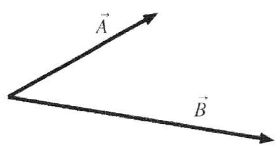
11 Work

11.2 Work and Kinetic Energy

11.3 Calculating and Using Work

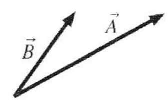
1. For each pair of vectors, is the sign of $\vec{A} \cdot \vec{B}$ positive (+), negative (–), or zero (0)?

a.



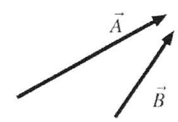
Sign = +

b.



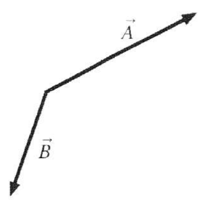
Sign = +

c.



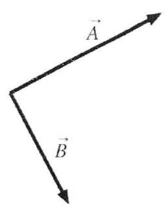
Sign = +

d.



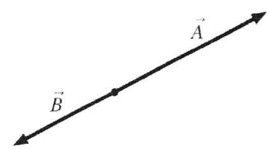
Sign = –

e.



Sign = 0

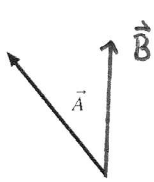
f.



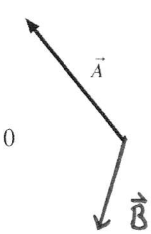
Sign = –

2. Each of the diagrams below shows a vector $\vec{A}$. Draw and label a vector $\vec{B}$ that will cause $\vec{A} \cdot \vec{B}$ to have the sign indicated.

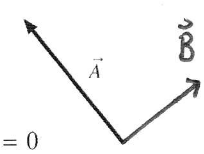
a. $\vec{A} \cdot \vec{B} > 0$



b. $\vec{A} \cdot \vec{B} < 0$



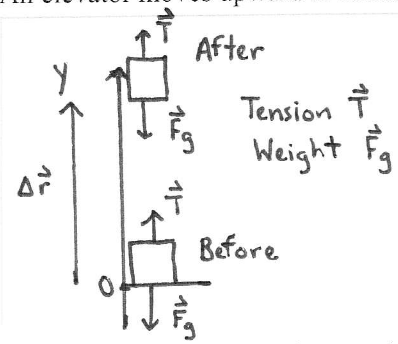
c. $\vec{A} \cdot \vec{B} = 0$



Exercises 3–10: For each situation:

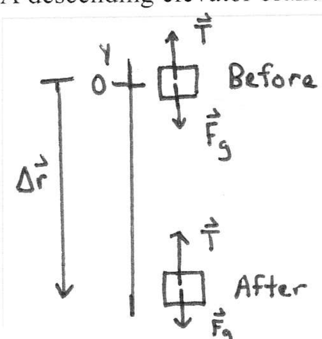
- Draw a before-and-after pictorial diagram.
- Draw and label the displacement vector $\Delta\vec{r}$ on your diagram.
- Draw a free-body diagram showing *all* forces acting on the object.
- Make a table beside your diagrams showing the sign (+, −, or 0) of (i) the work done by each force seen in your free-body diagram, (ii) the net work W_{net}, and (iii) ΔK, the object's change in kinetic energy.

3. An elevator moves upward at constant speed.



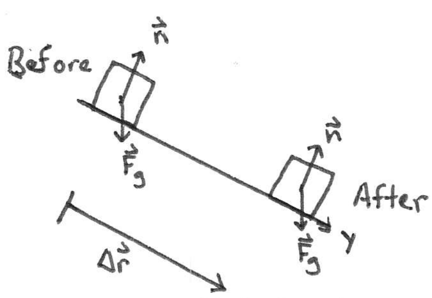
Force	Work	Sign
Tension	W_T	+
Weight	W_g	−
	W_{net}	0
	ΔK	0

4. A descending elevator brakes to a halt.



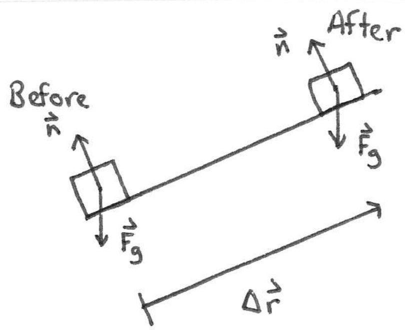
Force	Work	Sign
Tension	W_T	−
Weight	W_g	+
	W_{net}	−
	ΔK	−

5. A box slides down a frictionless slope.



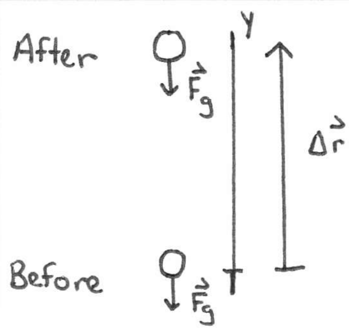
Force	Work	Sign
F_g	W_g	+
Normal	W_n	0
	W_{net}	+
	ΔK	+

6. A box slides up a frictionless slope.



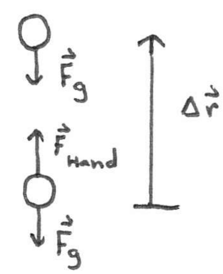
Force	Work	Sign
F_g	W_g	−
Normal	W_n	0
	W_{net}	−
	ΔK	−

7. A ball is thrown straight up. Consider the ball from one microsecond after it leaves your hand until the highest point of its trajectory.



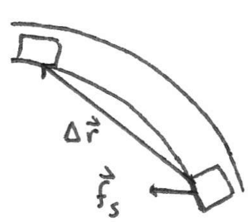
Force	Work	Sign
Weight	W_g	−
	W_{net}	−
	ΔK	−

8. You toss a ball straight up. Consider the ball from the instant you begin moving your hand until you release the ball.



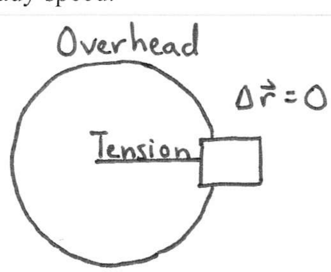
Force	Work	Sign
F_g	W_g	−
F_{Hand}	W_{Hand}	+
	W_{net}	+
	ΔK	+

9. A car turns a corner at constant speed.



Force	Work	Sign
friction	W_f	0
Weight	W_g	0
normal	W_n	0
	W_{net}	0
	ΔK	0

10. A flat block on a string swings once around a horizontal circle on a frictionless table. The block moves at steady speed.



Force	Work	Sign
Tension	W_T	0
Weight	W_g	0
Normal	W_n	0
	W_{net}	0
	ΔK	0

11. A 0.2 kg plastic cart and a 20 kg lead cart both roll without friction on a horizontal surface. Equal forces are used to push both carts forward a distance of 1 m, starting from rest. After traveling 1 m, is the kinetic energy of the plastic cart greater than, less than, or equal to the kinetic energy of the lead cart? Explain.

The kinetic energies are equal. Equal forces are applied over equal displacements so that the same work is done on each. Thus, the change in kinetic energy is the same. Because $K_i = 0$, $\Delta K = K_f$. (The plastic will be moving 10 times faster, however.)

12. Particle A has less mass than particle B. Both are pushed forward across a frictionless surface by equal forces for 1 s. Both start from rest.

a. Compare the amount of work done on each particle. That is, is the work done on A greater than, less than, or equal to the work done on B? Explain.

Because particle A has less mass, it will accelerate more and travel further during the one second push. Therefore, the work done on A is greater.

b. Compare the impulses delivered to particles A and B. Explain.

Because equal forces are applied for equal times, the impulses are the same.

c. Compare the final speeds of particles A and B. Explain.

A will be faster than B.

(1) For equal momenta, the lighter particle must be faster.

(2) The lighter particle has more kinetic energy because more work was done on it, making it faster.

(3) Particle A has a greater acceleration under the same force leading to a greater final speed.

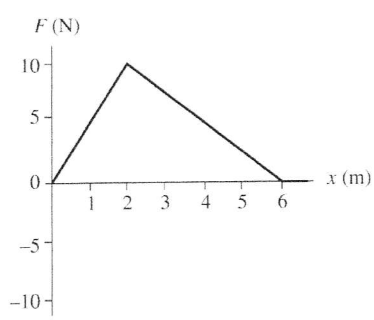
11.4 The Work Done by a Variable Force

13. In Chapter 9, we found a graphical interpretation of Δp as the area under the F-versus-t graph from an initial time t_i to a final time t_f. Provide an analogous graphical interpretation of ΔK, the change in kinetic energy.

Δk is the area under the F-versus-position graph. That area represents the net work done.

14. A particle moving along the x-axis experiences the forces shown below. How much work does each force do on the particle? What is each particle's change in kinetic energy?

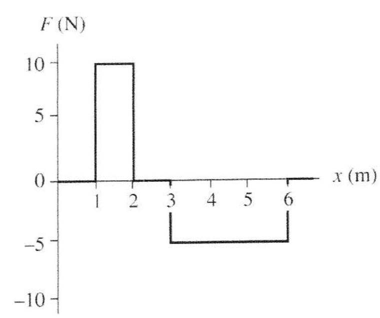
a.



$W =$ $\frac{1}{2}(10N)(6m) = 30J$

$\Delta K =$ $30J$

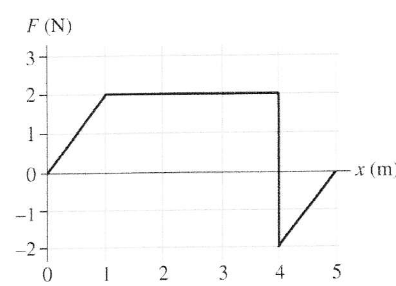
b.



$W =$ $10N(1m) - 5N(3m) = -5J$

$\Delta K =$ $-5J$

15. A 1 kg particle moving along the x-axis experiences the force shown in the graph. If the particle's speed is 2 m/s at $x = 0$ m, what is its speed when it gets to $x = 5$ m?



$K_i = \frac{1}{2}(1kg)(2\frac{m}{s})^2 = 2J$

$W_{net} = \frac{1}{2}(2N)(1m) + (2N)(3m) - \frac{1}{2}(2N)(1m)$
$= 6J$

$K_f = 8J = \frac{1}{2}mv_f^2$

$V_A = \sqrt{\frac{2(8J)}{1kg}} = \boxed{4\frac{m}{s}}$

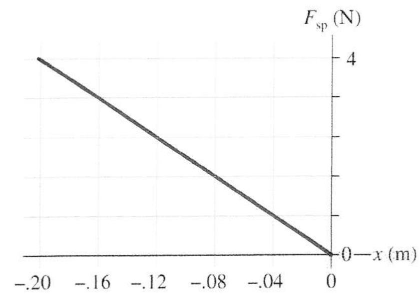
16. In Example 11.7 in the textbook, a compressed spring with a spring constant of 20 N/m expands from $x_0 = -20$ cm $= -0.20$ m to its equilibrium position at $x_1 = 0$ m.

a. Graph the spring force F_{sp} from $x_1 = -0.20$ m to $x_2 = -0$ m.

$F_{sp} = -K \Delta x$

$F_{spi} = -20\frac{N}{m}(0m - (-.20m)) = 4N$

$F_{sp} = 0$



b. Use your graph to determine ΔK, the change in a cube's kinetic energy when launched by a spring that has been compressed by 20 cm.

ΔK = Area under the curve

$= \frac{1}{2}(4N)(0 - (-.20m)) = 0.4J$

c. Use your result from part b to find the launch speed of a 100 g cube in the absence of friction. Compare your answer to the value found in the Example 11.7.

$K_f = \Delta K = \frac{1}{2} m v_{launch}^2$ $\quad v_{launch} = \sqrt{\frac{2K_f}{m}}$

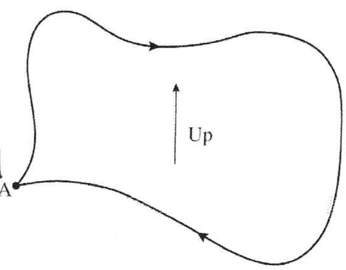
$= \sqrt{\frac{2(.4J)}{0.1kg}} = 2.83\frac{m}{s}$

The speed is slightly greater in the absence of friction.

11.5 Work and Potential Energy

11.6 Finding Force from Potential Energy

17. A particle moves in a vertical plane along a *closed* path, starting at A and eventually returning to its starting point. How much work is done on the particle by gravity? Explain.



No work was done by gravity. $W_{grav} = -mg\Delta y$
Here, $\Delta y = 0$. Any work done during a downward part of the motion was undone during the upward parts.

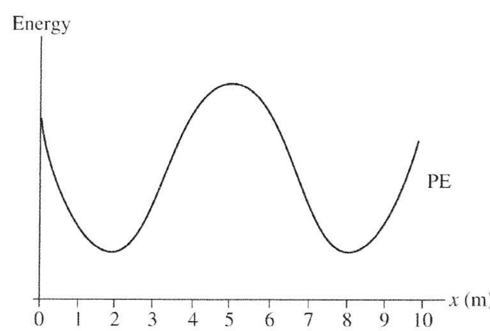
18. a. If the force on a particle at some point in space is zero, must its potential energy also be zero at that point? Explain.

No, the rate of change of potential energy with respect to position will be zero at that point, but the value of the potential energy is not known without specifying it at some reference point.

b. If the potential energy of a particle at some point in space is zero, must the force on it also be zero at that point? Explain.

No, the zero point for the potential energy is arbitrary. There will be a force present if the rate of change of the potential energy with position is non-zero.

19. The graph shows the potential-energy curve of a particle moving along the x-axis under the influence of a conservative force.



a. In which intervals of x is the force on the particle to the right?

From 0 to 2m, from 5 to 8m.
(wherever the slope is negative)

b. In which intervals of x is the force on the particle to the left?

From 2m to 5m, from 8m to 10m.
(wherever the slope is positive)

c. At what value or values of x is the magnitude of the force a maximum?

At $x=0$, $x=3.5$m, $x=6.5$m, and $x=10$m. The magnitude of the force is a maximum where the slope has its greatest magnitude.

d. What value or values of x are positions of stable equilibrium?

At the minima, $x = 2m$, $x = 8m$.

e. What value or values of x are positions of unstable equilibrium?

At the maxima, $x = 5m$.

f. If the particle is released from rest at $x = 0$ m, will it reach $x = 10$ m? Explain.

No. If released from rest, it will never have more energy than its initial potential energy and cannot reach a location where $U(x)$ is greater than its initial value. It will turn around at $\sim 3.5m$.

11.7 Thermal Energy

20. A ball of clay traveling at 10 m/s slams into a wall and sticks. What happened to the kinetic energy the clay had just before impact?

It was dissipated as thermal energy in the deformation of the ball of clay.

21. What energy transformations occur as a box slides up a gentle but slightly rough incline until stopping at the top?

Kinetic energy is transformed into potential energy to the extent the box moves upward, but also thermal energy is dissipated along the interface between the box and the rough incline.

11.8 Conservation of Energy

22. Give a *specific* example of a situation in which:

a. $W_{ext} \to K$ with $\Delta U = 0$ and $\Delta E_{th} = 0$.

Push a puck across a frictionless surface.
(system: puck)

$(W_{ext} = \vec{F} \cdot \Delta\vec{r} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$

b. $W_{ext} \to U$ with $\Delta K = 0$ and $\Delta E_{th} = 0$.

Lift a book at constant speed. (system: book/earth)

$(W_{ext} = \vec{F} \cdot \Delta y = mg\Delta y = \Delta U_{grav})$

c. $K \to U$ with $W_{ext} = 0$ and $\Delta E_{th} = 0$.

A ball thrown upward, just after release until reaching its peak. $(\frac{1}{2}mv^2 = mg\Delta y)$ (system: Ball/Earth) A puck sliding into a spring to its maximum compression. $(\frac{1}{2}mv^2 = \frac{1}{2}k\Delta x^2)$

d. $W_{ext} \to E_{th}$ with $\Delta K = 0$ and $\Delta U = 0$.

A box pushed at constant speed along a horizontal surface.
(system: box, surface)

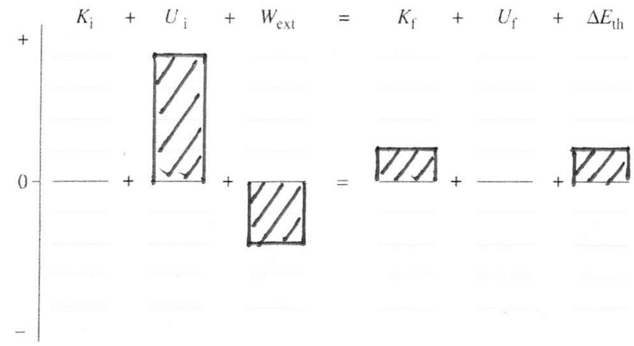
$(W_{ext} = \Delta E_{th})$

e. $U \to E_{th}$ with $\Delta K = 0$ and $W_{ext} = 0$.

A box slides at constant speed down a ramp with friction.
(system: box, surface, Earth)

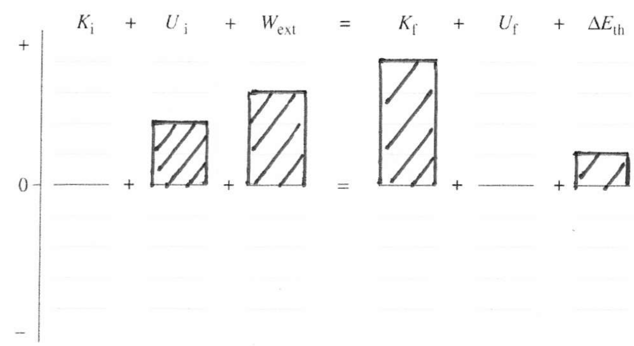
$(mg\Delta y = \Delta E_{th})$

23. A system loses 1000 J of potential energy. In the process, it does 500 J of work on the environment and the thermal energy increases by 250 J. Show this process on an energy bar chart.



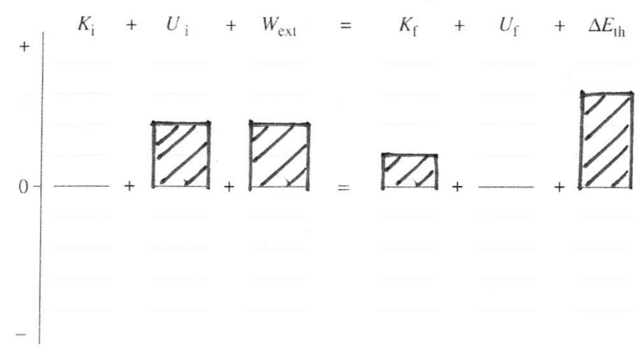
$K_f - K_i = 250\,\text{J}$ to balance the chart.

24. A system gains 1000 J of kinetic energy while losing 500 J of potential energy. The thermal energy increases by 250 J. Show this process on an energy bar chart.



$W_{ext} = 750\,\text{J}$ to balance the chart.

25. A box is sitting at the top of a ramp. An external force pushes the box down the ramp, causing it to slowly accelerate. Show this process on an energy bar chart.



The relative amount of energy of each type cannot be determined without more information.

11.9 Power

26. a. If you push an object 10 m with a 10 N force in the direction of motion, how much work do you do on it?

$$W = \vec{F} \cdot \Delta\vec{r} = 10\,\text{N} \cdot 10\,\text{m} = 100\,\text{J}$$

b. How much power must you provide to push the object in 1 s? In 10 s? In 0.1 s?

$$P = \frac{W_{ext}}{\Delta t}$$

In 1 s, $P = 100\,\text{W}$

In 10 s, $P = \frac{100\,\text{J}}{10\,\text{s}} = 10\,\text{W}$

In 0.1 s, $P = \frac{100\,\text{J}}{0.1\,\text{s}} = 1000\,\text{W}$