{m_2} - \frac{T}{m_2}$$

$$\frac{T}{m_1} - \mu_k g = g - \frac{T}{m_2}$$

$$\frac{T}{m_1} + \frac{T}{m_2} = g + \mu_k g$$

$$\frac{T m_2}{m_1 m_2} + \frac{T m_1}{m_1 m_2} = g (\mu_k + 1)$$

$$\frac{T m_1 + T m_2}{m_1 m_2} = g (\mu_k + 1)$$

$$T \left(\frac{m_1 + m_2}{m_1 m_2} \right) = g (\mu_k + 1)$$

$$T = \frac{m_1 m_2 g (\mu_k + 1)}{m_1 + m_2}$$

Part (e) What is the tension, T in Newtons?

Here, we simply need to solve the expression that we found in part (d) for the tension.

$$T = \frac{m_1 m_2 g (\mu_k + 1)}{m_1 + m_2}$$

$$T = \frac{7.1 \text{ kg} \cdot 8.1 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot (0.51 + 1)}{7.1 \text{ kg} + 8.1 \text{ kg}}$$

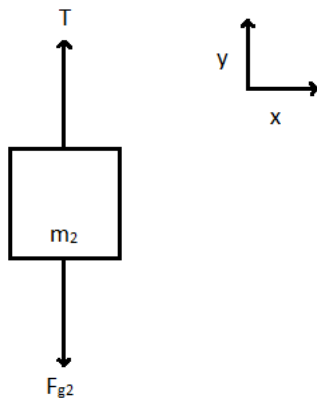
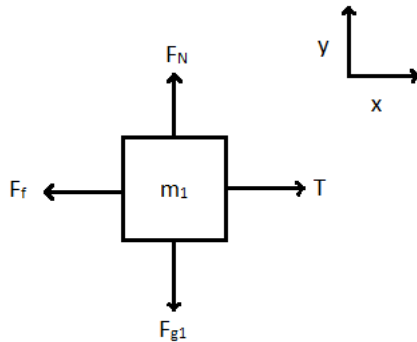
$$T = 56.046 \text{ N}$$

Problem 292 - 5.7.5 :

A block with mass $m_1 = 5.5$ kg rests on the surface of a horizontal table which has a coefficient of static friction of $\mu_s = 0.51$. This block is connected by a pulley system to another block which hangs vertically. The hanging block has mass m_2 .

Part (a) What is the minimum mass, $m_{2,\min}$, in kilograms, that will cause the system to move?

To begin, let's draw free-body diagrams for both blocks.



First, we'll look at the conditions for the first block moving. The block will begin to experience horizontal motion when the force of tension is barely above the friction. This gives us the following equation:

$$T = F_f$$

$$T = \mu_s F_N$$

To get the tension in terms of known variables, we need to find an expression for the normal force. Since block one will not experience any vertical motion, the normal gravitational force must be balanced.

$$F_N = F_{g1}$$

$$F_N = m_1 g$$

Plugging this into our expression for tension, we get:

$$T = \mu_s m_1 g$$

Now that we know the tension required to move the first block, let's look at the second block. The second block experiences a force of tension upwards and a gravitational force downwards. That means that the block will begin to move when the weight of the block is barely greater than the force of tension. This gives us the following condition for the block to move:

$$F_{g2} = T$$

$$m_{2,min} g = \mu_s m_1 g$$

$$m_{2,min} = \mu_s m_1$$

$$m_{2,min} = 0.51 \cdot 5.5 \text{ kg}$$

$$m_{2,min} = 2.805 \text{ kg}$$

Part (b) A second block with mass of 10 kg is placed on top of the first block. What is the new hanging mass, $m_{2,min}'$, in kilograms, that will cause the system to move?

As compared to part (a), all that has really changed is that the mass of block one has functionally increased. As such, we can use our result from part (a) while adding the additional mass.

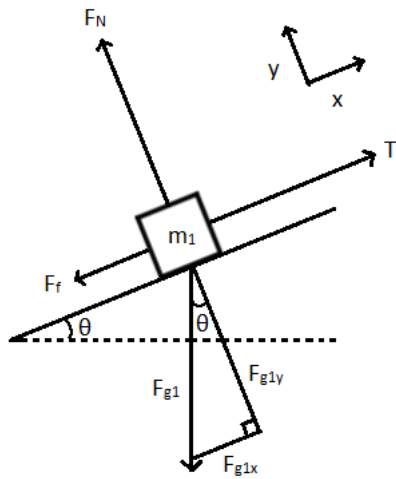
$$m_{2,min}' = \mu_s m_1'$$

$$m_{2,min}' = 0.51 \cdot (5.5 \text{ kg} + 10 \text{ kg})$$

$$m_{2,min}' = 7.905 \text{ kg}$$

Part (c) The table is inclined at an angle, θ , relative to the horizontal in such a way that the pulley is at the highest point and the 10 kg is removed from the system. What is the magnitude of the normal force, F_N , of the block?

Let's draw a new free body diagram for block one for the case where the table is inclined.



Using an x-y axis such that x is parallel to the surface of the table and y is perpendicular, we see that the net force in the y-direction must be zero. We can therefore write an expression for the net force in the y-direction.

$$F_{y,net} = F_N - F_{g1y}$$

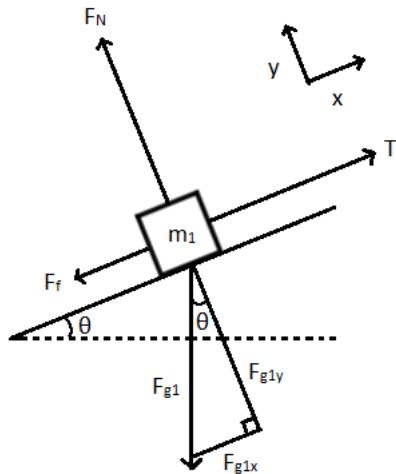
$$0 = F_N - m_1 g \cos(\theta)$$

$$-F_N = -m_1 g \cos(\theta)$$

$$F_N = m_1 g \cos(\theta)$$

Part (d) Write an equation for the tension, T , in terms of mass m_1 just before the block begins to slide. Note that the angle θ is not so large that the block would slide under its own weight (i.e. if the tension acting on mass m_1 were removed, the block would remain stationary based on the friction between the block and the table).

Let's begin by examining a free-body diagram for block one on the inclined table.



There are three forces acting on the block in the x-direction we have assigned: a force of friction acting in the negative direction, a component of the gravitational force acting in the negative direction, and a force of tension acting in the positive direction. The block will begin to slide when the force of tension overcomes the two forces in the negative direction. This gives us the following equation for the condition of motion:

$$T = F_f + F_{g1x}$$

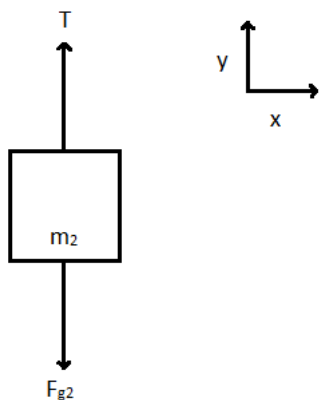
$$T = \mu_s F_N + m_1 g \sin(\theta)$$

Now, let's substitute the expression that we found in part (c) for the normal force.

$$T = \mu_s m_1 g \cos(\theta) + m_1 g \sin(\theta)$$

Part (e) Write an equation for the minimum mass m_2 necessary for the block to slide when the table is inclined.

Let's draw a free-body diagram for block two.



We see that there are only two forces acting on the block, a force of tension in the positive y-direction and a force of gravity in the negative y-direction. For the block to begin moving, the force of gravity must surpass the force of tension. The condition for the block to begin moving is therefore:

$$F_{g2} = T$$

$$m_2 g = T$$

We found an expression for the minimum value of tension for block one to begin moving in part (d). We can plug this in for tension to find an expression for the minimum mass m_2 the system begins to move.

$$m_2 g = \mu_s m_1 g \cos(\theta) + m_1 g \sin(\theta)$$

$$m_2 = \mu_s m_1 \cos(\theta) + m_1 \sin(\theta)$$

Part (f) How is the minimum mass m_2 for the inclined table different from that for the horizontal table considered in part (a)?

Intuitively, it seems like the minimum mass of block two would be higher in the case where the table was inclined, as the weight of block two would have to drag rather than just horizontally. To check this, let's look at the results of our equation from part (d) in the two extreme cases where θ is equal to 0° and 90° .

$$m_2 = \mu_s m_1 \cos(0^\circ) + m_1 \sin(0^\circ) \qquad m_2 = \mu_s m_1 \cos(90^\circ) + m_1 \sin(90^\circ)$$

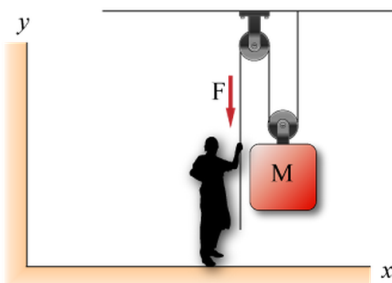
$$= \mu_s m_1 \qquad = m_1$$

In the case that $\theta = 0^\circ$, we see that the results are the same as they were in part (a). In the case that $\theta = 90^\circ$ we see that the mass of the second block must be higher if static friction is greater than one. As the coefficient of static friction is 0.51 in our case, we see that the required mass of block two is greater when $\theta = 90^\circ$. A between 0° and 90° , we know that mass two must be greater than it was in part (a). This means that mass two must be greater for the inclined table than it was for answer is therefore:

It is greater for the inclined table.

Problem 293 - 5.7.6 :

A man is attempting to lift a crate using a two part pulley system as shown in the image. The crate has mass $m_2 = 90$ kg, and the man has $m_1 = 75$ kg. He pulls downward on the rope with a force of magnitude $F = 659$ N.



Randomized Variables

$m_2 = 90$ kg
 $F = 659$ N

Part (a) Using T to describe the magnitude of the tension force, write an expression for the sum of the forces in the y -direction acting on the crate in terms of gravity and the variables provided.

Expression :
 $\Sigma F_y =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.
 $\alpha, \beta, \pi, \rho, \theta, a, b, c, d, F, g, h, i, j, k, m_1, m_2, P, S, T$

Part (b) Using the results from above, write an expression for the crate's vertical acceleration, a_c , in terms of T .

Expression :
 $a_c =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.
 $\alpha, \beta, \pi, \rho, \theta, a, b, c, d, F, g, h, i, j, k, m_1, m_2, P, S, T$

Part (c) What is the magnitude of the tension force, T in newtons?

Numeric : A numeric value is expected and not an expression.
 $T =$ _____

Part (d) What is the block's acceleration, a_c , in m/s^2 ?

Numeric : A numeric value is expected and not an expression.
 $a_c =$ _____

a)

$\Sigma F_y = 2T - Mg$ ← m_2

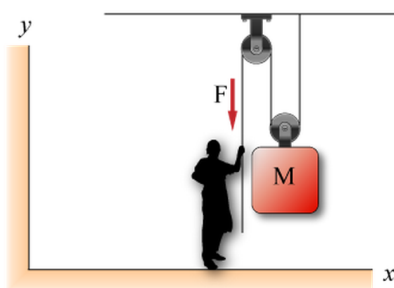
b) $\Sigma F_y = m_2 a_c \rightarrow a_c = 2 \frac{T}{m_2} - g$

c) The man is pulling the rope with a force F , so that's the tension on the rope. $T = 659$ N

d) $a_c \approx 4.83$ m/s²

Problem 294 - 5.7.6 (alt) :

A man is attempting to lift a crate using a two part pulley system as shown in the image. The crate has mass $m_2 = 90$ kg, and the man has $m_1 = 75$ kg. He pulls downward on the rope with a force of magnitude $F = 659$ N.



Part (a) Using T to describe the magnitude of the tension force, write an expression for the sum of the forces in the y -direction acting on the crate in terms of gravity and the variables provided.

Expression :

$\Sigma F_y =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \pi, \rho, \theta, a, b, c, d, F, g, h, i, j, k, m_1, m_2, P, S, T$

Part (b) Using the results from above, write an expression for the crate's vertical acceleration, a_c , in terms of T .

Expression :

$a_c =$ _____

Select from the variables below to write your expression. Note that all variables may not be required.

$\alpha, \beta, \pi, \rho, \theta, a, b, c, d, F, g, h, i, j, k, m_1, m_2, P, S, T$

Part (c) What is the magnitude of the tension force, T in newtons?

Numeric : A numeric value is expected and not an expression.

$T =$ _____

Part (d) What is the block's acceleration, a_c , in m/s^2 ?

Numeric : A numeric value is expected and not an expression.

$a_c =$

a)

$\Sigma F_y = 2T - Mg$ ← m_2

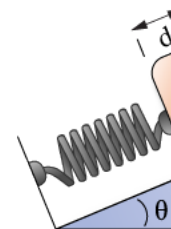
b) $\Sigma F_y = m_2 a_c \rightarrow a_c = 2 \frac{T}{m_2} - g$

c) The man is pulling the rope with a force F , so that's the tension on the rope. $T = 659 N$

d) $a_c \approx 4.83 m/s^2$

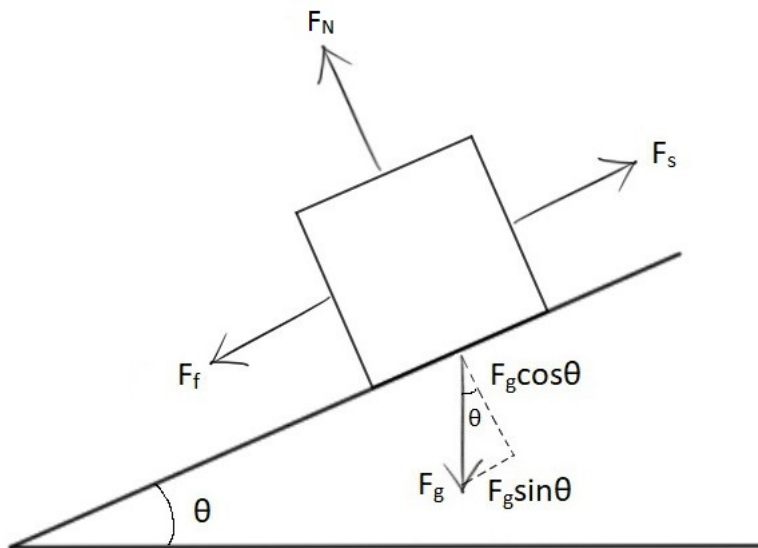
Problem 295 - 5.7.8 :

A block of mass $m = 110$ kg rests against a spring with a spring constant of $k = 510$ N/m on an inclined plane which makes an angle of θ degrees with the horizontal. Assume the spring has been compressed a distance d from its neutral position. Refer to the figure.



Part (a) Set your coordinates to have the x -axis along the surface of the plane, with up the plane as positive, and the y -axis normal to the plane, with out. Enter an expression for the normal force, F_N , that the plane exerts on the block (in the y -direction) in terms of defined quantities and g .

First, let's look at a free-body diagram of the problem.



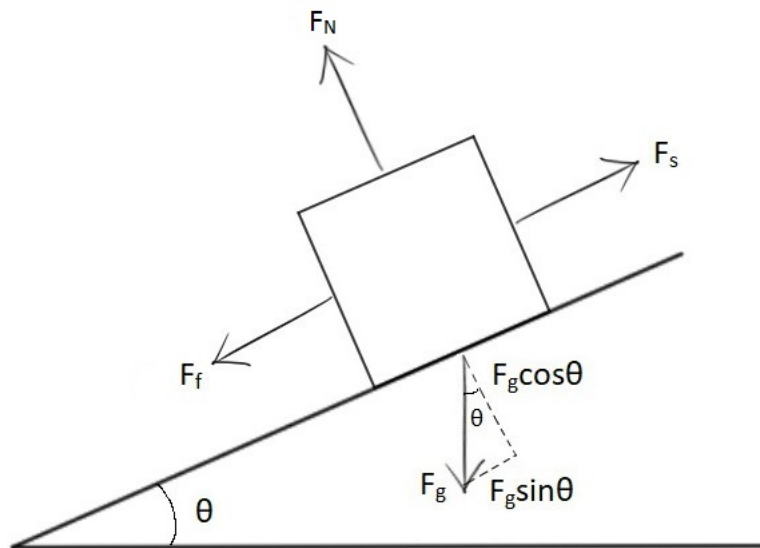
As you can see, the force of gravity can be decomposed using trigonometry into a pair of forces, one in the x -direction and one in the y -direction of the coordinate system. Now, let's consider the physics for a moment. The gravitational force in the y -direction is trying to pull the block through the ramp. In order for the block not to slide, it must exert a normal force that counteracts the force of gravity on the block in the y -direction. Therefore, we know that the normal force is equal to the y -component of the gravitational force.

$$F_N = F_g \cos(\theta)$$

$$F_N = mg \cos(\theta)$$

Part (b) Denoting the coefficient of static friction by μ_s , write an expression for the sum of the forces in the x -direction just before the block begins to slide. Use the defined quantities and g in your expression.

Let's look at a free-body diagram of the box.



As we see, the force of gravity can be decomposed into two forces that align with the coordinate plane we have defined for this problem. We will need to include acting in the x -direction as one of the forces acting on the block in the x -direction. We can also see that the forces of friction and gravity are acting on the block in that up the plane is positive and down the plane is negative according to our coordinate axis, we can sum the forces in the x -direction as

$$\Sigma F_x = F_s - F_f - F_g \sin(\theta)$$

Now, we will want to simplify these forces. We know that gravitational force is mass times the gravitational constant, the force of a spring is the spring constant times displacement, and the maximum force of static friction is the gravitational constant multiplied by the normal force (which we found in the previous part of this problem). Since we are looking for the angle just before it moves, static friction will be acting with its maximum possible force. With all this in mind, let's now write the sum of forces in terms of our permitted variables:

$$\Sigma F_x = kd - \mu_s mg \cos(\theta) - mg \sin(\theta)$$

Part (c) Assuming the plane is frictionless, what is the minimum angle in degrees the inclined plane can make before the block will move if the spring is compressed by a distance d ?

In part (b), we found an equation for the forces on the block just before it begins to move. By plugging values into that equation and solving for the angle when it can find the angle just before the block begins to move. Recall, however, that we are assuming that the plane is frictionless for this part of the problem, meaning that the friction force will be zero in our equation.

$$\Sigma F_x = kd - \mu_s mg \cos(\theta) - mg \sin(\theta)$$

$$0 = kd - 0 - mg \sin(\theta)$$

$$mg \sin(\theta) = kd$$

$$\sin(\theta) = \frac{kd}{mg}$$

$$\theta = \arcsin\left(\frac{kd}{mg}\right)$$

$$\theta = \arcsin\left(\frac{510 \text{ N/m} \cdot 0.1 \text{ m}}{110 \text{ kg} \cdot 9.81 \text{ m/s}^2}\right)$$

$$\theta = 2.71^\circ$$

Part (d) Assuming $\theta = 45$ degrees and the surface is frictionless, how far will the spring be compressed, d in meters?

For this problem, we will want to use the same strategy we did in part (c), except this time instead of plugging in a value for distance and assuming zero net force angle and assuming zero net force. As in part (c), we will once again begin with the equation for forces in the x-direction that we found in part (b) and are also as plane is frictionless.

$$\Sigma F_x = kd - \mu_s mg \cos(\theta) - mg \sin(\theta)$$

$$0 = kd - 0 - mg \sin(\theta)$$

$$-kd = -mg \sin(\theta)$$

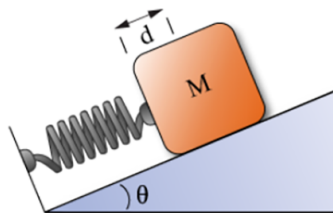
$$d = \frac{mg \sin(\theta)}{k}$$

$$d = \frac{110 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \sin(45^\circ)}{510 \text{ N/m}}$$

$$d = 1.496 \text{ m}$$

Problem 296 - 5.7.8 (alt) :

Problem 1: A block of mass $m = 260$ kg rests against a spring with a spring constant of $k = 905$ N/m on an inclined plane which makes an angle of θ degrees with the horizontal. Assume the spring has been compressed a distance d from its neutral position. Refer to the figure.



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Part (a) Set your coordinates to have the x-axis along the surface of the plane, with up the plane as positive, and the y-axis normal to the plane, with out of the plane as positive. Enter an expression for the normal force, F_N , that the plane exerts on the block (in the y-direction) in terms of defined quantities and g .

Expression :

$$F_N = \underline{\hspace{10em}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , μ_s , θ , d , g , k , m , t

Part (b) Denoting the coefficient of static friction by μ_s , write an expression for the sum of the forces in the x-direction just before the block begins to slide up the inclined plane. Use defined quantities and g in your expression

Expression :

$$\Sigma F_x = \underline{\hspace{10em}}$$

Select from the variables below to write your expression. Note that all variables may not be required.

$\cos(\alpha)$, $\cos(\phi)$, $\cos(\theta)$, $\sin(\alpha)$, $\sin(\phi)$, $\sin(\theta)$, α , β , μ_s , θ , d , g , k , m , t

Part (c) Assuming the plane is frictionless, what is the minimum angle in degrees the inclined plane can make before the block will move if the spring is compressed by 0.1 m?

Numeric : A numeric value is expected and not an expression.

$$\theta = \underline{\hspace{10em}}$$

Part (d) Assuming $\theta = 45$ degrees and the surface is frictionless, how far will the spring be compressed, d in meters?

Numeric : A numeric value is expected and not an expression.

$$d = \underline{\hspace{10em}}$$

a) (ignoring frictional forces)

$$\Sigma F_y = F_N - mg \cos \theta = 0, \quad \boxed{F_N = mg \cos \theta}$$

b) Assume static friction is down the ramp.

$$\Sigma F_x = F_s - F_f - F_{g_x} = kd - F_f - mg \sin \theta = 0$$

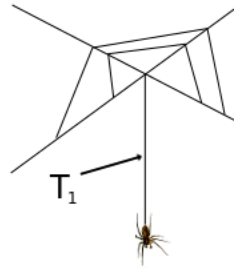
If F_f takes its maximum value we have $\boxed{\Sigma F_x = kd - \mu_s mg \cos \theta - mg \sin \theta}$

c) $\Sigma F_x = kd - mg \sin \theta = 0 \rightarrow \sin \theta = \frac{kd}{mg}, \quad \boxed{\theta \approx 2.02^\circ}$
(so assuming stationary)

d) $kd - mg \sin \theta = 0 \rightarrow d = \frac{mg \sin \theta}{k}, \quad \boxed{d \approx 2.0 \text{ m}}$

Problem 297 - 5.7.10 :

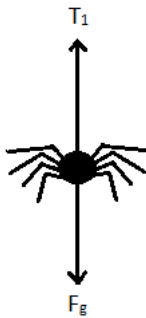
A spider of mass 7.75×10^{-5} kg hangs motionless from its web. Refer to the figures.



(a)

Part (a) Calculate the tension, in newtons, in a vertical strand of its web assuming the spider is only holding onto a single strand (see figure (a)).

Let's draw a free-body diagram.



Since the spider is stationary, the forces must be balanced in the y-direction. This means that the force of tension must be equal to the force of gravity. This allows an equation to solve for the force of tension:

$$T = F_g$$

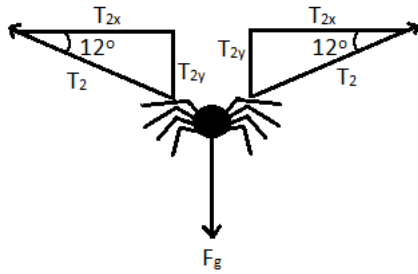
$$T = mg$$

$$T = 7.75 \cdot 10^{-5} \text{ kg} \cdot 9.8 \text{ m/s}^2$$

$$T = 0.0007595 \text{ N}$$

Part (b) The spider selects a horizontal strand of web and sits in its middle, causing it to sag at an angle of 12° below the horizontal at each end, and remains stationary (see figure (b)). Calculate the tension in the strand, in newtons.

Before we draw a free-body diagram, let's consider whether the force of tension will be the same on either side of the spider. Since the spider does not move left or right, the net force of tension to the left and right of the spider must be equal. However, since both threads make the same angle below the horizontal, the only way the horizontal forces on the spider can be balanced is if the tension in both threads is the same. With this in mind, let's draw a free-body diagram.



Since the spider is stationary, the downward force of gravity must be balanced by the upward forces of tension from the parts of the web to the left and right of the spider. The following equation is set up:

$$2T_{2y} = F_g$$

$$2T_2 \sin(12^\circ) = mg$$

$$T_2 = \frac{mg}{2\sin(12^\circ)}$$

$$T_2 = \frac{7.75 \cdot 10^{-5} \text{ kg} \cdot 9.8 \text{ m/s}^2}{2\sin(12^\circ)}$$

$$T_2 = 0.001826 \text{ N}$$

Problem 298 - 5.7.11 :

Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of 1.50 m/s²?

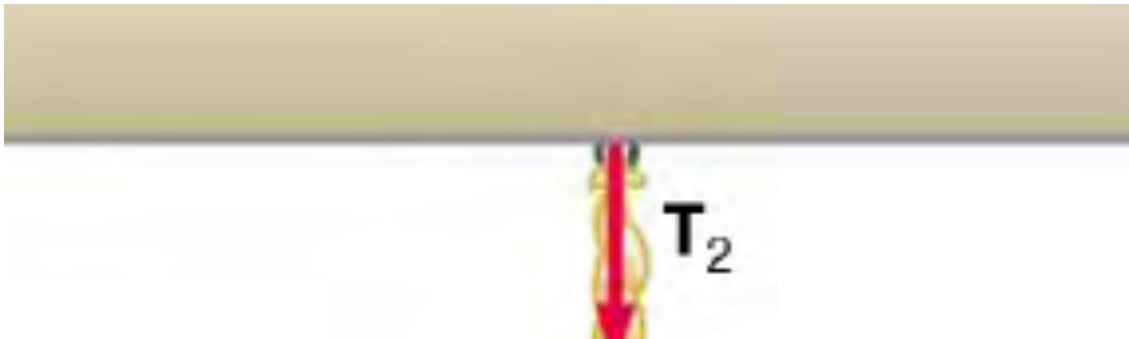
Solution (a) net $F = T - mg = ma = 0$ ($a = 0$ if constant speed)

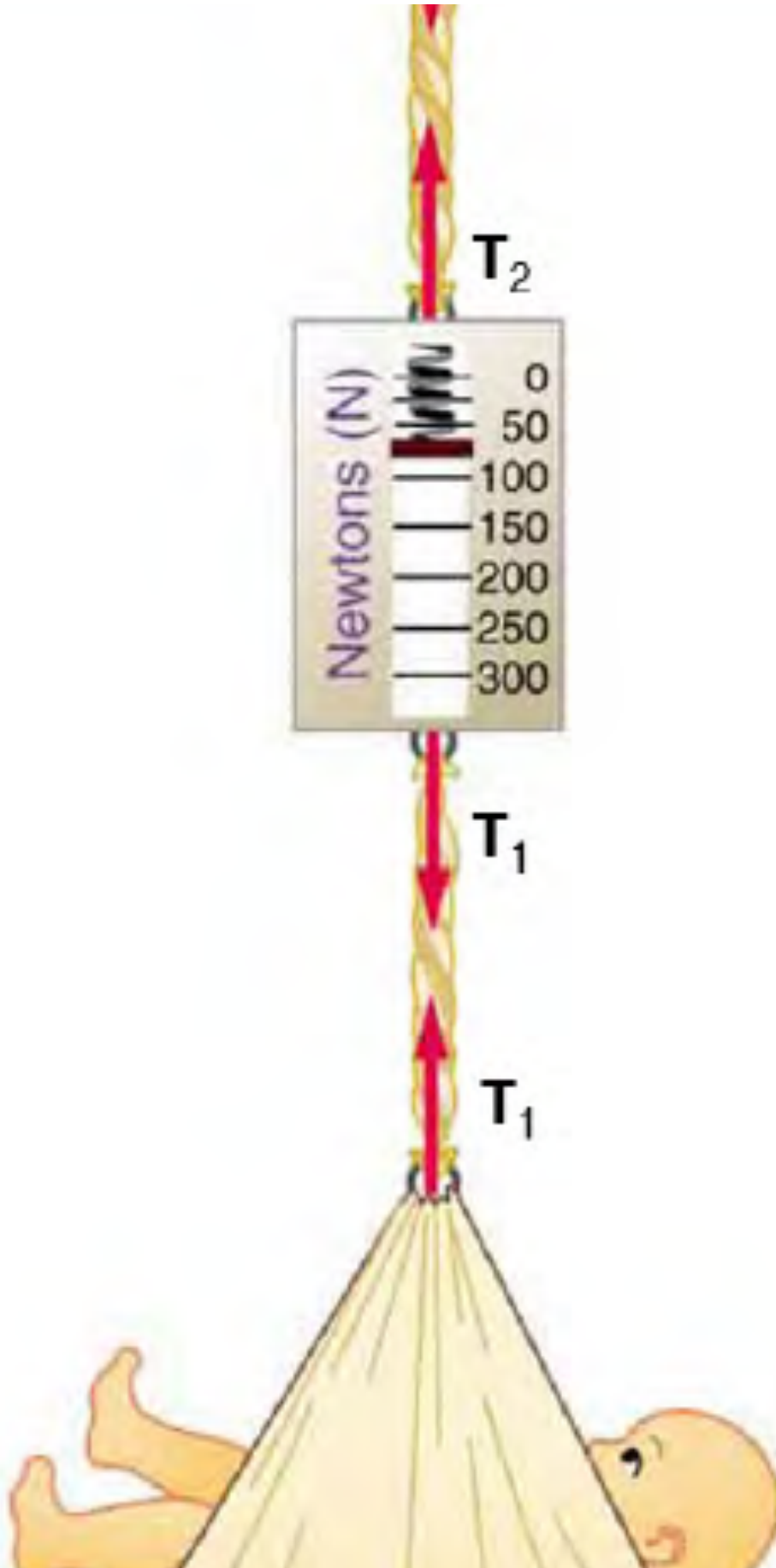
$$T = mg = (60.0 \text{ kg})(9.80 \text{ m/s}^2) = \underline{588 \text{ N}}$$

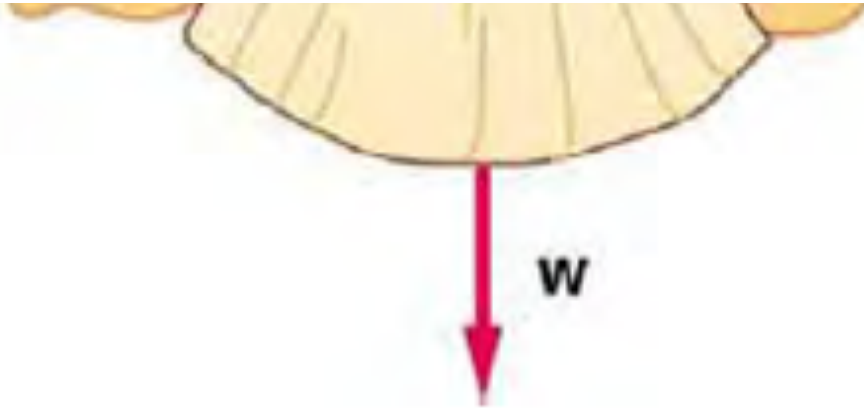
(b) net $F = T - mg = ma$

$$T = m(g + a) = (60.0 \text{ kg})(9.80 \text{ m/s}^2 + 1.50 \text{ m/s}^2) = \underline{678 \text{ N}}$$

Problem 299 - 5.7.12 :







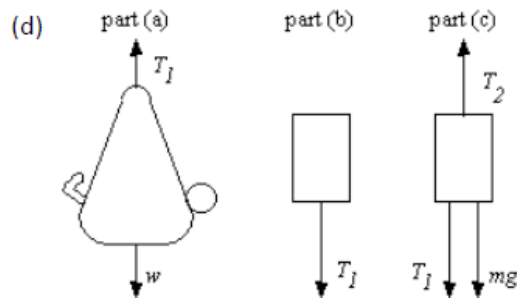
Consider the baby being weighed in the figure. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension T_1 in the cord attaching the baby to the scale? (c) What is the tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.

Solution

$$(a) \frac{55 \text{ N}}{9.80 \text{ m/s}^2} = \underline{5.6 \text{ kg}}$$

$$(b) \underline{55 \text{ N}}$$

$$(c) T_2 = T_1 + mg = 55 \text{ N} + (0.500 \text{ kg})(9.80 \text{ m/s}^2) = \underline{60 \text{ N}}$$

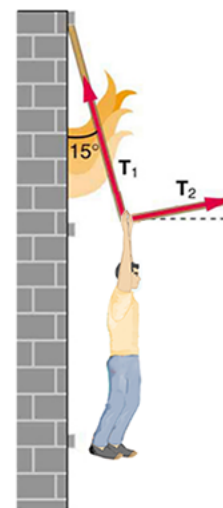


Problem 300 - 5.7.13 :

A 65 kg man is being pulled away from a burning building as shown in the figure.

Randomized Variables

$$m = 65 \text{ kg}$$



Part (a) Calculate the tension T_1 on the left side of the rope, in newtons, if the man is in static equilibrium.

This problem involves the application of a **free body diagram** and **Newton's second law** in two dimensions.

In this part, we are asked to find the tension, T_1 , in one of two ropes supporting a person in equilibrium. The person is subject to a total of three forces.

The solution involves 3 steps:

- resolving the forces on the person into their x and y components
- using a free body diagram and Newton's second law to determine the equations for equilibrium
- combining the equations for equilibrium to solve for T_1

Resolving Forces

Let's take the $+y$ direction *upward*, along the vertical, and the $+x$ direction to the right, along the horizontal. Then, our three forces can be written in terms of the

$$\vec{T}_1 = -T_1 \sin(\theta_1) \mathbf{i} + T_1 \cos(\theta_1) \mathbf{j}$$

$$\vec{T}_2 = T_2 \cos(\theta_2) \mathbf{i} + T_2 \sin(\theta_2) \mathbf{j}$$

$$\vec{w} = -mg \mathbf{j}$$

where $\theta_1 = 15^\circ$ and $\theta_2 = 20^\circ$.

Determining the Equations for Equilibrium

Because the person is in equilibrium, there is no acceleration. Hence, from Newton's second law, we know that the sum of the forces in each the x and y direction

$$\Sigma F_x = -T_1 \sin(\theta_1) + T_2 \cos(\theta_2) = 0$$

$$\Sigma F_y = T_1 \cos(\theta_1) + T_2 \sin(\theta_2) - mg = 0$$

Solving for T_1

While neither of the equations for equilibrium will give us T_1 individually, we can combine them to determine T_1 . (That is, have two equations that share two unknowns means we can use the equations together to determine each unknown!)

Let's rearrange the first equation to solve for T_2 ,

$$T_2 = T_1 \left(\frac{\sin(\theta_1)}{\cos(\theta_2)} \right)$$

Now, let's use this result in the second equation,

$$T_1 \cos(\theta_1) + \left(T_1 \left(\frac{\sin(\theta_1)}{\cos(\theta_2)} \right) \right) \sin(\theta_2) - mg = 0$$

Factoring out T_1 and switching the sines,

$$T_1 \left(\cos(\theta_1) + \left(\frac{\sin(\theta_2)}{\cos(\theta_2)} \right) \sin(\theta_1) \right) - mg = 0$$

$$T_1 (\cos(\theta_1) + \tan(\theta_2) \sin(\theta_1)) - mg = 0$$

Solving for T_1 ,

$$T_1 = \frac{mg}{(\cos(\theta_1) + \tan(\theta_2) \sin(\theta_1))}$$

Let's evaluate the result!

$$T_1 = \frac{(65 \text{ kg})(9.81 \text{ m/s}^2)}{\left(\cos\left(15^\circ\left(\frac{\pi}{180^\circ}\right)\right) + \tan\left(20^\circ\left(\frac{\pi}{180^\circ}\right)\right) \sin\left(15^\circ\left(\frac{\pi}{180^\circ}\right)\right) \right)}$$

$$= 601.484 \text{ N}$$

$$T_1 = 601.484 \text{ N}$$

Part (b) Calculate the tension T_2 on the right side of the rope, in newtons, if the man is in static equilibrium.

In this part, we are asked to find the tension, T_2 , in the second rope.

Using our equations for equilibrium,

$$\Sigma F_x = -T_1 \sin(\theta_1) + T_2 \cos(\theta_2) = 0$$

$$\Sigma F_y = T_1 \cos(\theta_1) + T_2 \sin(\theta_2) - mg = 0$$

where $\theta_1 = 15^\circ$ and $\theta_2 = 20^\circ$, we can use the first to solve for T_2 ,

$$T_2 = T_1 \left(\frac{\sin(\theta_1)}{\cos(\theta_2)} \right)$$

To avoid rounding errors, let's use the *expression* we found for T_1 in *Part (a)* rather than the numerical value,

$$T_2 = \left(\frac{mg}{(\cos(\theta_1) + \tan(\theta_2) \sin(\theta_1))} \right) \left(\frac{\sin(\theta_1)}{\cos(\theta_2)} \right)$$

Now, we can calculate the tension in the second rope!

$$T_2 = \left[\frac{(65 \text{ kg})(9.81 \text{ m/s}^2)}{\left(\cos\left(15^\circ\left(\frac{\pi}{180^\circ}\right)\right) + \tan\left(20^\circ\left(\frac{\pi}{180^\circ}\right)\right) \sin\left(15^\circ\left(\frac{\pi}{180^\circ}\right)\right) \right)} \right] \left[\frac{\sin\left(15^\circ\left(\frac{\pi}{180^\circ}\right)\right)}{\cos\left(20^\circ\left(\frac{\pi}{180^\circ}\right)\right)} \right]$$

$$= 165.666 \text{ N}$$

$$T_2 = 165.666 \text{ N}$$

Note that the second rope supports less of the person's weight than the first. Ask yourself: *Does this make sense, based on the orientation of the ropes?*

Problem 301 - 5.7.15 :

When starting a foot race, a **61.5** kg sprinter exerts an average force of **625** N backward on the ground for **0.55** s.

Randomized Variables

$$m = 61.5 \text{ kg}$$

$$f = 625 \text{ N}$$

$$t = 0.55 \text{ s}$$

Part (a) What is his final speed in m/s?

Newton's Third Law tells us that when the runner's foot pushes backwards on the ground, the ground exerts an equal and opposite force pushing forward to propel the runner. Using this information, we can use Newton's Second Law to find the acceleration of the runner.

$$F = ma$$

$$\frac{F}{m} = a$$

Now that we have an expression for the runner's acceleration, we can use a basic kinematic equation to find their final speed.

$$v = at$$

$$v = \left(\frac{F}{m}\right)t$$

$$v = \frac{625 \text{ N}}{61.5 \text{ kg}} \cdot 0.55 \text{ s}$$

$$v = 5.589 \text{ m/s}$$

Part (b) How far does he travel in meters?

To solve part (a), we had to find an expression for the acceleration of the sprinter. Because of this, we know the acceleration of the runner, that he has no initial velocity, and we know the time. With this information we can set up the following kinematic equation:

$$x = v_0t + \frac{1}{2}at^2$$

$$x = 0 + \frac{1}{2}\left(\frac{F}{m}\right)t^2$$

$$x = \frac{625 \text{ N}}{2 \cdot 61.5 \text{ kg}} \cdot (0.55 \text{ s})^2$$

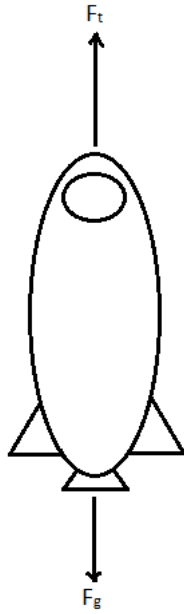
$$x = 1.537 \text{ m}$$

Problem 302 - 5.7.16 :

A large rocket has a mass of $1.75 \times 10^6 \text{ kg}$ at takeoff, and its engines produce a thrust of $3.5 \times 10^7 \text{ N}$.

Part (a) Find its initial acceleration, in meters per second squared, if it takes off vertically. Take up to be positive.

Let's begin by drawing a free-body diagram of the rocket.



To find the acceleration of the rocket, we must first find the net force acting on it. The net force is given by:

$$F_{net} = F_t - F_g$$

$$F_{net} = F_t - mg$$

Now that we have an expression for the net force, we can use Newton's Second Law to find the acceleration of the rocket.

$$F_{net} = ma$$

$$\frac{F_{net}}{m} = a$$

$$\frac{F_t - mg}{m} = a$$

$$a = \frac{3.5 \cdot 10^7 \text{ N} - 1.75 \cdot 10^6 \text{ kg} \cdot 9.8 \text{ m/s}^2}{1.75 \cdot 10^6 \text{ kg}}$$

$$a = 10.2 \text{ m/s}^2$$

Part (b) How long, in seconds, does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust?

First, let's convert 120 kilometers per hour to meters per second.

$$120 \text{ km/h} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{60 \text{ min}}{1 \text{ h}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = \left(120 \cdot \frac{3600}{1000}\right) \text{ m/s}$$

Now that we have found an expression for the velocity we are looking for, let's try to find a kinematic equation to solve for time. As we found the rocket's acceleration, that it has an initial velocity of zero, and what the final velocity we want is. With this information, we can set up the following kinematic equation:

$$v_f = v_i + at$$

$$v_f = 0 + \left(\frac{F_t - mg}{m} \right) t$$

$$\frac{v_f m}{F_t - mg} = t$$

$$t = \frac{(120 \cdot \frac{3600}{1000}) \text{ m/s} \cdot 1.75 \cdot 10^6 \text{ kg}}{3.5 \cdot 10^7 \text{ N} - 1.75 \cdot 10^6 \text{ kg} \cdot 9.8 \text{ m/s}^2}$$

$$t = 3.268 \text{ s}$$

Problem 303 - 5.7.18 :

A **2.1** kg fireworks shell is fired straight up from a mortar and reaches a height of **102** m. In this problem, take the upwards direction to be positive.

Part (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the magnitude of the shell's velocity when it leaves the mortar.

To find the velocity of the shell when it leaves the mortar, we will need to set up a kinematic equation. We know how far the shell travels, that the shell experiences due to gravity, and has a final velocity of zero when it reaches its highest point. We can therefore use the following kinematic equation to find the initial velocity:

$$v_f^2 = v_0^2 + 2ad$$

$$0 = v_0^2 - 2gh$$

$$2gh = v_0^2$$

$$\sqrt{2gh} = v_0$$

$$v_0 = \sqrt{2 \cdot 9.8 \text{ m/s}^2 \cdot 102}$$

$$v_0 = 44.712 \text{ m/s}$$

Part (b) The mortar itself is a tube **0.425** m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in part (a).

The average acceleration in the tube can be found by assuming that the acceleration of the shell in the tube is constant. To find this acceleration, we will need to set up a kinematic equation. For the shell in the tube, we know that the initial velocity will be zero, that the distance traveled will be the length of the tube, and that the final velocity is the answer from part (a). We can now set up the following kinematic equation to solve for the acceleration:

$$v_f^2 = v_0^2 + 2ad$$

$$(\sqrt{2gh})^2 = 0^2 + 2aL$$

$$2gh = 2aL$$

$$\frac{gh}{L} = a$$

$$a = \frac{9.8 \text{ m/s}^2 \cdot 102 \text{ m}}{0.425 \text{ m}}$$

$$a = 2352 \text{ m/s}^2$$

Part (c) What is the average force on the shell in the mortar? Express your answer in newtons.

To find the average force on the shell, we can use Newton's Second Law together with our results from part (b).

$$F = ma$$

$$F = m \left(\frac{gh}{L} \right)$$

$$F = \frac{mgh}{L}$$

$$F = \frac{2.1 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 102 \text{ m}}{0.425 \text{ m}}$$

$$F = 4939.2 \text{ N}$$

Problem 304 - 5.7.19 :

A **2.1**-kg fireworks shell is fired at an angle **14°** from the vertical from a mortar and reaches a height of **31** m. Take the upward direction to be positive for this problem.

Randomized Variables

$$m = 2.1 \text{ kg}$$

$$h = 31 \text{ m}$$

$$l = 0.425 \text{ m}$$

$$a = 14^\circ$$

Part (a) Neglecting air resistance (a poor assumption, but we will make it for this problem), calculate the shell's speed, in meters per second, when it leaves the mortar.

Using the equations of kinematics and our understanding of projectile motion, we can solve this problem. At the maximum height h , the vertical component of the shell's velocity is instantaneously zero m/s. Thus, we use the following kinematic equation, where the acceleration is that due to gravity, g .

$$v_y^2 = v_{0y}^2 - 2gh = 0$$

Solve for v_{0y} .

$$v_{0y}^2 = 2gh$$

$$v_{0y} = \sqrt{2gh}$$

Before we complete the solution, we remember that the vertical component of the shell's velocity can be found using trigonometry. Note: that the problem specifies the angle measured relative to the vertical direction, instead of being with respect to the horizontal, which is standard.

$$v_{0y} = v_0 \sin(90^\circ - a)$$

Now, complete the solution.

$$v_{0y} = v_0 \sin(90^\circ - a) = \sqrt{2gh}$$

$$\begin{aligned} v_0 &= \frac{\sqrt{2gh}}{\sin(90^\circ - a)} \\ &= \frac{\sqrt{2(9.80 \text{ m/s}^2)(31 \text{ m})}}{\sin(90^\circ - 14^\circ)} \end{aligned}$$

$$v_0 = 25.404 \text{ m/s}$$

Part (b) The mortar itself is a tube **0.425 m** long. Calculate the average acceleration of the shell, in meters per second squared, in the tube as it is propelled found in part (a).

We are asked to determine the average acceleration as the shell is being launched. From part (a), we know that it leaves with speed $v_0 = \frac{\sqrt{2gh}}{\sin(90^\circ - a)}$. Before the mortar had an initial velocity of zero m/s.

The acceleration can be found by knowing these velocities and the kinematic equations.

$$v_0^2 = 2al$$

$$a = \frac{v_0^2}{2l} = \frac{\left(\frac{\sqrt{2gh}}{\sin(90^\circ - a)}\right)^2}{2l} = \frac{\left(\frac{\sqrt{2(9.80 \text{ m/s}^2)(31 \text{ m})}}{\sin(90^\circ - 14^\circ)}\right)^2}{2(0.425 \text{ m})}$$

$$a = 759.261 \text{ m/s}^2$$

Part (c) What is the average net force, in newtons, acting on the shell in the mortar? Express your answer in newtons.

Newton's second law of motion applies to the acceleration of the shell in the mortar as it is being launched. In part (b), the acceleration was determined and the mortar problem statement.

$$F = ma = m \left(\frac{\sqrt{2gh}}{\sin(90^\circ - a)} \right)^2 = (2.1 \text{ kg}) \left(\frac{\sqrt{2(9.80 \text{ m/s}^2)(31 \text{ m})}}{\sin(90^\circ - 14^\circ)} \right)^2$$

$$F = 1594.447 \text{ N}$$

Problem 305 - 5.7.21 :

A 75.0 kg man stands on a bathroom scale in an elevator that accelerates uniformly upwards from rest to 30.0 m/s in 2.00 s.

Part (a) Calculate the scale reading in Newtons (The scale exerts an upward force on him equal to its reading.)

First, let's consider how a scale works. A scale reads the force pushing downward on it to find a person's weight. Newton's Third Law tells us that the scale exerts producing the normal force that the person standing on the scale experiences. The scale reading is therefore equal to the normal force that the man experiences. In force, let's begin by drawing a free-body diagram.



In order to proceed, we will need to find an expression for the net force the man experiences. In order to use Newton's Second Law to find the net force, we will find a kinematic equation and solve for his acceleration. Given that we know that the initial velocity of the elevator is zero along with its final velocity and the time it takes to reach that velocity, we can use the following kinematic equation:

$$v = at$$

$$\frac{v}{t} = a$$

Now, let's plug this result for acceleration into Newton's Second Law to get an expression for the net force the man experiences.

$$F_{net} = ma$$

$$F_{net} = \frac{mv}{t}$$

With this expression and our free-body diagram, we can now set up an equation for the sum of the forces that the man experiences and solve for the normal force

$$F_{net} = F - F_g$$

$$\frac{mv}{t} = F - mg$$

$$\frac{mv}{t} + mg = F$$

$$F = \frac{75.0 \text{ kg} \cdot 30.0 \text{ m/s}}{2.00 \text{ s}} + 75.0 \text{ kg} \cdot 9.8 \text{ m/s}^2$$

$$F = 1860 \text{ N}$$

Part (b) Calculate the scale reading compared with his weight.

A person's weight is given by the force that gravity exerts on them. Therefore, we can write the following expression for his weight:

$$w = mg$$

Now, let's divide the scale reading from part (a) by the man's actual weight to get the desired ratio.

$$F/w = \frac{\left(\frac{mv}{t} + mg\right)}{mg}$$

$$F/w = \frac{v}{tg} + 1$$

$$F/w = \frac{30.0 \text{ m/s}}{2.00 \text{ s} \cdot 9.8 \text{ m/s}^2} + 1$$

$$F/w = 2.53$$

Part (c) What is unreasonable about this?

As our answer in part (b) shows, you would experience roughly two and a half times your normal weight if you were in an elevator accelerating this fast. This is like carrying a weight equal to one and half times what you weigh on your shoulders! Obviously, no normal elevator would be designed like this for the sake of user-friendliness. The answer is:

The acceleration is much higher than any standard elevator.

Problem 306 - 5.7.21 (alt) :

Unreasonable Results A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

Solution (a)



Using $a = \frac{v - v_0}{t}$ gives:

$$a = \frac{v - v_0}{t} = \frac{30.0 \text{ m/s} - 0 \text{ m/s}}{2.00 \text{ s}} = 15.0 \text{ m/s}^2 \quad \text{net } F = F - w = ma,$$

$$F = ma + mg = m(a + g) = 75.0 \text{ kg}(15.0 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = \underline{1860 \text{ N}}.$$

$$\frac{F}{w} = \frac{m(a + g)}{mg} = \frac{15.0 \text{ m/s}^2 + 9.80 \text{ m/s}^2}{9.80 \text{ m/s}^2} = \underline{2.53}$$

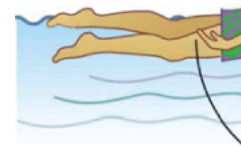
(b) The value (1860 N) is more force than you expect to experience on an elevator.

(c) The acceleration $a = 15.0 \text{ m/s}^2 = 1.53g$ is much higher than any standard elevator.

The final speed is too large (30.0 m/s is VERY fast)! The time of 2.00 s is not unreasonable for an elevator.

Problem 307 - 5.7.23 :

The **65**-kg swimmer in the figure starts a race with an initial velocity of **1.05** m/s and exerts an average force of **77.5** N backward with his arms during each 1.80 m long stroke.

**Part (a) What is his initial acceleration, in meters per square second, if water resistance is 42.5 N?**

Taking the direction of the swimmer's motion as the positive direction, we can find the net force on the swimmer by subtracting the force of resistance from the force exerted by the swimmer's arms.

$$F_{net} = F - F_r$$

We can now use Newton's Second Law to solve for the initial acceleration of the swimmer.

$$F_{net} = ma_1$$

$$F - F_r = ma_1$$

$$\frac{F - F_r}{m} = a_1$$

$$a_1 = \frac{77.5 \text{ N} - 42.5 \text{ N}}{65 \text{ kg}}$$

$$a_1 = 0.5385 \text{ m/s}^2$$

Part (b) What is the subsequent average resistance force, in newtons, from the water during the 4.75 s it takes him to reach his top velocity of 2.25 m/s?

Given that we know the initial and final velocities of the swimmer as well as the time it takes him to reach his maximum velocity, we can find his average acceleration using the kinematic equation:

$$v_f = v_i + a_2 t$$

$$v_f - v_i = a_2 t$$

$$\frac{v_f - v_i}{t} = a_2$$

Now that we have an equation for the average acceleration, we can find an expression for the average net force that the swimmer experiences using Newton's Second Law:

$$F_{net,2} = ma_2$$

$$F_{net,2} = m \cdot \frac{v_f - v_i}{t}$$

We can now set up an equation for the sum of forces on the swimmer to find the average force of resistance, F_2 .

$$F_{net,2} = F - F_2$$

$$m \cdot \frac{v_f - v_i}{t} = F - F_2$$

$$F_2 + m \cdot \frac{v_f - v_i}{t} = F$$

$$F_2 = F - m \cdot \frac{v_f - v_i}{t}$$

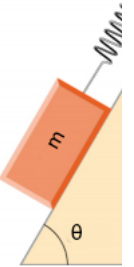
$$F_2 = 77.5 \text{ N} - 65 \text{ kg} \cdot \frac{2.25 \text{ m/s} - 1.05 \text{ m/s}}{4.75 \text{ s}}$$

$$F_2 = 61.079 \text{ N}$$

Problem 308 - 5.7.29 :

Full solution not currently available at this time.

Shown to the right is a block of mass $m = 20.5$ kg resting on a frictionless ramp inclined at $\theta = 51^\circ$ to the horizontal. The block is held by a spring that is stretched $d = 4.1$ cm.



Part (a) Write an equation for the force constant of the spring in terms of the variables from the problem statement (m , d , and θ). Use g for the gravitati

$$k = m g \sin(\theta)/d$$

Part (b) Calculate the force constant of the spring in newtons per meter.

$$k = 9.81 * m * \sin(\theta * \pi / 180) / (d / 100)$$

$$k = 9.81 * 20.5 * \sin(51 * \pi / 180) / (4.1 / 100)$$

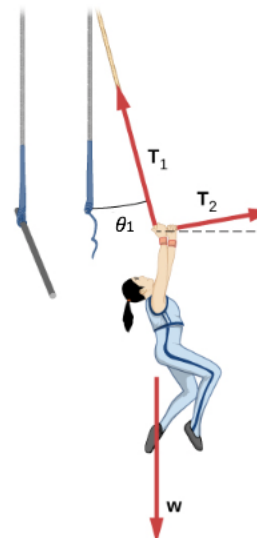
$$k = 3811.901$$

$$\text{Tolerance: } \pm 114.35703$$

Problem 309 - 5.7.30(sym) :

Full solution not currently available at this time.

After a mishap, a mass m circus performer clings to a trapeze which is being pulled to the side by another circus artist as displayed in the figure. The left rope (with tension T_1) is at an angle θ_1 to the vertical. The right rope (with tension T_2) is at an angle θ_2 from the horizontal. Assume the person is momentarily motionless.



Part (a) Write an equation for the net horizontal force on the performer

$\sum F_x$ in terms of the tensions T_1 and T_2 as well as the angles θ_1 and θ_2 . Let to the right be the positive x direction.

$$\sum F_x = T_2 \cos(\theta_2) - T_1 \sin(\theta_1)$$

Part (b) What is the sum of the forces in the y direction

$\sum F_y$ based on the tensions in ropes T_1 and T_2 , the mass m of the circus performer, the acceleration due to gravity g , and the angles θ_1 and θ_2 ? Let up be

$$\sum F_y = T_1 \cos(\theta_1) + T_2 \sin(\theta_2) - m g$$

Part (c) Write an equation for the tension in rope 1 (T_1) using variables from the problem statement together with g for acceleration due to gravity.

$$T_1 = m g / (\cos(\theta_1) + (\tan(\theta_2) \sin(\theta_1)))$$

Part (d) Write an equation for the tension in rope 2 (T_2) using variables from the problem statement together with g for acceleration due to gravity.

$$T_2 = m g \sin(\theta_1) / (\cos(\theta_1) + (\tan(\theta_2) \sin(\theta_1))) \cos(\theta_2)$$

Problem 310 - 5.7.32 :

Full solution not currently available at this time.

Displayed to the right are two carts connected by a cord that passes over a small frictionless pulley. Each cart rolls freely with negligible friction. Consider the mass of the cart to be $m_1 = 5.1$ kg and the angle the first cart sits on to be $\theta_1 = 30.5^\circ$, while the mass of the second cart as $m_2 = 20.1$ kg and the angle it sits on to be $\theta_2 = 48^\circ$.



Part (a) Considering the positive x direction to be pointing as it is in the image, write an equation for the acceleration that the two blocks experience in terms of block m_1 , the angle θ_1 , the mass of the second block m_2 , the angle θ_2 , and the acceleration due to gravity g .

$$a = (- m_1 g \sin(\theta_1) + m_2 g \sin(\theta_2)) / (m_1 + m_2)$$

Part (b) What is the acceleration of the carts in meters per square second?

$$a = 9.81 * (- m_1 * \sin(\theta_1/180 * \pi) + m_2 * \sin(\theta_2 * \pi/180)) / (m_1 + m_2)$$

$$a = 9.81 * (- 5.1 * \sin(30.5/180 * \pi) + 20.1 * \sin(48 * \pi/180)) / (5.1 + 20.1)$$

$$a = 4.807$$

$$\text{Tolerance: } \pm 0.14421$$

Part (c) What is the equation for tension in regards to the rope coming from the second block? Consider the mass of the block m_2 , the acceleration due to acceleration of the blocks found previously a .

$$T = m_2 g \sin(\theta_2) - m_2 a$$

Part (d) Calculate the tension in the rope T in newtons.

$$T = m_2 * 9.81 * \sin(\theta_2 * \pi/180) - m_2 * (9.81 * (-m_1 * \sin(\theta_1/180 * \pi) + m_2 * \sin(\theta_2 * \pi/180)) / (m_1 + m_2))$$

$$T = 20.1 * 9.81 * \sin(48 * \pi/180) - 20.1 * (9.81 * (-5.1 * \sin(30.5/180 * \pi) + 20.1 * \sin(48 * \pi/180)) / (5.1 + 20.1))$$

$$T = 49.909$$

$$\text{Tolerance: } \pm 1.49727$$

Problem 311 - 5.7.32(sym) :

Full solution not currently available at this time.

Displayed to the right are two carts connected by a cord that passes over a small frictionless pulley. Each cart rolls freely with negligible friction. Consider the mass of the cart to be m_1 and the angle the first cart sits on to be θ_1 , while the mass of the second cart as m_2 and the angle it sits on to be θ_2 .



Part (a) Considering the positive x direction to be pointing as it is in the image, write an equation for the acceleration that the two blocks experience in terms of block m_1 , the angle θ_1 , the mass of the second block m_2 , the angle θ_2 , and the acceleration due to gravity g .

$$F_{\text{net}} = (-m_1 g \sin(\theta_1) + m_2 g \sin(\theta_2)) / (m_1 + m_2)$$

Part (b) What is the equation for tension in regards to the rope coming from the second block? Consider the mass of the block m_2 , the acceleration due to gravity g , and the acceleration of the blocks found previously a .

$$T = m_2 g \sin(\theta_2) - m_2 a$$

Problem 312 - 5.7.33(sym) :

Full solution not currently available at this time.

Two blocks are connected by a string as shown. The inclination of the ramp is θ , while the masses of the blocks are m_1 and m_2 . Friction is negligible.



Part (a) Write an equation for the magnitude of the acceleration the two blocks experience. Give your equation in terms of m_1 , m_2 , θ , and the acceleration due to gravity g .

$$a = (m_2 g - m_1 g \sin(\theta)) / (m_1 + m_2)$$

Part (b) Write an equation for the tension in the string in terms of m_1 , the acceleration due to gravity g , and the acceleration of the two blocks a .

$$T = m_1 a + m_1 g \sin(\theta)$$

Problem 313 - 5.7.34 :

Full solution not currently available at this time.

Referring to the image to the right, the mass of block 1 is $m_1 = 3.1$ kg while the mass of block 2 is $m_2 = 8.1$ kg. The coefficient of friction between m_1 and the inclined surface is $\mu = 0.31$. The inclined surface is at an angle $\theta = 31^\circ$ above the horizontal.



Part (a) Write an equation for the magnitude of this system's acceleration. Use the variables from the problem statement together with g for acceleration write your equation.

$$a = (m_2 g - m_1 g \mu \cos(\theta) - m_1 g \sin(\theta)) / (m_1 + m_2)$$

Part (b) Calculate the magnitude of the system's acceleration in $\frac{m}{s^2}$.

$$a = 9.81 * (m_2 - m_1 * \mu * \cos(\theta * \pi / 180) - m_1 * \sin(\theta * \pi / 180)) / (m_1 + m_2)$$

$$a = 9.81 * (8.1 - 3.1 * 0.31 * \cos(31 * \pi / 180) - 3.1 * \sin(31 * \pi / 180)) / (3.1 + 8.1)$$

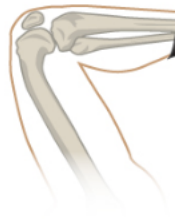
$$a = 4.975$$

$$\text{Tolerance: } \pm 0.14925$$

Problem 314 - 5.7.36 :

Full solution not currently available at this time.

A leg is suspended in a traction system as shown in the image to the right. The image below that shows a free-body diagram of the forces on pulley 4.



Part (a) What will the magnitude of force

\vec{F} keeping the pulley in place be in the free body diagram of pulley 4? Give your answer in terms of the other variables in the problem statement.

$$\vec{F} = 2 T \cos(\theta)$$

Part (b) Supposed that rather than a shinbone as shown in the image, instead the patient has a broken femur that must be placed in a traction setup. If \vec{F} is the tension in the cable attaching the leg to pulley 4, in what way could we increase the magnitude of \vec{F} in order to increase the force on the leg?

Decrease angle θ by moving the pulley left.

Problem 315 - 5.7.38 :**Full solution not currently available at this time.**

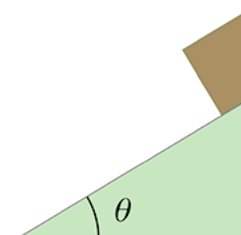
A car moves parallel to the x axis and its velocity v depends on position x as $v = kx^2$, where k is a constant.

Part (a) Write an expression for the net force on the car as a function of its mass m and the variables k and x .

$$\Sigma F = 2 m k x^3$$

Problem 316 - 5.7.39 :**Full solution not currently available at this time.**

A crate of mass $m = 0.75$ kg is on a frictionless incline that makes an angle of $\theta = 20.1$ degrees with horizontal. A force of magnitude $P = 7.01$ N is applied to the crate in a direction parallel and up the incline, as shown.



Part (a) Write an expression for the acceleration a_x of the crate, where positive x is up the incline. Your answer should be in terms of m , P , θ , and g (9.80

$$a_x = (P - m g \sin(\theta)) / m$$

Part (b) What is the acceleration of the crate in m/s^2 in the x direction defined in part (a)?

$$a_x = (p - 9.8 * m * \sin(\theta)) / m$$

$$a_x = (7.01 - 9.8 * 0.75 * \sin(20.1)) / 0.75$$

$$a_x = 5.979$$

$$\text{Tolerance: } \pm 0.17937$$

Part (c) What is the direction of the net force on the crate?

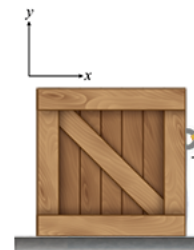
Up the incline

Part (d) What is the direction of the velocity of the crate?

It is not possible to say based on the information known.

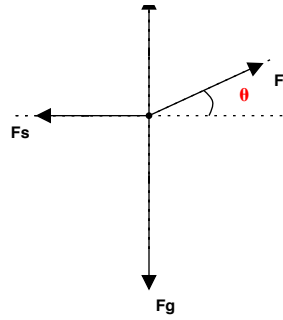
Problem 317 - 5.8.0 :**Full solution not currently available at this time.**

A crate sits on a rough surface. Using a rope, a man applies a force to the crate as shown in the figure. The force is not enough to move the crate however and it remains stationary. If necessary, use F_s for the force of static friction, and F_k as the force of kinetic friction.



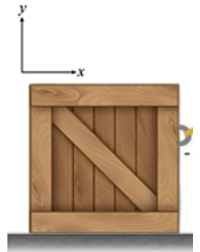
Part (a) Please use the interactive area below to draw the Free Body Diagram for the crate.

F_n
▲

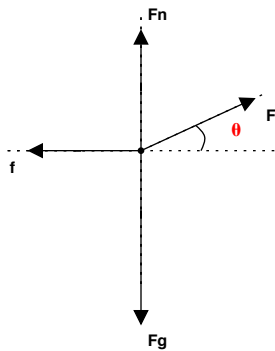
**Problem 318 - 5.8.0 (alt) :**

Full solution not currently available at this time.

A crate sits on a rough surface. Using a rope, a man applies a force to the crate as shown in the figure. The force is not enough to move the crate however and it remains stationary. Use f to represent the force of friction.

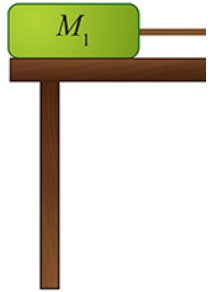


Part (a) Please use the interactive area below to draw the Free Body Diagram for the crate.

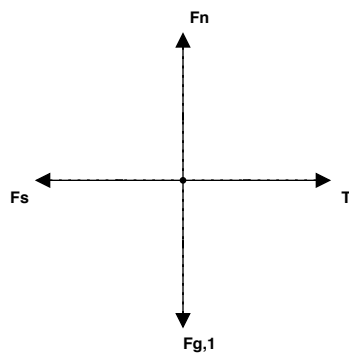
**Problem 319 - 5.8.3 :**

Full solution not currently available at this time.

Two blocks are tied together with a massless string that does not stretch and connected via a frictionless and massless pulley. Mass one, M_1 , rests on a table top and is stationary. Denote the force for static friction as F_s and the tension in the string by T .



Part (a) Please use the interactive area below to draw the Free Body Diagram for the mass M_1 .



Part (b) Please use the interactive area below to draw the Free Body Diagram for the mass M_2 .

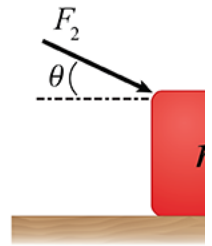


Problem 320 - 5.8.2 :

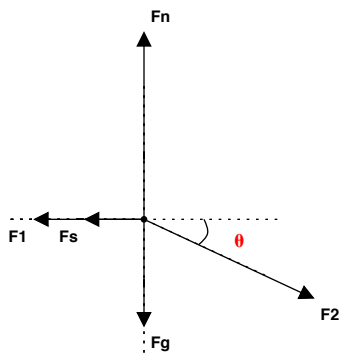
Full solution not currently available at this time.

A block with a mass of m rests on a rough surface and is subject to two forces acting on it. The first force is directed in the negative x -direction. The second acts on the body at an angle θ measured from horizontal, as shown. If necessary, use F_s and F_k for the forces of static and kinetic friction.

Note that $F_1 \ll F_2$.



Part (a) Please use the interactive area below to draw the Free Body Diagram for this block, assuming it is in static equilibrium.



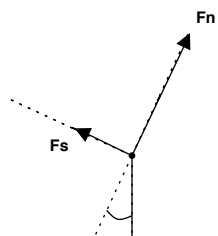
Problem 321 - 5.8.5 :

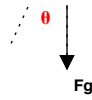
Full solution not currently available at this time.

Consider the block shown in the figure, which has a mass m and is sitting at rest on a ramp making an angle of θ with respect to the floor. Use F_s to denote the force of static friction.



Part (a) Please use the interactive area below to draw the Free Body Diagram for the block.

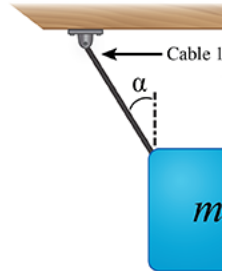




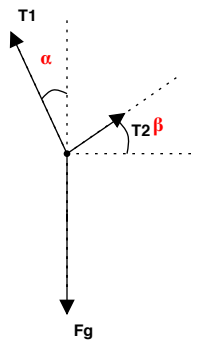
Problem 322 - 5.8.6 :

Full solution not currently available at this time.

A block having a mass of m is suspended via two cables as shown in the figure. We will label the tension in Cable 1 as T_1 and the tension in Cable 2 as T_2 at respective angles of α and β .



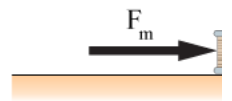
Part (a) Please use the interactive area below to draw the Free Body Diagram for the block.



Problem 323 - 5.8.8 :

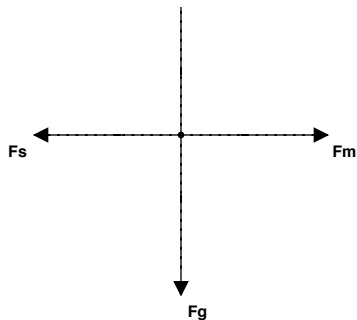
Full solution not currently available at this time.

A force, F_m , is applied to a stationary book that is resting on a rough horizontal tabletop. If necessary, use F_s for the force of static friction and F_k as the force of kinetic friction.



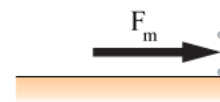
Part (a) Please use the interactive area below to draw the Free Body Diagram for the book.



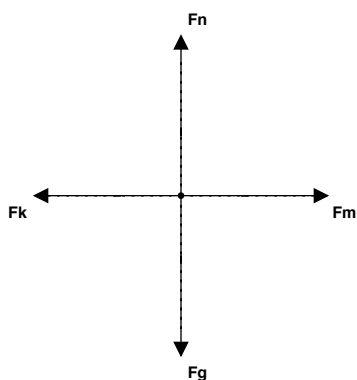
**Problem 324 - 5.8.9 :**

Full solution not currently available at this time.

A horizontal force, F_m , is applied to a book that is on a rough horizontal tabletop. As a result the book is moving with a constant velocity. If necessary, use F_s for the force of static friction and F_k as the force of kinetic friction.

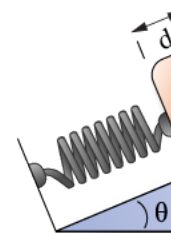


Part (a) Please use the interactive area below to draw the Free Body Diagram for the book.

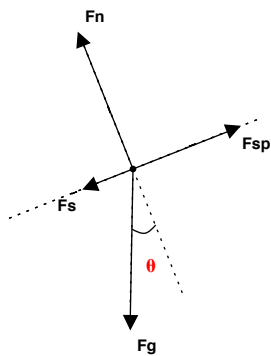
**Problem 325 - 5.8.12 :**

Full solution not currently available at this time.

A block of mass m rests against a spring with a spring constant of k on a rough inclined plane which makes an angle of θ degrees with the horizontal. The block compresses the spring a distance d from its neutral position, and friction is preventing the block from moving up the ramp. Use F_{sp} as the spring force, F_s for the force of static friction, and F_k as the force of kinetic friction.



Part (a) Please use the interactive area below to draw the Free Body Diagram for the block.



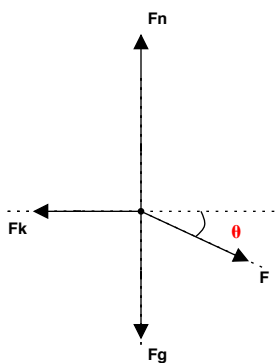
Problem 326 - 5.8.13 :

Full solution not currently available at this time.

A man pushes a block of ice across a frozen pond at a constant velocity. While the coefficients of static and kinetic friction for ice are low, they are not zero. Consider this problem to involve friction. If necessary, use F_s for the force of static friction, and F_k as the force of kinetic friction.



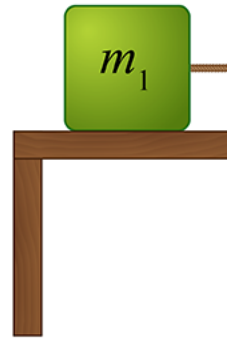
Part (a) Please use the interactive area below to draw the Free Body Diagram for the block of ice.



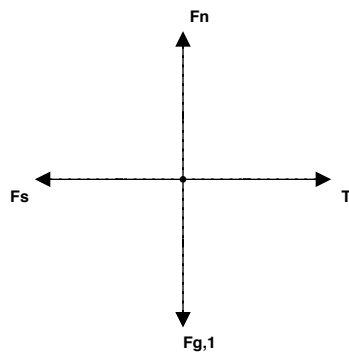
Problem 327 - 5.8.18 :

Full solution not currently available at this time.

Two blocks are tied together with a massless string that does not stretch and connected via a frictionless and massless pulley. Mass one, m_1 , sits stationary on table top. Denote the tension in the string as T , and the force of static friction as F_s .



Part (a) Please use the interactive area below to draw the Free Body Diagram for the mass m_1 .



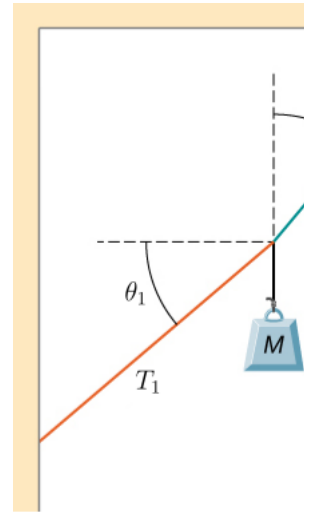
Part (b) Please use the interactive area below to draw the Free Body Diagram for the mass m_2 .



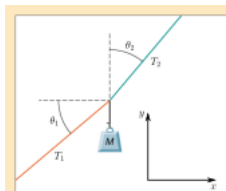
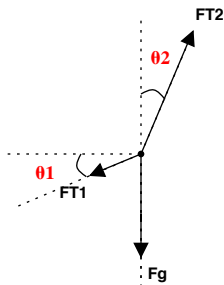
Problem 328 - 5.8.19 :

Full solution not currently available at this time.

Three ideal massless strings are connected in a single knot as shown in the figure below. The first string, with tension T_1 , extends from the knot at an angle θ_1 below the horizontal until it connects with the wall on the left. The second string, with tension T_2 , extends at an angle θ_2 with the vertical until it connects with ceiling towards the upper right. The remaining string connects the knot to a suspended weight with mass M . The system is in static equilibrium.



Part (a) Please use the interactive area below to draw the best Free Body Diagram to represent the forces on the knot where the three strings are connect



Part (b) Coordinate axes have been added to the figure. Enter an expression for F_{T1} , the force on the knot due to the tension T_1 terms of the standard unit vectors, \hat{i} , \hat{j} and \hat{k} .

$$F_{T1} = -T_1 \cos(\theta_1) \hat{i} - T_1 \sin(\theta_1) \hat{j}$$

Part (c) Enter an expression for F_{T2} , the force on the knot due to the tension T_2 . Express your answer in terms of the standard unit vectors, \hat{i} , \hat{j} and \hat{k} .

$$F_{T2} = T_2 \sin(\theta_2) \hat{i} + T_2 \cos(\theta_2) \hat{j}$$

Part (d) Enter an expression for F_g , the force on the knot due to the suspended weight. Express your answer in terms of the standard unit vectors, \hat{i} , \hat{j} and \hat{k} .

$$F_g = -M g \hat{j}$$

Part (e) Enter an expression for the magnitude of the net force on the knot in the x direction including explicit terms for the contributions of the separate

$$\Sigma F_x = -T_1 \cos(\theta_1) + T_2 \sin(\theta_2)$$

Part (f) Enter an expression for the magnitude of the net force on the knot in the y direction including explicit terms for the contributions of the separate

$$\Sigma F_y = -T_1 \sin(\theta_1) + T_2 \cos(\theta_2) - M g$$

Part (g) Give a numeric answer for the tension in the string to the lower left, T_1 for the case where $\theta_1 = 21^\circ$, $\theta_2 = 21^\circ$ and $M = 2.5$ kg.

$$T_1 = \text{mass} * 9.81 * \sin(\text{angle}2 * \pi / 180.) / (\cos(\text{angle}2 * \pi / 180.) * \cos(\text{angle}1 * \pi / 180.) - \sin(\text{angle}2 * \pi / 180.) * \sin(\text{angle}1 * \pi / 180.))$$

$$T_1 = 2.5 * 9.81 * \sin(21 * \pi / 180.) / (\cos(21 * \pi / 180.) * \cos(21 * \pi / 180.) - \sin(21 * \pi / 180.) * \sin(21 * \pi / 180.))$$

$$T_1 = 11.827$$

Tolerance: ± 0.35481

Part (h) Give a numeric answer for the tension in the string to the upper right, T_2 for the case where $\theta_1 = 21^\circ$, $\theta_2 = 21^\circ$ and $M = 2.5$ kg.

$$T_2 = \text{mass} * 9.81 * \cos(\text{angle}1 * \pi / 180.) / (\cos(\text{angle}2 * \pi / 180.) * \cos(\text{angle}1 * \pi / 180.) - \sin(\text{angle}2 * \pi / 180.) * \sin(\text{angle}1 * \pi / 180.))$$

$$T_2 = 2.5 * 9.81 * \cos(21 * \pi / 180.) / (\cos(21 * \pi / 180.) * \cos(21 * \pi / 180.) - \sin(21 * \pi / 180.) * \sin(21 * \pi / 180.))$$

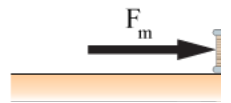
$$T_2 = 30.81$$

Tolerance: ± 0.9243

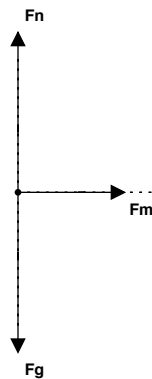
Problem 329 - 5.9.1 :

Full solution not currently available at this time.

A horizontal force, F_m , is applied to a book that is on a frictionless horizontal tabletop. As a result the book is accelerating to the right. If necessary, use F_s for the force of static friction and F_k as the force of kinetic friction.



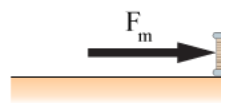
Part (a) Please use the interactive area below to draw the Free Body Diagram for the book.



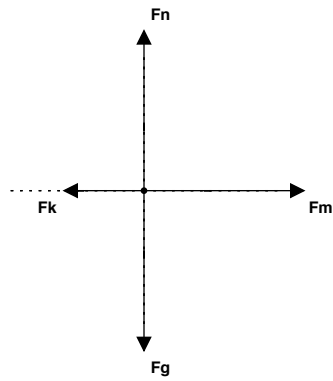
Problem 330 - 5.9.2 :

Full solution not currently available at this time.

A horizontal force, F_m , is applied to a book that is on a rough horizontal tabletop. As a result the book is accelerating to the right. If necessary, use F_s for the force of static friction and F_k as the force of kinetic friction.



Part (a) Please use the interactive area below to draw the Free Body Diagram for the book.

**Problem 331 - 5.9.3 :**

Full solution not currently available at this time.

A ball is dropped from a tall building. Use F_d to denote the drag force associated with air resistance for this problem.



Part (a) Please use the interactive area below to draw the Free Body Diagram for the ball during the period where it is still accelerating in the downward



Part (b) Please use the interactive area below to draw the Free Body Diagram for the ball assuming the ball has reached terminal velocity (i.e. it is falling



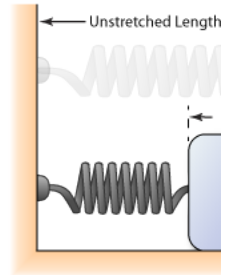


Problem 332 - 5.9.4 :

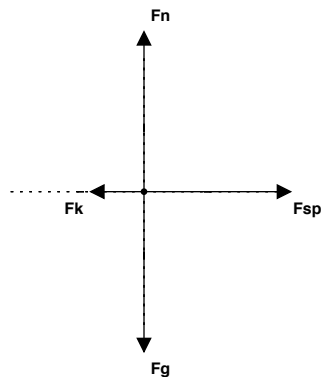
Full solution not currently available at this time.

A spring with a spring constant of k is initially compressed by a block a distance d . The block is on a rough horizontal surface with coefficient of kinetic friction μ_k , static friction μ_s , and has a mass of m . The block has just been released and is accelerating to the right.

Use F_{sp} as the spring force. If necessary, use F_s for the force of static friction and F_k as the force of kinetic friction.



Part (a) Please use the interactive area below to draw the Free Body Diagram for the block.

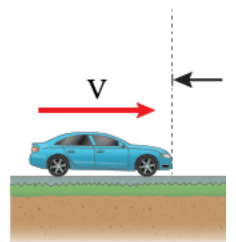


Problem 333 - 5.9.7 :

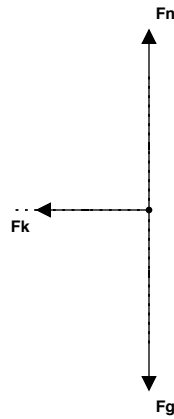
Full solution not currently available at this time.

A car is driving down the road and the driver sees a branch fall out of a tree onto the road, a distance d ahead. The driver applies the brakes in order to slow down and avoid hitting the branch. The driver applies maximum force to the brakes and the wheels stop, resulting in the car skidding (there is no ABS).

If necessary, use F_s for the force of static friction and F_k as the force of kinetic friction.



Part (a) Please use the interactive area below to draw the Free Body Diagram for the car.

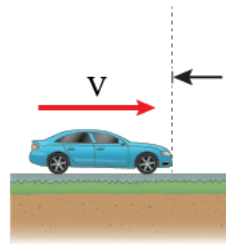


Problem 334 - 5.9.7 (alt) :

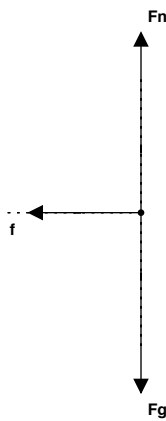
Full solution not currently available at this time.

A car is driving down the road and the driver sees a branch fall out of a tree onto the road, a distance d ahead. The driver applies the brakes in order to slow down and avoid hitting the branch. The driver applies maximum force to the brakes and the wheels stop, resulting in the car skidding (there is no ABS).

If necessary, use f for the force of friction.

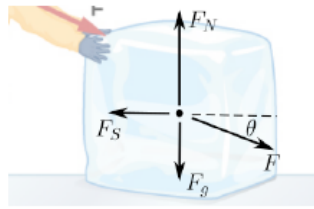


Part (a) Please use the interactive area below to draw the Free Body Diagram for the car.



Problem 335 - 5.9.8 :

The correct graph should resemble



with these pertinent features.

- Be certain to use static friction, F_S , not kinetic friction, F_K .
- The static friction vector, F_S , should point to the left with the same magnitude as the right-pointing horizontal component of the applied force, F .
- The length of the normal vector should equal the sum of the lengths of the weight vector and the vertical component of the applied force. (Pushing down on the block causes an increase in the normal force.)

Two forces contribute in the horizontal direction; the static friction, F_S , is purely horizontal, and the applied force, F , has a horizontal component.

These are combined as

$$\Sigma F_x = F \cos \theta - F_S$$

$$\Sigma F_x = F \cos \theta - \mu_S F_N$$

Notice the negative sign for the static friction because it is directed towards the left. The force of static friction has been replaced with an expression in terms of the coefficient of friction and the normal force. Note that the normal force does *not* have the same magnitude as the weight. The magnitude of the normal force will be obtained in a later step.

mass = 0.266 angle = 0.299

Three forces contribute in the vertical direction. They are the normal force, F_N , the weight, $F_g = mg$, and the vertical component of the applied force, F .

$$\Sigma F_y = F_N - F_g - F \sin \theta$$

$$\Sigma F_y = F_N - mg - F \sin \theta$$

Notice the negative signs for downward-acting weight and the downward-acting vertical component of the applied force.

mass = 0.266 angle = 0.299

There is no acceleration in the vertical direction, so the net force in the vertical direction sums to zero. It is then possible to solve for the normal force.

$$0 = F_N - mg - F \sin \theta$$

$$F_N = mg + F \sin \theta$$

Note: The normal force increases due to the downward component of the applied force.

mass = 0.266 angle = 0.299

The block is not sliding relative to the ice, so static friction applies. With no horizontal acceleration, the net force in horizontal direction sums to zero.

$$0 = F\cos\theta - F_S$$

Make the usual substitutions for the magnitude of the force of static friction.

$$0 = F\cos\theta - \mu_S F_N$$

We are now able to substitute for the magnitude of the normal force using the expression from the previous part.

$$\begin{aligned} 0 &= F\cos\theta - \mu_S(mg + F\sin\theta) \\ &= F\cos\theta - \mu_S mg - \mu_S F\sin\theta \end{aligned}$$

Perform a little algebra.

$$F(\cos\theta - \mu_S\sin\theta) = \mu_S mg$$

$$F = \frac{\mu_S mg}{\cos\theta - \mu_S\sin\theta}$$

Notes:

- The denominator has a maximum value of 1 when $\theta = 0^\circ$ and the applied force has its minimum value, $F = \mu_S mg$.
- The denominator decreases to zero when $\cot\theta = \mu_S$ implying that the minimum applied force increases towards infinity as the angle approaches this value. There comes a point where the block will not slide, and at this point the solution becomes unphysical.

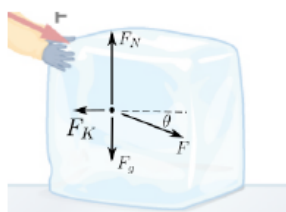
mass = 0.266 angle = 0.299

$$F = (0.0300)(0.266\text{kg}) \frac{(9.81\text{m/s}^2)}{(\cos(0.299^\circ) - 0.0300\sin(0.299^\circ))}$$

$$F = 0.0783 \text{ N}$$

mass = 0.266 angle = 0.299

The correct graph should resemble



with these pertinent features.

- Be certain to use kinetic friction, F_K , not static friction, F_S .
- The kinetic friction vector, F_K , should point to the left with a lesser magnitude than the right-pointing horizontal component of the applied force, F , because the block is accelerating to the right.
- The length of the normal vector should equal the sum of the lengths of the weight vector and the vertical component of the applied force. (Pushing down on the block causes an increase in the normal force.)

From the FBD in the Part (g), the net force in the horizontal direction becomes

$$\begin{aligned} \Sigma F_x &= F\cos(\theta) - F_K \\ &= F\cos(\theta) - \mu_K F_N \end{aligned}$$

The normal force is the same as obtained in the case with static friction.

$$F_N = mg + F\sin(\theta)$$

We may eliminate F_N which yields

$$\begin{aligned} \Sigma F_x &= F\cos(\theta) - \mu_K(mg + F\sin(\theta)) \\ &= F(\cos(\theta) - \mu_K\sin(\theta)) - \mu_K mg \end{aligned}$$

and

$$a = \frac{(\Sigma F_x)}{m}$$

$$a = \left(\frac{F}{m}\right)(\cos(\theta) - \mu_K\sin(\theta)) - \mu_K g$$

mass = 0.266 angle = 0.299

From Part (f),

$$F = 0.0783 \text{ N}$$

We use this value of the force when evaluating the acceleration just as static friction is broken.

$$a = (0.0783 \text{ N}) \frac{(\cos(0.299^\circ) - 0.0100\sin(0.299^\circ))}{(0.266\text{kg})} - (0.0100)(9.81\text{m/s}^2)$$

$$a = (0.1962 \text{ m/s}^2)$$

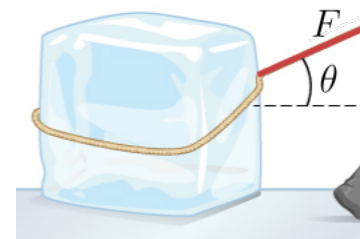
mass = 0.266 angle = 0.299

(Note: Simple javascript evaluation implmented for look and feel. Anything unhandled will just be displayed in full expression form.)

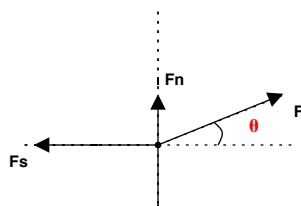
Problem 336 - 5.9.9 :

Full solution not currently available at this time.

A contestant in a winter sporting event pulls an m kg block of ice across a frozen lake by applying a force F at an angle θ above the horizontal as shown. Assume that the coefficient of static friction for ice on ice is 0.0300, and the coefficient of kinetic friction for the same is 0.0100. Let to the right be the positive x direction and up be the positive y direction for your equations.



Part (a) Please use the interactive area below to draw a free body diagram to represent the situation where the contestant is pulling, but the block has no F_s for the force of static friction and F_k for the force of kinetic friction if they are needed for your free body diagram.





Part (b) Enter an expression for the net force in the horizontal direction, ΣF_x . Your expression may include the normal force, F_N , the applied force, F , the angle above the horizontal, θ , and appropriate coefficients of friction. The friction coefficients are μ_K and μ_S .

$$\Sigma F_x = F \cos(\theta) - \mu_S F_N$$

Part (c) Enter an expression for the net force in the vertical direction, ΣF_y . Your expression may include the mass, m , acceleration due to gravity, g , the applied force, F , the angle of the applied force above the horizontal, θ , and appropriate coefficients of friction. The friction coefficients are μ_K and μ_S .

$$\Sigma F_y = F_N - m g + F \sin(\theta)$$

Part (d) Enter an expression for the magnitude of the normal force, F_N . Your expression may include the mass, m , the acceleration due to gravity, g , the angle of the applied force above the horizontal, θ .

$$F_N = m g - F \sin(\theta)$$

Part (e) Enter an expression for the magnitude of the applied force, F , at the threshold where static friction is at its maximum value. Your expression may include the mass, m , the acceleration due to gravity, g , the angle of the applied force above the horizontal, θ , and appropriate coefficients of friction. The friction coefficients are μ_K and μ_S .

$$F = \mu_S m g / (\cos(\theta) + \mu_S \sin(\theta))$$

Part (f) Obtain a numeric value, in newtons, for the magnitude of the maximum applied force, F , consistent with static friction when the force makes an angle of 15° above the horizontal and the mass of the block is 25 kg.

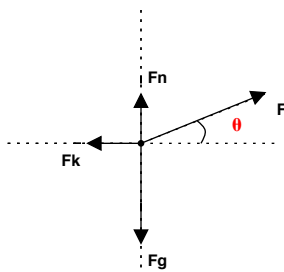
$$F = 0.0300 * \text{mass} * 9.81 / (\cos(\text{angle} * \text{pi}/180) + 0.0300 \sin(\text{angle} * \text{pi}/180))$$

$$F = 0.0300 * 25 * 9.81 / (\cos(15 * \text{pi}/180) + 0.0300 * \sin(15 * \text{pi}/180))$$

$$F = 7.556$$

$$\text{Tolerance: } \pm 0.22668$$

Part (g) Please use the interactive area below to draw a free body diagram to represent the situation where the contestant is pulling, and the block has just started to move. Use F_s for the force of static friction and F_k for the force of kinetic friction if they are needed for your free body diagram.



Part (h) Obtain an expression for the acceleration of the block corresponding to the free body diagram from part (g). Your expression may include the mass, m , the acceleration due to gravity, g , the applied force, F , the angle of the applied force above the horizontal, θ , and appropriate coefficients of friction. The friction coefficients are μ_K and μ_S .

$$a = F (\cos(\theta) + \mu_K \sin(\theta)) / m - \mu_K g$$

Part (i) Obtain a numeric value for the acceleration, a , in meters per squared seconds, when the mass of the block is 25 kg and the angle of the rope is 15° above the horizontal. Use the magnitude of the applied force, use the threshold value obtained in part (f).

$$a = (0.0300 * \text{mass} * 9.81 / (\cos(\text{angle} * \text{pi}/180) + 0.0300 \sin(\text{angle} * \text{pi}/180))) * (\cos(\text{angle} * \text{pi}/180) + 0.0100 * \sin(\text{angle} * \text{pi}/180)) / \text{mass} - (0.0100) * (9.81)$$

$$a = (0.0300 * 25 * 9.81 / (\cos(15 * \text{pi}/180) + 0.0300 * \sin(15 * \text{pi}/180))) * (\cos(15 * \text{pi}/180) + 0.0100 * \sin(15 * \text{pi}/180)) / 25 - (0.0100) * (9.81)$$

$$a = 0.1946$$

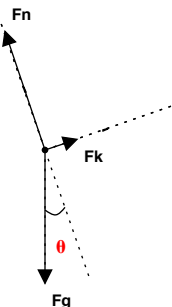
$$\text{Tolerance: } \pm 0.005838$$

Problem 337 - 5.9.10 :

Full solution not currently available at this time.

At a post office, a parcel that is an m kg box slides down a ramp inclined at an angle θ with the horizontal, as shown. The coefficient of kinetic friction between the box and the ramp is μ_K , and the coefficient of static friction for the same is μ_S . Notice that the coordinate axes have been chosen with the x axis directed up the incline, as shown.

Part (a) Please use the interactive area below to draw a free body diagram for the block. Use F_s for the force of static friction and F_k for the force of kinetic friction needed for your free body diagram.



Part (b) Input an expression for the net force in the x direction *consistent with the coordinate axes on the diagram*. Express your answer in terms of acceleration due to gravity, g , the angle of the incline, θ , the magnitude of the normal vector F_N , and appropriate coefficients of friction. The coefficient of kinetic friction between the box and the ramp is μ_K , and the coefficient of static friction for the same is μ_S .

$$\Sigma F_x = \mu_K F_N - m g \sin(\theta)$$

Part (c) Input an expression for the net force in the y direction *consistent with the coordinate axes on the diagram*. Express your answer in terms of acceleration due to gravity, g , the angle of the incline, θ , and the magnitude of the normal vector F_N .

$$\Sigma F_y = F_N - m g \cos(\theta)$$

Part (d) Input an expression for the magnitude of the normal force. Express your answer in terms of the mass of the box, m , the acceleration due to gravity, g , and the angle of the incline, θ .

$$F_N = m g \cos(\theta)$$

Part (e) Input an expression for the acceleration of the box in the x direction *consistent with the coordinate axes on the diagram*. Express your answer in terms of the mass of the box, m , the acceleration due to gravity, g , the angle of the incline, θ , and appropriate coefficients of friction. The coefficient of kinetic friction between the box and the ramp is μ_K , and the coefficient of static friction for the same is μ_S .

$$a = \mu_K g \cos(\theta) - g \sin(\theta)$$

Part (f) Enter a numeric answer for the *magnitude* of the acceleration in meters per squared seconds when the angle of the incline is 15° and the coefficient of kinetic friction is 0.035 .

$$|a| = 9.81 * (\sin(\text{angle} * \text{pi}/180.) - \text{coeffKineticFriction} * \cos(\text{angle} * \text{pi}/180.))$$

$$|a| = 9.81 * (\sin(15 * \text{pi}/180.) - 0.035 * \cos(15 * \text{pi}/180.))$$

$$|a| = 2.207$$

$$\text{Tolerance: } \pm 0.06621$$

Part (g) Using the numbers from the previous step, give a numeric answer for the time in seconds that elapse when the package, initially at rest, travels a distance of 2.25 m down the ramp.

$$d = \text{sqr}t(2.0 * \text{distance} / (9.81 * \text{abs}(\text{coeffKineticFriction} * \cos(\text{angle} * \text{pi}/180.) - \sin(\text{angle} * \text{pi}/180.))))$$

$$d = \text{sqr}t(2.0 * 2.25 / (9.81 * \text{abs}(0.035 * \cos(15 * \text{pi}/180.) - \sin(15 * \text{pi}/180.))))$$

$$d = 1.428$$

$$\text{Tolerance: } \pm 0.04284$$

Part (h) Using the numbers from the previous steps, give a numeric answer for the speed of the package at the moment it has traveled a distance of 2.25 m down the ramp.

$$v = \sqrt{2.0 * 9.81 * (\sin(\text{angle} * \pi/180.) - \text{coeffKineticFriction} * \cos(\text{angle} * \pi/180.)) * \text{distance}}$$

$$v = \sqrt{2.0 * 9.81 * (\sin(15 * \pi/180.) - 0.035 * \cos(15 * \pi/180.)) * 2.25}$$

$$v = 3.152$$

Tolerance: ± 0.09456