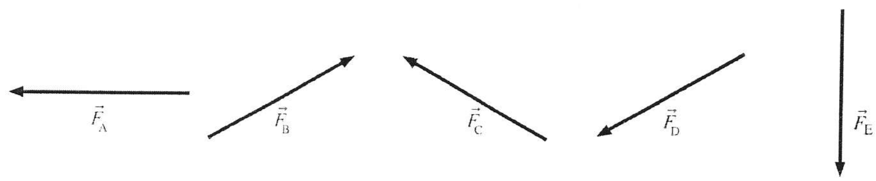
6 Dynamics I: Motion Along a Line

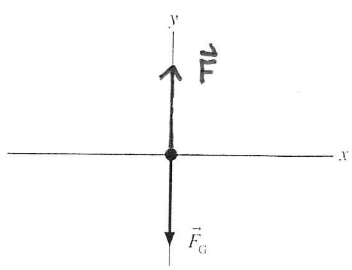
6.1 Equilibrium

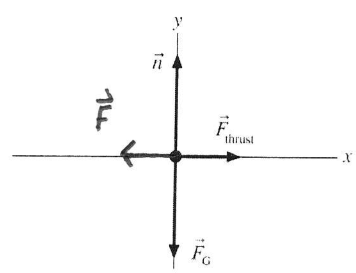
1. The vectors below show five forces that can be applied individually or in combinations to an object. Which forces or combinations of forces will cause the object to be in equilibrium?



$\vec{F}_B + \vec{F}_D$ or $\vec{F}_B + \vec{F}_C + \vec{F}_E$

2. The free-body diagrams show a force or forces acting on an object. Draw and label one more force (one that is appropriate to the situation) that will cause the object to be in equilibrium.





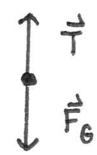
3. If you know all of the forces acting on a moving object, can you tell in which direction the object is moving? If the answer is Yes, explain how. If the answer is No, give an example.

No. If you know all of the forces, then you know the direction of the acceleration, not of the motion. For example, a car moving forward could have a net force forward if speeding up or backward if slowing down or no net force at all if moving at constant speed.

6.2 Using Newton's Second Law

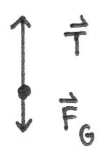
4. a. An elevator travels *upward* at a constant speed. The elevator hangs by a single cable. Friction and air resistance are negligible. Is the tension in the cable greater than, less than, or equal to the weight of the elevator? Explain. Your explanation should include both a free-body diagram and reference to appropriate physical principles.

Because the elevator is not accelerating, the net force on it must be zero. Therefore, the tension and weight must be equal in magnitude and opposite in direction.



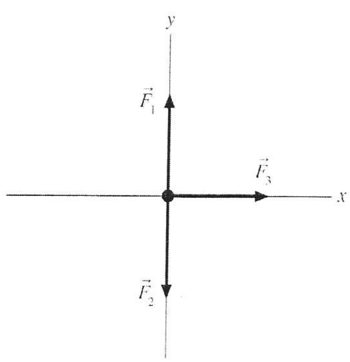
b. The elevator travels *downward* and is slowing down. Is the tension in the cable greater than, less than, or equal to the weight of the elevator? Explain.

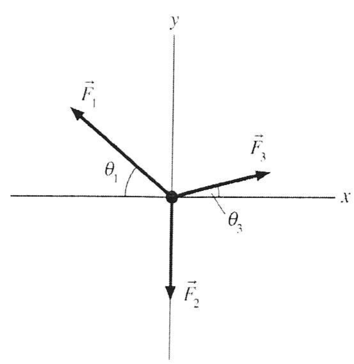
Because the elevator is slowing down, its acceleration is in the opposite direction from its motion. Therefore, the net force on the elevator is upward and the tension is greater than the weight.



Exercises 5–6: The figures show free-body diagrams for an object of mass m. Write the x- and y-components of Newton's second law. Write your equations in terms of the *magnitudes* of the forces F_1, F_2,... and any *angles* defined in the diagram. One equation is shown to illustrate the procedure.

5.





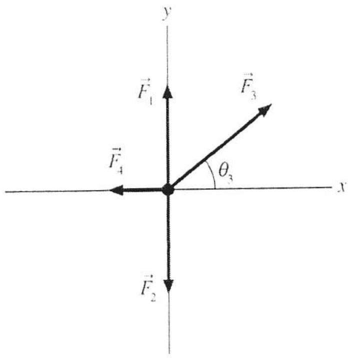
$ma_x = F_3$

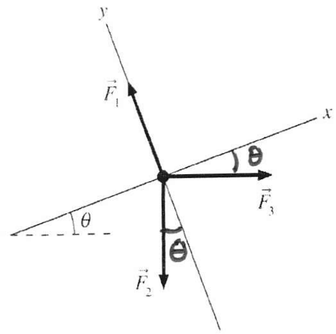
$ma_y = F_1 - F_2$

$ma_x = F_3 \cos\theta_3 - F_1 \cos\theta_1$

$ma_y = F_1 \sin\theta_1 + F_3 \sin\theta_3 - F_2$

6.





$ma_x = F_3 \cos\theta_3 - F_4$

$ma_y = F_1 + F_3 \sin\theta_3 - F_2$

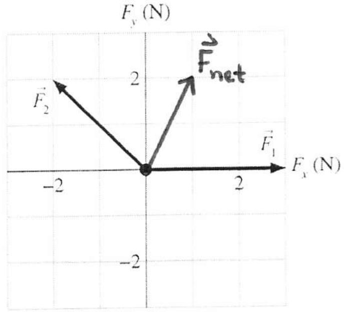
$ma_x = F_3 \cos\theta - F_2 \sin\theta$

$ma_y = F_1 - F_2 \cos\theta - F_3 \sin\theta$

Exercises 7–9: Two or more forces, shown on a free-body diagram, are exerted on a 2 kg object. The units of the grid are newtons. For each:

- Draw a vector arrow *on the grid,* starting at the origin, to show the net force $\vec{F}_{\text{net}}$.
- In the space to the right, determine the numerical values of the components a_x and a_y.

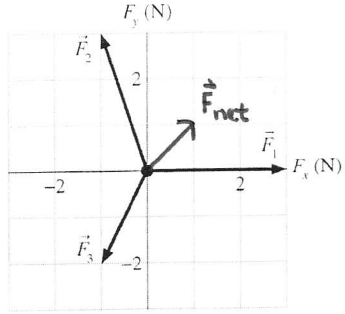
7.



$a_x = \frac{1}{2\,kg}(3N - 2N) = \frac{1}{2}\,m/s^2 = 0.5\,\frac{m}{s^2}$

$a_y = \frac{1}{2\,kg}(2N) = 1\,m/s^2$

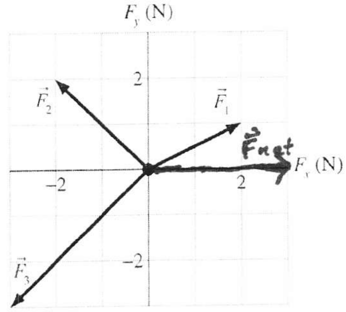
8.



$a_x = \frac{1}{2\,kg}(3N - 1N - 1N) = 0.5\,m/s^2$

$a_y = \frac{1}{2\,kg}(3N - 2N) = 0.5\,m/s^2$

9.

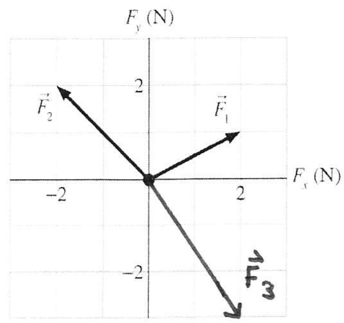


$a_x = \frac{1}{2\,kg}(2N - 2N - 3N) = 1.5\,m/s^2$

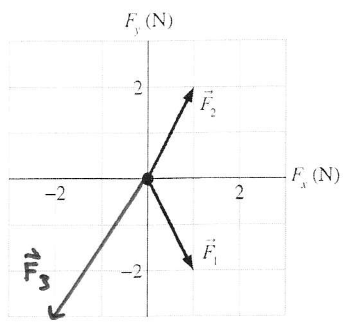
$a_y = \frac{1}{2\,kg}(1N + 2N - 3N) = 0\,m/s^2$

Exercises 10–12: Three forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ cause a 1 kg object to accelerate with the acceleration given. Two of the forces are shown on the free-body diagrams below, but the third is missing. For each, draw and label *on the grid* the missing third force vector.

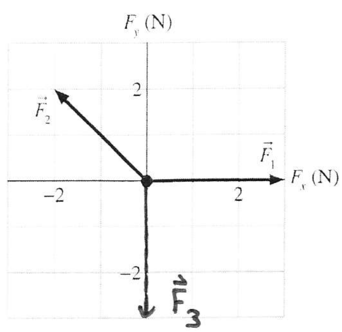
10. $\vec{a} = 2\hat{\imath}$ m/s^2



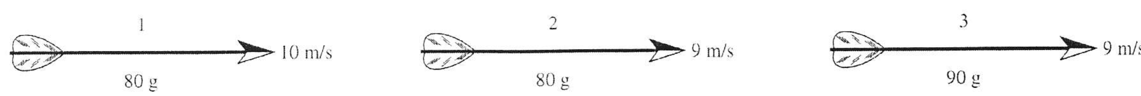
11. $\vec{a} = -3\hat{\jmath}$ m/s^2



12. The object moves with constant velocity.



13. Three arrows are shot horizontally. They have left the bow and are traveling parallel to the ground. Air resistance is negligible. Rank in order, from largest to smallest, the magnitudes of the *horizontal* forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ acting on the arrows. Some may be equal. Give your answer in the form A > B = C > D.



Order: 1 = 2 = 3

Explanation: After leaving the bow, there are no horizontal forces on the arrows (neglecting air resistance). The only force on the arrows is the downward force of gravity.

6.3 Mass, Weight, and Gravity

14. An astronaut takes his bathroom scales to the moon and then stands on them. Is the reading of the scales his weight? Explain.

The scales will read his weight on the moon. They will not read the weight that he has on Earth.

15. Suppose you attempt to pour out 100 g of salt, using a pan balance for measurement, while in an elevator that is accelerating upward. Will the quantity of salt be too much, too little, or the correct amount? Explain.

You will still pour the correct amount. Although the weight is increased in the elevator (which would lead to pouring too little on a spring scale), the pan balance compares the mass of the salt poured with the mass of the known 100g object. Both weights are affected by the acceleration in the same way.

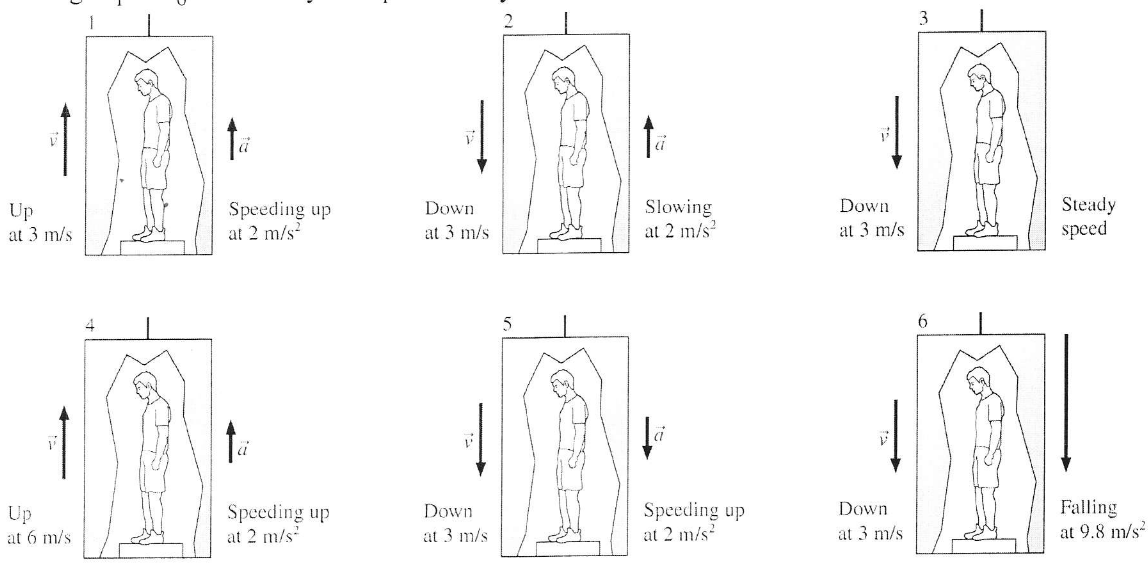
16. An astronaut orbiting the earth is handed two balls that are identical in outward appearance. However, one is hollow while the other is filled with lead. How might the astronaut determine which is which? Cutting them open is not allowed.

The force required to accelerate an object is proportional to its mass. ($\vec{F} = m\vec{a}$). Thus, the astronaut can determine which ball is hollow and which is filled with lead by shaking each or causing each to accelerate with a given force. The force required to accelerate the hollow ball is less due to its lower mass.

17. The terms "vertical" and "horizontal" are frequently used in physics. Give operational definitions for these two terms. An operational definition defines a term by how it is measured or determined. Your definition should apply equally well in a laboratory or on a steep mountainside.

Vertical can be defined by the line a plumb bob makes hanging down due to gravity. Horizontal can be defined by the surface of a liquid far from the edges of its container or by using a bubble level.

18. Suppose you stand on a spring scale in six identical elevators. Each elevator moves as shown below. Let the reading of the scale in elevator n be S_n. Rank in order, from largest to smallest, the six scale readings S_1 to S_6. Some may be equal. Give your answer in the form A > B = C > D.



Order: $S_1 = S_2 = S_4 > S_3 > S_5 > S_6$

Explanation: The scale reading reads your weight, which depends upon the magnitude and direction of your acceleration only, not your speed. Cases 1, 2 and 4 all involve upward acceleration so the scale reads your weight. Case 5 reads less than your weight because your acceleration is downward. For case 6, the scale reading S_6 will be zero.

6.4 Friction

19. A block pushed along the floor with velocity $\vec{v}_0$ slides a distance d after the pushing force is removed.

a. If the mass of the block is doubled but the initial velocity is not changed, what is the distance the block slides before stopping? Explain.

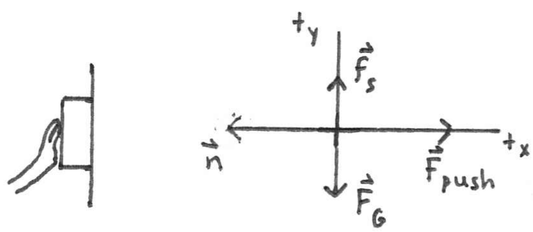
The block will slide the same distance d. The friction force is proportional to the mass, but the blocks response to that force is also proportional to the mass, so the acceleration is the same.

b. If the initial velocity of the block is doubled to $2\vec{v}_0$ but the mass is not changed, what is the distance the block slides before stopping? Explain.

The block will slide a distance of 4d. Because the acceleration is unchanged, it will take twice the time to lose twice the velocity. Because the average velocity is also doubled, the block will travel 4x further.

20. Suppose you press a book against the wall with your hand. The book is not moving.

a. Identify the forces on the book and draw a free-body diagram.



The forces are:

$F_x = F_{push} - n = 0$

$F_y = f_s - F_G = 0$

b. Now suppose you decrease your push, but not enough for the book to slip. What happens to each of the following forces? Do they increase in magnitude, decrease, or not change?

$\vec{F}_{push}$ decreases

$\vec{F}_G$ Same

$\vec{n}$ decreases

$\vec{f}_s$ Same

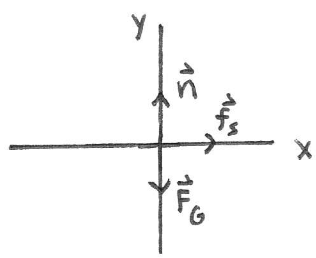
$\vec{f}_{s\,max}$ decreases

21. Consider a box in the back of a pickup truck.

a. If the truck accelerates slowly, the box moves with the truck without slipping. What force or forces act on the box to accelerate it? In what direction do those forces point?

The static friction force accelerates the box. The static friction force points in the same direction as the acceleration of the truck.

b. Draw a free-body diagram of the box.



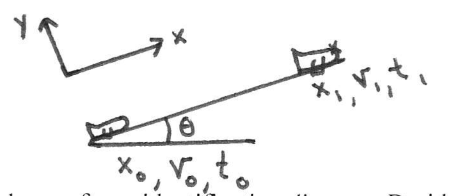
c. What happens to the box if the truck accelerates too rapidly? Explain why this happens, basing your explanation on physical models and the principles described in this chapter.

If the acceleration is very large then it may require a force on the box in the same direction that exceeds the maximum force that can be provided by static friction, $f_{s\,max} = \mu_s \vec{n}$. In this case, the block will tend to remain in place while the truck bed accelerates out from underneath it (leaving it to appear to slide backwards). Kinetic friction will accelerate the box, but at a lesser rate than the acceleration of the truck.

22. A small airplane of mass m must take off from a primitive jungle airstrip that slopes upward at a slight angle θ. When the pilot pulls back on the throttle, the plane's engines exert a constant forward force $\vec{F}_{\text{thrust}}$. Rolling friction is not negligible on the dirt airstrip, and the coefficient of rolling resistance is μ_r. If the plane's take-off speed is v_{off}, what minimum length must the airstrip have for the plane to get airborne?

PSS 6.2

a. Assume the plane takes off uphill to the right. Begin with a pictorial representation, as was described in Tactics Box 1.5. Establish a coordinate system with a tilted x-axis; show the plane at the beginning and end of the motion; define symbols for position, velocity, and time at these two points (six symbols all together); list known information; and state what you wish to find. $\vec{F}_{\text{thrust}}$, m, θ, μ_r, and v_{off} are presumed known, although we have only symbols for them rather than numerical values, and three other quantities are zero.



Known:

$x = 0$ $\quad \theta$

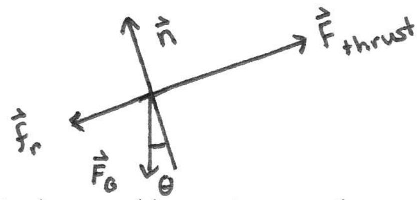
$t_0 = 0$ $\quad v_{off}$

$v_0 = 0$ $\quad m$

μ_r $\quad \vec{F}_{thrust}$

Find: x_1

b. Next, draw a force-identification diagram. Beside it, draw a free-body diagram. Your free-body diagram should use the same coordinate system you established in part a, and it should have 4 forces shown on it.



c. Write Newton's second law as two equations, one for the net force in the x-direction and one for the net force in the y-direction. Be careful finding the components of $\vec{F}_G$ (see Figure 6.2), and pay close attention to signs. Remember that symbols such as F_G or f_r or represent the *magnitudes* of vectors; you have to supply appropriate signs to indicate which way the vectors point. The right side of these equations have a_x and a_y. The motion is entirely along the x-axis, so what do you know about a_y? Use this information as you write the y-equation.

$$\Sigma F_x = F_{thrust} - F_G \sin\theta - F_K = m a_x$$

$$\Sigma F_y = n - F_G \cos\theta = 0$$

d. Now write the equation that characterizes the friction force on a rolling tire.

$$f_r = \mu_r n$$

e. Combine your friction equation with the y-equation of Newton's second law to find an expression for the magnitude of the friction force.

$$f_r = \mu_r F_G \cos\theta$$

f. Finally, substitute your answer to part e into the x-equation of Newton's second law, and then solve for a_x, the x-component of acceleration. Use $F_G = mg$ if you've not already done so.

$$F_{thrust} - mg\sin\theta - \mu_r mg\cos\theta = ma_x$$

$$a_x = \frac{F_{thrust}}{m} - g(\sin\theta + \mu_r\cos\theta)$$

g. With friction present, should the *magnitude* of the acceleration be larger or smaller than the acceleration of taking off on a frictionless runway? smaller

h. Does your expression for acceleration agree with your answer to part g? yes
Explain how you can tell. If it doesn't, recheck your work.

The frictional component has the opposite sign as F_{thrust}.

i. The force analysis is done, but you still have to do the kinematics. This is a situation where we know about velocities, distance, and acceleration but nothing about the time involved. That should suggest the appropriate kinematics equation. Use your acceleration from part f in that kinematics equation, and solve for the unknown quantity you're seeking.

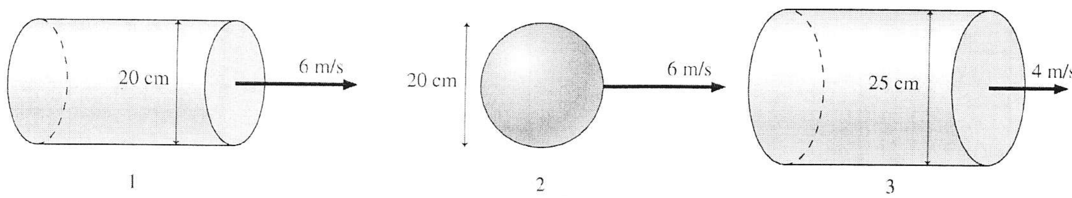
$v_{off}^2 = 2a(x_f)$ use $v_f^2 = v_i^2 + 2a\Delta x$ so

$$v_{off}^2 = 2a_x x_f \quad \text{or} \quad x_f = \frac{v_{off}^2}{2a_x} = \frac{v_{off}^2}{2\left[\frac{F_{thrust}}{m} - g(\sin\theta + \mu_r\cos\theta)\right]}$$

You've found a symbolic answer to the problem, one that you could now evaluate for a range of values of F_{thrust} or θ without having to go through the entire solution each time.

6.5 Drag

23. Three objects move through the air as shown. Rank in order, from largest to smallest, the three drag forces D_1, D_2, and D_3. Some may be equal. Give your answer in the form A > B = C > D.



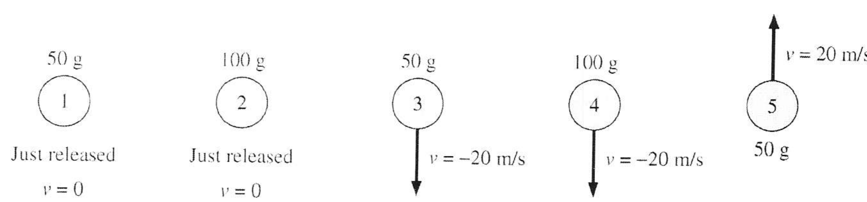
Order: $D_1 = D_2 > D_3$

Explanation: Using $D = \frac{AV^2}{4}$ or $D \propto r^2 v^2$, then $D_1 = D_2$ (same r, same v) and $r_3 = \frac{25}{20} r_1 = \frac{5}{4} r_1$; $V_3 = \frac{4}{6} V_1$ so

$A_3 V_3^2 = (\frac{5}{4})^2 (\frac{4}{6})^2 A_1 V_1$

$A_3 V_3^2 = (\frac{5}{6})^2 A_1 V_1$ so $D_3 < D_1$

24. Five balls move through the air as shown. All five have the same size and shape. Rank in order, from largest to smallest, the magnitude of their accelerations a_1 to a_5. Some may be equal. Give your answer in the form A > B = C > D.



Order: $a_5 > a_1 = a_2 > a_4 > a_3$

Explanation: a_5 is greatest because both the drag force and gravity are downward. $a_1 = a_2 = -g$ because there is no drag force if $v = 0$. The drag force is not proportional to the mass so the acceleration of ball 4 is greater than that of ball 3 because each experiences the same drag force, but ball 4 experiences a greater gravitational force.

25. A 1 kg wood ball and a 10 kg lead ball have identical shapes and sizes. They are dropped simultaneously from a tall tower.

a. To begin, assume that air resistance is negligible. As the balls fall, are the forces on them equal in magnitude or different? If different, which has the larger force? Explain.

The force on the lead ball is 10 times greater due to its 10 times greater mass. The Force of gravity is $-mg$.

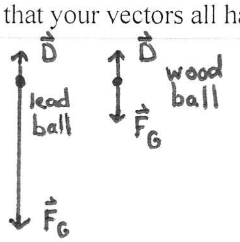
b. Are their accelerations equal or different? If different, which has the larger acceleration? Explain.

Equal. Though the force of gravity is 10 times larger on the 10 kg lead ball, its resistence to acceleration (inertia) is also 10 times greater. ($-mg = ma$ or $a = -g$ for both)

c. Which ball hits the ground first? Or do they hit simultaneously? Explain.

Simultaneously. The balls are dropped at the same time from the same height with the same acceleration. Therefore, they land at the same time.

d. If air resistance is present, each ball will experience the *same* drag force because both have the same shape. Draw free-body diagrams for the two balls as they fall in the presence of air resistance. Make sure that your vectors all have the correct *relative* lengths.



(Though the drag forces are equal speeds, the speeds will differ as explained in part e below.)

e. When air resistance is included, are the accelerations of the balls equal or different? If not, which has the larger acceleration? Explain, using your free-body diagrams and Newton's laws.

The lead ball has a greater acceleration. Because the drag force is independent of the mass, it will have less effect in reducing the acceleration due to gravity of the lead ball. Using Newton's 2nd Law $|a| = |F|/m = |F_G - D|/m = (mg - D)/m = g - D/m$. Thus, the larger mass of the lead ball leads to a smaller change in the magnitude of acceleration, (absent air resistance).

f. Which ball now hits the ground first? Or do they hit simultaneously? Explain.

The lead ball will hit the ground first because it has a greater magnitude acceleration.