

# Introduction to Vectors & Trigonometry

Start with "3 + 4" = ??

→→→→→ 3+4=5!

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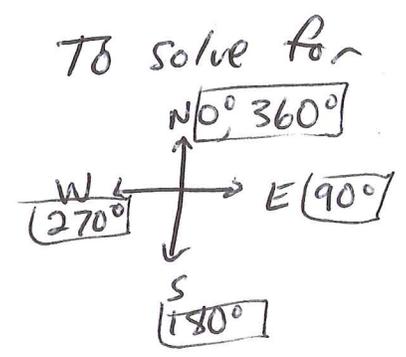
To determine quantities in a consistent manner, we need a set of rules to explain them universally. (Remember the puzzle activity?)

Many times in physics, we use VECTOR quantities; these have both a magnitude and a direction.

SCALAR quantities only have a magnitude.

For example: Velocity is a vector quantity, speed is a scalar quantity. - Is "45 m/s" an example of speed or velocity?  
- How about "38 miles per hour due East"?

Other examples of vector quantities are acceleration, force and displacement. To solve for directions, we'll need a standardized coordinate system.

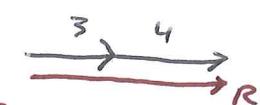


If we want to add vectors acting on an object at the same time (or "concurrently") we can do so several ways. Graphically, we add vectors by drawing them head to tail. The resultant of adding vectors together can be found by the tail of the first vector to the arrow tip of the last vector.

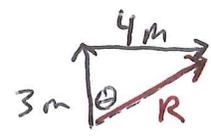
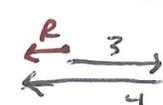
\* You'll need to know the Pythagorean Theorem, Trigonometry AND how to use your calculator!

Example 1:

$3m \rightarrow + 4m \rightarrow = \rightarrow ?$



$3m \rightarrow + \leftarrow 4m = \rightarrow ?$

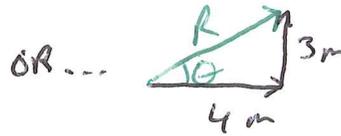


$R = \sqrt{3^2 + 4^2} = 5$

$\theta = \tan^{-1}(\frac{4}{3}) = 53^\circ$

So... 5m @ 53° East of North

↑ THESE ↓ SAY THE SAME THING!



$R = \sqrt{3^2 + 4^2} = 5$

$\theta = \tan^{-1}(\frac{3}{4}) = 37^\circ$

5m @ 37° North of East

NOTE: We can always just record things as angles from North (0 to 360 degrees) or interior angles measured from a frame of reference. So when something is 37° North of East, start on the EAST axis and head North 37° from the East axis.

\* What if we want to counteract multiple forces by applying a single force on an object? That single force will be equal in magnitude to the RESULTANT of all the forces, but exactly opposite in direction. That "counteracting force" is known as the EQUILIBRANT force.

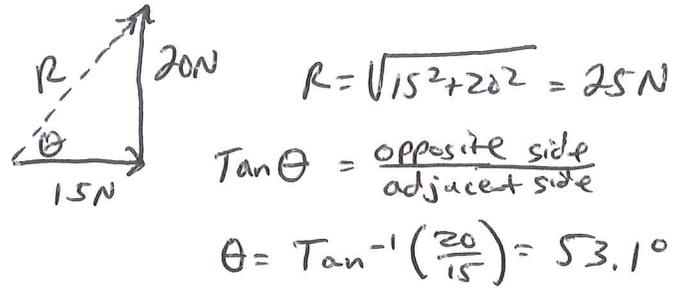
Do example with a chair being pulled by multiple people (or some one being pulled by ropes)

So an equilibrant force cancels out all the other forces acting on the object.

The idea of vector resolution is to break vectors into their vertical and horizontal components ... and then add the parts back together!

Sample 1: Assume you have a 15 Newton force pulling an object due East and a 20 Newton force acting concurrently (at the same time) pushing due North.

- Find a.) the resultant
- b.) the equilibrant.

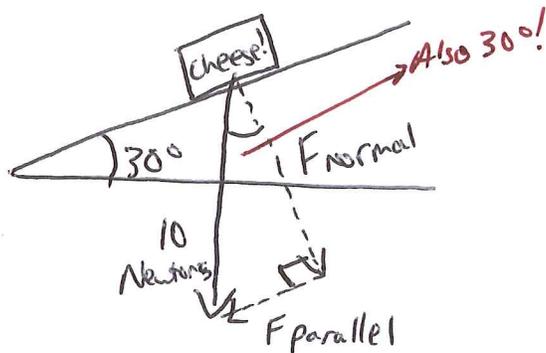


Note: ALWAYS draw a picture!!

- a.) Resultant is 25 Newtons @ 53° North of East
- b.) Equilibrant is 25 Newtons @ 53° South of West

Sample 2: Cheese on a hill! Assume a 10 Newton block of cheese is sitting on a hill with a slope of 30° from the horizontal. Find the components of the Cheese's weight that are perpendicular (or "normal") to the hill and parallel to the hill.

Again - Draw a picture!



Note: Think of extreme situations with a completely flat hill AND a completely vertical hill (a cliff)

$\text{Cosine } 30^\circ = \frac{F_{\text{normal}}}{10 \text{ Newtons}} \Rightarrow F_{\text{normal}} = 10 \cdot \text{Cosine } 30^\circ = 8.67 \text{ Newtons}$

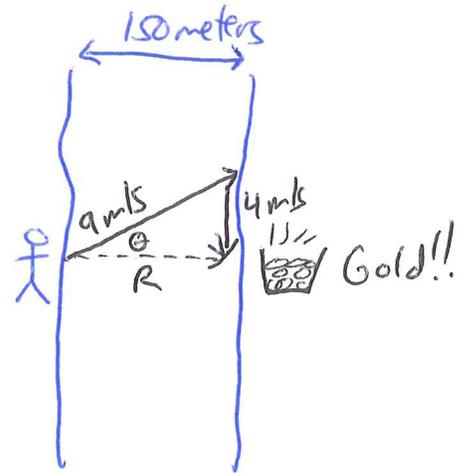
$\text{Sine } 30^\circ = \frac{F_{\text{parallel}}}{10 \text{ Newtons}} \Rightarrow F_{\text{parallel}} = 10 \cdot \text{sine } 30^\circ = 5.00 \text{ Newtons}$

### Sample 3 - A River Problem!

Hey - DRAW A PICTURE!

(4)

Assume you have a River (150m wide) with a current of 4 m/s flowing down stream. Also assume you have a swimmer who can swim 9 m/s in still water, and they want to reach a pot of Gold! directly across the river from where they start.



Note the resultant is not the hypotenuse of the triangle in this situation!

A.) At what angle should the person "aim" to end up directly across from their start?

$$\sin \theta = \frac{4 \text{ m/s}}{9 \text{ m/s}} \quad \theta = \sin^{-1}\left(\frac{4}{9}\right) = 26.4^\circ \text{ Upstream}$$

B.) What is the resultant velocity?  $A^2 + B^2 = c^2$  or ...

$$R = \sqrt{9^2 - 4^2} = 8.06 \text{ m/s due East}$$

$$B^2 = c^2 - A^2$$

C.) How much time will it take to cross the river?  
Velocity =  $\frac{\text{displacement}}{\text{time}} \Rightarrow \text{Time} = \frac{\text{Displacement}}{\text{Velocity}}$

$$T = \frac{150 \text{ meters}}{8.06 \text{ m/s}}$$

$$t = 18.6 \text{ seconds}$$

# Sample 4 - Solving with a Unit vector box

Three displacement vectors of:

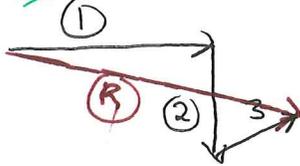
① 23 km east

② 15 km south

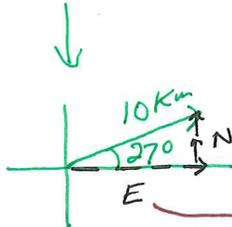
③ 10 km @ 27° North of East

	North	East
①	0	23 km
②	-15 km	0 km
③	$10 \text{ km} \sin 27^\circ = 4.54 \text{ km}$	$10 \text{ km} \cos 27^\circ = 8.91 \text{ km}$
<b>Overall</b>	<b>-10.5 km</b>	<b>31.9 km</b>

Overall



Draw a picture!



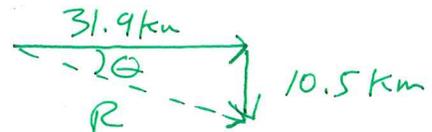
$$\sin \theta = \frac{N}{10 \text{ km}}$$

$$\cos \theta = \frac{E}{10 \text{ km}}$$

Now Draw a new triangle with the legs of that triangle from the "T chart".

$$R = \sqrt{(10.5)^2 + (31.9)^2}$$

$$R = \underline{\underline{33.6 \text{ km}}}$$



$$\theta = \tan^{-1} \left( \frac{10.5 \text{ km}}{31.9 \text{ km}} \right) = \underline{\underline{18^\circ}}$$

So the overall resultant is  
33.6 km @ 18° South of East