# 2 Kinematics in One Dimension

2.1 Uniform Motion

1. Sketch position-versus-time graphs (x versus t or y versus t) for the following motions. Include appropriate numerical scales along both axes. A small amount of computation may be necessary.

   a. A parachutist opens her parachute at an altitude of 1500 m. She then descends slowly to earth at a steady speed of 5 m/s. Start your graph as her parachute opens.


Slope $= -5 \frac{m}{s}$
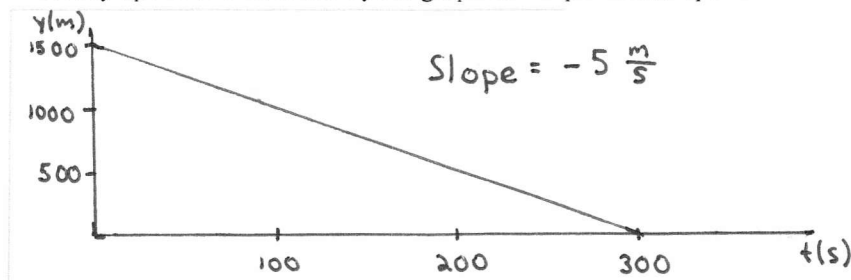
   b. Trucker Bob starts the day 120 miles west of Denver. He drives east for 3 hours at a steady 60 miles/hour before stopping for his coffee break. Let Denver be located at $x = 0$ mi and assume that the x-axis points to the east.
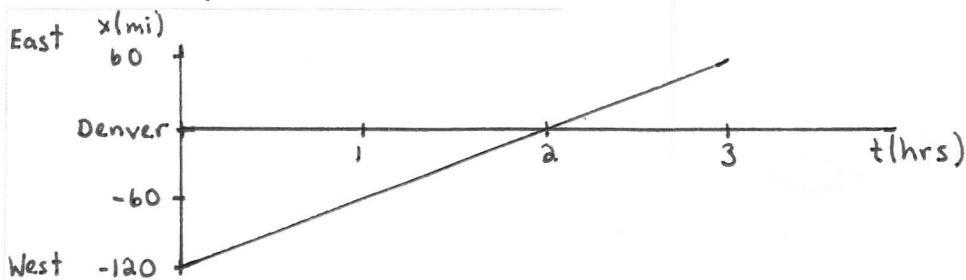
   c. Quarterback Bill throws the ball to the right at a speed of 15 m/s. It is intercepted 45 m away by Carlos, who is running to the left at 7.5 m/s. Carlos carries the ball 60 m to score. Let $x = 0$ m be the point where Bill throws the ball. Draw the graph for the *football*.
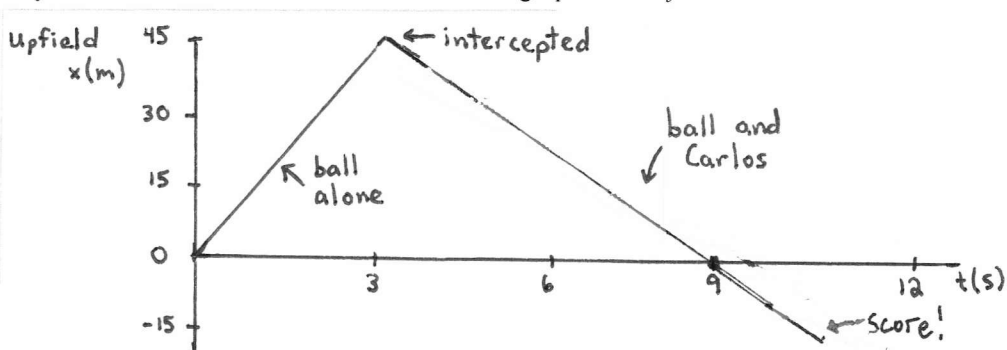
2. The figure shows a position-versus-time graph for the motion of objects A and B that are moving along the same axis.
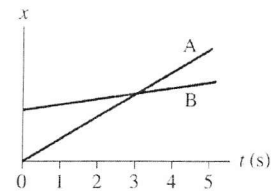

a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.

> At t=1s, the slope of the line for A is greater than that for object B. Therefore, object A's speed is greater. (Both are positive slopes.)

b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

> No, the speeds are never the same. Each has a constant speed (constant slope) and A's speed is always greater.

3. Interpret the following position-versus-time graphs by writing a short "story" about what is happening. Your stories should make specific references to the *speeds* of the moving objects, which you can determine from the graphs. Assume that the motion takes place along a horizontal line.

a.
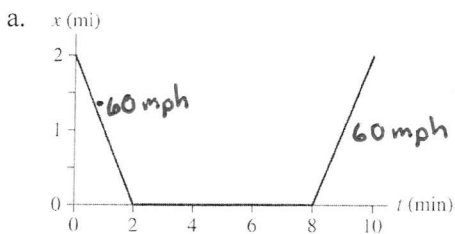
> On a quick trip on the interstate, Jimmy drives west for two miles at 60 mph, stops at an exit for 6 min, then returns east at 60 mph to where he started.

b.
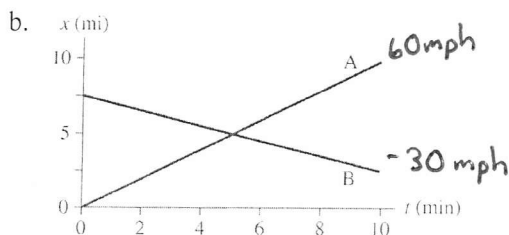
> A takes the interstate east for 10 mi at 60 mph. B starts 7.5 mi east and takes a parallel slower road west at 30 mph during the same time.

c.
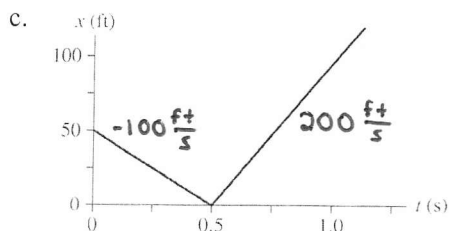
> Starting 10 ft in front of home plate, Bob pitched a change-up to Sammy at 100 ft/s. Sammy drove it straight back at him at 200 ft/s.

2.2 Instantaneous Velocity

4. Draw both a position-versus-time graph *and* a velocity-versus-time graph for an object at rest at $x = 1$ m.

Im   Constant Position

Zero Velocity
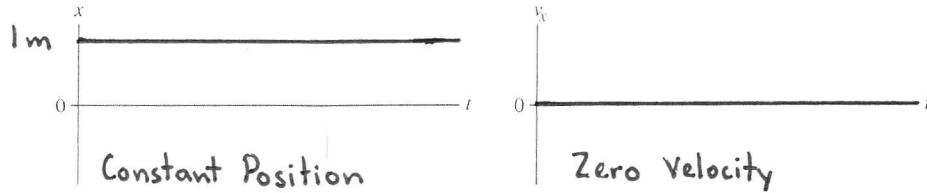
5. The figure shows the position-versus-time graphs for two objects, A and B, that are moving along the same axis.

   a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
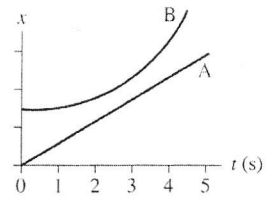
   A's speed is greater at $t=1s$.
   The slope of the tangent to B's
   curve at $t=1s$ is smaller than
   the slope of A's line.

   b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

   A and B have the same speed at just before $t= 3s$.
   At that time, the slope of the tangent to the curve
   representing B's motion is equal to the slope of
   the line representing A.

6. Below are six position-versus-time graphs. For each, draw the corresponding velocity-versus-time graph directly below it. A vertical line drawn through both graphs should connect the velocity v_s at time t with the position s at the *same* time t. There are no numbers, but your graphs should correctly indicate the *relative* speeds.

   a.

   b.

c.

d.

← identical → graphs

e.

f.
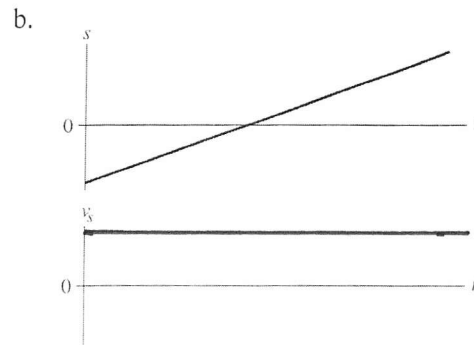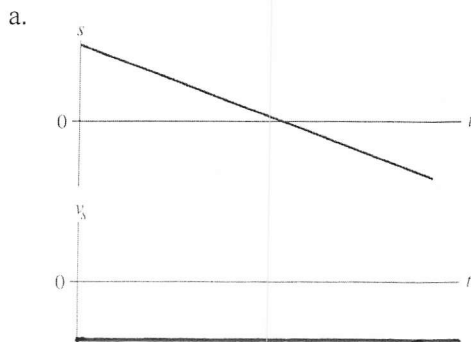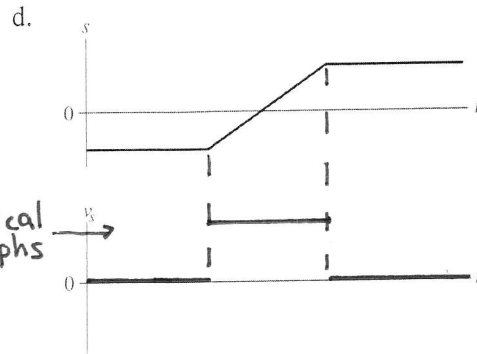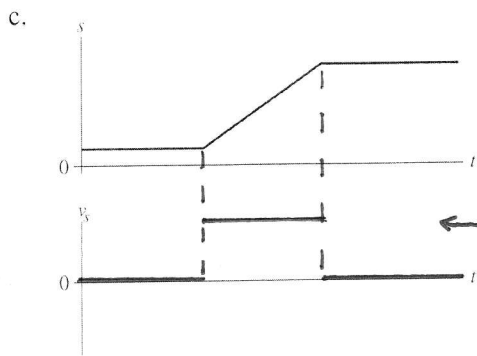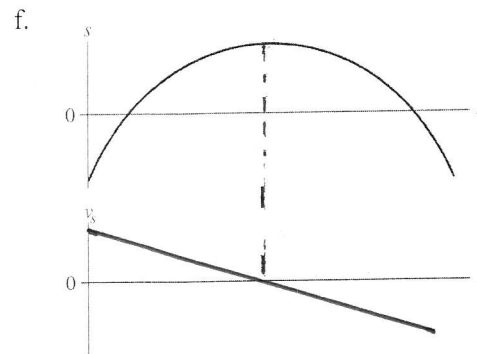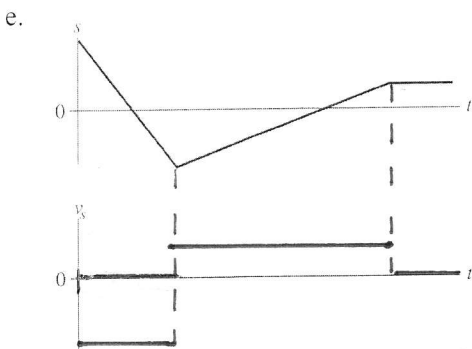
7. The figure shows a position-versus-time graph for a moving object. At which lettered point or points:

   a. Is the object *moving* the slowest? B

   b. Is the object moving the fastest? D

   c. Is the object at rest? A, C, E

   d. Does the object have a constant nonzero velocity? B, D
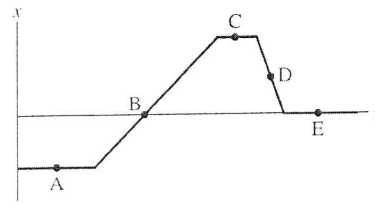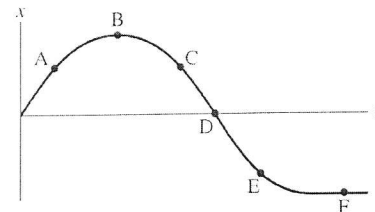
   e. Is the object moving to the left? D

8. The figure shows a position-versus-time graph for a moving object. At which lettered point or points:

   a. Is the object moving the fastest? D

   b. Is the object moving to the left? C, D, E

   c. Is the object speeding up? C

   d. Is the object slowing down? A, E

   e. Is the object turning around? B
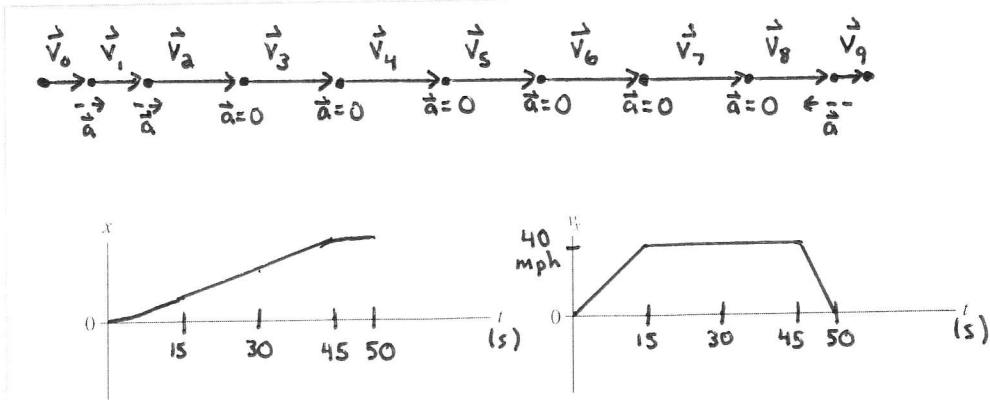
9. For each of the following motions, draw

- A motion diagam,
- A position-versus-time graph, and
- A velocity-versus-time graph.

a. A car starts from rest, steadily speeds up to 40 mph in 15 s, moves at a constant speed for 30 s, then comes to a halt in 5 s.

b. A rock is dropped from a bridge and steadily speeds up as it falls. It is moving at 30 m/s when it hits the ground 3 s later. Think carefully about the signs.
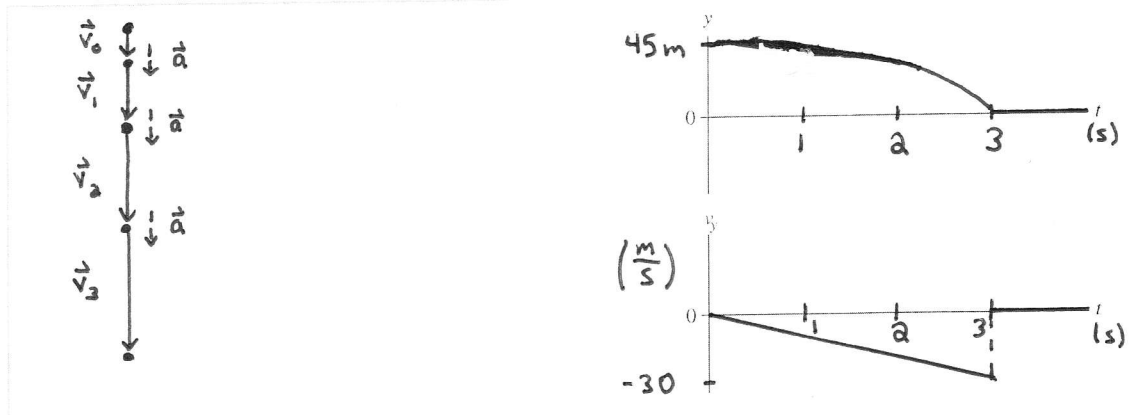
c. A pitcher winds up and throws a baseball with a speed of 40 m/s. One-half second later the batter hits a line drive with a speed of 60 m/s. The ball is caught 1 s after it is hit. From where you are sitting, the batter is to the right of the pitcher. Draw your motion diagram and graph for the *horizontal* motion of the ball.
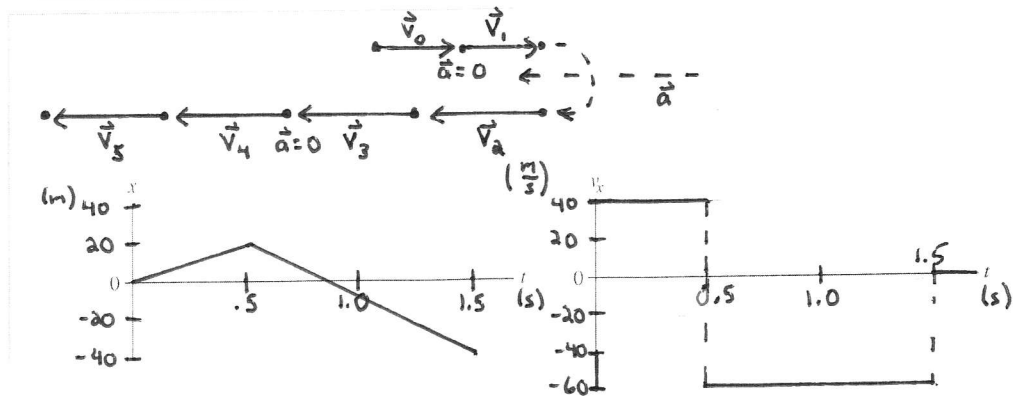
10. The figure shows six frames from the motion diagram of two moving cars, A and B.
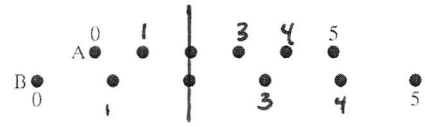
    a. Draw both a position-versus-time graph and a velocity-versus-time graph. Show the motion of *both* cars on each graph. Label them A and B.

    b. Do the two cars ever have the same position at one instant of time? If so, in which frame number (or numbers)?   Yes, at 2
    Draw a vertical line through your graphs of part a to indicate this instant of time.

    c. Do the two cars ever have the same velocity at one instant of time? If so, between which two frames?   No
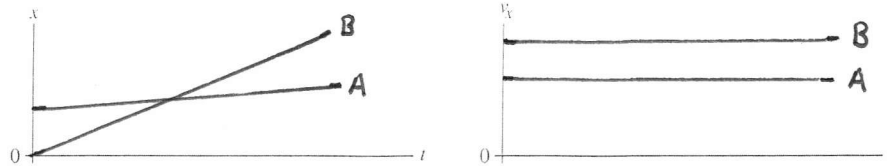
11. The figure shows six frames from the motion diagram of two moving cars, A and B.

    a. Draw both a position-versus-time graph and a velocity-versus-time graph. Show *both* cars on each graph. Label them A and B.

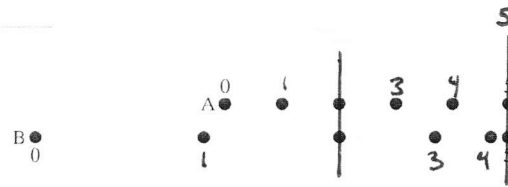   b. Do the two cars ever have the same position at one instant of time? If so, in which frame number (or numbers)?   Yes, at 2 and at 5
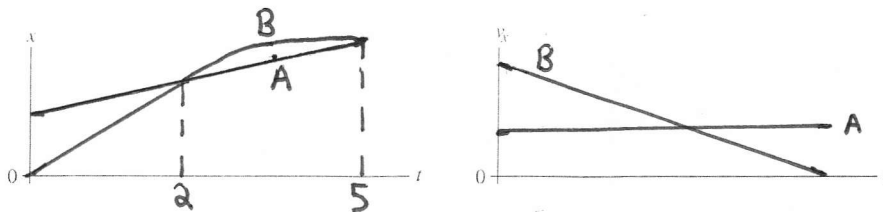   Draw a vertical line through your graphs of part a to indicate this instant of time.

    c. Do the two cars ever have the same velocity at one instant of time? If so, between which two frames?   Yes, from 3 to 4

2.3 Finding Position from Velocity

12. Below are shown four velocity-versus-time graphs. For each:
   • Draw the corresponding position-versus-time graph.
   • Give a written description of the motion.
   Assume that the motion takes place along a horizontal line and that $x_0 = 0$.

a.
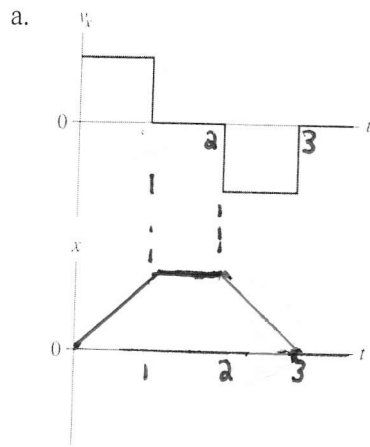
- Moving forward at constant speed for the first one-third.
- Remaining stationary for the second one-third of the time.
- Moving backward at the same speed for the final third.

b.
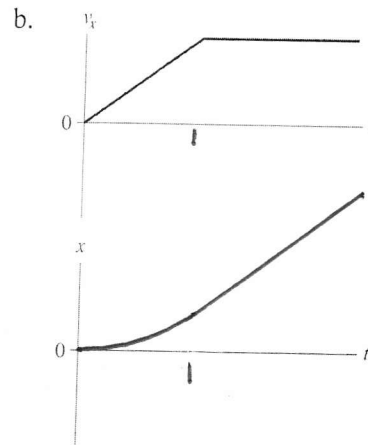
- Speeding up initially and then maintaining a constant speed after point 1.

c.
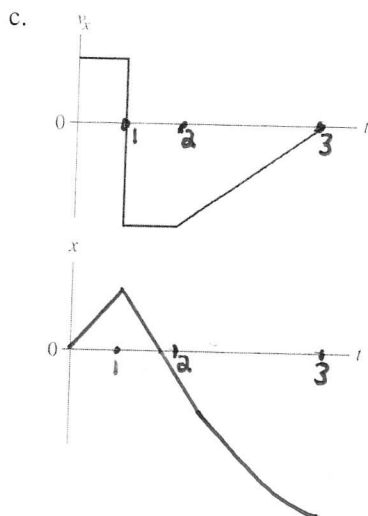
- Constant forward speed to 1.
- A greater speed backwards from 1 to 2.
- Slowing down while moving backwards from 2 to 3, and stopping at 3.

d.
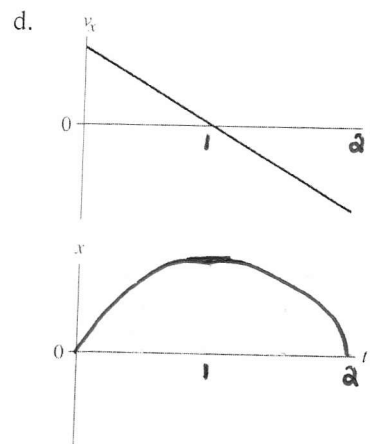
- Constant negative acceleration, slowing down until 1 and then turning around and speeding up to return to the starting point at 2.

13. The figure shows the velocity-versus-time graph for a moving object whose initial position is $x_0 = 20$ m. Find the object's position graphically, using the geometry of the graph, at the following times.

a. At $t = 3$ s.   Finding the area under the curve.

Use the rectangle marked ▨.

$$x(3s) = x_0 + v_x(0-3s)(3s)$$
$$= 20m + 10\tfrac{m}{s}(3s) = \boxed{50m}$$

b. At $t = 5$ s.

Add to the previous answer the area marked ⧄. This area can be found by adding the rectangle from 3s to 4s and 3/4 of that area for the portion from 4s to 5s. Or, equivalently,

$$x(5s) = 50m + v_x(3s-4s)(1s) + \tfrac{1}{2}(v_x(4s) + v_x(5s))(1s)$$
$$= 50m + 10 \text{ m/s}(1s) + \tfrac{1}{2}(10\text{ m/s} + 5\text{ m/s})(1s) = \underset{=\boxed{67.5\,m}}{50m+10m+7.5m}$$
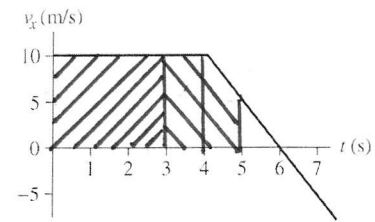
c. At $t = 7$ s.

Add to the previous answer the area of the triangle from 5s to 6s and **subtract** the area of the triangle from 6s to 7s. These areas are equal and opposite. $\tfrac{1}{2}5\text{ m/s}(1s) + \tfrac{1}{2}(-5\text{ m/s})(1s)$. So, $x(7s) = 67.5m + 2.5m - 2.5m = \boxed{67.5m}$

d. You should have found a simple relationship between your answers to parts b and c. Can you explain this? What is the object doing?

During the time from 5s-7s, the object is slowing while moving in the +x direction for 1s, then speeding up while moving in the -x direction for the 2nd-second. Because the acceleration is constant and the times are equal, the motion is symmetric. The object retraces its path in reverse. Time $t = 6s$ is a turning point.

2.4 Motion with Constant Acceleration

14. Give a specific example for each of the following situations. For each, provide:
    - A description, and
    - A motion diagram.

    a. $a_x = 0$ but $v_x \neq 0$.

    Traveling at constant velocity.

    b. $v_x = 0$ but $a_x \neq 0$.

    Turning around.

    This part of the motion diagram only shows $v_x = 0$, but $a_x < 0$.

    c. $v_x < 0$ and $a_x > 0$.

    Slowing down while moving in the negative direction.
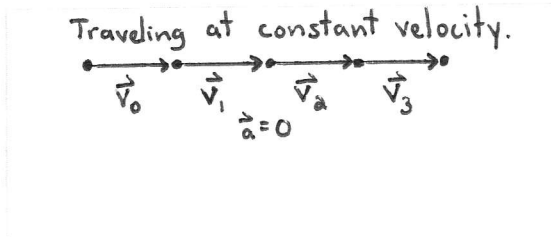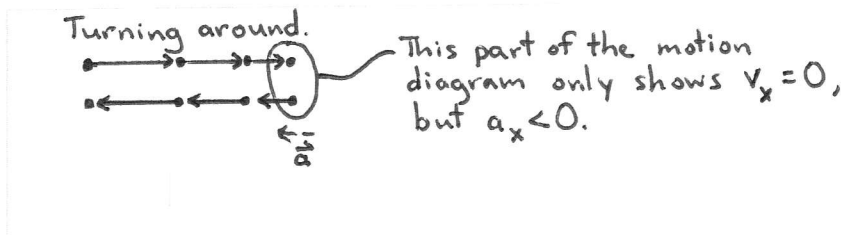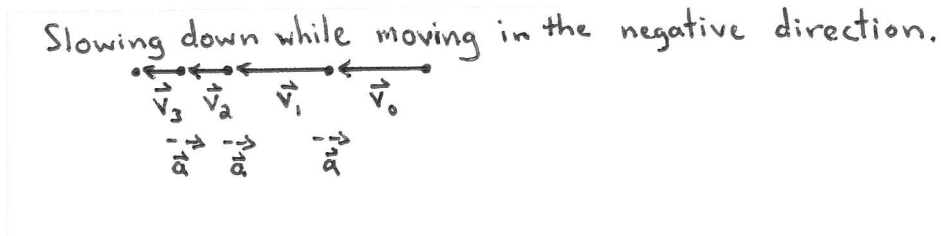
15. Below are three velocity-versus-time graphs. For each:
    - Draw the corresponding acceleration-versus-time graph.
    - Draw a motion diagram below the graphs.

    a.

    b.

    c.

    (a.) t_0 } No motion
    t_2

    t_1 $\vec{a}$ $\vec{a} = 0$

    (b.) t_0 $\vec{a} = 0$ $\vec{a} = 0$ t_1

    $\vec{a} = 0$ $\vec{a} = 0$ $\vec{a} = 0$ t_2 $\vec{a} -$

    Turning around

    (c.) t_0 $\vec{a}$ t_1 $\vec{a}$ t_2

    Stopped   turning point

    t_4 $\vec{a}$ $\vec{a}$ $\vec{a}$ t_3 $\vec{a}$ $\vec{a}$ $\vec{a}$

    $\vec{a} -$
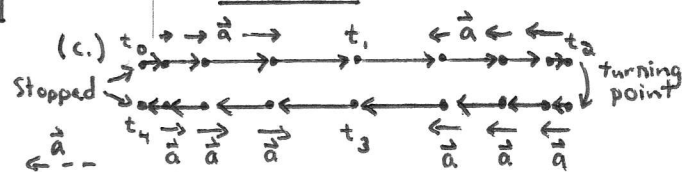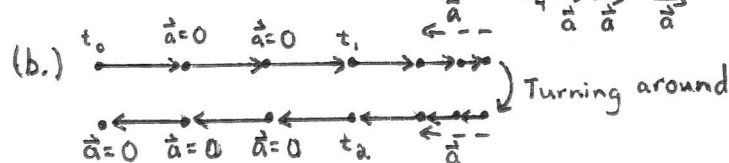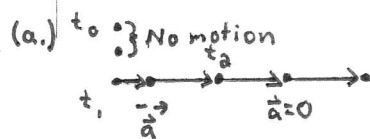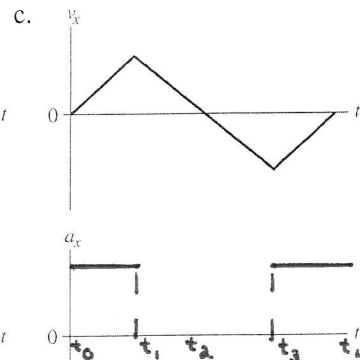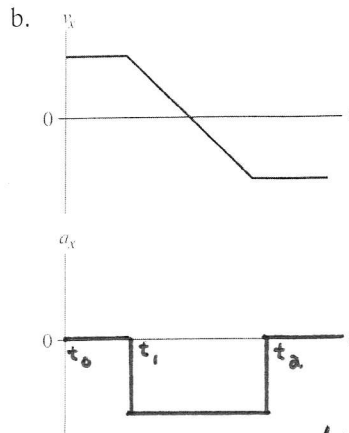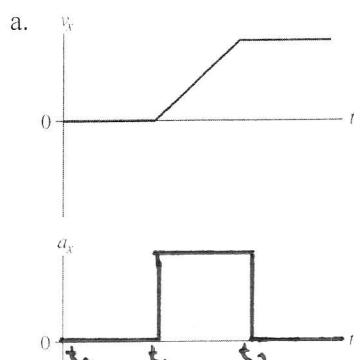
16. Below are three acceleration-versus-time graphs. For each, draw the corresponding velocity-versus-time graph. Assume that $v_{0x} = 0$.

a.

b.

c.
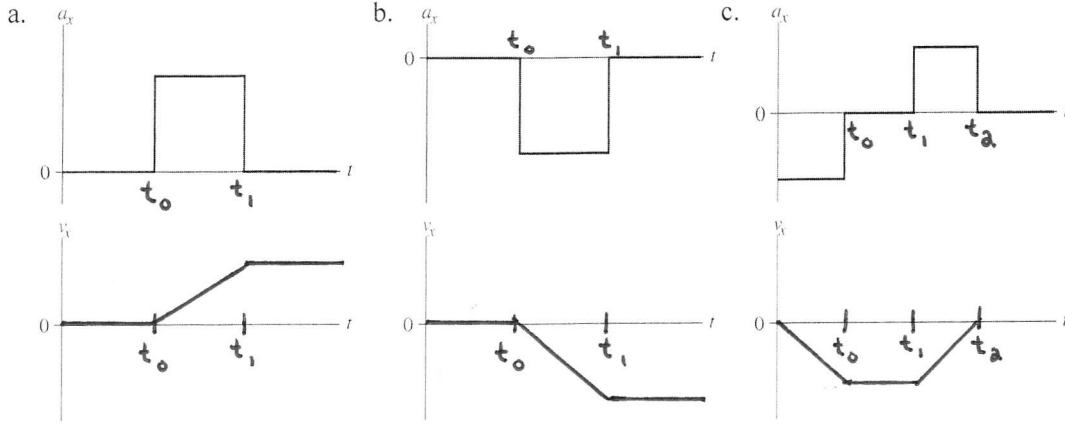
17. The figure below shows nine frames from the motion diagram of two cars. Both cars begin to accelerate, with constant acceleration, in frame 3.
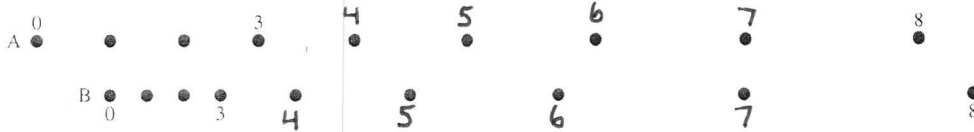
a. Which car has the larger initial velocity? _____A_____ The larger final velocity? _____B_____

b. Which car has the larger acceleration after frame 3? How can you tell?

B. B's acceleration must be greater to go from a smaller initial velocity to a larger final velocity. The change in spacing indicates this.

c. Draw position, velocity, and acceleration graphs, showing the motion of both cars on each graph. (Label them A and B.) This is a total of three graphs with two curves on each.
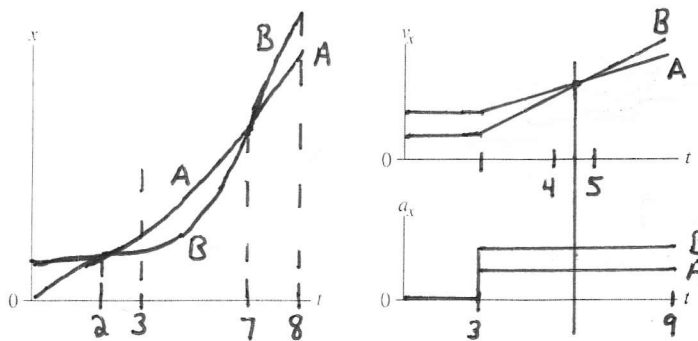
d. Do the cars ever have the same position at one instant of time? If so, in which frame?   2 and 7

e. Do the two cars ever have the same velocity at one instant of time?   Yes

If so, identify the *two* frames between which this velocity occurs.   4-5

Identify this instant on your graphs by drawing a vertical line through the graphs.

2.5 Free Fall

18. A ball is thrown straight up into the air. At each of the following instants, is the <u>magnitude</u> of the ball's acceleration greater than g, equal to g, less than g, or zero?

    a. Just after leaving your hand?   + g

    b. At the very top (maximum height)?   + g

    c. Just before hitting the ground?   + g

    Note
    $a_y = -g$
    for all

19. A rock is *thrown* (not dropped) straight down from a bridge into the river below.

    a. Immediately *after* being released, is the magnitude of the rock's acceleration greater than g, less than g, or equal to g? Explain.

    The magnitude of the acceleration while in free fall is equal to g at all times, independent of the initial velocity. The acceleration only tells how the velocity is changing.

    b. Immediately before hitting the water, is the magnitude of the rock's acceleration greater than g, less than g, or equal to g? Explain.
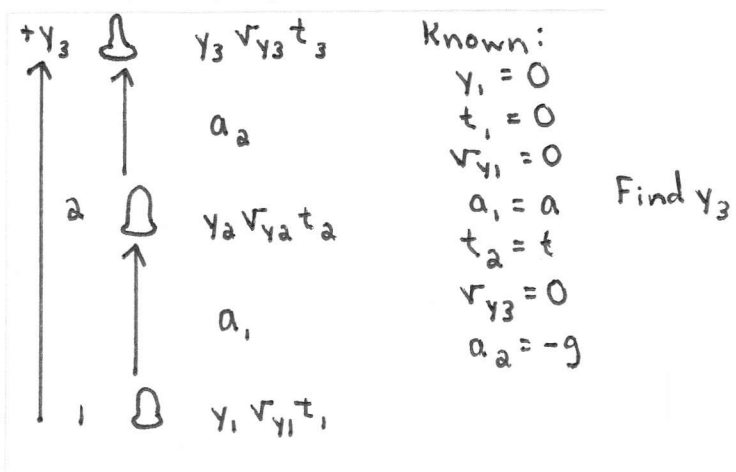
    The magnitude of the acceleration is still g because the rock is still in free fall. The speed is increasing at the same rate each instant, that is, by the same ΔV each second.

20. A model rocket is launched straight up with constant acceleration a. It runs out of fuel at time t.
PSS Suppose you need to determine the maximum height reached by the rocket. We'll assume that air
2.1 resistance is negligible.

  a. Is the rocket at maximum height the instant it runs out of fuel? No

  b. Is there anything other than gravity acting on the rocket after it runs out of fuel? No

  c. What is the name of motion under the influence of only gravity? projectile motion

  d. Draw a pictorial representation for this
     problem. You should have three identified
     points in the motion: launch, out of fuel,
     maximum height. Call these points 1, 2, and 3.

     • Using subscripts, define 11 quantities: y,
       v_y, and t at each of the three points, plus
       acceleration a_1 connecting points 1 and 2
       and acceleration a_2 connecting points 2
       and 3.
     • Identify 7 of these quantities as Knowns,
       either 0 or given symbolically in terms of
       a, t, and g. Be careful with signs!
     • Identify which one of the 4 unknown
       quantities you're trying to find.

$+y_3$ $y_3 \ v_{y3} \ t_3$ Known:
$y_1 = 0$
$t_1 = 0$
$v_{y1} = 0$
a_2
$a_1 = a$ Find y_3
$y_2 \ v_{y2} \ t_2$ $t_2 = t$
$v_{y3} = 0$
a_1
$a_2 = -g$
$y_1 \ v_{y1} \ t_1$

  e. This is a two-part problem. Write two kinematic equations for the first part of the motion to
     determine—again symbolically—the two unknown quantities at point 2.

$$v_{y2} = at$$
$$y_2 = \tfrac{1}{2}at^2$$

  f. Now write a kinematic equation for the second half of the motion that will allow you to find the
     desired unknown that will answer the question. Your equation should not contain the fourth un-
     known quantity. Just write the equation; don't solve it yet.

$$0 = (v_{y3})^2 = (v_{y2})^2 - 2g(y_3 - y_2)$$

  g. Now, substitute what you learned in part e into your equation of part f, do the algebra to solve for
     the unknown, and simplify the result as much as possible.

$$v_{y2}^2 = 2g(y_3 - y_2)$$
$$a^2 t^2 = 2g(y_3 - \tfrac{1}{2}at^2)$$
or $\dfrac{a^2 t^2}{2g} + \tfrac{1}{2}at^2 = y_3$

$$\frac{(a^2 + ga)t^2}{2g} = y_3$$

2.6 Motion on an Inclined Plane

21. A ball released from rest on an inclined plane accelerates down the plane at 2 m/s². Complete the table below showing the ball's velocities at the times indicated. Do *not* use a calculator for this; this is a reasoning question, not a calculation problem.
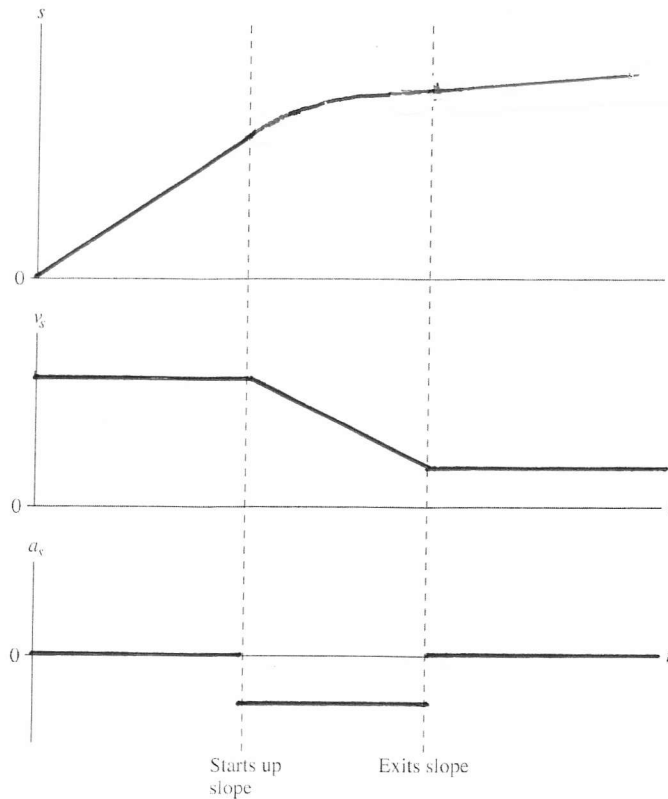
Time (s)	Velocity (m/s)
0	0
1	2 m/s
2	4 m/s
3	6 m/s
4	8 m/s
5	10 m/s

(If we define up the plane as the positive direction, then the velocities are all negative.)

22. A bowling ball rolls along a level surface, then up a 30° slope, and finally exits onto another level surface at a much slower speed.
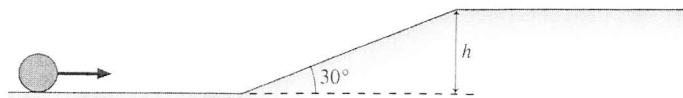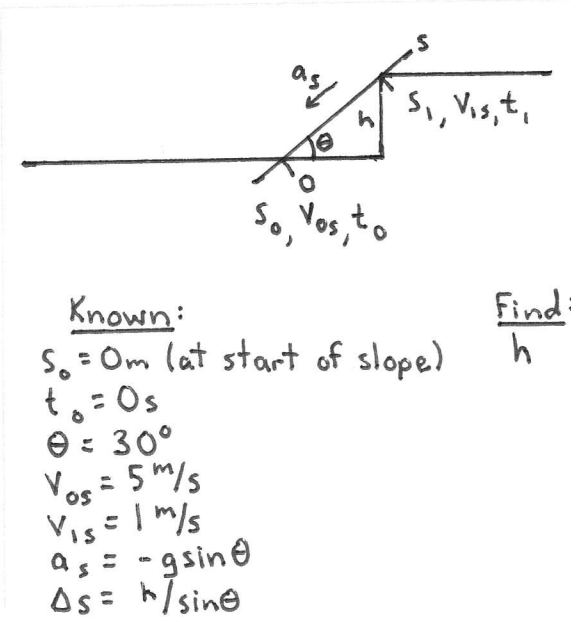

a. Draw position-, velocity-, and acceleration-versus-time graphs for the ball.

b. Suppose that the ball's initial speed is 5.0 m/s and its final speed is 1.0 m/s. Draw a pictorial representation that you would use to determine the height h of the slope. Establish a coordinate system, define all symbols, list known information, and identify desired unknowns.

Note: Don't actually solve the problem. Just draw the complete pictorial representation that you would use as a first step in solving the problem.

Known:
$s_0 = 0m$ (at start of slope)
$t_0 = 0s$
$\theta = 30°$
$v_{0s} = 5 \, m/s$
$v_{1s} = 1 \, m/s$
$a_s = -g\sin\theta$
$\Delta s = h/\sin\theta$

Find:
h

2.7 Instantaneous Acceleration

23. Below are two acceleration-versus-time curves. For each, draw the corresponding velocity-versus-time curve. Assume that $v_{0x} = 0$.

a.

b.