6

DYNAMICS I: MOTION ALONG A LINE

Conceptual Questions

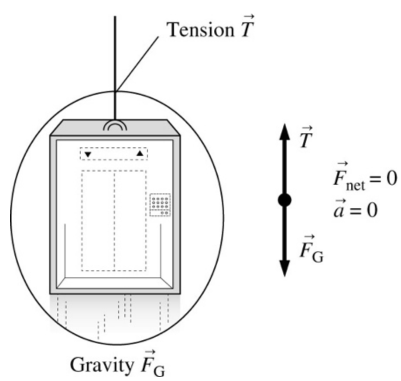
6.1. **(a)** Static equilibrium. The barbell is not accelerating and has a velocity of zero.
(b) Dynamic equilibrium. The girder is not accelerating but has a nonzero constant velocity.
(c) Not in equilibrium. Slowing down means the acceleration is not zero.
(d) Dynamic equilibrium. The plane is not accelerating but has a nonzero constant velocity.
(e) Not in equilibrium. The box slows down with the truck, so has a nonzero acceleration.

6.2. No. The ball is still changing its speed, and just momentarily has zero velocity.

6.3. Kat is closest to the correct statement, which should read "Gravity pulls down on it, but the table pushes it up so that the net force on the book is zero."

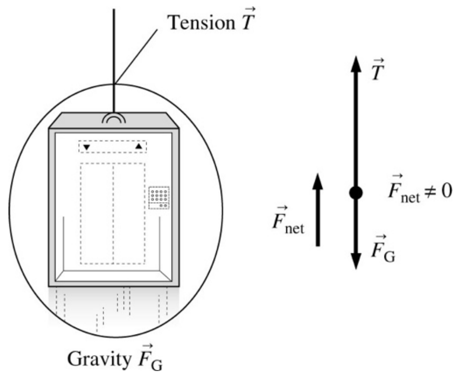
6.4. No, because the net force is not necessarily in the same direction as the motion. For example, a car using its brakes to slow its forward motion has a net force opposite its direction of motion.

6.5.



Equal. The tension in the cable is equal to the force of gravity, since the net force must be zero in order for the elevator to move with constant speed.

6.6.



Greater. Since the elevator is slowing down as it moves downward, it has an upward net force, so the tension must be greater than the gravitational force.

6.7. (a) False. The mass of an object is a measure of its inertia, which is the same regardless of location.
(b) True. The weight of an object is a measurement of how much force an object presses down on a surface with, and varies depending on location and whether the object is accelerating.
(c) False. Mass and weight describe very different things, as pointed out in parts (a) and (b).

6.8. Yes, the scale shows the astronaut's weight on the moon, since it shows how hard the astronaut is pressing down on the surface of the moon. His weight is different on the earth, of course.

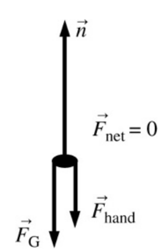
6.9. d > b = c > a. The net force on each ball is the gravitational force *mg*, so the net force on the balls is ranked by mass.

6.10. Correct. There will be a correct amount of salt in the pan balance since a pan balance measures mass, which is independent of any gravitational force or acceleration present.

6.11. Zero. The passenger's weight is zero once the box is launched since the passenger is in free fall (ignoring any air friction). While gravity still pulls the passenger down, a scale placed under his feet would not register any weight without a support under it.

6.12. The ball filled with lead is more massive. Since the balls are weightless, the astronaut must measure their inertia (mass) directly. One easy option is to move each ball side to side in turn. More force is required to change the more massive lead-filled ball's direction of motion.

6.13. Larger. A free-body diagram for the book is shown in the figure. The normal force of the table on the book is larger than its weight, since the net force is zero.



6.14. (a) $2(\Delta t)$. Your constant push F_x provides the net force, so the puck accelerates with constant acceleration $a_x = \frac{F_x}{m}$. From kinematics, with $v_{ix} = 0$,

$$. v = a_x \Delta t = \left(\frac{F_x}{m}\right)\Delta t.$$

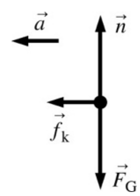
If the mass is doubled, the time must also be doubled to reach the same speed, so you must push for a time $2(\Delta t)$.

(b) $\sqrt{2}(\Delta t)$. From kinematics, with $x_i = 0$ and $v_{ix} = 0$,

$$d = \frac{1}{2}a_x(\Delta t)^2 = \frac{1}{2}\left(\frac{F_x}{m}\right)(\Delta t)^2$$

If *m* is doubled, then $(\Delta t)^2$ must be doubled, which means Δt is increased by a factor of $\sqrt{2}$. So you must push for a time $\sqrt{2}(\Delta t)$.

6.15.



(a) d. Kinetic friction $f_k = \mu_k n$ determines the horizontal acceleration $a = \frac{-f_k}{m}$, which slows down the block. From the free body diagram, $n = F_G = mg$. So $a = \frac{-(\mu_k mg)}{m} = -\mu_k g$, which is independent of mass. Note that changing the mass has no effect on the distance the block slides.

(b) 4d. From kinematics, with $v_{fx} = 0$,

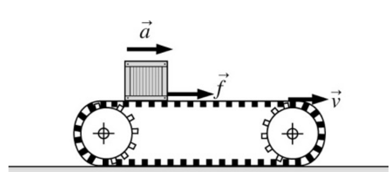
$$\left(a\frac{m}{s}\right)^2 = v_{0x}^2 - 2(\mu_k g)\Delta x$$

$$\Rightarrow \Delta x = \frac{v_{0x}^2}{2\mu_k g}$$

Thus $\Delta x \propto v_{0x}^2$, and we use proportional reasoning:

$$\frac{d}{v_{0x}^2} = \frac{\Delta x}{(2v_{0x})^2} \Rightarrow \Delta x = 4d$$

6.16. Yes, the friction force on a crate dropped on a conveyor belt speeds the crate up to the belt's speed.



6.17. North. The friction force on the crate is the only horizontal force and is responsible for speeding the crate up along with the truck. Therefore the friction force points in the same direction as the motion of the crate.

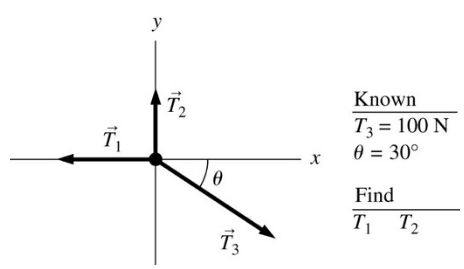
6.18. $a_e > a_a = a_b > a_d > a_c$. The balls all have the same cross-sectional area A. All of the balls have gravity pulling down, resulting in an acceleration g. The drag force $D = \frac{1}{4}Av^2$ results in an addition or subtraction to g of $\frac{1}{m}D$. For ball e, the drag force adds to gravity, resulting in a higher acceleration. $D = 0$ for balls a and b. The drag force opposes gravity for balls c and d, and since $m_d > m_c$, $a_d > a_c$.

Exercises and Problems

Section 6.1 Equilibrium

6.1. Model: We can assume that the ring is a single massless particle in static equilibrium.
Visualize:

Pictorial representation



Solve: Written in component form, Newton's first law is

$$(F_{net})_x = \sum F_x = T_{1x} + T_{2x} + T_{3x} = 0\text{ N} \quad (F_{net})_y = \sum F_y = T_{1y} + T_{2y} + T_{3y} = 0\text{ N}$$

Evaluating the components of the force vectors from the free-body diagram:

$$T_{1x} = -T_1 \quad T_{2x} = 0\text{ N} \quad T_{3x} = T_3\cos30°$$
$$T_{1y} = 0\text{ N} \quad T_{2y} = T_2 \quad T_{3y} = -T_3\sin30°$$

Using Newton's first law:

$$-T_1 + T_3\cos30° = 0\text{ N} \quad T_2 - T_3\sin30° = 0\text{ N}$$

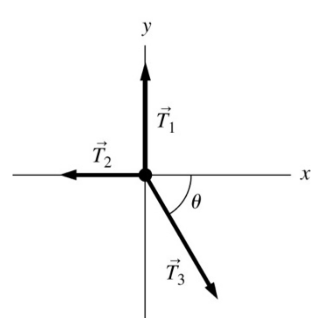
Rearranging:

$$T_1 = T_3\cos30° = (100\text{ N})(0.8666) = 86.7\text{ N} \quad T_2 = T_3\sin30° = (100\text{ N})(0.5) = 50.0\text{ N}$$

Assess: Since T_3 acts closer to the x-axis than to the y-axis, it makes sense that $T_1 > T_2$.

6.2. Model: We can assume that the ring is a particle.
Visualize:

Pictorial representation



This is a static equilibrium problem. We will ignore the weight of the ring, because it is "very light," so the only three forces are the tension forces shown in the free-body diagram. Note that the diagram *defines* the angle θ.

Solve: Because the ring is in equilibrium it must obey $F_{\text{net}} = 0\,\text{N}$. This is a vector equation, so it has both *x*- and *y*-components:

$$(F_{\text{net}})_x = T_3\cos\theta - T_2 = 0\,\text{N} \Rightarrow T_3\cos\theta = T_2$$
$$(F_{\text{net}})_y = T_1 - T_3\sin\theta = 0\,\text{N} \Rightarrow T_3\sin\theta = T_1$$

We have two equations in the two unknowns T_3 and θ. Divide the *y*-equation by the *x*-equation:

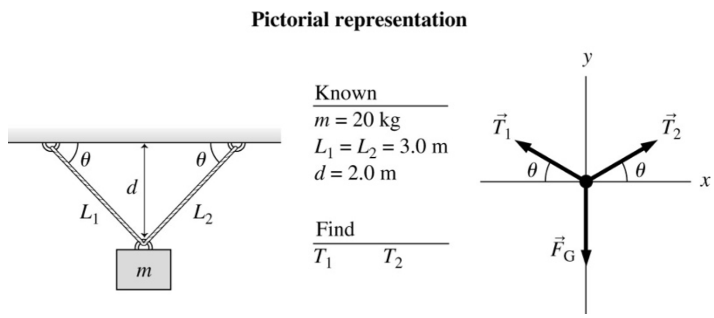
$$\frac{T_3\sin\theta}{T_3\cos\theta} = \tan\theta = \frac{T_1}{T_2} = \frac{80\,\text{N}}{50\,\text{N}} = 1.6 \Rightarrow \theta = \tan^{-1}(1.6) = 58°$$

Now we can use the *x*-equation to find

$$T_3 = \frac{T_2}{\cos\theta} = \frac{50\,\text{N}}{\cos 58°} = 94\,\text{N}$$

The tension in the third rope is 94 N directed 58° below the horizontal.

6.3. Model: We assume the speaker is a particle in static equilibrium under the influence of three forces: gravity and the tensions in the two cables.
Visualize:



Solve: From the lengths of the cables and the distance below the ceiling we can calculate θ as follows:

$$\sin\theta = \frac{2\,\text{m}}{3\,\text{m}} = 0.677 \Rightarrow \theta = \sin^{-1}0.667 = 41.8°$$

Newton's first law for this situation is

$$(F_{\text{net}})_x = \sum F_x = T_{1x} + T_{2x} = 0\,\text{N} \Rightarrow -T_1\cos\theta + T_2\cos\theta = 0\,\text{N}$$
$$(F_{\text{net}})_y = \sum F_y = T_{1y} + T_{2y} + w_y = 0\,\text{N} \Rightarrow T_1\sin\theta + T_2\sin\theta - w = 0\,\text{N}$$

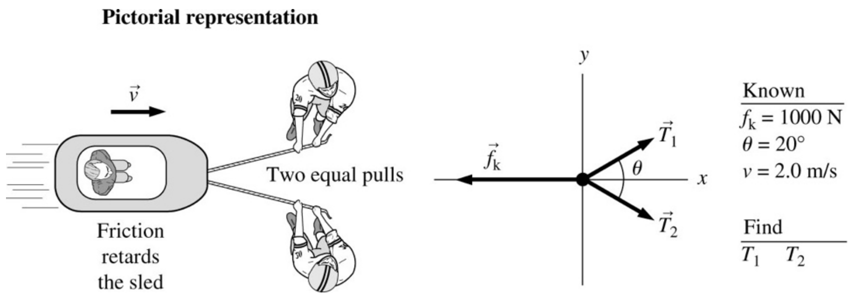
The *x*-component equation means $T_1 = T_2$. From the *y*-component equation:

$$2T_1\sin\theta = w \Rightarrow T_1 = \frac{w}{2\sin\theta} = \frac{mg}{2\sin\theta} = \frac{(20\,\text{kg})(9.8\,\text{m/s}^2)}{2\sin 41.8°} = \frac{196\,\text{N}}{1.333} = 147\,\text{N}$$

Assess: It's to be expected that the two tensions are equal, since the speaker is suspended symmetrically from the two cables. That the two tensions add to considerably more than the weight of the speaker reflects the relatively large angle of suspension.

6.4. Model: We can assume that the coach and his sled are a particle being towed at a constant velocity by the two ropes, with friction providing the force that resists the pullers.

Visualize:



Solve: Since the sled is not accelerating, it is in dynamic equilibrium and Newton's first law applies:

$$(F_{\text{net}})_x = \sum F_x = T_{1x} + T_{2x} + f_{kx} = 0\text{ N} \quad (F_{\text{net}})_y = \sum F_y = T_{1y} + T_{2y} + f_{ky} = 0\text{ N}$$

From the free-body diagram:

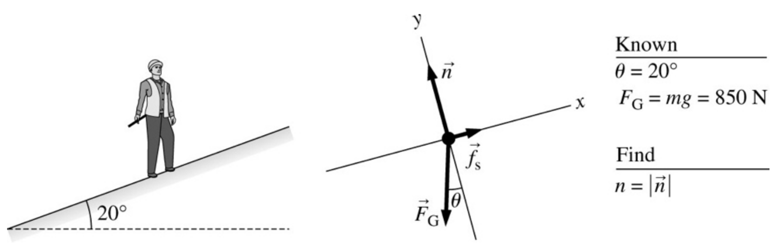
$$T_1\cos\left(\frac{1}{2}\theta\right) + T_2\cos\left(\frac{1}{2}\theta\right) - f_k = 0\text{ N} \quad T_1\sin\left(\frac{1}{2}\theta\right) - T_2\sin\left(\frac{1}{2}\theta\right) + 0\text{ N} = 0\text{ N}$$

From the second of these equations $T_1 = T_2$. Then from the first:

$$2T_1\cos10° = 1000\text{ N} \Rightarrow T_1 = \frac{1000\text{ N}}{2\cos10°} = \frac{1000\text{ N}}{1.970} = 508\text{ N} \approx 510\text{ N}$$

Assess: The two tensions are equal, as expected, since the two players are pulling at the same angle. The two add up to only slightly more than 1000 N, which makes sense because the angle at which the two players are pulling is small.

6.5. Model: Model the worker as a particle.
Visualize: In equilibrium the net force is zero in both directions. There must be a static friction force to keep her from sliding off.



Solve: We only need to examine the *y*-direction.

$$(\sum F)_y = n - mg\cos\theta = 0 \Rightarrow n = mg\cos\theta = (850\text{ N})(\cos20°) = 799\text{ N} \approx 800\text{ N}$$

Assess: A good way to assess solutions like this is to consider what happens in the limit as $\theta \to 0$ and as $\theta \to 90$. In the first case $n \to mg$ and in the second $n \to 0$ as expected.

Section 6.2 Using Newton's Second Law

6.6. Solve: (a) Applying Newton's second law to the diagram,

$$a_x = \frac{(F_{\text{net}})_x}{m} = \frac{2.0\text{ N} - 4.0\text{ N}}{2.0\text{ kg}} = -1.0\text{ m/s}^2 \quad a_y = \frac{(F_{\text{net}})_y}{m} = \frac{3.0\text{ N} - 3.0\text{ N}}{2.0\text{ kg}} = 0\text{ m/s}^2$$

(b) Applying Newton's second law to the diagram,

$$a_x = \frac{(F_{\text{net}})_x}{m} = \frac{4\text{ N} - 2\text{ N}}{2\text{ kg}} = 1.0\text{ m/s}^2 \qquad a_y = \frac{(F_{\text{net}})_y}{m} = \frac{3\text{ N} - 1\text{ N} - 2\text{ N}}{2\text{ kg}} = 0\text{ m/s}^2$$

6.7. Solve: (a) For the diagram on the left, three of the vectors lie along the axes of the tilted coordinate system. Notice that the angle between the 3 N force and the $-y$-axis is the same 20° by which the coordinates are tilted. Applying Newton's second law,

$$a_x = \frac{(F_{\text{net}})_x}{m} = \frac{5.0\text{ N} - 1.0\text{ N} - (3.0\sin 20°)\text{ N}}{2.0\text{ kg}} = 1.49\text{ m/s}^2 \approx 1.5\text{ m/s}^2$$

$$a_y = \frac{(F_{\text{net}})_y}{m} = \frac{2.82\text{ N} - (3.0\cos 20°)\text{ N}}{2.0\text{ kg}} = 0\text{ m/s}^2$$

(b) For the diagram on the right, the 2-newton force in the first quadrant makes an angle of 15° with the positive x-axis. The other 2-newton force makes an angle of 15° with the negative y-axis. The accelerations are

$$a_x = \frac{(F_{\text{net}})_x}{m} = \frac{(2.0\cos 15°)\text{ N} + (2.0\sin 15°)\text{ N} - 3.0\text{ N}}{2.0\text{ kg}} = -0.28\text{ m/s}^2$$

$$a_y = \frac{(F_{\text{net}})_y}{m} = \frac{1.414\text{ N} + (2.0\sin 15°)\text{ N} - (2.0\cos 15°)\text{ N}}{2.0\text{ kg}} = 0\text{ m/s}^2$$

6.8. Solve: We can use the constant slopes of the three segments of the graph to calculate the three accelerations. For t between 0 s and 2 s,

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{12\text{ m/s} - 0\text{ s}}{2\text{ s}} = 6\text{ m/s}^2$$

For t between 3 s and 6 s, $\Delta v_x = 0\text{ m/s}$, so $a_x = 0\text{ m/s}^2$. For t between 6 s and 8 s,

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{0\text{ m/s} - 12\text{ m/s}}{3\text{ s}} = 2\ 4\text{ m/s}^2$$

From Newton's second law, at $t = 1$ s we have

$$F_{\text{net}} = ma_x = (2.0\text{ kg})(6\text{ m/s}^2) = 12\text{ N}$$

At $t = 4\text{ s}$, $a_x = 0\text{ m/s}^2$, so $F_{\text{net}} = 0\text{ N}$. At $t = 7\text{ s}$,

$$F_{\text{net}} = ma_x = (2.0\text{ kg})(-4.0\text{ m/s}^2) = -8\text{ N}$$

Assess: The magnitudes of the forces look reasonable, given the small mass of the object. The positive and negative signs are appropriate for an object first speeding up, then slowing down.

6.9. Visualize: Assuming the positive direction is to the right, positive forces result in the object accelerating to the right and negative forces result in the object accelerating to the left. The final segment of zero force is a period of constant speed.
Solve: We have the mass and net force for all the three segments. This means we can use Newton's second law to calculate the accelerations. The acceleration from $t = 0\text{ s}$ to $t = 3\text{ s}$ is

$$a_x = \frac{F_x}{m} = \frac{4\text{ N}}{2.0\text{ kg}} = 2\text{ m/s}^2$$

The acceleration from $t = 3\text{ s}$ to $t = 5\text{ s}$ is

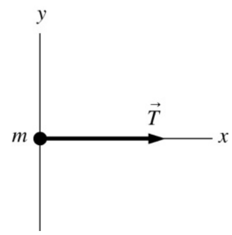
$$a_x = \frac{F_x}{m} = \frac{-2\text{ N}}{2.0\text{ kg}} = -1\text{ m/s}^2$$

The acceleration from $t = 5\text{ s}$ to 8 s is $a_x = 0\text{ m/s}^2$. In particular, $a_x(\text{at } t = 6\text{ s}) = 0\text{ m/s}^2$.
We can now use one-dimensional kinematics to calculate v at $t = 6\text{ s}$ as follows:

$$\begin{aligned} v &= v_0 + a_1(t_1 - t_0) + a_2(t_2 - t_0) \\ &= 0 + (2\text{ m/s}^2)(3\text{s}) + (-1\text{ m/s}^2)(2\text{ s}) = 6\text{ m/s} - 2\text{ m/s} = 4\text{ m/s} \end{aligned}$$

Assess: The positive final velocity makes sense, given the greater magnitude and longer duration of the positive F_1. A velocity of 4 m/s also seems reasonable, given the magnitudes and directions of the forces and the mass involved.

6.10. Model: We assume that the box is a particle being pulled in a straight line. Since the ice is frictionless, the tension in the rope is the only horizontal force.
Visualize:

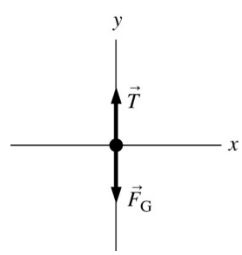


Solve: (a) Since the box is at rest, $a_x = 0\text{ m/s}^2$, and the net force on the box must be zero. Therefore, according to Newton's first law, the tension in the rope must be zero.
(b) For this situation again, $a_x = 0\text{ m/s}^2$, so $F_{\text{net}} = T = 0\text{ N}$.
(c) Here, the velocity of the box is irrelevant, since only a *change* in velocity requires a nonzero net force. Since $a_x = 5.0\text{ m/s}^2$,

$$F_{\text{net}} = T = ma_x = (50\text{ kg})(5.0\text{ m/s}^2) = 250\text{ N}$$

Assess: For parts (a) and (b), the zero acceleration immediately implies that the rope is exerting no horizontal force on the box. For part (c), the 250 N force (the equivalent of about half the weight of a small person) seems reasonable to accelerate a box of this mass at 5.0 m/s^2.

6.11. Model: We assume that the box is a point particle that is acted on only by the tension in the rope and the pull of gravity. Both the forces act along the same vertical line.
Visualize:



Solve: (a) Since the box is at rest, $a_y = 0\text{ m/s}^2$ and the net force on it must be zero:

$$F_{\text{net}} = T - F_G = 0\text{ N} \Rightarrow T = F_G = mg = (50\text{ kg})(9.8\text{ m/s}^2) = 490\text{ N}$$

(b) Since the box is rising at a constant speed, again $a_y = 0\text{ m/s}^2$, $F_{\text{net}} = 0\text{ N}$, and $T = F_G = 490\text{ N}$.
(c) The velocity of the box is irrelevant, since only a *change* in velocity requires a nonzero net force. Since $a_y = 5.0\text{ m/s}^2$,

$$F_{\text{net}} = T - F_G = ma_y = (50\text{ kg})(5.0\text{ m/s}^2) = 250\text{ N}$$
$$\Rightarrow T = 250\text{ N} + w = 250\text{ N} + 490\text{ N} = 740\text{ N}$$

(d) The situation is the same as in part (c), except that the rising box is slowing down. Thus $a_y = -5.0\text{ m/s}^2$ and we have instead

$$F_{\text{net}} = T - F_G = ma_y = (50\text{ kg})(-5.0\text{ m/s}^2) = -250\text{ N}$$
$$\Rightarrow T = -250\text{ N} + F_G = -250\text{ N} + 490\text{ N} = 240\text{ N}$$

Assess: For parts (a) and (b) the zero accelerations immediately imply that the gravitational force on the box must be exactly balanced by the upward tension in the rope. For part (c) the tension not only has to support the gravitational force on the box but must also accelerate it upward, hence, T must be greater than F_G. When the box accelerates downward, the rope need not support the entire gravitational force, hence, T is less than F_G.

6.12. Model: We assume that the block is a point particle that is acted on only the force shown.
Visualize: We apply Newton's second law in both parts.
Solve: (a) Since the net force is to the right the block is accelerating to the right, so it is speeding up in this case. The answer is A. **(b)** The net force, while decreasing, is still to the right, so the block continues to accelerate to the right and in this case continues to speed up. The answer is A.

Section 6.3 Mass, Weight, and Gravity

6.13. Model: Use the particle model for the woman.
Solve: (a) The woman's weight on the earth is

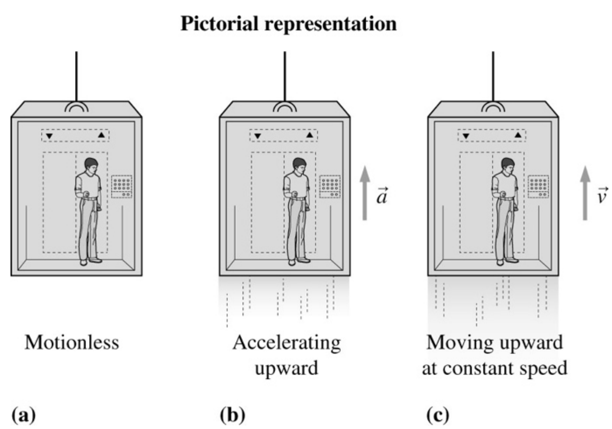
$$w_{\text{earth}} = mg_{\text{earth}} = (55\text{ kg})(9.80\text{ m/s}^2) = 540\text{ N}$$

(b) Since mass is a measure of the amount of matter, the woman's mass is the same on Mars as on the earth. Her weight on Mars is

$$w_{\text{Mars}} = mg_{\text{Mars}} = (55\text{ kg})(3.76\text{ m/s}^2) = 210\text{ N}$$

Assess: The smaller acceleration due to gravity on Mars reveals that objects are less strongly attracted to Mars than to the earth. Thus the woman's smaller weight on Mars makes sense.

6.14. Model: We assume that the passenger is a particle subject to two vertical forces: the downward pull of gravity and the upward push of the elevator floor. We can use one-dimensional kinematics and Equation 6.10.
Visualize:



Solve: (a) The weight is

$$w = mg\left(1 + \frac{a_y}{g}\right) = mg\left(1 + \frac{0}{g}\right) = mg = (60\text{ kg})(9.80\text{ m/s}^2) = 590\text{ N}$$

(b) The elevator speeds up from $v_{0y} = 0\text{ m/s}$ to its cruising speed at $v_y = 10\text{ m/s}$. We need its acceleration before we can find the apparent weight:

$$a_y = \frac{\Delta v}{\Delta t} = \frac{10\text{ m/s} - 0\text{ m/s}}{4.0\text{ s}} = 2.5\text{ m/s}^2$$

The passenger’s weight is

$$w = mg\left(1+\frac{a_y}{g}\right) = (590\text{ N})\left(1+\frac{2.5\text{ m/s}^2}{9.80\text{ m/s}^2}\right) = (590\text{ N})(1.26) = 740\text{ N}$$

(c) The passenger is no longer accelerating since the elevator has reached its cruising speed. Thus, $w = mg = 590$ N as in part (a).

Assess: The passenger’s weight is the gravitational force on the passenger in parts (a) and (c), since there is no acceleration. In part (b), the elevator must not only support the gravitational force but must also accelerate him upward, so it’s reasonable that the floor will have to push up harder on him, increasing his weight.

6.15. Model: We assume that the passenger is a particle acted on by only two vertical forces: the downward pull of gravity and the upward force of the elevator floor.

Visualize: The graph has three segments corresponding to different conditions: (1) increasing velocity, meaning an upward acceleration; (2) a period of constant upward velocity; and (3) decreasing velocity, indicating a period of deceleration (negative acceleration).

Solve: Given the assumptions of our model, we can calculate the acceleration for each segment of the graph and then apply Equation 6.10. The acceleration for the first segment is

$$a_y = \frac{v_1 - v_0}{t_1 - t_0} = \frac{8\text{ m/s} - 0\text{ m/s}}{2\text{ s} - 0\text{ s}} = 4\text{ m/s}^2$$

$$\Rightarrow w = mg\left(1+\frac{a_y}{g}\right) = mg\left(1+\frac{4\text{ m/s}^2}{9.80\text{ m/s}^2}\right) = (75\text{ kg})(9.80\text{ m/s}^2)\left(1+\frac{4}{9.80}\right) = 1035\text{ N}$$

For the second segment, $a_y = 0\text{ m/s}^2$ and the weight is

$$w = mg\left(1+\frac{0\text{ m/s}^2}{g}\right) = mg = (75\text{ kg})(9.80\text{ m/s}^2) = 740\text{ N}$$

For the third segment,

$$a_y = \frac{v_3 - v_2}{t_3 - t_2} = \frac{0\text{ m/s} - 8\text{ m/s}}{10\text{ s} - 6\text{ s}} = -2\text{ m/s}^2$$

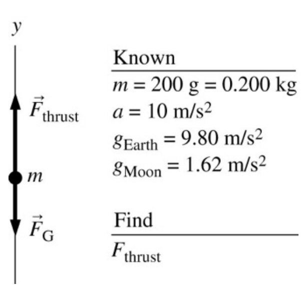
$$\Rightarrow w = mg\left(1+\frac{-2\text{ m/s}^2}{9.80\text{ m/s}^2}\right) = (75\text{ kg})(9.80\text{ m/s}^2)(1-0.2) = 590\text{ N}$$

Assess: As expected, the weight is greater than the gravitational force on the passenger when the elevator is accelerating upward and lower than normal when the acceleration is downward. When there is no acceleration the weight is the gravitational force. In all three cases the magnitudes are reasonable, given the mass of the passenger and the accelerations of the elevator.

6.16. Model: We assume the rocket is a particle moving in a vertical straight line under the influence of only two forces: gravity and its own thrust.

Visualize:

Pictorial representation



Solve: (a) Using Newton’s second law and reading the forces from the free-body diagram,

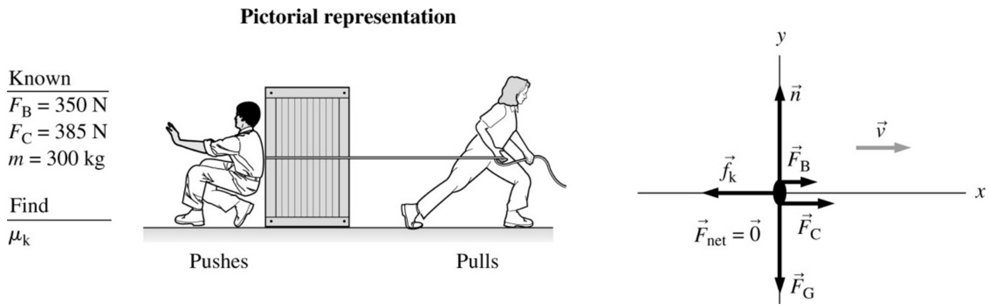
$$F_{\text{thrust}} - F_{\text{G}} = ma \Rightarrow F_{\text{thrust}} = ma + mg_{\text{Earth}} = (0.200\text{ kg})(10\text{ m/s}^2 + 9.80\text{ m/s}^2) = 3.96\text{ N}$$

(b) Likewise, the thrust on the moon is $(0.200\text{ kg})(10\text{ m/s}^2 + 1.62\text{ m/s}^2) = 2.32\text{ N}.$

Assess: The thrust required is smaller on the moon, as it should be, given the moon's weaker gravitational pull. The magnitude of a few newtons seems reasonable for a small model rocket.

Section 6.4 Friction

6.17. Model: We assume that the safe is a particle moving only in the *x*-direction. Since it is sliding during the entire problem, we can use the model of kinetic friction.
Visualize:



Solve: The safe is in equilibrium, since it's not accelerating. Thus we can apply Newton's first law in the vertical and horizontal directions:

$$(F_{net})_x = \sum F_x = F_B + F_C - f_k = 0\text{ N} \Rightarrow f_k = F_B + F_C = 350\text{ N} + 385\text{ N} = 735\text{ N}$$

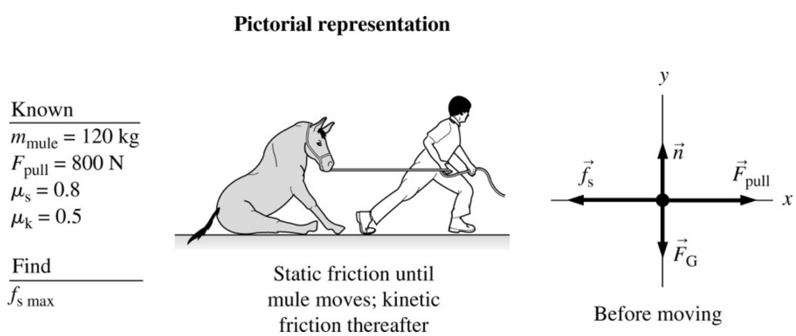
$$(F_{net})_y = \sum F_y = n - F_G = 0\text{ N} \Rightarrow n = F_G = mg = (300\text{ kg})(9.80\text{ m/s}^2) = 2.94\times 10^3\text{ N}$$

Then, for kinetic friction:

$$f_k = \mu_k n \Rightarrow \mu_k = \frac{f_k}{n} = \frac{735\text{ N}}{2.94\times 10^3\text{ N}} = 0.250$$

Assess: The value of $\mu_k = 0.250$ is hard to evaluate without knowing the material the floor is made of, but it seems reasonable.

6.18. Model: We assume that the mule is a particle acted on by two opposing forces in a single line: the farmer's pull and friction. The mule will be subject to static friction until (and if!) it begins to move; after that it will be subject to kinetic friction.
Visualize:

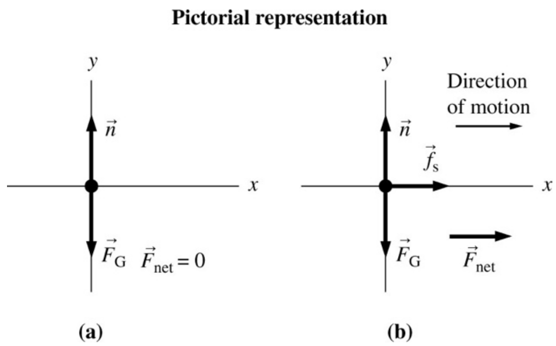


Solve: Since the mule does not accelerate in the vertical direction, the free-body diagram shows that $n = F_G = mg$. The maximum friction force is

$$f_{s\,max} = \mu_s mg = (0.8)(120\ \text{kg})(9.80\ \text{m/s}^2) = 940\ \text{N}$$

The maximum static friction force is greater than the farmer's maximum pull of 800 N; thus, the farmer will not be able to budge the mule.
Assess: Maybe the farmer could put something smoother under the mule.

6.19. Model: We will represent the crate as a particle.
Visualize:



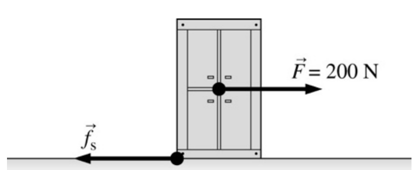
Solve: (a) When the belt runs at constant speed, the crate has an acceleration $a = 0\ \text{m/s}^2$ and is in dynamic equilibrium. Thus $F_{net} = 0$. It is tempting to think that the belt exerts a friction force on the crate. But if it did, there would be a *net* force because there are no other possible horizontal forces to balance a friction force. Because there is no net force, there cannot be a friction force. The only forces are the upward normal force and the gravitational force on the crate. (A friction force would have been needed to get the crate moving initially, but no horizontal force is needed to keep it moving once it is moving with the same constant speed as the belt.)
(b) If the belt accelerates gently, the crate speeds up without slipping on the belt. Because it is accelerating, the crate must have a net horizontal force. So *now* there is a friction force, and the force points in the direction of the crate's motion. Is it static friction or kinetic friction? Although the crate is moving, there is *no* motion of the crate relative to the belt. Thus, it is a *static* friction force that accelerates the crate so that it moves without slipping on the belt.
(c) The static friction force has a maximum possible value $(f_s)_{max} = \mu_s n$. The maximum possible acceleration of the crate is

$$a_{max} = \frac{(f_s)_{max}}{m} = \frac{\mu_s n}{m}$$

If the belt accelerates more rapidly than this, the crate will not be able to keep up and will slip. It is clear from the free-body diagram that $n = F_G = mg$. Thus,

$$a_{max} = \mu_s g = (0.5)(9.80\ \text{m/s}^2) = 4.9\ \text{m/s}^2$$

6.20. Model: Model the cabinet as a particle.
Visualize: In equilibrium the net force is zero.

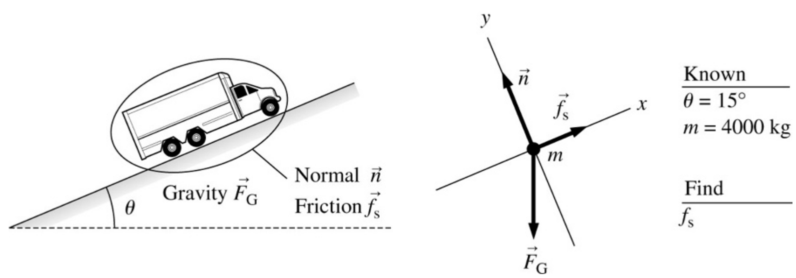


Solve: The cabinet is in static equilibrium, so the static frictional force must have the same magnitude as Bob's pulling force: 200 N.

Assess: A possible misconception is that $f_s = \mu n$ always. That value is the maximum possible value. If Bob pulled harder and harder and got up to $\mu n = 235\text{ N}$ then the cabinet would move. But the static frictional force can easily be less than this value.

6.21. Model: We assume that the truck is a particle in equilibrium, and use the model of static friction.

Visualize:



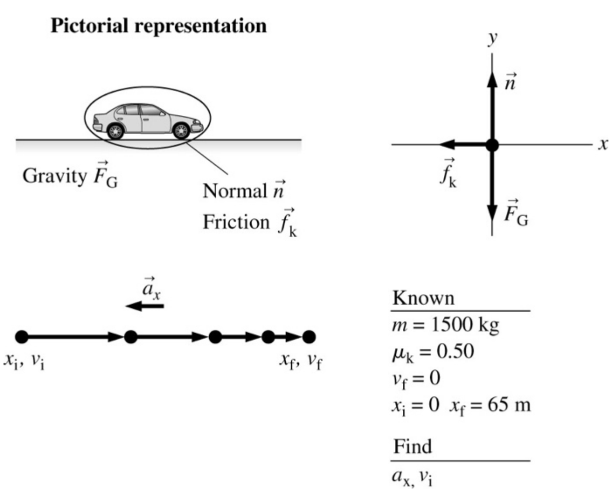
Solve: The truck is not accelerating, so it is in equilibrium, and we can apply Newton's first law. The normal force has no component in the x-direction, so we can ignore it here. For the other two forces:

$$(F_{net})_x = \sum F_x = f_s - (F_G)_x = 0\text{ N} \Rightarrow f_s = (F_G)_x = mg\sin\theta = (4000\text{ kg})(9.8\text{ m/s}^2)(\sin 15°) = 10{,}145\text{ N} \approx 10{,}000\text{ N}$$

Assess: The truck's weight (mg) is roughly 40,000 N. A friction force that is ≈ 25, of the truck's weight seems reasonable.

6.22. Model: The car is a particle subject to Newton's laws and kinematics.

Visualize:



Solve: Kinetic friction provides a horizontal acceleration which stops the car. From the figure, applying Newton's first and second laws gives

$$\sum F_x = -f_k = ma_x$$
$$\sum F_y = n - F_G = 0 \Rightarrow n = F_G = mg$$

Combining these two equations with $f_k = \mu_k n$ yields

$$a_x = -\mu_k g = -(0.50)(9.80\ \text{m/s}^2) = -4.9\ \text{m/s}$$

Kinematics can be used to determine the initial velocity.

$$v_f^2 = v_i^2 + 2a\Delta x \Rightarrow v_i^2 = -2a_x\Delta x$$

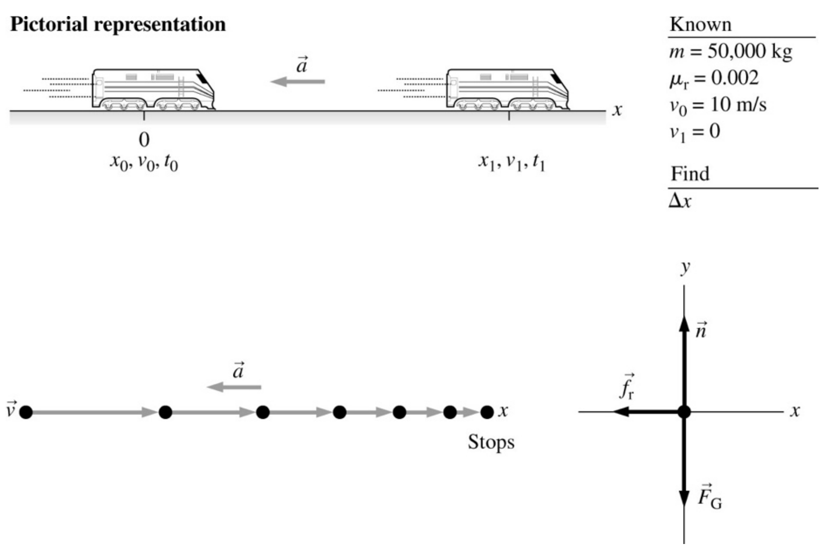
Thus

$$v_i = \sqrt{-2(-4.9\ \text{m/s}^2)(65\ \text{m} - 0\ \text{m})} = 25\ \text{m/s}$$

Assess: The initial speed of $25\ \text{m/s} \approx 56\ \text{mph}$ is a reasonable speed to have initially for a vehicle to leave 65-meter-long skid marks.

6.23. Model: We treat the train as a particle subject to rolling friction but not to drag (because of its slow speed and large mass). We can use the one-dimensional kinematic equations. Look up the coefficient of rolling friction in the table.

Visualize:



Solve: The locomotive is not accelerating in the vertical direction, so the free-body diagram shows us that $n = F_G = mg$. Thus,

$$f_r = \mu_r mg = (0.002)(50{,}000\ \text{kg})(9.80\ \text{m/s}^2) = 980\ \text{N}$$

From Newton's second law for the decelerating locomotive,

$$a_x = \frac{-f_r}{m} = \frac{-980\ \text{N}}{50{,}000\ \text{kg}} = -0.01960\ \text{m/s}^2$$

Since we're looking for the distance the train rolls, but we don't have the time:

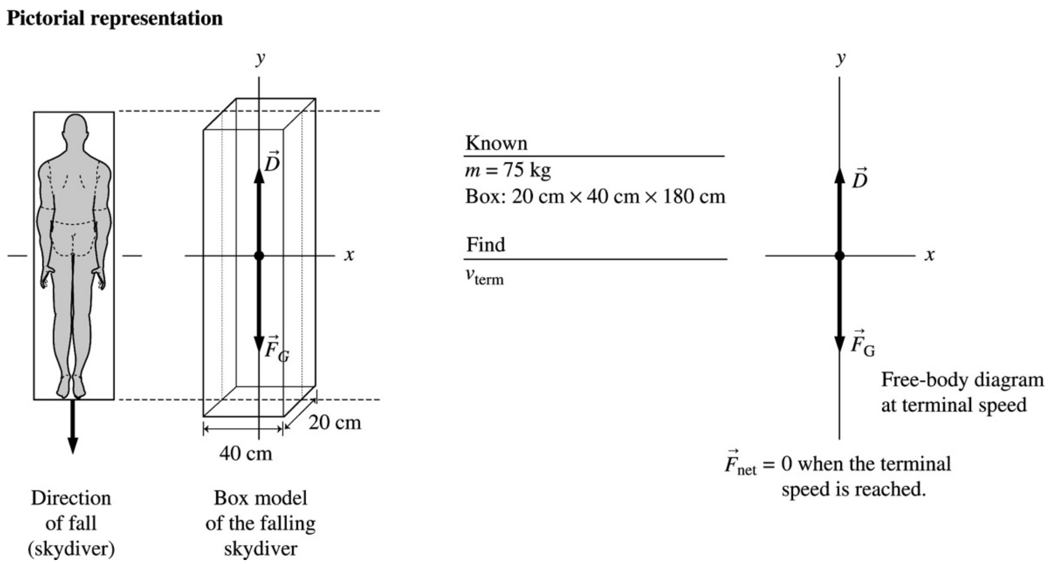
$$v_1^2 - v_0^2 = 2a_x(\Delta x) \Rightarrow \Delta x = \frac{v_1^2 - v_0^2}{2a_x} = \frac{(0\ \text{m/s})^2 - (10\ \text{m/s})^2}{2(-0.01960\ \text{m/s}^2)} = 2.55\times 10^3\ \text{m} \approx 2.6\times 10^3\ \text{m}$$

Assess: The locomotive's enormous inertia (mass) and the small coefficient of rolling friction make this long stopping distance seem reasonable.

Section 6.5 Drag

6.24. Model: We assume that the skydiver is shaped like a box and is a particle. But we will also model the diver as a cylinder falling end down to use $C = 0.8$.

Visualize:



The skydiver falls straight down toward the earth's surface, that is, the direction of fall is vertical. Since the skydiver falls feet first, the surface perpendicular to the drag has the cross-sectional area $A = 20\text{ cm} \times 40\text{ cm}$. The physical conditions needed to use Equation 6.16 for the drag force are satisfied. The terminal speed corresponds to the situation when the net force acting on the skydiver becomes zero.

Solve: The expression for the magnitude of the drag with v in m/s is

$$D \approx \tfrac{1}{2}C\rho Av^2 = 0.5(0.8)(1.2\text{ kg/m}^3)(0.20 \times 0.40)v^2\text{ N} = 0.038v^2\text{ N}$$

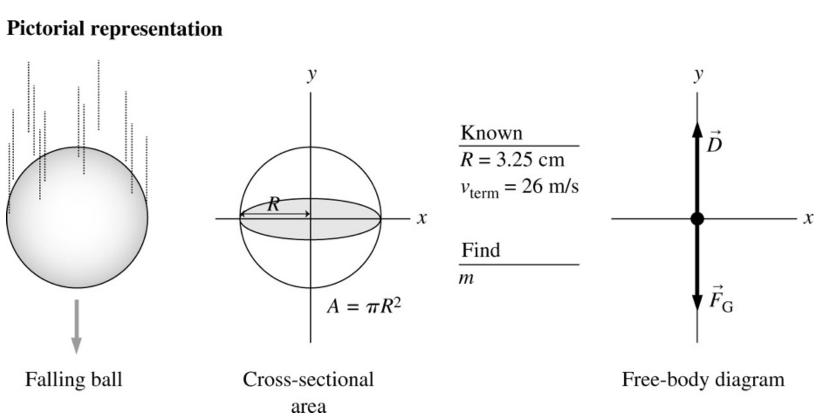
The gravitational force on the skydiver is $F_G = mg = (75\text{ kg})(9.8\text{ m/s}^2) = 735\text{ N}$. The mathematical form of the condition defining dynamical equilibrium for the skydiver and the terminal speed is

$$F_{net} = F_G + D = 0\text{ N}$$

$$\Rightarrow 0.038v_{term}^2\text{ N} - 735\text{ N} = 0\text{ N} \Rightarrow v_{term} = \sqrt{\frac{735}{0.038}} \approx 140\text{ m/s}$$

Assess: The result of the above simplified physical modeling approach and subsequent calculation, even if approximate, shows that the terminal velocity is very high. This result implies that the skydiver will be very badly hurt at landing if the parachute does not open in time.

6.25. Model: We will represent the tennis ball as a particle. The drag coefficient is 0.5.
Visualize:



The tennis ball falls straight down toward the earth's surface. The ball is subject to a net force that is the resultant of the gravitational and drag force vectors acting vertically, in the downward and upward directions, respectively. Once the net force acting on the ball becomes zero, the terminal velocity is reached and remains constant for the rest of the motion.

Solve: The mathematical equation defining the dynamical equilibrium situation for the falling ball is

$$F_{net} = F_G + D = 0\text{ N}$$

Since only the vertical direction matters, one can write:

$$\sum F_y = 0\text{ N} \Rightarrow \quad F_{net} = D - F_G = 0\text{ N}$$

When this condition is satisfied, the speed of the ball becomes the constant terminal speed $v = v_{term}$. The magnitudes of the gravitational and drag forces acting on the ball are:

$$F_G = mg = m(9.80\text{ m/s}^2)$$

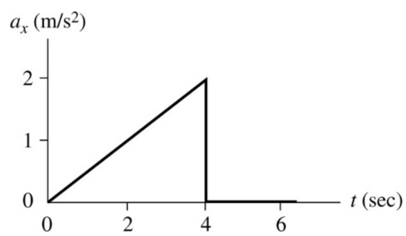
$$D \approx \frac{1}{2}(C\rho A v_{term}^2) = 0.5(0.5)(1.2\text{ kg/m}^3)(\pi R^2)v_{term}^2 = (0.3\pi)(0.0325\text{ m})^2(26\text{ m/s})^2 = 0.67\text{ N}$$

The condition for dynamic equilibrium becomes:

$$(9.80\text{ m/s}^2)m - 0.67\text{ N} = 0\text{ N} \Rightarrow m = \frac{0.67\text{ N}}{9.80\text{ m/s}^2} = 69\text{ g}$$

Assess: The value of the mass of the tennis ball obtained above seems reasonable.

6.26. Visualize:



We used the force-versus-time graph to draw the acceleration-versus-time graph. The peak acceleration was calculated as follows:

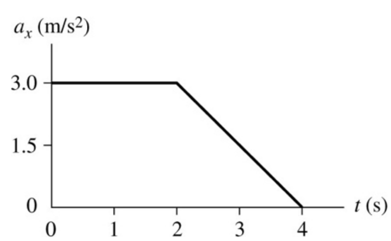
$$a_{max} = \frac{F_{max}}{m} = \frac{10\text{ N}}{5\text{ kg}} = 2\text{ m/s}^2$$

Solve: The acceleration is not constant, so we cannot use constant acceleration kinematics. Instead, we use the more general result that

$$v(t) = v_0 + \text{area under the acceleration curve from 0 s to } t$$

The object starts from rest, so $v_0 = 0\text{ m/s}$. The area under the acceleration curve between 0s and 6s is $\frac{1}{2}(4\text{ s})(2\text{ m/s}^2) = 4.0\text{ m/s}$. We've used the fact that the area between 4s and 6s is zero. Thus, at $t = 6\text{ s}$, $v_x = 4.0\text{ m/s}$.

6.27. Visualize:

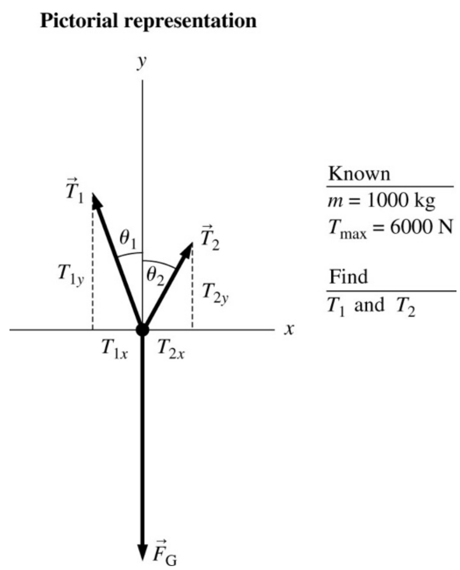


The acceleration is $a_x = F_x/m$, so the acceleration-versus-time graph has exactly the same shape as the force-versus-time graph. The maximum acceleration is $a_{max} = F_{max}/m = (6\text{ N})/(2\text{ kg}) = 3\text{ m/s}^2$.
Solve: The acceleration is not constant, so we cannot use constant-acceleration kinematics. Instead, we use the more general result that

$$v(t) = v_0 + \text{area under the acceleration curve from 0 s to } t$$

The object starts from rest, so $v_0 = 0\text{ m/s}$. The area under the acceleration curve between 0 s and 4 s is a rectangle $(3\text{ m/s}^2 \times 2\text{ s} = 6\text{ m/s})$ plus a triangle $\left(\frac{1}{2} \times 3\text{ m/s}^2 \times 2\text{ s} = 3\text{ m/s}\right)$. Thus $v_x = 9\text{ m/s}$ at $t = 4\text{ s}$.

6.28. Model: You can model the beam as a particle in static equilibrium.
Visualize:



Solve: Using Newton's first law, the equilibrium equations in vector and component form are:

$$\vec{F}_{net} = \vec{T}_1 + \vec{T}_2 + \vec{F}_G = \vec{0}\text{ N}$$
$$(F_{net})_x = T_{1x} + T_{2x} + F_{Gx} = 0\text{ N}$$
$$(F_{net})_y = T_{1y} + T_{2y} + F_{Gy} = 0\text{ N}$$

Using the free-body diagram yields:

$$-T_1\sin\theta_1 + T_2\sin\theta_2 = 0\text{ N} \qquad T_1\cos\theta_1 + T_2\cos\theta_2 - F_G = 0\text{ N}$$

The mathematical model is reduced to a simple algebraic system of two equations with two unknowns, T_1 and T_2. Substituting $\theta_1 = 20°$, $\theta_2 = 30°$, and $F_G = mg = 9800\text{ N}$, the simultaneous equations become

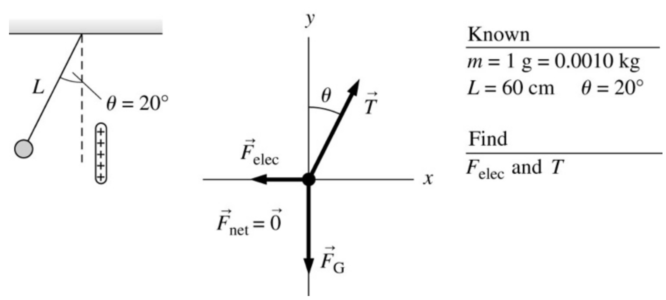
$$-T_1\sin 20° + T_2\sin 30° = 0\text{ N} \qquad T_1\cos 20° + T_2\cos 30° = 9800\text{ N}$$

You can solve this system of equations by simple substitution. The result is $T_1 = 6397\text{ N} \approx 6400\text{ N}$ and $T_2 = 4376\text{ N} \approx 4380\text{ N}$.
Assess: The above approach and result seem reasonable. Intuition indicates there is more tension in the left rope than in the right rope.

6.29. Model: The plastic ball is represented as a particle in static equilibrium.

Visualize:

Pictorial representation



Solve: **(a)** The electric force, like the weight, is a long-range force. So the ball experiences the contact force of the string's tension plus *two* long-range forces. The equilibrium condition is

$$(F_{net})_x = T_x + (F_{elec})_x = T\sin\theta - F_{elec} = 0\text{ N}$$
$$(F_{net})_y = T_y + (F_G)_y = T\cos\theta - mg = 0\text{ N}$$

We can solve the y-equation to get

$$T = \frac{mg}{\cos\theta} = \frac{(0.001\text{ kg})(9.8\text{ m/s}^2)}{\cos 20^\circ} = 0.0104\text{ N}$$

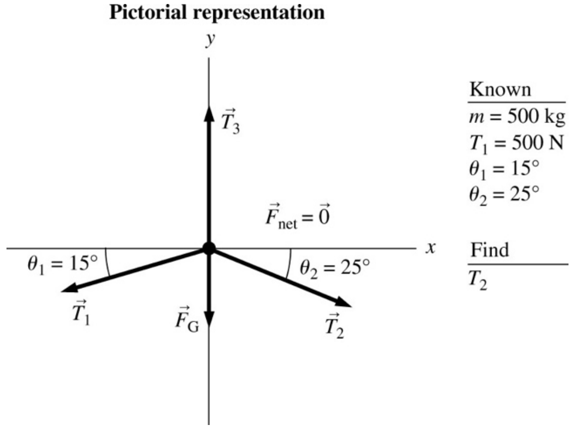
Substituting this value into the x-equation,

$$F_{elec} = T\sin\theta = (1.04\times10^{-2}\text{ N})\sin 20^\circ = 0.0036\text{ N}$$

(b) The tension in the string is $0.0104\text{ N} \approx 0.010\text{ N}.$

6.30. Model: The piano is in static equilibrium and is to be treated as a particle.
Visualize:

Pictorial representation



Solve: **(a)** Based on the free-body diagram, Newton's second law is

$$(F_{net})_x = 0\text{ N} = T_{1x} + T_{2x} = T_2\cos\theta_2 - T_1\cos\theta_1$$
$$(F_{net})_y = 0\text{ N} = T_{1y} + T_{2y} + T_{3y} + F_{Gy} = T_3 - T_1\sin\theta_1 - T_2\sin\theta_2 - mg$$

Notice how the force components all appear in the second law with *plus* signs because we are *adding* forces. The negative signs appear only when we *evaluate* the various components. These are two simultaneous equations in the two unknowns T_2 and T_3. From the x-equation we find

$$T_2 = \frac{T_1\cos\theta_1}{\cos\theta_2} = \frac{(500\text{ N})\cos 15^\circ}{\cos 25^\circ} = 533\text{ N}$$

(b) Now we can use the *y*-equation to find

$$T_3 = T_1\sin\theta_1 + T_2\sin\theta_2 + mg = 5.25\times10^3\ \text{N}$$

6.31. Model: We will represent Henry as a particle. His motion is governed by constant-acceleration kinematic equations.

Solve: (a) Henry undergoes an acceleration from 0 s to 2.0 s, constant velocity motion from 2.0 s to 10.0 s, and another acceleration as the elevator brakes from 10.0 s to 12.0 s. The weight is the same as the gravitational force during constant velocity motion, so Henry's weight $w = F_G = mg$ is 750 N. His weight is *less* than the gravitational force on him during the initial acceleration, so the acceleration is in a *downward* direction (negative *a*). Thus, the elevator's initial motion is *down*.

(b) Because the gravitational force on Henry is 750 N, his mass is $m = F_G/g = 76.5\ \text{kg} \approx 77\ \text{kg}.$

(c) The apparent weight during vertical motion is given by

$$w = mg\left(1+\frac{a}{g}\right) \Rightarrow a = g\left(\frac{w}{F_G}-1\right)$$

During the interval $0\ \text{s} \le t \le 2\ \text{s},$ the elevator's acceleration is

$$a = g\left(\frac{600\ \text{N}}{750\ \text{N}}-1\right) = -1.96\ \text{m/s}^2$$

At $t = 2\ \text{s},$ Henry's position is

$$y_1 = y_0 + v_0\Delta t_0 + \frac{1}{2}a(\Delta t_0)^2 = \frac{1}{2}a(\Delta t_0)^2 = -3.92\ \text{m}$$

and his velocity is

$$v_1 = v_0 + a\Delta t_0 = a\Delta t_0 = -3.92\ \text{m/s}$$

During the interval $2\ \text{s} \le t \le 10\ \text{s}, a = 0\ \text{m/s}^2.$ This means Henry travels with a constant velocity $v_1 = -3.92\ \text{m/s}.$ At $t = 10\ \text{s}$ he is at position

$$y_2 = y_1 + v_1\Delta t_1 = -35.3\ \text{m}$$

and he has a velocity $v_2 = v_1 = -3.92\ \text{m/s}.$ During the interval $10\ \text{s} \le t \le 12.0\ \text{s},$ the elevator's acceleration is

$$a = g\left(\frac{900\ \text{N}}{750\ \text{N}}-1\right) = +1.96\ \text{m/s}^2$$

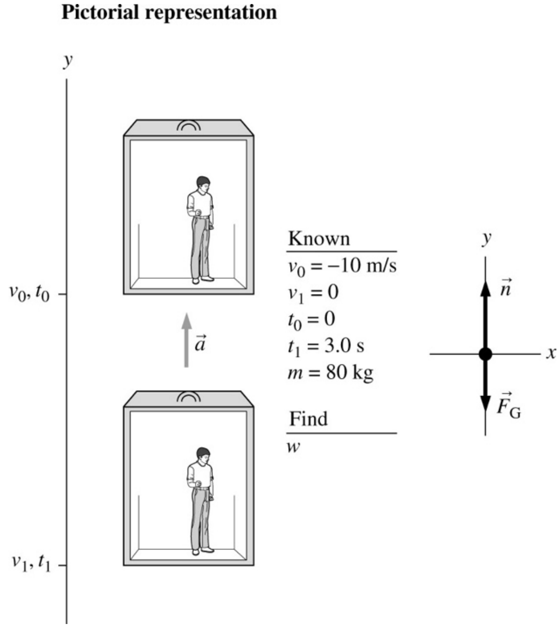
The upward acceleration vector slows the elevator and Henry feels heavier than normal. At $t = 12.0\ \text{s}$ Henry is at position

$$y_3 = y_2 + v_2(\Delta t_2) + \frac{1}{2}a(\Delta t_2)^2 = -39.2\ \text{m}$$

Thus Henry has traveled distance $39.2\ \text{m} \approx 39\ \text{m}.$

6.32. Model: We'll assume Zach is a particle moving under the effect of two forces acting in a single vertical line: gravity and the supporting force of the elevator.

Visualize:



Solve: (a) Before the elevator starts braking, Zach is not accelerating. His weight is

$$w = mg\left(1+\frac{a}{g}\right) = mg\left(1+\frac{0\text{ m/s}^2}{g}\right) = mg = (80\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$$

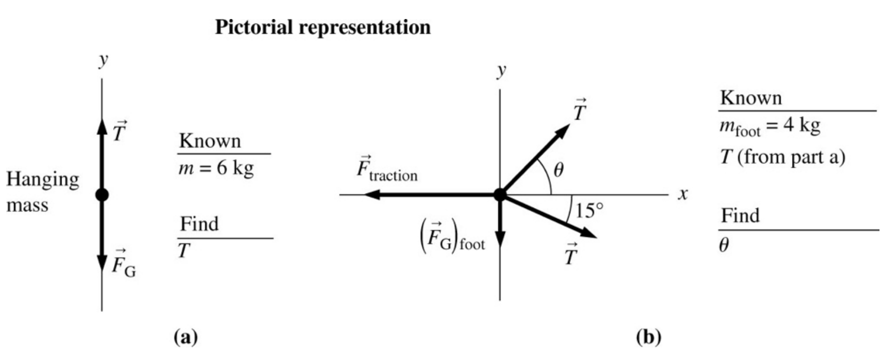
Zach's weight is 7.8×10^2 N.
(b) Using the definition of acceleration,

$$a = \frac{\Delta v}{\Delta t} = \frac{v_1 - v_0}{t_1 - t_0} = \frac{0-(-10)\text{ m/s}}{3.0\text{ s}} = 3.33\text{ m/s}^2$$

$$\Rightarrow w = mg\left(1+\frac{a}{g}\right) = (80\text{ kg})(9.80\text{ m/s}^2)\left(1+\frac{3.33\text{ m/s}^2}{9.80\text{ m/s}^2}\right) = (784\text{ N})(1+0.340) = 1050\text{ N}$$

Now Zach's weight is $1.05\times 10^3\text{ N} \approx 1.1\text{ kN}$.
Assess: While the elevator is braking, it not only must support the gravitational force on Zach but must also push upward on him to decelerate him, so his weight is greater than the gravitational force.

6.33. Model: We can assume the foot is a single particle in equilibrium under the combined effects of gravity, the tensions in the upper and lower sections of the traction rope, and the opposing traction force of the leg itself. We can also treat the hanging mass as a particle in equilibrium. Since the pulleys are frictionless, the tension is the same everywhere in the rope. Because all pulleys are in equilibrium, their net force is zero. So they do not contribute to T.
Visualize:



Solve: **(a)** From the free-body diagram for the mass, the tension in the rope is

$$T = F_G = mg = (6\text{ kg})(9.80\text{ m/s}^2) = 58.8\text{ N} \approx 59\text{ N}$$

(b) Using Newton's first law for the vertical direction on the pulley attached to the foot,

$$(F_{net})_y = \sum F_y = T\sin\theta - T\sin 15^\circ - (F_G)_{foot} = 0\text{ N}$$

$$\Rightarrow \sin\theta = \frac{T\sin 15^\circ + (F_G)_{foot}}{T} = \sin 15^\circ + \frac{m_{foot}g}{T} = 0.259 + \frac{(4\text{ kg})(9.80\text{ m/s}^2)}{58.8\text{ N}} = 0.259 + 0.667 = 0.926$$

$$\Rightarrow \theta = \sin^{-1} 0.926 = 67.8^\circ \approx 68^\circ$$

(c) Using Newton's first law for the horizontal direction,

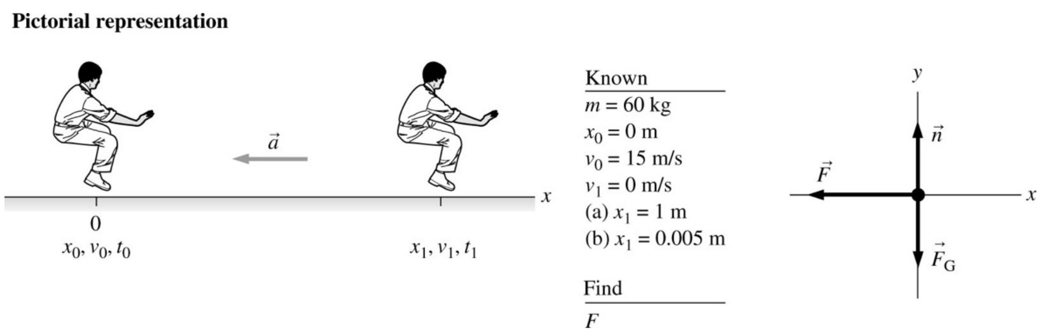
$$(F_{net})_x = \sum F_x = T\cos\theta + T\cos 15^\circ - F_{traction} = 0\text{ N}$$

$$\Rightarrow F_{traction} = T\cos\theta + T\cos 15^\circ = T(\cos 67.8^\circ + \cos 15^\circ)$$

$$= (58.8\text{ N})(0.3778 + 0.9659) = (58.8\text{ N})(1.344) = 79\text{ N}$$

Assess: Since the tension in the upper segment of the rope must support the foot and counteract the downward pull of the lower segment of the rope, it makes sense that its angle is larger (a more direct upward pull). The magnitude of the traction force, roughly one-tenth of the gravitational force on a human body, seems reasonable.

6.34. Model: We can assume the person is a particle moving in a straight line under the influence of the combined decelerating forces of the air bag and seat belt or, in the absence of restraints, the dashboard or windshield.
Visualize:



Solve: **(a)** In order to use Newton's second law for the passenger, we'll need the acceleration. Since we don't have the stopping time:

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) \Rightarrow a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)} = \frac{0\text{ m}^2/\text{s}^2 - (15\text{ m/s})^2}{2(1\text{ m} - 0\text{ m})} = -112.5\text{ m/s}^2$$

$$\Rightarrow F_{net} = F = ma = (60\text{ kg})(-112.5\text{ m/s}^2) = -6750\text{ N}$$

The net force is 6750 N to the left.
(b) Using the same approach as in part (a),

$$F = ma = m\frac{v_1^2 - v_0^2}{2(x_1 - x_0)} = (60\text{ kg})\frac{0\text{ m}^2/\text{s}^2 - (15\text{ m/s})^2}{2(0.005\text{ m})} = -1{,}350{,}000\text{ N}$$

The net force is 1,350,000 N to the left.
(c) The passenger's weight is $mg = (60\text{ kg})(9.8\text{ m/s}^2) = 588\text{ N}$. The force in part (a) is 11.5 times the passenger's weight. The force in part (b) is 2300 times the passenger's weight.
Assess: An acceleration of 11.5g is well within the capability of the human body to withstand. A force of 2300 times the passenger's weight, on the other hand, would surely be catastrophic.

6.35. Visualize: All the motion is in the horizontal (i.e., x)direction. Acceleration is the second derivative of position.

Solve: The first derivative is $v = \frac{dx}{dt} = (6t^2 - 6t)\text{ m/s}.$ The second derivative is $a = \frac{dv}{dt} = (12t - 6)\text{ m/s}^2.$ Apply Newton's second law: $F = ma = (2.0\text{ kg})((12t - 6)\text{ m/s}^2).$ Plug in the two values for *t*.

(a)

$$F|_{0\text{s}} = (2.0\text{ kg})((12(0\text{ s}) - 6)\text{ m/s}^2) = -12\text{ N}$$

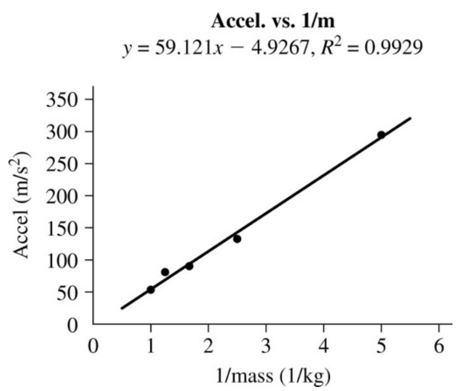
(b)

$$F|_{1\text{s}} = (2.0\text{ kg})((12(1\text{ s}) - 6)\text{ m/s}^2) = 12\text{ N}$$

Assess: The net force changed direction between $t = 0\text{s}$ and $t = 1\text{s}.$

6.36. Visualize: We'll use $v_f^2 = v_i^2 + 2a\Delta s$ to find the acceleration of the balls, which will be inversely proportional to the mass of the balls. $\Delta s = 15\text{ cm}$ and $v_i = 0$ in each case.

Solve: Newton's second law relates mass, acceleration, and net force: $a = F\frac{1}{m}.$ If we graph a vs. $\frac{1}{m}$ then the slope of the straight line should be the size of the piston's force.

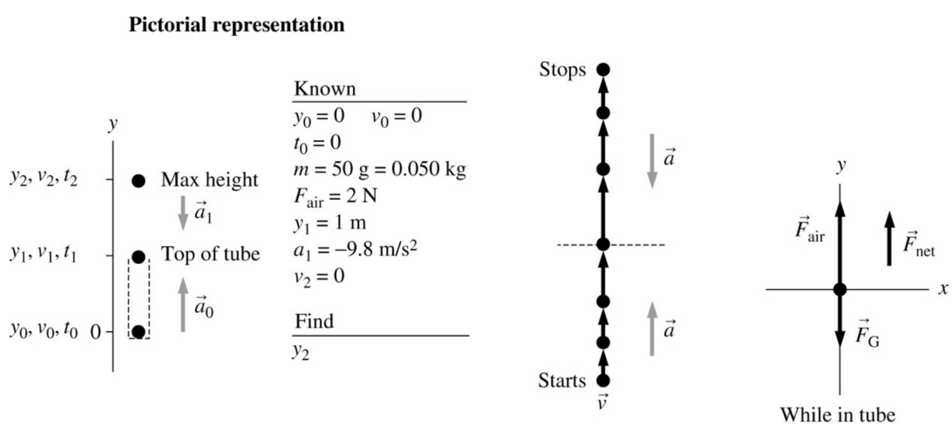


We see that the linear fit is very good. The slope is $59.12\text{ N} \approx 59\text{ N};$ this is the size of the piston's force.

Assess: We are glad to see that the intercept of our line looks very small, even though we don't have a ball the inverse of whose mass is zero.

6.37. Model: The ball is represented as a particle that obeys constant-acceleration kinematic equations.

Visualize:



Solve: This is a two-part problem. During part 1 the ball accelerates upward in the tube. During part 2 the ball undergoes free fall $(a = -g).$ The initial velocity for part 2 is the final velocity of part 1, as the ball emerges from the tube. The free-body diagram for part 1 shows two forces: the air pressure force and the gravitational force. We need only the y-component of Newton's second law:

$$a_y = a = \frac{(F_{\text{net}})_y}{m} = \frac{F_{\text{air}} - F_G}{m} = \frac{F_{\text{air}}}{m} - g = \frac{2\text{ N}}{0.05\text{ kg}} - 9.80\text{ m/s}^2 = 30.2\text{ m/s}^2$$

We can use kinematics to find the velocity v_1 as the ball leaves the tube:

$$v_1^2 = v_0^2 + 2a(y_1 - y_0) \quad \Rightarrow \quad v_1 = \sqrt{2ay_1} = \sqrt{2(30.2\text{ m/s}^2)(1\text{ m})} = 7.77\text{ m/s}$$

For part 2, free-fall kinematics $v_2^2 = v_1^2 - 2g(y_2 - y_1)$ gives

$$y_2 - y_1 = \frac{v_1^2}{2g} = 3.1\text{ m}$$

6.38. Model: Model the rocket as a particle. Assume the mass of the rocket is constant so the acceleration is constant. Assume the rocket starts from rest. Neglect air resistance.

Visualize: We'll use $v_f^2 = v_i^2 + 2a\Delta y$ to find the speed of the rocket. The net force is $F_{\text{thrust}} - mg$.

Solve:

(a) $\Delta y = h, \quad v_i = 0, \quad a = F_{\text{net}}/m$

$$v_f^2 = v_i^2 + 2a\Delta y = 2\left(\frac{F_{\text{thurst}} - mg}{m}\right)h$$

For v as a function of h we have:

$$v(h) = v_f = \sqrt{2\left(\frac{F_{\text{thurst}}}{m} - g\right)h}$$

(b) For $h = 85\text{ m}$

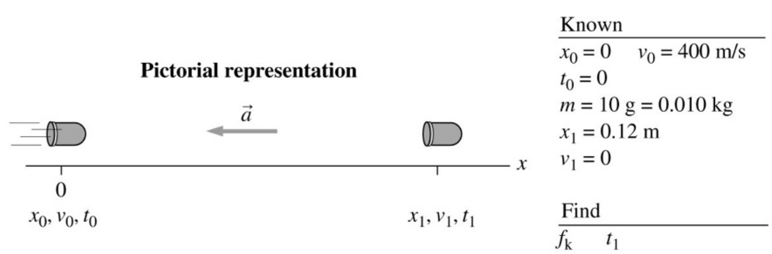
$$v = \sqrt{2\left(\frac{9.5\text{ N}}{0.35\text{ kg}} - 9.8\text{ m/s}^2\right)(85\text{ m})} = 54\text{ m/s}$$

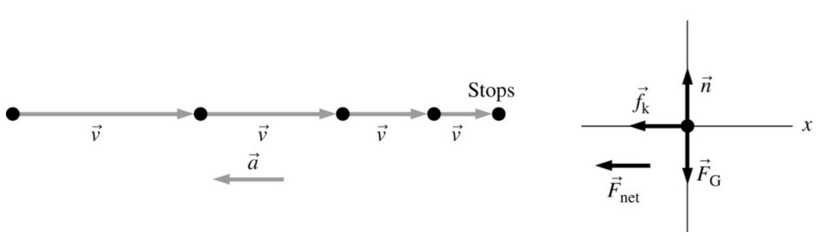
Assess: This 54 m/s speed seems reasonable for a model rocket.

Some of our assumptions would not be good approximations for large fast rockets that go very high: air resistance wouldn't be negligible, and the fuel expended reduces the mass of the rocket which increases the acceleration. At very high altitudes (where air resistance no longer has an effect) even g decreases slightly.

6.39. Model: We will represent the bullet as a particle.

Visualize:





Solve: (a) We have enough information to use kinematics to find the acceleration of the bullet as it stops. Then we can relate the acceleration to the force with Newton's second law. (Note that the barrel length is not relevant to the problem.) The kinematic equation is

$$v_1^2 = v_0^2 + 2a\Delta x \Rightarrow a = -\frac{v_0^2}{2\Delta x} = -\frac{(400\ \text{m/s})^2}{2(0.12\ \text{m})} = -6.67\times10^5\ \text{m/s}^2$$

Notice that a is negative, in agreement with the vector $\vec{a}$ in the motion diagram. Turning to forces, the wood exerts two forces on the bullet. First, an upward normal force that keeps the bullet from "falling" through the wood. Second, a retarding frictional force $\vec{f}_k$ that stops the bullet. The only horizontal force is $\vec{f}_k$, which points to the left and thus has a negative *x*-component. The *x*-component of Newton's second law is

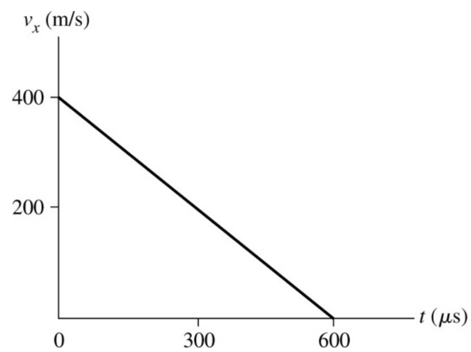
$$(F_{net})_x = -f_k = ma \Rightarrow f_k = -ma = -(0.01\ \text{kg})(-6.67\times10^5\ \text{m/s}^2) = 6670\ \text{N} \approx 6700\ \text{N}$$

Notice how the signs worked together to give a positive value of the magnitude of the force.

(b) The time to stop is found from $v_1 = v_0 + a\Delta t$ as follows:

$$\Delta t = -\frac{v_0}{a} = 6.00\times10^{-4}\ \text{s} = 600\ \mu\text{s}$$

(c)

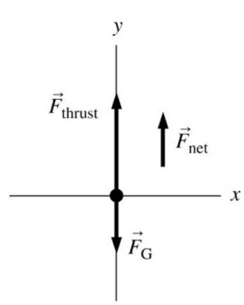


Using the above kinematic equation, we can find the velocity as a function of *t*. For example at $t = 60\ \mu\text{s}$,

$$v_x = 400\ \text{m/s} + (-6.667\times10^5\ \text{m/s}^2)(60\times10^{-6}\ \text{s}) = 360\ \text{m/s}$$

6.40. Model: Represent the rocket as a particle that follows Newton's second law.

Visualize:



Solve: (a) The *y*-component of Newton's second law is

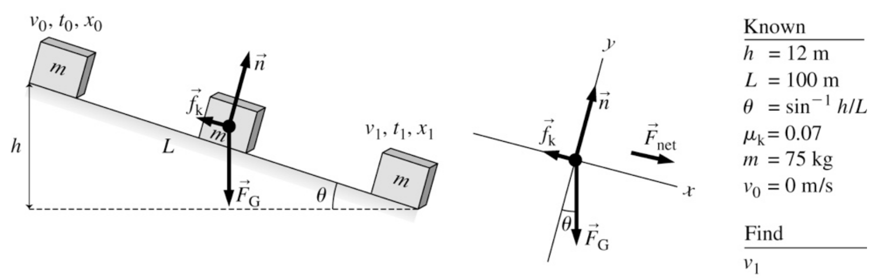
$$a_y = a = \frac{(F_{net})_y}{m} = \frac{F_{thrust} - mg}{m} = \frac{3.0\times10^5\ \text{N}}{20{,}000\ \text{kg}} - 9.80\ \text{m/s}^2 = 5.2\ \text{m/s}^2$$

(b) At 5000 m the acceleration has increased because the rocket mass has decreased. Solving the equation of part (a) for *m* gives

$$m_{5000\text{ m}} = \frac{F_{\text{thrust}}}{a_{5000\text{ m}} + g} = \frac{3.0\times 10^5\text{ N}}{6.0\text{ m/s}^2 + 9.80\text{ m/s}^2} = 1.9\times 10^4\text{ kg}$$

The mass of fuel burned is $m_{\text{fuel}} = m_{\text{initial}} - m_{5000\text{ m}} = 1.0\times 10^3\text{ kg}.$

6.41. Model: Model the object as a particle. Neglect air resistance.
Visualize: We'll use $v_1^2 = v_0^2 + 2a\Delta x$ to find the speed of the object. Since $v_0 = 0,\ v_1 = \sqrt{2a_xL}.$
We'll also use Newton's second law in both directions in order to find $a_x.$



Solve:
(a)

$$\sum F_y = n - mg\cos\theta = 0 \Rightarrow n = mg\cos\theta$$
$$\sum F_x = mg\sin\theta - f_k = ma_x$$
$$mg\sin\theta - \mu_k n = ma_x$$
$$mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

Cancel the *m*.

$$a_x = g(\sin\theta - \mu_k\cos\theta)$$

Now put this back in to the equation for $v_1.$

$$v_1 = \sqrt{2a_xL} = \sqrt{2\left[g(\sin\theta - \mu_k\cos\theta)\right]L} = \sqrt{2g(h - \mu_k\sqrt{L^2 - h^2})}$$

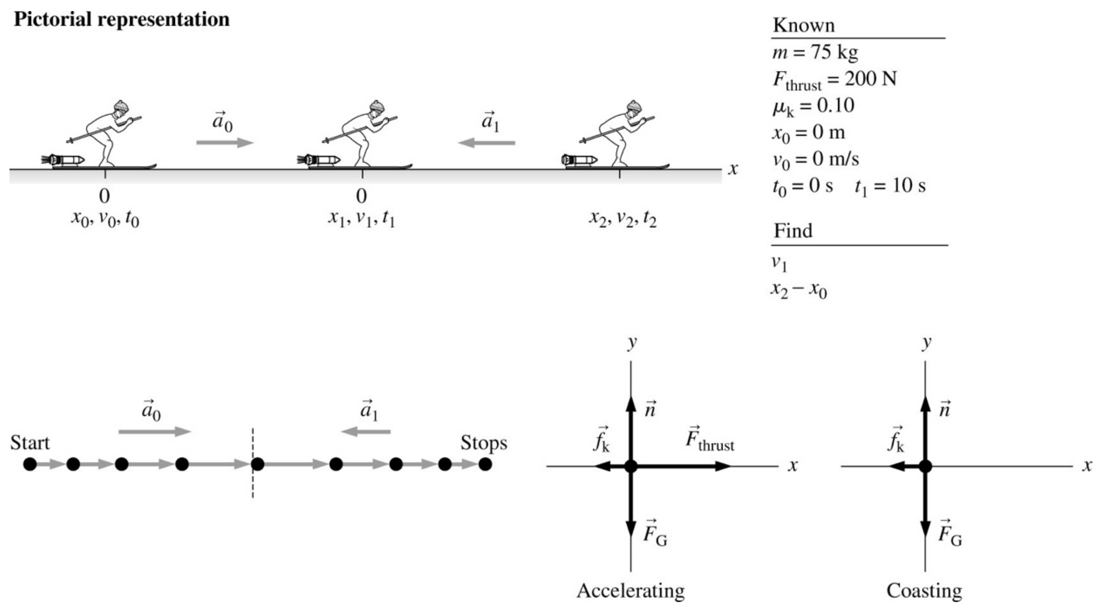
(b) For $h = 12\text{ m},\ L = 100\text{ m},\ \mu_k = 0.07$ we have

$$v_1 = \sqrt{2(9.8\text{ m/s}^2)\left((12\text{ m} - 0.07)\sqrt{(100\text{ m})^2 - (12\text{ m})^2}\right)} = 9.949\text{ m/s} \approx 9.9\text{ m/s}$$

Assess: Sam's mass was extra unneeded information because *m* cancels out of the equation for $v_1.$ Any skier, regardless of their mass, would achieve the same speed at the bottom of the same hill with the same $\mu_k.$

6.42. Model: We assume that Sam is a particle moving in a straight horizontal line under the influence of two forces: the thrust of his jet skis and the resisting force of friction on the skis. We can use one-dimensional kinematics.

Visualize:



Solve: (a) The friction force of the snow can be found from the free-body diagram and Newton's first law, since there's no acceleration in the vertical direction:

$$n = F_G = mg = (75\text{ kg})(9.80\text{ m/s}^2) = 735\text{ N} \Rightarrow f_k = \mu_k n = (0.10)(735\text{ N}) = 73.5\text{ N}$$

Then, from Newton's second law:

$$(F_{net})_x = F_{thrust} - f_k = ma_0 \Rightarrow a_0 = \frac{F_{thrust} - f_k}{m} = \frac{200\text{ N} - 73.5\text{ N}}{75\text{ kg}} = 1.687\text{ m/s}^2$$

From kinematics:

$$v_1 = v_0 + a_0 t_1 = 0\text{ m/s} + (1.687\text{ m/s}^2)(10\text{ s}) = 16.9\text{ m/s}$$

(b) During the acceleration, Sam travels to

$$x_1 = x_0 + v_0 t_1 + \frac{1}{2}a_0 t_1^2 = \frac{1}{2}(1.687\text{ m/s}^2)(10\text{ s})^2 = 84\text{ m}$$

After the skis run out of fuel, Sam's acceleration can again be found from Newton's second law:

$$(F_{net})_x = -f_k = -73.5\text{ N} \Rightarrow a_1 = \frac{F_{net}}{m} = \frac{-73.5\text{ N}}{75\text{ kg}} = -0.98\text{ m/s}^2$$

Since we don't know how much time it takes Sam to stop:

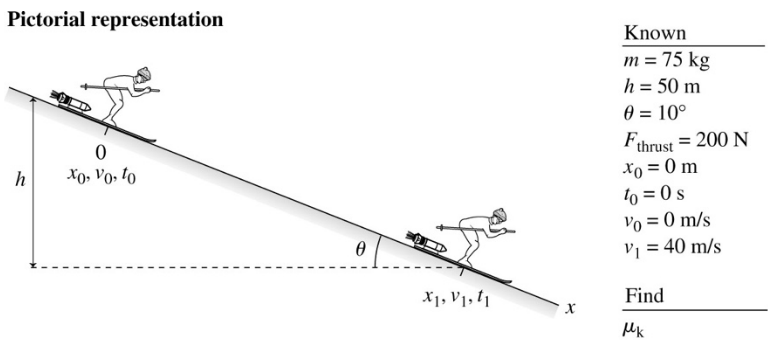
$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow x_2 - x_1 = \frac{v_2^2 - v_1^2}{2a_1} = \frac{0\text{ m}^2/\text{s}^2 - (16.9\text{ m/s})^2}{2(-0.98\text{ m/s}^2)} = 145\text{ m}$$

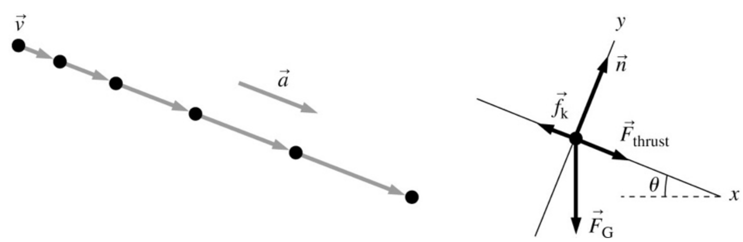
The total distance traveled is $(x_2 - x_1) + x_1 = 145\text{ m} + 84\text{ m} = 229\text{ m}$.

Assess: A top speed of 16.9 m/s (roughly 40 mph) seems quite reasonable for this acceleration, and a coasting distance of nearly 150 m also seems possible, starting from a high speed, given that we're neglecting air resistance.

6.43. Model: We assume Sam is a particle moving in a straight line down the slope under the influence of gravity, the thrust of his jet skis, and the resisting force of friction on the snow.

Visualize:





Solve: From the height of the slope and its angle, we can calculate its length:

$$\frac{h}{x_1 - x_0} = \sin\theta \Rightarrow x_1 - x_0 = \frac{h}{\sin\theta} = \frac{50\text{ m}}{\sin 10°} = 288\text{ m}$$

Since Sam is not accelerating in the y-direction, we can use Newton's first law to calculate the normal force:

$$(F_{net})_y = \sum F_y = n - F_G\cos\theta = 0\text{ N} \Rightarrow n = F_G\cos\theta = mg\cos\theta = (75\text{ kg})(9.80\text{ m/s}^2)(\cos 10°) = 724\text{ N}$$

One-dimensional kinematics gives us Sam's acceleration:

$$v_1^2 = v_0^2 + 2a_x(x - x_0) \Rightarrow a_x = \frac{v_1^2 - v_0^2}{2(x_1 - x_2)} = \frac{(40\text{ m/s})^2 - 0\text{ m}^2/\text{s}^2}{2(288\text{ m})} = 2.78\text{ m/s}^2$$

Then, from Newton's second law and the equation $f_k = \mu_k n$:

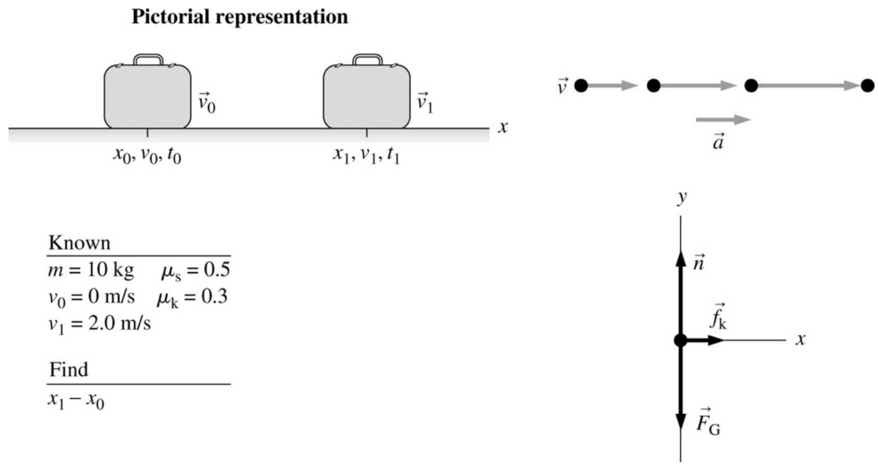
$$(F_{net})_x = \sum F_x = F_G\sin\theta + F_{thrust} - f_k = ma_x$$

$$\Rightarrow \mu_k = \frac{mg\sin\theta + F_{thrust} - ma}{n} = \frac{(75\text{ kg})(9.80\text{ m/s}^2)(\sin 10°) + 200\text{ N} - (75\text{ kg})(2.78\text{ m/s}^2)}{724\text{ N}} = 0.165$$

Assess: This coefficient seems a bit high for skis on snow, but not impossible.

6.44. Model: We assume the suitcase is a particle accelerating horizontally under the influence of friction only.

Visualize:



Solve: Because the conveyor belt is already moving, friction drags your suitcase to the right. It will accelerate until it matches the speed of the belt. We need to know the horizontal acceleration. Since there's no acceleration in the vertical direction, we can apply Newton's first law to find the normal force:

$$n = F_G = mg = (10\text{ kg})(9.80\text{ m/s}^2) = 98.0\text{ N}$$

The suitcase is accelerating, so we use μ_k to find the friction force

$$f_k = \mu_k mg = (0.3)(98.0\text{ N}) = 29.4\text{ N}$$

We can find the horizontal acceleration from Newton's second law:

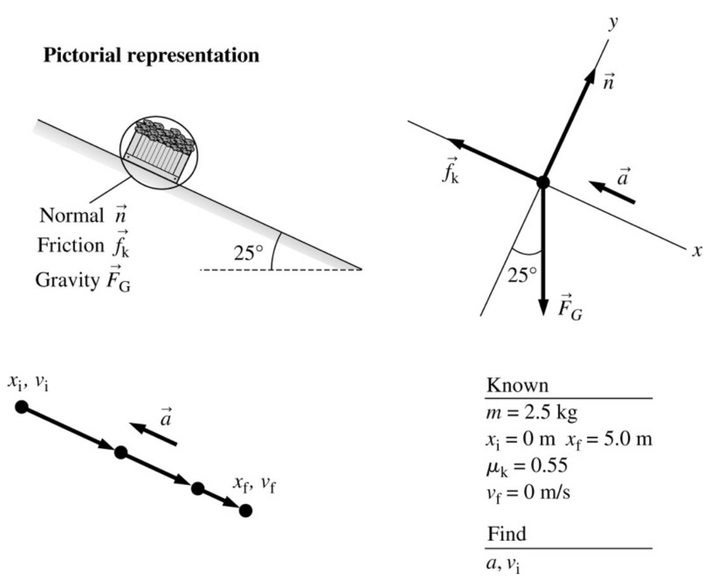
$$(F_{net})_x = \sum F_x = f_k = ma \Rightarrow a = \frac{f_k}{m} = \frac{29.4\text{ N}}{10\text{ kg}} = 2.94\text{ m/s}^2$$

From one of the kinematic equations:

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) \Rightarrow x_1 - x_0 = \frac{v_1^2 - v_0^2}{2a} = \frac{(2.0\text{ m/s})^2 - (0\text{ m/s})^2}{2(2.94\text{ m/s}^2)} = 0.68\text{ m}$$

The suitcase travels 0.68 m before catching up with the belt and riding smoothly.
Assess: If we imagine throwing a suitcase at a speed of 2.0 m/s onto a motionless surface, 0.68 m seems a reasonable distance for it to slide before stopping.

6.45. Model: The box of shingles is a particle subject to Newton's laws and kinematics.
Visualize:



Solve: Newton's laws can be used in the coordinate system in which the direction of motion of the box of shingles defines the +*x*-axis. The angle that F_G makes with the –*y*-axis is 25°.

$$(\sum F)_x = F_G \sin 25° - f_k = ma$$

$$(\sum F)_y = n - F_G \cos 25° = 0 \Rightarrow n = F_G \cos 25°$$

We have used the observation that the shingles do not leap off the roof, so the acceleration in the *y*-direction is zero. Combining these equations with $f_k = \mu_k n$ and $F_G = mg$ yields

$$mg \sin 25° - \mu_k mg \cos 25° = ma$$

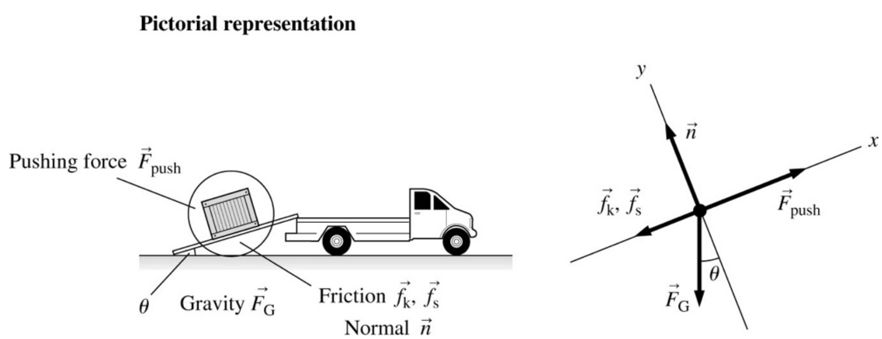
$$\Rightarrow a = (\sin 25° - \mu_k \cos 25°)g = -0.743 \text{ m/s}^2$$

where the minus sign indicates the acceleration is directed up the incline. The required initial speed to have the box come to rest after 5.0 m is found from kinematics.

$$v_f^2 = v_i^2 + 2a\Delta x \Rightarrow v_i^2 = -2(-0.743 \text{ m/s}^2)(5.0 \text{ m}) \Rightarrow v_i = 2.7 \text{ m/s}$$

Assess: To give the shingles an initial speed of 2.7 m/s requires a strong, determined push, but is not beyond reasonable.

6.46. Model: We will model the box as a particle, and use the models of kinetic and static friction.
Visualize:



The pushing force is along the +*x*-axis, but the force of friction acts along the –*x*-axis. A component of the gravitational force on the box acts along the –*x*-axis as well. The box will move up if the pushing force is at least equal to the sum of the friction force and the component of the gravitational force in the *x*-direction.
Solve: Let's determine how much pushing force you would need to keep the box moving up the ramp at steady speed. Newton's second law for the box in dynamic equilibrium is

$$(F_{net})_x = \sum F_x = n_x + (F_G)_x + (f_k)_x + (F_{push})_x = 0 \text{ N} - mg \sin\theta - f_k + F_{push} = 0 \text{ N}$$

$$(F_{net})_y = \sum F_y = n_y + (F_G)_y + (f_k)_y + (F_{push})_y = n - mg \cos\theta + 0 \text{ N} + 0 \text{ N} = 0 \text{ N}$$

The *x*-component equation and the model of kinetic friction yield:

$$F_{push} = mg \sin\theta + f_k = mg \sin\theta + \mu_k n$$

Let us obtain *n* from the *y*-component equation as $n = mg\cos\theta,$ and substitute it in the above equation to get

$$F_{push} = mg \sin\theta + \mu_k \, mg \cos\theta = mg(\sin\theta + \mu_k \cos\theta)$$

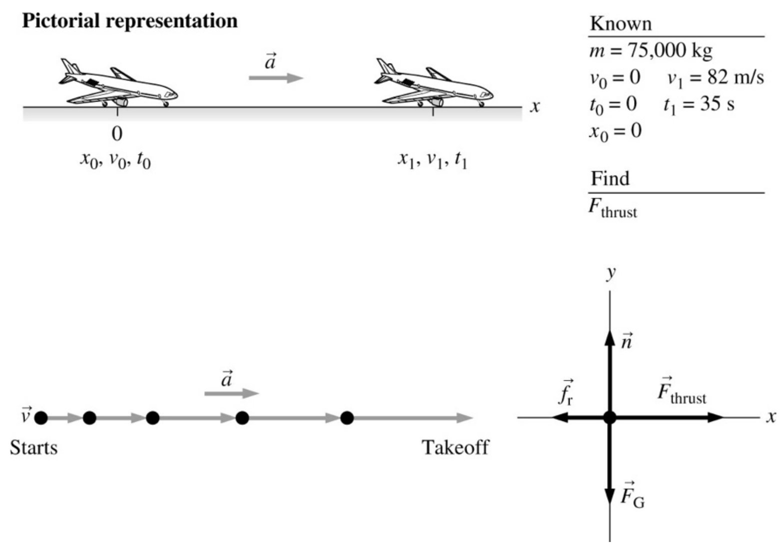
$$= (100 \text{ kg})(9.80 \text{ m/s}^2)(\sin 20° + 0.60 \cos 20°) = 888 \text{ N}$$

The force is less than your maximum pushing force of 1000 N. That is, once in motion, the box could be kept moving up the ramp. However, if you stop on the ramp and want to start the box from rest, the model of static friction applies. The analysis is the same except that the coefficient of static friction is used and we use the maximum value of the force of static friction. Therefore, we have

$$F_{push} = mg(\sin\theta + \mu_s \cos\theta) = (100 \text{ kg})(9.80 \text{ m/s}^2)(\sin 20° + 0.90 \cos 20°) = 1160 \text{ N}$$

Since you can push with a force of only 1000 N, you can't get the box started. The big static friction force and the weight are too much to overcome.

6.47. Model: We assume that the plane is a particle accelerating in a straight line under the influence of two forces: the thrust of its engines and the rolling friction of the wheels on the runway. We can use one-dimensional kinematics.
Visualize:



Solve: We can use the definition of acceleration to find a, and then apply Newton's second law. We obtain:

$$a = \frac{\Delta v}{\Delta t} = \frac{82\text{ m/s} - 0\text{ m/s}}{35\text{ s}} = 2.34\text{ m/s}^2$$

$$(F_{\text{net}}) = \sum F_x = F_{\text{thrust}} - f_r = ma \Rightarrow F_{\text{thrust}} = f_r + ma$$

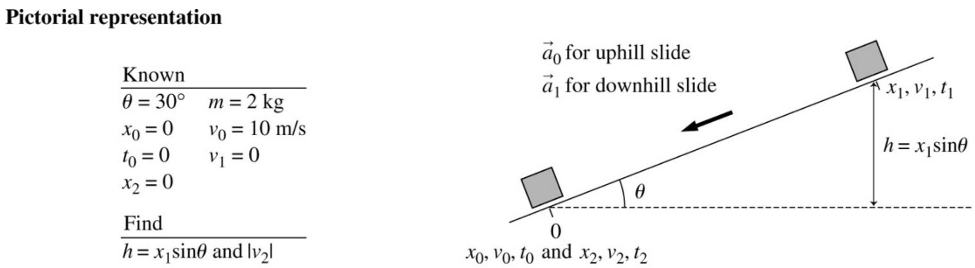
For rubber rolling on concrete, $\mu_r = 0.02$ (Table 6.1), and since the runway is horizontal, $n = F_G = mg$. Thus:

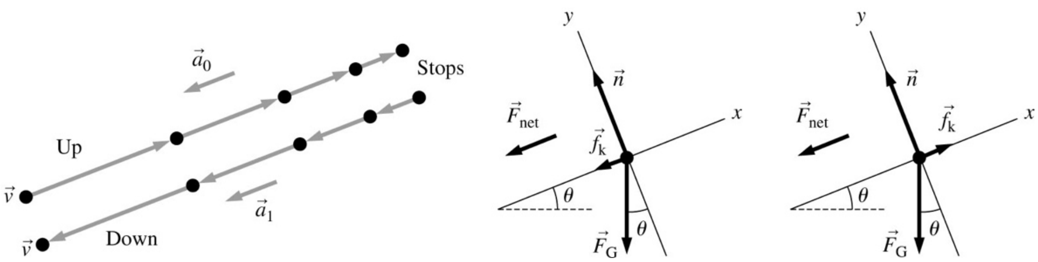
$$F_{\text{thrust}} = \mu_r F_G + ma = \mu_r mg + ma = m(\mu_r g + a)$$
$$= (75{,}000\text{ kg})[(0.02)(9.8\text{ m/s}^2) + 2.34\text{ m/s}^2] = 190{,}000\text{ N}$$

Assess: It's hard to evaluate such an enormous thrust, but comparison with the plane's mass suggests that 190,000 N is enough to produce the required acceleration.

6.48. Model: We will represent the wood block as a particle, and use the model of kinetic friction and kinematics. Assume $w \sin \theta > f_s$, so it does not hang up at the top.

Visualize:





The block ends where it starts, so $x_2 = x_0 = 0$ m. We expect v_2 to be negative, because the block will be moving in the $-x$-direction, so we'll want to take $|v_2|$ as the final speed. Because of friction, we expect to find $|v_2| < v_0$.

Solve: (a) The friction force is opposite to v, so f_k points down the slope during the first half of the motion and up the slope during the second half. F_G and n are the only other forces. Newton's second law for the upward motion is

$$a_x = a_0 = \frac{(F_{\text{net}})_x}{m} = \frac{-F_G\sin\theta - f_k}{m} = \frac{-mg\sin\theta - f_k}{m}$$

$$a_y = 0\text{ m/s}^2 = \frac{(F_{\text{net}})_y}{m} = \frac{n - F_G\cos\theta}{m} = \frac{n - mg\cos\theta}{m}$$

The friction model is $f_k = \mu_k n$. First solve the y-equation to give $n = mg\cos\theta$. Use this in the friction model to get $f_k = \mu_k mg\cos\theta$. Now substitute this result for f_k into the x-equation:

$$a_0 = \frac{-mg\sin\theta - \mu_k mg\cos\theta}{m} = -g(\sin\theta + \mu_k\cos\theta) = -(9.8\text{ m/s}^2)(\sin 30° + 0.20\cos 30°) = -6.60\text{ m/s}^2$$

Kinematics now gives

$$v_1^2 = v_0^2 + 2a_0(x_1 - x_0) \Rightarrow x_1 = \frac{v_1^2 - v_0^2}{2a_0} = \frac{0\text{ m}^2/\text{s}^2 - (10\text{ m/s})^2}{2(-6.60\text{ m/s}^2)} = 7.6\text{ m}$$

The block's height is then $h = x_1\sin\theta = (7.6\text{ m})\sin 30° = 3.8\text{ m}$.

(b) For the return trip, f_k points up the slope, so the x-component of the second law is

$$a_x = a_1 = \frac{(F_{\text{net}})_x}{m} = \frac{-F_G\sin\theta + f_k}{m} = \frac{-mg\sin\theta + f_k}{m}$$

Note the sign change. The y-equation and the friction model are unchanged, so we have

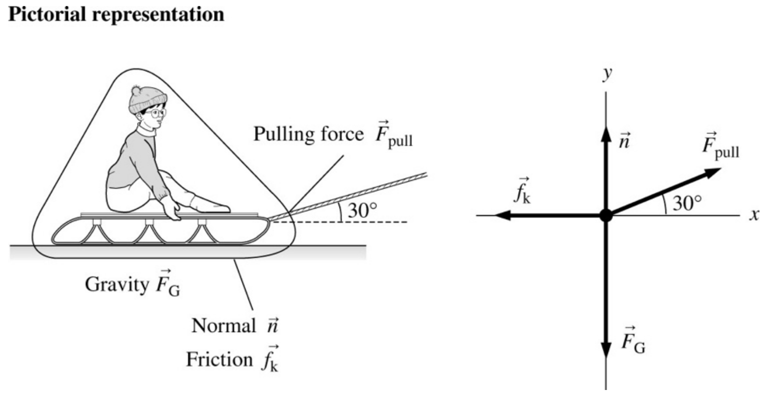
$$a_1 = -g(\sin\theta - \mu_k\cos\theta) = -3.20\text{ m/s}^2$$

The kinematics for the return trip are

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow v_2 = \sqrt{-2a_1x_1} = \sqrt{2(-3.20\text{ m/s}^2)(-7.6\text{ m})} = -7.0\text{ m/s}$$

Notice that we used the *negative* square root because v_2 is a *velocity* with the vector pointing in the $-x$-direction. The final *speed* is $|v_2| = 7.0$ m/s.

6.49. Model: We will model the sled and friend as a particle, and use the model of kinetic friction because the sled is in motion.
Visualize:



The net force on the sled is zero (note the constant speed of the sled). That means the component of the pulling force along the $+x$-direction is equal to the magnitude of the kinetic force of friction in the $-x$-direction. Also note that $(F_{net})_y = 0\text{ N},$ since the sled is not moving along the y-axis.
Solve: Newton's second law is

$$(F_{net})_x = \sum F_x = n_x + (F_G)_x + (f_k)_x + (F_{pull})_x = 0\text{ N} + 0\text{ N} - f_k + F_{pull}\cos\theta = 0\text{ N}$$
$$(F_{net})_y = \sum F_y = n_y + (F_G)_y + (f_k)_y + (F_{pull})_y = n - mg + 0\text{ N} + F_{pull}\sin\theta = 0\text{ N}$$

The x-component equation using the kinetic friction model $f_k = \mu_k n$ reduces to

$$\mu_k n = F_{pull}\cos\theta$$

The y-component equation gives

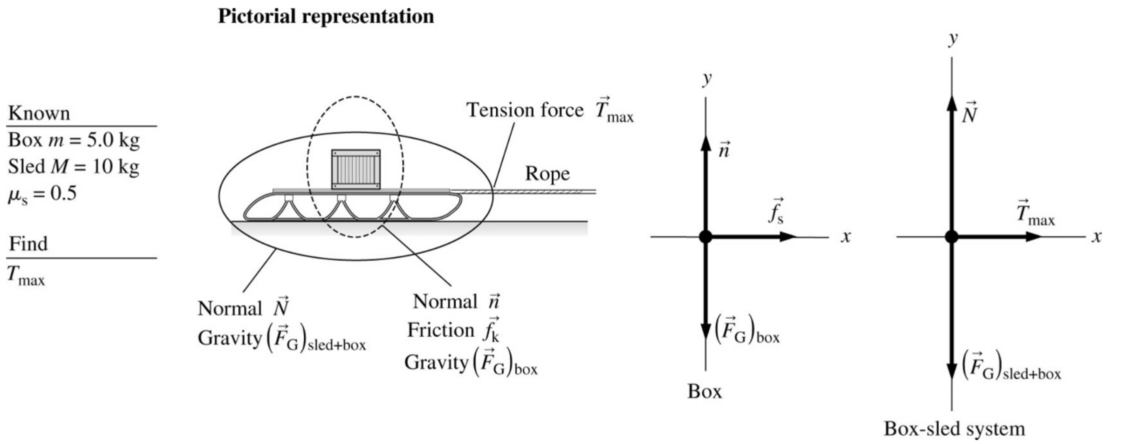
$$n = mg - F_{pull}\sin\theta$$

We see that the normal force is smaller than the gravitational force because F_{pull} has a component in a direction opposite to the direction of the gravitational force. In other words, F_{pull} is partly lifting the sled. From the x-component equation, μ_k can now be obtained as

$$\mu_k = \frac{F_{pull}\cos\theta}{mg - F_{pull}\sin\theta} = \frac{(75\text{ N})(\cos 30^\circ)}{(60\text{ kg})(9.80\text{ m/s}^2) - (75\text{ N})(\sin 30^\circ)} = 0.12$$

Assess: A quick glance at the various μ_k values in Table 6.1 suggests that a value of 0.12 for μ_k is reasonable.

6.50. Model: Model the small box as a particle and use the model of static friction. The acceleration of the small box must be the same as the acceleration of the large box in order for it not to slip.
Visualize: First use Newton's second law in both directions on the small box. The force that is responsible for the small box's acceleration is the static friction force. We use this to determine a_{max}. Then we use Newton's second law on the the two-box system.



Solve:
(a)

$$\sum F_y = n - mg = 0 \Rightarrow n = mg$$
$$\sum F_x = f_s = ma_x$$
$$(f_s)_{max} = \mu_s n = \mu_s mg = ma_{max}$$
$$a_{max} = \mu_s g$$

Now consider the two-box system.

$$\sum F_x = T_{max} = (M + m)a_{max}$$

Put these together to arrive at

$$T_{max} = (M + m)\mu_s g$$

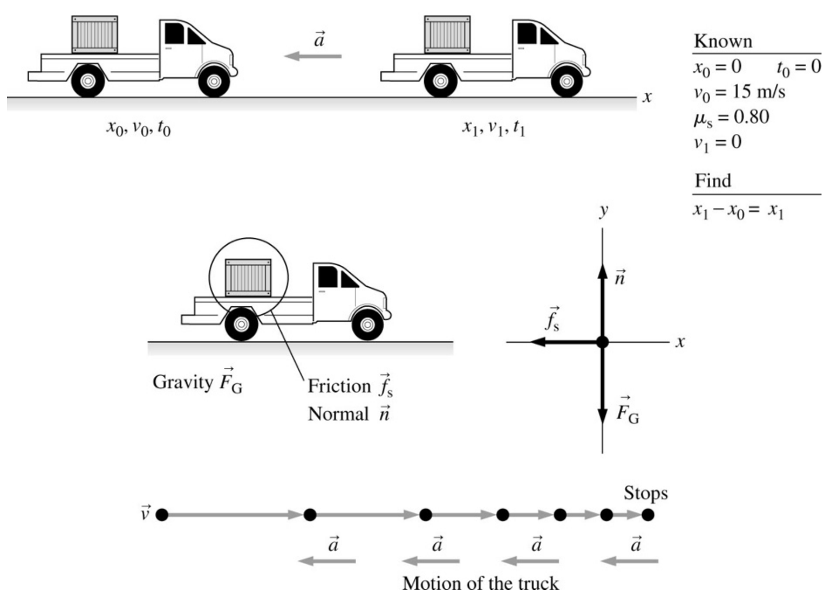
(b) Insert the known values for M and m, and look up μ_s for wood on wood in the table.

$$T_{max} = (10\text{ kg} + 5\text{ kg})(0.5)(9.8\text{ m/s}^2) = 73.5\text{ N} \approx 74\text{ N}$$

Assess: Check the reasonableness of our answer by examining the dependence of T_{max} on μ_s: if the small box were glued to the large box $(\mu_s \to \infty)$ then one could pull on the rope with any tension desired; if the friction between the two boxes were zero then one could not pull at all without causing the small box to slip. We expect a similar dependence on g.

6.51. Model: Model the steel cabinet as a particle. It touches the truck's bed, so only the steel bed can exert contact forces on the cabinet. As long as the cabinet does not slide, the acceleration a of the cabinet is equal to the acceleration of the truck.
Visualize: First use Newton's second law in both directions on the cabinet. The force that is responsible for the small box's acceleration is the static friction force. We use this to determine a.

Pictorial representation



Solve:
(a)

$$\sum F_y = n - mg = 0 \Rightarrow n = mg$$

$$\sum F_x = 2\, f_s = ma_x$$

$$-(f_s)_{max} = -\mu_s n = -\mu_s mg = ma_x$$

$$a_x = -\mu_s g$$

Now use the kinematic equation $v_1^2 = v_0^2 + 2a\Delta x$ where $\Delta x = d_{min}$ and $v_1 = 0.$

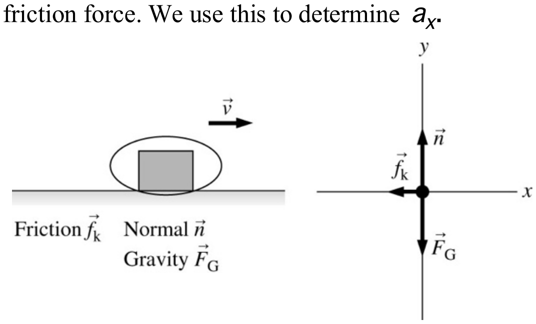
$$d_{min} = \frac{v_1^2 - v_0^2}{2a_x} = \frac{-v_0^2}{2(-\mu_s g)} = \frac{v_0^2}{2(\mu_s g)}$$

(b) Insert the known value for v_0 and look up μ_s for steel on steel in the table.

$$d_{min} = \frac{(15\text{ m/s})^2}{2(0.80)(9.8\text{ m/s}^2)} = 14.35\text{ m} \approx 14\text{ m}$$

Assess: Check the reasonableness of our answer by examining the dependence of T_{max} on μ_s: if the cabinet were glued to the truck $(\mu_s \to \infty)$ then one could stop in an arbitrarily small distance without the cabinet slipping; if the friction between the cabinet and truck were zero then $d_{min} = \infty$ and there is no minimum stopping distance without causing the cabinet to slip.

6.52. Model: Model the block as a particle.
Visualize: First use Newton's second law in both directions on the block. The force that is responsible for the small box's acceleration is the kinetic friction force. We use this to determine a_x.



Solve:

$$\sum F_y = n - mg = 0 \Rightarrow n = mg$$

$$\sum F_x = -f_k = ma_x$$

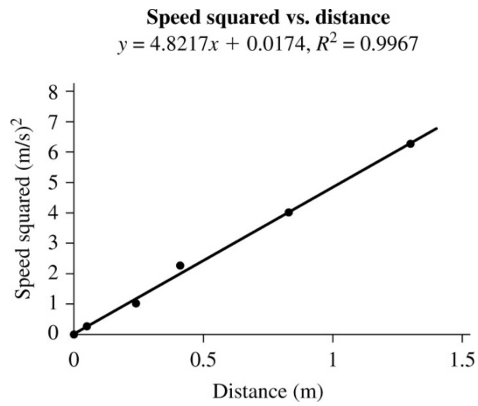
$$-(f_k) = -\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

Now use the kinematic equation $v_1^2 = v_0^2 + 2a\Delta x$ where $v_1 = 0$ and Δx is the sliding distance.

$$v_0^2 = -2a_x\Delta x = -2(-\mu_k g)\Delta x = 2\mu_k g\Delta x$$

This says that a graph of v_0^2 vs. Δx would be a straight line with a slope of $2\mu_k g$.



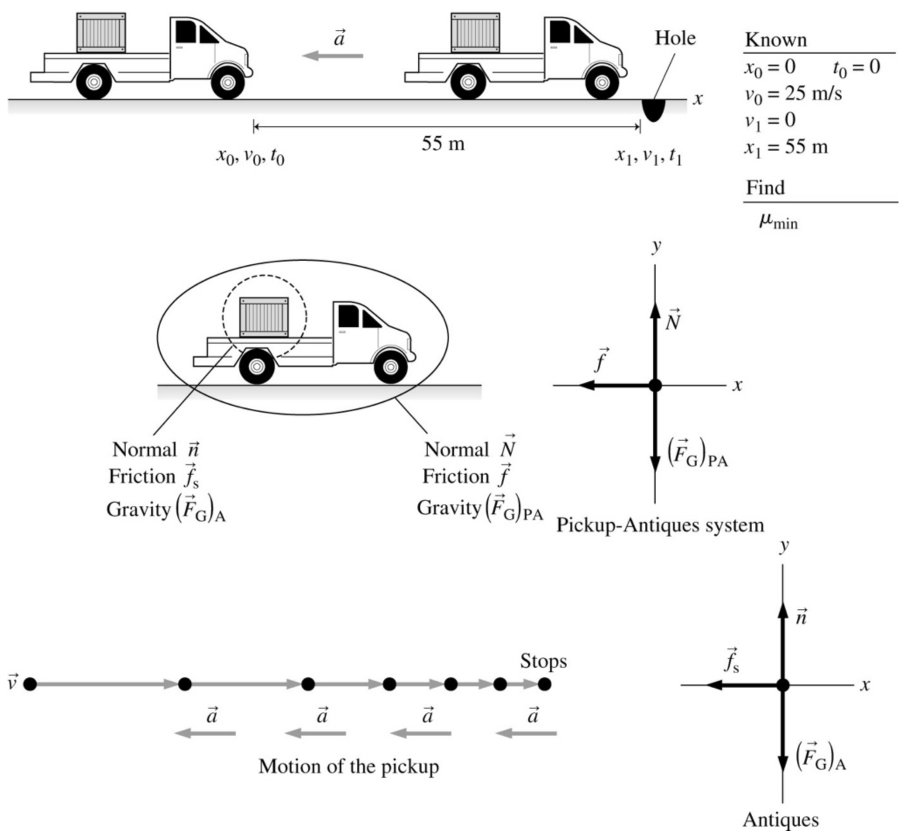
We see that the linear fit is very good and that the slope is 4.82 m/s^2.

$$2\mu_k g = 4.82 \text{ m/s}^2 \Rightarrow \mu_k = \frac{\text{slope}}{2g} = \frac{4.82 \text{ m/s}^2}{2(9.8 \text{ m/s}^2)} = 0.246 \approx 0.25$$

Assess: Our answer for wood on smooth metal is higher than we expected because the table gives μ_k for wood on wood as 0.20. We expected the intercept of our graph to be small; in fact, we included (0,0) in the data table. The mass of the block canceled out and so was unnecessary information.

6.53. Model: The antiques $(\text{mass} = m)$ in the back of your pickup $(\text{mass} = M)$ will be treated as a particle. The antiques touch the truck's steel bed, so only the steel bed can exert contact forces on the antiques. The pickup-antiques system will also be treated as a particle, and the contact force on this particle will be due to the road.
Visualize:

Pictorial representation



Solve: (a) We will find the smallest coefficient of friction that allows the truck to stop in 55 m, then compare that to the known coefficients for rubber on concrete. For the pickup-antiques system, with mass $m + M$, Newton's second law is

$$(F_{net})_x = \sum F_x = N_x + ((F_G)_{PA})_x + (f)_x = 0\text{ N} + 0\text{ N} - f = (m+M)a_x = (m+M)a$$
$$(F_{net})_y = \sum F_y = N_y + ((F_G)_{PA})_y + (f)_y = \text{N} - (m+M)g + 0\text{ N} = 0\text{ N}$$

The model of static friction is $f = \mu N$, where μ is the coefficient of friction between the tires and the road. These equations can be combined to yield $a = -\mu g$. Since constant-acceleration kinematics gives $v_1^2 = v_0^2 + 2a(x_1 + x_0)$, we find

$$a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)} \Rightarrow \mu_{min} = \frac{v_0^2}{2g(x_1 - x_0)} = \frac{(25\text{ m/s})^2}{(2)(9.8\text{ m/s}^2)(55\text{ m})} = 0.58$$

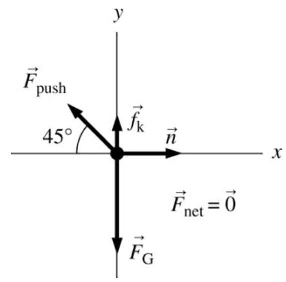
The truck cannot stop if μ is smaller than this. But both the static and kinetic coefficients of friction, 1.00 and 0.80 respectively (see Table 6.1), are larger. So the truck can stop.

(b) The analysis of the pickup-antiques system applies to the antiques, and it gives the same value of 0.58 for μ_{min}. This value is smaller than the given coefficient of static friction $(\mu_s = 0.60)$ between the antiques and the truck bed. Therefore, the antiques will not slide as the truck is stopped over a distance of 55 m.

Assess: The analysis of parts (a) and (b) are the same because mass cancels out of the calculations. According to the California Highway Patrol Web site, the stopping distance (with zero reaction time) for a passenger vehicle traveling at 25 m/s or 82 ft/s is approximately 43 m. This is smaller than the 55 m over which you are asked to stop the truck.

6.54. Model: The box will be treated as a particle. Because the box slides down a vertical wood wall, we will also use the model of kinetic friction.

Visualize:



Solve: The normal force due to the wall, which is perpendicular to the wall, is here to the right. The box slides down the wall at constant speed, so $a = 0$ and the box is in dynamic equilibrium. Thus, $F_{net} = 0$. Newton's second law for this equilibrium situation is

$$(F_{net})_x = 0\text{ N} = n - F_{push}\cos 45°$$

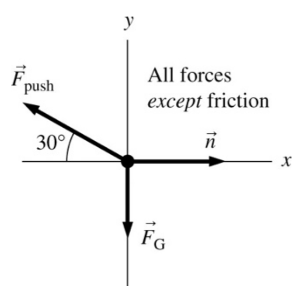
$$(F_{net})_y = 0\text{ N} = f_k + F_{push}\sin 45° - F_G = f_k + F_{push}\sin 45° - mg$$

The friction force is $f_k = \mu_k n$. Using the *x*-equation to get an expression for *n*, we see that $f_k = \mu_k F_{push}\cos 45°$. Substituting this into the *y*-equation and using Table 6.1 to find $\mu_k = 0.20$ gives,

$$\mu_k F_{push}\cos 45° + F_{push}\sin 45° - mg = 0\text{ N}$$

$$\Rightarrow F_{push} = \frac{mg}{\mu_k \cos 45° + \sin 45°} = \frac{(2.0\text{ kg})(9.80\text{ m/s}^2)}{0.20\cos 45° + \sin 45°} = 23\text{ N}$$

6.55. Model: Use the particle model for the block and the model of static friction.
Visualize:



Solve: The block is initially at rest, so initially the friction force is static friction. If the 12 N push is too strong, the box will begin to move up the wall. If it is too weak, the box will begin to slide down the wall. And if the pushing force is within the proper range, the box will remain stuck in place. First, let's evaluate the sum of all the forces *except* friction:

$$\sum F_x = n - F_{push}\cos 30° = 0\text{ N} \Rightarrow n = F_{push}\cos 30°$$

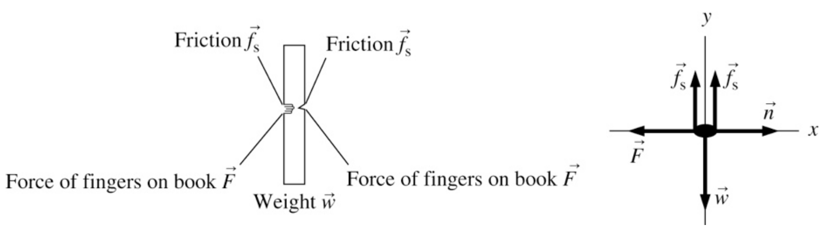
$$\sum F_y = F_{push}\sin 30° - F_G = F_{push}\sin 30° - mg = (12\text{ N})\sin 30° - (1\text{ kg})(9.8\text{ m/s}^2) = -3.8\text{ N}$$

In the first equation we utilized the fact that any motion is parallel to the wall, so $a_x = 0\text{ m/s}^2$. These three forces add up to $-3.8\hat{j}$ N. This means the static friction force will be able to prevent the box from moving if $f_s = +3.8\hat{j}$ N. Using the *x*-equation and the friction model we get

$$(f_s)_{max} = \mu_s n = \mu_s F_{push}\cos 30° = 5.2\text{ N}$$

where we used $\mu_s = 0.5$ for wood on wood. The static friction force f_s needed to keep the box from moving is *less* than $(f_s)_{max}$. Thus the box will stay at rest.

6.56. Visualize: The book is in static equilibrium so the net force is zero. The maximum static frictional force the person can exert will determine the heaviest book he can hold.
Solve: Consider the free-body diagram below. The force of the fingers on the book is the reaction force to the normal force of the book on the fingers, so is exactly equal and opposite the normal force on the fingers.

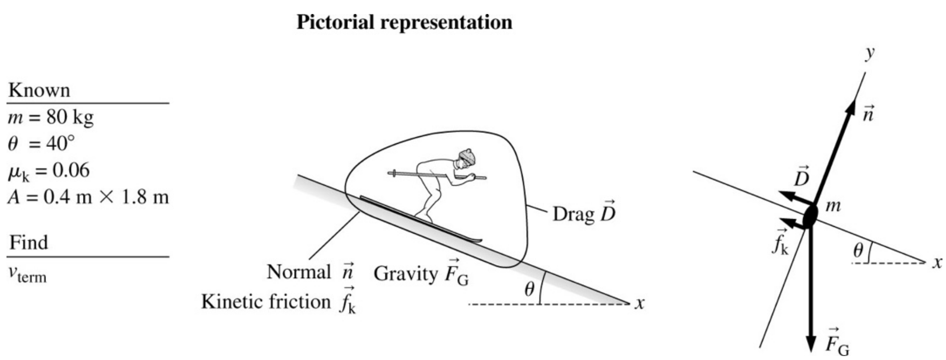


The maximal static friction force will be equal to $f_{s\,max} = \mu_s n = (0.80)(6.0\text{ N}) = 4.8\text{ N}$. The frictional force is exerted on both sides of the book. Considering the forces in the y-direction, we have that the weight supported by the maximal frictional force is

$$w = f_{s\,max} + f_{s\,max} = 2f_{s\,max} = 9.6\text{ N}$$

Assess: Note that the force on both sides of the book are exactly equal also because the book is in equilibrium.

6.57. Model: We will model the skier along with the wooden skis as a particle of mass m. The snow exerts a contact force and the wind exerts a drag force on the skier. We will therefore use the models of kinetic friction and drag. Assume the skier is a cylinder end-forward so that $C = 0.8$.
Visualize:



We choose a coordinate system such that the skier's motion is along the +x-direction. While the forces of kinetic friction f_k and drag D act along the $-x$-direction opposing the motion of the skier, the gravitational force on the skier has a component in the +x-direction. At the terminal speed, the net force on the skier is zero as the forces along the +x-direction cancel out the forces along the $-x$-direction.
Solve: Newton's second law and the models of kinetic friction and drag are

$$(F_{net})_x = \sum F_x = +(F_G)_x + (f_k)_x + (D)_x = mg\sin\theta - f_k - \frac{1}{2}C\rho Av^2 = ma_x = 0\text{ N}$$
$$(F_{net})_y = \sum F_y = n_y + (F_G)_y = n - mg\cos\theta = 0\text{ N}$$
$$f_k = \mu_k n$$

These three equations can be combined together as follows:

$$(1/2)C\rho Av^2 = mg\sin\theta - f_k = mg\sin\theta - \mu_k n = mg\sin\theta - \mu_k mg\cos\theta$$

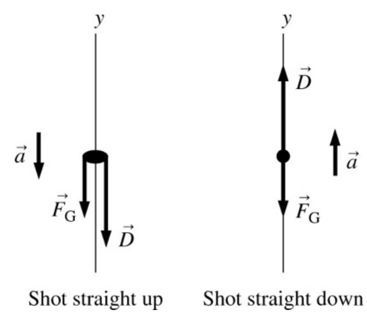
$$\Rightarrow v_{term} = \left(mg\frac{\sin\theta - \mu_k\cos\theta}{\frac{1}{2}C\rho A} \right)^{1/2}$$

Using $\mu_k = 0.06$ and $A = 1.8\ \text{m} \times 0.40\ \text{m} = 0.72\ \text{m}^2$, we find

$$v_{term} = \left[(80\ \text{kg})(9.8\ \text{m/s}^2)\left(\frac{\sin 40° - 0.06\cos 40°}{\frac{1}{2}(0.8)(1.2\ \text{kg/m}^3)(0.72\ \text{m}^2)} \right) \right]^{1/2} = 37\ \text{m/s}$$

Assess: A terminal speed of 37 m/s corresponds to a speed of ≈ 82 mph. This speed is reasonable but high due to the steep slope angle of 40° and a small coefficient of friction.

6.58. Model: The ball is a particle experiencing a drag force and traveling at twice its terminal velocity.
Visualize:



Solve: (a) An object falling at greater than its terminal velocity will slow down to its terminal velocity. Thus the drag force is greater than the force of gravity, as shown in the free-body diagrams. When the ball is shot straight up,

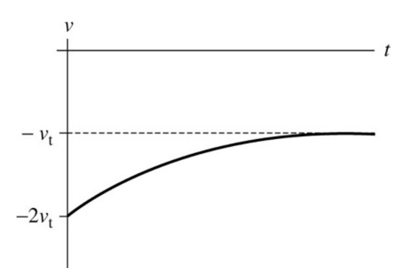
$$(\sum F)_y = ma = -(F_G + D) = -\left(mg + \frac{1}{2}C\rho Av^2 \right) = -mg - \frac{1}{2}C\rho A(2v_{term})^2 =$$

$$-mg - \frac{1}{2}C\rho A\left(4\frac{2mg}{C\rho A} \right) = -mg - (4mg) = -5mg$$

Thus $a = -5g$, where the minus sign indicates the downward direction. We have used Equations 6.16 for the drag force and 6.19 for the terminal velocity.
(b) When the ball is shot straight down,

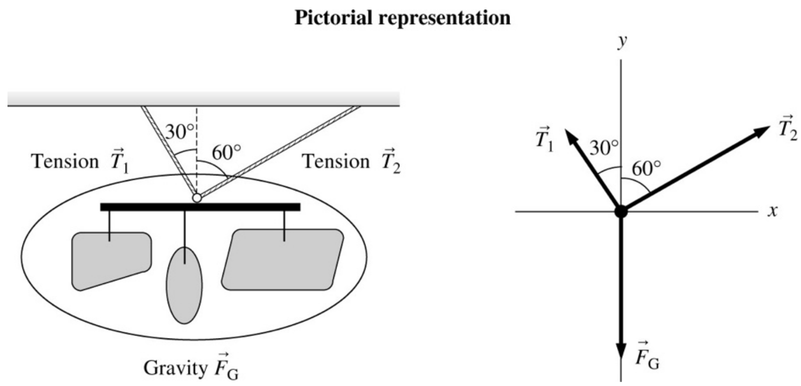
$$(\sum F)_y = ma = D - F_G = \frac{1}{2}C\rho A(2v_{term})^2 - mg = \frac{1}{2}C\rho A\left(4\frac{2mg}{C\rho A} \right) - mg = 3mg$$

Thus $a = 3g$, this time directed upward.
(c)



The ball will slow down to its terminal velocity, slowing quickly at first, and more slowly as it gets closer to the terminal velocity because the drag force decreases as the ball slows.

6.59. Model: We will model the sculpture as a particle of mass *m*. The ropes that support the sculpture will be assumed to have zero mass.
Visualize:



Solve: Newton's first law in component form is

$$(F_{net})_x = \sum F_x = T_{1x} + T_{2x} + F_{Gx} = -T_1 \sin 30° + T_2 \sin 60° + 0\text{ N} = 0\text{ N}$$
$$(F_{net})_y = \sum F_y = T_{1y} + T_{2y} + F_{Gy} = -T_1 \cos 30° + T_2 \cos 60° - F_G = 0\text{ N}$$

Using the *x*-component equation to obtain an expression for T_1 and substituting into the *y*-component equation yields:

$$T_2 = \frac{F_G}{\dfrac{(\sin 60°)(\cos 30°)}{\sin 30°} + \cos 60°} = \frac{500\text{ lbs}}{2} = 250\text{ lbs}$$

Substituting this value of T_2 back into the *x*-component equation,

$$T_1 = T_2 \frac{\sin 60°}{\sin 30°} = 250\text{ lbs} \frac{\sin 60°}{\sin 30°} = 433\text{ lbs}$$

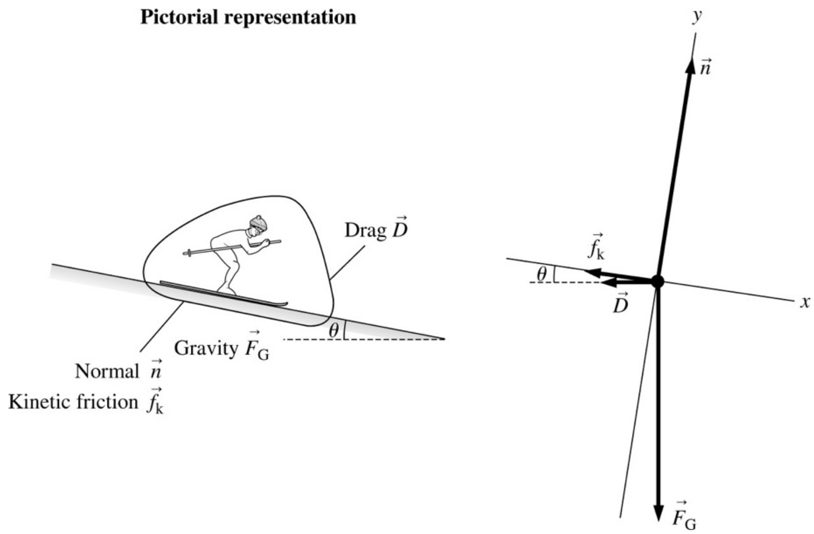
We will now find a rope size for a tension force of 433 lbs, that is, the diameter of a rope with a safety rating of 433 lbs. Since the cross-sectional area of the rope is $\frac{1}{4}\pi d^2$, we have

$$d = \left[\frac{4(433\text{ lbs})}{\pi(4000\text{ lbs/inch}^2)}\right]^{1/2} = 0.371\text{ inch}$$

Any diameter larger than 0.371 inch will ensure a safety rating of at least 433 lbs. The rope size corresponding to a diameter of 3/8 of an inch will therefore be appropriate.
Assess: If only a single rope were used to hang the sculpture, the rope would have to support a gravitational force of 500 lbs. The diameter of the rope for a safety rating of 500 lbs is 0.399 inches, and the rope size jumps from a diameter of 3/8 to 4/8 of an inch. Also note that the gravitational force on the sculpture is distributed in the two ropes. It is the sum of the *y*-components of the tensions in the ropes that will equal the gravitational force on the sculpture.

6.60. Model: We will model the skier as a particle, and use the model of kinetic friction.

Visualize:



Solve: Your best strategy, if it's possible, is to travel at a very slow *constant* speed $(a = 0 \text{ so } F_{net} = 0)$. Alternatively, you want the smallest positive a_x. A negative a_x would cause you to slow and stop. Let's find the value of μ_k that gives $F_{net} = 0$.

Newton's second law for the skier and the model of kinetic friction are

$$(F_{net})_x = \sum F_x = n_x + (F_G)_x + (f_k)_x + (D)_x = 0 + mg\sin\theta - f_k - D\cos\theta = 0\text{ N}$$
$$(F_{net})_y = \sum F_y = n_y + (F_G)_y + (f_k)_y + (D)_y = n - mg\cos\theta + 0\text{ N} - D\sin\theta = 0\text{ N}$$
$$f_k = \mu_k n$$

The x- and y-component equations are

$$f_k = +mg\sin\theta - D\cos\theta \quad n = mg\cos\theta + D\sin\theta$$

From the model of kinetic friction,

$$\mu_k = \frac{f_k}{n} = \frac{mg\sin\theta - D\cos\theta}{mg\cos\theta + D\sin\theta} = \frac{82\text{ kg}(9.8\text{ m/s}^2)\sin15^\circ - (50\text{ N})\cos15^\circ}{82\text{ kg}(9.8\text{ m/s}^2)\cos15^\circ + (50\text{ N})\sin15^\circ} = 0.20$$

Yellow wax with $\mu_k = 0.20$ is perfect.

6.61. Model: The astronaut is a particle oscillating on a spring.

Solve: (a) The position versus time function $x(t)$ can be used to find the velocity versus time function $v(t) = \frac{dx}{dt}$. We have

$$v(t) = \frac{d}{dt}\{(0.30\text{ m})\sin((\pi\text{ rad/s})t)\} = (0.30\pi\text{ m/s})\cos((\pi\text{ rad/s})t)$$

This can then be used to find the acceleration $a(t) = \frac{dv}{dt}$.

$$a(t) = \frac{dv}{dt} = -(0.30\pi^2\text{ m/s}^2)\sin((\pi\text{ rad/s})t)$$

Newton's second law yields a general expression for the force on the astronaut.

$$F_{net}(t) = ma(t) = -(75\text{ kg})(0.30\pi^2\text{ m/s}^2)\sin((\pi\text{ rad/s})t)$$

Evaluating this at $t = 1.0\text{ s}$ gives $F_{\text{net}}(1.0\text{ s}) = 0\text{ N},$ since $\sin(\pi) = 0.$
(b) Evaluating at $t = 1.5\text{ s},$

$$F_{\text{net}} = -22.5\pi^2\text{ N}\sin\left(\frac{3\pi}{2}\right) = 2.2\times 10^2\text{ N}$$

Assess: The force of 220 N is only one-third of the astronaut's weight on earth, so is easy for her to withstand.

6.62. Solve: Using $a_x = \dfrac{dv_x}{dt},$ we express Newton's second law as a differential equation, which we then use to solve for v_x.

$$F_x = m\frac{dv_x}{dt} \Rightarrow dv_x = \frac{F_x}{m}dt = \frac{ct}{m}dt$$

Integrating from the initial to final conditions for each variable of integration,

$$\int_{v_{0x}}^{v_x} dv_x = \frac{c}{m}\int_0^t t\,dt \Rightarrow v_x - v_{0x} = \frac{ct^2}{2m}$$

Thus

$$v_x = v_{0x} + \frac{ct^2}{2m}$$

6.63. Model: Model the object as a particle. The acceleration is not constant so we can't use the kinematic equations. All the motion is in the *x*-direction.
Visualize: Divide *F* by *m* to get *a* and then integrate twice. The constants of integration are both zero because of the initial conditions.
Solve:

$$a_x(t) = \frac{F_x}{m} = \frac{F_0}{m}\left(1 - \frac{t}{T}\right)$$

(a)

$$v_x(t) = \int a_x dt = \frac{F_0}{m}\int\left(1 - \frac{t}{T}\right)dt = \frac{F_0}{m}\left(t - \frac{t^2}{2T}\right) + v_0 = \frac{F_0}{m}\left(t - \frac{t^2}{2T}\right)$$

$$v_x(T) = \frac{F_0}{m}\left(T - \frac{T^2}{2T}\right) = \frac{F_0}{m}\frac{T}{2}$$

(b)

$$x(t) = \int v_x dt = \frac{F_0}{m}\int\left(t - \frac{t^2}{2T}\right)dt = \frac{F_0}{m}\left(\frac{t^2}{2} - \frac{t^3}{6T}\right) + x_0 = \frac{F_0}{m}\left(\frac{t^2}{2} - \frac{t^3}{6T}\right)$$

$$x(T) = \frac{F_0}{m}\left(\frac{T^2}{2} - \frac{T^3}{6T}\right) = \frac{F_0}{m}\frac{T^2}{3}$$

Assess: It seems reasonable that the velocity after time *T* would increase with *T* and that the position at time *T* would increase with T^2.

6.64. Model: Model the object as a particle. The acceleration is not constant so we can't use the kinematic equations. All the motion is in the *x*-direction.
Visualize: Divide *F* by *m* to get *a* and then integrate twice. The constants of integration are both zero because of the initial conditions.
Solve:

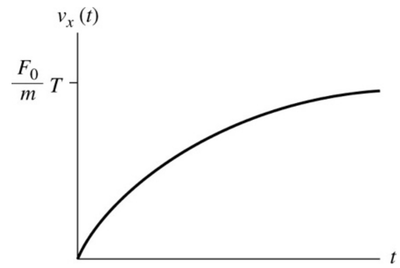
$$a_x(t) = \frac{F_x}{m} = \frac{F_0}{m}(e^{-t/T})$$

(a)

$$v_x(t) = \int a_x dt = \frac{F_0}{m}\int\left(e^{-\frac{t}{T}}\right)dt = \frac{F_0}{m}(-T)\left(e^{-\frac{t}{T}}\right) + C$$

The constant of integration is not zero. $v(0) = 0 \Rightarrow C = \frac{F_0}{m}(T)$

$$v_x(t) = \frac{F_0}{m}(-T)\left(e^{-\frac{t}{T}}\right) + \frac{F_0}{m}(T) = \frac{F_0}{m}(T)\left(1 - e^{-\frac{t}{T}}\right)$$



(b) After a very long time the decaying exponential term is close to zero so $v_x(t) \to \frac{F_0}{m}T.$

Assess: It seems reasonable that the velocity after time T would increase with T and that the position at time T would increase with T^2.

6.65. Model: Use the linear model of drag. Assume the microorganisms are swimming in water at $20°C$.

Visualize: The viscosity of water is $\eta = 1.0\times 10^{-3}\ \text{N}\cdot\text{s/m}^2$ at $20°C$.

Solve:

(a)

$$\sum F = F_{\text{prop}} - D = 0 \Rightarrow F_{\text{prop}} = 6\pi\eta Rv$$

For a paramecium

$$F_{\text{prop}} = 6\pi(1.0\times 10^{-3}\ \text{N}\cdot\text{s/m}^2)(50\times 10^{-6}\ \text{m})(0.0010\ \text{m/s}) = 9.4\times 10^{-10}\ \text{N}$$

For an *E. coli* bacterium

$$F_{\text{prop}} = 6\pi(1.0\times 10^{-3}\ \text{N}\cdot\text{s/m}^2)(1.0\times 10^{-6}\ \text{m})(30\times 10^{-6}\ \text{m/s}) = 5.7\times 10^{-13}\ \text{N}$$

(b)

$$a = \frac{F_{\text{prop}}}{m} = \frac{F_{\text{prop}}}{\rho V} = \frac{F_{\text{prop}}}{\rho\frac{4}{3}\pi R^2}$$

For a paramecium

$$a = \frac{9.4\times 10^{-10}\ \text{N}}{(1000\ \text{kg/m}^3)\frac{4}{3}\pi(50\times 10^{-6}\ \text{m})^3} = 1.8\ \text{m/s}^2$$

For an *E. coli* bacterium

$$a = \frac{5.7\times 10^{-13}\ \text{N}}{(1000\ \text{kg/m}^3)\frac{4}{3}\pi(1.0\times 10^{-6}\ \text{m})^3} = 135\ \text{m/s}^2$$

Assess: The two accelerations are within a factor of two of each other.

6.66. Model: Use the linear model of drag.
Visualize: The viscosity of air is $\eta = 2.0\times10^{-5}\ \text{N}\cdot\text{s/m}^2$ at $25°\text{C}$. The density of dust is $\rho = 2700\ \text{kg/m}^3$.
Solve:
(a) At terminal speed the net force is zero.

$$\sum F_y = D - mg = 0 \Rightarrow 6\pi\eta R v_{\text{term}} = mg$$

$$v_{\text{term}} = \frac{mg}{6\pi\eta R}$$

(b)

$$v_{\text{term}} = \frac{\rho V g}{6\pi\eta R} = \frac{\rho\left(\frac{4}{3}\pi R^3\right)g}{6\pi\eta R} = \frac{\rho\frac{4}{3}R^2 g}{6\eta} =$$

$$\frac{(2700\ \text{kg/m}^3)\frac{4}{3}(25\times10^{-6}\ \text{m})^2(9.8\ \text{m/s}^2)}{6(2.0\times10^{-5}\ \text{N}\cdot\text{s/m}^2)} = 0.18375\ \text{m/s}$$

$$\Delta t = \frac{\Delta y}{v_{\text{term}}} = \frac{300\ \text{m}}{0.18375\ \text{m/s}} = 1633\ \text{s} = 27\ \text{min}$$

Assess: 27 min sounds like a long time, but isn't too surprising for dust 300 m in the air.

6.67. Solve: (a) A 1.0 kg block is pulled across a level surface by a string, starting from rest. The string has a tension of 20 N, and the block's coefficient of kinetic friction is 0.50. How long does it take the block to move 1.0 m?
(b) Newton's second law for the block is

$$a_x = a = \frac{(F_{\text{net}})_x}{m} = \frac{T - f_k}{m} = \frac{T - \mu_k n}{m} \qquad a_y = 0\ \text{m/s}^2 = \frac{(F_{\text{net}})_y}{m} = \frac{n - F_G}{m} = \frac{n - mg}{m}$$

where we have incorporated the friction model into the first equation. The second equation gives $n = mg$. Sub-stituting this into the first equation gives

$$a = \frac{T - \mu_k mg}{m} = \frac{20\ \text{N} - 4.9\ \text{N}}{1.0\ \text{kg}} = 15.1\ \text{m/s}^2$$

Constant acceleration kinematics gives

$$x_1 = x_0 + v_0\Delta t + \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}a(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2x_1}{a}} = \sqrt{\frac{2(1.0\ \text{m})}{15.1\ \text{m/s}^2}} = 0.36\ \text{s}$$

6.68. Solve: (a) A 15,000 N truck starts from rest and moves down a 15° hill with the engine providing a 12,000 N force in the direction of the motion. Assume the frictional force between the truck and the road is very small. If the hill is 50 m long, what will be the speed of the truck at the bottom of the hill?
(b) Newton's second law is

$$\sum F_y = n_y + F_{Gy} + f_y + E_y = ma_y = 0$$

$$\sum F_x = n_x + F_{Gx} + f_x + E_x = ma_x \Rightarrow 0\ \text{N} + F_G\sin\theta + 0\ \text{N} + 12{,}000\ \text{N} = ma$$

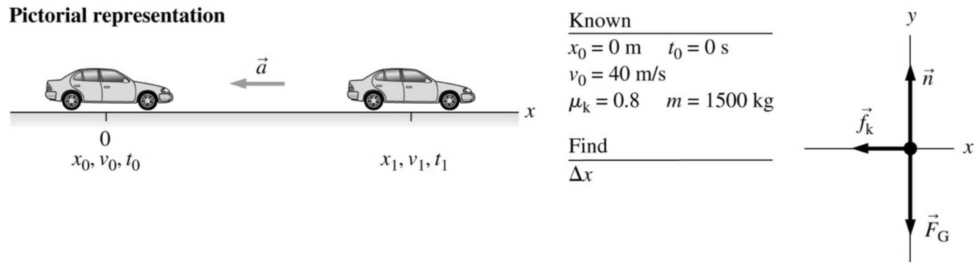
$$\Rightarrow a = \frac{mg\sin\theta + 12{,}000\ \text{N}}{m} = \frac{(15{,}000\ \text{N})\sin15° + 12{,}000\ \text{N}}{(15{,}000\ \text{N}/9.8\ \text{m/s}^2)} = 10.4\ \text{m/s}^2$$

where we have calculated the mass of the truck from the gravitational force on it. Using the constant-acceleration kinematic equation $v_x^2 - v_0^2 = 2ax$,

$$v_x^2 = 2a_x x = 2(10.4\ \text{m/s}^2)(50\ \text{m}) \Rightarrow v_x = 32\ \text{m/s}$$

6.69. Solve: (a) A driver traveling at 40 m/s in her 1500 kg auto slams on the brakes and skids to rest. How far does the auto slide before coming to rest?

(b)



(c) Newton's second law is

$$\sum F_y = n_y + (F_G)_y = n - mg = ma_y = 0\text{ N} \qquad \sum F_x = -0.80n = ma_x$$

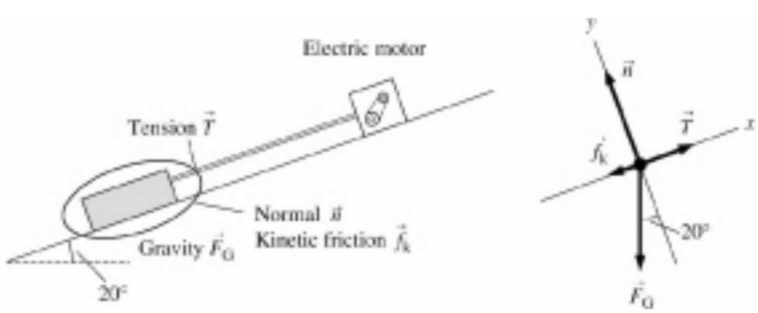
The y-component equation gives $n = mg = (1500\text{ kg})(9.8\text{ m/s}^2)$. Substituting this into the x-component equation yields

$$(1500\text{ kg})a_x = -0.80(1500\text{ kg})(9.8\text{ m/s}^2) \Rightarrow a_x = (-0.80)(9.8\text{ m/s}^2) = -7.8\text{ m/s}^2$$

Using the constant-acceleration kinematic equation $v_1^2 = v_0^2 + 2a\Delta x$, we find

$$\Delta x = -\frac{v_0^2}{2a} = -\frac{(40\text{ m/s})^2}{2(-7.8\text{ m/s}^2)} = 102\text{ m}$$

6.70. Solve: (a) A 20.0 kg wooden crate is being pulled up a 20° wooden incline by a rope that is connected to an electric motor. The crate's acceleration is measured to be 2.0 m/s^2. The coefficient of kinetic friction between the crate and the incline is 0.20. Find the tension T in the rope.
(b)



(c) Newton's second law for this problem in the component form is

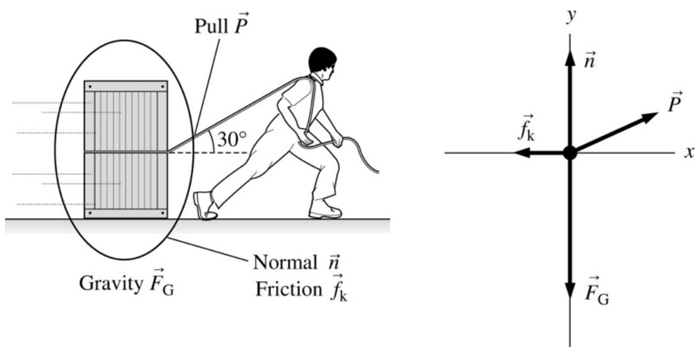
$$(F_{net})_x = \sum F_x = T - 0.20n - (20\text{ kg})(9.80\text{ m/s}^2)\sin 20° = (20\text{ kg})(2.0\text{ m/s}^2)$$

$$(F_{net})_y = \sum F_y = n - (20\text{ kg})(9.80\text{ m/s}^2)\cos 20° = 0\text{ N}$$

Solving the y-component equation, $n = 184.18\text{ N}$. Substituting this value for n in the x-component equation yields $T = 144\text{ N}$.

6.71. Solve: (a) You wish to pull a 20 kg wooden crate across a wood floor $(\mu_k = 0.20)$ by pulling on a rope attached to the crate. Your pull is 100 N at an angle of $30°$ above the horizontal. What will be the acceleration of the crate?

(b)



(c) Newton's equations and the model of kinetic friction are

$$\sum F_x = n_x + P_x + (F_G)_x + f_x = 0\text{ N} + (100\text{ N})\cos 30° + 0\text{ N} - f_k = (100\text{ N})\cos 30° - f_k = ma_x$$

$$\sum F_y = n_y + P_y + (F_G)_y + f_y = n + (100\text{ N})\sin 30° - mg - 0\text{ N} = ma_y = 0\text{ N}$$

$$f_k = \mu_k n$$

From the y-component equation, $n = 150\text{ N}$. From the x-component equation and using the model of kinetic friction with $\mu_k = 0.20$,

$$(100\text{ N})\cos 30° - (0.20)(150\text{ N}) = (20\text{ kg})a_x \Rightarrow a_x = 2.8\text{ m/s}^2$$

6.72. Model: The acceleration of the block is not constant before it gets to L; it increases until L and is then constant (with increasing v).

Visualize: Since the coefficient of friction is a function of the roughness of the two surfaces, it is understandable that it could be a function of x and not t.

Solve:

(a) Use the chain rule.

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = v_x\frac{dv_x}{dx}$$

(b)

$$\sum F_x = F_0 - f_k = F_0 - \mu_k mg = F_0 - \mu_0\left(1 - \frac{x}{L}\right)mg = ma_x$$

$$a_x = \frac{F_0}{m} + \mu_0 g\left(\frac{x}{L} - 1\right)$$

Now examine the result in part (a).

$$\int a_x dx = \int v_x dv_x$$

$$\int \frac{F_0}{m} + \mu_0 g\left(\frac{x}{L} - 1\right)dx = \int v_x dv_x$$

$$\frac{F_0}{m}x + \mu_0 g\left(\frac{x^2}{2L} - x\right) = \frac{1}{2}v_x^2 + C$$

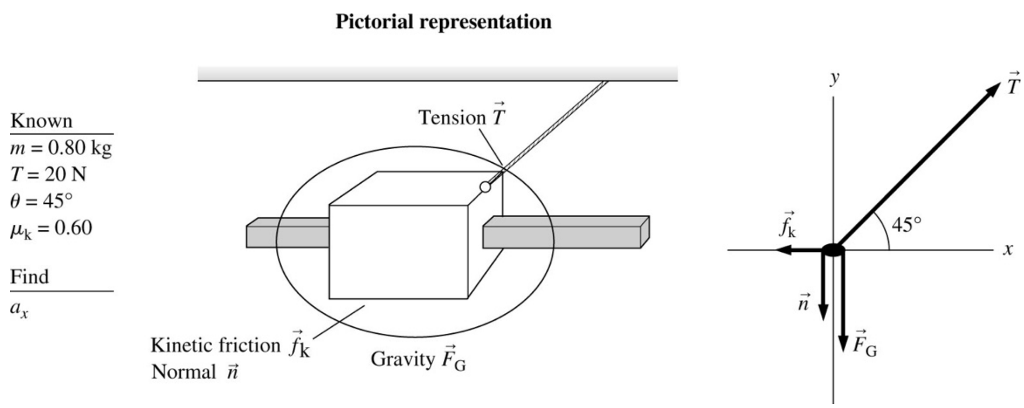
The constant of integration C is zero because $v_x = 0$ at $x = 0$.

$$v_x(x) = \sqrt{2\left[\frac{F_0}{m}x + \mu_0 g\left(\frac{x^2}{2L} - x\right)\right]}$$

$$v_x(L) = \sqrt{2\left[\frac{F_0}{m}L + \mu_0 g\left(\frac{L^2}{2L} - L\right)\right]} = \sqrt{2\left[\frac{F_0}{m}L + \mu_0 g\left(-\frac{L}{2}\right)\right]} = \sqrt{L\left(\frac{2F_0}{m} - \mu_0 g\right)}$$

Assess: Check dependencies; we expect $v_x(L)$ to increase with *L* and decrease with increasing *m*, μ_0, and *g*.

6.73. Model: We will model the shuttle as a particle and assume the elastic cord to be massless. We will also use the model of kinetic friction for the motion of the shuttle along the square steel rail.
Visualize:



Solve: The upward tension component $T_y = T\sin 45° = 14.1\text{ N}$ is larger than the gravitational force on the shuttle. Consequently, the elastic cord pulls the shuttle up against the rail and the rail's normal force pushes downward. Newton's second law in component form is

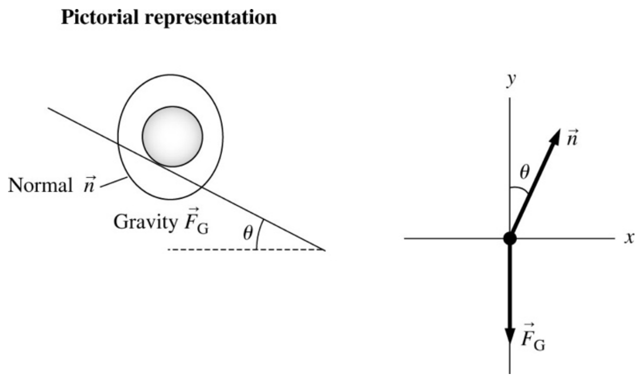
$$(F_{\text{net}})_x = \sum F_x = T_x + (f_k)_x + (n)_x + (F_G)_x = T\cos 45° - f_k + 0\text{ N} + 0\text{ N} = ma_x = ma_x$$
$$(F_{\text{net}})_y = \sum F_y = T_y + (f_k)_y + (n)_y + (F_G)_y = T\sin 45° + 0\text{ N} - n - mg = ma_y = 0\text{ N}$$

The model of kinetic friction is $f_k = \mu_k n$. We use the *y*-component equation to get an expression for *n* and hence f_k. Substituting into the *x*-component equation and using the value of μ_k in Table 6.1 gives us

$$a_x = \frac{T\cos 45° - \mu_k(T\sin 45° - mg)}{m}$$
$$= \frac{(20\text{ N})\cos 45° - (0.60)[+(20\text{ N})\sin 45° - (0.800\text{ kg})(9.80\text{ m/s}^2)]}{0.800\text{ kg}} = 13\text{ m/s}^2$$

Assess: The *x*-component of the tension force is 14.1 N. On the other hand, the net force on the shuttle in the *x*-direction is $ma_x = (0.800\text{ kg})(13.0\text{ m/s}^2) = 10.4\text{ N}.$ This value for *ma* is reasonable since a part of the 14.1 N tension force is used up to overcome the force of kinetic friction.

6.74. Model: Assume the ball is a particle on a slope, and that the slope increases as the *x*-displacement increases. Assume that there is no friction and that the ball is being accelerated to the right so that it remains at rest on the slope.
Visualize: Although the ball is on a slope, it is accelerating to the right. Thus we'll use a coordinate system with horizontal and vertical axes.



Solve: Newton's second law is

$$\sum F_x = n\sin\theta = ma_x \qquad \sum F_y = n\cos\theta - F_G = ma_y = 0\text{ N}$$

Combining the two equations, we get

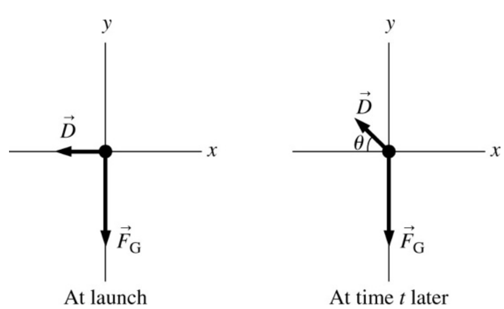
$$ma_x = \frac{F_G}{\cos\theta}\sin\theta = mg\tan\theta \Rightarrow a_x = g\tan\theta$$

The curve is described by $y = x^2$. Its slope a position x is $\tan\theta$, which is also the derivative of the curve. Hence,

$$\frac{dy}{dx} = \tan\theta = 2x \Rightarrow a_x = (2x)g$$

(b) The acceleration at $x = 0.20\text{ m}$ is $a_x = (2)(0.20)(9.8\text{ m/s}^2) = 3.9\text{ m/s}^2$.

6.75. Visualize:



Solve: (a) The horizontal velocity as a function of time is determined by the horizontal net force. Newton's second law as the x-direction gives

$$(F_{\text{net}})_x = ma_x = -D\cos\theta = -bv\cos\theta = -bv_x$$

Note that D points opposite to v, so the angle θ with the x-axis is the same for both vectors, and the x components of both vectors have the same $\cos\theta$ term. As the particle changes direction as it falls, the evolution of the horizontal motion depends only on the horizontal component of the velocity.
Thus

$$m\frac{dv_x}{dt} = -bv_x$$

Separating and integrating, $\displaystyle\int_{v_0}^{v_x(t)} \frac{dv_x}{v_x} = 2\,\frac{b}{m}\int_0^t dt$

$$\Rightarrow \ln\left(\frac{v_x(t)}{v_0}\right) = -\frac{b}{m}t$$

Solving,

$$v_x(t) = v_0 e^{-\frac{bt}{m}} = v_0 e^{-\frac{6\pi\eta Rt}{m}}$$

(b) The time to reach $v(t) = \frac{1}{2}v_0$ is found by solving for the time when

$$\frac{1}{2}v_0 = v_0 e^{-\frac{6\pi\eta Rt}{m}}$$

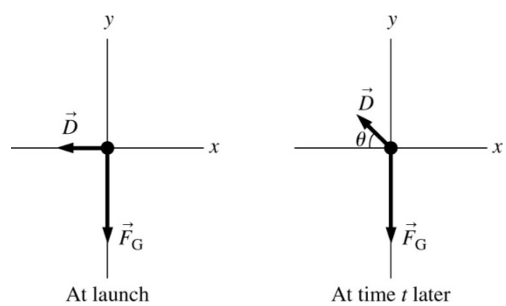
Hence

$$t = \frac{m\ln(2)}{6\pi\eta R}$$

With $\eta = 1.0\times10^{-3}\ \mathrm{Ns/m^2}$, $R = 2.0\times10^{-2}\ \mathrm{m}$, and $m = 0.033\ \mathrm{kg}$, we get $t = 61\ \mathrm{s}$.

Assess: The magnitude of the acceleration is $a_x = \frac{6\pi\eta R}{m}v = (1.1\times10^{-2}\ \mathrm{s^{-1}})v_x$. This is a small fraction of the velocity, so a time of about one minute to slow to half the initial speed is reasonable.

6.76. Visualize:



Solve: (a) Using the chain rule, $a_x = \frac{dv_x}{dt} = \left(\frac{dv_x}{dx}\right)\left(\frac{dx}{dt}\right) = v_x\frac{dv_x}{dx}$.

(b) The horizontal motion is determined by using Newton's second law in the horizontal direction. Using the free-body diagram at a later time *t*,

$$(F_{\text{net}})_x = ma_x = -D\cos\theta = -bv\cos\theta = -bv_x$$

Note that since D points opposite to v, the angle θ with the *x*-axis is the same for both vectors, and the *x*-components of both vectors have the same $\cos\theta$ term. Thus

$$ma_x = mv_x\frac{dv_x}{dx} = -bv_x$$

Separating and integrating, $\displaystyle\int_{v_0}^{v_x(x)} dv_x = -\frac{b}{m}\int_{x_0}^{x(t)} dx$

$$\Rightarrow v_x(x) - v_0 = -\frac{b}{m}(x(t) - x_0)$$

Solving with $x_0 = 0$,

$$v_x(x) = v_0 - \frac{b}{m}x = v_0 - \frac{6\pi\eta R}{m}x$$

(c) The marble stops after traveling a distance *d* when $v_x(d) = 0$.

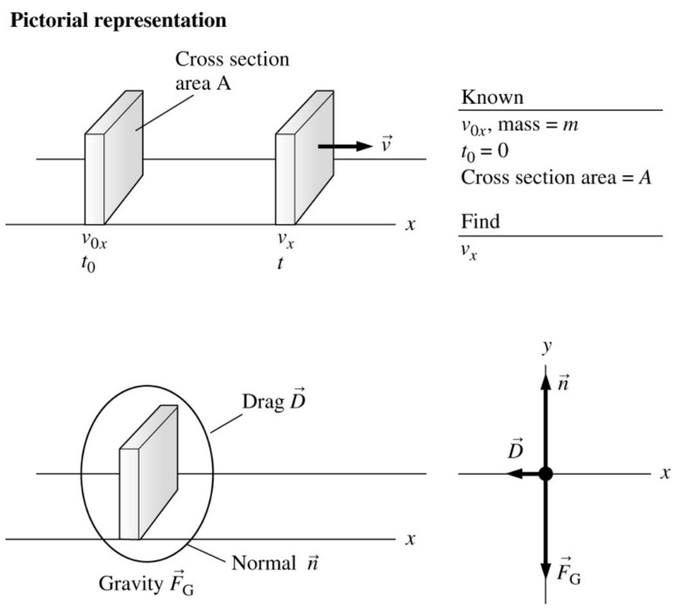
Hence
$$v_0 = \frac{6\pi\eta R}{m}d$$
$$\Rightarrow d = \frac{mv_0}{6\pi\eta R}$$

Using $v_0 = 10$ cm/s, $R = 0.50$ cm, $m = 1.0\times10^{-3}$ kg, and using $\eta = 1.0\times10^{-3}$ Ns/m,

$$d = \frac{(1.0\times10^{-3}\text{ kg})(0.10\text{ m/s})}{6\pi(1.0\times10^{-3}\text{ Ns/m}^2)(5.0\times10^{-3}\text{ m})} = 1.1\text{ m}$$

Assess: The equation for *d* indicates that a marble with a faster initial velocity travels a farther distance.

6.77. Model: We will model the object as a particle, and use the model of drag.
Visualize:



Solve: (a) We cannot use the constant-acceleration kinematic equations since the drag force causes the acceleration to change with time. Instead, we must use $a_x = dv_x/dt$ and integrate to find v_x. Newton's second law for the object is

$$(F_{\text{net}})_x = \sum F_x = D = -\frac{1}{2}C\rho Av_x^2 = ma_x = m\frac{dv_x}{dt}$$

This can be written

$$\frac{dv_x}{v_x^2} = \frac{C\rho A}{2m}dt$$

We can integrate this from the start $(v_{0x}\text{ at }t = 0)$ to the end $(v_x\text{ at }t)$:

$$\int_{v_{0x}}^{v_x}\frac{dv_x}{v_x^2} = \frac{C\rho A}{2m}\int_0^t dt \Rightarrow -\frac{1}{v_x} + \frac{1}{v_{0x}} = \frac{C\rho A}{2m}t$$

Solving for v_x gives

$$v_x = \frac{v_{0x}}{1 + C\rho A v_{0x} t/2m}$$

(b) Using $A = (1.6\text{ m})(1.4\text{ m}) = 2.24\text{ m}^2$, $v_{0x} = 20\text{ m/s}$, and $m = 1500\text{ kg}$, we get

$$v_x = \frac{20\text{ m/s}}{1 + \dfrac{(0.35)(1.2\text{ kg/m}^3)(2.24\text{ m}^2)t(20\text{ m/s})}{2 \times 1500}} = \frac{20}{1 + 0.006272t} \Rightarrow t = \left(\frac{1}{0.006272}\right)\left(\frac{20\text{ m/s}}{v_x} - 1\right)$$

where *t* is in seconds. We can now obtain the time *t* for $v = 10\text{ m/s}$:

$$t = \left(\frac{1}{0.006272}\right)\left(\frac{20\text{ m/s}}{10\text{ m/s}} - 1\right) = 159.44\left(\frac{20}{10} - 1\right) = 160\text{ s}$$

When $v_x = 5\text{ m/s}$, then $t = 480\text{ s}$.

(c) If the only force acting on the object was kinetic friction with, say, $\mu_k = 0.05$, that force would be (0.05) (1500 kg) $(9.8\text{ m/s}^2) = 735\text{ N}$. The drag force at an average speed of 10 m/s is $D = \frac{1}{4}(2.24)(10)^2\text{ N} = 56\text{ N}$. We conclude that it is not reasonable to neglect the kinetic friction force.