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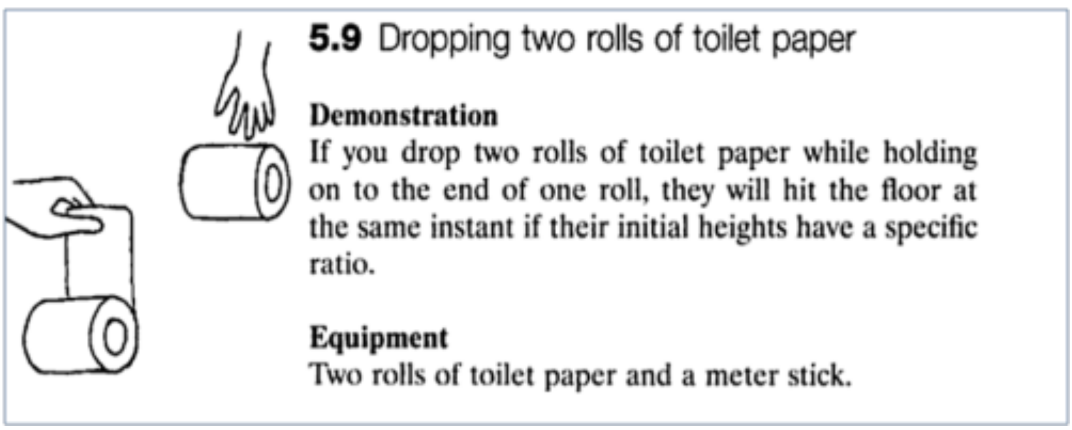
APLUSPHYSICS

UNROLLING TOILET PAPER

DECEMBER 27, 2010 | 4 COMMENTS

In his Dec. 17 Action-Reaction blog post titled "Falling Rolls," one of my heroes of physics instruction, Frank Noschese, details an exercise from Robert Ehrlich's book Why Toast Lands Jelly-Side Down.



5.9 Dropping two rolls of toilet paper

Demonstration
If you drop two rolls of toilet paper while holding on to the end of one roll, they will hit the floor at the same instant if their initial heights have a specific ratio.

Equipment
Two rolls of toilet paper and a meter stick.

The exercise, a rotational motion problem that challenges students to find the ratio of heights at which you can drop two identical toilet paper rolls, one dropped regularly, the other dropped by holding onto the end of the paper and letting it unroll, such that the two rolls hit the ground at the same time. It's a terrific, easy-to-replicate and demonstrate problem that pulls together a great number of rotational motion skills –> finding the moment of inertia, applying the parallel-axis theorem, identifying forces and torques from free body diagrams, and converting angular acceleration to linear acceleration. My students dove into the challenge with zest!

To begin the exercise, we set our variables (H=height for dropped roll, h=height for unrolled roll, r = inner diameter, R = outer diameter), then identified the time it takes for the dropped roll to hit the ground using standard kinematics:

$$t_{drop} = \sqrt{\frac{2H}{g}}$$

Next, we did the same thing for the unrolling toilet paper roll:

$$t_{unroll} = \sqrt{\frac{2h}{a}}$$

Of course, if we want them to hit at the same time, the times must be equal, therefore we can show:

$$\frac{H}{h} = \frac{g}{a}$$

Obviously, what we really need to focus our efforts on is finding the linear acceleration of the unrolling roll. To save ourselves some time, we started by looking up the moment of inertia for a cylinder:

$$I = \frac{1}{2}M(r^2 + R^2)$$

Using the parallel-axis theorem to account for the unrolled roll rotating about its outer radius we find:

$$I = \frac{1}{2}M(r^2 + R^2) + MR^2 = \frac{1}{2}M(r^2 + 3R^2)$$

Next, we can use a free body diagram to identify the net torque on the roll as MgR, and use Newton's 2nd Law for Rotational Motion to find the angular acceleration:

$$\tau_{net} = I\alpha \Rightarrow \alpha = \frac{\tau_{net}}{I} = \frac{MgR}{0.5*M(r^2+3R^2)} = \frac{2gR}{r^2+3R^2}$$

Since linear acceleration can be found from angular acceleration multiplied by the radius of rotation (R):

$$a = \alpha R = \frac{2gR^2}{r^2+3R^2}$$

Finally, since we're looking for the ratio of the dropped height to the unrolled height:

$\frac{H}{h} = \frac{g}{a} = \frac{g}{\frac{2gR^2}{r^2+3R^2}} = \frac{r^2+3R^2}{2R^2} = \frac{3}{2} + \frac{r^2}{2R^2}$

This conflicts with the results from Noschese's class, where they derived $\frac{H}{h} = 2 + \frac{r^2}{R^2}$, however, their demonstration based on their results is very convincing. Let's take a look at the difference in ratios using the two derivations:

For a toilet paper roll of inner diameter .0095m and outer diameter R=.035m (our school rolls from the janitor supply closet):

$\frac{H}{h} = 2 + \frac{r^2}{R^2} = 2 + \frac{.0095m^2}{.035m^2} = 2.074$

$\frac{H}{h} = \frac{3}{2} + \frac{r^2}{2R^2} = 1.5 + \frac{.0095m^2}{2*.035m^2} = 1.54$

It appears that our discrepancies aren't just differing mathematical representations of the same formula, but that we have a significant difference in our derivations.

In looking over our assumptions, we assumed no air resistance, and also that the unrolling toilet paper roll rotates about its outer radius (is this really true)? I wonder what assumptions were made in Noschese's class that may account for these differences. It will be interesting to get his class's perspective on the problem, and provides a great practical study for our students of different approaches to a problem, and the importance of understanding the ramifications of assumptions made in beginning a problem solving exercise!

Update: it appears our calculations are correct. Check out our high-speed video confirmation!

Slow Motion Toilet Paper Falling

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