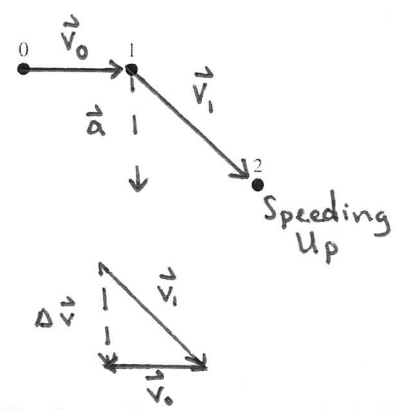
4 Kinematics in Two Dimensions

4.1 Acceleration

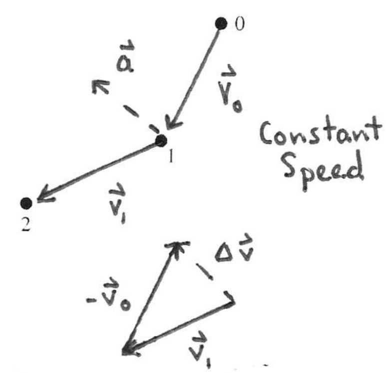
Exercises 1–2: The figures below show an object's position in three successive frames of film. The object is moving in the direction $0 \rightarrow 1 \rightarrow 2$. For each diagram:

- Draw and label the initial and final velocity vectors $\vec{v}_0$ and $\vec{v}_1$. Use **black**. (solid lines)
- Use the steps of Figures 4.2 and 4.3 to find the change in velocity $\Delta\vec{v}$.
- Draw and label $\vec{a}$ next to dot 1 on the motion diagram. Use **red**. (dashed lines)
- Determine whether the object is speeding up, slowing down, or moving at a constant speed. Write your answer beside the diagram.

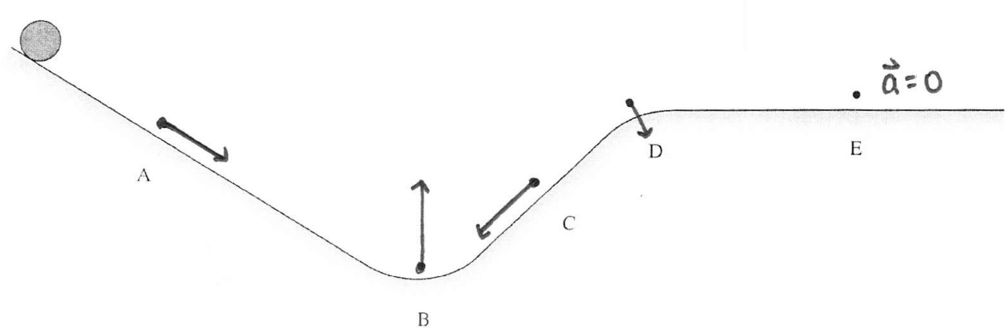
1.



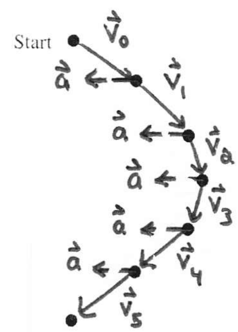
2.



3. The figure shows a ramp and a ball that rolls along the ramp. Draw vector arrows on the figure to show the ball's acceleration at each of the lettered points A to E (or write $\vec{a} = \vec{0}$, if appropriate).
Hint: At each point is the ball changing speed, changing direction, or both?



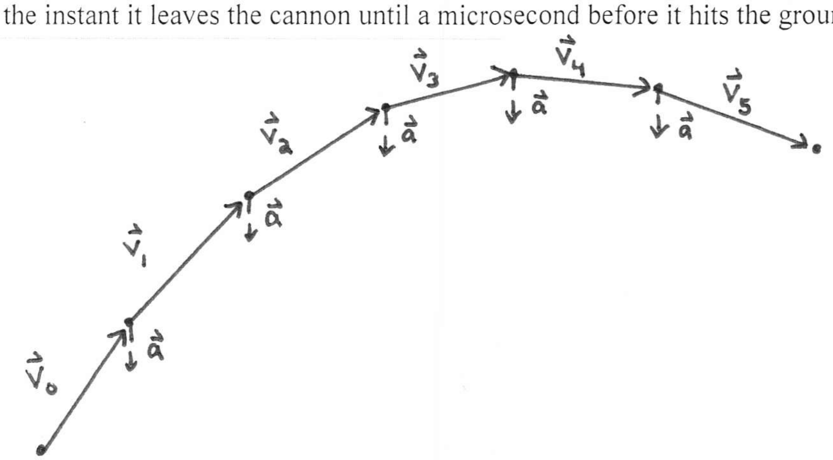
4. Complete the motion diagram for this trajectory, showing velocity and acceleration vectors.



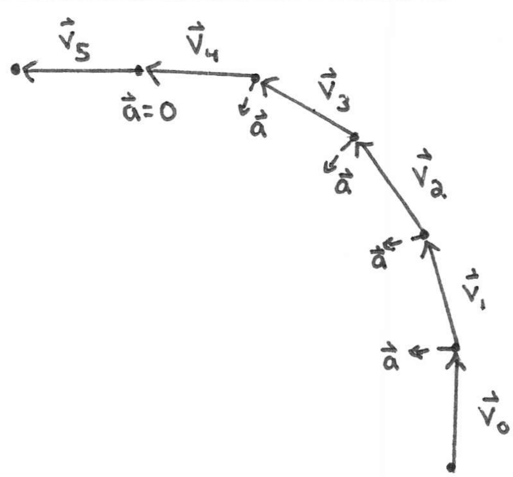
Exercises 5–6: Draw a complete motion diagram for each of the following.

- Draw and label the velocity vectors $\vec{v}$. Use **black**. (solid lines)
- Draw and label the acceleration vectors $\vec{a}$. Use **red**. (dashed lines)

5. A cannon ball is fired from a Civil War cannon up onto a high cliff. Show the cannon ball's motion from the instant it leaves the cannon until a microsecond before it hits the ground.

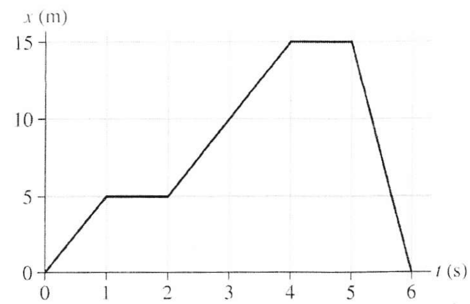


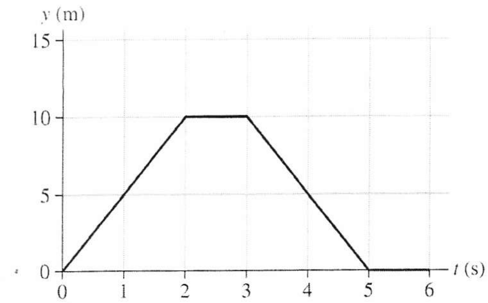
6. A plane flying north at 300 mph turns slowly to the west without changing speed, then continues to fly west. Draw the motion diagram from a viewpoint above the plane.

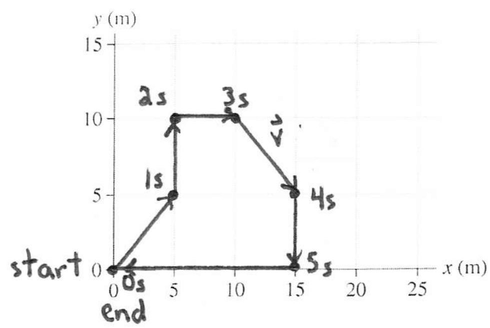


4.2 Two-Dimensional Kinematics

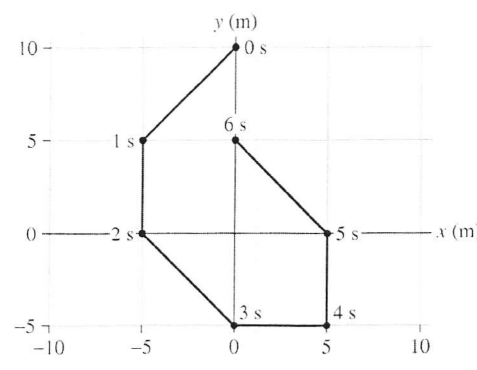
7. A particle moving in the xy-plane has the x-versus-t graph and the y-versus-t graphs shown below. Use the grid to draw a y-versus-x graph of the trajectory.

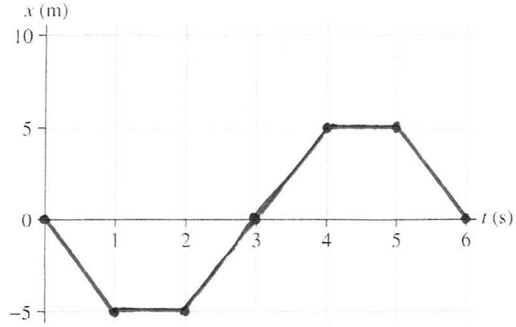


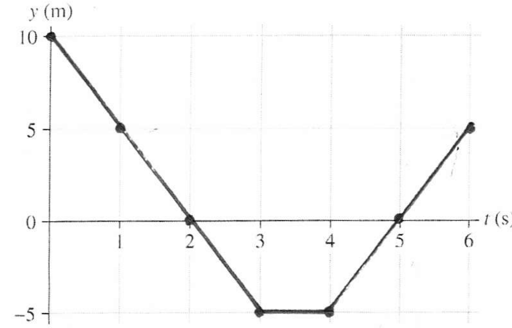




8. The trajectory of a particle is shown below. The particle's position is indicated with dots at 1-second intervals. The particle moves between each pair of dots at constant speed. Draw x-versus-t and y-versus-t graphs for the particle.

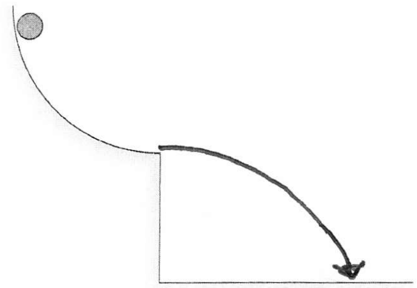




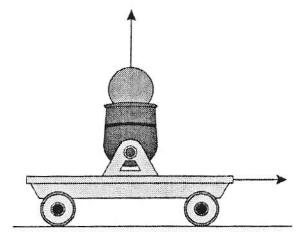


4.3 Projectile Motion

9. The figure shows a ball that rolls down a quarter-circle ramp, then off a cliff. Sketch the ball's trajectory from the instant it is released until it hits the ground.



10. a. A cart that is rolling at constant velocity fires a ball straight up. When the ball comes back down, will it land in front of the launching tube, behind the launching tube, or directly in the tube? Explain.

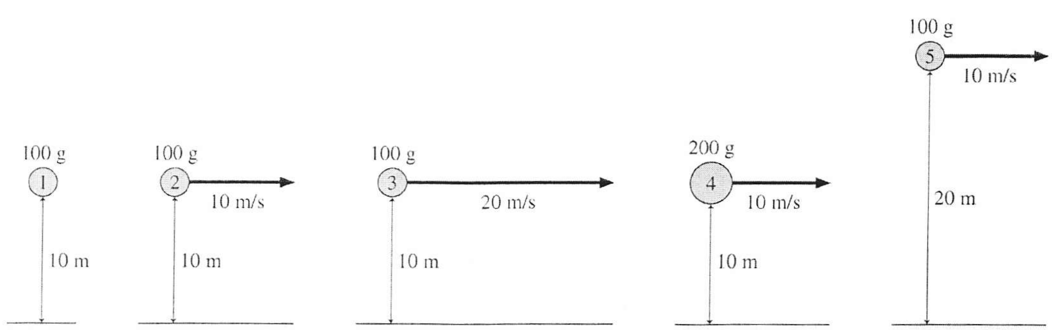


The ball will land directly in the cart. The ball is already moving horizontally with the cart when launched. Though the force on the ball when launched is vertical, the ball retains the same constant horizontal velocity as the cart and lands in it.

b. Will your answer change if the cart is accelerating in the forward direction? If so, how?

If the cart is accelerating, then the ball will land behind the cart. The ball's horizontal velocity when launched is the same as the cart's at that moment, but the cart is speeding up and gets ahead of the ball.

11. Rank in order, from shortest to longest, the amount of time it takes each of these projectiles to hit the ground. Ignore air resistance. (Some may be simultaneous.)



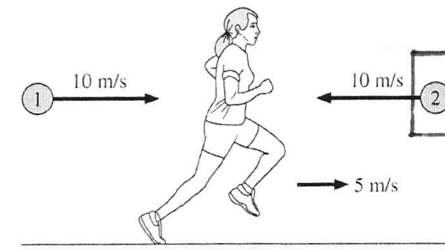
Order: 1 = 2 = 3 = 4, 5

Explanation:

None of the projectiles has any initial velocity in the y-direction. The time to fall depends only on the height above the ground. Balls 1-4 begin at the same height.

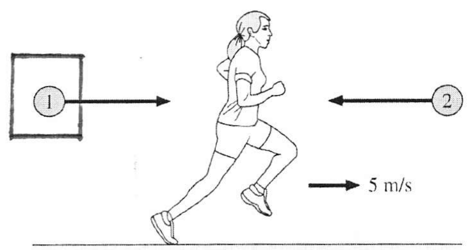
4.4 Relative Motion

12. Anita is running to the right at 5 m/s. Balls 1 and 2 are thrown toward her at 10 m/s by friends standing on the ground. According to Anita, which ball is moving faster? Or are both speeds the same? Explain.



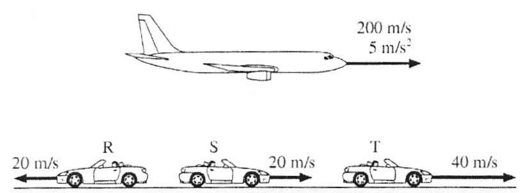
According to Anita, ball 2 is moving at $-10\,m/s - 5\,m/s = -15\,m/s$.
Ball 1 is moving at $10\,m/s - 5\,m/s = 5\,m/s$.
Ball 2 is faster.

13. Anita is running to the right at 5 m/s. Balls 1 and 2 are thrown toward her by friends standing on the ground. According to Anita, both balls are approaching her at 10 m/s. Which ball was thrown at a faster speed? Or were they thrown with the same speed? Explain.



Ball 1 was thrown at $15\,m/s$.
Ball 2 was thrown at $-5\,m/s$.
Thus, according to Anita, ball 2 is moving at $-5\,m/s - 5\,m/s = -10\,m/s$ and ball 1 is moving at $15\,m/s - 5\,m/s = 10\,m/s$.

14. Ryan, Samantha, and Tomas are driving their convertibles at a steady speed. At the same instant, they each see a jet plane with an instantaneous velocity of 200 m/s and an acceleration of 5 m/s^2 relative to the ground. Rank in order, from largest to smallest, the jet's *speed* v_R, v_S, and v_T according to Ryan, Samantha, and Tomas. Explain.



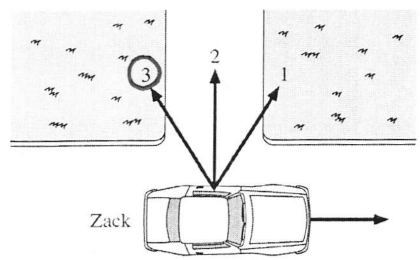
$v_R > v_S > v_T$

The jet's speed, according to Ryan, is $200\,m/s - (-20\,m/s) = 220\,m/s$.
The jet's speed, according to Samantha, is $200\,m/s - 20\,m/s = 180\,m/s$.
The jet's speed, according to Tomas, is $200\,m/s - 40\,m/s = 160\,m/s$.

15. An electromagnet on the ceiling of an airplane holds a steel ball. When a button is pushed, the magnet releases the ball. The experiment is first done while the plane is parked on the ground, and the point where the ball hits the floor is marked with an X. Then the experiment is repeated while the plane is flying level at a steady 500 mph. Does the ball land slightly in front of the X (toward the nose of the plane), on the X, or slightly behind the X (toward the tail of the plane)? Explain.

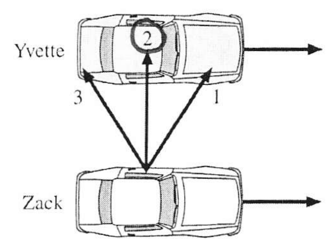
The ball still lands on the X. In the second experiment, the ball's initial velocity relative to the plane is still zero. Although someone at rest on the ground would perceive both the ball and the plane to be moving past at 500 mph, to someone on the plane the ball still falls straight down.

16. Zack is driving past his house. He wants to toss his physics book out the window and have it land in his driveway. If he lets go of the book exactly as he passes the end of the driveway, should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain. (Ignore air resistance.)



Throw 3. When Zack throws the book, it also has the forward motion of the car. If he wants the book to follow path 2 into the driveway, he needs to include a backward component to the velocity from his throw that is equal and opposite to the forward velocity of the car.

17. Yvette and Zack are driving down the freeway side by side with their windows rolled down. Zack wants to toss his physics book out the window and have it land in Yvette's front seat. Should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain. (Ignore air resistance.)



Throw 2. Yvette's car is not moving relative to Zack's car. From the perspective of an observer at rest with respect to the freeway, the book already has a forward velocity equal to Yvette's and, therefore, only needs to be thrown straight outward.

4.5 Uniform Circular Motion

4.6 Velocity and Acceleration in Uniform Circular Motion

18. a. The crankshaft in your car rotates at 3000 rpm. What is the frequency in revolutions per second?

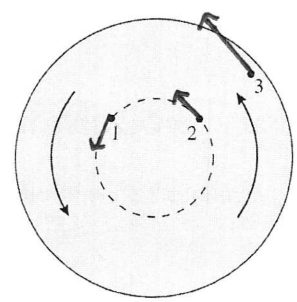
$$\frac{3000\text{ rev}}{1\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = 50\frac{\text{rev}}{\text{s}}$$

b. A record turntable rotates at 33.3 rpm. What is the period in seconds?

$$\frac{33.3\text{ rev}}{1\text{min}} \times \frac{1\text{min}}{60\text{s}} = 0.555\frac{\text{rev}}{\text{s}}$$

$$T = \frac{1}{0.555}\text{ s} = 1.8\text{ s}$$

19. The figure shows three points on a steadily rotating wheel.
   a. Draw the velocity vectors at each of the three points
   b. Rank in order, from largest to smallest, the angular velocities ω_1, ω_2, and ω_3 of these points.



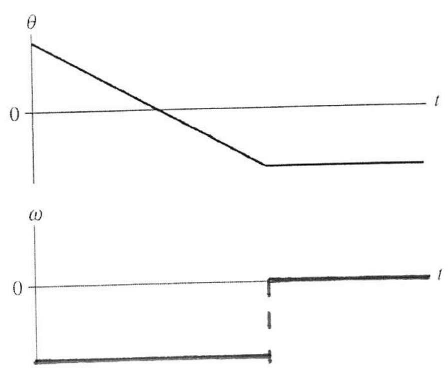
Order: $\omega_1 = \omega_2 = \omega_3$
Explanation:
Each point traverses the same angle in the same time. All points on the wheel rotate with the same period.

c. Rank in order, from largest to smallest, the speeds v_1, v_2, and v_3 of these points.

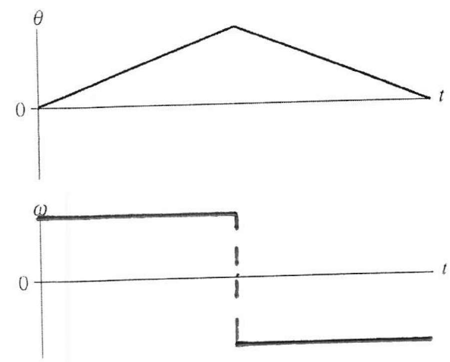
Order: $v_3 > v_1 = v_2$
Explanation:
$v_t = r\omega$. Point 3 is further away from the axis of rotation and so has a greater speed than points 1 and 2, which are at the same radius r as each other.

20. Below are two angular position-versus-time graphs. For each, draw the corresponding angular velocity-versus-time graph directly below it.

a.

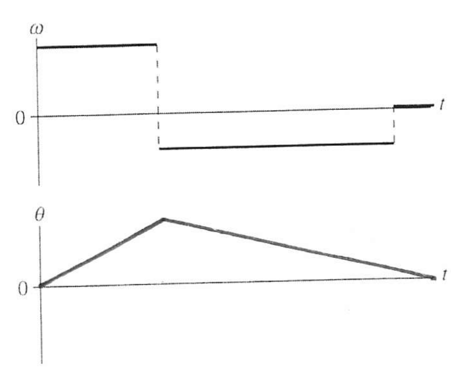


b.

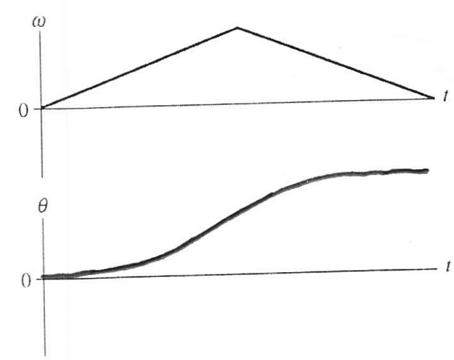


21. Below are two angular velocity-versus-time graphs. For each, draw the corresponding angular position-versus-time graph directly below it. Assume $\theta_0 = 0$ rad.

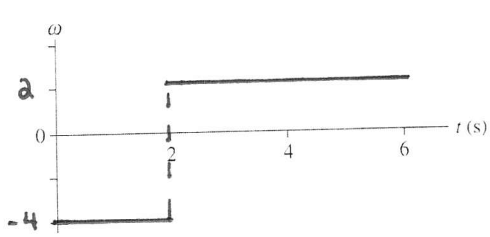
a.

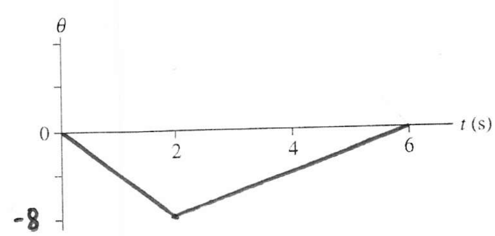


b.



22. A particle in circular motion rotates clockwise at 4 rad/s for 2 s, then counterclockwise at 2 rad/s for 4 s. The time required to change direction is negligible. Graph the angular velocity and the angular position, assuming $\theta_0 = 0$ rad.





23. A particle rotates in a circle with $a_r = 8\ \text{m/s}^2$. What is a_r if

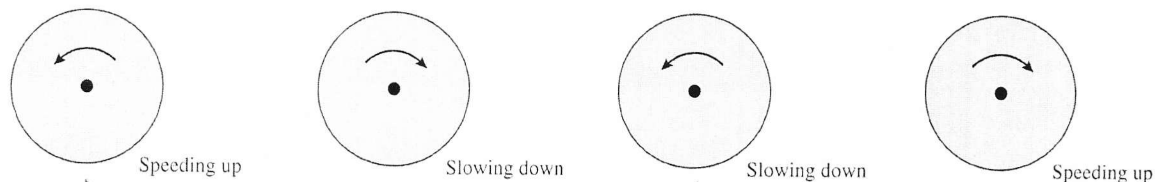
a. The radius is doubled without changing the angular velocity? $a_r = 16\ \text{m/s}^2$

b. The radius is doubled without changing the particle's speed? $a_r = 4\ \text{m/s}^2$

c. The angular velocity is doubled without changing the circle's radius? $a_r = 32\ \text{m/s}^2$

4.7 Nonuniform Circular Motion and Angular Acceleration

24. The following figures show a rotating wheel. Determine the signs (+ or –) of ω and α.

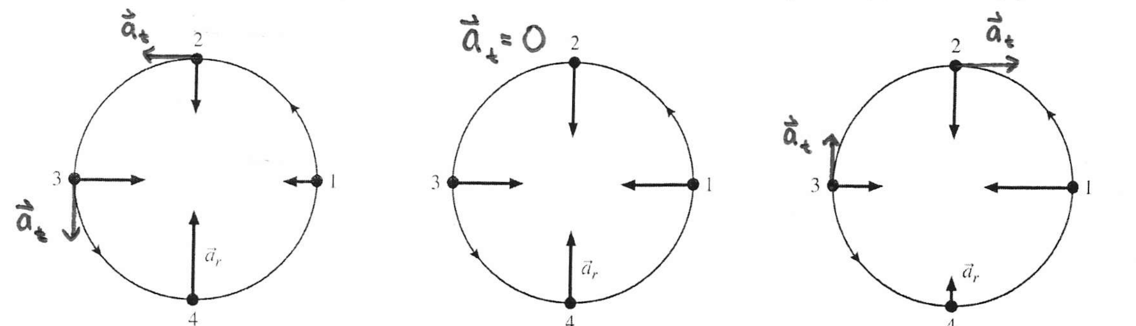


	Speeding up	Slowing down	Slowing down	Speeding up
ω	+	–	+	–
α	+	+	–	–

25. The figures below show the radial acceleration vector $\vec{a}_r$ at four successive points on the trajectory of a particle moving in a counterclockwise circle.

a. For each, draw the tangential acceleration vector $\vec{a}_t$ at points 2 and 3 or, if appropriate, write $\vec{a}_t = \vec{0}$.

b. Determine if the particle's angular acceleration α is positive (+), negative (–), or zero (0).



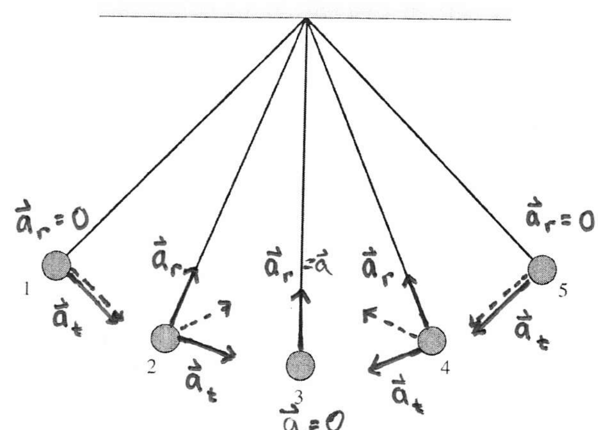
$\alpha =$ + $\alpha =$ 0 $\alpha =$ –

26. A pendulum swings from its end point on the left (point 1) to its end point on the right (point 5). At each of the labeled points:

a. Use a **black** pen or pencil to draw and label the vectors $\vec{a}_r$ and $\vec{a}_t$ at each point. Make sure the length indicates the relative size of the vector.

b. Use a **red** pen or pencil to draw and label the total acceleration vector $\vec{a}$.

——— Black
– – – Red

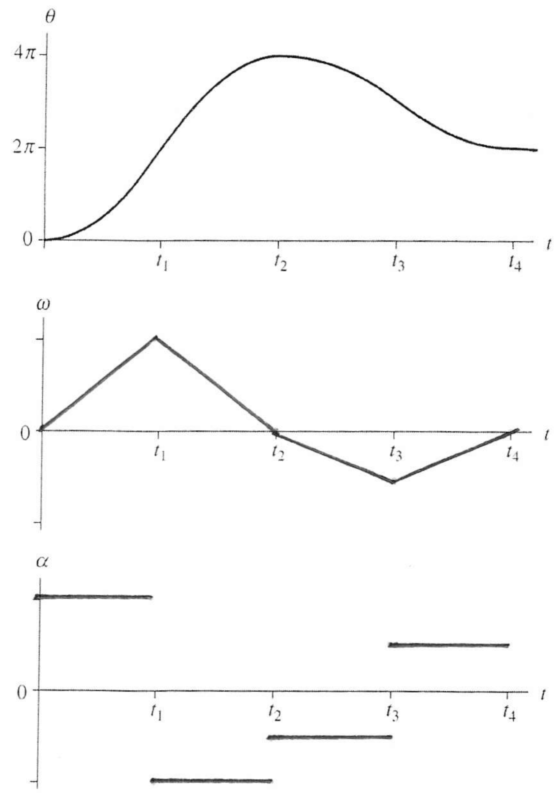


27. The figure shows the θ-versus-t graph for a particle moving in a circle. The curves are all sections of parabolas.

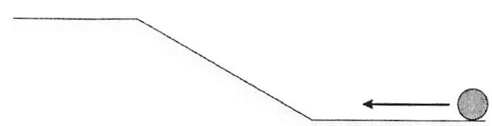
   a. Draw the corresponding ω-versus-t and α-versus-t graphs. Notice that the horizontal tick marks are equally spaced.

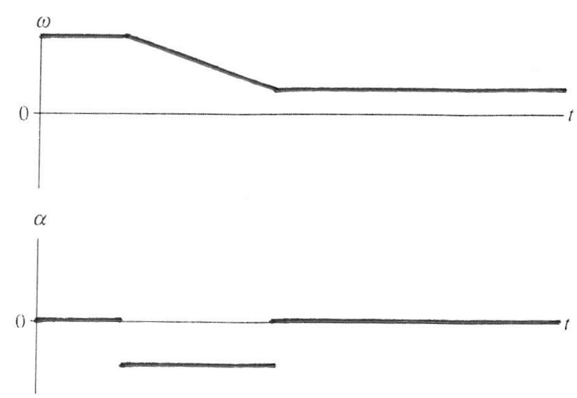
   b. Write a description of the particle's motion.

The particle starts from rest and steadily speeds up ccw around the circle for one full cycle, then slows down steadily for a second revolution at which time it stops and reverses direction to steadily speed up at a slower rate for half the circle. It then slows for the final half cycle and stops after traveling cw for one cycle.



28. A wheel rolls to the left along a horizontal surface, up a ramp, then continues along the upper horizontal surface. Draw graphs for the wheel's angular velocity ω and angular acceleration α as functions of time.





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