Rotational Motion

Arc Length **S = θ r**
Circumference **C = 2π r** ***Circumference is an <u>Arc Length</u>***

1 revolution = 1 rotation = 360 degrees = *<u>2π radians</u>*

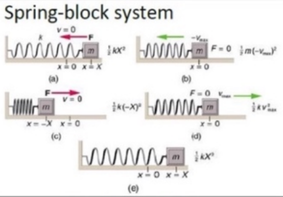
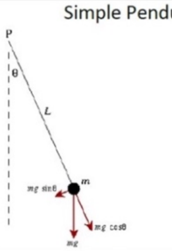
Constant Acceleration	
Linear Kinematic Equations	**Rotational Kinematic Equations**
Units (m, m/s, m/s², s)	**Units (rad, rad/s, rad/s², s)**
$V = V_0 + at$	$\omega = \omega_0 + \alpha t$
$X = X_0 + V_0 t + \frac{1}{2} a t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
$V^2 = {V_0}^2 + 2a\Delta X$	$\omega^2 = {\omega_0}^2 + 2\alpha\Delta\theta$
$\Delta X = \frac{1}{2}(V + V_0)\, t$	$\Delta\theta = \frac{1}{2}(\omega + \omega_0)\, t$

How linear (tangential) velocity relates to angular speed → $v = \omega * r$

How linear (tangential) acceleration relates to angular acceleration → $a = \alpha * r$

$$\omega = \omega_0 + \alpha t \quad \rightarrow \quad (\omega * r) = (\omega_0 * r) + (\alpha * r)t \quad \rightarrow \quad v = v_0 + at$$

Period is the time to complete one cycle

Type of simple harmonic oscillator	Angular frequency & Period equations	How do we get there?
Spring-block system 	$\omega = \sqrt{\frac{k}{m}}$ and $T = 2\pi\sqrt{\frac{m}{k}}$ Where m is the mass of the block in $[kg]$ and k is the spring constant in $\left[\frac{N}{m}\right]$	$\Sigma F = ma$ $F_{Spring} = ma$ $kx = ma$ **For SHM (Simple Harmonic Motion)** $a = \omega^2 x$ $kx = m(\omega^2 x)$ $\omega^2 = \frac{k}{m}$ **therefore** $\omega = \sqrt{\frac{k}{m}}$
Simple Pendulum 	$\omega = \sqrt{\frac{g}{L}}$ and $T = 2\pi\sqrt{\frac{L}{g}}$ Where g is the gravitational acceleration at that location in $\left[\frac{m}{s^2}\right]$ and L is the length of the pendulum from the pivot point to the center of mass of the bob in $[m]$.	$\sum \tau = r \times F_{\perp} = I\alpha$ $F_{\perp} = F_g sin\theta$ creating a negative (clockwise) torque $I = mL^2$ for a point mass a distance L away from the pivot $\alpha = -\omega^2\theta$ for SHM in rotation $-Lmgsin\theta = mL^2(-\omega^2\theta)$ For small angles $sin\theta \approx \theta$, called the Small Angle Approximation, therefore: $\frac{g}{L} = \omega^2$ therefore $\omega = \sqrt{\frac{g}{L}}$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Period of a Spring

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

Period of a Pendulum

$$T_p = 2\pi\sqrt{\frac{l}{g}}$$

In order to change direction you have to have a Net Force (ΣF) acting on the object. Things moving in a circle must have a force directed to the center of the circle (can be that force could be caused by gravity, friction, tension, ect.

Newton's 2nd Law
$\Sigma F = ma$

Rotational version of Newton's 2nd Law
$\Sigma \tau = I\alpha$

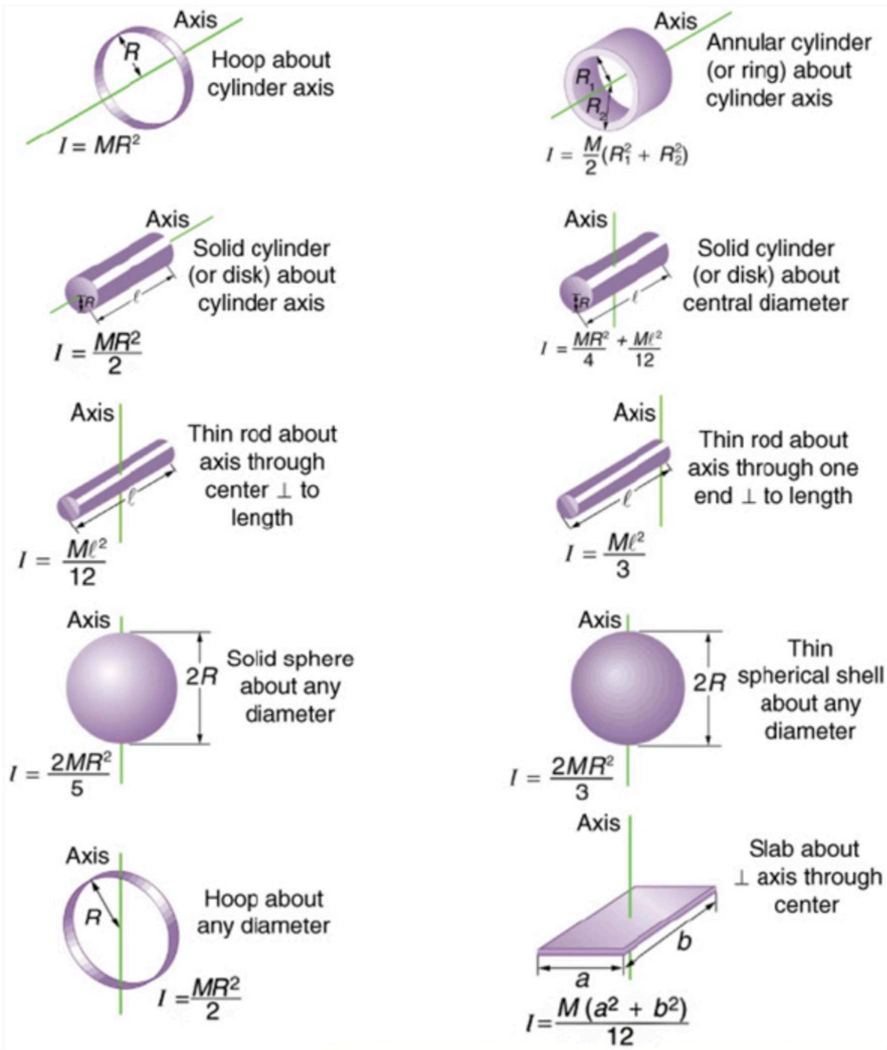
$$\tau = torque, I = moment\ of\ inertia, \alpha = angular\ acceleration$$
$$\tau = r_{\perp}F$$

Torque is caused when you are a distance (radius) from rotation point (center of circle) and you apply a force. Torque's units are Newton*meters.
Opening a door you push (force) on the doorknob of the door (distance from center).

Moment of inertia is like mass, it is some version of $I = \_\_MR^2$

A single mass rotating around something has a Moment of Inertia of $I = MR^2$

DO NOT NEED TO MEMORIZE they would give you this information.



The more of the mass that is located a radius away the closer to $\boldsymbol{I = MR^2}$ the Moment of Inertia becomes. The closer to the center the smaller the fraction in front of $\boldsymbol{MR^2}$**.** The bigger $\boldsymbol{I}$, is the harder it is to start or stop rotating something is.

Momentum	Angular momentum	Change of Angular Momentum
$\boldsymbol{p = mv}$	$\boldsymbol{L = I\omega}$	$\boldsymbol{\Delta L = \tau \Delta t}$
Kinetic Energy	Rotational Kinetic Energy	
$\boldsymbol{K = \frac{1}{2}mv^2}$	$\boldsymbol{K = \frac{1}{2}I\omega^2}$	